AMATH 383 Final Paper:
Nash Equilibria and Modeling of People's Response to COVID-19

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March 12, 2020

Abstract

Nash Equilibrium and the scenario of the Prisoner's Dilemma are well-known in the world of game theory and economics. Although it is still controversial if John Nash, the Princeton mathematician's contributions to game theory have great influence outside of the field of economics, the mathematics behind the model are essential to understand the subject and therefore used to analyze the effects of certain social behaviors.

In this paper, we analyze the effects of people's response to communicative diseases like COVID-19. We briefly introduce the subject of game theory and modeling of Nash Equilibrium first. Then, we will use the mathematical model behind Nash Equilibria to describe the outcomes of people's decision making in response to the disease. With the support of game theory and the Nash Equilibria, the model along with different scenarios and its social science implications are explained by simulations.

1 Introduction to Games and Nash Equilibria

In a world of information, competitions exist in every aspect of our society. From a company's strategies used in an industry to an individual's move to bargain at the public market, decision makers usually use available information to come up with the best strategies to maximize their profit. The subject of game theory was therefore established. A major breakthrough of modern game theory was his first paper "Non-Cooperative Games" published by John Nash in 1950, which provided a broader mathematical model to generalize non-cooperative games. From that point on, the framework of Nash equilibria of competitive games had gone through major improvements by multiple other scholars and was being more generalized, but the original models of Nash Equilibrium is still prevalent today.

Let us first introduce games. According to a textbook of game theory by Dario Bauso[1], a game in strategic form can be represented as a tuple $\langle N, (A_i)_{i \in N}, (\pi_i)_{i \in N} \rangle$, where

1. $N = \{1, 2, \ldots, n\}$ is the set of players,
2. $A_i = \{(a_i)_{i \in N}\}$ is the set of actions by player i,
3. π_i is the payoff of player i in the game by playing a_i.

We also denote $a_{-i} = (a_j)_{j \in N, j \neq i}$ as the action profile of all players except player i. Since all players are payoff maximizers, the objective of player i in this game is to maximize π_i.

In John Nash's paper, he argued that "a finite non-cooperative game always has at least one equilibrium point."[2] What he means by a equilibrium point is a profile of optimal strategies of an n-person non-cooperative game where no player in the game can increase

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[1] Bauso, Dario. *Game Theory: Models, Numerical Methods and Applications*. Foundations and Trends® in Systems and Control: Vol. 1: No. 4 (2014): 379-522. http://dx.doi.org/10.1561/2600000003
[2] Nash, John. "Non-Cooperative Games." *Annals of Mathematics*, Second Series, 54, no. 2 (1951): 286-95. Accessed March 11, 2020. doi:10.2307/1969529.

her payoff by changing to any other strategies, given other players' strategies are fixed. In mathematical notations, we denote the equilibrium point $(a_1^*,\ a_2^*, \ldots,\ a_n^*)$, where

$$\pi_i(a_i^*,\ a_{-i}^*) \geq \pi_i(a_i,\ a_{-i}^*),\ \ \forall a_i \in A_i,\ \ \forall i \in N.$$

In Nash Equilibrium, we can also define a best response set $\mathfrak{B}(a_{-i})$, where given all other player's strategy profile a_i, player i chooses from a set of best responses that can maximize π_i. All players in the game play a strategy from their own set of best responses, so $a_i^* \in \mathfrak{B}(a_{-i}^*),\ \forall i \in N.$

However, there does not always exist a pure strategy Nash Equilibrium where a player's one action gives strictly greater payoff than taking the other action. Then, people begin to use a mixed strategy with possibility p of taking one action, and $(1-p)$ of taking the other. We will discuss this scenario in the later sections.

2 An Example of Prisoner's Dilemma

In this project, we begin with an example of Prisoner's Dilemma[3] to better understand Nash Equilibrium, and then expand and apply to the COVID-19 case. In this game, we have two players I and II, and each of them has a profile of two actions they can choose, cooperate or refuse. If they both choose to cooperate, then each will get a payoff of 3, so $(\pi_I,\ \pi_{II}) = (3, 3)$. If one player chooses to cooperate, and the other refuses, then the payoffs are either (1,4) or (4,1) since the one who chooses to refuse cooperation can benefit from attacking the other. If they both refuse to cooperate, then the payoffs $(\pi_I,\ \pi_{II}) = (2, 2)$. The game can be represented in a strategic form as the following:

[3] Ross, Don. "Game Theory." *The Stanford Encyclopedia of Philosophy* (Winter 2019 Edition), Edward N. Zalta (ed.). Accessed March 11, 2020. https://plato.stanford.edu/archives/win2019/entries/game-theory/.

		Player II	
		Cooperate	Refuse
Player I	Cooperate	3,3	1,4
	Refuse	4,1	2,2

What strategies would both players choose? Since both players are payoff maximizers in a non-cooperative game, each will choose the strategies that always give them the most payoff given the other player's strategy. For player I, we want to maximize π_I. If player I chooses to cooperate, then

$$\pi_I(cooperate,\ a_{-I}) = 3 \text{ if } a_{-I} = a_{II} = cooperate,$$

$$\pi_I(cooperate,\ a_{-I}) = 1 \text{ if } a_{-I} = a_{II} = refuse.$$

If player I chooses to refuse, then

$$\pi_I(refuse,\ a_{-I}) = 4 \text{ if } a_{-I} = a_{II} = cooperate,$$

$$\pi_I(refuse,\ a_{-I}) = 2 \text{ if } a_{-I} = a_{II} = refuse.$$

Since $\pi_I(refuse,\ a_{-I}) = (4,\ 2) > (3,\ 1)\ = \pi_I(cooperate,\ a_{-I})$ for all actions taken by players II, then $a_I^* = refuse.$ Similarly, $\pi_{II}(refuse,\ a_{-II}) = (4,\ 2) > (3,\ 1)\ \pi_{II}(cooperate,\ a_{-II})$ for all actions taken by players I, then $a_{II}^* = refuse.$ Therefore, the Nash Equilibrium of this game is $(a_I^*, a_{II}^*) = (refuse,\ refuse)$, and the optimal payoff is $(\pi_I^*, \pi_{II}^*) = (2,\ 2)$. The name Prisoner's Dilemma is given because this game actually has a payoff $(3,\ 3)$, which is strictly better than the equilibrium payoff $(2,\ 2)$. However, neither of the players is willing to deviate from the Nash Equilibrium strategies given the other player's optimal strategy is fixed.

3 Application of Nash Equilibrium in Modeling COVID-19 Response

In the late December of 2019, a novel coronavirus outbreak started in China. Since the disease can be fatal and is highly contagious, it is recommended that people take protective actions to contain the spread of the disease such as wearing a facemask, washing and sanitizing hands regularly, and avoiding crowded areas. However, due to different reasons like lack of attention, politics, finance and aesthetics, some people choose not to take actions like wearing a mask. As a result, the disease has spread to many regions around the globe.

Many researchers today focus on the modeling of the number of infections based on actions taken by the public, but it is hard to predict what actions people are going to take. Therefore, game theory provides a useful tool in predicting the actions taken by them based on different scenarios. Based on the discussion earlier, we first start with a case where there are only two people in a public area. They can choose whether or not to wear a facemask.

In this scenario, the amount of infected people is relatively low in the whole society, and the demand for facemasks is low (say the price is \$2 each). The possibility of both people getting infected is relatively low without wearing masks at this point (0.01), but if the person gets infected, then she loses \$100 for medical treatments. If both people choose not to wear a facemask, then both people's payoff is $0.01 \times \$100 = -1$. If one of them wears a facemask, she spends \$2 to buy a mask, so the payoff π is -2. And since we only consider two people in this case, the other person's possibility of infection falls nearly to 0 after the other takes protection, so her payoff is 0. We denote this scenario in the following strategic form:

		Person 2: Wear	Person 2: Not Wear
Person 1	Wear	-2, -2	-2, 0
	Not Wear	0, -2	-1, -1

Therefore, we have

$$\pi_i(wear,\ a_{-i}) = -2 \text{ if } a_{-i} = wear\ or\ not\ wear,$$

$$\pi_i(not\ wear,\ a_{-i}) = -1 \text{ if } a_{-i} = not\ wear,$$

$$\pi_i(not\ wear,\ a_{-i}) = 0 \text{ if } a_{-i} = wear.$$

In summary, $\pi_i(not\ wear,\ a_{-i}) = (-1, 0) > (-2,\ -2) = \pi_i(wear,\ a_{-i})$ for both people. As a result, neither of them will wear a mask.

This simple case of a two-people game gives us a general idea what actions people will take in the beginning of the disease outbreak. In reality, though, there are a lot more than two people to consider, and the number of infections will climb as time goes. We will further discuss the n-person games of different scenarios in the next section and simulate the outcomes.

4 Outcomes in an N-person society

In the sections above, we discussed the possible outcomes between two individuals using the Nash Equilibrium. However, in a real society people can interact with many others and the Nash Equilibrium is too complicated to be represented in a standard strategic form if the number of people is large. So we decided to make some simplifications and use a software to simulate the outcomes with the Monte Carlos method[4]. Here are some observations:

1. People make decisions based more on the current state rather than possible decisions of other people's.
2. They can only interact with people around them directly.
3. They are rational but only care about their own benefits.

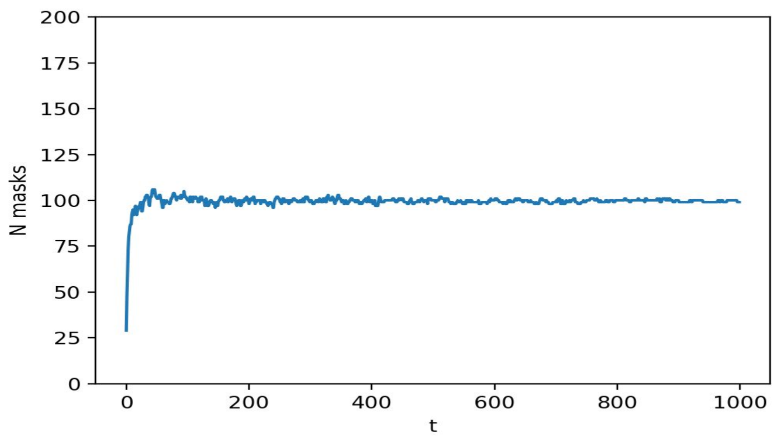
With these assumptions in mind, we are able to build a simple model of an N-person society. Imagine people are standing in a line, and they decide whether to wear a mask based only on the current state of the person in front of them (i.e. whether that person is wearing a mask or not). We represent this case as a matrix in Nash Equilibrium here again, with some modifications:

		Person 2: Wear	Person 2: Not Wear
Person 1	Wear	-2, -2	-2, 0
	Not Wear	0, -2	-10, -10

This is the case where the disease has become an epidemic, and wearing a mask is costly (i.e. price is high due to demands, possible discrimination by others, etc.). In addition, people don't think masks are effective. In this case, there is no pure strategy Nash Equilibrium in the game since $\pi_i(wear, a_{-i}) = (-2, \ -2)$ is not strictly greater than

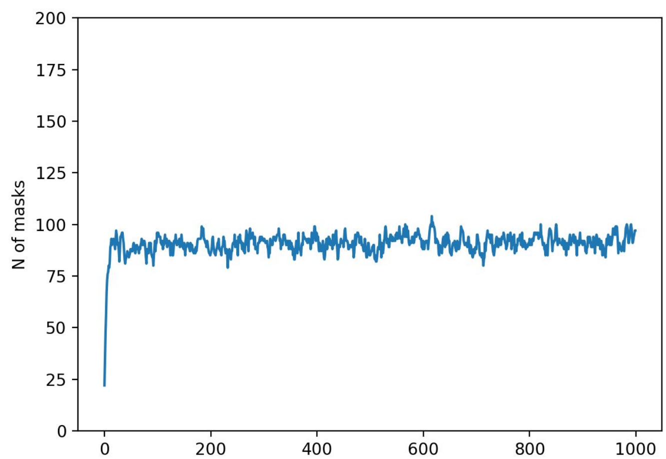
[4] Bohm, Gerhard, and Günter Zech. (2010). *Introduction to Statistics and Data Analysis for Physicists*, Third Revised Edition: 107-129.

$\pi_i(wear, a_{-i}) = (0, -10)$. Therefore, people begin to use mixed strategies. People care about their health conditions, so they would look at the person in front of them and decide whether to wear a mask based on this strategic form. They always make the choice with less cost. We create a model of two hundred people, and simulate their behaviors with discrete time. Assume there are thirty random people who decide to change their current decision on each time slice, and they do so if their cost will be lowered by this change. Assume no one is wearing a mask initially. Below is the result:



We noticed that as time goes by about half of the population is wearing a mask. This result is as expected: if two people are both wearing masks or not wearing masks, one changes the current state; if only one of them is wearing a mask, they don't change their current state.

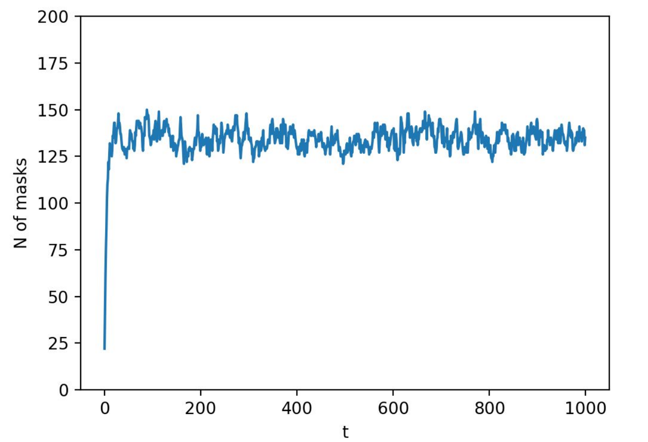
This is rather a boring case in that people's behaviors follow a certain pattern and thus the equilibrium can be easily predicted: maximum population, zero, or half the population. In real life, people are not quite predictable, and there are some randomness when they make decisions. So we assign a possibility when people try to change their current states and use Markov chain in the simulation. Imagine a situation where a person is wearing a mask but the person in front of him is not, and he is deciding whether to take his mask off or not. His current cost is -2, but when he takes his mask off the cost becomes -10. We calculate probability -10/(-2-10) = ⅚, but remember that the strategic form gives the cost, and this probability describes the unwillingness of this person to change his current state. As a result, the possibility for this person to change his state is 1-⅚=⅙.



The behavior is similar to the simulation above, but we noticed that this time the equilibrium is around 90. What’s even more interesting is when we change the off diagonal values of the cost matrix to

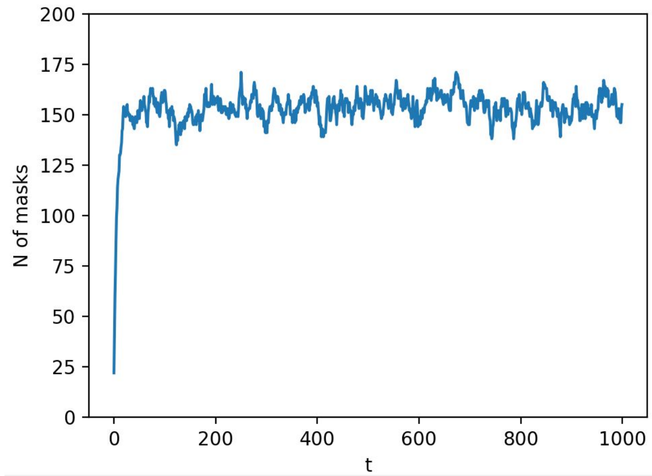
		Person 2: Wear	Person 2: Not Wear
Person 1	Wear	-2, -2	-2, -3
	Not Wear	-3, -2	-10, -10

In this case the people start to believe that wearing masks are effective and the risks of getting infected is high when not wearing masks, and the government is probably forcing people to wear masks in an epidemic. Then, the results is shown below:



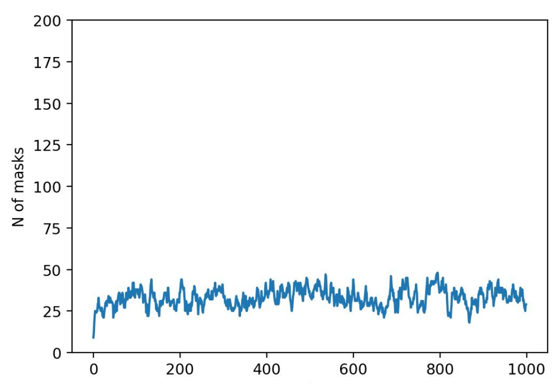
Notice that the equilibrium jumps to around 135. Then, when we change the matrix again to:

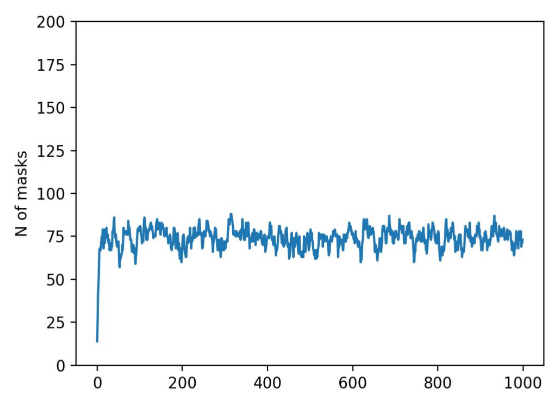
		Person 2: Wear	Person 2: Not Wear
Person 1	Wear	-2, -2	-2, -6
	Not Wear	-6, -2	-10, -10

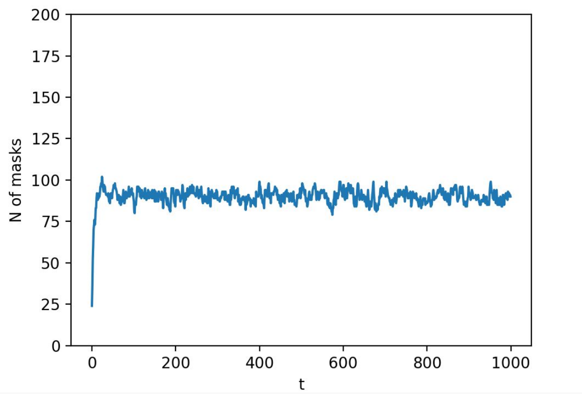


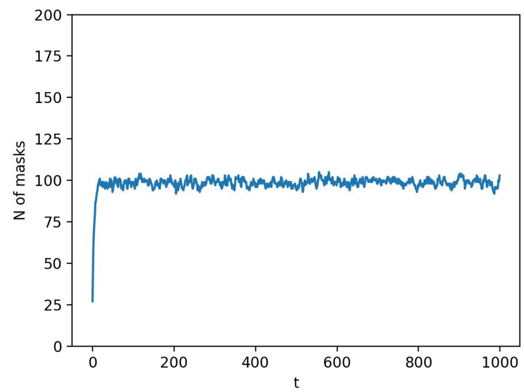
The equilibrium jumps to around 150. This suggests that if people somehow get punished more for not wearing masks, they tend to wear masks.

Besides the situations above, we are also interested in how the severity of the disease affects people's decisions. This time we fix the off-diagonal values to (-2, 0)and set the lower-right corner values to -1, -3, -10, -100 respectively. And our results are shown below:









These are cases where people do not believe in the effectiveness of the mask and the government is not regulating. The first plot is when the severity is low, and very few people are wearing masks. As the severity increases, more and more people decide to wear masks, but the limit is half of the population. When the limit is reached, the number of people wearing masks does not increase no matter how severe the disease becomes.

5 Improvement

The models we built above are mostly based on symmetric, simultaneous games and assumptions we made about people's decisions. If we consider repeated games where a person meets a lot of people at different times, there are other results we would need to consider. This improvement needs a deeper understanding of game theory and requires the use of extensive form, where we need contribution of more people with the background of game theory. Also, we could have implemented the model of spread disease discussed in class in order to model the number of people infected over time, so we could get a better estimation of the probability people get infected.

In terms of the results, conducting surveys of people's mask wearing decisions may help improve the reliability of the outcomes. Since this project is more about mechanistic modeling of mathematics, this improvement is out of the scope of our discussion.

6 Conclusion

In this paper, we first introduced and discussed the concept of game theory, Nash Equilibrium and the Prisoner's Dilemma. Then, we applied the mathematical model behind Nash Equilibrium to predict people's decisions on wearing masks in an epidemic with simplification. After making some assumptions based on the Nash Equilibrium, we used Monte Carlo method to simulate an N-person society. If only two people are present at the time, by game theory, there will usually be an equilibrium in which both people refuse to wear masks. Even though that is the best solution, it does not seem like a good practice to contain a disease. However, when we simulate the N-person society, we noticed that both the motive(belief in effectiveness of masks, government regulations, etc.)to wear masks and the severity of the disease can affect the outcome. When the motive is fixed, the severity of the disease leads to more people wearing masks, with a maximum of half of the population. But if we increase the motive, the equilibrium approaches to almost the total population. This suggests that a more efficient way to convince people to wear masks or take other protective actions is to, instead of exaggerating the severity, educate people on the effectiveness of them in containing communicable diseases. If possible, strict laws and government regulations may help stimulate preventive actions. Even though our model is simple compared to the real world situations, it still gives some expected qualitative predictions.

References

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Appendix: Python Codes for the Monte Carlo Simulation

```
import numpy as np
import matplotlib.pyplot as plt


def flips(s):
    num = 0
    if(s == 1):
        num = 0
    else:
        num = 1
    return num

N = 200
t = 1000

PA = np.array([[-2,-2],[0,-10]])
state1 = np.ones(N);
num = np.zeros(t)
for i in range(t):
    peo = np.random.randint(0,199,30)
    for j in peo:

        current = PA[int(state1[j]),int(state1[j+1])]
        state1[j] = flips(int(state1[j]))
        trial = PA[int(state1[j]), int(state1[j+1])]

        p = 1-trial/(current+trial)
        r = np.random.rand()

        if(r>p):
            state1[j] = flips(state1[j])
    num[i] = N-sum(state1)

```