

Bubble Sort

Bubble sort is a simple sorting algorithm. It repeatedly compares adjacent elements and swaps them if they are in the wrong order. This algorithm is easy to implement but inefficient for large lists.

Pseudocode:

BUBBLE-SORT(A)

for i from 0 to n-1
 for j from 0 to n-2
 if $A[j] > A[j+1]$ swap $A[j]$ and $A[j+1]$
return A

```
In [1]: def bubble_sort(a):  
        n=len(a)  
        for i in range(n):  
            for j in range(n-1-i):  
                if a[j]>a[j+1]:  
                    temp=a[j];a[j]=a[j+1];a[j+1]=temp  
        return a
```

```
In [2]: arr = [5,3,1,4,2]  
  
        result = bubble_sort(arr)  
  
        print(result)
```

[1, 2, 3, 4, 5]

```
In [3]: arr = [9,7,5,3,1]  
  
        print(bubble_sort(arr))
```

[1, 3, 5, 7, 9]

Complexity Analysis

Time Complexity: Worst case: $O(n^2)$ Best case: $O(n^2)$ Space Complexity: $O(1)$

Quick Sort

Quick sort is a divide and conquer sorting algorithm. It selects a pivot element and partitions the array into two parts: elements smaller than the pivot and elements greater than the pivot. Then it recursively sorts the subarrays.

Pseudocode:

QUICK-SORT(A)

if length(A) <= 1 return A

pivot = A[0] create empty list LEFT create empty list RIGHT

for each element x in A[1:] if x <= pivot add x to LEFT else add x to RIGHT

return QUICK-SORT(LEFT) + pivot + QUICK-SORT(RIGHT)

```
In [4]: import random

def quick_sort_random(a):
    if len(a)<=1:
        return a
    pivot=random.choice(a)
    left=[]
    equal=[]
    right=[]
    for x in a:
        if x<pivot:
            left.append(x)
        elif x>pivot:
            right.append(x)
        else:
            equal.append(x)
    return quick_sort_random(left)+equal+quick_sort_random(right)

def quick_sort_middle(a):
    if len(a)<=1:
        return a
    first=a[0]
    middle=a[len(a)//2]
    last=a[-1]
    pivot=sorted([first,middle,last])[1]
    left=[]
    equal=[]
    right=[]
    for x in a:
        if x<pivot:
            left.append(x)
        elif x>pivot:
            right.append(x)
        else:
            equal.append(x)
    return quick_sort_middle(left)+equal+quick_sort_middle(right)
```

```
In [5]: arr = [8,3,1,7,0,10,2]

print(quick_sort_random(arr))
print(quick_sort_middle(arr))
```

```
[0, 1, 2, 3, 7, 8, 10]
[0, 1, 2, 3, 7, 8, 10]
```

```
In [6]: arr = [5,4,3,2,1]
```

```
print(quick_sort_random(arr))
print(quick_sort_middle(arr))
```

```
[1, 2, 3, 4, 5]
[1, 2, 3, 4, 5]
```

Complexity Analysis

Time Complexity: Best case: $O(n \log n)$ Average case: $O(n \log n)$ Worst case: $O(n^2)$ Space Complexity: $O(n)$

Merge Sort

Merge sort is a divide and conquer sorting algorithm. It splits the list into two halves, sorts each half, and merges them.

Pseudocode:

MERGE-SORT(A)

if length(A) \leq 1 return A

split A into LEFT and RIGHT LEFT = MERGE-SORT(LEFT) RIGHT = MERGE-SORT(RIGHT)

return MERGE(LEFT, RIGHT)

```
In [7]: def merge_sort(a):
        if len(a) <= 1: return a
        mid = len(a) // 2
        left = merge_sort(a[:mid])
        right = merge_sort(a[mid:])
        result = []
        while left and right:
            if left[0] <= right[0]:
                result.append(left.pop(0))
            else:
                result.append(right.pop(0))
        result += left
        result += right
        return result
```

```
In [8]: print(merge_sort([5,3,1,4,2]))
        print(merge_sort([9,7,5,3,1]))
```

```
[1, 2, 3, 4, 5]
[1, 3, 5, 7, 9]
```

Complexity Analysis

Time Complexity: $T(n) = 2T(n/2) + \Theta(n)$ By Master Theorem (Case 2): $T(n) = \Theta(n \log n)$ Space Complexity: $O(n)$

Heap Sort

Heap sort builds a max-heap and repeatedly swaps the largest element with the last element. After each swap, it restores the heap property.

Pseudocode:

HEAP-SORT(A) BUILD-MAX-HEAP(A) for end from $n-1$ down to 1 swap $A[0]$ and $A[\text{end}]$ SIFT-DOWN($A, 0, \text{end}-1$) BUILD-MAX-HEAP(A)

for i from parent of last node down to 0 SIFT-DOWN($A, i, n-1$) SIFT-DOWN($A, \text{start}, \text{end}$) $\text{root} = \text{start}$

while root has left child within range $\text{child} = \text{left child}$ if right child exists and is larger $\text{child} = \text{right child}$ if $A[\text{child}] > A[\text{root}]$ swap $A[\text{root}]$ and $A[\text{child}]$ $\text{root} = \text{child}$ else break

```
In [11]: def heap_sort(a):
n=len(a)
def sift_down(start,end):
    root=start
    while True:
        child=2*root+1
        if child>end:
            break
        if child+1<=end and a[child]<a[child+1]:
            child=child+1
        if a[root]<a[child]:
            temp=a[root]
            a[root]=a[child]
            a[child]=temp
            root=child
        else:
            break
    start=(n-2)//2
    while start>=0:
        sift_down(start,n-1)
        start-=1
    end=n-1
    while end>0:
        temp=a[0]
        a[0]=a[end]
        a[end]=temp
        sift_down(0,end-1)
        end-=1
    return a
```

```
In [12]: print(heap_sort([5,3,1,4,2]))
print(heap_sort([9,7,5,3,1]))
```

```
[1, 2, 3, 4, 5]
[1, 3, 5, 7, 9]
```

Complexity Analysis

Time Complexity: $O(n \log n)$ Space Complexity: $O(1)$

Sorting Algorithms Summary

Bubble Sort Time: $O(n^2)$ Space: $O(1)$

Quick Sort (Random / Median-of-three) Best: $O(n \log n)$ Average: $O(n \log n)$
Worst: $O(n^2)$ Space: $O(n)$

Merge Sort Time: $O(n \log n)$ Space: $O(n)$

Heap Sort Time: $O(n \log n)$ Space: $O(1)$

```
In [13]: import random
def test_sort(fn,name):
    tests=[[5,3,1,4,2],[9,7,5,3,1],[3,1,2,1,3,0]]
    for n in [10,50]:
        random.seed(n)
        tests.append([random.randint(-100,100) for _ in range(n)])
    for a in tests:
        if fn(a[:])!=sorted(a):
            print(name,"FAIL")
            return
    print(name,"OK")
```

```
In [14]: test_sort(bubble_sort,"Bubble")
test_sort(quick_sort_random,"Quick random")
test_sort(quick_sort_middle,"Quick middle")
test_sort(merge_sort,"Merge")
test_sort(heap_sort,"Heap")
```

```
Bubble OK
Quick random OK
Quick middle OK
Merge OK
Heap OK
```

```
In [ ]:
```