THE QUADRATIC FORMULA   #27

You have used factoring and the Zero Product Property to solve quadratic equations. You can solve <u>any</u> quadratic equation by using the **QUADRATIC FORMULA.**

$$\text{If } ax^2 + bx + c = 0, \qquad \text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

For example, suppose $3x^2 + 7x - 6 = 0$. Here $a = 3$, $b = 7$, and $c = -6$. Substituting these values into the formula results in:

$$x = \frac{-(7) \pm \sqrt{7^2 - 4(3)(-6)}}{2(3)} \Rightarrow x = \frac{-7 \pm \sqrt{121}}{6} \Rightarrow x = \frac{-7 \pm 11}{6}$$

Remember that non-negative numbers have both a positive and negative square root. The sign $\pm$ represents this fact for the square root in the formula and allows us to write the equation <u>once</u> (representing two possible solutions) until later in the solution process.

Split the numerator into the two values: $\qquad x = \frac{-7 + 11}{6}$ or $x = \frac{-7 - 11}{6}$

Thus the solution for the quadratic equation is: $\qquad x = \frac{2}{3}$ or -3.

Example 1

Solve $x^2 + 3x - 2 = 0$ using the quadratic formula.

First, identify the values for a, b, and c. In this case they are 1, 3, and -2, respectively. Next, substitute these values into the quadratic formula.

$$x = \frac{-(3) \pm \sqrt{3^2 - 4(1)(-2)}}{2(1)} \Rightarrow x = \frac{-3 \pm \sqrt{17}}{2}$$

Then split the numerator into the two values: $x = \dfrac{-3 + \sqrt{17}}{2}$ or $x = \dfrac{-3 - \sqrt{17}}{2}$

Using a calculator, the solution for the quadratic equation is: $x = 0.56$ or -3.56.

Example 2

Solve $4x^2 + 4x = 3$ using the quadratic formula.

To solve any quadratic equation it must first be equal to zero. Rewrite the equation as $4x^2 + 4x - 3 = 0$. Identify the values for a, b, and c: 4, 4, and -3, respectively.

Substitute these values into the quadratic formula.

$$x = \frac{-(4) \pm \sqrt{4^2 - 4(4)(-3)}}{2(4)} \Rightarrow x = \frac{-4 \pm \sqrt{64}}{8} \Rightarrow x = \frac{-4 \pm 8}{8}$$

Split the numerator into the two values: $x = \frac{-4+8}{8}$ or $x = \frac{-4-8}{8}$, so $x = \frac{1}{2}$ or $-\frac{3}{2}$.

Use the quadratic formula to solve each of the following equations.

1. $x^2 - x - 6 = 0$ 2. $x^2 + 8x + 15 = 0$

3. $x^2 + 13x + 42 = 0$ 4. $x^2 - 10x + 16 = 0$

5. $x^2 + 5x + 4 = 0$ 6. $x^2 - 9x + 18 = 0$

7. $5x^2 - x - 4 = 0$ 8. $4x^2 - 11x - 3 = 0$

9. $6x^2 - x - 15 = 0$ 10. $6x^2 + 19x + 15 = 0$

11. $3x^2 + 5x - 28 = 0$ 12. $2x^2 - x - 14 = 0$

13. $4x^2 - 9x + 4 = 0$ 14. $2x^2 - 5x + 2 = 0$

15. $20x^2 + 20x = 1$ 16. $13x^2 - 16x = 4$

17. $7x^2 + 28x = 0$ 18. $5x^2 = -125x$

19. $8x^2 - 50 = 0$ 20. $15x^2 = 3$

Answers

1. $x = -2, 3$

2. $x = -5, -3$

3. $x = -7, -6$

4. $x = 2, 8$

5. $x = -4, -1$

6. $x = 3, 6$

7. $x = -\frac{4}{5}, 1$

8. $x = -\frac{1}{4}, 3$

9. $x = -\frac{3}{2}, \frac{5}{3}$

10. $x = -\frac{3}{2}, -\frac{5}{3}$

11. $x = -4, \frac{7}{3}$

12. $x = \frac{1 \pm \sqrt{113}}{4}$

13. $x = \frac{9 \pm \sqrt{17}}{8}$

14. $x = 2, \frac{1}{2}$

15. $x = \frac{-20 \pm \sqrt{480}}{40} = \frac{-5 \pm \sqrt{30}}{10}$

16. $x = \frac{16 \pm \sqrt{464}}{26} = \frac{8 \pm 2\sqrt{29}}{13}$

17. $x = -4, 0$

18. $x = -25, 0$

19. $x = -\frac{5}{2}, \frac{5}{2}$

20. $x = \frac{\pm \sqrt{5}}{5}$