

Fundamental Algorithm Techniques

Problem Set #3

Due: October 25, 2025

Problem 1 (Fibonacci Super Fast!). 1. *compute Fibonacci with the relation:*

$$\begin{bmatrix} F_{n+1} \\ F_n \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

2. *This can also be expressed as:*

$$\begin{bmatrix} F_{n+1} \\ F_n \end{bmatrix} = \left(\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \right)^{n/2} \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

use Master Theorem to discuss the complexity of this decomposition and show, explain why time complexity is then $\log_2(n)$.

Problem 2 (Levenshtein (Edit) Distance). *Compute the edit distance between the strings: "SATURDAY" and "SUNDAY" using a tabular approach.*

Problem 3 (0/1 Knapsack Algorithm!). 1. *Why is Knapsack not greedy Algo., why dynamical programming?*

2. *Solve the Knapsack Algorithm for the course example.*

3. *Can you get space complexity to $\mathcal{O}(W)$?*