

Fundamental Algorithm Techniques

Problem Set #1

Due: October 4, 2025

Problem 1 (Simplest Divide and Conquer on Sparse Vector). *Consider a sparse vector of size n with only 0's and a single 1:*

$$v = [0, 0, \dots, 1, \dots, 0]$$

- (a) *Write pseudo recursive code that perform **just** the binary divide and conquer method without creating a copy of the vector and runs until vector fully decomposed into sizes 1.*
 - *function $\text{divide}(v, \dots, 2)$*
 - *function $\text{divide}(v, \dots, m)$ for any $2 \ll m$*
- (b) *Analyse complexity of above divide and conquer: $m=2, m=3, \dots$ tertiary division, which $T(n)$ and which \mathcal{O} ?*
- (c) *Next, once the division has reach sizes of 1, we collect the unique 1 and its position.*
 - *Evaluate the cost $f(n)$*
 - *What is the recurrence relation $T(n)$*
 - *is complexity now $\mathcal{O}(\log(n))$ or $\mathcal{O}(n)$ (use master Theorem)*

Problem 2 (Multiplication in basis 10). *Similar to Paesant multiplication, but in basis 10. it is just the first multiplication you learned at school.*

represent x, y in \mathbb{N}^+ with the vectors/arrays X and Y , such that $x = \sum_{i=0}^{n_x} X[i] \cdot 10^i$, $y = \sum_{j=0}^{n_y} Y[j] \cdot 10^j$.

1. *write a working multiplication code*
2. *tweak code such that results can be larger than the limit of your standard integer*
3. *Explain how above multiplication can be described with the divide and conquer recursion to find $T(n) \approx 4T(\frac{n}{2})$ or Karatsuba algorithm: $T(n) \approx 3T(\frac{n}{2})$, finding with Master Theorem resp. $\mathcal{O}(n^2)$ and $\mathcal{O}(n^{1.585})$ **hard***

$$\text{Hint 1: } x \cdot y = (x_1 \cdot 10^{n/2} + x_0) (y_1 \cdot 10^{n/2} + y_0)$$

$$\text{Hint 2: } x \cdot y = z_2 \cdot 10^n + z_1 \cdot 10^{n/2} + z_0$$

4. *$\sum_{i=1}^n i$ or $n+n-1+\dots+2+1$ can be computed by one application of school multiplication: $n! = \frac{1}{2} \text{mult}(v, w)$ with which v, w ? **hard** but simple math...*