Imperial College
Business School

Lecture 4:

Trade size and market depth

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Readings

Core

- Chapter 4 of textbook

Recommended

- Kyle, Albert S., 1985 Continuous auctions and insider trading, *Econometrica* 53, Sections 1, 2 and 6 only, pp. 1315-1320, 1333-1335 only.

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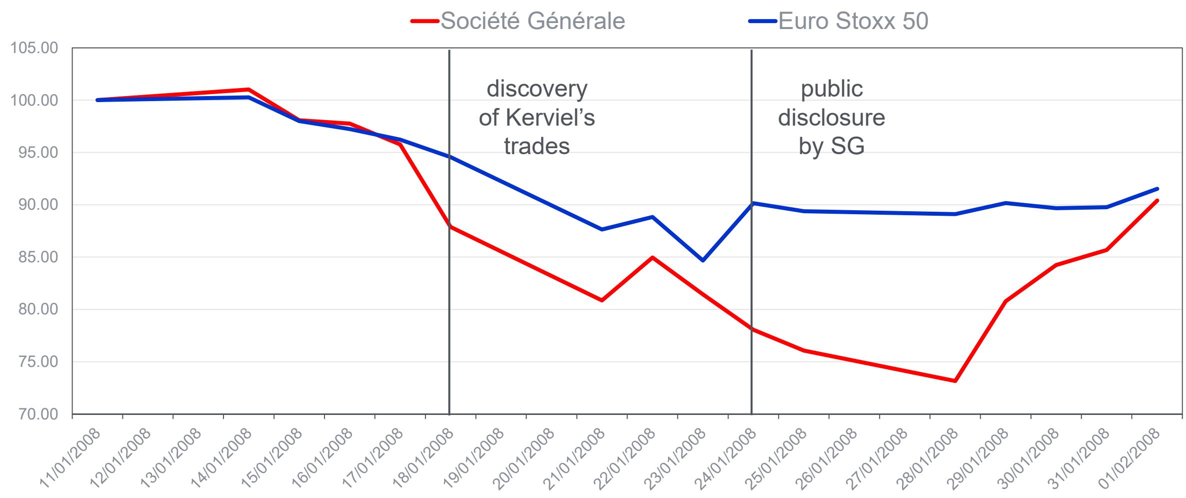
Outline

- How does trade size affect prices?
- What is market depth?
- How do traders choose their trade size?
- We shall address these questions in the context of call markets:
  1. with informative order flow: Kyle's model
     - under perfect and imperfect competition
  2. with risk averse market makers
     - again, under perfect and imperfect competition

Société Générale, January 2008

- Friday 18 January: Société Générale discovers that employee Jerome Kerviel has accumulated massive positions in European equity derivatives $\Rightarrow$ losses = €1.5 bn
- 21 January: bank starts executing small sales
- 22 January: even though sales less than 10% of daily volume, stock markets decline steeply $\Rightarrow$ Fed intervenes
- 24 January: SG discloses the news publicly; its losses = €4.9 bn
- Episode illustrates three facts:
  - large intended trades, yet gradual and cautious sales
  - secrecy, for fear of unsettling market
  - even so, steep price drop: derivative markets have limited depth, even though they are very liquid

Stock price of Société Générale and Euro Stoxx 50 index



Depth vs. spread: two dimensions of liquidity

- A market can be very liquid for small trades, but not for large ones: "it lacks depth"
- Why?
  - Large orders may be much **more informative** than small ones, being more likely placed by informed traders
  - Filling a large order exposes market makers to **greater inventory risk**: this depends on their total risk bearing capacity (number of market makers also matters)
  - Market makers may have **market power**: if they do, they may offer less depth to large order placers
- To capture these points, we must allow for variable order *size* (not just order *direction*)
- **Depth** = size of order that can be filled at a given deviation of price from its pre-existing level

Depth and asymmetric information

1. Market depth under asymmetric information

- Model by Kyle (1985): as in Glosten-Milgrom, there are two types of traders:
  - uninformed traders, who place random order $u \sim N(0, \sigma_u^2)$
  - informed traders, who observe $v \sim N(\mu, \sigma_v^2)$ and place order with size (x) that depends on v : $x = X(v)$
- But now orders are not executed one by one. They are batched and submitted as a single net trade q:

$$q \equiv x + u$$

order from informed trader ("insider")

order from uninformed trader ("noise trader")

Market makers

- Risk neutral, competitive market makers:
  - do not observe *v*, and cannot distinguish *x* (informed orders) from *u* (noise)
  - post price-quantity schedules based on expectation of *v,* conditional on the total net trade *q*
- They try to <u>infer *v* from *q*</u> = *x* + *u* or from the price *p*
- In equilibrium *p* and *q* will be linearly related:

$$p = \mu + \lambda q$$

where λ **= price impact** of a market order $\Rightarrow$ **1/λ = depth**

- In equilibrium, λ depends on *order flow informativeness,* which drives dealers' inference about *v*

Solving for equilibrium: four steps

- **Market maker's inference**: explain how market makers infer *v* from orders *q*, for a *given* (assumed) strategy of informed traders *x*=*X*(*v)*
- **Pricing function**: derive the market makers' pricing function *p*(*q*), assuming perfect competition among them (zero expected profits)
- **Order placement by informed traders**: find the parameters of the *X*(*v)* function that maximize expected profits, for a given pricing function of market makers
- **Nash equilibrium**: find the parameters that make the two best responses *p*(*q*) and *X*(*v*) mutually consistent

Market makers' inference

- The insider trades $x = X(v) = \beta(v-\mu)$

  for some $\beta > 0$

- Market makers do not observe x, but total demand q:

$$q = x + u = \beta(v-\mu) + u, \quad \text{with } q \sim N(0, \beta^2\sigma_v^2 + \sigma_u^2)$$

- Their estimate of v is:

$$E(v|q) = \mu + \underbrace{\frac{\text{cov}(v,q)}{\text{var}(q)}}_{\substack{\text{OLS}\\ \text{regression}\\ \text{coefficient}}} q = \mu + \underbrace{\frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}}_{\alpha} q$$

What does order flow informativeness α depend on?

- Informativeness of order flow α is
  - Increasing in σ_v^2 (variance of the "signal")
  - decreasing in σ_u^2 (variance of the "noise")
- It is non-monotonic in β (aggressiveness of trading by insiders): first increasing, then decreasing in β
- Why?
  - aggressive trading by insiders makes their orders more informative
  - it also inflates trades for given v $\rightarrow$ α must be scaled down when insiders' trading is very aggressive

Perfect competition among market makers

- Zero expected profits for market makers:

$$\boxed{p(q) = \mathrm{E}(v|q)} = \mu + \underbrace{\frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}}_{\lambda = \text{price pressure}} q \qquad \boxed{\Rightarrow \lambda = \alpha}$$

- So the call market implicitly features a bid-ask spread that is increasing in the quantity traded:

$$S(q) = 2\lambda q$$

- Note: p depends on **total** order flow $q \Rightarrow$ (i) depth = $1/\lambda$; (ii) execution risk for any given order (as q is random)!

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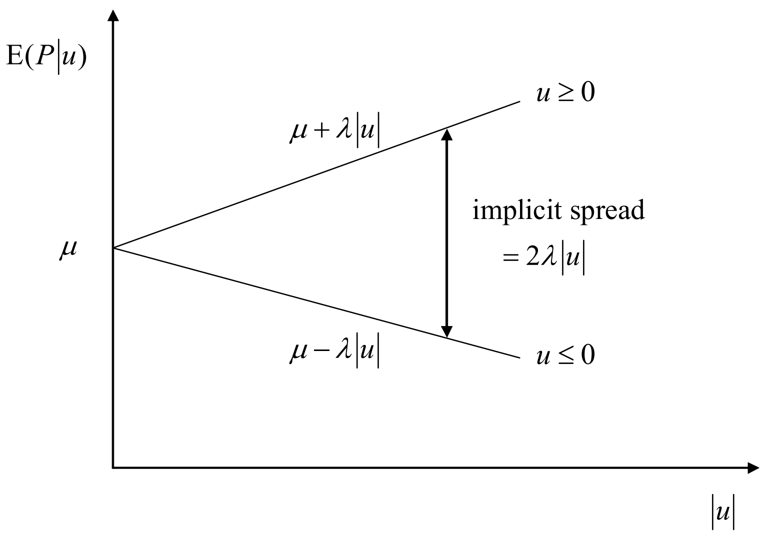
Expected price and noise trade size



depth depends inversely on price pressure parameter λ

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Endogenous insider behavior

- So far β (intensity of insider's trading) was taken as given
- But the insider chooses x to maximize expected profits:

$$\max_x \mathrm{E}\left[x \cdot (v-p)\right] \qquad = x \cdot \left(v - \left(\mu + \lambda \mathrm{E}(q \mid x)\right)\right)$$

- The first order condition yields the "optimal aggressiveness" β:

$$v - \mu = 2\lambda x \qquad = x \cdot \left(v - \left(\mu + \lambda x\right)\right)$$

$$\Rightarrow x = \underbrace{\frac{1}{2\lambda}}_{=\beta}(v-\mu)$$

aggressiveness of insider's orders β is inversely related to price pressure $\lambda \Rightarrow$ it is proportional to market depth $1/\lambda$.

We derived the best responses of the two players

Market maker set prices:

$$\lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}$$

The insider responds with:

$$\beta = \frac{1}{2\lambda}$$

The values of β and λ that solve these 2 equations make the best response of the market makers (1st equation) and that of the informed trader (2nd equation) mutually consistent: **Nash equilibrium**

Solving for the Nash equilibrium

Recall the 2 equations: $$\lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2+\sigma_u^2}, \qquad \beta = \frac{1}{2\lambda}$$

From the 1st eq.: $$\frac{1}{\lambda} = \frac{\beta^2\sigma_v^2+\sigma_u^2}{\beta\sigma_v^2} = 2\beta \text{ by using the 2nd eq.}$$

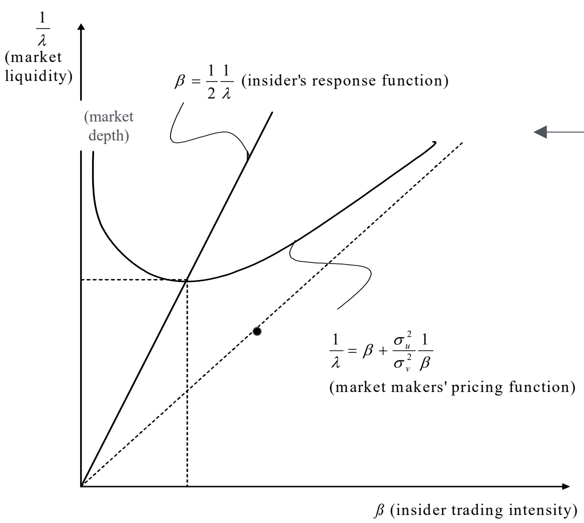
$$\Rightarrow \beta^2\sigma_v^2+\sigma_u^2 = 2\beta^2\sigma_v^2 \quad \Rightarrow \sigma_u^2 = \beta^2\sigma_v^2 \quad \Rightarrow \boxed{\beta = \frac{\sigma_u}{\sigma_v}}$$

From the 2nd eq. again: $$\lambda = \frac{1}{2\beta} \quad \Rightarrow \boxed{\lambda = \frac{\sigma_v}{2\sigma_u}}$$

Graphically

In **equilibrium** market **depth** is at **minimum**.

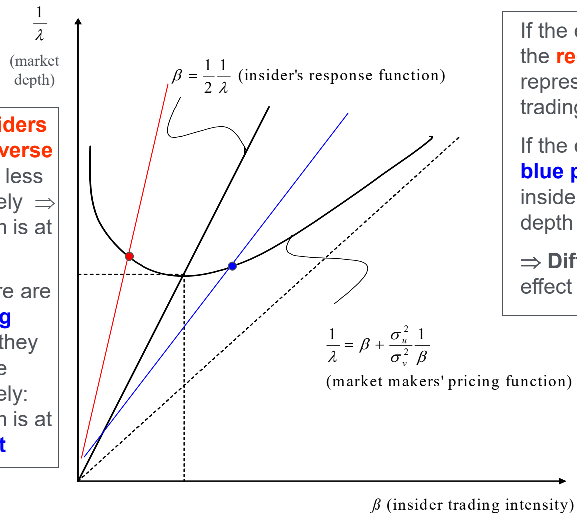
Anti-insider regulation steepens β (λ) ⇒ increases market depth.



increasing: insider is more aggressive when market makers offer more depth

non-monotonic: market maker can offer depth when insider is **either not** very aggressive **or so** aggressive as to give away much information

Other scenarios are possible



When **insiders** are **risk-averse** they trade less aggressively ⇒ equilibrium is at **red point**

When there are **competing insiders**, they trade more aggressively: equilibrium is at **blue point**

If the equilibrium is at the **red point**, repressing insider trading raises depth

If the equilibrium is at **blue point**, repressing insider trading reduces depth

⇒ **Difficult** to predict effect in general!

Equilibrium market depth

- The equilibrium value of λ determines the price response to the total net order q:

$$p(q) = \mu + \lambda q = \mu + \frac{\sigma_v}{2\sigma_u} \cdot q$$

- The equilibrium market depth, $1/\lambda = 2\sigma_u/\sigma_v$, is
  - increasing in the variability of noise trading σ_u: the greater noise trading volume, the more market makers earn to compensate losses with insiders
  - decreasing in the variability of the stock value σ_v: the greater the insider's informational advantage, the more market makers are afraid of losing money to insiders

How much money do insiders make on average?

- Insider's expected profit, *conditional* on *v*:

$$\mathrm{E}\left[x\cdot(v-p)\,|\,v\right]=x\cdot\left[v-\mathrm{E}(p\,|\,x)\right] \qquad =x\cdot\left[v-(\mu+\lambda x)\right]$$

$$=\underbrace{\beta\left(v-\mu\right)}_{x}\cdot[v-\mu-\overbrace{\lambda\,\underbrace{\beta\left(v-\mu\right)}_{x}}^{1/2}]$$

$$=\frac{1}{2}\beta\left(v-\mu\right)^2 \qquad =\frac{1}{2}\frac{\sigma_u}{\sigma_v}\left(v-\mu\right)^2$$

- So it is increasing in σ_u (as noise trading allows the insider to "hide") and in the distance of *v* from its mean μ (which measures the insider's information advantage)

Ex ante, how much money do insiders expect to make?

- Consider the insider's expected profit, before he observes the asset value *v*, that is, unconditionally.
- We must take the expectation of the previous expression over the asset value *v*:

$$\mathrm{E}\left[x\cdot(v-p)\right]=\frac{1}{2}\frac{\sigma_u}{\sigma_v}\mathrm{E}\left(v-\mu\right)^2 \qquad =\frac{1}{2}\frac{\sigma_u}{\sigma_v}\sigma_v^2 \qquad =\frac{1}{2}\sigma_v\sigma_u$$

- Increasing in σ_v: "information is money"
- Increasing in σ_u: noise trading allows the insider to "hide"
- Insider's expected profit = average noise trader's costs

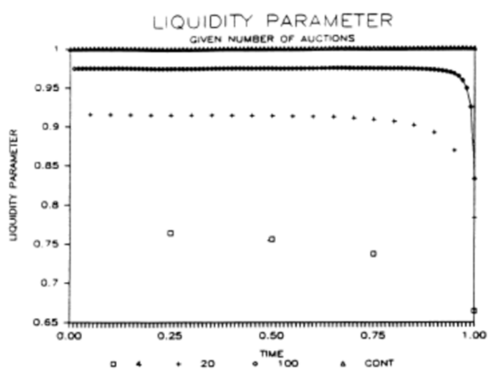
Kyle (1985): dynamic case

- Suppose that there are two trading rounds.

- Then the informed investor's problem is more complex:
  1. he can repeatedly exploit his signal
  2. however, each trade permanently impacts the price and therefore reduces the profit that the informed investor can expect from future trades

- Solving the equilibrium: $\{\beta_t\}$, $\{\lambda_t\}$ are jointly determined at each date (no recursive solution)

Key findings for the dynamic case



Kyle (1985) shows that, in discrete time, illiquidity
- decreases over time
- increases as the number of trading rounds increases

Instead, in continuous time, illiquidity is constant at each date

Market Depth and Imperfect Competition

Imperfectly competitive market making

- Now consider a call auction where K risk neutral market makers compete by submitting supply schedules $y^k=Y^k(p)$, for k = 1,…, K
- Look for rational expectations equilibrium, i.e.:
  - market makers maximize expected profits, given their beliefs about the security's value, and the (assumed) behavior of (i) the informed trader *and* (ii) their competitors
  - their beliefs are rational (=verified in equilibrium)
  - the market clears

Updating and market clearing conditions

- As before, assume that the order flow *q* is informative about the value of the security *v*:

$$E(v|q) = \mu + \alpha q$$

- The market clears when the market makers' total supply equals the customers' order flow *q*:

$$\sum_{k=1}^{K} Y^k(p) = q$$

- Note that by conditioning on *p*, each market maker can effectively condition on *q*

Imperfectly competitive equilibrium

- Derive the optimal (profit-maximizing) response of market maker *k*, assuming that each competitor *j* submits an *identical linear* supply function:

$$y^j = Y^j(p) = \phi(p - \mu)$$

- Market maker *k*'s optimal response $y^k = Y^k(p)$ also depends on his estimate $\mathrm{E}(v|q)$
- In equilibrium, the slope of y^k must equal the slope ϕ assumed for his competitors, which yields:

$$p = \mu + \underbrace{\alpha \frac{K-1}{K-2}}_{\lambda} q$$

Note that:

$$\frac{K-1}{K-2} > 1$$

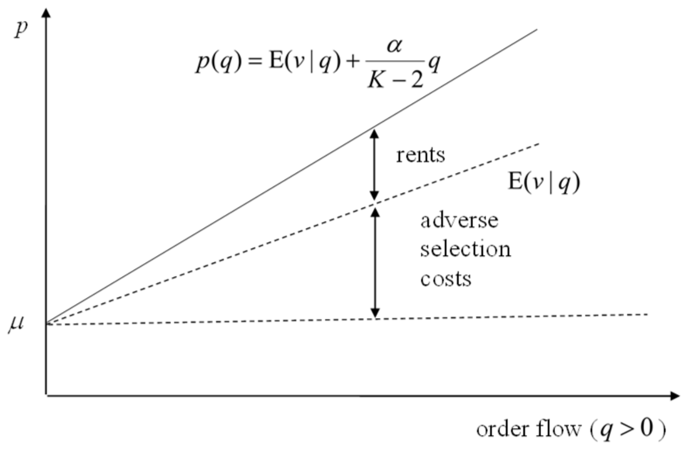
Imperfectly competitive equilibrium (2)

- So under imperfect competition the price "overreacts" to the order flow relative to perfect competition:

$$p = \underbrace{\mu + \alpha q}_{\substack{\text{competitive} \\ \text{price} = \mathrm{E}(v|q)}} + \underbrace{\frac{1}{K-2}\alpha q}_{\text{markup}} \qquad \boxed{\Rightarrow \lambda > \alpha}$$

- The markup is positive for *K*>2 and is inversely related to the number of competing market makers *K*
- The markup goes to 0 as *K* goes to ∞ : perfect competition!

Dealers' quotes: perfect vs. imperfect competition



2. Market depth with risk averse market makers

- *K* risk averse market makers *k = 1,...,K*, with utility:

$$\mathrm{E}[U(w^k)] = \mathrm{E}(w^k) - \frac{\bar{\rho}}{2}\,\mathrm{var}(w^k)$$

final wealth

and budget constraint:

stock supply

initial inventory

$$w^k = v(\underbrace{z^k - y^k}_{\text{final inventory}}) + \underbrace{py^k + c^k}_{\text{final cash}}$$

initial cash

- The security has value with mean μ and variance σ_v^2
- Each market maker posts a net supply schedule
- He is **competitive**, *i.e.* takes *p* as given

Individual optimization

- Market maker k chooses y^k so as to maximize his utility under the budget constraint. Replace the constraint in $\mathrm{E}[U(w^k)]$:

$$\mu(z^k - y^k) + py^k + c^k - \frac{\bar{\rho}}{2}\sigma_v^2 (z^k - y^k)^2$$

and compute the first order condition:

$$p - \mu + \bar{\rho}\sigma_v^2 (z^k - y^k) = 0$$

- So the individual supply function of market maker *k* is:

$$y^k = \frac{p - \mu}{\bar{\rho}\sigma_v^2} + z^k$$

Competitive equilibrium

- The market clearing condition is:

$$\sum_{k=1}^{K} y^k = q$$

- So the equilibrium price is:

$$p = \mu - \rho\sigma_v^2 Z + \rho\sigma_v^2 q$$

where

$$Z = \sum_{k=1}^{K} z^k \quad \text{and} \quad \rho = \frac{\bar{\rho}}{K}$$

the “risk aversion of the market” is inversely related to number of dealers K: each supplies 1/K of q

Competitive equilibrium (2)

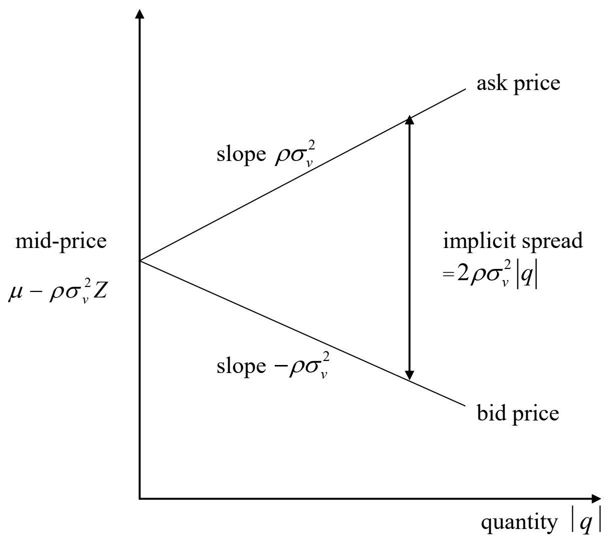
- Since

$$p = \underbrace{\mu - \rho\sigma_v^2 Z}_{\text{midquote } m} + \rho\sigma_v^2 q = m + \rho\sigma_v^2 q$$

the trading cost $p - m$ is increasing in the

- risk of the security
- size of total net trade q
- market makers’ “collective” risk aversion $\rho \Rightarrow$ decreasing in their number K (= risk bearing capacity of the market)

Graphically



depth depends inversely on (i) risk σ_v^2 and (ii) the **market** risk-bearing capacity ρ

Imperfectly competitive dealers

- Suppose dealers are (i) imperfectly competitive; (ii) equally risk averse
- There is Nash equilibrium in which each dealer *k* submits the following supply function, assuming that his competitors submit *identical linear* supply functions:

$$y^k = Y^k(p) = \phi(p-\mu) + \psi z^k$$

- Market maker *k*'s optimal response y^k takes into account that the price responds to his supply y^k in equilibrium:

$$y^k + \underbrace{(K-1)\phi(p-\mu) + \psi \sum_{j \neq k} z^j}_{\text{supply by } k\text{'s competitors}} = q$$

Deriving the Nash equilibrium: three steps

1. Dealer k maximizes his mean-variance objective *considering the response of the equilibrium price to his trade* $\Rightarrow$ obtain his **supply function** (best response) $Y^k(p)$
2. By **symmetry** the coefficients of dealer k's supply must **equal** those of his competitors' (conjectured) supplies $\Rightarrow$ obtain his **Nash equilibrium supply** function:

$$Y^k(p) = \frac{K-2}{K-1}\left[\frac{p-\mu}{\bar{\rho}\sigma_v^2} + z^k\right]$$

3. Impose **market clearing**:

$$\sum_{k=1}^{K} Y^k(p) = q$$

Nash equilibrium price

- Equilibrium price (for K >2):

$$p = \mu - \rho\sigma_v^2 Z + \boxed{\frac{K-1}{K-2}}\rho\sigma_v^2 q \qquad \boxed{= 1 + \underbrace{\frac{1}{K-2}}_{\text{markup}}}$$

where

$$Z = \sum_{k=1}^{K} z^k \quad \text{and} \quad \rho = \frac{\bar{\rho}}{K}$$

- So the price "overreacts" to the order flow compared to perfect competition (case where $K \to \infty$)
- The markup is inversely related to the number of competing market makers K
- Here K adds to market depth for 2 reasons: 1) more risk bearing capacity; 2) more competition

Risk sharing in the competitive case

- How do dealers share risk? Do they share risk efficiently?
- Suppose q=0. If all dealers have the same risk aversion. efficient risk sharing requires each to have a final position Z/K: so risk is shared equally among dealers
- This is indeed the case in the competitive equilibrium since after trading, a dealer's position is:

$$z^k - y^k(p^*) = \frac{Z}{K}$$

- This yields a Pareto optimal allocation among dealers after trading $\Rightarrow$ no need for re-trading

Risk sharing in the imperfectly competitive case

- In a non-competitive equilibrium, a dealer's position after trading is a weighted average of his initial position z^k and Z/K:

$$z^k - y^k(p^*) = \frac{1}{K-1} z^k + \frac{K-2}{K-1}\frac{Z}{K}$$

- So risk allocation after one trading round is not Pareto- optimal
- Why? Dealers trade "too little": they reduce their supply or demand so as to limit the adverse price reaction they trigger
- If the market reopens, they would want to trade again after the first trading round

Key takeaways

- Large orders generally affect prices more than small orders: markets can offer great liquid for small trades but lack depth
- The price impact of large orders is an inverse measure of market depth
- Depth is lower
  - the more informative is the order flow, i.e., the higher the signal to noise ratio
  - the greater is asset risk and market makers’ risk aversion, and the lower their number
  - the greater is the market power of liquidity suppliers, and thus (again) the fewer they are