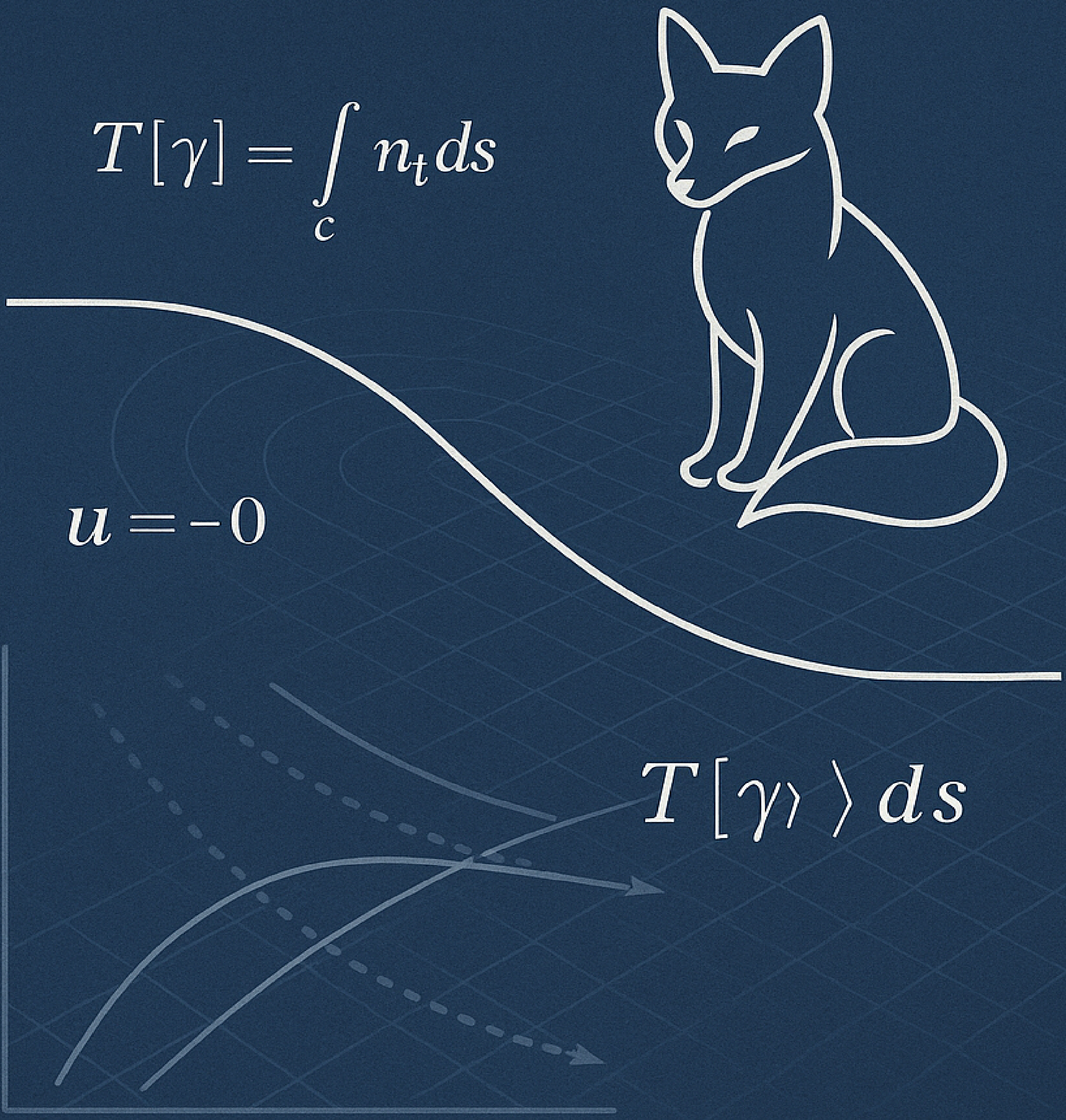
Principle of Temporal Economy



The Principle of Temporal Economy

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Abstract

We introduce the *Principle of Temporal Economy (PTE)*: when proper time is spatially non-uniform, physical evolution is naturally organized by stationarity of accumulated travel time in the local time-rate landscape, while familiar energy-based descriptions arise as secondary, regime-dependent encodings. In the Time Gradient / Time Field language we represent fractional time-rate curvature by the dimensionless field $u = \Delta\tau/\tau$ (weak field $u = \Phi/c^2$) and define an effective temporal index $n_t = 1 + u$. For fixed endpoints, stationary paths of the functional $T[\gamma] = \int_\gamma n_t \, ds/c$ yield a Snell-type temporal-geodesic law, linking least-time cycloids (brachistochrone structure) and gravitational bending within a single variational statement. At the field level, the vacuum condition $\nabla^2 u = 0$ is interpreted as harmonic smoothing of time-rate under boundary constraints. The framework highlights integrated observables—accumulated delay $\Delta\tau = \int u \, ds/c$ and phase shift $\Delta\varphi = \omega \, \Delta\tau$—and provides a schematic visualization program in which macroscopic relaxation paths (e.g., crack-like trajectories) trace temporal-gradient streamlines. PTE is empirically non-trivial in regimes where integrated $\Delta\tau$ (and hence $\Delta\varphi = \omega\Delta\tau$) can be large despite weak local gradients, as for extended low-density profiles.

Following the variational intuition of Pierre Louis Maupertuis and Euler, PTE treats least action as an economy principle and proposes that, in a time-rate landscape, this economy is naturally expressed as stationarity of accumulated proper time (time-of-flight) under constraints.

Okilmes: the gravity of understanding.

Contents

1 Motivation

Classical dynamics admits many equivalent formulations, but it is commonly interpreted through energy-centric intuition. However, there exists a robust and well-studied class of variational problems in which the extremized quantity is *time*. The brachistochrone provides the archetype: under uniform gravity, the minimum-time curve between two fixed points is a cycloid.

That same least-time structure reappears sometimes explicitly (optical Fermat paths), sometimes implicitly (geodesic selection, integrated phase). The repeated emergence of cycloidal and cycloid-like forms across mechanics, optics, and planetary geology motivates a unified interpretation: the Principle of Temporal Economy, which treats the restoration of temporal coherence as the primary variational driver, and energy-minimization as a derived, regime-dependent description.

Scope. This paper focuses on variational structure and integrated observables; it does not attempt a full elastic fracture mechanics model.

Non-claim. PTE is proposed as an interpretive variational hierarchy and modeling guide; it introduces no new adjustable constants and preserves standard weak-field predictions under $u = \Phi/c^2$.

Notation

We use $\tau(x)$ for proper-time field, $u = \Delta\tau/\tau$ for dimensionless time-rate curvature (weak-field $u = \Phi/c^2$), and $n_t = 1 + u$ for the effective temporal index. Here $\Delta\tau/\tau$ denotes a fractional deviation of time-rate relative to a reference flow, not the Laplacian operator. $\Delta\tau$ denotes a finite accumulated proper-time offset (or fractional deviation relative to a reference flow); the Laplacian operator is written exclusively as ∇^2 throughout.

2 Statement of the Principle

Principle of Temporal Economy (PTE):

> In any physical system where proper time is non-uniform, physically realized trajectories are stationary points of an appropriate travel-time functional built from the local time-rate field, under fixed boundary conditions, causal connectivity, and continuity constraints.

This principle is scale-independent and does not rely on a specific force law. In the weak-field case we take

$$T[\gamma] = \int_\gamma (1 + u)\,\frac{ds}{c},$$

with

$$u = \frac{\Delta\tau}{\tau}.$$

Here $\Delta\tau/\tau$ denotes the fractional deviation of proper-time rate relative to a reference flow.

3 Time Field Theory Link to Time as a Physical Field

We treat proper time $\tau(x)$ as a physical scalar field. Its dimensionless curvature is defined as

$$u(x) = \frac{\Delta\tau}{\tau}. \to u = \frac{\Phi}{c^2}. \tag{1}$$

In the weak-field limit this recovers the gravitational potential. Spatial gradients of u correspond to gradients in clock rate.

4 Laplacian Dynamics as Temporal Smoothing

In vacuum, the Newtonian potential satisfies the Laplace equation,

$$\nabla^2\Phi = 0 \quad \Leftrightarrow \quad \nabla^2 u = 0, \tag{2}$$

where $u = \Delta\tau/\tau$ and, in the weak-field regime, $u = \Phi/c^2$.

Rather than being introduced as an *energy-minimization rule*, the Laplacian condition can be read as a *coherence constraint* on the time-rate landscape: u is harmonic between boundaries and sources, i.e. it adopts the smoothest admissible interpolation consistent with the imposed constraints. Equivalently, u is the stationary point of the Dirichlet (roughness) functional

$$\mathcal{D}[u] = \frac{1}{2}\int |\nabla u|^2 \, dV, \tag{3}$$

which yields $\nabla^2 u = 0$ in vacuum under fixed boundary conditions.

This interpretation motivates treating vacuum time-rate structure as a harmonic smoothing process in which gradients redistribute under constraints. Energy-based intuition can be recovered as a derived language once one identifies energetic densities with quadratic forms in ∇u in the appropriate limit.

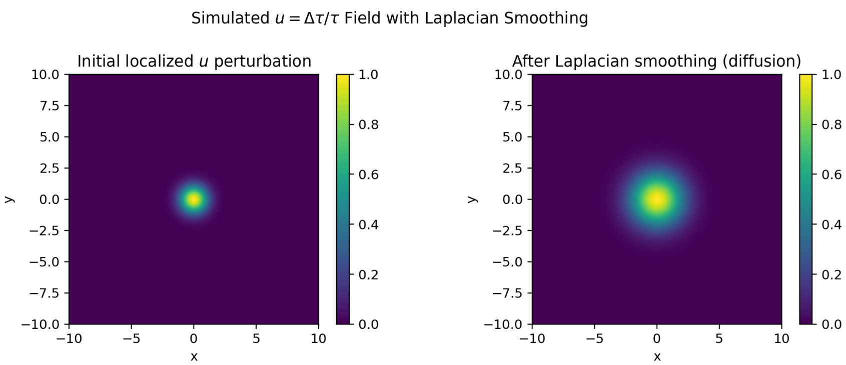

Figure 1: Simulated temporal-curvature field $u = \Delta\tau/\tau$ undergoing Laplacian smoothing from a localized perturbation. In the PTE lens, $\nabla^2 u = 0$ is read as harmonic coherence of the time-rate landscape between boundaries, rather than as a primitive force statement.

5 Cycloids as Temporal Geodesics

We define a temporal geodesic as a path that extremizes accumulated proper-time delay in a non-uniform time field. Cycloids arise as solutions to the brachistochrone problem because they minimize travel time. Under PTE, cycloids are interpreted as *temporal geodesics* where paths of extremal proper time through a non-uniform time field. The cycloid emerges as the solution to the brachistochrone problem because it minimizes travel time under gravity. Under PTE, this result generalizes:

- Cycloids represent **temporal geodesics**.
- They are the natural relaxation paths of systems evolving in a non-uniform time field.

This could explain their appearance in:

- Classical mechanics
- Optical refraction
- Fracture propagation under gravity
- Large-scale geological patterns

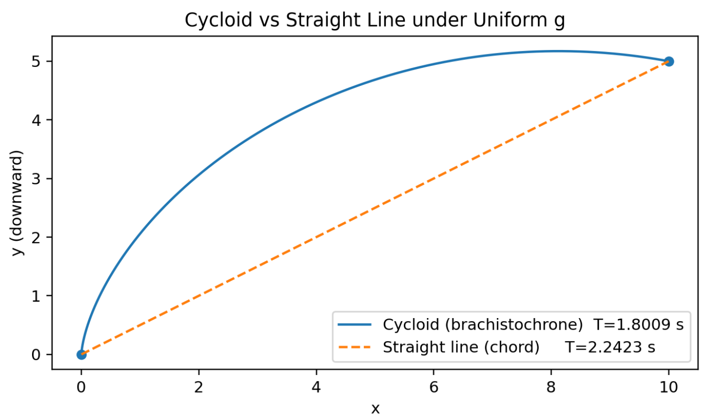

Figure 2: Cycloid (brachistochrone) versus straight-line chord under uniform gravitational acceleration. The cycloid extremizes travel time, illustrating the time-first variational signature used in the Principle of Temporal Economy.

6 Planetary Fracture Geometry

Surface fractures on icy moons (most prominently *Europa*) include long, arc-shaped *cycloidal* crack chains that are widely interpreted as tensile cracks propagating through a *time-varying tidal stress field.* [5, 7] Related Europa-style, cycloid-like arcuate rifts have also been discussed for *Enceladus.* [8]

Within PTE, these morphologies are reinterpreted as macroscopic records of relaxation in a non-uniform time-rate landscape:

- Time-varying tidal forcing (and associated gravity-loaded stresses) is represented as a modulation of the temporal-curvature field $u = \Delta\tau/\tau$.
- Crack tips follow paths of fastest temporal relaxation, modeled schematically as temporal-gradient streamlines (e.g. $d\mathbf{r}/ds \propto -\nabla u$) under boundary and continuity constraints.
- Curving, cycloid-like trajectories emerge naturally when propagation occurs through a changing effective field, consistent with the observed chained-arc morphology.

This interpretation does not replace elastic fracture mechanics; rather, it provides a time-first variational lens in which the observed geometry is read as a trajectory selected by temporal coherence (integrated $\Delta\tau$) rather than by a purely static energy-minimization picture. PTE is offered as a trajectory-selection lens layered on top of fracture mechanics, not a substitute constitutive model.

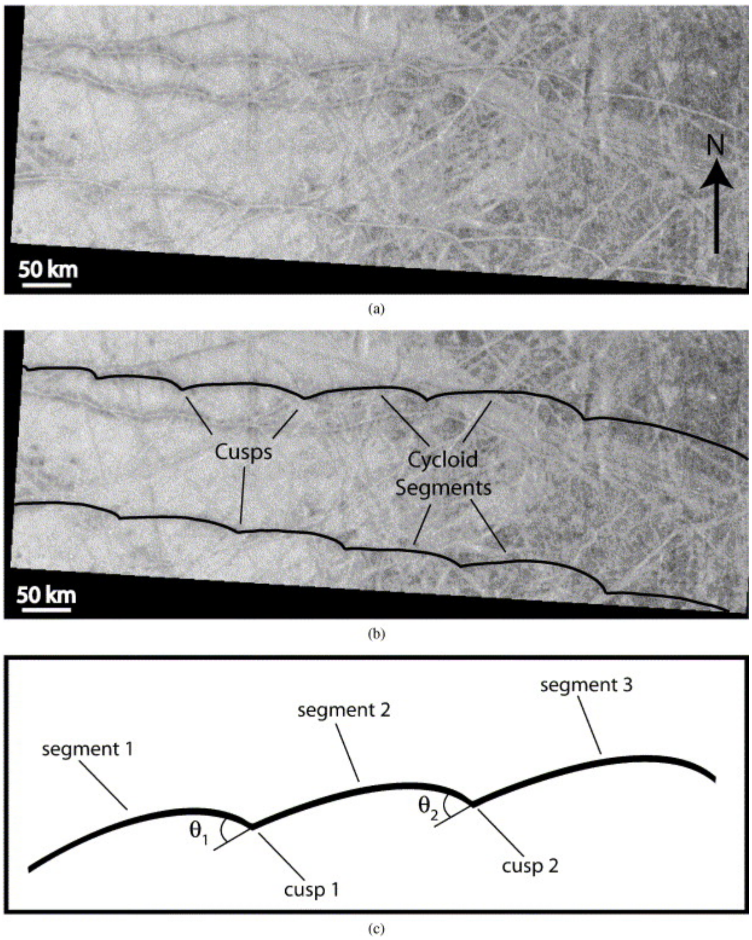

Figure 3: **Cycloidal fracture chain on Europa.** (a) Galileo image mosaic showing arcuate, cuspate lineaments at ~50 km scale. (b) Interpreted trace highlighting cusps and cycloid segments. (c) Schematic definition of segment-to-segment cusp geometry (angles θ_i). *After* Marshall & Kattenhorn (2005), based on Galileo imagery; reproduced/adapted with appropriate credit.

7 Energy as a Derived Quantity

Within this framework, the dimensionless temporal curvature is

$$u \equiv \frac{\Delta\tau}{\tau}. \tag{4}$$

The natural energetic quantity associated with u is the *specific* energy (energy per unit mass)

$$\varepsilon \equiv c^2 u, \tag{5}$$

which has units of $\mathrm{J\,kg^{-1}}$ and may be identified with the weak-field potential scale via $u = \Phi/c^2$, hence $\varepsilon = \Phi$.

For a particle of rest mass m, the corresponding energy scale is

$$E = m\,c^2\,u = m\,\varepsilon. \tag{6}$$

In the weak-field limit this reproduces the standard gravitational potential energy change $E \simeq m\Phi$ and is consistent with gravitational redshift to leading order.

8 Conceptual Implications

Viewed through the PTE time-first lens, several *interpretive* and *modeling* implications follow:

- **Space as derived description:** spatial geometry can be treated as an effective representation of underlying time-rate structure (i.e. gradients and curvature in $u = \Delta\tau/\tau$).
- **Gravity as time-rate gradient:** gravitational acceleration is naturally read as motion along temporal-index gradients ($n_t = 1+u$), so "attraction" is recast as refraction/relaxation in a non-uniform time flow.
- **Interference as phase bookkeeping:** wave interference emphasizes temporal phase alignment because phase accumulates with proper time ($\Delta\varphi = \omega\,\Delta\tau$), making integrated time-delay the primary control variable for coherent addition.
- **Quantization as loop consistency:** discrete spectra can be interpreted as consistency conditions for closed phase accumulation (loop closure) in a temporally curved field, without asserting that time itself is fundamentally discrete.

9 Minimal Variational Links (Equations)

This section pins PTE to standard variational physics using the smallest possible bridge.

Temporal Geodesic Lemma (PTE). Let $u(x) = \Delta\tau/\tau$ be the dimensionless time-curvature field, and define an effective temporal index

$$n_t(x) \equiv 1 + u(x)$$

(weak-field: $|u| \ll 1$). For any path γ connecting fixed endpoints, define the temporal–optical functional

$$T[\gamma] = \int_\gamma n_t(x)\,\frac{ds}{c}.$$

Then the physically realized trajectory is a stationary path of T ($\delta T = 0$). In piecewise-smooth n_t media, the stationarity condition implies a Snell-type refraction law:

$$n_t \sin\theta = \text{const along the ray},$$

where θ is the angle between the path tangent and the local normal of an n_t-interface (or equivalently, between the tangent and ∇n_t in a smooth medium). In the Newtonian weak-field identification $u = \Phi/c^2$, this yields bending toward slower-time regions and reduces at leading order to

$$\vec{g} \propto -\nabla u.$$

In a smooth medium (no sharp interfaces), the corresponding ray equation can be written compactly as

$$\frac{d}{ds}\left(n_t\,\hat{t}\right) = \nabla n_t,$$

where s is arclength and $\hat{t}$ is the unit tangent to the path. This makes the "refraction by ∇u" explicit: spatial gradients in time-rate act exactly like an index gradient bending rays.

9.1 Time-as-action in the optical/mechanical sense

A travel-time functional is

$$T[\gamma] = \int_\gamma \frac{ds}{v(x)}.$$

Extremizing T (Fermat/brachistochrone logic) yields the Euler–Lagrange condition for the path γ. In a constant gravitational field with energy conservation

$$v(y) = \sqrt{2gy}$$

(after a vertical drop from rest), one obtains the cycloid as the minimizing curve.

9.2 Proper time as the relativistic action

In GR, free-fall is an extremum of proper time:

$$\delta\tau = 0, \qquad \tau[\gamma] = \int_\gamma \sqrt{-g_{\mu\nu}\,dx^\mu dx^\nu}.$$

In the weak-field limit,

$$\frac{d\tau}{dt} \approx 1 + \frac{\Phi}{c^2},$$

so spatial variations in Φ are literally spatial variations in clock-rate.

9.3 TFT/TGT identification: gravity is time-curvature

Define the dimensionless temporal curvature

$$u(x) \equiv \frac{\Delta\tau}{\tau} = \frac{\Phi(x)}{c^2}.$$

A minimal "time-first" path functional consistent with this is a time-of-flight through a non-uniform time field:

$$T[\gamma] \approx \int_\gamma \Big(1 + u(x)\Big) \frac{ds}{c}.$$

Interpretation: $1 + u$ plays the role of an *effective temporal index* (a local slow-down factor). Extremizing T gives the path rule; gradients in u bend trajectories. To first order, the associated acceleration follows the time-gradient:

$$\vec{g} \propto -\nabla u \equiv -\nabla\left(\frac{\Delta\tau}{\tau}\right),$$

which is equivalent to the Newtonian relation $\vec{g} = -\nabla\Phi$ under $\Phi = c^2 u$.

9.4 Laplacian smoothing as temporal economy

In vacuum, the potential is harmonic:

$$\nabla^2\Phi = 0 \quad \Leftrightarrow \quad \nabla^2 u = 0.$$

This can be obtained by minimizing the Dirichlet functional

$$\mathcal{D}[u] = \int |\nabla u|^2 \, dV, \qquad \delta\mathcal{D} = 0 \ \Rightarrow \ \nabla^2 u = 0.$$

Under PTE, $\mathcal{D}$ is interpreted not as "energy preference" in isolation, but as the field's *smoothest admissible redistribution of time-rate*, i.e. the most coherent (least rough) time landscape compatible with sources and boundaries.

10 Minimal Empirical Distinction

Time-first optimization predicts that observables depend on integrated temporal curvature rather than local force alone.

10.1 Accumulated time delay

$$\Delta\tau = \int u(x(s)), \frac{ds}{c}. \tag{7}$$

10.2 Phase shift

For a probe of angular frequency ω,

$$\Delta\varphi = \omega, \Delta\tau. \tag{8}$$

This applies directly to interferometric experiments.

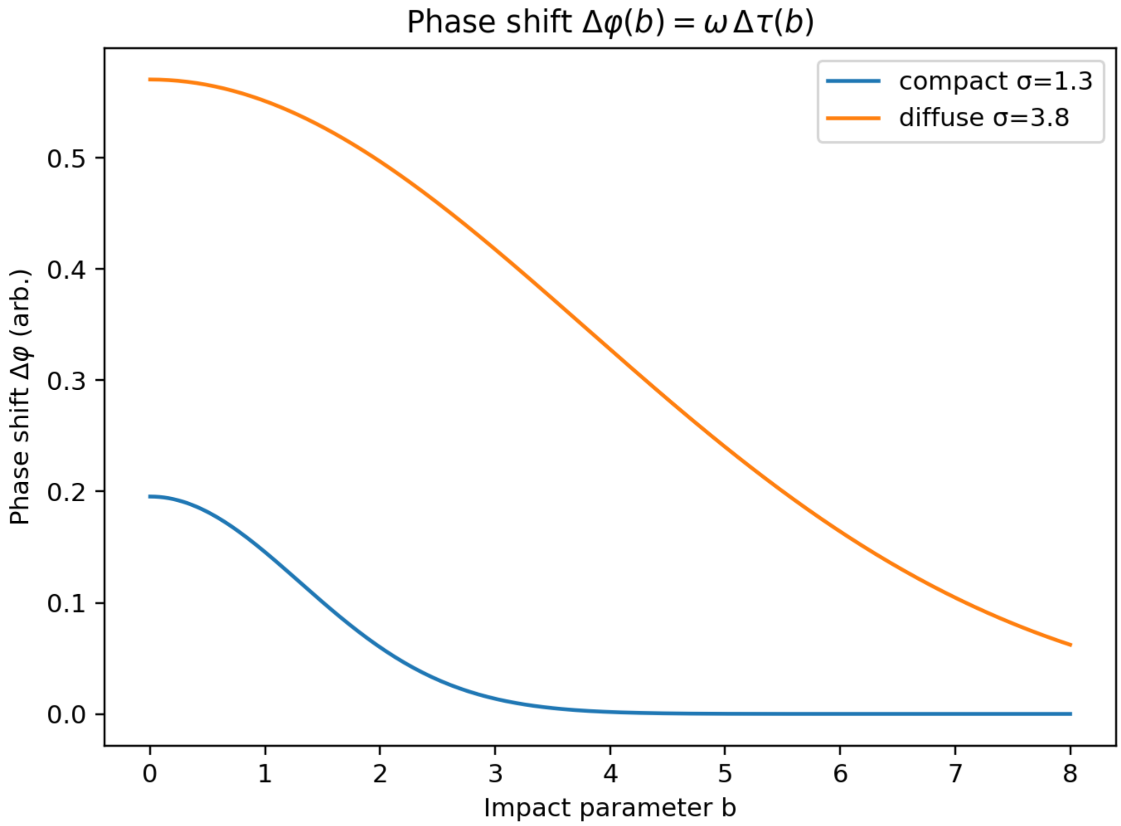

Figure 4: Phase shift $\Delta\varphi = \omega\,\Delta\tau$ as a function of impact parameter b for compact versus diffuse time-curvature profiles (schematic).

10.3 Discriminators (what would make PTE non-trivial)

PTE becomes empirically distinct when integrated time-curvature competes with local-gradient intuition. Three practical discriminators are: (i) phase/clock observables scaling with $\Delta\tau = \int u\,ds/c$ even where $|\nabla u|$ is weak, (ii) extended low-density sources producing measurable cumulative $\Delta\varphi = \omega\Delta\tau$ at large impact parameter, and (iii) trajectory families that align with temporal-gradient streamlines under changing boundary constraints (e.g. time-varying tides) where static energy-minimization heuristics fail.

11 Paths Traced as Streamlines of The Time Field Theory

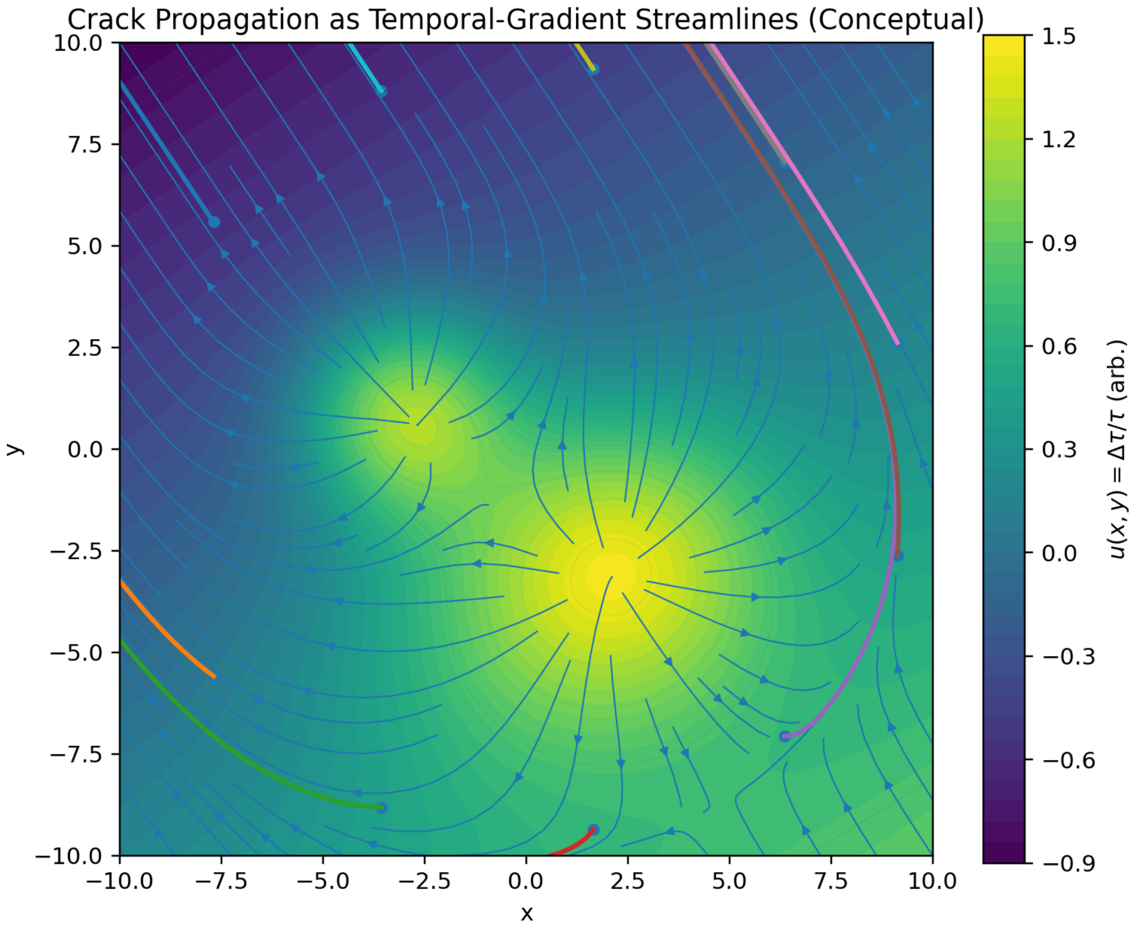

Figure 5: Conceptual crack propagation paths traced as streamlines of the temporal-gradient field (normalized $-\nabla u$) over a curved-surface proxy. The plotted curves illustrate how fracture-like trajectories can be modeled as temporal-relaxation paths in a non-uniform time field.

11.1 How Figure 5 was generated (Visualization in TFT Language).

Define a simple *temporal stress landscape*

$$u(x, y),$$

optionally smoothing it via diffusion:

$$\partial_t u = \kappa \nabla^2 u.$$

Compute spatial gradients

$$\nabla u$$

(or the tangential gradient $\nabla_{\parallel} u$ when restricted to a surface).

Seed crack tips

$$\mathbf{r}_0$$

along a chosen boundary or arc. Crack-like relaxation paths can be modeled as streamlines of the temporal-gradient field. Choosing arclength parameter s, we take the crack-tip direction to follow steepest descent of the time-curvature landscape:

$$\frac{d\mathbf{r}}{ds} = -\frac{\nabla u(\mathbf{r})}{\|\nabla u(\mathbf{r})\| + \epsilon}, \tag{9}$$

where $\epsilon > 0$ prevents numerical instability near critical points. Using a stable numerical integrator (e.g., RK4) to obtain smooth crack trajectories. The minus sign enforces temporal relaxation toward lower u (reduced temporal mismatch), consistent with the "descent" interpretation used throughout.

We interpret crack advance as a temporal-relaxation path on a surface. Define the dimensionless temporal curvature

$$u(x) = \frac{\Delta\tau(x)}{\tau}, \qquad n_t(x) = 1 + u(x),$$

and treat the crack tip as a particle whose direction follows steepest descent of time-curvature. On the surface, the propagation direction is

$$\hat{\mathbf{t}}(s) = -\frac{\nabla_{\parallel} u(\mathbf{r})}{\|\nabla_{\parallel} u(\mathbf{r})\| + \epsilon}, \qquad \frac{d\mathbf{r}}{ds} = \hat{\mathbf{t}}(s).$$

Numerically: (i) construct a smooth $u(x, y)$ field (basic sources + optional Laplacian smoothing), (ii) compute ∇u, (iii) seed multiple crack tips on a boundary, (iv) integrate the ODE using RK4 to obtain crack-like trajectories. The resulting curves visualize "time seeking coherence": paths align with temporal-gradient streamlines rather than explicit energy-minimizing fracture mechanics.

12 Toward an Explicit Action Principle (PTE Formalization)

The previous sections use a time-first language to motivate PTE. Here we package that intuition into a minimal variational structure that (i) reproduces the temporal-geodesic lemma for trajectories and (ii) clarifies why Laplacian behavior appears naturally as temporal smoothing.

Minimal variational skeleton:

$$\mathcal{J}[u, \gamma] = \frac{1}{2}\int |\nabla u|^2 \, dV \; - \; \int s\, u \, dV \; + \; \lambda \int_{\gamma} (1 + u)\, \frac{ds}{c}.$$

12.1 Two coupled extremization problems

PTE separates into two complementary variational problems:

1. a *trajectory principle*: for a given time-rate landscape $u(x)$ (equivalently $n_t(x) = 1 + u$), physical paths are selected by stationarity of accumulated travel time;
2. a *field principle*: for given boundary conditions (and possible sources), the time-rate landscape relaxes toward the smoothest admissible configuration.

Analogy (optics): rays extremize travel time for a fixed refractive index, while the medium determines the index. PTE uses the same logic, but with a *temporal index* rather than an optical one.

12.2 Trajectory action: least-time in the temporal index

Define the temporal travel-time functional

$$T[\gamma] = \int_\gamma n_t(x)\,\frac{ds}{c}, \qquad n_t(x) = 1 + u(x), \tag{10}$$

which plays the role of a time-of-flight action. Stationarity $\delta T = 0$ yields the temporal-geodesic condition (Snell-type refraction in piecewise media, and the smooth-medium ray equation already stated in the lemma). *Analogy (Fermat):* n_t acts like an index that encodes how "expensive" it is (in time) to traverse a region. Paths bend toward slower-time regions in the same way light bends toward higher optical index.

12.3 Field action: temporal coherence as smoothing

To formalize the statement "time seeks coherence," we introduce the minimal vacuum functional that penalizes roughness in the time-rate landscape:

$$\mathcal{D}[u] = \frac{1}{2}\int |\nabla u|^2\,dV. \tag{11}$$

Under fixed boundary conditions, stationarity gives the Euler–Lagrange condition

$$\delta\mathcal{D} = 0 \quad \Rightarrow \quad \nabla^2 u = 0, \tag{12}$$

which is precisely the Laplacian condition interpreted in this manuscript as harmonic redistribution of time-rate rather than an energy-minimization axiom. *Analogy (membrane / soap film):* u behaves like the height of a stretched sheet pinned at the boundary. The sheet settles into the smoothest surface compatible with its constraints. In the same sense, $\mathcal{D}$ measures the "roughness" of time-rate; minimizing it produces the most coherent time landscape.

12.4 Adding sources without introducing free structure

In matter-dominated regions, the simplest controlled extension is a linear coupling to a source term $s(x)$:

$$\mathcal{D}_s[u] = \frac{1}{2}\int |\nabla u|^2\,dV - \int s(x)\,u(x)\,dV, \tag{13}$$

whose stationarity yields a Poisson-type equation

$$\nabla^2 u = s(x). \tag{14}$$

Under the weak-field identification $u = \Phi/c^2$ and Newtonian Poisson equation $\nabla^2\Phi = 4\pi G\rho$, a natural choice is

$$s(x) = \frac{4\pi G}{c^2}\,\rho(x), \tag{15}$$

so that the standard field equation is recovered as a time-rate coherence equation with sources.

12.5 A minimal joint action (field + path)

For conceptual completeness, one may combine field coherence and trajectory time into a single objective:

$$\mathcal{J}[u,\gamma] = \underbrace{\frac{1}{2}\int |\nabla u|^2\,dV - \int s\,u\,dV}_{\text{field coherence}} + \lambda \underbrace{\int_\gamma (1+u)\,\frac{ds}{c}}_{\text{trajectory time}}, \tag{16}$$

where λ is a bookkeeping weight (e.g. enforcing the path term as a constraint or selecting a family of coupled solutions). Varying γ at fixed u reproduces the temporal-geodesic condition; varying u at fixed sources and boundary conditions reproduces Laplacian smoothing in vacuum (and Poisson-type behavior with sources).

This is not claimed to be the unique or final action. It is the smallest variational skeleton consistent with PTE: time-rate smooths under constraints, and trajectories are selected by least-time in the resulting temporal index.

12.6 Euler Lagrange Variations

Start from the functional

$$\mathcal{J}[u,\gamma] = \frac{1}{2}\int |\nabla u|^2\,dV - \int s\,u\,dV + \lambda\int_\gamma (1+u)\,\frac{ds}{c}.$$

Variation with respect to the field $u(x)$.
Taking $u \to u + \delta u$ and integrating by parts,

$$\delta\mathcal{J} = \int \Big(-\nabla^2 u - s\Big)\,\delta u\,dV + \lambda\int_\gamma \delta u\,\frac{ds}{c}.$$

Stationarity for arbitrary δu yields the field equation

$$\nabla^2 u = -s$$

in the bulk, together with a line-source contribution along γ,

$$\nabla^2 u = -s - \frac{\lambda}{c}\,\delta_\gamma,$$

where δ_γ is a Dirac measure supported on the path.

Variation with respect to the path γ. Varying the curve while holding endpoints fixed gives

$$\delta \int_\gamma (1+u)\, ds = 0,$$

which implies the ray (geodesic) equation

$$\frac{d}{ds}\Big[(1+u)\,\hat{t}\Big] = \nabla u,$$

where s is arclength and $\hat{t}$ is the unit tangent.

Interpretation. The coupled system is therefore:

$$\boxed{\begin{aligned} \nabla^2 u &= -s && \text{(temporal field)} \\ \frac{d}{ds}\Big[(1+u)\,\hat{t}\Big] &= \nabla u && \text{(trajectory)} \end{aligned}}$$

The first equation enforces smooth redistribution of time-curvature, while the second states that paths refract along gradients of clock-rate.

12.7 Reinterpreting least action

Standard mechanics selects dynamics by stationarity of an action functional. PTE suggests a hierarchy in which the most primitive functional is time-of-flight in the *temporal metric* (encoded by n_t), while conventional actions can be read as effective encodings of how systems trade geometry, phase, and curvature to reduce temporal mismatch.

In short: *least action can be read as least-time once the correct temporal index is identified.* This provides a disciplined bridge between classical variational physics, proper-time extremization, and the integrated observables emphasized elsewhere in the manuscript.

PTE is therefore not proposed as a new force law, but as a re-ordering of explanation: least action is interpreted as least-time in the appropriate temporal index, and familiar energetic descriptions arise as derived conveniences.

13 Conclusion

The Principle of Temporal Economy reframes a wide class of physical behavior as the consequence of a single organizing tendency: the restoration of temporal coherence. In this view, the primary optimized quantity is not energy, distance, or a particular force functional, but accumulated proper-time structure, represented by the dimensionless time-rate curvature field $u = \Delta\tau/\tau$ (with the weak-field correspondence $u = \Phi/c^2$).

This time-first perspective clarifies why least-time geometries recur across domains. The cycloid, historically known as the brachistochrone solution, is not merely a special curve in classical mechanics but a signature of time-of-flight stationarity in an effective temporal index. Likewise, the Laplacian character of vacuum gravitational fields is interpreted here as harmonic smoothing of time-rate a coherence condition that can be derived from a minimal roughness functional, while the familiar "energy-minimizing" intuition appears as a secondary, regime-dependent translation.

Most importantly, PTE connects three descriptive levels without altering established laws: (i) classical least-time paths, (ii) relativistic proper-time extremization, and (iii) macroscopic relaxation patterns (e.g. fracture trajectories) that can be modeled schematically as streamlines of temporal gradients. Within this unification, "least action" may be read as least-time in the appropriate temporal metric: the action functions as a bookkeeping device for how systems trade curvature, phase, and geometry to reduce temporal mismatch.

The result is not a new force law but a proposed hierarchy of explanation. Energy, force, and geometry remain valid and predictive, yet appear as derived languages describing the same deeper constraint: time-rate structure interpolates and relaxes under boundary conditions and causal connectivity. The practical criterion for non-triviality is therefore empirical: PTE matters precisely in regimes where integrated proper-time observables (e.g. $\Delta\tau$ and $\Delta\varphi = \omega\,\Delta\tau$) provide cleaner discrimination than local-gradient intuition. In that sense, PTE is offered as a minimal bridge between classical variational physics, general relativistic time structure, and observable relaxation patterns in natural systems.

This paper's goal is to make that bridge explicit enough to compute, visualize, and test.

Outlook

Future work proceeds along deliberately separated tracks:

(1) Quantitative phenomenology. The conceptual crack–streamline picture will be upgraded into a data-facing model by (i) importing mapped fracture traces on icy bodies, (ii) fitting a low-parameter u landscape consistent with boundary conditions, and (iii) testing whether temporal-gradient geodesics reproduce observed cycloid-like arcs more naturally than elastic-only heuristics. This will clarify what aspects of fracture geometry are explained by temporal structure alone and what requires additional mechanical constraints.

(2) Laboratory observables. Because PTE emphasizes integrated proper time, the cleanest tests target accumulated quantities rather than instantaneous forces. We will prioritize phase- and time-sensitive measurements (atom and slow-neutron interferometry, precision clock comparisons) and explore long-baseline scenarios in which extended, low-density time curvature can yield a measurable $\Delta\tau$ even when local gradients are weak.

(3) Variational closure. We will refine a closed formulation in which trajectories arise as stationary paths of the temporal functional $T[\gamma]$, while the field $u = \Delta\tau/\tau$ follows a coherence principle that reduces to harmonic behavior in vacuum and admits physically interpretable source terms.

(4) Open computational reference. An open implementation will (i) generate u landscapes under boundary constraints, (ii) integrate temporal geodesics/streamlines on planar and spherical shells, and (iii) output predicted traces, deflection curves, and $\Delta\tau$ / $\Delta\varphi$ maps for direct comparison with data.

(5) Validation plan. Two side-by-side benchmarks will be emphasized: (i) *planetary morphology* (fit-and-compare against elastic baselines) and (ii) *precision phase tests* (predictions for $\Delta\varphi = \omega\,\Delta\tau$ and related time-delay observables), focusing on regimes where integrated time curvature is large despite weak local gradients.

The aim is not to assert new physics prematurely, but to provide clear criteria for when "least action" may be read as least-time in an appropriate temporal metric and, crucially, to identify where this reinterpretation is purely explanatory versus where it becomes experimentally discriminating.

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