SOLUCION PREGUNTA #1

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- LA DISTANCIA MEDIA ENTRE DOS SEÑALES PERIODICAS $x_1(t) \in \mathbb{R}, \mathbb{C}$ Y $x_2(t) \in \mathbb{R}, \mathbb{C}$; SE PUEDE EXPRESAR A PARTIR DE LA POTENCIA MEDIA DE LA DIFERENCIA ENTRE ELLAS

$$d(x_1, x_2) = \bar{P}_{x_1 - x_2} = \lim_{T \to \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 \, dt$$

- SEA $x_1(t)$ Y $x_2(t)$ DOS SEÑALES COMO SE MUESTRA A CONTIUACION

$$x_1(t) = A e^{j w_0 t}$$

$$x_2(t) = B e^{j 5 w_0 t}$$

- CON $w_0 = \frac{2\pi}{T}$; $T, A, B, \in \mathbb{R}^+$, DETERMINE LA SEÑAL ENTRE LAS DOS SEÑALES

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- SEGUN LA INTEGRAL DE POTENCIA SE TIENE QUE:

$$\frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 \, dt = \frac{1}{T} \Big[\underbrace{\int_T |x_1(t)|^2 \, dt}_{E_{x_1}} - 2 \int_T x_1(t) x_2(t) \, dt + \cdots$$

$$\cdots + \underbrace{\int_T |x_2(t)|^2 \, dt}_{E_{x_2}} \Big]$$

$$\boxed{= \bar{P}_{x_1} - \frac{2}{T} \int_T x_1(t)\, x_2(t) \, dt + \bar{P}_{x_2}}$$

- PARA $\bar{P}_{x_1}$ SE TIENE QUE, COMO $x_1(t) \in \mathbb{C} \Rightarrow |x_1(t)|^2 = x_1(t)\, x_1(t)^*$

$$\bar{P}_{x_1} = \frac{1}{T} \int_T |x_1(t)|^2 \, dt = \frac{A^2}{T} \int_0^T e^{j w_0 t} \cdot e^{-j w_0 t} \, dt = \frac{A^2}{T} \int_0^T e^0 \, dt$$

$$= \frac{A^2}{T} \int_0^T dt = \frac{A^2}{T} \left[t \Big|_0^T \right] = \frac{A^2}{\cancel{T}} [\cancel{T}]$$

$$\therefore \boxed{\bar{P}_{x_1} = A^2}$$

• PARA $\bar{P}_{x_2}$ SE TIENE QUE, COMO $x_2(t) \in \mathbb{C} \Rightarrow |x_2(t)|^2 = x_2(t)\, x_2(t)^*$

$$\bar{P}_{x_2} = \frac{1}{T}\int_T |x_2(t)|^2\, dt = \frac{B^2}{T}\int_0^T e^{j5w_0t} \cdot e^{-j5w_0t}\, dt$$

$$= \frac{B^2}{T}\int_0^T dt = \frac{B^2}{T}\left[t \Big|_0^T \right] = \frac{B^2}{\cancel{T}}\left[\cancel{T}\right]$$

$$\boxed{\therefore \bar{P}_{x_2} = B^2}$$

• PARA EL OTRO TERMINO

$$-\frac{2}{T}\int_T x_1(t)\, x_2(t)\, dt = -\frac{2}{T}\int_0^T A e^{jw_0t} \cdot B e^{j5w_0t}\, dt$$

$$= -\frac{2}{T}\int_0^T A e^{j\frac{2\pi}{T}t} \cdot B e^{j\frac{5\cdot 2\pi}{T}t}\, dt$$

$$= \frac{-2}{T}\int_0^T A\cdot B \cdot e^{j\frac{12\pi}{T}\cdot t}\, dt = \frac{-2}{T} A\cdot B \int_0^T e^{j\frac{12\pi}{T}t}\, dt$$

$$= \frac{-2}{T} A\cdot B \left[\frac{T}{j12\pi} e^{j\frac{12\pi}{T}t} \right]_0^T$$

$$= \frac{-2}{T} A\cdot B \left[\frac{T}{j12\pi} e^{j\frac{12\pi}{\cancel{T}}\cancel{T}} - \frac{T}{j12\pi} \cancelto{1}{e^{j\frac{12\pi}{T}\cdot 0}} \right]$$

• COMO $e^{j12\pi} = \cancelto{1}{\cos(12\pi)} + j\,\cancelto{0}{\operatorname{sen}(12\pi)} = 1$

$$= \frac{-2}{T} A\cdot B \left[\cancelto{0}{\frac{T}{j12\pi} - \frac{T}{j12\pi}} \right]$$

$$\boxed{= \frac{-2}{T} A\cdot B\, [0] = 0}$$

- POR LO TANTO DE LA INTEGRAL QUEDA

$$\frac{1}{T}\int_{T} |x_1(t) - x_2(t)|^2 \, dt = \bar{P}_{x_1} - 0 + \bar{P}_{x_2}$$

$$\boxed{= \bar{P}_{x_1} + \bar{P}_{x_2} = A^2 + B^2}$$

- PARA LO CUAL SE OBTIENE $d(x_1, x_2)$

$$d(x_1, x_2) = \lim_{T \to \infty} A^2 + B^2$$

$$\boxed{\therefore \; d(x_1, x_2) = A^2 + B^2}$$

SOLUCION PREGUNTA #2

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• CUAL ES LA SEÑAL OBTENIDA EN TIEMPO DISCRETO AL UTILIZAR UN CONVERSOR ANALOGO DIGITAL CON FRECUENCIA DE MUESTREO DE 5 KHz, APLICADO A LA SEÑAL CONTINUA

$x(t) = 3 \cos 1000\pi t + 5 \operatorname{sen} 2000\pi t + 10 \cos 11000\pi t$

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HACIENDO $t = n/F_s \rightarrow F_s = 5000$

$x_1[n] = 3 \cos \left[1000\pi \frac{n}{5000} \right]$

$x_1[n] = 3 \cos \left[\frac{\pi}{5} n \right] \rightarrow \boxed{\Omega_1 = \frac{\pi}{5}}$

$x_2[n] = 5 \operatorname{sen} \left[2000\pi \frac{n}{5000} \right]$

$x_2[n] = 5 \operatorname{sen} \left[\frac{2\pi}{5} n \right] \rightarrow \boxed{\Omega_2 = \frac{2\pi}{5}}$

$x_3[n] = 10 \cos \left[11000\pi \frac{n}{5000} \right]$

$x_3[n] = 10 \cos \left[\frac{11\pi}{5} n \right] \rightarrow \boxed{\Omega_3 = \frac{11\pi}{5}} \rightarrow$ COPIA

• COMO $\Omega_3 > \pi$, HAY QUE HALLAR LA ORIGINAL

$$\boxed{\Omega_{OR3} = \Omega_3 - 2\pi = \frac{\pi}{5}}$$

• REEMPLAZANDO

$x[n] = 3 \cos \left[\frac{\pi}{5} n \right] + 5 \operatorname{sen} \left[\frac{2\pi}{5} n \right] + 10 \cos \left[\frac{\pi}{5} n \right]$

$$\boxed{\therefore\ x[n] = 13 \cos \left[\frac{\pi}{5} n \right] + 5 \operatorname{sen} \left[\frac{2\pi}{5} n \right]}$$

• COMO SE GENERA ALIAS, APLICAMOS NYQUIST PARA UNA DISCRETIZACION APROPIADA

$F_s' \geq 2 F_{MAX} \rightarrow \boxed{F_{MAX} = \frac{w_3}{2\pi} = \frac{11000\pi}{2\pi} = 5500}$

$F_s' \geq 2(5500)$

$$\boxed{F_s' \geq 11000}$$

• LA SEÑAL OBTENIDA USANDO LA Fs' ES:

$$x[n] = 3\cos\left[1000\pi \frac{n}{11000}\right] + 5\,\text{sen}\left[2000\pi \frac{n}{11000}\right]$$

$$+ 10\cos\left[11000\pi \frac{n}{11000}\right]$$

$$\therefore\ x[n] = 3\cos\left[\frac{\pi}{11} n\right] + 5\,\text{sen}\left[\frac{2\pi}{11} n\right] + 10\cos[\pi n]$$

SOLUCION PREGUNTA #3

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$$x(t) = 20\left[\cos(t/3) + \cos(t/4)\right]$$

$$\frac{w_1}{w_2} = \frac{1}{3} \div \frac{1}{4} = \frac{4}{3}$$

$$\boxed{\begin{aligned} f_1 &= \frac{w_1}{2\pi} = \frac{1}{6\pi} \\ f_2 &= \frac{w_2}{2\pi} = \frac{1}{8\pi} \end{aligned}}$$

$$T_1 = 6\pi$$

$$T_2 = 8\pi$$

$$T = MCM\{6, 8\} = 24$$

$$\boxed{T = 24\pi}$$