Quantum-Inspired Evolutionary Algorithms for High-Dimensional Multi-Objective Optimization (MaOO)

Below is a compact “mini-proposal” style explainer you can use to frame the topic, carve out contributions, and plan experiments.

1) Problem space in a nutshell

- **Goal:** Optimize **many conflicting objectives** (often 4–20+) over **high-dimensional decision spaces** (hundreds–thousands of variables).
- **Output:** A *set* of trade-off solutions approximating the **Pareto front** (no solution is strictly better in all objectives).
- **Why it’s hard:**
  - **Curse of dimensionality** → distance metrics lose contrast; diversity maintenance and convergence both degrade.
  - **Many-objective setting** → Pareto dominance gets weak (most solutions become non-dominated), making selection pressure tricky.
  - **Expensive evaluations** → real problems (e.g., simulation-based engineering, AutoML) can take minutes–hours per evaluation.

2) Quantum-inspired (QI) idea: probabilistic superposition + rotation search

“Quantum-inspired” ≠ running on a quantum computer. You **simulate** principles from quantum computing to improve exploration/exploitation:

- **Q-bit representation:** Each binary decision variable is a Q-bit with state $[\alpha, \beta]$ s.t. $|\alpha|^2 + |\beta|^2 = 1$. Measuring yields 0 with prob $|\alpha|^2$, 1 with $|\beta|^2$. A solution is sampled by “measuring” all Q-bits.
  - For real variables, use **QI binning** or **Gray coding** with Q-bits per dimension, or continuous analogues (amplitude vectors).
- **Rotation/Update operators:** Inspired by quantum gates: update $[\alpha, \beta]$ using a **rotation angle** $\Delta\theta$ guided by superior exemplars (leaders) to increase the probability of sampling their schema.
- **Population as probability model:** Instead of storing one fixed solution, each “individual” encodes a **distribution** over many candidate bitstrings → **implicit diversity** and **compact modeling** (like EDAs but lighter).
- **Interference-like effects:** Carefully designed correlated updates can encourage constructive combinations of schemata without premature fixation.

Why QI helps in high-D:
Probabilistic superposition keeps many possibilities “alive” cheaply; rotation steps give a **directional bias** without committing too early. This tends to scale better than hard-point mutation/crossover when selection pressure is weak.

3) Multi-objective mechanics to combine with QI

- **Dominance-based:** NSGA-II style ranking/crowding (weak for many objectives).
- **Decomposition:** MOEA/D / RVEA—convert M-objective problem into K scalar subproblems via weight/reference vectors.
- **Indicator-based:** Hypervolume, ε, R2 indicators to drive selection (careful with HV cost as objectives grow).
- **Preference-based:** Reference points, knee region biasing, or decision-maker priors to regain selection pressure.

Key metrics: Hypervolume (HV), Inverted Generational Distance (IGD/IGD+), ε-indicator, spacing, coverage. For MaOO, **IGD** and **R2** are commonly used due to HV complexity.

4) What a QI-MOEA looks like

1. **Initialize** Q-bit arrays (or continuous amplitudes) with broad superposition.
2. **Sample** a population by measuring each Q-bit string.
3. **Evaluate** objectives (and constraints).
4. **Select guides** (leaders) via decomposition/indicator rules.
5. **Rotate/Update** Q-bits toward leaders; adapt $\Delta\theta$ based on improvement signals, diversity, and constraint feasibility.
6. **Periodically re-measure** to generate new candidate sets; maintain archives/reference vectors.
7. **Stop** when budget is exhausted; return the nondominated archive.

5) High-impact research angles (contribution ideas)

- **A. Decomposition-aware QI updates:**
  Couple Q-bit rotations to **reference vectors** (RVEA/MOEA/D). Each subproblem updates amplitudes toward its *local* elite while sharing information across neighbors to preserve global diversity.
- **B. Adaptive rotation via performance indicators:**
  Learn $\Delta\theta$ from **online IGD (or R2)** improvements; larger steps where progress stalls, smaller near estimated Pareto manifold. Could use bandits for step-size control.
- **C. Correlated amplitude modeling:**
  Move beyond independent Q-bits with **low-rank linkage** or block rotations (QI-CMA-like) to capture variable dependencies in high-D without full covariance costs.
- **D. Constraint-aware phase shifts:**
  Use "phase" (sign of α, β) or auxiliary amplitudes to encode **feasibility likelihood**; rotate differently for feasible vs. infeasible elites (penalty-free feasibility pressure).
- **E. Hybrid surrogates:**
  Train **cheap surrogate models** (radial basis, random forests, tiny NNs) on the measured population; use **two-stage sampling**: exploit surrogate for wide Q-bit updates; verify elites with the true evaluator (trust-region style).
- **F. Many-objective selection that scales:**
  Replace expensive HV with **R2**, **IGD+**, or **decomposition**; design **QI-niching** that steers amplitudes to under-covered reference directions dynamically.
- **G. Dynamic & noisy objectives:**
  Keep **broad superposition** as a hedge; when drift/noise detected, widen amplitudes before re-focusing—an elegant QI answer to non-stationarity.

6) Benchmarks & baselines

- **Synthetic suites:** DTLZ (esp. DTLZ1–7 with 10–20 objectives), WFG (deceptive, nonseparable, bias), MaF (many-objective focus). Scale decision variables to 100–1000+.
- **Baselines:** NSGA-III, MOEA/D, RVEA, IBEA, HypE/SMS-EMOA (for ≤5 objs), SPEA2, R2-IBEA, MO-CMA-ES/LM-CMA variants.
- **Real problems (good demos):**
  - **Feature selection** and **AutoML hyper-parameter tuning** (accuracy, latency, model size, energy).
  - **Engineering design:** truss/airfoil (weight, stress, drag), process optimization.
  - **Scheduling/portfolio:** risk–return–cost–carbon.
  - **Neural architecture search:** accuracy, FLOPs, params, inference time.

7) Experimental protocol (robust & fair)

- **Scalability sweeps:** objectives $M \in \{5, 10, 15, 20\}$, dimensions $D \in \{100, 500, 1000\}$.
- **Budgets:** fixed evaluation counts (e.g., 50k–200k), plus expensive-evaluation regime (surrogate ablations).
- **Statistical rigor:** ≥30 independent runs; report median + IQR; **Wilcoxon** / **Friedman + Nemenyi** tests.
- **Ablations:** remove each proposed QI component (adaptive $\Delta\theta$, correlated blocks, decomposition coupling, surrogate) to quantify contribution.
- **Anytime curves:** IGD/HV over evaluations; coverage of reference directions; archive size growth.
- **Visualization:** parallel coordinates for objectives, 2D projections (PCA/t-SNE/UMAP) of decision vectors, attainment surfaces for M≤3.

8) Theoretical angles (optional but valuable)

- **Convergence in probability:** Show that under diminishing $\Delta\theta$ and sufficient exploration, sampling probability mass concentrates near Pareto-optimal regions.
- **Diversity lower bounds:** With decomposition, prove minimal coverage of weight vectors given non-zero exploration amplitude.
- **Complexity:** Per-generation cost dominated by evaluations; update cost $O(ND)$ for independent Q-bits, $O(NDk)$ for block-correlated (k = block size).

9) Design details that often decide results

- **Encoding real variables:** (i) multi-bit fixed-point per variable; (ii) sampled from a QI-parameterized Gaussian/Logit (continuous analogue); (iii) **mixed** discrete/continuous via heterogenous Q-blocks.
- **Adaptive sampling rate:** Don't "measure" all Q-bits every gen; keep partial resampling to maintain memory.
- **Leader selection:** Use **reference-vector neighborhoods**; rotate toward *multiple* leaders with weights to avoid mode collapse.
- **Constraint handling:** feasibility first, ε-constraint schedules, or penalty coefficients learned from archive statistics.
- **Restart/widening:** If stagnation, increase amplitude entropy on under-covered subproblems only.

10) Expected outcomes & risks

- **Outcomes:** Better **IGD/R2** than mainstream MaOO baselines under tight evaluation budgets; smoother scaling with D and M; strong results on AutoML/feature-selection.
- **Risks:** Indicator bias (e.g., poor knee coverage), HV intractability when M is large, surrogate drift.
- **Mitigations:** Multi-indicator selection, knee-region boosters, trust-region surrogates with uncertainty checks.

11) Practical tooling

- **Libraries:** `pymoo` (Python) and `jMetalPy` are great for quick baselines & metrics; PlatEMO (MATLAB) has many MaOO algorithms.
- **Reproducibility:** Fix RNG seeds, publish YAML configs, dump archives each checkpoint; release code + full logs.

A concrete thesis statement (you can adapt)

> *“We develop a decomposition-driven quantum-inspired evolutionary algorithm that uses adaptive, indicator-guided rotation of correlated Q-bit blocks to efficiently approximate many-objective Pareto fronts in high-dimensional spaces. Across DTLZ/WFG/MaF and real AutoML problems, our method delivers superior IGD/R2 under strict evaluation budgets, supported by ablations and convergence-in-probability analysis.”*