

Problem 2: (Fibonacci)

1. matrix exponentiation computers.

$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n$ in $O(n^2)$ multiplication
 $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ using binary exponentiation
Yielding f_n directly.

2. The recurrence $T(n) = T(n/2) + O(1)$ fits
case 1 of the master theorem: $T(n) = O(\log n)$.
The decomposition halves n each step, requiring
only $O(\log n)$ matrix multiplications. Hence
time complexity $\Theta(\log n)$.

Subject _____ Date _____

Problem 2 (Levenshtein (Edit) Distance)

Edit distance between "Saturday" and "Sunday"

= 3

Operations:

SATURDAY

SUNDAY

Changes: A → U, T → N, insert R?

wait, check:

Alignment:

SATURDAY

SUNDAY

Substitute: A → U (a) substitute T → N (a)
R (a), align A, align Y. Delete

So 3 edits

Problem 3 (0/1 knapsack Algorithm)

1. Greedy fails because choosing highest value / weight ratio may not yield optimal yield optimal solution, due to integer constraints and leftover capacity.
Dynamic programming guarantees optimal by considering all subsets.

Solve

2. Solve for Course Ex (weights and value)
: DP table of size $(n+1) \times (w+1)$.
fill using :

$$dp[i][w] = \max \{ dp[i-1][w], dp[i-1][w-w_i] + v_i \}$$

3. Yes, Space complexity reduced to $O(w)$ by using a 1D array updated right-to-left for each time.