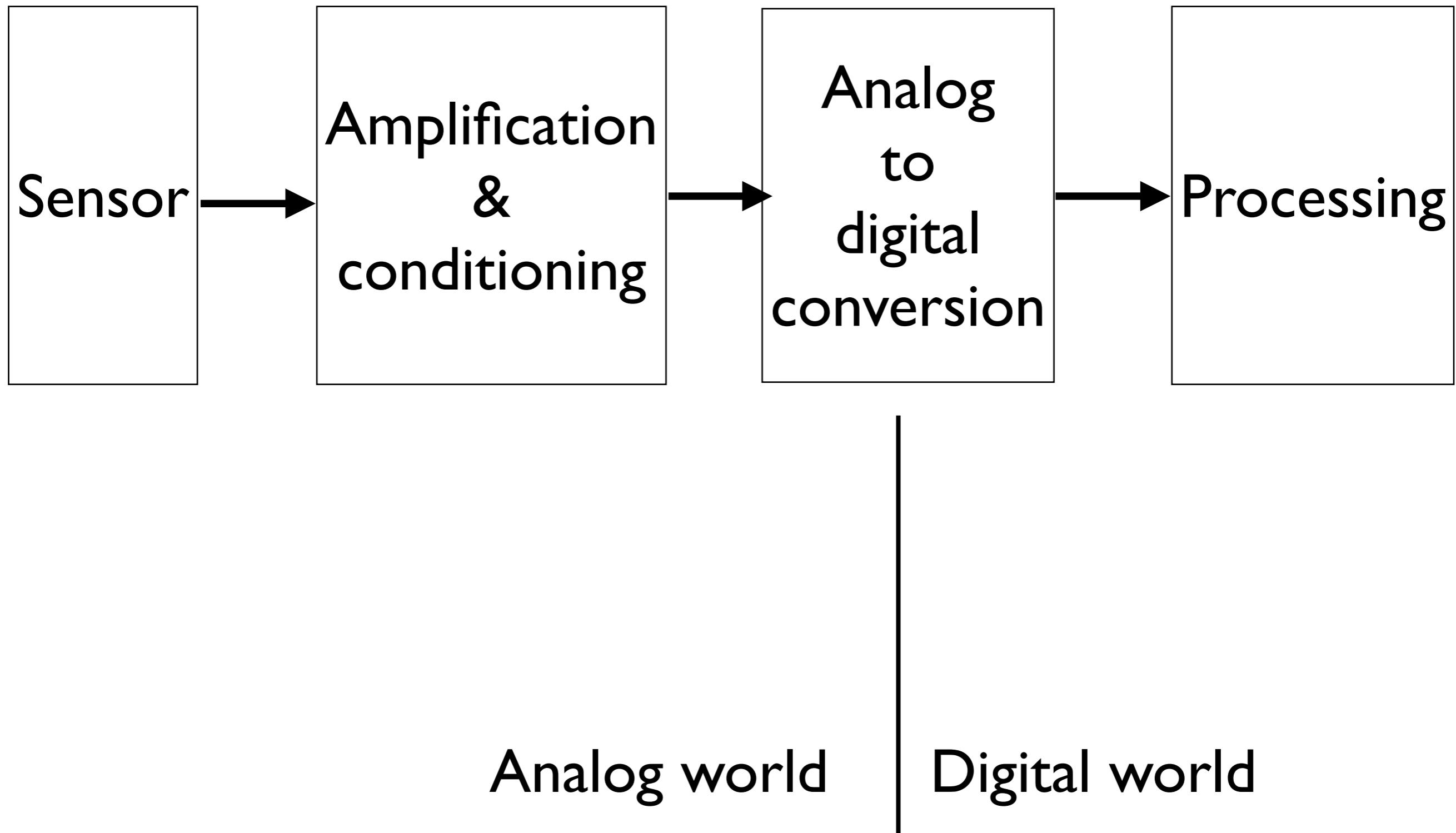


# Lecture I

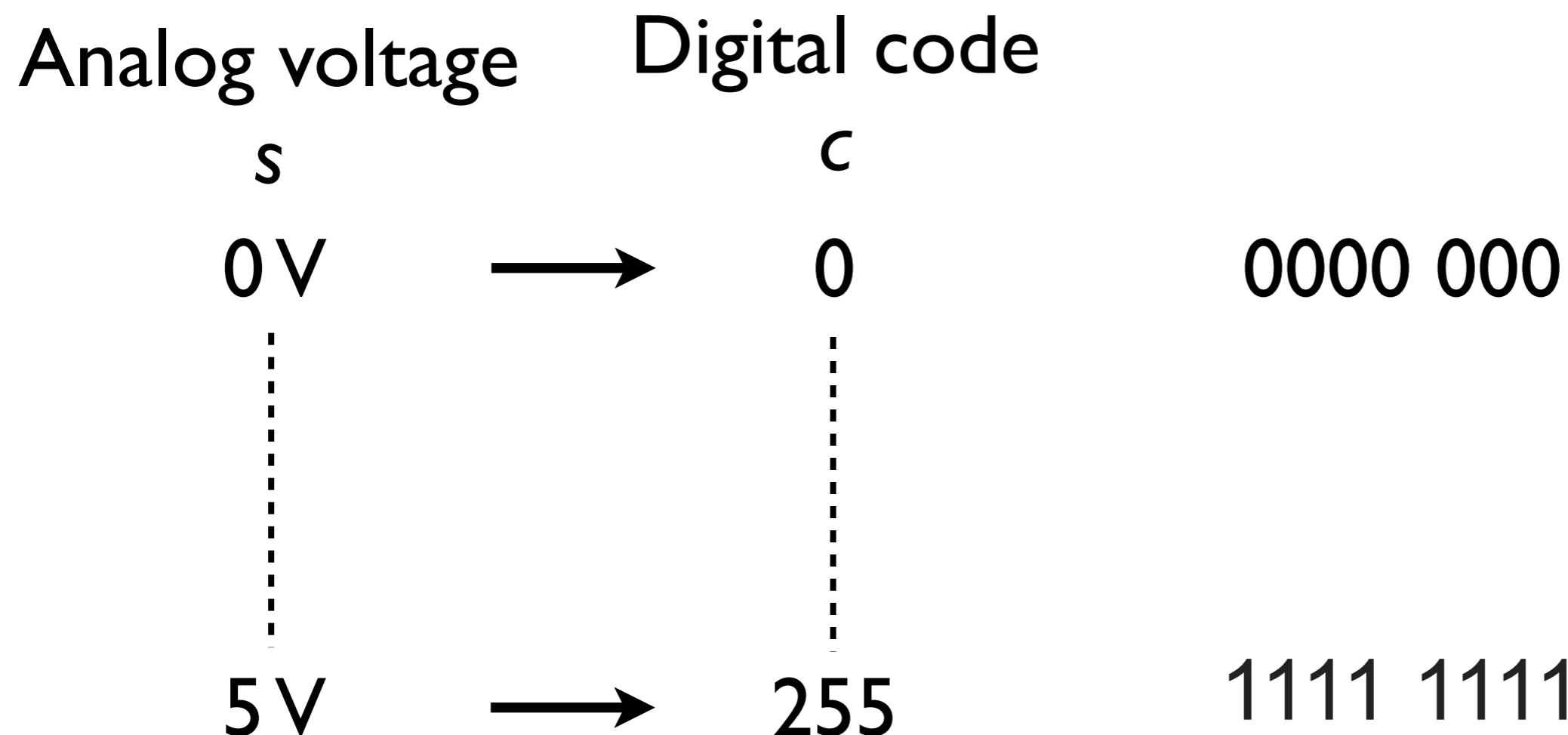
- Sampling and quantization
  - AD converters
- Digital oscilloscope

# The measurement chain



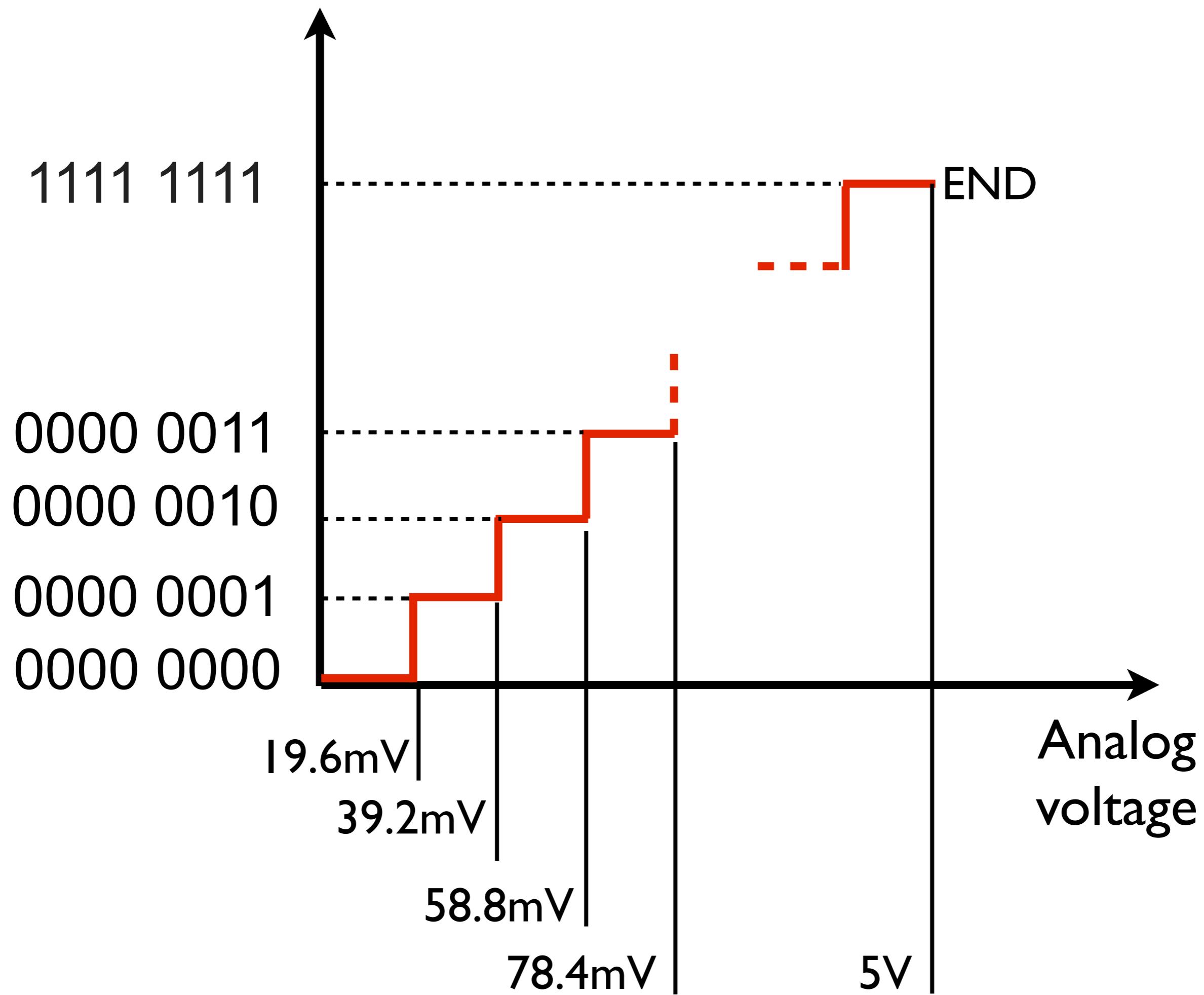
# What's the difference between analog and digital data?

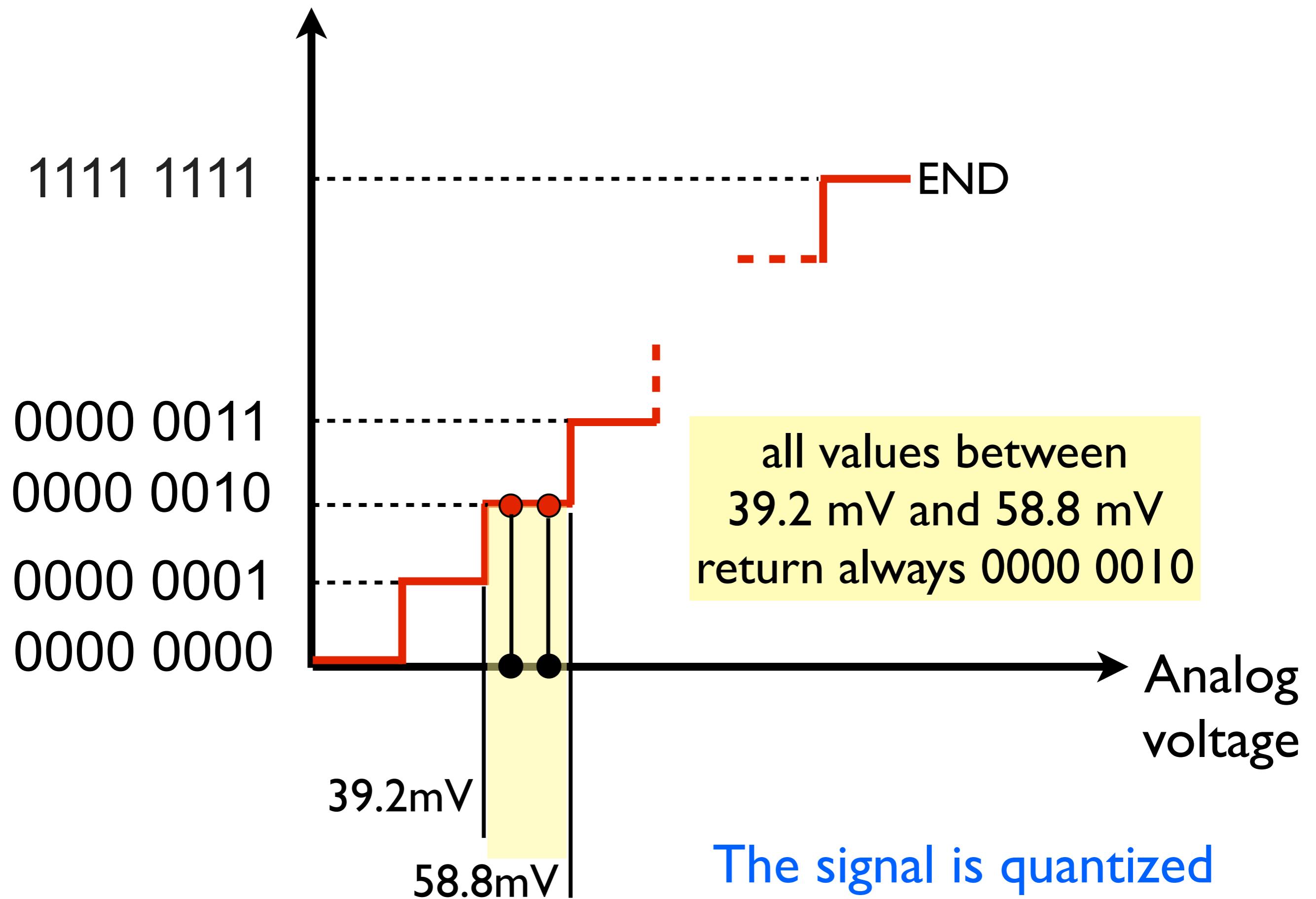
## I - Amplitude

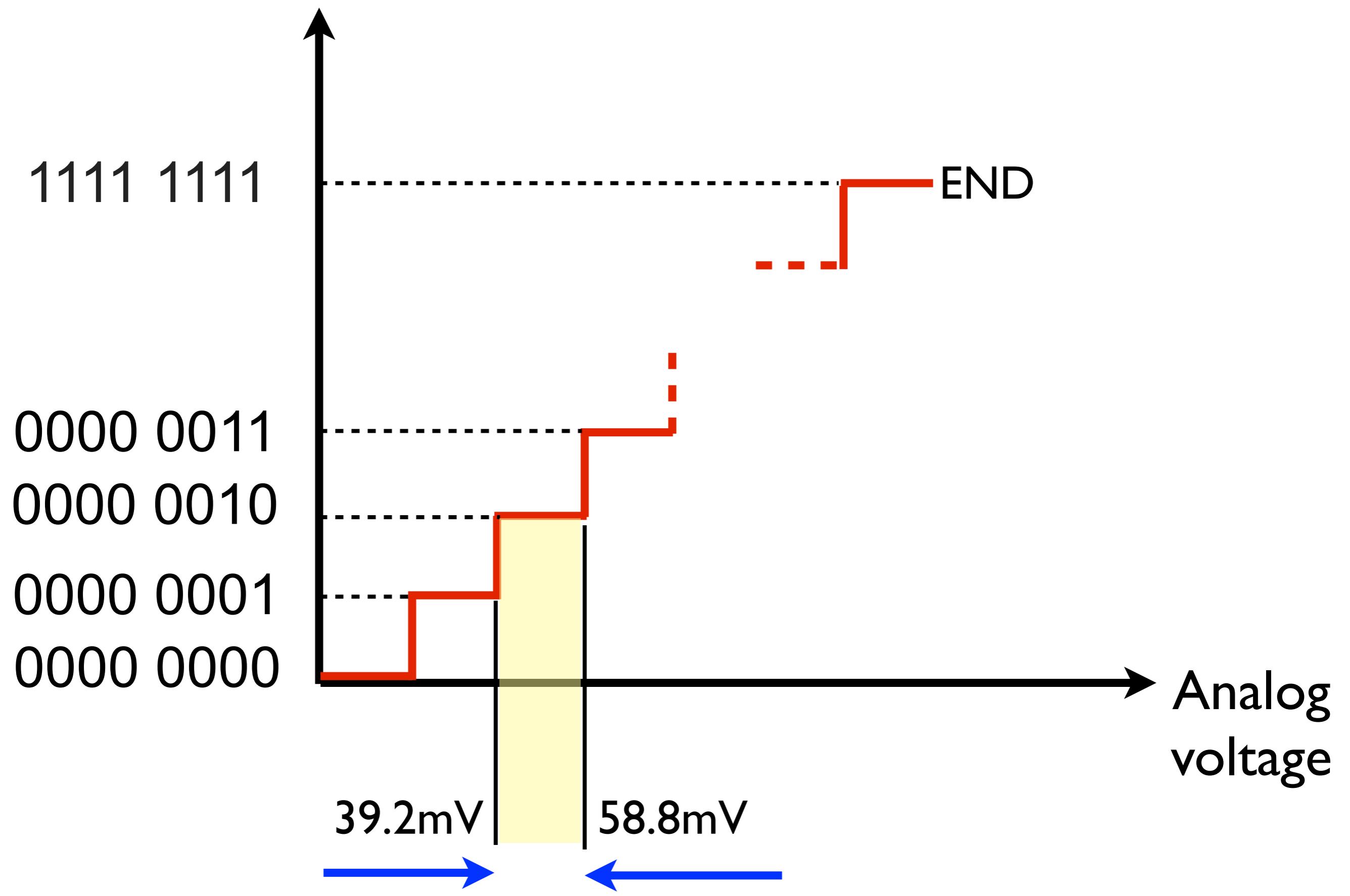


# ANALOG TO DIGITAL CONVERSION

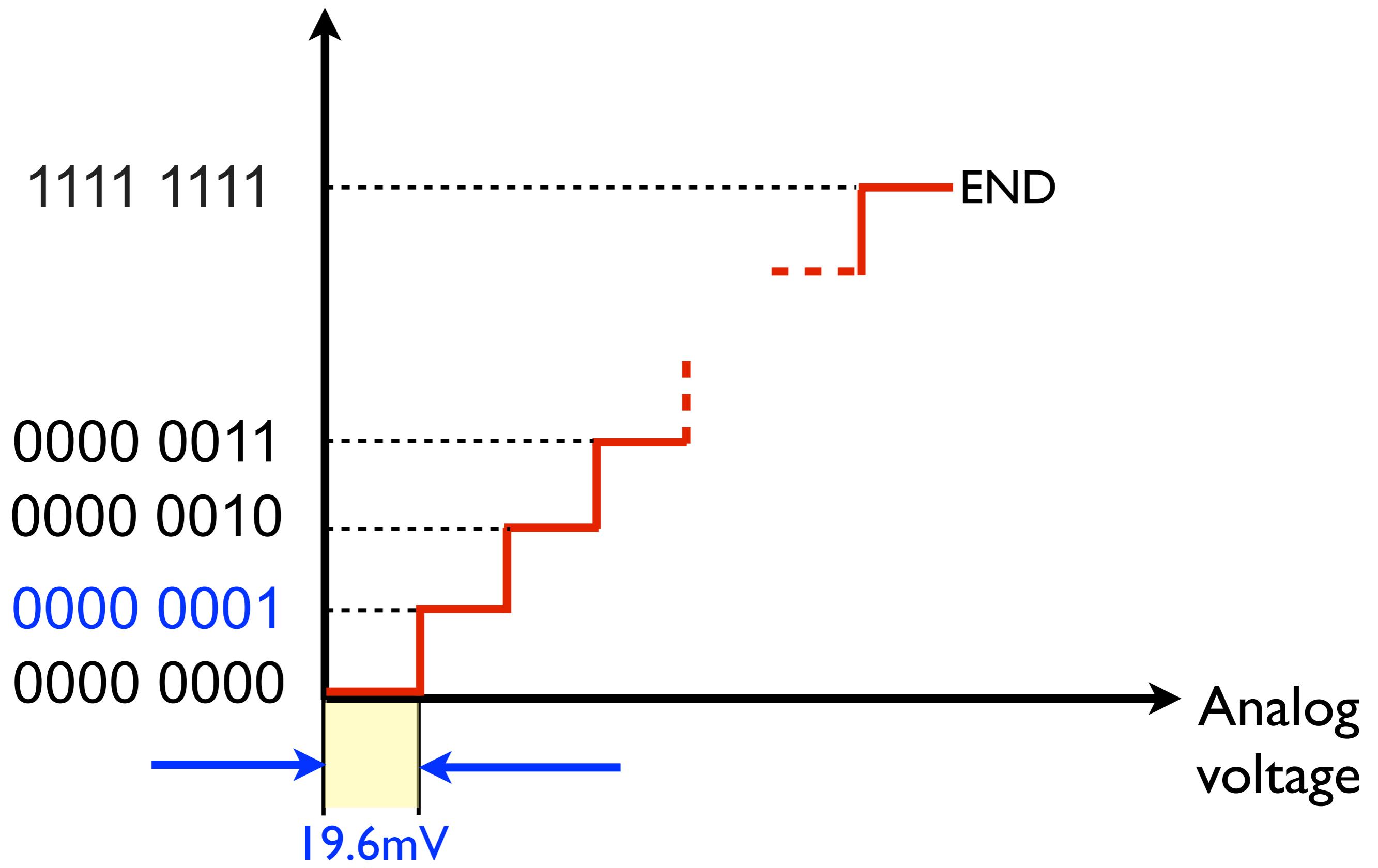
Analog voltage	Digital code
0V	0 0000 0000
0.84V	43 0010 1011
5V	255 1111 1111







minimum voltage we can discriminate  $58.8 - 39.2 = 19.6 \text{ mV}$



LSB = least significant bit

**Total Parts: 452**

	Resolution (bits)	Input Chan.	Conv. Rate (ksps)	Data Bus	Diff/S.E. Input	Internal	External	
						V <sub>REF</sub> (V)	V <sub>REF</sub> (V)	
						nominal	min	max
<input checked="" type="checkbox"/> MAX11166 NEW! 16-Bit 500ksps ±5V SAR ADC with Internal/External Reference in 3mm x 3mm Package	16	1	500	SPI	Both	4.096	2.5	5
<input checked="" type="checkbox"/> MAX11167 NEW! 16-Bit 250ksps ±5V SAR ADC with Internal/External Reference in 3mm x 3mm Package	16	1	250	SPI	Both	4.096	2.5	5
<input checked="" type="checkbox"/> MAX11120 1Msps 4 channel 8 bit Analog-to-Digital Converter with SampleSet	8	4	1000	SPI	Both	-	1	3.6
<input checked="" type="checkbox"/> MAX11121 1Msps 4 channel 10 bit Analog-to-Digital Converter with SampleSet	10	4	1000	SPI	Both	-	1	3.6
<input checked="" type="checkbox"/> MAX11122 1Msps 4 channel 12bit Analog-to-Digital Converter with SampleSet	12	4	1000	SPI	Both	-	1	3.6
<input checked="" type="checkbox"/> MAX11123 1Msps 8 channel 8 bit Analog-to-Digital Converter with SampleSet	8	8	1000	SPI	Both	-	1	3.6

0000 0000 0000 (resolution)      0000 0000  
 :    :  
 | | | | | | | | | | | | | | | | | | | |

from 0 to  $4095 = 2^{12} - 1$

number of bits  
(resolution)  
 8  
 10  
 12  
 8

full scale voltage  
 $V_{fs} (=V_{REF})$

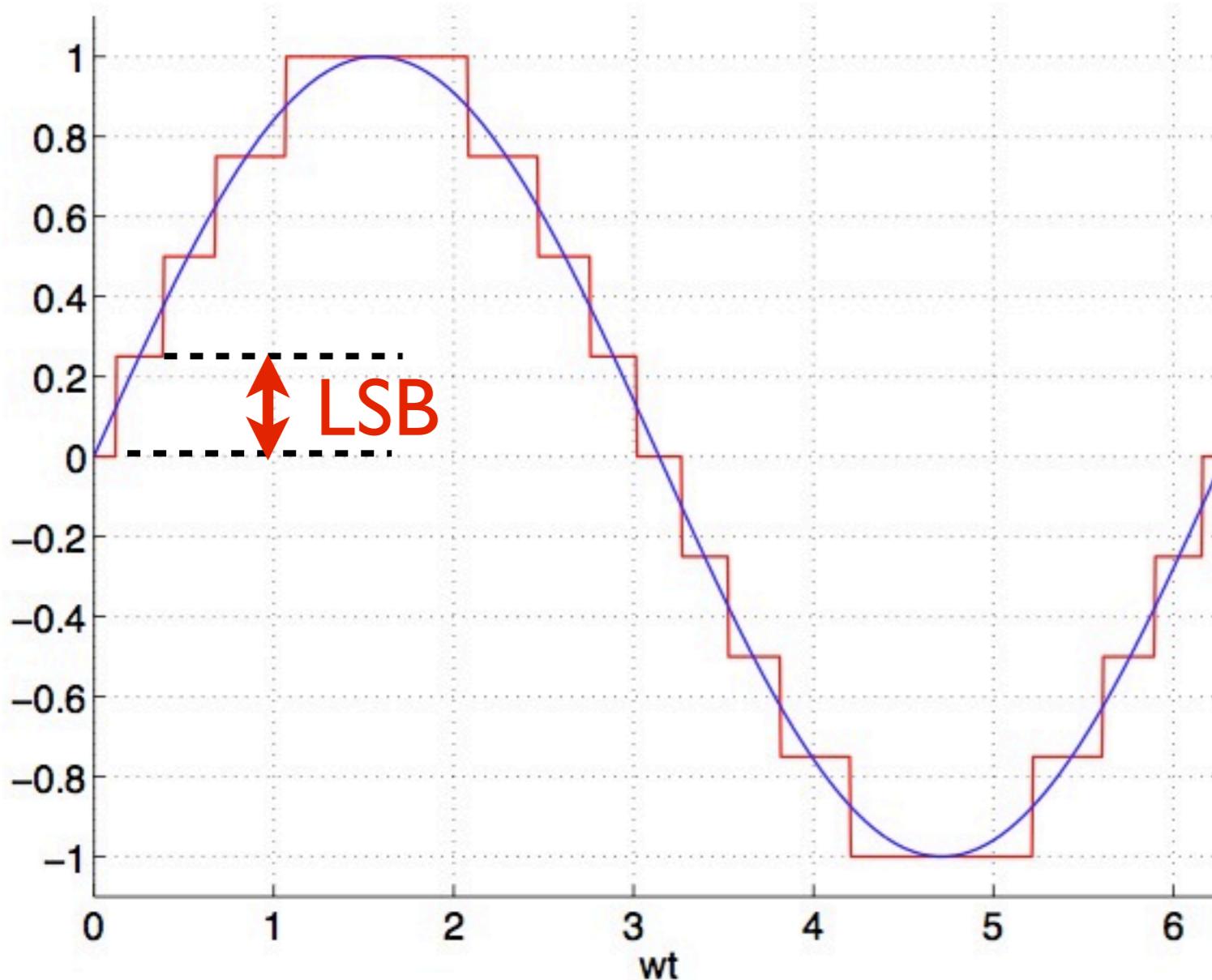
from 0 to  $255 = 2^8 - 1$

$n$  = number of bits

$V_{fs}$  = full scale voltage

$$LSB = \frac{V_{fs}}{2^n}$$

# How does LSB affect the digital signal?



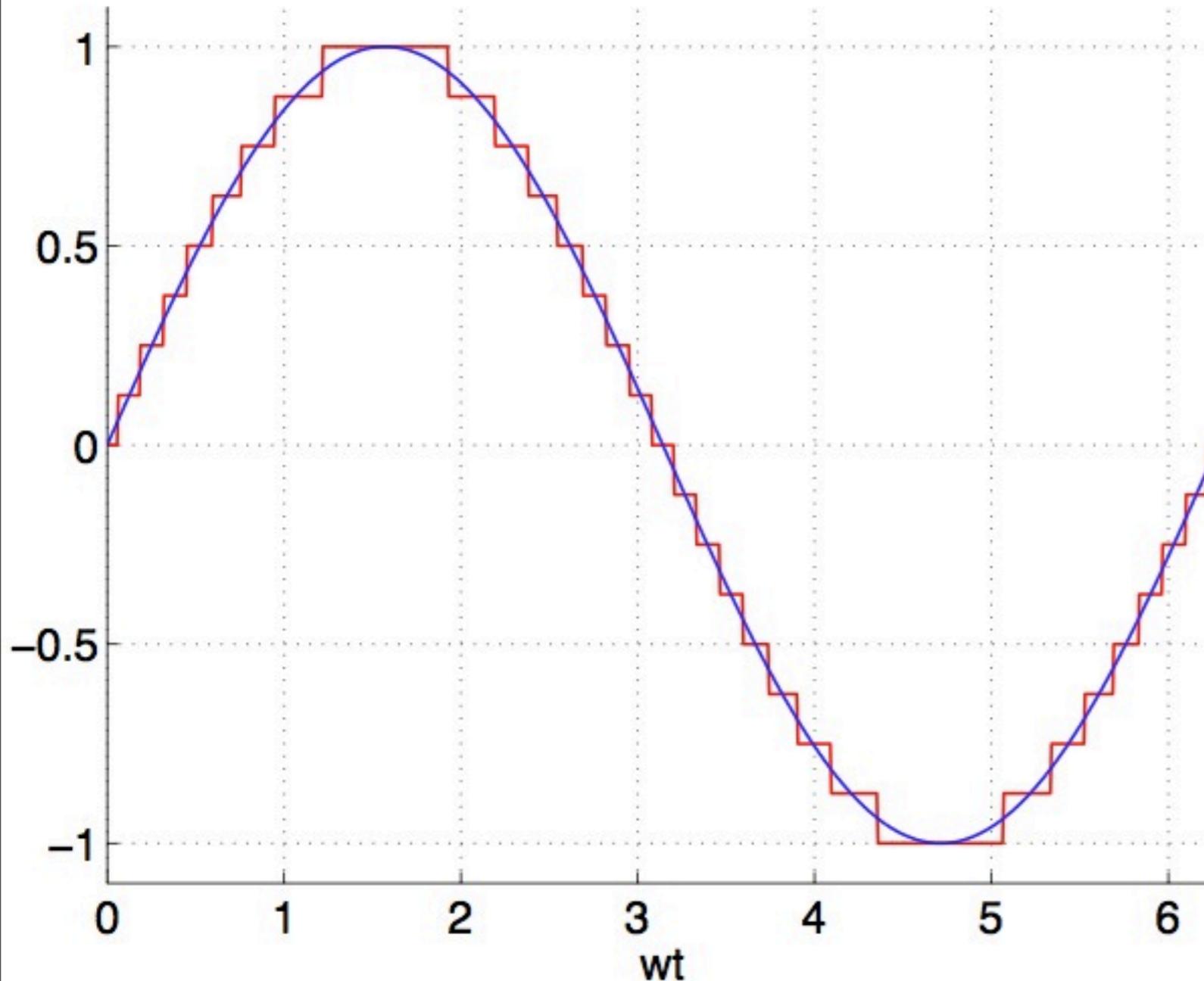
3 bits ADC

$$2^3 = 8$$

Maximum voltage  
range 2V

$$\text{LSB} = 2V/8 = 0.25V$$

# How does LSB affect the digital signal?

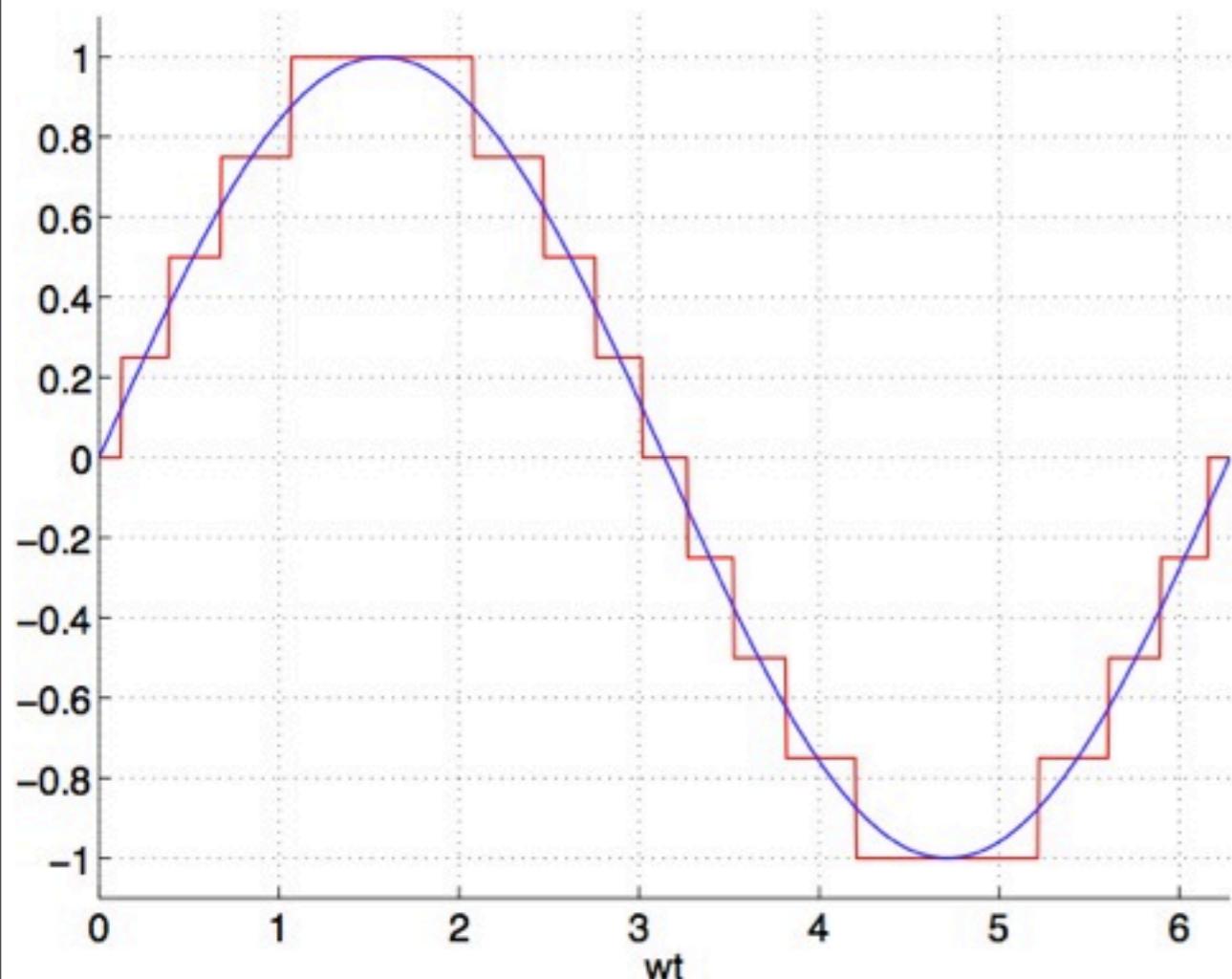


4 bits ADC

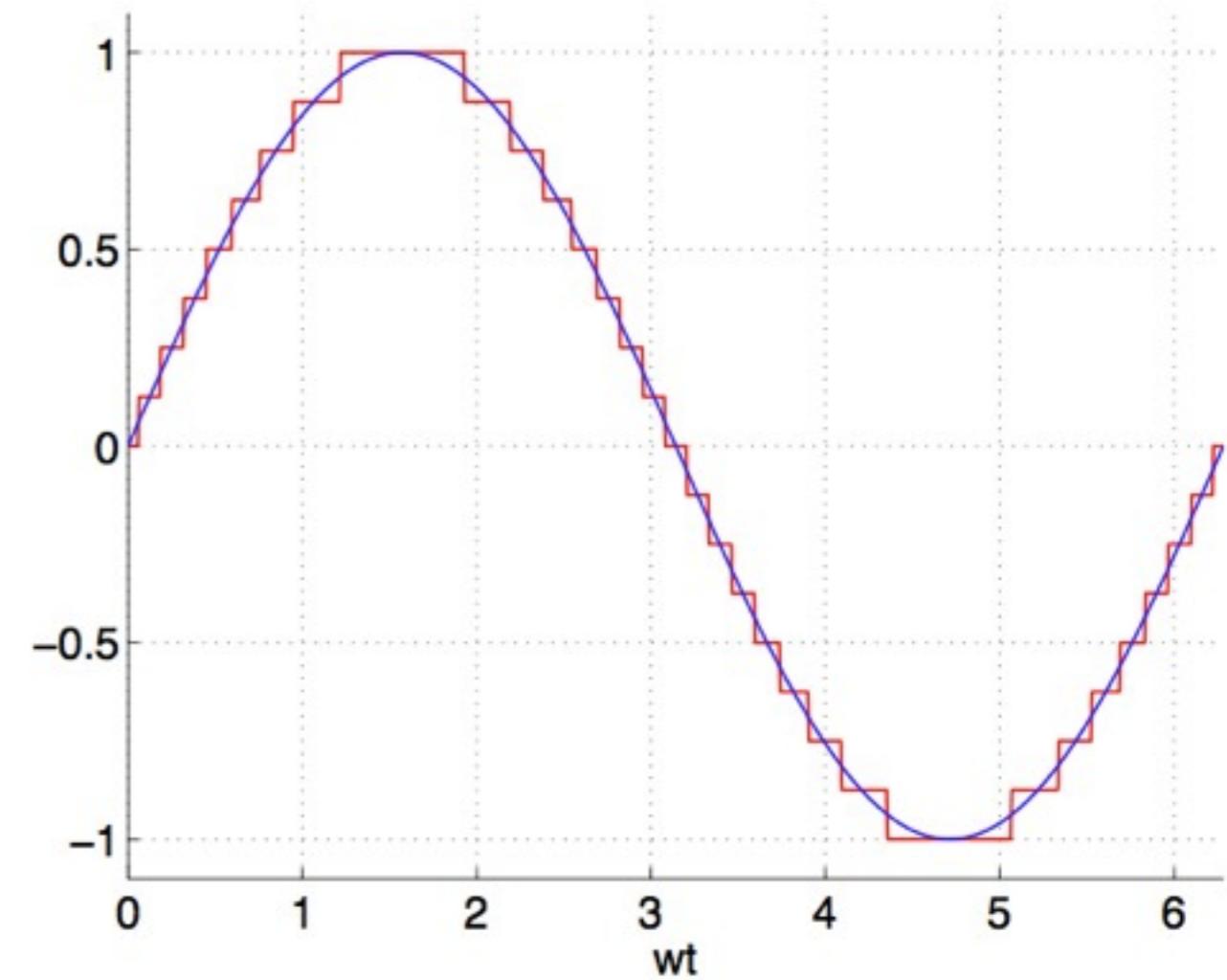
$$2^4 = 16$$

$$\text{LSB} = 2V/16 = 0.125V$$

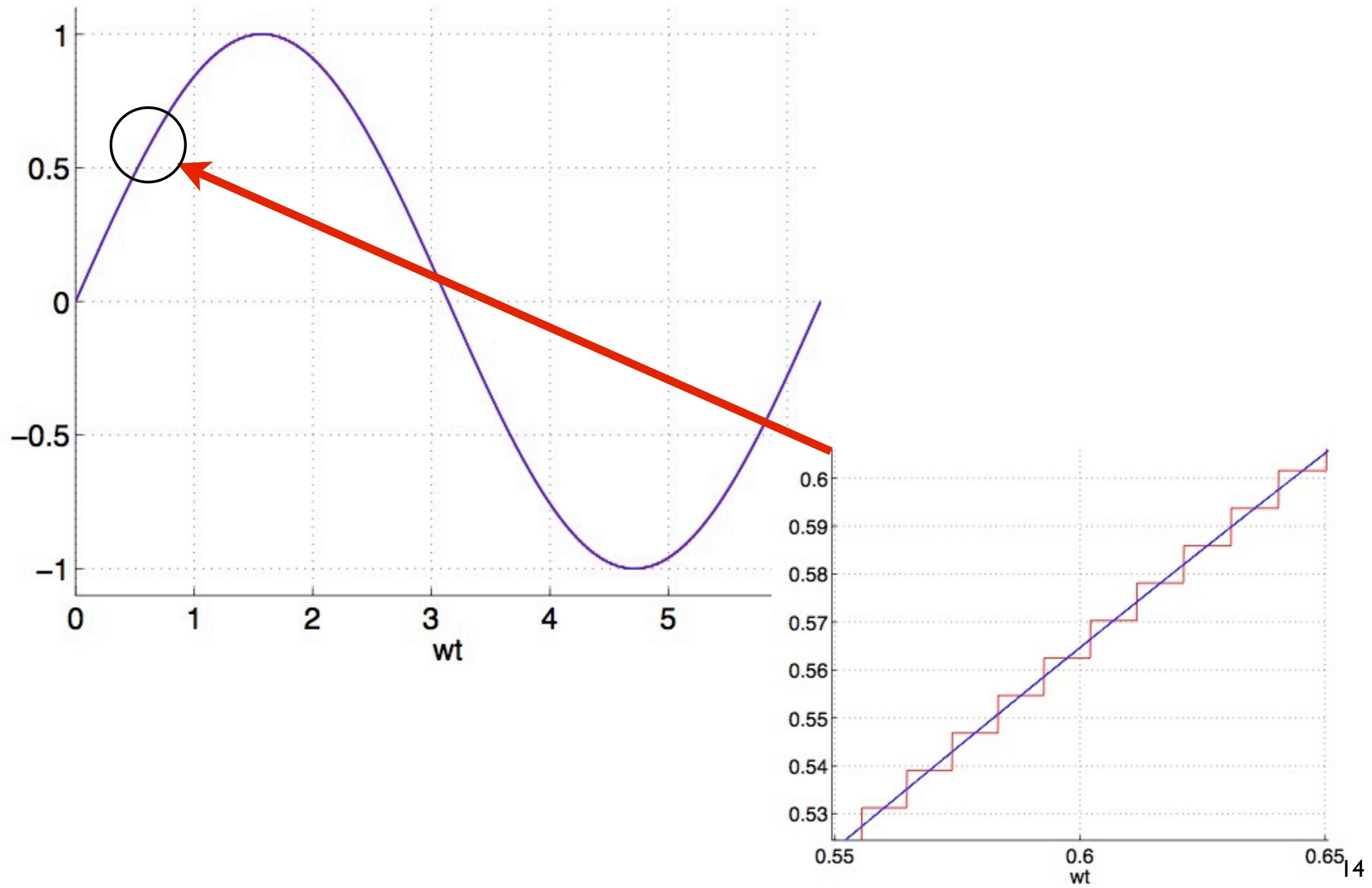
3 bits ADC



4 bits ADC

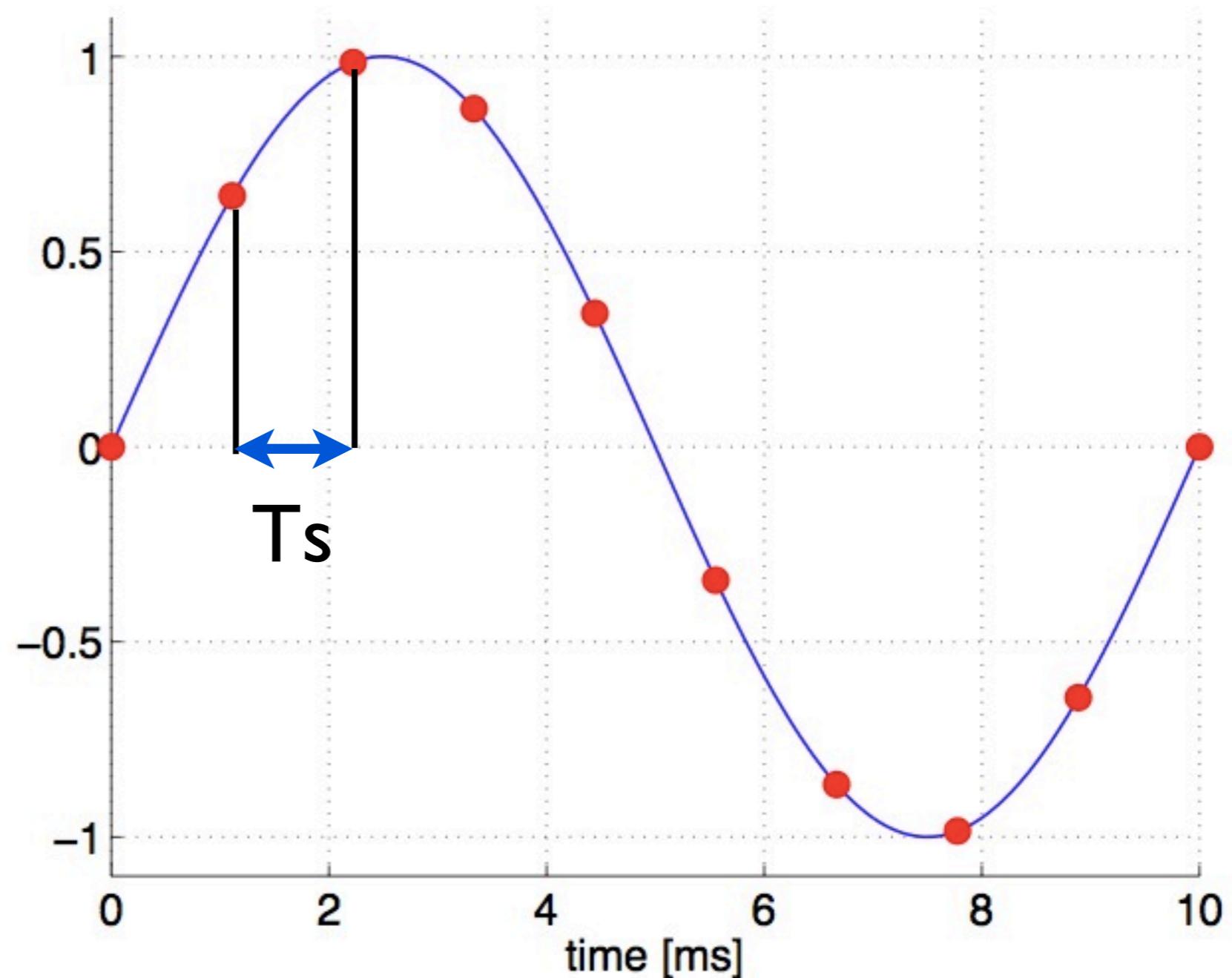


# 8 bits ADC



# What's the difference between analog and digital data?

- 1 - Amplitude
- 2 - Time



Quantization  
on time

# DEFINITIONS

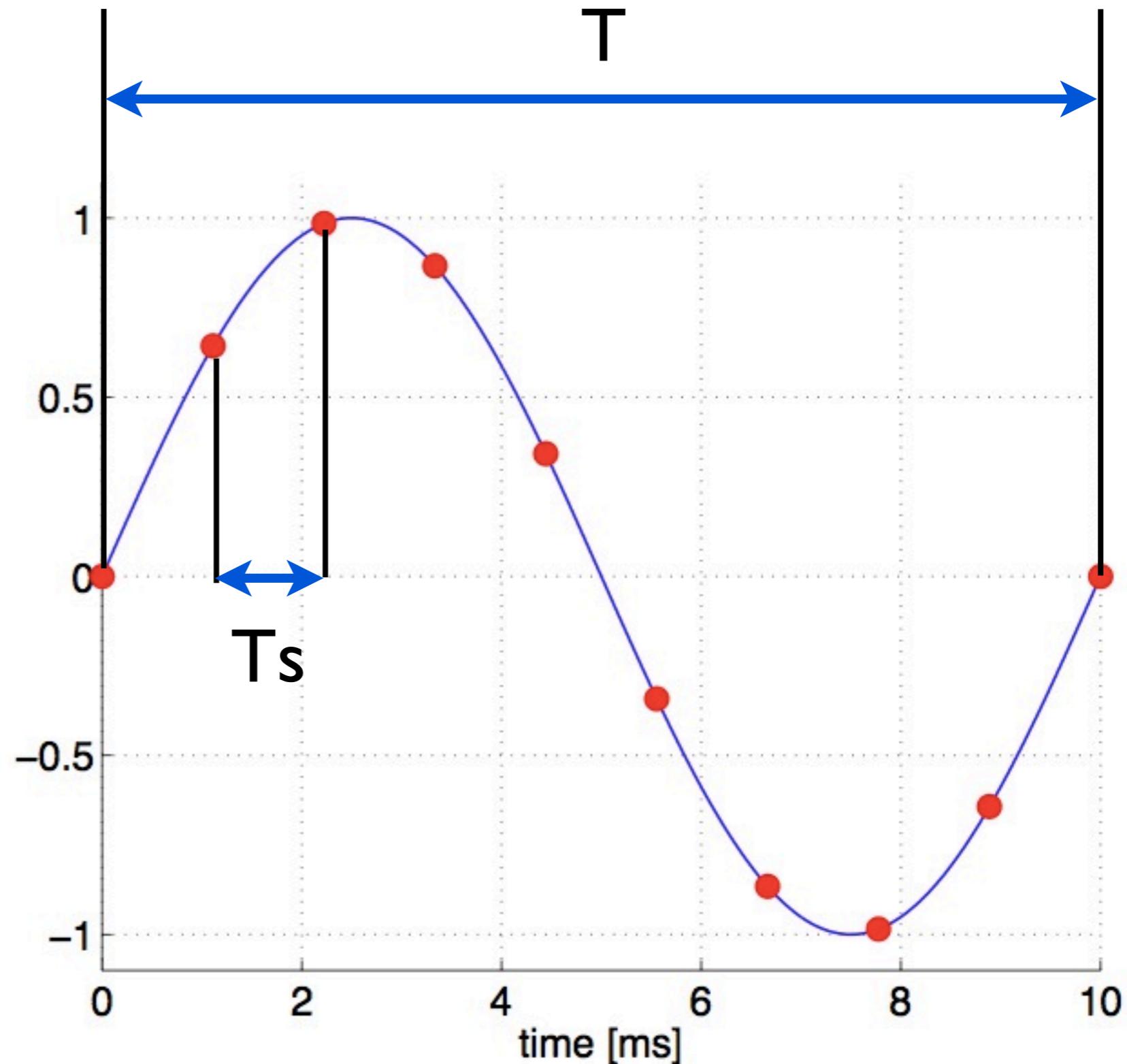
$T = 10 \text{ ms}$   
period of the signal

$f = 100 \text{ Hz}$   
frequency of the signal

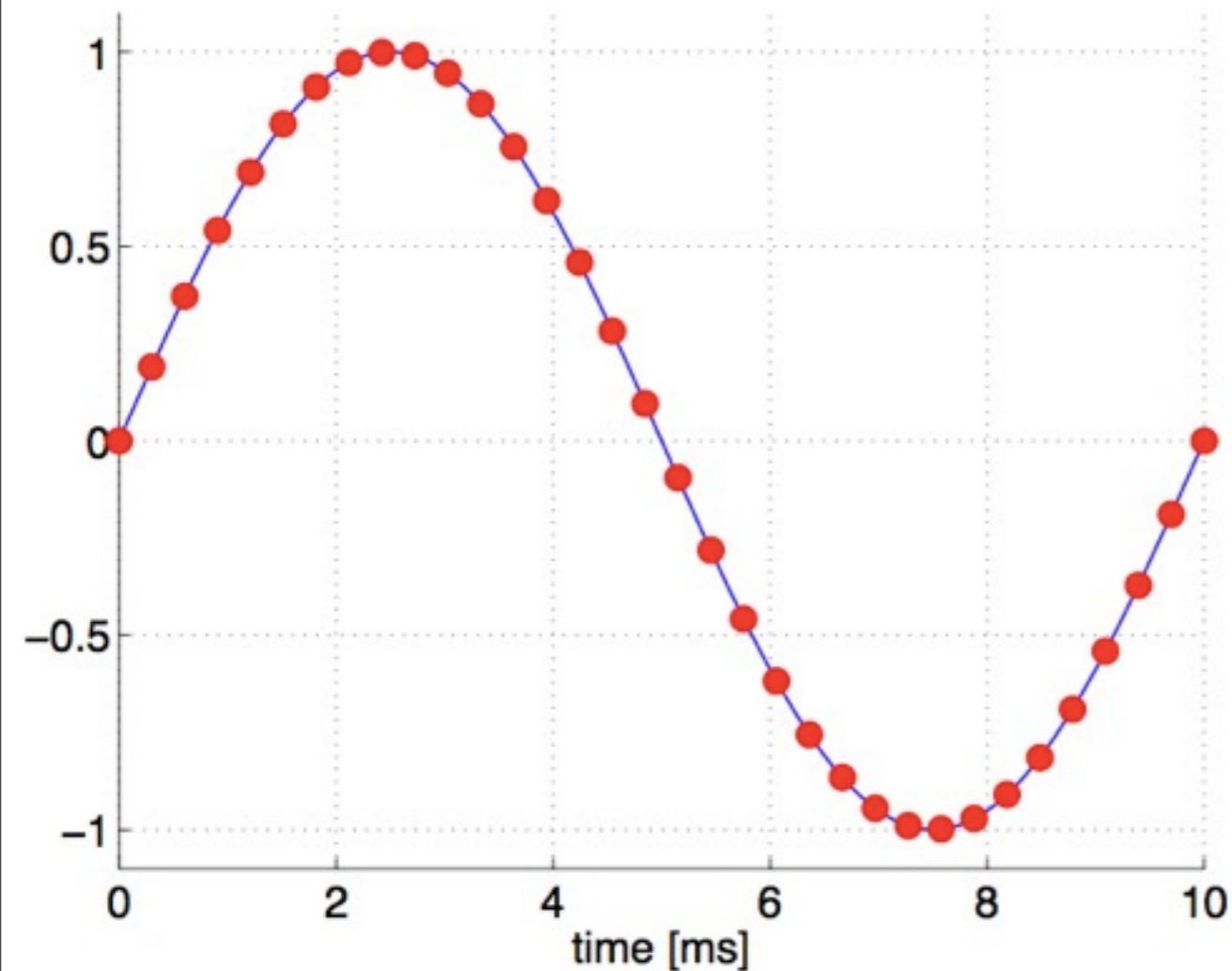
$n = 9$   
number of samples per period

$T_s = T / n = 10 \text{ ms} / 9$   
sampling time

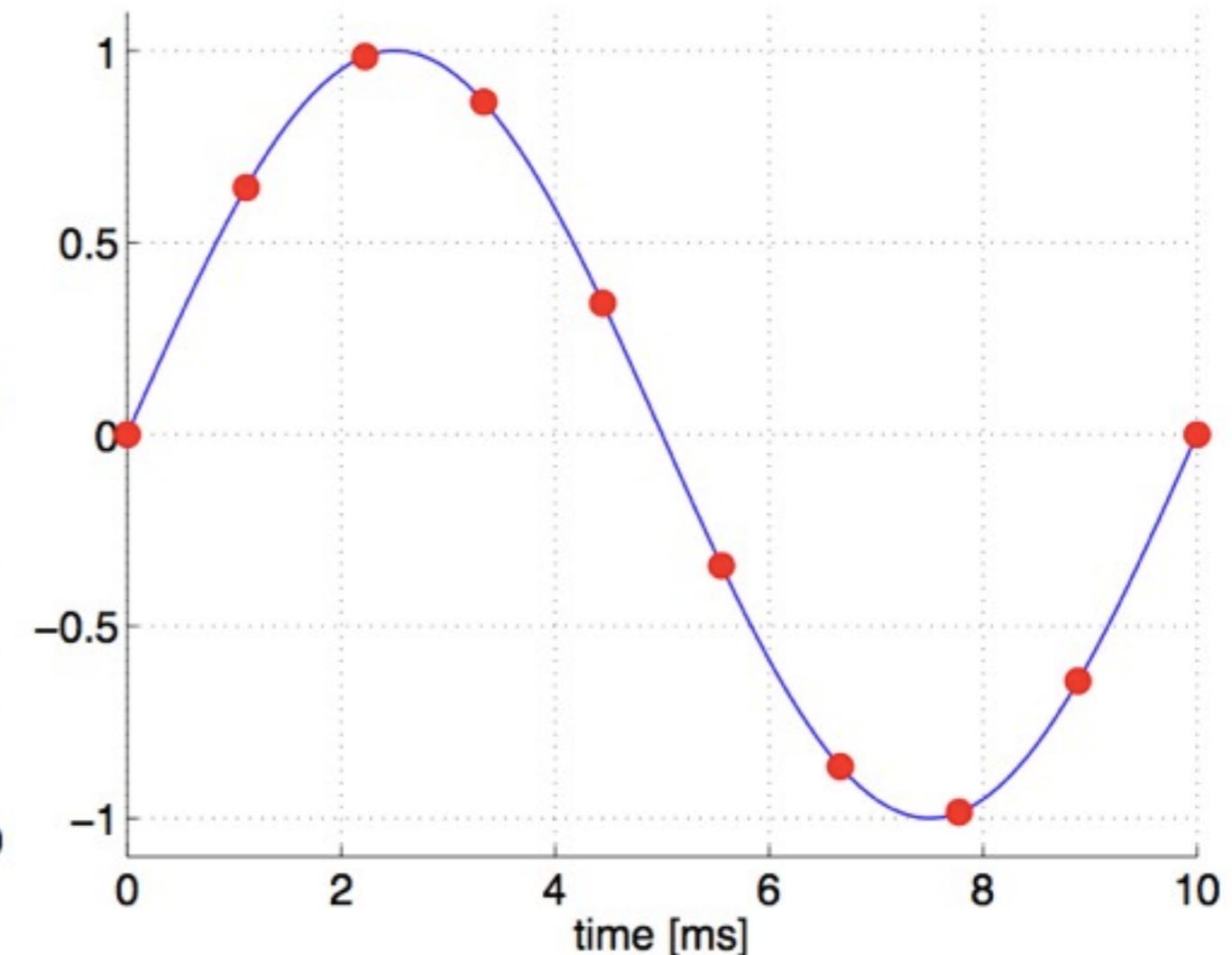
$f_s = 1/T_s$   
sampling frequency



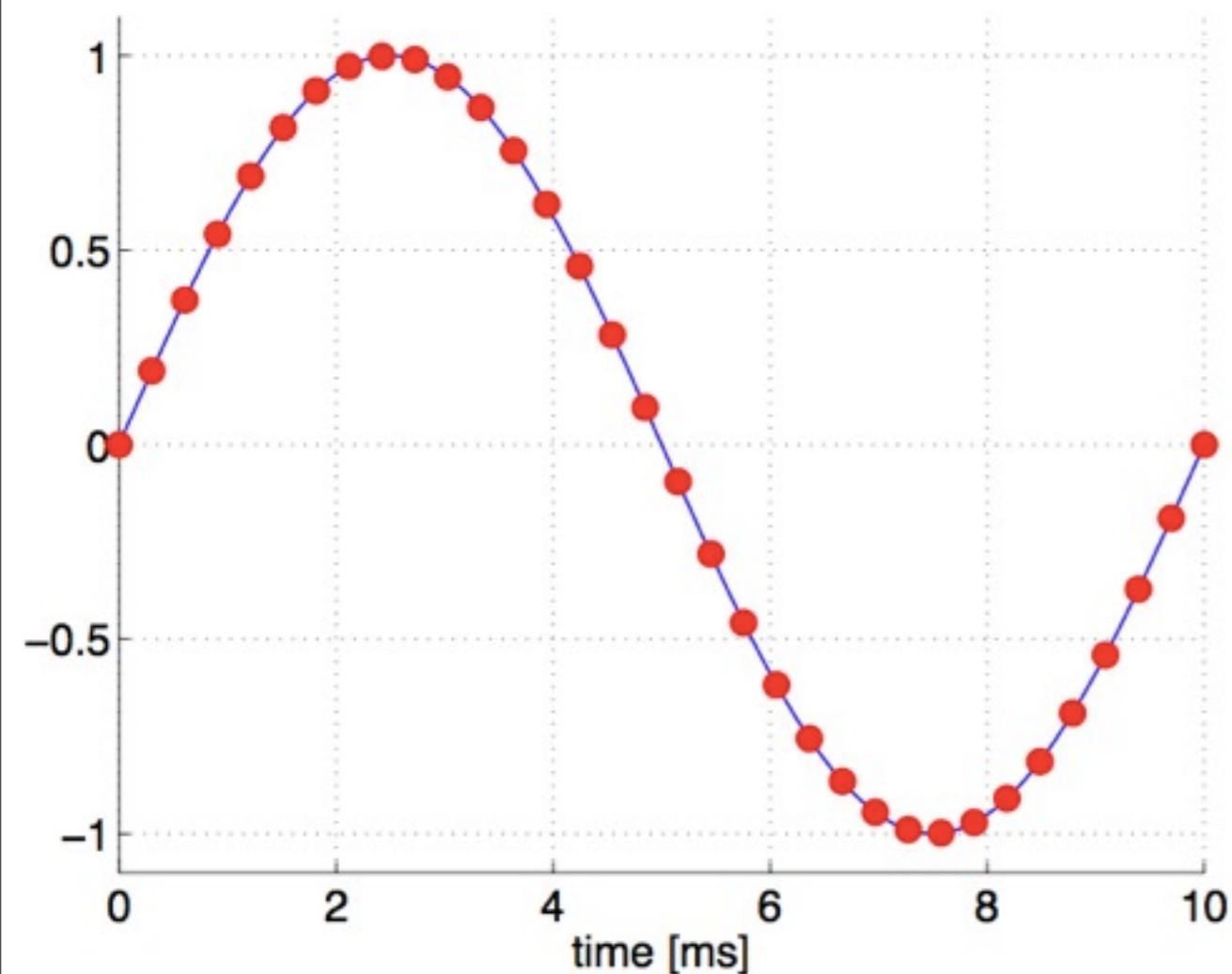
# Which is better?



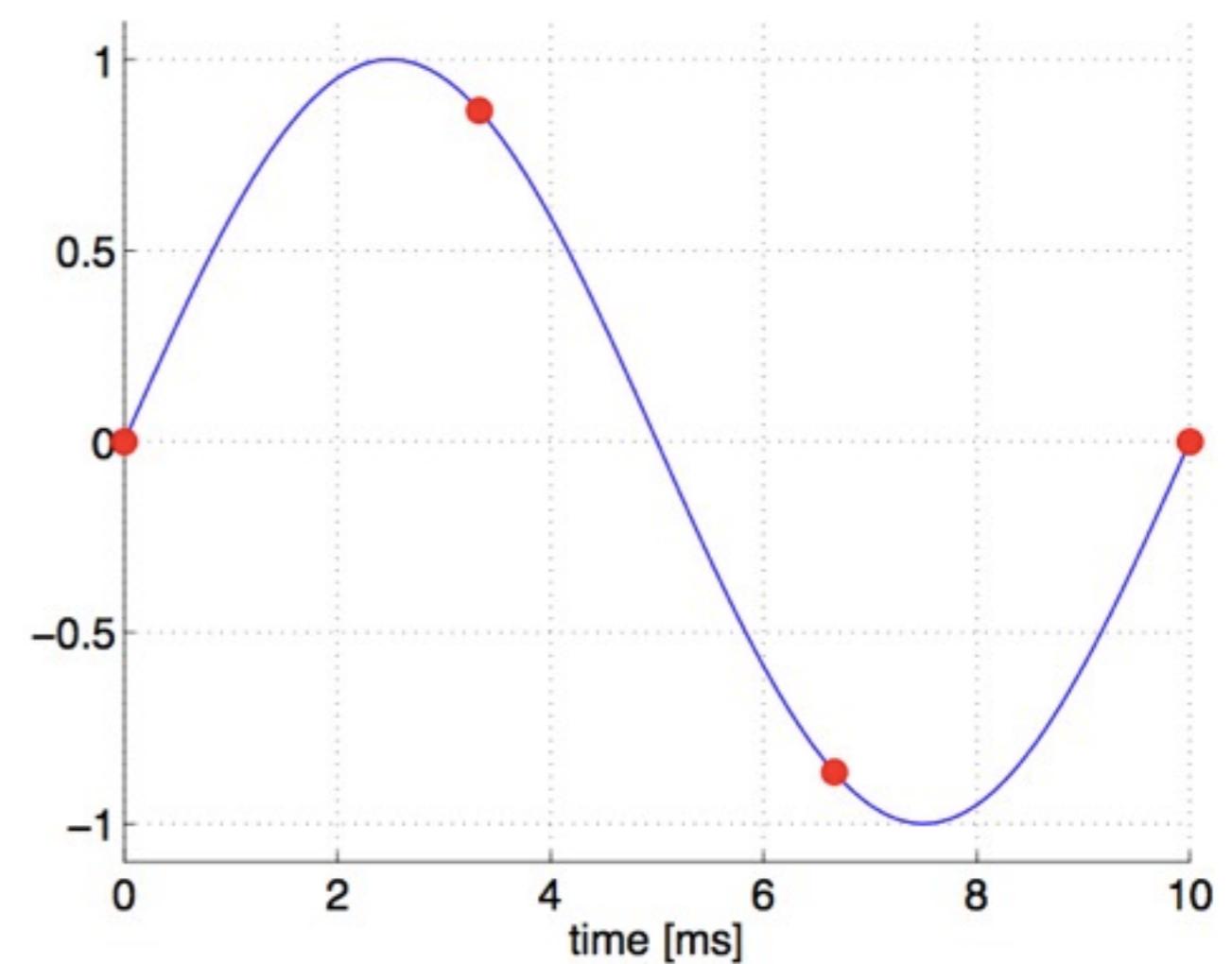
$n=34$



$n=10$



$n=33$



$n=3$

In fact, 3 points are enough...

$$s = A \sin(\omega t + \varphi) + k$$

3 unknowns

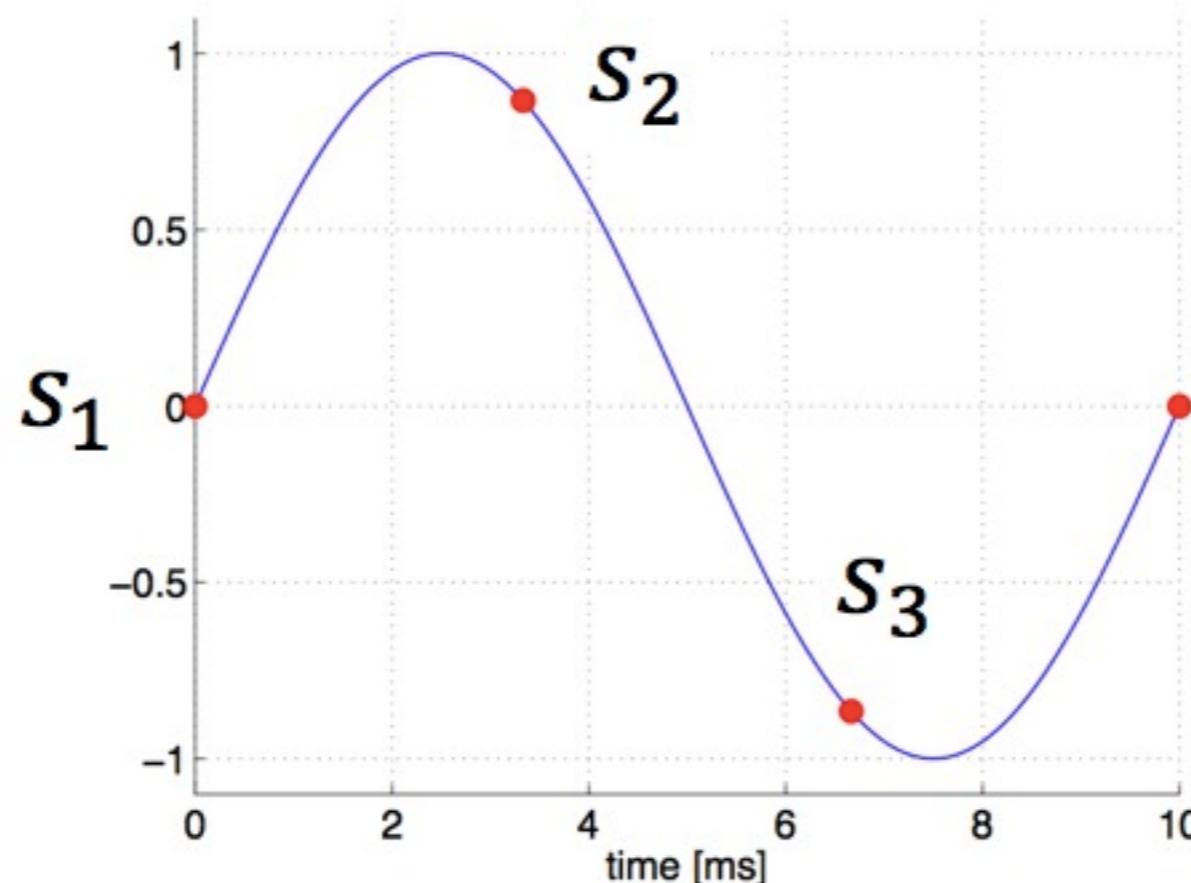
- **A**: amplitude
- **$\varphi$** : phase
- **k**: offset

3 equations

$$s_1 = s(t_1) = A \sin(\omega t_1 + \varphi) + k$$

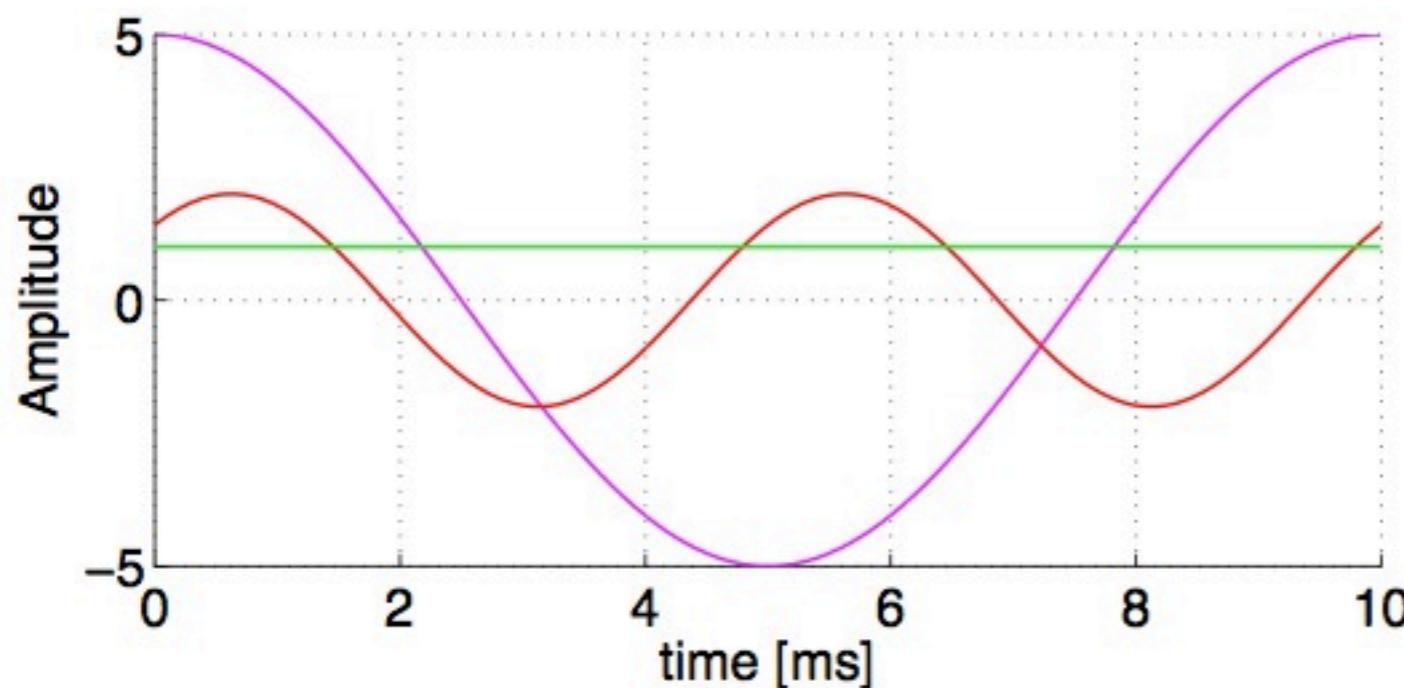
$$s_2 = s(t_2) = A \sin(\omega t_2 + \varphi) + k$$

$$s_3 = s(t_3) = A \sin(\omega t_3 + \varphi) + k$$

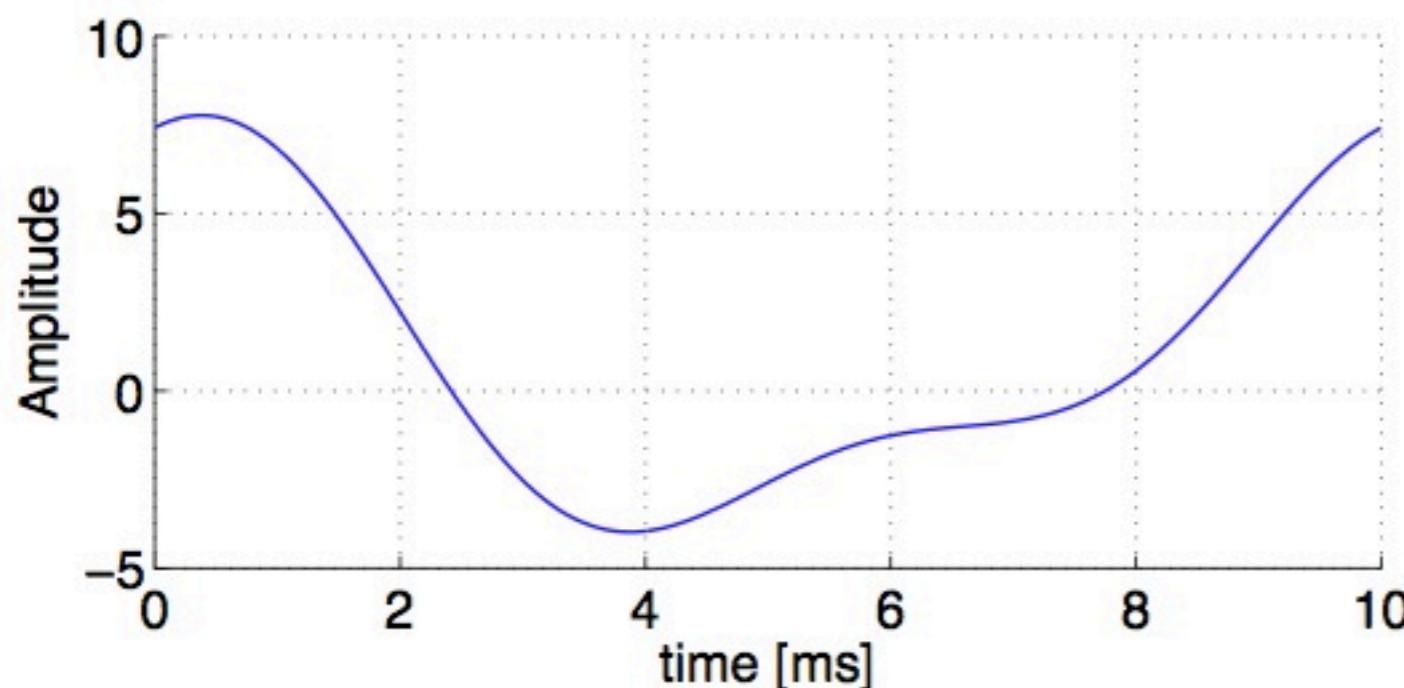


# What if we have multiple harmonics?

$$s = 5 \cos(1\omega t + 0) + 2 \cos\left(2\omega t - \frac{\pi}{2}\right) + 1$$



— first harmonic  
— second harmonic  
— dc value



— total

# What if we have multiple harmonics?

$$s = 5 \cos(1\omega t + 0) + 2 \cos\left(2\omega t - \frac{\pi}{2}\right) + 1$$

$$s = A_1 \cos(1\omega t + \varphi_1) + A_2 \cos(2\omega t + \varphi_2) + k$$

dc value:  $k$

1<sup>st</sup> harmonic:  $A_1 \quad \varphi_1$

2<sup>nd</sup> harmonic:  $A_2 \quad \varphi_2$

.....

N<sup>th</sup> harmonic:  $A_N \quad \varphi_N$

If the signal has  
N harmonics  
we need

n=2N+1 samples

If the signal has  $N$  harmonics we need

$$n=2N+1 \text{ samples per period } T$$

Sampling period:  $T_s = T / (2N+1)$

Sampling frequency:  $f_s = 1 / T_s = (2N+1) / T$

but  $1/T = f$  frequency of the signal

so

$$f_s = (2N+1) f$$

If the signal has  $N$  harmonics we need

$n=2N+1$  samples per period  $T$

$$f_s = (2N+1) f$$

$f_s = 2N f$  is **NOT** enough

because we would get  $2N$  sample, and we need  $2N+1$

Therefore

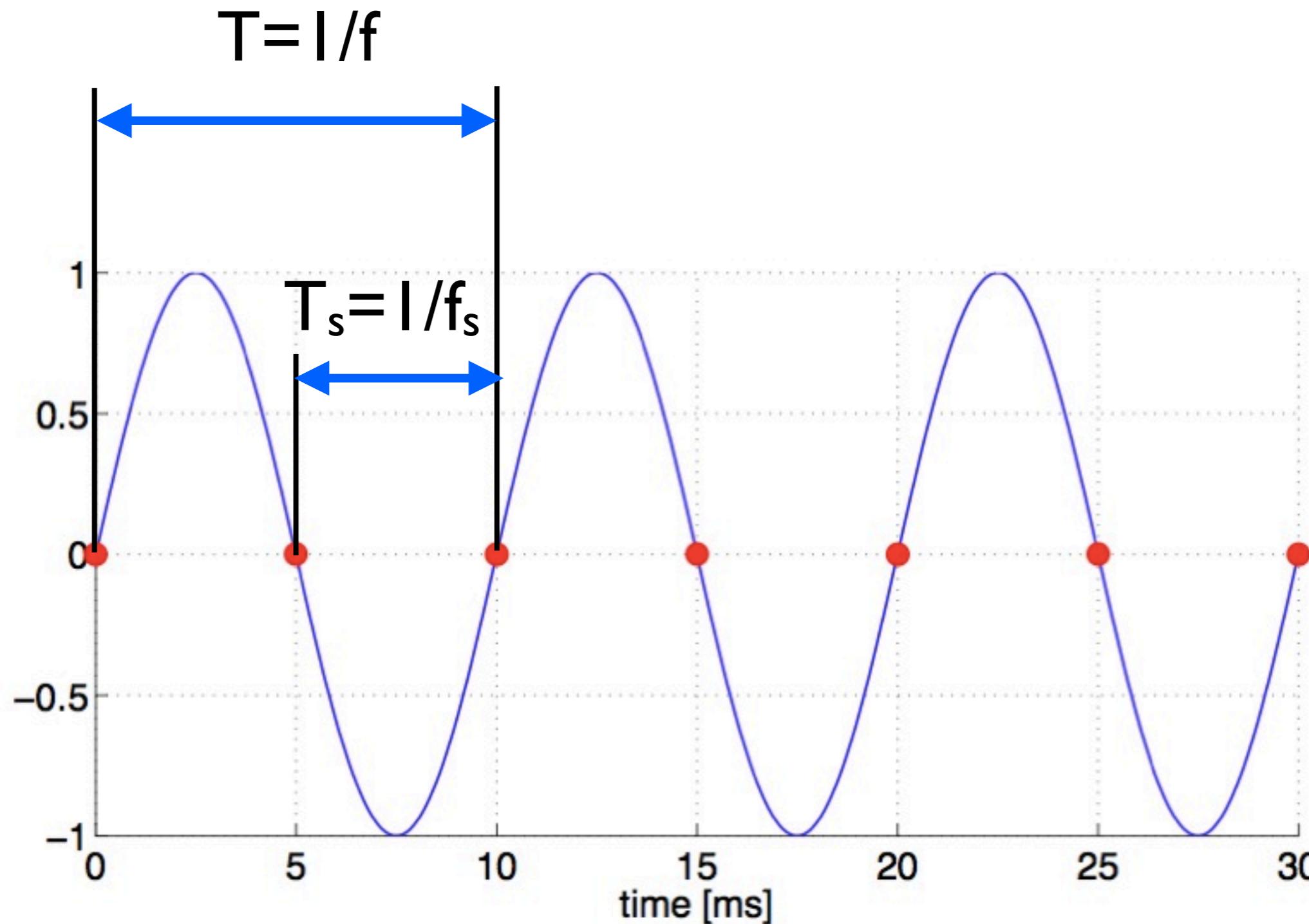
$$f_s > 2N f$$

that is

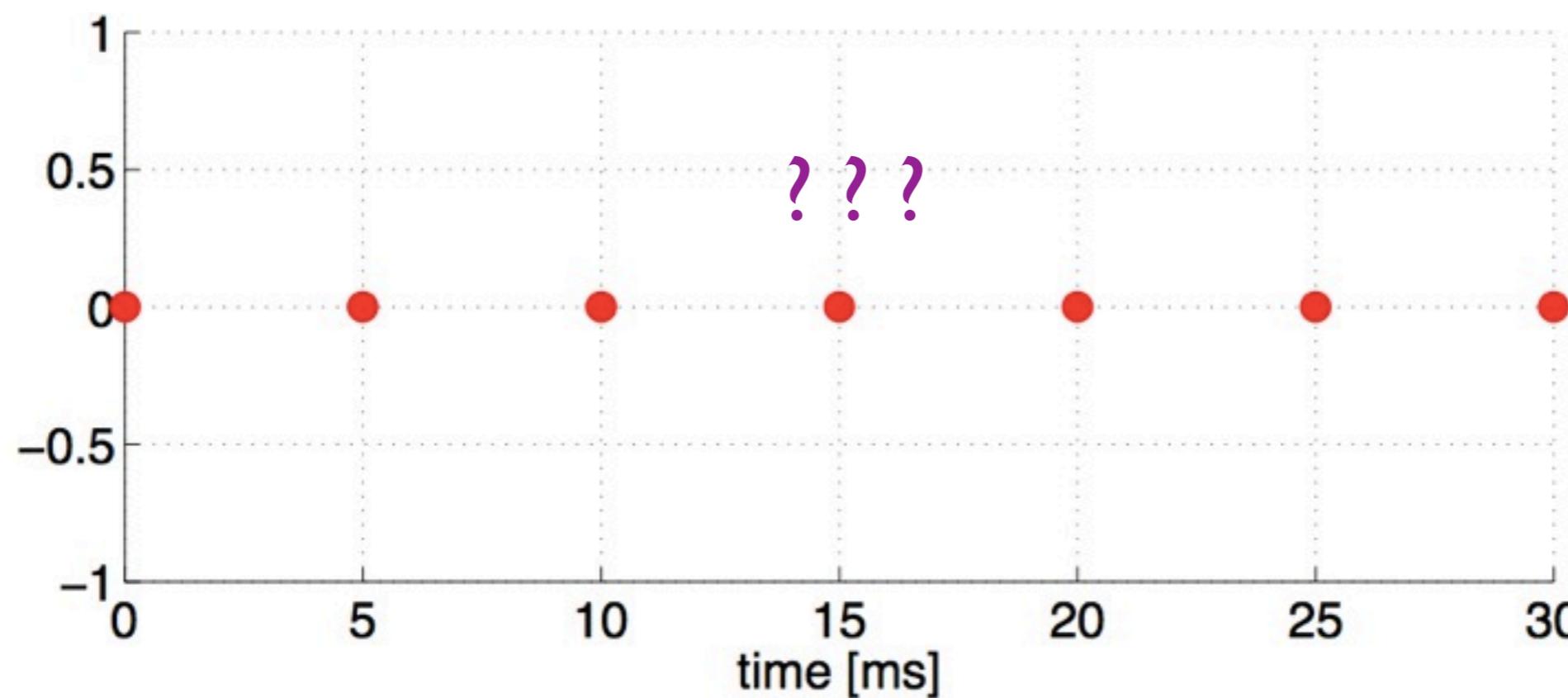
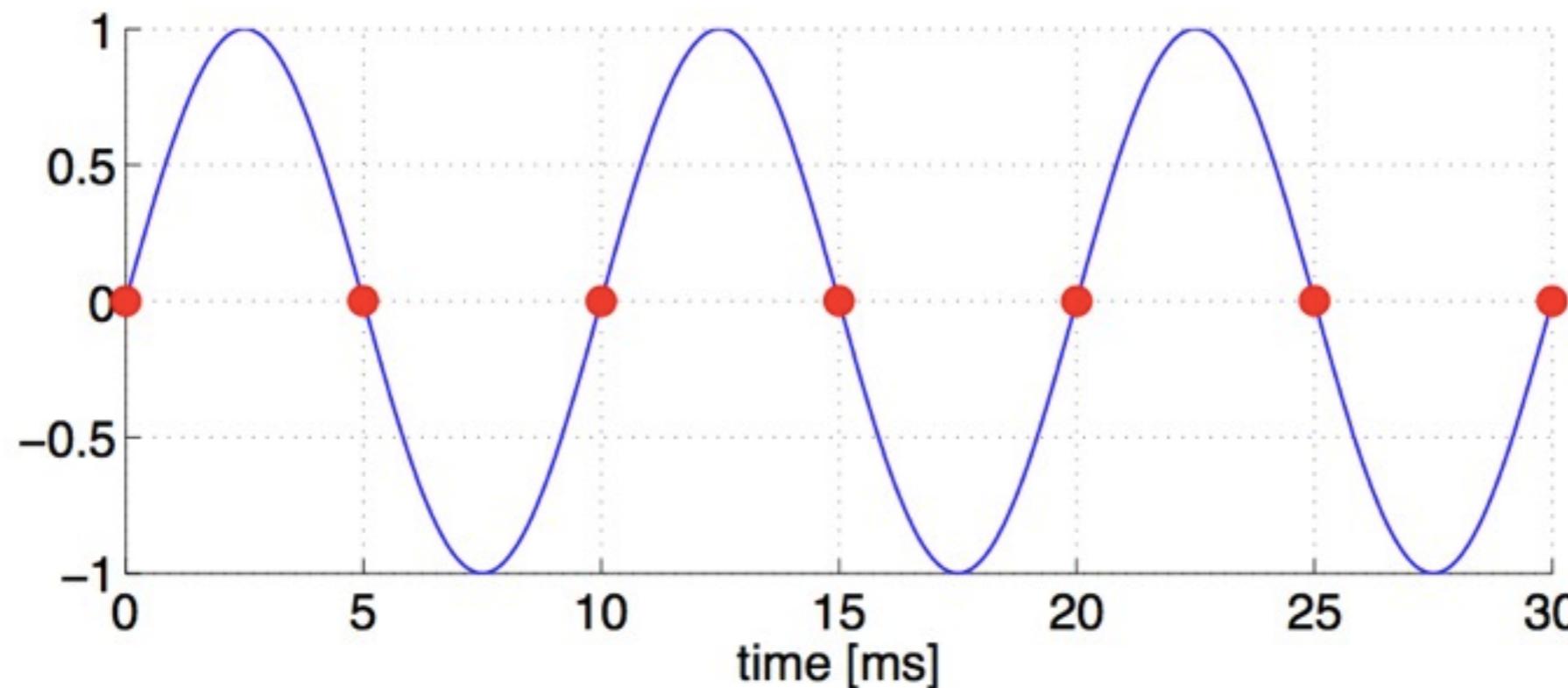
$$f_s > 2 f_N$$

$f_N = N f$  is the maximum harmonic in the signal

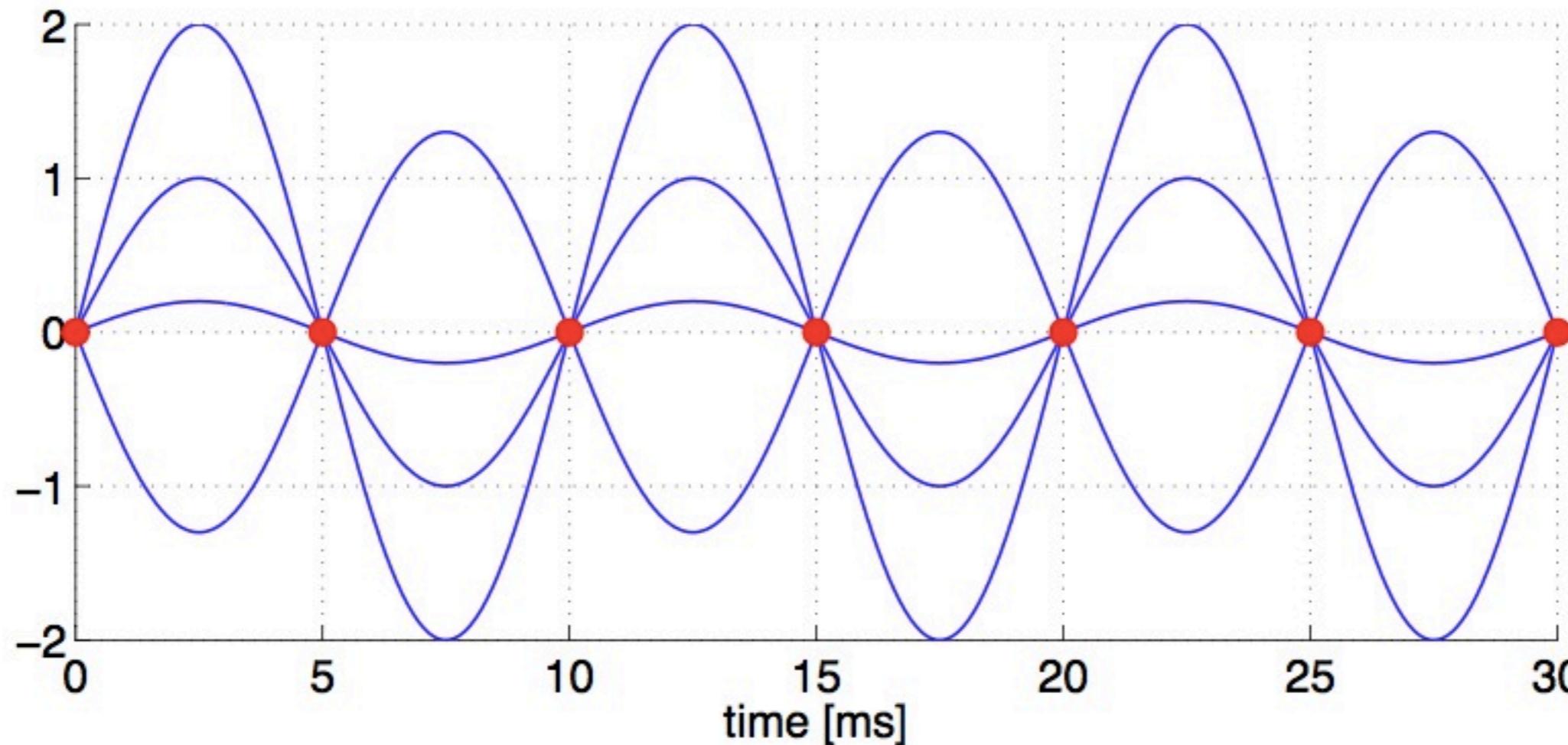
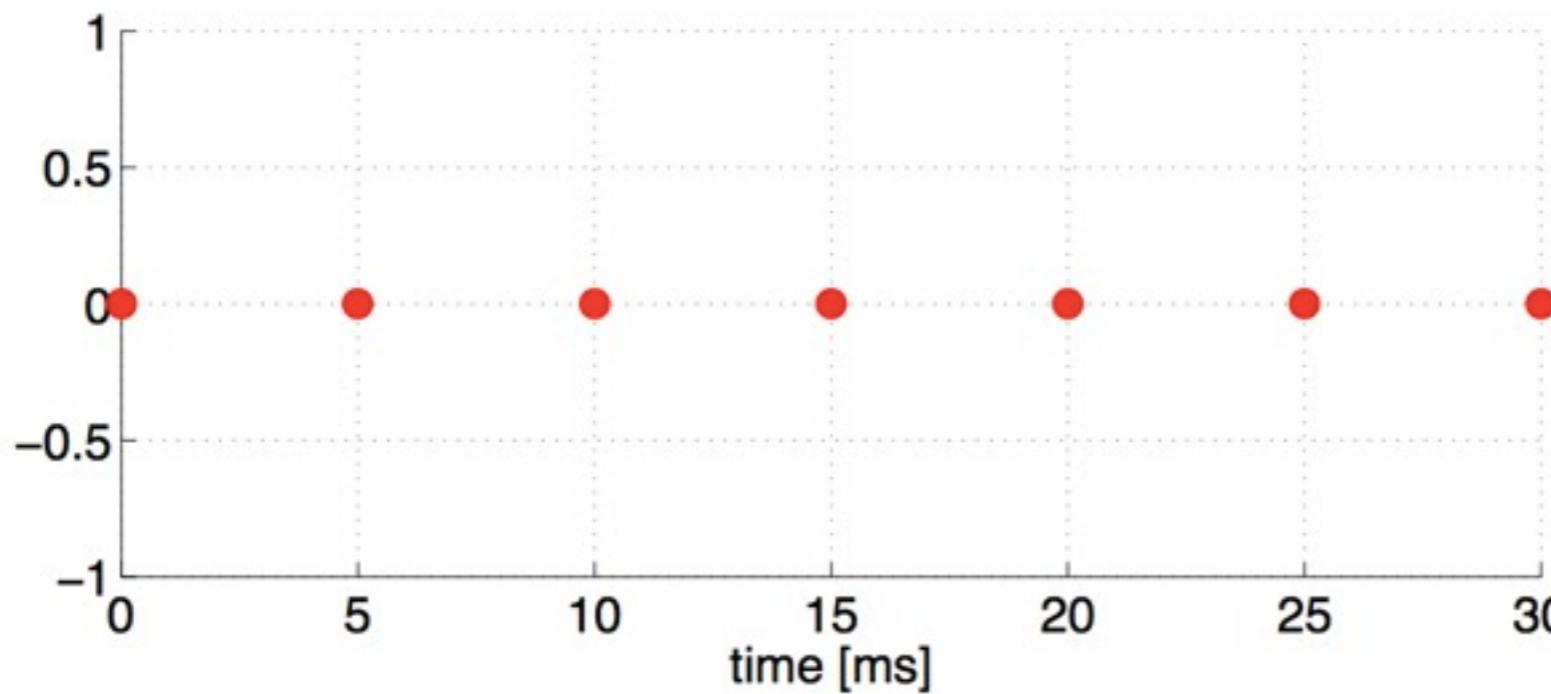
What happens if  $f_s = 2f$



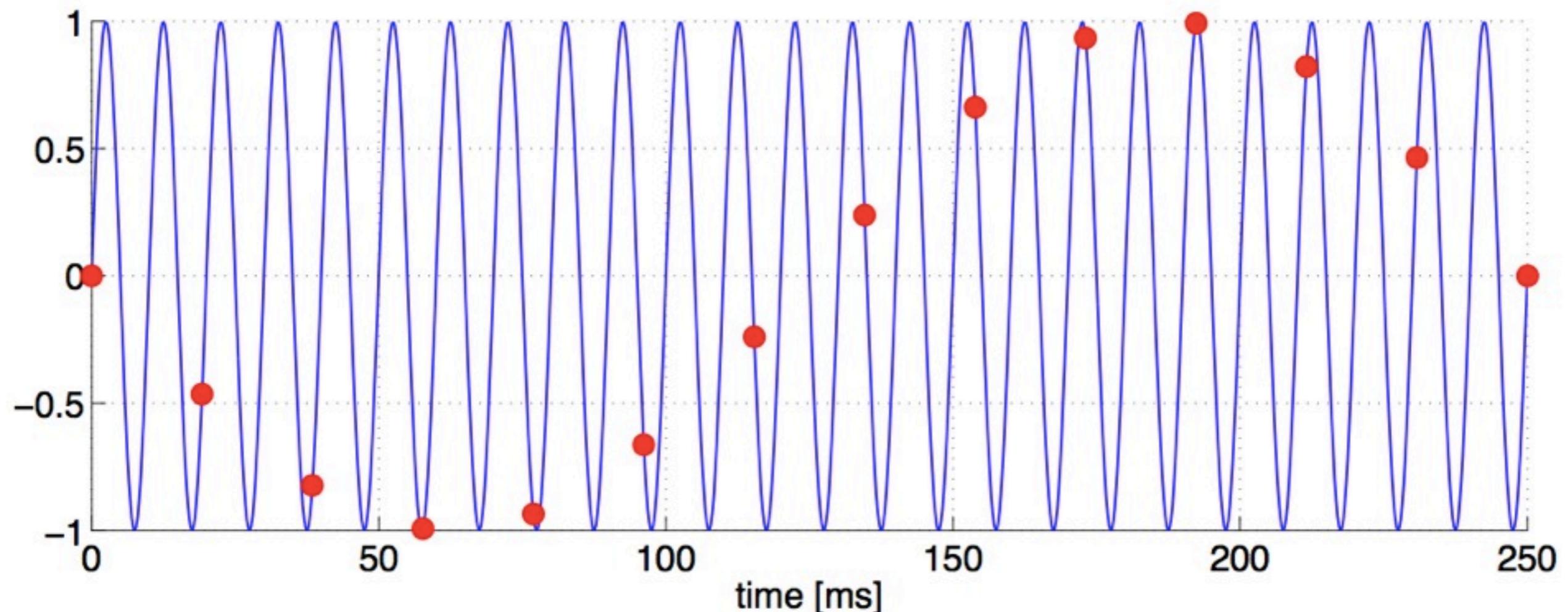
# What happens if $f_s=f$



# What happens if $f_s=f$

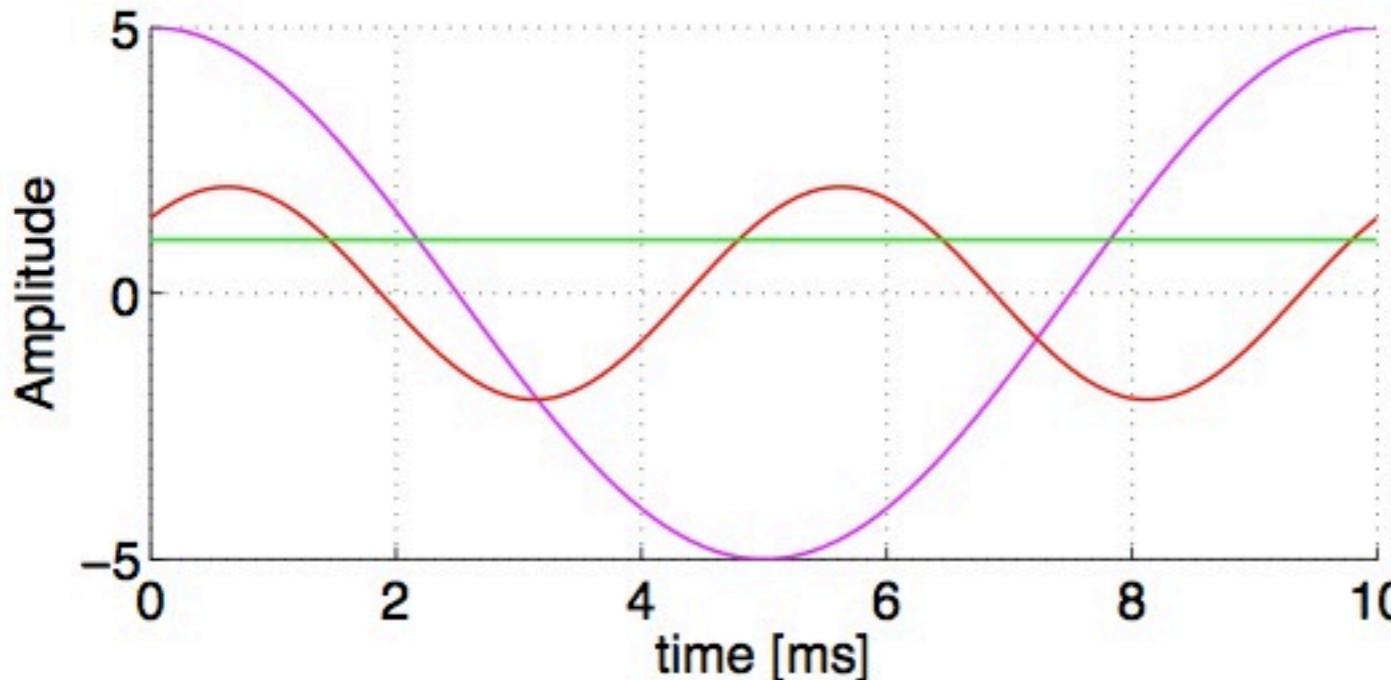


# What might happen if $f_s$ is very low

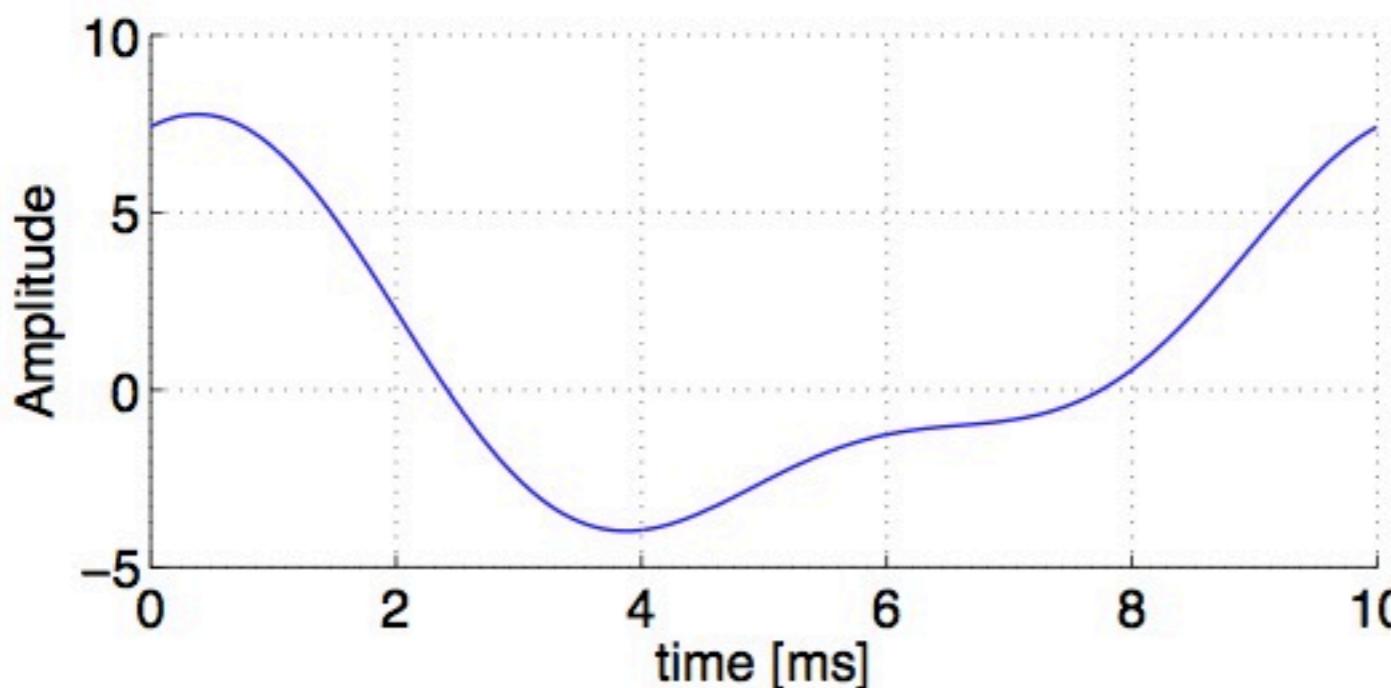


ALIASING

$$s = 5 \cos(1\omega t + 0) + 2 \cos\left(2\omega t - \frac{\pi}{2}\right) + 1$$



- first harmonic
- second harmonic
- dc value

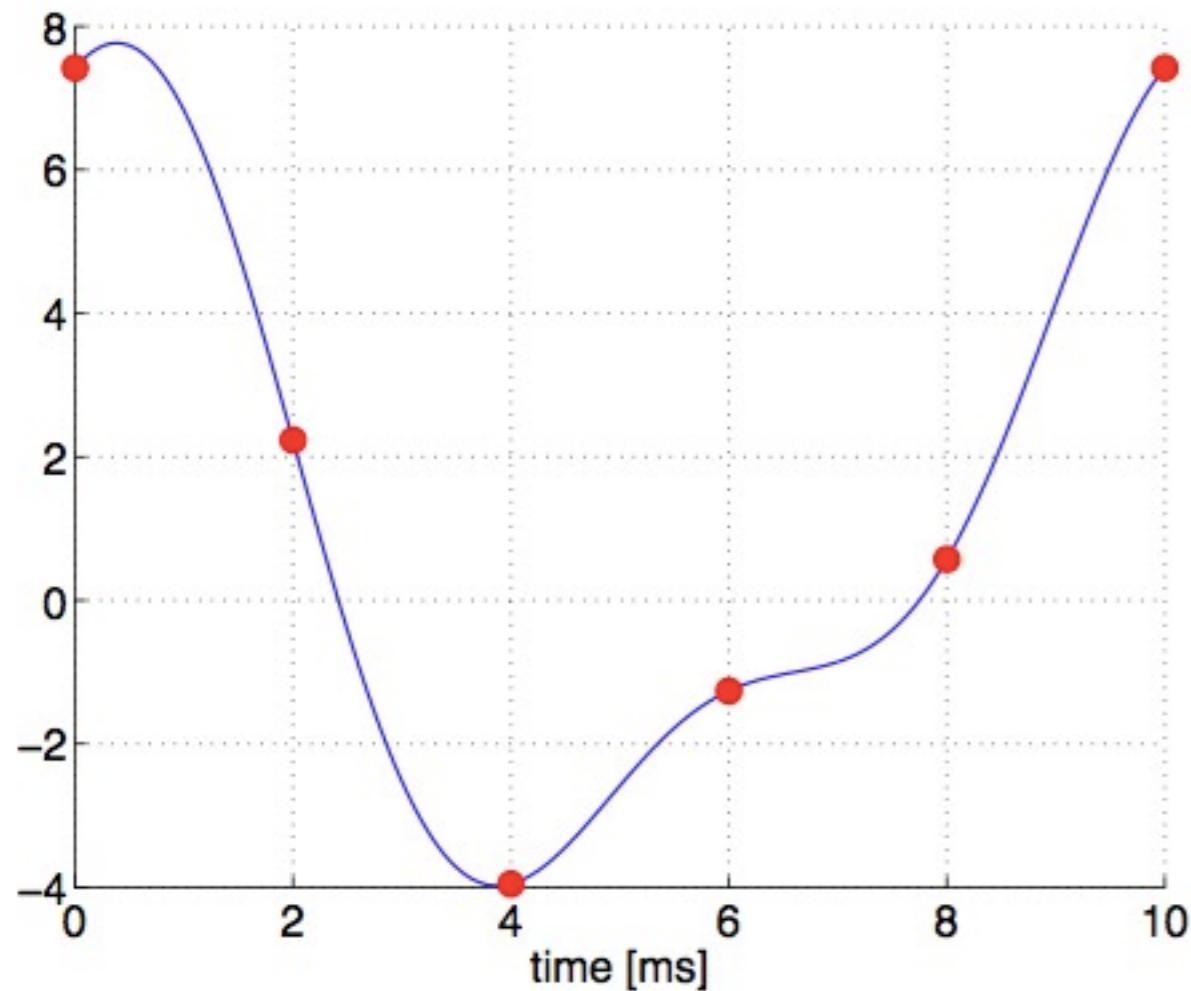


- total

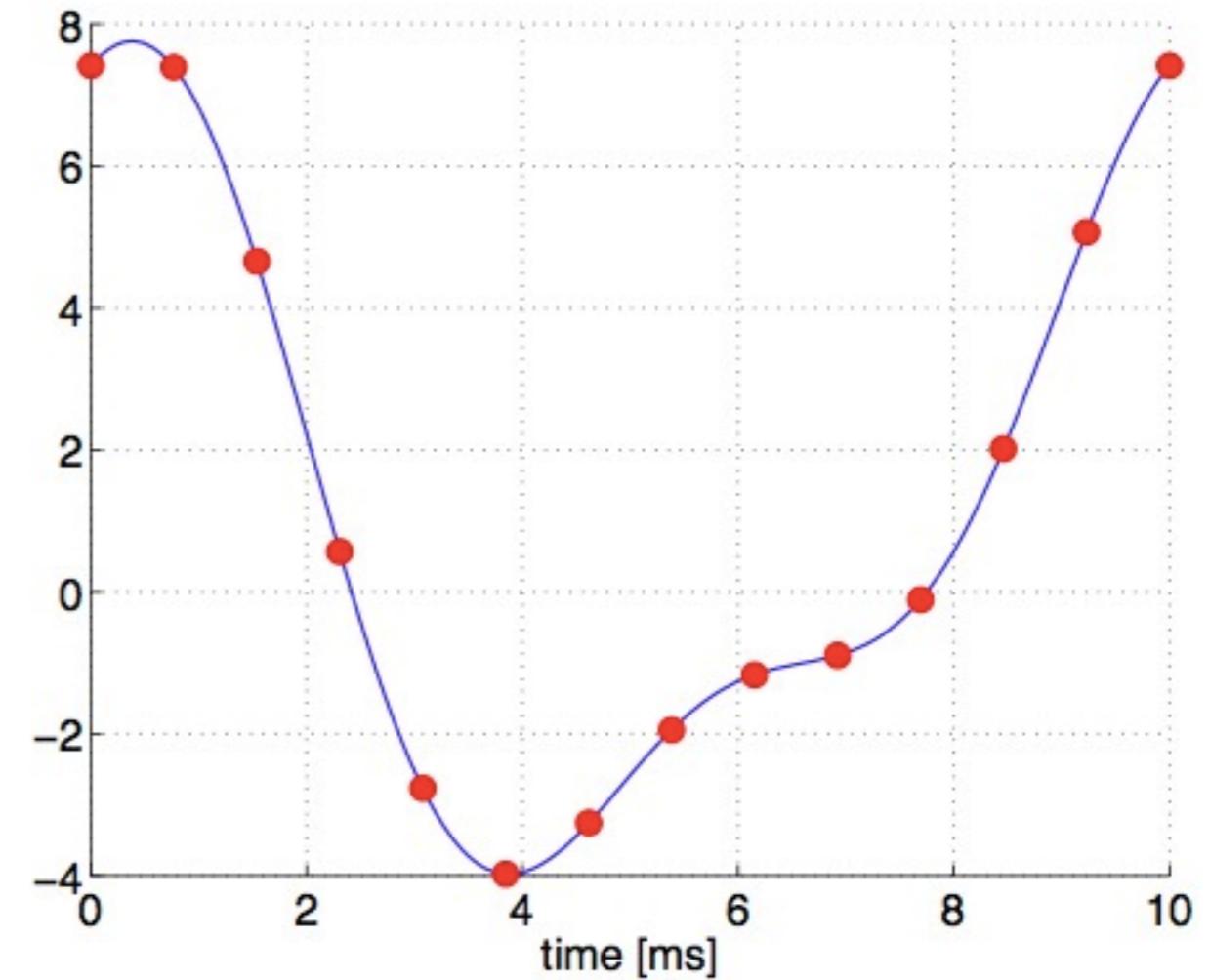
$N=2$   
 $n=2N+1 = 5$

We need at least 5 samples per period

# Oversampling

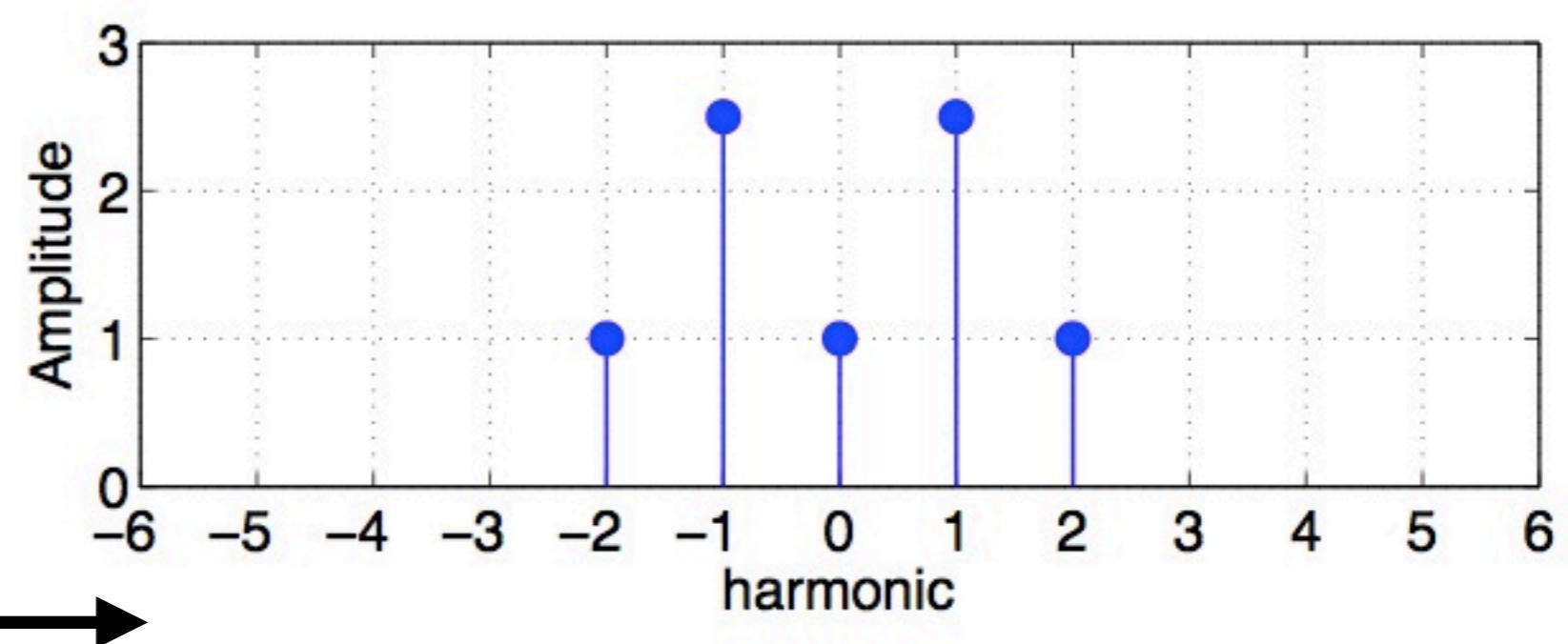
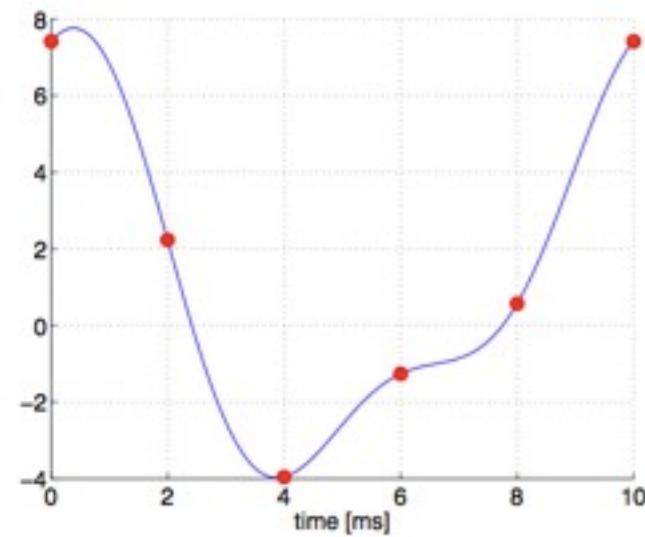


$n=5$

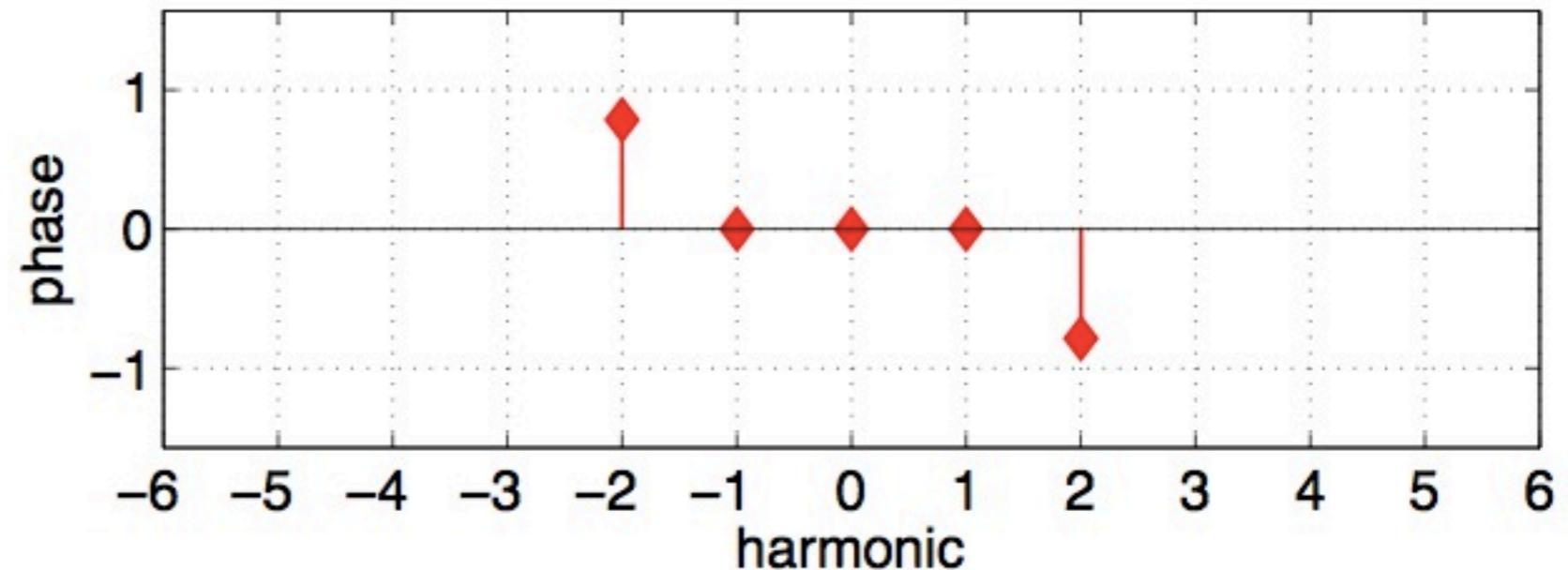


$n=13$

$n=5$

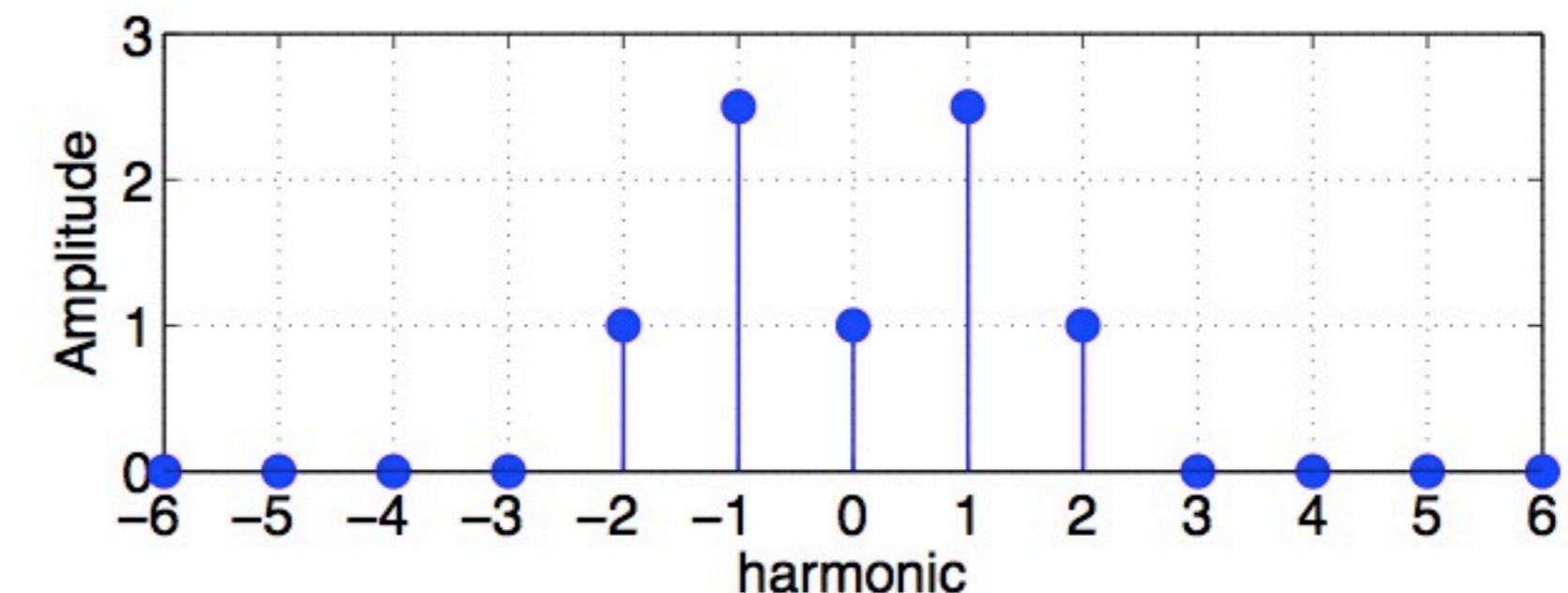
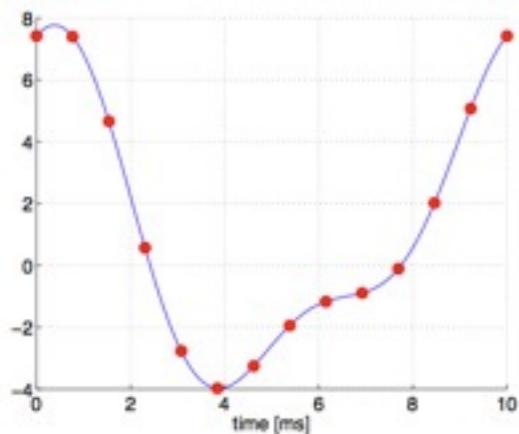


DFT



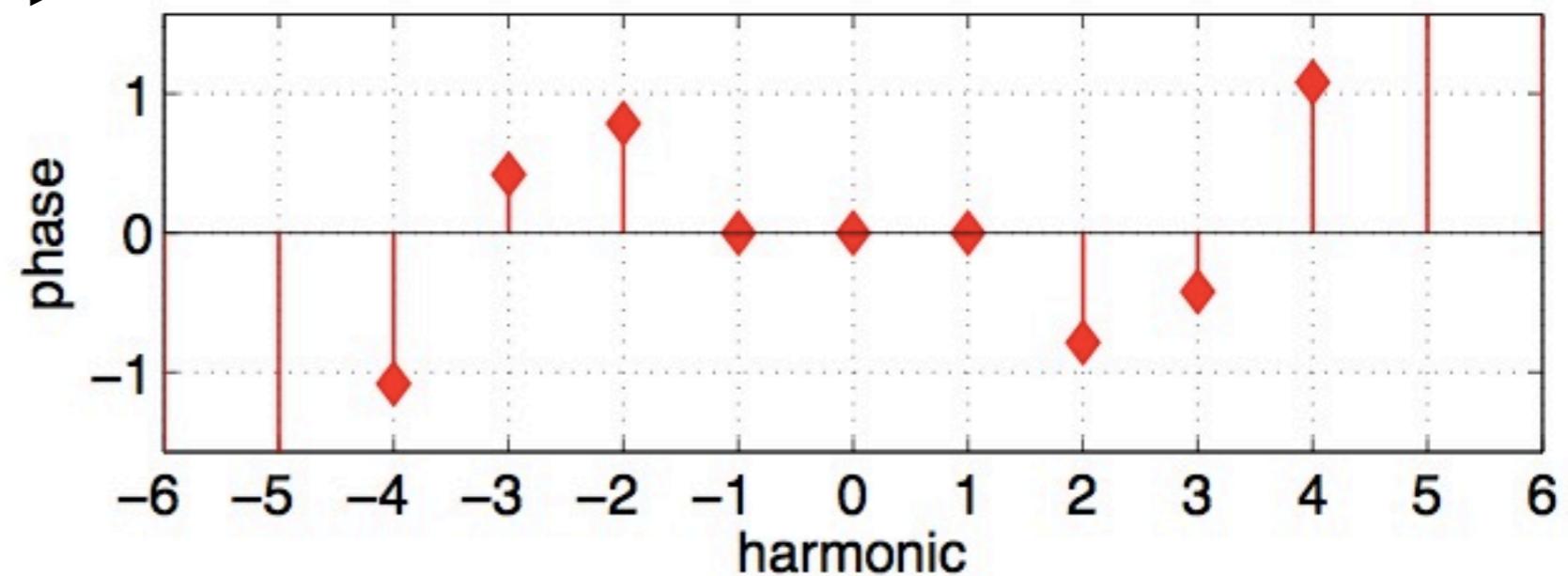
$$s = 5 \cos(1\omega t + 0) + 2 \cos\left(2\omega t - \frac{\pi}{2}\right) + 1$$

$n=13$



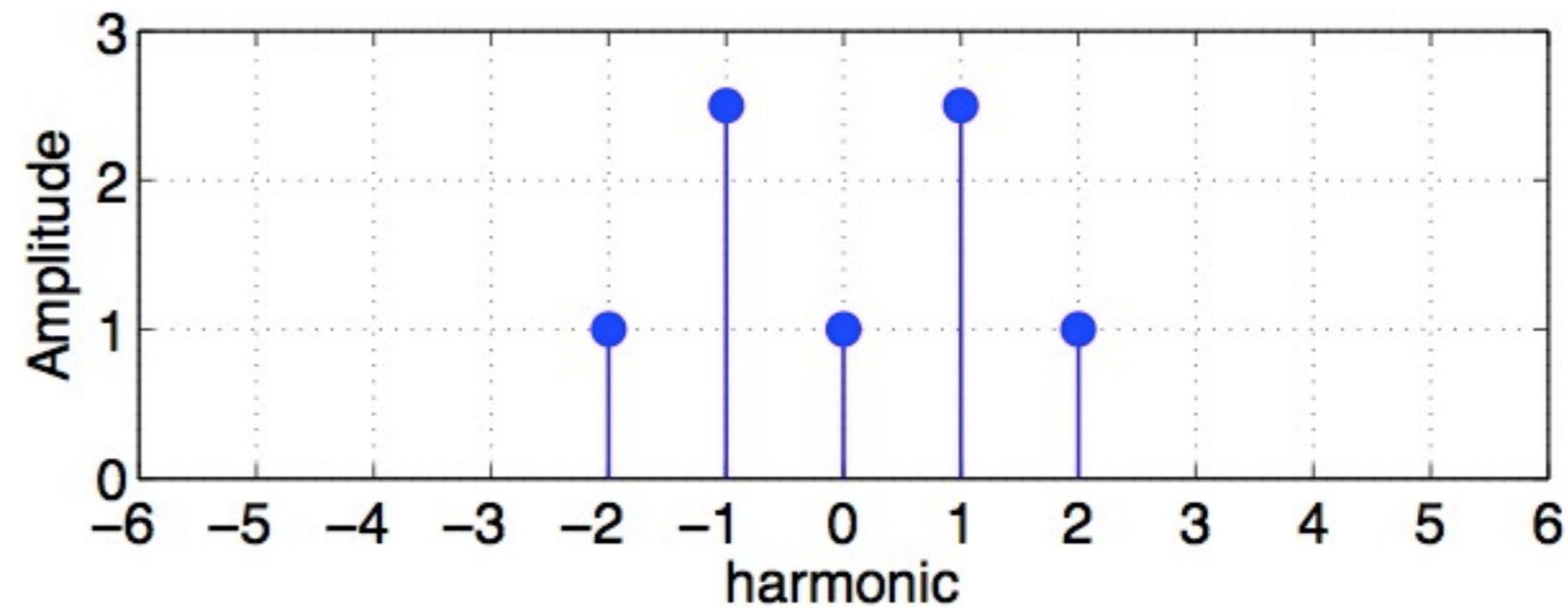
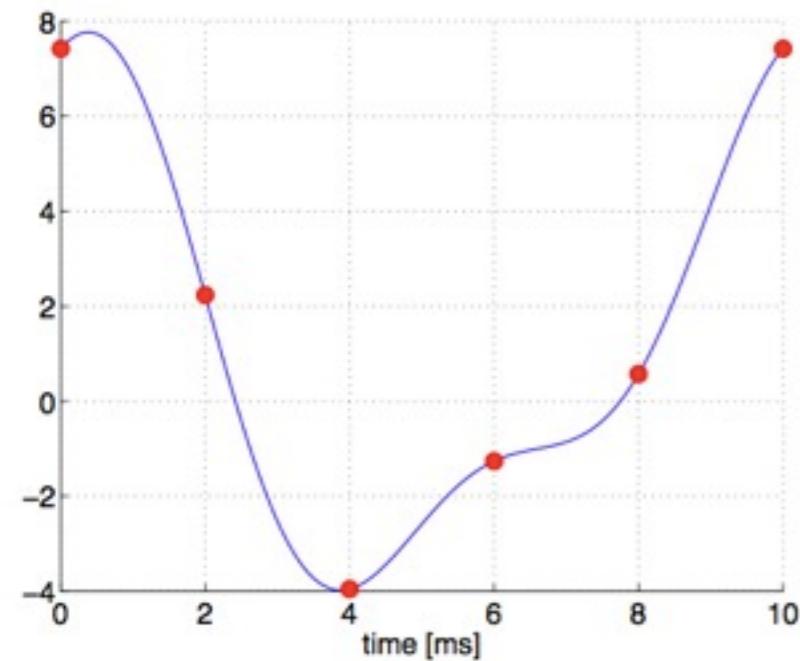
→

DFT

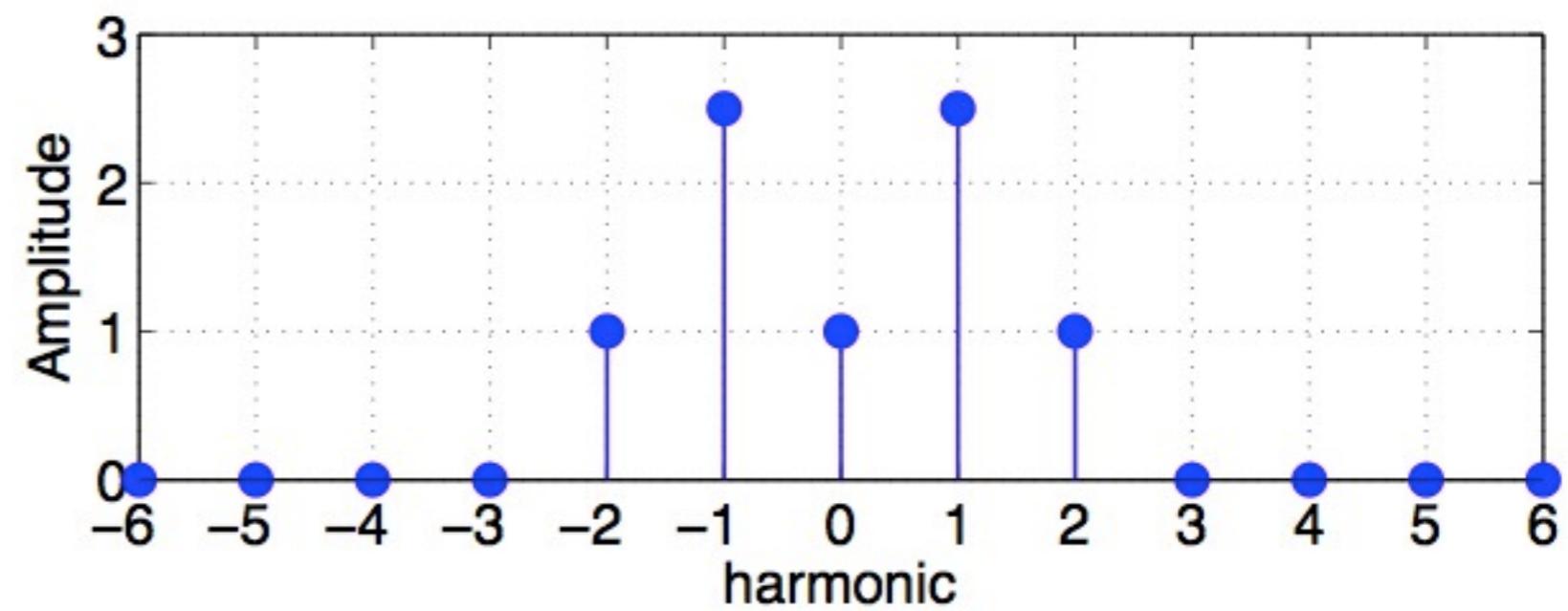
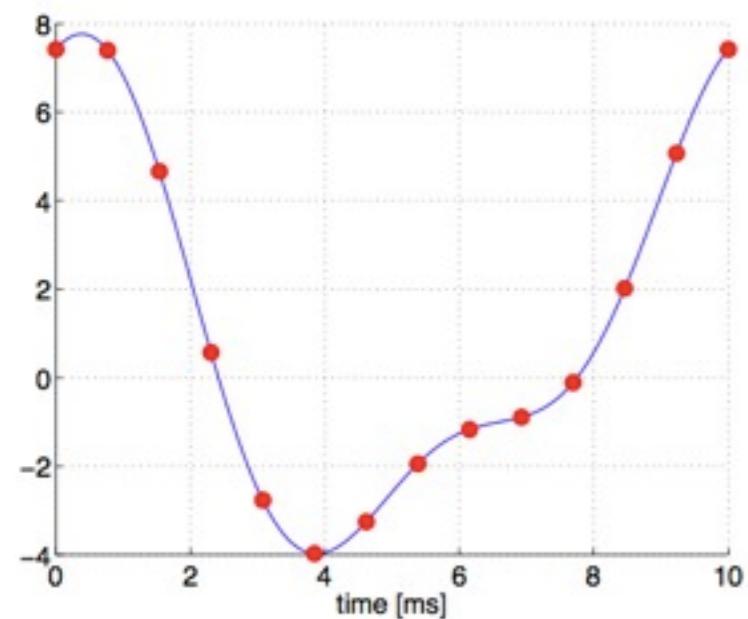


$$s = 5 \cos(1\omega t + 0) + 2 \cos\left(2\omega t - \frac{\pi}{2}\right) + 1$$

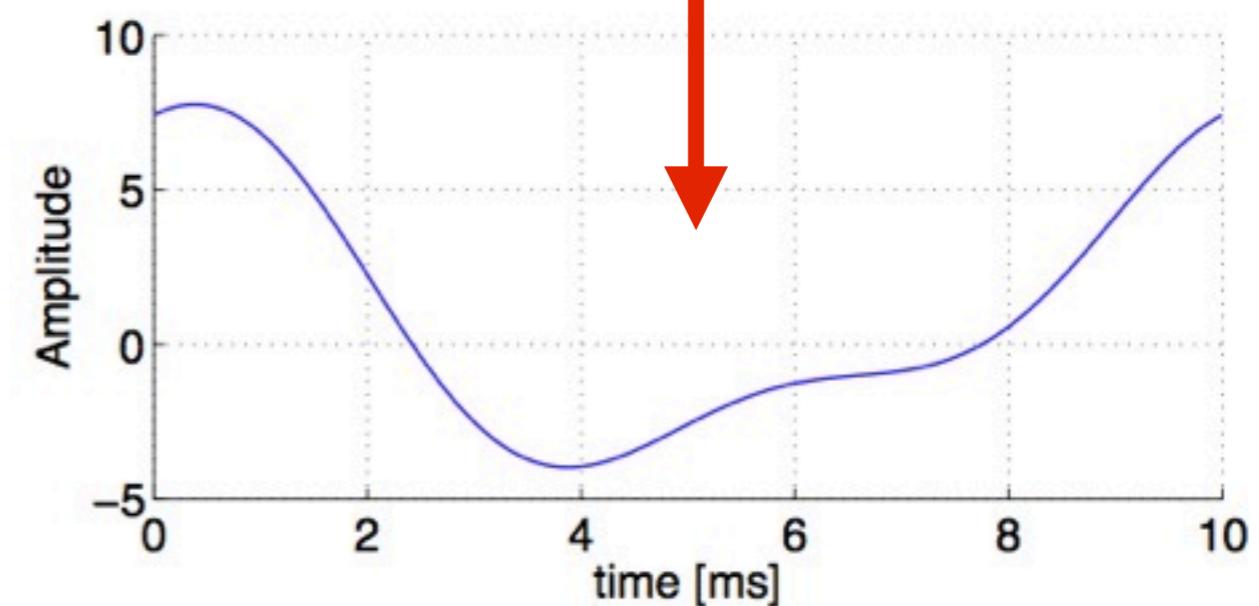
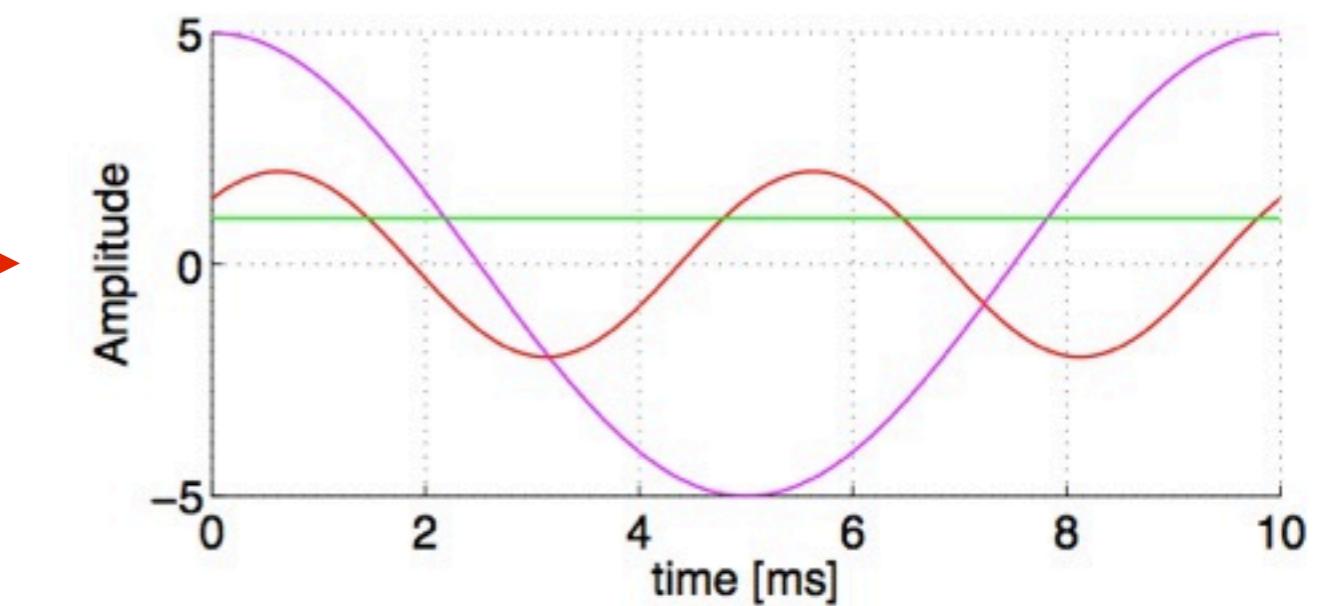
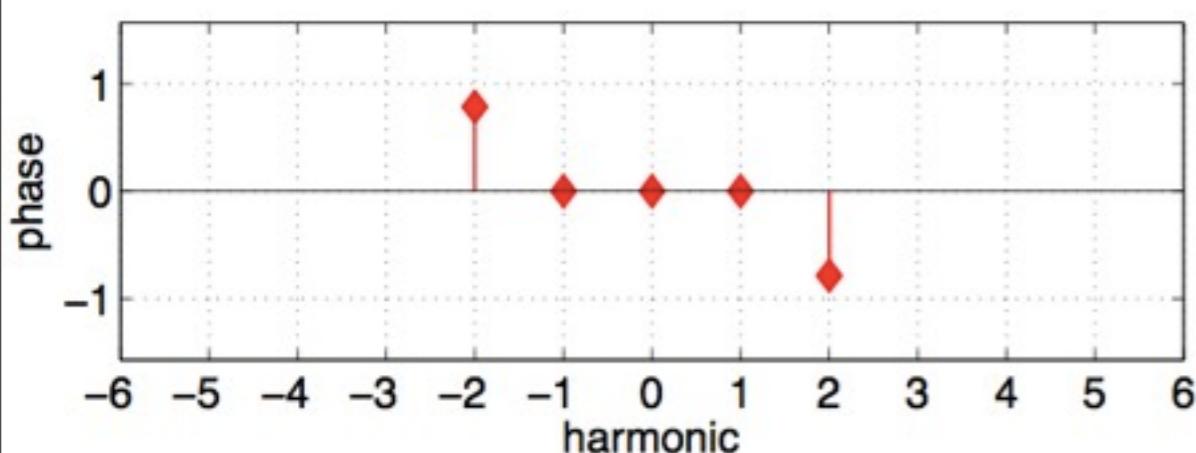
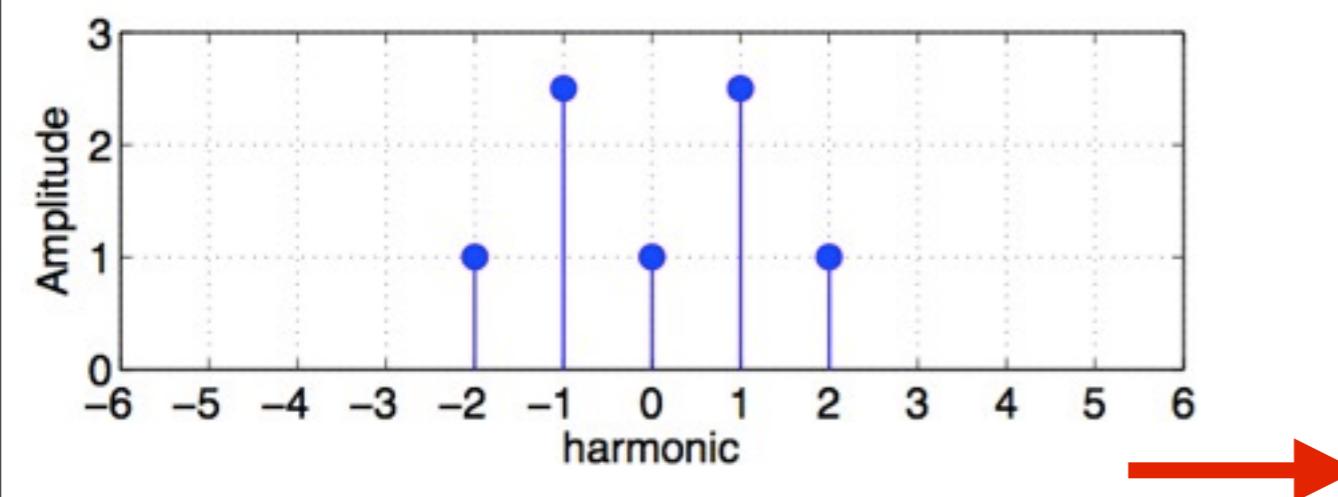
$n=5$



$n=13$

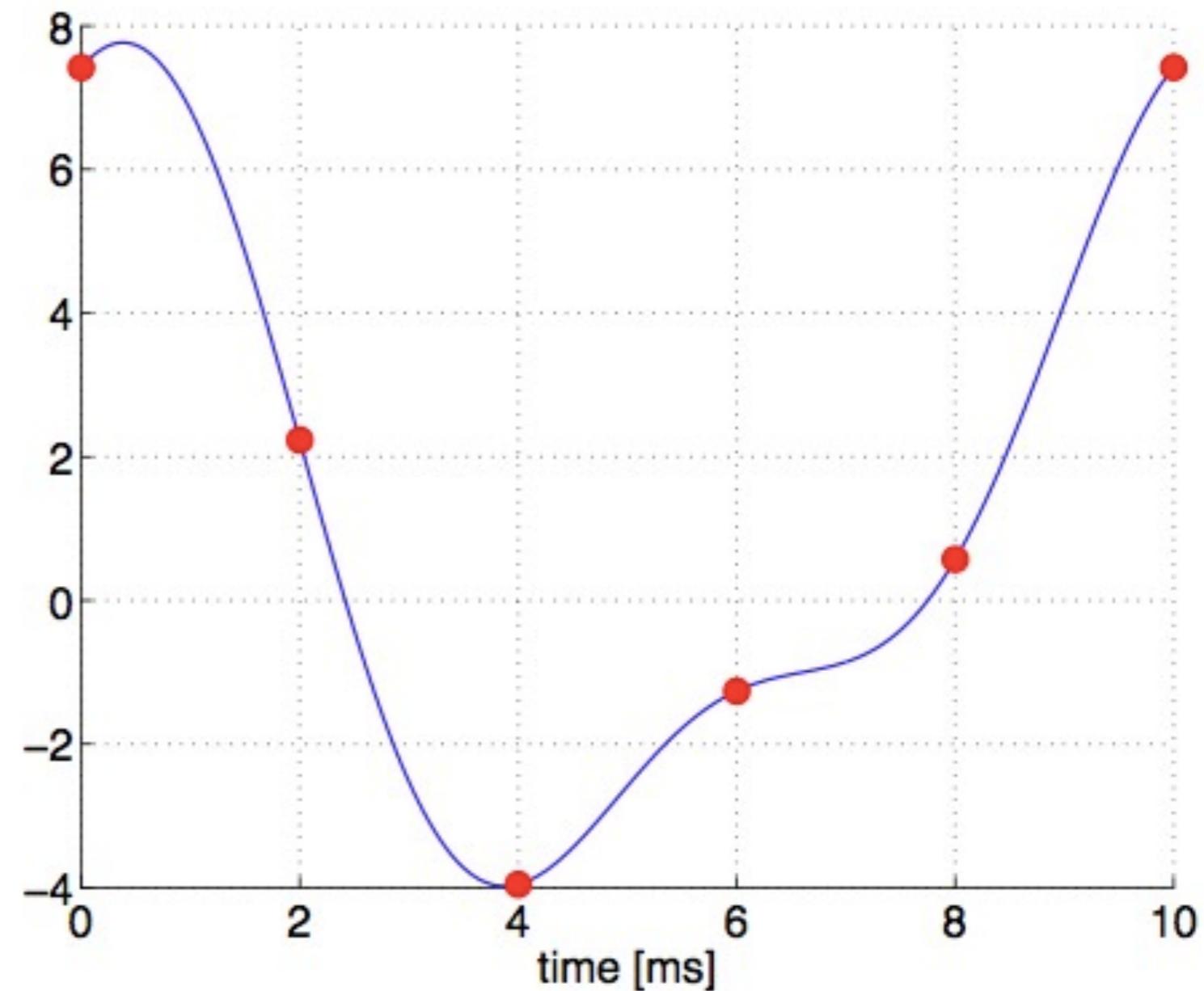


# Reconstruction of the signal

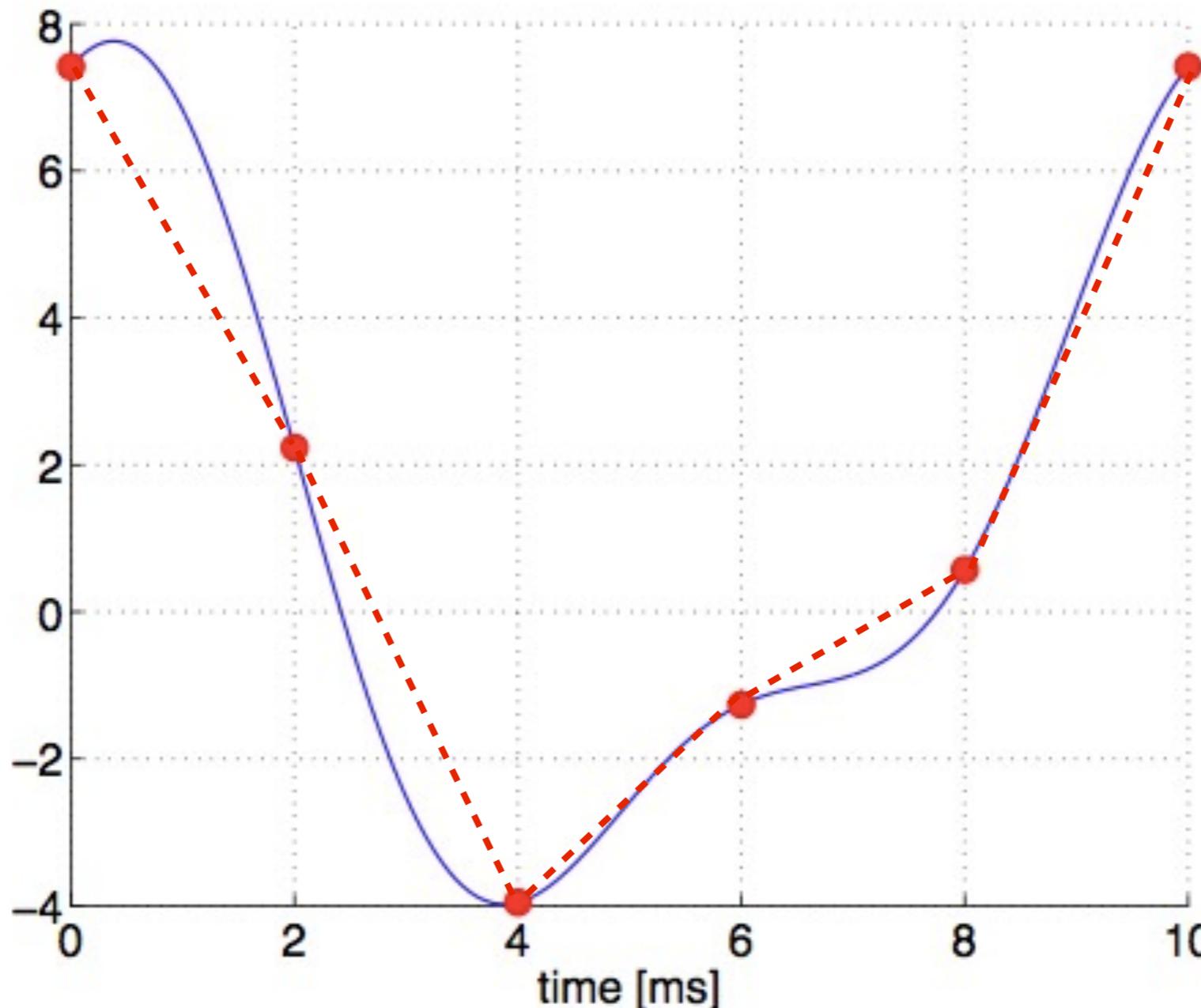


# Reconstruction of the signal

1. Sampling
2. samples
3. DFT
4. spectrum
5. amplitude and phase of the harmonics
6.  $s(t) = \sum_{i=1}^N A_i \cos(i\omega t + \varphi_i) + k$
7. we can calculate  $s(t)$  at any value of  $t$

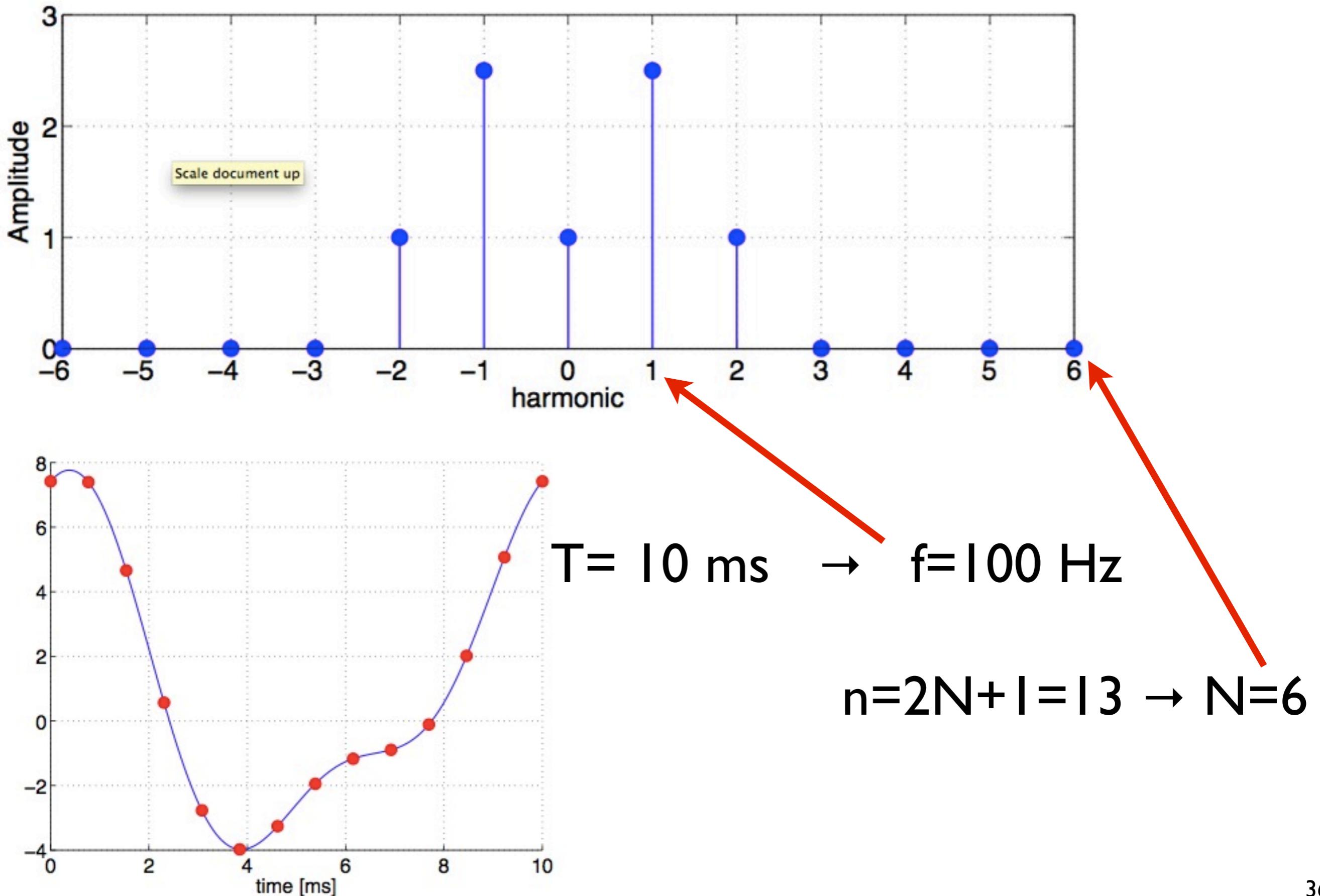


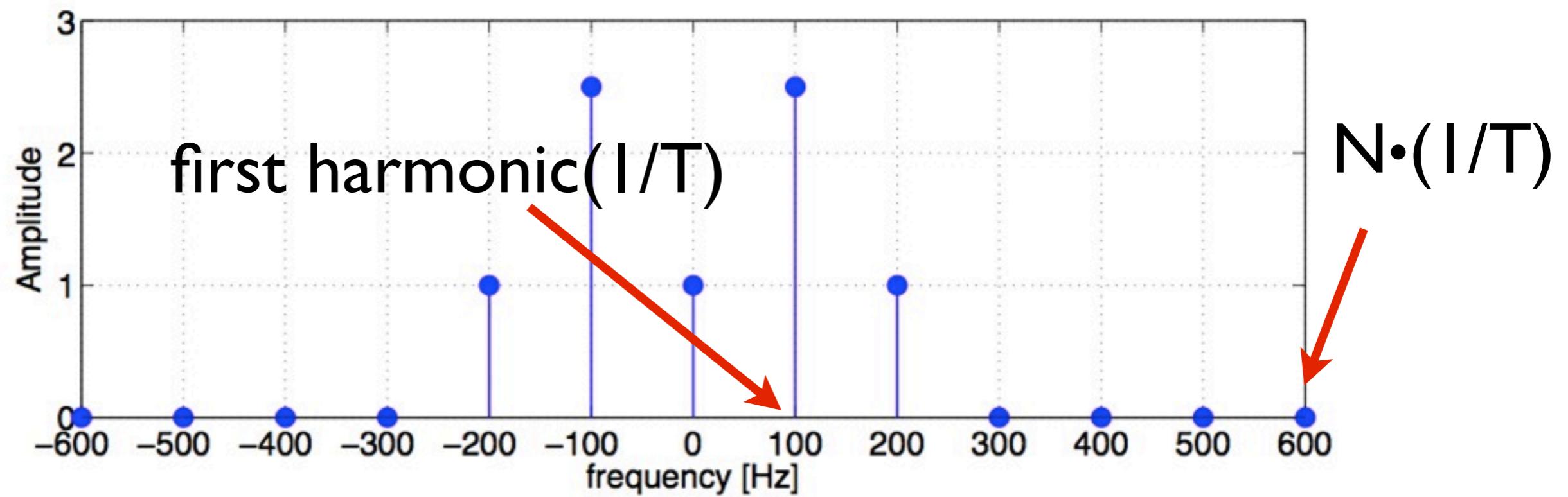
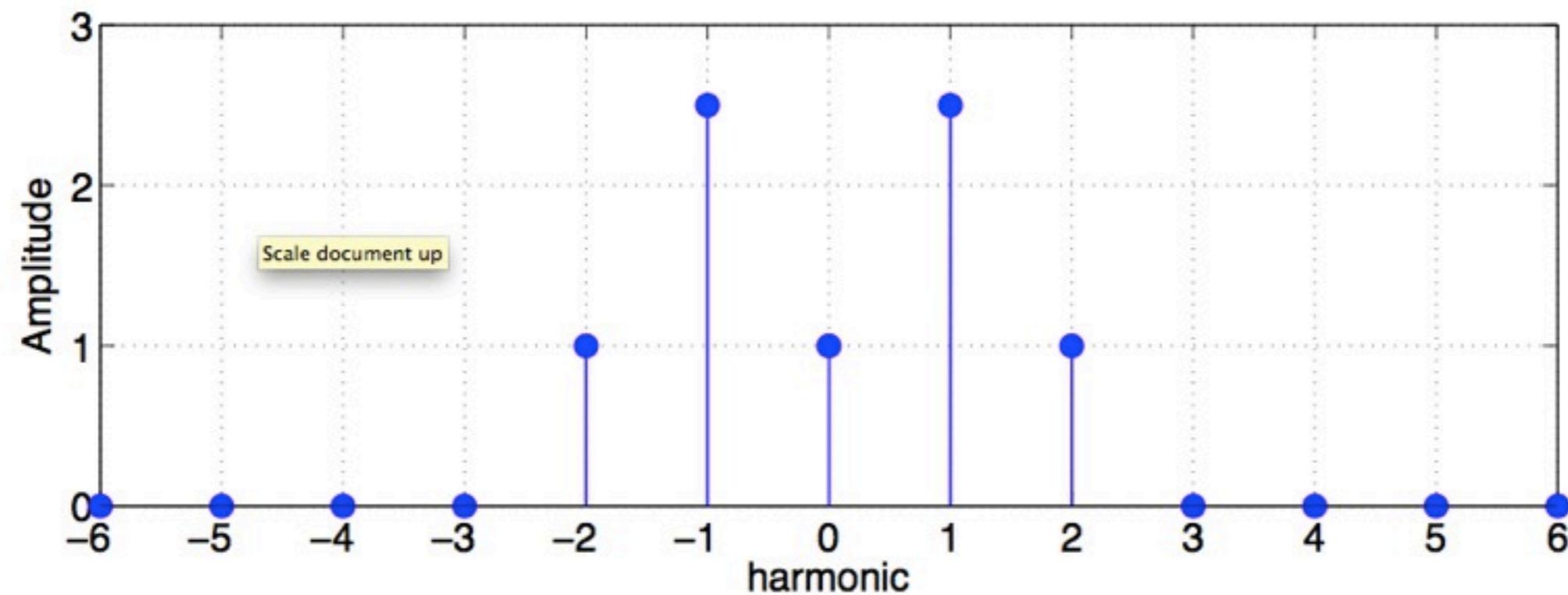
# Reconstruction of the signal is **NOT** interpolation



Reconstruction of the signal can calculate perfectly  
any point of the signal, interpolation cannot

# Let's watch the spectrum closer

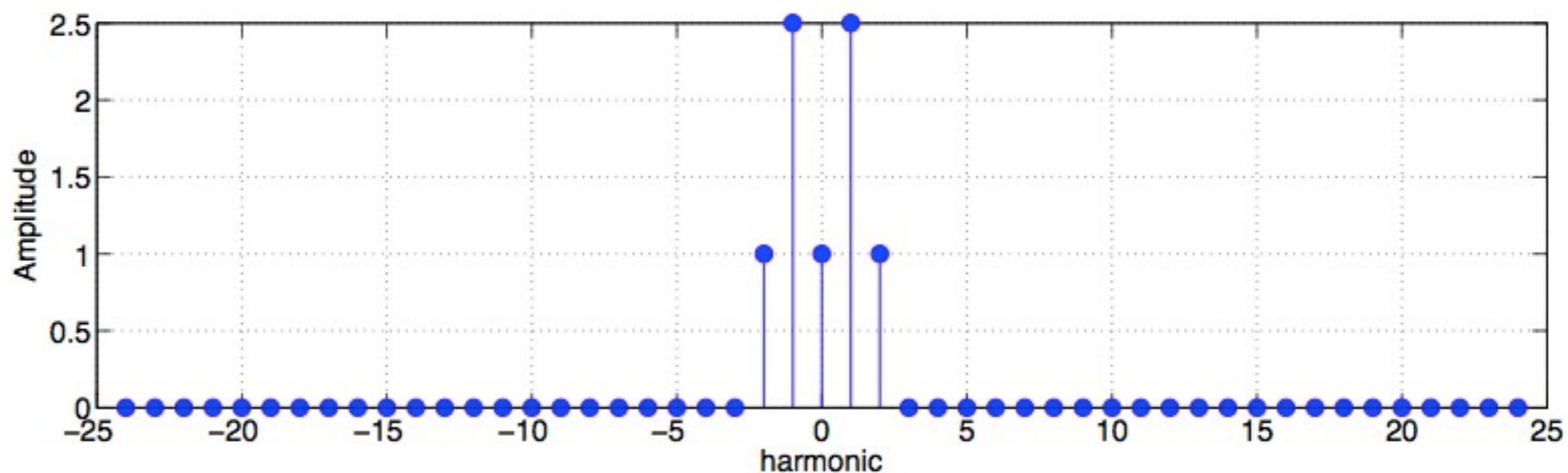
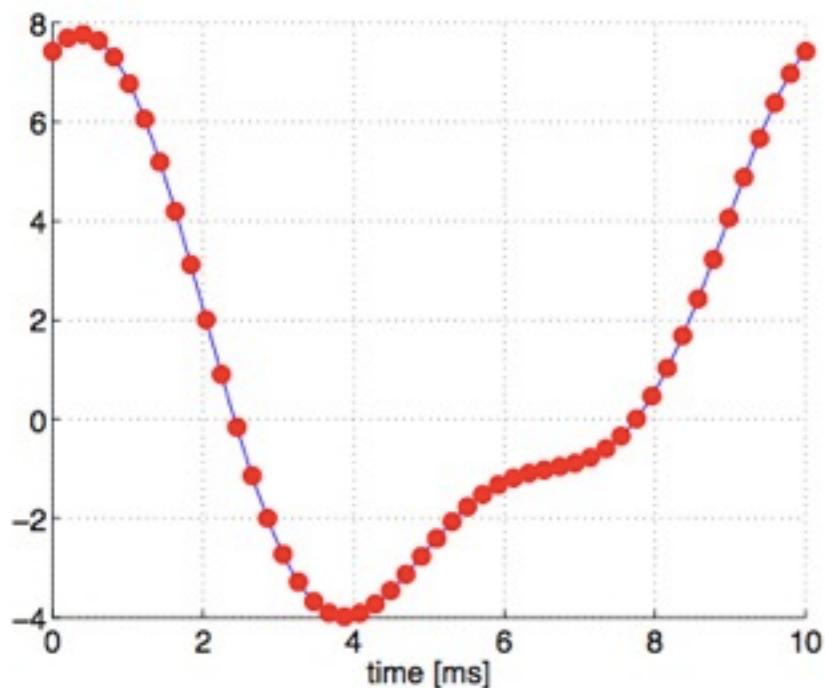




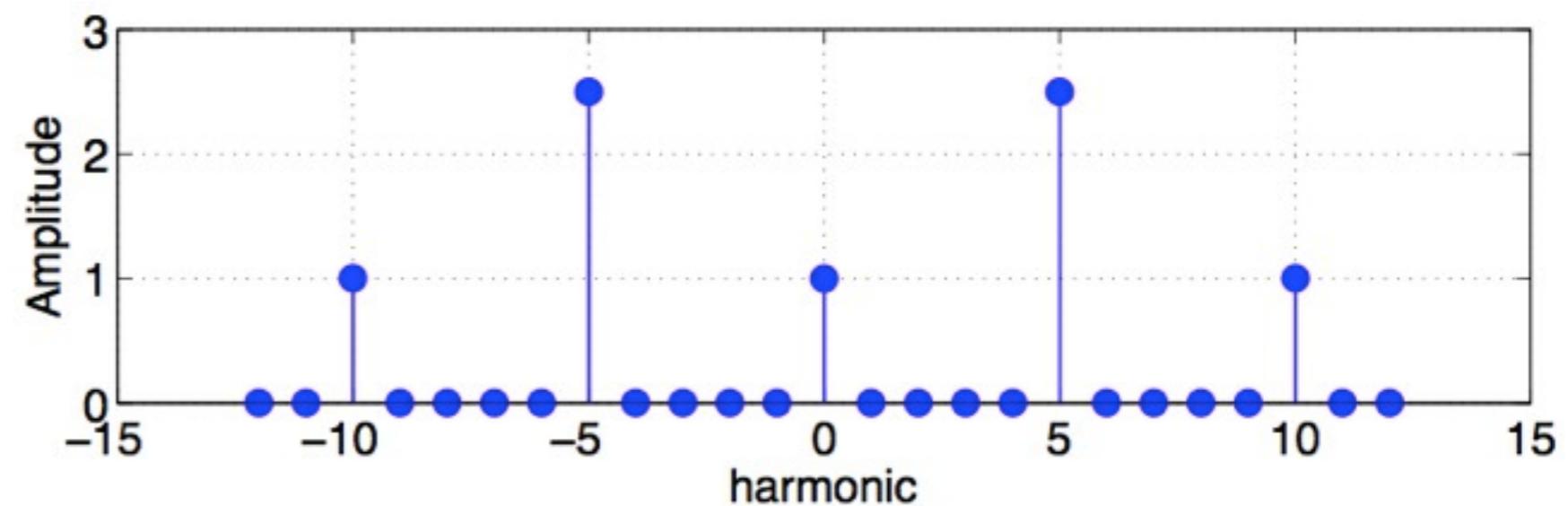
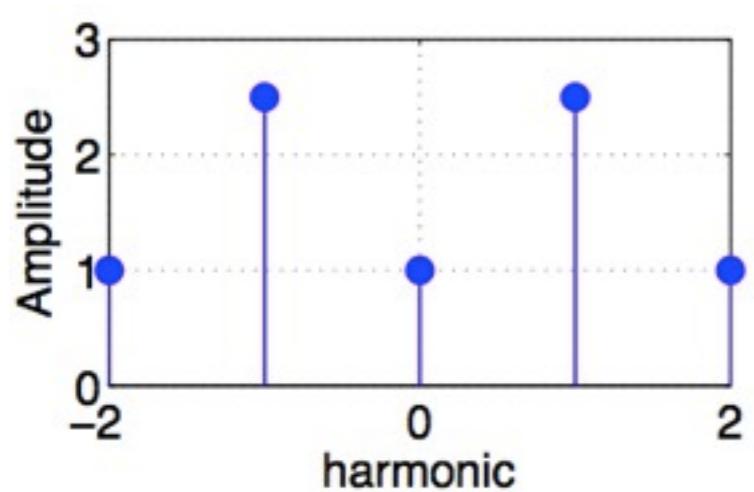
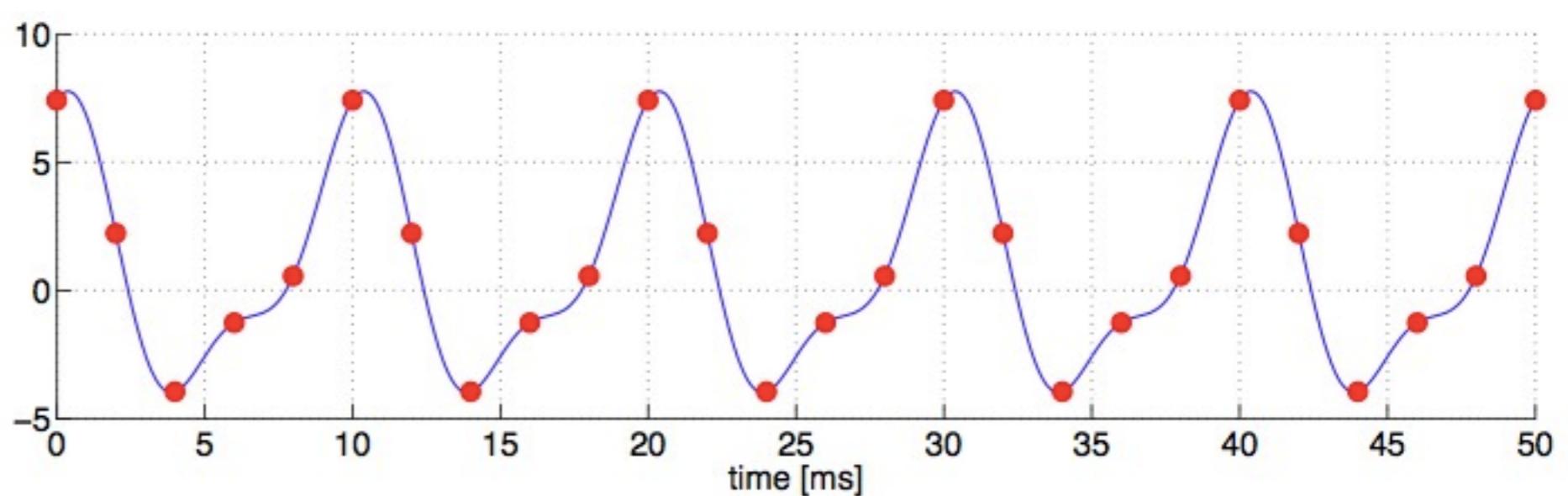
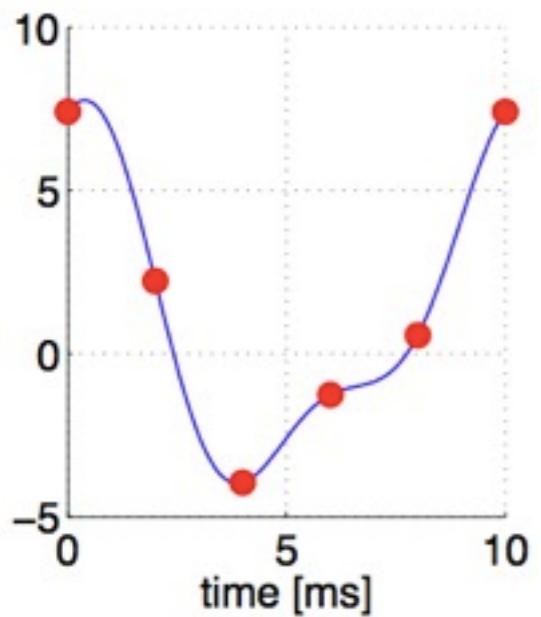
# What happens if...

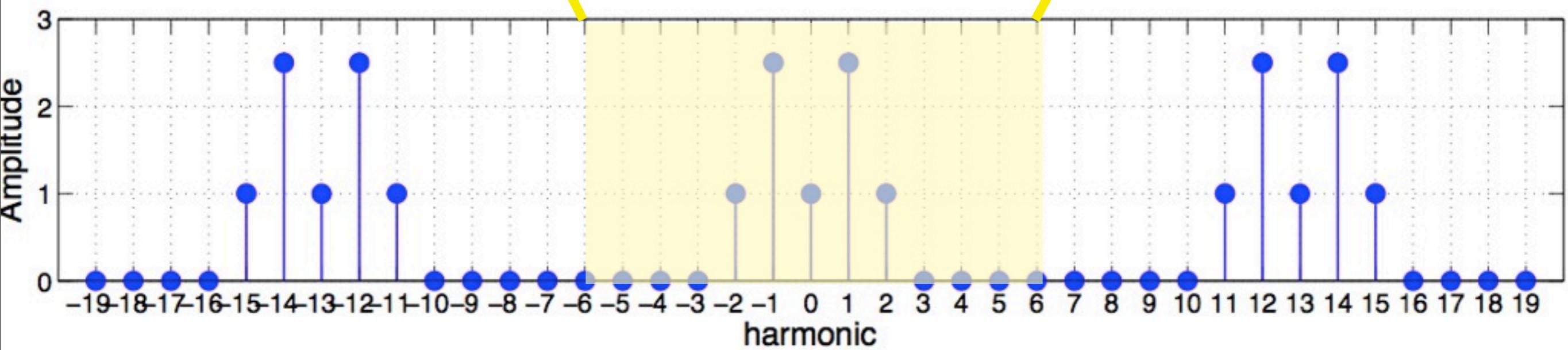
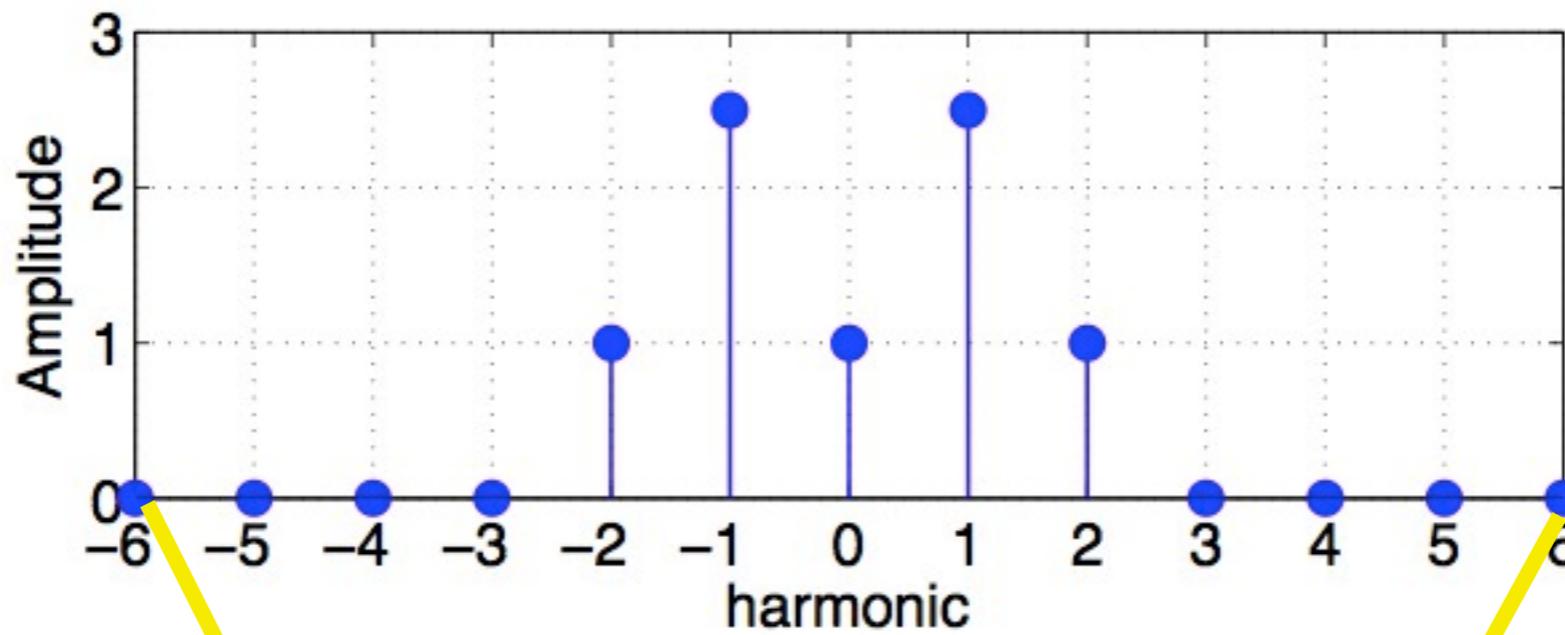
we increase the sampling frequency  $f_s$

$f_s \uparrow$        $T_s \downarrow$        $n=2N+1 \uparrow$        $N \uparrow$

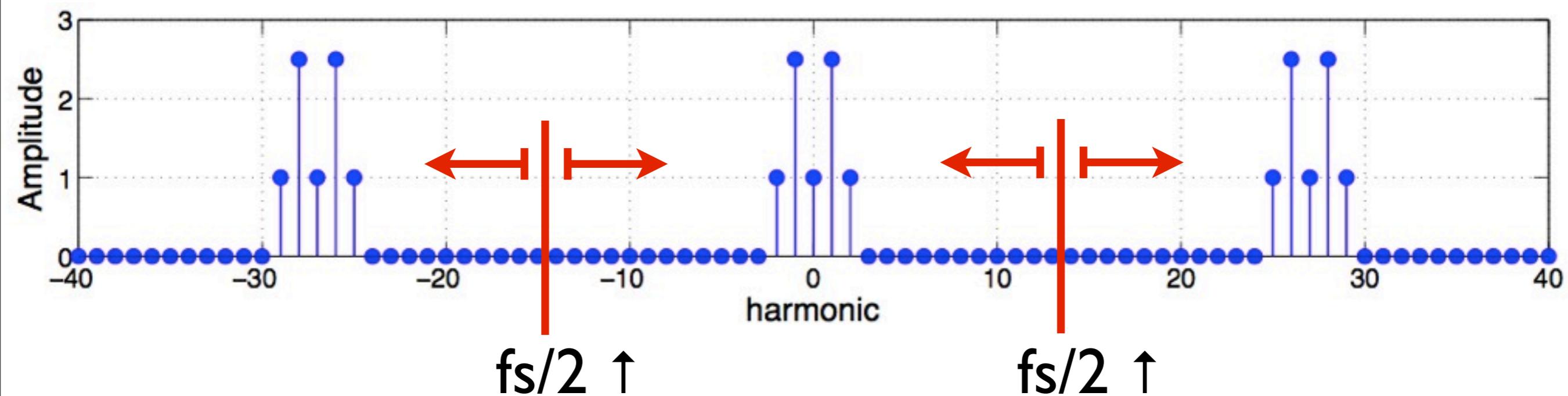


What happens if...  
we increase the acquiring time  
and  $f_s$  does not change

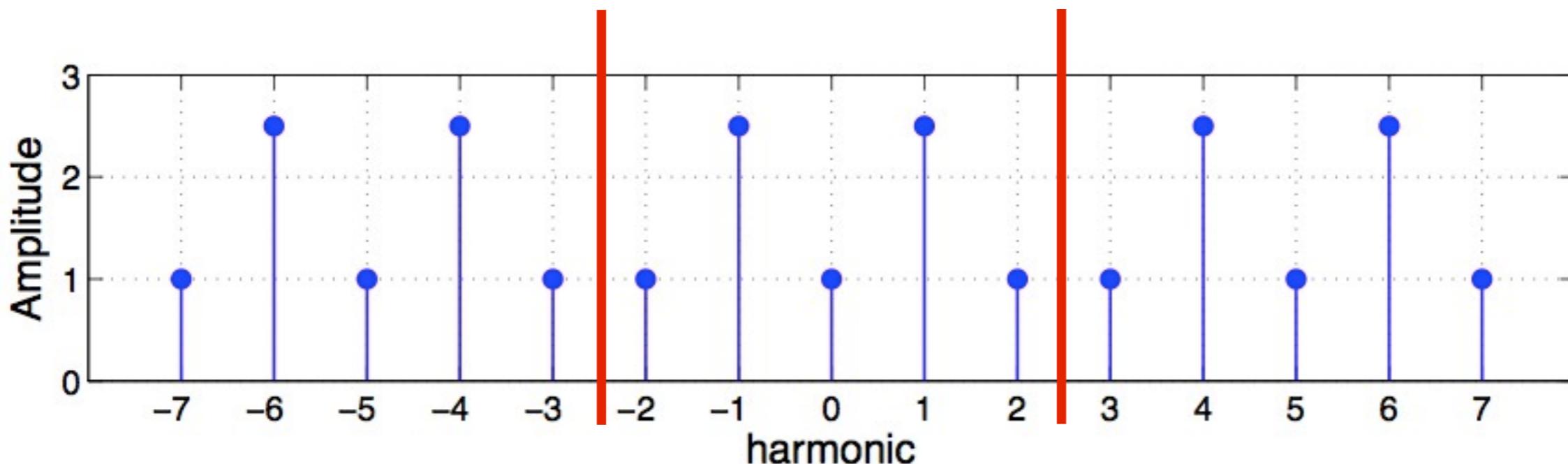




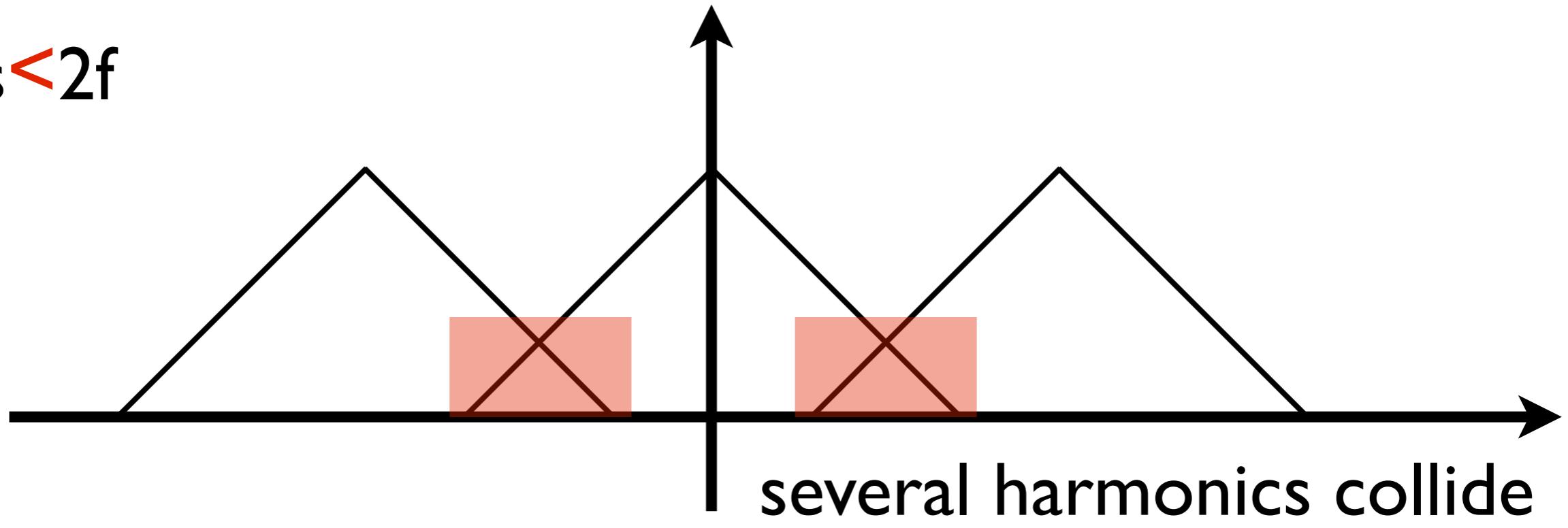
# Large sampling frequency pushes the spectra apart



The minimum  $fs$  to avoid “collision” of spectra is  $fs > 2f$

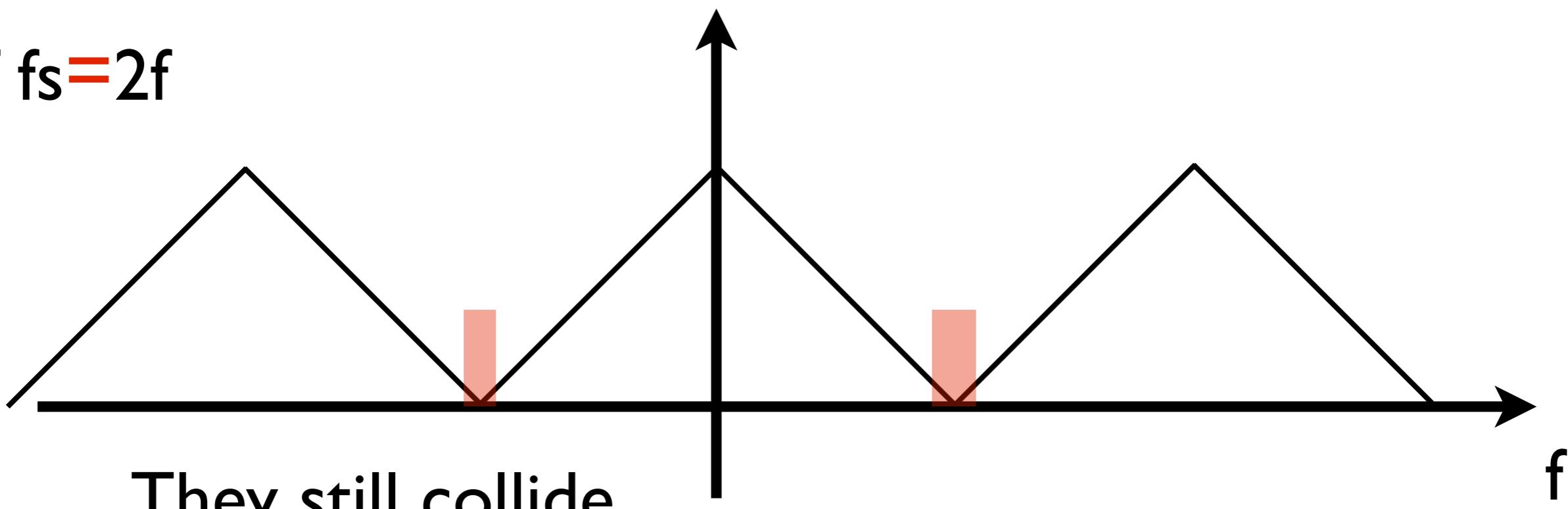


if  $f_s < 2f$



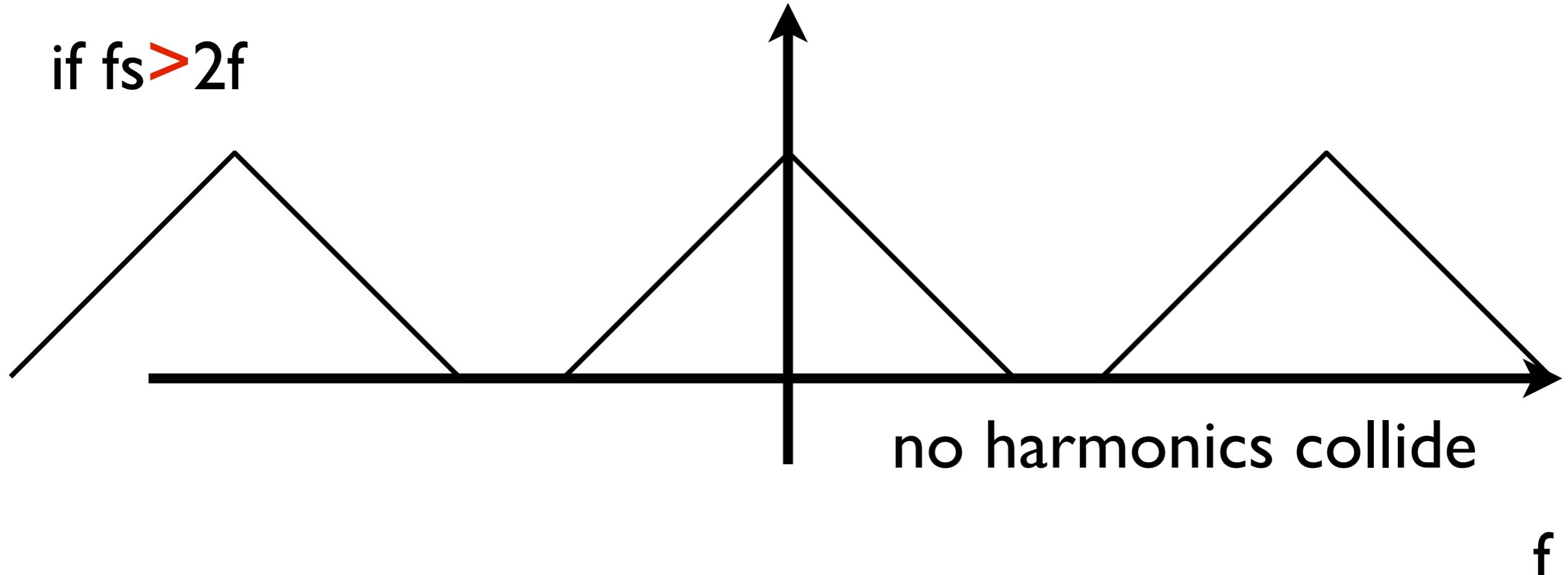
several harmonics collide

if  $f_s = 2f$



They still collide

if  $f_s > 2f$



# ANALOG TO DIGITAL CONVERTERS

Two main parameters:

- resolution (number of bits)
- sampling frequency

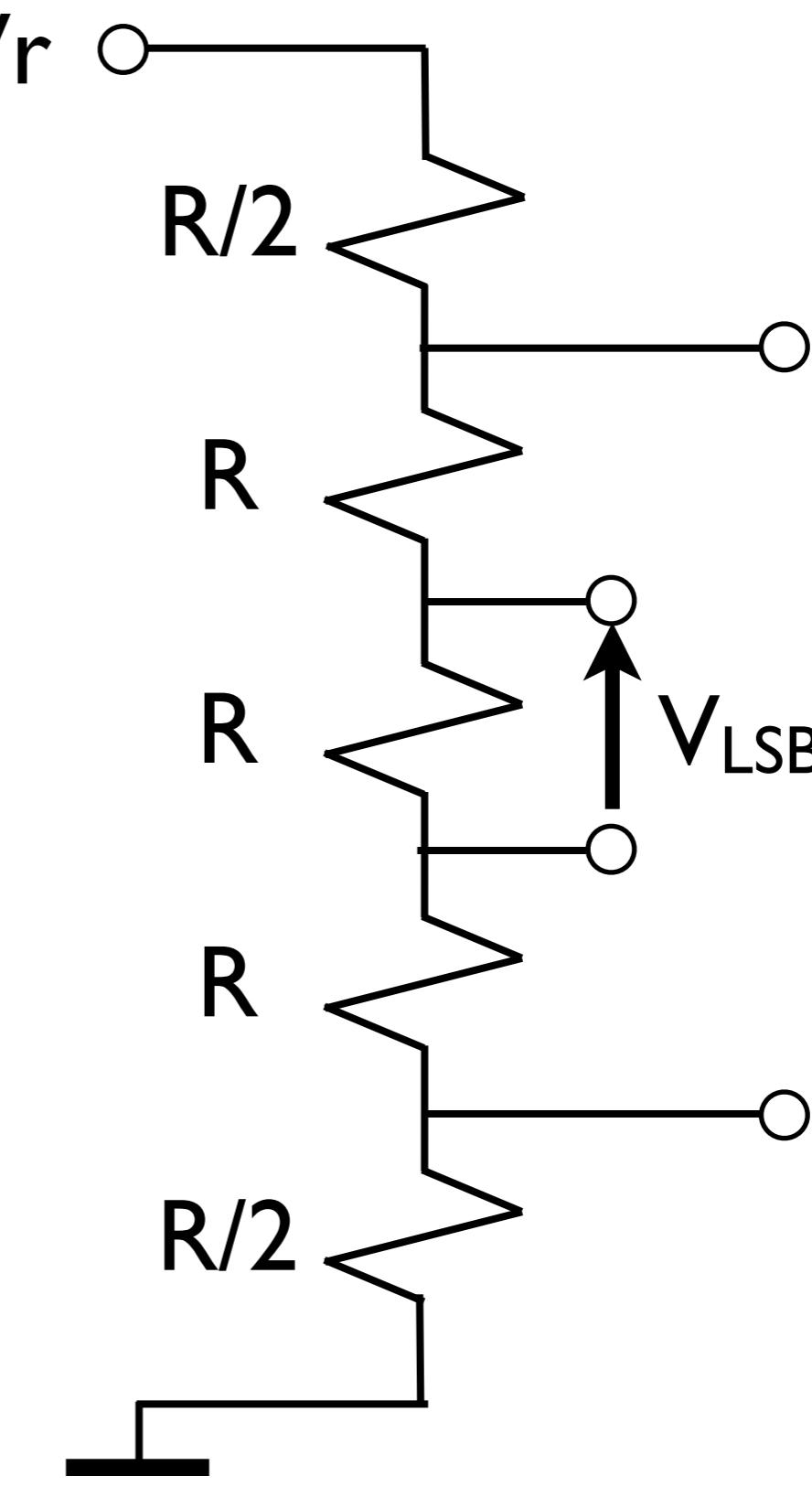
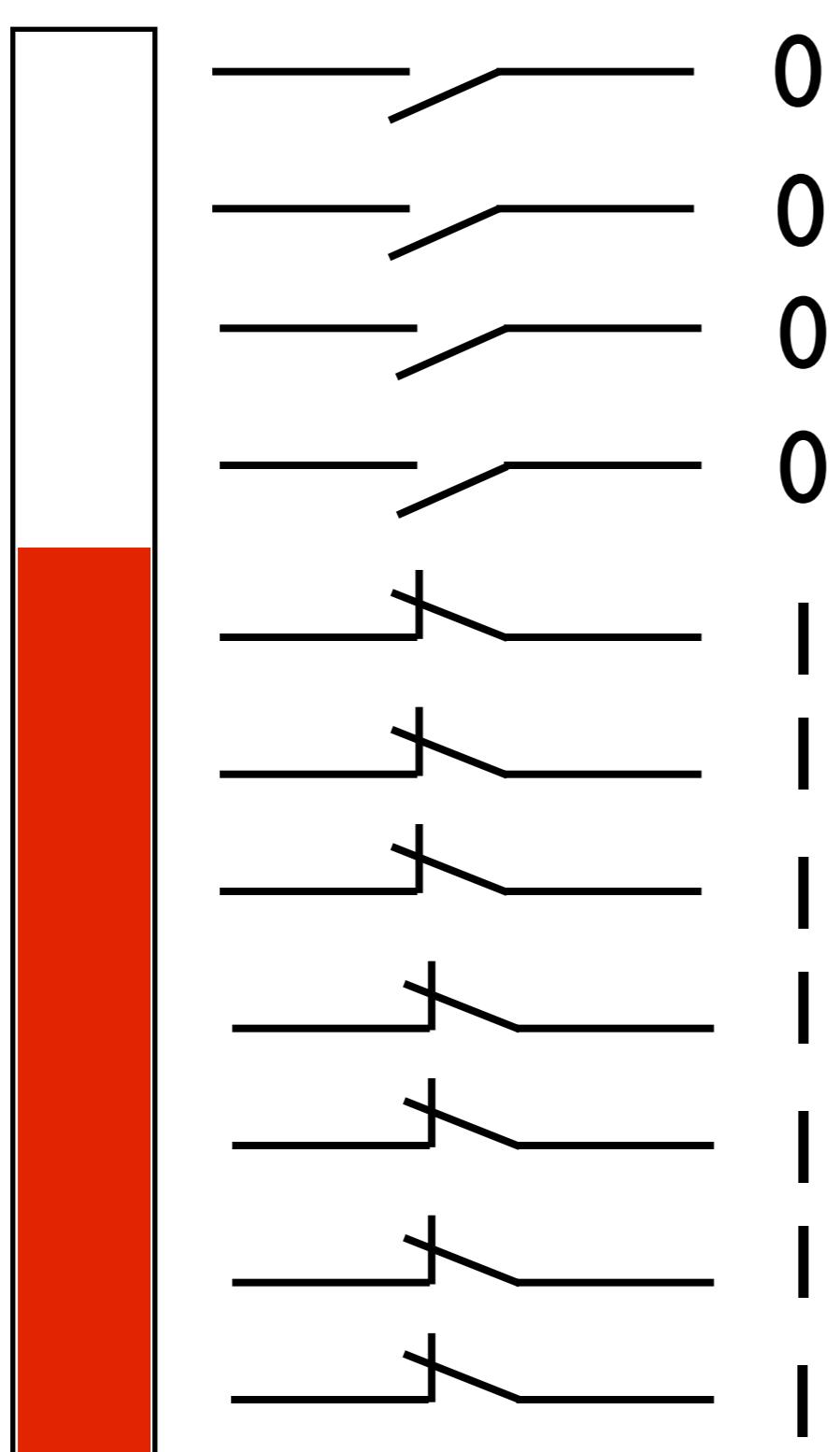
FLASH

Successive approximation

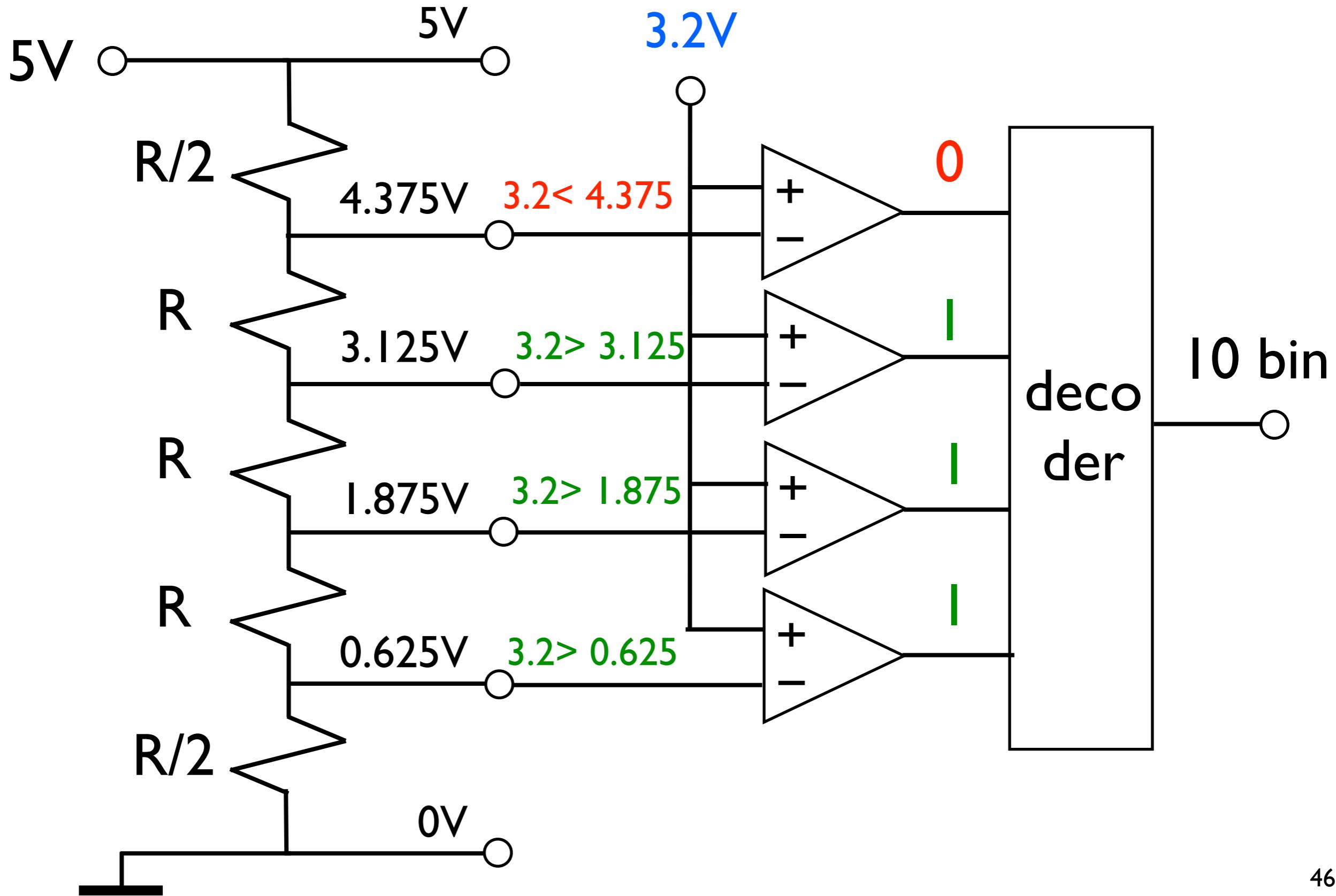
Dual slope integration

Sigma delta

# FLASH



# FLASH



## FLASH

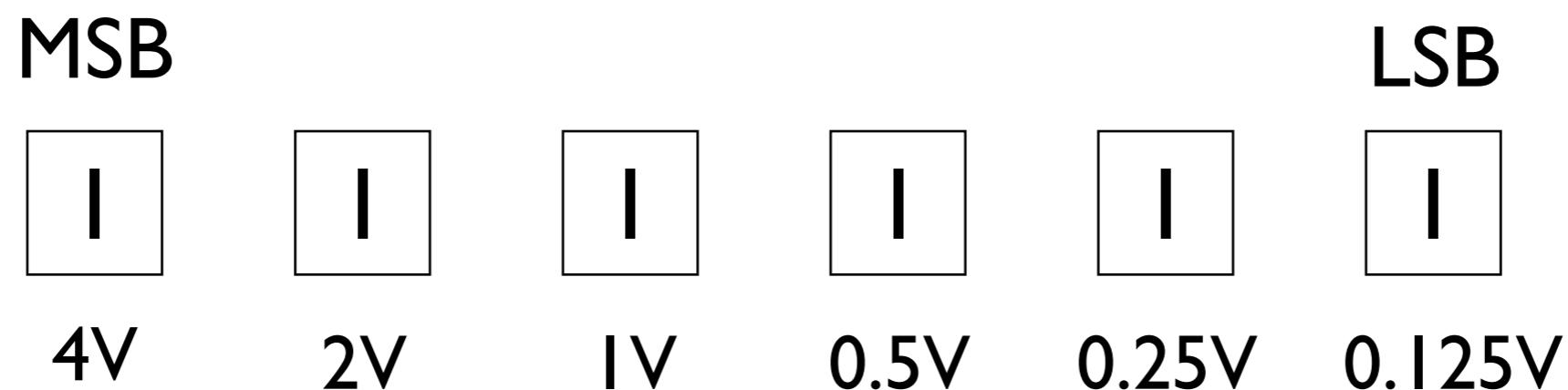
- Very high speed (even 1 GHz)
- Poor resolution (6 to 8 bits)

# SUCCESSIVE APPROXIMATIONS

## Principle

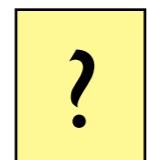
$V_{ref}=8V$

Resolution 6 bits



# SUCCESSIVE APPROXIMATIONS

MSB



4V

?

2V

?

IV

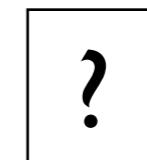
?

0.5V

?

0.25\

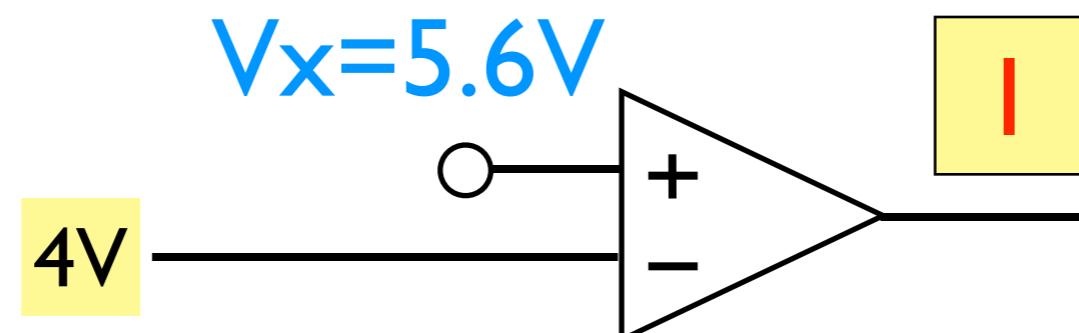
LSB



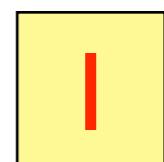
0.125V

I want to convert  $V_x=5.6V$

# STEP I



MSB



4V

?

2V

?

IV

?

0.5V

?

0.25W

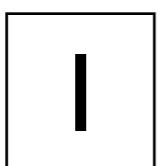
LSB



0.125V

# SUCCESSIVE APPROXIMATIONS

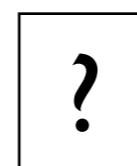
MSB



4V



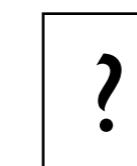
2V



IV

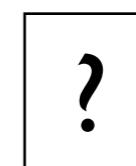


0.5V



0.25V

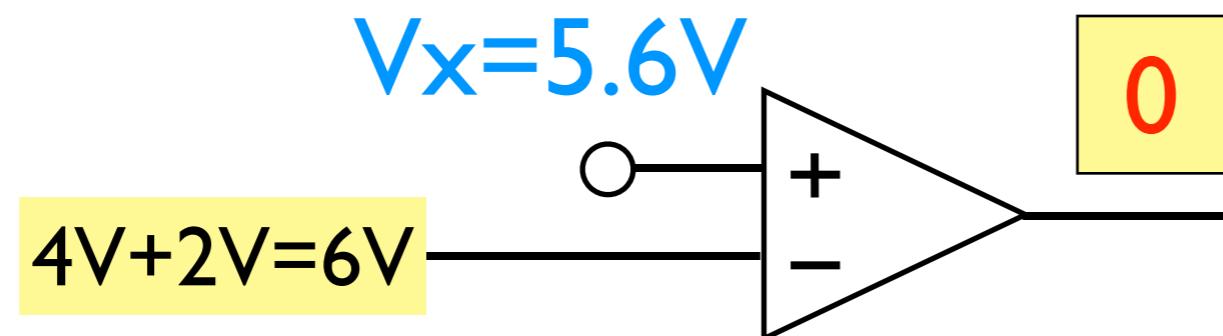
LSB



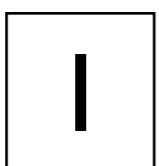
0.125V

# STEP 2

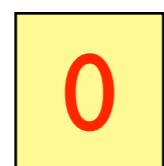
V<sub>x</sub>=5.6V



MSB



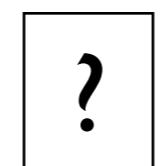
4V



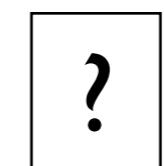
2v



IV

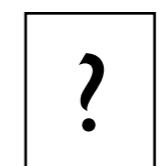


0.5V



.25%

LSB



0.125V

# SUCCESSIVE APPROXIMATIONS

MSB

1

4V

0

2V

?

IV

?

0.5V

?

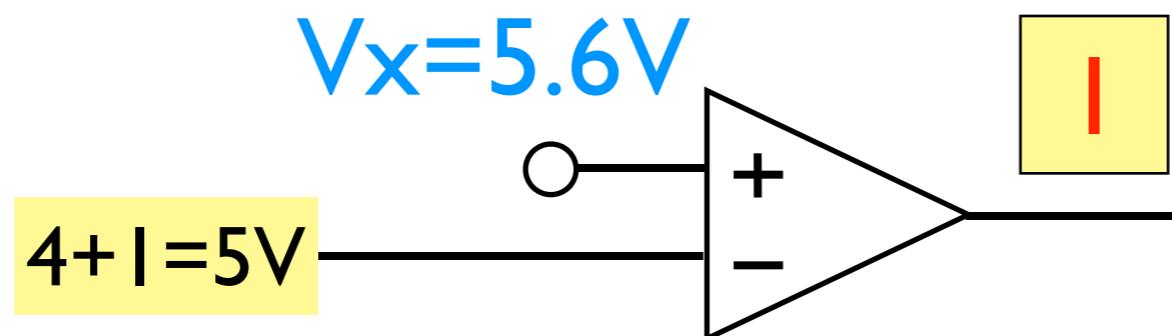
0.25

# LSB

?

0.125V

# STEP 3



MSB

1

4V

0

2v

1

IV

?

0.5V

?

25

LSB

# SUCCESSIVE APPROXIMATIONS

MSB

1

4V

0

2V

1

IV

?

0.5V

?

0.25V

# LSB

?

0.125V

# STEP 4

$V_x = 5.6V$

$4 + I + 0.5 = 5.5V$

MSB

1

4V

0

2v

1

IV

1

0.5V

LSB

?

0.125V

# SUCCESSIVE APPROXIMATIONS

MSB

1

4V

0

2V

1

IV

1

0.5V

?

0.25V

LSB

?

0.125V

# STEP 5

$V_x = 5.6V$

$4 + 1 + 0.5 + 0.25 = 5.75V$

MSB

1

4V

0

2v

1

IV

0

0.25

# LSB

?

0.125V

# SUCCESSIVE APPROXIMATIONS

MSB

1

4V

0

2V

1

IV

1

0.5V

0

0.25\

LSB

?

0.125V

# STEP 6

$V_x = 5.6V$

$4 + 1 + 0.5 + 0.125 = 5.625V$

MSB

1

4V

0

2v

1

IV

1

0.5V

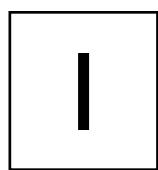
LSB

0

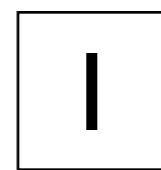
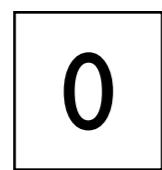
0.125V

# SUCCESSIVE APPROXIMATIONS

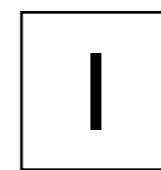
MSB



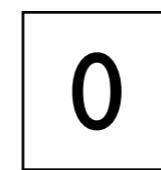
4V



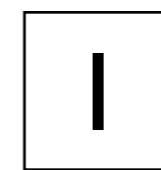
1V



0.5V

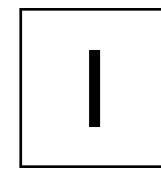


LSB

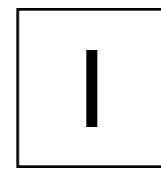
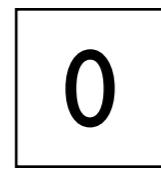


0.125V

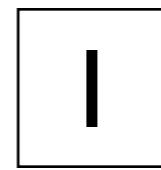
5.625V



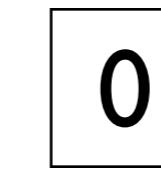
4V



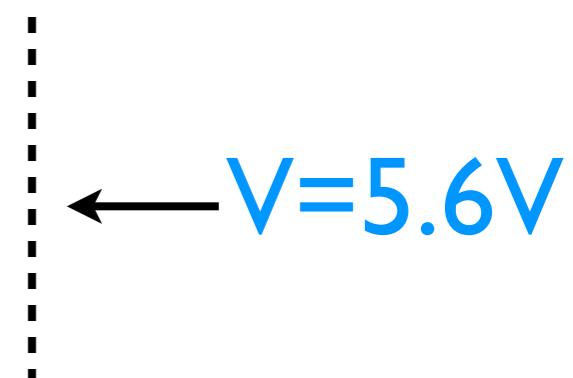
1V



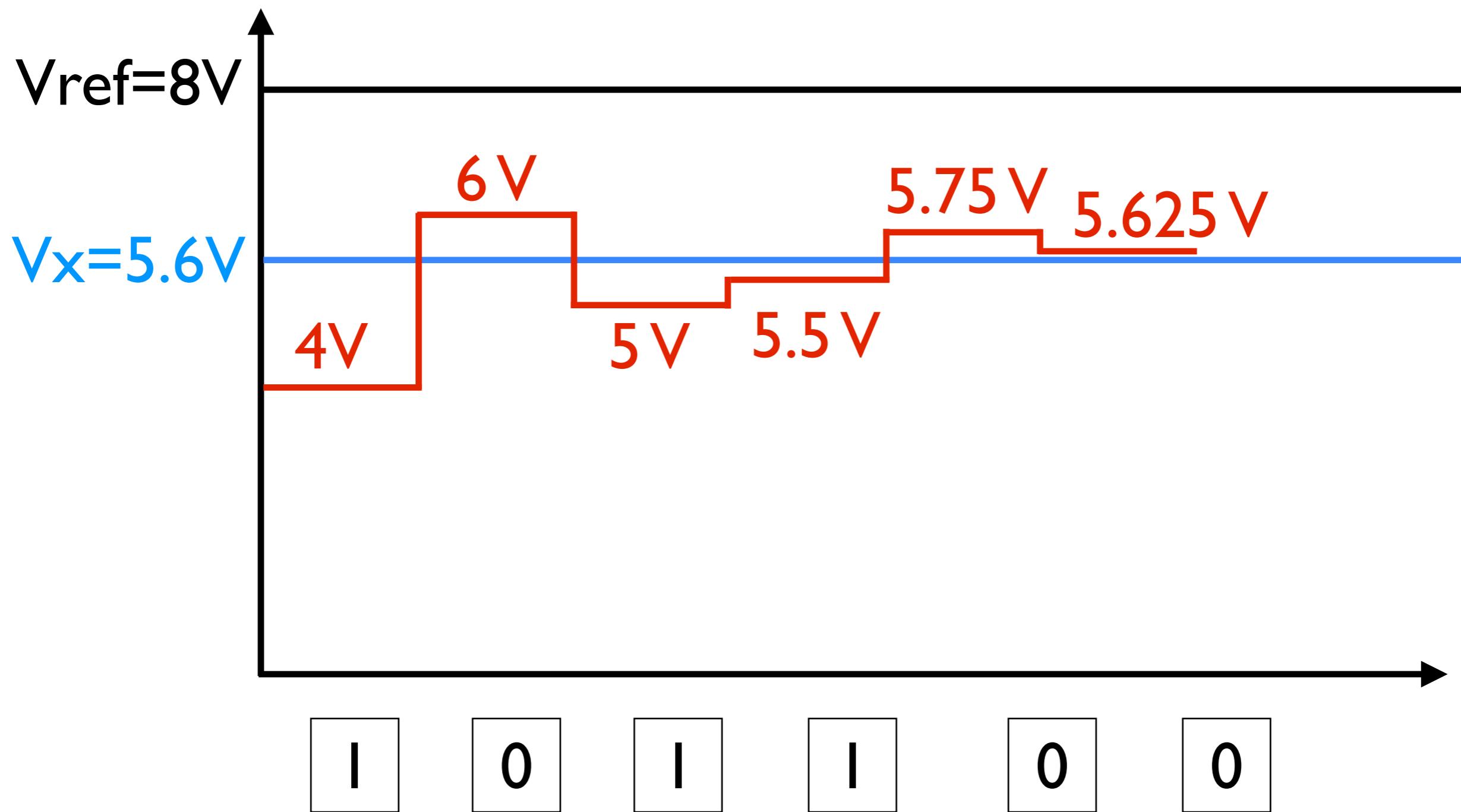
0.5V



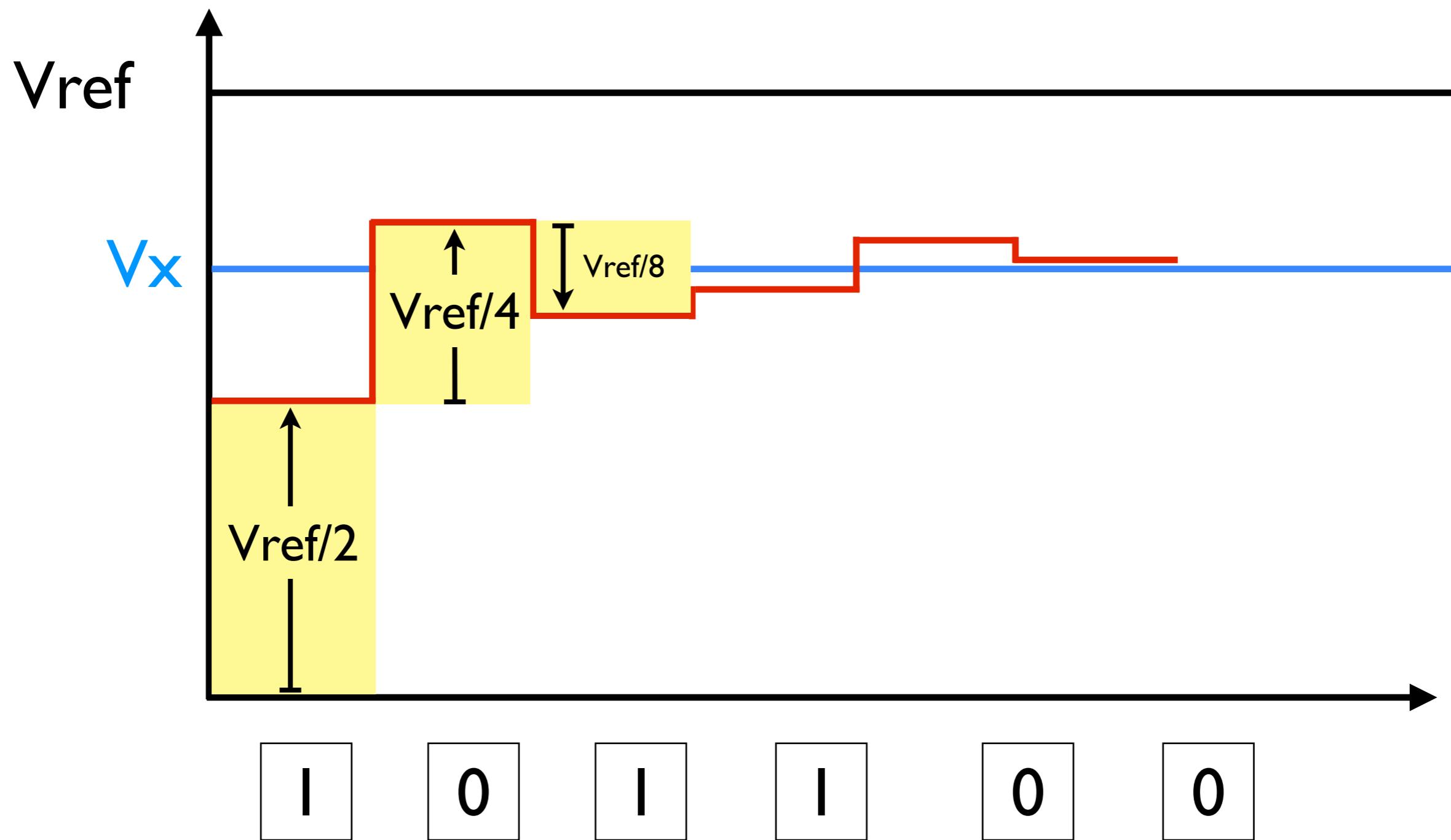
5.5V



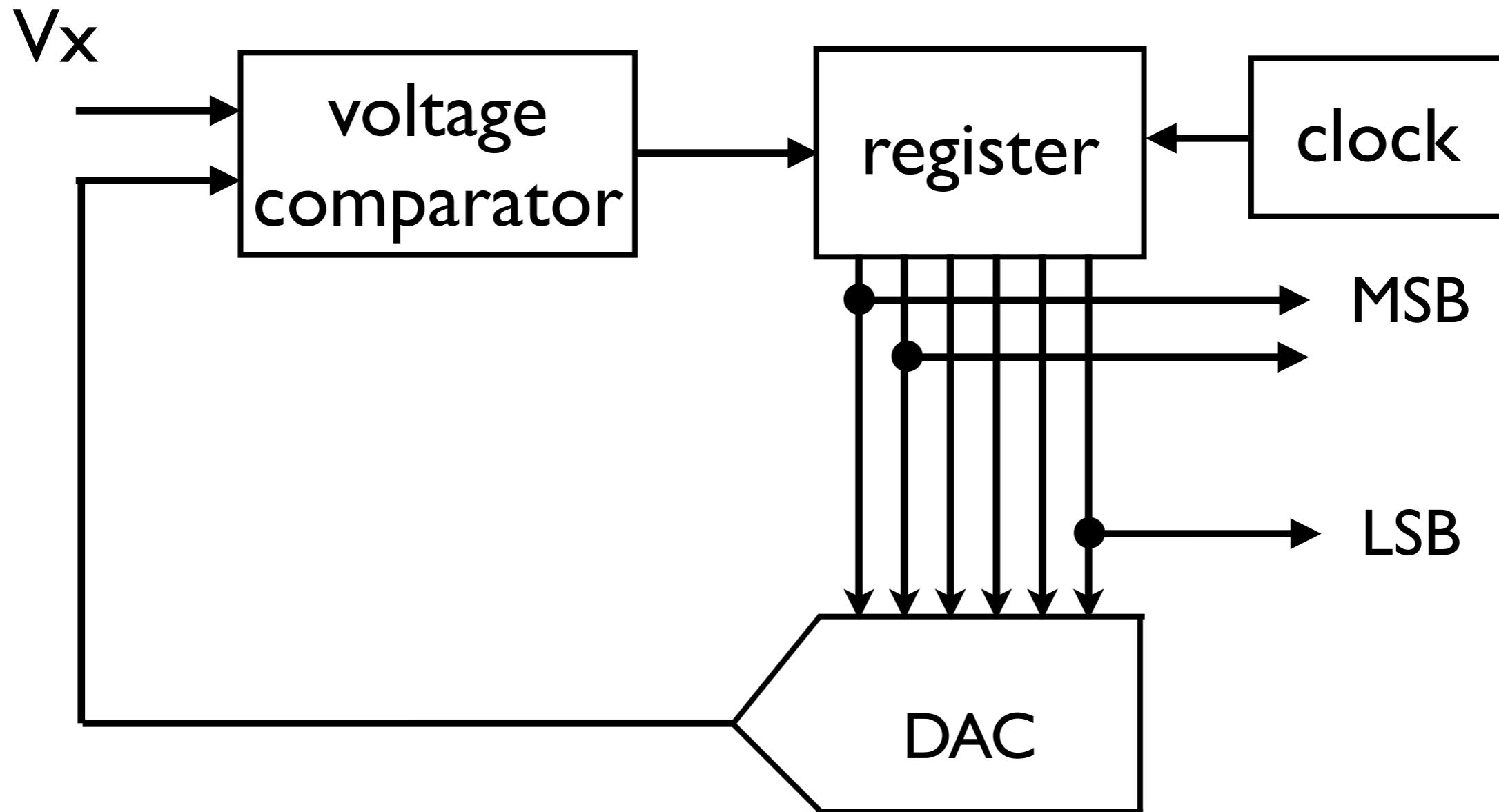
# SUCCESSIVE APPROXIMATIONS



# SUCCESSIVE APPROXIMATIONS

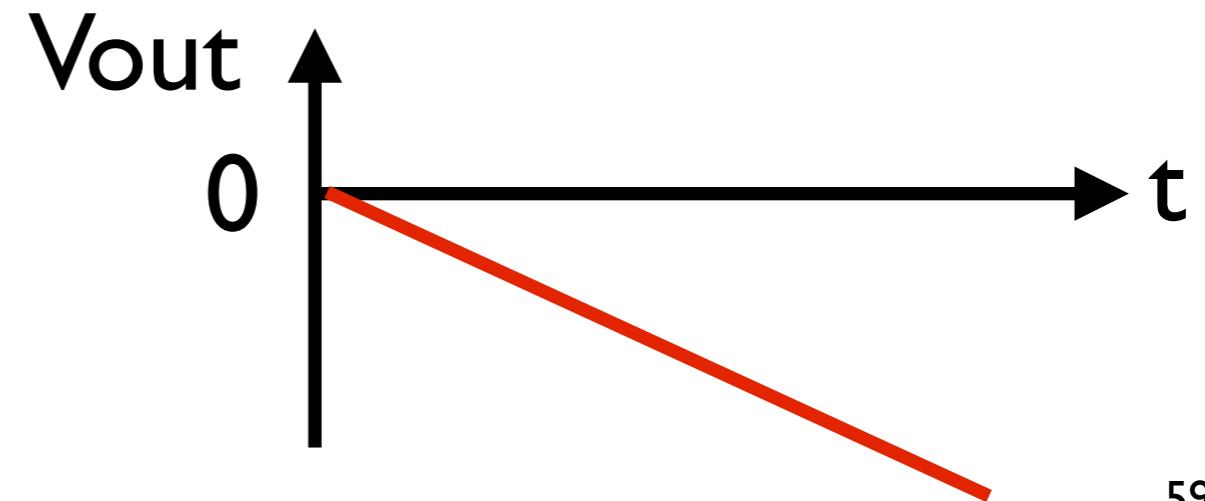
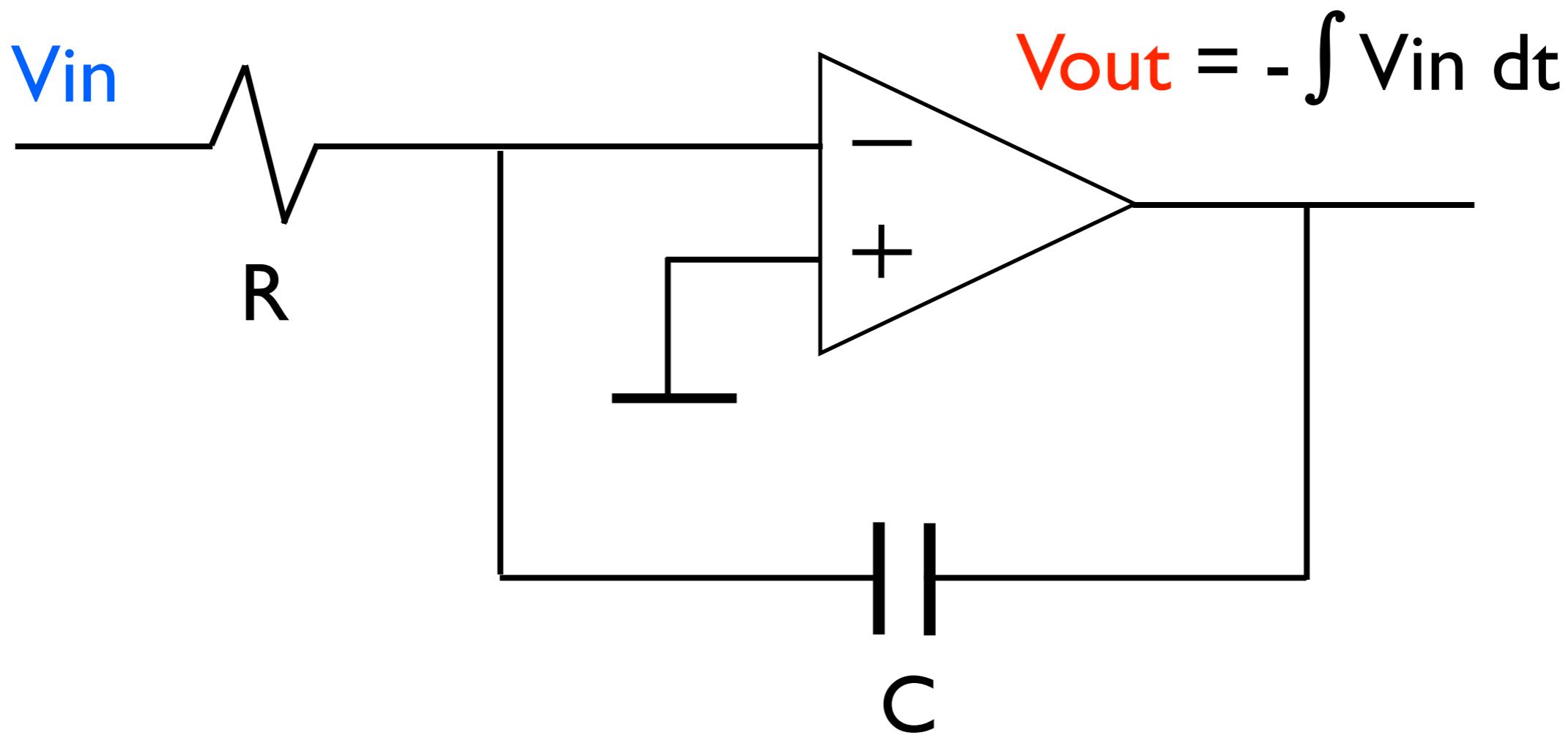


# SUCCESSIVE APPROXIMATIONS



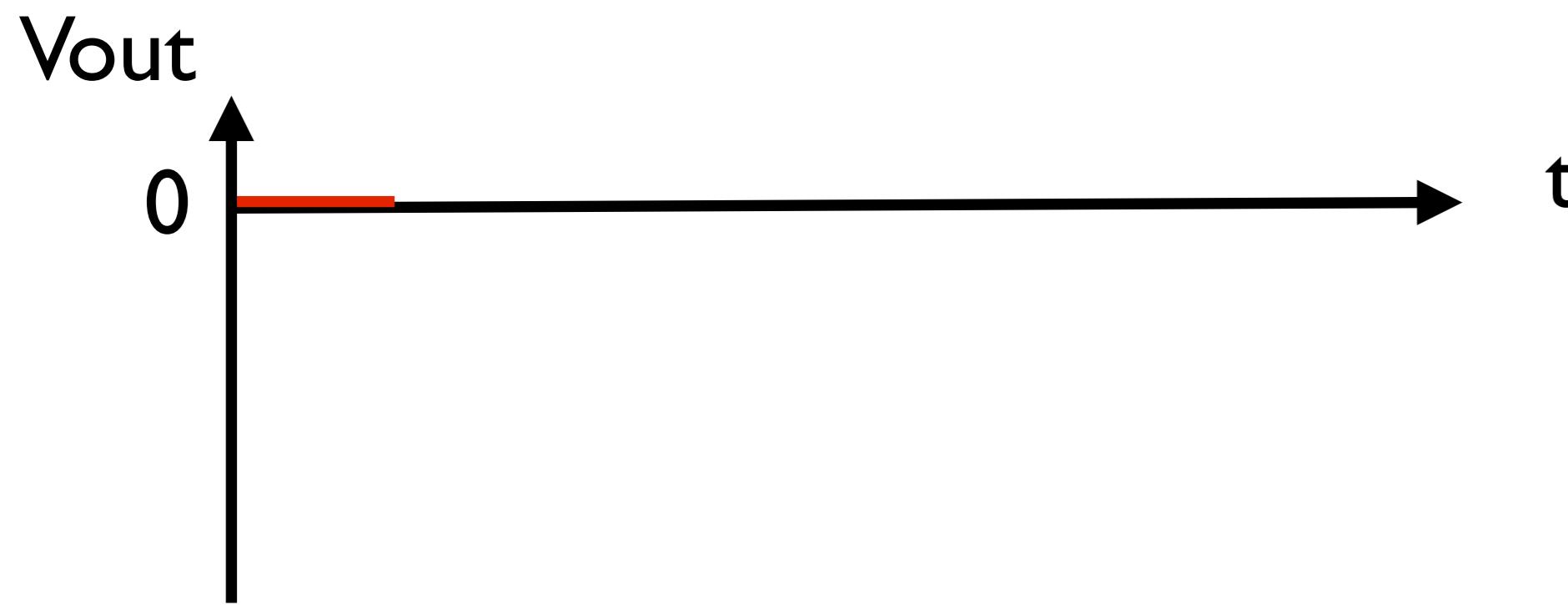
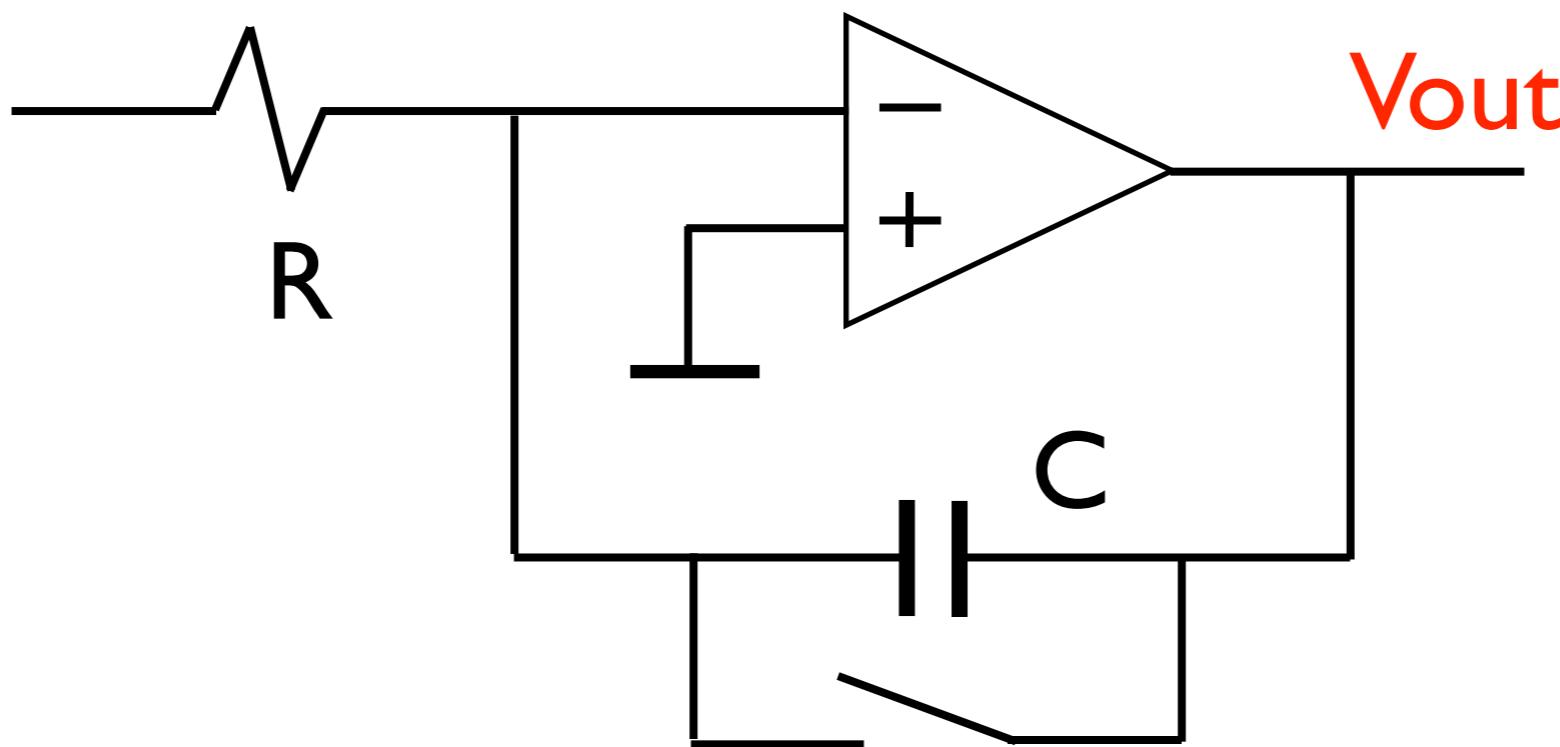
Intermediate speed (1 to 10 MHz)  
Intermediate resolution (8-16 bits)

# DUAL SLOPE INTEGRATION ADC



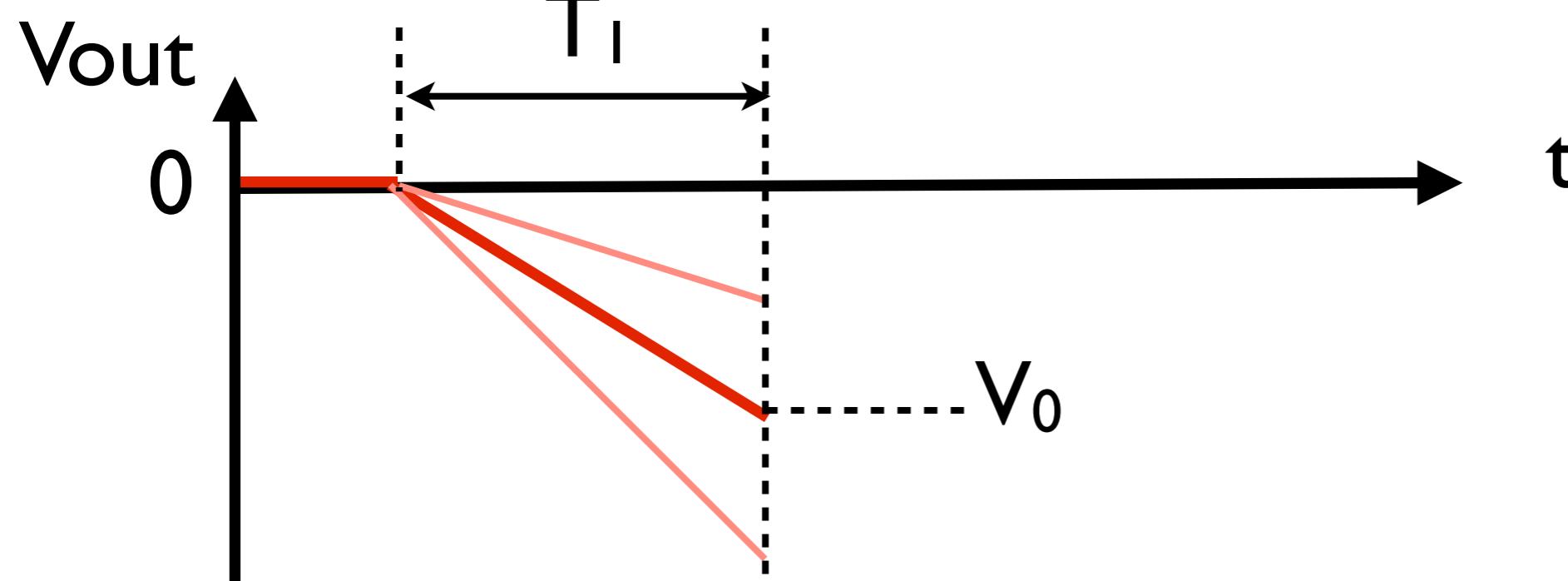
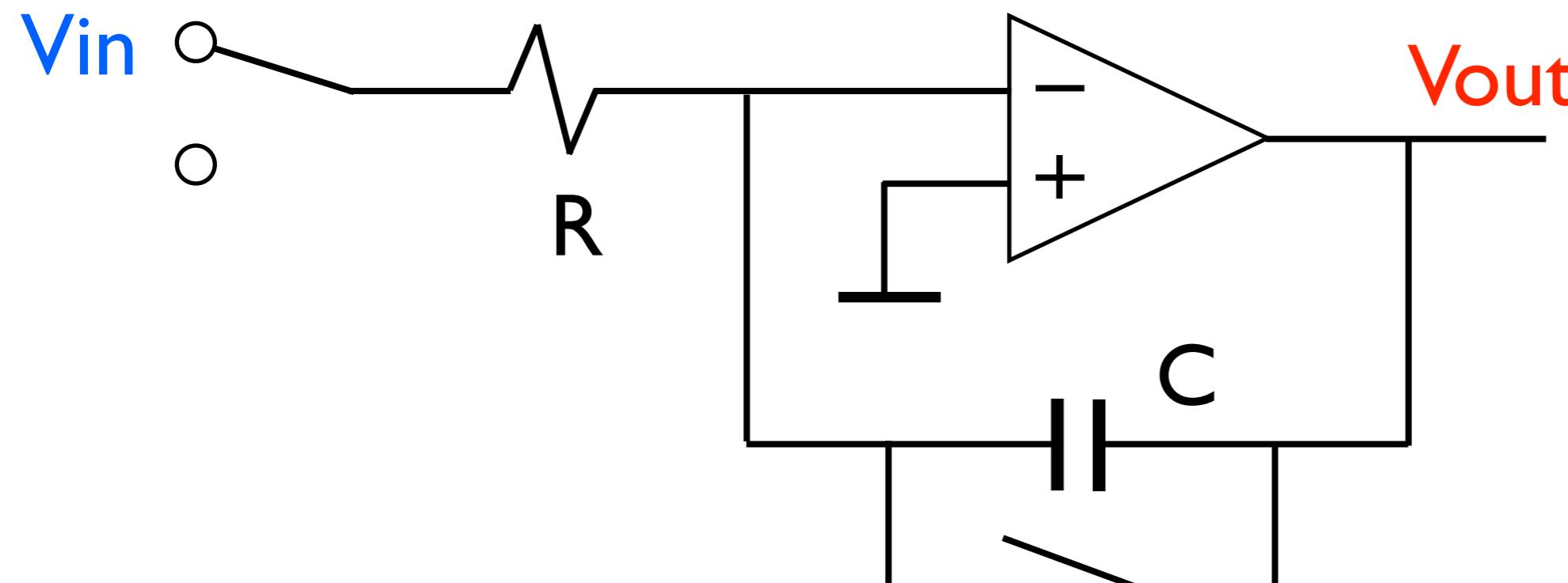
# DUAL SLOPE INTEGRATION ADC

STEP 0



# DUAL SLOPE INTEGRATION ADC

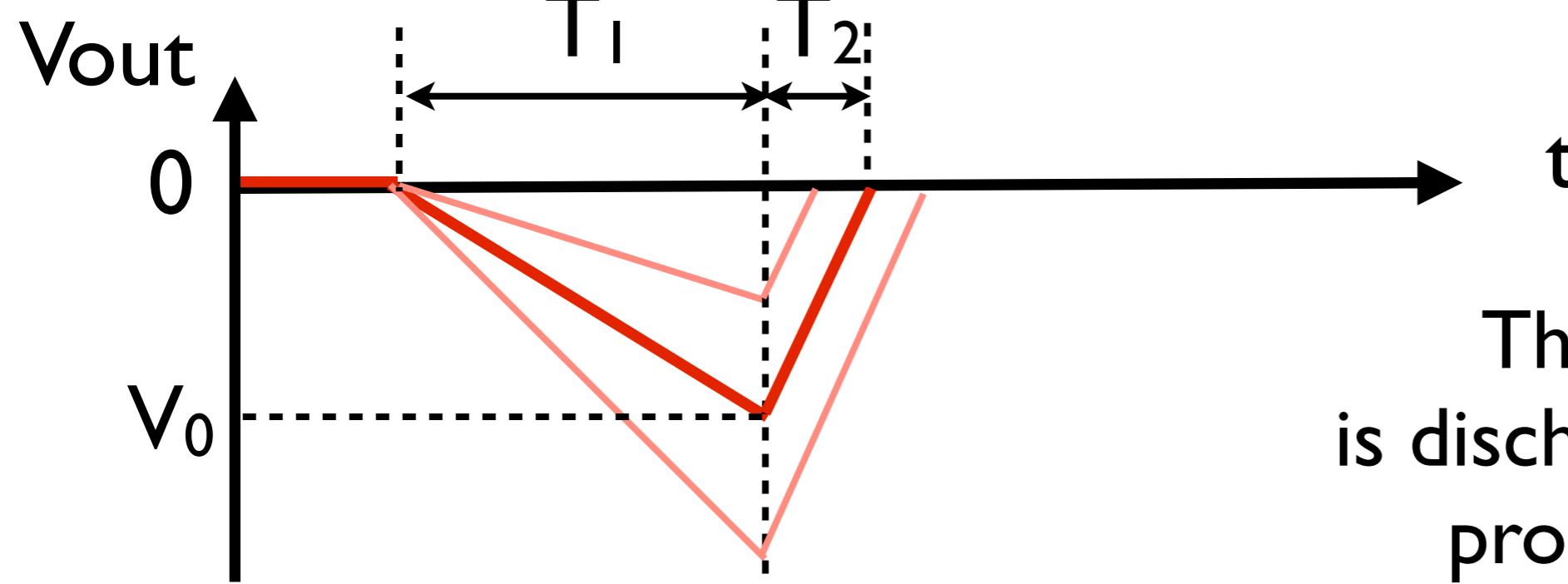
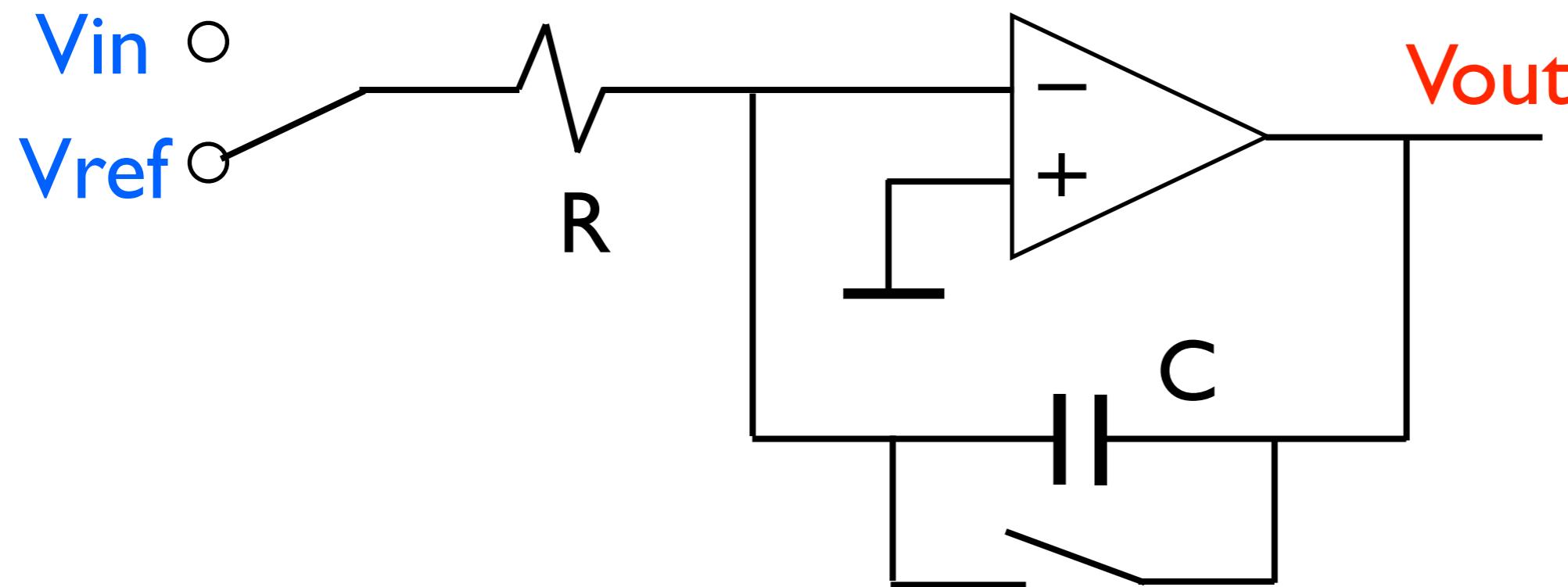
STEP I



$T_I$  is constant  
the higher is  $V_{in}$   
the higher is  $V_0$

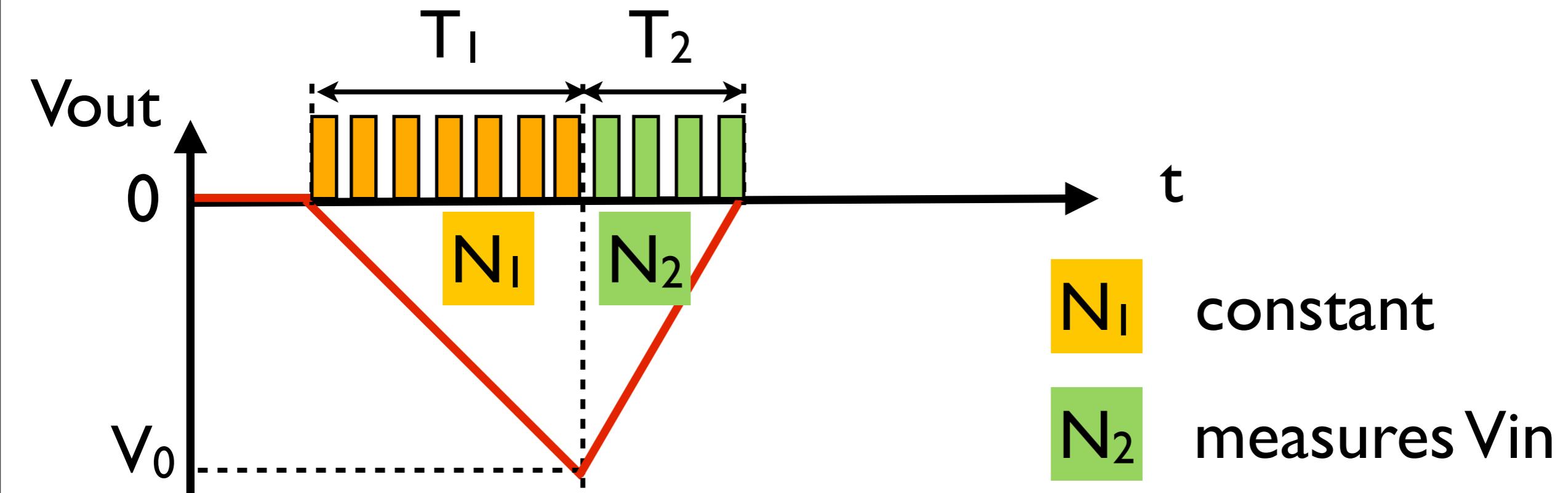
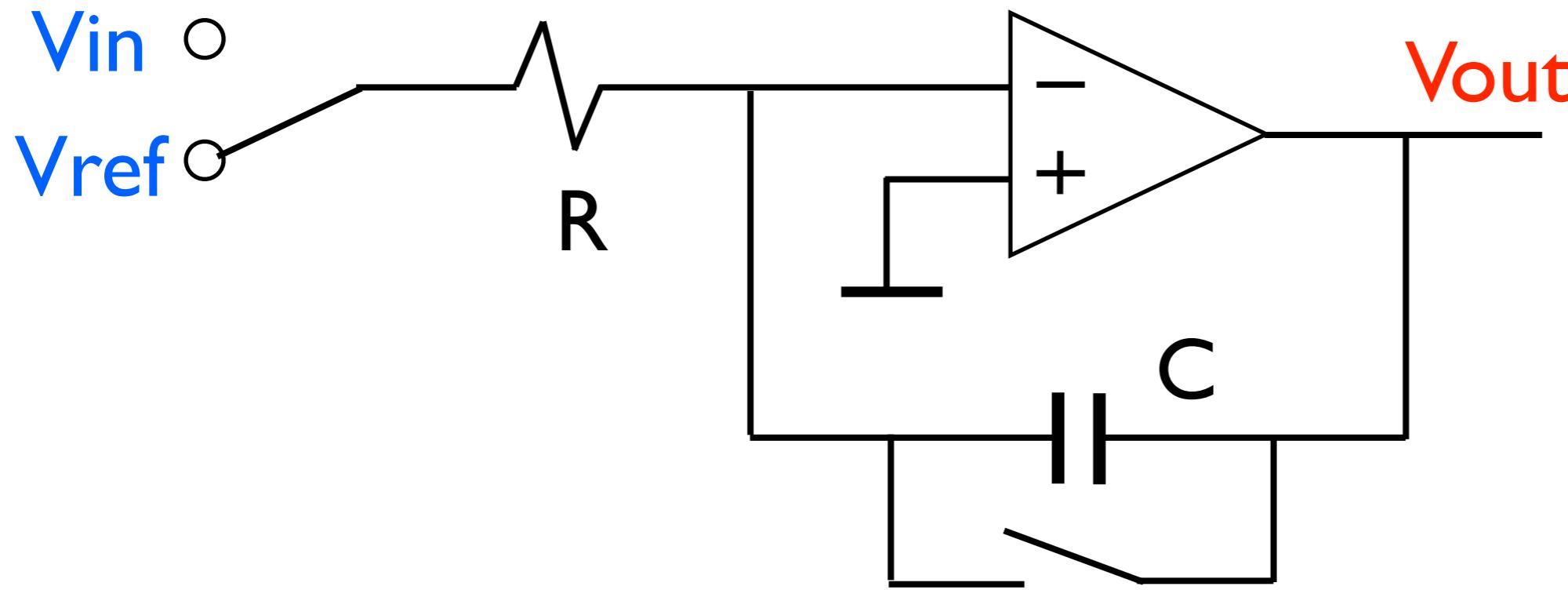
# DUAL SLOPE INTEGRATION ADC

STEP I



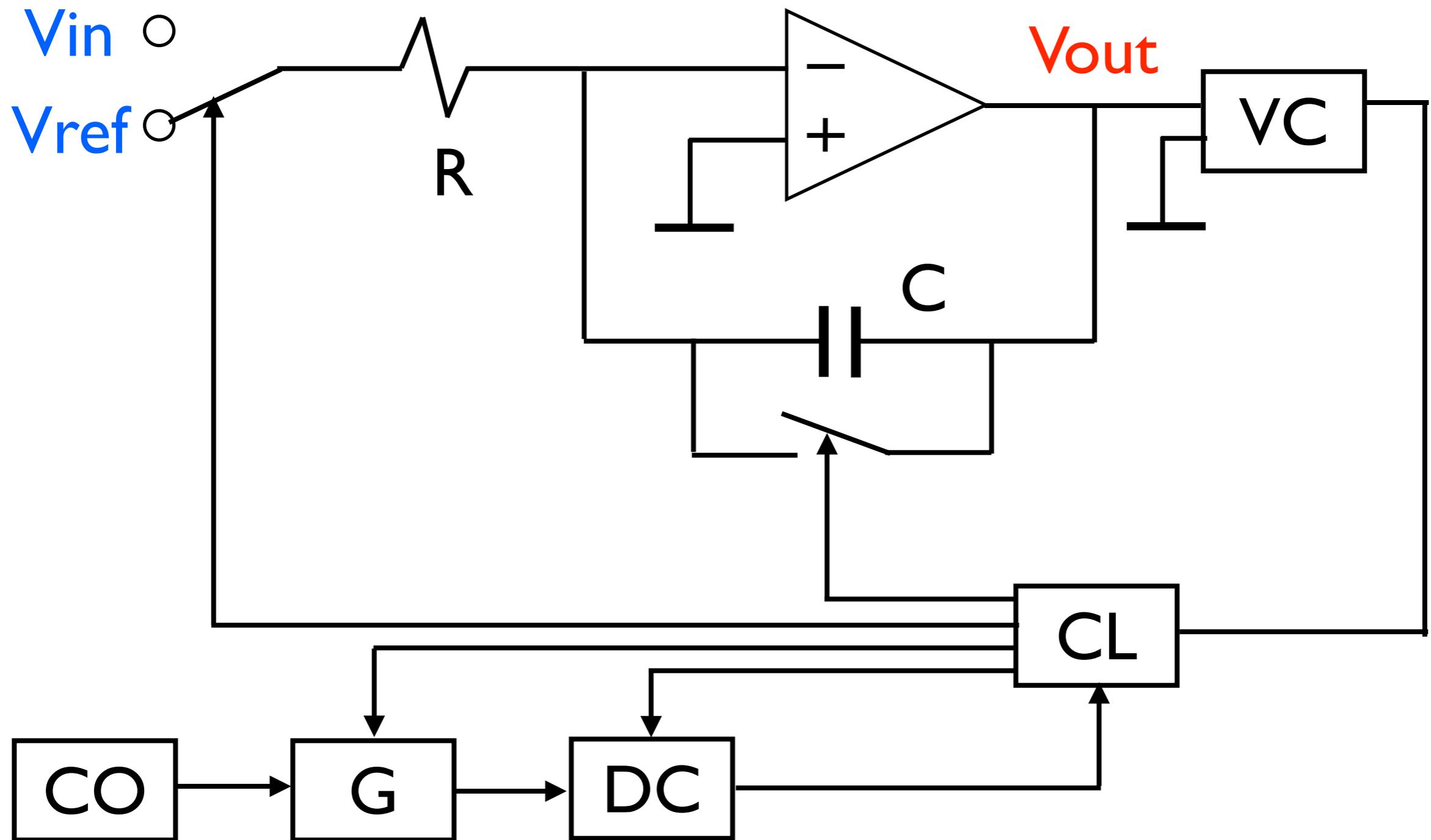
The capacitor  $C$   
is discharged in a time  $T_2$   
proportional to  $V_0$

# DUAL SLOPE INTEGRATION ADC



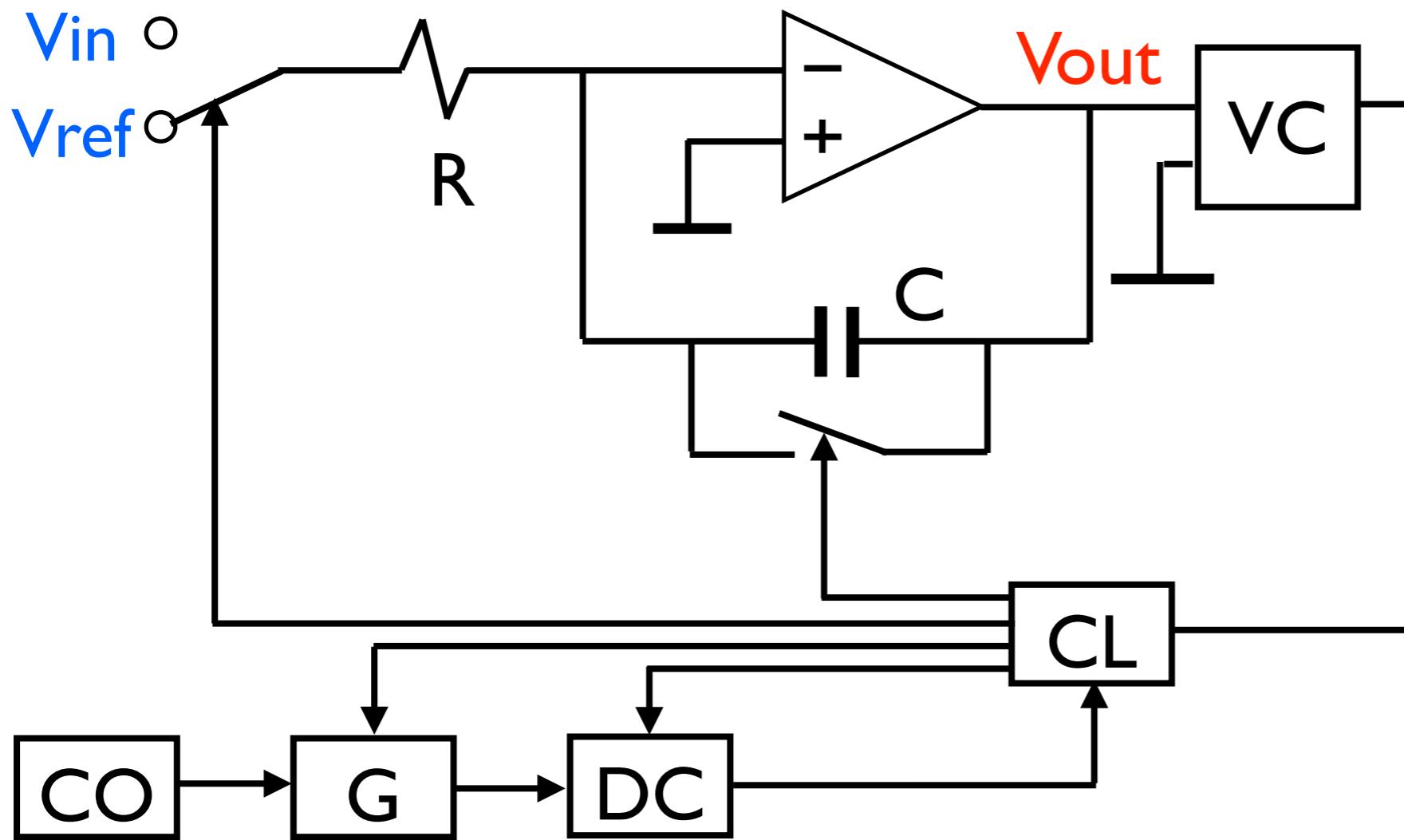
# DUAL SLOPE INTEGRATION ADC

## Detailed structure



CO= crystal oscillator; G=gate; DC=decimal counter;  
CL=control logic; VC=voltage comparator

# DUAL SLOPE INTEGRATION ADC



Very precise, but also very slow  
Used in multimeters

# SIGMA DELTA

Basic idea:

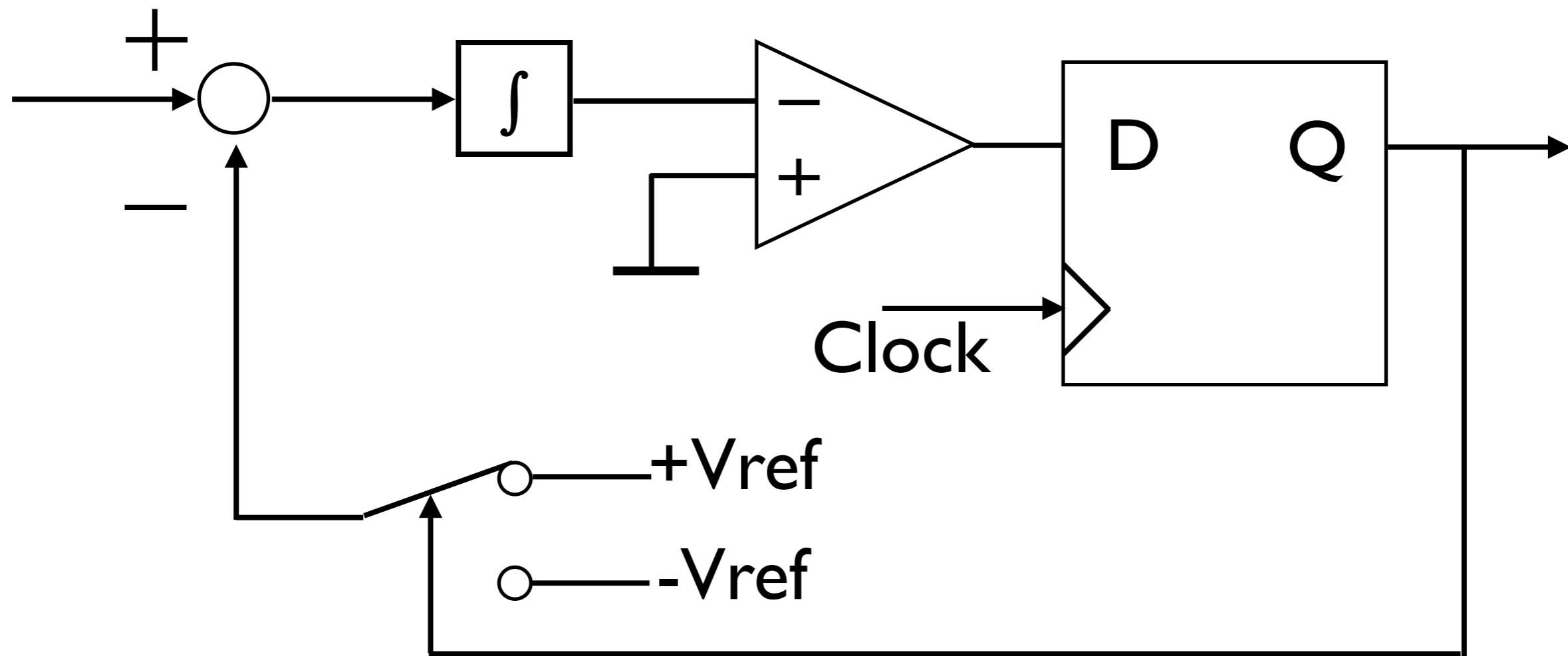
sample the signal with  
low resolution  
and high sampling frequency

then you can work on the data and  
obtain  
high resolution  
with low sampling frequency

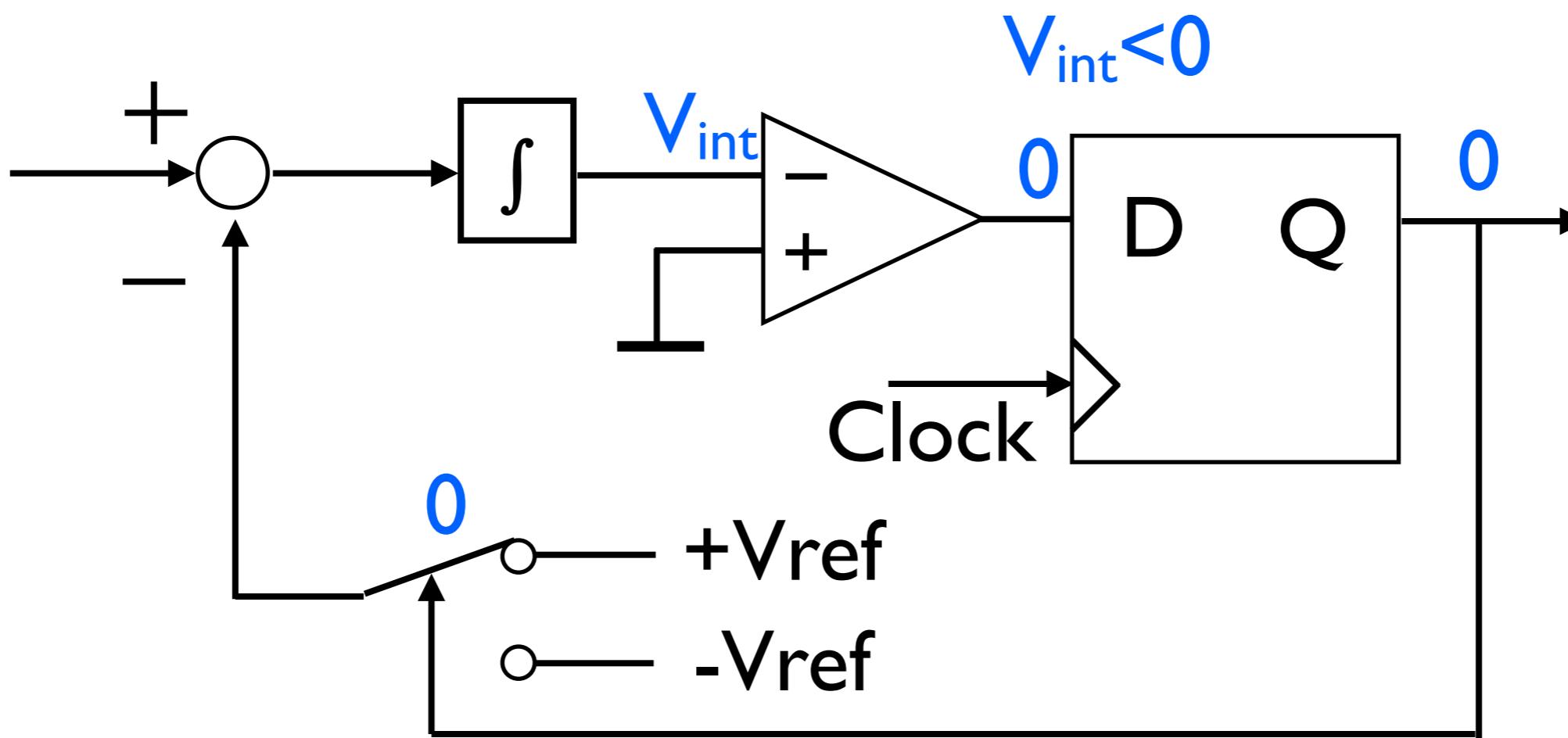
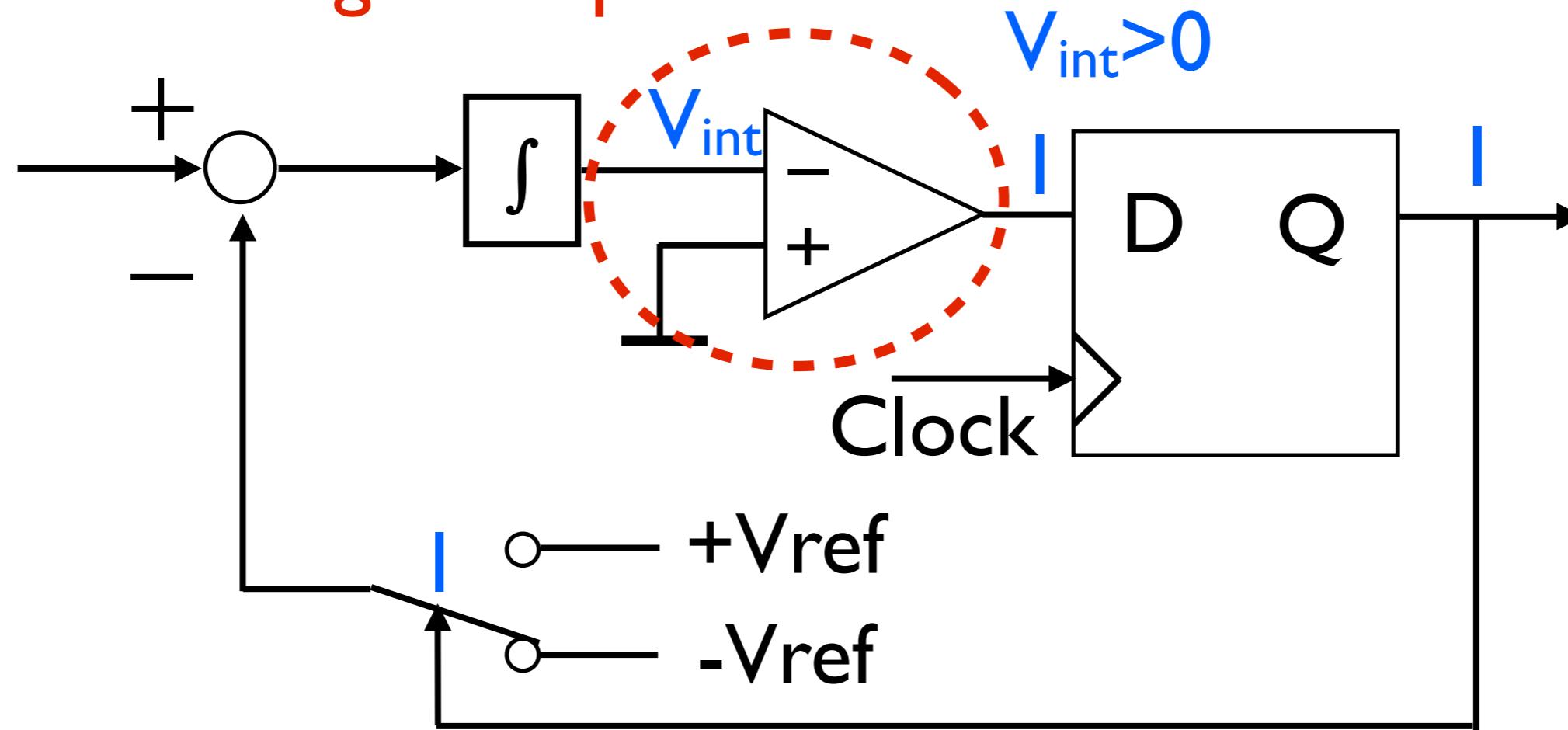
# SIGMA DELTA

One bit sampling with ultra high sampling frequency

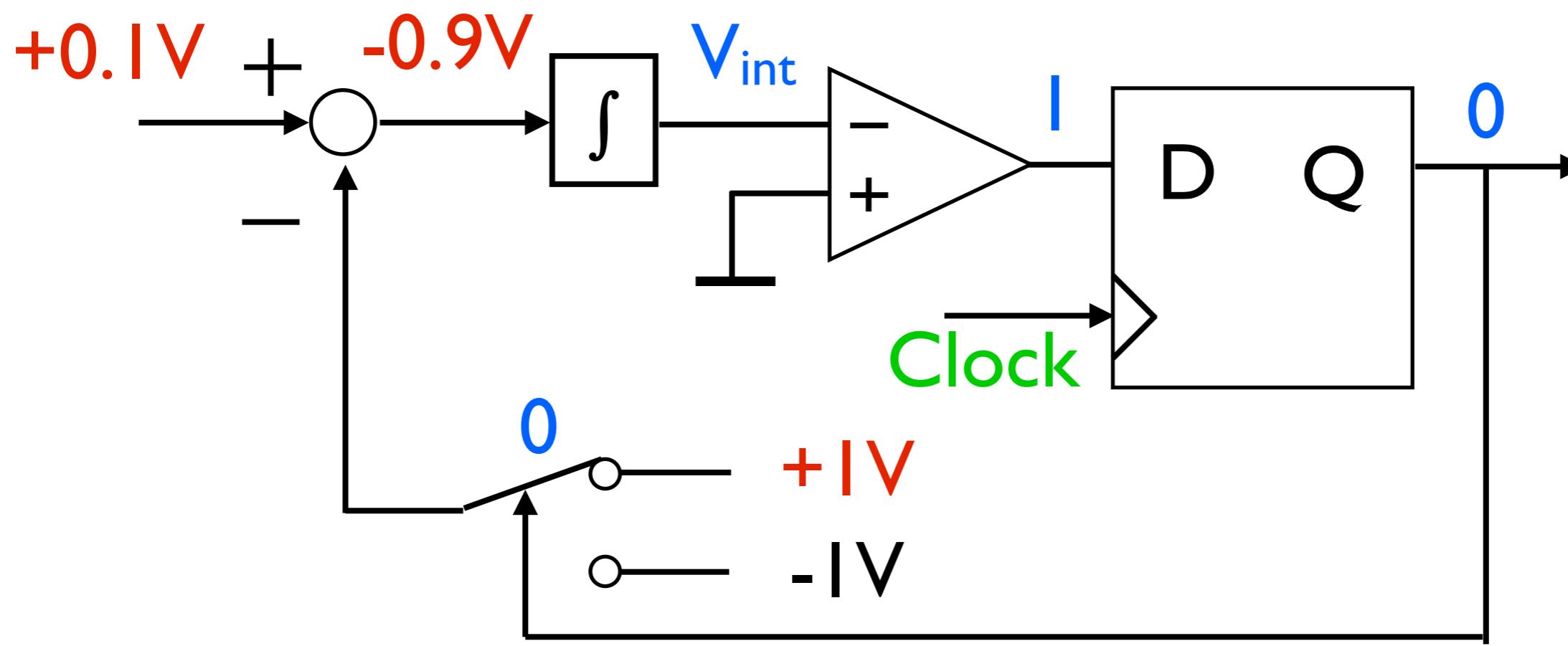
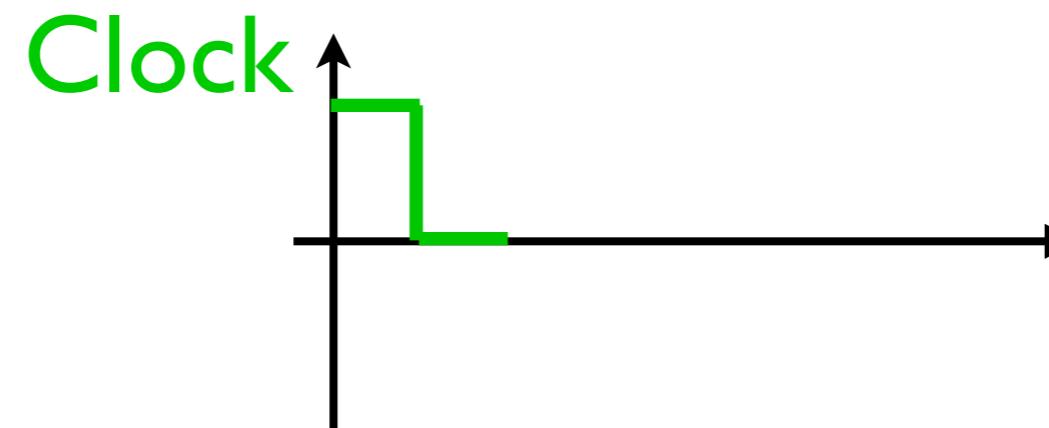
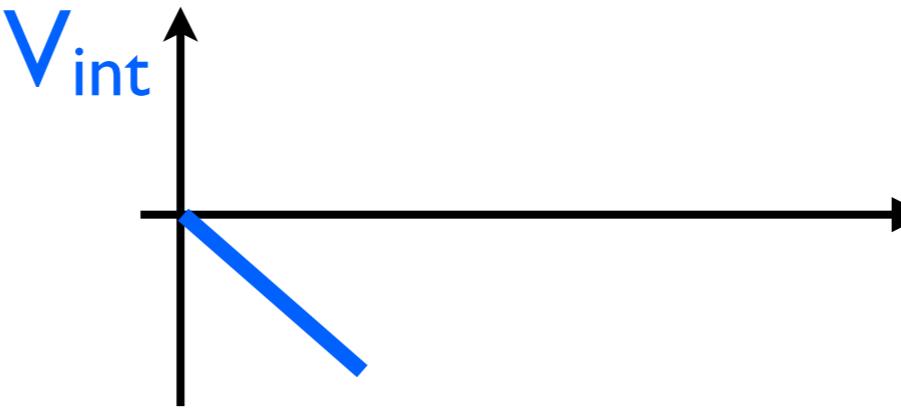
Structure

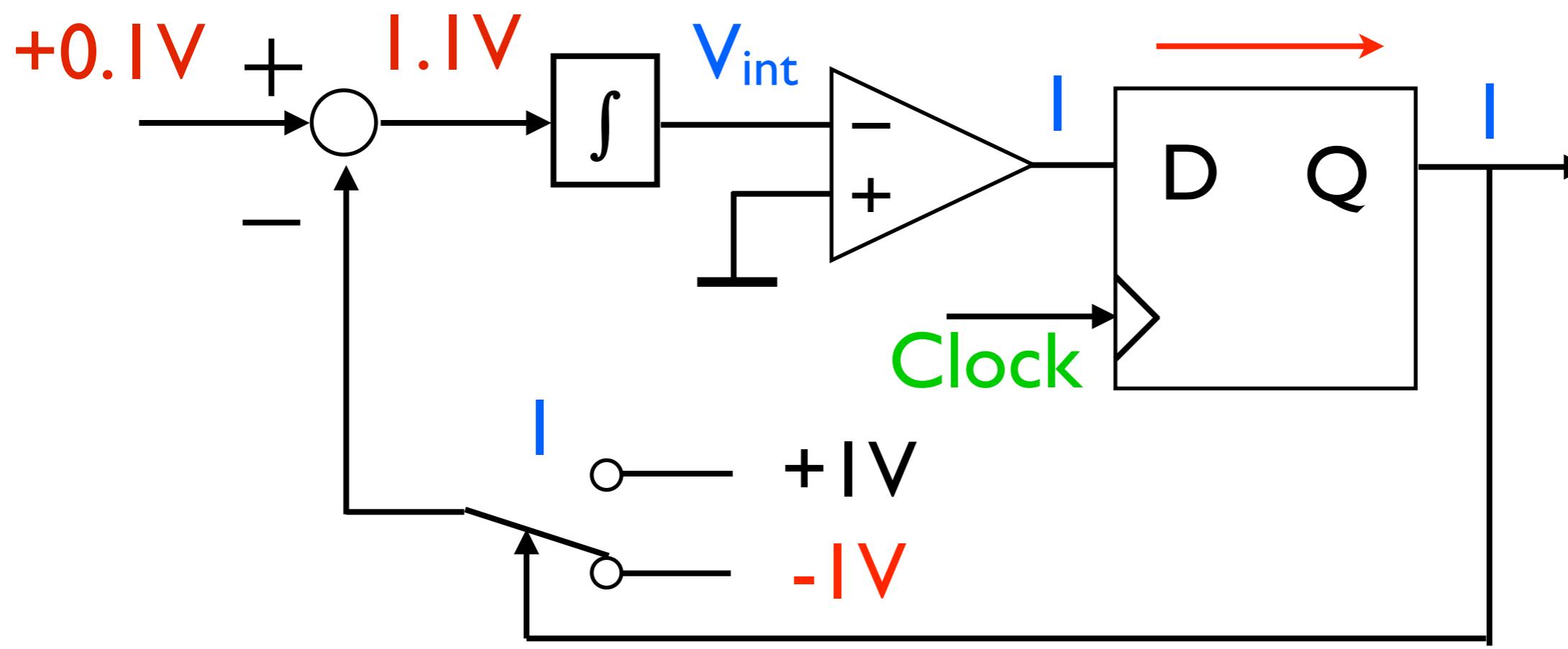
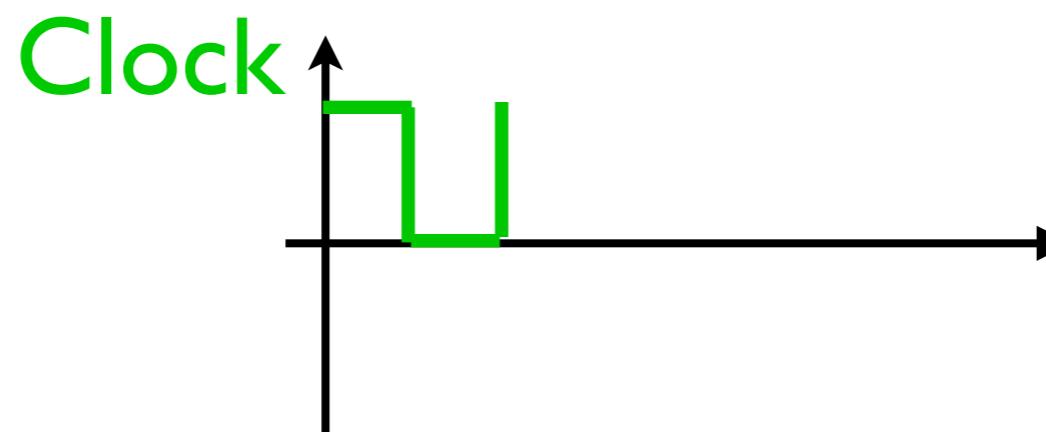
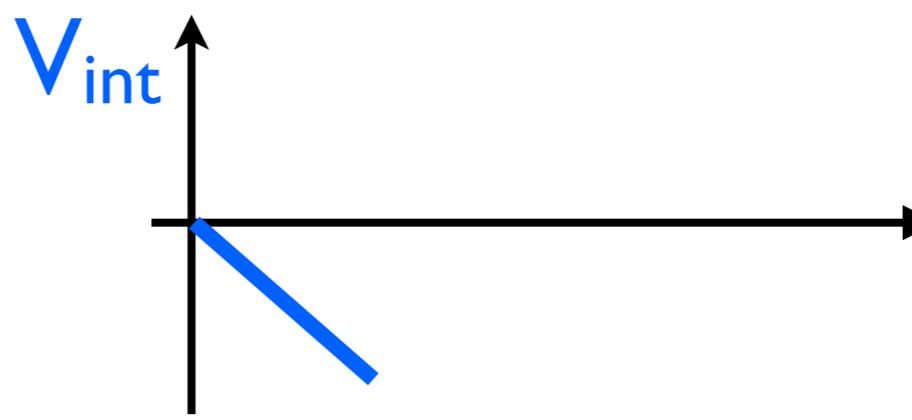


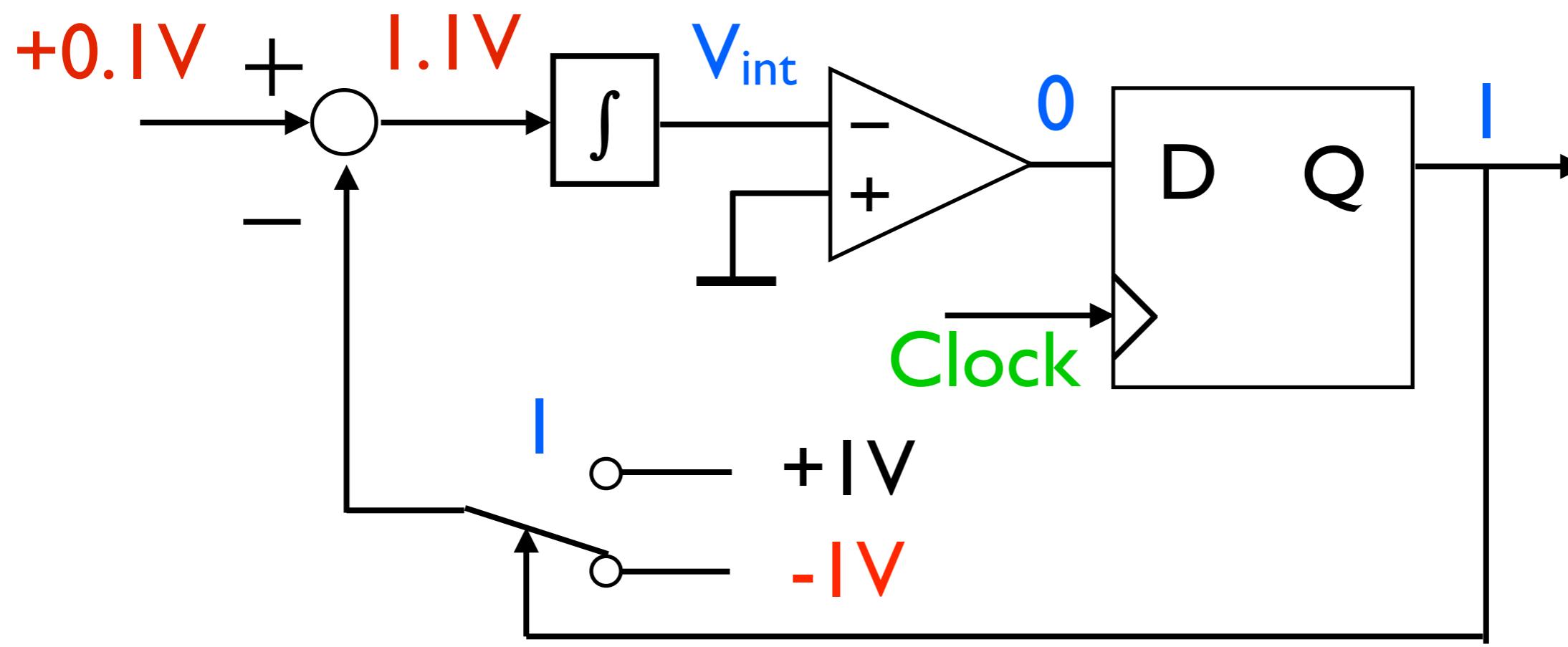
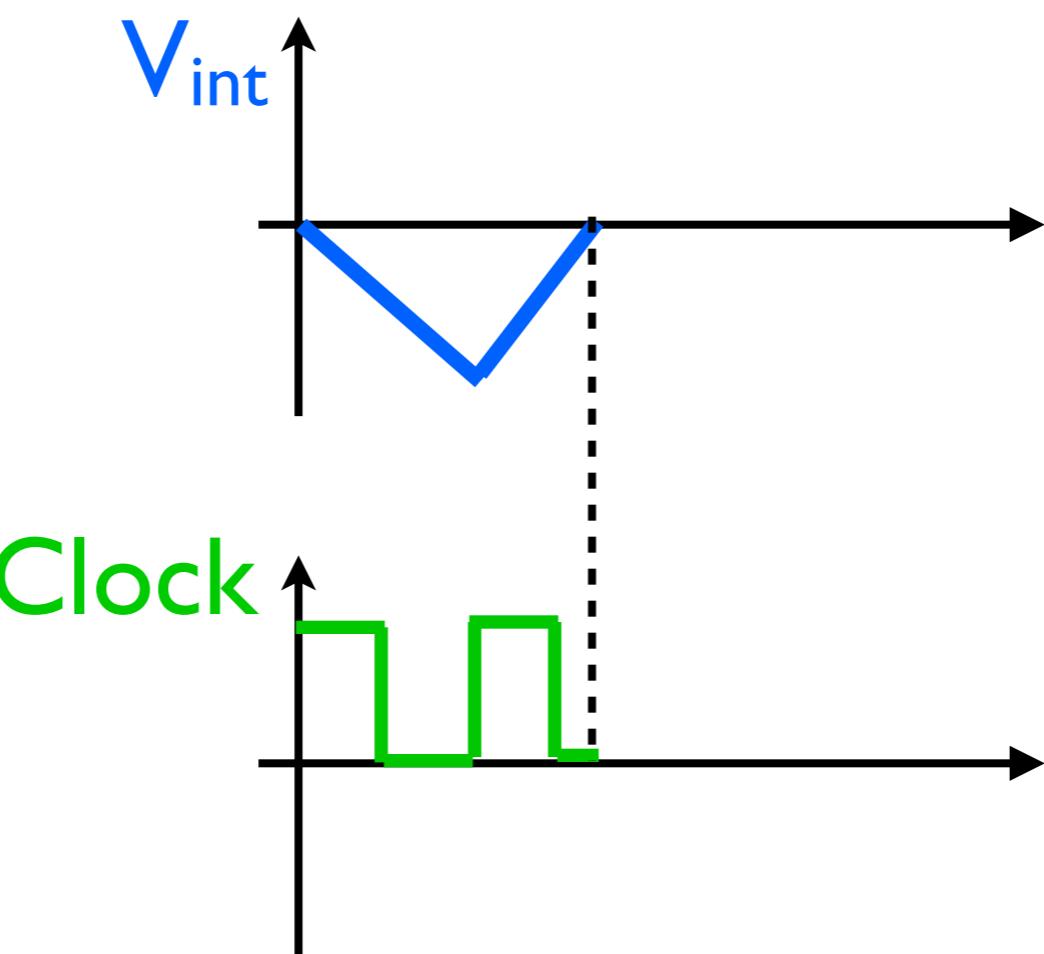
# Voltage comparator

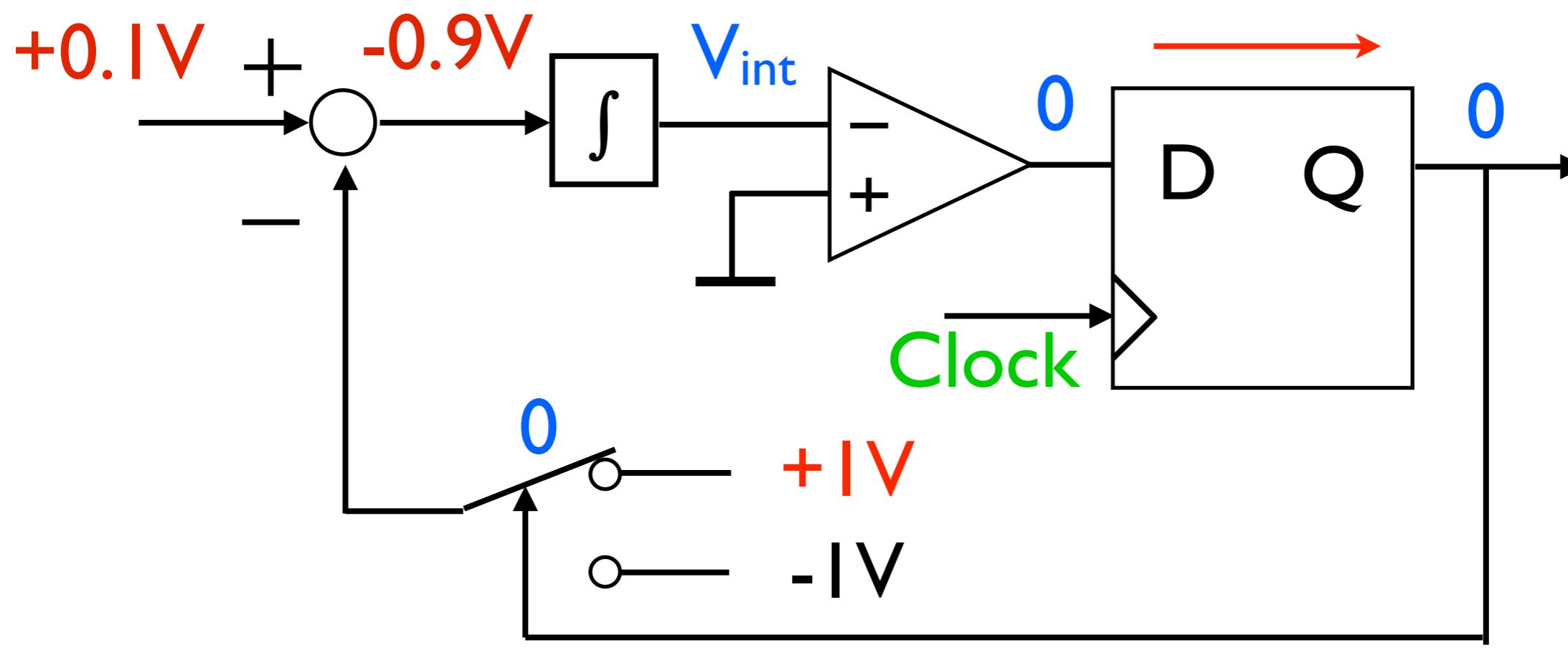
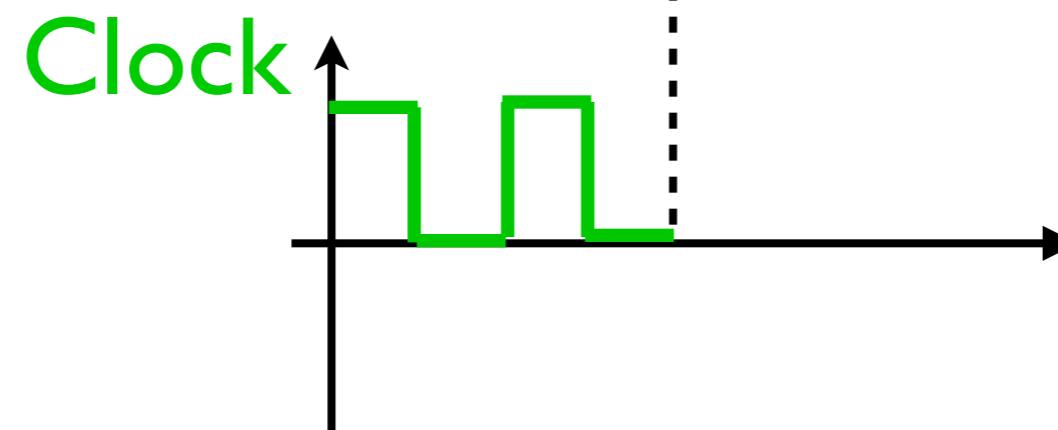
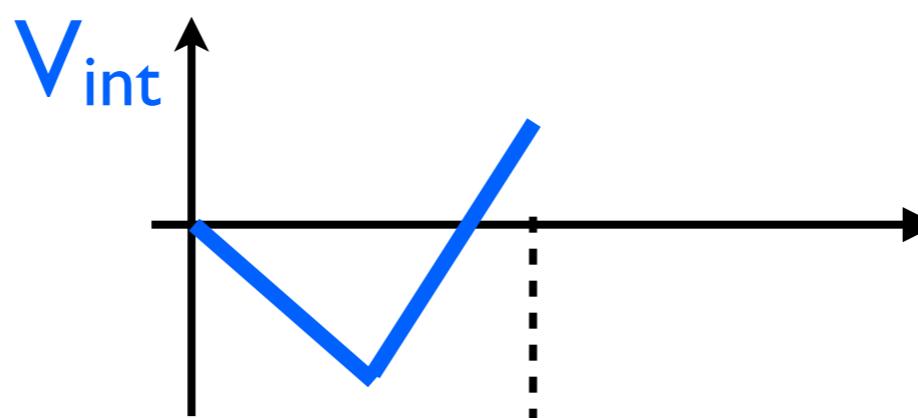


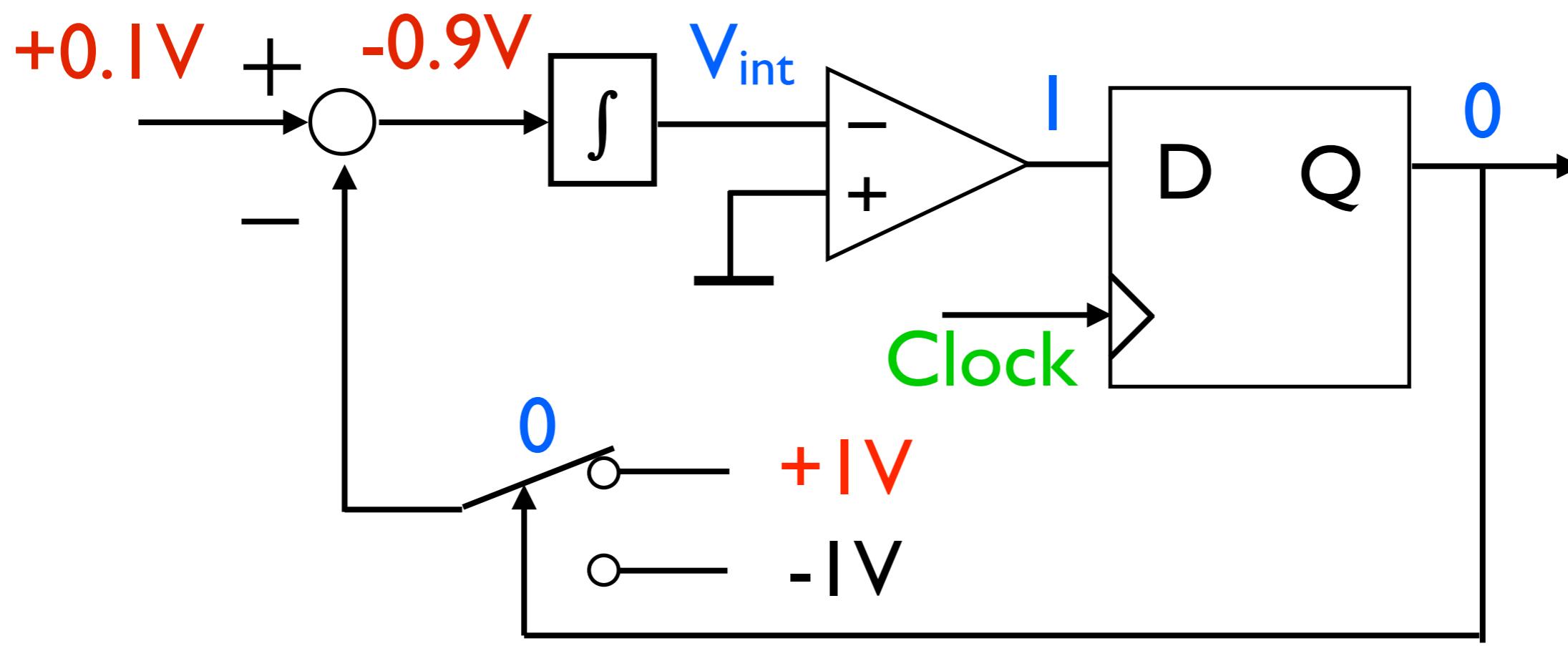
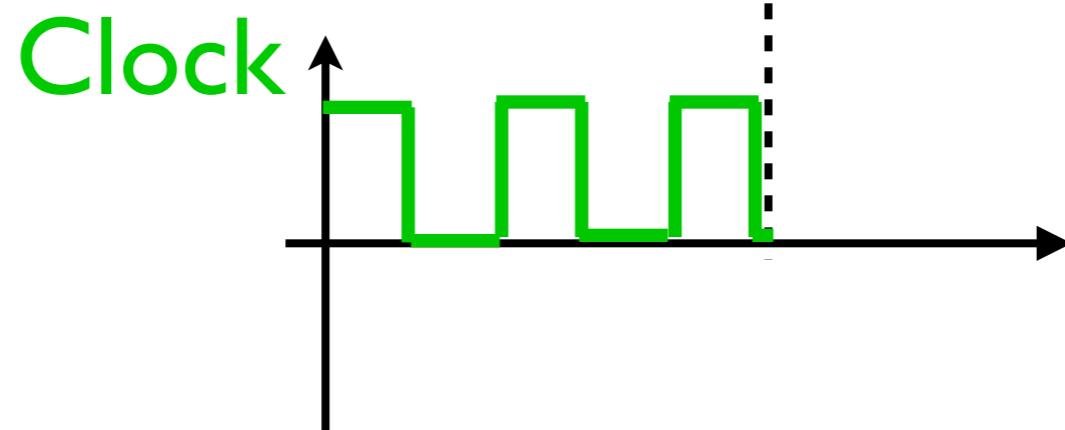
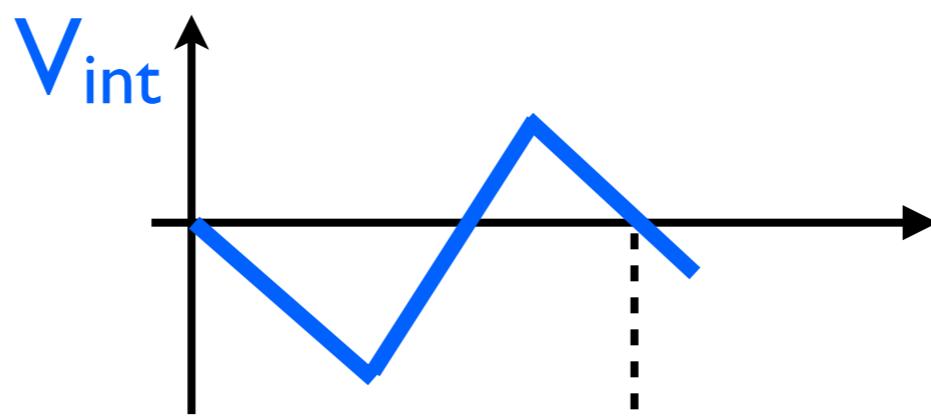
Initial condition:  
integrator=0  
 $Q=0$

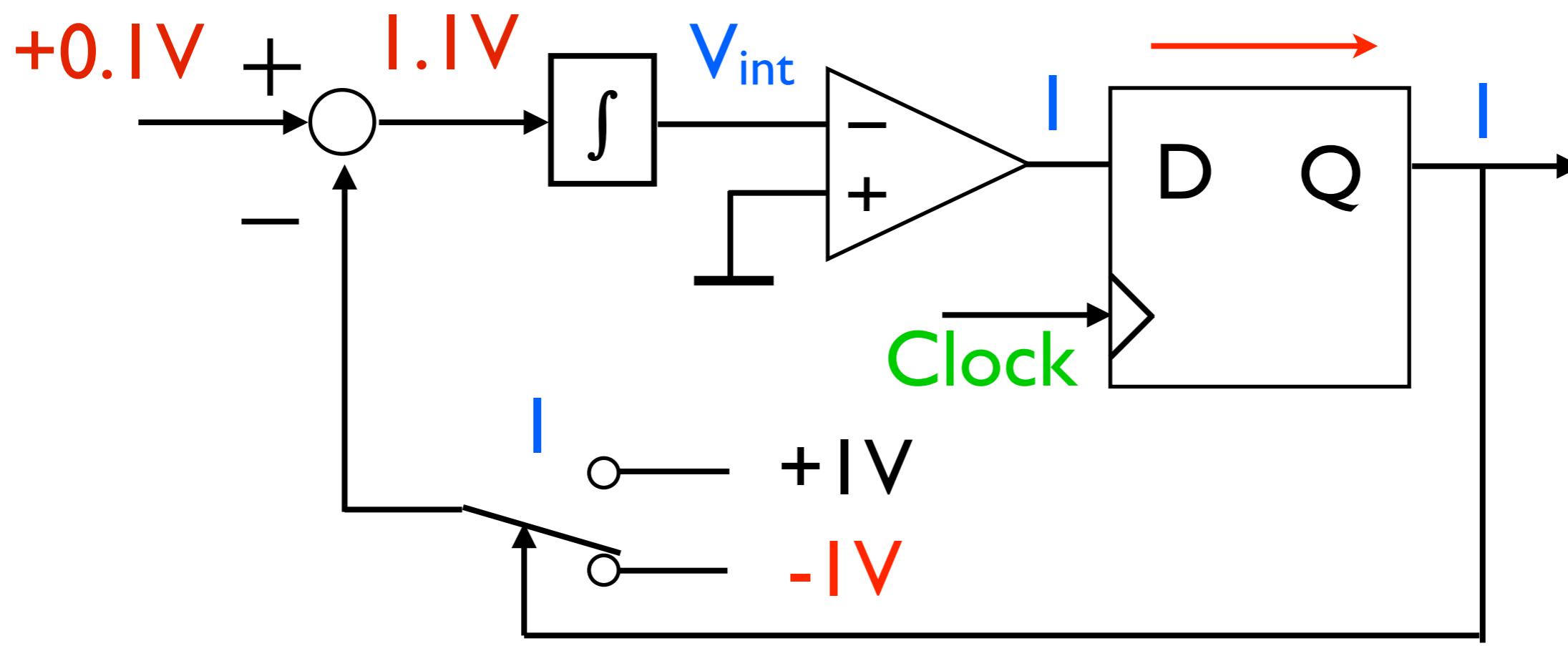
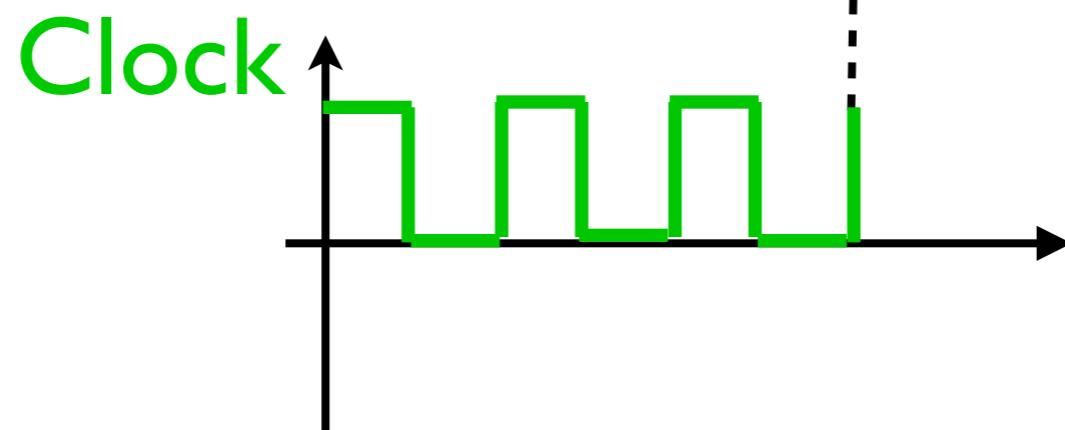
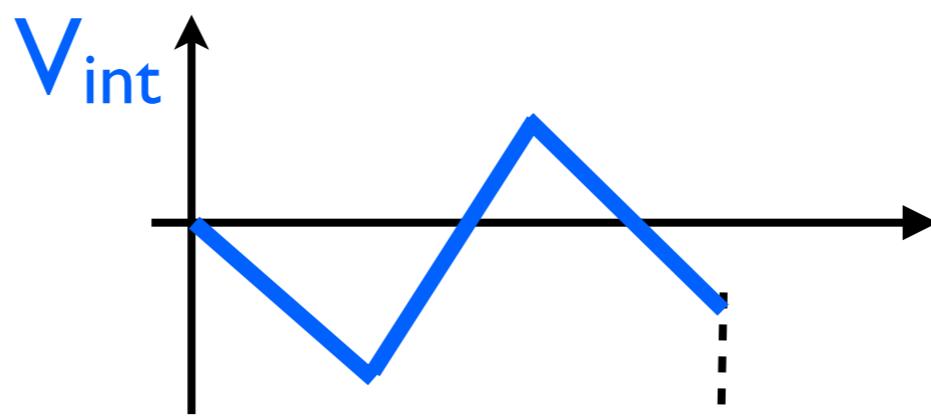


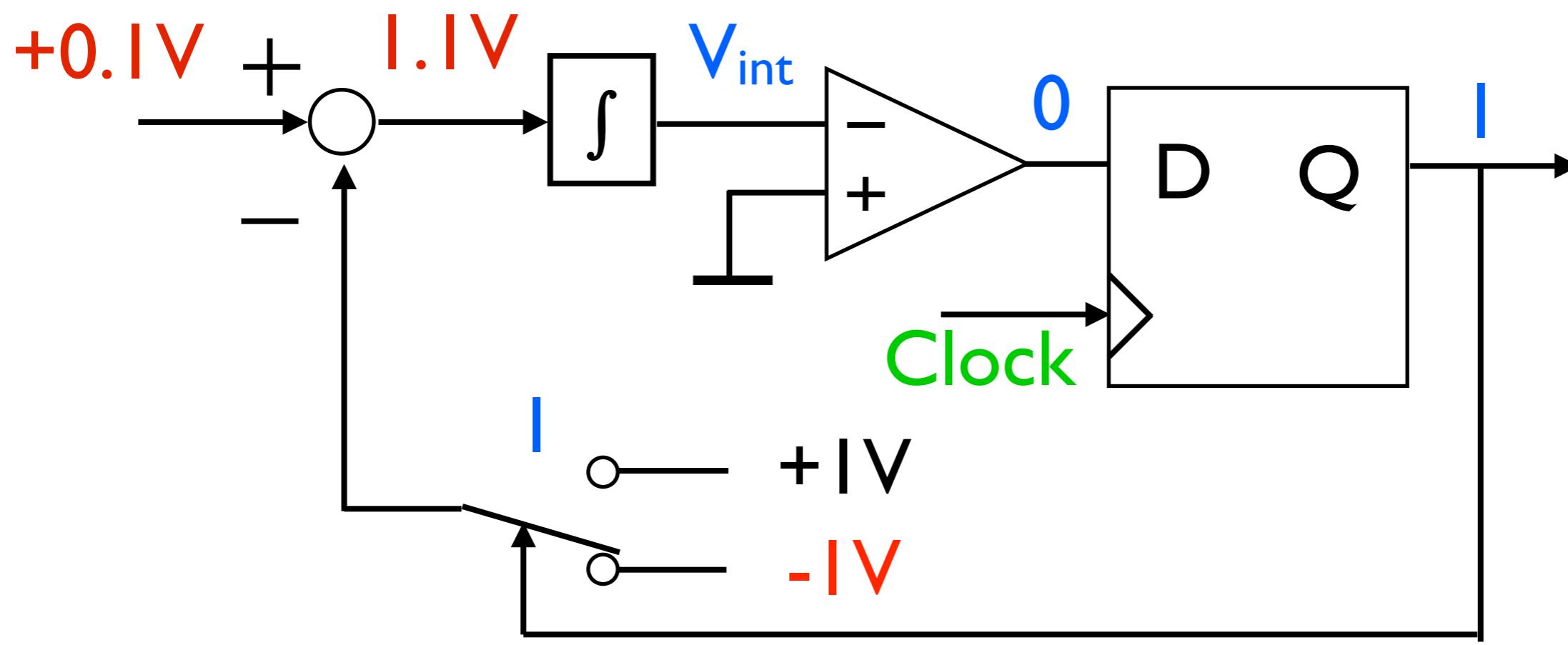
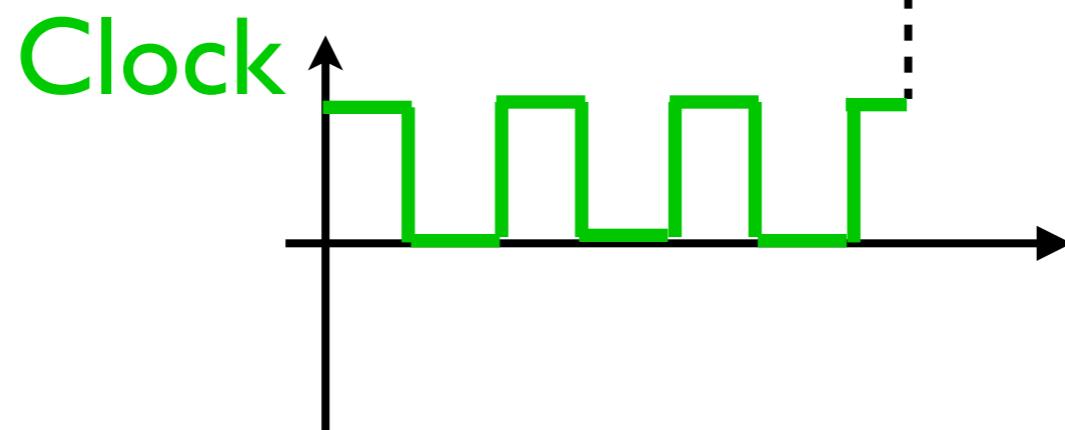
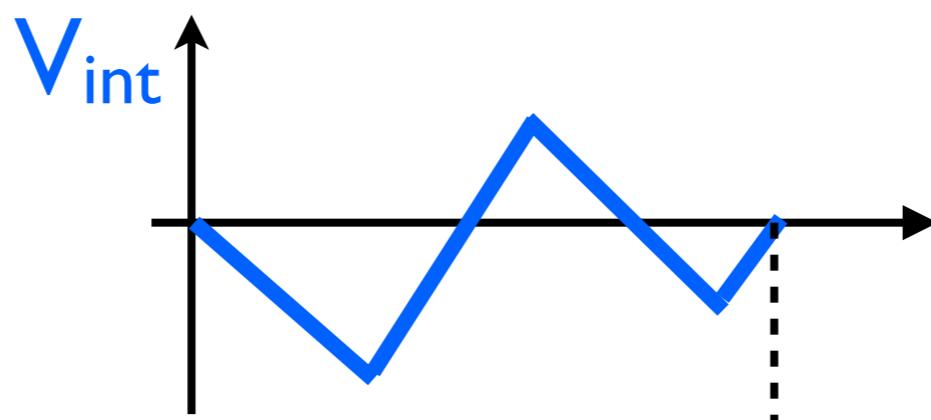


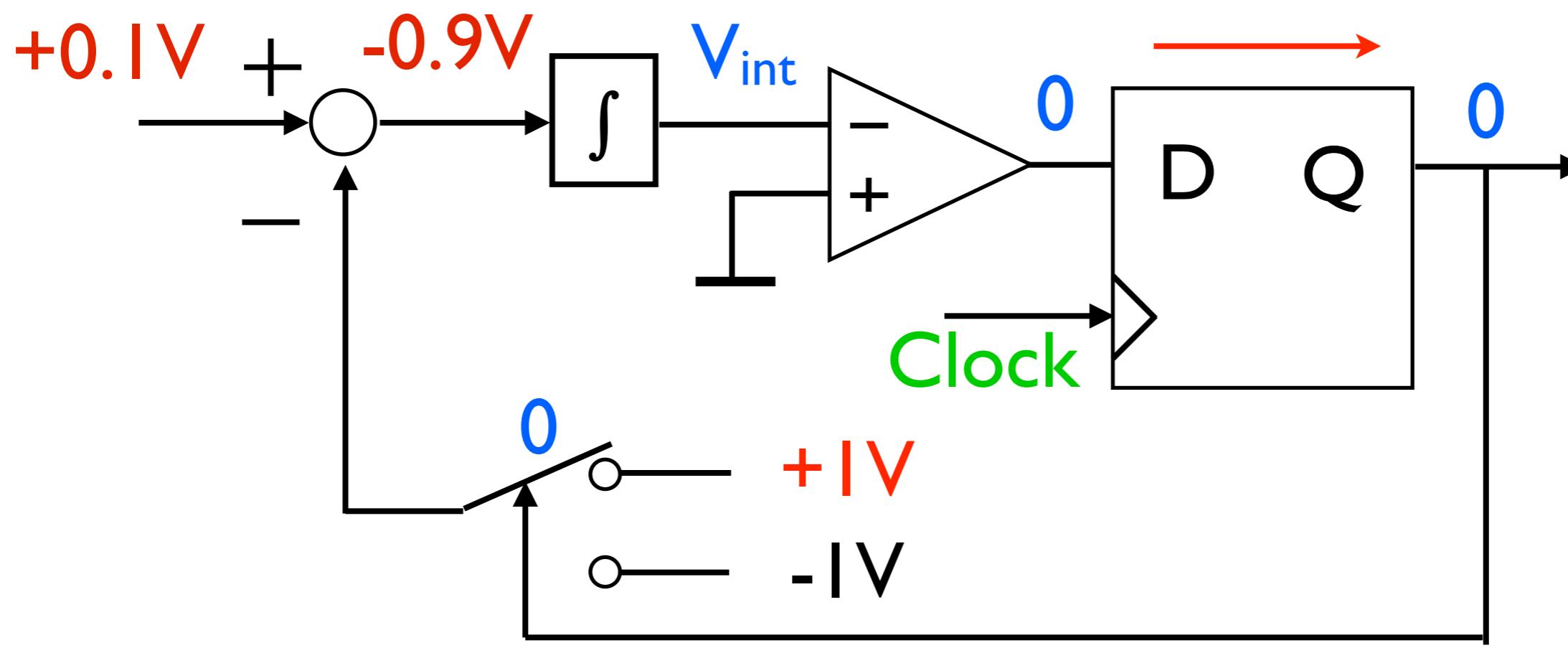
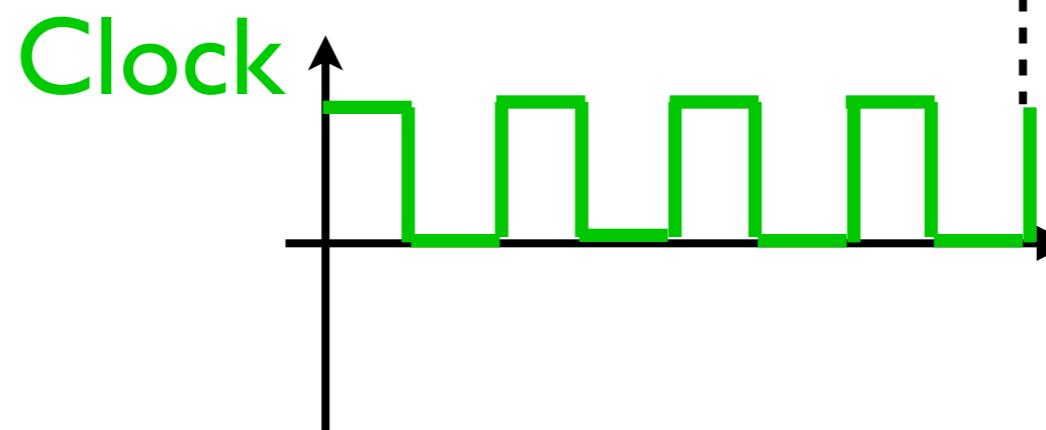
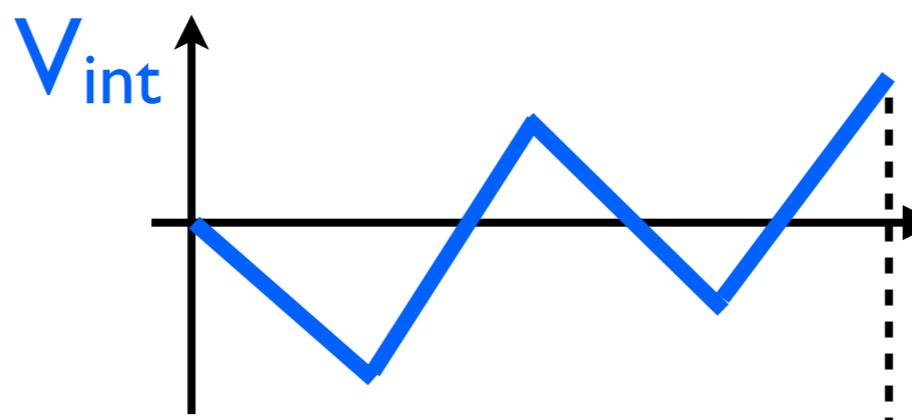


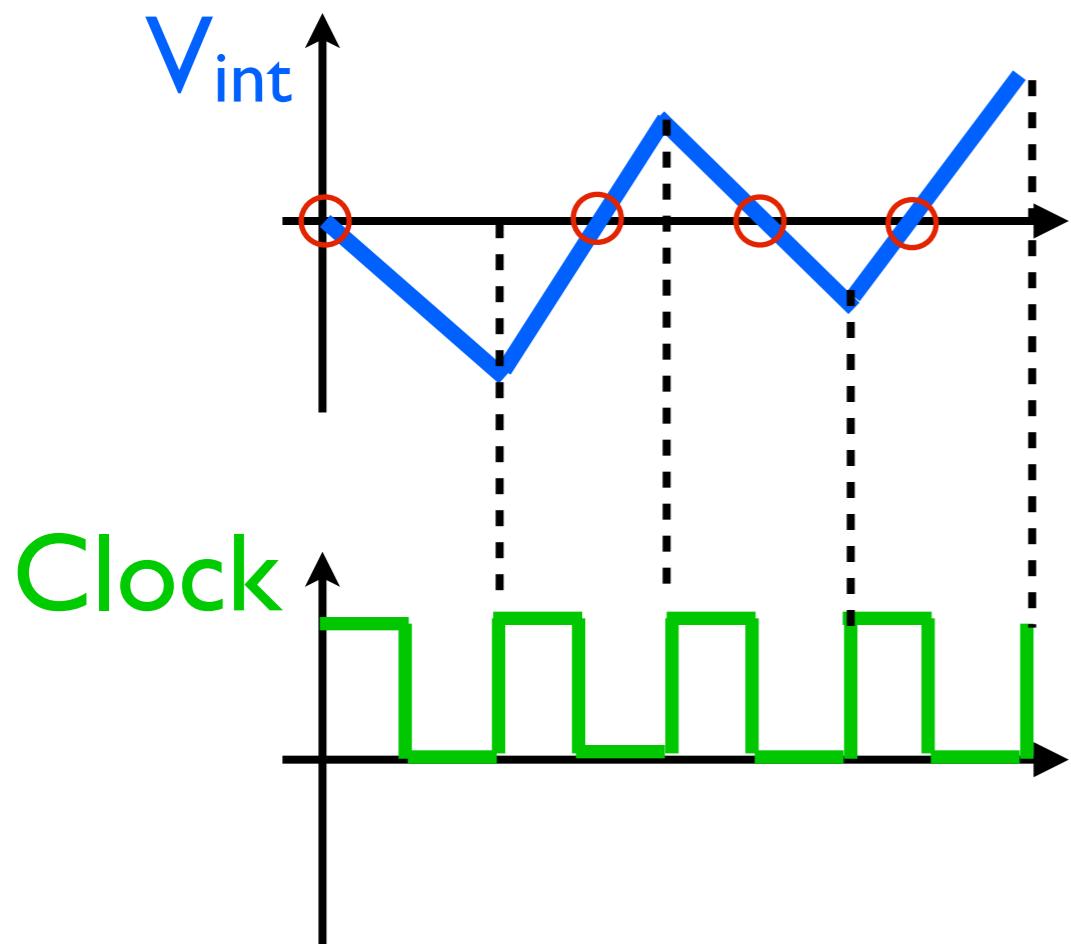




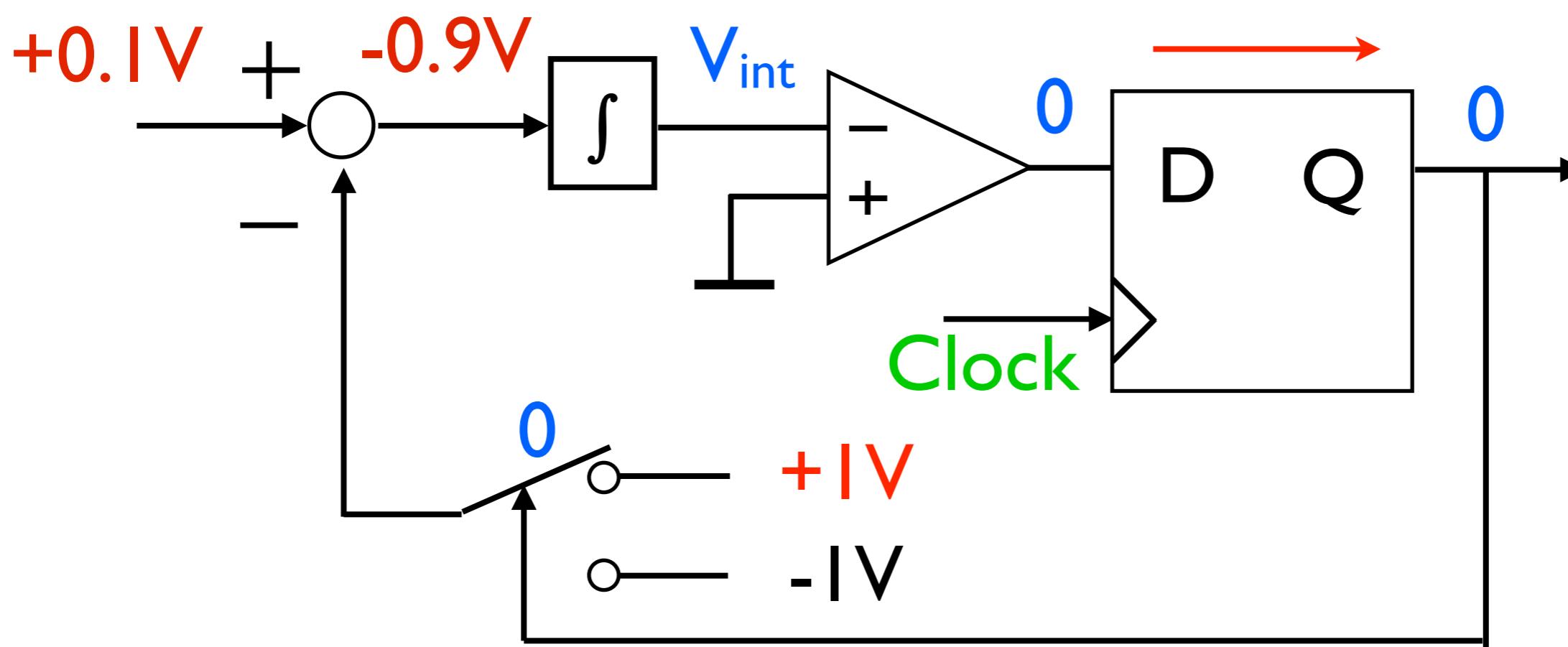








**Q** inverts its status at  
every rising edge following  
a change of sign of  $V_{int}$



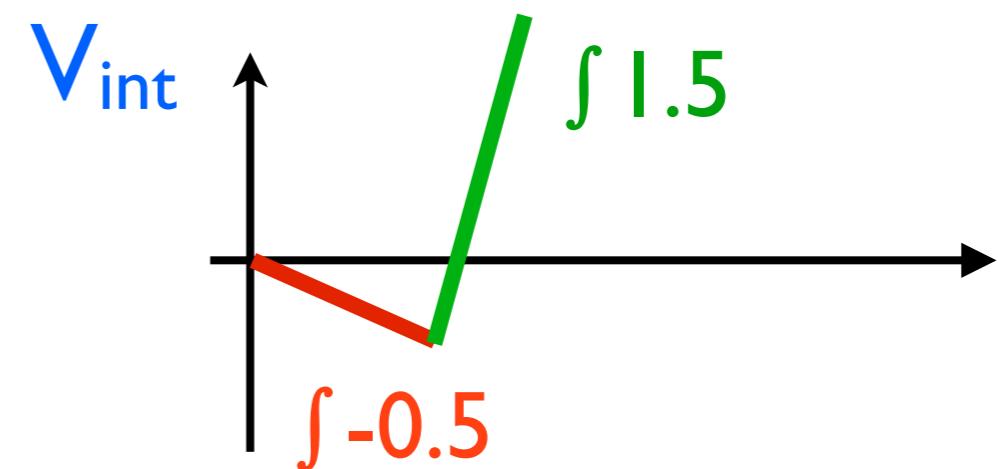
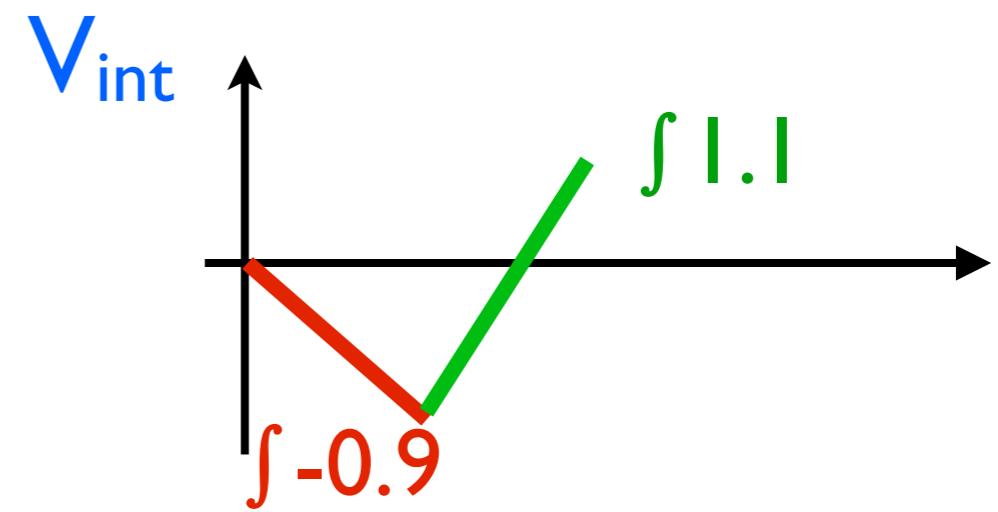
Ex.  $V_{ref} = 1V$

$$V_i = 0.1V$$

$$\begin{aligned} V_{diff} &= 1.1 \\ V_{diff} &= -0.9 \end{aligned}$$

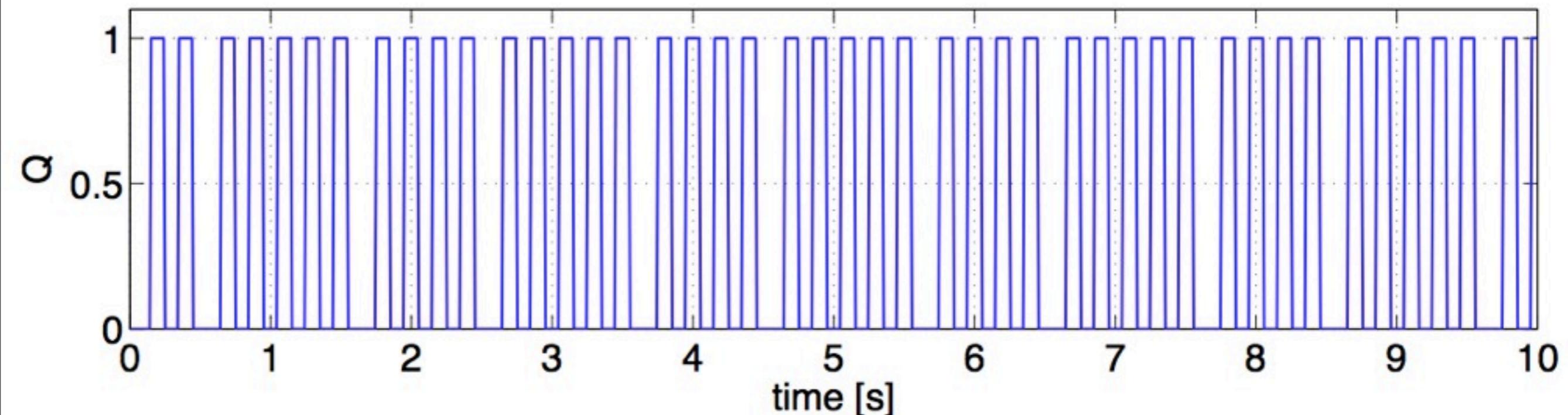
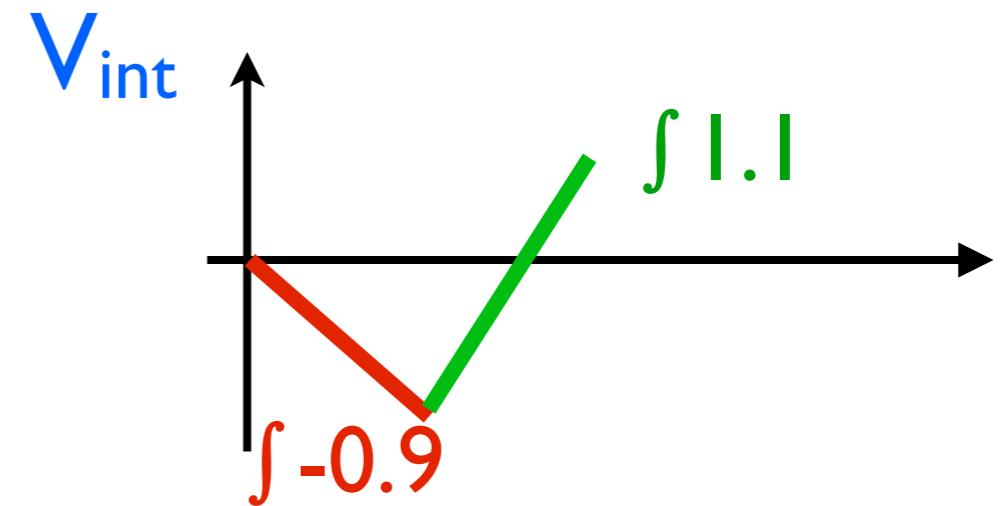
$$V_i = 0.5V$$

$$\begin{aligned} V_{diff} &= 1.5 \\ V_{diff} &= -0.5 \end{aligned}$$



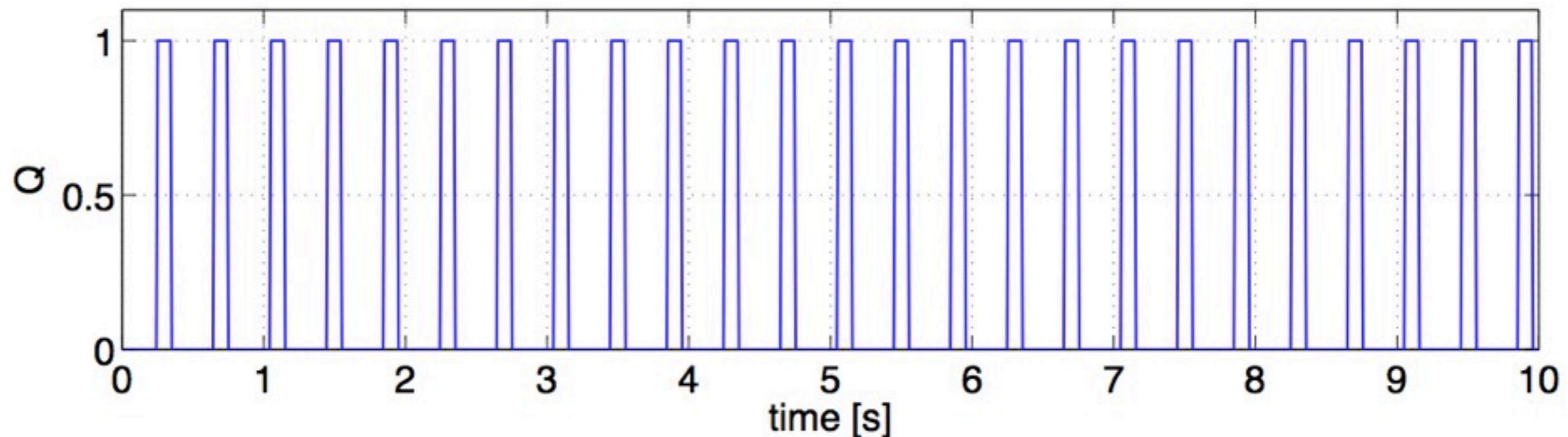
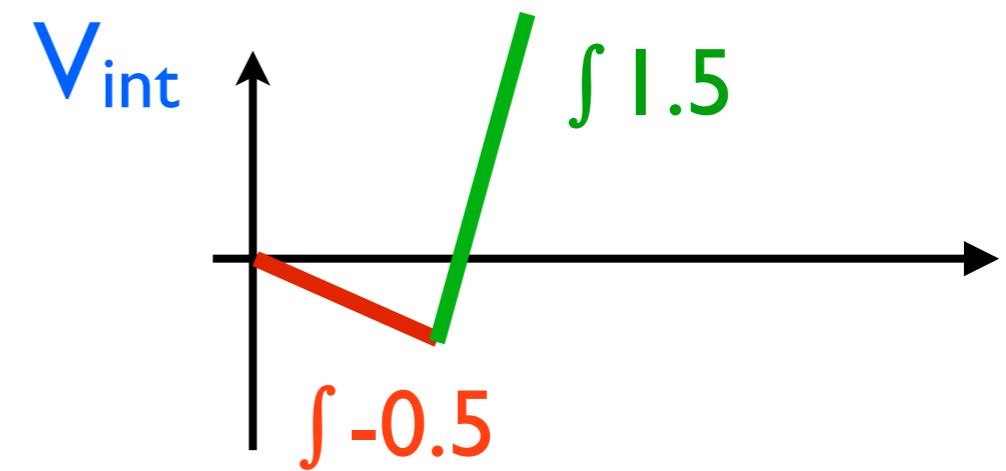
$V_i = 0.1V$

$V_{diff} = 1.1$   
 $V_{diff} = -0.9$

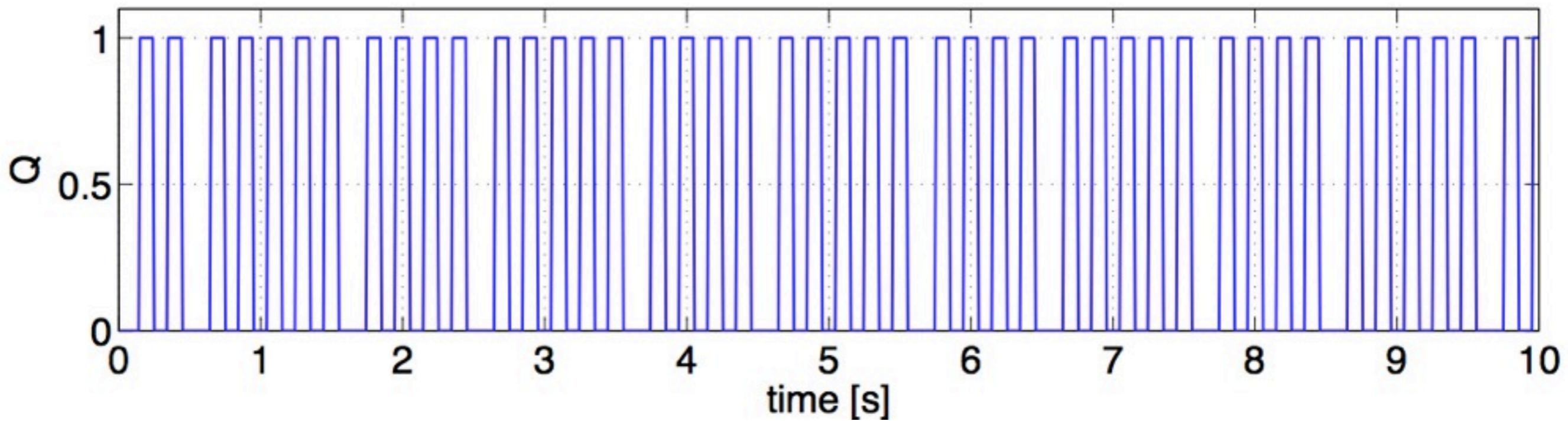


$V_i = 0.5V$

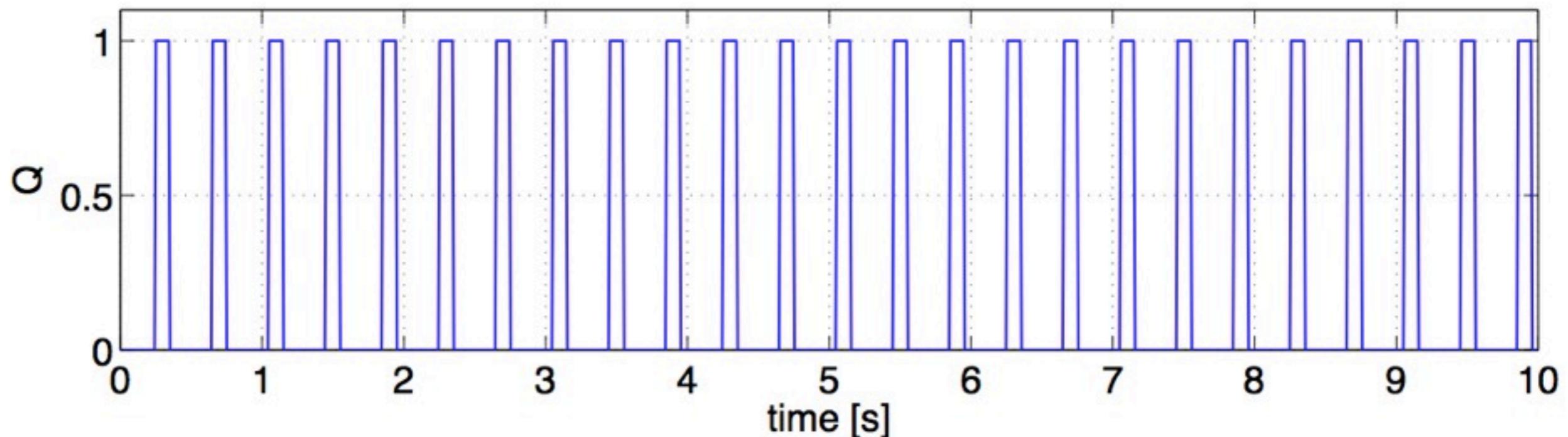
$V_{diff} = 1.5$   
 $V_{diff} = -0.5$



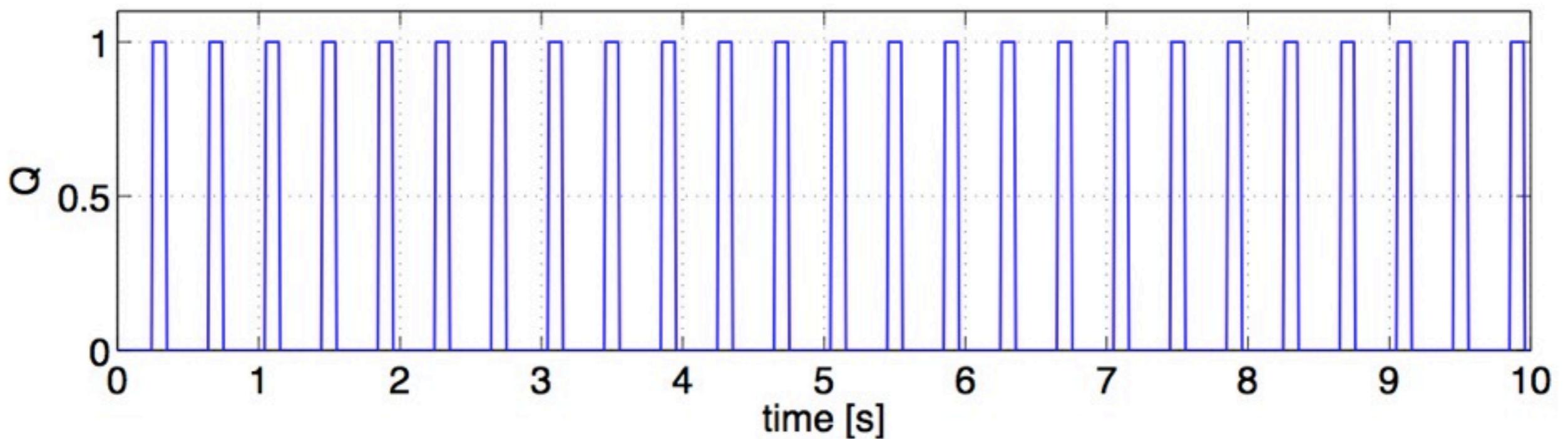
$V_i=0.1V$



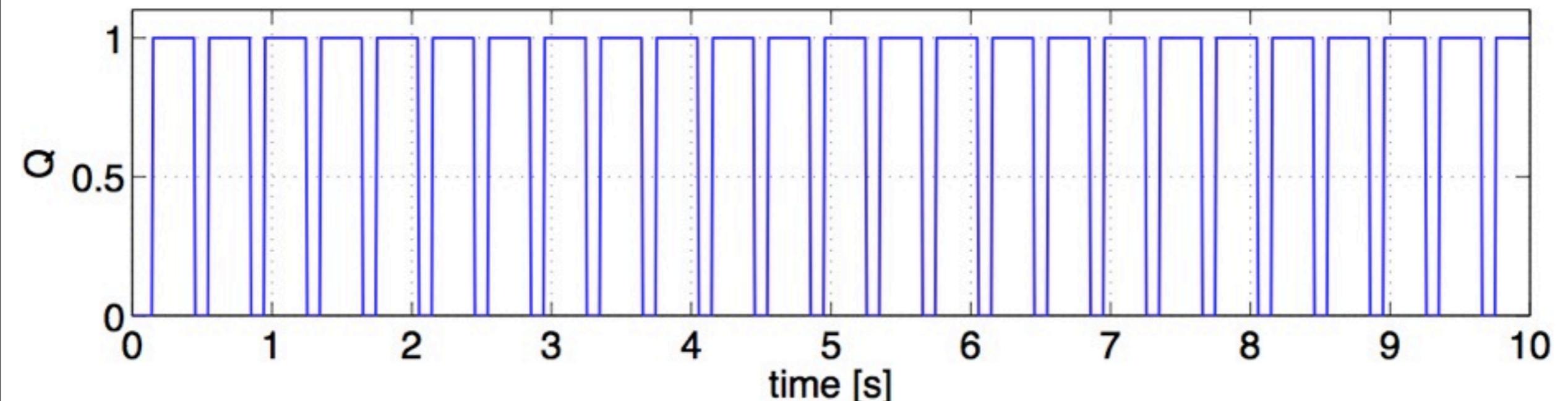
$V_i=0.5V$



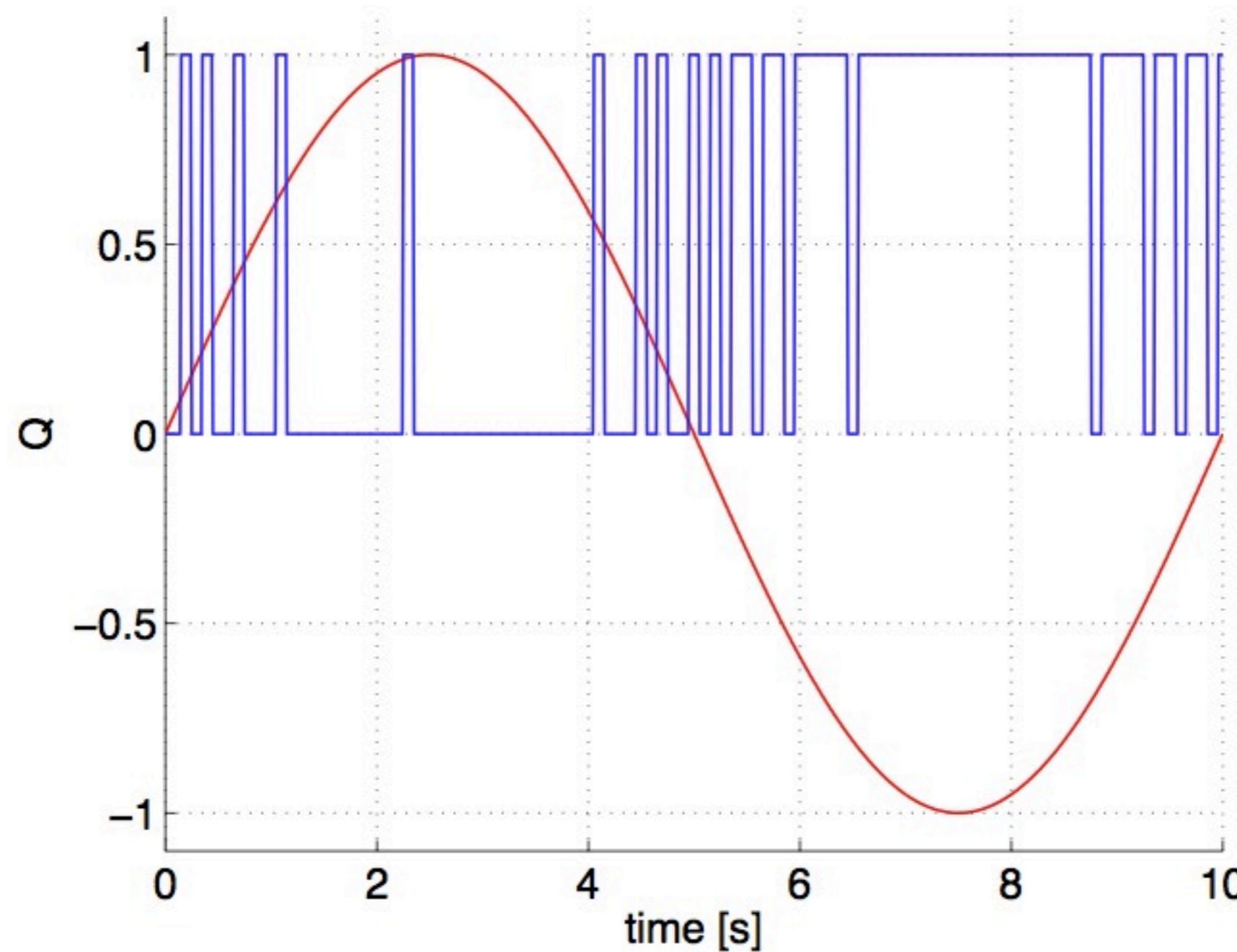
$V_i=0.5V$



$V_i=-0.5V$

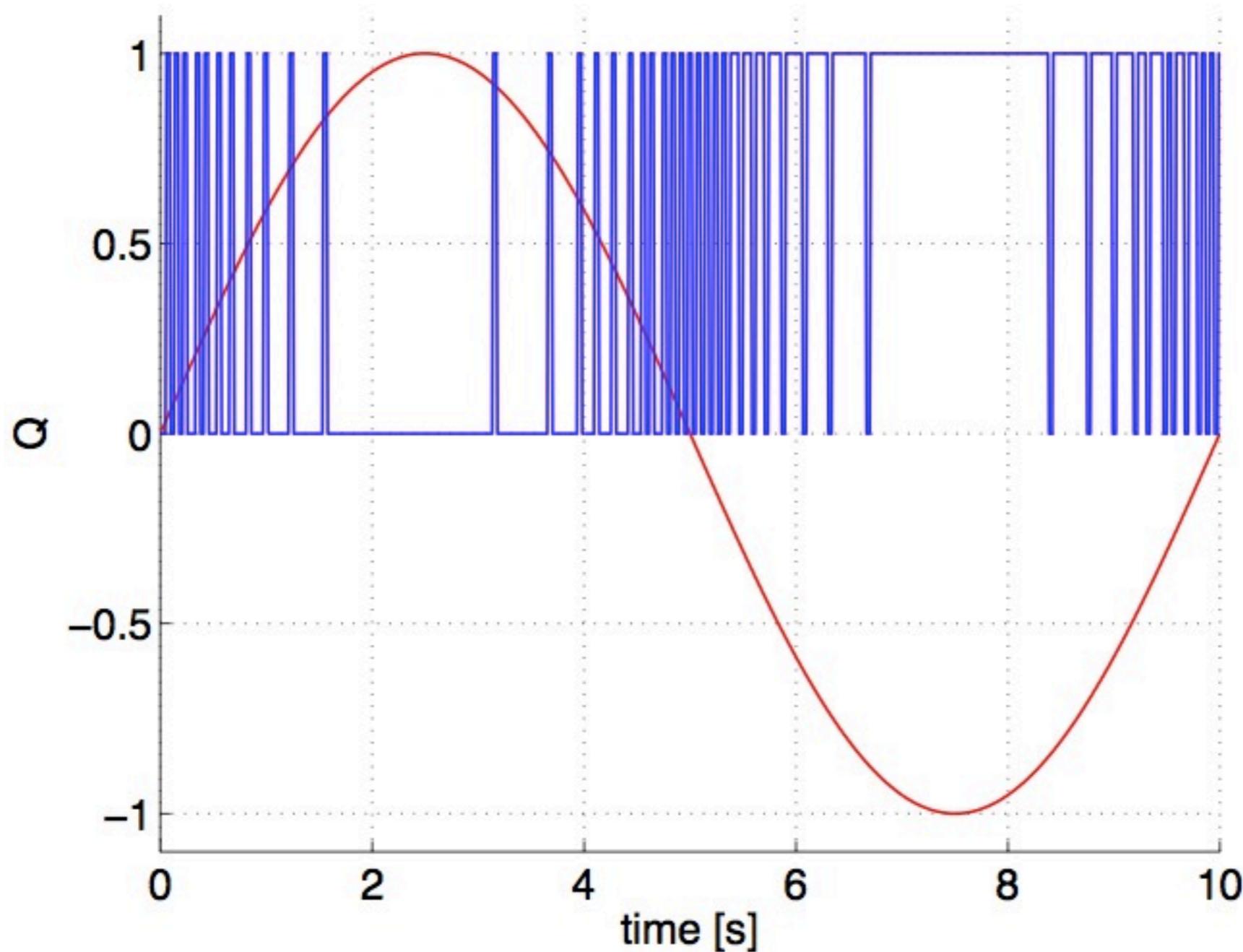


# Converting a Sine wave



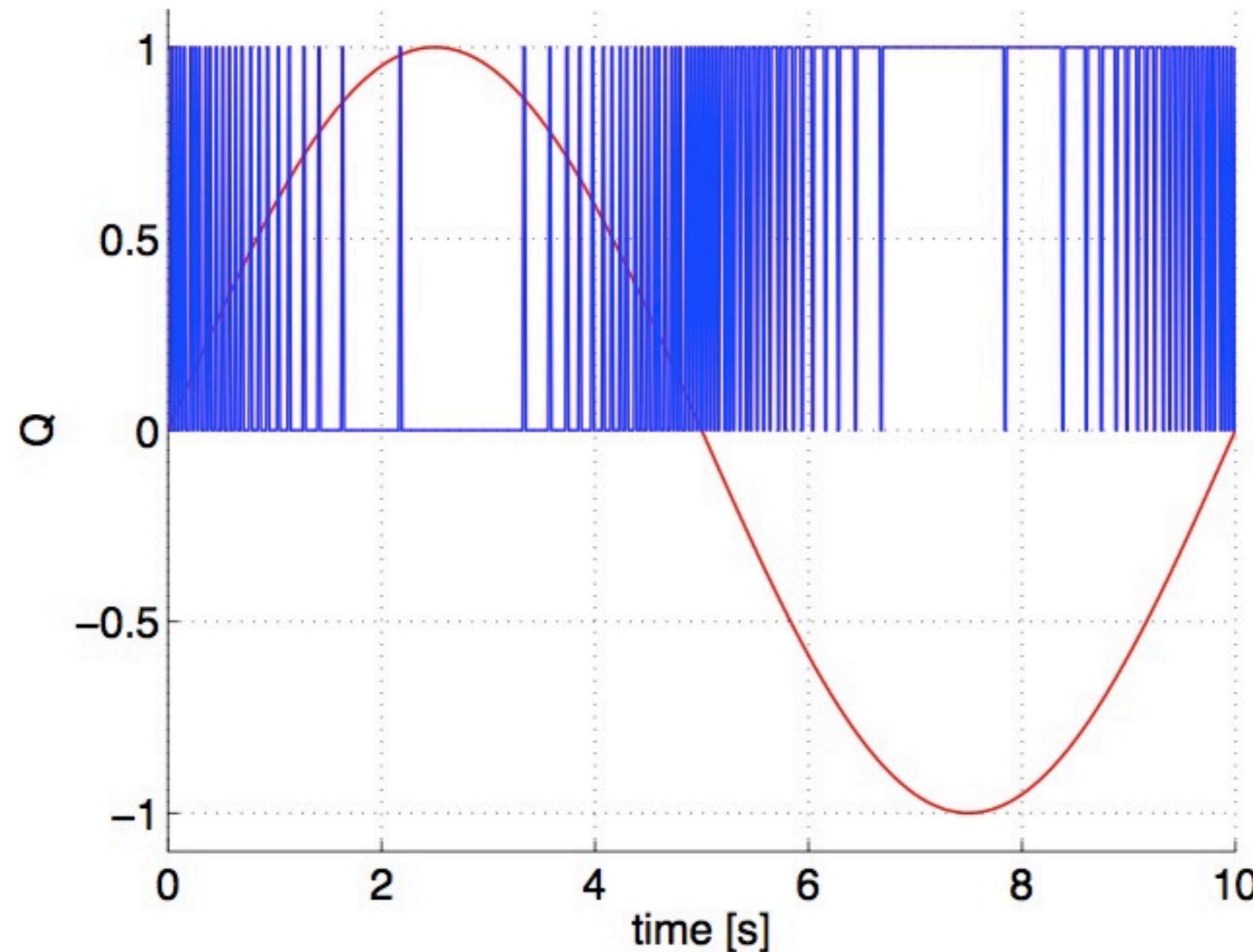
# Converting a Sine wave

higher clock rate



# Converting a Sine wave

even higher clock rate

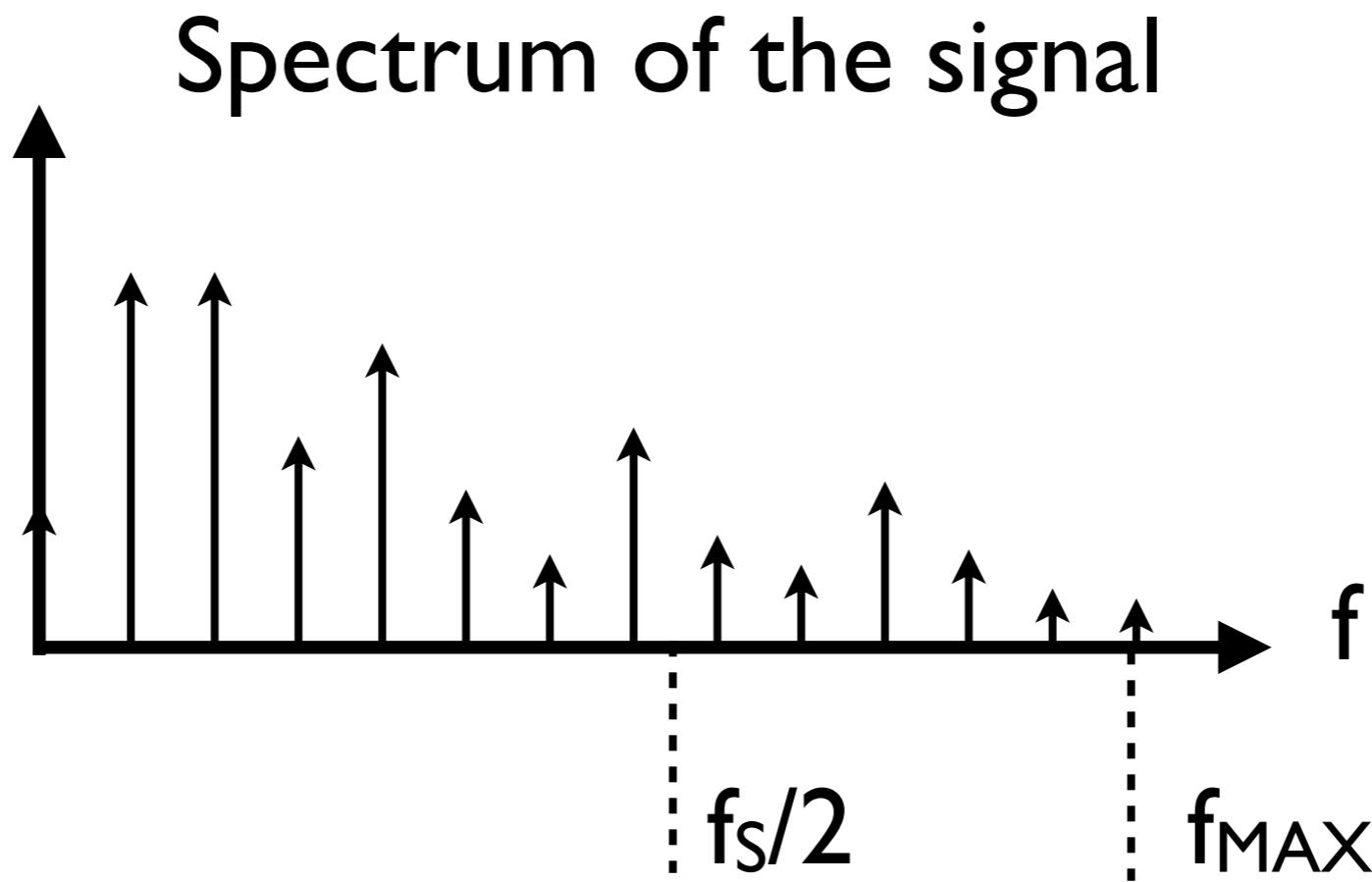


# SIGMA DELTA

One bit sampling with ultra high sampling frequency  
High speed is transformed into high resolution

Total Parts: 452	Resolution (bits)	Input Chan.	Conv. Rate (ksps)	Budgetary Price
Matching Parts: 5	ADC		max	See Notes
Current Selections:	= 24	-	-	-
<input checked="" type="checkbox"/> <b>MAX11200</b> 24-bit, 1-channel, Ultra-Low Power Delta Sigma ADC	24	1	0.48	\$2.65 @1k
<input checked="" type="checkbox"/> <b>MAX11201</b> 24-bit, 1-channel, Ultra-Low Power Delta Sigma ADC	24	1	0.12	\$2.55 @1k
<input checked="" type="checkbox"/> <b>MAX11202</b> 24-bit, 1-channel, Ultra-Low Power Delta Sigma ADC	24	1	0.12	\$2.45 @1k
<input checked="" type="checkbox"/> <b>MAX11210</b> 24-bit, 1-channel, Ultra-Low Power Delta Sigma ADC with Programmable Gain	24	1	0.48	\$2.85 @1k

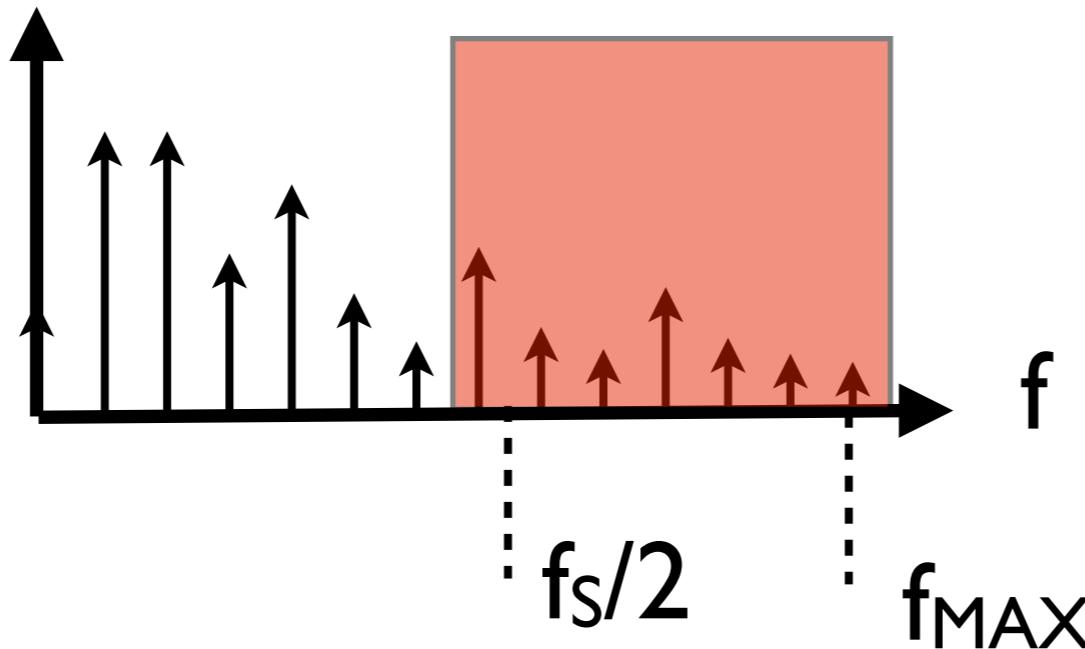
What if the sampling frequency of the ADC is  
NOT high enough?



It should be  $fs/2 > f_{MAX}$  but it is  $fs/2 < f_{MAX}$

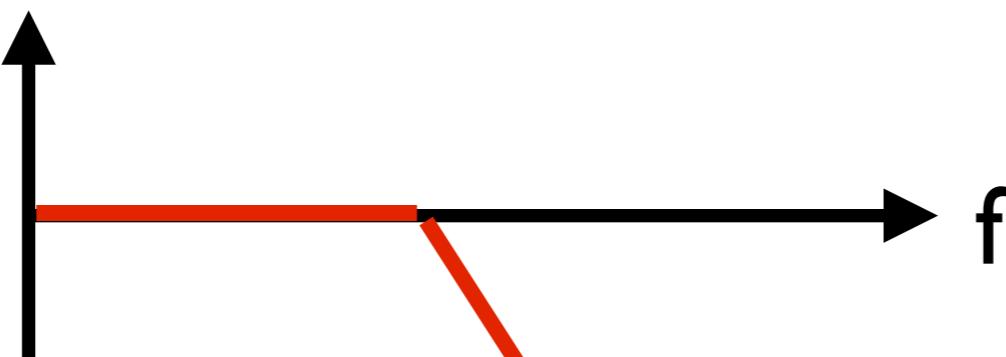
# ANTI-ALIASING FILTER

Spectrum  
of the  
signal

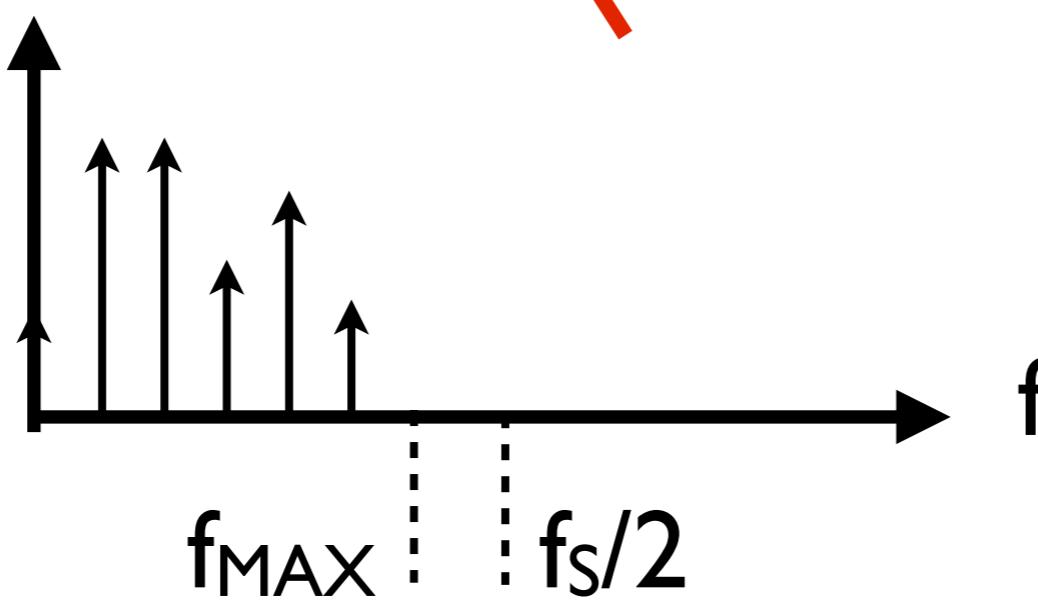


$f_s/2 < f_{MAX}$   
**BAD**

low pass  
filter

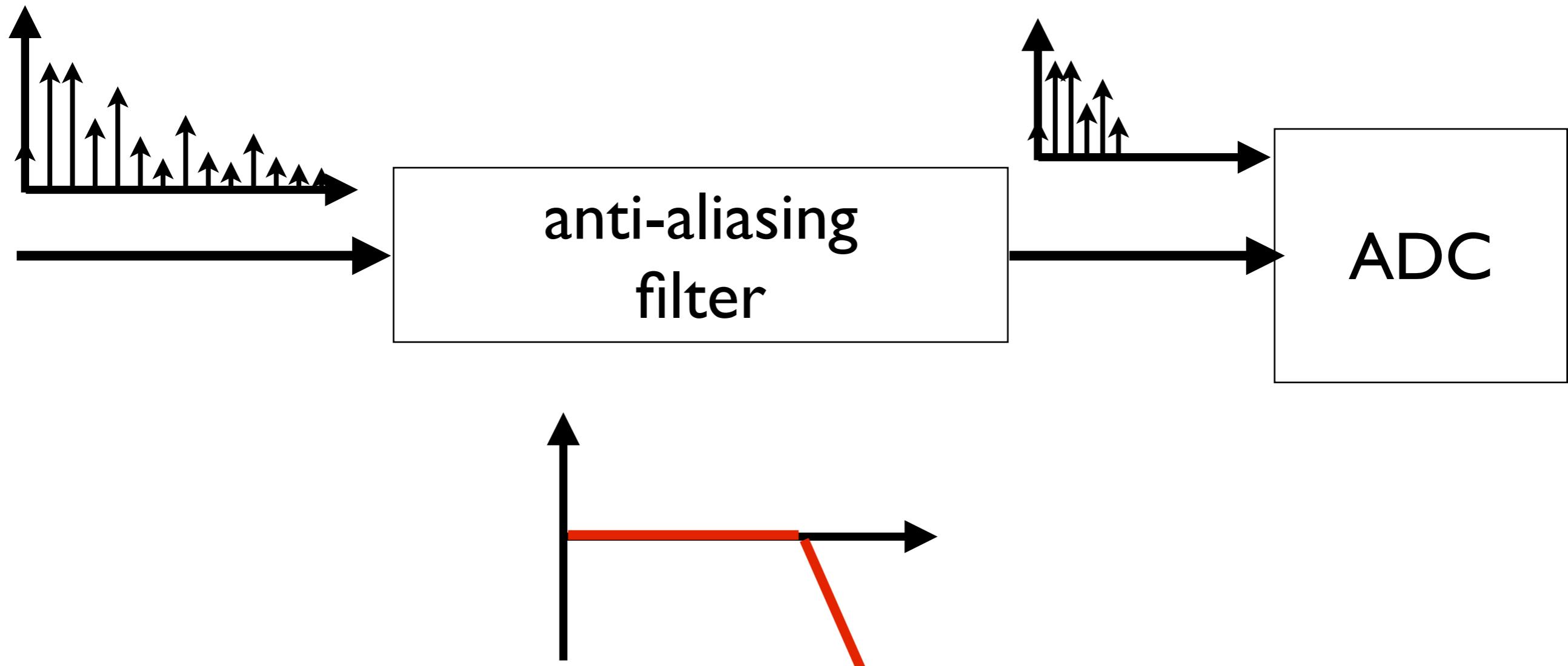


Spectrum  
to



$f_s/2 > f_{MAX}$   
**GOOD**

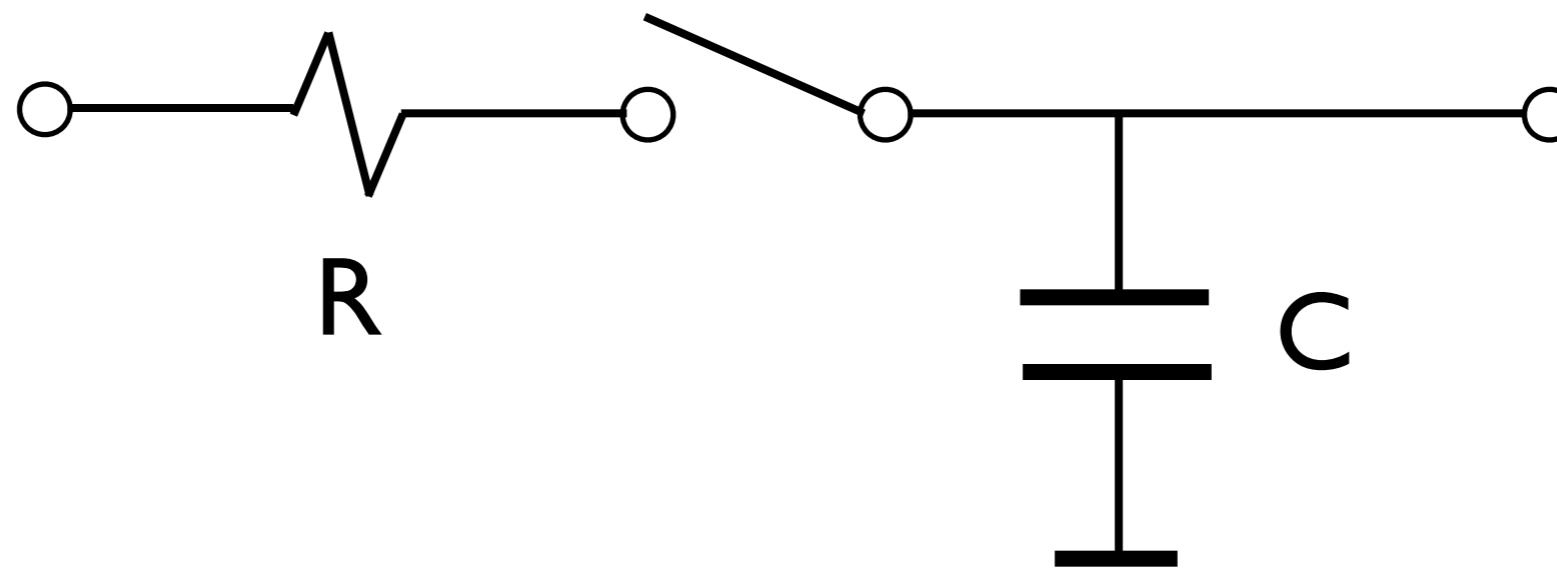
# ANTI-ALIASING FILTER



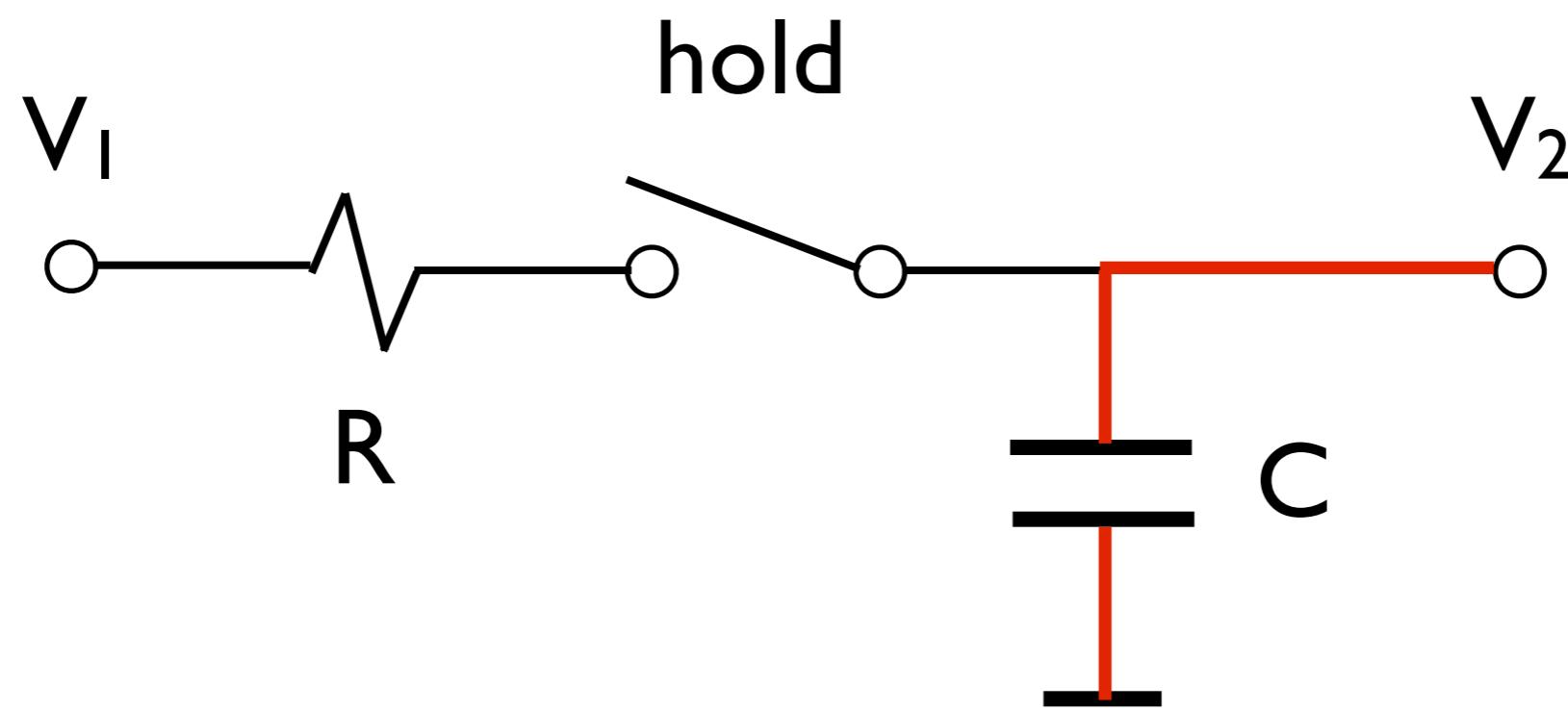
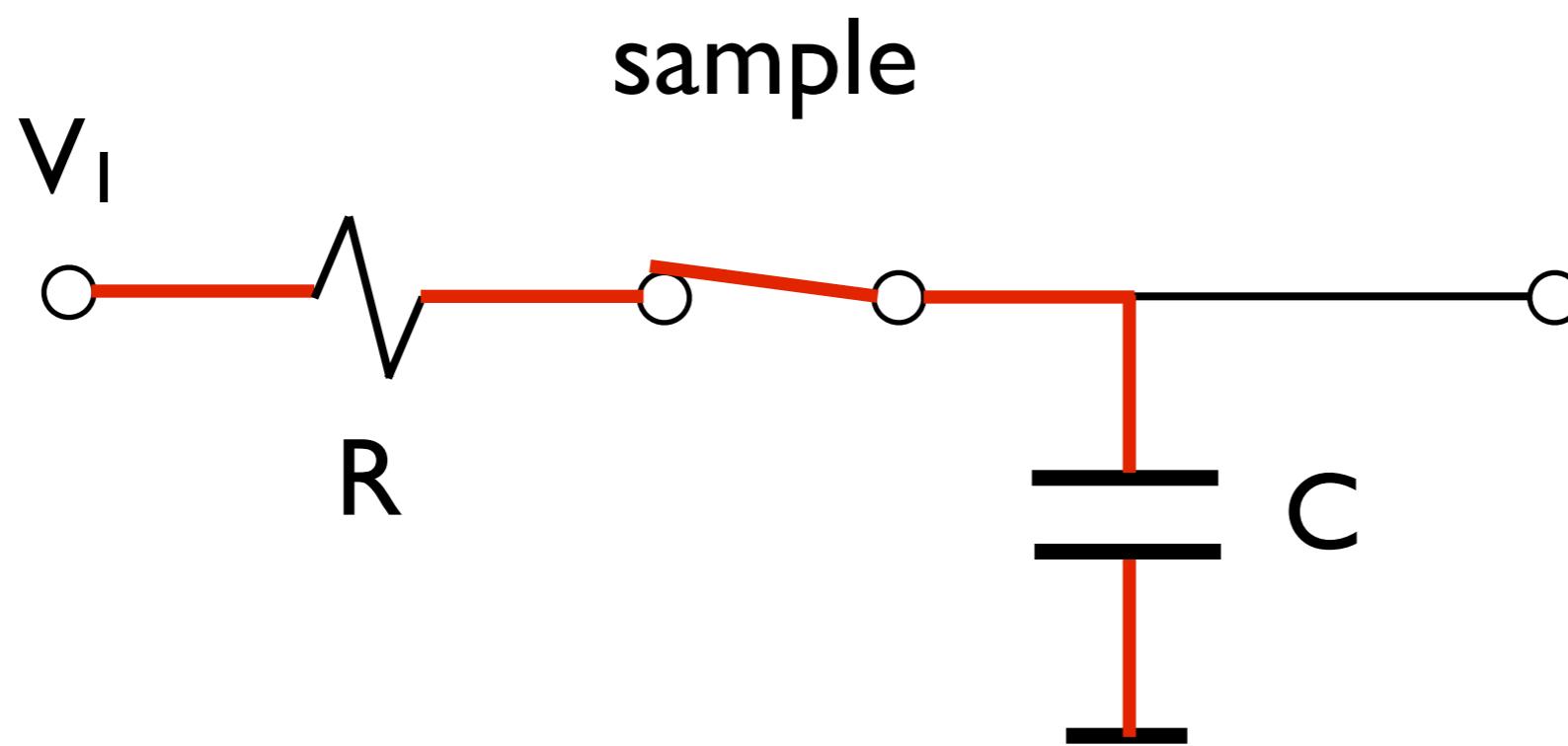
# SAMPLE & HOLD

We need to keep the voltage constant during the conversion time

Basic idea

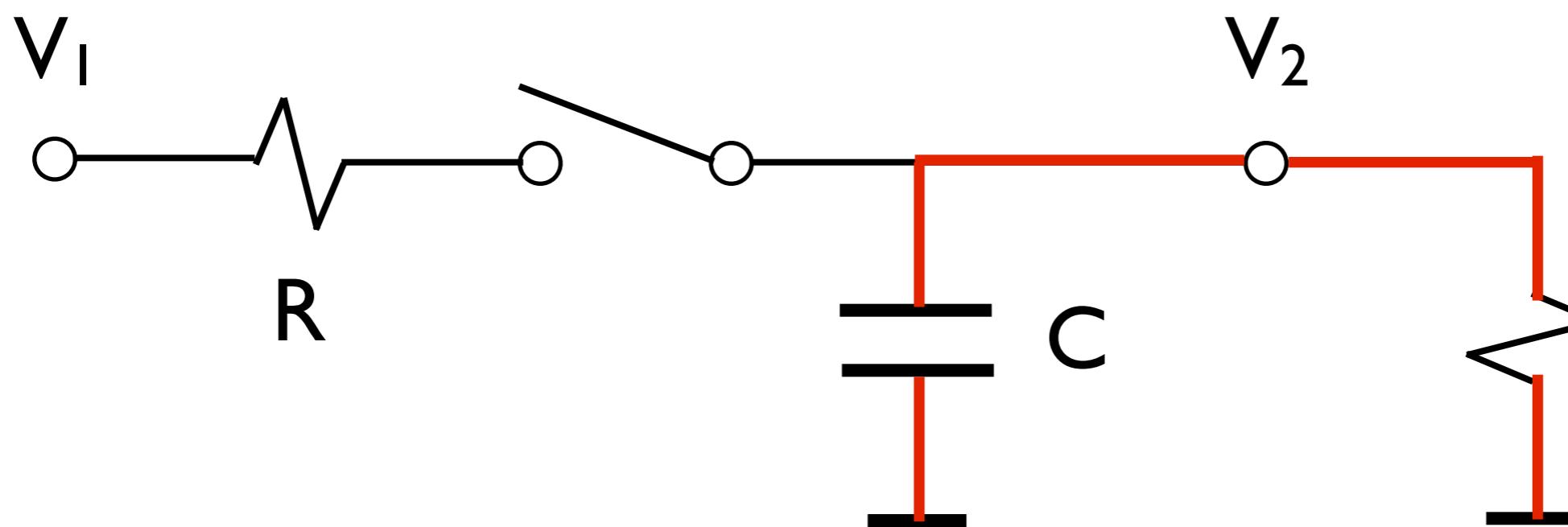


# SAMPLE & HOLD



# SAMPLE & HOLD

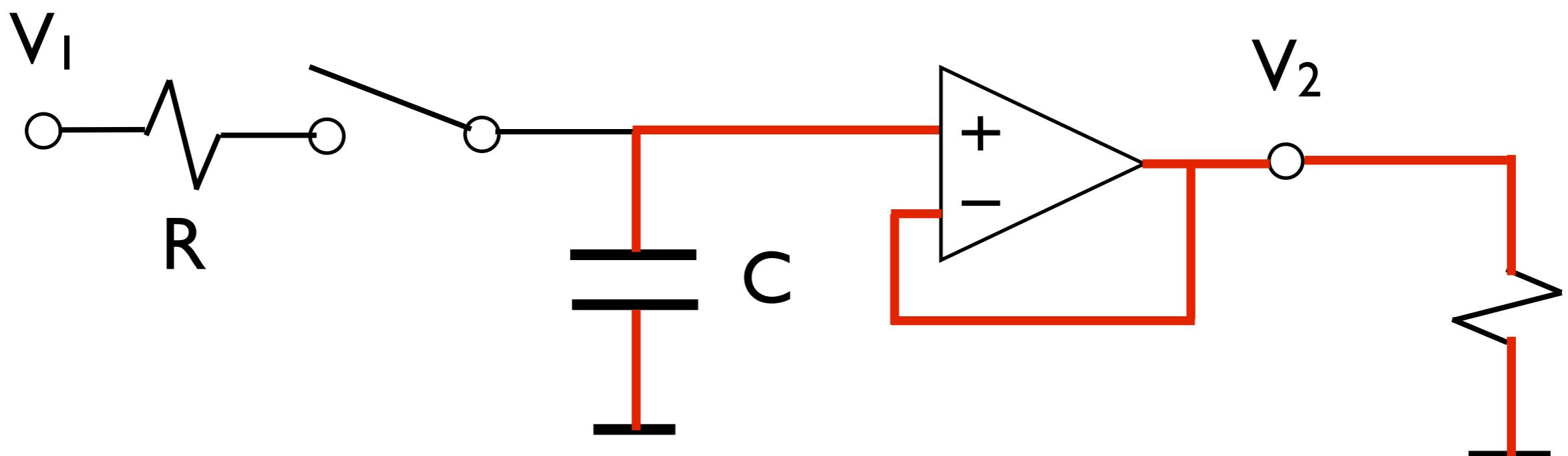
Problem: the capacitor will discharge



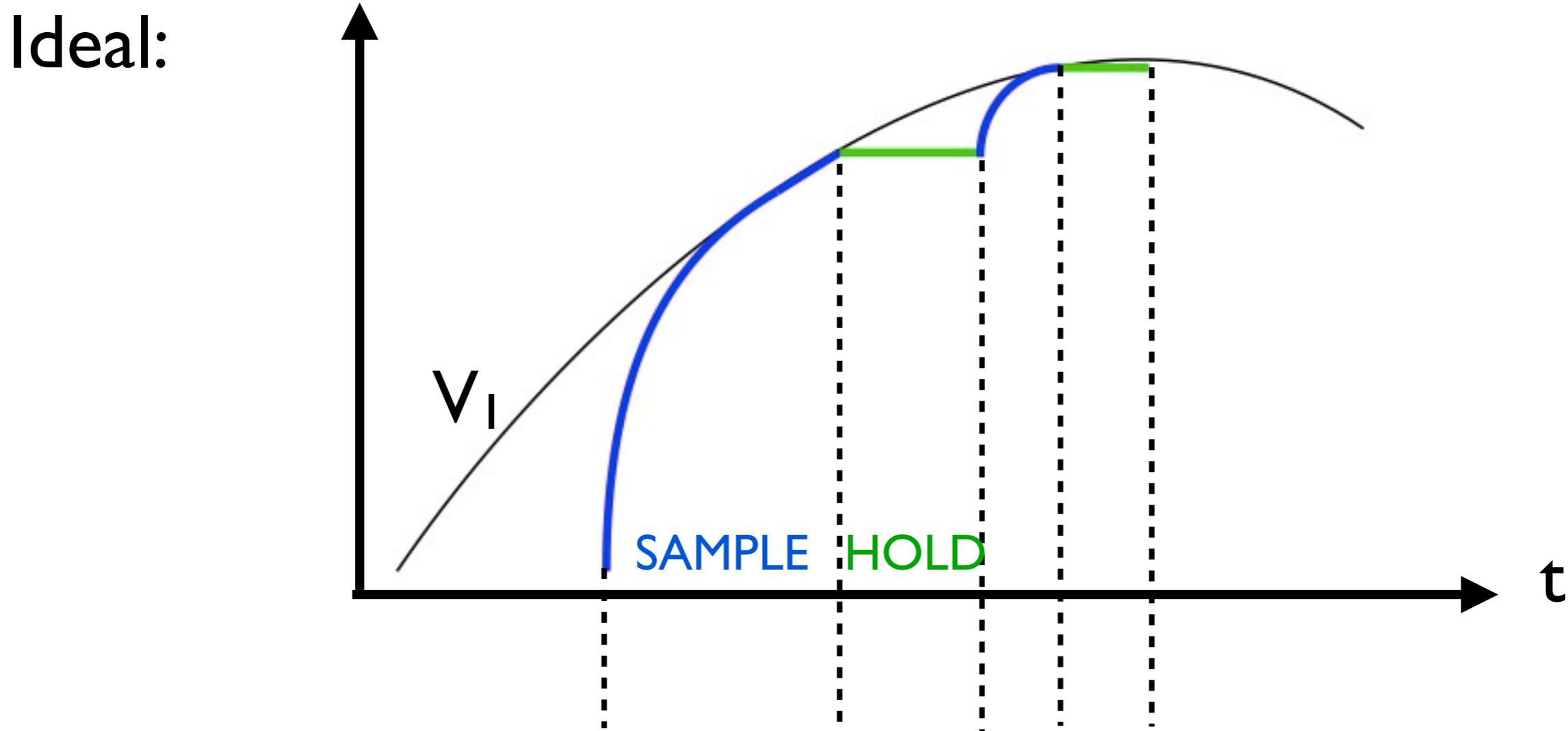
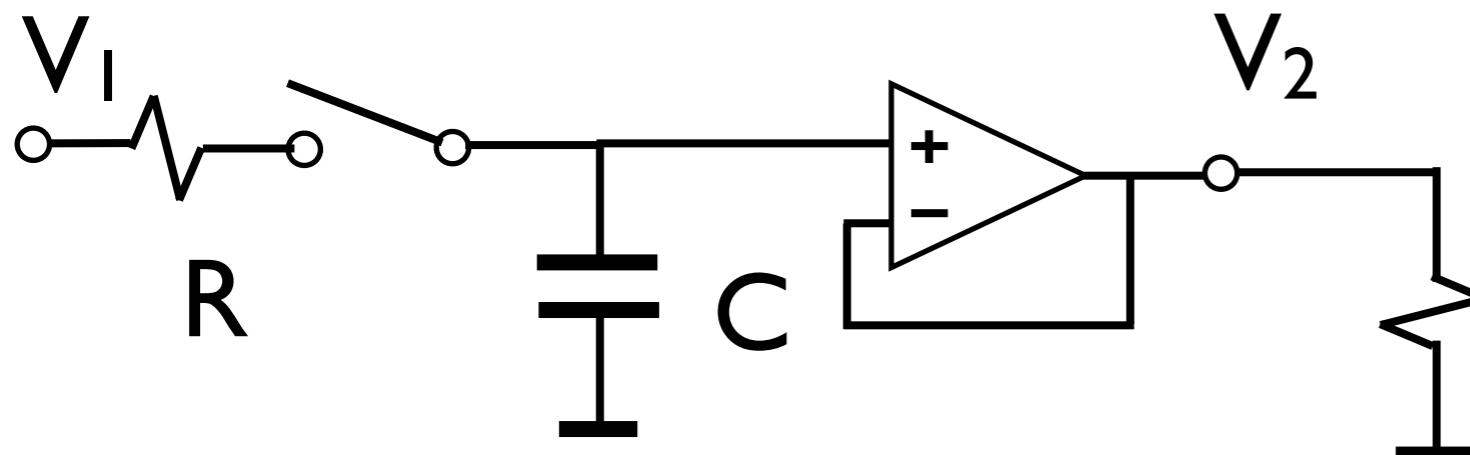
Input impedance of  
ADC

# SAMPLE & HOLD

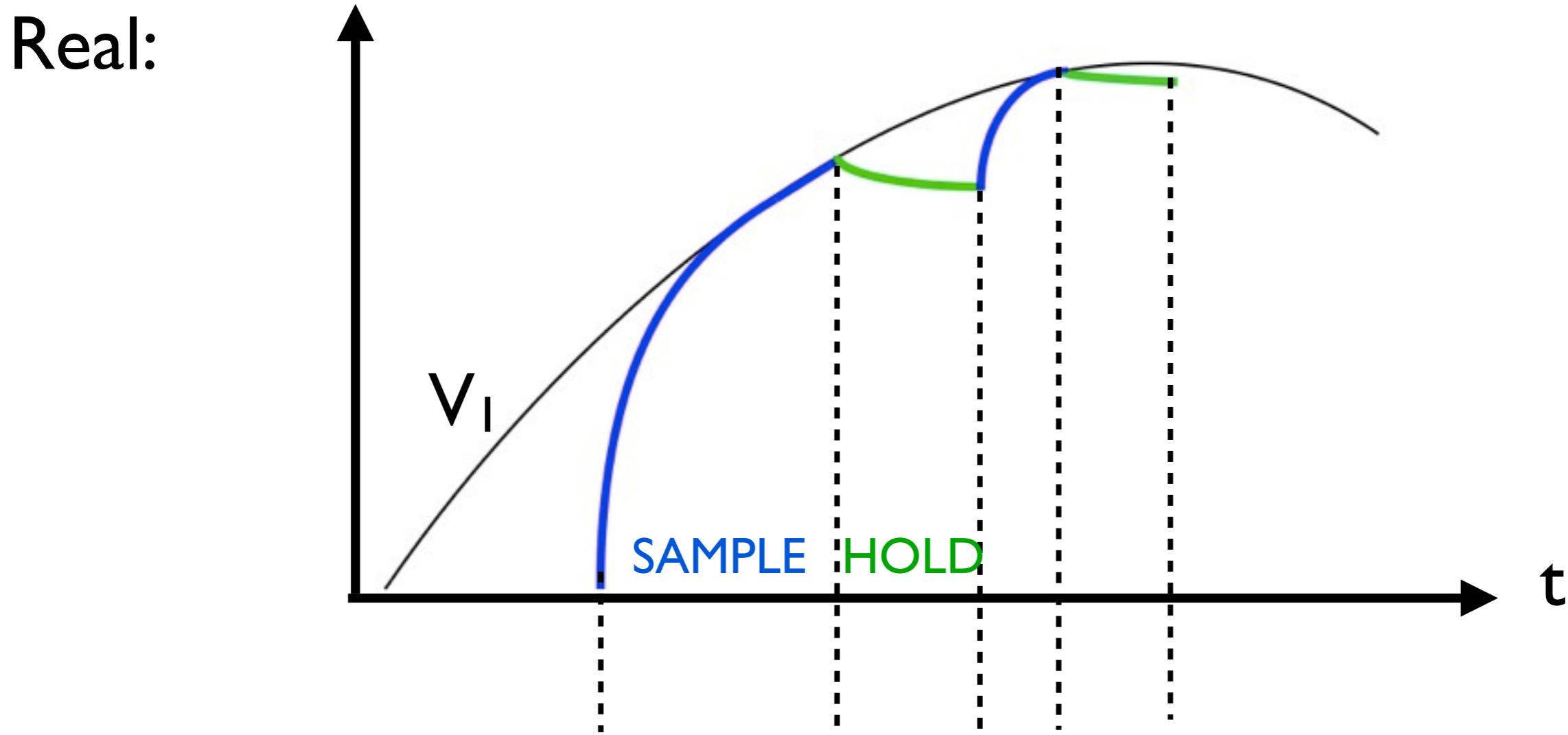
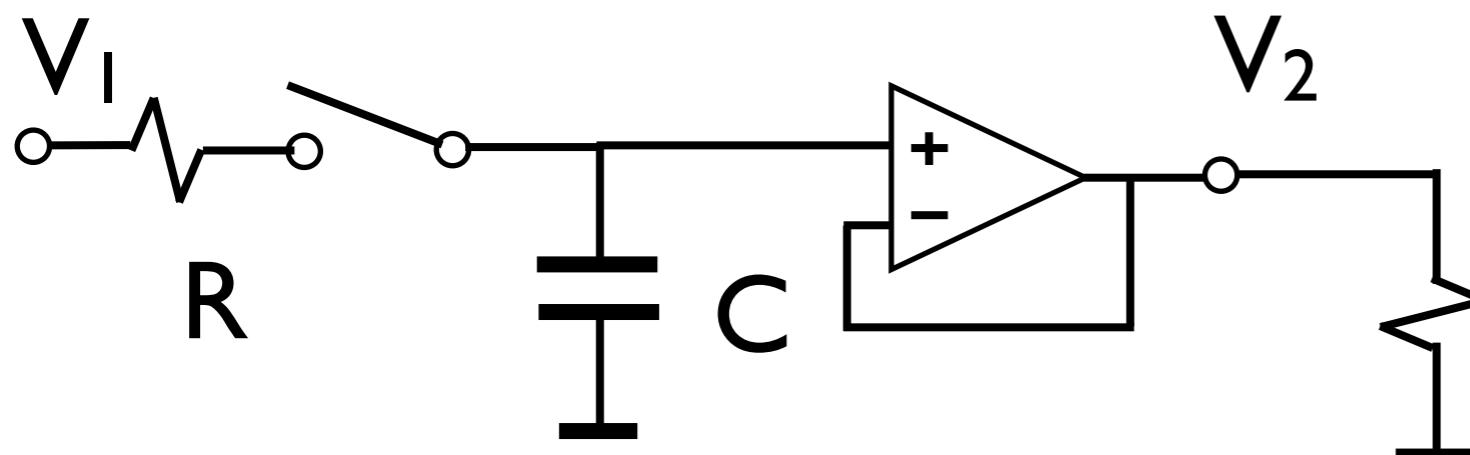
Solution: we use a voltage follower



# SAMPLE & HOLD



# SAMPLE & HOLD



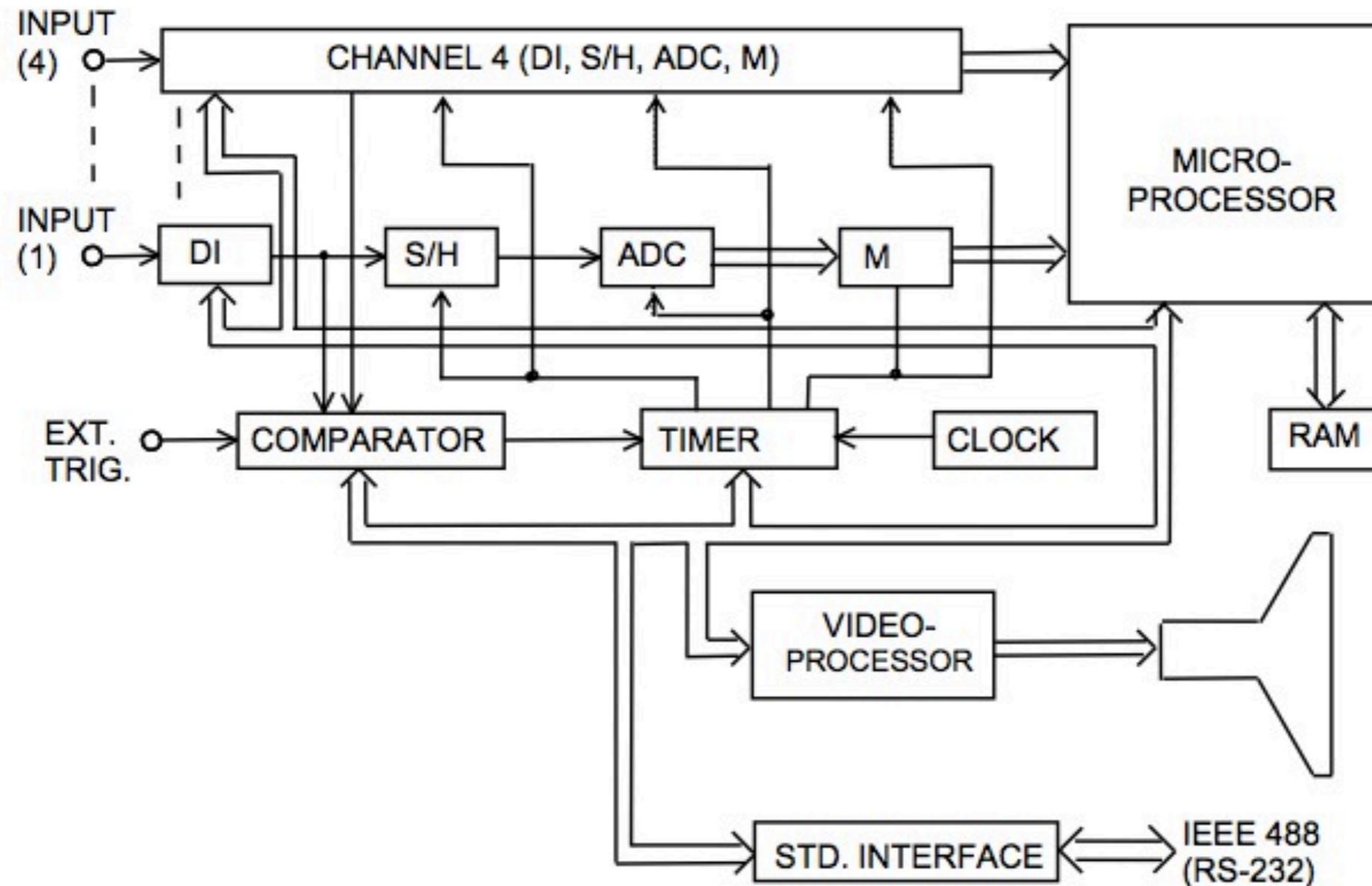
# DIGITAL OSCILLOSCOPE



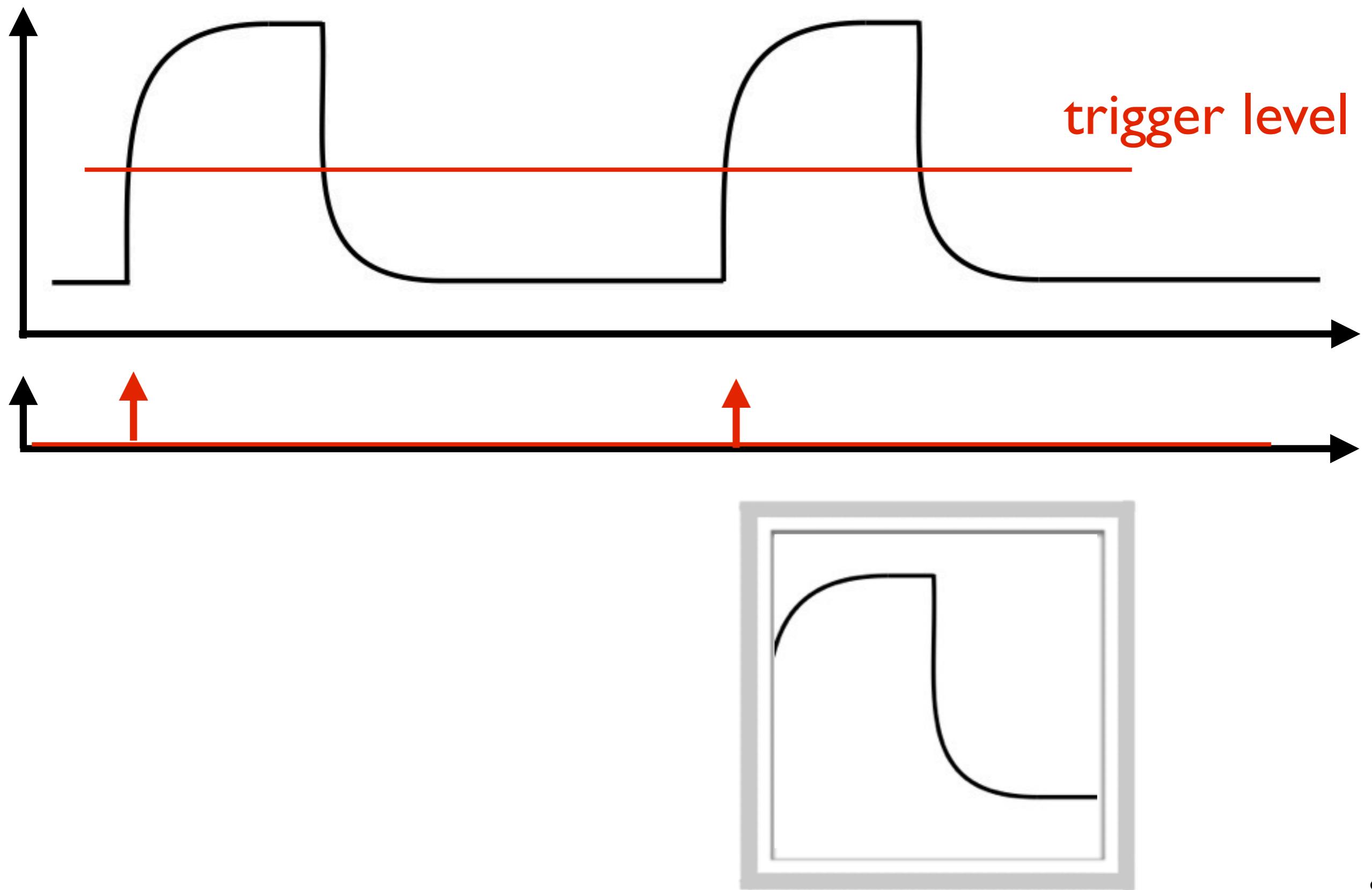
- memory
- computation of many parameters of the waveform
- smart computation such as FFT
- storage of waveform
- pre triggering

# DIGITAL OSCILLOSCOPE

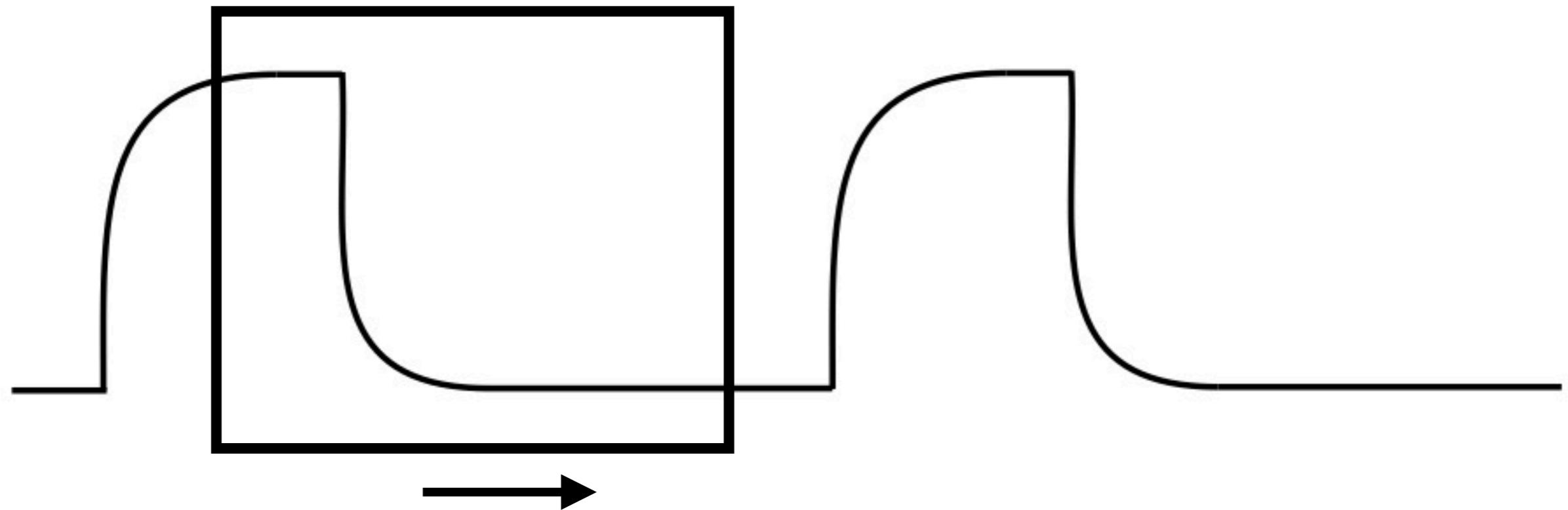
## Structure



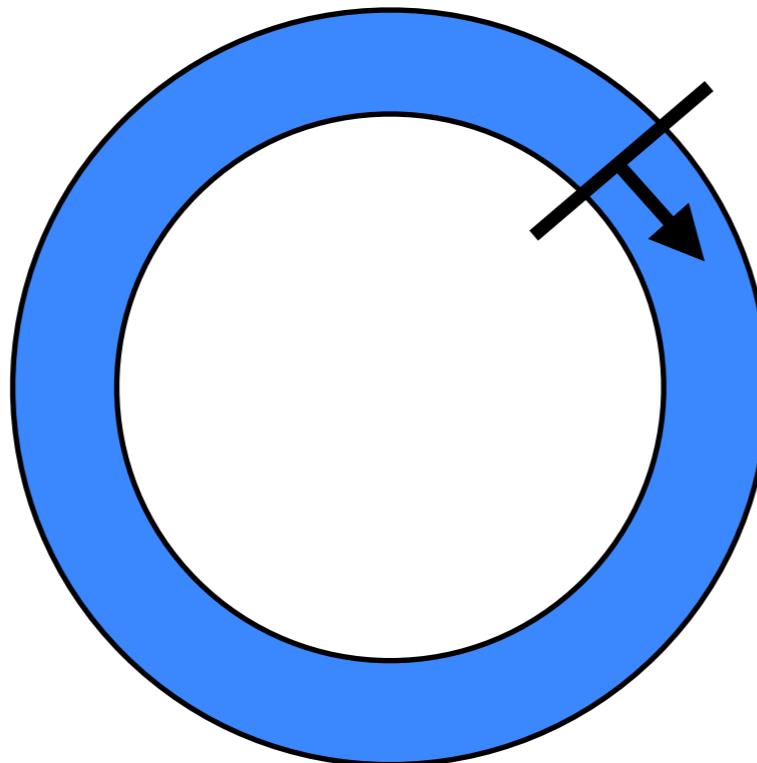
# Trigger in analog oscilloscopes



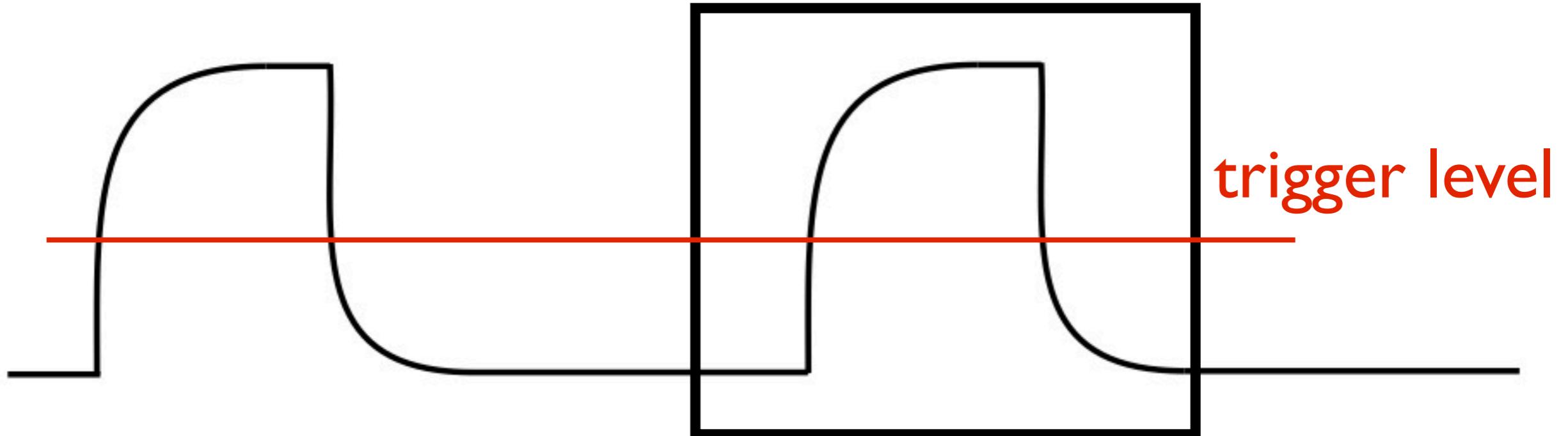
# Trigger in digital oscilloscopes



CIRCULAR  
MEMORY



# Trigger in digital oscilloscopes: pre triggering

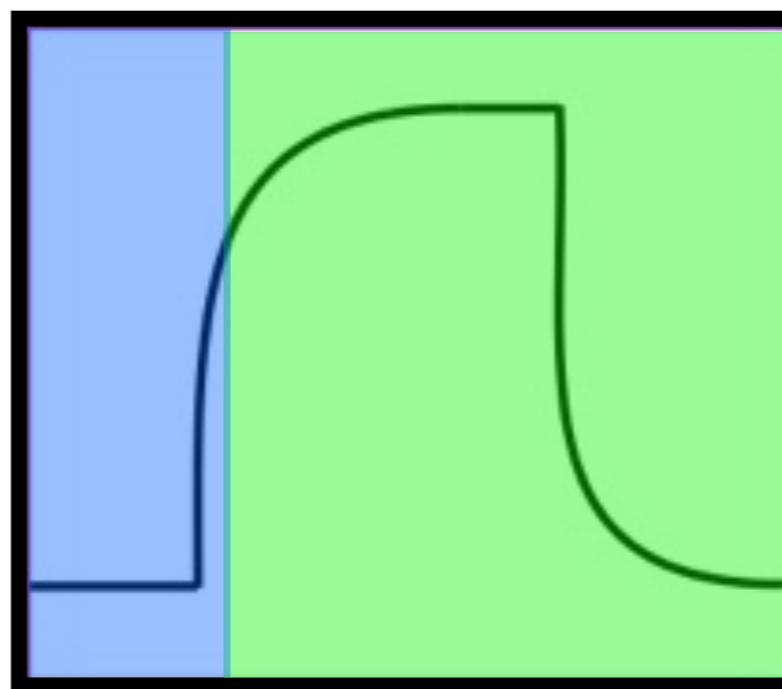


Ex. 1000 samples memory

Pre triggering 20%

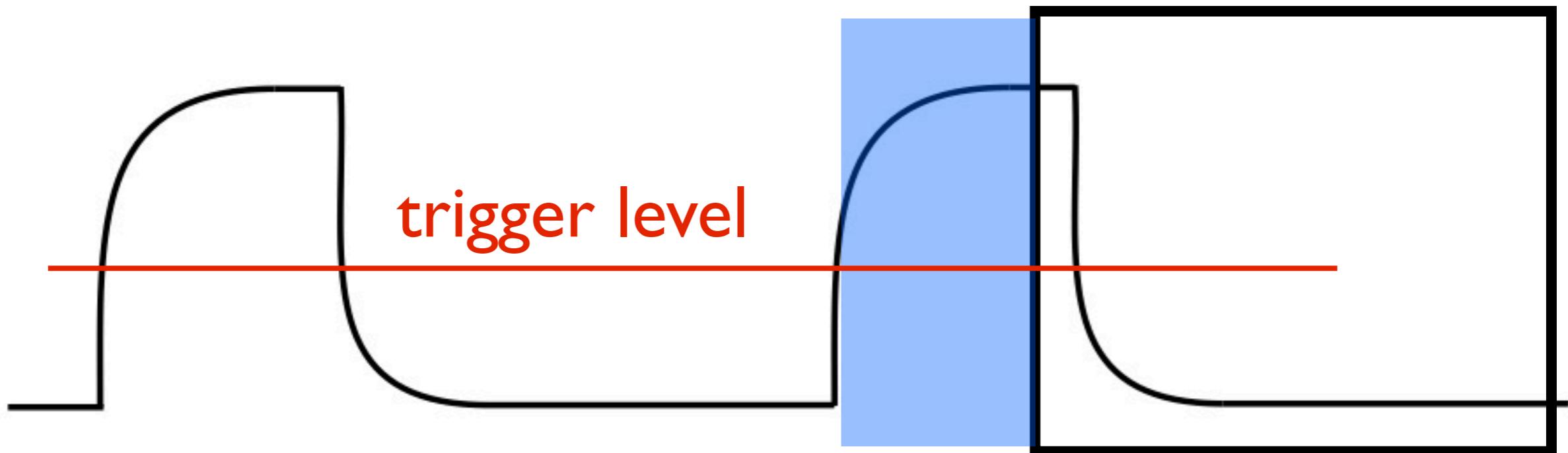
OS stores 800 samples after triggering then stops

200 samples  
before trigger



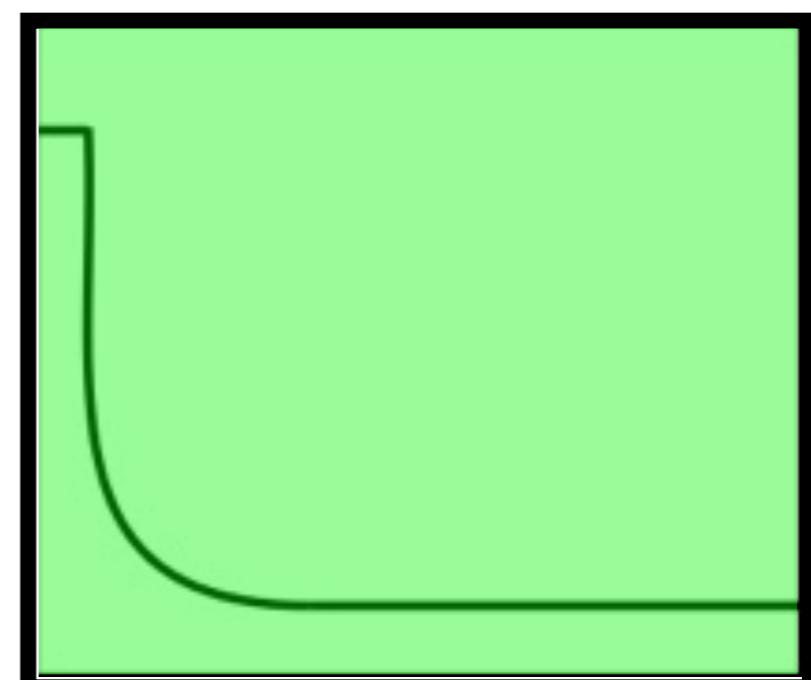
800 samples  
after trigger  
100

# Trigger in digital oscilloscopes: delay

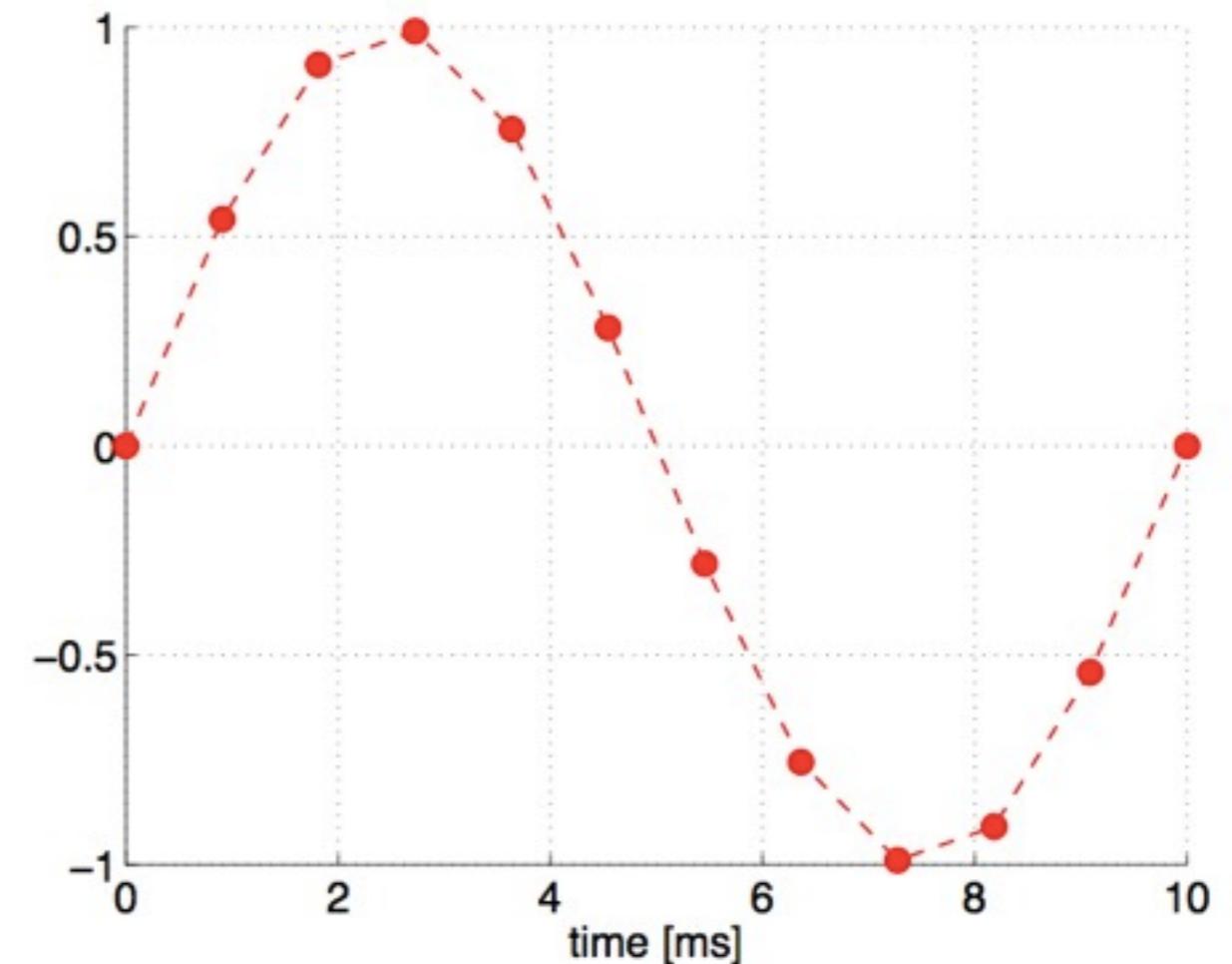
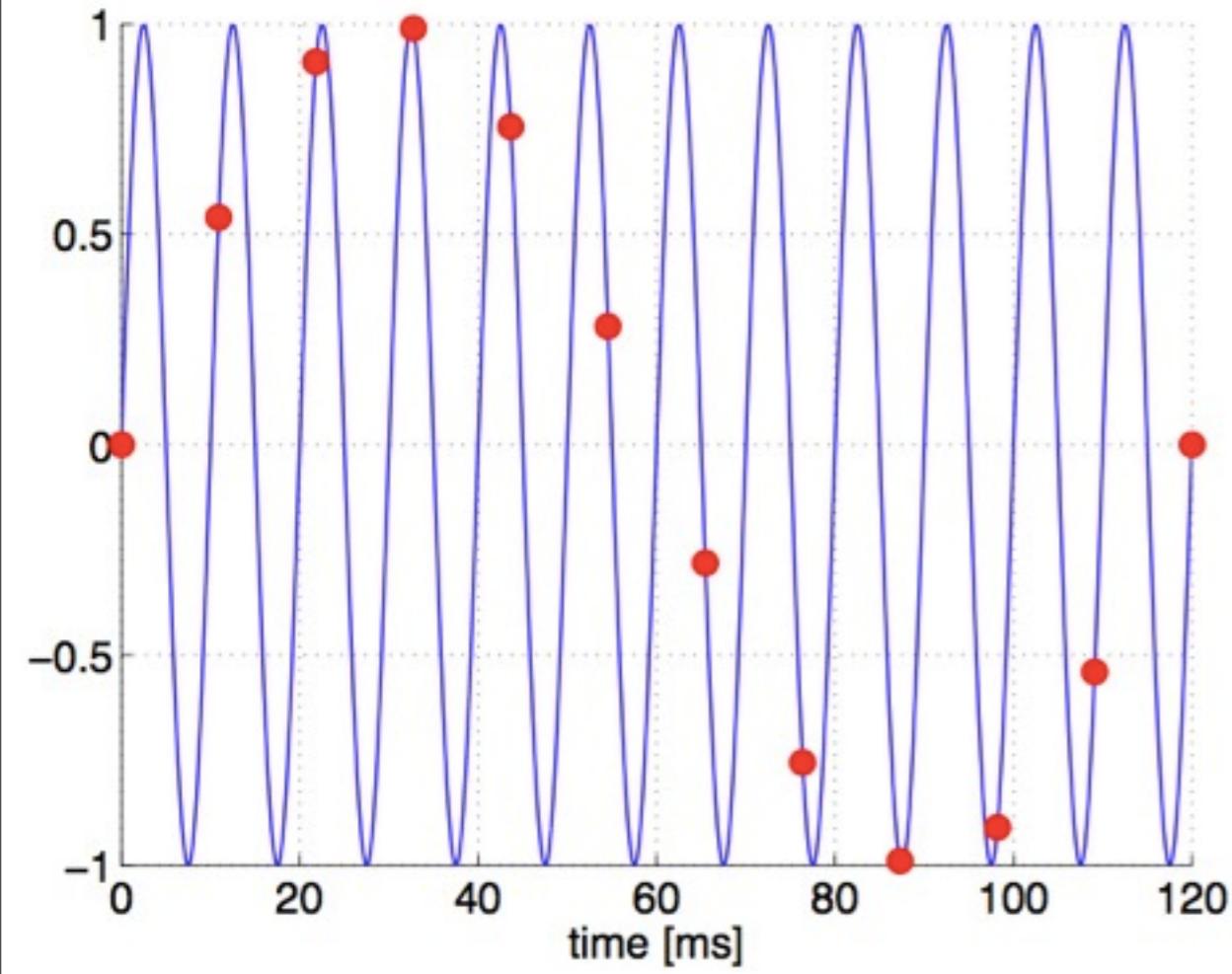


200 acquired  
then overwritten

Ex. 1000 samples memory  
Delay 20%  
OS stores 1200 samples  
after triggering then stops



## - Equivalent time acquisition



The signal must be periodic

## - Random equivalent time acquisition