

# Fundamental Algorithm Techniques

## Problem Set #5

Review on March 02, 2026

**Problem 1** (Graph and Tree Definitions, 5/10 pts). *Prove that the following definitions are all equivalent:*

1. A tree is a connected acyclic graph.
2. A tree is one component of a forest. (A forest is an acyclic graph.)
3. A tree is a connected graph with at most  $V - 1$  edges.
4. A tree is a minimally connected graph; removing any edge disconnects the graph.
5. A tree is an acyclic graph with at least  $V - 1$  edges.
6. A tree is a maximally acyclic graph; adding an edge between any two vertices creates a cycle.
7. A tree is a graph that contains a unique path between each pair of vertices.

**Problem 2** (Sparse representation of graphs, 5/10 pts). *Given the following CSC (Compressed Sparse Column) representations for two graphs on vertices  $\{A, B, C, D, E\}$  (indexed as  $A \rightarrow 0$ ,  $B \rightarrow 1$ ,  $C \rightarrow 2$ ,  $D \rightarrow 3$ ,  $E \rightarrow 4$ ):*

**Graph 1 (undirected):**

```
col_pointers = [0, 2, 5, 8, 11, 12]
row_indices = [1, 2, 0, 2, 3, 0, 1, 3, 1, 2, 4, 3]
values = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
```

**Graph 2 (directed):**

```
col_pointers = [0, 0, 2, 4, 5, 7]
row_indices = [0, 3, 0, 1, 2, 1, 3]
values = [1, 1, 1, 1, 1, 1]
```

For each graph, reconstruct: (a) the adjacency matrix, and (b) a clear diagram of the graph (layout and style of your choice), (c) what is the unique cycle in directed graph (Describe it as e.g.  $X \rightarrow Y \rightarrow Z \rightarrow X$ ).