# Continuous-time quantum walk-based ansätze on neutral atom hardware

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A key use of near-term quantum computers is as analogue simulators, matching the action of some virtual or abstracted system onto a program executed on physical quantum hardware. This work explores continuous-time quantum walks (CTQW) on constrained graphs of independent set configurations and implements these abstract walks on analog-mode neutral-atom hardware. First, we explore variational state preparation protocols with limited controls through the lens of CTQW and optimal control to prepare nontrivial target states in non-separable Hilbert spaces. Next, we match these virtual walk dynamics to physical execution on analog-mode neutral atom hardware by leveraging the Rydberg blockade phenomenon. We analyze the convergence and scaling in the preparation of these known states as an indicator of the upper bounds on the quantum speedup mechanisms predicted by ideal CTQWs, showing that these signatures persist even when executed on noisy hardware. Finally, we introduce noise mitigation methods using Bayesian postprocessing. This paper demonstrates the ability to prepare nontrivial entangled states using quantum walks in constrained subspaces, and that nonequilibrium dynamics on constrained independent set supspaces is feasible on cloud-accessible analog-mode neutral-atom quantum computers.

### I. INTRODUCTION

Quantum search and optimization are key applications for near-term quantum computers [1]. Algorithms for unstructured or spatial search and the Quantum Approximate Optimization Algorithm (QAOA) exemplify the unique properties of quantum computing: superposition to explore exponentially many computational pathways in quantum parallel, entanglement to encode a cost landscape, and interference to amplify desired states [2, 3]. These algorithms typically utilize an alternating sequence of unitaries, where the first diagonal unitary phase-encodes the cost or validity of each solution state, and the second mixing unitary drives amplitude transfer between these states. This alternating structure can be naturally understood through the framework of continuous-time quantum walks (CTQWs), where coherent dynamics of quantum states traversing a graph underpin the convergence behavior of the algorithm.

CTQWs are a foundational concept in quantum algorithm design, providing a universal platform for computation and achieving exponential speedup over classical methods in some instances [4, 5]. Indeed, extension of the original QAOA framework of unconstrained optimization over binary variables to constrained problems and problem spaces that map to higher-order permutation groups has found that CTQW-inspired mixers can offer highly-improved convergence and be less prone to vanishing gradients, by restricting the search to a space of valid solutions or capturing key features of the cost landscape [6–9].

More generally, CTQW models operate in a non-adiabatic regime and can saturate the Mandelstam-Tamm quantum speed limit [10, 11], making them especially well-suited for describing and analyzing variational heuristics driven by non-equilibrium dynamics [12] and shortcuts to adiabaticity [13].

Experimentally, CTQWs have been realized in the position basis across a variety of platforms, including walks over line graphs using photonic waveguides [14], neutral atoms [15], and trapped ions [16], as well as two-dimensional grid graphs using superconducting qubit arrays [17, 18], and [19] implementing a spatial quantum search on large-scale two-dimensional lattices of strontium atoms. Beyond spatial walks, variational circuits that implement CTQWs in the computational basis have been demonstrated with photonic waveguides [20], and CTQW-like dynamics have been observed in quantum many-body scar dynamics in Rydberg chains [21] on neutral atom systems [22].

In this work, we implement CTQW based ansätze for constrained combinatorial spaces on QuEra's Aquila neutral-atom platform [23] by mapping abstract walk generators onto the Rydberg Hamiltonian under blockade constraints. We focus on the preparation of two classes of benchmark states—computational-basis product states and coherent states in the symmetric subspace of the walk generator—and analyze their convergence properties under CTQW-informed variational optimization schemes. By comparing ideal CTQW dynamics, noiseless Rydberg emulation, and experimental results, we assess the extent to which characteristic amplification and interference effects of efficient CTQW-based ansätze survive on noisy intermediate-scale hardware. In Section II, we introduce the phase-walk ansätze, performance metrics used in later analysis, the representative target states, and the variational optimization strategies. In Section III, we describe the mapping of the ansätze to the Rydberg Hamiltonian on Aquila. Results of state preparation on hardware are compared with the scaling predicted by ideal CTQW dynamics and noiseless Rydberg emulation in Section IV. In Section V, we apply quenches on the prepared states as an indicator of coherent state preparation. Finally, in Section VI, we

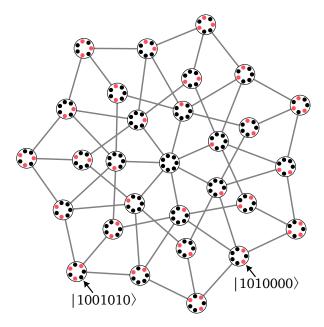


FIG. 1. An example walk graph for independent set constrained subspace walks. Each vertex is an independent set in a ring of 7 vertices labeled by bitstring z and representing basis vector  $|z\rangle$ , with red dots indicating inclusion of that physical vertex in the set. There is an edge between vertices iff the Hamming distance of the bitstring label is 1.

summarize our conclusions. Overall, our results establish constrained-subspace CTQWs as a framework for entangled state preparation on analog-mode neutral-atom processors, and demonstrate that CTQW-based ansätze for known states can exhibit the super-quadratic convergence characteristic of efficient quantum-walk protocols.

### II. PHASE-WALK ANSÄTZE

A Continuous-Time Quantum Walk (CTQW) [24] is the coherent evolution of a quantum state under the unitary time-evolution operator

$$\hat{U}_W(\tau) = \exp(-i\tau\,\hat{\mathcal{G}}),\tag{1}$$

where the generator

$$\hat{\mathcal{G}} = \sum_{(a,b)\in\mathcal{E}} w_{ab} |a\rangle\langle b|, \tag{2}$$

is obtained from the (possibly weighted) adjacency matrix of a quantum walk graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  with vertex set  $\mathcal{V}$ , edge set  $\mathcal{E}$ , and real, symmetric weights  $w_{ab} = w_{ba}$ . Each vertex is matched to a basis state of a quantum state  $a \leftrightarrow |a\rangle$ , and the terms of the graph adjacency matrix are matched to the generator.

In the context of algorithms for search and optimization, CTQWs are often paired with a diagonal unitary that adds phase to the graph vertices weighted by  $\phi_a = \gamma c_a$ , defined by a (variational) scaling parameter  $\gamma$  and the generator

$$\hat{C} = \sum_{a \in \mathcal{V}} c_a |a\rangle\langle a|. \tag{3}$$

Together the two unitaries form a parameterized phase-walk ansatz for which a common task is to prepare some target or resource state by alternating between the walk (or "mixer") generator  $\hat{\mathcal{G}}$  and phasor generator  $\hat{C}$  applied to some fiducial initial state  $|\psi_0\rangle$ 

$$|\psi\rangle = \prod_{q=1}^{p} e^{-i\tau_q \hat{\mathcal{G}}} e^{-i\gamma_q \hat{\mathcal{C}}} |\psi_0\rangle, \tag{4}$$

for 2p variational parameters  $\{\vec{\gamma}, \vec{\tau}\}\$  – as exemplified by the quantum approximate optimization algorithm [2].

A common representation of basis states is binary bitstrings  $a \in \mathcal{Z} = \{0,1\}^n$ , where each bit  $z_i$  represents the state of qubit i. In this work, we define  $|\psi_0\rangle$  as an initial walk from the all-zeros state,

$$|\psi_0\rangle \equiv |\tau_0\rangle = e^{-i\tau_0\hat{\mathcal{G}}}|0\rangle.$$
 (5)

The phasor generator can be written as some function  $c_a \equiv f(z)$ , typically a low-order polynomial to conform to the locality constraints of quantum hardware. Here, we choose a first-order function  $f(z) = \sum_i c_i z_i$  so that the phasor generator can be efficiently written in terms of Pauli matrices

$$\hat{C} = \frac{1}{2} \sum_{i=1}^{N} c_i \left( \mathbb{I} - \hat{\sigma}_z^{(i)} \right). \tag{6}$$

Similarly, the walk graph is often formed by edges between vertices whose labels differ by a Hamming distance of one or two [2, 8, 9, 25]:

$$d_h(z^a, z^b) \equiv ||z^a - z^b||_1, \tag{7}$$

Hamming-distance-based constructions yield graphs that are efficiently implementable and typically highly symmetric (generally vertex-transitive) [8, 9]. If edges connect labels with  $d_h=1$  over all  $2^N$  possible bitstrings, then the walk graph is the N-dimensional hypercube. A Hamming-distance-1 graph walk generator can be efficiently written as  $\hat{\mathcal{G}} = \sum_i \hat{\sigma}_x^{(i)}$ , and small Hamming distances correspond to low-depth polynomial functions over Pauli operators.

### A. Walks in constrained subspaces

Frameworks for optimization with constraints restrict the vertices to a subset of valid solutions and construct the walk generator so that it preserves this reduced search space as an invariant subspace [6, 9, 25]. The resulting CTQW is often non-separable and inherits the symmetries of the constraint, while breaking others. As we detail in Section III, such walks are the focus of this work as neutral-atom platforms offer unique opportunities for their efficient implementation. We constrain the full space of length-N bitstrings according to an underlying constraint graph G=(V,E) on N vertices. The walk vertices  $\mathcal V$  are then independent-set configurations of G,

$$\mathcal{V} = \left\{ z \in \{0, 1\}^N \mid z_i + z_j \le 1 \ \forall (i, j) \in E(G) \right\}, \quad (8)$$

with edges between vertices in V having  $d_h = 1$ . The resulting walk graph is a subgraph of the hypercube, and the walk generator can be written efficiently as

$$\hat{\mathcal{G}} = \sum_{i=1}^{N} \hat{\mathcal{P}} \,\hat{\sigma}_x^{(i)} \,\hat{\mathcal{P}},\tag{9}$$

where  $\hat{\sigma}_x^{(i)}$  is the Pauli-X operator on qubit i, and  $\hat{\mathcal{P}}$  is the projector onto  $\mathcal{V}$  (equivalently, the independent-set subspace of the constraint graph G).

In particular, we choose the constraint graph G to be the cycle graph on N vertices (i.e., a ring with nearest-neighbor edges). The resulting walk graph is a Lucas cube [26], shown in Fig. 1. We make this choice because its structural properties are relatively well understood, supporting efficient enumeration and generation of  $\mathcal{V}$ , and reduced-dimensional representations of  $\mathcal{G}$ , at large N (which is not the case for all independent-set subspaces [27]), while remaining suitably complex to produce non-trivial dynamics. Specifically, the number of vertices of the Lucas cube is exponentially smaller than that of the hypercube at the same N, yet still grows exponentially as  $|\Lambda_N| \sim \varphi^N$ , where  $\varphi = (1 + \sqrt{5})/2$  and, as it preserves the dihedral symmetry of G, it retains a relatively small but structurally rich dihedrally symmetric sector that grows like  $B_N^{(D)} \sim \varphi^N/(2N)$  [28].

### B. State preparation

A key application of quantum walks is to prepare defined target states  $|\psi\rangle$ . Commonly, this is some unknown but well-defined state, such as a product state that maximizes the value of some low-order polynomial objective C(z), such as in QAOA [2]. Similarly, the states could be defined concretely as a resource state with an efficient description, such as the GHZ state  $|\psi\rangle = (|00\cdots00\rangle + |11\cdots11\rangle)/\sqrt{2}$ . Given the extensive literature on QAOA and its variants to solve combinatorial objectives C(z), this work focuses instead on preparing concretely defined states and characterizing the capacity of variational quantum walks to prepare these target states on blockaded subspaces [29].

Given some defined target state  $|\psi\rangle$ , a variational op-

timizer chooses p(N+1) parameters  $\{\vec{\gamma}, \tau\}$  of the variational ansatz state  $|\gamma, \tau\rangle$  to maximize the overlap with the target state

$$\mathrm{MAX}_{\gamma,\tau}: \left| \langle \gamma, \tau | \psi \rangle \right|^2. \tag{10}$$

Or, similarly, minimize the statistical divergence between the target distribution  $P(z) = |\langle z|\psi\rangle|^2$  and the variational distribution  $Q(z|\gamma,\tau) = |\langle z|\gamma,\tau\rangle|^2$ .

### C. Target states

For the constraint graph of the N vertex ring, we chose two classes of target states. The first class are product states, e.g.  $|\psi\rangle = |0010101\rangle$  represented by some bitstring z. While these states are trivial and unentangled, it may be nontrivial to prepare such a state with a quantum walk, given the constrained global mixer term that can only be broken by a locally tuned value of the phasor  $\gamma_i$ .

The second class consists of superpositions over the symmetric subspace. An N-vertex ring has both translational and reflective symmetries invariant under the dihedral group  $D_N$ ; accordingly, these symmetry groups coincide with the automorphism group of the constraint graph G – the set of vertex relabelings that preserve the graph's adjacency. The symmetric subspace is therefore spanned by equal-weight superpositions of all states labelled by bitstrings related by cyclic rotations or reflections,

$$|[z]\rangle = \frac{1}{\sqrt{|[z]|}} \sum_{u \in [z]} |u\rangle,\tag{11}$$

where  $[z] = \{g \cdot z : g \in D_N\}$  is the dihedral orbit of a representative bitstring  $z^1$ .

We refer to these states as *bracelet states*, borrowing from the combinatorial term for equivalence classes of bitstrings under cyclic rotations and reflections [30].

### D. Ansätze performance

Phase-walk ansätze are heuristic methods for preparing approximate solutions to combinatorial problems.

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\begin{split} & \big| \big[ 010010001 \big] \big\rangle = \frac{1}{\sqrt{18}} \big( |010010001\rangle + |010010100\rangle + |010001010\rangle + \\ & |010001001\rangle + |010100100\rangle + |010100010\rangle + |101001000\rangle + \\ & |101000100\rangle + |100101000\rangle + |100100010\rangle + |100010100\rangle + \\ & |100010010\rangle + |001010001\rangle + |001010010\rangle + |001001010\rangle + \\ & |001000101\rangle + |000101001\rangle + |000100101\rangle \big). \end{split}
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<sup>&</sup>lt;sup>1</sup> The first instance in which the symmetric subspace contains states whose entire orbit cannot all be generated by rotations alone occurs at N=9, where

Outside a few special cases, formal guarantees are limited, so benchmarking is primarily empirical and problem-dependent [2, 6, 31]. In this context, an informative metric is the *amplification* of desirable solutions relative to a uniform baseline over the accessible subspace

$$A(|\mathcal{V}|, p) = \frac{|\mathcal{V}| P(z^*, p)}{|z^*|}, \tag{12}$$

where  $P(z^*, p)$  is the probability of preparing the target  $|z^*\rangle$  at depth p [8, 9] and  $|z^*|$  is the cardinality of the target set of basis states. Effective ansätze are expected to closely follow a power-law growth in  $|\mathcal{V}|$ ,

$$A(|\mathcal{V}|, p) = c |\mathcal{V}|^{\alpha}, \tag{13}$$

with the exponent  $\alpha$  as the key performance indicator [32, 33]. Because the single-shot success scales as  $|\mathcal{V}|^{\alpha-1}$ ,  $\alpha$  induces an effective polynomial speedup order n,

$$n = \frac{1}{1 - \alpha},\tag{14}$$

which can be understood as a proxy for query-complexity scaling (e.g., at  $\alpha = \frac{1}{2}$  the scaling is Grover-like).

For a particular phase-walk ansatze to offer the possibility of quantum-speedup given the overheads of optimization, sampling, and classical post-processing, it should ideally demonstrate two key features. First, at fixed depth p, the amplification factor A should grow with problem size as a high-order polynomial in  $|\mathcal{V}|$ . Second, at fixed  $|\mathcal{V}|$ , the amplification should itself increase polynomially with circuit depth p. In both instances, orders n > 2 indicate the leveraging of global phase-structure to accelerate convergence [8, 34].

In the following, we probe these criteria through the preparation of known target states, seeking a reliable protocol for CTQW implementation and an empirical view of potential quantum speedup possible within the degrees of freedom of our ansätze when applied non-trivial problem instances.

#### E. Variational optimization

### 1. Product states

If the constraint graph G is the null graph, then the walk Hamiltonian is the N-dimensional hypercube and the time evolution remains fully separable. With p=1, the state is

$$|\psi\rangle = \bigotimes_{k=0}^{N-1} e^{-i\tau_1 \sigma_x^k} e^{-i\gamma c_k \sigma_z^k} e^{-i\tau_0 \sigma_x^k} |0\rangle.$$
 (15)

Setting the X-rotation angles to  $(\tau_0, \tau_1) \equiv \{\pi/4, 3\pi/4\}$  (mod  $2\pi$ ) (in any order) and choosing  $\gamma = \pm \pi$  (mod  $2\pi$ ) and  $c_k = z_k^*$  enables the preparation of any

computational-basis state<sup>2</sup>. However, when a nontrivial connected constraint graph is introduced, each  $\sigma_x^k$  becomes a neighbor-conditioned multi-qubit gate, so the evolution is no longer separable. Nevertheless, the walk ansatz can still prepare the exact state. Given a phasor unitary

$$\hat{U}_{z^*}(\gamma)|z\rangle = \exp\left(-i\gamma \sum_{i:z_i^*=1} z_i\right)|z\rangle \tag{16}$$

as a sum of Pauli-z terms that are non-zero in the target bitstring, three protocols are possible. The first is a Trotterized adiabatic protocol that uses perturbatively small parameters  $\gamma, \tau \ll 1$ , exact in the  $p \to \infty$  limit [35]. The second is a Trotterized mask protocol with  $\gamma, \tau \ll 1$  but  $\gamma \gg \tau$  to "pin" the zero qubits in the zero state with a large effective Z field. The third is a fast-forward limit [13] with  $\gamma = \pi$  and  $\tau_0 + p\tau_1 = \frac{\pi}{2}$ , which can be shown to be exact by the following argument. First, the walk Hamiltonian can be split by  $U_{z^*}$  into two components:

$$\mathcal{G} = \mathcal{G}_+ + \mathcal{G}_-,\tag{17}$$

where  $\mathcal{G}_{-}$  flips  $h_{z^*}(z)$  and  $\mathcal{G}_{+}$  preserves it. Then,

$$\mathcal{G}_{\pm} = \frac{1}{2} \left( \mathcal{G} \pm U_{z^*} \mathcal{G} U_{z^*} \right), \tag{18}$$

Here the +1 eigenspace of  $U_{z^*}$ , which contains the ground state  $|0\rangle$  and  $|z^*\rangle$ , is the *positive* subspace and the -1 eigenspace the *negative* subspace.

The interleaved  $U_{z^*}$  effectively toggles a rotation between the negative and positive subspaces,

$$R_{\pm}(\tau) = e^{-i\tau(\mathcal{G}_{+} \pm \mathcal{G}_{-})},\tag{19}$$

which together produce constructive interference on the positive subspace. To illustrate this effect, consider the even-depth (2p'=p) ansatz,

$$|\tau_0, \tau_1\rangle = [R_-(\tau_1)R_+(\tau_1)]^{p'}|\psi_0\rangle.$$
 (20)

By BCH expansion to second-order, a single pair is,

$$R_{-}(\tau_{1})R_{+}(\tau_{1}) = \exp\left[-2i\tau_{1}\mathcal{G}_{+} - \tau_{1}^{2}[\mathcal{G}_{+}, \mathcal{G}_{-}] + O(\tau_{1}^{3})\right].$$
(21)

The linear  $\mathcal{G}_{-}$  terms cancel, so to leading order the dynamics are generated by  $\mathcal{G}_{+}$  with the effective time  $p'\tau_{1}$ .

Projection of  $\exp(-i2\tau_1\mathcal{G}_+)$  onto the positive subspace

<sup>&</sup>lt;sup>2</sup> Geometrically, the single-qubit gates perform an X-Z-X rotation sequence on the Bloch sphere of each qubit. Starting from +z (i.e.,  $|0\rangle$ ), an X-rotation by  $\tau_0 = \pi/4$  moves the Bloch vector to -y. Then, if  $c_k = 1$ , the intermediate Z-rotation moves the vector to -y. Finally, an X-rotation by  $\tau_1 = 3\pi/4$  drives vectors on +y back to +z (i.e.,  $|0\rangle$ ) and those on -y to -z (i.e.,  $|1\rangle$ ).

yields an effective evolution over states

$$|v_j\rangle = \frac{1}{\sqrt{|S_j|}} \sum_{z \in S_j} |z\rangle,$$
 (22)

where  $S_j$  is the set of bitstrings in the positive subspace with Hamming weight j (so  $|v_0\rangle = |0\rangle$  and  $|v_k\rangle = |z^*\rangle$ ). The effective Hamiltonian J is the SU(2) spin-k/2 chain Hamiltonian (permutation-symmetric noninteracting spins 1/2) expressed in the symmetric Dicke basis, which has non-zero entries only on the first off-diagonals:

$$J_{j,j+1} = J_{j+1,j} = \sqrt{(k-j)(j+1)},$$
 (23)

since each state in  $S_j$  has k-j forward neighbors in  $S_{j+1}$  and each state in  $S_{j+1}$  has j+1 backward neighbors in  $S_j$ . The lowest-order coupling between  $|0\rangle$  and  $|z^*\rangle$  then occurs at order k with strength,

$$J_{0\to z^*} = \left(\prod_{j=0}^{k-1} J_{j,j+1}\right)^{1/k}.$$
 (24)

and, since J inherits the bipartite structure of  $\mathcal{G}$ ,

$$\langle v_k | e^{-iT_{\text{eff}}J} | v_0 \rangle \approx \begin{cases} -i \sin\left(T_{\text{eff}}J_{0\to z^*}\right), & k \text{ odd,} \\ 1 - \cos\left(T_{\text{eff}}J_{0\to z^*}\right), & k \text{ even,} \end{cases}$$
 (25)

which, in either case, gives

$$\operatorname{Prob}(|z^*\rangle) \propto \sin^2(T_{\text{eff}}J_{0\to z^*}),$$

maximized at  $T_{\text{eff}} = \frac{\pi}{2J_{0\to z^*}}$ , with leakage proportional to  $||\tau_1^2[\mathcal{G}_+, \mathcal{G}_-]||$ .

As  $T_{\rm eff}$  is inversely proportional to  $J_{0\to z^*}$ , at low k and p' we seek an initial state that reduces the second-order leakage. Let  $|w_1\rangle = \frac{1}{\beta_-}\mathcal{G}_-|v_0\rangle$ , with  $\beta_- = ||\mathcal{G}_-|v_0\rangle||$ , and  $\beta_+ = ||\mathcal{G}_+|v_0\rangle||$ . The state prepared by an initial walk over  $\mathcal{G}$  is

$$|\psi_0\rangle = |v_0\rangle - i\tau_0 (\beta_+|v_1\rangle + \beta_-|w_1\rangle) + \mathcal{O}(\tau_0^2).$$
 (26)

To first order, the amplitude driving the chain is reduced by

$$\cos \phi = \frac{1}{\sqrt{1 + (\frac{\beta -}{\beta_+} \tau_0)^2}},\tag{27}$$

Consequently, the coupling  $J_{0\to z^*}$  is attenuated to the effective coupling  $J_{\text{eff}}(\tau_0) = J_{0\to z^*}\cos(\phi)$ , with

$$T_{\text{eff}} = \tau_0 + p' \, \tau_1 = \frac{\pi}{2 J_{\text{eff}}(\tau_0)}.$$
 (28)

Minimization of the second-order leakage leads to a unique positive-valued solution for  $\tau_0$  and  $\tau_1$ . The second-order term  $-\tau_1^2[\mathcal{G}_+,\mathcal{G}_-]$  returns amplitude from  $|w_1\rangle$  to the positive subspace proportional to

 $\tau_1^2 \kappa_{\rm ret} \sin \phi$  and leaks amplitude from  $|v_1\rangle$  proportional to  $\tau_1^2 \kappa_{\rm leak} \cos \phi$ , where  $\kappa_{\rm ret}$  and  $\kappa_{\rm leak}$  are graph-dependent constants. These channels are balanced when

$$\frac{\kappa_{\text{leak}}}{\kappa_{\text{rot}}} \tan \phi \approx 1.$$
 (29)

As

$$\tan \phi = \frac{\beta_- \tau_0}{\sqrt{1 + \tau_0^2 \beta_+^2}},\tag{30}$$

to first-order the optimal leakage-reducing  $\tau_0$  is

$$\tau_0^* = \frac{\kappa}{\sqrt{\beta_- - \kappa^2 \beta_+}},\tag{31}$$

where  $\kappa = \kappa_{\rm leak}/\kappa_{\rm ret}$ , and

$$\tau_1^* = \frac{1}{p'} \left[ \frac{\pi}{2J_{\text{eff}}(\tau_0^*)} - \tau_0^* \right]. \tag{32}$$

For odd p, Eq. (32) still holds, but the absence of a full +/- pair leaves a linear  $\mathcal{G}_-$  contribution. The resulting leakage scales as  $\mathcal{O}(\tau_1 - \tau_0)$ , rather than  $\mathcal{O}(\tau_1^2)$  as in the even (paired) case. In practice, this can be mitigated by further attenuating  $J_{\text{eff}}$  via a larger  $\tau_0$ . More generally, while higher-order terms contribute significantly if  $\tau_0, \tau_1 \not\ll 1$ , near  $(\tau_0^*, \tau_1^*)$ , quadratic curvature from second-order terms still dominates both the linear odd-p residual leakage and any higher-order corrections. The result is a locally smooth and convex objective function – with local optimization initiated at  $(\tau_0^*, \tau_1^*)$  reliably converging to the unique optimum<sup>3</sup>.

Figure 2 shows the result of running Nelder-Mead from the analytic starting point  $(\tau_0^*, \tau_1^*)$  for system sizes N=5to 23 at depths p = 1 to 5. In every case the solver converges to an optima in a single basin. For the largest system N=23, the worst-case success probability over all bit-string targets at p = 1 is only 0.267 (occurring at Hamming weight h = 8), but this minimum probability rises to 0.925 by p = 5. Conversely, the h = 11state achieves a probability of 0.901 at p = 1 and essentially unity at p=5. At fixed N the probability profile as a function of Hamming weight is well described by a skewed quadratic, with its lowest point near  $h \approx |N/4|$ and monotonic increase towards both the all-zero and maximum Hamming weight. The  $(\tau_0, \tau_1)$  for the optima depicted in the right panel are listed in II. Over p=1to 3 the effective walk time  $T_{\text{eff}}$ , as given by Eq. (28), approaches  $\pi/2$ .

<sup>&</sup>lt;sup>3</sup> For the unconstrained hypercube,  $[\mathcal{G}_+, \mathcal{G}_-] = 0$ . Consequently  $R_-(\tau_1)R_+(\tau_1) = \exp(-2i\tau_1\,\mathcal{G}_+)$  exactly, and perfect transfer is trivial at p'=1 with  $\tau_0=0$ .

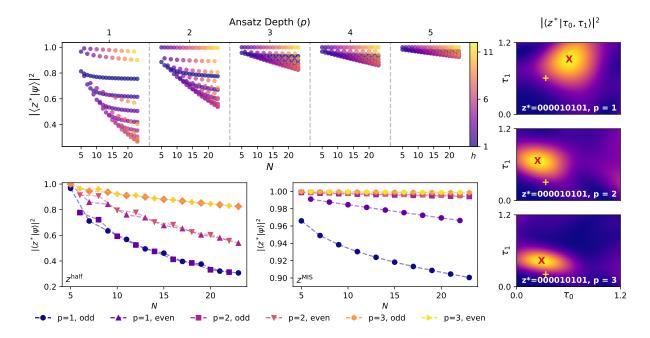


FIG. 2. Success probability of preparing product states  $|z^*\rangle = |0^{N-2h}(01)^h\rangle$ , where h=1 to  $\lfloor N/2 \rfloor$ , using optimized walk times  $\tau_0$  and  $\tau_1$ , for systems sizes N=5 to 23. The top-left panel shows success probability across ansatz depths p=1 to 5 colored by Hamming weight (h), while the bottom-left panel highlights the reference states  $z^{\text{half}}$  and  $z^{\text{MIS}}$  (see Section II C). The right panels show success probability as a function of  $\tau_0$  and  $\tau_1$  for the N=9 instance of  $z^{\text{half}}$  at depths p=1, 2, and 3. The optimal points (given in Table II) are marked with a red 'X', and theoretical predictions from Eqs. (31) and (32) is indicated by a yellow '+'.

#### 2. Bracelet states

The optimization strategy for preparation of bracelet states is guided by two practical considerations, as expanded in Section III. First, we aim to minimize the total evolution time and, second, utilize the hardware-native realization of the phasor unitary

$$\hat{U}_h(\gamma) = e^{-i\gamma\hat{n}},\tag{33}$$

where  $\hat{n} = \sum_{i=1}^{N} |1\rangle_i \langle 1|_i$  applies a uniform phase to all computational basis states with the same Hamming weight.

The initial stage sweeps the walk generator alone,

$$|\psi(\tau)\rangle = e^{-i\tau\mathcal{G}}|0\rangle \tag{34}$$

at discrete times  $\tau_j = j\Delta\tau$  for  $0 < \tau_j \le \tau_{\rm max}$ . At each sample we measure the population of the target bracelet subspace  $|\langle [z^*]|\psi(\tau_j)\rangle|^2$  and locate the local maxima  $\tau_{\rm peak}^{(m)}$  higher than  $1/|D_N|$ , as demonstrated in the top panel of Fig. 3.

Starting from the second identified peak, we fix the total walk duration to  $\tau_{\rm tot} = \tau_{\rm peak}^{(m)}$  and set the ansatz depth and walk times to,

$$p = \left\lfloor \frac{\tau_{\text{tot}}}{\tau_{\text{min}}^{\text{hw}}} \right\rfloor - 2, \qquad \tau = \frac{\tau_{\text{tot}}}{p+1}$$
 (35)

where  $\tau_{\min}^{\text{hw}}$  is the minimum allowed walk time ( $\sim$  0.4) and the -2 guards against quantization error arising through hardware constraints so the walk time are fixed and only the phasor unitary  $\vec{\gamma} = \{\gamma_k\}_{k=1}^p$  remain variational. As the resulting landscape is non-convex, we optimize these with the derivative-free COBYLA optimizer initialized at  $\gamma_k = 0$ , stopping the scan over peaks when there is a decrease in success probability or the range of identified peaks is exhausted.

We discard the first peak because the state is concentrated in the  $C_N$ -symmetric (translation-invariant) subspace. Since the phasor unitary depends on Hamming weight, it acts as  $e^{i\gamma h}\mathbb{I}$  on each weight-h sector and therefore cannot distinguish distinct  $D_N$ -symmetric components. The total walk time  $\tau_{\rm tot}$  must be long enough to accumulate relative phase between  $D_N$ -symmetric components at fixed h before optimization over  $\vec{\gamma}$  can appreciably improve the success probability.

More generally, the required  $\tau_{\rm eff}$  for high-fidelity preparation of  $|[z^*]\rangle$  is explainable with a frequency-resolution model. Let  $\{(\lambda_r,|r\rangle)\}_{r=1}^d$  be the eigenpairs of the walk generator  $\mathcal G$  restricted to the  $D_N$ -invariant subspace [36]. Because the only interleaved control is the phasor unitary, resolvability of the target state is governed by spectral gaps  $\lambda_{rs} = |\lambda_r - \lambda_s|$  between eigenmodes  $r, s \in \{1,\ldots,d\}$  that both have appreciable support on the same Hamming-weight sector as the target [37]. Consequently, control under variation of  $\vec{\gamma}$  requires order-unity accumulated phase,  $\lambda_{rs} \tau_{\rm eff} = \Theta(1)$ .

We encode this threshold by introducing the dimensionless constant  $\kappa = \Theta(1)$  and enforcing  $\lambda_{rs}\tau_{\text{eff}} \geq \kappa$ . Among resolvable pairs, we then define the resolvable spectral minimum,

$$\Delta_{\min}(\kappa) = \min_{\lambda_{rs} \ge \kappa/\tau_{\text{eff}}} \lambda_{rs}, \tag{36}$$

and the model predicts that this low-frequency component sets the timescale,

$$\tau_{\rm eff} \propto \frac{1}{\Delta_{\rm min}(\kappa)}.$$
(37)

To identify which gaps are associated with the target Hamming-weight,  $h^* = h([z^*])$ , we expand each eigenmode  $|r\rangle$  in the bracelet basis  $\{|[z]\rangle\}$  and compute its weight in the target sector

$$u_r = \sum_{h([z])=h^*} |\langle [z]|r|\rangle|^2 \in [0,1],$$
 (38)

with  $\Delta_{\min}(\kappa)$  chosen from the eigenmodes with non-zero weight.

The bottom panel of Fig. 3 shows that, across all distinct targets  $|[z^*]\rangle$  at N=5 to 12, the accumulated walk time  $\tau_{\rm eff}$  scales linearly with  $1/\Delta_{\rm min}(\kappa^*)$  with a single fixed  $\kappa^*$ . Within each N, targets that are a maximal independent set consistently exhibit the smallest  $\tau_{\rm eff}$  and the largest  $\Delta_{\rm min}$ . This aligns with prior work on scarred quantum walks [38], which places these states on or near a small, low-degree hypercube-like subgraph weakly coupled to the rest of the graph – with dynamics confined to this subgraph producing sparser, quasi-regular spectra with larger internal gaps, resulting in faster preparation.

# III. RYDBERG ATOM ARRAYS

Neutral atom quantum computers are a promising modality for various applications, from gate-based computing [39, 40], fault-tolerant algorithms [41–44], and Hamiltonian simulation [45–47]. In a neutral atom computer, individual atoms, such as Rubidium 87, are trapped in optical lattices with the qubit encoded into the electronic states of valence electrons [48]. Quantum operations are implemented by manipulating the electronic state of the atom using finely tuned lasers. Entanglement is mediated by the Rydberg state, a highly excited orbital that strongly interacts via a Van der Waals interaction with nearby atoms in the Rydberg state.

In this work, we focus on analog mode neutral atom computers, and specifically that of Aquila, QuEra Computing's cloud analog Hamiltonian simulator [23]. Here, the qubit is encoded into a ground state  $|g\rangle = |5S\frac{1}{2}\rangle$  and Rydberg state  $|r\rangle = |70S\frac{1}{2}\rangle$  of Rb87. Due to adjacent Rydberg states constantly interacting, an analog mode computer implements the time-dependent dynamics of

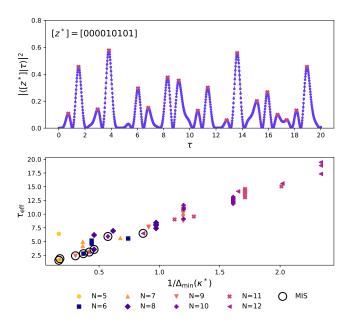


FIG. 3. Bracelet state preparation. Top: Population of the N=9 target bracelet state  $|[z^{\mathrm{half}}]\rangle$  during a continuous-time quantum walk from  $|0\rangle$ , sampled at  $\tau_j=j\,\Delta\tau$  with  $\Delta\tau=0.02$ , over  $\tau\in[0,20]$ . Local maxima  $\tau_{\mathrm{peak}}^{(m)}$  above the threshold  $1/|D_N|$  are marked by a pink 'X', which serve as fixed parameters in variational optimization of phases  $\gamma$ . Bottom: Accumulated walk time  $\tau_{\mathrm{eff}}$  required to reach success probability  $\geq 0.98$  against the inverse resolvable spectral minimum  $1/\Delta_{\min}(\kappa^*)$  (see Eqs. (36) and (37)) for all distinct targets  $|[z^*]\rangle$  with  $N=5,\ldots,12$ . Here  $\Delta_{\min}(\kappa)$ . We fix  $\kappa^*\approx 7.2\pm 0.4$  from a stability plateau by maximizing mean(r)-2 std(r) over sliding  $\kappa$  windows and taking the window center. Maximumindependent-set (MIS) targets are circled in black.

an Ising-like Hamiltonian

$$H(t) = \frac{\Omega(t)}{2} \sum_{i} e^{i\phi(t)} |g_{i}\rangle\langle r_{i}| + e^{-i\phi(t)} |r_{i}\rangle\langle g_{i}|$$
$$-\sum_{i} (\Delta(t) + w_{i}\delta(t)) |r_{i}\rangle\langle r_{i}|$$
$$+\sum_{i,j} V_{i,j} |r_{i}r_{j}\rangle\langle r_{i}r_{j}|. \quad (39)$$

The first term is a Rabi drive, which coherently drives each atom between the ground and Rydberg state. The second term is the detuning, which is a Pauli-Z-like term that applies an energy penalty to the Rydberg state. The detuning is separated into a global detuning  $\Delta(t)$  that acts on every qubit uniformly, and a local detuning  $w_i\delta(t)$  that acts on individual atoms i with weights  $w_i$  and time-dependent value  $\delta(t)$ . The third term is the interaction  $V_{ij} = C_6/|\vec{x}_i - \vec{x}_j|^6$  between atoms at positions  $\vec{x}_i$  and  $\vec{x}_j$  respectively. In systems such as Aquila used in this work, the typical Rabi frequency is  $\sim 2.5 \mathrm{MHz}$  and the characteristic inter-atomic distance is  $\sim 8\mu\mathrm{m}$ , where up to 256 atoms can be arbitrarily positioned in 2d space.

The goal of hardware implementation is to abstract away the physical Hamiltonian so that it optimally matches the generator of continuous time quantum walks

$$\mathcal{T}e^{-i\hbar\int_0^T H(t)dt} \approx \mathcal{T}e^{-i\int_0^\tau G(\tau)d\tau},$$
 (40)

where  $G(t) = X(t)\hat{\mathcal{G}} + Z_i(t)\hat{\mathcal{C}}$  is the generator of quantum walks in the independent set subspace spanned by the projector  $\mathcal{P}$ ; X(t) and Z(t) are piecewise constant pulses that recreate the alternating ansatz. The states can be identified with bitstrings by matching  $|g\rangle \mapsto |0\rangle$  and  $|r\rangle \mapsto |1\rangle$ . The Rabi drive can be matched to the  $\sigma_x$  term by rescaling time  $t = 2\tau/\Omega$ .

A phase jump of the Rabi drive  $\phi(t) = -d\phi\Theta(t - t_0)$  can implement a global phase jump  $Z_i(t) = d\phi\delta(t - t_0)$  like that used in state preparation. Local detuning can be used to implement local phase jumps by quickly turning off the Rabi drive, then implementing a fast pulse of local detuning  $w_i\delta(t)$  on the target atoms such that  $d\phi_i = w_i \int_0^T \delta(t) dt$ . On Aquila, this is most naturally done with a 100 ns triangle pulse with an area of the maximum phase jump, as shown in Fig. 4.

The projection to the independent set subspace can be implemented with a key feature of the Rydberg atom Hamiltonian: the **Rydberg blockade**. Due to the strong sixth power of the interaction with the distance between atoms, two atoms close together with a distance less than the **blockade radius**  $|\vec{x}_i - \vec{x}_j| < r_b$  will have an energy of the doubly-excited Rydberg state that is much higher than any other scale in the problem. These doubly-excited states can then be integrated out of the dynamics, resulting in an effective subspace of all excitations in the space of independent sets [22, 49]. The graph connectivity is defined by a unit disk graph where the radius is analogous to the blockade radius of the Hamiltonian.

The blockade radius is defined by the Rabi frequency, detuning, and  $C_6$  coefficient. The dynamic blockade radius, as used for encoding blockaded dynamics, is  $r_d \equiv (C_6/\Omega)^{1/6}$ . The static blockade radius, as used for encoding the maximum independent sets into Ising ground states [49], is  $r_s = (C_6/\Delta)^{1/6}$ . For Aquila,  $\Omega$  has a maximum of 15.8rad/ $\mu$ s and  $C_6 = 5420503 \ \mu m^6 {\rm rad}/\mu$ s, so that the dynamic blockade radius is  $r_d \approx 8.367 \mu {\rm m}$ .

To preserve the independent set condition, adjacent atoms must be close together to ensure that they are well within the blockade radius. However, to preserve independent set dynamics, nearby atoms outside of the unit disk radius must be far enough apart to ensure that the  $1/R^6$  interaction tail does not spuriously add unwanted interactions. These two factors can be balanced by considering that there is no singular unit disk radius; instead, there is a range of radii that generate the same unit disk graph. The minimum unit disk radius  $r_{min}$  is the maximum distance between any two vertices connected by an edge; the maximum unit disk radius  $r_{max}$  is the minimum distance between any two vertices not connected by an edge; as an example, consider Fig. 11. To best preserve

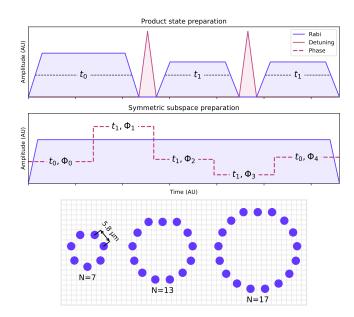


FIG. 4. Representative Analog programs to implement quantum walk state preparation. **Top** plots the analog waveforms for product state preparation. Purple is the waveform for the Rabi frequency, which implements the quantum walk Hamiltonian where the total area of each trapezoid implements a walk of unitless time  $\tau_i$ . Red is the waveform for the local detuning, which implements the phasor Z term on a subset of sites; each triangle has an integrated area of  $\pi/2$ . The Rabi term is turned off during the phase accumulation to avoid any non-commutative terms. Middle plots the analog waveforms for bracelet state preparation. The red dashed line is the Rabi phase, which is U(1) gauge equivalent to the phasor via phase jumps. Bottom plots the atom positions for 7, 13, and 17 atoms, with a background grid spacing of  $2\mu m$ . Observe that the top of each circle and bottom of N=13 and 17 are flattened to conform to the row constraints of Aquila [23].

dynamics, the energy associated with the blockade radius should be much larger (smaller) than the minimum (maximum) distance between atoms

$$\frac{C_6}{r_{\min}^6} \gg \frac{C_6}{r_{\rm b}^6} \gg \frac{C_6}{r_{\max}^6}.$$
(41)

More specifically, the perturbative Schrieffer-Wolff corrections arising from finite blockade energies must be much smaller than the energy-scale of the Rabi drive. This condition can be maximally satisfied by choosing the blockade radius to be  $r_{\rm b} = \eta \sqrt{r_{\rm min} r_{\rm max}}$  where  $\eta \approx 0.936$  for the atom geometries explored in this work. For more details, see Appendix B and the supplemental of [50].

The space of independent sets of an N-vertex nearest-neighbor ring can be represented by an N-atom ring of atoms in a 2d atom array. Given the ring where each atom is a distance D from the origin,  $r_{\min} = 2D\sin(\pi/N)$ 

and  $r_{\text{max}} = 2D\sin(2\pi/N)$ . The distance is

$$D = \frac{r_d}{2\eta \sqrt{\sin(\pi/N)\sin(2\pi/N)}} \approx r_d \times \frac{1}{\sqrt{2}} \times \frac{N}{2\pi}.$$
 (42)

In aggregate, it is possible to physically reproduce the state prepared by the phase-walk ansatz (see Eq. (4)) by implementing a piecewise-constant, time-dependent evolution with respect to a walk generator

$$|\psi\rangle = \mathcal{T} \exp\left\{-i\int_0^T \left[X(\lambda)\hat{\mathcal{G}} + Z(\lambda)\hat{C}\right]d\lambda\right\} |\psi_0\rangle, (43)$$

where  $X(\lambda)$  and  $Z(\lambda)$  are non-overlapping, piecewiseconstant waveforms whose fragments have area  $\tau_q$  or  $\gamma_q$ , with an equivalent generator of the Rydberg-atom Hamiltonian for a ring of blockaded atoms by matching the independent-set subspace to the Rydberg-blockade subspace and setting the ring radius as above based on the dynamic blockade radius, using the average Rabi-drive amplitude. The global detuning is set to zero throughout. Global phase shifts are implemented via rotating-frame phase shifts of the Rabi drive <sup>4</sup>. Local phase shifts are implemented via the local detuning term by choosing a waveform  $\delta(t)$  such that  $\int_0^T w_i \delta(t) dt = \phi_i$  in the physical interval [0,T]. Global  $\sigma_x$  accumulation is implemented by choosing a waveform  $\Omega(t)$  such that  $\int_0^T \Omega(t)dt = 2\tau$  over the minimized physical interval [0,T]. The blockade radius and thus the distance between each atom is set by the average of  $\Omega(t)$  and thus varies slightly depending on the ansatz. Waveforms and positions are minimally relaxed to conform to slew rate and atom placement constraints inherent to Aquila [23]. An example bloqade program implementing a single walk step is shown in Box I, and a representative program is shown in Fig. 4.

### IV. RESULTS: PREPARATION OF PRODUCT AND BRACELET STATES

We present results for the preparation of the product states  $z^{\rm half}$  and  $z^{\rm MIS}$ , together with their corresponding bracelet states  $[z^{\rm half}]$  and  $[z^{\rm MIS}]$ , as described in Section II C, using the two phase-walk ansätze introduced in Section II E. These are realized in three scenarios: the "perfect" CTQW dynamics, its approximation by the Rydberg Hamiltonian in noiseless emulation, and experimental implementation on Aquila, sharing the same variational parameters across each case. Our comparison focuses on the success probability and amplification relative to a uniform baseline, contrasting the observed amplification as a function of both the cardinality of the invariant (blockaded) subspace  $|\mathcal{V}|$  and the ansatz depth p,

```
import bloqade.analog as ba # QuEra's SDK
# --- Input parameters ---
N = 20 # Set the number of atoms
tau = 2*np.pi*1.2345 # Unitless evolution time
Omega = 2*np.pi*2.5 # Aquila's Rabi frequency
# Evolution time in usec
evolution_time = tau / Omega
# Dynamic blockade radius in um
d = (5_420_503/0mega) ** (1/6)
D = d/(2*np.sqrt(
        np.sin(np.pi/N)*np.sin(2*np.pi/N)))
D *= 1.0 # Optionally add a variational
# fudge factor to optimize the blockade radius.
theta = np.linspace(0,2*np.pi,N+1)[0:N]
positions = np.array([D*np.sin(theta),
                      D*np.cos(theta)]).T
positions = [tuple(q) for q in positions]
positions = ba.atom_arrangement.\\
    ListOfLocations (positions)
              ba.constant(
              duration=evolution_time,
              value=Omega)
detuning = ba.constant(
              duration=evolution_time,
              value=0.0)
program = ba.rydberg_h(
              atoms_positions=positions,
              amplitude=rabi_drive,
              detuning=detuning)
# Run using emulation
data = program.bloqade.python().run(100)
# Run using Aquila via BraKet
data2 = program.braket.aquila().run_async(100)
```

TABLE I. A code snippet outlining a simple bloqade.analog program, which implements a single quantum walk step for a nearest neighbor blockaded ring. The parameters at the top define the program: N defines the number of atoms in the ring; phi sets the total (unitless) evolution time of the quantum walk; Omega defines the Rabi frequency as set by the capabilities of Aquila. Note that this program does not fit Aquila's hardware constraints, such as row spacing and rise time, which have been excluded for simplicity.

following the power-law scaling relationships introduced in Section II D. Particular attention is given to the extent to which the scaling behavior predicted by the ideal CTQW is qualitatively reproduced on noisy hardware.

The unitless walk times  $\tau$  are mapped to physical hardware time by maximizing the Rabi frequency of  $\Omega=15.8\mathrm{rad}/\mu\mathrm{s}$ . Evolution under the walk generator is implemented with a hardware-minimum rise and fall time of  $0.05\mu\mathrm{s}$  [23]. Consequently, there are four regimes of Rabi drive pulse length. For walk times  $\tau<0.40$ , the pulse is triangular with a constant duration of  $0.10~\mu\mathrm{s}$ . Between  $0.40 \leq \tau < 0.59$  the pulse is also triangular, with a duration of  $\frac{4\tau}{15.8}~\mu\mathrm{s}$ , increasing from 0.10 to  $0.15~\mu\mathrm{s}$ . Between  $0.59 \leq \tau \leq 0.79$ , the optimal pulse

<sup>&</sup>lt;sup>4</sup> Note that global phase shifts can be done equivalently through a time-integrated global detuning, but this requires a longer physical evolution time that yields in a lower fidelity result.

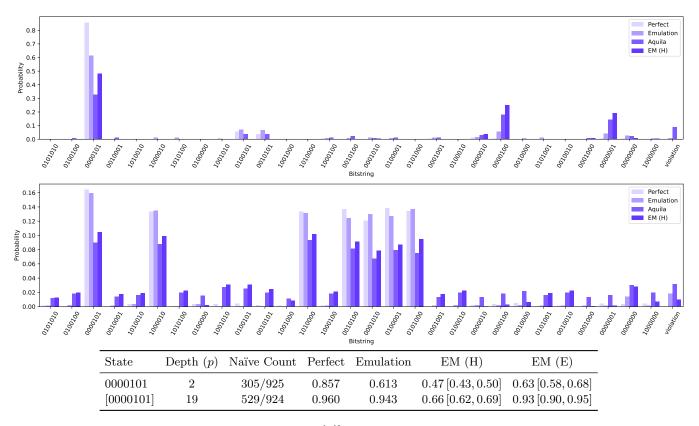


FIG. 5. Probability distributions for preparation of the  $z^{\rm half}$  product state (top panel) and the corresponding bracelet state (middle panel) at N=7, obtained from numerical simulation of the ideal CTQW dynamics ("Perfect"), noiseless emulation of the Rydberg Hamiltonian ("Emulation"), the raw probability distribution from 1000 shots on Aquila, and the reconstructed distribution ("EM") that accounts for measurement error. The table below summarizes the corresponding success probabilities: the **State** column labels the target state; **Ansatz Depth** is the number of walk layers; the **Naïve count** column gives the fraction of bitstrings matching the target directly from raw counts; the **Perfect** column reports the preparation probability under ideal CTQW dynamics; the **Emulation** column shows the same quantity using noiseless Rydberg-atom emulation. The **EM** (**H**) column gives the success probability from Aquila hardware data after mitigation of measurement errors, while **EM** (**E**) reports the corresponding value from 1000 emulated shots convolved with the noisy measurement channel. Data for other target states are shown in tables II and III.

is a reduced-amplitude trapezoid of duration 0.15  $\mu s$ . Beyond  $\tau=0.79$ , the pulse duration grows linearly as  $0.127\,\tau+0.05~\mu s$ .

The blockaded subspace is encoded into atom positions encoded as a ring, with nearest-neighbor distances given by Eq. (42). Atom positions are rounded to the nearest  $0.1\mu$ m, and the tops and bottoms of the rings are flattened to conform to the  $2\mu$ m row spacing constraints of Aquila [23]. Some example atom positions are shown in Fig. 4. All experimental data were acquired through Aquila using QuEra's exclusive access mode during the month of August 2025, and all shots were post-selected on fully filled arrays.

There is added complexity when using Aquila due to a measurement error of  $\sim 7\%$  in misidentification of the Rydberg state as ground and, when using local detuning, an additional  $\sim 10\%$  error in misidentification of the ground state, which renders naive frequentist bitstring counting unreliable for large bitstrings. To address this, we employ a Bayesian postprocessing method based on

expectation maximization (EM, see Appendix A), which reconstructs the pre-measurement probability distribution by modeling readout as an asymmetric bit-flip channel, with the probability of the target state taken from the reconstructed distribution unless otherwise noted. Uncertainties in the estimated probabilities are quantified using a nonparametric bootstrap to obtain 95% confidence intervals. In Section IV A, we fit the amplification A to Eq. (13) via weighted nonlinear least squares, taking the 95% CI for  $\alpha$  from the fit covariance under the constraint  $\alpha < 1$ . The intervals are propagated through  $n = 1/(1-\alpha)$ , with  $n \to \infty$  (reported as " $\geq$ ") when the upper bound reaches one.

Figure 5 presents the unprocessed probability distributions for the  $z^{\rm half}$  product and bracelet states at N=7 across the three considered scenarios, together with the success probabilities obtained from 1000 shots in noiseless emulation convolved with the asymmetric bit-flip channel, and from the same number of shots on Aquila. Here, and for all other considered target states (see tables , II

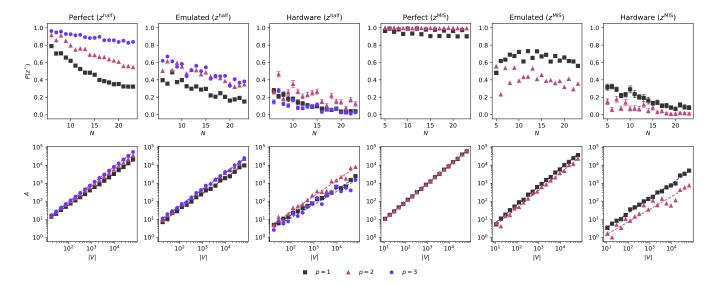


FIG. 6. Comparison of success probabilities  $P(z^*)$  (top row) and amplification factors  $A = |\mathcal{V}| P(z^*)$  (bottom row) for  $z^{\text{half}}$  (left panels) and  $z^{\text{MIS}}$  (right panels) across CTQW (Perfect), noiseless Rydberg emulation, and Aquila hardware. Results are shown at depths p = 1, 2, 3 for  $z^{\text{half}}$  and p = 1, 2 for  $z^{\text{MIS}}$ . Amplification plots include power-law fits to Eq. (13) (dashed lines). For  $z^{\text{half}}$  targets, CTQW fits yield  $c = \{1.099, 1.112, 1.015\}$  with exponents  $\alpha = \{0.878 \pm 0.006, 0.935 \pm 0.003, 0.981 \pm 0.002\}$  ( $R^2 = \{0.9870, 0.9997, 0.9997\}$ ); emulation gives  $c = \{0.632, 0.719, 0.785\}$  with  $\alpha = \{0.877 \pm 0.009, 0.934 \pm 0.006, 0.930 \pm 0.006\}$  ( $R^2 = \{0.9881, 0.9961, 0.9959\}$ ); and hardware yields  $c = \{0.595, 0.463, 0.418\}$  with  $\alpha = \{0.729 \pm 0.041, 0.876 \pm 0.025, 0.734 \pm 0.051\}$  ( $R^2 = \{0.9326, 0.9374, 0.7889\}$ ). For  $z^{\text{MIS}}$  targets, CTQW remains near-linear with  $c = \{0.995, 1.000\}$  and  $\alpha = \{0.996 \pm 0.001, 1.000 \pm 0.000\}$  ( $R^2 = \{0.9972, 1.0000\}$ ); emulation gives  $c = \{0.641, 0.507\}$  with  $\alpha = \{1.000 \pm 0.004, 0.967 \pm 0.006\}$  ( $R^2 = \{0.9838, 0.9857\}$ ); and hardware yields  $c = \{0.512, 0.285\}$  with  $\alpha = \{0.820 \pm 0.029, 0.703 \pm 0.067\}$  ( $R^2 = \{0.9724, 0.9250\}$ ).

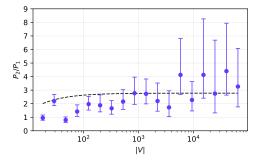


FIG. 7. Ratio of success probabilities for preparation of  $z^{\text{half}}$  at depth p=2  $(P_2)$  relative to depth p=1  $(P_1)$  on Aquila. The black dashed line shows the corresponding ratio for a noiseless restricted depth Grover's search over a database of size  $|\mathcal{V}|$  with one marked element.

and III), the reconstructed distributions obtained from the scrambled emulation results agree with the true values within the 95% confidence intervals.

# A. Product state preparation

The walk parameters for the target  $z^{\text{half}}$  and  $z^{\text{MIS}}$  products states are obtained through ideal simulation of a quantum walk generator, that is, a walk without Rydberg interactions, blockade violations, and finite rise times.

They are then directly transferred, with no further optimization, to the time-dependent Rydberg Hamiltonian as described above. The local phasor unitary (see Eq. (16)) is encoded into a minimum-duration triangle pulse with a 50ns rise and fall time commensurate with the constraints of Aquila [23], and a peak value of  $\Delta_{\rm max} = \phi/50 {\rm ns} \le 62 {\rm rad}/us$ 

In Fig. 6, the  $z^{\rm half}$  targets show higher scaling exponents for both the CTQW reference and noiseless emulation. Via Eq. (14), the CTQW fits correspond to effective polynomial speedup orders  $n=\{8.17\,[7.80-8.58],\ 15.3\,[14.6-16.2],\ 53.1\,[48.3-59.0]\}$ , with emulation following closely at p=1 and 2 with  $n=\{8.15\,[7.57-8.82],\ 15.1\,[13.8-16.7]\}$ , but diverging at p=3 with  $n=14.4\,[13.2-15.7]$ . On hardware, the same qualitative behavior is observed, albeit diminished: the scaling exponent increases from p=1  $(n=3.70\,[3.21-4.35])$  to p=2  $(n=8.07\,[6.69-10.1])$  but declines at depth p=3  $(n=3.76\,[3.15-4.66])$ .

Figure 7 shows the ratio of success probabilities at p=2 relative to p=1. Although variability and uncertainty preclude a full fitting-based analysis, the increase in success probability appears to follow a generally rising trend with N which, due to the proportional increase in amplification with  $|\mathcal{V}|$ , exceeds the convergence gain predicted by an ideal noiseless restricted-depth Grover search [34] for all even  $N \geq 16$  and for odd N=23.

For  $z^{\text{MIS}}$  targets, the CTQW fits remain tightly clustered around unity across all depths. At p=1, the effective

tive order is n=247 [200-322], and at p=2 within the limits of numerical precision, the amplification scales directly proportional to  $|\mathcal{V}|$  over the range of considered N. This can be explained by noting that in  $\mathcal{G}$  (here a "Lucas" cube [26]), the subspaces defined by odd and even qubits each induce a hypercube of dimension  $\lfloor N/2 \rfloor$ , containing the antipodal pair  $0^N \leftrightarrow 1^N$  [38], for which the hypercube graph is known to exhibit perfect state transfer [11]. Emulated Rydberg dynamics show similarly strong scaling at depth p=1, with  $n \geq 261(\alpha \to 1)$ , while depth p=2 yields a reduced effective order of n=30.5 [25.9–37.3].On hardware, amplification follows n=5.56 [4.79 – 6.63] at p=1, dropping to n=3.37 [2.75 – 4.34] at p=2.

Overall, these results are consistent with the superlinear amplification scaling in  $|\mathcal{V}|$  expected of an efficient phase-walk ansatz, and to a lesser extent with increasing depth. The phasor unitary relies on local detuning, which carries a shot-to-shot per-site coherent error of roughly 10%, leading to deviations from the intended  $\pm \pi$  phase shifts that compound across iterations. This is likely the primary reason for the breakdown of increased convergence at higher p, in addition to the effects of incoherent errors from longer program times. It is therefore notable that the  $z^{\text{half}}$  targets still show improved convergence at p=2. For the  $z^{\text{MIS}}$  targets, the CTQW dynamics predict convergence near unity across the range of N. Consequently, the decrease in amplification from p=1 to p=2in both emulated and experimentally implemented Rydberg dynamics is consistent with saturation of CTQWbased convergence at p=1, where further iterations offer little additional amplification and are outweighed by the loss in success probability from accumulated phase errors.

# B. Bracelet state preparation

In bracelet state preparation, we find that direct parameter transfer from the ideal CTQW-based ansatz to its approximation via the Rydberg Hamiltonian is unreliable due to the effect of van der Waals interactions over  $\tau_{\rm eff}$ , which is significantly longer than the  $\tau$  used in product state preparation. These interactions introduce an additional positive phase through Rydberg-Rydberg couplings, which contributes coherent phase error to the walk evolution and breaks the  $-\gamma \equiv \gamma$  equivalence that holds for the ideal CTQW. However, we find that this effect can be mitigated through joint optimization of  $\tau_{\rm eff}$  and  $\gamma$ . By taking as the objective the mean success probability

$$\frac{1}{2} \left( P_{\mathrm{CTQW}}([z^*]) + P_{\mathrm{Ryd}}([z^*]) \right),$$

we are able to identify parameter sets that achieve high convergence simultaneously in both the CTQW and Rydberg cases, with the global phasor (see Eq. (33)) encoded into a piecewise constant phase profile of the Rabi drive.

Compared to the CTQW-derived parameters, the jointly optimal parameters systematically require a

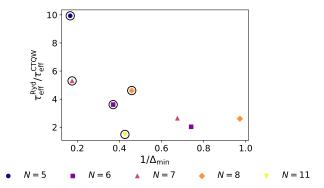


FIG. 8. Comparison between the ideal CTQW-based bracelet-state preparation ansatz and its implementation via the Rydberg Hamiltonian for states  $[z^{\rm half}]$  and  $[z^{\rm MIS}]$  (circled in black) that achieved  $\frac{1}{2}\left(P_{\rm CTQW}([z^*]) + P_{\rm Ryd}([z^*])\right) \geq 0.8$  under joint parameter optimization. The horizontal axis shows the CTQW-predicted inverse resolvable spectral minimum  $1/\Delta_{\rm min}$  (also shown in Fig. 3), while the vertical axis shows the ratio of optimal walk times found through the procedure described in Section II E 2.

longer walk time. In Fig. 8, the ratio of the jointly-optimal cumulative walk time  $\tau_{\rm eff}^{\rm Ryd}$  over the CTQW optimal  $\tau_{\rm eff}^{\rm CTQW}$  against the CTQW-predicted  $1/\Delta_{\rm min}$  instances reveals that target states with smaller spectral gaps have the largest inflation in  $\tau_{\rm eff}$  (up to  $\sim 10\times$ ), while more "resolvable" targets cluster around a factor of  $\sim 1.5\text{-}2.5$ . This behavior indicates that van-der-Waals-induced phases compress the effective gap, thus requiring a longer effective walk time to resolve the target. The  $[z^{\rm MIS}]$  states are more strongly impacted than the  $[z^{\rm half}]$  targets, which is consistent with their higher Rydberg density and the correspondingly faster accumulation of interaction-induced phases.

In Fig. 9, we see that success probability for preparation of  $[z^{\text{half}}]$  and  $[z^{\text{MIS}}]$  on hardware decreases approximately linearly with  $\tau_{\rm eff}$ , which most strongly impact the  $[z^{half}]$  targets. Nevertheless, the amplification of  $[z^{\text{MIS}}]$  is qualitatively consistent with power-law scaling in  $|\mathcal{V}|$ , with the even N instances positively offset from the odd, which may be explained by the even case corresponding to a superposition over two states. In contrast, the odd cases span an N-fold superposition. In contrast, while emulated results for the  $[z^{half}]$  are also consistent with a power-law scaling, the hardware results do not reproduce this trend, due to the accumulation of error associated with the longer required cumulative walk time. Although, since circuit depth scales linearly with  $\tau_{\rm eff}$  in this setting, unlike in product-state preparation, we cannot preclude the existence of alternative parameter sets that achieve equivalent or superior performance at shorter effective walk times.

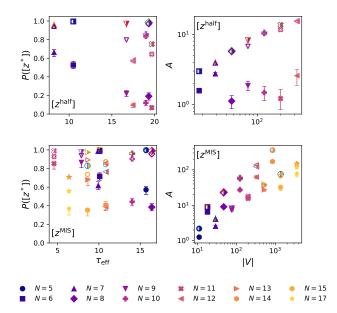


FIG. 9. Performance for bracelet state preparation of  $[z^{\text{half}}]$  (top) and  $[z^{\text{MIS}}]$  (bottom) on Aquila (filled markers outlined in black, error bars denote 95% CI), compared with noiseless emulation (hollow markers) and ideal CTQW dynamics (half-filled markers). The left panels show success probability as a function of cumulative walk time  $\tau_{\text{eff}}$ , while the right panels show the amplification of the target state as a function of  $|\mathcal{V}|$ .

### V. RESULTS: COHERENT QUENCHES OF BRACELET STATES

While the projected probability distribution  $P(z) = |\langle z|\psi\rangle|^2$  is a reasonable proxy to the fidelity of a state, it is incomplete. Notably, measurements in the Z basis ignore all phase information in the wavefunction, which makes the translationally averaged incoherent mixture

$$\rho_{\rm inc}([z]) = \frac{1}{|[z]|} \sum_{u \in [z]} |u\rangle\langle u|, \tag{44}$$

and the coherent bracelet state

$$\rho_{\mathrm{coh}}([z]) = |[z]\rangle\langle[z]| = \frac{1}{|[z]|} \sum_{u,v \in [z]} |u\rangle\langle v|. \tag{45}$$

have identical populations  $(|\langle z|\rho_{\rm inc}|z\rangle|^2 = |\langle z|\rho_{\rm coh}|z\rangle|^2)$ , even though there are nontrivial off-diagonal matrix elements in  $\rho_{\rm inc}$  and (if prepared exactly) none in  $\rho_{\rm inc}$ .

We implement quenches from  $|[z^{\text{half}}]\rangle$  and, for comparison, from the representative product state  $|z^{\text{half}}\rangle$ , using the resulting dynamics as an indicator of bracelet-state preparation, as the post-quench evolution of the coherent bracelet state diverges from that of its incoherent counterpart. The two states are prepared as shown in Fig. 5 and the protocol extended by adding an extra Rabi pulse of duration  $t = 2\tau/\Omega$ .

Figure 10 illustrates that, at short evolution times, the

dynamics of these two states are equivalent, but at longer times, interference effects cause the distributions to diverge. The coherent and incoherent perfect walk dynamics (gray) show the earliest divergence and clearly distinguish the coherent and incoherent states. Emulated neutral atom dynamics (black) follow the perfect walk dynamics, as expected from parameter matching the virtual and physical systems. However, at longer times the similarity diverges due to perturbative blockade violations and  $R^{-6}$  Rydberg interactions outside of the blockade radius (see App. B). This suggests that, while for short times, such as state preparation, the two dynamics approximately match. For longer evolution times, the dynamic blockade radius approximation begins to break down, and the Rydberg atom Hamiltonian can no longer accurately represent CTQW on independent set subspaces.

We find that results from Aguila qualitatively track the dynamics of the noiseless emulation. The most prominent difference in the observed dynamics being the higher population a  $\tau \approx 5.8$  in the  $|[z^{\text{half}}]\rangle$  quench (lower panel), which is consistent with the predicted dynamics of the coherent bracelet state. However, there is notable divergence from the noiseless prediction. Most clearly, in the quench from  $|[z^{\text{half}}]\rangle$  between  $\tau \approx 1.9$  and 3.2, two closely spaced peaks merge into a single broader maximum, consistent with the accumulation of dephasing error. For the incoherent quench, the 95% CI overlaps the ideal curve for most  $\tau$ , but it has a much broader interval relative to the coherent case - which could result from use of local detuning during the initial preparation stage (as discussed in Section IV A). Overall, although the experimental timescales ( $\leq 2\mu s$ ) lie within the Rabi-drive coherence time ( $\approx 5\mu s$ ), the deviations are not inconsistent with accumulated incoherent error over this window. We leave a deeper investigation of the specific contributing factors to future work.

### VI. CONCLUSION

This work has presented the first implementation and characterization of continuous-time quantum walk (CTQW) based ansätze on neutral atom hardware. By transferring the phase-walk structure from abstract models to execution under the Rydberg Hamiltonian, we established a direct link between ideal CTQW dynamics and their realization on an noisy intermediate-scale quantum (NISQ) device. Using known target states as controlled testbeds, we probed how constructive interference, amplification, and entanglement emerge in constrained Hilbert spaces subject to realistic hardware limitations. Variational optimization and errormitigation were carried out using Pawsey's "Setonix" and "Ella" supercomputing clusters, while hardware execution was performed on QuEra Computing's cloudaccessible, analog-mode neutral-atom processor Aquila.

Unlike earlier approaches [2] that used quantum walks

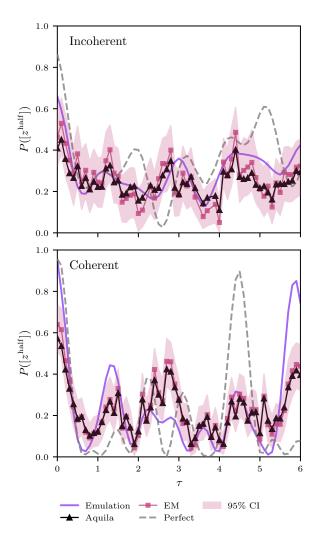


FIG. 10. Quenches for target state  $|z^{\rm half}\rangle = |[000101]\rangle$  under incoherent product state preparation (top) and bracelet state preparation (bottom), plotting the probability of measuring  $|z^{\rm half}\rangle$  in the dihedrally symmetric subspace of  $\mathcal G$ . Aquila data averaged over 100 shots per  $\tau$ .

as variational routines for preparing unknown states, here we target nontrivial but efficiently describable entangled states [29] chosen as benchmarks for assessing interference and amplification on NISQ hardware. In particular, we focus on bracelet states  $|[z]\rangle$  that respect the dihedral symmetry of an  $N\text{-}\mathrm{vertex}$  ring, and product states  $|z\rangle$  that must nonetheless have nontrivial dynamics due to the Hilbert space. Their entanglement does not arise from an explicit mixer term, but from the non-separable structure of the independent-set Hilbert space itself. We find that this constrained space, together with restricted local controls, is sufficient to support variational ansätze capable of preparing such structured target states.

We demonstrate a key use of quantum processors as analogue simulators of abstract models: the dynamics of CTQWs on independent-set graphs can be mapped directly onto the nonequilibrium dynamics of the Rydberg Hamiltonian. The independent-set constraint is naturally enforced by the Rydberg blockade, enabling a hardware representation of nonequilibrium dynamics on constrained subspaces that has previously only been explored in the context of quantum scar dynamics [22]. Despite substantial measurement errors, we find that Aquila can reproduce the same target states as the abstract CTQW system through direct parameter transfer with little to no further optimization. In the case of unentangled product-state targets, hardware circuits reproduce the hallmark amplification through constructive interference of efficient CTQW-inspired protocols. Scaling analysis shows super-linear growth in success probability with system size, and effective polynomial orders that are consistent with the super-quadratic convergence bounds expected from theory. Although this does not constitute evidence of an algorithmic speedup, the analysis demonstrates that the mechanisms underpinning such speedups are already observable on NISQ neutral atom hardware.

While this work is an initial step in implementing CTQW on neutral atom hardware, there are many future directions to explore. Instead of directly optimizing to some efficiently describable target state, future work could instead use the quantum walks framework to prepare resource states for hybrid optimization [51] or machine learning tasks [52]. Furthermore, this work focused on the walk graph formed by independent sets of an N-vertex ring; future work could implement different walk graphs induced by other independent set graph constraints or Hamiltonian engineering techniques [53] to explore quantum walks on tailored subspaces.

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#### ACKNOWLEDGEMENTS

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### Appendix A: Bayesian inference for distribution reconstruction

A central challenge in analyzing coherent state preparation is measurement error, where a 0 is mis-detected as a 1 or vice versa. This appendix details our use of the expectation maximization (EM) algorithm, a Bayesian inference procedure, to reconstruct the underlying distribution from measured bitstring data. Our approach is related to recent iterative methods for error mitigation [55], which employ histogram-based unfolding. Here, we instead construct a computationally tractable model by parameterizing only the blockaded subspace  $\mathcal{V}$  directly and regularizing contributions from its complement.

### 1. Bayesian inference

Bayesian inference provides a framework to address uncertainties arising through noisy measurement [56, 57]. Given observed data Z and model parameters  $\phi$ , Bayes

$$P(\phi \mid Z) \propto P(Z \mid \phi) P(\phi),$$
 (A1)

where the *prior* distribution  $P(\phi)$  is the model before data collection,  $P(Z \mid \phi)$  is the *likelihood* of the observing Z given parameters  $\phi$ , and the posterior distribution  $P(\phi \mid Z)$  describes how plausible the values of  $\phi$  are given Z. Obtaining parameters that best explain the observed data then occurs through maximization of the likelihood.

### Likelihood under measurement error

Measurement error on Aquila is well described by an asymmetric bit-flip channel acting independently on each qubit [23]. The probability of recording a bitstring zgiven pre-measurement string s is

$$K(z \mid s) = \prod_{j=1}^{N} P(z_j \mid s_j),$$
 (A2)

where j indexes the bits of z and s, which essentially convolves the pre-measurement probabilities with the transition matrix

$$P(z \mid s) = \begin{pmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{pmatrix},$$

where  $P_{ab}$  is the probability for the transition  $a \to b$ ,  $P_{01} = 1 - P_{00}$  and  $P_{10} = 1 - P_{11}$ .

As our aim is to construct the pre-measurement distribution over the subspace of graph vertices  $\mathcal{V}$ , we consider a model with parameters  $\phi_{|\mathcal{V}|} = \{\phi^k\}_{k=1}^{|\mathcal{V}|}$  for each possible originating bitstring  $s^k \in \mathcal{V}$ , and a reduced set of parameters  $\phi^{\perp} = \{\phi_j^{\perp}\}_{j=1}^N$  for bitstrings

 $s \in \mathcal{V}^{\perp} = \{0,1\}^N \setminus \mathcal{V}$ . Here each  $\phi_i^{\perp}$  models the probability that bit j of an observed bitstring is 1 in  $\mathcal{V}^{\perp}$ , using independent Bernoulli parameters,

$$P_{\text{out}}(s; \boldsymbol{\phi}^{\perp}) = \prod_{j=1}^{N} \left(\phi_{j}^{\perp}\right)^{s_{j}} \left(1 - \phi_{j}^{\perp}\right)^{1 - s_{j}}, \quad s \in \mathcal{V}^{\perp}.$$
(A3)

a choice informed by our assumption of independent single-qubit noise channels. The full parameter set is

$$\phi = \phi_{|\mathcal{V}|} \oplus \phi^{\perp} = (\phi^1, \dots, \phi^{|\mathcal{V}|}, \phi_1^{\perp}, \dots, \phi_N^{\perp}). \tag{A4}$$

Combining the contributions from  $\mathcal{V}$  and  $\mathcal{V}^{\perp}$ , the model probability of measuring bitstring  $z^i$  is

$$m_i(\phi) = \sum_{k=1}^{|\mathcal{V}|} \phi^k L_{k,i} + L_i^{\perp}(\phi^{\perp}),$$
 (A5)

where  $L_{k,i} = K(z^i \mid s^k)$  is the likelihood for  $s^k \in \mathcal{V}$ , and the likelihood for  $s \in \mathcal{V}^{\perp}$  is

$$L_i^{\perp}(\phi^{\perp}) = \sum_{s \in \mathcal{V}^{\perp}} K(z^i \mid s) P_{\text{out}}(s; \phi^{\perp}).$$
 (A6)

#### **Expectation Maximization**

The expectation–maximization (EM) algorithm is an iterative method for likelihood maximization. The algorithm alternates between two steps, which are guaranteed to monotonically converge to a local optimum of the likelihood [58]. The first is an "E-step" that computes responsibilities – the posterior probability that each component of the model generated observation  $z^i$ :

$$\rho_{k,i}^{(t)} = \frac{\phi^{k(t)} L_{k,i}}{m_i(\phi^{(t)})} \qquad \rho_{\perp,i}^{(t)} = \frac{L_i^{\perp} \left(\phi^{\perp(t)}\right)}{m_i(\phi^{(t)})}, \tag{A7}$$

with  $\sum_{k=1}^{|\mathcal{V}|} \rho_{k,i}^{(t)} + \rho_{\perp,i}^{(t)} = 1$ The second is the "M-step", which updates the parameters using the average of the responsibilities over all observations, which for  $s^k \in \mathcal{V}$  is:

$$\phi^{k(t+1)} = \frac{1}{|Z|} \sum_{i=1}^{|Z|} \rho_{k,i}^{(t)}.$$
 (A8)

where |Z| is the number of observed bitstrings. Updating  $\phi^{\perp}$  requires estimation of the expected pre-measurement bit values. For bit j this is given by:

$$\phi_j^{\perp (t+1)} = \frac{\sum_{i=1}^{|Z|} \rho_{\perp,i}^{(t)} \mathbb{E}[T_j \mid z_j^i] + \alpha}{\sum_{i=1}^{|Z|} \rho_{\perp,i}^{(t)} + \alpha + \beta}, \quad (A9)$$

where  $T_j$  denotes the true pre-measurement value of bit j, and  $\rho_{\perp,i}^{(t)}$  is the responsibility of  $\mathcal{V}^{\perp}$  for  $z^i$  at iteration

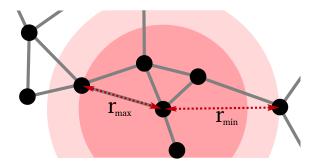


FIG. 11. An example unit disk graph to illustrate  $r_{\min}$  and  $r_{\max}$ . Given a unit disk graph of vertices (black) at positions  $\vec{x}_i$  and edges (grey) there is a range of unit disk radii  $r_{\max} \leq r_{\text{ud}} \leq r_{\min}$  which produce the same graph.  $r_{\max}$  is the maximum distance between any two vertices connected by an edge (equivalently, the minimum unit disk radius, pink).  $r_{\min}$  is the minimum distance between any two vertices not connected by an edge (equivalently, the maximum unit disk radius, light pink).

t. The strength of the Rydberg blockade effect leads to a small contribution from  $\mathcal{V}^{\perp}$ , so  $\alpha$  and  $\beta$  are introduced as regularizing terms (a Beta $(\alpha, \beta)$  prior [56]) on each  $\phi_j^{\perp}$  to prevent the estimate from collapsing to exactly 0 or 1. The expectation value  $\mathbb{E}[T_j \mid z_j^i]$  is computed by inversion of the bit-flip channel (Eq. (A2)):

$$P(T_j = 1 \mid z_j^i) = \frac{\phi_j^{\perp} P_{1, z_j^i}}{\phi_j^{\perp} P_{1, z_j^i} + (1 - \phi_j^{\perp}) P_{0, z_j^i}}.$$
 (A10)

In this work, we choose an "uninformative" prior, initializing  $\phi_{\mathcal{V}}$  as a uniform distribution and  $\phi^{\perp}$  to a uniform baseline with an uniform Beta(1,1) prior [57]. Parameters for the bit-flip channel are  $(P_{00}, P_{11}) = (0.99, 0.93)$  under standard operation [23] and  $(P_{00}, P_{11}) = (0.90, 0.93)$  when using local detuning. We use the convergence criterion

$$\|\phi_{\mathcal{V}}^{(t+1)} - \phi_{\mathcal{V}}^{(t)}\|_{1} + \frac{1}{N} \|\phi^{\perp (t+1)} - \phi^{\perp (t)}\|_{1} < \epsilon, \quad (A11)$$

where we set  $\epsilon = 10^{-8}$ . Uncertainty in the estimated probability of measuring a given target  $z^*$ , taken from the optimized  $\phi$ , is computed using a nonparametric bootstrap to a 95% confidence interval. We choose this approach as it adapts well to model complexity and finite-sample variability with minimal assumptions about the underlying distribution [59].

# Appendix B: Blockade subspace guarantees from perturbation theory

This appendix details the derivation of the scaling of the positions to best match the blockade radius  $r_b$ , which may be derived from perturbation theory. In the limit of large interaction strength between atoms within the blockade radius, we may derive the first-order correction and perturbative Hamiltonian from a virtual excitation to the blockaded space. By balancing the contribution from these first-order corrections and long-range  $r^6$  interactions outside of the blockade radius, the error Hamiltonian is minimized, maximizing the effective projective Hamiltonian evolution.

For each edge between vertices in the unit disk graph, consider the 2-qubit subspace

$$|gg\rangle \quad |gr\rangle \quad \stackrel{\longleftrightarrow}{\leftrightarrow} \quad |rr\rangle \quad \stackrel{\longleftrightarrow}{\leftrightarrow} \quad |rg\rangle \qquad (B1)$$

where the labels represent Hamiltonian terms. Given a definition of Pauli matrices  $\sigma_x \equiv |g\rangle\langle r| + |r\rangle\langle g|$  and  $n \equiv |r\rangle\langle r|$ , the effective error Hamiltonian is [50]

$$H_{\text{err}} = \sum_{\text{edges }(i,j)} \frac{-|\Omega|^2}{4V_{ij}} \left( \sigma_x^i \sigma_x^j + \sigma_y^i \sigma_y^j + n_i + n_j \right) + \sum_{\text{--edges }(i,j)} V_{ij} n_i n_j.$$
(B2)

The first term is the sum over the perturbative corrections from a virtual excitation to the doubly excited subspace, which results in a hopping term  $|gr\rangle \leftrightarrow |rg\rangle$  and Stark shift. The second term is the sum over all van der Waals interactions outside the blockade radius.

Minimizing the contribution of the error Hamiltonian is equivalent to minimizing its norm

$$||H_{\text{err}}|| = \sum_{\text{edges }(i,j)} \frac{|\Omega|^4}{4V_{ij}^2} + \sum_{\neg \text{ edges }(i,j)} V_{ij}^2.$$
 (B3)

Defining the Blockade radius  $|\Omega| \equiv C_6/r_b^6$  and assume that the contribution from blockaded edges comes from  $n_b$  edges per vertex at a maximum distance  $r_{\text{max}}$ , and the contribution from unblockaded edges comes from  $n_u$  edges per vertex at a minimum distance of  $r_{\text{min}}$ ;  $n_b$  and  $n_u$  can be scaled to include more weakly interacting edges by rescaling the sum. Then, the norm of the error is

$$\frac{||H_{\text{err}}||}{N} = \frac{n_b}{4} \left(\frac{C_6}{r_b^6}\right)^4 \left(\frac{r_{\text{max}}^6}{C_6}\right)^2 + n_u \left(\frac{C_6}{r_{\text{min}}^6}\right)^2.$$
 (B4)

Given a scaling  $\lambda$  between virtual coordinates  $\overline{x}$  and physical atom positions x as  $x=\lambda\overline{x}$ , the error can be minimized by minimizing with respect to  $\lambda$  as  $\partial_{\lambda}||H||=0$  to find

$$r_b = \eta \sqrt{r_{\min} r_{\max}}, \qquad \eta = \left(\frac{n_b}{4n_u}\right)^{1/24}.$$
 (B5)

Although the prefactor scales weakly with the 1/24th power, as shown in Fig. 12, we find that its inclusion tangibly improves the CTQW fidelity at small  $\tau$ . For the 1d chain,  $n_b=2$  and  $n_u\approx 2.220$ , so the prefactor shifts the blockade radius by a factor of 0.939.

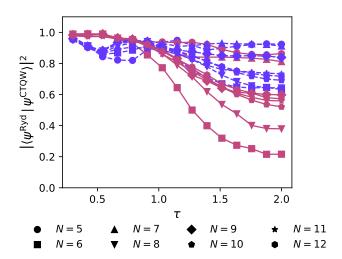


FIG. 12. Emulated fidelity for approximation of a CTQW with the  $|\psi^{\text{CTQW}}\rangle$  with walk time  $\tau$  by evolution under the Rydberg atom Hamiltonian  $|\psi^{\text{R}}\rangle$  (see Section III), shown with (red, solid lines) and without (purple, dashed lines) inclusion of the perturbative correction factor  $\eta$  for blockaded rings of size  $N=5,\ldots,12$ . Over this range the prefactor takes values  $\eta=(0.849,0.875,0.893,0.905,0.914,0.920,0.924,0.927)$ , asymptotically approaching 0.939.

For a 2d King's lattice,  $n_b = 2.25$  and  $n_u \approx 7.791$ , so the prefactor shifts the blockade radius by a factor of 0.940. The magnitude of the minimized error term is

$$\frac{||H_{\rm err}||}{N} = \sqrt{n_b n_u} \left(\frac{C_6^2}{r_b^{12}}\right) \left(\frac{r_{\rm max}}{r_{\rm min}}\right)^6.$$
 (B6)

Reducing the ratio between the maximum distance between vertices connected by a unit disk edge, and the minimum distance between vertices not connected by a unit disk edge will reduce the error. For the 1d chain,  $r_{\rm max}/r_{\rm min}=1/2$ , so  $||H_{\rm err}||/N\approx (0.181\Omega)^2$ . For the 2d king's graph,  $r_{\rm max}/r_{\rm min}=1/\sqrt{2}$ , so  $||H_{\rm err}||/N\approx (0.723\Omega)^2$ .

#### Appendix C: State Preparation Results

Tables II and III summarize the numerical and hardware results obtained for the preparation of product states and bracelet states, respectively. In both cases, the **State** column identifies the target configuration, where N denotes the number of vertices in the constraint graph and  $|\mathcal{V}|$  the number of vertices in the corresponding walk graph  $\mathcal{G}$ . The **Depth** (p) column records the number of ansatz layers. For product state preparation (Table II), the variationally optimised parameters are the fiducial and walk times  $\tau_0$  and  $\tau_1$ , and  $J_{\text{eff}}$  is the effective coupling (see Eq. (28)) computed from the walk times. For bracelet states (Table III), the relevant parameters are the effective evolution time  $\tau_{\text{eff}} = \tau p$  and the phase jumps  $\gamma$ .

The Naïve count column gives the fraction of bitstrings matching the target state directly from raw shot data. The Perfect column reports the corresponding preparation probability under ideal CTQW dynamics, while the Emulation column shows the same quantity obtained from noiseless Rydberg-atom emulation. Hardware performance is reported in the EM (H) column, which shows the success probability obtained from Aquila experiments after measurement-error mitigation (see Appendix A), together with the 95% CI. As a check for consistency, the EM (E) column presents values obtained from 1000 emulated shots convolved with the noisy measurement channel.

N	V	State	Depth $(p)$	$\tau_0$	$\tau_1$	$J_{ m eff}$	Naïve Count	Perfect	Emulation	EM (H)	EM (E)
5	11	00101	1	0.532	1.146	0.936	278/954	0.964	0.481	0.32 [0.28, 0.35]	0.48 [0.44, 0.51]
5 5	11 11	00101 00101	2	0.330 $0.225$	0.642 $0.456$	0.973 $0.987$	166/948 127/955	1.000 1.000	0.554 $0.503$	0.15 [0.12, 0.18] 0.08 [0.05, 0.10]	$0.56 [0.52, 0.59] \\ 0.50 [0.45, 0.54]$
6	18	000101	1	0.606	0.960	1.003	444/1882	0.789	0.396	0.28 [0.25, 0.30]	0.40 [0.35, 0.44]
6	18	000101	2	0.268	0.655	0.995	456/1874	0.915	0.504	0.26[0.24, 0.29]	0.51[0.46, 0.55]
6	18	000101	3	0.275	0.435	0.995	314/1870	0.963	0.620	0.15 [0.12, 0.17]	0.59 [0.54, 0.64]
7 7	29 29	0000101 0000101	$\frac{1}{2}$	0.639 $0.226$	$0.878 \\ 0.663$	1.035 $1.012$	336/1884 632/1855	$0.704 \\ 0.857$	0.356 $0.613$	0.21 [0.18, 0.23] 0.47 [0.43, 0.50]	0.38 [0.34, 0.43] 0.63 [0.58, 0.68]
7	29	0000101	3	0.302	0.423	0.999	454/1862	0.947	0.669	0.28 [0.25, 0.30]	0.69 [0.64, 0.73]
8	47	00010101	1	0.578	0.992	1.001	351/1857	0.706	0.487	0.23[0.21, 0.26]	0.45[0.40, 0.50]
8	$\frac{47}{47}$	00010101 00010101	2 3	0.285 $0.264$	0.649 $0.439$	0.992 $0.994$	314/1859 255/1902	0.911 $0.957$	0.511 $0.614$	0.19[0.16, 0.21] 0.12[0.10, 0.15]	0.51 [0.46, 0.55] 0.63 [0.58, 0.68]
9	76	00010101	1	0.204 $0.615$	0.908	1.032	144/929	0.657	0.374	0.18 [0.15, 0.22]	0.39 [0.35, 0.43]
9	76	000010101	2	0.250	0.656	1.006	361/1841	0.849	0.596	0.26 [0.23, 0.29]	0.60[0.55, 0.65]
9	76	000010101	3	0.289	0.428	0.998	202/1841	0.929	0.551	0.10 [0.08, 0.12]	0.54 [0.49, 0.60]
10 10	123 123	0000010101 0000010101	$\frac{1}{2}$	0.636 $0.223$	0.861 $0.661$	1.049 1.016	276/1822 $238/944$	0.620 $0.796$	0.397 $0.552$	0.18 [0.16, 0.20] 0.35 [0.31, 0.39]	0.40[0.34, 0.44] 0.54[0.49, 0.59]
10	123	0000010101	3	0.305	0.422	1.000	247/1844	0.926	0.584	0.17 [0.15, 0.19]	0.61 [0.55, 0.66]
11	199	00000010101	1	0.650	0.831	1.061	115/914	0.564	0.326	0.14[0.11,0.17]	0.31[0.27, 0.36]
11	199	00000010101	2	0.202	0.665	1.025	333/1828	0.748	0.474	0.27 [0.24, 0.29]	0.52 [0.46, 0.57]
11 12	199 322	00000010101 000001010101	3 1	0.317 $0.620$	0.417 $0.885$	1.002 $1.044$	135/1812 172/1821	0.913 $0.525$	0.442 $0.298$	0.08 [0.06, 0.10] 0.13 [0.11, 0.15]	0.42[0.37, 0.47] 0.25[0.21, 0.30]
12	322	000001010101	2	0.241	0.657	1.011	252/1792	0.761	0.514	0.21 [0.18, 0.24]	0.52 [0.47, 0.58]
12	322	000001010101	3	0.296	0.426	0.999	123/1800	0.919	0.512	0.07 [0.05, 0.09]	0.56 [0.50, 0.60]
13	521	0000001010101	1	0.635	0.852	1.056	135/1765	0.483	0.324	0.11 [0.08, 0.13]	0.32 [0.27, 0.37]
13 13	$\frac{521}{521}$	0000001010101 0000001010101	2 3	0.221 $0.308$	$0.660 \\ 0.421$	1.019 1.001	253/1784 122/1802	0.755 $0.892$	$0.510 \\ 0.562$	0.23[0.20, 0.26] 0.09[0.07, 0.11]	$0.51 [0.46, 0.58] \\ 0.54 [0.48, 0.59]$
14	843	00000001010101	1	0.646	0.829	1.065	132/1806	0.482	0.288	0.09 [0.07, 0.11]	0.29 [0.25, 0.35]
14	843	00000001010101	2	0.205	0.664	1.025	279/1804	0.689	0.463	0.25[0.22, 0.28]	0.43[0.38,0.50]
14	843	00000001010101	3	0.317	0.417	1.002	120/1814	0.882	0.500	0.09 [0.07, 0.11]	0.52 [0.47, 0.59]
15 15	1364 $1364$	000000001010101 000000001010101	1 2	0.654 $0.191$	0.812 $0.667$	$\frac{1.071}{1.031}$	$\frac{121}{1756}$ $\frac{245}{1767}$	0.459 $0.684$	0.296 $0.485$	0.10 [0.08, 0.12] 0.27 [0.24, 0.29]	0.31 [0.26, 0.35] 0.52 [0.48, 0.60]
15	1364	000000001010101	3	0.324	0.414	1.003	126/1776	0.884	0.544	0.11 [0.09, 0.13]	0.53 [0.48, 0.60]
16	2207	0000000101010101	1	0.635	0.847	1.060	78/1754	0.399	0.221	0.07[0.05, 0.09]	0.27[0.23, 0.32]
16	$\frac{2207}{2207}$	0000000101010101	2	0.220	0.660	1.020	153/1749	0.664	0.439	0.15 [0.12, 0.17]	0.49 [0.44, 0.56]
16 17	3571	0000000101010101 00000000101010101	3 1	0.309 $0.644$	0.420 $0.828$	1.001 1.067	40/1727 $74/1740$	0.896 $0.385$	0.412 $0.206$	0.03 [0.02, 0.04] 0.06 [0.04, 0.08]	0.42[0.37, 0.49] 0.22[0.18, 0.26]
17	3571	00000000101010101	2	0.207	0.663	1.025	106/1700	0.662	0.440	0.10 [0.08, 0.12]	0.42 [0.37, 0.49]
17	3571	00000000101010101	3	0.317	0.417	1.002	60/1741	0.856	0.446	0.06[0.04, 0.08]	0.49[0.43, 0.53]
18 18	5778 5778	000000000101010101 000000000101010101	1 2	0.651 $0.195$	0.814 $0.665$	1.072 $1.030$	66/1706 166/1711	$0.370 \\ 0.637$	0.246 $0.419$	0.05 [0.04, 0.07] 0.22 [0.19, 0.24]	$0.24 [0.20, 0.28] \\ 0.45 [0.40, 0.51]$
18	5778	000000000101010101	3	0.323	0.414	1.003	52/1685	0.857	0.442	0.06 [0.04, 0.07]	0.39 [0.34, 0.45]
19	9349	0000000000101010101	1	0.657	0.802	1.076	67/1713	0.350	0.204	0.07 [0.05, 0.08]	0.21[0.17, 0.26]
19	9349	0000000000101010101	2	0.184	0.667	1.034	118/1704	0.622	0.386	0.15 [0.12, 0.18]	0.47[0.41, 0.52]
19	9349 15 127	0000000000101010101 0000000001010101010	3 1	0.328 $0.643$	0.412 $0.828$	1.004 1.068	56/1678 40/1698	0.856 $0.352$	0.446 $0.161$	0.07 [0.05, 0.09] 0.03 [0.02, 0.05]	0.48 [0.41, 0.51] 0.20 [0.16, 0.24]
	15 127	00000000010101010101	2	0.208	0.662	1.025	112/1753	0.608	0.356	0.14 [0.12, 0.17]	0.35 [0.31, 0.44]
	15127	00000000010101010101	3	0.316	0.417	1.002	23/1721	0.837	0.331	0.02[0.01, 0.04]	0.36[0.30, 0.41]
	24 476	000000000010101010101		0.649	0.815	1.073	35/1708	0.324	0.175	0.03 [0.01, 0.04]	0.19 [0.14, 0.22]
	$24\ 476$ $24\ 476$	000000000010101010101 00000000001010101		0.198 $0.322$	0.664 $0.415$	1.029 1.003	$\frac{58/1726}{30/1722}$	0.558 $0.851$	0.319 $0.407$	$0.07 [0.05, 0.09] \\ 0.02 [0.01, 0.04]$	$0.36 [0.31, 0.42] \\ 0.41 [0.35, 0.46]$
	39 603	0000000000010101010101		0.655	0.804	1.077	39/1696	0.322	0.192	0.04 [0.02, 0.05]	0.21 [0.16, 0.24]
	39 603	000000000001010101010		0.189	0.666	1.033	106/1719	0.557	0.335	0.17[0.14, 0.20]	0.44[0.38,0.49]
	39 603	0000000000001010101010		0.326	0.413	1.004	13/1691	0.827	0.368	0.010 [0.000, 0.019]	0.43 [0.37, 0.48]
	$64\ 079$ $64\ 079$	000000000000101010101 00000000000010101010101		0.659 $0.180$	$0.796 \\ 0.668$	1.080 1.036	33/1652 64/1631	0.322 $0.546$	0.150 $0.346$	0.04 [0.02, 0.05] 0.13 [0.10, 0.15]	0.17 [0.13, 0.20] 0.47 [0.39, 0.49]
	34 079	000000000000101010101		0.330	0.411	1.004	19/1667	0.838	0.382	0.02 [0.01, 0.04]	0.41 [0.35, 0.46]
6	18	010101	1	0.439	1.202	0.957	534/1880	0.990	0.617	0.32[0.29, 0.35]	0.60[0.56, 0.64]
6 7	18 29	010101 0010101	2 1	$0.340 \\ 0.506$	0.632 $1.153$	0.980 $0.947$	74/860 475/1889	0.998 $0.954$	0.232 $0.633$	0.06 [0.03, 0.08] 0.29 [0.26, 0.32]	0.22 [0.18, 0.26] 0.63 [0.59, 0.66]
7	29	0010101	2	0.332	0.639	0.975	157/860	0.999	0.536	0.18 [0.14, 0.21]	0.52 [0.47, 0.56]
8	47	01010101	1	0.439	1.201	0.958	353/1862	0.981	0.690	0.22[0.19, 0.25]	0.73[0.68, 0.77]
8	47	01010101	2	0.340	0.632	0.980	57/821	0.998	0.364	0.07[0.05, 0.10]	0.40[0.36, 0.44]
9 9	76 76	001010101 001010101	1 2	0.492 $0.334$	0.637	$0.950 \\ 0.977$	342/1821 115/840	0.930 $0.996$	0.675 $0.540$	0.23[0.20, 0.26] 0.14[0.11, 0.17]	0.68 [0.64, 0.72] 0.53 [0.48, 0.57]
10	123	0101010101	1	0.439	1.201	0.958	202/933	0.991	0.722	0.29 [0.25, 0.33]	0.73 [0.68, 0.77]
10	123	0101010101	2	0.340	0.632	0.980	54/809	0.998	0.390	0.07[0.04, 0.10]	0.40[0.35,0.44]
11	199	00101010101 00101010101	1 2	0.483	1.166	0.952	143/884	0.934	0.612	0.23 [0.20, 0.27] 0.07 [0.04, 0.10]	$0.64 [0.59, 0.68] \\ 0.42 [0.38, 0.47]$
11 12	$\frac{199}{322}$	010101010101	1	$0.335 \\ 0.439$	0.636 $1.201$	0.977 $0.958$	56/819 262/1789	0.995 $0.990$	$0.432 \\ 0.731$	0.07 [0.04, 0.10] 0.20 [0.17, 0.23]	0.42[0.38, 0.47] 0.77[0.71, 0.80]
12	322	010101010101	2	0.340	0.632	0.980	48/834	0.998	0.436	0.06 [0.04, 0.09]	0.42 [0.37, 0.47]
13	521	0010101010101	1	0.477	1.170	0.953	100/844	0.926	0.653	0.19[0.15,0.23]	0.66[0.61, 0.70]
13 14	521 843	0010101010101 01010101010101	2 1	0.336	0.635 $1.201$	0.978	77/814 98/896	0.997	0.530	0.12 [0.08, 0.15] 0.16 [0.12, 0.20]	$0.54 [0.49, 0.58] \\ 0.70 [0.65, 0.75]$
14	843	01010101010101	2	0.439 $0.340$	0.632	0.958 $0.980$	34/828	0.980 $0.996$	0.729 $0.409$	0.16 [0.12, 0.20]	0.40 [0.35, 0.45]
15	1364	001010101010101	1	0.472	1.174	0.954	156/1760	0.904	0.671	0.13 [0.11, 0.15]	0.67 [0.62, 0.71]
15	1364	001010101010101	2	0.336	0.635	0.978	46/800	0.998	0.456	0.08[0.05, 0.11]	0.49  [0.44, 0.54]
16 16	$\frac{2207}{2207}$	0101010101010101	$\frac{1}{2}$	0.439	1.201	0.958	157/1776	0.978	0.689	0.14[0.11, 0.17] 0.03[0.02, 0.04]	$0.71 [0.65, 0.75] \\ 0.42 [0.37, 0.48]$
16 17	3571	0101010101010101 00101010101010101	1	$0.340 \\ 0.468$	0.632 $1.177$	0.980 $0.955$	61/2576 $114/1769$	0.997 $0.905$	0.388 0.611	0.03[0.02, 0.04] 0.10[0.08, 0.12]	0.42[0.37, 0.48] 0.64[0.58, 0.67]
17	3571	00101010101010101	2	0.337	0.635	0.978	80/2644	0.992	0.396	0.04 [0.03, 0.05]	0.40 [0.35, 0.46]
18	5778	010101010101010101	1	0.439	1.201	0.958	108/1716	0.968	0.675	0.10 [0.08, 0.13]	0.67 [0.61, 0.72]
18	5778	010101010101010101 0010101010101010101	2	0.340	0.632	0.980	32/2558	0.997	0.362	0.01 [0.00, 0.02] 0.08 [0.06, 0.10]	0.39 [0.34, 0.45]
19 19	9349 9349	001010101010101010101 00101010101010101	1 2	$0.465 \\ 0.337$	1.179 $0.634$	0.955 $0.978$	84/1708 18/1671	$0.907 \\ 0.997$	0.621 $0.386$	0.08 [0.06, 0.10] 0.006 [0.000, 0.017]	$0.67 [0.61, 0.70] \\ 0.40 [0.35, 0.46]$
	15 127	01010101010101010101	1	0.439	1.201	0.958	66/1736	0.964	0.658	0.06 [0.04, 0.09]	0.67 [0.62, 0.72]
20 1	15127	0101010101010101010101	2	0.340	0.632	0.980	22/2576	0.995	0.317	0.008[0.000, 0.017]	0.34[0.30,0.40]
	24 476	00101010101010101010101		0.463	1.181	0.955	103/1736	0.901	0.617	0.11 [0.09, 0.14]	0.67 [0.61, 0.70]
	24 476 39 603	001010101010101010101 01010101010101010		0.337 $0.439$	0.634 $1.201$	0.979 $0.958$	$\frac{30}{2534}$ $\frac{73}{1718}$	0.995 $0.974$	$0.411 \\ 0.612$	0.02 [0.01, 0.03] 0.08 [0.06, 0.11]	$0.41 [0.36, 0.47] \\ 0.64 [0.57, 0.67]$
	39 603	010101010101010101010		0.340	0.632	0.980	25/2537	0.993	0.292	0.015 [0.006, 0.024]	0.30 [0.26, 0.38]
23 6	$64\ 079$	00101010101010101010101 001010101010101		0.461	1.183	0.956	58/1630	0.902	0.561	0.08[0.06,0.10]	0.60[0.54, 0.63]
	64079		01 2	0.337	0.634	0.979	19/2520	0.995	0.355	0.012[0.004, 0.021]	0.38[0.33, 0.44]

TABLE II. Numerical and hardware results for  $|z^{\rm half}\rangle$  and  $|z^{\rm MIS}\rangle$  product states, as detailed at the start of C. Highlighted rows emphasize the target state of Fig. 5

N	V	State	Depth (p)	$ au_{ m eff}$	γ	Naïve Count	Perfect	Emulation	EM (H)	EM (E)
5	11	[00101]	38	15.708	$\begin{array}{c} 0.494,\ 0.074,\ -0.008,\ 0.019,\ -0.023,\ 0.211,\ -0.046\\ 0.009,\ 0.388,\ 0.012,\ 0.056,\ -0.075,\ -0.040,\ 0.019\\ 0.031,\ 0.015,\ -0.283,\ 0.709,\ 0.051,\ -0.011,\ -0.174\\ 0.034,\ 0.034,\ 0.020,\ 0.035,\ -0.043,\ 0.033,\ -0.163\\ 0.110,\ -0.170,\ -0.048,\ -0.122,\ 0.116,\ -0.029,\ 0.008\\ 0.064,\ -0.225,\ 0.107 \end{array}$	460/947	1.000	0.994	0.57 [0.52, 0.61]	0.97 [0.94, 0.99]
6	18	[000101]	25	10.485	$\begin{array}{c} 0.109,\ 0.230,\ 0.002,\ 0.087,\ -0.285,\ 0.556,\ 0.141 \\ 0.108,\ 0.578,\ -0.244,\ 0.191,\ -0.060,\ -0.106,\ -0.304 \\ -0.077,\ -0.108,\ -0.330,\ -0.162,\ -0.014,\ 0.526,\ 0.155 \\ 0.591,\ 0.284,\ -0.266,\ 0.104 \end{array}$	453/940	0.996	0.994	0.53 [0.49, 0.57]	0.98 [0.95, 0.99]
7	29	[0000101]	19	8.234	-0.286, -0.024, 0.156, 0.335, 0.451, 1.000, 0.814 -0.008, -0.623, -0.354, 0.564, 0.230, 0.066, -0.640 -0.225, -0.349, -0.173, -0.221, -0.486	529/924	0.960	0.943	0.66 [0.62, 0.69]	0.93 [0.90, 0.95]
8	47	[00010101]	47	19.320	$\begin{array}{l} 0.123,\ 0.300,\ -0.245,\ 1.259,\ 0.720,\ -0.236,\ -0.194\\ -0.003,\ -0.098,\ 0.006,\ 1.157,\ 0.730,\ -0.447,\ -0.100\\ -0.068,\ 0.092,\ -0.244,\ 0.405,\ 0.095,\ 0.168,\ 0.118\\ -0.369,\ -0.277,\ -0.255,\ -0.318,\ -0.342,\ -0.078,\ -0.090\\ -0.005,\ 0.066,\ -0.451,\ -0.102,\ 0.352,\ 0.252,\ -0.023\\ 0.609,\ 0.361,\ -0.013,\ -0.287,\ -0.389,\ -0.114,\ -0.537\\ 0.009,\ -0.276,\ -0.126,\ 0.509,\ 0.043 \end{array}$	190/938	0.984	0.970	0.19 [0.15, 0.23]	0.94 [0.91, 0.96]
9	76	[000010101]	41	16.708	$\begin{array}{l} -0.026, -0.205, -0.368, -0.118, -0.337, -0.132, 0.319 \\ -0.064, -0.066, 0.981, -0.275, -0.109, -0.011, -0.037 \\ -0.208, -0.522, 0.994, 0.205, -0.071, 0.078, -0.050 \\ 0.201, 0.255, -0.030, -0.116, 0.037, 0.011, -0.025 \\ 0.166, 0.012, 0.270, -0.203, 0.059, 0.273, -0.044 \\ 1.012, 0.049, 0.288, -0.183, 0.799, 0.206 \end{array}$	181/934	0.970	0.797	0.22 [0.19, 0.25]	0.78 [0.74, 0.81]
10	123	[0000010101]	47	18.979	$\begin{array}{l} 1.204, -0.112, \ 0.011, -0.081, \ 0.427, \ 0.420, -0.922 \\ 0.020, \ 0.077, \ -0.402, \ 0.323, \ 0.834, \ -0.081, \ 0.417 \\ -0.013, \ -0.368, \ 1.169, \ 0.916, \ -0.199, \ 0.341, \ 0.284 \\ -0.046, \ -0.087, \ -0.264, \ 0.085, \ -0.548, \ 0.349, \ -0.217 \\ 0.102, \ 0.125, \ 0.135, \ 0.057, \ -0.022, \ 0.076, \ -0.371 \\ -0.178, \ -0.153, \ 0.306, \ 0.758, \ -0.036, \ 0.006, \ 0.004 \\ -0.035, \ 0.002, \ -0.039, \ 0.100, \ -0.020 \end{array}$	98/907	0.859	0.835	0.12 [0.09, 0.15]	0.81 [0.76, 0.82]
11	199	[00000010101]	48	19.670	$\begin{array}{c} 0.002,\ 0.274,\ 0.363,\ 0.147,\ 0.039,\ 0.054,\ -0.247\\ -0.058,\ -0.068,\ 0.066,\ 0.711,\ -0.184,\ 0.104,\ 0.929\\ 0.098,\ 0.051,\ 0.191,\ -0.083,\ -0.227,\ 0.185,\ -0.208\\ -0.173,\ -0.173,\ -0.106,\ 0.004,\ -0.031,\ -0.322,\ -0.285\\ -0.218,\ -0.120,\ -0.197,\ -0.131,\ -0.287,\ -0.149,\ -0.044\\ -0.446,\ -0.271,\ -0.210,\ -0.290,\ -0.067,\ -0.157,\ -0.007\\ -0.069,\ 0.141,\ 0.056,\ -0.234,\ -0.177,\ 0.332 \end{array}$	65/909	0.751	0.643	0.07 [0.05, 0.09]	0.61 [0.56, 0.64]
12	322	[000001010101]	43	17.459	$\begin{array}{c} 0.983,\ 0.241,\ 0.820,\ 0.094,\ 0.044,\ 0.996,\ 0.136\\ -0.012,\ 0.072,\ 0.547,\ 0.165,\ 0.120,\ -0.004,\ -0.083\\ 0.487,\ -0.094,\ 0.271,\ -0.016,\ 0.020,\ 0.281,\ 0.409\\ 0.319,\ 0.681,\ -0.069,\ -0.061,\ 0.887,\ -0.056,\ -0.238\\ -0.505,\ -0.036,\ 0.073,\ -0.192,\ 0.535,\ -0.234,\ 0.073\\ -0.534,\ -0.750,\ -0.393,\ -0.169,\ -0.522,\ -0.436,\ 0.019\\ 0.472 \end{array}$	68/909	0.580	0.568	0.10 [0.07, 0.12]	0.56 [0.50, 0.58]
5	11	[00101]	38	15.708	$\begin{array}{c} 0.494,\ 0.074,\ -0.008,\ 0.019,\ -0.023,\ 0.211,\ -0.046\\ 0.009,\ 0.388,\ 0.012,\ 0.056,\ -0.075,\ -0.040,\ 0.019\\ 0.031,\ 0.015,\ -0.283,\ 0.709,\ 0.051,\ -0.011,\ -0.174\\ 0.034,\ 0.034,\ 0.020,\ 0.035,\ -0.043,\ 0.033,\ -0.163\\ 0.110,\ -0.170,\ -0.048,\ -0.122,\ 0.116,\ -0.029,\ 0.008\\ 0.064,\ -0.225,\ 0.107 \end{array}$	460/947	1.000	0.994	0.57 [0.52, 0.61]	0.97 [0.94, 0.99]
6	18	[010101]	24	10.095	$\begin{array}{c} 0.921,\ 0.081,\ -0.386,\ -0.371,\ -0.530,\ 0.084,\ 0.064\\ 0.035,\ -0.321,\ 0.254,\ 0.245,\ 0.035,\ -0.056,\ 0.248\\ 0.065,\ 0.009,\ -0.068,\ 0.102,\ 0.016,\ 0.103,\ 0.353\\ 0.396,\ -0.422,\ -0.587 \end{array}$	521/934	0.996	0.986	0.72 [0.67, 0.75]	0.98 [0.95, 0.98]
7	29	[0010101]	24	9.925	$\begin{array}{c} 0.244,\ -0.004,\ -0.228,\ 0.175,\ 0.096,\ 0.023,\ -0.277 \\ -0.143,\ -0.022,\ -0.300,\ 1.014,\ 0.060,\ -0.009,\ -0.176 \\ -0.094,\ -0.212,\ 0.061,\ 0.056,\ 0.343,\ 0.190,\ 0.000 \\ -0.164,\ -0.121,\ -0.242 \end{array}$	456/936	0.988	0.988	0.62 [0.58, 0.66]	0.96 [0.93, 0.98]
8	47	[01010101]	40	16.398	$\begin{array}{c} 0.826, -0.028, -0.126, -0.179, \ 0.098, -0.082, -0.030 \\ -0.202, -0.426, -0.028, \ 0.686, \ 0.090, -0.212, \ 0.150 \\ 0.202, -0.386, -0.154, \ 0.122, \ 0.044, \ 0.225, -0.001 \\ 0.182, \ 0.127, \ -0.315, \ -0.191, \ 0.481, \ 0.259, \ -0.473 \\ 0.095, \ 0.977, \ 1.219, -0.070, \ 0.359, -0.027, \ 0.213 \\ -0.088, -0.018, \ 0.212, \ 0.141, -0.035 \end{array}$	259/920	0.990	0.957	0.38 [0.35, 0.42]	0.95 [0.91, 0.96]
9	76	[001010101]	18	7.884	$\begin{array}{c} 1.435,\ 0.430,\ 0.101,\ 0.216,\ -0.042,\ 0.105,\ -0.295 \\ 0.352,\ -0.023,\ 0.099,\ -0.002,\ 0.212,\ 0.417,\ -0.052 \\ -0.118,\ 0.243,\ -0.634,\ -1.151 \end{array}$	542/895	0.976	0.938	0.87 [0.81, 0.88]	0.92 [0.88, 0.93]
10	123	[0101010101]	34	14.027	$\begin{array}{llllllllllllllllllllllllllllllllllll$	272/911	0.960	0.903	0.44 [0.40, 0.48]	0.88 [0.84, 0.90]
11	199	[00101010101]	10	4.572	1.577, 0.240, -0.008, 0.129, -0.432, 0.005, -0.759 -0.388, -0.622, -1.846	513/913	0.990	0.929	0.85 [0.79, 0.86]	0.93 [0.88, 0.93]
12	322	[010101010101]	26	10.825	$\begin{array}{c} 1.178,\ 0.160,\ -0.266,\ -0.031,\ 0.387,\ 0.053,\ 0.220 \\ 0.115,\ -0.098,\ -0.315,\ -0.460,\ -0.357,\ -0.380,\ -0.076 \\ -0.167,\ -0.070,\ 0.228,\ 0.123,\ 0.036,\ 0.485,\ 0.331 \\ 0.258,\ 0.252,\ 0.086,\ -0.358,\ -1.016 \end{array}$	213/883	0.765	0.856	0.39 [0.35, 0.43]	0.83 [0.79, 0.85]
13	521	[0010101010101]	20	8.674	$\begin{array}{c} -1.181, \ -0.232, \ -0.033, \ 0.031, \ 0.086, \ 0.149, \ -0.183 \\ 0.216, \ -0.231, \ 0.038, \ -0.242, \ 0.014, \ -0.107, \ 0.246 \\ 0.217, \ 0.356, \ 0.689, \ 0.745, \ 0.809, \ 1.815 \end{array}$	369/893	0.974	0.893	0.68 [0.62, 0.70]	0.89 [0.83, 0.89]
14	843	[01010101010101]	26	10.825	$\begin{array}{c} -1.159, -0.479, -0.159, -0.025, -0.041, 0.027, 0.270\\ 0.414, 0.449, 0.621, 0.533, 0.429, 0.264, -0.251\\ 0.199, 0.416, 0.132, 1.116, 1.318, 0.926, 0.774\\ 0.951, 0.156, 0.191, 1.008, 1.032 \end{array}$	218/913	0.845	0.875	0.41 [0.36, 0.44]	0.86 [0.81, 0.87]
15	1364	[001010101010101]	20	8.624	1.492, 0.149, 0.268, 0.127, 0.035, -0.067, -0.073 0.060, -0.261, -0.003, 0.059, -0.452, 0.259, -0.545 -0.513, 0.329, 1.458, -0.224, 0.320, 0.192	183/876	0.830	0.737	0.35 [0.29, 0.37]	0.74 [0.67, 0.75]
17	3571	[001010101010101010	1] 15	6.383	$\begin{array}{c} 0.972,\ 1.184,\ 0.297,\ 1.054,\ -0.300,\ 1.005,\ -0.772 \\ -0.410,\ -0.740,\ -0.186,\ 0.058,\ 0.397,\ 0.054,\ 0.015 \\ -0.583 \end{array}$	182/876	0.708	0.557	0.37 [0.30, 0.38]	0.52 [0.45, 0.53]

TABLE III. Numerical and hardware results for bracelet states  $|[z^{\text{half}}]\rangle$  and  $|[z^{\text{MIS}}]\rangle$  product states, as detailed at the start of Appendix C.