Imperial College
Business School

Lecture 3b

Price dynamics and liquidity - II

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Outline

- Bid-ask spread and price dynamics with:
  - order processing costs
  - rents due to imperfectly competitive dealers
  - inventory holding costs: risk-averse dealers
- Putting it all together: contributions to price dynamics of
  - (i) adverse selection
  - (ii) order processing costs and rents
  - (iii) inventory risk

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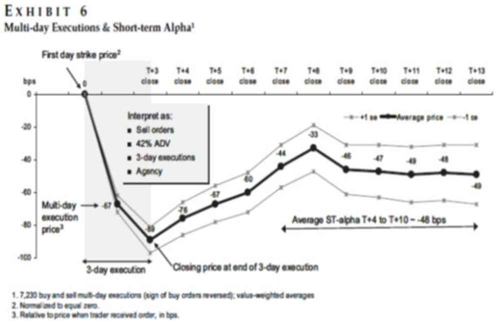
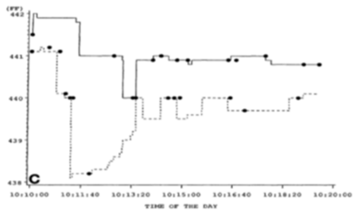
Stylized fact

- Trades in financial markets have both a permanent and transient effects on prices
- Remember the return reversals in Kraus and Stoll (1972). Same in the graphs on the right, drawn from more recent studies
- Is informed order flow sufficient to explain this? No!



Source: Cai and Sofianos (2006)



Source: Biais et al. (1995)

Price impact of orders under asymmetric information

- To see this, recall that in the previous lecture we showed that under asymmetric information the price is given by:

$$p_t = \mu_t = E(v|\Omega_{t-1}, d_t) = \mu_{t-1} + s(d_t)d_t$$

- So, under those assumptions, the price is a random walk, and the short-run impact $s(d_t)d_t$ is expected to persist permanently:

$$p_t = \mathrm{E}\left[\mathrm{E}\left(p_{t+\tau}|\Omega_{t+\tau-1}, d_{t+\tau}\right)|\Omega_{t-1}, d_t\right)\right] = E(p_{t+\tau}|\Omega_{t-1}, d_t)$$

- Hence, with asymmetric information alone, orders do not generate mean reversion in prices

Return reversals

- So, what is missing from the model with informed order flow? Can we enrich it so as to capture price reversals?
- We can, if we allow for
  - order processing costs (due to real costs of executing trades)
  - inventory holding costs (due to risk borne by market makers)
- This is what we turn to now

Order processing costs

Order processing costs

- Processing an order requires time and money:
  - exchange, clearing and settlement fees
  - cost of paperwork and back-office work, etc.
- Some of these order processing costs (OPC) are
  - on a per-share (or per-dollar traded) basis: variable cost
  - on a per-transaction basis: fixed cost
- Here we assume *variable* OPC: γ per share
  - but many results generalize to per-dollar and per-transaction OPC

Bid-ask spread with order processing costs

- Suppose that orders are informative *and* OPC are γ per share
- The bid-ask spread must compensate dealers both (i) for their losses to informed traders and (ii) for the OPC:

$$\left.\begin{aligned} a_t &= \mu_{t-1} + \gamma + s_t^a \\ b_t &= \mu_{t-1} - \gamma - s_t^b \end{aligned}\right\} \Rightarrow S_t = \underbrace{2\gamma}_{\text{OPC}} + \underbrace{s_t^a + s_t^b}_{\substack{\text{adverse} \\ \text{selection}}}$$

- Can we measure the relative importance of these two components of the spread?

Price dynamics with order processing costs

- From these ask and bid prices, the transaction price is:

$$\boxed{p_t = \mu_{t-1} + s(d_t)d_t + \gamma d_t}$$

- Since the expectation of the stock's fundamental is still:

$$\mu_t = \mu_{t-1} + s(d_t)d_t,$$

  the transaction price deviates from the stock's fundamental value by the size of the OPC:

$$p_t = \mu_t + \gamma d_t$$

Short-run price impact of a buy order

- How much does the transaction price deviate from the *midquote* (not the fundamental) after a buy order?
- The midquote is

$$m_t = \frac{a_t + b_t}{2} \quad = \mu_{t-1} + \frac{s_t^a - s_t^b}{2}$$

- So the short-term impact of the buy order is

$$p_t - m_t = \overbrace{\underbrace{\mu_{t-1} + s_t^a}_{\mu_t^+} + \gamma}^{a_t} - m_t = \frac{s_t^a + s_t^b}{2} + \gamma > 0$$

Long-run price impact of a buy order

- What is the effect of the same order after *T* periods?
- The expected price is

$$\mathrm{E}(p_{t+T}) = \underbrace{\mu_{t-1} + s_t^a}_{\mu_t^+} + \gamma\, \underbrace{\mathrm{E}(d_{t+T} \mid \Omega_{t-1}, d_t)}_{0}$$

- So the long-term impact is expected to be:

$$\mathrm{E}(p_{t+T}) - m_t = \underbrace{\mu_{t-1} + s_t^a}_{\mu_t^+} - m_t = \frac{s_t^a + s_t^b}{2}$$

in the LR, the OPC component γ vanishes: only the informational one persists

Rents or order processing costs?

- γ may not just capture costs, but also oligopoly profits (per share traded) accruing to dealers:

$$a_t = \mu_{t-1} + s_t^a + \overbrace{\underbrace{\gamma^p}_{\text{rent per share}} + \underbrace{\gamma^c}_{\text{OPC}}}^{\gamma}$$

- Same on the bid side
- So, the OPC component of the bid-ask spread may also include non-competitive rents!

Inventory holding costs

Inventory holding costs

- Assume no informed traders ($\pi = 0$) and no OPC ($\gamma = 0$): still, a bid-ask spread emerges as compensation for inventory risk
- After filling orders, dealers end up with unbalanced portfolios ("inventories"): short after buys, long after sells
- If risk-averse, they want to be paid to bear inventory risk ("holding costs") $\Rightarrow$ bid-ask spread
- Dealer's wealth after supplying y_t shares out of inventory z_t:

$$w_{t+1} = \overbrace{p_{t+1} z_{t+1}}^{\text{final inventory}} + \overbrace{c_{t+1}}^{\text{final cash}}$$

$$= \underbrace{p_{t+1}}_{v} \underbrace{(z_t - y_t)}_{z_{t+1}} + \underbrace{c_t + p_t y_t}_{c_{t+1}} = v z_t + c_t + \underbrace{(p_t - v) y_t}_{\text{profits}}$$

Static model with risk-averse dealer

- Consider a 2-period model: $t = 0$ and $t+1 = 1$
- Suppose the dealer has a mean-variance objective function:

$$U = \mathrm{E}(w_1) - \frac{\rho}{2}\ \mathrm{var}(w_1)$$

- His objective as a function of the sale y_0 is

$$U(y_0) = \mathrm{E}(v)z_0 + c_0 + [p_0 - \mathrm{E}(v)]y_0 - \frac{\rho}{2}(z_0 - y_0)^2\ \mathrm{var}(v)$$

$$= \mu_0 z_0 + c_0 + \underbrace{(p_0 - \mu_0)y_0}_{\text{expected profits}} - \frac{\rho}{2}\underbrace{(z_0 - y_0)^2\sigma_\varepsilon^2}_{\text{inventory risk}}$$

Dealer's supply function and price quotes

- The first-order condition yields the dealer's inverse supply function (price required to supply y_0 shares):

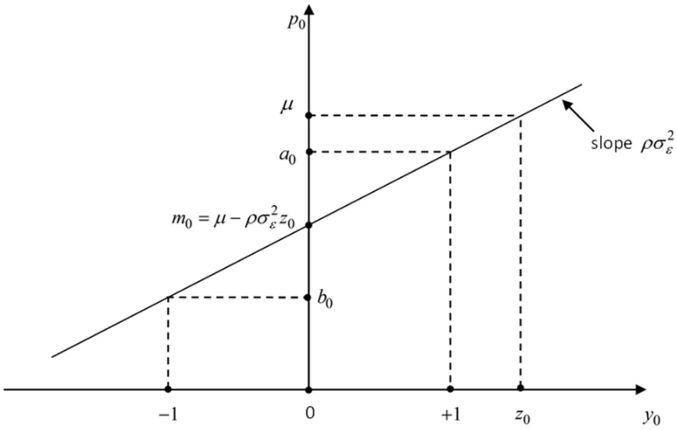
$$p_0 = \mu_0 + \rho\sigma_\varepsilon^2(y_0 - z_0)$$

- In equilibrium, the dealer's supply y_0 equals investors' order q_0:

$$p_0 = \underbrace{\mu_0 - \rho\sigma_\varepsilon^2 z_0}_{m_0} + \rho\sigma_\varepsilon^2 q_0 = \begin{cases} a_0 = m_0 + \rho\sigma_\varepsilon^2|q_0| & \text{if } q_0 > 0 \\ b_0 = m_0 - \rho\sigma_\varepsilon^2|q_0| & \text{if } q_0 < 0 \end{cases}$$

- Bid-ask spread: $S_t = 2\rho\sigma_\varepsilon^2|q_0|$ $\Rightarrow$ increasing in
  (i) dealers' **risk aversion**
  (ii) fundamental **volatility**
  (iii) trade **size**

Dealer's quotes for a unit buy or sell order : $|q_t|$ =1



Price pressure

- The midquote m_0 differs from the expected value μ_0 as it is the dealer's marginal valuation of the asset given his current inventory z_0:

$$m_0 = \mu_0 - \rho\sigma_\varepsilon^2 z_0$$

- **Intuition:**
  - if the dealer is long ($z_0 > 0$), his marginal valuation is lower than μ_0 because extra shares add to his risk exposure
  - if he is short ($z_0 < 0$), his marginal valuation is higher than μ_0 because extra shares lower his risk exposure

$\Rightarrow$ **price pressure**: orders affect the dealers' midquote because they affect his inventory, hence his marginal valuation $\Rightarrow$ over time, **midquotes and inventories are mirror images** of each other

Three main testable implications of inventory model

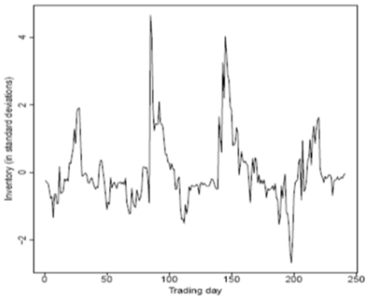
1. The price pressure of an order is inversely related to the inventory imbalance produced by the order: dealers with long (short) positions are willing to trade at lower (higher) prices
2. Receiving offsetting orders helps dealers unwind their positions ⇒ accelerate inventory reversion to target
3. Prices are also mean-reverting, at a speed that reflects that of reversion in inventories (mirror images)

1. Price pressure is inversely related to inventory position

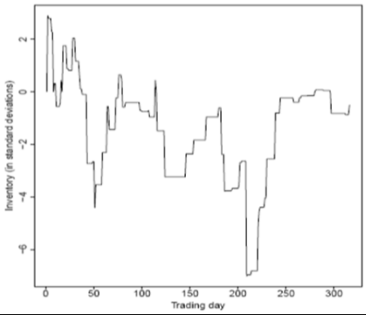
- For stocks listed on the NYSE, Hendershott and Menkveld (2014) estimate the price pressure per unit of inventory
- They find that the higher dealers' inventories, the lower the midquote: their relationship features a negative coefficient
- Larger in absolute value for small-cap stocks: a $100,000 inventory results in 1.01% price pressure for small stocks and 0.02% for large stocks
- As there is less trading in small stocks, it takes longer to unwind inventories and dealers remain exposed to risk for longer: they find that price pressure lasts longer for small-caps

2. Offsetting orders allow mean reversion in inventories

- Daily inventory of the dollar position of a block market making desk of a major broker-dealer for the Apple stock (source: Duffie, 2012)



- Daily inventory of a market making desk of a major broker-dealer for a single investment-grade corporate bond (source: Duffie, 2012)

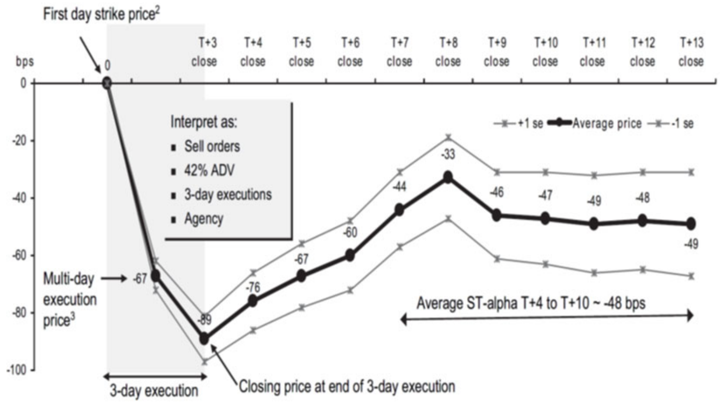


What determines the speed of mean reversion?

- Mean reversion is faster when
  - the market is more active (e.g., large-cap vs. small cap, Apple stock vs. corporate bonds): less than 1 day for liquid stocks vs. 2 months for some small stocks
  - dealers manage to elicit inventory-reducing orders: Reiss and Werner (1998) and Hansch, Naik, and Viswanathan (1998) find that dealers with long positions are more likely to execute buy market orders; those with short positions, are more likely to execute sell market orders
  - there are no regulatory limits on the size of dealers' positions, such as short-sales constraints or margin constraints on leverage

3. Mean reversion in prices mirrors that in inventories

- Mean reversion in prices is especially strong for large stock orders executed over 3 days (from Cai and Sofianos, 2006, seen at start of this class)



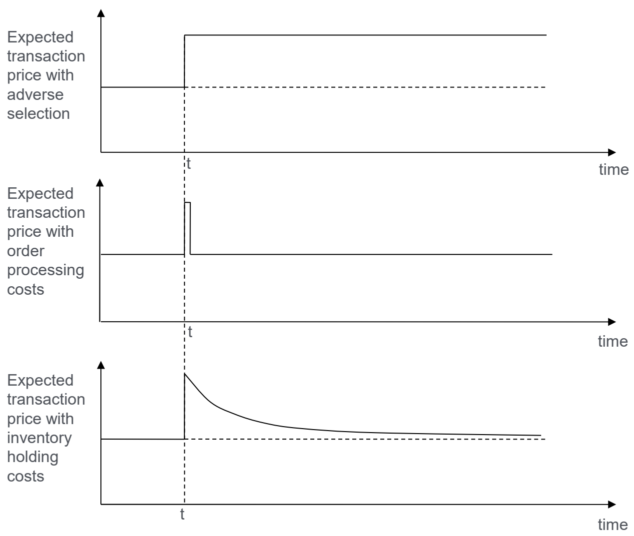
- Mean reversion in prices is faster in more active markets, in sync with the greater speed of mean reversion of inventories: price pressure for NYSE stocks lasts longer for small-cap stocks, whose trading is less frequent (Hendershott and Menkveld, 2014)

The full picture

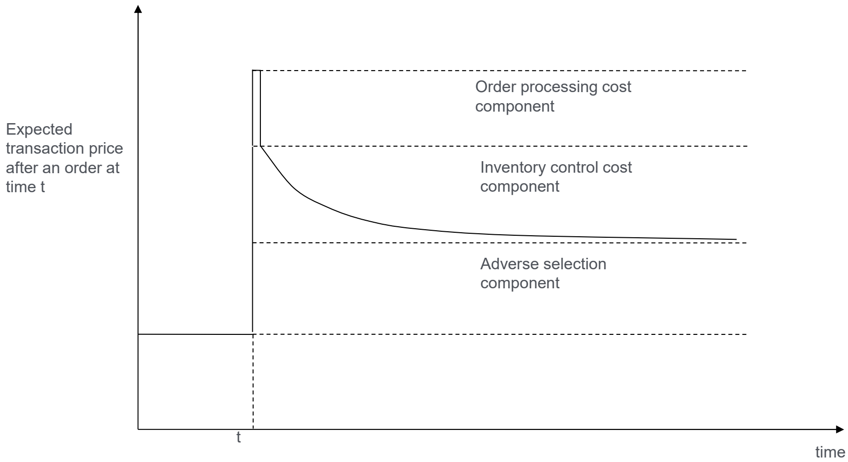
The full picture: effects of a buy order

- **Adverse selection:**
  - Short run price impact = long run price impact >0
- **Order processing costs:**
  - Short run price impact > 0
  - Long and medium run price impact = 0
- **Inventory holding costs:**
  - Short run price impact > 0
  - Medium run price impact > 0 but declining
  - Long run price impact = 0

Graphically: separately



Graphically: all together



Key takeaways

- The response of asset prices to orders often features some reversion to the initial level, which is inconsistent with models of informed trading
- Order-processing costs and inventory holding costs can account for such patterns, the first at high frequency, the second at lower frequencies
- Inventory holding costs also account for the fact that
  - midquotes are negatively related to dealers' inventories
  - their respective dynamics after large orders are “mirror images” of each other