Relative Positioning System Derivation 05/17/2017

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This is the mathematical derivation for the Relative Positioning System that estimates relative robot positions from mutual distance measurements. It takes in distances between robots and estimates their relative positions in the Cartesian plane. To do so, it performs gradient descent on their relative distance equations. It requires at least 5 robots to converge.

The system will take in a triangular matrix of measured distances:

$$\begin{pmatrix} 0 & & & & & \\ d_{21} & 0 & & & & \\ d_{31} & d_{32} & 0 & & & \\ d_{41} & d_{42} & d_{43} & 0 & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ d_{N1} & d_{N2} & d_{N3} & d_{N4} & \cdots & 0 \end{pmatrix} \tag{1}$$

Then, it will compute the square of each distance:

$$M_{measured} = \begin{pmatrix} 0 & & & & & \\ d_{21}^2 & 0 & & & & \\ d_{31}^2 & d_{32}^2 & 0 & & & \\ d_{41}^2 & d_{42}^2 & d_{43}^2 & 0 & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ d_{N1}^2 & d_{N2}^2 & d_{N3}^2 & d_{N4}^2 & \cdots & 0 \end{pmatrix} \tag{2}$$

The distance squared is known to relate to the robot positions by the Euclidian distance:

$$d_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2 \tag{3}$$

Using the gradient descent to solve this nonlinear system, we need a random initial guess for each position:

$$estimate = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ \vdots & \vdots \\ x_N & y_N \end{pmatrix} \tag{4}$$

Then, we use the guess to compute the estimate square distances matrix $M_{estimate}$, as in Equation 2. With $M_{measured}$ and $M_{estimate}$, we compute the estimation error:

$$e = M_{measured} - M_{estimate} \tag{5}$$

Then, we need to compute the error gradient. Luckily, the derivative is simple, so we have:

$$E = d_{ij}^2 - (x_i - x_j)^2 + (y_i - y_j)^2 \tag{6}$$

$$\frac{dE}{dx_i} = 2(x_j - x_i) \tag{7}$$

$$\frac{dE}{dx_j} = 2(x_i - x_j) \tag{8}$$

$$\frac{dE}{dy_i} = 2(y_j - y_i) \tag{9}$$

$$\frac{dE}{dy_j} = 2(y_i - y_j) \tag{10}$$

So the gradient will be

$$\nabla E = 2\langle x_j - x_i, x_i - x_j, y_j - y_i, y_i - y_j \rangle \tag{11}$$

And the gradient descent step will be

$$-\, stepsize \cdot e \cdot \nabla E \tag{12}$$