1)

1. $p \to q$ (premise)
2. $\neg q$ (Premise)
3. p (assume)
4. q ($\to e$ 1,3)
5. $\bot$ ($\neg e$ 2,3)
6. $\neg p$ ($\neg i$ 3-5)

3.

1. $\neg(p \vee \neg p)$ (assume)
2. p (assume)
3. $p \vee \neg p$ ($\vee_i$ 2)
4. $\bot$ ($\neg e$ 1,4)
5. $\neg p$ (PBC 2-4)
6. $p \vee \neg p$ ($\vee_i$ 5)
7. $\bot$ ($\neg e$ 1,6)

4) 1. $p \to q$ (premise)
2. $p \lor \neg p$ (LEM)
3. p (assume)
4. q ($\to e$ 1)
5. $q \lor \neg p$ ($\lor i$ 4)
6. $\neg p$ (assume)
7. $q \lor \neg p$ ($\lor i$ 7)
8. $q \lor \neg p$ ($\lor e$ 2, 3-5, 6-7)

5 ~~1. $((q \to r) \to ((\neg q \to \neg p) \to (p \to r))$~~ (assume)
2. $(q \to r)$ (assume)
~~3. $((\neg q \to \neg p) \to (p \to r))$~~ ($\to e$ 1 2)
4. $(\neg q \to \neg p)$ (assume)
5. p (assume)
6. $\neg\neg q$ (MT 4 5)
7. q ($\neg e$ 6)
8. r ($\to e$ 2 8)
9. $p \to r$ ($\to i$ 5-8)
10. $(\neg q \to \neg p) \to (p \to r)$ ($\to i$ 5-9)
11. $(q \to r) \to ((\neg q \to \neg p) \to (p \to r))$ ($\to i$ 2-10)

6. 1. $(p \land q) \to r$ (premise)
2 $r \to s$ (premises)
3. $q \land \neg s$ (premise)
4. $\neg s$ ($\land e$ 3)
5. q ($\land e$ 3)
6. $\neg r$ (MT 2,4)
7. $\neg(p \land q)$ (mt 1,6)
8 $\neg p$ ($\land e$ 7 5)

7) $r(x)$: x is red
$b(x)$: x is in box
$c(x)$: x is cat
$d(x)$: x is dog
$g(x,y)$: x is girl that won y
$p(x)$: x is prize
$gm(x,y)$ x is y grandmother
$f(x,y)$ x is y father
$U(x)$ x is uncle
$A(x)$ x is aunt

a. $\forall(x)\ r(x) \to b(x)$
b. $\forall(x)\ (\neg r(x) \to \neg b(x)) \wedge (r(x) \to b(x))$
c. $\forall(x)\ (c(x) \to \neg d(x)) \wedge (d(x) \to \neg c(x))$
d. $\forall(x)\ \exists(y)\ g(y,x)$
e. $\exists(x)\ \forall(y)\ g(x,y)$
f. $\forall_{x,y,z}\ g(x,y) \to f(y,z)$
g. $\forall_x\ U(x) \to \neg A(x)$

8. a. $\exists_x (P(y,z) \wedge (\forall_y (\neg Q(y,x) \vee P(y,z))))$ $\phi[w/x]$
b $\exists_x (P(f(x),z) \wedge (\forall_y (\neg Q(y,x) \vee P(y,z))))$ $\phi[f(x)/y]$
c $\exists_x (P(y, g(y,z)) \wedge (\forall_y (\neg Q(y,x) \vee P(y, g(y,z)))))$ $\phi[g(y,z)/z]$

9. a 1. $(x=0) \vee ((x+x)>0)$ (premise)
2. $x=0$ ($\vee e$ 1)
3. $y=x+x$ (assume)
4. $y=0+x$ ($= e$ 1)
5. $x+x>0$ ($\vee e$ 1)
6. $y>0$ ($= e$ 3,5)
7. $y>0 \vee y=0+x$ ($\vee$, 4,6)
8. $y=x+x \to ((y>0) \vee (y=0+x))$ ($\to$, 3-7)

b 1. $\forall_x P(x)$ (premise)
2. x_0 $P(x_0)$ ($\forall_x e$ 1)
3. $\forall_y (P_y)$ ($\forall$, 2)

c
1. $\forall_x \phi$ (premise)
2. x_0 ϕ ($\forall_x e$ 1)
3. $\exists_x \phi$ ($\exists_x i$ 2)

d
1. $\neg\forall_x P(x)$ premise
2. x_0 $\neg P(x_0)$ ($\forall e$ 1)
3. $\exists_x \neg P(x)$ ($\exists_x i$ 2)
4. $\exists x$ $\neg P(x)$

e
1. $\forall_x (P(x) \to Q(x))$ (premise)
2. x_0 $P(x_0) \to Q(x_0)$ ($\forall_x e$ 1)
3. $\neg Q(x_0)$ (assume)
4. $\neg P(x_0)$ (MT 2 3)
5. $\neg Q(x_0) \to \neg P(x_0)$ ($\to i$ 3-4)
6. $\forall_x \neg Q(x) \to \forall_x \neg P(x)$ ($\forall_x i$ 2-5)

f
1. $\forall_x (P(x) \to \neg Q(x))$ (premise)
2. x_0 $P(x_0) \to \neg Q(x_0)$ ($\forall e$ 1)
3. $Q(x_0)$ (Assume)
4. $\neg P(x_0)$ (MT 2 3)
5. $Q(x_0) \wedge \neg P(x_0)$ ($\wedge$ 3,4)
6. $\neg \exists_x (P(x) \wedge Q(x))$ ($\exists_i$ 3-5)

1a Every binary that has a prefix is not a prefix of it's prefix

This holds true for the case where $x=y$

You can say the entire string is prefix.

If def requires different, just add ε to end

11 ~4 hours

13. Step 1

1. $p \to (q \to r)$ (premise)
2. $p \wedge \neg r$ (premise)
3. p ($\wedge e_1$ 2)
4. $(q \to r)$ ($\to e$ 1,3)
5. $\neg r$ ($\wedge e_2$ 2)
6. $\neg q$ (MT 4 5)

Step 2

p	q	r	$p \wedge \neg r$	$p \to (q \to r)$	$\neg q$
F	F	F			
F	F	T			
	⋮				
T	F	F	T	T	T
	⋮				

Step 3

File in zip

Step 4

Q13.a15 in zip File

Step 5

1. q (assumption)
2. $p \to (q \to r)$ (premise)
3. $p \wedge \neg r$ (premise)
4. p ($\wedge e_1$ 3)
5. $\neg r$ ($\wedge e_2$ 3)
6. $q \to r$ ($\to e$ 4 2)
7. $\neg q$ (MT 5,6)
8. $\perp$ ($\neg e$ 7, 1)