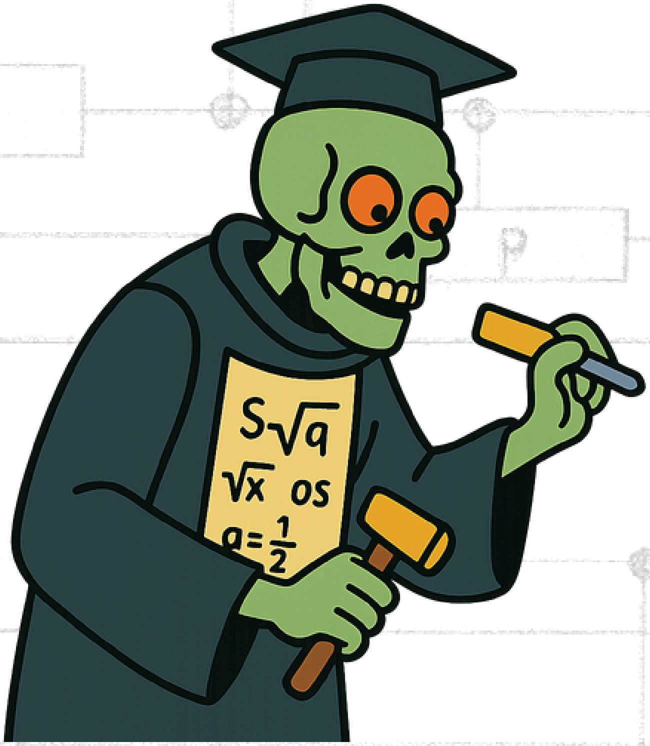
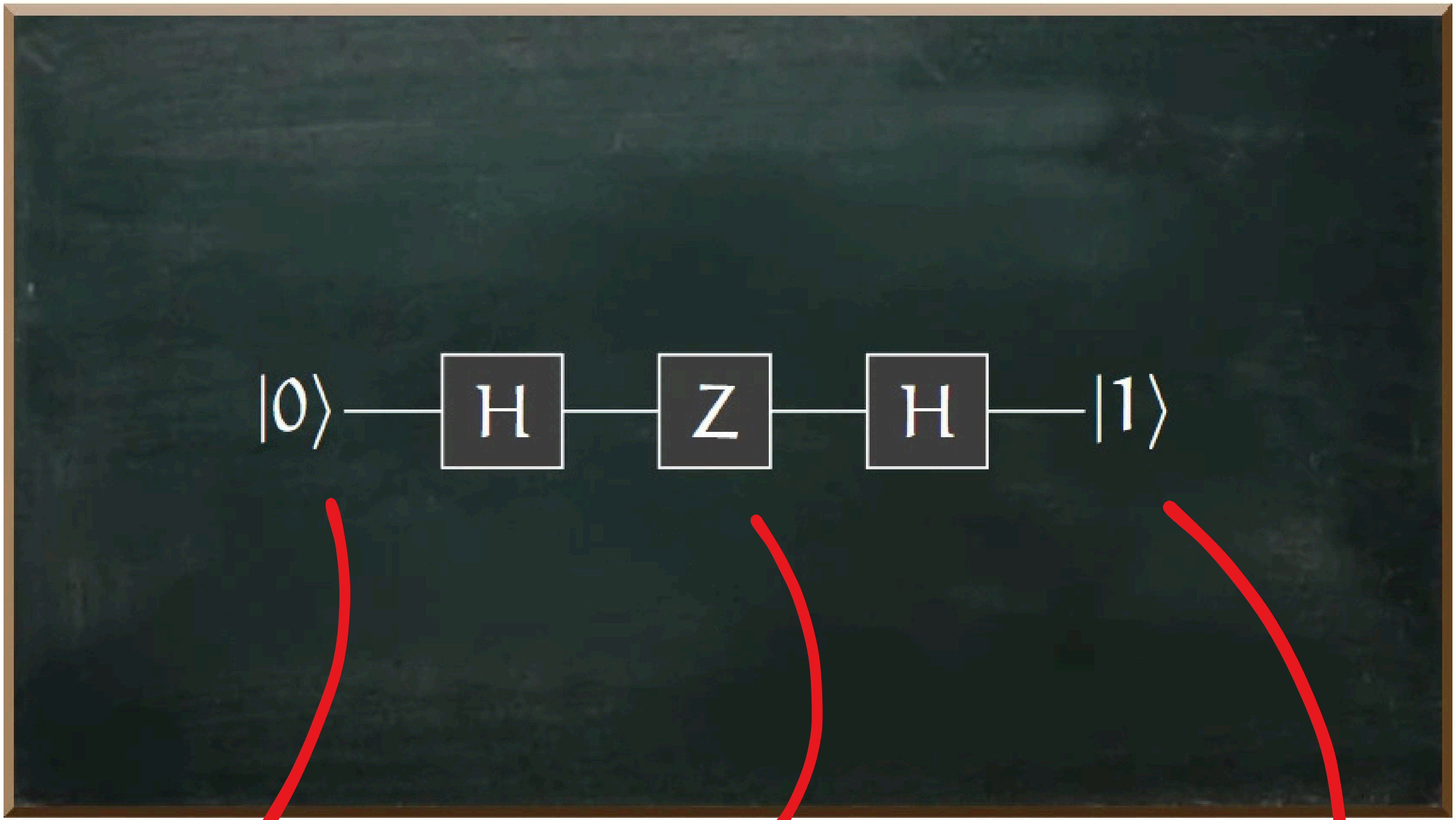


Quantum Computing Isn't Weird. It Is Just Matrix Multiplication

And this is what makes it pretty weird

https://www.quantum-machine-learning.com

Quantum Circuit Diagrams Convey a False Mental Model



Initial state

Quantum Gates

Output state

- A circuit diagram suggests step-by-step updates
- But! A quantum circuit is not a list of sequential commands

Quantum Circuit Constructions Feel Like a Series of Instructions

hzh.py

```
import numpy as np
from qiskit import QuantumCircuit
from qiskit.quantum_info import Statevector

qc = QuantumCircuit(1)

qc.h(0)
qc.z(0)
qc.h(0)

psi_out = Statevector.from_instruction(qc)

print("Statevector:", np.array(psi_out))
print("Probabilities:", psi_out.probabilities())
```

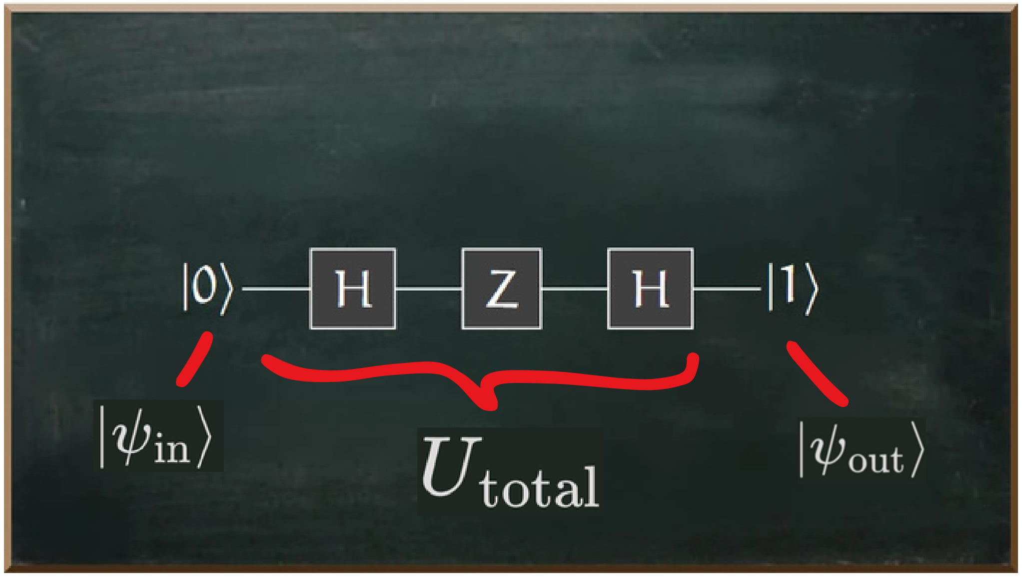
Define the space for the computation

Apply gates
order matters!

from INSTRUCTION?

- Creating a quantum circuit is pretty linear
- The order of gates matter
- We even create the statevector from the instructions?!

A Quantum Circuit Defines A Quantum Operator



$$|\psi_{\text{out}}\rangle = U_{\text{total}}|\psi_{\text{in}}\rangle$$

- A quantum circuit is only a quantum operator composed from other quantum operators

$$U_{\text{total}} = HZH$$

The Computation Is Nothing But Matrix Multiplication

... and they act on quantum state vectors!

$$U_{\text{total}}|\psi_{\text{in}}\rangle$$

$$|\psi_{\text{in}}\rangle = \begin{bmatrix}1\\0\end{bmatrix}$$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$$

$$Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$$

$$U_{\text{total}} = HZH$$

$$|\psi_{\text{out}}\rangle = \left(\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\right)\begin{bmatrix}1\\0\end{bmatrix}$$

The Computation Is Nothing But Matrix Multiplication

$$|\psi_{\text{out}}\rangle = \left(\overset{H}{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}} \overset{Z}{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}} \overset{H}{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}} \right) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Column vector
(in Dirac notation)

Matrix

Column vector

$$|\psi_{\text{in}}\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Dirac notation

Matrix multiplication is not commutative

Mulitplying two matrices gives a matrix again

Changing the order of matrices in a multiplication changes the result!

$$HZH \neq ZHH \neq HHZ$$

Applying the circuit operator

Multiplying a matrix with a column vector gives a column vector

$$|\psi_{\text{out}}\rangle = U_{\text{total}}|\psi_{\text{in}}\rangle$$

How About Measurement?

$$\langle i|\psi_{\text{out}}\rangle = \underbrace{\langle i|U_{\text{total}}|\psi_{\text{in}}\rangle}$$

A row vector corresponding to the Basis state $|i\rangle$.

When multiplied with a Quantum State Vector, it computes the projection Amplitude onto outcome i.

For example, basis state $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$:

$$\langle 0|\psi_{\text{out}}\rangle = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \alpha$$

the complex amplitude of basis state $|0\rangle$

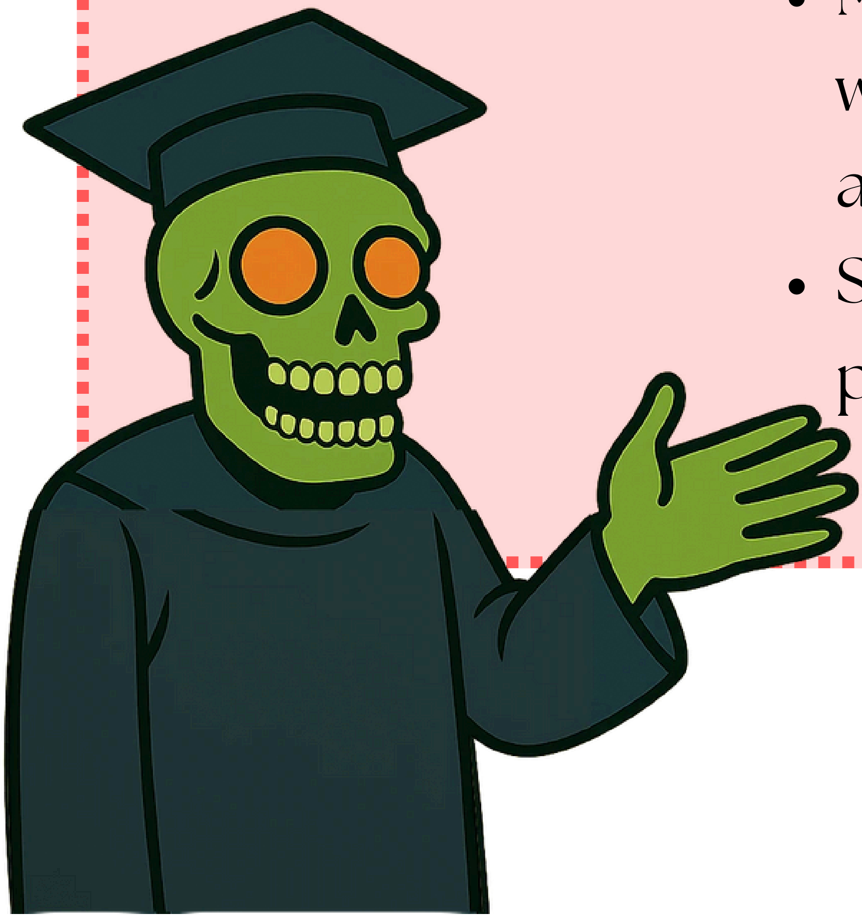
$$P(0) = |\langle 0|\psi_{\text{out}}\rangle|^2 = |\alpha|^2$$

The absolute square of the amplitude gives the probability of measuring the qubit in the corresponding basis state

Nothing But Linear Algebra

From beginning to end, quantum computation is only Linear Algebra. It is a sequence of multiplications.

- The circuit composes quantum operators by multiplying their matrices
- Applying the circuit multiplies the circuit matrix with the vector to produce amplitudes.
- Measurement multiplies row vectors with the final state to extract amplitudes.
- Squaring those Amplitudes produces probabilities.



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