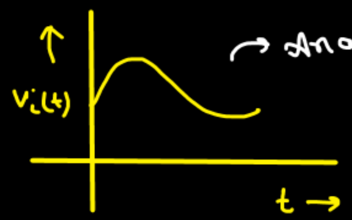
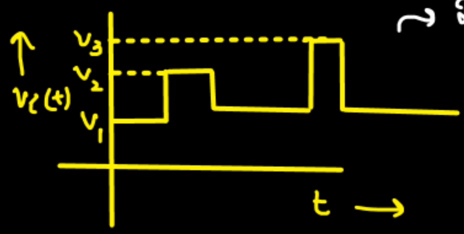
# Digital Electronics

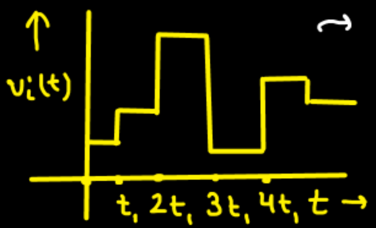
## * Digital v/s Analog signals:-



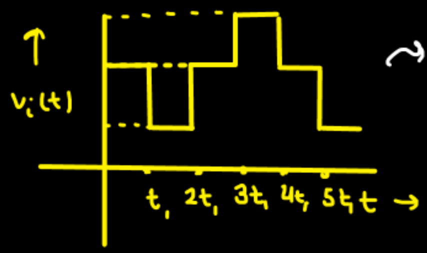

→ Analog signal




→ Digital signal



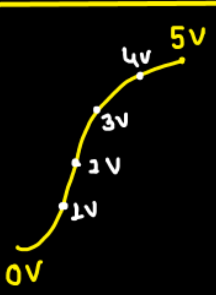

→ → Analog signal




→ Digital signal

* Continuous Amplitude → Analog signals
* Discrete Amplitude → Digital Signals

## Biggest Advantage of Digital over Analog:-




Analog

$\xrightarrow[\text{Magnitude}]{\text{Discretized}}$

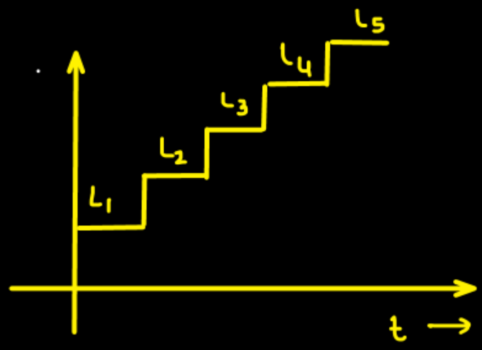
$L_1 \rightarrow$ 0V - 1V

$L_2 \rightarrow$ 1V - 2V

$L_3 \rightarrow$ 2V - 3V

$L_4 \rightarrow$ 3V - 4V

$L_5 \rightarrow$ 4V - 5V




## Transmitting a Signal:-

**Analog:-**

Tr 1.72V → [ Wire ] → Rx 1.75V

+0.03V

**Digital:-**

Tr $L_2$ [1.72V] → [ Wire ] → Rx $L_2$ [1.75V]

+0.03

- Better in noise performance.
- Easy to store

## Binary Digits, Logic Level and Digital waveform:-

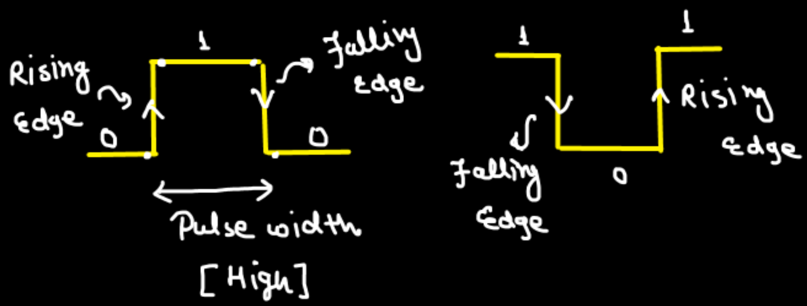
★ Generally, in Digital system there are only two possible states.

High
[1, +5V]

Low
[0, 0V]

Digital systems such as computers can only understand '1' & '0'. This two state number system is known as "binary number system". and its two digits '0' and '1' are called bits.

## A Digital waveform:-

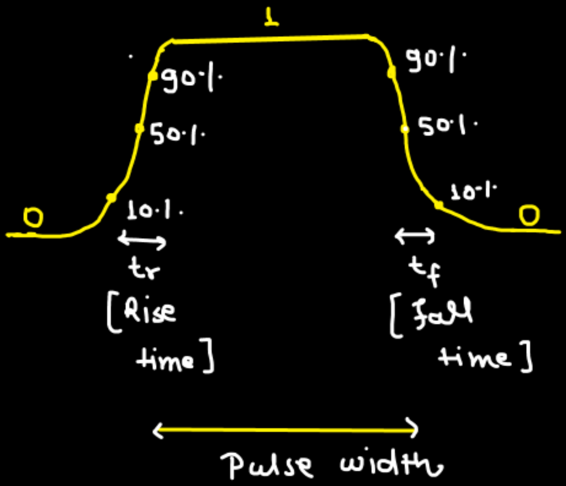



★ **Rise time** :- Time taken to go from 0 to 1. [10% → 90%]

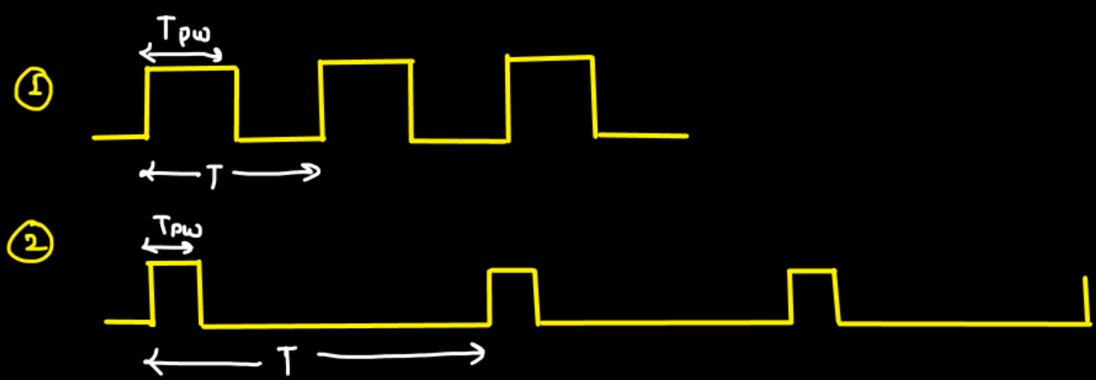
★ **Fall time** :- Time taken to come from 1 to 0. [90% → 10%]

ideally, Rise time = fall time = 0 sec.

But Practically, digital waveforms will have some finite rise and fall time




## Periodic Digital waveforms :-




**Time period (T) :-** The time window at which the waveform is being repeated.

## Non-periodic Digital waveforms:-

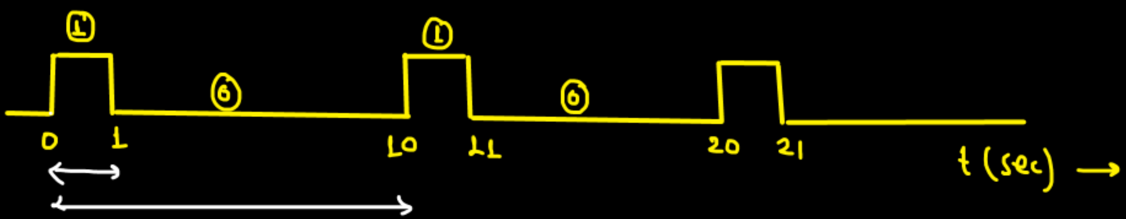
★ **Duty cycle:-**

Ratio of pulse width to the time period.

$$\text{Duty cycle} = \frac{T_{pw}}{T} \times 100\,\%$$

$$\boxed{f = \frac{1}{T}}$$

Q.




For the given waveform, find

(a) Pulse width = 1 sec.

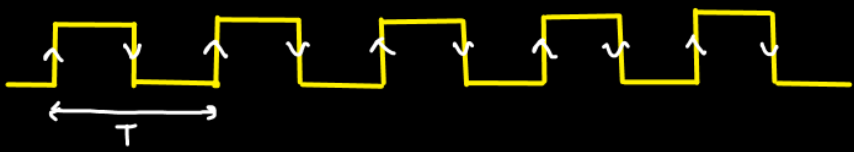
(b) Time period = 10 sec.

(c) Frequency = $\frac{1}{10}$ = 0.1 Hz

(d) Duty cycle = $\frac{1}{10} \times 100\,\% = 10\,\%$

## Clock :-

An electronic logic signal which oscillates between a High and a low state at a constant freq. and ideally have a duty cycle of 50%.




Time period of the clock = T

Frequency of the clock = $f = \frac{1}{T}$

## ★ Data Sequence :-

The Electronic representation of an information in terms of bits.

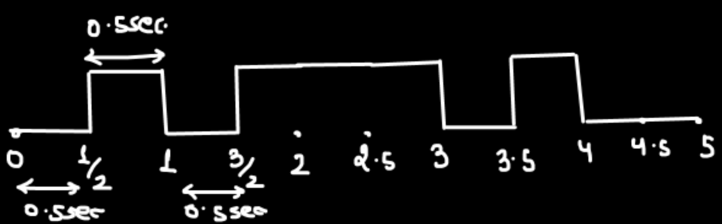
↳ A random data sequence → 0101110100

★ Data rate → 2 bps [ bits per second ]

2bps :-

1 sec. → 2 bits

1 bits → $\frac{1}{2}$ sec.



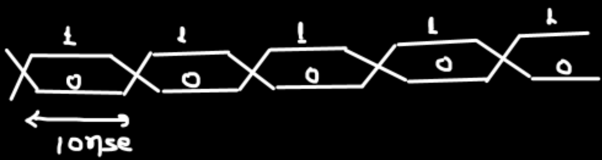
Q. Make a clock and data sequence given that clock frequency and data rate are 100 MHz and 100Mbps [Take any random data sequence]

↳ **Clock:-**

$f = 100MHz = 10^8 Hz$

$T = 10^{-8}$ sec. = 10nsec.


**Data:-** 1011/0111/1110 ....



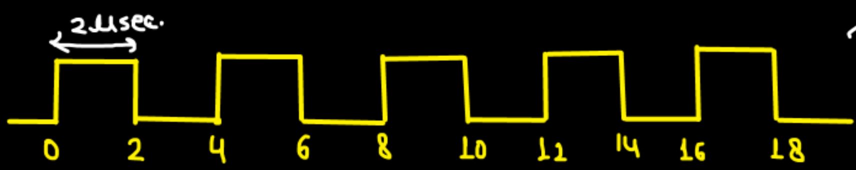

data rate = $10^8$ bps

1sec. → $10^8$ bits

1 bit → $10^{-8}$ sec.

→ 10nsec.

Q. A data line is shown.




t (μs) →

→ NOT a clock

→ Data sequence

Find the data rate [in Mbps] ___ .

0101010101

↳

1 bit → 2μsec

1 sec. → $\frac{1}{2\mu}$

1sec. → 0.5Mbits

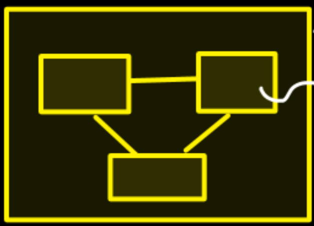
Data rate = 0.5Mbps ✓

$\frac{1}{4\mu}$ = 0.25 Mbps X X

## Building a digital System:-

### Logic Gates:-

Basic Building block of any digital system.




**Basic Gates:-** AND, OR, NOT, Buffer

**Universal Gates:-** NAND, NOR

**Arithmatic Gates:-** XOR, XNOR

Binary operation $\neq$ Boolean Algebra

### * Basic Boolean Algebra:-

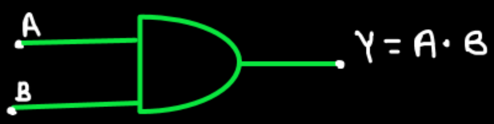
- $1+0=1$
- $1+1=1$
- $0+0=0$
- $0+1=1$
- $1\cdot0=0$
- $0\cdot1=0$
- $1\cdot1=1$
- $0\cdot0=0$
- $1+0+0+1+0=1$
- $1\cdot0\cdot0\cdot1\cdot0\cdot1=0$
- $1+A=1$
- $A\cdot0=0$
- $\overline{(\overline{A})}=A$
- $\overline{0}=1$
- $\overline{1}=0$

1. AND Gate :-

Output will be logic High when

- all the inputs are logic High

Symbol :-

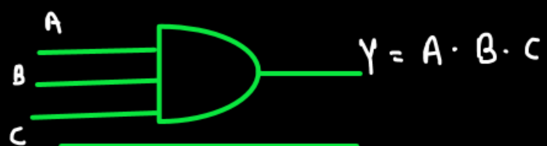



Two i/p AND GATE

Truth Table :-

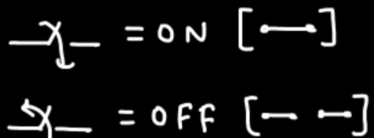
[The table which consists all the possible comb$^n$ of i/ps & their respective outputs]

| A | B | Y = A · B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |



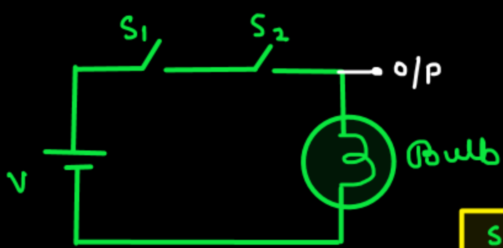

3-i/p AND Gate

| A | B | C | Y = A · B · C |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |




For $n$ - inputs, total possible combination = $2^n$

★ Switch representation of 2-i/p AND GATE




ON → 1

OFF → 0

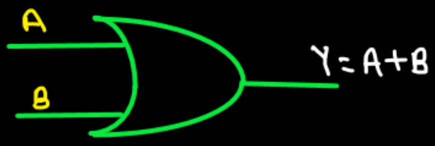
Series combination of switches $S_1$ and $S_2$ = $S_1 \cdot S_2$

| $S_1$ | $S_2$ | Bulb = $S_1 \cdot S_2$ |
|---|---|---|
| ON | ON | ON ← |
| ON | OFF | OFF |
| OFF | ON | OFF |
| OFF | OFF | OFF |

## 2. OR Gate:-

Output will be logic High when - any of the inputs is logic High

### Symbol:-




Two i/p OR GATE

### Truth Table:-

[The table which consists all the possible combⁿ of i/ps & their respective outputs]

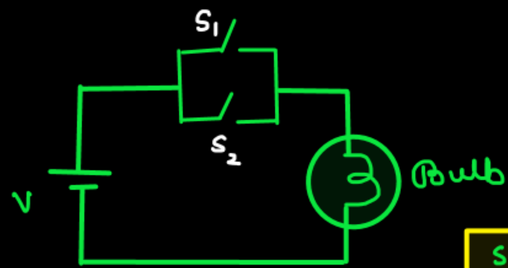
| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |


3-i/p OR Gate

| A | B | C | Y = A+B+C |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

For $n$ - inputs, total possible combination = $2^n$

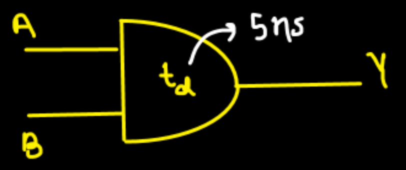
* $\boxed{\text{Switch representation of 2-i/p OR GATE}}$




parallel combination of switches $S_1$ and $S_2 = S_1 + S_2$

| $S_1$ | $S_2$ | Bulb |
|---|---|---|
| ON | ON | ON |
| ON | OFF | ON |
| OFF | ON | ON |
| OFF | OFF | OFF |

## Concept of Delay:-

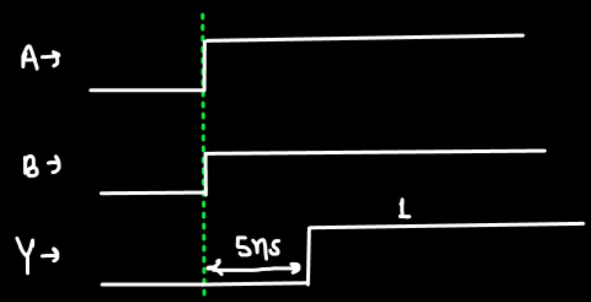



Delay of AND Gate ($t_d$) = 5ns

For $t>0$, $A=1$, $B=1$ ; For $t<0$, $A=0$, $B=0$

Draw the timing diagram of Y. $Y = A \cdot B$.

→

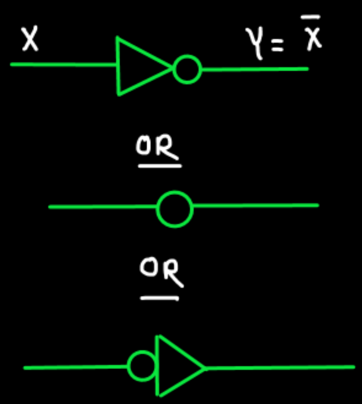



## 3. NOT GATE / Inverter:-

o/p will be logic High when i/p is logic low.

o/p will be logic low when i/p is logic High.

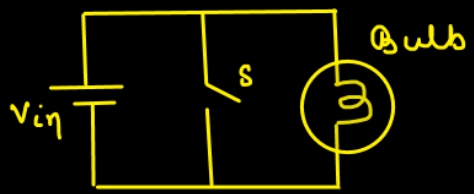
Symbol:-




Truth Table:-

| X | Y |
|---|---|
| 0 | 1 |
| 1 | 0 |

## Switch Representation:-

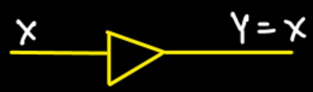



| S | Bulb |
|---|---|
| ON | OFF |
| OFF | ON |

## 4. Buffer:-

↳ O/P will be Logic High when i/p is logic High.
O/P will be Logic Low when i/p is logic Low.

Symbol:-

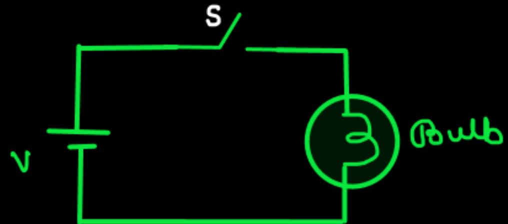



Truth Table:-

| X | Y |
|---|---|
| 0 | 0 |
| 1 | 1 |

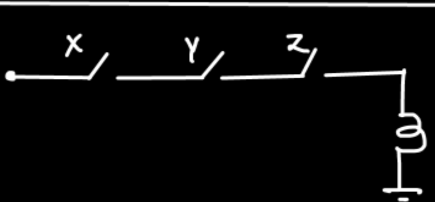
↳ Buffer is used to introduce delay and avoid loading effect.

## Switch Representation :-




| S | Bulb |
|---|---|
| ON | ON |
| OFF | OFF |

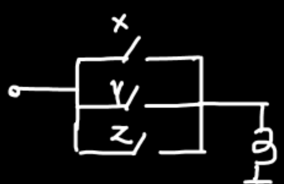
## Conclusion of switch Representation :-

① 

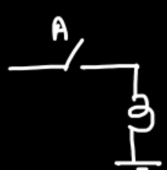

$\Rightarrow X \cdot Y \cdot Z$

③ 


$\Rightarrow \bar{A}$

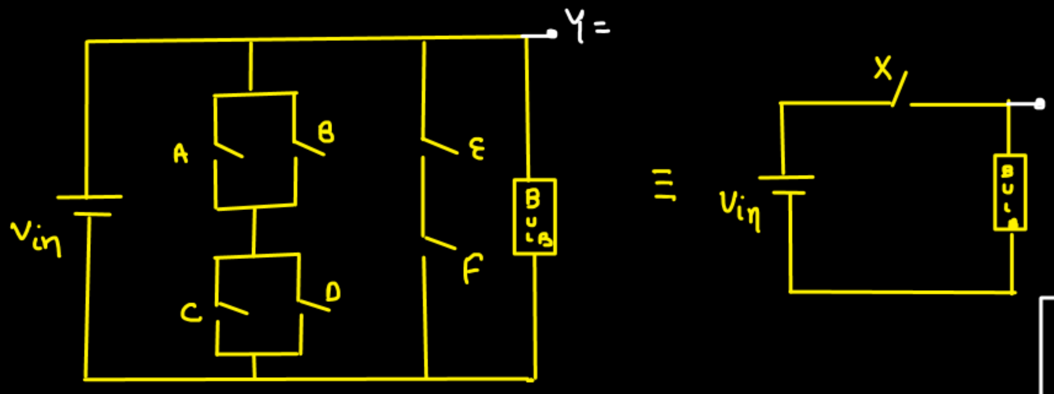
② 


$\Rightarrow X + Y + Z$

④ 


$\Rightarrow A$

Q. If both circuits are equivalent then find X in terms of A, B, C, D, E and F.

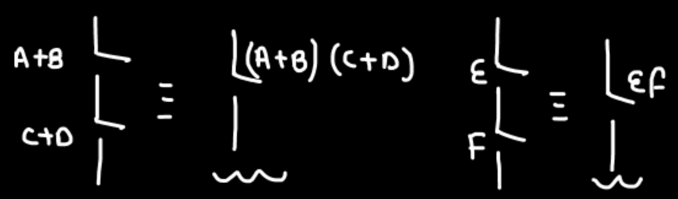



$Y = X$

$Y = \overline{(A+B)(C+D) + EF}$

$$\boxed{X = \overline{(A+B)(C+D) + EF}}$$

↳

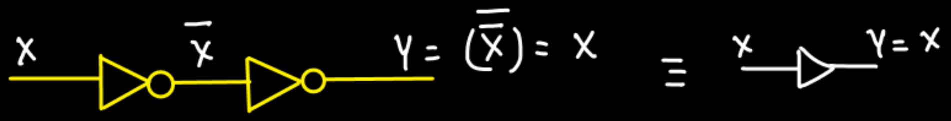



o/p = Y

$(A+B)(C+D) + EF$

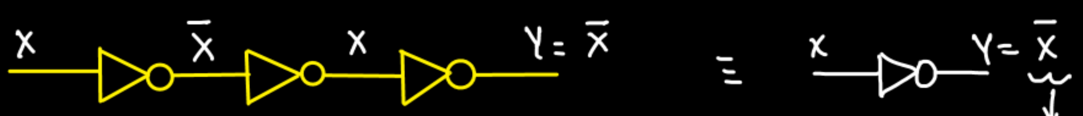
$\overline{(A+B)(C+D) + EF}$

✲ Two NOT Gates in cascade :-




Even number of NOT Gates in cascade ⇒ $Y = X$

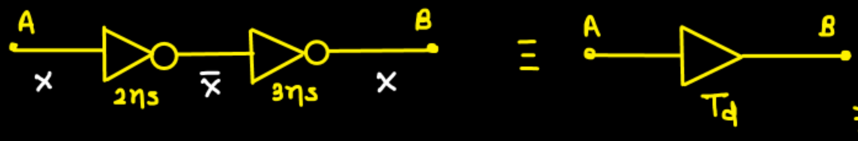
✲ Three NOT Gates in cascade :-




complement of $X$

odd number of NOT Gates in cascade ⇒ $Y = \overline{X}$

Q.




find $T_d$ = ?

$= 5\,ns$

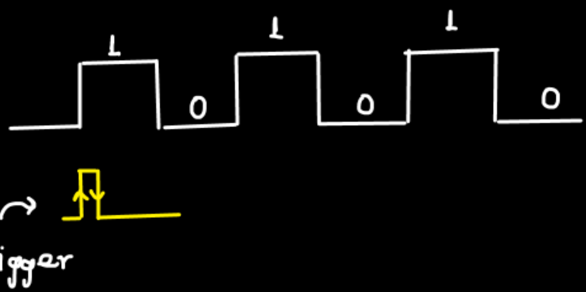
## ⇒ Concept of Multivibrators:-

Electronic circuits which implement two state system like oscillators timer and flip-flops.

### ① Astable multivibrator:-

NOT Stable

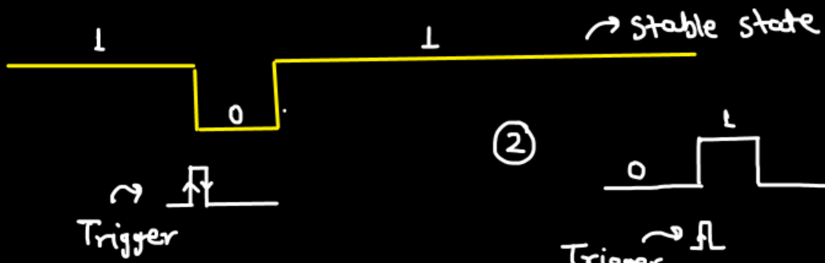
↳ No stable state.



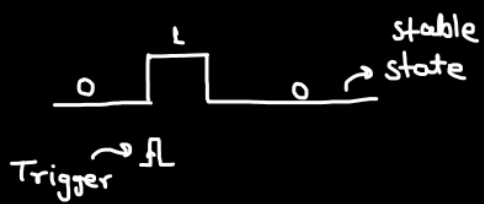

### ② Monostable Multivibrator:-

↳ Only one stable state.

Eg. → ①

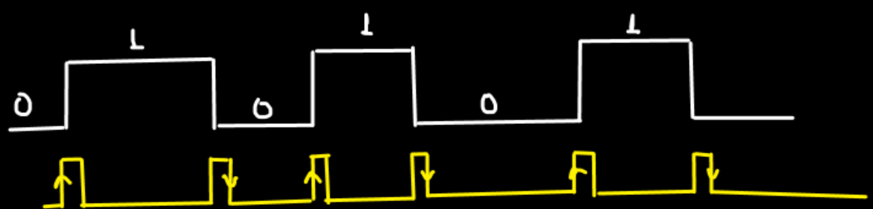



②

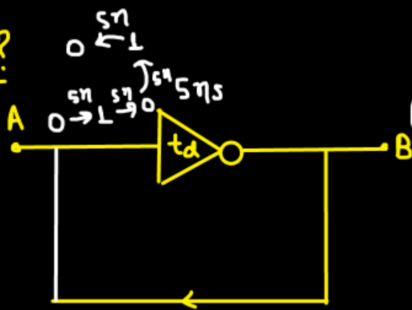



### ③ Bistable Multivibrators:-

↳ Two stable state



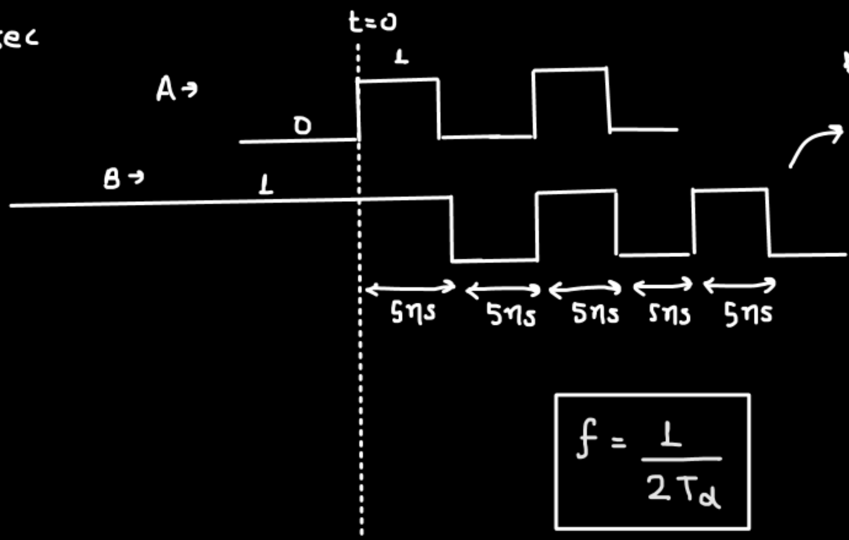
What if ?




$\textcircled{1} \xrightarrow{5n} 0 \xrightarrow{5n} 1 \xrightarrow{5n} 0$

output

100sec




$$\boxed{f = \frac{1}{2T_d}}$$

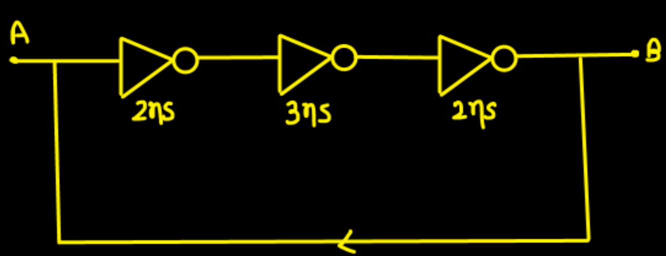
** Astable Multivibrator [Ring oscillator]

→ freq. of oscillations = ?

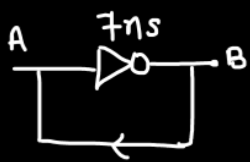
$T = 10ns$

$f = \frac{1}{T} = \frac{1}{10 \times 10^{-9}} = 100MHz$ = Ans

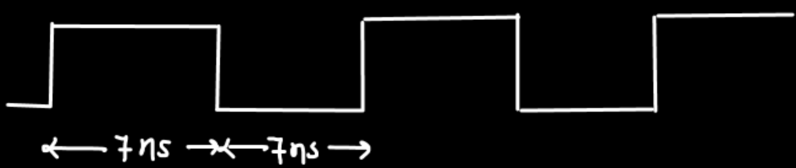
Eg →




≡



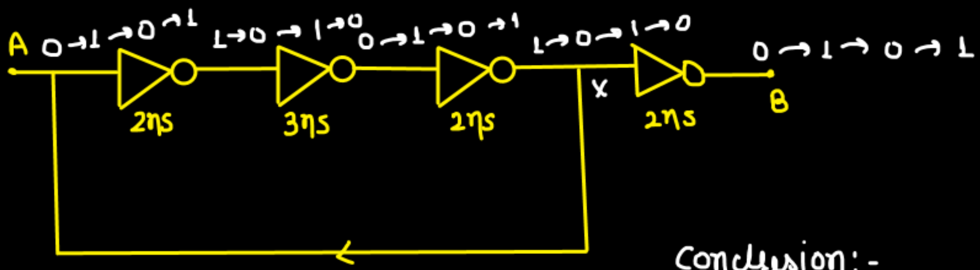

→

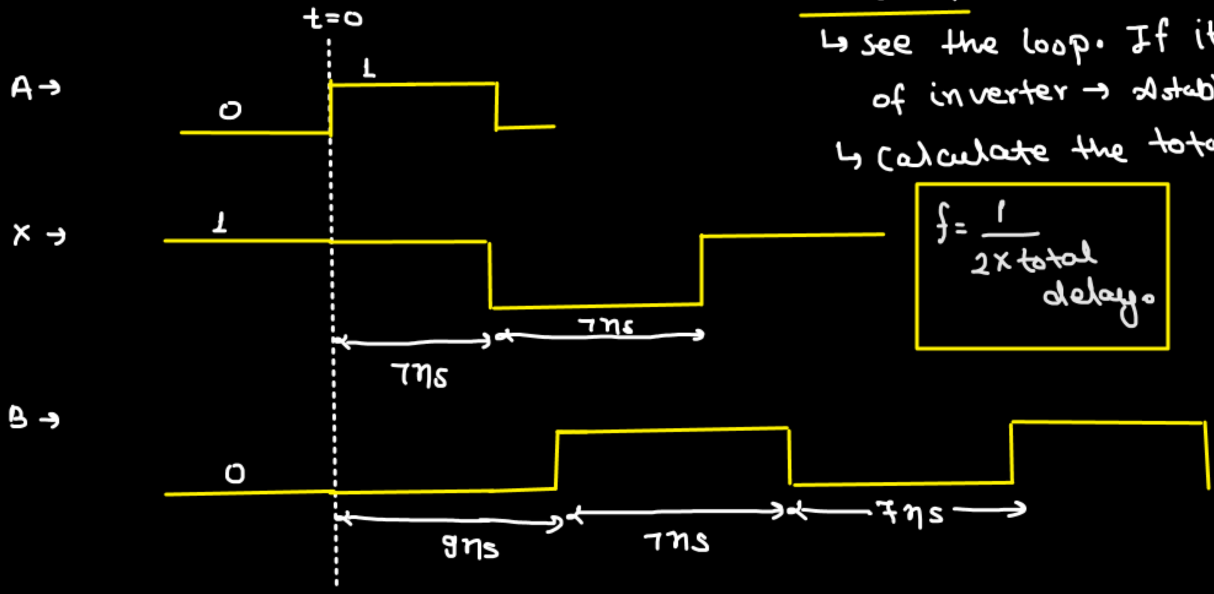



$T = 14ns$

$f = \frac{1}{14} \times 10^{9} = 71.42MHz$

Eg. →




Conclusion :-

↳ see the loop. If it has odd no. of inverter → Astable Multivibrator

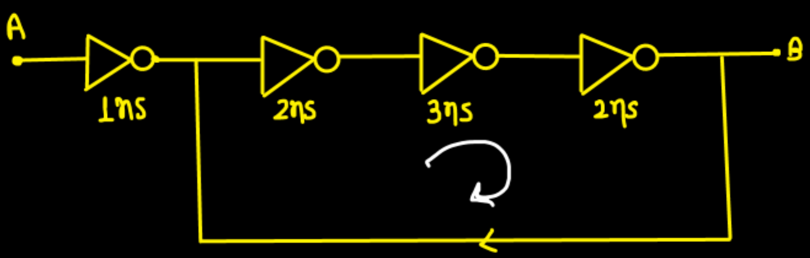
↳ Calculate the total delay in loop.

$$\boxed{f = \frac{1}{2 \times \text{total delay}}}$$

$$f = \frac{1}{2 \times 7n}$$

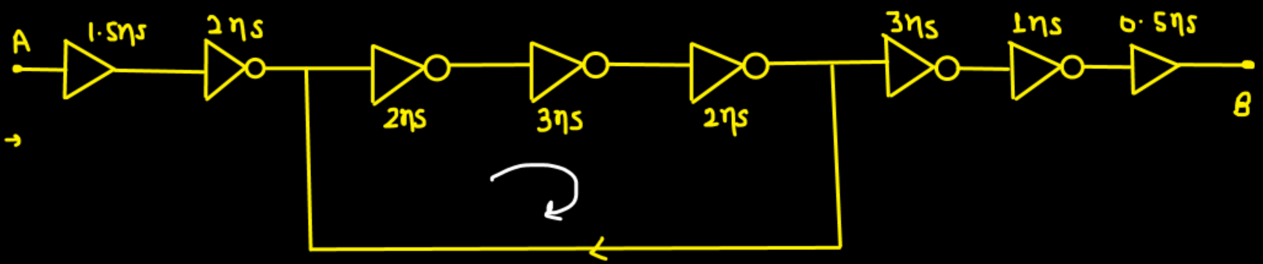
$$= 71.42\,MHz$$

Eg. →




↳ $f = \frac{1}{2 \times 7n} = 71.42\,MHz$

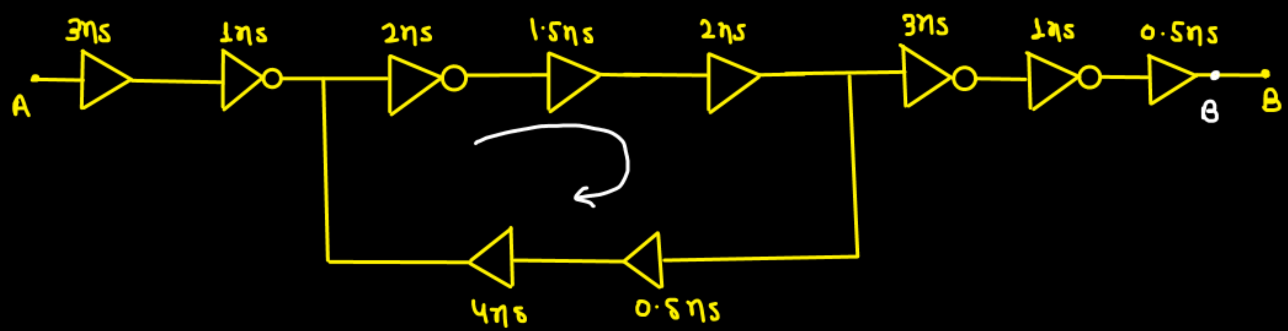
Eg →




Freq. of oscillations?

↳ $f = \frac{1}{2 \times 7n} = 71.42\,MHz$

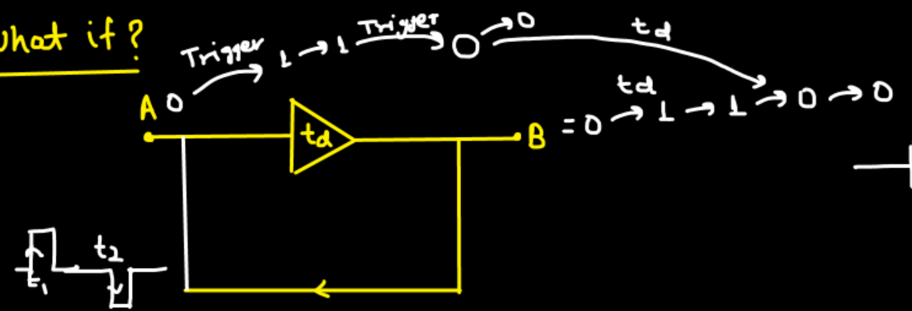
Q.



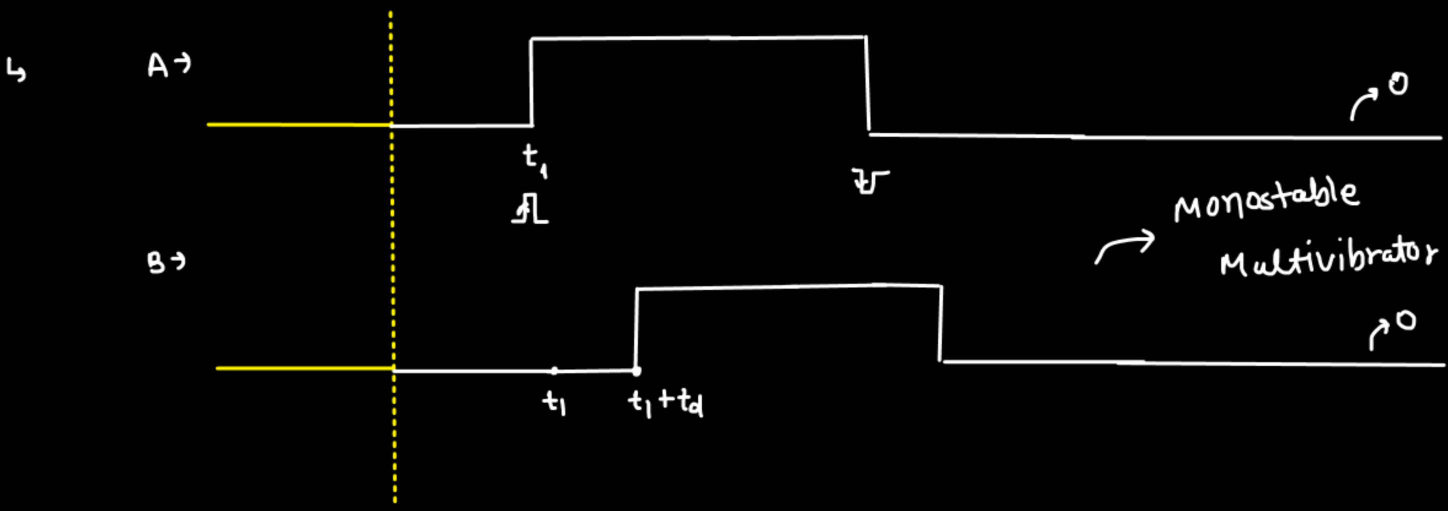

Freq. of oscillations →

↳ $f = \frac{1}{2 \times 10n} = \frac{10^8}{2} = 50\,MHz$ ANS.

## What if ?



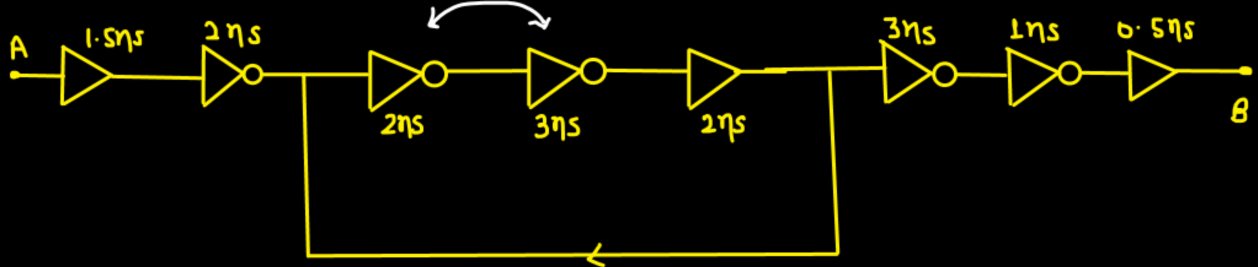

↳ Even no. of inverters




↳ Monostable Multivibrator

Eg. →

Even no. of inverters ⇒ M. M.




Frequency of oscillations?

⇓

$f = 0 \Rightarrow$ no oscillations

N.B.:-

## De-Morgan's Theorem:-

1) $\overline{A\cdot B} = \bar{A} + \bar{B}$

2) $\overline{A+B} = \bar{A}\cdot\bar{B}$

* $\overline{A+B+C} = \bar{A}\cdot\bar{B}\cdot\bar{C}$

* $\overline{A\cdot B\cdot C} = \bar{A}+\bar{B}+\bar{C}$

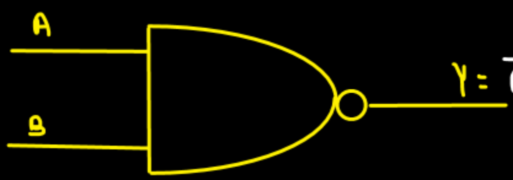
## * Some importants points:-

↳ $A\cdot 1 = A$ ↳ $A\cdot A = A$ ↳ $A + A = A$ ↳ $A + 1 = 1$

↳ $A\cdot 0 = 0$ ↳ $A\cdot\bar{A} = 0$ ↳ $A + 0 = A$ ↳ $A + \bar{A} = 1$

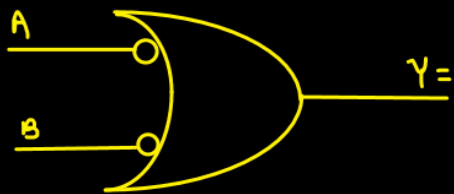
## 4. NAND GATE:-

[AND + NOT]




$Y = \overline{AB} = \bar{A} + \bar{B}$

≡

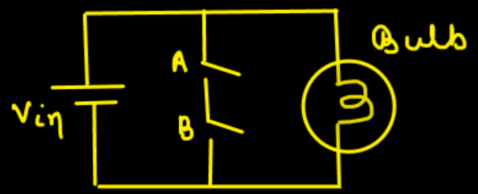



$Y = \bar{A} + \bar{B}$

| A | B | $Y = \overline{AB}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

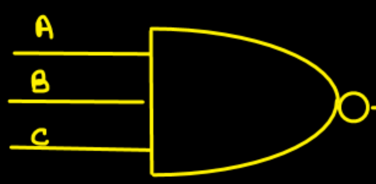
NAND Gate = Bubbled OR Gate

## Switching Circuit:-



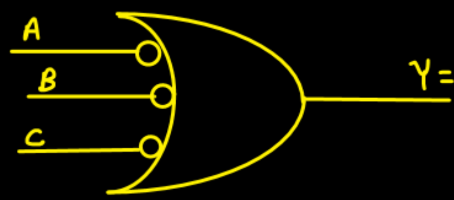

$Y = \overline{AB}$

*



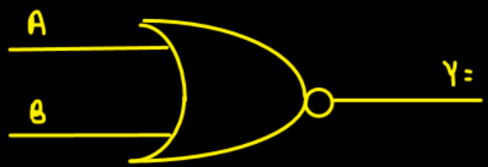

$Y = \overline{ABC} \equiv$

$= \bar{A} + \bar{B} + \bar{C}$



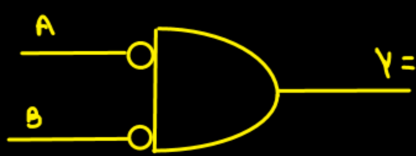

$Y = \bar{A} + \bar{B} + \bar{C}$

## 5. NOR GATE:-




$Y = \overline{A+B} = \bar{A} \cdot \bar{B}$

$\equiv$

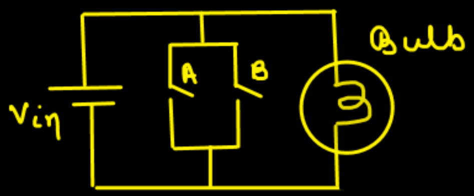



$Y = \bar{A} \cdot \bar{B}$

| A | B | $Y = \overline{A+B}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

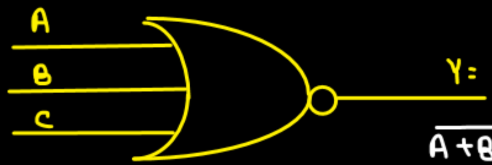
NOR GATE = Bubbled AND

## Switching Circuit :-




$\overline{A+B} = Y$

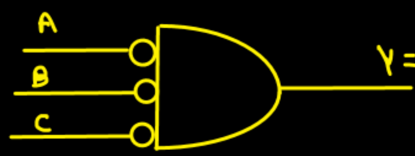
*




$\overline{A+B+C}$

$= \bar{A}\cdot\bar{B}\cdot\bar{C}$

$\equiv$




$Y = \bar{A}\cdot\bar{B}\cdot\bar{C}$

## * Why NAND and NOR GATEs are called Universal GATE?

↳ With the help of AND, OR, NOT GATEs combined, you can design any boolean function.

↳ But using NAND or NOR GATE alone you can design all the basic Gates [AND, OR, NOT]

AND, OR, NOT } ⇒

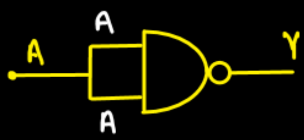
NAND or NOR } ⇒ Alone enough to implement any boolean function

⇓

Universal Gate

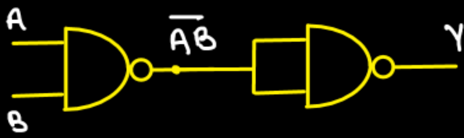
## ✯ NAND Gate as Universal Gate:-

### ① NOT Gate:-




$Y = \overline{A \cdot A} = \bar{A}$

### ② AND Gate:-

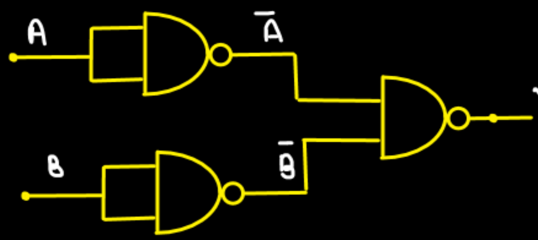



$Y = [\overline{\overline{AB} \cdot \overline{AB}}]$

$= (\overline{\overline{AB}}) = AB$

### ③ OR Gate:-

$Y = A+B$

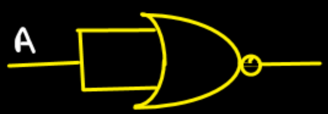



$Y = (\overline{\bar{A} \cdot \bar{B}}) = \bar{\bar{A}} + \bar{\bar{B}}$

$= \underline{\underline{A+B}}$

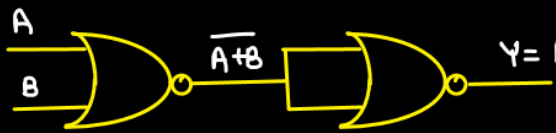
## ✯ NOR Gate as Universal Gate:-

### ① NOT Gate:-




$Y = \overline{A+A} = \bar{A}$

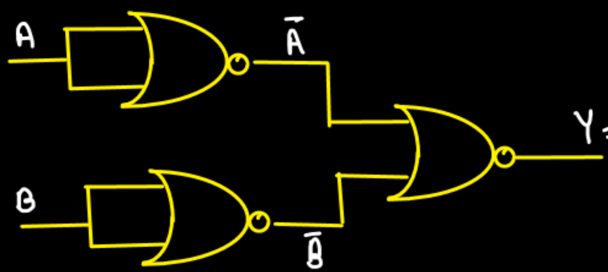
### ② OR Gate:-




$Y = A+B$

### ③ AND Gate:-

$Y = AB$




$Y = (\overline{\bar{A} + \bar{B}}) = \bar{\bar{A}} \cdot \bar{\bar{B}}$

$= A \cdot B$

## Arithmatic Gates:-

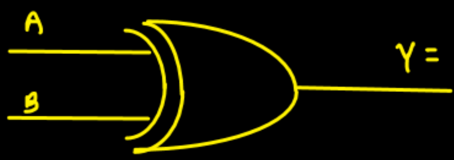
$AB = BA$

$A+B = B+A$

### 1. Ex-OR Gate | XOR Gate:-

↳ o/p is logic High when both i/p are different.

↳ o/p is logic low when both i/p are same.




$Y = \bar{A}B + A\bar{B} = A\bar{B} + B\bar{A} = A \oplus B$

| A | B | $\bar{A}$ | $\bar{B}$ | $\bar{A}B$ | $A\bar{B}$ | $Y = A\oplus B = \bar{A}B + A\bar{B}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 |

$0 \oplus 0 = 0$

$0 \oplus 1 = 1$

$1 \oplus 0 = 1$

$1 \oplus 1 = 0$

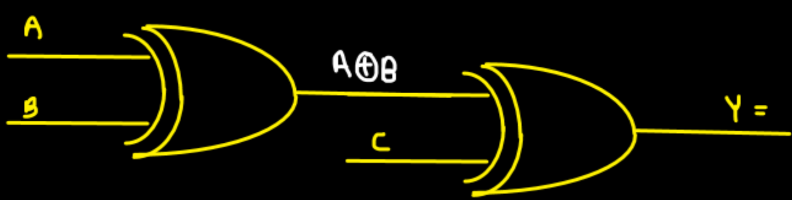
→ Non-equivalence detector

[only two i/p XOR]

**NOTE:-**

3-i/p XOR GATE doesn't exist.


$Y = A \oplus B \oplus C$ } X Morris Mano




$Y = A \oplus B \oplus C$

★ Solve:-

① $0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 = 0$

② $0 \oplus 0 \oplus 0 = 0$

③ $1 \oplus 1 \oplus 1 = 0 \oplus 1 = 1$

④ $1 \oplus 1 \oplus 1 \oplus 1 = 0$

⑤ $1 \oplus 0 \oplus 1 \oplus 1 \oplus 0 \oplus 0 = 1$

⑥ $1 \oplus 1 \oplus 0 \oplus 1 \oplus 1 \oplus 0 = 0$

XOR Gate = odd no. of 1's detector

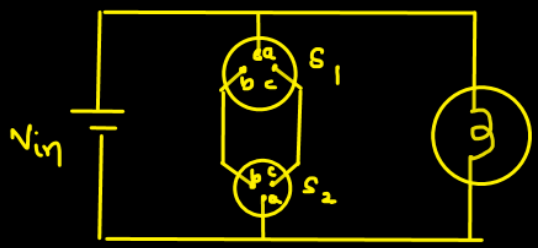
## Switching Ckt:-

### ★ Two way switch:-


switch S

when a is connected to b → ON (1)

a is connected to c → OFF (0)




| $S_1$ | $S_2$ | $Y = S_1 \oplus S_2$ |
|---|---|---|
| ON | ON | OFF |
| ON | OFF | ON |
| OFF | ON | ON |
| OFF | OFF | OFF |

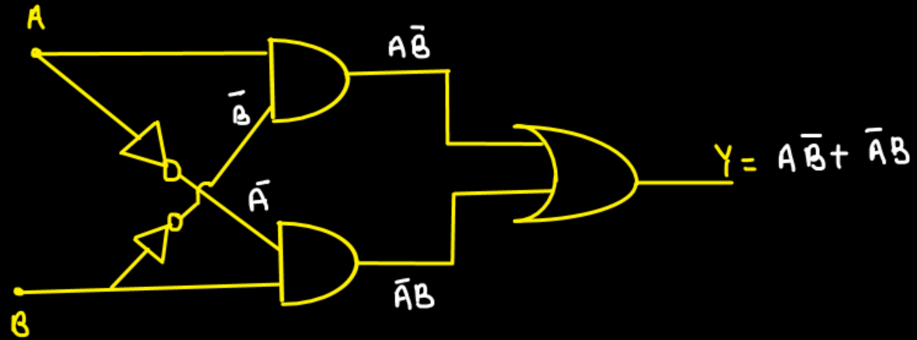
## ↳ Some Properties:-

① $A \oplus B = B \oplus A$

② $A \oplus B \oplus C = (A \oplus B) \oplus C = A \oplus (B \oplus C)$

## ↳ XOR Gate from basic Gates:-

$$Y = \bar{A}B + A\bar{B}$$



★ **Important observations:-**

① $A \oplus A = A\bar{A} + \bar{A}A = 0 + 0 = 0 \Rightarrow A \oplus A = 0$

② $A \oplus 0 = A \cdot \bar{0} + \bar{A} \cdot 0 = A \cdot 1 + 0 = A \Rightarrow A \oplus 0 = A$

③ $A \oplus 1 = A \cdot \bar{1} + \bar{A} \cdot 1 = A \cdot 0 + \bar{A} = \bar{A} \Rightarrow A \oplus 1 = \bar{A}$

④ $A \oplus \bar{A} = A \cdot A + \bar{A} \cdot \bar{A} = A + \bar{A} = 1 \Rightarrow A \oplus \bar{A} = 1$

⑤ If $A \oplus B = C$

then
$$A \oplus C = A \oplus (A \oplus B)$$
$$= A \oplus A \oplus B$$
$$= 0 \oplus B$$
$$= \underline{B}$$

$\Rightarrow$ if $A \oplus B = C$

then (i) $A \oplus C = B$

(ii) $C \oplus B = A$

★ **Find:-**

① $A \oplus A \oplus A \oplus A = 0 \oplus 0 = 0$

② $A \oplus A \oplus A \oplus A \oplus A \oplus A = 0 \oplus 0 \oplus 0 = 0$

③ $A \oplus A \oplus A = 0 \oplus A = A$

④ $A \oplus A \oplus A \oplus A \oplus A = 0 \oplus 0 \oplus A = 0 \oplus A = A$

**Conclusion:-**

$A \oplus A \oplus A \oplus A \oplus \ldots\ldots$

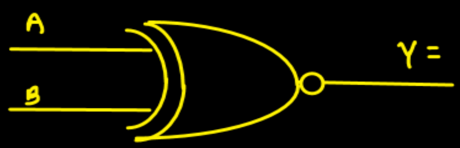
Even no. of A $= 0$

odd no. of A $= A$

1. **Ex-NOR Gate / XNOR Gate:-** [XOR + NOT]

↳ o/p is logic Low when both i/p are different.

↳ o/p is logic low when both i/p are same.




$Y = \overline{A \oplus B} = A \odot B = \bar{A}\bar{B} + AB$

| A | B | $\bar{A}$ | $\bar{B}$ | $\bar{A}\bar{B}$ | AB | $Y = A \odot B = \bar{A}\bar{B} + AB$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 |

→ equivalence detector [only two i/p XNOR]

* Solve:-

① $1 \odot 1 \odot 1 = 1$

② $1 \odot 1 \odot 1 \odot 1 = 1$

③ $0 \odot 0 \odot 0 = 0$

④ $0 \odot 0 \odot 0 \odot 0 = 1$

$\boxed{\text{XNOR GATE = Even no. of 0's detector}}$

⑤ $1 \odot 0 \odot 0 \odot 1 \odot 0 = 0$

⑥ $1 \odot 0 \odot 1 \odot 0 \odot 1 = 1$

XOR Gate = odd no. of 1's detector

XNOR Gate = Even no of 0's detector

## ★ Interesting observation :-

| A | B | C | $A\oplus B$ | $A\odot B$ | $A\oplus B\oplus C$ | $A\odot B\odot C$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 |

$A\oplus B = \overline{A\odot B}$

$A\oplus B\oplus C = A\odot B\odot C$

## Conclusion :-

★ For Even no of inputs -

XOR = $\overline{\text{XNOR}}$

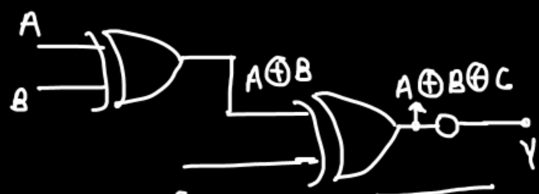
For odd no. of inputs -

XOR = XNOR

Eg. →

1. $A\oplus B = \overline{A\odot B}$
2. $A\oplus B\oplus C = A\odot B\odot C$
3. $A\oplus B\oplus C\oplus D = \overline{A\odot B\odot C\odot D}$
4. $A\oplus B\oplus C\oplus D\oplus E = A\odot B\odot C\odot D\odot E$

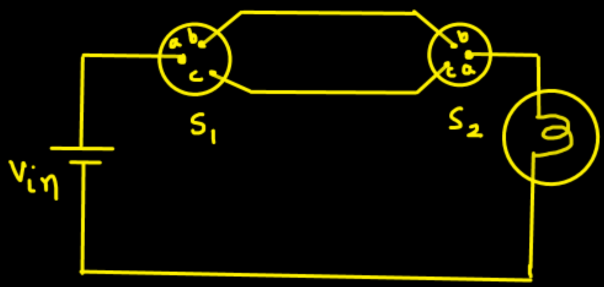
$\overline{A\oplus B\oplus C} \neq A\odot B\odot C$




$Y = \overline{A\oplus B\oplus C}$

$= (A\oplus B)\odot C$

## Switching ckt:-




$a \to b \equiv ON$

$a \to c \equiv OFF$

| $S_1$ | $S_2$ | $Y = A \odot B$ |
|---|---|---|
| ON | ON | ON |
| ON | OFF | OFF |
| OFF | ON | OFF |
| OFF | OFF | ON |

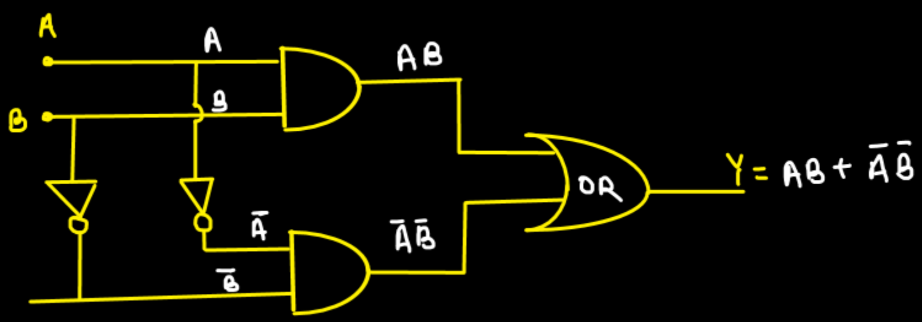
## Some Properties:-

① $A \odot B = B \odot A$

② $[A \odot B] \odot C = A \odot [B \odot C]$

## * XNOR Gate using basic Gates:-

$$Y = \bar{A}\bar{B} + AB$$




## * Important observations:-

① $A \odot A = 1$

② $A \odot 0 = \bar{A}$

③ $A \odot 1 = A$

④ $A \odot \bar{A} = 0$

⑤ If $A \odot B = C$, then $A \odot C = B$

$B \odot C = A$

then $A \odot C = A \odot (A \odot B)$

$= A \odot A \odot B = 1 \odot B$

$= B$

★ Find:-

① $A \odot A = 1$

② $A \odot A \odot A = A$

③ $A \odot A \odot A \odot A = 1$

④ $A \odot A \odot A \odot A \odot A = A$

Conclusion:-

$A \odot A \odot A \odot A \odot A \ldots\ldots$

If no. of i/p are odd $= A$

If no of i/p are even $= 1$

↳ Important Properties:-

① $\bar{A} \oplus B = \bar{A}\bar{B} + (\bar{\bar{A}})B = \bar{A}\bar{B} + AB = A \odot B = \overline{A \oplus B}$

② $\bar{A} \oplus \bar{B} = \overline{\overline{A \oplus B}} = A \oplus B$

③ $\bar{A} \odot B = (\bar{\bar{A}})\bar{B} + \bar{A}B = A\bar{B} + \bar{A}B = A \oplus B = \overline{A \odot B}$

④ $\bar{A} \odot \bar{B} = \overline{\overline{A \odot B}} = A \odot B$

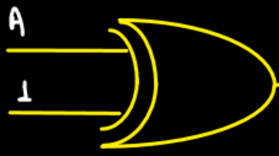
Eg. $\bar{A} \odot B \oplus C$

$= \bar{A} \odot X \qquad \{X = B \oplus C\}$

$= \overline{A \odot X} = A \oplus X$

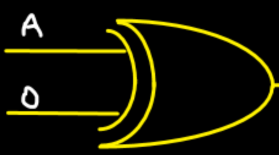
$= A \oplus B \oplus C$

★ NOT Gate and buffer using XOR and XNOR:-



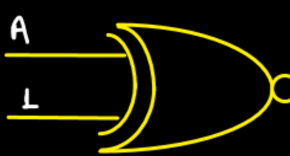

$Y = A \oplus 1 = \bar{A}$

↪ NOT GATE



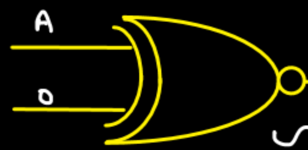

$Y = A \oplus 0 = A$

↪ Buffer




$Y = A \odot 1 = A$

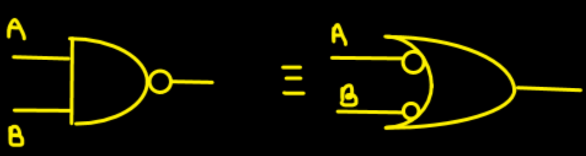
↪ Buffer

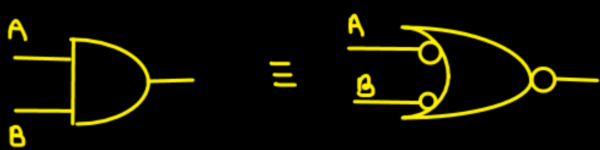


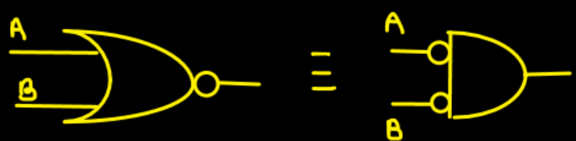

$Y = A \odot 0 = \bar{A}$

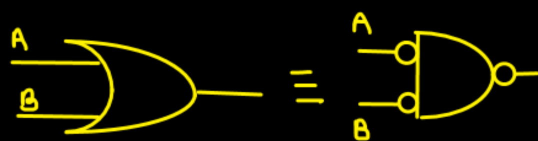
↪ NOT GATE

★ Logic Gates and their Equivalent Gates:-

① 


② 


③ 


④ 

## ✱ Sum of Product [SOP] & Product of Sum [POS]:-

① SOP:-

$$AB + CD, \quad AB + BC + CD$$

② POS:-

$$(A+B)(C+D), \quad (A+B)(B+C)(C+D)$$

## ⇒ Implementation using minimum no. of 2-i/p NAND/NOR GATE:-

↳ Minimum no. of NAND Gate:-

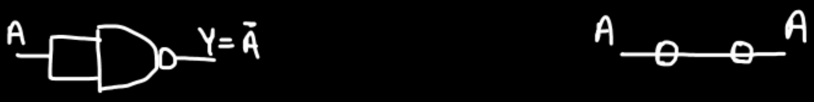
① Minimize the function. [Try converting it in SOP]

② Implement the function using Basic Gates.

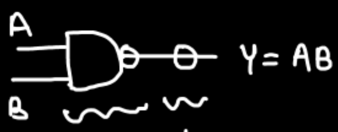
③ Convert basic Gates into NAND Gates.

| Basic Gates | No. of NAND Gates Required |
|---|---|
| NOT | 1 |
| AND | 2 |
| OR | 3 |

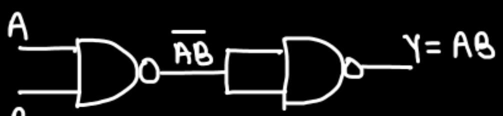
① $\bar{A}$



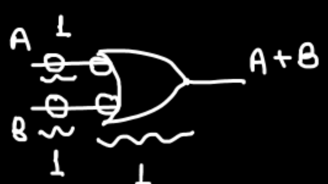

② AB



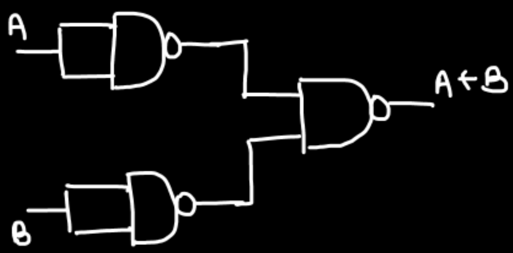

$1 + 1 = 2$



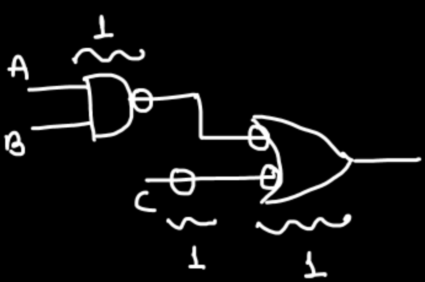

③ A+B



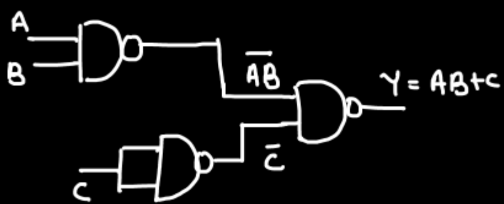

$1+1+1= 3$



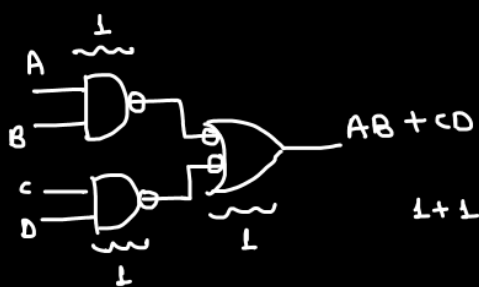

④ AB+C




$1+1+1=3$



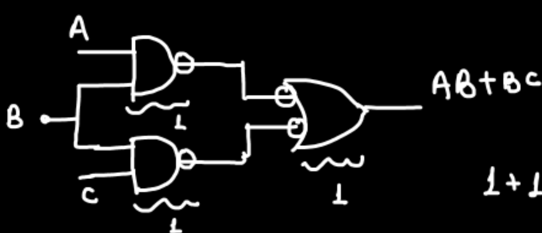

⑤ AB+ CD




$1+1+1=3$

⑥ AB + BC

$= B(A+C)$



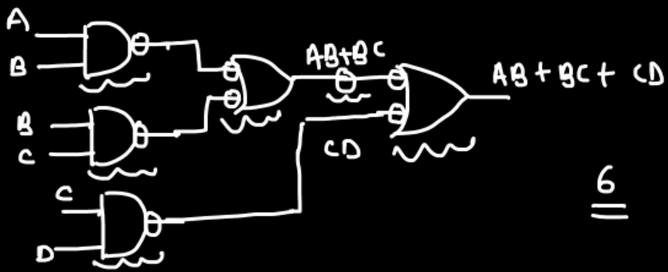

$1+1+1 = 3$

⑦ $\overset{3}{\widetilde{AB+BC}}+CD = B(A+C)+CD$

$\overset{3}{\tilde{X}}+CD$, where $\underbrace{X+CD}_{3}$

$= 3+3$

$= \underline{\underline{6}}$




$\underline{\underline{6}}$

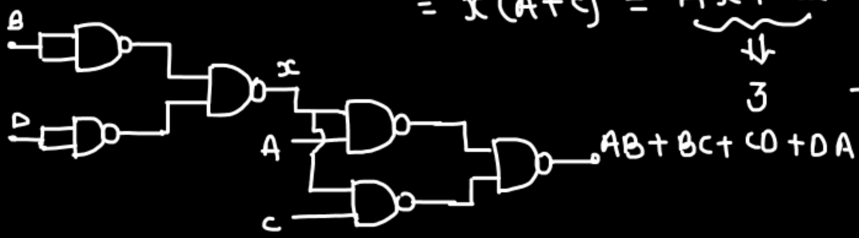
⑧ $AB+BC+CD+DA$

$= B(A+C)+D(A+C)$

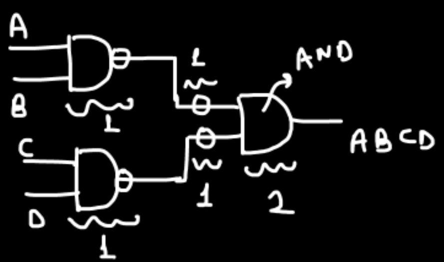
$= \underbrace{(B+D)}(A+C)$

$= X(A+C) = \underbrace{AX+CX}_{3} \quad \{\underbrace{X=B+D}_{3}\}$

$3 + 3 = \underline{\underline{6}}$




⑨ ABCD




$= \underline{\underline{6}}$

**

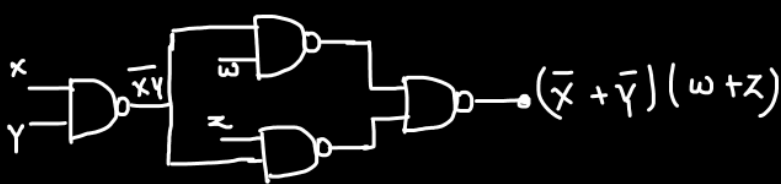
⑩ $(\bar{X}+\bar{Y})(W+Z) = (\overline{XY})(W+Z)$

$= A(W+Z)$

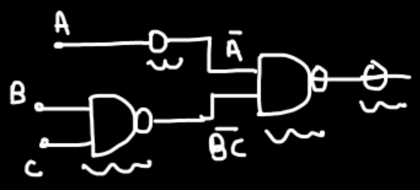
$= \underbrace{WA+ZA}_{3}$

$\{\underbrace{A=\overline{XY}}_{1}\}$

$1+3 = \underline{4}$



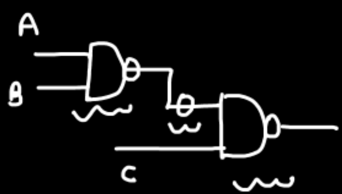
(11) $\bar{A}\bar{B} + \bar{A}\bar{C} = \bar{A}(\bar{B}+\bar{C}) = \bar{A}(\overline{BC})$




$\underline{\underline{4}}$

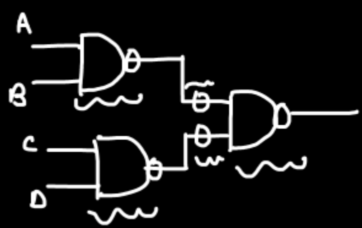
(12) $\overline{AB} + \overline{BC} = \bar{X} + \bar{Y} = \overline{XY} = \overline{AB \cdot BC} = \overline{ABC}$

$= \bar{A} + \bar{B} + \bar{B} + \bar{C} = \bar{A} + \bar{B} + \bar{C} = \overline{ABC}$



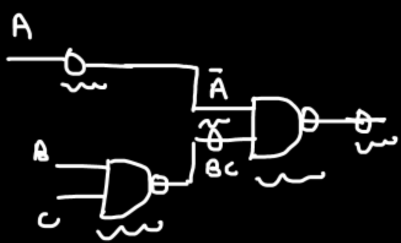

$1 + 1 + 1 = \underline{\underline{3}}$

(13) $\overline{AB} + \overline{CD} = \overline{ABCD}$




$= \underline{\underline{5}}$

(14) $\bar{A}BC$




$= \underline{\underline{5}}$

★★★ (15) $(\bar{x}+y)(\bar{w}+z) = \bar{x}\bar{y} + \bar{x}z + y\bar{w} + zy$

$$= \overline{\overline{\underbrace{(\bar{x}+y)}_{A}\,\underbrace{(\bar{w}+z)}_{B}}}$$

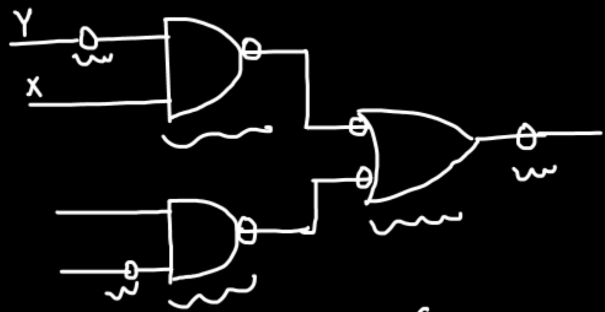
$$= \overline{\overline{(\bar{x}+y)} + \overline{(\bar{w}+z)}}$$

$$= \underbrace{\overline{x\cdot\bar{y} + w\bar{z}}}_{3+1+1+1=6}$$

NOTHING CLICKS

⇓

Take $=$




$= \underline{\underline{6}}$ ANS

---

↳ <u>Minimum no. of NOR Gate:-</u>

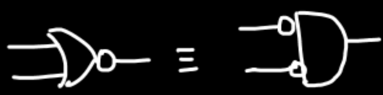
① Minimize the function. [Try converting it in <u>POS</u>]

② Implement the function using Basic Gates.

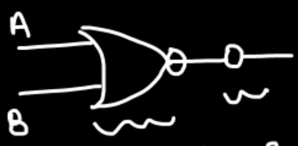
③ Convert basic Gates into NOR Gates.

| Basic Gates | No. of NOR Gates Required |
|---|---|
| NOT | 1 |
| OR | 2 |
| AND | 3 |

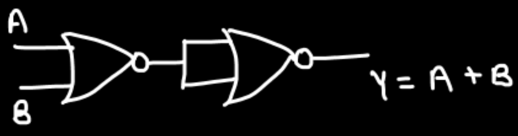
① $\bar{A}$

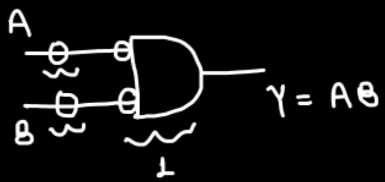

② A+B

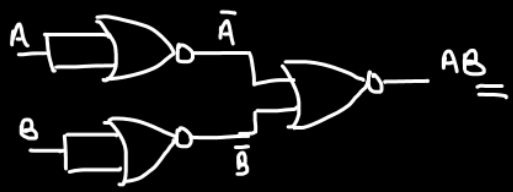



1 + 1 = 2



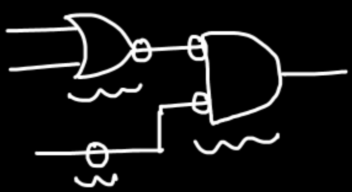

③ AB



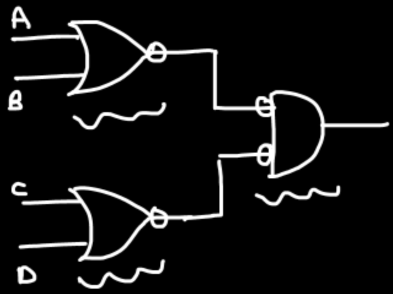

1+1+1 = 3

④ (A+B) C



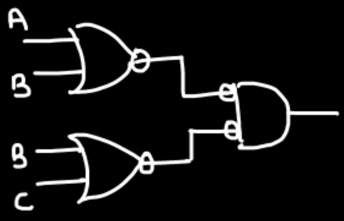
= 3

⑤ (A+B)(C+D)



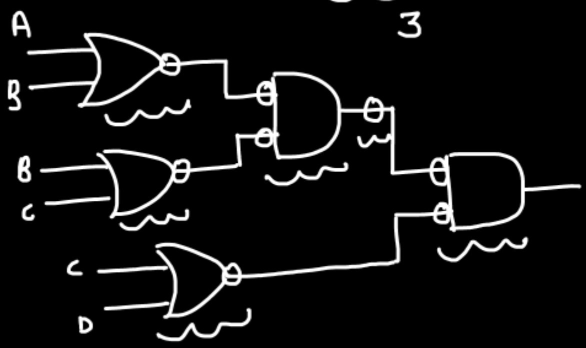

= 3

⑥ $(A+B)(B+C) = 3$



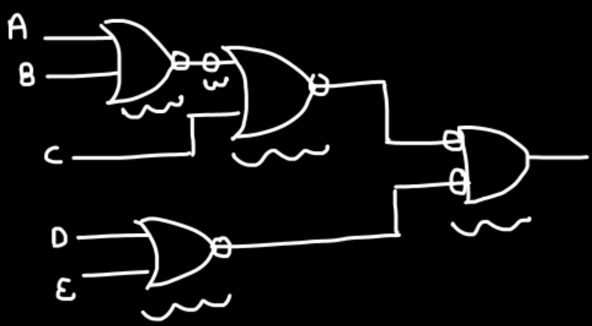

$= 3$

⑦ $(A+B)(B+C)(C+D) = \underbrace{x^{3}[(C+D)]}_{3} = 3+3 = 6$



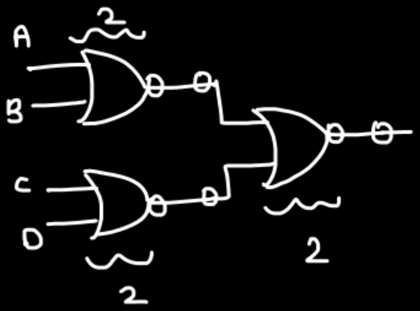

$= 6$

⑧ $(A+B+C)(D+E)$




$= 5$

⑨ $A+B+C+D$



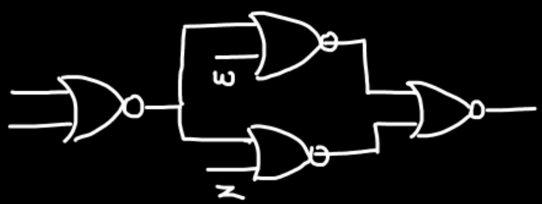

$2+2+2=6$

⑩ $\bar{X}\bar{Y}+WZ = (\overline{X+Y}) + WZ = A+WZ$

$= (A+W)(A+Z)$

$\{A = \overline{X+Y}\}$

$3+1 = \underline{\underline{4}}$ Ans.

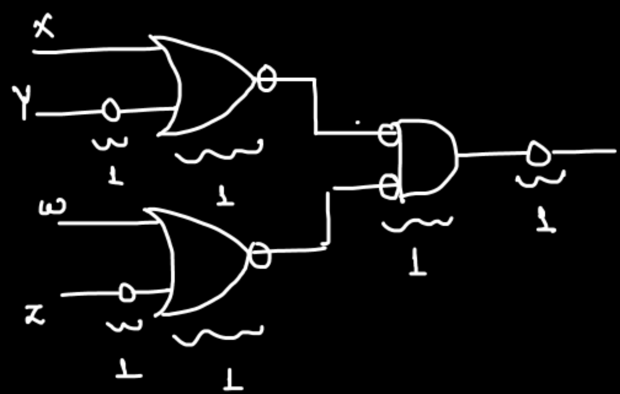



$= \underline{\underline{4}}$

$$\boxed{\begin{aligned}(A+B)(A+C) &= A + AC + AB + BC \\ &= A[1+C+B] + BC \\ &= A+BC \\ A+BC &= (A+B)(A+C)\end{aligned}}$$

⑪ $\bar{X}Y+\bar{W}Z = \overline{\overline{[\bar{X}Y+\bar{W}Z]}} = \overline{(\overline{\bar{X}Y})\cdot(\overline{\bar{W}Z})}$

$= \overline{(X+\bar{Y})(W+\bar{Z})}$




$= \underline{\underline{6}}$

## * Important Points:-

① $A \oplus B = \bar{A}B + A\bar{B} = (\bar{A}+\bar{B})(A+B)$

② $A \odot B = \overline{A \oplus B} = \bar{A}\bar{B} + AB = (\bar{A}+B)(A+\bar{B})$

## * Implementing XOR and XNOR Gate using NAND:-

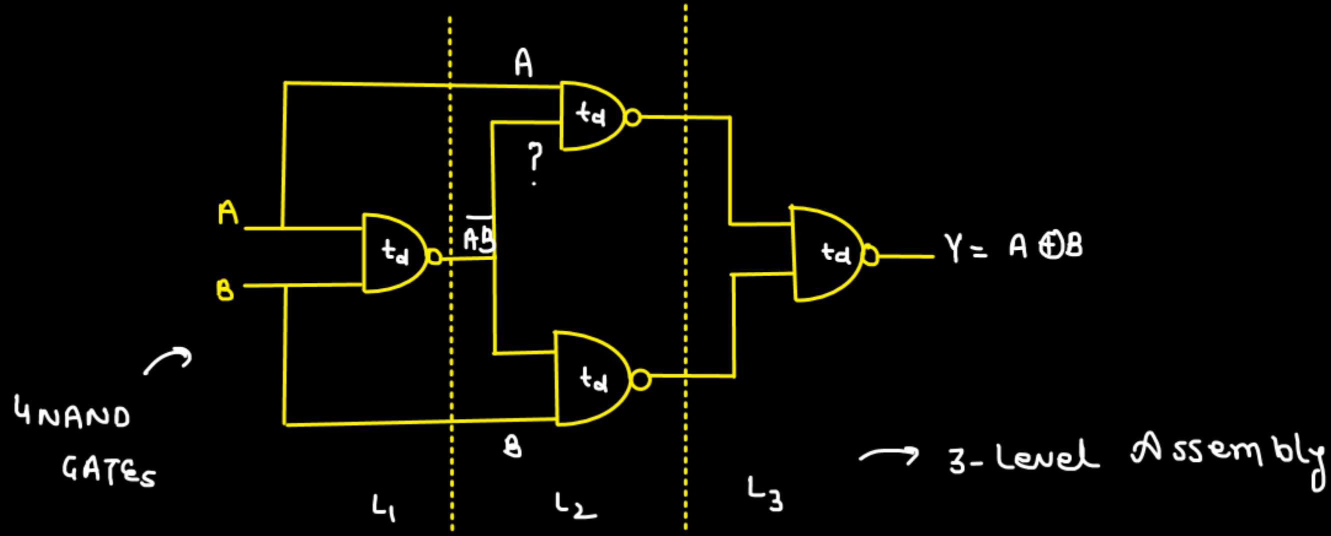
XOR:-

$$A \oplus B = \bar{A}B + A\bar{B} = (\bar{A}+\bar{B})(A+B) = (\overline{AB})(A+B)$$

$$= X(A+B) \quad \{X = \underbrace{\overline{AB}}_{1}\}$$

$$= \underbrace{AX + BX}_{3}$$

$$3+1 = \underline{4}$$




XOR using NAND

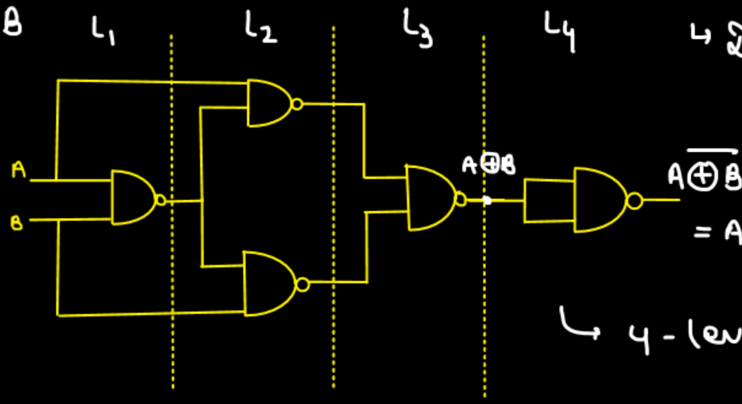
* Propagation Delay of one NAND Gate = $t_{pd}$

Total propagation Delay = $3t_{pd}$

## XNOR:-

$$A \odot B = \overline{A \oplus B} = (\bar{A}+B)(A+\bar{B}) = \bar{A}\bar{B} + AB$$

**Way-1:-** $\overline{A \oplus B}$




5 NAND GATES

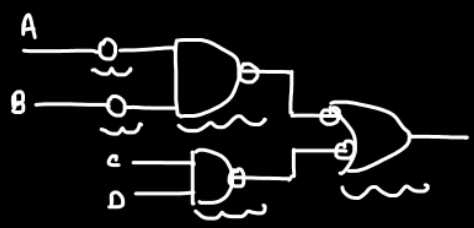
↳ Delay of each NAND = $t_{pd}$

Total propagation Delay = $4t_{pd}$

↳ 4-level Assembly

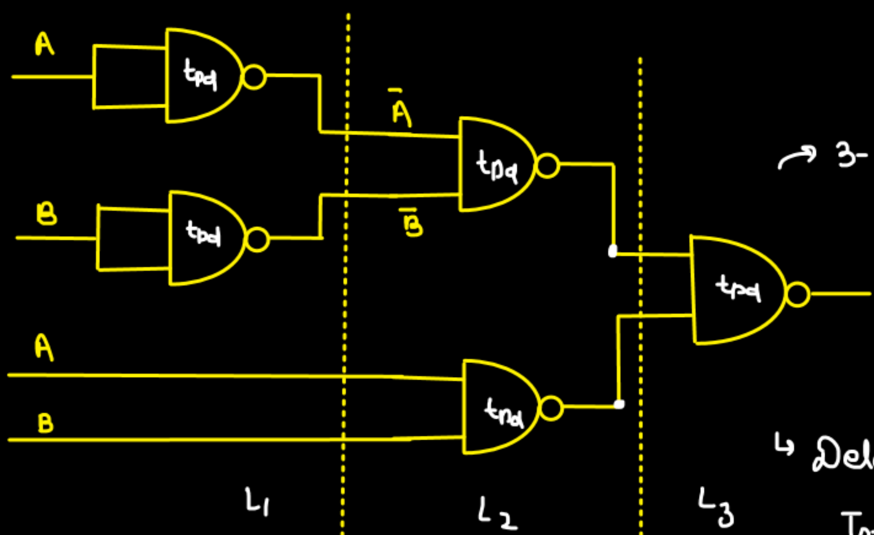
**Way-2:-**

$$Y = A \odot B = \underbrace{\bar{A}^{1}\,\bar{B}^{1} + AB}_{3}$$




$= \underline{\underline{5}}$

t=0




→ 3-level Assembly

↳ Delay of Each NAND = $t_{pd}$

Total propagation delay $= 3t_{pd}$

**XNOR using NAND**

## Implementing XNOR and XOR using NOR:-

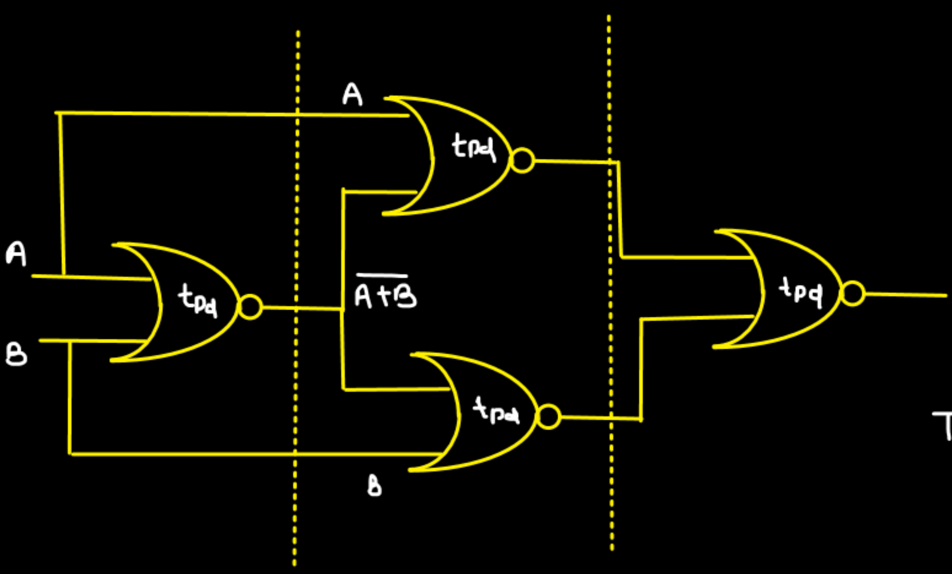
↳ **XNOR:-**

$$Y = A \odot B = \bar{A}\bar{B} + AB = \overline{(A+B)} + AB \quad \overset{1}{}$$

$$= X + AB \qquad \{X = \overline{A+B}\}$$

$$= \underbrace{(X+A)(X+B)}_{3} \qquad \{P + QR = (P+Q)(P+R)\}$$

$$3 + 1 = \underline{\underline{4}}$$




$A \odot B = \overline{A \oplus B}$

Total delay = $3t_{pd}$

**XNOR using NOR**

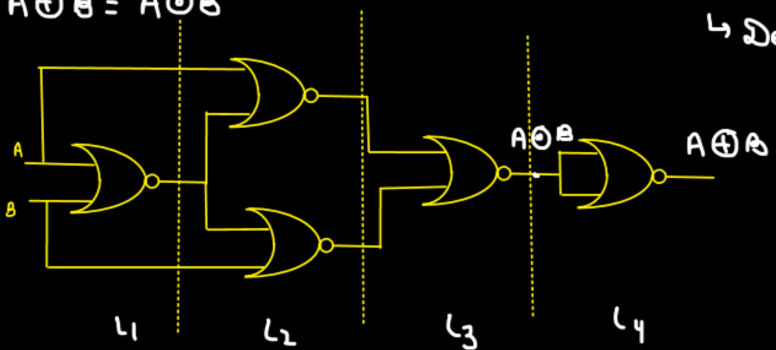
↳ XOR:-

$Y = A \oplus B = \bar{A}B + A\bar{B}$

Way-1:-

$A \oplus B = \overline{A \odot B}$

↳ level 4 Assembly

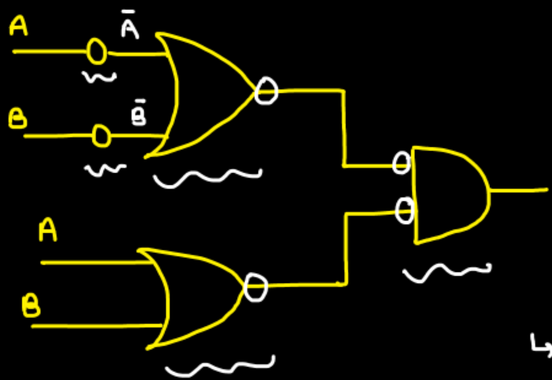



↳ Delay of each NOR = td

Total propagation delay = 4 tpd

Way-2:-

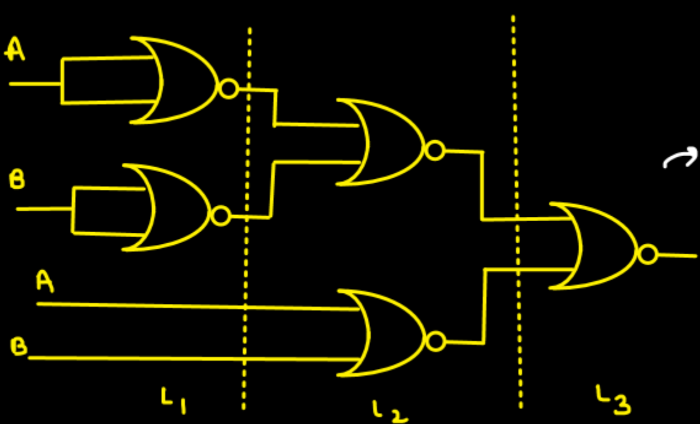
$$y = A \oplus B = \bar{A}B + A\bar{B} = \underbrace{(\bar{A}+\bar{B})(A+B)}_{3} = 3+1+1 = \underline{\underline{5}}$$




↳ Delay of each NOR = td

Total propagation delay = 3td

→ level-3 Assembly



Q. Two i/p XOR Gate and XNOR is implemented using min. no of two i/p NOR Gate and NAND Gate respectively. Find minimum possible propagation Delay.

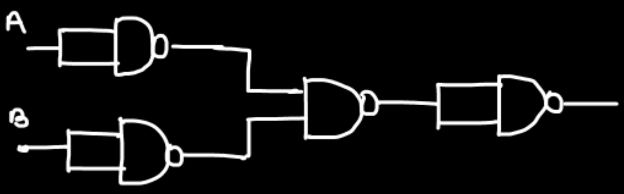
↳ XOR Gate ⟵ NOR ⇒ 5 Gates = level - 3 = 3tpd

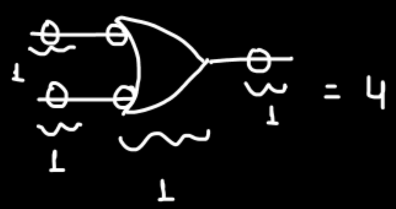
XNOR ⟵ NAND ⇒ 5 Gates = level - 3 = 3tpd

4 tpd ✗

★ NOR Gate Using NAND:-

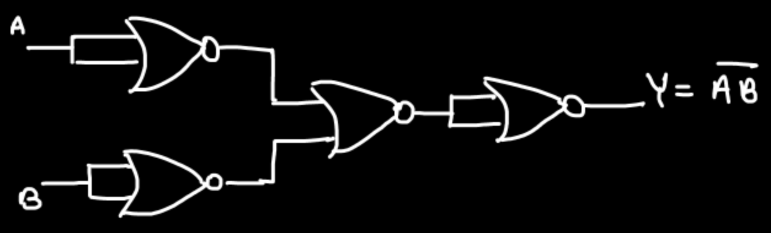
$Y = \overline{A+B}$

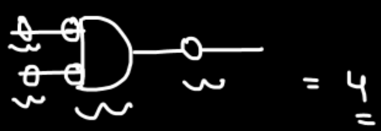



★ NAND Gate using NOR Gate:-

$Y = \overline{AB}$



| Implementation of Gate | Min. no of NAND Gates required | Min. no of NOR Gates required |
|---|---|---|
| NOT | 1 | 1 |
| AND | 2 | 3 |
| OR | 3 | 2 |
| NOR | 4 | 1 |
| NAND | 1 | 4 |
| XOR | 4 | 5 |
| XNOR | 5 | 4 |

NOR → XNOR

⇒ Interesting Concept of Universal Gate :-

Universal Gate :-

Alone Enough to implement any boolean expression.

Q. Are there only two universal Gates? [NAND, NOR]

↳ Generally, there are only two universal Gates

but

↓

one of these

AND + NOT
or
OR + NOT

* If any logic circuit can give "AND" or "OR" and "NOT" both together then that ckt can be used as universal Gate [with the condn that "1" and "0" are present]

☆ NAND → Enough to implement any boolean expression.

NOR → " " " " " "

AND + NOT → " " " " " "

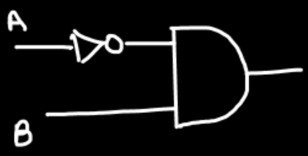
OR + NOT → " " " " " "

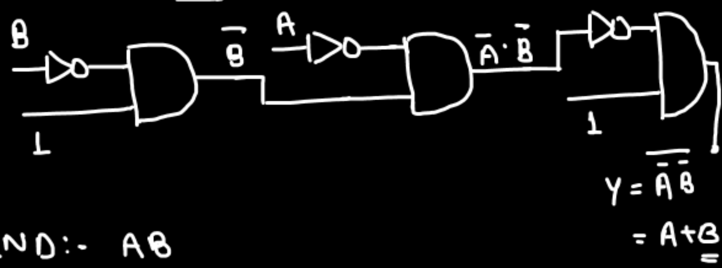
AND + OR + NOT → " " " " " "

OR → $A+B = \overline{\overline{A+B}} = \overline{\bar{A}\cdot\bar{B}}$

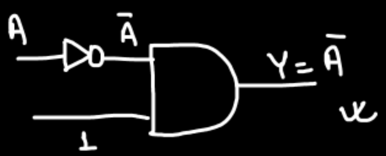
Eg → ① $\bar{A}B$ → universal Gate?

→ OR
→ AND
→ NOT

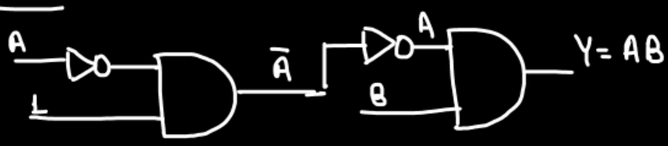


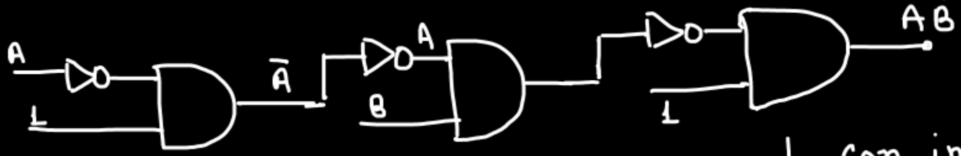

NOT →




AND :- AB

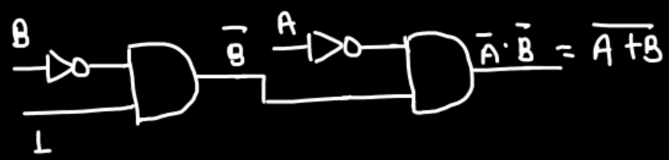



NAND :-

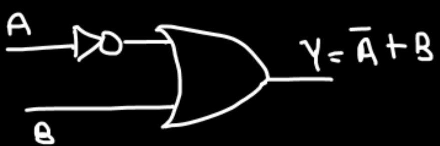



→ can implement any boolean expression

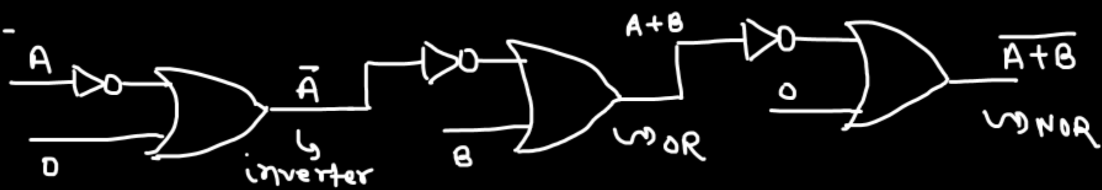
NOR :-




② $\bar{A} + B$




NOR :-



MSQ

Q. Which of those are universal Gates?

(a) NOR

~~(b)~~

$\overline{\bar{A}+\bar{B}} = \overline{\overline{AB}} = AB$ "NOT" X

(c)

$\overline{\bar{A}+B} = A\bar{B}$

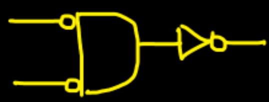
(D)

$\overline{\bar{A}\cdot B} = A+\bar{B}$

(E)

$\bar{A}\cdot\bar{B} = \overline{A+B}$

~~(f)~~



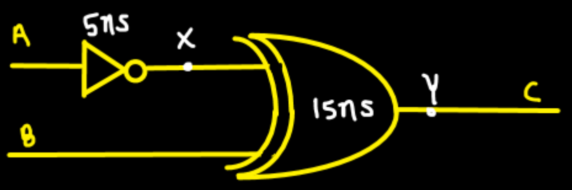
$\overline{\bar{A}\cdot\bar{B}} = A+B$

| | |
|---|---|
| Q.49 | Consider a Boolean gate (D) where the output $Y$ is related to the inputs $A$ and $B$ as, $Y = A + \bar{B}$, where + denotes logical OR operation. The Boolean inputs '0' and '1' are also available separately. Using instances of only D gates and inputs '0' and '1', __________ (select the correct option(s)). GATE-2022 |
| (A) | NAND logic can be implemented |
| (B) | OR logic cannot be implemented |
| (C) | NOR logic can be implemented |
| (D) | AND logic cannot be implemented |

NAND ✓
NOR ✓
XOR ✓
AND ✓
XNOR ✓
OR ✓
NOT ✓

⇒ Timing diagrams of different Digital circuits involving elements which are having non-zero delay:-

Q.




$X = \bar{A}$

$Y = X \oplus B$

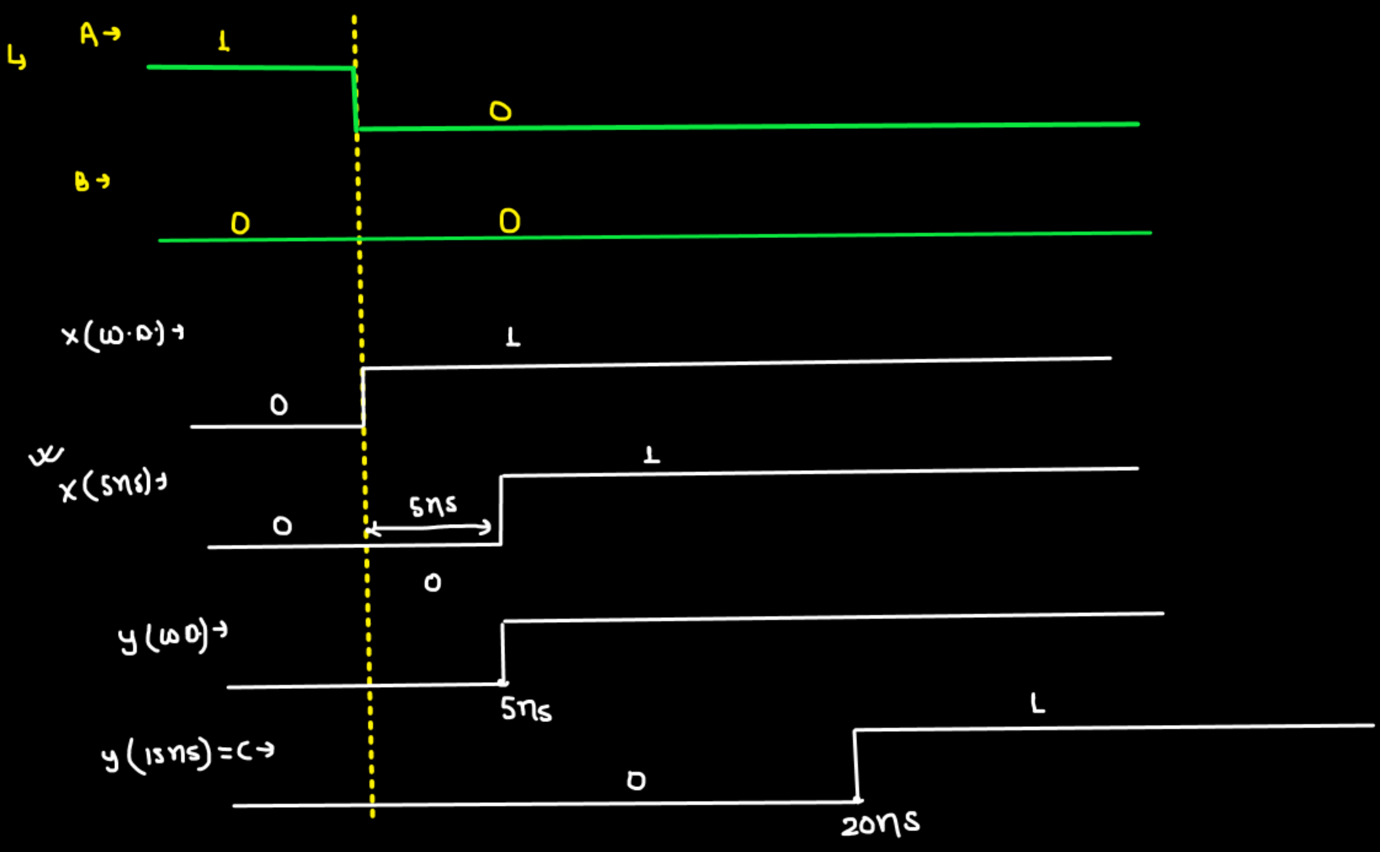
For $t<0$ :- $A = 1$, $B = 0$

For $t>0$ :- $A = 0$, $B = 0$

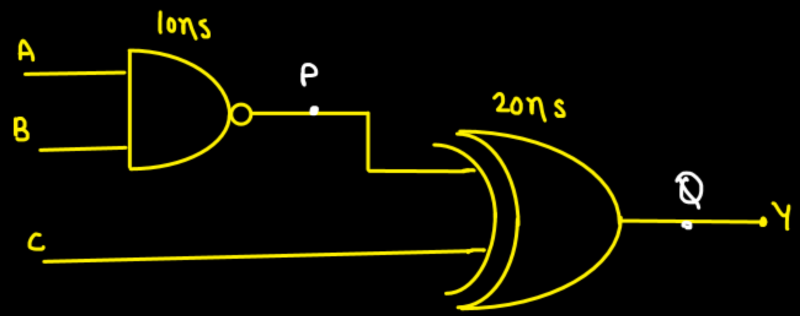
NOT Gate has 5ns delay and XOR Gate has 15ns delay.

Draw the timing diagram of C.

↳



Q.

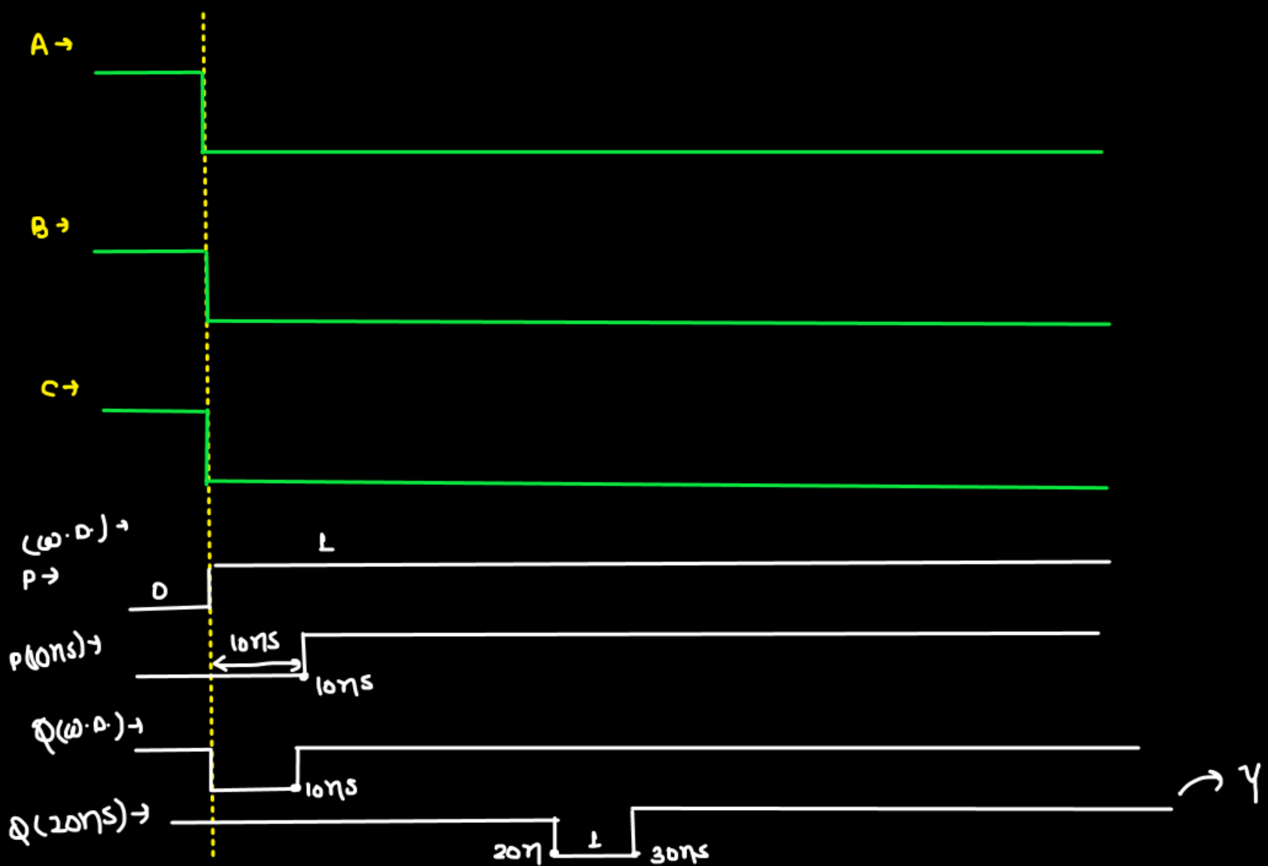



$P = \overline{AB}$

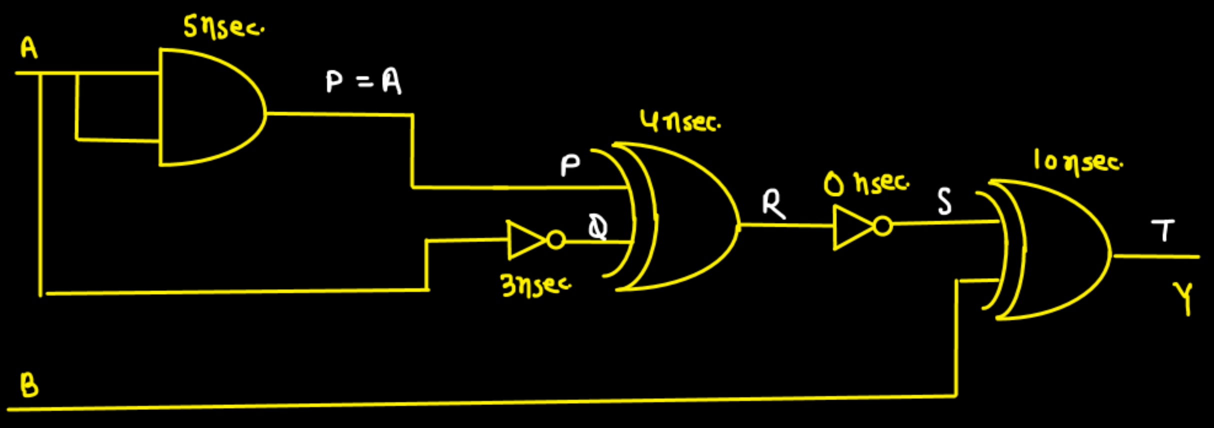
$Q = P_d \oplus C$

All i/p are changing from "1" to "0" at t=0

Draw Timing diagram of Y.



Q.



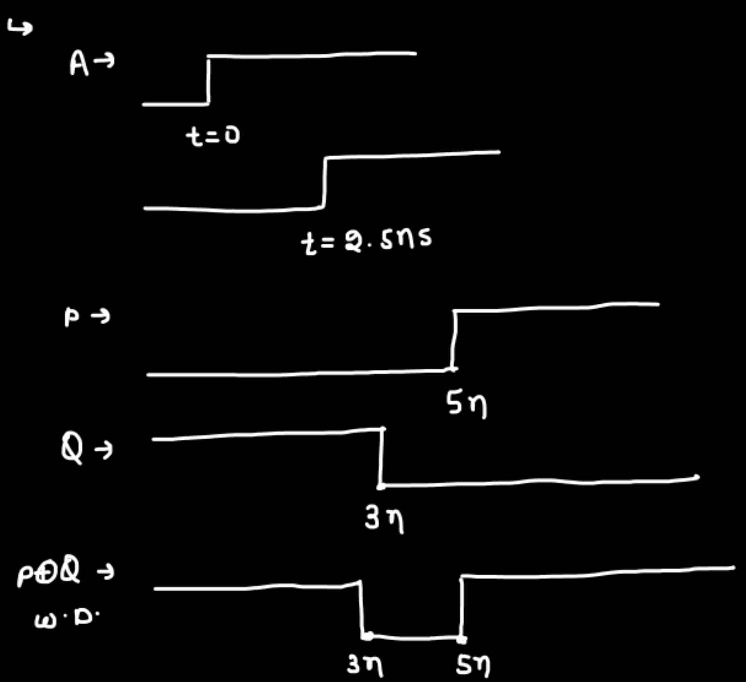

A is changing from "0" to "1" at $t = 0$ nsec

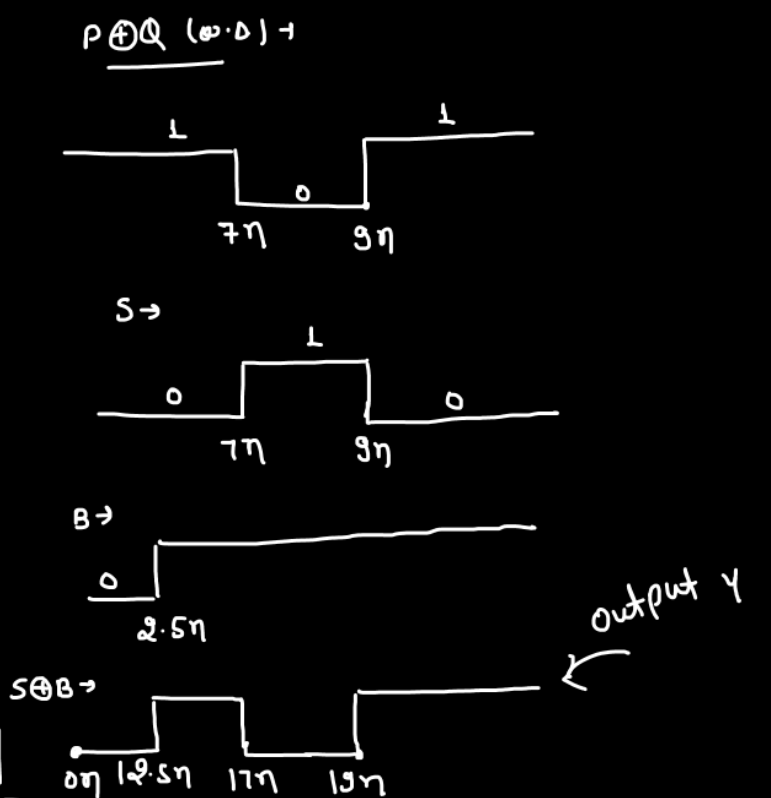
B is changing from "0" to "1" at $t = 2.5$ nsec.

Draw Timing diagram.

Find the time [$t > 0$] for which o/p is zero.
window

↳




Zero ⇒ $0 \rightarrow 12.5n$ [$12.5n + 2n = 14.5ns$] =

$17n \rightarrow 19n$

# Boolean Algebra

**Boolean Addition:-**

$0+0=0$ , $1+0=1$ , $0+1=1$ , $1+1=1$

**Boolean Multiplication:-**

$0\cdot0=0$ , $1\cdot0=0$ , $0\cdot1=0$ , $1\cdot1=1$

Q. Determine the values of A, B, C, D such that

(i) $A\bar{B}C\bar{D}=1$ $\Rightarrow$ $A=1$, $B=0$, $C=1$, $D=0$

(ii) $A+\bar{B}+\bar{C}+D=0$ $\Rightarrow$ $A=0$, $B=1$, $C=1$, $D=0$

⇒ **Laws of boolean Algebra:-**

(i) **Commutative Laws:-**

$A+B=B+A$

$AB=BA$

$A\oplus B=B\oplus A$

$A\odot B=B\odot A$

(ii) **Associative Laws:-**

$(A+B)+C=A+(B+C)$

$A(BC)=(AB)C$

$(A\oplus B)\oplus C=A\oplus(B\oplus C)$

$(A\odot B)\odot C=A\odot(B\odot C)$

(iii) **Distributive Law:-**

$A(B+C)=AB+AC$

$A(B\oplus C)=AB\oplus AC$

* $\{A\oplus(B+C)\neq(A\oplus B)+(A\oplus C)\}$

## Rules of boolean Algebra:-

$A + 0 = A$

$A + 1 = 1$

$A \cdot 0 = 0$

$A \cdot 1 = A$

$A + A = A$

$A + A + A + \dots + A = A$

$A \cdot A = A$

$A \cdot A \cdot A \cdot A \cdot A \cdot \dots \cdot A = A$

$A + \bar{A} = 1$

$A \cdot \bar{A} = 0$

$\bar{\bar{A}} = A$

$A + AB = A[1+B] = A[1]$
$= A$

---

$(A+B)(A+C) = A + AC + AB + BC$
$= A[1 + C + B] + BC$

$\boxed{(A+B)(A+C) = A + BC}$

$\boxed{A + BC = (A+B)(A+C)}$

$A + \bar{A}C = (A+\bar{A})(A+C) = A + C$

$A \oplus \underbrace{\bar{A}C}_{B} = A(\overline{\bar{A}C}) + (\bar{A}C)(\bar{A})$
$= A(A + \bar{C}) + \bar{A}C$
$= A + A\bar{C} + \bar{A}C$
$= A[1 + \bar{C}] + \bar{A}C$
$= A + \bar{A}C = A + C$

$\boxed{X + \bar{X}Y = X + Y}$

$\boxed{X \oplus \bar{X}Y = X + Y}$

* $X \oplus Y \oplus XY = X \oplus [Y(1 \oplus X)] = X \oplus Y\bar{X} = X \oplus \bar{X}Y$

$= X + Y$

* $\boxed{X \oplus Y \oplus XY = X + Y}$

Q. Find

① $(A+B)(\bar{A}+B) = (B+A)(B+\bar{A}) = B + A\cdot\bar{A} = B$

② $(A+C+B)(A+D+B) = (A+B+C)(A+B+D) = A+B+CD$

③ $\bar{A}+AB = \bar{A}+(\bar{\bar{A}})B = \bar{A}+B$

④ $\bar{A}+A\bar{B} = \bar{B}+\bar{A} = \overline{AB}$

⑤ $PQ + \overline{PQ}RS = PQ + RS$

⑥ $AB + \bar{A}B + A\bar{B} + \bar{A}\bar{B}$

$= B(A+\bar{A}) + \bar{B}(A+\bar{A})$

$= B(1) + \bar{B}(1)$

$= B + \bar{B}$

$= \underline{\underline{1}}$

* $AB + BC + C\bar{A} = \bar{A}C + AB$

Proof:-

$AB + C\bar{A} + BC[A+\bar{A}]$

$= AB + C\bar{A} + ABC + \bar{A}BC$

$= AB[1+C] + \bar{A}C[1+B]$

$= AB + \bar{A}C$

↳ $\bar{A}\bar{B} + \bar{B}\bar{C} + \bar{C}A = \bar{A}\bar{B} + \bar{C}A$

↳ $AB + B\bar{C} + CA = AC + \bar{C}B$

↳ $\bar{A}\bar{B} + \bar{B}\bar{C} + CA$ = NOT Applicable

↳ $A\bar{B} + BC + CA = A\bar{B} + BC$

↳ $\bar{A}\bar{B} + \bar{B}\bar{C} + CA$

$= \bar{B}(\bar{A} + \bar{C}) + CA$

$= \bar{B}(\overline{CA}) + CA$

$= \bar{B} + AC$

↳ $(A+B)(B+C)(C+\bar{A}) = (A+B)(\bar{A}+C)$

Proof:-

$(A+B)(\bar{A}+C)(B+C+A\cdot\bar{A})$

$= (A+B)(\bar{A}+C)(B+C+A)(B+C+\bar{A})$

$= (A+B)(A+B+C)(\bar{A}+C)(\bar{A}+C+B) = (A+B)(\bar{A}+C)$

↳ $(A+\bar{B})(B+C)(C+A) = (A+\bar{B})(B+C)$

↳ $(A+B)(B+C)(\bar{C}+A) = (B+C)(\bar{C}+A)$

↳ $(\bar{A}+\bar{B})(B+\bar{C})(\bar{C}+\bar{A}) = (\bar{A}+\bar{B})(B+\bar{C})$

↳ $(\bar{A}+\bar{B})(\bar{B}+\bar{C})(C+\bar{A}) = (\bar{B}+\bar{C})(C+\bar{A})$

↳ $\bar{A}B + \bar{A}C + \bar{B}C = \bar{A}B + \bar{B}C$

↳ $\bar{B}\bar{C} + AB + A\bar{C} = \bar{B}\bar{C} + AB$

Q. Simplify :-

(i) $A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C = A\bar{B}C + \bar{A}C[B+\bar{B}] = A\bar{B}C + \bar{A}C$

$= C[\bar{A} + A\bar{B}]$

$= C[\bar{A}+\bar{B}]$

$= C\,\overline{AB}$

(ii) $\bar{A}B + \bar{A}B\bar{C} + \bar{A}BDE + \bar{A}B\bar{C}D\bar{E} =$

$\bar{A}B[1+\bar{C}+DE+\bar{C}D\bar{E}]$

$= \bar{A}B$

(iii) $AB + \bar{A}C + \underline{BCDEF} =$

$AB + \bar{A}C + BCDEF(A+\bar{A})$

$= AB + \bar{A}C + ABCDEF + \bar{A}BCDEF$

$= AB[1+CDEF] + \bar{A}C[1+BDEF] = AB + \bar{A}C$

(iv) $ABC + B\bar{C}D + \underline{ABD} = ABC + B\bar{C}D$

$AB \cdot BD = ABD$

↳ De-Morgan's Theorem :-

↳ $\overline{AB} = \bar{A} + \bar{B}$

↳ $\overline{A+B} = \bar{A}\cdot\bar{B}$

↳ $\overline{ABC} = \bar{A}+\bar{B}+\bar{C}$

↳ $\overline{A+B+C} = \bar{A}\cdot\bar{B}\cdot\bar{C}$

↳ $\overline{ABCD} = \bar{A}+\bar{B}+\bar{C}+\bar{D}$

↳ $\overline{A+B+C+D} = \bar{A}\cdot\bar{B}\cdot\bar{C}\cdot\bar{D}$

## Simplify:-

↳ $\overline{\bar{A}\cdot B\cdot\bar{C}} = A+\bar{B}+C$

↳ $\overline{(A+B)\cdot(C+D)} = \overline{(A+B)} + \overline{(C+D)} = \bar{A}\cdot\bar{B} + \bar{C}\cdot\bar{D}$

↳ $\overline{\bar{A}\cdot\bar{B}\cdot C\cdot D} = A+B+\bar{C}+\bar{D}$

↳ $\overline{(A+\bar{B}+C+\bar{D})} + \overline{ABC\bar{D}} = \bar{A}B\bar{C}D+\bar{A}+\bar{B}+\bar{C}+D = \bar{A}[1+BC\bar{D}]+\bar{B}+\bar{C}+D$

$= \bar{A}+\bar{B}+\bar{C}+D$

$= \overline{ABC\bar{D}}$

↳ $\bar{A}\bar{B}C + \overline{(A+B+\bar{C})} + \bar{A}\bar{B}\bar{C}D =$

$\bar{A}\bar{B}C + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D = \bar{A}\bar{B}(C+\bar{C}D) = \bar{A}\bar{B}(C+D)$

$= \bar{A}\bar{B}C + \bar{A}\bar{B}D$

↳ $A\bar{B}C+\bar{A}BC+\bar{A}\bar{B}C = A\bar{B}C + \bar{A}C[B+\bar{B}] = A\bar{B}C+\bar{A}C$

$= C[\bar{A}+A\bar{B}] = C[\overline{AB}]$

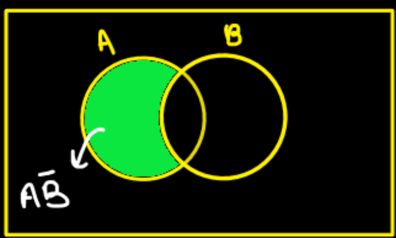
## Venn diagrams:-

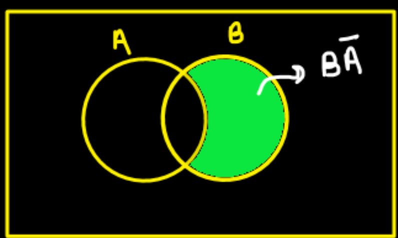
### Two variable:-

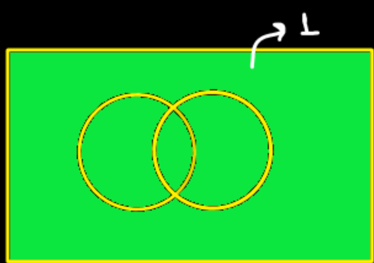
$A+B = A\bar{B} + \bar{A}B + AB$

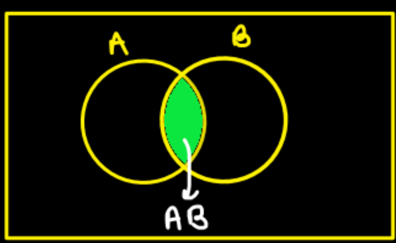
$A\bar{B}+\bar{A}B+AB+\bar{A}\bar{B} = (A+B) + \overline{(A+B)}$

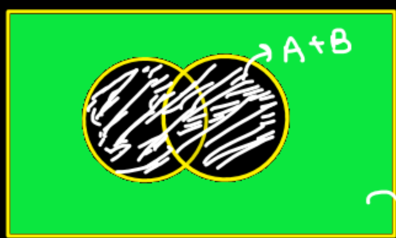
$= 1$



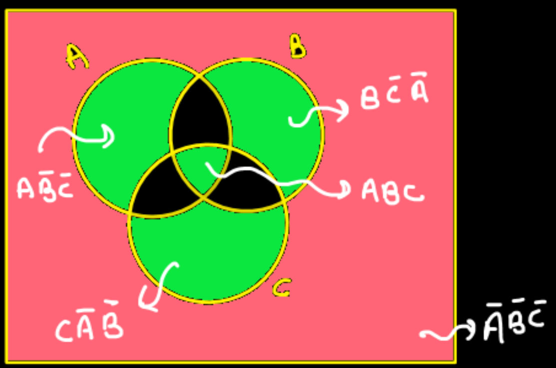



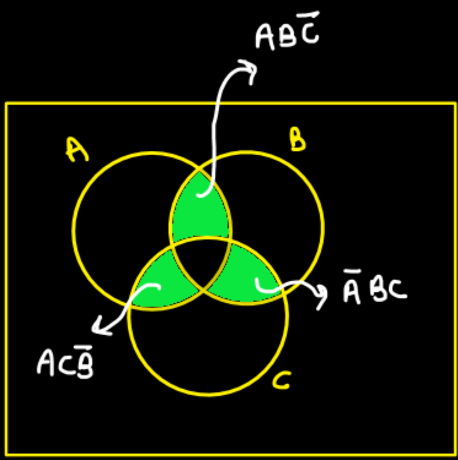



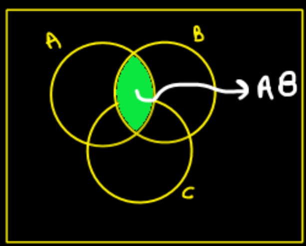




$\bar{A}\cdot\bar{B} = \overline{(A+B)}$

## Three variable :-

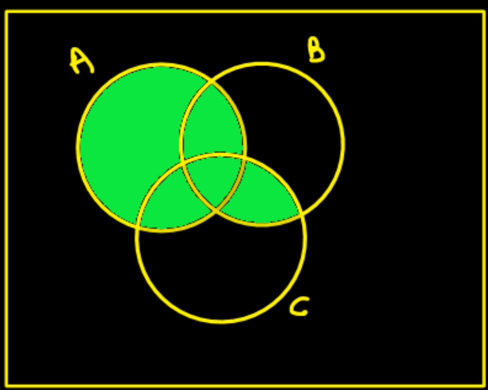



$$AB = AB\bar{C} + ABC$$
$$= AB[C + \bar{C}]$$
$$= AB$$

Q. Shaded Area Represents →




$$A + BC\bar{A}$$
$$= A + \bar{A}BC$$
$$= A + BC$$

↳ **Distinct logical expression :-**

same: A, A + AB

$A + AB$
$= A[1 + B]$
$= A$

A
AB
A + B

All Three different [Distinct logical expression]

Why A and A+AB are same?

Same Truth Table

| A | B | A | A + AB |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 |

## ★ Total no. of logical expressions with variables:-

**Q.** Suppose you are given 1 variable. How many "distinct logical expressions" you can make?

↳

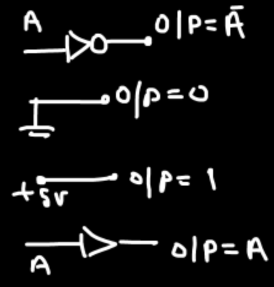

$A$

$\bar{A}$

$0$

$1$

$= \underline{\underline{4}}$




★ For two variables?


↳ Variable A and B can have $2^2 = 4$ possible combinations.

| A (2) | B (2) |
|---|---|
| 0 | 0 |
| 0 | 1 |
| 1 | 0 |
| 1 | 1 |

$2 \times 2 = 4$

With respect to each combination, two O/P ["0" or "1"] are possible.

| | A | B | 0,1 | $f_0$ (0) | $f_1$ ($\bar{A}\bar{B}$) | $f_2$ ($\bar{A}$) | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ | - - - - - - - - - - | $f_{15}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 0 | | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | | 1 |
| 2 | 0 | 1 | | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | | 1 |
| 2 | 1 | 0 | | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | | 0 |
| 2 | 1 | 1 | | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | | 1 |

$2 \times 2 \times 2 \times 2 = 2^4 = 2^{2^2} = 16$

↳ with 2 variables → 16 possible distinct logical expressions.

★ with three variables:-

$\overset{\nearrow 2}{A} \quad \overset{\nearrow 2}{B} \quad \overset{\nearrow 2}{C} = 2\times2\times2 = 8$

| | A | B | C |
|---|---|---|---|
| 2 → | 0 | 0 | 0 |
| 2 → | 0 | 0 | 1 |
| 2 → | 0 | 1 | 0 |
| 2 → | 0 | 1 | 1 |
| 2 → | 1 | 0 | 0 |
| 2 → | 1 | 0 | 1 |
| 2 → | 1 | 1 | 0 |
| 2 → | 1 | 1 | 1 |

$2\times2\times2\times2\times2\times2\times2\times2 = 2^8$

$= 2^{2^3}$

↳ For $n$ variable $= 2^{2^n}$ distinct logical / boolean expression will be there.

★ Literal:-

↳ Each variable in a logical expression.

Eg. → (i) $AB + CD + BD$ → L.C. = 6

(ii) $AB + \bar{A}B + DE + FG$ → L.C = 8

(iii) $(A+B+C)(D+E+F)$ → L.C. = 6

★ Taking $\boxed{\text{Dual}}$ of a logical Expression :-

(i) '+' ↔ '.'

(ii) '0' ↔ '1'

(iii) Don't complement the literals.

Eg. (i) $f = 0$

$\hookrightarrow f^D = 1$

(ii) $f = 1$

$\hookrightarrow f^D = 0$

(iii) $f = A+B$

$f^D = AB$

(iv) $f = AB$

$f^D = A+B$

(v) $f = (A+B)\cdot(BC) + CD$

$f^D = [AB + (B+C)]\cdot[C+D]$

***(vi) $f = AB + BC + CA$

$f^D = (A+B)\cdot(B+C)\cdot(C+A)$

$= (AB + AC + B + BC)(A+C)$

$= [B(A+1+C) + AC](A+C)$

$= (B+AC)(A+C)$

$= AB + BC + AC + AC = \underline{AB+BC+CA}$

$f^D = AB + BC + CA$

$\boxed{f^D = f}$ $\rightsquigarrow$ Self dual $f^n$

---

N.B.- (i) If $f^D = f$

then $f$ is called "Self dual" function

(ii) $(f^D)^D = f$

* Taking complement of a function "f" using duality :-

Complement of $f$ $(f^c)$ :-

$$f \xrightarrow[\text{dual}]{\text{Take}} f^D \xrightarrow[\text{each Literal Individually}]{\text{Compliment}} f^c$$

Q. Take compliment of $f = \{(a+\bar{b}c)d + \bar{e}\}f + \bar{g}$

↳ complement $\left[\{\bar{a}\cdot(b+\bar{c})+\bar{d}\}\cdot e+\bar{f}\right]\cdot g$

Q. Take compliment of $f = AB+BC+CA$ → self dual $f^n$

$f^D = f$

$\bar{f} = \overline{AB} + \overline{BC} + \overline{CA}$

$= (\bar{A}+\bar{B})(\bar{B}+\bar{C})(\bar{C}+\bar{A})$

★ $n$ variable → $2^{2^n}$ Total possible Boolean $f^n$ → $2^{2^{n-1}}$ are self dual.

★ 3 variable → $2^{2^3} = 2^8 = 256$ Total possible boolean $f^n$ → $2^{2^2} = 2^4 = 16$ are self dual

# Assignment-1

MSQ

Q. $Y = A \oplus B \oplus C \oplus D \oplus E \oplus F$

Which of the statements are true?

(a) Y can be used as odd no. of 1's detector.

(b) Y can be used as even no of 0's detector.

(c) The number of possible input combinations for Y=1 is 32.

(d) ~~The~~ number of " " " for Y=0 is 32.

↳

$Y = A \oplus B \oplus C \oplus D \oplus E \oplus F$ ← XOR operation

↳ odd no of 1's detector

$$Y = \overset{2\times}{A} \oplus \overset{2\times}{B} \oplus \overset{2\times}{C} \oplus \overset{2\times}{D} \oplus \overset{2\times}{E} \oplus \overset{2}{F} = 1 \qquad (= 64)$$

only one "1"

only three "1"s

only five "1"s

$${}^6C_1 + {}^6C_3 + {}^6C_5 = 6 + 20 + 6 = 32$$

Total combination = 64

zero = 32 times

Q. $Y = (A \odot B) + (B \odot C) + (C \odot A)$

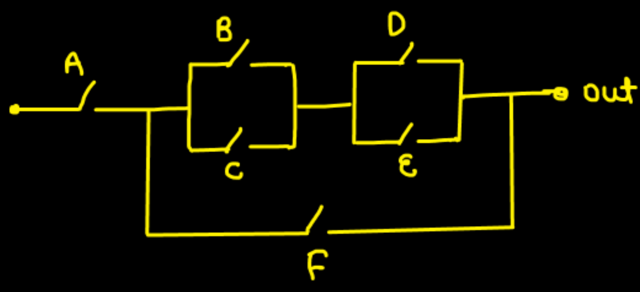
Total no. of NAND Gates required to implement Y is -

↳ $Y = \bar{A}\bar{B} + AB + \bar{B}\bar{C} + BC + \bar{C}\bar{A} + CA$

$= AB + BC + CA + \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{C}\bar{A}$ $\{AB + BC + CA$ is self dual$\}$

$= Y + \bar{Y} = \underline{\underline{1}}$ → zero NAND GATES required

Q.




which boolean $f^n$ this ckt represent?

↳ $[(B+C)\cdot(D+E) + F] \cdot A$

Q. $AB + \overline{AB}C + A$ is equivalent to

(a) $A+C$ (b) $B+C$ (c) $A+B$ (d) $A+B+C$

→ $AB + \overline{AB}C = AB + C$

↳ $AB + C + A = A[1+B] + C = A + C$

Q. $(\bar{A}+B)(\bar{A}+\bar{C}) + \bar{A}\bar{B} + \bar{A}\bar{C}$ reduced expression is

(a) $A + B\bar{C}$ (b) $\bar{A} + \bar{B}C$ (c) $\bar{A} + B\bar{C}$ (d) $\bar{A} + B\bar{C} + A\bar{B}$

↳ $\bar{A} + B\bar{C} + \bar{A}\bar{B} + \bar{A}\bar{C}$ $\{(X+Y)(X+Z) = X + YZ\}$

$= \bar{A} + B\bar{C} + \bar{A}\bar{B}$

$= \bar{A}[1+\bar{B}] + B\bar{C} = \bar{A} + B\bar{C}$

Q. $Y = A[B + \bar{C}(\overline{AB + A\bar{C}})]$

Reduced expression will have literals $\underline{2}$.

↳ $Y = A[B + \bar{C}\{(\bar{A}+\bar{B})(\bar{A}+C)\}]$

$= A[B + \bar{C}(\bar{A} + \bar{B}C)] = A[B + \bar{C}\bar{A}] = AB \to 2$

Q. Boolean Expression Y will reduce to-

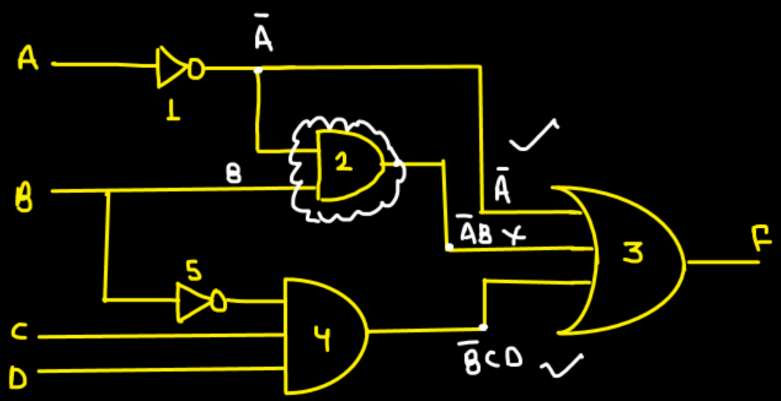
$$Y = \overline{[A+\bar{B}+C]} + \overline{[\bar{A}+\bar{B}+C]} + \overline{[A+\bar{B}+\bar{C}]} + ABC$$

(a) A (b) B (c) C (d) A+B+C

↳ $\bar{A}\cdot B\bar{C} + AB\bar{C} + \bar{A}BC + ABC$

$B\bar{C}[A+\bar{A}] + BC[A+\bar{A}] = B\bar{C} + BC = B[C+\bar{C}] = B$

Q.




Which of the gate is redundant?

(a) 1 (b) 2 (c) 4 (d) 5

⇒ $F = \bar{A} + \bar{A}B + \bar{B}CD$

$= \bar{A}[1+B] + \bar{B}CD$

$= \bar{A} + \bar{B}CD$

Q. $Y = PQR\bar{S} + PQRST + PQ\bar{R}S + PQ\bar{R}\bar{S} + PQ\bar{R}ST + PQRS + PQ\bar{R}\bar{S}T$ will be reduced to.

(a) PQ      (c) PQRST

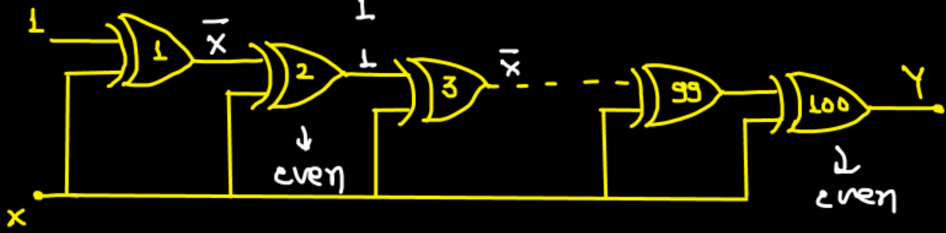
(b) P+Q      (d) P+Q+R+S

$\bar{A}B + A\bar{B} + AB + \bar{A}\bar{B} = 1$

↳ $PQ[R\bar{S} + RST + \bar{R}S + \bar{R}\bar{S} + \bar{R}ST + RS + \bar{R}\bar{S}T]$

$= PQ[\underbrace{R\bar{S} + \bar{R}S + RS + \bar{R}\bar{S}}_{1} + RST + \bar{R}ST + \bar{R}\bar{S}T] = PQ$

Q.




O/P Y is -

(a) X      (b) $\bar{X}$

(c) 1      (d) 0

$\bar{x} \oplus x = 1$

Q. A bulb in a staircases has two switches, one switch being at the ground floor and the other one at the first floor. The bulb can be turned ON and also can be turned OFF by one of the switches irrespective of the state of the other switch. The logic of switching of the bulb resembles

1 → ON , 0 → OFF

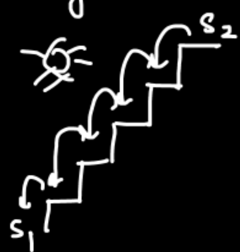
↳ (i) going up


| $S_1$ | $S_2$ | Y |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

| $S_1$ | $S_2$ | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

XOR GATE

(ii) coming down




| $S_1$ | $S_2$ | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |

$\boxed{\text{XOR GATE}}$

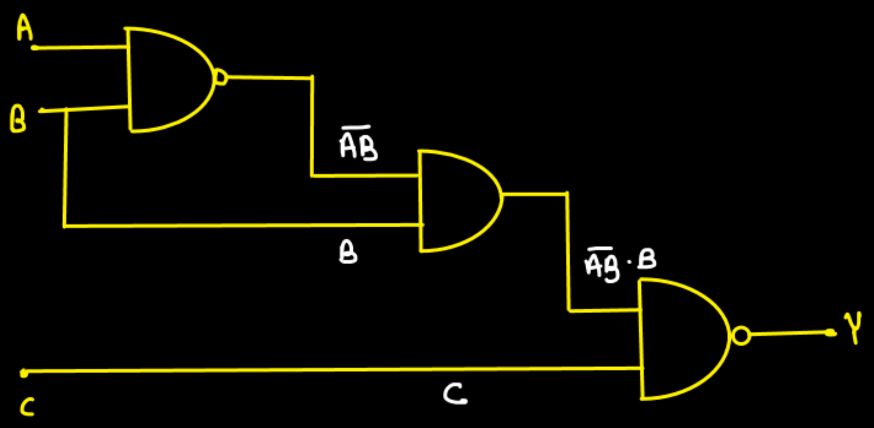
↳ Staircase switch

↳ Non-equivalence detector

↳ odd no. of 1's detector

MSQ

Q.




Output Y is →

(a) $\overline{A\bar{B}\bar{C}}$

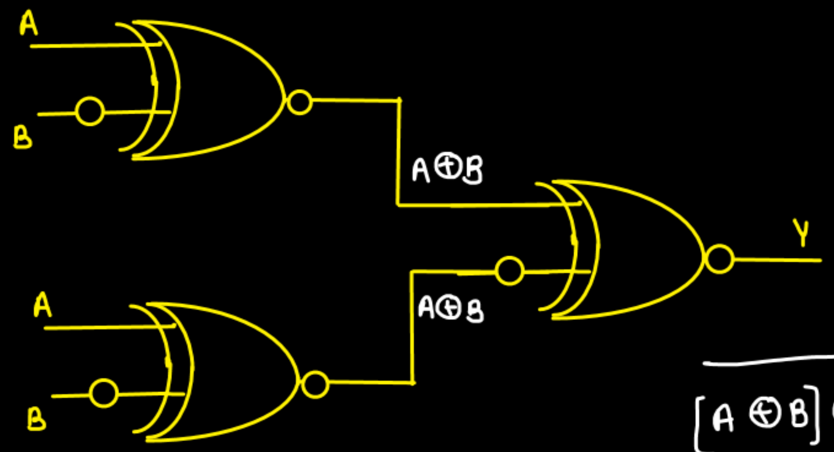
(b) $\bar{A}+B+C$

(c) $\overline{\bar{A}BC}$

(d) $A+\bar{B}+\bar{C}$

$$Y=\overline{(\bar{A}+\bar{B})\cdot B\cdot C}$$
$$=\overline{\bar{A}B\cdot C}$$
$$=A+\bar{B}+\bar{C}$$

Q.




O/P Y is -

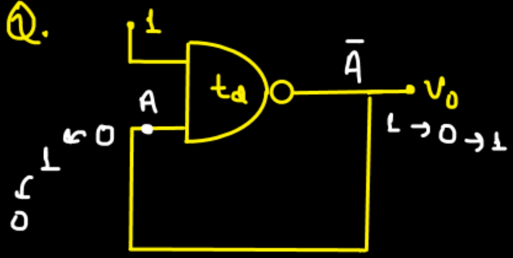
(A) $A\oplus B$ (B) $A\odot B$ (C) 0 (D) 1

$$\overline{[A\oplus B]\oplus[\overline{A\oplus B}]}=[A\odot B]\oplus[A\oplus B]$$
$$=0$$

Q. $Y = (A \oplus B)(B \odot C) \cdot C$

which combination of $(A, B, C)$ will make $Y = 1$

(a) 1, 0, 1  (b) 0, 0, 1  (c) 1, 1, 1  (D) 0, 1, 1

Q. 


The shown NAND Gate has a finite (non-zero) delay $t_d$.

The circuit acts as-

(a) Astable Multivibrator

(b) Bistable Multivibrator

(c) Monostable Multivibrator

(D) None

Q. $\bar{B}C + AB + A\bar{C}$ equals to

(a) $[A + \bar{B}][\bar{A} + \bar{C}]$

(b) $(A + \bar{B})(A + C)$

(c) $A + B$

(d) None

↳ $\bar{B}C + AB + A\bar{C}$

$= \bar{B}C + A[B + \bar{C}]$

$= \bar{B}C + A[\overline{\bar{B}C}]$

$= X + A\bar{X}$

$= X + A \quad = A + \bar{B}C \quad = (A + \bar{B})(A + C)$

Q. $A \oplus (\bar{A} + B)$ equals -

(a) 1 (b) 0 (c) $\overline{AB}$ (d) None

$A \oplus (B+C) \neq (A \oplus B) + (A \oplus C)$

↳ $A \oplus [\bar{A}+B] = [A \oplus \bar{A}] + A \oplus B$
$= 1 + [A \oplus B]$
$= 1$ ✗

$A \oplus [\bar{A}+B] = A \cdot A\bar{B} + (\bar{A}+B)\bar{A}$
$= A\bar{B} + \bar{A} + \bar{A}B$
$= \bar{A} + A\bar{B} = \bar{A} + \bar{B} = \overline{AB}$

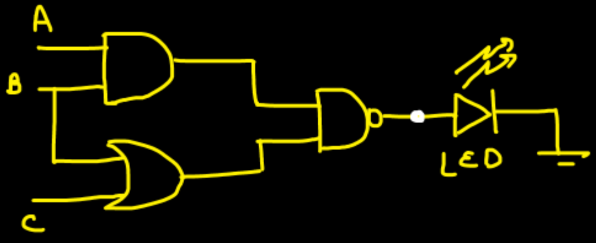
Q. Assuming that the variables and their complements are available, then the minimum number of 2-input NOR Gate needed to implement two input XOR Gate is 3.

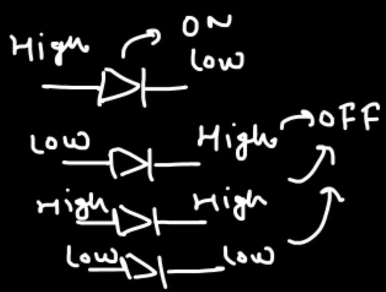
↳ $Y = \bar{A}B + A\bar{B} = (\bar{A}+\bar{B})(A+B)$

$A, B, \bar{A}, \bar{B} \rightarrow$ Available

$(C+D)\ (A+B) \longrightarrow \underline{\underline{3}}$

Q. MSQ




Possible combination of A, B, C to turn ON the LED-

(a) A=0, B=0, C=1

(b) A=1, B=0, C=0

~~(c)~~ A=1, B=1, C=0

~~(d)~~ A=1, B=1, C=1

Q. If $A \oplus B = C$

then $\bar{B} \oplus C$ will be -

(a) $A$ ~~(b)~~ $\bar{A}$ (c) $AB$ (d) $1$

$A \oplus B = C$

$B \oplus C = A$

↳ $\bar{B} \oplus C = \overline{B \oplus C} = \bar{A}$

Q. $\left[\overline{\overline{\bar{A} \oplus B} \odot \bar{C}}\right]$ equals to

(a) $A \oplus B \oplus C$ ~~(b)~~ $A \odot B \odot C$ (c) $A \odot B \oplus C$ (d) None

↳ $\overline{\overline{\bar{A} \oplus B} \odot \bar{C}} = \overline{\overline{[A \oplus B]} \odot C} = \overline{Y \odot C} = Y \oplus C = \overline{A \oplus B} \oplus C$

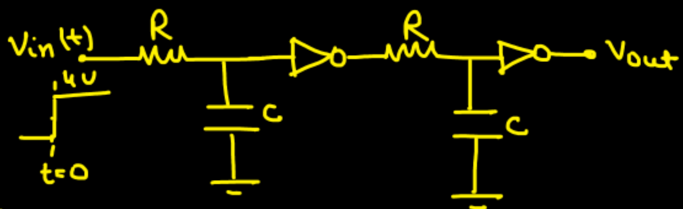
$= [\overline{A \oplus B}] \oplus C$

$= A \odot B \oplus C$ ✓

$= A \odot C \oplus B$ ✓ $= A \oplus B \odot C$ ✓

Q. The inverters are driven off a supply of 2V and their threshold points are at mid supply.

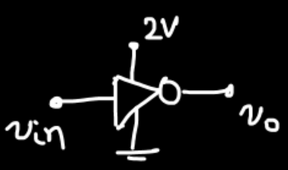
At what point of time $v_o(t)$ goes to $V_{DD} = 2V$




RC = 1 sec.

$v_{in}(t) = 4u(t)$

↳



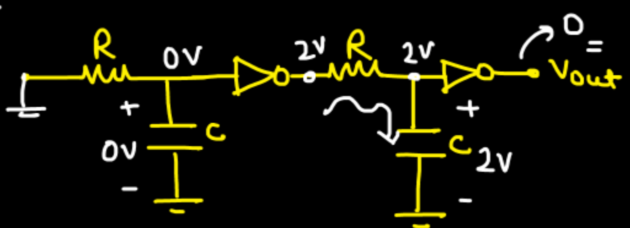

$V_T = 2/2 = 1V$

$\boxed{v_{in} > 1 \Rightarrow v_o = 0V}$

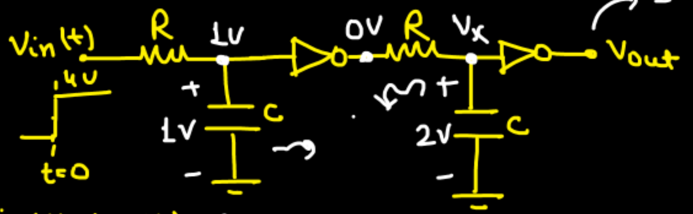
$\boxed{1V}$ → Threshold

$\boxed{v_{in} < 1}$ $\Rightarrow \boxed{v_o = 2V}$

**t<0:-**




**t>0:-**




$v_{in}(t) = 4u(t)$

$4[1-e^{-t/1}] = 1$

$t_1 = \ln(4/3)$ sec.

$2e^{-t/1} = 1$

$t_2 = \ln(2)$

$\ln(4/3) + \ln(2) = \ln(8/3)$ ANS.

when $V_1$ goes down to

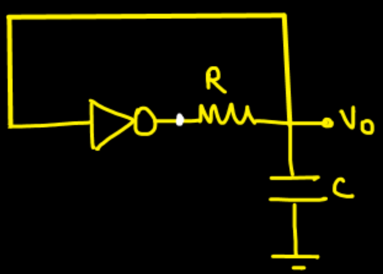
0.999999999 V

$\Downarrow$

Vout goes to 2V

---

**Q.**
**MSQ**




which of the statement is true?

(a) The circuit works as Astable multivibrator.

~~(b) " " " " Monostable " .~~

~~(c) The circuit works as bistable multivibrator.~~

(d) The circuit produces a periodic square wave if RC value is very low.

## Representation of boolean $f^n$

### ↳ Basics of Number System:-

① **Decimal:-**

[Base 10] = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Eg. → $(729)_{10} = \underbrace{9\times10^0}_{\text{Least sigficant Digit}} + 2\times10^1 + \underbrace{7\times10^2}_{\text{Most signficant Digit}}$

MSD (7), LSD (9)

② **Binary:-**

[Base 2] = 0, 1 → bits

Eg. → $(110)_2 = 0\times2^0 + 1\times2^1 + 1\times2^2$

MSB (first 1), LSB (0)

Eg. → $101 = 1\times2^2 + 0\times2^1 + 1\times2^0 = 4+0+1 = 5$

$111 = 1\times2^2 + 1\times2^1 + 1\times2^0 = 4+2+1 = 7$

8 4 2 1

$1101 = 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0 = 8+4+0+1 = 13$

32 16 8 4 2 1

$111001 = 32+16+8+1 = 57$

↳ $2^{n-1}$ --- $2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$b_n$ --- $b_5\ b_4\ b_3\ b_2\ b_1$

MSB ($b_n$), LSB ($b_1$)

**2-Variables:-**

| D (Decimal) | $b_2$ (MSB) | $b_1$ (LSB) |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 2 | 1 | 0 |
| 3 | 1 | 1 |

$00 = 0\times2^0 + 0\times2^1 = 0$

$01 = 0\times2^1 + 1\times2^0 = 1$

$10 = 1\times2^1 + 0\times2^0 = 2$

$11 = 2+1 = 3$

**3-variables:-**

| D | $b_2$ (4) | $b_1$ (2) | $b_0$ (1) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 |
| 7 | 1 | 1 | 1 |

**4-Variables:-**

| D | $b_3$ (8) | $b_2$ (4) | $b_1$ (2) | $b_0$ (1) |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 0 |
| 3 | 0 | 0 | 1 | 1 |
| 4 | 0 | 1 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 |
| 6 | 0 | 1 | 1 | 0 |
| 7 | 0 | 1 | 1 | 1 |
| 8 | 1 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 |
| 10 | 1 | 0 | 1 | 0 |
| 11 | 1 | 0 | 1 | 1 |
| 12 | 1 | 1 | 0 | 0 |
| 13 | 1 | 1 | 0 | 1 |
| 14 | 1 | 1 | 1 | 0 |
| 15 | 1 | 1 | 1 | 1 |

**n-variable:-**

$D \Rightarrow 2^n - 1$ = max^m decimal possible

⇒ **Product Term:-** Product of Literals.

Eg. → $AB$, $\bar{A}B$, $A\bar{B}$, $ABC$, $A\bar{B}C$, $ABCD$, $A\bar{B}\bar{C}D$

⇒ **Sum Term:-** Sum of Literals.

Eg → $A+B$, $\bar{A}+B$, $A+\bar{B}$, $\bar{A}+\bar{B}+\bar{C}$, $\bar{A}+B+\bar{C}$, $A+B+\bar{C}+D$

## ⇒ Minterm and Maxterm.

↳ **Minterm:-**

↳ A logical $\boxed{\text{AND}}$ operation including all variables, where each variable can be itself or its complement.

? ↳ o/p of the minterm function is $\boxed{1}$.

↳ Minterm is represented by $\boxed{m}$.

↳ $\boxed{1 \rightarrow \text{Normal form}}$, $\boxed{0 \rightarrow \text{complemented form.}}$

Eg.→

| D | A | B | Min-term | Expression |
|---|---|---|---|---|
| 0 | 0 | 0 | $\bar{A}\cdot\bar{B}$ | → $m_0$ |
| 1 | 0 | 1 | $\bar{A}\cdot B$ | → $m_1$ |
| 2 | 1 | 0 | $A\cdot\bar{B}$ | → $m_2$ |
| 3 | 1 | 1 | $A\cdot B$ | → $m_3$ |

⇒ Total 4 min-terms

↳ 3-variables:-

| D | A | B | C | Minterm | Expression |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $\bar{A}\bar{B}\bar{C}$ → | $m_0$ |
| 1 | 0 | 0 | 1 | $\bar{A}\bar{B}C$ → | $m_1$ |
| 2 | 0 | 1 | 0 | $\bar{A}B\bar{C}$ → | $m_2$ |
| 3 | 0 | 1 | 1 | $\bar{A}BC$ → | $m_3$ |
| 4 | 1 | 0 | 0 | $A\bar{B}\bar{C}$ → | $m_4$ |
| 5 | 1 | 0 | 1 | $A\bar{B}C$ → | $m_5$ |
| 6 | 1 | 1 | 0 | $AB\bar{C}$ → | $m_6$ |
| 7 | 1 | 1 | 1 | $ABC$ → | $m_7$ |

⇒ Total 8 min-terms

Eg. → $f(\overset{MSB}{w}, x, y, \overset{LSB}{z}) = \bar{w}\cdot x\cdot y\cdot \bar{z}$

Expression for min-term ?

↳ $\bar{w}\, x\, y\, \bar{z}$

$$\underset{8\,4\,2\,1}{0110} = 4+2=6 \xrightarrow[\text{term}]{\text{min-}} m_6$$

★ $n$ - variable = $2^n$ minterms

⇒ <u>writing output of a digital system using min-terms:-</u>

Q. Truth Table for a digital s/s is as follow:-

| D | A | B | Output = y = f(A, B) | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | → $\bar{A}\bar{B}$ | → $m_0$ |
| 1 | 0 | 1 | 0 | → $\bar{A}B$ | → $m_1$ |
| 2 | 1 | 0 | 1 | → $A\bar{B}$ | → $m_2$ |
| 3 | 1 | 1 | 0 | → $AB$ | → $m_3$ |

Find $y = f(A, B) = ?$

↳ $y = (\bar{A}\bar{B}\cdot 1) + (\bar{A}\cdot B\cdot 0) + (A\cdot\bar{B}\cdot 1) + (A\cdot B\cdot 0) = m_0 + m_2$

$= \bar{A}\,\bar{B} + A\bar{B}$ $\qquad = \Sigma m(0, 2)$ ANS.

$= \bar{B}[\bar{A}+A] = \bar{B}$ ANS.

Q. Repeat the same Question for this Truth Table.

| D | A | B | C | $y=f(A,B,C)$ | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | $\rightarrow \bar{A}\bar{B}\bar{C}$ |
| 1 | 0 | 0 | 1 | 0 | |
| 2 | 0 | 1 | 0 | 0 | |
| 3 | 0 | 1 | 1 | 1 | $\rightarrow \bar{A}BC$ |
| 4 | 1 | 0 | 0 | 1 | $\rightarrow A\bar{B}\bar{C}$ |
| 5 | 1 | 0 | 1 | 0 | |
| 6 | 1 | 1 | 0 | 0 | |
| 7 | 1 | 1 | 1 | 1 | $\rightarrow ABC$ |

↳ $Y = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC$

$Y = \Sigma m(0,3,4,7)$

↳ Concept of don't Care (X) :-

Q. Find $y = f(A,B,C)$ if truth Table is given :-

| | A (MSB) | B | C (LSB) | $y=f(A,B,C)$ | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | $\rightarrow \bar{A}\bar{B}\bar{C}$ |
| 1 | 0 | 0 | 1 | 0 | |
| 2 | 0 | 1 | 0 | 0 | |
| 3 | 0 | 1 | 1 | 1 | $\rightarrow \bar{A}BC$ |
| | 1 | 0 | X | 1 | |
| 6 | 1 | 1 | 0 | 1 | $\rightarrow AB\bar{C}$ |
| 7 | 1 | 1 | 1 | 0 | |

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}BC + AB\bar{C} + \underbrace{A\bar{B}\bar{C} + A\bar{B}C}_{\underbrace{A\bar{B}[C+\bar{C}]}_{A\bar{B}}}$$

| A B C | Y | |
|---|---|---|
| 1 0 0 | $\rightarrow 1$ | $\rightarrow A\bar{B}\bar{C}$ |
| 1 0 1 | $\rightarrow 1$ | $\rightarrow A\bar{B}C$ |

$Y = \bar{A}\bar{B}\bar{C} + \bar{A}BC + AB\bar{C} + A\bar{B}$

$= m_0 + m_3 + m_6 + m_5 + m_4$

$= \Sigma m(0,3,4,5,6)$

* Sum of all minterms = 1

**Q.** Find $y = f(A,B,C,D)$ if Truth - Table is given below.

| D | A | B | C | $y=f(A,B,C)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | D |
| 5 | 1 | 0 | 1 | $\bar{D}$ |
| 6 | 1 | 1 | 0 | 0 |
| 7 | 1 | 1 | 1 | 1 |

$Y = f(\overset{MSB}{A}, B, C, \overset{LSB}{D})$

$A \rightarrow$ MSB

$D \rightarrow$ LSB

$\hookrightarrow$ $Y = \underbrace{\bar{A}\bar{B}\bar{C}\cdot 1}_{\substack{m_0 \times \\ m_0 + m_1}} + \underbrace{\bar{A}BC\cdot 1}_{\substack{m_3 \times \\ m_6 + m_7}} + \underbrace{ABC}_{m_{14}+m_{15}} + \underbrace{A\bar{B}\bar{C}\cdot D}_{m_9} + \underbrace{A\bar{B}C\bar{D}}_{m_{10}}$

**Q.** $y = f(A,B,C,D) = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + ABC \rightarrow$ SOP $\downarrow$ min-term

Find the min-terms.

$\hookrightarrow$ $m_0 + m_1 + m_6 + m_7 + m_9 + m_{10} + m_{14} + m_{15}$

$= \Sigma m(0, 1, 6, 7, 9, 10, 14, 15)$

↳ Maxterm :-

↳ A logical $\boxed{OR}$ operation including all variables, where each variable can be itself or its complement.

↳ o/p of the minterm function is $\boxed{0.}$

↳ Maxterm is represented by $\boxed{M.}$

↳ $\boxed{0 \rightarrow \text{Normal form}}$ , $\boxed{1 \rightarrow \text{complemented form.}}$

Eg. →

| D | A | B | Max-term | Expression |
|---|---|---|---|---|
| 0 | 0 | 0 | $A + B$ | $M_0$ |
| 1 | 0 | 1 | $A + \bar{B}$ | $M_1$ |
| 2 | 1 | 0 | $\bar{A} + B$ | $M_2$ |
| 3 | 1 | 1 | $\bar{A} + \bar{B}$ | $M_3$ |

↳ 3-variables:-

| D | A | B | C | Max-term | Expression |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $A+B+C$ | $M_0$ |
| 1 | 0 | 0 | 1 | $A+B+\bar{C}$ | $M_1$ |
| 2 | 0 | 1 | 0 | $A+\bar{B}+C$ | $M_2$ |
| 3 | 0 | 1 | 1 | $A+\bar{B}+\bar{C}$ | $M_3$ |
| 4 | 1 | 0 | 0 | $\bar{A}+B+C$ | $M_4$ |
| 5 | 1 | 0 | 1 | $\bar{A}+B+\bar{C}$ | $M_5$ |
| 6 | 1 | 1 | 0 | $\bar{A}+\bar{B}+C$ | $M_6$ |
| 7 | 1 | 1 | 1 | $\bar{A}+\bar{B}+\bar{C}$ | $M_7$ |

Eg. → $f(w, x, y, z) = \bar{w} + x + y + \bar{z}$

Expression for max-term ?

↳ $\overset{8}{1}\,0\,0\,\overset{1}{1} = 9 \Rightarrow M_9$

★ $n$-variable = $2^n$ Maxterms

⇒ **Writing output of a digital system using max-terms:-**

Q. Truth Table for a digital s/s is as follow:-

| A | B | Output = $y = f(A, B)$ | |
|---|---|---|---|
| 0 | 0 | 1 | → $A + B$ |
| 0 | 1 | 0 ✓ $M_1$ | → $A + \bar{B}$ |
| 1 | 0 | 1 | → $\bar{A} + B$ |
| 1 | 1 | 0 ✓ $M_3$ | → $\bar{A} + \bar{B}$ |

Find $y = f(A, B) = ?$

↳ $\boxed{\text{min-terms:-}}$

$Y = \bar{A}\bar{B} + A\bar{B} = \Sigma m(0, 2)$ ✓

$= \bar{B}$ ✓

$\boxed{\text{max-terms}}$

$Y = (A + B + 1) \cdot (A + \bar{B} + 0)(\bar{A} + B + 1)(\bar{A} + \bar{B} + 0)$

$= 1 \cdot (A + \bar{B}) \cdot 1 \cdot (\bar{A} + \bar{B}) = (A + \bar{B})(\bar{A} + \bar{B})$

$Y = M_1 \cdot M_3 = \Pi M(1, 3)$ ✓ $= A\bar{B} + \bar{B}\bar{A} + \bar{B}$ ✓

$= \bar{B}$

Q. Repeat the same Question for this Truth Table.
[with C being MSB and A being LSB]

| D | A (L) | B | C (M) | $y = f(A,B,C)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 4 | 0 | 0 | 1 | 0 |
| 2 | 0 | 1 | 0 | 0 |
| 6 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 0 |
| 3 | 1 | 1 | 0 | 0 |
| 7 | 1 | 1 | 1 | 1 |

↳ $Y = \pi M(2,4,5,3) = \pi M(2,3,4,5)$

$Y = (A+B+\bar{C})\cdot(A+\bar{B}+C)\cdot(\bar{A}+B+\bar{C})\cdot(\bar{A}+\bar{B}+C)$

↳ Concept of don't care (X) :-

Q. Find $y = f(A,B,C)$ if truth table is given :-

| | A | B | C | $y = f(A,B,C)$ |
|---|---|---|---|---|
| | 0 | 0 | 0 | 1 |
| | 0 | 0 | 1 | 0 |
| | 0 | 1 | 0 | 0 |
| | 0 | 1 | 1 | 1 |
| | 1 | 0 | X | 0 |
| | 1 | 1 | 0 | 1 |
| | 1 | 1 | 1 | 1 |

1 0 1 → 0

1 0 0 → 0

↳ $Y = (A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+\bar{C})(\bar{A}+B+C)$

$= (A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B)$

* Product of all max-terms = 0

Q. find $y = f(A,B,C,D)$ if Truth-Table is given below.

| D | A | B | C | $y = f(A,B,C)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | D |
| 5 | 1 | 0 | 1 | $\bar{D}$ |
| 6 | 1 | 1 | 0 | 0 |
| 7 | 1 | 1 | 1 | 1 |

↳ $Y = (A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+C+D)(\bar{A}+B+\bar{C}+\bar{D})(\bar{A}+\bar{B}+C)$

$= \cancel{M_1} \quad \cancel{M_2}$

$= M_2 \cdot M_3 \cdot M_4 \cdot M_5 \cdot M_8 \cdot M_{11} \cdot M_{12} \cdot M_{13} = \pi M(2,3,4,5,8,11,12,13)$

Q. $y[A,B,C,D] = (A+B+C)\cdot(\bar{A}+B+C)\cdot(\bar{A}+B+C+\bar{D}) \rightarrow$ POS ↷ maxterms

↳

$M_0$ 0000 $\quad M_8$ 1000 $\quad$ 1001 $M_9$

$M_1$ 0001 $\quad$ ~~1001~~

$\Rightarrow \pi M(0,1,8,9) = y$

↳ Q. $f(A,B,C) = \Sigma m(0,2,4)$ ; write in terms of max-terms.

→ 3 variables ⇒ max decimal = 7

$\pi M(1,3,5,6,7)$

↳ Q. $f(A,B,C,D) = \pi M(0,2,3,5,7)$ ; write in min-terms.

↳ max decimal = 15

$\Sigma m(1,4,6,8,9,10,11,12,13,14,15)$

⇒ **Sum of Product and Product of Sum :-**

(i) **SOP :-**

Product terms "OR"ed together.

Eg. → $ABC + \bar{A}\bar{B}$ , $A\bar{B} + BCD$

(ii) **POS :-**

Sum terms "AND"ed together

Eg → $(A+B+C)(D+E)$ , $(A+\bar{B}+C)(\bar{D}+E)$

⇒ **Standard / canonical SOP / POS form :-**

All the variables should be present in each product / sum term. [Either in normal form or complemented]

Eg. → $f(A, B, C)$ is given as -

(i) $(A+B)(B+\bar{C})$ → NOT standard POS

(ii) $(A+B+C)(A+B+\bar{C})(A+B+\bar{C})$ → Standard POS

(iii) $AB + B\bar{C}$ → NOT standard SOP

(iv) $ABC + AB\bar{C} + \bar{A}BC$ → Standard SOP

which of them is standard SOP / POS ?

Q. For the given $f^n$, write min-terms and maxterms.

Then write standard SOP and POS form.

(a) $f(A,B,C) = A + \bar{B}C$

(b) $f(A,B,C) = AB + \bar{A}C$

(c) $f(A,B,C) = A + B + \bar{C}$

↳ (a) $f(A,B,C) = A + \bar{B}C \rightarrow$ SOP

SOP:-

$f(A,B,C) = A + \bar{B}C$

$\bar{B}C$

001, 101

min-terms → 100, 101, 110, 111

$A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC = A[\bar{B}\bar{C} + \bar{B}C + B\bar{C} + BC] = A$

Standard SOP:-

$f(A,B,C) = A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC + A\bar{B}C + \bar{A}\bar{B}C$

$= A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC + \bar{A}\bar{B}C \rightarrow$ SOP

= 100, 101, 110, 111, 001

= 4, 5, 6, 7, 1

$= \Sigma m(1,4,5,6,7) \rightarrow$ minterms

$= \pi M(0,2,3) \rightarrow$ maxterms

000 → $A+B+C$

010 → $A+\bar{B}+C$

011 → $A+\bar{B}+\bar{C}$

Standard POS:-

$f(A,B,C) = (A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})$

Standard POS (Mtd 2)-

$$f(A,B,C) = A + \bar{B}C \qquad \{X+YZ = (X+Y)(X+Z)\}$$

$$= (A+\bar{B})(A+C)$$

$$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+B+C)(A+\bar{B}+C)$$

$$f(A,B,C) = (A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+B+C) \rightarrow \text{std POS}$$

$$010,\ 011,\ 000 \quad = \pi M(0,2,3)$$

(b) $f(A,B,C) = AB + \bar{A}C \rightarrow \text{SOP}$

Standard SOP:-

$$= ABC + AB\bar{C} + \bar{A}BC + \bar{A}\bar{B}C$$

| | | | | |
|---|---|---|---|---|
| 111 | 110 | 011 | 001 | $= \Sigma m(1,3,6,7)$ |
| 7 | 6 | 3 | 1 | $= \pi M(0,2,4,5)$ |

Standard POS:-

$$\pi M(0,2,4,5) = f(A,B,C)$$

$$000,\ 010,\ 100,\ 101$$

$$f(A,B,C) = (A+B+C)(A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+B+\bar{C})$$

Mtd 2:-

$$f(A,B,C) = AB + \bar{A}C = (AB+\bar{A})(AB+C)$$

$$= (A+\bar{A})(\bar{A}+B)(C+A)(C+B)$$

$$= (\bar{A}+B)(A+C)(B+C)$$

$$f(A,B,C) = (\bar{A}+B+C)(\bar{A}+B+\bar{C})(A+B+C)(A+\bar{B}+C)(A+B+C)(\bar{A}+B+C)$$

$$f(A,B,C) = (A+B+C)(A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+B+\bar{C})$$

(c) $f(A,B,C) = (A+B+\bar{C})$ → standard POS ✓

$001 \rightarrow M(1) \Rightarrow$ maxterm

$\Sigma m(0,2,3,4,5,6,7) \Rightarrow$ minterms

000, 010, 011, 100, 101, 110, 111

$f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$ ✓

↳ standard SOP

M-2 [standard SOP]

$$f(A,B,C) = A+B+\bar{C}$$

$$= A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C} + AB\bar{C}$$

$$= A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}\bar{B}\bar{C}$$

$$= 4, 5, 6, 7, 2, 3, 0 = \Sigma m(0,2,3,4,5,6,7)$$

Q. What is the function $Y = A + \bar{B}C$ in POS form

~~(a) $M_0 + M_2 + M_3$~~ ~~(b) $M_0 M_2 M_3$~~ (c) $M_0 M_1 M_2 M_3$

(d) $M_0 + M_1 + M_2 + M_3$

↳ $Y = A + \bar{B}C = (A+\bar{B})(A+C) = (A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+B+C)$

$$\underbrace{010}_{2} \quad \underbrace{011}_{3} \quad \underbrace{000}_{0}$$

$$= \Pi M(0,2,3)$$

Q. A three-variable table has high output for the inputs 010, 011 and 110. The boolean expression can be written as-

↳

010 → 1 ✓
011 → 1 ✓ → minterms
110 → 1 ✓ SOP

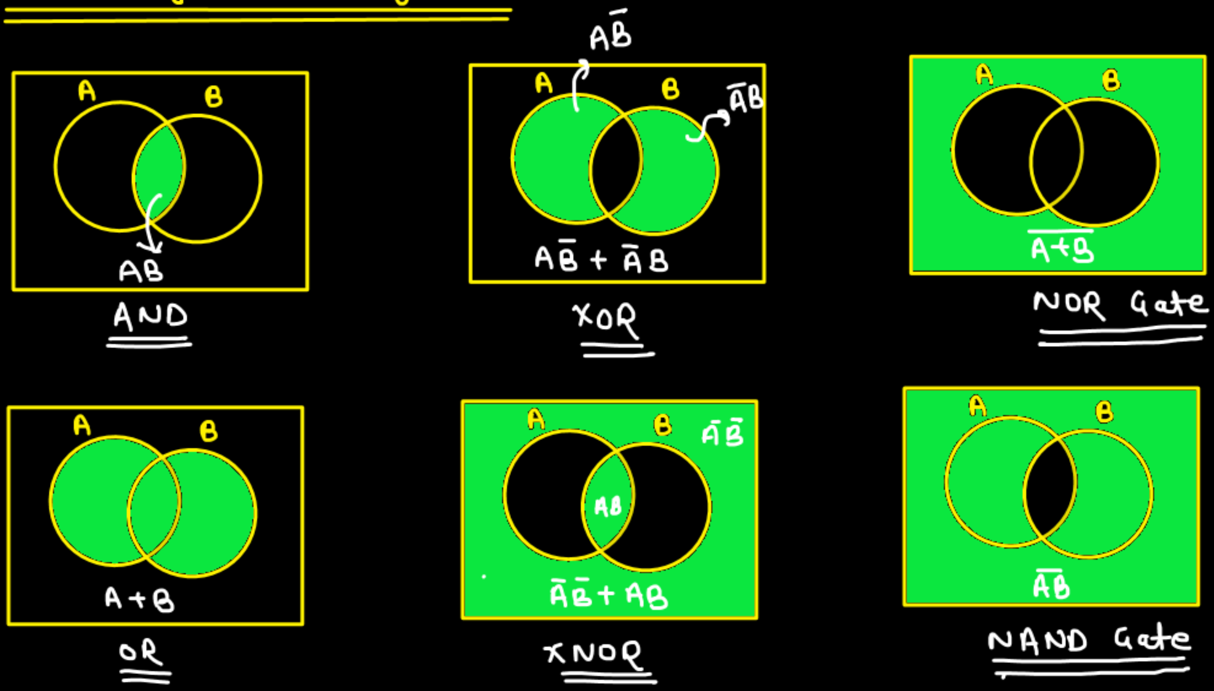
$Y = \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C}$

$= \bar{A}B + AB\bar{C}$

$= B[\bar{A} + A\bar{C}] = \bar{A}B + \bar{A}C$ ✓

⇒ Min-terms through Logic Gates:-

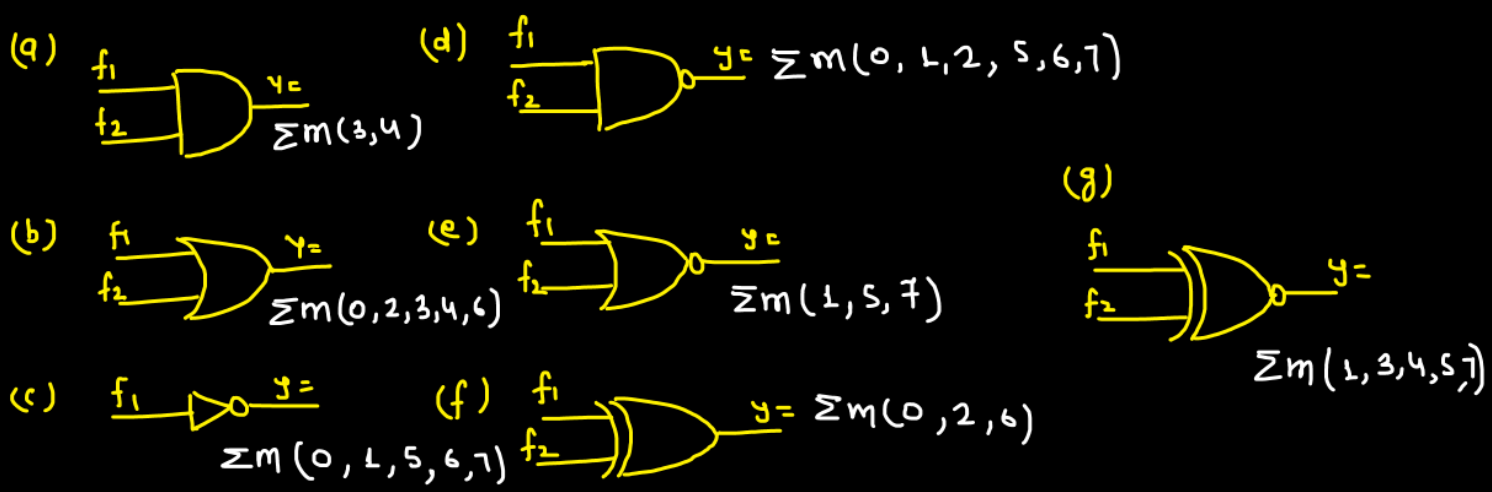
★ Revisiting venn diagrams:-



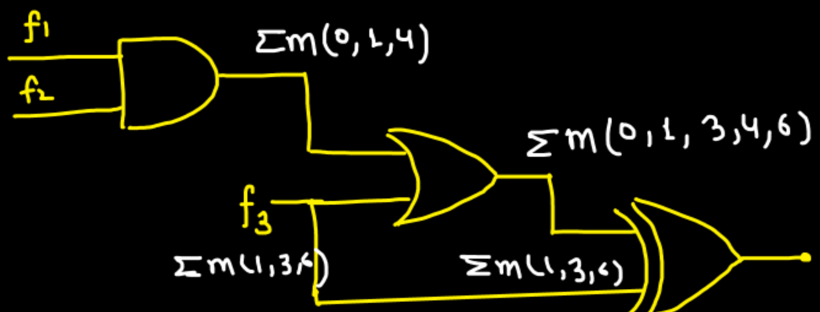

Q. Find output y if $f_1$ and $f_2$ are given:-

$$f_1 = \Sigma m(2, \underline{3}, \underline{4})$$

$$f_2 = \Pi M(1, 2, 5, 7) = \Sigma m(0, \underline{3}, \underline{4}, 6)$$



Q. Find y.




$Y = \Sigma m(0,4)$

$= \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$

$= \bar{B}\bar{C}$

$f_1 = \Pi M(2,3,5) = \Sigma m(0,1,4,6,7)$

$f_2 = \Pi M(3,6,7) = \Sigma m(0,1,2,4,5)$

$f_3 = \Sigma m(1,3,6) = \Sigma m(1,3,6)$

# K-Maps (Karnaugh-Maps)

↳ Graphical Method to minize boolean expressions.

↳ For $n$ variables, there will be $2^n$ cells.

★ **2-variables:-** 4 cells

A → MSB

B → LSB

$f(A,B)$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 00 $_0$ | 01 $_1$ → cell |
| 1 | 10 $_2$ | 11 $_3$ |

| A \ B | $\bar{B}$ 0 | B 1 |
|---|---|---|
| $\bar{A}$ 0 | $_0$ | $_1$ |
| A 1 | $_2$ | $_3$ |

SOP

↓

min-terms

| A \ B | B 0 | $\bar{B}$ 1 |
|---|---|---|
| A 0 | $_0$ | $_1$ |
| $\bar{A}$ 1 | $_2$ | $_3$ |

POS

Q. $f(A,B) = \Sigma m(1,2)$

Fill the K-map cells.

↳

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 (0) | 1 (1) |
| 1 | 1 (2) | 0 (3) |

$f(A,B) = \Sigma m(1,2)$

$= \Pi M(0,3)$

3-Variables :- $2^3 = 8$ cells

☆☆☆

A → MSB

C → LSB

$f(A,B,C)$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 (0) | 001 (1) | 011 (3) | 010 (2) |
| 1 | 100 (4) | 101 (5) | 111 (7) | 110 (6) |

(a)

| BC \ A | 0 | 1 |
|---|---|---|
| 00 | 0 | 4 |
| 01 | 1 | 5 |
| 11 | 3 | 7 |
| 10 | 2 | 6 |

(b)

| C \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 (0) | 010 (2) | 110 (6) | 100 (4) |
| 1 | 001 (1) | 011 (3) | 111 (7) | 101 (5) |

(c)

Q. $f(A,B,C) = \prod M(2,3,5,7) = \sum m(0,1,4,6)$

Fill the K-map cells.

↳

$A \rightarrow MSB$

$C \rightarrow LSB$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $1_0$ | $1_1$ | $0_3$ | $0_2$ |
| 1 | $1_4$ | $0_5$ | $0_7$ | $1_6$ |

| A \ BC | $\bar{B}\bar{C}$ | $\bar{B}C$ | $BC$ | $B\bar{C}$ |
|---|---|---|---|---|
| $\bar{A}$ | $1_0$ | $1_1$ | $_3$ | $_2$ |
| $A$ | $1_4$ | $_5$ | $_7$ | $1_6$ |

SOP

min-terms

| A \ B+C | $B+C$ | $B+\bar{C}$ | $\bar{B}+\bar{C}$ | $\bar{B}+C$ |
|---|---|---|---|---|
| $A$ | $_0$ | $_1$ | $0_3$ | $0_2$ |
| $\bar{A}$ | $_4$ | $0_5$ | $0_7$ | $_6$ |

POS

## 4-Variables:-

$A \rightarrow MSB$

$D \rightarrow LSB$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 3 | 2 |
| 01 | 4 | 5 | 7 | 6 |
| 11 | 12 | 13 | 15 | 14 |
| 10 | 8 | 9 | 11 | 10 |

(a)

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 4 | 12 | 8 |
| 01 | 1 | 5 | 13 | 9 |
| 11 | 3 | 7 | 15 | 11 |
| 10 | 2 | 6 | 14 | 10 |

(b)

Q. $f(A,B,C,D) = AB\bar{C}D + A\bar{B}C + ABCD + \bar{A}BCD \rightarrow$ SOP

$= 1101, 1010, 1011, 1111, 0111$

Fill the K-maps cell.

$\Sigma m(13, 10, 11, 15, 7)$

↳

| AB \ CD | $\bar{C}\bar{D}$ | $\bar{C}D$ | $CD$ | $C\bar{D}$ |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ | $_{0}$ | $_{1}$ | $_{3}$ | $_{2}$ |
| $\bar{A}B$ | $_{4}$ | $_{5}$ | $1_{7}$ | $_{6}$ |
| $AB$ | $_{12}$ | $1_{13}$ | $1_{15}$ | $_{14}$ |
| $A\bar{B}$ | $_{8}$ | $_{9}$ | $1_{11}$ | $1_{10}$ |

| A+B \ C+D | $C+D$ | $C+\bar{D}$ | $\bar{C}+\bar{D}$ | $\bar{C}+D$ |
|---|---|---|---|---|
| $A+B$ | $0_{0}$ | $0_{1}$ | $0_{3}$ | $0_{2}$ |
| $A+\bar{B}$ | $0_{4}$ | $0_{5}$ | $_{7}$ | $0_{6}$ |
| $\bar{A}+\bar{B}$ | $0_{12}$ | $_{13}$ | $_{15}$ | $0_{14}$ |
| $\bar{A}+B$ | $0_{8}$ | $0_{9}$ | $_{11}$ | $_{10}$ |

SOP

Q. Truth Table is given for $f(A,B,C,D)$

Fill the K-maps cells

| A (8) | B (4) | C (2) | D (1) | Y |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

↳

| AB \ CD | $\bar{C}\bar{D}$ | $\bar{C}D$ | $CD$ | $C\bar{D}$ |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ | $_{0}$ | $_{1}$ | $_{3}$ | $1_{2}$ |
| $\bar{A}B$ | $_{4}$ | $1_{5}$ | $_{7}$ | $1_{6}$ |
| $AB$ | $_{12}$ | $1_{13}$ | $_{15}$ | $_{14}$ |
| $A\bar{B}$ | $_{8}$ | $1_{9}$ | $1_{11}$ | $_{10}$ |

SOP

$\Sigma m(2, 5, 6, 9, 11, 13)$

Q. Truth Table is given for $f(A, B, C, D)$
Fill the K-maps cells.

| A (8) | B (4) | C (2) | D (1) | Y |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 ✓ |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 ✓ |
| 0 | 1 | 0 | 0 | 0 ✓ |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 ✓ |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 ✓ |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 ✓ |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 ✓ |
| 1 | 1 | 1 | 1 | 1 |

$\pi M(1, 3, 4, 7, 10, 12, 14)$

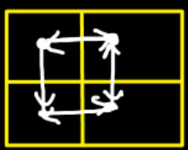
| A+B \ C+D | $C+D$ | $C+\bar{D}$ | $\bar{C}+\bar{D}$ | $\bar{C}+D$ |
|---|---|---|---|---|
| $A+B$ | 0 | 0 (1) | 0 (3) | 2 |
| $A+\bar{B}$ | 0 (4) | 5 | 0 (7) | 6 |
| $\bar{A}+\bar{B}$ | 0 (12) | 13 | 15 | 0 (14) |
| $\bar{A}+B$ | 8 | 9 | 11 | 0 (10) |

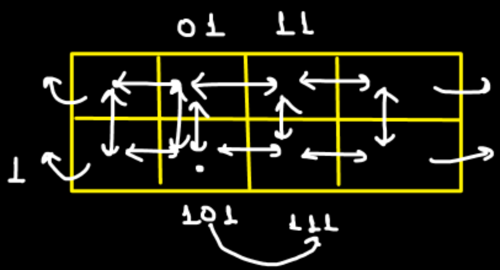
POS

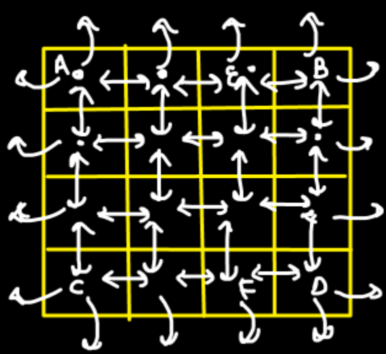
★ Rules to be followed in K-maps for minimization:-

(i) Club the adjacent cells.

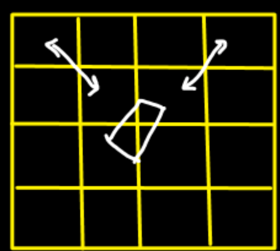
↳ adjacent cells ?





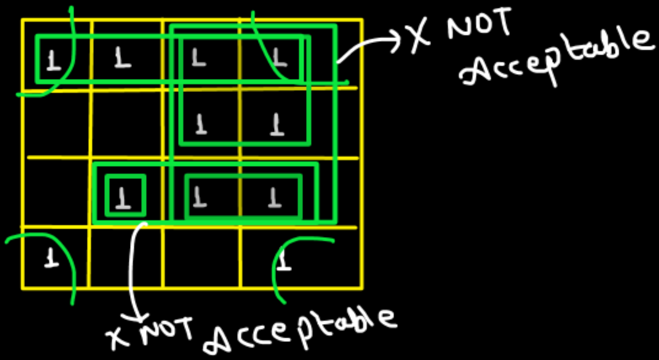




(ii) Don't club diagonal cells.

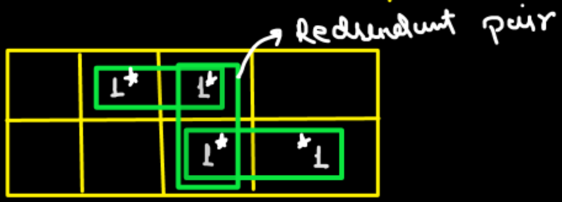


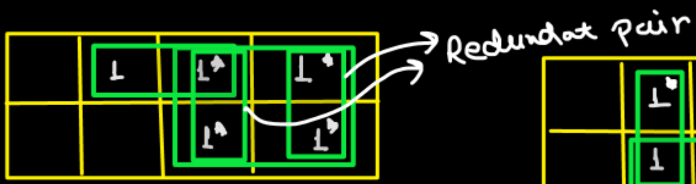
[N.B. → Between two adjacent cells in K-maps, there will be a difference of 1-bit only]

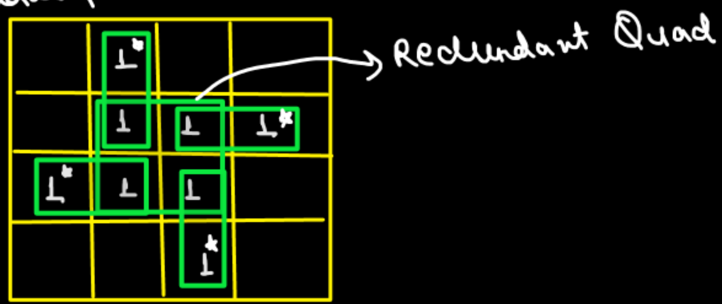
(iii) Clubbing can be done in order of 1, 2, 4, 8 --- $2^n$ cells.

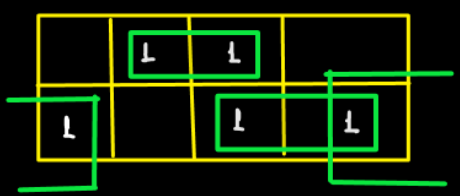


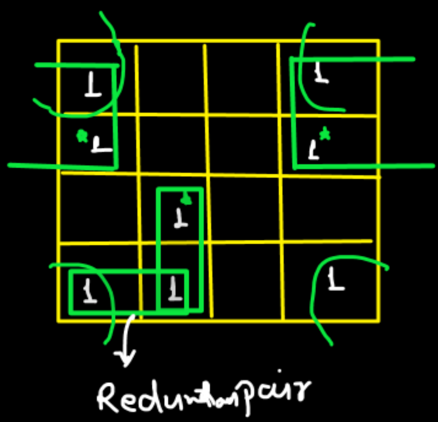

(iv) Make the best possible pairs. Avoid redundant pairs. No. of pairs should be as min. as possible.





(v) <u>Effectively we don't care</u> :- [x]

| | | | |
|---|---|---|---|
| 0 | 0 | 1 | x |
| 0 | 0 | 1 | 1 |

→ 1

| | | | |
|---|---|---|---|
| 0 | 0 | 1 | x |
| 0 | 0 | 1 | 0 |

→ 0

(vi) Write down the boolean Expression.

[Target ⇒ minimum literal count]

Q. $f(A, B) = \Sigma m(0, 3)$ ; write boolean expression.

↳

| A \ B | $\bar{B}$ | $B$ |
|---|---|---|
| $\bar{A}$ | $\boxed{1}_0$ | $_1$ |
| $A$ | $_2$ | $\boxed{1}_3$ |

$Y = \bar{A}\bar{B} + AB$

Q. $f(A, B) = \Sigma m(0, 1, 2)$

| A \ B | $\bar{B}$ | $B$ |
|---|---|---|
| $\bar{A}$ | $1_0$ | $1_1$ |
| $A$ | $1_2$ | $_3$ |

$Y = \bar{A} + \bar{B} = \overline{AB}$

Q.3 $f(A,B) = \prod M(1,2)$

↳

| $A \backslash B$ | $B$ | $\bar{B}$ |
|---|---|---|
| $A$ | $_0$ | $0_1$ |
| $\bar{A}$ | $0_2$ | $_3$ |

$Y = (\bar{A}+B)\cdot(\bar{B}+A)$

Q.4 $f(A,B) = \prod M(0,1,2)$

↳

| $A \backslash B$ | $B$ | $\bar{B}$ |
|---|---|---|
| $A$ | $0_0$ | $0_1$ |
| $\bar{A}$ | $0_2$ | $_3$ |

$Y = (A)\cdot(B) = AB$

Q. $f(A,B) = \Sigma m(0,3) + \Sigma d(1)$

↳

| $A \backslash B$ | $\bar{B}$ | $B$ |
|---|---|---|
| $\bar{A}$ | $1_0$ | $X_1$ |
| $A$ | $_2$ | $1_3$ |

$Y = \bar{A}+B$ ⟶ SOP

Q. $f(A,B) = \prod M(0,2) \cdot \prod d(1)$

| $A \backslash B$ | $B$ | $\bar{B}$ |
|---|---|---|
| $A$ | $0_0$ | $X_1 \rightarrow 1$ |
| $\bar{A}$ | $0_2$ | $_3$ |

POS

$Y = B$

Q. $f(A,B,C) = \Sigma m(2,3,5)$

↳

| $A \backslash BC$ | $\bar{B}\bar{C}$ | $\bar{B}C$ | $BC$ | $B\bar{C}$ |
|---|---|---|---|---|
| $\bar{A}$ | $_0$ | $_1$ | $1_3$ | $1_2$ |
| $A$ | $_4$ | $1_5$ | $_7$ | $_6$ |

SOP

$Y = B \cdot \bar{A} + A\bar{B}C$

Q. $f(A,B,C) = \Sigma m(0,2,3,4,6,7)$

↳

| $A \backslash BC$ | $\bar{B}\bar{C}$ | $\bar{B}C$ | $BC$ | $B\bar{C}$ |
|---|---|---|---|---|
| $\bar{A}$ | 1 | | 1 | 1 |
| $A$ | 1 | | 1 | 1 |

SOP :-

$Y = B + \bar{C}$

Q. $f(A,B,C) = \Pi M(1,2,5,6)$

↳

| $A \backslash B+C$ | $B+C$ | $B+\bar{C}$ | $\bar{B}+\bar{C}$ | $\bar{B}+C$ |
|---|---|---|---|---|
| $A$ | $_0$ | $0_1$ | $_3$ | $0_2$ |
| $\bar{A}$ | $_4$ | $0_5$ | $_7$ | $0_6$ |

POS

$Y = (\bar{B}+C) \cdot (B+\bar{C})$

Q. $f(A,B,C) = \Pi M(0,2,3,6,7)$

↳

| $A \backslash B+C$ | $B+C$ | $B+\bar{C}$ | $\bar{B}+\bar{C}$ | $\bar{B}+C$ |
|---|---|---|---|---|
| $A$ | 0 | | 0 | 0 |
| $\bar{A}$ | | | 0 | 0 |

POS

$Y = \bar{B} \cdot (C+A)$

Q. $f(A,B,C) = \sum m(0,1,5,6) + \sum d(3,7)$

| A \ BC | $\bar{B}\bar{C}$ | $\bar{B}C$ | $BC$ | $B\bar{C}$ |
|---|---|---|---|---|
| $\bar{A}$ | 1 | 1 | X | |
| $A$ | | 1 | X | 1 |

SOP

$$Y = C + AB + \bar{A}\bar{B}$$

Q. $f(A,B,C) = \prod M(1,3,6,7) \prod d(2,4)$

| A \ BC | $B+C$ | $B+\bar{C}$ | $\bar{B}+\bar{C}$ | $\bar{B}+C$ |
|---|---|---|---|---|
| $A$ | | 0 | 0 | X |
| $\bar{A}$ | X | | 0 | 0 |

1

POS

$$Y = \bar{B} \cdot (A+\bar{C})$$

Q. $f(A,B,C,D) = \sum m(0,1,3,4,5,6,7,13,15)$

| AB \ CD | $\bar{C}\bar{D}$ | $\bar{C}D$ | $CD$ | $C\bar{D}$ |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ | 1 (0) | 1 (1) | 1 (3) | 2 |
| $\bar{A}B$ | 1 (4) | 1 (5) | 1 (7) | 1 (6) |
| $AB$ | 12 | 1 (13) | 1 (15) | 14 |
| $A\bar{B}$ | 8 | 9 | 11 | 10 |

SOP

$$Y = B \cdot D + D \cdot \bar{A} + \bar{C}\bar{A}$$
$$= \bar{A}\bar{C} + \bar{A} \cdot D + B \cdot D$$

Q. $f(A,B,C,D) = \sum m(1,5,6,7,11,12,13,15)$

| AB \ CD | $\bar{C}\bar{D}$ | $\bar{C}D$ | $CD$ | $C\bar{D}$ |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ | | 1 | | |
| $\bar{A}B$ | | 1 | 1 | 1 |
| $AB$ | 1 | 1 | 1 | |
| $A\bar{B}$ | | | 1 | |

SOP :-

$$Y = \bar{A}\bar{C}D + \bar{A}BC + ACD + AB\bar{C}$$

Q. $f(A,B,C,D) = \prod M(0,2,3,4,8,9,10,14)$ ↳ maxterms

| $A+B \backslash C+D$ | $C+D$ | $C+\bar{D}$ | $\bar{C}+\bar{D}$ | $\bar{C}+D$ |
|---|---|---|---|---|
| $A+B$ | 0 | | 0 | 0 |
| $A+\bar{B}$ | 0 | | | |
| $\bar{A}+\bar{B}$ | | | | 0 |
| $\bar{A}+B$ | 0 | 0 | | 0 |

POS:-

$$Y = (A+C+D)(A+B+\bar{C})(\bar{A}+\bar{C}+\bar{D})(\bar{A}+B+C)$$

Q. $f(A,B,C,D) = \prod M(1,3,5,8,9,12,14,15)$

| $A+B \backslash C+D$ | $C+D$ | $C+\bar{D}$ | $\bar{C}+\bar{D}$ | $\bar{C}+D$ |
|---|---|---|---|---|
| $A+B$ | | 0 | 0 | |
| $A+\bar{B}$ | | 0 | | |
| $\bar{A}+\bar{B}$ | 0 | | 0 | 0 |
| $\bar{A}+B$ | 0 | 0 | | |

N.B.- using k-map you can two different reduced boolean expressions with same l.c.

$$Y = (A+C+\bar{D})(A+B+\bar{D})(\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+D)$$

OR

$$Y = (A+C+\bar{D})(A+B+\bar{D})(\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+C+D)$$

Q. $f(A,B,C,D) = \sum m(4,5,7,12,14,15) + \sum d(3,8,10)$

↳

| $AB \backslash CD$ | $\bar{C}\bar{D}$ | $\bar{C}D$ | $CD$ | $C\bar{D}$ |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ | | | X | |
| $\bar{A}B$ | 1 | 1 | 1 | |
| $AB$ | 1 | | 1 | 1 |
| $A\bar{B}$ | X | | | X |

SOP

$$Y = B\bar{C}\bar{D} + \bar{A}BD + ABC$$

Q. $f(A,B,C,D) = \prod M(0,1,2,3,4,5) \cdot \prod d(10,11,12,13,14,15)$

↳

| $A+B \backslash C+D$ | $C+D$ | $C+\bar{D}$ | $\bar{C}+\bar{D}$ | $\bar{C}+D$ |
|---|---|---|---|---|
| $A+B$ | 0 | 0 | 0 | 0 |
| $A+\bar{B}$ | 0 | 0 | | |
| $\bar{A}+\bar{B}$ | X | X | X | X |
| $\bar{A}+B$ | | | X | X |

$$Y = (A+C)(A+B)$$

OR

$$Y = (A+C)(B+\bar{C})$$

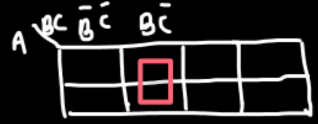
Q. You are given an $n$-variable logical expression. In K-map, you have done grouping of $2^k$ cells. What will be the literal count of that particular group?

[given:- $n > k$, $k \in$ whole number]

(a) $2^n - 2^k$ (b) 0 (c) 1 (d) $n - k$

↳ $n = 3$, $k = 1$

3 variable, 2 cells grouping



$Y = B\bar{C}$

L.C. = 2
= 3 − 1
= $n - k$

| Var. | cell group | L.C. |
|---|---|---|
| $n$ | $2^0$ | $n-0$ |
| $n$ | $2^1$ | $n-1$ |
| $n$ | $2^2$ | $n-2$ |
| ⋮ | | |
| $n$ | $2^k$ | $n-k$ |

Q. K-map for $f(A, B, C, D)$ is given. Draw K-map for $f'(A, B, C, D)$

| AB \ CD | | | | |
|---|---|---|---|---|
| | 0 | 1 | 1 | 0 |
| | 0 | 0 | 1 | 0 |
| | 0 | 1 | 1 | 0 |
| | 1 | 0 | 0 | 1 |

$f(A, B, C, D) = \Sigma m(1, 3, 7, 8, 10, 13, 15)$

$f' = \Sigma m(0, 2, 4, 5, 6, 9, 11, 12, 14)$

↳

| AB \ CD | | | | |
|---|---|---|---|---|
| | 1 | 0 | 0 | 1 |
| | 1 | 1 | 0 | 1 |
| | 1 | 0 | 0 | 1 |
| | 0 | 1 | 1 | 0 |

Q. Map the given logical expressions in K-maps and find reduced logical expression.

(i) $f(A,B,C) = \bar{A} + BC$

(ii) $f(A,B,C) = \bar{A} + AC$

(iii) $f(A,B,C) = AC + BC$

(iv) $f(A,B,C) = (A+C)(B+C)$

(v) $f(A,B,C,D) = ABC + CD + ABD$

(vi) $f(A,B,C,D) = \bar{A}(\bar{B}C + \bar{B}\bar{C} + BCD) + \bar{B}\bar{D}(A+C)$

$f_d(A,B,C,D) = \bar{A}B(\bar{C}D + C\bar{D}) + ACD$ ↝ Don't care

↳

① 

| A \ BC | $\bar{B}\bar{C}$ | $\bar{B}C$ | $BC$ | $B\bar{C}$ |
|---|---|---|---|---|
| $\bar{A}$ | 1 | 1 | 1 | 1 |
| $A$ | | | 1 | |

$\bar{A} + BC$ ↝ SOP ↝ Reduced Boolean Expression

② 

| A \ BC | $\bar{B}\bar{C}$ | $\bar{B}C$ | $BC$ | $B\bar{C}$ |
|---|---|---|---|---|
| $\bar{A}$ | 1 | 1 | 1 | 1 |
| $A$ | | 1 | 1 | |

$\bar{A} + AC$

$Y = \bar{A} + C$ ↝ R.B.E.

③ 

| A \ BC | $\bar{B}\bar{C}$ | $\bar{B}C$ | $BC$ | $B\bar{C}$ |
|---|---|---|---|---|
| $\bar{A}$ | | | 1 | |
| $A$ | | 1 | 1 | |

$AC + BC$ ↝ R.B.E.

④ 

| | $B+C$ | $B+\bar{C}$ | $\bar{B}+\bar{C}$ | $\bar{B}+C$ |
|---|---|---|---|---|
| $A$ | 1 | | | 1 |
| $\bar{A}$ | 1 | | | |

$(A+C)(B+C)$ ↝ POS ↝ R.B.E.

(v)

| AB \ CD | $\bar{C}\bar{D}$ | $\bar{C}D$ | $CD$ | $C\bar{D}$ |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ | | | 1 | |
| $\bar{A}B$ | | | 1 | |
| $AB$ | | 1 | 1 | 1 |
| $A\bar{B}$ | | | 1 | |

$ABC + CD + ABD$ → SOP → R.B.E.

(vi)

| AB \ CD | $\bar{C}\bar{D}$ | $\bar{C}D$ | $CD$ | $C\bar{D}$ |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ | 1 | 1 | 1 | 1 |
| $\bar{A}B$ | | X | | X |
| $AB$ | | | 1 | |
| $A\bar{B}$ | 1 | | X | 1 |

$\bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + ABCD + A\bar{B}\bar{D} + \bar{B}C\bar{D}$

$\bar{A}(\bar{B}C + \bar{B}\bar{C} + BCD) + \bar{B}\bar{D}(A+C)$

$\bar{A}B(\bar{C}D + C\bar{D}) + ACD$ → Don't care

$\bar{A}B\bar{C}D + \bar{A}BC\bar{D} + ACD$

$Y = \bar{A}\bar{B} + ACD + A\bar{B}\bar{D}$ → R.B.E.

Q. K-maps for two logical expression $f_1$ and $f_2$ is given.

$f_1$:

| A \ BC | | | | |
|---|---|---|---|---|
| 0 | 0 | 1 (1) | 0 | 0 |
| 1 | 0 | 1 (5) | 0 | 1 (6) |

$f_2$:

| A \ BC | | | | |
|---|---|---|---|---|
| 0 | 1 (0) | 1 (1) | 0 | 0 |
| 1 | 1 (4) | 0 | 0 | 1 (6) |

Perform the following operations -

(a) $f_1 f_2$

(b) $f_1 + f_2$

(c) $f_1 \oplus f_2$

(d) $f_1 \odot f_2$

$f_1 = \Sigma m(1, 5, 6)$

$f_2 = \Sigma m(0, 1, 4, 6)$

M-I

(a) $f_1 \cdot f_2 = \Sigma m(1, 6)$

(b) $f_1 + f_2 = \Sigma m(0, 1, 4, 5, 6)$

(c) $f_1 \oplus f_2 = \Sigma m(0, 4, 5)$ x

(d) $f_1 \odot f_2 = \Sigma m(1, 2, 3, 6, 7)$

↳ $f_1 \cdot f_2$

| | | | |
|---|---|---|---|
| 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 |

$f_1$

| A \ BC | | | | |
|---|---|---|---|---|
| | 0 | 1 (1) | 0 | 0 |
| | 0 | 1 (5) | 0 | 1 (6) |

$f_2$

| A \ BC | | | | |
|---|---|---|---|---|
| | 1 (0) | 1 (1) | 0 | 0 |
| | 1 (4) | 0 | 0 | 1 (6) |

$\Rightarrow f_1 \cdot f_2$

*** Cell to cell operation is performed.

↳

| A \ BC | | | | |
|---|---|---|---|---|
| | 1 | 0 | 0 | 0 |
| | 1 | 1 | 0 | 0 |

$f_1 \oplus f_2$

$\Sigma m(0,4,5)$

## 5-variable K-map:-

5-variable = 32 cells

$f(A, B, C, D, E) \rightarrow$ (A: MSB, E: LSB)

A = 0

| BC \ DE | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 00000 (0) | 00001 (1) | 3 | 2 |
| 01 | 4 | 5 | 7 | 6 |
| 11 | 12 | 13 | 15 | 14 |
| 10 | 8 | 9 | 11 | 10 |

9 → 01001

A = 1

| BC \ DE | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 16 | 17 | 19 | 18 |
| 01 | 20 | 21 | 23 | 22 |
| 11 | 28 | 29 | 31 | 30 |
| 10 | 24 | 25 | 27 | 26 |

25 → 11001

9 → 01001
25 → 11001
adjacent cells

Q. $f(A,B,C,D,E) = \sum m(3,4,6,11,12,14,17,20,22,$
$25,28,30)$ ⇝ SOP (min term)

↳

| $BC \backslash DE$ | $\bar{D}\bar{E}$ | $\bar{D}E$ | $DE$ | $D\bar{E}$ |
|---|---|---|---|---|
| $\bar{B}\bar{C}$ | | | 1 | |
| $\bar{B}C$ | 1 | | | 1 |
| $BC$ | 1 | | | 1 |
| $B\bar{C}$ | | | 1 | |

$A=0 \rightarrow \bar{A}$

$\bar{A}(C\bar{E} + \bar{C}DE)$

$\bar{A}C\bar{E} + \bar{A}\bar{C}DE$

| $BC \backslash DE$ | $\bar{D}\bar{E}$ | $\bar{D}E$ | $DE$ | $D\bar{E}$ |
|---|---|---|---|---|
| $\bar{B}\bar{C}$ | | 1 | | |
| $\bar{B}C$ | 1 | | | 1 |
| $BC$ | 1 | | | 1 |
| $B\bar{C}$ | | 1 | | |

$A=1 \rightsquigarrow A$

$A(C\bar{E} + \bar{C}\bar{D}E)$

$AC\bar{E} + A\bar{C}\bar{D}E$

$Y = \bar{A}C\bar{E} + \bar{A}\bar{C}DE$
$+ AC\bar{E} + A\bar{C}\bar{D}E$

$Y = C\bar{E}(A+\bar{A})$
$+ \bar{A}\bar{C}DE$
$+ A\bar{C}\bar{D}E$

$Y = C\bar{E} + \bar{A}\bar{C}DE$
$+ A\bar{C}\bar{D}E$

Q. $f(A,B,C,D,E) = \bar{A}\bar{B}\bar{D} + BCE + CD\bar{E} + AB\bar{D}$

Draw K-map.

↳

| $BC \backslash DE$ | $\bar{D}\bar{E}$ | $\bar{D}E$ | $DE$ | $D\bar{E}$ |
|---|---|---|---|---|
| $\bar{B}\bar{C}$ | 1 | 1 | | |
| $\bar{B}C$ | 1 | 1 | | 1 |
| $BC$ | | 1 | 1 | 1 |
| $B\bar{C}$ | | | | |

$A=0 \rightsquigarrow \bar{A}$

| $BC \backslash DE$ | $\bar{D}\bar{E}$ | $\bar{D}E$ | $DE$ | $D\bar{E}$ |
|---|---|---|---|---|
| $\bar{B}\bar{C}$ | | | | |
| $\bar{B}C$ | | | | 1 |
| $BC$ | 1 | 1 | 1 | 1 |
| $B\bar{C}$ | 1 | 1 | | |

$A=1 \rightsquigarrow A$

Q. $f(A,B,C,D,E) = \Pi M(3,5,6,9,10,11,13,19,21,22,23,25,26,27,29)$ → Max-term

POS

↳

**A = 0 → A**

| $B+C$ \ $D+E$ | $D+E$ | $D+\bar E$ | $\bar D+\bar E$ | $\bar D+E$ |
|---|---|---|---|---|
| $B+C$ | | | 0 | |
| $B+\bar C$ | | 0 | | 0 |
| $\bar B+\bar C$ | | 0 | | |
| $\bar B+C$ | | 0 | 0 | 0 |

**A = 1 → $\bar A$**

| | $D+E$ | $D+\bar E$ | $\bar D+\bar E$ | $\bar D+E$ |
|---|---|---|---|---|
| $B+C$ | | | 0 | |
| $B+\bar C$ | | 0 | 0 | 0 |
| $\bar B+\bar C$ | | 0 | | |
| $\bar B+C$ | | 0 | 0 | 0 |

↳

$$(\bar C+D+\bar E)(\bar B+C+\bar E)(\bar B+C+\bar D)(C+\bar D+\bar E)(\bar A+B+\bar C+\bar D)(B+\bar C+\bar D+E)$$

* K-maps of XOR Gate, XNOR Gate and their complements:-

↳ $f(A,B) = \bar A B + A\bar B$ → XOR Gate

| A \ B | $\bar B$ | $B$ |
|---|---|---|
| $\bar A$ | 0 | 1 |
| $A$ | 1 | 0 |

↳ $f(A,B) = A \odot B =$

| A \ B | | |
|---|---|---|
| | 1 | 0 |
| | 0 | 1 |

$\overline{A\oplus B} = A \odot B$

↳ $A \backslash BC$

| | | | |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |

$\rightsquigarrow A\oplus B\oplus C = A\odot B\odot C$

$\downarrow$

$A \backslash BC$

| | | | |
|---|---|---|---|
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |

$\rightsquigarrow \overline{A\oplus B\oplus C} = \overline{A\odot B\odot C} = A\oplus B\odot C$

$= A\odot B\oplus C$

$= B\odot A\oplus C = C\odot A\oplus B$

$= C\odot B\oplus A$

↳ $AB \backslash CD$

| | | | |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |

$\rightsquigarrow A\oplus B\oplus C\oplus D$

$AB \backslash CD$

| | | | |
|---|---|---|---|
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |

$\overline{A\oplus B\oplus C\oplus D}$

$= A\odot B\odot C\odot D$

Q. The K-map given below shows:-

$BC \backslash DE$

| | | | |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |

$\rightsquigarrow B\oplus C\oplus D\oplus E$

$A=0 \rightsquigarrow \bar{A}$

$BC \backslash DE$

| | | | |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |

$\rightsquigarrow B\oplus C\oplus D\oplus E$

$A=1 \rightsquigarrow A$

(a) $A\odot[B\oplus C\oplus D\oplus E]$ ✓(b) $B\oplus C\oplus D\oplus E$ (c) both (d) None

↳ $\bar{A}[B\oplus C\oplus D\oplus E] + A[B\oplus C\oplus D\oplus E]$

$(B\oplus C\oplus D\oplus E)(A+\bar{A}) = B\oplus C\oplus D\oplus E$

**Q.** The K-map given below shows:-

**MSQ**

$BC \backslash DE$

| | | | |
|---|---|---|---|
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |

$A = 1$

$BC \backslash DE$

| | | | |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |

$A = 0$

(a) $A \oplus B \oplus C \oplus D \oplus E$ (b) $A \odot B \odot C \odot D \odot E$ (c) $A \oplus B \odot C \odot D \odot E$

(D) None

↳ $A\left[\overline{B \oplus C \oplus D \oplus E}\right] + \bar{A}\left[B \oplus C \oplus D \oplus E\right]$

$x\bar{y} + \bar{x}y = x \oplus y = A \oplus B \oplus C \oplus D \oplus E = A \odot B \odot C \odot D \odot E$

---

**Q.** The K-map given below shows:-

**MSQ**

$BC \backslash DE$

| | | | |
|---|---|---|---|
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |

$A = 0$

$BC \backslash DE$

| | | | |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |

$A = 1$

(a) $A \odot B \oplus C \oplus D \oplus E$ (b) $A \oplus B \odot C \odot D \odot E$ (c) $A \oplus B \odot C \oplus D \oplus E$

(d) $A \odot B \oplus C \odot D \odot E$

↳ $\bar{A}\left[\overline{B \oplus C \oplus D \oplus E}\right] + A\left[B \oplus C \oplus D \oplus E\right]$

$\bar{x}\bar{y} + xy = x \odot y = A \odot B \oplus C \oplus D \oplus E = \overline{A \oplus B \oplus C \oplus D \oplus E} = A \odot B \odot C \odot D \oplus E$

$= A \odot B \oplus C \oplus D \oplus E$

↳ $\overline{A\oplus B} = \bar{A}\oplus B = A\odot B$

↳ $A\oplus B\oplus C = A\odot B\odot C$

↳ $\overline{A\oplus B\oplus C} = A\odot B\oplus C = A\oplus B\odot C = A\oplus C\odot B = B\odot A\oplus C$

↳ $\overline{A\oplus B\oplus C\oplus D} = \bar{A}\oplus(B\oplus C\oplus D) = A\odot(B\oplus C\oplus D) = A\odot B\odot C\odot D$

$= \overline{A\oplus B}\oplus(C\oplus D) = A\oplus B\odot C\oplus D$

$= \overline{A\oplus B\oplus C}\oplus D = A\odot B\oplus C\oplus D$

↳ $A\oplus B\oplus C\oplus D\oplus E = A\odot B\odot C\odot D\odot E$

↳ $\overline{A\oplus B\oplus C\oplus D\oplus E} = A\odot B\odot C\odot D\oplus E = A\oplus B\oplus C\oplus D\odot E$

$= A\odot B\oplus C\oplus D\oplus E$

$= A\oplus B\odot C\odot D\odot E$

Q. $Y = A\odot B\odot C\odot D\oplus E\oplus F\oplus G$ will be equivalent to

✓ (a) $\overline{A\oplus B\oplus C\oplus D\oplus E\oplus F\oplus G}$

✓ (b) $A\odot B\odot C\odot D\odot E\odot F\oplus G$

✓ (c) $A\odot B\oplus C\oplus D\oplus E\oplus F\oplus G$

(D) $A\odot B\odot C\odot D\odot E\odot F\odot G$

↳

✓ (a) $\overline{\overline{\overline{A\oplus B\oplus C\oplus D\oplus E\oplus F\oplus G}}} = \overline{A\oplus B\oplus C\oplus D\oplus E\oplus F\oplus G}$ ✓

✓ (b) $\overline{\overline{\overline{\overline{\overline{A\oplus B\oplus C\oplus D\oplus E\oplus F\oplus G}}}}} = \overline{A\oplus B\oplus C\oplus D\oplus E\oplus F\oplus G}$ ✓

✓ (c) $\overline{A\oplus B\oplus C\oplus D\oplus E\oplus F\oplus G}$ ✓

(D) $A\oplus B\oplus C\oplus D\oplus E\oplus F\oplus G$

## Number System

- **Decimal** ☆ → Base 10
  - [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
  - Eg. $[93]_{10}$ ✓
  - Eg. $[127]_{10}$ ✓
  - Eg. $[A32]_{10}$ ✗
- **Binary** ☆☆ → Base 2
  - [0, 1]
  - Eg $[010]_2$ ✓
  - Eg. $[100]_2$ ✓
  - Eg. $[210]_2$ ✗
  - Eg $[720]_2$ ✗
- **Octal** → Base 8
  - [0, 1, 2, 3, 4, 5, 6, 7]
  - Eg → $[723]_8$ ✓
  - Eg → $[982]_8$ ✗
  - Eg → $[777]_8$ ✓
  - Eg → $[72.32]_8$ ✓
- **Hexadecimal** → Base 16
  - [0, 1, 2 --- , 9, A, B, C, D, E, F]
  - Eg → $[923]_{16}$ ✓
  - $[DEF]_{16}$ ✓
  - $[D7F]_{16}$ ✓
  - $[H9D]_{16}$ ✗
- **Any Number with base r.**
  - [0, 1, 2 --- r-1]
  - Eg → $[\alpha\beta\gamma]_r$ ✓
  - $r > \alpha$
  - $r > \beta$
  - $r > \gamma$

## ① Decimal Number System :-

↳ Base 10

↳ [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]

Eg → $[729.32]_{10}$ ✓

$[8D2.F3]_{10}$ ✗

$[794.3B]_{10}$ ✗

Eg. → ① $[2743]_{10} = 3\times10^0 + 4\times10^1 + 7\times10^2 + 2\times10^3 = 2000 + 700 + 40 + 3 = [2743]_{10}$

$10^1 \quad 10^0 \quad 10^{-1}$

② $[43.5]_{10} = 3\times10^0 + 4\times10^1 + 5\times10^{-1} = 3 + 40 + 0.5 = 43.5$ ✓

③ $[973256.5374] = 9\times10^5 + 7\times10^4 + 3\times10^3 + 2\times10^2 + 5\times10^1 + 6\times10^0$
$+ 5\times10^{-1} + 3\times10^{-2} + 7\times10^{-3} + 4\times10^{-4}$

## ② Binary Number System:-

↳ Base 2

↳ [0, 1]

Eg. → $(110.0101)_2$ ✓

$(110.201)_2$ ✗

$(310.21101)_2$ ✗

Eg. → ① $[\overset{2^3}{1}\overset{2^2}{1}\overset{2^1}{0}\overset{2^0}{1}]_2 = 1\times2^0 + 0\times2^1 + 1\times2^2 + 1\times2^3 = 1+0+4+8 = (13)_{10}$

② $[101.\underset{2^{-1}}{1}\underset{2^{-2}}{1}] = 1\times2^0 + 0\times2^1 + 1\times2^2 + 1\times2^{-1} + 1\times2^{-2} = (5.75)_{10}$

③ $[1101011.101] = 1+2+8+32+64+\frac{1}{2}+\frac{1}{8} = (107.625)_{10}$

## ③ Octal Number System:-

↳ Base 8

↳ [0, 1, 2, 3, 4, 5, 6, 7]

Eg. → $[724]_8$ ✓

$[872.78]_8$ ✗

$[971.33]_8$ ✗

Eg. → ① $[6472]_8 = 2\times8^0 + 7\times8^1 + 4\times8^2 + 6\times8^3 = [3386]_{10}$

② $[1011]_8 = 1\times8^0 + 1\times8^1 + 0\times8^2 + 1\times8^3 = [521]_{10}$

③ $[\underset{8^4}{6}\underset{8^3}{7}\underset{8^2}{3}\underset{8^1}{1}\underset{8^0}{6}.\underset{8^{-1}}{3}\underset{8^{-2}}{4}\underset{8^{-3}}{9}]_8 =$

## ④ Hexadecimal Number System:-

$[923\cdot FE]_{16}$ ✓

$[92G\cdot HE]_{16}$ ×

$[KLP\cdot 67]_{16}$ ×

↳ Base 16

↳ [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F]

Eg.→ $[953]_{16} = 3\times16^0 + 5\times16^1 + 9\times16^2 = [2387]_{10}$

$[C549DF]_{16} = 15\times16^0 + 13\times16^1 + 9\times16^2 + 4\times16^3 + 5\times16^4 + 12\times16^5 = [\quad]_{10}$

$[7429F\cdot DB7]_{16} =$

$16^4\ 16^3\ 16^2\ 16^1\ 16^0\quad 16^{-1}\ 16^{-2}\ 16^{-3}$

# Conversions:-

① Any Base to decimal

Eg.→ (i) $[\overset{5^2}{3}\overset{5^1}{4}\overset{5^0}{2}\cdot\overset{5^{-1}}{1}\overset{5^{-2}}{0}]_5 = (3\times25)+(4\times5)+(2\times1)+(1\times5^{-1})+(0\times5^{-2}) = [97\cdot2]_{10}$

(ii) $[A27F\cdot C]_{16} = (10\times16^3)+(2\times16^2)+(7\times16^1)+(15\times16^0)+(12\times16^{-1})$
$= [41599\cdot75]_{10}$

(iii) $[8762]_9 = (8\times9^3)+(7\times9^2)+(6\times9^1)+(2\times9^0)$
$= [6455]_{10}$

② Decimal to any base

Eg.→ $[732\cdot625]_{10} = [\ ?\ ]_2 = [\ ?\ ]_{16} = [\ ?\ ]_8 = [\ ?\ ]_r$

## Understand the Method by Example:-

Eg.→ $(73.25)_{10} = (\ ?\ )_2$

$\underbrace{73}$ → Integer Part

| | | |
|---|---|---|
| 2 | 73 | Rem |
| 2 | 36 | 1 |
| 2 | 18 | 0 |
| 2 | 9 | 0 |
| 2 | 4 | 1 |
| 2 | 2 | 0 |
| | 1 | 0 |

$1 < 2$

$(1001001)_2 = (73)_{10}$

$.25 \times 2 = 0.50$

$.50 \times 2 = 1.00$

$.00 \times 2 = 0.00$

$.00 \times 2 = 0.00$

$(.01)_2 = (.25)_{10}$

$(73.25)_{10} = (1001001.01)_2$

Eg.→ $(739.26)_{10} = (\quad)_8$

| | | |
|---|---|---|
| 8 | 739 | |
| 8 | 92 | 1 |
| 8 | 11 | 4 |
| | 1 | 3 |

$(1341.2050)_8$

$.26 \times 8 = 2.08$

$.08 \times 8 = 0.64$

$.64 \times 8 = 5.12$

$.12 \times 8 = 0.96$

$\vdots$

Eg. $(7932.67)_{10} = (\quad\quad)_{16}$

↳

| | | |
|---|---|---|
| 16 | 7932 | |
| 16 | 495 | 12 → C |
| 16 | 30 | 15 → F |
| | 1 | 14 → E |

$.67 \times 16 = 10.72 = A.72$

$.72 \times 16 = 11.52 = B.52$

$.52 \times 16 = 8.32$

$\vdots$

$\therefore (1EFC.AB8)$

Eg. $(8914.73)_{10} = (\quad\quad)_{6}$

| | | |
|---|---|---|
| 6 | 8914 | |
| 6 | 1485 | 4 |
| 6 | 247 | 3 |
| 6 | 41 | 1 |
| 6 | 6 | 5 |
| | 1 | 0 |

$.73 \times 6 = 4.38$

$.38 \times 6 = 2.28$

$.28 \times 6 = 1.68$

$\vdots$

$(105134.421)_6$

$= 4 + 18 + 36 + 1080 + 7776 + 4\times6^{-1} + 2\times6^{-2} + 1\times6^{-3}$

$= (8914.726)_{10}$

$\approx (8914.73)_{10}$

## ★ Conversion from any base to another base :-

Eg. → $[341.12]_5 = [\ ?\ ]_6$

Base 10

$= 1 + (4\times5) + (3\times25) + (1\times5^{-1}) + (2\times5^{-2})$

$= [96.28]_{10} = [\ ?\ ]_6$

$$\begin{array}{r|ll} 6 & 96 & \\ \hline 6 & 16 & 0 \\ \hline & 2 & 4 \end{array}$$

$.28\times6 = 1.68$

$.68\times6 = 4.08$

$.08\times6 = 0.48$

$\vdots$

$[341.12]_5$

$\shortparallel$

$[240.140]_6$ Ans.

## ★ Special Case :-

↳ **Binary to $2^n$ Number System :-** $[n \in N]$

Eg. → $(10110110010110.110110)_2 = (\ ?\ )_8 = (\ ?\ )_{16} = (\ ?\ )_4$

$2^3 \qquad 2^4 \qquad 2^2$

↳ (a) $(\qquad)_2 = (\qquad)_{2^3}$ → 3-bits clubbed

$(26626.66)_8$

(b) $(\quad)_2 = (\qquad)_{2^4}$ → 4-bits clubbed

$= (2096.D8)_{16}$

(c) $(\quad)_2 = (\qquad)_{2^2}$

$= (2312112.312)_4$

421
111

## ↳ $2^n$ number System to binary :-

Eg. $(742 \cdot 32)_8 = (\ ?\ )_2$

↳ $(742 \cdot 32)_8 = (742 \cdot 32)_{2^3}$ → Each number should be denoted in 3-bits

$(\underleftrightarrow{111}\ \underleftrightarrow{100}\ \underleftrightarrow{010} \cdot \underleftrightarrow{011}\ \underleftrightarrow{010})_2$

$= (111100010 \cdot 01101)_2$ ✓

Eg. → $(7EB2 \cdot A4)_{16} = (\ ?\ )_2$

↳ $16 = 2^4$ → Each no. in 4 bits

$(\underleftrightarrow{0111}\ \underleftrightarrow{1110}\ \underleftrightarrow{1011}\ \underleftrightarrow{0010} \cdot \underleftrightarrow{1010}\ \underleftrightarrow{0100})_2$

$= (111\ 1110\ 1011\ 0010 \cdot 101001)_2$

Eg. $(3021 \cdot 32)_4 = (\ ?\ )_2$

$4 = 2^2$ → Each no. in 2 bits

$(11\ 00\ 10\ 01 \cdot 11\ 10)_2$

## ★ Conversion from $2^{n_1}$ to $2^{n_2}$ :-

Eg. → $(734.26)_8 = (\,?\,)_4$

↳ Binary ↑

$(\ \ )_8 = (0111011100.010110)_2 = (\,?\,)_4$

$4 = 2^{2}$

↳ $(13130.112)_4$ ANS

## ★ Summary :-

① $[abcdef.ghi]_r = [\,?\,]_{10}$

$\Rightarrow f\times r^0 + e\times r^1 + d\times r^2 + c\times r^3 + b\times r^4 + a\times r^5 + g\times r^{-1} + h\times r^{-2} + i\times r^{-3}$

② $[746.32]_{10} = [\,?\,]_r$

↳

| r | 746 | R |
|---|---|---|
| r | p | q |
| r | m | n |
| | k | l |

$k < r$

$.32 \times r = I_1 . \alpha$

$.\alpha \times r = I_2 . \beta$

$.\beta \times r = I_3 . \gamma$

⋮

$[klnq.\alpha\beta\gamma]$

③ $[\underbrace{101}_{n\text{-bits}}\underbrace{101}_{n\text{-bits}}\underbrace{110}_{n\text{-bits}} \cdot \underbrace{101}_{n\text{-bits}}\underbrace{100}_{n\text{-bits}}\underbrace{100}_{n\text{-bits}}]_2 = [\,?\,]_{2^n}$

↳ $[556 \cdot 544]_8$

④ $[613 \cdot 54]_{2^n} = [\,?\,]_2$

$\Rightarrow [\underbrace{00110}_{n\text{-bits}}\ \underbrace{00001}_{n\text{-bits}}\ \underbrace{00011}_{n\text{-bits}} \cdot \underbrace{00101}_{n\text{-bits}}\ \underbrace{00100}_{n\text{-bits}}]_2$ for base 32

Q. For a Particular Number system

$$24 + 17 = 40$$

Find the base of the system.

↳ $(24)_r + (17)_r = (40)_r$

$\Rightarrow 4 \times r^0 + 2 \times r^1 + 7 \times r^0 + 1 \times r^1 = 0 \times r^0 + 4 \times r^1$

$\Rightarrow 4 + 2r + 7 + r = 0 + 4r \quad \Rightarrow \boxed{r = 11}$

Q. $\sqrt{41} = 5$. Find the base.

↳ $41 = 25$ ✗

First target → convert into decimal ⇒ Then perform calculation.

$\sqrt{1 + 4r} = 5 \times r^0 \quad \Rightarrow \sqrt{1 + 4r} = 5 \quad \Rightarrow 1 + 4r = 25 \quad \Rightarrow r = 6$ ANS.

Q. In a particular Number System; one of the roots of $x^2 - 11x + 22 = 0$ is 3. Find the base.

↳

root $= (3)_r$

Multiplication of roots $= (22)_r$

Addition of roots $= (11)_r$

Let the other root be $(\alpha)_r$ $\quad \{(\alpha)_r = (x)_{10}\}$

↳ $(3)_r (\alpha)_r = (22)_r$

$3x = 2 + 2r$ — ①

↳ $(3)_r + (\alpha)_r = (11)_r$

$3 + x = 1 + r$ — ②

$x = r - 2$

$\frac{2+2r}{3} = r - 2$

$2 + 2r = 3r - 6$

Ans- $\boxed{r = 8}$

$ax^2 + bx + c = 0$

$\alpha + \beta = -b/a$

$\alpha\beta = c/a$

$\alpha$ and $\beta \rightarrow$ roots

$(\alpha)_8 = (6)_{10}$

$\boxed{\alpha = 6}$ → Base 8

Other root:-

$3 + x = 1 + r$

$3 + x = 1 + 8$

$\boxed{x = 6}$

Decimal

Q. $(43)_x = (y3)_8$

Find all possible combinations of $x$ and $y$?

↳

$3x^0 + 4x^1 = 3 + 8y$

$3 + 4x = 3 + 8y$

$\boxed{y = x/2}$

$(43)_x$ $\quad x > 4 \Rightarrow x = [5, 6 \ldots]$ ∈ N

$(y3)_8$ $\quad y < 8 \Rightarrow y = [7, 6, 5 \ldots 0]$ ∈ N

$x = 5 \Rightarrow y = 5/2$ ✗ ; $x = 6 \Rightarrow y = 3$ ✓ ; $x = 7 \Rightarrow y = 7/2$ ✗ ; $x = 8 \Rightarrow y = 4$ ✓

$x = 9 \Rightarrow y = 9/2$ ✗ ; $x = 10 \Rightarrow y = 5$ ✓ , $x = 11 \Rightarrow y = 11/2$ ✗ ; $x = 16 \Rightarrow y = 8$ ✗

$\boxed{(x, y) = (6,3)\ (8,4)\ (10,5)\ (12,6)\ (14,7)}$

Q. Find number of 1's present in binary presentation of

$(3 \times 512) + (7 \times 64) + 3 = (?)_{10} = (?)$

↳

$N = (3 \times 512) + (7 \times 64) + 3$

$= (3 \times 8^3) + (7 \times 8^2) + (0 \times 8^1) + (3 \times 8^0)$

$= (3703)_8 = (\,?\,)_2$

$8 = 2^3$

$= (011\,111\,000\,011)_2$

$\Rightarrow$ 7 1's are there.

Q. $M \times 32768 + J \times 1024 + 3 = (\,?\,)_{32}$

↳ $M \times (32)^3 + J \times (32)^2 + (0 \times 32^1) + (3 \times 32^0)$

$= (MJ03)_{32}$

Q. $\frac{41}{3} = 13$ in some number system. Find the base.

↳ $\frac{(41)_r}{(3)_r} = (13)_r$

$\frac{4r+1}{3} = 3 + r$

$\Rightarrow 4r + 1 = 9 + 3r$

$\boxed{r = 8}$

MSQ

Q. $\frac{55}{5} = 11$ ; possible base is

~~(a) 10~~ ~~(b) 3~~ ~~(c) 5~~ (d) 6 ✓ (e) 7 ✓ (f) 298 ✓

↳

$$\frac{(55)_r}{(5)_r} = (11)_r \qquad \boxed{r > 5}$$

$$\frac{5r+5}{5} = r+1$$

$\boxed{5r+5 = 5r+5}$ → Always true

For any base $r > 5 \Rightarrow \frac{55}{5} = 11$

## Arithmatic Operations:-

### ① Decimal Addition:-

(a)
$$\begin{array}{r} [23]_{10} \\ +[72]_{10} \\ \hline 95 \end{array}$$

(b)
$$\begin{array}{r} \overset{1}{7}4 \\ +88 \\ \hline 162 \end{array}$$

$4+8 = 12-10 = 2$

$1+7+8 = 16-10 = 6$

### ② Decimal Subtration:-

(a)
$$\begin{array}{r} 52 \\ -21 \\ \hline 31 \end{array}$$

(b) $-1$, $+10$ → Base
$$\begin{array}{r} \overset{6}{7}\,4 \\ -36 \\ \hline 38 \end{array}$$

$$\begin{array}{r} 4 \\ -6 \\ \hline ? \end{array}$$

$10+4 = 14$
$$\begin{array}{r} 14 \\ -6 \\ \hline 8 \end{array}$$

Arithmatic Calculations

Convert to decimal

Do the Calculation

Revert back

Standard Calculation

N.B.:- Standard Calculations in binary number s/s is important.

For other bases, we can do the calculation by converting it to decimal.

Eg. → ① $(227)_8 + (562)_8 = (\ ?\ )_8$

M-I :- Converting into decimal:-

$(227)_8 = 7 + 2\times8 + 2\times64 = (151)_{10}$

$(562)_8 = 2 + 6\times8 + 5\times64 = +(370)_{10}$

$(521)_{10} = (?)_8$

| | | |
|---|---|---|
| 8 | 521 | |
| 8 | 65 | 1 |
| 8 | 8 | 1 |
| | 1 | 0 |

$\Rightarrow (1011)_8$ Ans.

$$
\begin{array}{r}
{\scriptstyle 1\,1} \\
227 \\
{\scriptstyle 1}\;+562 \\
\hline
1011 \\
\hline
\end{array}
$$

$7+2=9-8=1$

$6+2+1=9-8=1$

$1+2+5=8-8=0$

② $(1.2)_4 + (2.3)_4 = (?)_2$

M-I: $(1.2)_4 = 1 + 2\times 4^{-1} = (1.5)_{10}$

$(2.3)_4 = 2 + 3\times 4^{-1} = (2.75)_{10}$

$$
\begin{array}{r}
+\;(1.5)_{10} \\
(2.75)_{10} \\
\hline
(4.25)_{10}
\end{array}
$$

$$
\begin{array}{r|l}
2 & 4 \\
\hline
2 & 2 \quad 0 \\
\hline
 & 1 \quad 0
\end{array}
$$

$.25\times 2 = 0.50$

$.50\times 2 = 1.00$

$.00\times 2 = 0.00$

$(100.01)_2 \Rightarrow$ Ans.

M-II

$$
\begin{array}{r}
{\scriptstyle 1}\;\;\;\; \\
1.2 \\
{\scriptstyle 1}+2.3 \\
\hline
(10.1)_4 \\
\hline
\end{array}
$$

$2+3=5-4=1$

$2+1+1=4-4=0$

$(10.1)_{2^2} = (?)_2$

$(0100.01)_2$

$= (100.01)_2$

## ⇒ Arithmatic operations in binary Number System:-

### ① Binary Addition:-

(a)
$$\begin{array}{r} 1011 \\ +\,0100 \\ \hline 1111 \end{array}$$

(b)
$$\begin{array}{r} {\scriptstyle 1\,1}\phantom{0000} \\ 111011 \\ +\,111000 \\ \hline 1110011 \end{array}$$

$1+1 = 2-2 = 0$

$1+1+1 = 3-2 = 1$

$1+1+1 = 3-2 = 1$

(c)
$$\begin{array}{r} {\scriptstyle 1\,1\,1}\phantom{0} \\ 0011 \\ +\,0101 \\ \hline 1000 \end{array}$$

(d)
$$\begin{array}{r} {\scriptstyle 1\,1\,1}\phantom{00} \\ 1111 \\ +\,1111 \\ \hline 11110 \end{array}$$

$$\left\{ \begin{array}{l} 1+1 = (2)_{10} = (10)_2 \\ 1+1+1 = (3)_{10} \\ \qquad = (11)_2 \end{array} \right\}$$

### ② Binary Subtraction:-

Eg.-

(i)
$$\begin{array}{r} 1011 \\ -\,1001 \\ \hline 0010 \end{array}$$

(ii)
$$\begin{array}{r} {\scriptstyle 0\;+2}\phantom{00} \\ 11010 \\ -\,10110 \\ \hline 00100 \end{array}$$

(iii)
$$\begin{array}{r} {\scriptstyle +2\;+2}\phantom{00000} \\ 1101010 \\ 0110000 \\ \hline 0111010 \end{array}$$

## ③ Binary Multiplication :-

Eg. ① 

```
        1 0 1 0
      × 1 0 1
    -----------
      1 0 1 0
    0 0 0 0 ×
  1 0 1 0 × ×
  -------------
  1 1 0 0 1 0
  -------------
```

② 

```
          1 1 1 1
        × 1 1 1 1
    ---------------
  2=10 1
          1 1 1 1
  3=11  1 1 1 1 ×
  3=11 1 1 1 1 × ×
  2=10 1 1 1 1 × × ×
  -----------------
  1 1 1 0 0 0 0 1
  -----------------
```

2 ⇒ 10

1+1+1+1 ⇒ 4 ⇒ 100

10 = 2

2+1+1+1+1 ⇒ 6 ⇒ 110

3+1+1 ⇒ 5 ⇒ 101

2+1 ⇒ 3 ⇒ 11

## ④ Binary division :-

* decimal division

$\frac{4824}{6} = ?$

```
6 ) 4824 ( 804
    48↓
    ----
    0024
      24
    ----
    0000
    ----
```

Eg. - ① $\frac{(11011)_2}{(11)_2} = ?$

```
11 ) 11011 ( 1001
     11↓↓↓
     -----
     00011
        11
     -----
     00000
     -----
```

Q. $[723]_8 - [564]_8 = [\ ?\ ]_8$ → convert decimal ↓ Revert back $\underbrace{\qquad\qquad}$ ON YOUR OWN

↳
$$\begin{array}{r} 7\ 2\ 3 \\ -\ 5\ 6\ 4 \\ \hline 1\ 3\ 7 \\ \hline \end{array}$$
(+8 +8)

Q. $[ADD]_{16} + [DAD]_{16} = [\ ?\ ]_{16} = [?]_2$

$$\begin{array}{r} {}^{1}\ {}^{1}\ \\ A\ D\ D \\ +\ D\ A\ D \\ \hline (1\ 8\ 8\ A)_{16} \\ \hline \end{array}$$

$[0001\ 1000\ 1000\ 1010]$

D → 13
A → 10
B → 11
C → 12
E → 14
F → 15

$26 - 16 = 10$
$24 - 16 = 8$
$24 - 16 = 8$

Q. $(974B)_{16} - (587C)_{16} = (\ ?\ )_{16}$

↳
$$\begin{array}{r} 9\ 7\ 4\ B \\ -\ 5\ 8\ 7\ C \\ \hline 3\ E\ C\ F \\ \hline \end{array}$$
(+16 +16 +16)

$11 + 16 = 27 - 12 = 15 \Rightarrow F$
$16 + (4-1) = 16 + 3 = 19 - 7 = 12 = C$
$6 + 16 = 22 - 8 = 14 = E$

Q. $(10w1z)_2 \times (15)_{10} = (Y0101100 1)_2$ ; $w, Y, z = ?$

↳ M-I

$(z + 2 + 4w + 16) \times 15 = 256Y + 64 + 16 + 8 + 1$

$\boxed{15z + 270 + 60w = 256Y + 89}$

↳ Hit and Trial $\boxed{w = Y = z = 1}$

★ $\boxed{w, z, Y < 2}$

$w \in (0, 1)$
$z \in (0, 1)$
$y \in (0, 1)$

M-II

1 0 ω 1 Z
× 1 1 1 1 1 → $(15)_{10}$

1 0 ω 1 Z
1 0 ω 1 Z ×
1 0 ω 1 Z × ×
1 0 ω 1 Z × ×

Z

$(Y01011001)_2$

$\boxed{Z=1}$

2 2 1
1 0 ω 1 1
2 1 0 ω 1 1
2 1 0 ω 1 1
Y=1 1 1 0 ω 1 1
1 0 ω 1 1

$\boxed{1}$ 0 1 0 1 1 0 0 1

1+ω+1+1

if ω=0
3=11

if $\boxed{ω=1}$
4=100

5=101

$\boxed{\text{Codes}}$

↓

Group of Symbols used to represent number, letters or words.

Eg. → FISH = # * • α
GATE = 2 7 4 5

In digital design, we have $\boxed{\text{binary}}$ ** , non-binary and Alphanumeric codes.

## * Binary Codes:-

① **BCD Codes:-** [Binary coded Decimal]

$\boxed{\text{BCD Codes}}$

- Weighted Code
  [specific positional weight of each bit]
  - ** Positively weighted BCD [8 4 2 1]
  - Negatively weighted BCD [8 4 −2 −1]
- Non-weighted codes
  ↓
  Excess-3 BCD

* In BCD codes, $\boxed{\text{each decimal digit}}$ is represented by 4-bit binary code.

** If direct "BCD code" is written consider it to be $\boxed{\text{8421 code.}}$

Eg. → ① $(7)_{10} = (\ ?\ )_{BCD}$

$= (0111)_{BCD}$ [8 4 2 1]

② $(15)_{10} = \cancel{(1111)_{BCD}} = (0001\ 0101)_{BCD}$ ✓

③ $(319)_{10} = (0011\ 0001\ 1001)_{BCD}$

## Excess-3 Code :-
(XS-3)

8 4 2 1

It's a 4-bit code which can be derived from BCD code $\boxed{\text{by adding "3" to each coded Number.}}$

Eg. → (i) $(0)_{10} = (\ ?\ )_{XS-3}$

$0 + 3 = 3$

$(0011)_{XS-3}$

(ii) $(27)_{10} = (\ ?\ )_{XS-3}$

5 10 $= (0101\ 1010)$

(iii) $(532)_{10} = (\ ?\ )_{XS-3}$

+3 +3 +3

$(8\ 6\ 5) = (1000\ 0110\ 0101)_{XS-3}$

(iv) $(99)_{10} = (1100\ 1100)_{XS-3}$

12 12

---

★ Binary v/s BCD

↳ $(10)_{10} = (1010)_2 = (0001\ 0000)_{BCD}$

Binary

★ All decimal digits in different BCD codes :-

| D | 8 4 2 1 | XS-3 | 8 4 -2 -1 |
|---|---|---|---|
| 0 | 0 0 0 0 | 0 0 1 1 | 0 0 0 0 |
| 1 | 0 0 0 1 | 0 1 0 0 | 0 1 1 1 |
| 2 | 0 0 1 0 | 0 1 0 1 | 0 1 1 0 |
| 3 | 0 0 1 1 | 0 1 1 0 | 0 1 0 1 |
| 4 | 0 1 0 0 | 0 1 1 1 | 0 1 0 0 |
| 5 | 0 1 0 1 | 1 0 0 0 | 1 0 1 1 |
| 6 | 0 1 1 0 | 1 0 0 1 | 1 0 1 0 |
| 7 | 0 1 1 1 | 1 0 1 0 | 1 0 0 1 |
| 8 | 1 0 0 0 | 1 0 1 1 | 1 0 0 0 |
| 9 | 1 0 0 1 | 1 1 0 0 | 1 1 1 1 |

Eg:-

① $(697)_{10}$ 		8421 		8 4 -2 -1

8 4 2 1 → $(0110\ 1001\ 0111)_{8421}$

XS-3 → $(1001\ 1100\ 1010)_{XS-3}$

8 4 -2 -1 → $(1010\ 1111\ 1001)_{84-2-1}$

② The state of 12-bit register is $\boxed{100010010111}$.
what is its content if it represents

(a) Three decimal digits in BCD. [8421]

(b) Three decimal digits in XS-3 code.

↳ (a) $(897)_{10}$

(b) 897 
 -3 -3 -3 		$(564)_{10}$

---

*

| D | 8 4 2 1 | XS - 3 | 8 4 -2 -1 |
|---|---|---|---|
| 0 | 0 0 0 0 | 0 0 1 1 | 0 0 0 0 |
| 1 | 0 0 0 1 | 0 1 0 0 | 0 1 1 1 |
| 2 | 0 0 1 0 | 0 1 0 1 | 0 1 1 0 |
| 3 | 0 0 1 1 | 0 1 1 0 | 0 1 0 1 |
| 4 | 0 1 0 0 | 0 1 1 1 | 0 1 0 0 |
| 5 | 0 1 0 1 | 1 0 0 0 | 1 0 1 1 |
| 6 | 0 1 1 0 | 1 0 0 1 | 1 0 1 0 |
| 7 | 0 1 1 1 | 1 0 1 0 | 1 0 0 1 |
| 8 | 1 0 0 0 | 1 0 1 1 | 1 0 0 0 |
| 9 | 1 0 0 1 | 1 1 0 0 | 1 1 1 1 |

0 0 1 1
↓ ↓ ↓ ↓
1 1 0 0

All bits Individually complemented

weighted
↓
8421, 84-2-1

Non-weighted
↓
XS-3, Gray code

XS-3 and 84-2-1 codes are self $\boxed{\text{complementry codes.}}$

8421 codes are $\boxed{\text{not self}}$ complementry codes.

* **Gray Codes:-**

↳ Unweighted codes

↳ Unit distance code [K-maps]

* **Binary to Grey conversion:-**

B ⟶ G

$(7)_{10} = (?)_{Gray}$

$(7)_{10} = (111)_2$

1 1 1 → Binary

↓ ↓ ↓

(1 0 0) → Gray

B → G

BG ⇒ Busy

$(111)_2$ → Binary

(100) → Gray

* Convert $(010010)_2$ to Gray code.

↳

0 1 0 0 1 0

↓ ↓ ↓ ↓ ↓ ↓

0 1 1 0 1 1 → Gray

B → G

$\boxed{\text{MSB comes as it is}}$

* **Gray to binary conversion:-**

G → B (GB) Ganda Baccha

Eg. → ① Convert $(10110)_{gray}$ to binary.

1 0 1 1 0

1 1 0 1 1 ⇒ $(11011)_2$

② $(10111)_{gray} = (?)_2$

1 0 1 1 1

1 1 0 1 0

⇒ $(11010)_2$

| D | Binary $b_3$ | $b_2$ | $b_1$ | $b_0$ | Gray $G_3$ | $G_2$ | $G_1$ | $G_0$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 3 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 4 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 5 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 6 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 7 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| 8 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 10 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 11 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 13 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 15 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

$(1101100101)_2$

$\downarrow$

$\underline{(1011010111)_{Gray}}$

Q. What all are unused cases of 4-bits in BCD and XS-3 codes?

↳

| D | 8 4 2 1 | XS-3 | |
|---|---|---|---|
| 0 | 0000 | 0011 | |
| 1 | 0001 | 0100 | |
| 2 | 0010 | 0101 | |
| 3 | 0011 | 0110 | |
| 4 | 0100 | 0111 | |
| 5 | 0101 | 1000 | |
| 6 | 0110 | 1001 | |
| 7 | 0111 | 1010 | |
| 8 | 1000 | 1011 | |
| 9 | 1001 | 1100 | |

$(10)_{10} \rightarrow (1010)_{BCD}$ ✗

$(0001\ 0000)_{BCD}$ ✓

| unused in BCD | unused in XS-3 |
|---|---|
| 1010 | 0000 |
| 1011 | 0001 |
| 1100 | 0010 |
| 1101 | 1101 |
| 1110 | 1110 |
| 1111 | 1111 |

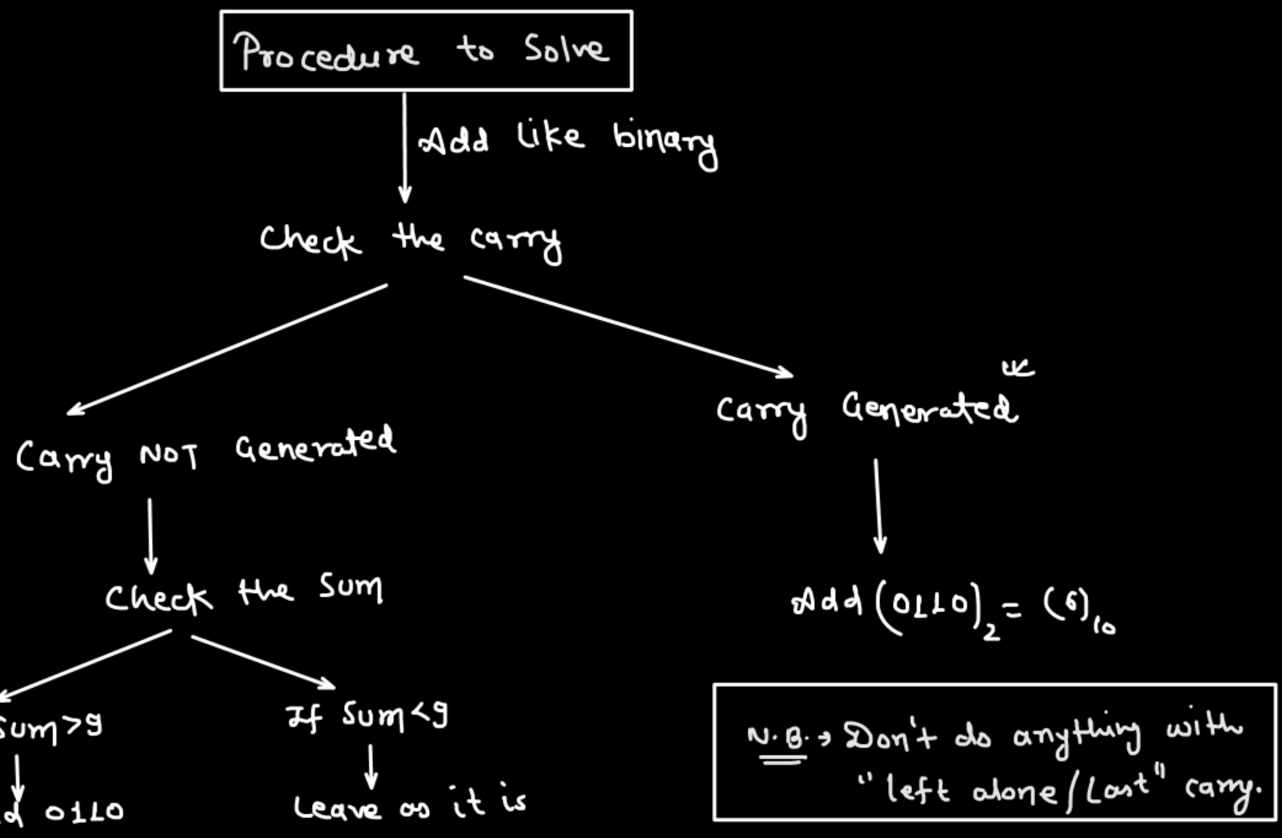
9 1001 1100 1111

## BCD Addition:-

**Rules:-**

1. Add like binary.
2. If sum $\leq 9$ → Valid case
3. If sum $\geq 9$ or carry is generated → Invalid case
4. Valid case → leave as it is

   Invalid case → Add 6 (0110)
5. If carry alone is left, don't do anything with carry.



Eg. →

① $(2)_{10}$ $\xrightarrow{BCD}$ 0010
$+(3)_{10}$     0011
$(5)_{10}$     □ 0101

BCD of 5 = 0101

② $(21)_{10}$ $\xrightarrow{BCD}$ 0010 0001
$+(22)_{10}$     0010 0010
43     □ 0100 0011

③ $(521)_{10}$ → 0101 0010 0001
$+(343)_{10}$     0011 0100 0011
864     1000 0110 0100
8 6 4

④ 521.41 → 0101 0010 0001 . 0100 0001
+343.32     0011 0100 0011 . 0011 0010
864.73     1000 0110 0100 . 0111 0011

★ Carry Involved :-

① 9 → 1001
+8     + 1000
17     $\boxed{1}$ 0001 ↷ 11X
0110
0001 0111
1 7

② 92 → 1001 0010
+81     + 1000 0001
173     1 0001 0011
0110
0001 0111 0011

③ $(99)_{10}$ + $(89)_{10}$ = $(188)_{10}$

1001 1001
1000 1001
1 0010 0010
0110 0110
0001 1000 1000

④ 999.99 + 999.99 = 1999.98

1001 1001 1001 . 1001 1001
1001 1001 1001 . 1001 1001
1 0011 0011 0011 . 0011 0010
0110 0110 0110 . 0110 0110
0001 1001 1001 1001 . 1001 1000
1 9 9 9 . 9 8

* Sum > 9

① 9 + 1 = 10

1001
0001
1010 → [10>9]
0110
$\underbrace{0001}_{1}$ $\underbrace{0000}_{0}$

② 29 + 31 = 60

0010 1001
0011 0001
0101 1010
0110
0110 0000

③ 437 + 214 = 651

0100 0011 0111
+ 0010 0001 0100
0110 0100 1011
0110
0110 0101 0001
6 5 1

## * Sum > 9 and Carry also generated :-

① 

$$
\begin{array}{rr}
 & 76 \\
+ & 94 \\
\hline
 & 170
\end{array}
\qquad
\begin{array}{rrr}
 & 0111 & 0110 \\
 & 1001 & 0100 \\
\hline
 & 1\,0000 & 1010 \; (>9) \\
 & 0110 & 0110 \\
\hline
0001 & 0111 & 0000 \\
1 & 7 & 0
\end{array}
$$

② 

$$
\begin{array}{r}
947 \\
738 \\
\hline
1685 \\
\hline
\end{array}
\qquad
\begin{array}{rrrr}
 & 1001 & 0100 & 0111 \\
 & 0111 & 0011 & 1000 \\
\hline
 & 1\,0000 & 0111 & 1111 \\
 & 0110 & & 0110 \\
\hline
0001 & 0110 & 1000 & 0101 \\
1 & 6 & 8 & 5
\end{array}
$$

## Excess-3 Addition :-

- ↳ Add the numbers like binary.
- ↳ If carry = 1 ⇒ Add 3 (0011)
  
  If carry = 0 ⇒ Subtract 3 (0011)
- ↳ If carry alone is left, ⇒ Add 3 in carry.
- ↳ Results will be in XS-3 code.

Eg. → $(5)_{10} + (4)_{10} = (9)_{10}$

$$\begin{array}{r} 1000 \\ +\,0111 \\ \hline 1111 \\ -\,0011 \\ \hline 1100 \end{array}$$

→ xs-3 of $(9)_{10}$

Eg. → $(9)_{10} + (9)_{10} = (18)_{10}$

$$\begin{array}{rr} & 1100 \\ & 1100 \\ \hline 0001 & 1000 \\ 0011 & 0011 \\ \hline 0100 & 1011 \end{array}$$

→ xs-3 of $(18)_{10}$

③ $99 + 99 = 198$

$$\begin{array}{rrr} & 1100 & 1100 \\ & 1100 & 1100 \\ \hline 0001 & 1001 & 1000 \\ 0011 & 0011 & 0011 \\ \hline 0100 & 1100 & 1011 \\ 4 & 12 & 11 \end{array}$$

→ xs-3 of (198)

④ $441.58 + 531.49 = 973.02$

$$\begin{array}{rrr.rr} 0111 & 0111 & 0100 & 1000 & 0110 \\ 1000 & 0110 & 0100 & 0111 & 1100 \\ \hline 1111 & 1101 & 1001 & 0000 & 0010 \\ -0011 & -0011 & -0011 & 0011 & 0011 \\ \hline 1100 & 1010 & 0110 & 0011 & 0101 \\ 12 & 10 & 6 & 3 & 5 \end{array}$$

xs-3 ⟸ $[973.02]_{10}$

# Signed Number Representation

① Signed - Magnitude Representation

② r's complement Representation [r → Base]

③ (r-1)'s complement Representation

| | Binary | Decimal | Octal | Hexadecimal |
|---|---|---|---|---|
| (r-1)'s c → | 1's c | 9's c | 7's | F' c |
| r's c → | 2's c | 10's c | 8'c | G' c |

## ↳ ① Sign - Magnitude Representation :-

Q. Express $(24)_{10}$ in binary representation / Unsigned Number representation.

↳ $(11000)_2$ → Total 5 bits required

Sign - Magnitude
↓ ↓
± Modulus

[ bit 1 → -ve
[ bit 0 → +ve

* MSB bit will show sign and Rest of the bits will be modulus.

Q. Represent $(+24)_{10}$ in Signed-Magnitude Representation. (SMR)

↳ $(\underset{\text{sign}}{0}\ \underbrace{11000}_{\text{magnitude}})_{SMR}$ → Total 6 bits required

Q. Represent $(-24)_{10}$ in SMR.

↳ $\underbrace{1}_{-ve}11000$

Q. $(11011)_2$ which decimal number is that?

↳ $(-11)_{10}$ ✗ [where is mentioned that it is SMR?]

$(11011)_2 = (27)_{10}$ ✓

Q. $N = (11011)_2$
The given Number is represented in <u>SMR</u>.
which number is that?

↳ $\underbrace{1}\ \underbrace{1011}$

$(-11)_{10}$ ✓

Q. Express $(+24)_{10}$ in SMR in 8-bits.

↳ 00011000

Q. Express $(-24)_{10}$ in SMR in 8-bits.

10011000

## ★ Range of binary Number representation / Unsigned - NR :-

| $b_3$ | $b_2$ | $b_1$ | D |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 2 |
| 0 | 1 | 1 | 3 |
| 1 | 0 | 0 | 4 |
| 1 | 0 | 1 | 5 |
| 1 | 1 | 0 | 6 |
| 1 | 1 | 1 | 7 |

n-bits :- $0$ to $2^n - 1$

## ★ Range of Sign - Magnitude Representation :-

sign bit → $b_3$

| $b_3$ | $b_2$ | $b_1$ | D |
|---|---|---|---|
| 0 | 0 | 0 | +0 |
| 0 | 0 | 1 | +1 |
| 0 | 1 | 0 | +2 |
| 0 | 1 | 1 | +3 |
| 1 | 0 | 0 | −0 |
| 1 | 0 | 1 | −1 |
| 1 | 1 | 0 | −2 |
| 1 | 1 | 1 | −3 |

$\boxed{-0 \neq +0}$

$-3 \rightarrow +3$

$$-\left[\frac{2^n - 2}{2}\right] \text{ to } +\left[\frac{2^n - 2}{2}\right]$$

★★ $\boxed{-[2^{n-1} - 1] \text{ to } +[2^{n-1} - 1]}$

Q. Represent $(-F27A)_{16}$ in SMR.

↳ $(\underbrace{1}\ 1111\,0010\,0111\,1010)_{SMR}$

Q. Represent $(-702)_8$ in SMR.

↳ $(\underbrace{1}\ 111000010)_{SMR}$

Q. Represent $(-43)_5$ in SMR.

$(\ )_5 \rightarrow (\ )_2$ ?

$43 = 3 + 4 \times 5 = (23)_{10}$

$(-23)_{10} = (1\ 10111)_{SMR}$

★ (r-1)'s complement Representation:-

10 → 9's CR

8 → 7's CR

16 → F's CR

2 → 1's CR

Q. 9's complement Representation of $(27)_{10}$

↳

$$\begin{array}{r} 99 \\ -27 \\ \hline (72)_{10} \end{array}$$

Q. 7's CR of $(35)_8$ ⇒

$$\begin{array}{r} 77 \\ 35 \\ \hline (42)_8 \end{array}$$

Q. F's CR of $(A23)_{16}$ ⇒

$$\begin{array}{r} FFF \\ A23 \\ \hline (5DC)_{8} \end{array}$$

★

## $\boxed{\text{1's complement Representation}}$

↓

Check if Number is +ve or -ve

If +ve

↓

write SMR

↳ for +ve Numbers

SMR = 1's CR = 2's CR

eg. → $(+21)_{10}$ in 1's CR

↳ $(010101)_2$ → 1's CR = SMR = 2's CR

If -ve

↓

write SMR

↓

(i) keep sign bit as it is

(ii) subtract Mag. bits from (111....)

OR

Take complement of each Mag. bit

---

Q. $(-21)_{10}$ in 1's CR :-

↳ SMR = 1 10101

1's CR = 101010

$$\begin{array}{r} 11111 \\ -10101 \\ \hline 01010 \end{array}$$

16 8 4 2 1

Q. $(+21)_{10}$ in 1's CR in 8- bits.

↳ SMR = 1's CR = 2's CR = $(00010101)_2$

Q. $(-21)_{10}$ in 1's CR in 8- bits.

↳ SMR = 10010101

↓

1's CR = 11101010

eg. → $(-28)_{10}$ in 1 CR →

① SMR → 1 11100 ↓ 1's CR = 100011 $\}$ 6 - bits

② SMR = 10011100 ↓ 1's CR = 11100011 $\}$ → 8-bits

Q. $N = (11011)_2$

N is represented in SMR.

which Number is that?

↳ $\overset{\ }{1}\ \overset{8}{1}\ 0\ \overset{2}{1}\ \overset{1}{1}$

↓

$(-11)_{10}$

* $N = (11011)_2$ ⇝ Represented in SMR

↓ 1's complement

$(10100)_2$ ⇝ 1's c of {N [in SMR]}

↓ 1's complement

$(11011)_2$ ⇝ 1's c of [1's of { N [in SMR] }] = N { is SMR }

---

Q. $N = (10100)_2$

N corresponds to the 1's complement of →

↳ 10100

1's CR ↓

11011

$\boxed{(-11)_{10}}$ ⇝ decimal

Ans.

$(-11)_{10}$ take 1's CR

SMR → 11011

1's CR → 10100

**Q.** The following numbers are 1's CR of some number. Find those numbers.

↳ sign

| | | |
|---|---|---|
| 0 0 0 | | → +0 |
| 0 0 1 | | → +1 |
| 0 1 0 | | → +2 |
| 0 1 1 | | → +3 |
| 1 0 0 | → 1 1 1 | → −3 |
| 1 0 1 | → 1 1 0 | → −2 |
| 1 1 0 | → 1 0 1 | → −1 |
| 1 1 1 | → 1 0 0 | → −0 |

[ For +ve numbers SMR = 1's C = 2's ]

$\boxed{\text{For 1's CR} \quad -0 \neq +0}$

Range → $-[2^{n-1}-1]$ to $+[2^{n-1}-1]$ { same as SMR }

---

★ $\boxed{\text{2's complement Representation}}$

↓

Check if Number is +ve or -ve

If +ve

↓

Write SMR

↳ For +ve Numbers

SMR = 1's CR = 2's CR

Eg. → $(+21)_{10}$ in 2's CR

↳ 010101

If -ve

↓

Write SMR

↓

P.T.O.

If -ve →

(i) write SMR

(ii) keep sign bit as it is.

(iii) write 1's complement and add $(1)_2$

$\boxed{\text{OR}}$

(i) write SMR

(ii) Keep sign bit as it is

(iii) Start writing bits from right side as it is until you encounter the first "1". Take complement of each bit afterwards.

Eg. → $(-21)_{10}$ in 2's complement Representation.

M-I

(i) SMR → 110101

1's CR → 101010

+1

2's CR → 101011

M-II

SMR → 110101 ←

↓

2's CR → 101011

Eg. → $(-14)_{10}$ in 2's CR:-

SMR:→ 11110

1's CR:→ 10001

+1

2's CR 10010

M-II

SMR:- 11110 ←

2's CR :→ 10010

Q. $(-21)_{10}$ in 2's CR in 8-bits

↳ SMR :- 10010101

2's CR → 11101011

N.B. - 2's complement of [2's complement of x (in S-M)] = x [in SM]

Q. $(-14)_{10}$ in 2's CR in 8-bits

SMR :- 10001110

2's CR :- 11110010

1's CR :- 11110001

Q. 11001, 1001 and 111001 corresponds to 2's complement of →

↳ (a) 11001 ⟶ 2's complement

10111

$\underbrace{1}\ \underbrace{0111}$

$= -7$

2's of [2's of x [in SM]] = x [in SM]

(b) 1001 ⟶ 2's C

1111

$= -7$

(c) 111001 ⟶ 2's C

100111

$= -7$

★ Revision Examples:-

① $(+18)_{10}$ in:-

(i) SMR → 010010

(ii) 1's C → 010010

(iii) 2's C → 010010

② $(+18)_{10}$ in 8-bits:-

(i) SMR → 00010010

(ii) 1's C → 00010010

(iii) 2's C → 00010010

③ $(-18)_{10}$ in

(i) SMR:- 110010

(ii) 1's C:- 101101

(iii) 2's C:- 101110

④ $(-18)_{10}$ in 8-bits:-

(i) SMR:- 10010010

(ii) 1's C → 11101101

(iii) 2's C → 11101110

↳ Interesting to notice:-

★ For 1's complement:-

$(-18)_{10}$ in 6-bits = 101101

in 8 bits = 11101101

$(+18)_{10}$ in 6-bits = 010010

in 8-bits = 00010010

★ For 2's complement:-

$(-18)_{10}$ in 6-bits = 101110

in 8-bits = 11101110

$(+18)_{10}$ in 6-bits = 010010

in 8-bits = 00010010

conclusion:-

For 1's and 2's complement:-

~~1~~101101 = 101101 = ~~11~~1 01101 = ~~111~~101101 - - - -

OR

101110 = 11101110 = 1101110 = 11101110 - - - -

Q. 2's complement of x is 10111011.
what will be 2's complement of x is 12-bits

↳ 111110111011

Q. 1's complement of x is 10101101.
what will be 1's complement of x in 12-bits.

↳ 111110101101

Q. 1's complement of x is ~~111111~~101101.
The number x in decimal is →

↳ 1's CR = 101101 → 1's C

010010
$= (-18)_{10}$

N.B. → For '0' there is only one representation in 2's C.
It will be the same as 2's CR of $(+0)_2$

↳ SMR for +0 = 00
1's CR +0 = 00
2's CR +0 = 00

↳ SMR of −0 = 10
1's CR −0 = 11
2's CR −0 = same as 2's CR +0 = 00

* 
| |
|---|
| −0 = 0 → for 2's CR |
| −0 ≠ 0 ⇒ for SMR and 1's CR |

★ Special case in 2's complement:-

Q. For $(-4)_{10}$

SMR :→ 1100 → 4-bits

1's CR → 1011 → 4-bits

2's CR → $\cancel{1}100$

= 100

2's of $(-4)_{10}$ = 100 → 3-bits

$2^2$

For $(-8)_{10}$

SMR:→ 11000 → 5 bits

1's CR → 10111 → 5-bits

2's CR → $\cancel{1}1000$

1000 → 4 bits

2's CR of $(-8)_{10}$ = 1000

$2^3$

★ Special case

| 2's CR | No. of zero's in 2's CR | Decimal no. |
|---|---|---|
| 100 | 2 | $-2^2 = (-4)_{10}$ |
| 1000 | 3 | $-2^3 = (-8)_{10}$ |
| 100 ... n time | n | $(-2^n)_{10}$ |

Q. $N = (10000)_2$

(a) x is SMR of N. Find x

(b) y is 1's CR of N. Find y

(c) z is 2's CR of N. Find z

↳ (a) $(-0)_{10} = x$

(b) 10000 → 1's

11111 = $(-15)_{10} = y$

(c) $N = (10000)_2$

$= -2^4 = (-16)_{10} = z$

Q. The following numbers are 2's CR of some number. Find those numbers.

$\boxed{-0 = +0}$

$000 = +0 = -0$
$001 = +1$
$010 = +2$
$011 = +3$
$100 = -2^2 = -4$
$101 \Rightarrow 111 = -3$
$110 \Rightarrow 110 = -2$
$111 \Rightarrow 101 = -1$

3-bits

$-4 \rightarrow 3$

Range ⇒ $-[2^{n-1}]$ to $+[2^{n-1}-1]$

## * Range of Signed Number Representation:-

| bits | SMR | 1's CR | 2's CR |
|---|---|---|---|
| 3 | $-3$ to $+3$ | $-3$ to $+3$ | $-4$ to $3$ |
| 4 | $-7$ to $+7$ | $-7$ to $+7$ | $-8$ to $7$ |
| ⋮ | ⋮ | ⋮ | ⋮ |
| n | $-(2^{n-1}-1)$ to $+(2^{n-1}-1)$ | $-(2^{n-1}-1)$ to $+(2^{n-1}-1)$ | $-2^{n-1}$ to $+(2^{n-1}-1)$ |

## Complement Arithmatic:-

1's complement Arithmatic

↓

Represent the no. in SMR and get 1's complement

↓

Perform Binary Addition

| Carry NOT Generated | Carry Generated |
| --- | --- |
| ↓ | * $\boxed{\text{Add the carry to the results}}$ |
| Answer is in 1's CR. | ↓ |
| | Answer is in 1's CR |

If Answer = +ve ⇒ 1's CR = SMR = Normal form

* If you perform the operation using $\boxed{\text{min. no. of bits}}$ then

| if carry Gen. | if Carry NOT Gen. |
| --- | --- |
| ↓ | ↓ |
| Answer +ve | Answer -ve |

Eg. → $(26)_{10}$
$-(14)_{10}$
$(12)_{10}$

$\xrightarrow{1\text{'s CR}}$ $(+26)_{10} + (-14)_{10} = (\ ?\ )_{10}$

SMR [+26] = 011010

1's CR [+26] = 011010

SMR [−14] = 101110

1's CR [−14] = 110001

```
  011010
+ 110001
---------
 1 001011
       +1
---------
  001100  ↝ 1's CR = SMR
          ↳ +12
```

Eg. → $(14)_{10}$
$-(26)_{10}$
$-(12)_{10}$

→ $(+14)_{10} + (-26)_{10} = (\ ?\ )_{10}$

1's CR $(+14)_{10}$ = 001110

SMR $(-26)_{10}$ = 111010

1's CR $(-26)_{10}$ = 100101

```
  001110
  100101
---------
  110011  ↝ 1's CR of Ans.
```

$(-12)_{10}$ ← 101100 ↝ SMR of Ans. ($\xleftarrow{1\text{'s CR}}$ 110011)

## Complement Arithmatic:-

| 1's CR → Add carry |
|---|
| 2's CR → Ignore Carry |

2's complement Arithmatic

↓

Represent the no. in SMR and get 2's complement

↓

Perform Binary Addition

- Carry NOT Generated
  ↓
  Result is in 2's CR
- Carry Generated
  ↓
  * $\boxed{\text{Ignore the carry}}$
  ↓
  Result is 2's CR

If Answer = +ve ⇒ 2's CR = SMR = Normal form

* If you perform the operation $\boxed{\text{using min. no. of bits}}$ then

if carry Gen
↓
Answer +ve

if carry NOT Gen
↓
Answer −ve

---

Q. $(26)_{10} - (14)_{10} = (12)_{10}$ $\xrightarrow{\text{2's CR}}$ $(+26)_{10} + (-14)_{10} = (?)_{10}$

(+26) in SMR = 011010

(+26) in 2's CR = 011010

(−14) in SMR = 101110

(−14) in 2's CR = 110010

```
  011010
  110010
---------
(1)001100  → 2's CR = SMR
```

Ignore

↓

$(+12)_{10}$

Q. $(14)_{10} - (26)_{10} = (-12)_{10}$ $\xrightarrow{\text{2's CR}}$

(+14) in 2's CR = 001110

(−26) in SMR = 111010

(−26) 2's CR = 100110

```
  001110
  100110
---------
  110100  → 2's CR
```

110100 → 2's CR

↓ 2's CR

101100 → SMR = $(-12)_{10}$

Q. Perform the given operation using 1's CR and 2's CR

(i) $(28)_{10} - (12)_{10}$

(ii) $(12)_{10} - (28)_{10}$

↳ $(+28)_{10}$ in SMR → 011100

1's → 011100

2's → 011100

$(+12)_{10}$ is SMR = 001100

1's C = 001100

2's C = 001100

$(-28)_{10}$ in SMR → 111100

1's → 100011

2's → 100100

$(-12)_{10}$ in SMR = 101100

1's CR = 110011

2's CR = 110100

---

(i) $(+28)_{10} + (-12)_{10} = (?)_{10}$

1's CR:-

```
  011100
  110011
  ------
[1]001111
      +1
  ------
  010000  → 1's CR = SMR = normal form
  +ve
```

$(+16)_{10}$

2's CR:-

```
  011100
  110100
  ------
[1]010000  → 2's CR = SMR
Ignore  +ve          = N.F.
```

$+16$

(ii) $(+12)_{10} + (-28)_{10} = (?)$

```
  001100
  100011
  ------
  101111  → 1's CR
  ↓ 1's CR
  110000  → SMR
```

$(-16)_{10}$

2's CR:-

```
  001100
  100100
  ------
  110000  → 2's CR
```

SMR: 110000 $\xleftarrow{2's}$ 110000

$(-16)_{10}$

10000 $\xleftarrow{2's}$ $(-2^4)_{10} = -(16)$

* Concept of overflow :- [2's CR]

Q. Solve using 2's CR :-

① $(5)_{10}$ $-(6)_{10}$ = $(-1)_{10}$ ⇒ $(+5)_{10} + (-6)_{10}$ ⇒

```
  0101
  1010
  ----
  1111  → 2's CR
  ----
   ↓ 2's CR
  1001 = (-1)10  → correct Ans.
```

4 2 1
+6 = 110
1110
↓
1010

② $(6)_{10}$ $(-5)_{10}$ = $(1)_{10}$ ⇒ $(+6)_{10} + (-5)_{10}$

```
   0110
   1011
   ----
 [1]0001  → 2's CR = SMR
 ↓
 Ignore    (+1)10 → correct Ans.
```

1101
1011

③ $(6)_{10}$ $+(5)_{10}$ = $(11)_{10}$ $\xrightarrow[\text{4-bits}]{\text{using}}$

```
  0110
  0101
  ----
  1011  → 2's CR
  ----
   ↓ 2's CR
  1101  (-5)10 → Incorrect
  ↳ SMR
```

↓

overflow has occurred

④ $(-6)_{10}$ $-(5)_{10}$ = $-(11)_{10}$ $\xrightarrow[\text{4-bits}]{\text{Using}}$

```
   1010
   1011
   ----
 [1]0101  → 2's CR = SMR
 ↓
 Ignore  (+5)10 → Incorrect
```

Range of no. in 2's CR using 4-bits $= -(2^{4-1})$ to $+(2^{4-1}-1)$

$= -8$ to $+7$ [+11 and -11 doesn't lie in this range]

③ $(6)_{10}$ $\xrightarrow[\text{5-bits}]{\text{using}}$ 00110

$+(5)_{10}$ 00101

$(11)_{10}$ 01011 → 2's CR = SMR

$(+11)_{10}$ → correct

④ $(-6)_{10}$ $\xrightarrow[\text{5-bits}]{\text{Using}}$ 11010

$-(5)_{10}$ 11011

$-(11)_{10}$ $\boxed{1}$ 10101 → 2's CR

Ignore ↓ 2's CR

11011 → $(-11)_{10}$ → correct

Range of no. in 2's CR using 5 bits $= -[2^{5-1}]$ to $+[2^{5-1}-1]$

$= -16$ to $+15$ [+11 and −11 lies in this range]

Conclusion:-

↳ If add$^n$ of two positive numbers results in −ve number OR add$^n$ of two negative numbers results in +ve number ⇒ overflow has occurred.

Q. If the following operations are performed using 2's CR using min no. of bits possible. Then which of these will result in overflow?

Range of 2's CR

3-bits ⇒ −4 to +3

4-bits ⇒ −8 to +7

(a) 3 + 2 = 5 (3b, 3b, 4b) ~~(b)~~ +3 − 2 = 1 (3b, 3b, 3b) ~~(c)~~ 2 − 3 = −1 (d) −3 − 2 = −5

~~(E)~~ 4 + 2 = 6 (4b, 4b, 4b) (F) −4 − 2 = −6 (3b, 3b, 4b) ~~(G)~~ 4 − 2 ~~(H)~~ 2 − 4

Let's take example of 3 and 2 again:-

① $+3$ $-2$ → overflow doesn't occur

$$\begin{array}{r} +3 \\ -2 \\ \hline 1 \\ \hline \end{array} \qquad \begin{array}{r} 1 \leftarrow c_{in} \\ 011 \\ 110 \\ \hline 1\,001 \\ \hline \end{array}$$

Cout

cin ⊕ Cout

$= 1 \oplus 1 = 0$

② $2$ $-3$ → over flow doesn't occur

$$\begin{array}{r} 2 \\ -3 \\ \hline -1 \\ \hline \end{array} \qquad \begin{array}{r} \boxed{0} \\ 001 \\ 101 \\ \hline \boxed{0}\,110 \\ \hline \end{array}$$

cin ⊕ Cout

$0 \oplus 0 = 0$

③ $3$ $+2$ → over flow occurs →

$$\begin{array}{r} 3 \\ +2 \\ \hline 5 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ 011 \\ 010 \\ \hline \boxed{0}\,101 \\ \hline \end{array}$$

→ 2's cR

cin ⊕ Cout

$= 1 \oplus 0$

$= 1$

④ $-3$ $-2$ → over flow occurs →

$$\begin{array}{r} -3 \\ -2 \\ \hline -5 \\ \hline \end{array} \qquad \begin{array}{r} \boxed{0} \\ 101 \\ 110 \\ \hline \boxed{1}\,011 \\ \hline \end{array}$$

→ 2's cR

cin → carry into the MSB bit

Cout → carry out of the MSB bit

cin ⊕ Cout

$= 0 \oplus 1$

$= 1$

↳ cin ⊕ Cout $= 0$ → no overflows

cin ⊕ Cout $= 1$ → overflows occurs

Condition for overflow:-

$$\begin{array}{l} \boxed{C_{in}} \text{ (MSB bits)} \\ A_n\, A_{n-1}\, A_{n-2} \,\cdots\cdots\, A_3\, A_2\, A_1 \rightsquigarrow 2\text{'s c} \\ B_n\, B_{n-1}\, B_{n-2} \,\cdots\cdots\, B_3\, B_2\, B_1 \rightsquigarrow 2\text{'s c} \\ \hline \boxed{C_{out}}\; S_n\, S_{n-1}\, S_{n-2} \,\cdots\cdots\, S_3\, S_2\, S_1 \rightsquigarrow 2\text{'s c} \end{array}$$

(i) $C_{in} \oplus C_{out} = 1$

(ii) $\bar{A}_n \bar{B}_n S_n + A_n B_n \bar{S}_n$

$\rightarrow$ overflow occured

Q. Two Numbers are presented in $\boxed{\text{binary}}$.

$P = 1010$ $\qquad$ $Q = 11001$

Perform $P-Q$ without converting the numbers to decimal.

↳

$$P-Q \Rightarrow \begin{array}{r} 01010 \\ -11001 \\ \hline ?0001 \end{array}$$ ↝ using binary subtraction

Use 2's CR:-

$P-Q = (+P) + (-Q)$

$+P \rightarrow 001010$ ↝ SMR = 2's CR

$-Q \rightarrow 111001$ ↝ SMR

$\quad\quad 100111$ ↝ 2's CR

$$\begin{array}{r} 001010 \\ 100111 \\ \hline 110001 \end{array}$$ ↝ 2's CR

$\Rightarrow 101111 = (-15)_{10}$

(+P) in 1's CR = 001010

(−Q) in SMR = 111001

1's CR = 100110

$$\begin{array}{r} 001010 \\ 100110 \\ \hline 110000 \end{array}$$ ⤷ 1's CR

↓ 1's CR

101111 ⤷ SMR

---

Q. The subtraction of a binary number Y, from another binary number X, done by adding 2's complement of Y to X, results in binary number without overflow. This implies that the result is -

(a) Negative and in normal form

(b) Negative and in 2's CR.

? (c) positive and in normal form.

? (d) positive and in 2's CR.

X − Y

X + (2's c of −Y)

| +ve | −ve |
|---|---|
| ↓ | ↓ |
| 2's CR | 2's CR |
| ↓ | |
| SMR, 2's CR | |
| 1's CR, normal | |

Y → 01100 ← binary

+Y → 01100 ⤷ SMR = 2's CR

−Y → 11100 ⤷ SMR

10100 ⤷ 2's CR

## Assignment-3

Q. The minimum decimal equivalent of the number $(71A)_x$ is →

↳ $x > A$ ; $x > 10$ {min base → B}

$(71A)_B$

$10 \times 11^0 + 1 \times 11^1 + 7 \times 11^2 = (868)_{10}$

Q. The number of 1's present binary representation of

$13 \times 4096 + 7 \times 16 - 9$

↳ $(\underbrace{13}_{D} \times 16^3) + (0 \times 16^2) + (7 \times 16^1) + (0 \times 16^0) - (9)_{10}$

$(D070)_{16}$
$-(9)_{10}$
$(\quad)$

$(D070)_{16} - (9)_{10} = (D070)_{16} - (9)_{16}$

$$\begin{array}{r} D07\overset{+16}{0} \\ -9 \\ \hline (D067)_{16} \end{array}$$

$(D067)_{16}$ ; $16 = 2^4$

$(1101\ 0000\ 0110\ 0111)_2$

$= 8$ 1's

Q. Find the value of base x.

$(21)_x = \log_2 (11202)_x$

(a) 3 (b) 4 (c) 5 (d) 6

↳ $1 + 2x = \log_2 [2 + 2x^2 + x^3 + x^4]$

put $x = 3$

$7 = \log_2 [2 + 18 + 27 + 81]$

$= \log_2 [128] = \log_2 2^7 = 7$

**Q. Write down 1's CR and 2's CR of**

$(-23.50)_{10}$

↳ 23.50 in binary:-

$(10111.10)_2$

| | | |
|---|---|---|
| 2 | 23 | |
| 2 | 11 | 1 |
| 2 | 5 | 1 |
| 2 | 2 | 1 |
| | 1 | 0 |

$.50 \times 2 = 1.00 \; 1$

$.00 \times 2 = 0.00$

$.00 \times 2 = 0.00$

$(-23.50)$ in SMR:-

$(\underset{\text{↳sign}}{1}\,10111.10)_2$

1's CR:- $(10100.01)_2$

2's CR $(101000.10)_2$

**Question:-**

① If 2's CR of Q is → 1101101

what will be 2's CR of −Q = ?

↳ 2's CR of Q → 1101101

↓ 2's CR

$\underbrace{1}_{-ve}\underbrace{010011}_{\text{Magnitude}}$ ↳ SMR of Q

−Q ↓

Ans: $\underbrace{0010011}_{+ve}$ ↳ SMR of −Q

SMR = 2's CR

② If 2's CR of Q is → 011010100

2's CR of −Q = ?

↳ Q → $\underbrace{0}_{+ve}\underbrace{11010100}_{\text{Mgn.}}$ ↳ 2's CR = SMR

−Q → 111010100 ↳ SMR

↓ 2's C

2's CR of (−Q) = 100101100 ↳ Ans

Q. P and Q are represented in signed 2's CR.

P = 1110 1101

Q = 1110 0110

if Q is subtracted from P.

The value obtained in signed 2's complement form is →

↳

$Y = P - Q$

$Y = (+P) + (-Q)$

2's CR of Q

= 11100110

$\downarrow$ 2's CR

10011010 → SMR of Q

sign   Mgn.

00011010 → SMR of −Q = 2's CR of −Q

2's CR of +P = 11101101

2's CR of −Q = ?

```
 11101101
 00011010
---------
[1]00000111 → 2's CR  Ans.
```

M-II:-

P = 11101101

$\downarrow$ 2's CR

10010011 → SMR

= −19

Q = 11100110

$\downarrow$ 2's CR

10011010 → SMR

= −26

P − Q = −19 + 26

= 7

$\downarrow$ SMR

0111 = 2's CR

Q. 2's CR of 16-bit number is $(FFFF)_{16}$. The magnitude of the numbe in decimal is →

↳

$$(1111\,1111111111\,11)_2 \rightsquigarrow 2\text{'s CR}$$

$$\downarrow 2\text{'s CR}$$

$$\underbrace{1000000000000001}_{=-1} \rightsquigarrow \text{SMR}$$

$$\rightsquigarrow \text{Mg.} \Rightarrow (+1)$$

Q. In the following series the same Integer is expressed in some different number system 0111, 21, 13, x, 11 - - - -
what is x?

↳ Let first base is 2.

Hit and Trial →

$(0111)_2 = 7$

$(21)_3 = 1 + (2\times3) = 7$

$(13)_4 = 3+4 = (7)_{10}$

$(x)_5 \leftarrow = (7)_{10}$

$(11)_6 = 1+6 = 7$

$(7)_{10} = (x)_5$

$= (12)_5$

$\boxed{x = 12}$

| | |
|---|---|
| 5 | 7 |
| | 1 2 |

Q. Find the result of the following oprⁿ in 2's CR.

$(1101)_2 + (31)_4 + (121)_5$

↳

$1+4+8+1+12+1+10+25$

$= (62)_{10} = (111110)_2 \rightarrow$ Binary

SMR = 2's CR = (0111110)

Ans.

| | | |
|---|---|---|
| 2 | 62 | |
| 2 | 31 | 0 |
| 2 | 15 | 1 |
| 2 | 7 | 1 |
| 2 | 3 | 1 |
| | 1 | 1 |

Q. Find Equivalent Hexadecimal representation of

$(110101)_{Gray}$

↳ $G \rightarrow B$

$110101$

$\downarrow$

$(100110)_2$

$0(0100110)_2$

$(26)_{16}$

Q. Find the resultant Gray code representation of →

$(9B)_{16} - (3F)_{16}$

↳

$9B \rightsquigarrow 11+16=27$

$-3F \rightsquigarrow 15$

$5C \rightsquigarrow 12$

$(5C)_{16} \downarrow$ Binary

$(01011100)_2$

$B \rightarrow G$

$(01110010)_{gray}$ Ans.

Q. Perform the following opreation and tell the no. of 1's present in the binary equivalent of the result.

$(11111111)_2 \div (5)_{10}$

↳ M–I

$(255)_{10} \div (5)_{10} = 51$

$= (110011)_2$

no. of 1's = 4

| | | |
|---|---|---|
| 2 | 51 | |
| 2 | 25 | 1 |
| 2 | 12 | 1 |
| 2 | 6 | 0 |
| 2 | 3 | 0 |
| | 1 | 1 |

M–II

$$
\begin{array}{r}
110011 \\
101\overline{)11111111} \\
101 \\
\hline
101 \\
101 \\
\hline
0111 \\
101 \\
\hline
101 \\
101 \\
\hline
0
\end{array}
$$

Q. In a single byte, How many different signed decimal values can be stored?

↳ Byte = 8 - bits

Unsigned $= 2^8 = 256$ [Range:- 0 to 255]

Range:- [signed]

$$SMR = -[2^7 - 1] \text{ to } +[2^7 - 1] = -127 \text{ to } +127$$

$$= \underbrace{-127, -126, \ldots, -1, -0, +0, +1, +2, \ldots, +127}_{256}$$

$$\boxed{Ans = 256 = 2^n}$$

Q. Given that largest n-bit binary number requires "d" digits in decimal representation, which of the following expression between n and d is correct?

~~(A)~~ $d = 2^n$ ~~(B)~~ $n = 2^d$ ~~(C)~~ $d < n \log_{10} 2$ (D) $d > n \log_{10} 2$

↳ $n = 1$ → 0, → 1 $d = 1$

$n = 3$ 000 = 0, 001 = 1, ⋮, 111 = 7 $d = 1$

$n = 2$ 00 → 0, 01 → 1, 10 → 2, 11 → 3 $d = 1$

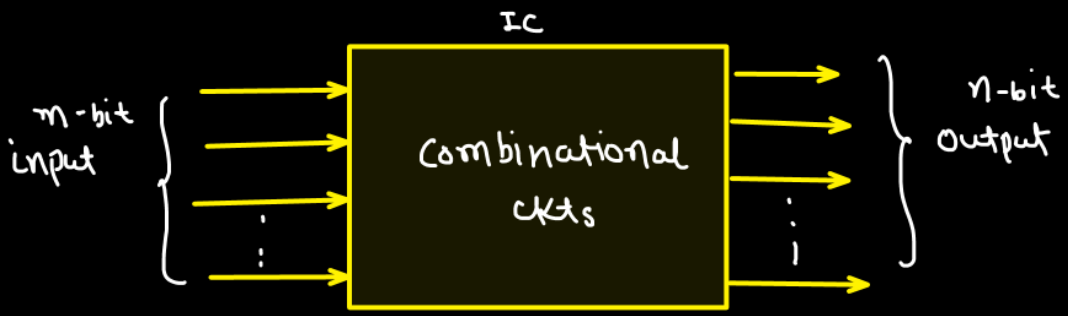
$n = 4$ 000 = 0, 001 = 1, 010 = 2, ⋮, 111 ⋮ 10 $d = 2$

# Combinational Circuits

↓

Interconnection of Logic Gates

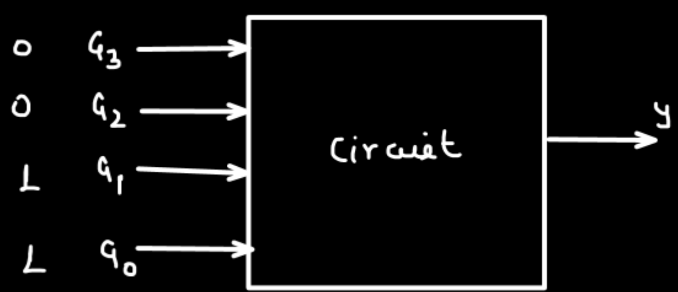
↳ Present output depends on present i/p only.




## Design Examples:-

Q. Design a circuit to detect decimal no. 5 to 10 in a 4-bit Gray code.

↳



| D | $G_3$ | $G_2$ | $G_1$ | $G_0$ | Y |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 1 | 0 |
| 3 | 0 | 0 | 1 | 0 | 0 |
| 4 | 0 | 1 | 1 | 0 | 0 |
| 5 | 0 | 1 | 1 | 1 | 1 |
| 6 | 0 | 1 | 0 | 1 | 1 |
| 7 | 0 | 1 | 0 | 0 | 1 |
| 8 | 1 | 1 | 0 | 0 | 1 |
| 9 | 1 | 1 | 0 | 1 | 1 |
| 10 | 1 | 1 | 1 | 1 | 1 |
| 11 | 1 | 1 | 1 | 0 | 0 |
| 12 | 1 | 0 | 1 | 0 | 0 |
| 13 | 1 | 0 | 1 | 1 | 0 |
| 14 | 1 | 0 | 0 | 1 | 0 |
| 15 | 1 | 0 | 0 | 0 | 0 |

Using K-map:-

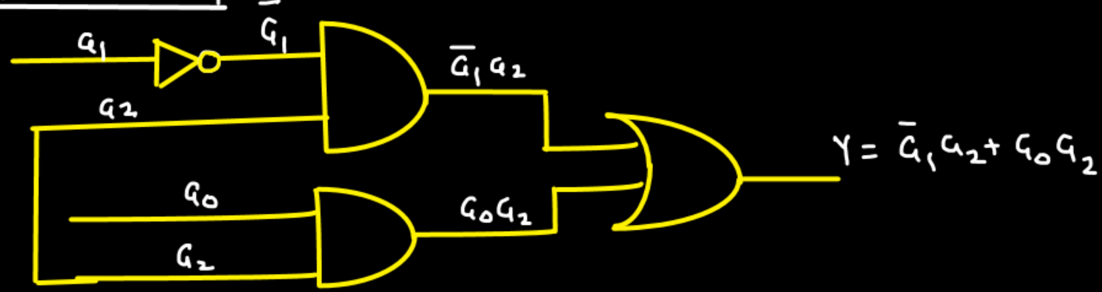
$G_1G_0$ → LSB

MSB ← $G_3G_2$

| $G_3G_2$ \ $G_1G_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | |
| 01 | 1 | 1 | 1 | |
| 11 | 1 | 1 | 1 | |
| 10 | | | | |

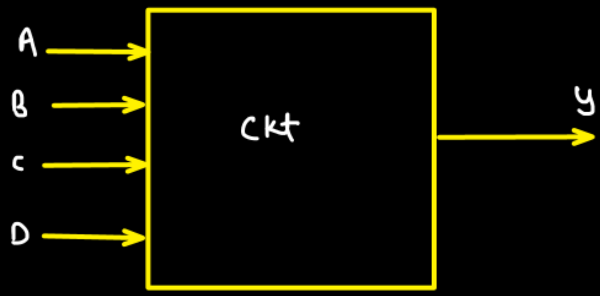
$$Y = G_2\bar{G}_1 + G_0G_2$$

circuit implementation:-



Q. Design a circuit to detect the decimal number 1, 3, 5, 7, 9 in a 4-bit 5211 BCD code Input.

↳




| 5<br>A | 2<br>B | 1<br>C | 1<br>D | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

1, 3, 5, 7, 9

K-Map :-

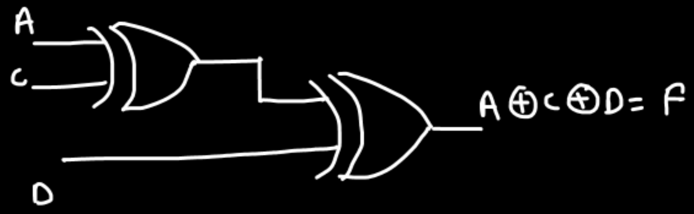
| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | 1 | | 1 |
| 01 | | 1 | | 1 |
| 11 | 1 | | 1 | |
| 10 | 1 | | 1 | |

$$F = \bar{A}\bar{C}D + A\bar{C}\bar{D} + ACD + \bar{A}C\bar{D}$$
$$= \bar{A}(\bar{C}D + C\bar{D}) + A(\bar{C}\bar{D} + CD)$$
$$= \bar{A}(C \oplus D) + A(\overline{C \oplus D})$$

$$F = A \oplus C \oplus D$$

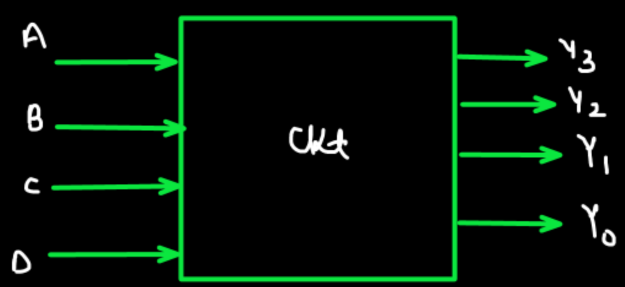
$$= A \odot C \odot D$$

**Circuit Implementation:-**




Q. Design a combinational ckts which produces 2's complement of a 4-bit binary number. Ignore sign bit.

↳



| D | A | B | C | D | $Y_3$ | $Y_2$ | $Y_1$ | $Y_0$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 2 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 3 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 4 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 6 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 7 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 8 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 10 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 11 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 12 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 13 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |

$y_3 = \Sigma(1,2,3,4,5,6,7,8)$

$y_2 = \Sigma(1,2,3,4,9,10,11,12)$

$y_1 = \Sigma(1,2,5,6,9,10,13,14)$

$y_0 = D$

$Y_3 \rightarrow$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | 1 | 1 | 1 |
| 01 | 1 | 1 | 1 | 1 |
| 11 | | | | |
| 10 | 1 | | | |

$y_3 = A\bar{B}\bar{C}\bar{D} + \bar{A}B + \bar{A}D + \bar{A}C$

$Y_2 \rightarrow$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | 1 | 1 | 1 |
| 01 | 1 | | | |
| 11 | 1 | | | |
| 10 | | 1 | 1 | 1 |

$y_2 = B\bar{C}\bar{D} + \bar{B}D + \bar{B}C$

$Y_1 \rightarrow$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | 1 | | 1 |
| 01 | | 1 | | 1 |
| 11 | | 1 | | 1 |
| 10 | | 1 | | 1 |

$Y_1 = \bar{C}D + C\bar{D}$

$= C \oplus D$

$Y_0 \rightarrow$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | |
| 01 | | | | |
| 11 | | | | |
| 10 | | | | |

$y_0 = D$

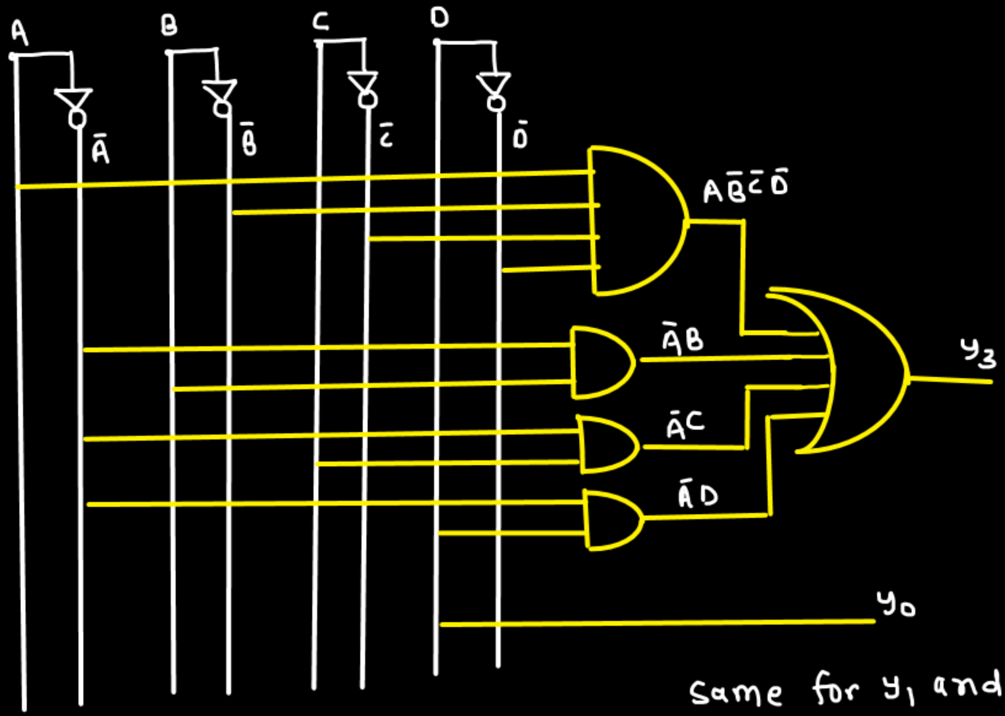
$$y_3 = A\bar{B}\bar{C}\bar{D} + \bar{A}B + \bar{A}C + \bar{A}D$$

$$y_2 = \bar{B}D + \bar{B}C + B\bar{C}\bar{D}$$

$$y_1 = C \oplus D$$

$$y_0 = D$$

## Circuit Implementation:-




Same for $y_1$ and $y_2$

Q. Design a combinational ckts which produces 1's complement of a 4-bit binary number. Ignore sign bit.

↳

↳

| D | A | B | C | D | $Y_3$ | $Y_2$ | $Y_1$ | $Y_0$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 3 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 4 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 5 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| 6 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 7 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 8 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 9 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 11 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 12 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 13 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 14 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 15 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |

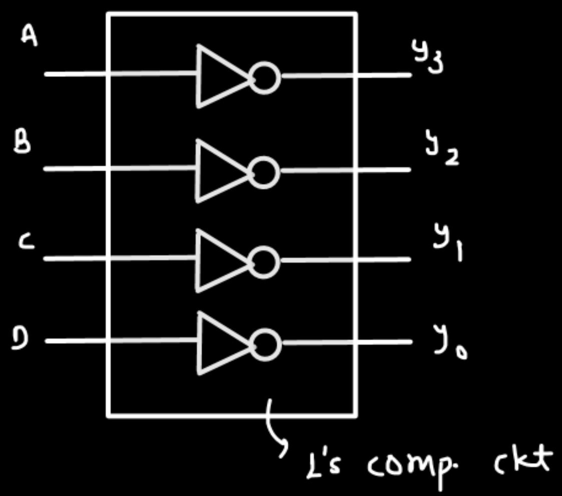
$y_3 = \bar{A}$

$y_2 = \bar{B}$

$y_1 = \bar{A}$

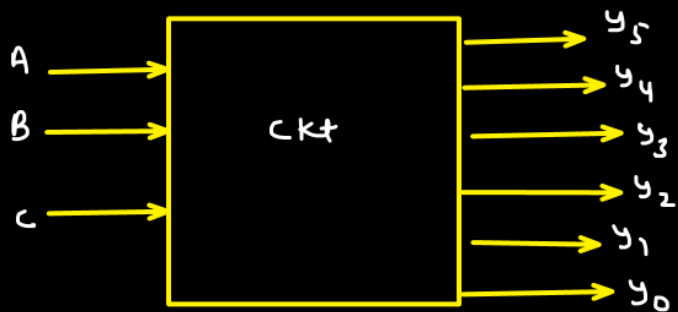
$y_0 = \bar{D}$

ckt Implementation:-




Q. Design a combinational ckt which take 3-bit binary input and produces o/p equal to the square of the input in binary form.

↳



|  | A | B | C | $Y_5$ (32) | $Y_4$ (16) | $Y_3$ (8) | $Y_2$ (4) | $Y_1$ (2) | $Y_0$ (1) |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 4 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 5 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 6 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 7 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |

$y_2 = \bar{A}B\bar{C} + AB\bar{C}$

$\boxed{y_2 = B\bar{C}}$

$\boxed{y_1 = 0}$

$\boxed{y_0 = C}$

$y_5 = AB\bar{C} + ABC$

$\boxed{y_5 = AB}$

$y_4 = A\bar{B}\bar{C} + A\bar{B}C$

$\boxed{y_4 = A\bar{B}}$

$y_3 = \bar{A}BC + A\bar{B}C$

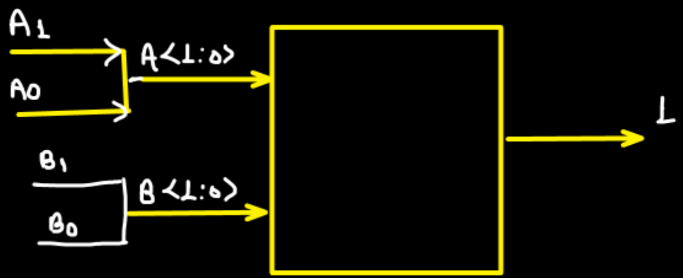
$\boxed{y_3 = (A \oplus B)C}$

Q. Design a combinational circuit which takes two 2-bit binary input and produces a single output. The o/p will be high when the decimal equivalent of the binary sum of the i/p is

(a) odd number

(b) Even number

↳



| D | $\overset{2}{A_1}$ | $\overset{1}{A_0}$ | $\overset{2}{B_1}$ | $\overset{1}{B_0}$ | $Y_a$ (odd) | $Y_b$ (even) | Sum |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 2 | 0 | 0 | 1 | 0 | 0 | 1 | 2 |
| 3 | 0 | 0 | 1 | 1 | 1 | 0 | 3 |
| 4 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 5 | 0 | 1 | 0 | 1 | 0 | 1 | 2 |
| 6 | 0 | 1 | 1 | 0 | 1 | 0 | 3 |
| 7 | 0 | 1 | 1 | 1 | 0 | 1 | 4 |
| 8 | 1 | 0 | 0 | 0 | 0 | 1 | 2 |
| 9 | 1 | 0 | 0 | 1 | 1 | 0 | 3 |
| 10 | 1 | 0 | 1 | 0 | 0 | 1 | 4 |
| 11 | 1 | 0 | 1 | 1 | 1 | 0 | 5 |
| 12 | 1 | 1 | 0 | 0 | 1 | 0 | 3 |
| 13 | 1 | 1 | 0 | 1 | 0 | 1 | 4 |
| 14 | 1 | 1 | 1 | 0 | 1 | 0 | 5 |
| 15 | 1 | 1 | 1 | 1 | 0 | 1 | 6 |

$Y_a = \overline{Y_b}$

$Y_a = \Sigma m(1,3,4,6,9,11,12,14)$

K-map:-

$y_a \rightarrow$

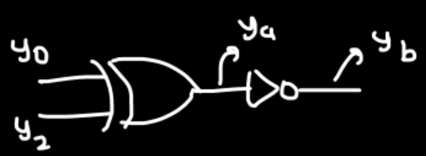
| $y_3y_2 \backslash y_1y_0$ | $\bar{y}_1\bar{y}_0$ 00 | $\bar{y}_1 y_0$ 01 | $y_1 y_0$ 11 | $y_1\bar{y}_0$ 10 |
|---|---|---|---|---|
| $\bar{y}_3\bar{y}_2$ 00 | | 1 | 1 | |
| $\bar{y}_3 y_2$ 01 | 1 | | | 1 |
| $y_3 y_2$ 11 | 1 | | | 1 |
| $y_3\bar{y}_2$ 10 | | 1 | 1 | |

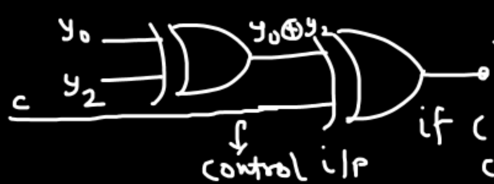
| $y_3y_2 \backslash y_1y_0$ | $\bar{y}_1\bar{y}_0$ 00 | $\bar{y}_1 y_0$ 01 | $y_1 y_0$ 11 | $y_1\bar{y}_0$ 10 |
|---|---|---|---|---|
| $\bar{y}_3\bar{y}_2$ 00 | | | | |
| $\bar{y}_3 y_2$ 01 | | | | |
| $y_3 y_2$ 11 | | | | |
| $y_3\bar{y}_2$ 10 | | | | |

$y_a = y_0\bar{y}_2 + \bar{y}_0 y_2$

$$\boxed{y_a = y_0 \oplus y_2}$$

$$\boxed{y_b = y_0 \odot y_2}$$




Final o/p [Y]

if $c = 0 \Rightarrow Y = y_0 \oplus y_2$

$c = 1 \Rightarrow Y = y_0 \odot y_2$

Q. For a 3-bit input design a combination ckt which detects:-

(a) majority no. of 1's or minority no. of 0's

(b) majority no. of 0's or minority no. of 1's.

↳

| D | A | B | C | $Y_a$ | $Y_b$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 2 | 0 | 1 | 0 | 0 | 1 |
| 3 | 0 | 1 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 | 0 |
| 6 | 1 | 1 | 0 | 1 | 0 |
| 7 | 1 | 1 | 1 | 1 | 0 |

$Y_a$ → majority of 1's

$Y_b$ → majority no. of 0

$y_a = \Sigma m(3, 5, 6, 7)$

| A \ BC | $\bar{B}\bar{C}$ | $\bar{B}C$ | $BC$ | $B\bar{C}$ |
|---|---|---|---|---|
| $\bar{A}$ | | | 1 | |
| A | | 1 | 1 | 1 |

$Y = BC + AC + AB$

$$\boxed{Y = AB + BC + CA}$$ → self dual function

$y_a \rightarrow$

| $A \backslash B+C$ | $B+C$ | $B+\bar{C}$ | $\bar{B}+\bar{C}$ | $\bar{B}+C$ |
|---|---|---|---|---|
| $A$ | 0 | 0 | | 0 |
| $\bar{A}$ | 0 | | | |

$$\boxed{\begin{aligned} y &= (A+B)(B+C)(C+A) \\ y &= AB + BC + CA \end{aligned}}$$

| $A \backslash BC$ | $\bar{B}\bar{C}$ | $\bar{B}C$ | $BC$ | $B\bar{C}$ |
|---|---|---|---|---|
| $\bar{A}$ | | | 1 | |
| $A$ | | 1 | 1 | 1 |

$$y = AB + A\bar{B}C + \bar{A}BC$$
$$= AB + C[A\bar{B} + \bar{A}B]$$

$$\left.\begin{aligned} y &= AB + C[A \oplus B] \\ y &= AC + B[A \oplus C] \\ y &= BC + A[B \oplus C] \\ y &= AB + BC + CA \\ y &= (A+B)(B+C)(C+A) \\ y &= \Sigma m(3,5,6,7) \end{aligned}\right\} \rightarrow \text{Self dual } f^n$$

dual

| $A \backslash BC$ | $\bar{B}\bar{C}$ | $\bar{B}C$ | $BC$ | $B\bar{C}$ |
|---|---|---|---|---|
| $\bar{A}$ | | | 1 | |
| $A$ | | 1 | 1 | 1 |

$$y = BC + A\bar{B}C + AB\bar{C}$$
$$= BC + A[\bar{B}C + B\bar{C}] = BC + A[B \oplus C]$$

$y_b \rightarrow$

$$y_b = \bar{y}_a$$
$$y_a = AB + BC + CA$$
$\hookrightarrow$ self dual $f^n$

$$\left.\begin{aligned} y_b &= \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{C}\bar{A} \\ y_b &= (\bar{A}+\bar{B})(\bar{B}+\bar{C})(\bar{C}+\bar{A}) \\ y_b &= \bar{A}\bar{B} + \bar{C}(A \oplus B) \\ y_b &= \bar{A}\bar{C} + \bar{B}(A \oplus C) \\ y_b &= \bar{B}\bar{C} + \bar{A}(B \oplus C) \\ y_b &= \Sigma m(0,1,2,4) \end{aligned}\right\}$$

$$\boxed{\begin{aligned} & f \longrightarrow f^C \\ & \hookrightarrow f^D \nearrow \text{Ind. com.} \end{aligned}}$$

$$y_a = AB + C[A \oplus B]$$
$$y_b = \bar{y}_a$$
$$= \bar{A}\bar{B} + \bar{C}[\bar{A} \oplus \bar{B}]$$
$$= \bar{A}\bar{B} + \bar{C}(A \oplus B)$$

Conclusion :-

| Majority of 1's detector | Majority no of 0's detector |
|---|---|
| $\Sigma(3,5,6,7)$ | $\Sigma(0,1,2,4)$ |
| $= AB + BC + CA$ | $= \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{C}\bar{A}$ |
| $= (A+B)(B+C)(C+A)$ | $= (\bar{A}+\bar{B})(\bar{B}+\bar{C})(\bar{C}+\bar{A})$ |
| $= AB + C(A \oplus B)$ | $= \bar{A}\bar{B} + \bar{C}(A \oplus B)$ |
| $= AC + B(A \oplus C)$ | $= \bar{A}\bar{C} + \bar{B}(A \oplus C)$ |
| $= BC + A(B \oplus C)$ | $= \bar{B}\bar{C} + \bar{A}(B \oplus C)$ |