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Business School

Lecture 3a

Price dynamics and liquidity - I

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Reading

- Reading: Chapter 3, Foucault, Pagano and Röell, Market Liquidity, 2nd edition, 2024.

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Outline

- Intraday price fluctuations and order flow
- Price dynamics in frictionless markets:
  - Efficient Market Hypothesis
- Price dynamics with informative order flow:
  - Glosten-Milgrom model
  - Bid-ask spread
  - Time series properties of prices
  - Price discovery

Intraday price fluctuations

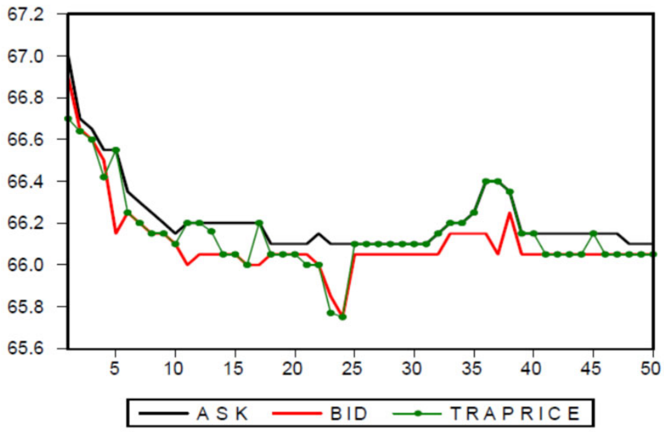
Intraday price fluctuations and order flow

- According to asset pricing theory, prices are determined by news about fundamentals:
  - future cash flows
  - discount factors
- But over very short intervals of time, prices appear to fluctuate "too much":
  - response to order flow: price falls with sell orders and rises with buy orders
  - returns are negatively correlated

Trading in AGF on Euronext

Time t	Trade size $(\lvert q_t \rvert)$	Price (p_t)	Direction (d_t)	Bid (b_t)	Ask (a_t)
90604	20	66.70	-1	66.90	67.00
90611	25	66.64	-1	66.65	66.70
90626	18	66.60	-1	66.60	66.65
90718	273	66.42	-1	66.50	66.55
90736	27	66.55	+1	66.15	66.55
91803	100	66.25	-1	66.25	66.35
91937	267	66.20	-1	66.20	66.30
92308	12	66.15	-1	66.15	66.25
92331	157	66.15	-1	66.15	66.20
92338	30	66.10	-1	66.10	66.15
92626	1,000	66.20	+1	66.00	66.20
93010	1	66.20	+1	66.05	66.20
93054	24	66.16	+1	66.05	66.20
93440	90	66.05	-1	66.05	66.20
93539	6	66.05	-1	66.05	66.20
93610	1,000	66.00	-1	66.00	66.20
93614	15	66.20	+1	66.00	66.20
93956	75	66.05	-1	66.05	66.10

Serial correlation of price changes = – 0.45



Prices adjust in the direction of the order flow: e.g., most of the first 10 trades are generated by market sell orders

Frictionless benchmark

Frictionless benchmark: no bid-ask spread

- Competitive and risk neutral market makers ⇒ zero expected profits
- All information is public ⇒ investors are not more informed than market makers
- No order processing costs in market making ⇒ no need to set prices so as to cover such costs

But "zero expected profits" implies
bid price = ask price = fundamental value μ_t
⇒ **zero bid-ask spread!**

… and the Efficient Market Hypothesis (EMH) holds

- So prices reflects all current fundamental information Ω_t :

$$p_t = \mathrm{E}\left(v \mid \Omega_t\right) \equiv \mu_t$$

- By the Law of Iterated Expectations,

$$\mathrm{E}\left[\underbrace{\mathrm{E}(v \mid \Omega_{t+1})}_{p_{t+1}} \mid \Omega_t\right] = \underbrace{\mathrm{E}\left(v \mid \Omega_t\right)}_{p_t} = \mu_t$$

- So p_t is the *best predictor* of p_{t+1} ⇒ price is a *martingale* ⇒ returns $p_{t+1} - p_t = \varepsilon_{t+1}$ are unpredictable using current information Ω_t ⇒ they are serially uncorrelated

With frictions, bid-ask spreads may appear because ...

- **Private information** is reflected in order flow
- Market making involves **order processing costs**
- Dealers are risk averse $\Rightarrow$ **inventory holding costs**
- They must generate revenue to compensate for the
  i. adverse selection costs
  ii. real resource costs
  iii. risks that they bear
- When imperfectly competitive, dealers also earn
  iv. **rents**

next!

Informed trading

How is information incorporated into prices?

- Prices are set by market makers (liquidity suppliers = sell side):
  - dealers
  - other speculators (anyone placing limit orders)
- They gather information Ω_t from:
  - public announcements, news, etc.
  - observing the order flow, insofar as some orders are driven by private information

Informative order flow: Glosten-Milgrom model

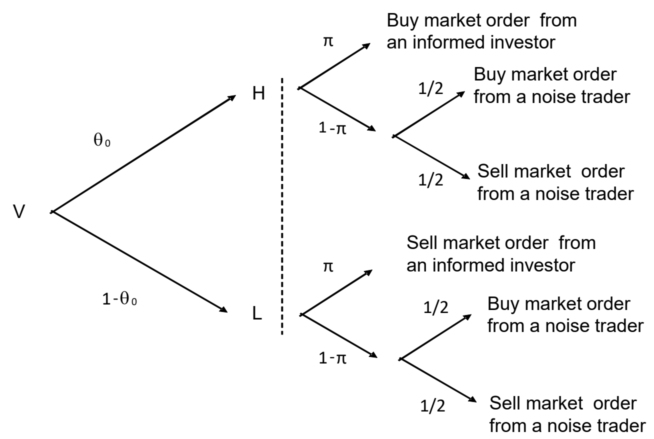
- **Simplest example: high-low fundamental value and unit trade size**
- Security's value: $$v = \begin{cases} v^H & \text{with prob. } 1/2 \\ v^L & \text{with prob. } 1/2 \end{cases}$$
- Order flow at time t: d_t = 1 (buy order) or d_t = –1 (sell order)
- Market maker's expected value *after* observing *t*th order:

$$\underbrace{\mu_t}_{\substack{\text{expected} \\ \text{value at t}}} \equiv \mathrm{E}(v \mid \Omega_{t-1}, d_t) = \underbrace{\theta_t}_{\substack{\text{probability of} \\ \textbf{high} \text{ value at t}}} v^H + (\underbrace{1-\theta_t}_{\substack{\text{probability of} \\ \textbf{low} \text{ value at t}}}) v^L$$

Glosten-Milgrom model (2)

- At each time *t*, a single trader arrives on the market:
  - **with probability π** he is **informed**: he knows v $\Rightarrow$ buys 1 unit if the value is higher than the ask (*i.e.* $v = v_H > a$) and sells 1 unit if it is lower than the bid (*i.e.* if $v = v_L < b$)
  - **with probability 1–π** he is an **uninformed** ("noise" or "liquidity" trader): he does not know the security's true value $\Rightarrow$ buys or sells 1 unit with probability ½ each
- **Market makers ignore** if the trader is informed or not: asymmetric information $\Rightarrow$ **adverse selection**
- They are **risk-neutral** and **perfectly competitive**

Glosten-Milgrom model: order arrival process



Learning from order flow

- The distribution of the order flow depends on the true value of the asset:

$$\Pr(d_t = 1 | v_H) = \frac{1+\pi}{2} \text{ and } \Pr(d_t = -1 | v_H) = \frac{1-\pi}{2}$$

$$\Pr(d_t = 1 | v_L) = \frac{1-\pi}{2} \text{ and } \Pr(d_t = -1 | v_L) = \frac{1+\pi}{2}$$

- Hence, dealers can learn about the value of the asset from the order flow

Bid and ask prices

- **Competition** among **risk-neutral** market makers pushes their **expected profits** down to **zero**

  $\Rightarrow$ ask and bid prices equal their estimate of the security's value *conditional on a buy or a sell order* :

$$a_t = \mu_t^+ \equiv E(v | \Omega_{t-1}, d_t > 0)$$

$$b_t = \mu_t^- \equiv E(v | \Omega_{t-1}, d_t < 0)$$

- **Intuitively:** $E(v|\Omega_{t-1}, d_t > 0) \; > \; E(v|\Omega_{t-1}, d_t < 0)$

- **Key point**: d_t can be used to forecast v because the distribution of d_t depends on v (previous slide)

Setting the first ask price in the trading day

- Ask is set conditional on arrival of a buy order ($d_1 > 0$)
- … so that dealers' expected profits per share are zero:

$$\boxed{\frac{\pi}{2}}(a_1 - v^H) + \boxed{\frac{1-\pi}{2}}(a_1 - \mu_0) = 0$$

probability of buy from informed

dealer's "gain" from informed buyer

probability of buy from uninformed

dealer's gain from uninformed buyer

prob(informed|buy)

$$\Rightarrow \quad a_1 = \mu_0 + \frac{\boxed{\frac{\pi}{2}}}{\boxed{\frac{\pi}{2} + \frac{1-\pi}{2}}}(v^H - \mu_0) \quad = \mu_0 + \pi(v^H - \mu_0)$$

probability of buy order

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First ask and bid prices in the trading day

- Dealers' initial estimate of v is its unconditional mean:
- So: $$\mu_0 = \frac{v^H + v^L}{2}$$

$$a_1 = \mu_0 + \pi(v^H - \mu_0) = \mu_0 + \boxed{\frac{\pi}{2}(v^H - v^L)}$$

ask-side half spread s_1^a

- Symmetrically on the bid side:

$$b_1 = \mu_0 + \pi(v^L - \mu_0) = \mu_0 - \boxed{\frac{\pi}{2}(v^H - v^L)}$$

bid-side half spread s_1^b

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First bid-ask spread in the trading day

- Bid-ask spread at t =1:

$$\boxed{S_1} = a_1 - b_1$$
$$= s_1^a + s_1^b$$
$$= \frac{\pi}{2}(v^H - v^L) + \frac{\pi}{2}(v^H - v^L)$$
$$= \boxed{\pi}\boxed{(v^H - v^L)}$$

increasing in proportion of informed traders

… and in their informational advantage (range of variation of v)

- The spread just covers dealers' losses to informed traders
- Cost of adverse selection is ultimately borne by liquidity traders $\Rightarrow$ markets are often organized/regulated to alleviate informational asymmetries

Ask price at any time *t* in the trading day

- Ask is set conditional on arrival of a buy order (q_t > 0)
- … so that dealers' expected profits per share are zero:

$$\boxed{\pi\theta_{t-1}}(a_t - v^H) + \boxed{\frac{1-\pi}{2}}(a_t - \mu_{t-1}) = 0$$

probability of buy from informed

dealer's "gain" from informed buyer

probability of buy from uninformed

dealer's gain from uninformed buyer

$$\Rightarrow \quad a_t = \mu_{t-1} + \frac{\boxed{\pi\theta_{t-1}}}{\boxed{\pi\theta_{t-1} + \frac{1-\pi}{2}}}(v^H - \mu_{t-1})$$

prob(informed|buy)

probability of buy order

Ask price, bid price and spread at any time *t*

- But since

we have: $$\mu_{t-1} = \theta_{t-1} v^H + (1-\theta_{t-1}) v^L$$

- Symmetrically: $$a_t = \mu_{t-1} + \underbrace{\frac{\pi\theta_{t-1}(1-\theta_{t-1})}{\pi\theta_{t-1} + (1-\pi)/2}(v^H - v^L)}_{s_t^a}$$

$$b_t = \mu_{t-1} - \underbrace{\frac{\pi\theta_{t-1}(1-\theta_{t-1})}{\pi(1-\theta_{t-1}) + (1-\pi)/2}(v^H - v^L)}_{s_t^b}$$

$$\boxed{S_t = 2\pi\theta_{t-1}(1-\theta_{t-1})\left(\frac{1}{2\pi\theta_{t-1} + 1 - \pi} + \frac{1}{2\pi(1-\theta_{t-1}) + 1 - \pi}\right)(v^H - v^L)}$$

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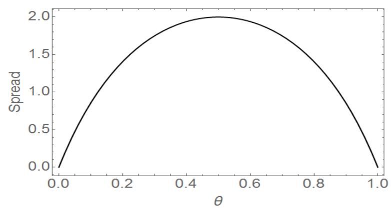
Determinants of bid-ask spreads

- In this case too, the bid-ask spread S_t is increasing in
  - the range of variation of the security's value ($v^H - v^L$)
  - the likelihood of informed trading (π)
- Now S_t is also affected by the belief θ_t that $v = v^H$:
  - the closer θ_t is to 1 or 0, the less uncertainty about v
  - uncertainty is maximal for $\theta = ½ \Rightarrow$ so is bid-ask spread S_t:



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Price dynamics

- The sequence of transaction prices is still a **martingale**
- **Why?**
- If market makers are competitive, risk neutral and equally informed about the history of the order flow, then

$$p_t = E\left(v \mid \Omega_t\right)$$

where **Ω_t now includes the direction of the time-t order**

- So, price changes are serially uncorrelated, as in a frictionless market $\Rightarrow$ adverse selection is consistent with **semi-strong efficiency**: one cannot predict returns using public information

Price dynamics (2)

- To show this, recall that the ask (bid) price is the expected value of v conditional on a buy (sell) order:

$$\boxed{p_t} = \begin{cases} a_t = \mu_t^+ = \mu_{t-1} + \dfrac{\pi\theta_{t-1}(1-\theta_{t-1})}{\pi\theta_{t-1} + (1-\pi)/2}(v^H - v^L) \\ b_t = \mu_t^- = \mu_{t-1} - \dfrac{\pi\theta_{t-1}(1-\theta_{t-1})}{\pi(1-\theta_{t-1}) + (1-\pi)/2}(v^H - v^L) \end{cases}$$

$$\boxed{= \mu_t} = \begin{cases} \mu_{t-1} + s_a \\ \mu_{t-1} - s_b \end{cases} \boxed{= \mu_{t-1} + s(d_t)d_t} \Rightarrow \boxed{\text{price is random walk with innovation } s(d_t)\, d_t}$$

- E.g., if $d_t = +1$, the expectation of v is revised from μ_{t-1} to $\mu_t = a_t \Rightarrow$ new quotes a_t and b_t will bracket the previous fundamental

Orders $\Rightarrow$ revision of beliefs $\Rightarrow$ revision of quotes

- The belief at time t–1 is $\theta_{t-1} = \Pr(v^H \mid \Omega_{t-1})$, and at t is $\theta_t^{+} = \Pr(v^H \mid \Omega_t)$
- If a buy ($B \Leftrightarrow d_t = 1$) arrives at t, the belief is updated with Bayes' rule:

$$\theta_t^+ = \frac{\Pr(B \mid v^H)\overbrace{\Pr(v^H \mid \Omega_{t-1})}^{\theta_{t-1}}}{\Pr(B)} = \underbrace{\frac{\pi \cdot 1 + (1-\pi)\frac{1}{2}}{\pi\theta_{t-1} + (1-\pi)\frac{1}{2}}}_{\text{fraction } > 1} \cdot \theta_{t-1} = \frac{\frac{1+\pi}{2}}{\pi\theta_{t-1} + \frac{1-\pi}{2}} \cdot \theta_{t-1}$$

- Hence the new estimate of the value of the asset becomes:

$$\mu_t^+ = \theta_t^+ v^H + (1-\theta_t^+)v^L > \mu_{t-1} = \theta_{t-1} v^H + (1-\theta_{t-1})v^L$$

Dealers' quotes play two roles

- The deviation of the ask (bid) quote from the previous estimate of the stock value μ_{t-1} plays two roles:
  - allow the dealer to earn a revenue to cover the expected losses made by trading with informed investors:

$$a_t - \mu_{t-1} > 0,\ b_t - \mu_{t-1} < 0 \Rightarrow \text{bid-ask spread}$$

  - update the previous quotes in line with the informational content of a buy (sell) order:

$$\mu_t^+ - \mu_{t-1} > 0,\ \mu_t^- - \mu_{t-1} < 0 \Rightarrow \text{quote updating}$$

- The two coincide, since (if dealers are competitive) $a_t = \mu_t^+,\ b_t = \mu_t^-$

Price discovery and informational efficiency

- **How fast can dealers discover the true value of the asset?** Can pricing errors $(p_t - v)^2$ be large for a long time?
- Recall that we may have **3 forms of informational efficiency:**
  - Weak form: current prices reflect all information contained in past returns/prices
  - Semi strong form: current prices reflect all public information (including that contained in the order flow)
  - Strong form: current prices reflect all public and private information
- **If $\boxed{\pi < 1}$, the market is semi-strong form efficient but not strong-form efficient** since:

$$p_t = \mu_t = E(v|\Omega_{t-1}, d_t) \text{ but } E(v|\Omega_{t-1}, d_t) \neq v$$

Speed of price discovery: example

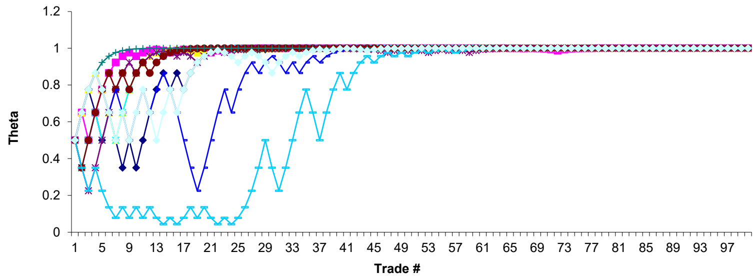
- 100 periods = 100 orders
- $\pi = 0.3$
- $\theta_0 = 0.5$
- $v_H = 102$
- $v_L = 98$
- True value is v_H
- 10 rounds of simulations, with randomly drawn sequences of buy and sell orders
- Use previous expression to derive the evolution of beliefs ($\Rightarrow$ dynamics of prices) for each of the 10 sequences of orders

Example: two consecutive sell orders at *t* = 1 and *t* = 2

- Suppose that $d_1 = d_2 = -1$ (two consecutive sell market orders)
- What are the quotes? What are the price dynamics?
  - **Date1:**
    - Quotes: a_1 =100.6 and b_1 = 99.4
    - Trade takes place at $p_1 = b_1$ = 99.4
    - Dealers' belief that $v=v^H$ becomes θ_1 = 35%
  - **Date 2:**
    - Quotes: a_2= 100 and b_2 = 98.9
    - Trade takes place at $p_2 = b_2$= 98.9
    - Dealers' beliefs after the trade that $v=v^H$ becomes θ_2 = 22%
  - **So orders move prices and beliefs in their own direction**

Example: simulating the dynamics of dealer beliefs

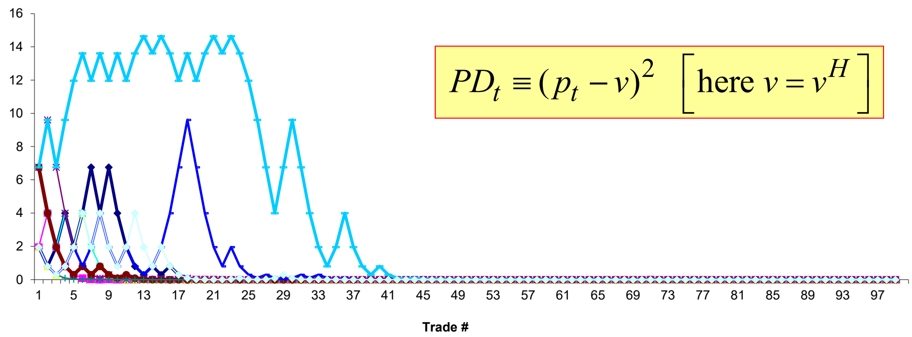
Figure 1 : Dealers beliefs: each curve shows the evolution of dealers' beliefs in each run (10 runs of 100 orders)



Over time, the order flow reveals the true value: **price discovery** ⇒ the market **becomes** strong-form efficient (even private info. in prices)

Simulation: dynamics of the (squared) pricing errors

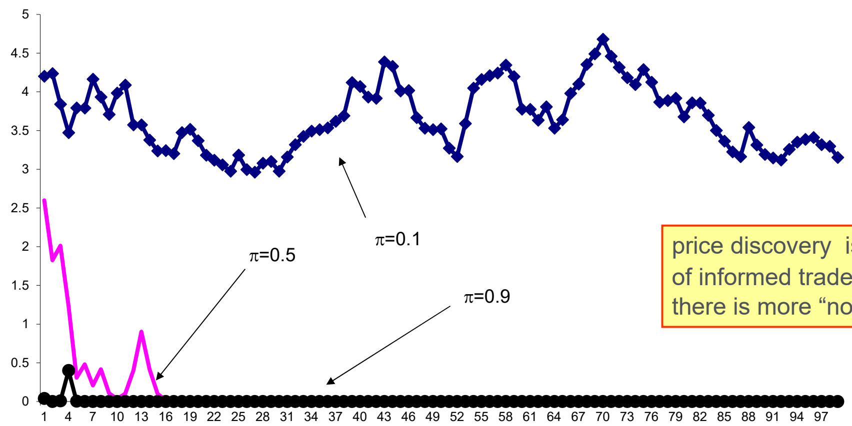
Figure 2: Pricing errors: each curve shows the evolution of the pricing error in each run (10 runs)



$$PD_t \equiv (p_t - v)^2 \quad \left[\text{here } v = v^H\right]$$

Simulation: price discovery for different values of π

Figure 3. Squared pricing error, averaged over 10 simulations for π = 10%, 50% and 90%



price discovery is slower when the fraction of informed traders π is lower, *i.e.* when there is more "noise" in the order flow

$\Rightarrow$ trade-off: higher π lowers liquidity but speeds up price discovery

Illiquidity and informational efficiency

- **Hence, when π increases, illiquidity (i.e., the bid-ask spread) increases – but the speed of price discovery increases as well**:

1. There is a **trade-off** between liquidity and informational efficiency, at least in the short run

2. Informational efficiency and illiquidity are **distinct** notions: the market features a bid-ask spread and yet prices are semi-strong form efficient in Glosten and Milgrom (1985)

Volatility

- Recall that price dynamics is dictated by:

$$\boxed{p_t = \mu_t = \mu_{t-1} + s(d_t)d_t}$$

$$\Rightarrow \Delta p_t = \Delta \mu_t = s(d_t)d_t$$

$$\Rightarrow \text{var}(\Delta p_t) = \text{var}\left(s(d_t)d_t\right)$$

- So return volatility is increasing in
  - (i) the bid-ask spread and
  - (ii) the uncertainty about the order flow direction
- Both may change during the trading day or around special events, such as a takeover

Key takeaways

- Intraday price fluctuations are correlated with the order flow, which should not occur if markets were frictionless
- Asymmetric information between market makers and some traders can account both for the bid-ask spread and for the fact that orders move prices in their direction
- In this setting, the more informative is the order flow,
  - the more it moves prices, generating more illiquidity (higher bid-ask spread) and increasing volatility
  - the greater the speed at which private information is impounded in transaction prices, *i.e.* the faster is price discovery