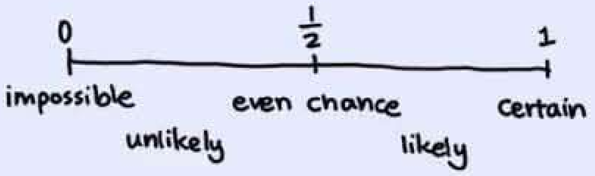
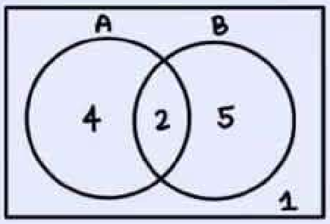
Probability 1

Probability: How likely an event will occur

$$P(\text{event}) = \frac{\text{number of favourable}}{\text{total number of outcomes}}$$



Venn diagram



Two-way

	A	A'	
B	2	5	7
B'	4	1	5
	6	6	12

Range = maximum value − minimum value

Interquartile range (IQR) = upper quartile (Q_3) − lower quartile (Q_1)

IQR is the range of the middle 50% of data.

Theoretical probability: the actual chance or likelihood that an event will occur when an experiment takes place.

E.g. the theoretical probability of rolling a 5 is $P(5) = \frac{1}{6} = 0.1\dot{6} = 16.\dot{6}\%$

Experimental probability: an estimate of probability using a large number of trials.

E.g. a die is rolled 600 times and show a 5 on 99 occasions, the experimental probability is $P(5) \approx \frac{99}{100} = 0.165 = 16.5\%$

Note: if the number of trials is large, the experimental probability should be very close to theoretical probability.

Expected number = P (event) × number of trials

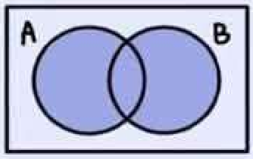
E.g. Roll die 36 times, expected number of 5s is $\frac{1}{6} \times 36 = 6$

Sample space - the list of all possible outcomes

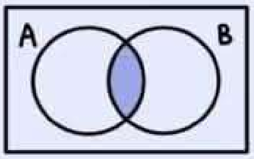
E.g. when rolling a die, the sample space is $\{1, 2, 3, 4, 5, 6\}$

Unions and Intersections

union $A \cup B$ (A or B)

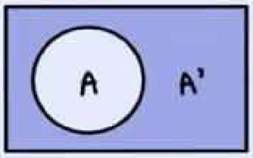


intersection $A \cap B$ (A and B)

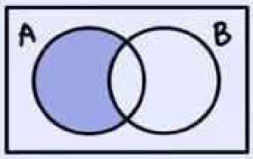


Complement of A is A'

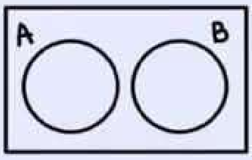
$P(\text{not } A) = 1 - P(A)$



A only is $A \cap B'$



Mutually exclusive events $A \cap B = \emptyset$



Addition Rule

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are mutually exclusive:

$P(A \cap B) = 0$ and

$P(A \cup B) = P(A) + P(B)$

Independent Events

$P(A|B) = P(A)$

$P(A \cap B) = P(A) \times P(B)$

Mode: the most common value (there can be more than one).

Mean: average value $= \frac{\text{sum of all value}}{\text{number of value}}$

Median: the middle value of data.

Outlier: a value that is not in the vicinity of the rest of the data.

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KARL PEARSON

Statistics

MEASURES OF CENTRAL TENDENCY

Generally, average of a distribution is a measure of the middle or expected value of the distribution. Such values are known as **measures of central tendency**. These are as follows:

(a) Mathematical Average such as (i) Arithmetic Mean, (ii) Geometric Mean and (iii) Harmonic Mean

(b) Positional Average such as (i) Median, (ii) Mode, (iii) Quartiles, (iv) Deciles and (v) Percentiles

ARITHEMETIC MEAN

Individual Observations. $\bar{x} = \dfrac{\Sigma x}{N}$, Σx = sum of items, N = number of observations

Short-cut Method. $\bar{x} = A + \dfrac{\Sigma d}{N}$, $d = x - A$

Discrete Series. $\bar{x} = \dfrac{\Sigma f(x)}{N}$, f = frequency, N = total frequency.

Short-cut Method. $\bar{x} = A + \dfrac{\Sigma fd}{N}$, $d = x - A$, N = total frequency.

Continuous Series. *Direct Method:*

$\bar{x} = \dfrac{\Sigma fm}{N}$, m = mid-values of various classes, N = total frequency.

Short-cut Method. $\bar{x} = A + \frac{\Sigma fd}{N}$, A = assumed mean $d = m - A$, N = Total frequency.

Step Deviation Method. $\bar{x} = A + \frac{\Sigma fd'}{N} \times c$, A = assumed mean, $d' = \frac{m - A}{c}$, c = class size, m = class marks.

Combined Mean. If a series of N observations consists of two components with means $\bar{x}_1$ and $\bar{x}_2$ and number of items N_1 and N_2, then the combined mean is:

$$\bar{x}_{12} = \frac{N_1\bar{x}_1 + N_2\bar{x}_2}{N_1 + N_2}$$

MEDIAN

Firstly, we arrange the data in increasing or decreasing order.

Individual Observation. If N is odd, then Med = size of $[(N + 1)/2]$th item.

If N is even, Med = average of $(N/2)$th and $[(N/2) + 1]$th items.

Discrete Series. Firstly, arrange the data in ascending or descending order, find cumulative frequency, then

$$\text{Med} = \text{size of } \frac{N+1}{2} \text{th item}$$

Continuous Series. Firstly, find the median class by using

Med = size of $\frac{N+1}{2}$ th item, then apply

$$\text{Med} = l + \frac{(N/2) - c.f.}{f} \times i,$$

where

l = lower limit of the median class

$c.f.$ = cumulative frequency of the class preceding the median class

f = frequency of the median class

i = class interval of the median class

QUARTILES, DECILES AND PERCENTILES

The technique for determining quartiles, deciles and percentiles is the same as that of median.

In the case of individual observation and discrete series:

$$Q_2 = \text{size of } \frac{N+1}{4} \text{th item}$$

$$Q_3 = \text{size of } \frac{3(N+1)}{4} \text{th item}$$

$$D_1 = \text{size of } \frac{N+1}{10} \text{th item}$$

$$D_7 = \text{size of } \frac{7(N+1)}{10} \text{th item}$$

$$P_1 = \text{size of } \frac{N+1}{100} \text{th item}$$

$$P_{89} = \text{size of } \frac{89(N+1)}{100} \text{th item}$$

In the case of continuous series:

Firstly, find the class in which quartiles, deciles or percentiles lie by using

$$Q_1 = \text{size of } \frac{N}{4} \text{th item}$$

$$Q_3 = \text{size of } \frac{3N}{4} \text{th item}$$

$$D_1 = \text{size of } \frac{N}{10} \text{th item}$$

$$D_7 = \text{size of } \frac{7N}{10} \text{th item}$$

$$P_1 = \text{size of } \frac{N}{100} \text{th item}$$

$$P_{89} = \text{size of } \frac{89N}{100} \text{th item}$$

Then applying the formula for measuring the magnitude and interpolating the measurement, we get

$$Q_1 = l + \frac{(N/4) - c.f.}{f} \times i, \quad Q_3 = l + \frac{(3N/4) - c.f.}{f} \times i;$$

$$D_1 = l + \frac{(N/10) - c.f.}{f} \times i, \quad D_7 = l + \frac{(7N/10) - c.f.}{f} \times i;$$

$$P_1 = l + \frac{(N/100) - c.f.}{f} \times i, \quad P_{89} = l + \frac{(89N/100) - c.f.}{f} \times i$$

where

l = lower limit of the class in which Q_1, Q_3, D_1, D_7, P_1, P_{89} lie

$c.f.$ = cumulative frequency below the class in which these lie

f = frequency of these classes

i = class interval

MODE

It is that value of the variable which occurs itself the greatest number of times. Firstly, determine the modal class by grouping method or by inspection. Then we calculate mode by applying the formula

$$\text{Mode} = l_1 + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times i$$

where

i = class interval of the modal class

l_1 = lower limit of the modal class

Δ_1 = difference between the frequency of the modal class the frequency of the pre-modal class (ignoring signs)

Δ_2 = difference between the frequency of the modal class and the frequency of the post-modal class (ignoring sings)

$$\therefore \quad M = l_1 + \frac{|f_1 - f_0|}{|f_1 - f_0| + |f_1 - f_2|} \times i$$

When mode is ill defined,

Mode = 3 Median – 2 Mean

GEOMETRIC MEAN

$$\text{G.M.} = (x_1, x_2, \ldots, x_n)^{1/n}$$
$$= \text{antilog}\left(\frac{\Sigma \log x}{N}\right)$$

Discrete Series: G.M. = antilog $\left(\frac{\Sigma f \log x}{N}\right)$

Continuous Series: G.M. = antilog $\frac{\Sigma f \log m}{N}$, where m is the mid-value of various classes, N = total frequency.

HARMONIC MEAN

$$\text{H.M.} = \frac{N}{\Sigma \frac{1}{x_i}} = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}, \; x_i \neq 0$$

Discrete Series: H.M. $= \frac{N}{\Sigma(f/\text{m})}$, N = total frequency.

Continuous Series: H.M. $= \frac{N}{\Sigma(f/\text{m})}$, where m is the mid-point of various classes, N = Total frequency.

DISPERSION

Dispersion may be defined as the extent of the scatteredness of items around a measure of central tendency.

Methods of Measuring Dispersion

The following are the methods of measuring dispersion:

(i) Range
(ii) Semi-interquartile Range or Quartile Deviation
(iii) Mean Deviation
(iv) Standard Deviation

(i) *Range*

It is the difference between the highest and the lowest values in the series, i.e. Range = $x_h - x_l$

where x_h is the highest value and x_l is the lowest value.

(ii) *Quartile Deviation*

$$\text{Q.D.} = \frac{1}{2}(Q_3 - Q_1)$$

$$\text{Coefficient of Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

(iii) *Mean Deviation*

It is the mean of the absolute value of the deviations from some measure of central tendency.

M.D. $= \frac{\Sigma|D|}{N}$, where $|D|$ stands for the deviations from mean or median.

Coefficient of M.D. $= \frac{\text{Mean Deviation}}{\text{Mean}}$ (when deviations are taken from the mean)

Coefficient of M.D. $= \frac{\text{Mean Deviation}}{\text{Median}}$ (when deviations are taken from the median)

Discrete Series: M.D. $= \frac{\Sigma f|D|}{N}$, where $|D|$ denotes the deviations of the items from the median (or mean) ignoring signs.

Continuous Series: M.D. $= \frac{\Sigma f|D|}{N}$, where $|D|$ denotes the deviations of the mid-values of the classes from the mean (or median) ignoring signs.

STANDARD DEVIATION

The arithmetic mean of the square of deviations of the variable value from its actual arithmetic mean is known as **variance** and its square root is known as **standard deviation**.

Individual Observation. *Direct Method I:*

$$\sigma = \sqrt{\frac{\Sigma x^2}{N}}, \; x = x - \overline{x}$$

Direct Method II: $\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$

Discrete Series. *Direct Method I:*

$$\sigma = \sqrt{\frac{\Sigma fx^2}{N}}, \; x = x - \overline{x}$$

Direct Method II: $\sigma = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2}$

Short-cut Method: $\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$

where

d = deviations of items from assumed mean

$N = \Sigma f$

Step Deviation Method.

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times C$$

where $d' = \frac{X - A}{C}$, i.e. deviations of the items from the assumed mean divided by the common factor C.

Continuous Series. $\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2}$, $d' = \frac{m - A}{C}$

Variance = σ^2

Coefficient of variation $= \frac{\sigma}{\text{Mean}} \times 100$

COMBINED STANDARD DEVIATION

If there are two sets of observations containing n_1 and n_2 items with respective mean $\bar{x}_1$ and $\bar{x}_2$ and standards deviations σ_1 and σ_2, then the mean $\bar{x}$ and the standard deviations of the $n_1 + n_2$ observations, taken together, are

$$\bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$\sigma^2 = \frac{1}{n_1 + n_2}[n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)]$$

$$d_1 = \bar{x}_{12} - \bar{x}_1,\ d_2 = \bar{x}_{12} - \bar{x}_2$$

MORE PROPERTIES

(1) For a symmetrical distribution

Mean $\pm$ 0.6745	covers 50% of items
Mean $\pm$ 1σ	covers 68.27% of items
Mean $\pm$ 2σ	covers 95.45% of items
Mean $\pm$ 3σ	covers 99.93% of items

(2) The sum of squares of deviations of items in the series from their arithmetic mean is minimum.

(3) $\text{Q.D.} = \frac{2}{3}\sigma$

$\text{M.D.} = \frac{4}{5}\sigma$, $\quad 4\sigma = 5\text{M.D.} = 6\text{Q.D.}$

$\text{Q.D.} = \frac{5}{6}\text{M.D.}$

SOLVED QUESTIONS

1. The mean of the series $1, 2, 4, 8, 16, \ldots, 2^n$ is

(a) $\frac{2^n - 1}{n}$ (b) $\frac{2^n - 1}{n+1}$

(c) $\frac{2^{n+1} - 1}{n+1}$ (d) $\frac{2^n + 1}{n}$

Ans. (c)

Solution: Given: series $1, 2, 4, 8, 16, \ldots, 2^n$.

We know that the mean of the given series

$$= \frac{1 + 2 + 4 + 8 + 16 + \cdots + 2^n}{n+1}$$

$$= \frac{1 + 2^1 + 2^2 + 2^3 + \cdots + 2^n}{n+1}$$

$$= \frac{2^{n+1} - 1}{(n+1)(2-1)} = \frac{2^{n+1} - 1}{n+1}$$

2. The average of n numbers $x_1, x_2, x_3, \ldots, x_n$ is M. If x_1 is replaced by x, then the new average is

(a) $M - x_1 + x$ (b) $\frac{(n-1)M - x_1 + x}{n}$

(c) $\frac{(n-1)M - x_1 + x}{n}$ (d) $\frac{nM - x_1 + x}{n}$

Ans. (d)

Solution: Given: Numbers $x_1, x_2, x_3, \ldots, x_n$ and their average = M. We know that the average of n numbers

$$M = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

or $M + \frac{x}{n} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$

or $\frac{nM + x}{n} = \frac{x_1 + x_2 + x_3 + \cdots + x_n + x}{n}$

or $\frac{nM + x}{n} = \frac{x_1}{n} + \frac{x_2 + x_3 + \cdots + x_n + x}{n}$

or $\dfrac{nM + x - x_1}{n} = \dfrac{x_2 + x_3 + \cdots + x_n + x}{n}$

or new average

$$= \left(\frac{x_2 + x_3 + \cdots + x_n + x}{n}\right) = \frac{nM - x_1 + x}{n}$$

3. The arithmetic mean for the following frequency distribution:

Class	0–8	8–16	16–24	24–32	32–40	40–48
Frequency	8	7	16	24	15	7

is

(a) 24.40 (b) 25.40
(c) 26.20 (d) 28.20

Ans. (b)

Solution: Given data

Class	0–8	8–16	16–24	24–32	32–40	40–48
Frequency	8	7	16	24	15	7

We know that mid-values of class (x) = 4, 12, 20, 28, 36, 44

We also know that $\Sigma f = 8 + 7 + 16 + 24 + 15 + 7 = 77$

Let the assumed mean be (A) = 28 and h = 8

Therefore, $\left(u = \dfrac{x - A}{h}\right) = \dfrac{4 - 28}{8}, \dfrac{12 - 28}{8}, \dfrac{20 - 28}{8},$

$\dfrac{28 - 28}{8}, \dfrac{36 - 28}{8}, \dfrac{44 - 28}{8} = -3, -2, -1, 0, 1, 2$

and $\Sigma fu = 8 \times (-3) + 7(-2) + 16 \times (-1) + 24 \times (-0) + (15 \times 1) + (7 \times 2)$

$$= -24 - 14 - 16 + 0 + 15 + 14 = -25$$

Therefore, the arithmetic mean of the data

$$= A + h\frac{\Sigma fu}{\Sigma f} = 28 + 8\left(-\frac{25}{77}\right) = 28 - \frac{200}{77} = 25.40$$

4. In the frequency distribution of the discrete data given below, the frequency x against value 0 is missing.

Variable x:	0	1	2	3	4	5
Frequency f:	x	20	40	40	20	4

If the mean is 2.5, then the missing frequency x will be

(a) 0 (b) 1
(c) 3 (d) 4

Ans. (d)

Solution: Given: Mean $(\bar{x}) = 2.5$ and discrete data

Variable x:	0	1	2	3	4	5
Frequency f:	x	20	40	40	20	4

We know that

$$\Sigma f = x + 20 + 40 + 40 + 20 + 4 = 124 + x$$

We also know that

$$\Sigma fx = (0 \times x) + (1 \times 20) + (2 \times 40) + (3 \times 40) + (4 \times 20) + (5 \times 4)$$

$$= 0 + 20 + 80 + 20 = 320$$

Therefore, mean $(\bar{x}) = \dfrac{\Sigma fx}{\Sigma f} = \dfrac{320}{124 + x}$ or $2.5 = \dfrac{320}{124 + x}$

or $124 + x = \dfrac{320}{2.5} = 128$ or $x = 128 - 124 = 4$

5. If a variate takes values $a, ar, ar^2, \ldots, ar^{n-1}$, which of the relations between means hold?

(a) $AH = G^2$ (b) $\dfrac{A + H}{2} = G$
(c) $A > G > H$ (d) $A = G = H$

Ans. (a)

Solution: $A = \dfrac{a(1 + r + r^2 + \cdots + r^{n-1})}{n} = \dfrac{a}{n}\dfrac{1 - r^n}{1 - r}$

$$G = (a \cdot ar \cdot ar^2 \ldots ar^{n-1})^{1/n}$$

$$= a \cdot r^{\frac{1+2+\cdots(n-1)}{n}} = ar^{\frac{n-1}{2}}$$

$$\frac{1}{H} = \frac{1}{na}\left(1 + \frac{1}{r} + \frac{1}{r^2} + \cdots + \frac{1}{r^{n-1}}\right) = \frac{1}{na} \cdot \frac{1 - (1/r^n)}{1 - (1/r)}$$

$$\Rightarrow H = \frac{na(1 - r)r^{n-1}}{1 - r^n}$$

$$\therefore AH = a^2 r^{n-1} = G^2$$

6. The mean weight of 150 persons in a group is 60 kg. The mean weight of men in the group is 70 kg and that of the women is 55 kg. The number of men and women are

(a) 50 and 100 (b) 60 and 90
(c) 100 and 50 (d) 75 and 75

Ans. (a)

Solution: Number of persons = 150; their mean weight = 60 kg; mean weight of men $(\bar{x}_1) = 70$ kg and mean weight of women $(\bar{x}_2) = 55$ kg.

Let n_1 and n_2 be the number of men and number of women respectively.

We know that the total number of persons $(n_1 + n_2) = 150$ or $n_2 = 150 - n_1$.

We also know that the mean weight of all persons

$$(\bar{x}) = \frac{(n_1\bar{x}_1 + n_2\bar{x}_2)}{n_1 + n_2}$$

or $$60 = \frac{70n_1 + 55n_2}{150}$$

or $70n_1 + 55n_2 = (60\times150) = 9000$

or $14n_1 + 11n_2 = 1800$ or $14n_1 + 11(150 - n_1) = 1800$

or $14n_1 + 1650 - 11n_1 = 1800$

or $3n_1 = (1800 - 1650) = 150$

or $n_1 = 50$ and $n_2 = 100$

7. The median of the observations 12, 17, 3, 14, 5, 8, 7, 15 is

(a) 12 (b) 14
(c) 8 (d) 10

Ans. (d)

Solution: Given observations are 12, 17, 3, 14, 5, 8, 7, 15.
We know that the number of observations (n) = 8.
Arranging the given observations in the ascending order, we get 3, 5, 7, 8, 12, 14, 15 and 17.
Since the number of observations is even, therefore their median

$= \text{Arithmetic mean of } \left(\frac{n}{2}\right)\text{th and } \left(\frac{n}{2}+1\right)\text{th observations}$

$= \text{Arithmetic mean of } \left(\frac{8}{2}\right)\text{th and } \left(\frac{8}{2}+1\right)\text{th observations}$

= Arithmetic mean of 4th and 5th observations

$= \frac{(8+12)}{2} = \frac{20}{2} = 10$

8. The marks obtained by 10 students in an examination are 22, 26, 14, 30, 18, 11, 35, 41, 12, 32. What is the median mark?

(a) 14 (b) 22
(c) 20 (d) 24

Ans. (d)

Solution: Number of students (n) = 10 and marks obtained by them = 22, 26, 14, 30, 18, 11, 35, 41, 12, 32
Arranging the given marks in the ascending order, we get 11, 12, 14, 18, 22, 26, 30, 32, 35, 41.
Since the number of students is even, therefore median of their marks

$= \text{Arithmetic mean of } \left(\frac{n}{2}\right)\text{th and } \left(\frac{n+2}{2}\right)\text{th marks}$

$= \text{Arithmetic mean of } \left(\frac{10}{2}\right)\text{th and } \left(\frac{10+2}{2}\right)\text{th marks}$

= Arithmetic mean of 5th and 6th marks

$= \frac{22+26}{2} = 24$

9. In a moderately skewed distribution, the values of mean and median are 5 and 6 respectively. Then the value of mode is

(a) 8 (b) 11
(c) 16 (d) 21

Ans. (a)

Solution: Given mean = 5 and median = 6
We know that in moderately skewed distribution, mode = (3 median – 2 mean)
$= (3\times6) - (2\times5) = 18 - 10 = 8$

10. If the difference between mean and mode is 63, then the difference between mean and median is

(a) 21 (b) 31.5
(c) 48.5 (d) 50.9

Ans. (a)

Solution: Given: Mean – mode = 63
We know that mode = 3 median – 2 mean
Let mean > median > mode
Therefore, mean – mode = 3 (mean – median)
or 63 = 3 (mean – median) or mean – median = 21
Therefore, difference between mean and median = 21.

11. Which of the following mode formulae is applicable for the continuous series?

(a) $\text{mode} = l + \left(\frac{f_1}{f - f_1 - f_2}\right)\times h$

(b) $\text{mode} = l + \left(\frac{f - f_1}{f - f_1 - f_2}\right)\times h$

(c) $\text{mode} = l + \left(\frac{f - f_1}{2f - f_1 - f_2}\right)\times h$

(d) $\text{mode} = l + \left(\frac{2f - f_1}{f - f_1 - f_2}\right)\times h$

Ans. (c)

Solution: We know that the mode value of a distribution is that value of the variable for which the frequency is maximum.
We also know that in the case of a discrete frequency distribution, the value of mode is determined by the method of a continuous series.
We also know that for a continuous series,

$$\text{mode} = l + \left(\frac{f - f_1}{2f - f_1 - f_2}\right)\times h$$

where l = lower limit of modal class, h = width of the modal class, f_1 = frequency of the class preceding the modal class, f_2 = frequency of the class following the modal class and f = frequency of the modal class.

12. The weighted A.M. of the first n natural numbers whose weights are equal to the corresponding numbers is equal to

(a) $2n+1$ (b) $\frac{1}{2}(2n+1)$
(c) $\frac{1}{3}(2n+1)$ (d) $\frac{2n+1}{6}$

Ans. (c)

Solution: The required weighted mean is

$$\overline{x}=\frac{1\cdot 1+2\cdot 2+3\cdot 3+\cdots+n\cdot n}{1+2+3+\cdots+n}$$

$$=\frac{[n(n+1)(2n+1)]/6}{[n(n+1)]/2}=\frac{2n+1}{3}$$

13. The A.M. of n numbers of a series is $\overline{x}$. If the sum of the first $(n-1)$ terms is k, then the nth number is

(a) $\overline{x}-k$ (b) $n\overline{x}-k$
(c) $\overline{x}-nk$ (d) $n\overline{x}-nk$

Ans. (b)

Solution: Let the n numbers be $x_1, x_2, \ldots, x_n$. Then

$$\overline{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$$

$$\Rightarrow \quad \overline{x}=\frac{x_1+x_2+\cdots+x_{n-1}+x_n}{n}$$

$$\Rightarrow \quad \overline{x}=\frac{k+x_n}{n} \qquad [\because \; x_1+x_2+\cdots+x_{n-1}=k]$$

$$\Rightarrow \quad x_n=n\overline{x}-k$$

14. The mean of a set of numbers is $\overline{x}$. If each number is divided by 3, then the new mean is

(a) $\overline{x}$ (b) $\overline{x}+3$
(c) $3\overline{x}$ (d) $\dfrac{\overline{x}}{3}$

Ans. (d)

Solution: Let $x_1, x_2, \ldots, x_n$ be n numbers. Then $\overline{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$.

If each number is divided by 3, then the new mean $\overline{y}$ is given by

$$\overline{y}=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{3}\right)=\frac{1}{3}\left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)=\frac{\overline{x}}{3}$$

15. The mean and S.D. of 63 children on an arithmetic test are respectively 27.6 and 7.1. To them are added a new group of 26 children who had less training and whose mean is 19.2 and S.D. is 6.2. The values of the combined groupsdiffer from the original as to (i) the mean and (ii) the S.D. by

(a) 2.5, 0.7 (b) 2.3, 0.8
(c) 1.5, 0.9 (d) none of these

Ans. (a)

Solution: Mean of S.D. of the combined group is

$$m=\frac{63\times 27.6+26\times 19.2}{63+26}=25.1$$

Thus, A.M. is decreased by $27.6-25.1=2.5$

$$\sigma^2=\frac{63\times(7.1)^2+26\times(6.2)^2}{89}+\frac{63(25.1-27.6)^2+26(25.1-19.2)^2}{89}$$

$$\Rightarrow \sigma = 7.8 \text{ approx.}$$

EXERCISE SET

1. The mean of n items is $\overline{x}$. If the first item is increased by 1, second by 2 and so on, then the new mean is

(a) $\overline{x}+n$ (b) $\overline{x}+\dfrac{n}{2}$
(c) $\overline{x}+\dfrac{n+1}{2}$ (d) none of these

2. If g_1 and g_2 are the geometric means of two series of n_1 and n_2 items. Then the G.M. of the series obtained on combining them is

(a) $[(g_1)^{n_1}\cdot(g_2)^{n_2}]^{\frac{1}{n_1+n_2}}$ (b) $(g_1\,g_2)^{\frac{n_1}{n_1+n_2}}$
(c) $(g_1\,g_2)^{\frac{n_2}{n_1+n_2}}$ (d) $(g_1\,g_2)^{\frac{n_1 n_2}{n_1+n_2}}$

3. If a variable takes discrete values $x+4$, $x-\dfrac{7}{2}$, $x-\dfrac{5}{2}$, $x-3$, $x-2, x+\dfrac{1}{2}$; $x-\dfrac{1}{2}$, $x+5 (x>0)$, then the median is

(a) $x-\dfrac{5}{4}$ (b) $x-\dfrac{1}{2}$
(c) $x-2$ (d) $x+\dfrac{5}{4}$

4. If the coefficient of correlations between x and y is 0.28, covariance between x and y is 7.6 and the variance of x is 9, then the S.D. of y series is

(a) 9.8 (b) 10.1
(c) 9.05 (d) 10.05

5. If $\overline{x}$ is the mean of $x_1, x_2, x_3, \ldots, x_n$, then the mean of $(x_1+a), (x_2+a), \ldots, (x_n+a)$ is

(a) $\overline{x}+2a$ (b) $\overline{x}+a$
(c) $\overline{x}$ (d) $2\overline{x}+a$

6. The median from the data:

Class	*Frequency*
110–120	6
120–130	25
130–140	48
140–150	72
150–160	116
160–170	60
170–180	38
180–190	22
190–200	3

is

(a) 153.79 (b) 153.91
(c) 165.18 (d) 165.93

7. If mean, mode and S.D. of a frequency distribution are 45, 49 and 4, respectively, then Karl Pearson's coefficient of skewness is

(a) –1 (b) 1
(c) 0 (d) –(1/2)

8. The mean and S.D. of 1, 2, 3, 4, 5, 6 is

(a) $\frac{7}{2}, \sqrt{\frac{35}{12}}$ (b) $\frac{7}{2}, \sqrt{3}$

(c) 3, 3 (d) $3, \frac{35}{12}$

9. The mean of 5 numbers is 18. If one number is excluded, their mean becomes 16, then the excluded number is

(a) 18 (b) 25
(c) 26 (d) 30

10. If the mean of the distribution

x	1	2	3	4	5
f	4	5	k	1	2

is 2.6, then the value of k is

(a) 8 (b) 10
(c) 12 (d) 18

11. A group of 10 items has mean 6. If the mean of 4 of these items is 7.5, then the mean of the remaining items is

(a) 6.5 (b) 5.5
(c) 4.5 (d) 5.0

12. The regression lines of x on y and y on x are $x = 4y + 5$ and $y = kx + 4$ respectively. The limits of k are given by

(a) $0 < k < \frac{1}{4}$ (b) $-1 < k < \frac{1}{2}$

(c) $-1 < k < 1$ (d) none of these

13. If a variable takes values 0, 1, 2,..., n with frequencies proportional to $q^n, nC_1pq^{n-1}, nC_2p^2q^{n-2},\ldots, p^n$, where $p+q=1$, then the mean is

(a) np (b) nq
(c) npq (d) np^2

14. The mean age of a combined group of men and women is 25 years. If the mean age of the group of men is 26 and that of the group of women is 21, then the percentages of men and women in the group are

(a) 46, 60 (b) 80, 20
(c) 20, 80 (d) 60, 40

15. The means of two samples of size 200 and 300 were found to be 25 and 10, respectively. Their standard deviations were 3 and 4, respectively. The variance of the combined sample of size 500 is

(a) 64 (b) 65.2
(c) 67.2 (d) 64.2

16. The mean of 9 observations is 100 and that of 6 observations is 80, then the combined mean of all 15 observations is

(a) 100 (b) 92
(c) 90 (d) 80

17. One set containing 5 numbers has mean 8 and variance 18 and the second set containing 3 numbers has mean 8 and variance 24. Then the variance of the combined set of numbers is

(a) 42 (b) 20.25
(c) 18 (d) none of these

18. A person purchases 1 kg of tomatoes from each of the 4 places at the rate of 1 kg, 2 kg, 3 kg and 4 kg per rupee, respectively. On the average, if he has purchased x kg of tomatoes per rupee, then the value of x is

(a) 2.5 (b) 1.92
(c) 2 (d) none of these

Assertion-Reason Type Questions

Each of these questions contains two statements: Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below:

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.
(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.
(c) Statement I is true; Statement II is false.
(d) Statement I is false; Statement II is true.

19. Suppose two groups of scores A and B are such that $A = (x, x+2, x+4)$ and $B = (x-2, x+2, x+6)$.

Statement I: Group B has more variability than group A.

Statement II: The value of mean for group B is more than that of group A.

20. **Statement I:** The algebraic sum of the deviations of 20 observations measured from 30 is 2. The mean value of the observation is 30.

Statement II: The sum of deviations from their mean is zero.

21. **Statement I:** The mean of the squares of the first n natural numbers is $\frac{(n+1)(2n+1)}{6}$.

Statement II: $\Sigma n = \frac{n(n+1)}{2}$.

22. **Statement I:** The standard deviation of the variable $\frac{ax+b}{c}$ is $\left|\frac{a}{c}\right|\alpha$.

Statement II: The standard deviation of a linear equation is $\sigma \times |\text{coefficient of } x|$.

Previous Years' Questions

23. In a class of 100 students, there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class are 72, then what is the average marks of the girls?

(a) 73 (b) 65
(c) 68 (d) 74 **[AIEEE 2002]**

24. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set

(a) is decreased by 2
(b) is two times the original median
(c) remains the same as that of the original set
(d) is increased by 2 **[AIEEE 2003]**

25. In an experiment with 15 observations on x, the following results were available $\Sigma x^2 = 2830$ and $\Sigma x = 170$. One observation 20 was replaced by 30 which was correct. The corrected variance is

(a) 80.33 (b) 78.00
(c) 188.66 (d) 177.33

[AIEEE 2003]

26. Consider the following statements:

(i) Mode can be computed from histogram
(ii) Median is not independent of change of scale
(iii) Variance is independent of change of origin and scale

Which of these is/are correct?

(a) Only (i) (b) Only (ii)
(c) Only (i) and (ii) (d) Only (i), (ii) and (iii)

[AIEEE 2004]

27. In a series of $2n$ observations, half of them equal a and the remaining half equal $-a$. If the standard deviation is 2, then $|a|$ equals

(a) $1/n$ (b) $\sqrt{2}$
(c) 2 (d) $\sqrt{2}/n$

[AIEEE 2004]

28. Let $x_1, x_2, \ldots, x_x$ be n observations such that $\Sigma x_i^2 = 400$ and $\Sigma x_i = 80$. Then, a possible value of n among the following is

(a) 12 (b) 9
(c) 18 (d) 15

[AIEEE 2005]

29. If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately

(a) 24.0 (b) 25.5
(c) 20.5 (d) 22.0

[AIEEE 2005]

30. Suppose a population A has 100 observations 101, 102, …, 200 and another population B has 100 observations 151, 152,…, 250. If V_A and V_B represent the variances of the two populations respectively, then V_A/V_B is

(a) 9/4 (b) 4/9
(c) 2/3 (d) 1 **[AIEEE 2006]**

31. The average mark of boys in a class is 52 and that of girls is 42. The average mark of boys and girls combined is 50. The percentage of boys in the class is

(a) 40% (b) 20%
(c) 80% (d) 60%

[AIEEE 2007]

32. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives the possible values of a and b?

(a) $a = 3$, $b = 4$ (b) $a = 0$, $b = 7$
(c) $a = 5$, $b = 2$ (d) $a = 1$, $b = 6$

[AIEEE 2008]

33. If the mean deviation of numbers $1, 1+d, 1+2d, \ldots, 1+100d$ from their mean is 255, then d is equal to

(a) 10.0 (b) 20.0
(c) 10.1 (d) 20.2

[AIEEE 2009]

34. **Statement I:** The variance of the first n even natural numbers is $(n^2 - 1)/4$.

Statement II: The sum of the first n natural numbers is $n(n + 1)/2$ and the sum of the squares of the first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.

(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

[AIEEE 2009]

35. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is
(a) 5/2 (b) 11/2
(c) 6 (d) 13/2

[AIEEE 2010]

36. The number of observations in a group is 40. If the average of the first 10 is 4.5 and that of the remaining 30 is 3.5, then the average of the whole group is
(a) 1/5 (b) 15/4
(c) 4 (d) 8

[AMU 1992, Delhi CEE 1996]

37. The mean of the values 0, 1, 2,..., n having corresponding weights ${}^nc_0, {}^nc_1, {}^nc_2, \ldots, {}^nc_n$ respectively is
(a) $\dfrac{2^n}{n+1}$ (b) $\dfrac{2^{n+1}}{n(n+1)}$
(c) $\dfrac{n+1}{2}$ (d) $\dfrac{n}{2}$

[AMU 1990, Ph. CET 1998]

38. The mean deviation of the numbers 3, 4, 5, 6, 7 is
(a) 0 (b) 1.2
(c) 5 (d) 25

[AMU 1992]

39. If the regression coefficients are 0.8 and 0.2, then the value of the coefficient of correlation is
(a) 0.16 (b) 0.4
(c) 0.04 (d) 0.164 **[DCE 2003]**

40. The coefficient of correlation (r) and the two regression coefficients b_{yx} and b_{xy} are related as
(a) $r = \dfrac{b_{xy}}{b_{yx}}$ (b) $r = b_{xy} \times b_{yx}$
(c) $r = b_{xy} + b_{yx}$ (d) $r = (\text{sign } b_{yx})\sqrt{b_{xy} b_{yx}}$

[DCE 2003]

41. If the variable takes the values 0, 1, 2,..., n with frequencies proportional to the binomial coefficients $C(n, 0), C(n, 1), C(n, 2), \ldots, C(n, n)$ respectively, then the variance of the distribution is
(a) n (b) $\sqrt{n}/2$
(c) $n/2$ (d) $n/4$ **[DCE 2004]**

42. The angle between two lines of regression is given by
(a) $\tan^{-1}\left[\dfrac{b_{xy} - (1/b_{yx})}{1 - (b_{xy}/b_{yx})}\right]$ (b) $\tan^{-1}\left(\dfrac{b_{yx} \cdot b_{xy} - 1}{b_{yx} + b_{xy}}\right)$
(c) $\tan^{-1}\left[\dfrac{b_{xy} - (1/b_{yx})}{1 + (b_{xy}/b_{yx})}\right]$ (d) $\tan^{-1}\left(\dfrac{b_{yx} - b_{xy}}{1 + b_{yx}.b_{xy}}\right)$

[DCE 2005]

43. If two lines of regression are $3\overline{x} - 2\overline{y} + 1 = 0$ and $2\overline{x} - \overline{y} - 2 = 0$, then $(\overline{x}, \overline{y})$ is
(a) (8, 5) (b) (5, 8)
(c) (5, 5) (d) (8, 8) **[DCE 2006]**

44. The two lines of regression are given by $3x + 2y = 26$ and $6x + y = 31$. The coefficient of correlation between x and y is
(a) –1/3 (b) 1/3
(c) –1/2 (d) 1/2 **[AMU 2003]**

45. If $\Sigma x = 15$, $\Sigma y = 36$, $\Sigma xy = 110$, $n = 5$, then cov(x, y) equals
(a) 1/5 (b) –1/5
(c) 2/5 (d) –2/5 **[AMU 2004]**

46. The mean deviation and coefficient of mean deviation from the data
Weight (in kg) 54, 50, 40, 42, 51, 45, 47, 55, 57 is
(a) 0.0900 (b) 0.0956
(c) 0.0056 (d) 0.0946 **[AMU 2005]**

47. If the standard deviation of a variable x is σ, then the standard deviation of another variable $(ax + b)/c$ is
(a) $(\sigma a + b)/c$ (b) $\sigma a/c$
(c) σ (d) none of these

48. The mode of the series 3, 4, 2, 6, 1, 7, 6, 7, 6, 8, 9, 5 is
(a) 5 (b) 6
(c) 7 (d) 8 **[AMU 2007]**

49. The median from the table

Value	7	8	10	9	11	12	1
Frequency	2	1	4	5	6	1	[illegible]

is
(a) 100 (b) 10
(c) 110 (d) 1110 **[2009 AMU]**

50. For the arithmetic progression $a, (a+d), (a+2d), (a+3d), \ldots, (a+2nd)$, the mean deviation from the mean is
(a) $\dfrac{n(n+1)d}{2n-1}$ (b) $\dfrac{n(n+1)d}{2n+1}$
(c) $\dfrac{n(n-1)d}{2n+1}$ (d) $\dfrac{(n+1)d}{2}$

[Kerala CEE 2006]

51. The intersecting point of two regression lines is
(a) $(\bar{x}, 0)$ (b) $(0, \bar{y})$ (c) $(0, 0)$ (d) $(\bar{x}, \bar{y})$
[Kerala CEE 2004]

52. If the two lines of regression are $4x+3y+7=0$ and $3x+4y+8=0$, then the means of x and y are
(a) $\frac{-4}{7}, \frac{-11}{7}$ (b) $\frac{-4}{7}, \frac{11}{7}$ (c) $\frac{4}{7}, \frac{-11}{7}$ (d) 4, 7
[BCECE 2005]

53. For the given data, the calculation corresponding to all value of variates (x, y) is the following $\Sigma(x-\bar{x})^2 = 36$, $\Sigma(y-\bar{y})^2 = 25$, $\Sigma(x-\bar{x})(y-\bar{y}) = 20$.
The Kerl Pearson's correlation coefficient is
(a) 0.2 (b) 0.5 (c) 0.66 (d) 0.33
[Jamia Millia Islamia 2006]

54. The corelation coefficient between x and y from the following data $\sum x = 40$, $\sum y = 50$, $\sum xy = 220$, $\sum x^2 = 200$ $\sum y^2 = 262$, $n = 10$ is
(a) 0.89 (b) 0.76 (c) 0.91 (d) 0.98
[MP PET 2006]

55. In a bivariate data, $\sum x = 30$, $\sum y = 400$, $\sum x^2 = 196$, $\sum xy = 850$, $n = 10$. The regression coefficient of y on x is
(a) –3.1 (b) –3.2 (c) –3.3 (d) –3.4
[MP PET 2007]

56. If the lines of regression are $3x+12y=19$ and $3y+9x=46$, then r_{xy} will be
(a) 0.289 (b) –0.289 (c) 0.209 (d) none of these
[MP PET 2008]

57. If $x_1, x_2, \ldots, x_{18}$ are the observations such that $\sum_{j=1}^{18}(x_j-8) = 9$ and $\sum_{j=1}^{18}(x_j-8)^2 = 45$, then the standard deviation of these observations is
(a) $\sqrt{81/34}$ (b) 5 (c) $\sqrt{5}$ (d) 3/2
[J & K CET 2004]

58. A scientist is weighing each of 30 fishes. Their mean weight worked out to be 30 g and the standard deviation to be 2 g. Later, it was found that the measuring scale was misaligned and always underreported every fish weight by 2 g. The correct mean and standard deviation (in gram) of fishes are respectively
(a) 32, 2 (b) 32, 4 (c) 28, 2 (d) 28, 4
[AIEEE 2011]

59. Let $x_1, x_2, \ldots, x_n$ be n observations and let $\bar{x}$ be their arithematic mean and σ^2 be their variance.
Statement 1: Variance of $2x_1, 2x_2, \ldots, 2x_n$ is $4\sigma^2$.
Statement 2: Arithmetic mean of $2x_1, 2x_2, \ldots, 2x_n$ is $4\bar{x}$.
(a) Statement 1 is false, statement 2 is true
(b) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1
(c) Satement 1 is true, statement 2 is true, statement 2 is not a correct explanation for statement 1
(d) Satement 1 is true, statement 2 is false
[AIEEE 2012]

ANSWERS

Exercise Set

1. (c)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (a)	**8.** (a)	**9.** (c)	**10.** (a)
11. (d)	**12.** (a)	**13.** (a)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (b)	**19.** (c)	**20.** (d)
21. (c)	**22.** (a)	**23.** (b)	**24.** (c)	**25.** (b)	**26.** (c)	**27.** (c)	**28.** (c)	**29.** (a)	**30.** (d)
31. (c)	**32.** (a)	**33.** (c)	**34.** (d)	**35.** (b)	**36.** (b)	**37.** (d)	**38.** (b)	**39.** (b)	**40.** (d)
41. (d)	**42.** (b)	**43.** (b)	**44.** (c)	**45.** (c)	**46.** (b)	**47.** (b)	**48.** (b)	**49.** (b)	**50.** (b)
51. (d)	**52.** (a)	**53.** (c)	**54.** (c)	**55.** (c)	**56.** (b)	**57.** (d)	**58.** (a)	**59.** (d)	

HINTS AND SOLUTIONS—EXERCISE SET

1. Let $x_1, x_2, \ldots, x_n$ be n items.

Then $\bar{x} = \frac{1}{n}\sum x_i$

Let $y_1 = x_1+1$, $y_2 = x_2+2$, $y_3+3, \ldots, y_n = x_n+n$. Then the mean of the new series is

$$\frac{1}{n}\sum_{n=1}^{n} y_i = \frac{1}{n}\sum_{n=1}^{n}(x_i+i) = \frac{1}{n}\sum_{n=1}^{n} x_i + \frac{1}{n}(1+2+3+\cdots+n)$$

$$\bar{y} = \bar{x} + \frac{1}{n}\cdot\frac{n(n+1)}{2} = \bar{x} + \frac{n+1}{2}$$

2. Let $g_1 = (x_1x_2\ldots x_{n_1})^{1/n_1}$ and $g_2 = (y_1y_2\ldots y_{n_2})^{1/n_2}$. If g is the G.M. of the combined series, then

$$g = [(x_1x_2\ldots x_{n_1})\times(y_1y_2\ldots y_{n_2})]^{1/(n_1+n_2)}$$
$$= (g_1^{n_1}\cdot g_2^{n_2})^{1/(n_1+n_2)}$$

3. Arranging the given values in ascending order of magnitude, we get

$$x-\frac{7}{2},\ x-3,\ x-\frac{5}{2},\ x-2,\ x-\frac{1}{2},\ x+\frac{1}{2},\ x+4,\ x+5$$

There are 8 observations in this series. Therefore, Median = A.M. of 4th and 5th observations

$$= \text{A.M. of } (x-2) \text{ and } (x-1/2)$$

$$= \frac{x-2+x-(1/2)}{2} = x - \frac{5}{4}$$

4. $\rho = \dfrac{\operatorname{cov}(x, y)}{\sigma_x \sigma_y}$

$\Rightarrow\ 0.28 = \dfrac{7.6}{\sigma_y \times 3}$

$\Rightarrow\ \sigma_y = 9.05$

5. Given mean of numbers $(x_1, x_2, x_3, \ldots, x_n) = \bar{x}$

We know that mean $(\bar{x}) = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$

Let the mean of $(x_1 + a), (x_2 + a), \ldots, (x_n + a) = \bar{x}$

Therefore, mean

$$(\bar{x}) = \frac{(x_1+a)+(x_2+a)+\cdots+(x_n+a)}{n}$$

$$= \frac{(x_1+x_2+\cdots+x_n)+na}{n}$$

$$= \frac{x_1+x_2+\cdots+x_n}{n} + a = (\bar{x}) + a$$

6.

Class interval	f	*c.f.*
110–120	6	6
120–130	25	31
130–140	48	79
140–150	72	151
150–160	116	267
160–170	60	327
170–180	38	365
180–190	22	387
190–200	3	390

$$N = \Sigma f = 390$$

Median = size of (N/2)th item = size of (390/2) = 195th item.

$\therefore$ Median class = 150 – 160

$l = 150,\ h = 10,\ f = 116,\ (N/2) = 195,\ C = 151$

$$\text{Median } = l + \frac{(N/2) - C}{f} \times h = 150 + \frac{195-151}{116} \times 10$$

$$= 150 + \frac{440}{116} = 150 + \frac{110}{29} = 150 + 3.79$$

$$= 153.79$$

7. Coefficient of skewness $= \dfrac{\text{Mean} - \text{Mode}}{\sigma}$

$$= \frac{45-49}{4} = \frac{-4}{4} = -1$$

8. Mean $\bar{x} = \dfrac{1+2+3+4+5+6}{6} = \dfrac{21}{6} = \dfrac{7}{2}$

$\therefore$ S.D. $= \sigma = \sqrt{\dfrac{\Sigma x^2}{N} - \left(\dfrac{\Sigma x}{N}\right)^2}$

$$= \sqrt{\frac{1}{6}(1+4+9+16+25+36) - \frac{49}{4}}$$

$$= \sqrt{\frac{91}{6} - \frac{49}{4}} = \sqrt{\frac{182-147}{12}} = \sqrt{\frac{35}{12}}$$

9. Initial numbers = 5; initial mean = 18; final numbers = 4 and final mean = 16.

We know that the sum of the intial numbers = (5 × 18) = 90

Similarly, the sum of the final numbers $= (4 \times 16) = 64$

Therefore, excluded number $= (90 - 64) = 26$

10. $\Sigma fx = 4 + 10 + 3k + 4 + 10 = 3k + 28$

$\Sigma f = 12 + k$

Mean $= \dfrac{\Sigma fx}{\Sigma f}$

$\Rightarrow\ 2.6 = \dfrac{3k+28}{12+k} \qquad \Rightarrow\ \dfrac{13}{5} = \dfrac{3k+28}{12+k}$

$\Rightarrow\ 156 + 13k = 15k + 140$

$\Rightarrow\ 2k = 16 \qquad \Rightarrow\ k = 8$

11. Total of 10 items $= 10 \times 6 = 60$

Total of 4 items $= 4 \times 7.5 = 30$

Mean of the remaining 6 items $= \dfrac{60-30}{6} = 5 = 5.0$

12. We have $b_{yx} = k$ and $b_{xy} = 4$

$$\Rightarrow \qquad r^2 = b_{yx} \cdot b_{xy} = 4k$$

Now since r^2 is the square of the real quantity, it should be non-negative. Also since $0 < r^2 < 1$, we have $0 < 4k < 1$ or $0 < k < (1/4)$.

13. Mean $= \dfrac{(0 \cdot q^n + 1 \cdot {}^nC_1 pq^{n-1} + 2\,{}^nC_2 p^2 q^{n-2} + \cdots + n \cdot p^n)k}{k(q^n + {}^nC_1 pq^{n-1} + {}^nC_2 p^2 q^{n-2} + \cdots + p^n)}$

$$= \frac{{}^nC_1 pq^{n-1} + 2 \cdot {}^nC_2 p^2 q^{n-2} + \cdots + n \cdot p^4}{(q+p)^n}$$

$$= npq^{n-1} + 2 \cdot \frac{n(n-1)}{2} p^2 q^{n-2} + \cdots + n \cdot p^n$$

$$(\because\ q + p = 1)$$

$$= np[q^{n-1} + (n-1)pq^{n-2} + \cdots + p^{n-1}]$$

$$= np \cdot (q+q)^{n-1} = np(1)^{n-1} = np$$

$\therefore$ Mean $= np$

14. Let n_1 and n_2 be the number of men and women respectively, in a group. According to the given condition,

$$\frac{n_1\times 26+n_2\times 21}{n_1+n_2}=25$$

$\Rightarrow\ 26n_1+21n_2=25n_1+25n_2$

$\Rightarrow\ n_1=4n_2 \qquad \Rightarrow\ \frac{n_1}{n_2}=\frac{4}{1}$

$\Rightarrow\ \frac{n_1}{n_2}=\frac{80}{20}$

15. Combined mean $\bar{x}=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$

$$=\frac{200\times 25+300\times 10}{500}=16$$

Let $d_1=\bar{x}_1-\bar{x}=25-16=9$

$d_2=\bar{x}_2-\bar{x}=10-16=-6$

We know that $\sigma^2=\frac{n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)}{n_1+n_2}$

$$=\frac{200(9+81)+300(16+36)}{500}$$

$$=\frac{33{,}600}{500}=67.2$$

16. Given: Number of first observations (n_1) = 9 and its mean $(\bar{x}_1)=100$;

Number of second observations (n_2) = 6 and its mean $(\bar{x}_2)=80$.

We know that combined mean

$$(\bar{x})=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}=\frac{(9\times 100)+(6\times 80)}{(9+6)}$$

$$=\frac{900+480}{15}=\frac{1380}{15}=92$$

17. Here $n_1=5,\ \bar{x}_1=8,\ \sigma_1^2=18$

$n_2=3,\ \bar{x}_2=8,\ \sigma_2^2=24$

$\bar{x}_{12}$ = combined mean $=\frac{5\times 8+3\times 8}{5+3}=\frac{64}{8}=8$

Combined variance $=\frac{n_1(\sigma_1^2+D_1^2)+n_2(\sigma_2^2+D_2^2)}{n_1+n_2}$

where $D_1=\bar{x}_1-\bar{x}_{12},\ D_2=\bar{x}_2-\bar{x}_{12}$

$\therefore\ D_1=8-8;\ D_2=8-8=0$

Combined variance $=\frac{5(18)+3(24)}{5+3}$

$$=\frac{90+72}{8}=\frac{162}{8}=20.25$$

18. Since we are given the rate per rupees, harmonic mean willgive the correct answer

$$\text{H.M.}=\frac{4}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}=\frac{4\times 12}{12+6+4+3}$$

$$=\frac{48}{25}=1.92\ \text{kg per rupee}$$

19. Since $A=(x,\ x+2,\ x+4)$ and $B=(x-2,\ x+2,\ x+6)$

$\therefore$ Mean of $A=\frac{x+x+2+x+4}{3}=x+2$

and mean of $B=\frac{x-2+x+2+x+6}{3}=x+2$

Hence, group B has more variability than group A.

20. Since deviations $=\Sigma(x_i-A)$

$\Rightarrow\ 2=\Sigma x_i-20\times 30$

$\Rightarrow\ \Sigma x_i=602$

$\therefore$ Mean $=\frac{602}{20}=30.1$

21. Required mean $=\frac{\Sigma n^2}{n}=\frac{n(n+1)(2n+1)}{6n}$

$$=\frac{(n+1)(2n+1)}{6}$$

Statement II is also a true statement.
Hence, option (c) is correct.

22. Since $\sigma\left(\frac{ax+b}{c}\right)=\left|\frac{a}{c}\right|\sigma$

Hence, both statements are true and Statement II is a correct explanation for Statement I.

23. Since the total number of students = 100 and the number of boys = 70,

$\therefore$ Number of girls $=(100-70)=30$.

Now, the total marks of 100 students $=100\times 72=7200$

And total marks of 70 boys $=70\times 75=5250$

$\Rightarrow$ Total marks of 30 girls $=7200-5250=1950$

$\therefore$ Average marks of 30 girls $=\frac{1950}{30}=65$.

24. Since the median is the 5th term and the increase is made in the last four terms, hence the median remains unchanged.

25. Given: $N=15$

$$\Sigma x^2=2830,\ \Sigma x=170$$

One observation 20 was replaced by 30, then

$$\Sigma x^2=2830-400+900=3330$$

$$\sum x = 170 - 20 + 30 = 180$$

$\therefore$ Variance, $\sigma^2 = \dfrac{\sum x^2}{N} - \left(\dfrac{\sum x}{N}\right)^2$

$$= \frac{3330}{15} - \left(\frac{180}{15}\right)^2 = 222 - 144 = 78$$

26. It is true that mode can be computed from histogram and median is not independent of change of scale. But variance is independent of change of origin and not of scale.

27. In the $2n$ observations, half of them are equal to a and the remaining half are equal to $-a$. Then, the mean of the total $2n$ observations is equal to zero.

$\therefore$ S.D. $= \sqrt{\dfrac{\Sigma(x-\bar{x})^2}{N}}$

$\Rightarrow 2 = \sqrt{\dfrac{\Sigma x^2}{2n}}$

$\Rightarrow 4 = \dfrac{\Sigma x^2}{2n} = \dfrac{2na^2}{2n}$

$\Rightarrow a^2 = 4$

$\Rightarrow |a| = 2$

28. Given $\sum x_i^2 = 400$ and $\sum x_i = 80$ since $\sigma^2 \geq 0$

$\Rightarrow \dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i}{n}\right)^2 \geq 0$

$\Rightarrow \dfrac{400}{n} - \dfrac{6400}{n^2} \geq 0$

$\Rightarrow n \geq 16$

$\therefore n = 18$

29. Given that mean = 21 and median = 22
Using the relations
Mode = 3 Median – 2 Mean

$\therefore$ Mode $= 3(22) - 2(21)$

$= 66 - 42 = 24$

30. Since variance is independent of change of origin, hence, the variance of the observations 101, 102,..., 200 is the same as the variance of the observations 151, 152,..., 250.

$\therefore \quad V_A = V_B$

$\Rightarrow \quad \dfrac{V_A}{V_B} = 1$

31. Let the number of boys and girls be x and y.

$\therefore 52x + 42y = 50(x+y)$

$\Rightarrow 52x + 42y = 50x + 50y$

$\Rightarrow 2x = 8y \qquad \Rightarrow x = 4y$

$\therefore$ Total number of students in the class

$= x + y = 4y + y = 5y$

$\therefore$ Required percentage of boys

$= \dfrac{4y}{5y} \times 100\% = 80\%$

32. According to the given condition,

$$6.80 = \frac{\left[\begin{array}{r}(6-a)^2 + (6-b)^2 + (6-8)^2 \\ +(6-5)^2 + (6-10)^2\end{array}\right]}{5}$$

$\Rightarrow 34 = (6-a)^2 + (6-b)^2 + 4 + 1 + 16$

$\Rightarrow (6-a)^2 + (6-b)^2 = 13 = 9 + 4 = 3^2 + 2^2$

$\Rightarrow a = 3,\ b = 4$

33. $\bar{x} = \dfrac{\text{Sum of quantities}}{n} = \dfrac{n/2(a+l)}{n}$

$= \dfrac{1}{2}(1 + 1 + 100d) = 1 + 50d$

M.D. $= \dfrac{1}{n}\Sigma|x_i - \bar{x}|$

$\Rightarrow 255 = \dfrac{1}{101}(50d + 49d + \cdots + d + 0 + d + \cdots + 50d]$

$= \dfrac{2d}{101}\left(\dfrac{50 \times 51}{2}\right)$

$\Rightarrow d = \dfrac{255 \times 101}{50 \times 51} = 10.1$

34. Statement II is true.
Statement I: Sum of n even natural numbers = n (n + 1)

Mean $(\bar{x}) = \dfrac{n(n+1)}{n} = n + 1$

Variance $= \left[\dfrac{1}{n}\Sigma(x_i)^2\right] - (\bar{x})^2$

$= \dfrac{1}{n}[2^2 + 4^2 + \cdots + (2n)^2] - (n+1)^2$

$= \dfrac{1}{n}2^2[1^2 + 2^2 + \cdots + n^2] - (n+1)^2$

$= \dfrac{4}{n}\dfrac{n(n+1)(2n+1)}{6} - (n+1)^2$

$= \dfrac{(n+1)[2(2n+1) - 3(n+1)]}{3}$

$= \dfrac{(n+1)(n-1)}{3} = \dfrac{n^2-1}{3}$

$\therefore$ Statement I is false.

35. $\sigma_x^2 = 4$ and $\sigma_y^2 = 5$

Also $\bar{x} = 2$ and $\bar{y} = 4$

Now, $\dfrac{\sum x_i}{5} = 2 \Rightarrow \sum x_i = 10$

$\dfrac{\sum y_i}{5} = 4 \Rightarrow \sum y_i = 20$

Since $\sigma_x^2 = \dfrac{1}{5}(\sum x_i^2) - (\bar{x})^2$

$\Rightarrow \sum x_i^2 = 40.$

Similarly $\sum y_i^2 = 105$

$\therefore \quad \sigma_z^2 = \dfrac{1}{10}(\sum x_i^2 + \sum y_i^2) - \left(\dfrac{\bar{x}+\bar{y}}{2}\right)^2$

$= \dfrac{1}{10}(40+105) - 9$

$= \dfrac{55}{10} = \dfrac{11}{2}$

36. $\dfrac{x_1 + x_2 + \cdots + x_{10}}{10} = 4.5$

$\Rightarrow x_1 + x_2 + \cdots + x_{10} = 45$

and $\dfrac{x_{11} + x_{12} + \cdots + x_{40}}{30} = 3.5$

$\Rightarrow x_{11} + x_{12} + \cdots + x_{40} = 105$

$\therefore \quad x_1 + x_2 + \cdots + x_{40} = 105$

$\therefore \quad \dfrac{x_1 + x_2 + \cdots + x_{40}}{40} = \dfrac{150}{40} = \dfrac{15}{4}$

37. Required mean

$$= \frac{0 \cdot {}^nC_0 + 1 \cdot {}^nC_1 + 2 \cdot {}^nC_2 + \cdots + n \cdot {}^nC_n}{{}^nC_0 + {}^nC_1 + \cdots + {}^nC_n}$$

$$= \frac{n \cdot 2^{n-1}}{2^n} = \frac{n}{2}$$

$[\because \ 1 \cdot {}^nC_1 + 2 \cdot {}^nC_2 + \cdots + n\,{}^nC_n = n \cdot 2^{n-1}]$

38. Mean $\bar{x} = \dfrac{3+4+5+6+7}{5} = 5,\ N = 5$

x	$\lvert x-\bar{x}\rvert$
3	2
4	1
5	0
6	1
7	2

$$\sum \lvert x - \bar{x} \rvert = 6$$

$\therefore$ Mean deviation from the mean

$$= \frac{\sum \lvert x - \bar{x} \rvert}{N} = \frac{6}{5} = 1.2$$

39. $b_{yx} = 0.8,\ b_{xy} = 0.2$

Then $r = \sqrt{b_{yx} b_{yx}} = \sqrt{(0.8)(0.2)} = \sqrt{0.16}$

$\Rightarrow r = 0.4$

40. We know that

$$r = \pm\sqrt{b_{yx} \cdot b_{xy}}$$

Also we know that signs of r, $b_{xy} \cdot b_{yx}$ are all the same.

$\therefore \quad r = (\text{sign of } b_{yx})\sqrt{b_{yx} \cdot b_{xy}}$

41. Now, $\mu_1' = \dfrac{\sum_{r=0}^{n} r \cdot {}^nC_r}{\sum_{r=0}^{n} {}^nC_r} = \dfrac{n \cdot 2^{n-1}}{2^n} = \dfrac{n}{2}$

$$\mu_2' = \frac{\sum_{r=0}^{n} r^2 \cdot {}^nC_r}{\sum_{r=0}^{n} {}^nC_r} = \frac{n(n-1)}{2^n} \cdot 2^{n-2} + \frac{n}{2}$$

$$= \frac{n(n-1)}{4} + \frac{n}{2}$$

$\therefore$ Variance $\mu_2 = (\mu_2') - (\mu_1')^2 = \dfrac{n(n-1)}{4} + \dfrac{n}{2} - \dfrac{n^2}{4} = \dfrac{n}{4}$

42. The slopes of the lines of regression of y on x and x on y are $m_1 = b_{yx}$ and $m_2 = 1/b_{xy}$, respectively. Therefore, the angle between them is given by

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{b_{yx} - (1/b_{xy})}{1 + (b_{yx}/b_{xy})}$$

$$\Rightarrow \qquad \theta = \tan^{-1}\left(\frac{b_{yx} \times b_{xy} - 1}{b_{yx} \times b_{xy}}\right)$$

43. Given lines are $3\bar{x} - 2\bar{y} + 1 = 0$ (i)

and $2\bar{x} - \bar{y} - 2 = 0$ (ii)

On solving Eqs. (i) and (ii), we get $\bar{x} = 5,\ \bar{y} = 8$.

44. Given $3x + 2y = 26$

$\Rightarrow y = -\dfrac{3}{2}x + 13$

and $6x + y = 31$

$$\Rightarrow\ x=-\frac{1}{6}y+\frac{13}{6}$$

$$\therefore\ r=-\sqrt{\left(\frac{-3}{2}\right)\left(\frac{-1}{6}\right)}$$

$$\Rightarrow\ r=-\frac{1}{2}$$

45. $\operatorname{cov}(x, y)=\frac{1}{n}\Sigma xy-\bar{x}\,\bar{y}$

$$=\frac{1}{5}(110)-\left(\frac{15}{5}\right)\left(\frac{36}{5}\right)=\frac{2}{5}$$

46. On arranging the terms in increasing order of magnitude, we have 40, 42, 45, 47, 50, 51, 54, 55, 57
Number of terms, $N = 9$

$\therefore$ Median $=\left(\frac{9+1}{2}\right)$th term = 5th term = 50 kg

Weight (kg)	*Deviation from median* (*d*)	$\lvert d\rvert$
40	−10	10
42	−8	8
45	−5	5
47	−3	3
50	0	0
51	1	1
54	4	4
55	5	5
57	7	7
		$\lvert d\rvert=43$

M.D. from median $=\frac{43}{9}=4.78$ kg

$\therefore$ Coefficient of M.D. from median

$$=\frac{\text{M.D.}}{\text{median}}$$

$$=\frac{4.78}{50}=0.0956$$

47. Standard deviation does not depend on origin but it depends on scale, so

$$\frac{ax+b}{c}=\frac{ax}{c}+\frac{b}{c}$$

$\Rightarrow$ Standard deviation of $\frac{ax+b}{c}$ is $\frac{a\sigma}{c}$.

48. In the given distribution, 6 occurs most of the times. Hence, mode of the series = 6.

49. Arranging the terms in increasing order, we have

Value (x)	*Frequency* (f)	*Cumulative frequency* (*c.f.*)
7	2	2
8	1	3
9	5	8
10	4	12
11	6	18
12	1	19
13	3	22

$\because\ N=22$

$\therefore$ Median number $=\frac{N+1}{2}=11.5$

which comes under the cumulative frequency 12. The corresponding value of x will be the median, i.e. median = 10.

50. The mean of the series $a, a+d, \ldots, +a+2d$ is

$$\bar{x}=\frac{1}{2n+1}(a+a+d+a+2d+\cdots+a+2nd)$$

$$=\frac{1}{2n+1}\left[\frac{2n+1}{2}(a+a+2nd)\right]=a+nd$$

$\therefore$ Mean deviation from mean

$$=\frac{1}{2n+1}\sum_{r=0}^{2n}\lvert(a+rd)-(a+nd)\rvert$$

$$=\frac{1}{2n+1}\sum_{r=0}^{2n}(r-n)d$$

$$=\frac{1}{2n+1}2d(1+2+\cdots+n)$$

$$=\frac{n(n+1)}{2n+1}d$$

51. The intersecting point of two regression lines is on the mean, i.e. $(\bar{x}, \bar{y})$.

52. $4\bar{x}+3\bar{y}+7=0$ (i)

and $3\bar{x}+4\bar{y}+8=0$ (ii)

On solving Eqs. (i) and (ii), we get

$$\bar{x}=-\frac{4}{7}\quad\text{and}\quad\bar{y}=-\frac{11}{7}$$

53. $r_{xy}=\dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2\,\Sigma(y-\bar{y})^2}}$

$$=\frac{20}{\sqrt{36\times25}}=\frac{2}{3}=0.66$$

54. Correlation coefficient

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum x^2 - (\sum x^2)]}\sqrt{[n\sum y^2 - (\sum y^2)]}}$$

$$= \frac{10(220) - 40 \times 50}{\sqrt{10(200) - (40)^2}\sqrt{10(262) - (50)^2}}$$

$$= \frac{200}{20 \times 10.954} = \frac{200}{219.08} = 0.91$$

55. $\text{cov}(x, y) = \frac{\sum xy}{n} - \frac{\sum x}{n} \cdot \frac{\sum y}{n} = \frac{1}{10}(850) - \left(\frac{30}{10}\right)\left(\frac{400}{10}\right)$

$= 85 - 120 = -35$

and $\text{var}(x) = \sigma_x^2 = \frac{1}{n}\sum x^2 - \left(\frac{\sum x}{n}\right)^2$

$= \frac{196}{10} - \left(\frac{30}{10}\right)^2 = 10.6$

$\therefore \quad b_{yx} = \frac{\text{cov}(x, y)}{\text{var}(x)} = \frac{-35}{10.6} = -3.3$

56. Let us assume that the line of regression y on x be $3x + 12y = 19$ and x on y be $3y + 9x = 46$.

$\therefore \quad b_{yx} = -\frac{3}{12}$ and $b_{xy} = -\frac{3}{9} = -\frac{1}{3}$

$\therefore \quad r_{xy} = -\sqrt{b_{yx} \times b_{xy}} = -\sqrt{\left(\frac{3}{12}\right) \times \left(\frac{1}{3}\right)}$

$= -\sqrt{\frac{1}{12}} = -\sqrt{0.083}$

$= -0.289$

57. Standard deviation $= \sqrt{\frac{\sum_{j=1}^{18}(x_j - 8)^2}{n} - \left(\frac{\sum_{j=1}^{18}(x_j - 8)}{n}\right)^2}$

$= \sqrt{\frac{45}{18} - \left(\frac{9}{18}\right)^2} = \sqrt{\frac{45}{18} - \frac{1}{4}} = \sqrt{\frac{81}{36}}$

$= \frac{9}{6} = \frac{3}{2}$

58. There is no change in the mean deviation if each observation is increased by a constant number while the mean is increased by that constant number.
Hence, A.M. = 30 + 2 + 32, M.D. = 2

59. $\sigma^2 = \sum \frac{x_i^2}{n} - \left(\sum \frac{x_i}{n}\right)^2$

Variance of $2x_1, 2x_2, \ldots, 2x_n$

$$= \sum \frac{(2x_i)^2}{n} - \left(\sum \frac{2x_i}{n}\right)^2 = 4\left[\sum \frac{x_i^2}{n} - \left(\sum \frac{x_i}{n}\right)^2\right]$$

$= 4\sigma^2$

Statement 1 is true.

A.M. of $2x_1, 2x_2, \ldots, 2x_n = \frac{2x_1 + 2x_2 + \cdots + 2x_n}{n}$

$= 2\left(\frac{x_1 + x_2 + \cdots + x_n}{n}\right) = 2\bar{x}$

Statement 2 is false.

Alternative method

Statement 2 is false because the arithmetic mean of $\frac{2x_1 + 2x_2 + 2x_3 + \cdots + 2x_n}{n}$ is $2\bar{x}$.

MULTIPLE CHOICE TYPE QUESTIONS—LEVEL 1

1. Mode of the data 3, 2, 5, 2, 3, 5, 6, 6, 3, 5, 2, 5 is
(a) 6 (b) 4 (c) 5 (d) 3

2. The median of 10, 14, 11, 9, 8, 12, 6 is
(a) 10 (b) 12 (c) 14 (d) 11

3. Mode of the distribution

[redacted]	4	5	6	7	8
[redacted]	3	5	10	6	1

is

(a) 6 (b) 10
(c) 8 (d) none of these

4. The quartile deviation of the daily wages (in ₹) of 7 persons given as 12, 7, 15, 10, 17, 19, 25 is
(a) 145 (b) 5 (c) 9 (d) 4.5

5. The median of the following data 11, 29, 17, 21, 13, 31, 39, 19 is
(a) (21 + 13)/12 (b) 19
(c) 21 (d) 20

6. If the mode of a distribution is 18 and the mean is 24, then the median is
(a) 18 (b) 24 (c) 22 (d) 21

7. The mode of the following distribution

[redacted]	40	43	46	49	52	55
[redacted]	5	8	16	9	7	3

is

(a) 40 (b) 46
(c) 55 (d) none of these

8. The median value of the following frequency distribution

[redacted]	153–159	160–166	167–173	174–180	181–187
[redacted]	2	4	6	4	4

is equal to
(a) 169 (b) 171 (c) 173 (d) 175

9. If in a moderately skewed distribution, the values of mode and mean are 6λ and 9λ respectively, then the value of the median is
(a) 8λ (b) 7λ (c) 6λ (d) 5λ

10. The mode and median of the data 82, 98, 73, 71, 43, 82, 90 are
(a) 80, 90 (b) 82, 82 (c) 77, 98 (d) none of these

MULTIPLE CHOICE TYPE QUESTIONS—LEVEL 2

1. Which of the following is not a measure of central tendency?
(a) mean (b) median
(c) mode (d) range

2. The weighted mean of the first n natural numbers whose weights are equal to the squares of the corresponding numbers is
(a) $\frac{n+1}{2}$ (b) $\frac{3n(n+1)}{2(2n+1)}$
(c) $\frac{(n+1)(2n+1)}{6}$ (d) $\frac{n(n+1)}{2}$

3. The mean of the following frequency table is 50.

Class	*Frequency*
0–20	17
20–40	f_1
40–60	32
60–80	f_2
80–100	19
Total	120

The missing frequencies are
(a) 28, 24 (b) 24, 36
(c) 38, 28 (d) none of these

4. The geometric mean of 1, 2, $2^2, 2^3, \ldots, 2^n$ is
(a) $2^{2/n}$ (b) $2^{n/2}$ (c) $2^{(n-1)/2}$ (d) $2^{(n+1)/2}$

5. If $\bar{x}$ is the mean of a distribution, then $\sum f_i(x_i - \bar{x}) =$
(a) 0 (b) M.D.
(c) S.D. (d) none of these

6. The variance of the first n natural numbers is
(a) $\frac{n^2-1}{12}$ (b) $\frac{n^2-1}{6}$
(c) $\frac{n^2+1}{6}$ (d) $\frac{n^2+1}{12}$

7. If $\bar{x}_1$ and $\bar{x}_2$ are means of two distributions such that $\bar{x}_1 < \bar{x}_2$ and $\bar{x}$ is the mean of the combined distribution, then
(a) $\bar{x} < \bar{x}_1$ (b) $\bar{x} > \bar{x}_2$
(c) $\bar{x} = \frac{\bar{x}_1 + \bar{x}_2}{2}$ (d) $\bar{x}_1 < \bar{x} < \bar{x}_2$

8. The A.M. of n observations is $\bar{x}$. If the sum of $n - 5$ observations is a, then the mean of the remaining 5 observations is
(a) $\frac{n\bar{x}+a}{5}$ (b) $\frac{n\bar{x}-a}{5}$
(c) $n\bar{x}+a$ (d) none of these

9. A car completes the first half of its journey with a velocity v_1, and the rest half with velocity v_2. Then, the average velocity of the car for the whole journey is
(a) $\frac{v_1+v_2}{2}$ (b) $\sqrt{v_1 v_2}$
(c) $\frac{2v_1v_2}{v_1+v_2}$ (d) none of these

10. Which one of the following measures is the most suitable one of central location for computing intelligence of students?
(a) mode (b) A.M.
(c) G.M. (d) median

ANSWERS

Level 1

1. (c) **2.** (a) **3.** (a) **4.** (d) **5.** (d) **6.** (c) **7.** (b) **8.** (b) **9.** (a) **10.** (b)

Level 2

1. (d) **2.** (b) **3.** (a) **4.** (b) **5.** (a) **6.** (a) **7.** (d) **8.** (b) **9.** (c) **10.** (d)