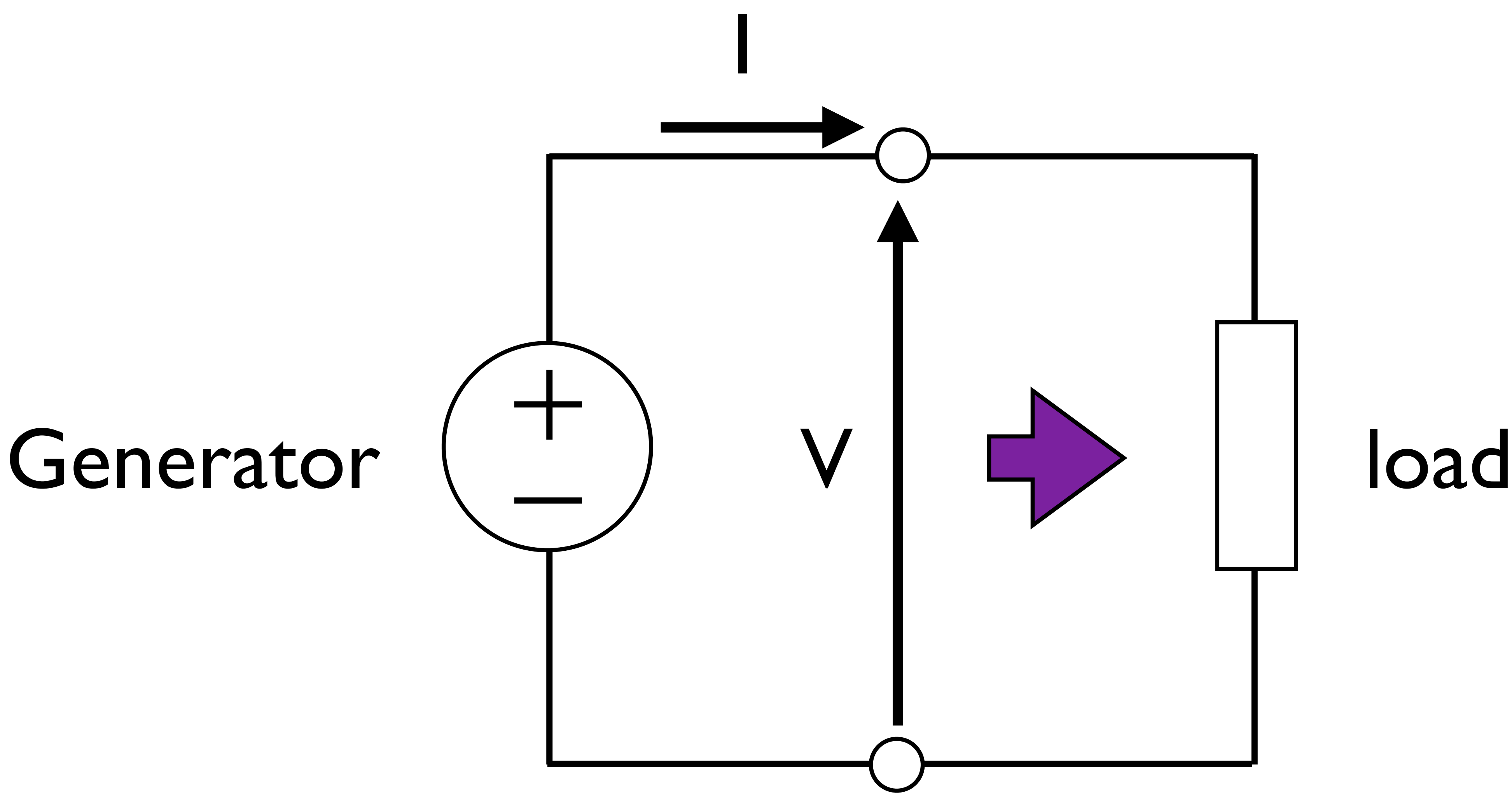
Lecture 3

- Measurement of RMS value
- Measurement of Power and Energy

DC Power

$$P = \text{Energy/Time} = V \cdot I$$



WHY $P = V \cdot I$?

1 volt = work necessary to move a 1 columb

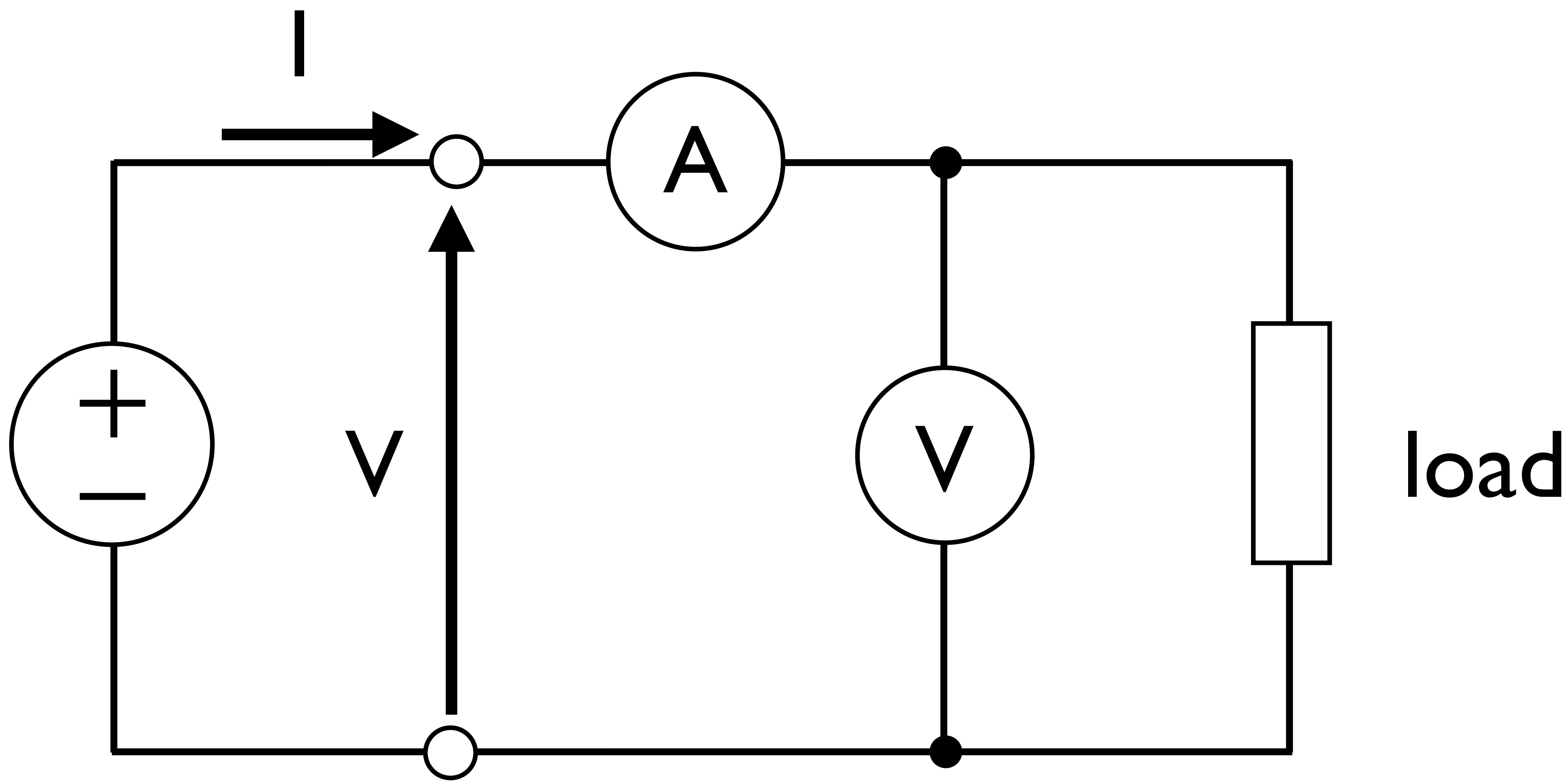
volt= work/charge $[V]=[J]/[C]$

1 ampere = 1 columb of change moving in 1 second

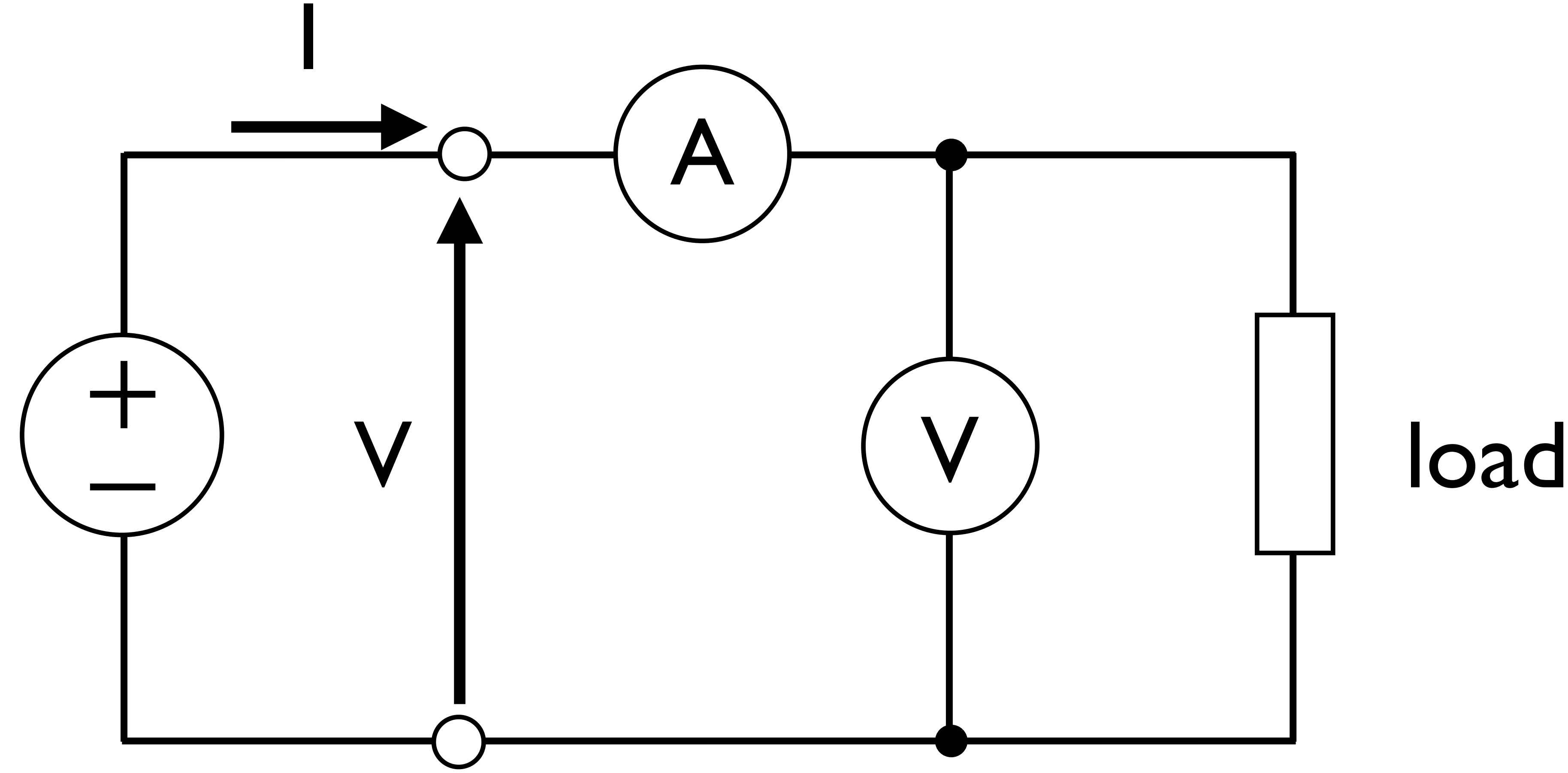
ampere = charge/time $[A] = [C]/[s]$

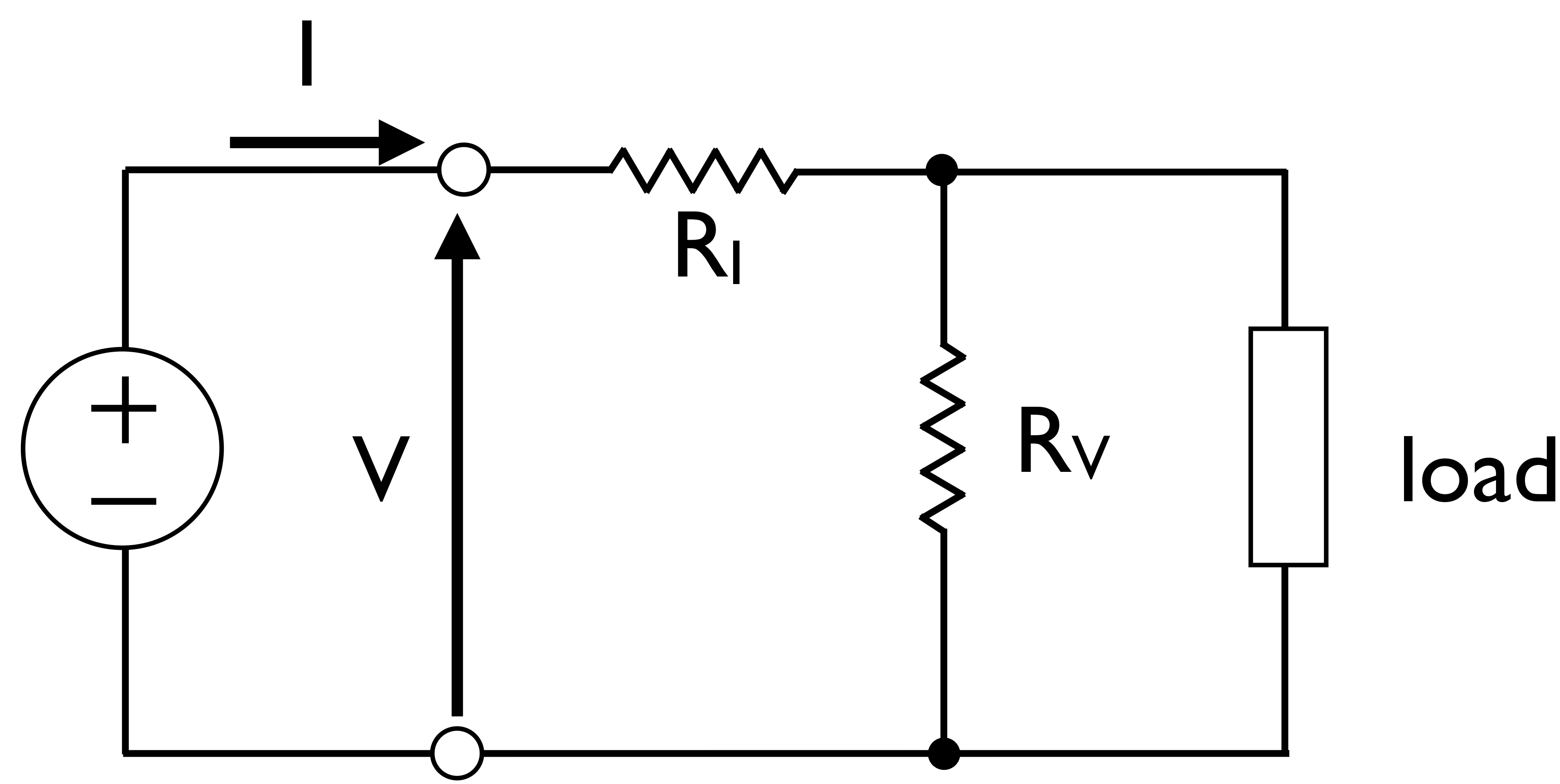
$P = V \cdot I$ = work/charge $\cdot$ charge/time =work/time

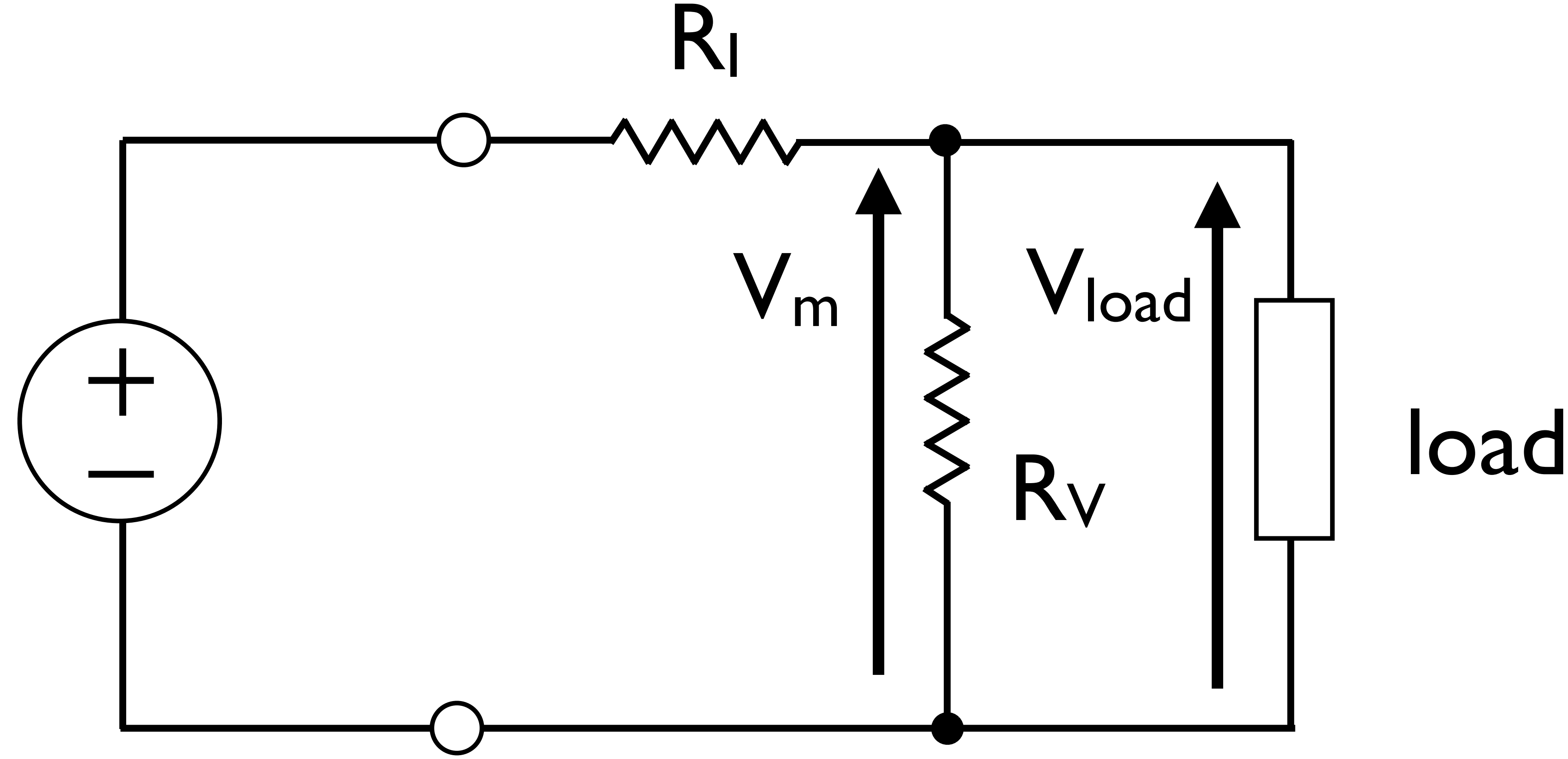
$$P = V \cdot I$$



$P = V \cdot I$



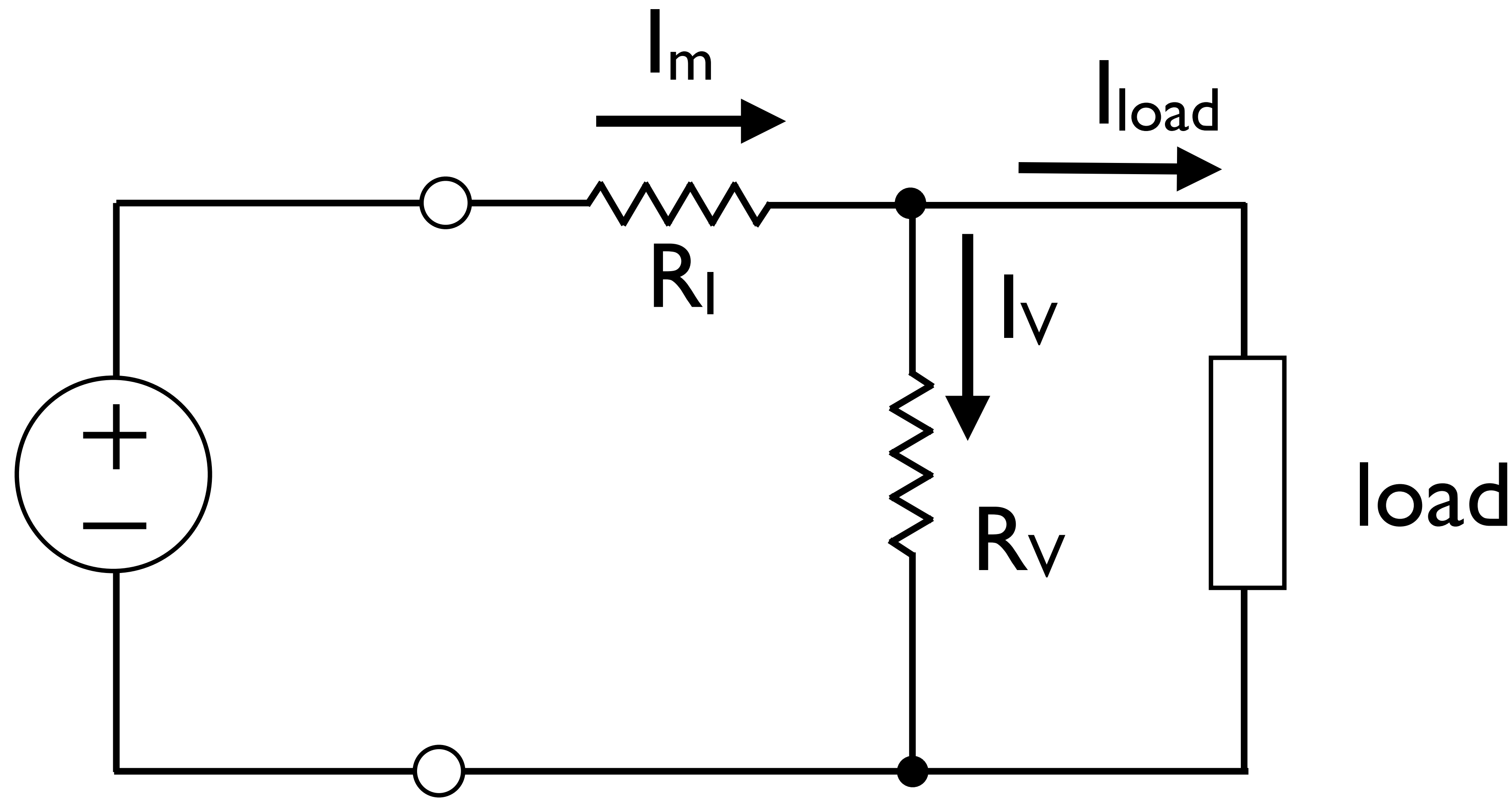




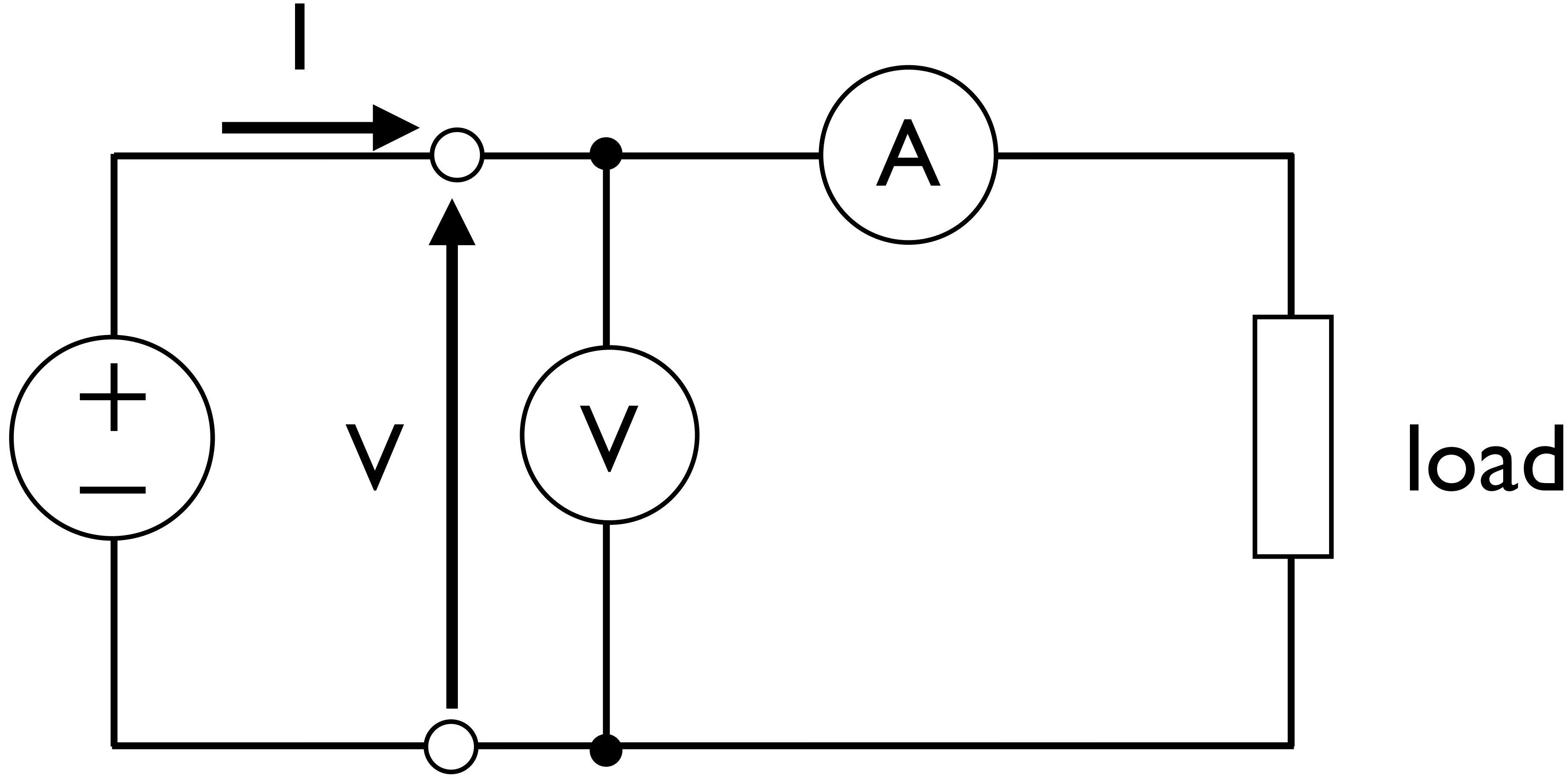
Correction of methodic error

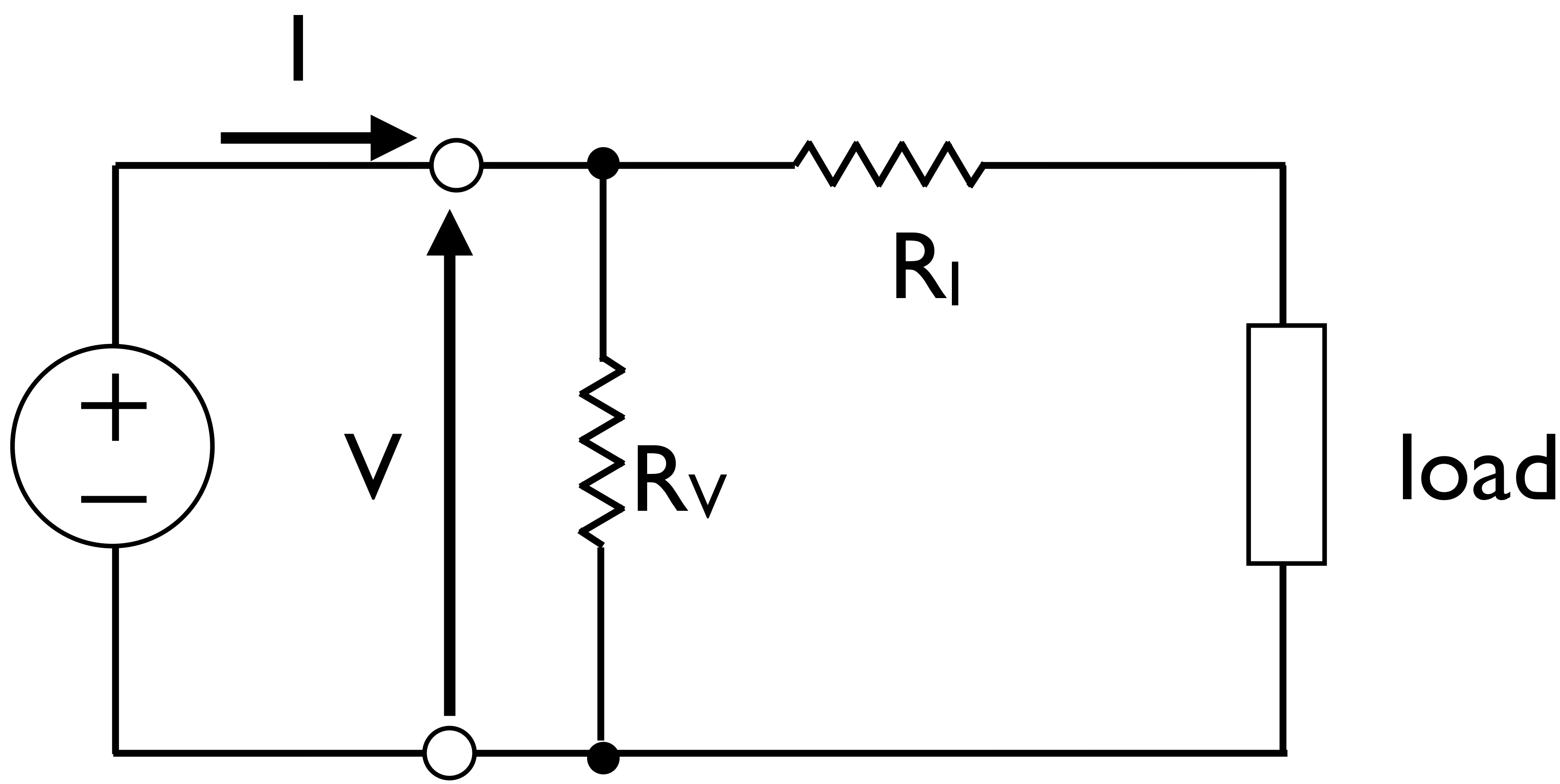
$$P = V_m \cdot I_{load}$$
$$= V_m \cdot (I_m - I_V)$$

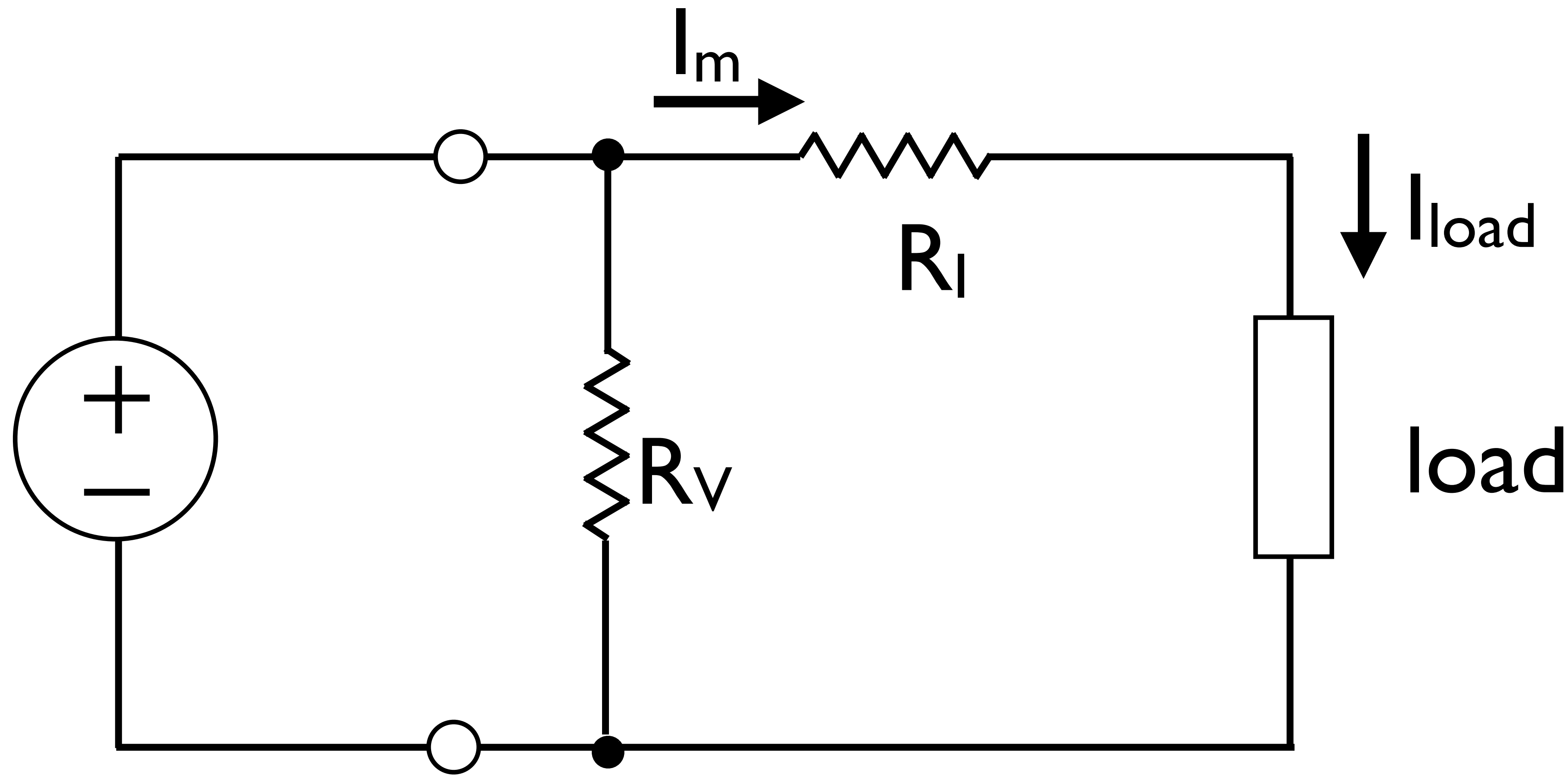
$$I_V = V_m / R_V$$



$P = V \cdot I$



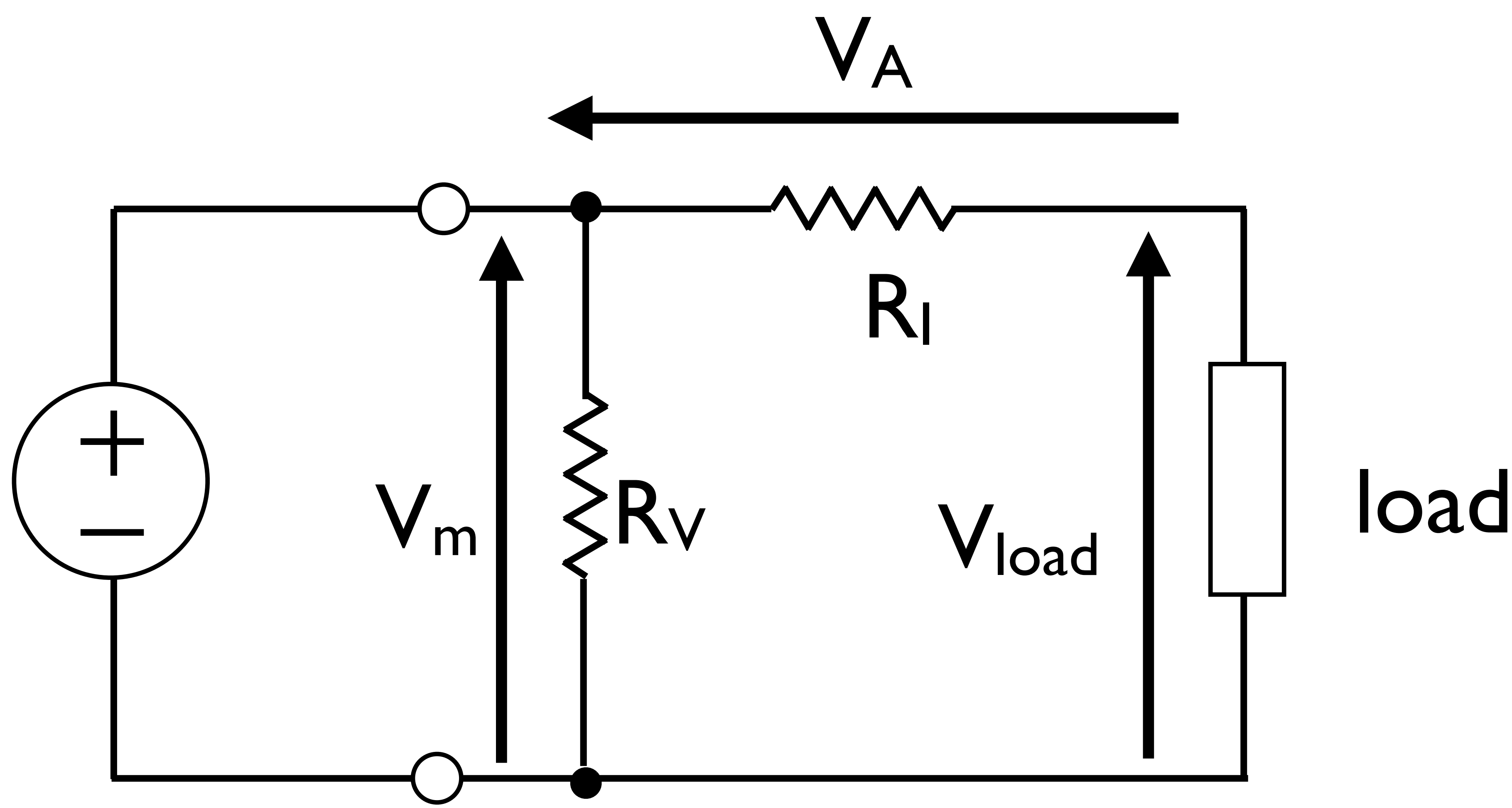




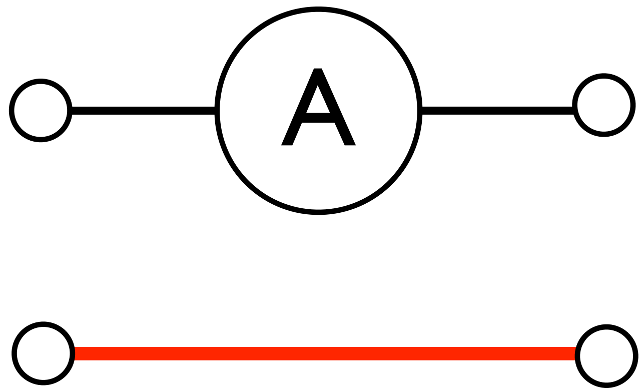
Correction of methodic error

$$P = V_{load} \cdot I_{load}$$
$$= (V_m - V_A) \cdot I_m$$

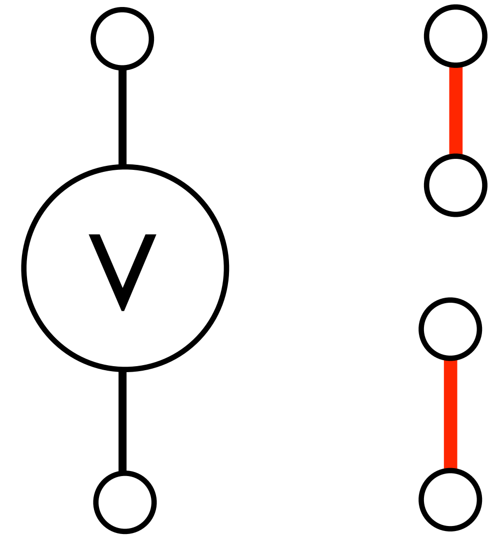
$$V_A = I_A \cdot R_A$$



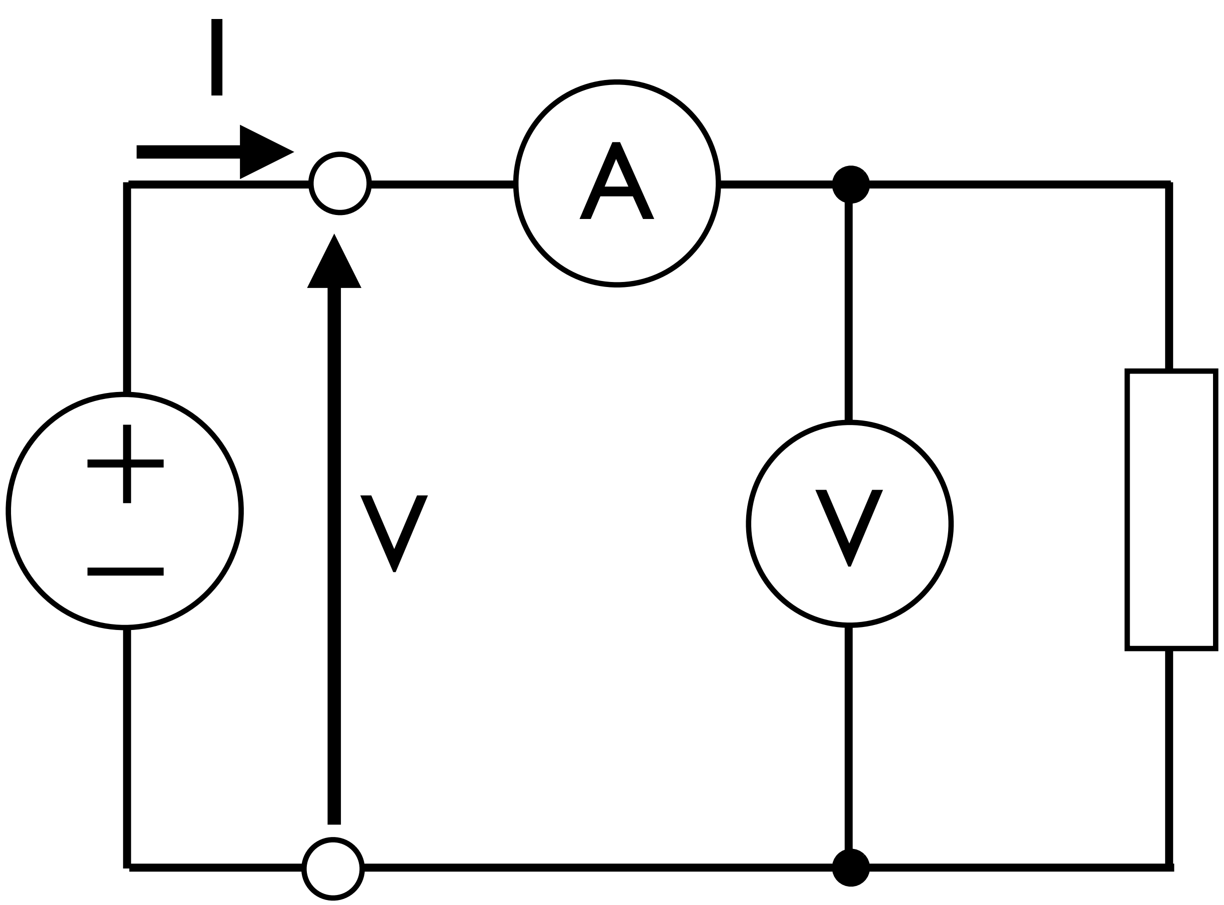
Ideally

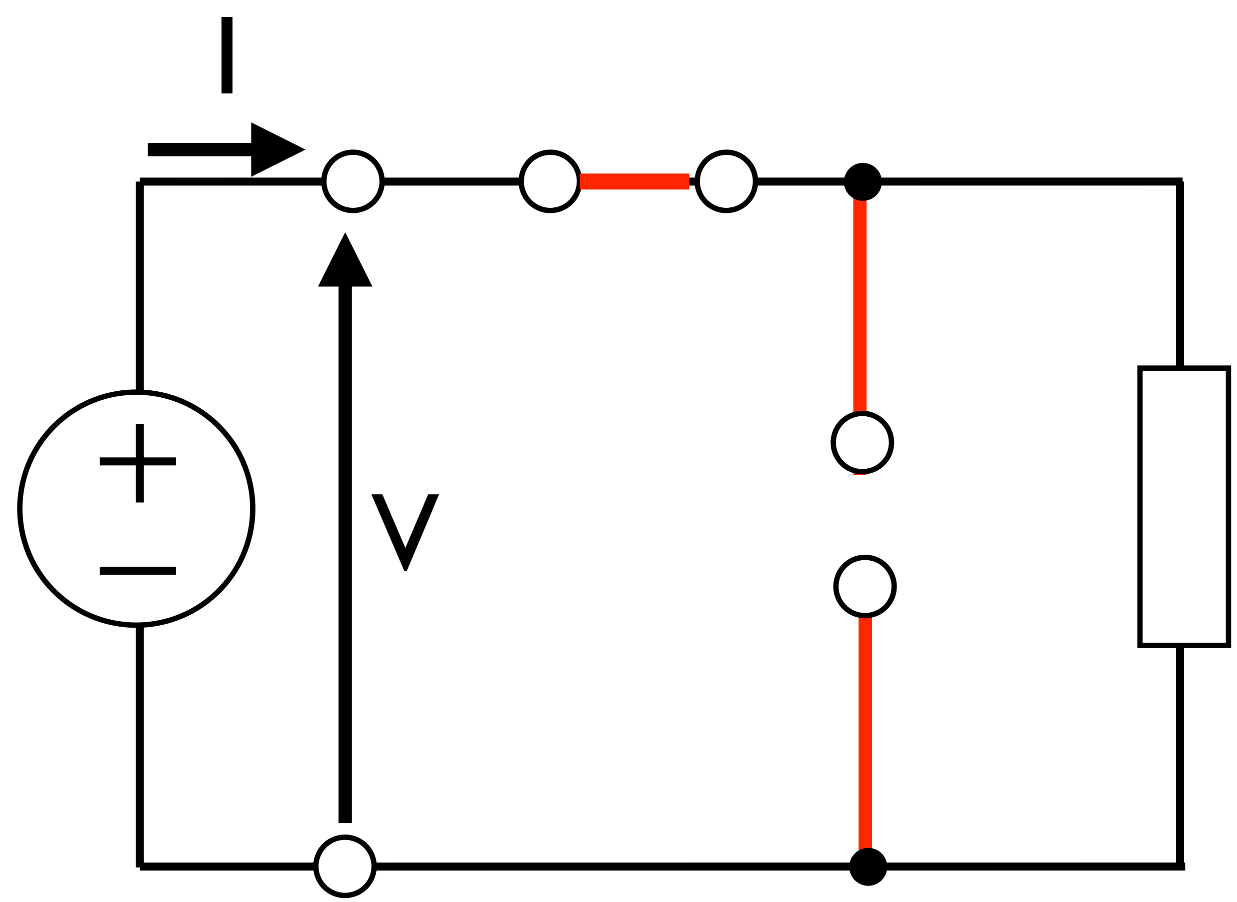


$R_A=0$

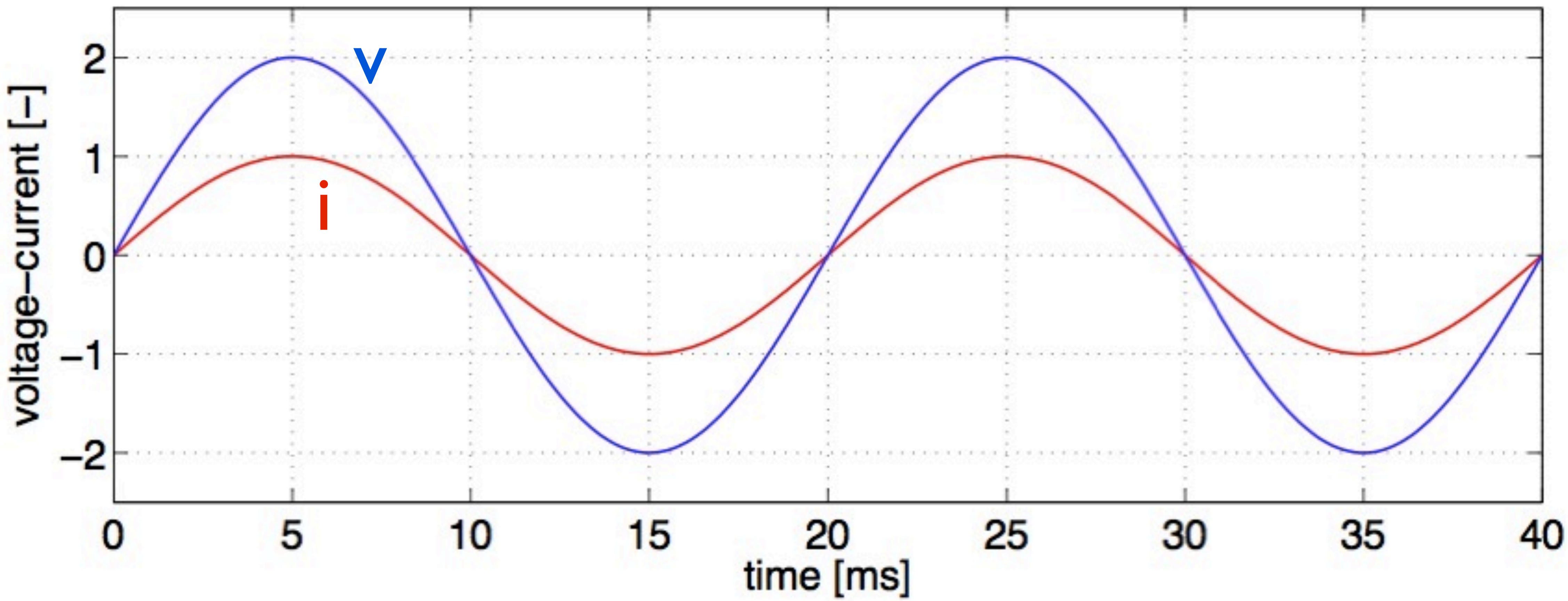


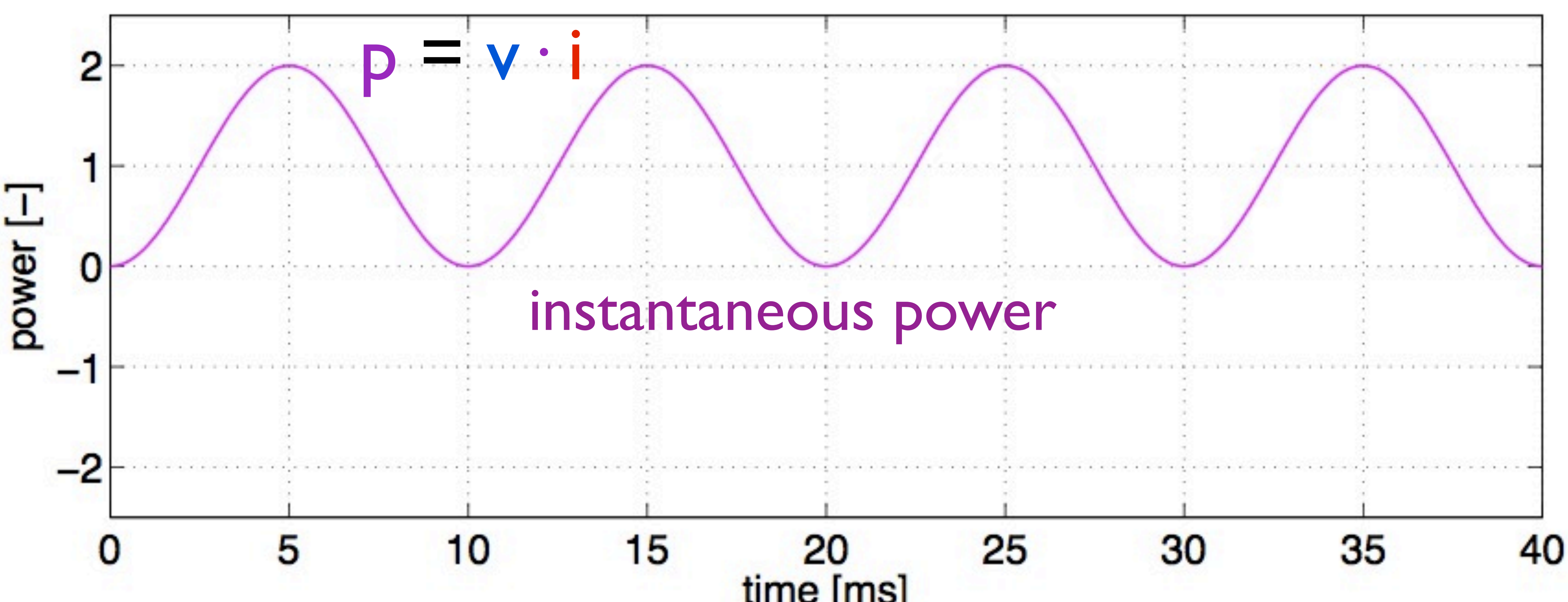
$R_V=\infty$



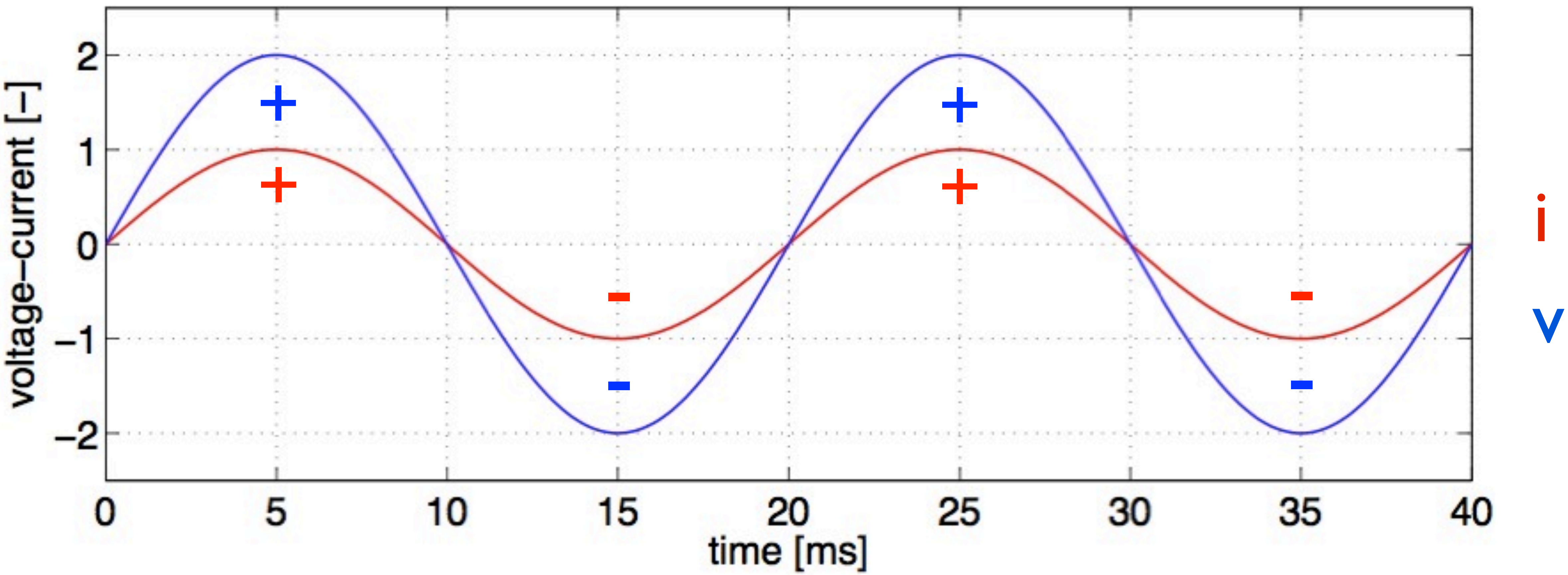


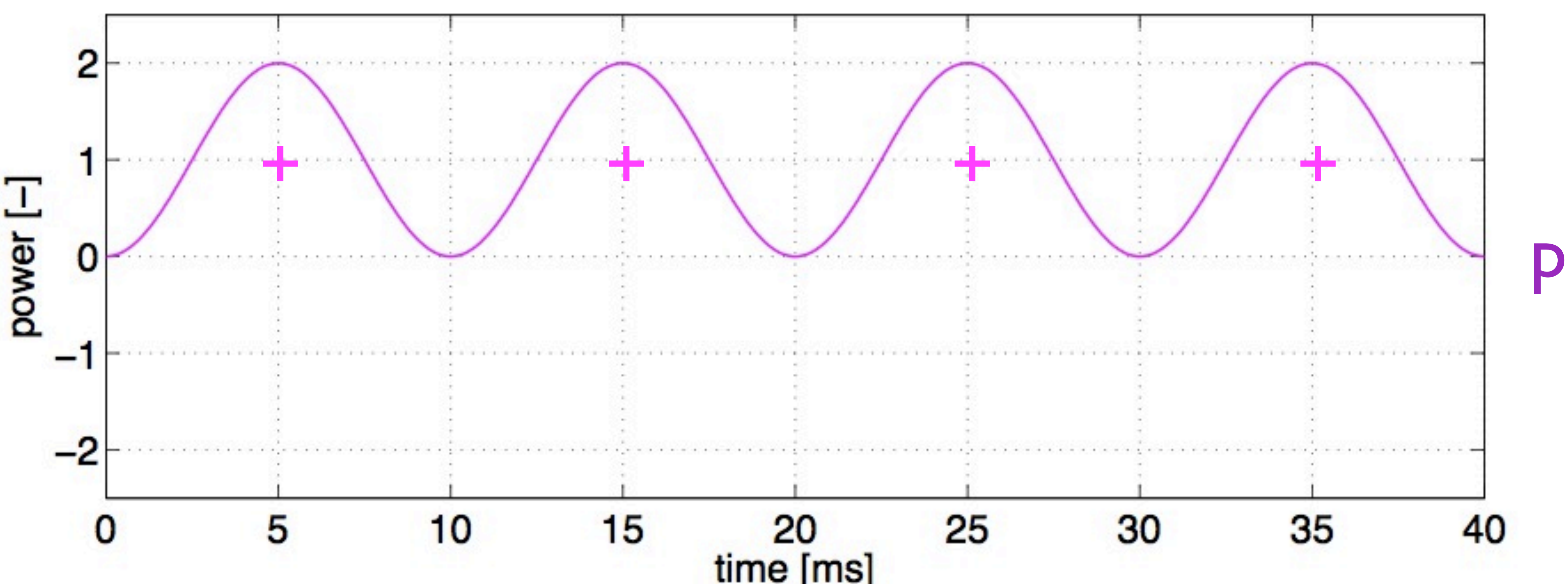
AC POWER


v
i
voltage–current [–]
time [ms]

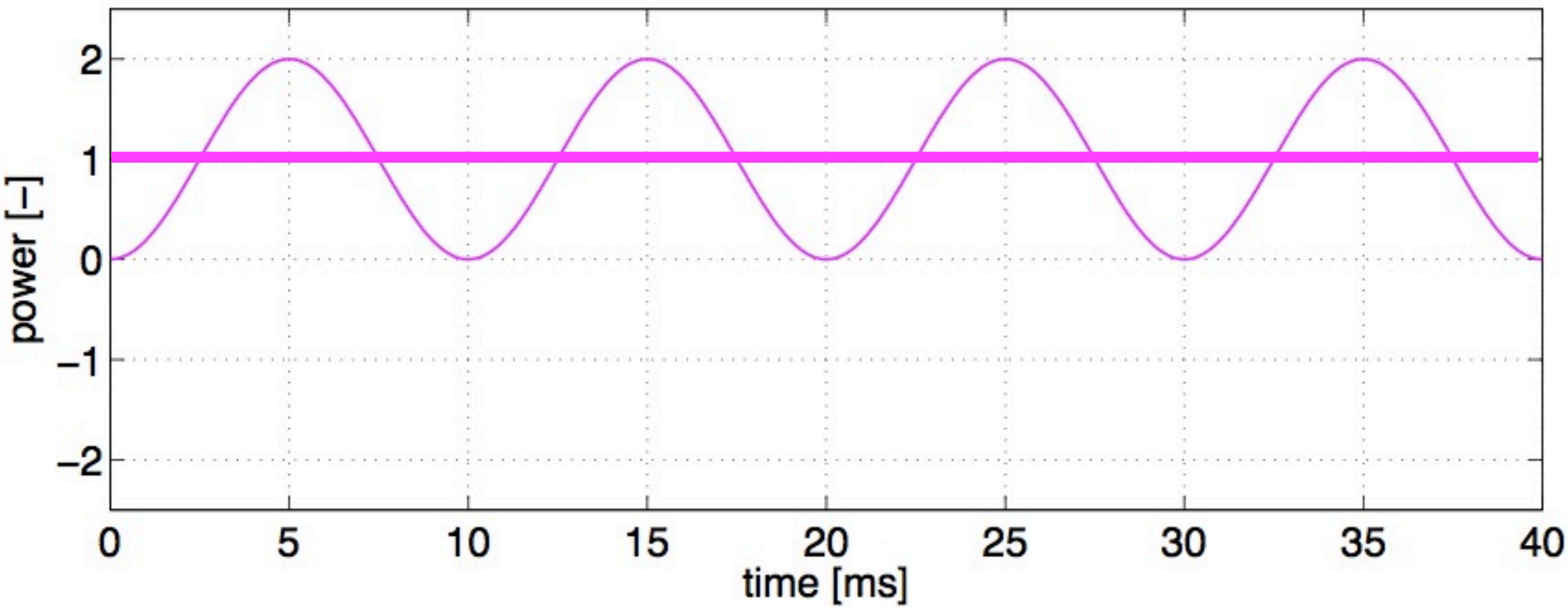

p = v · i
instantaneous power
power [–]
time [ms]

AC POWER



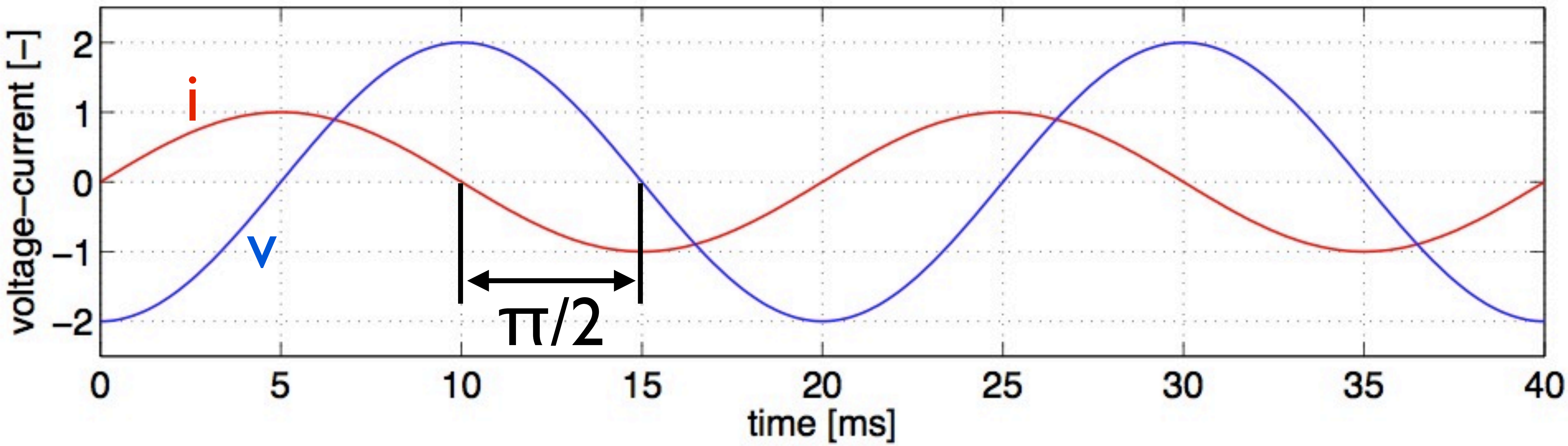


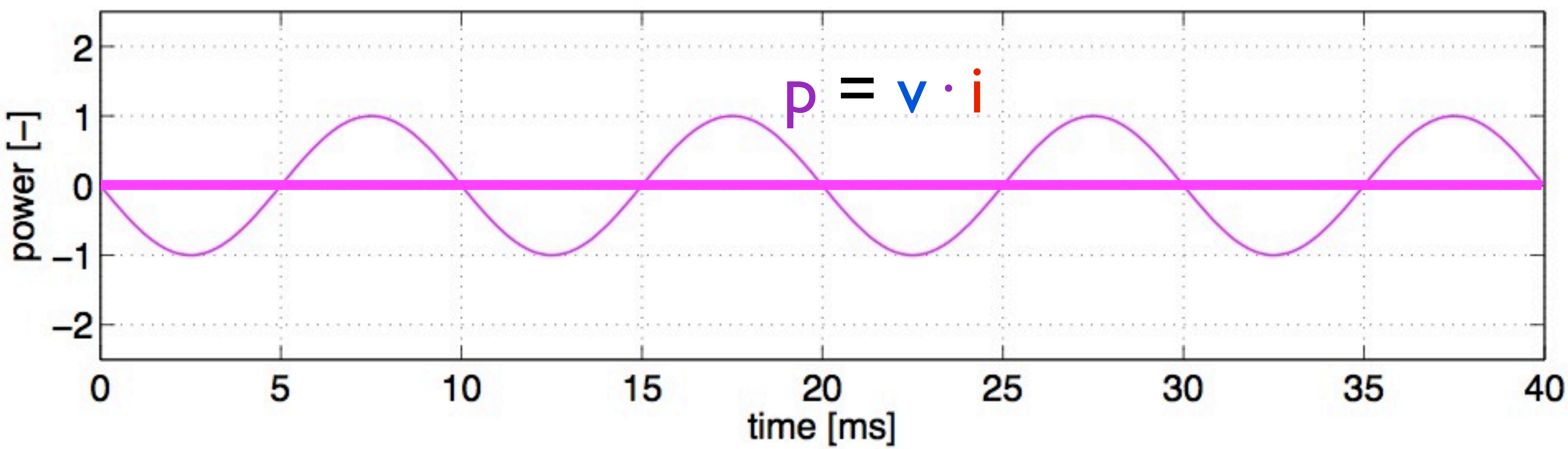
The average value of the instantaneous power is the effective power we are interested in



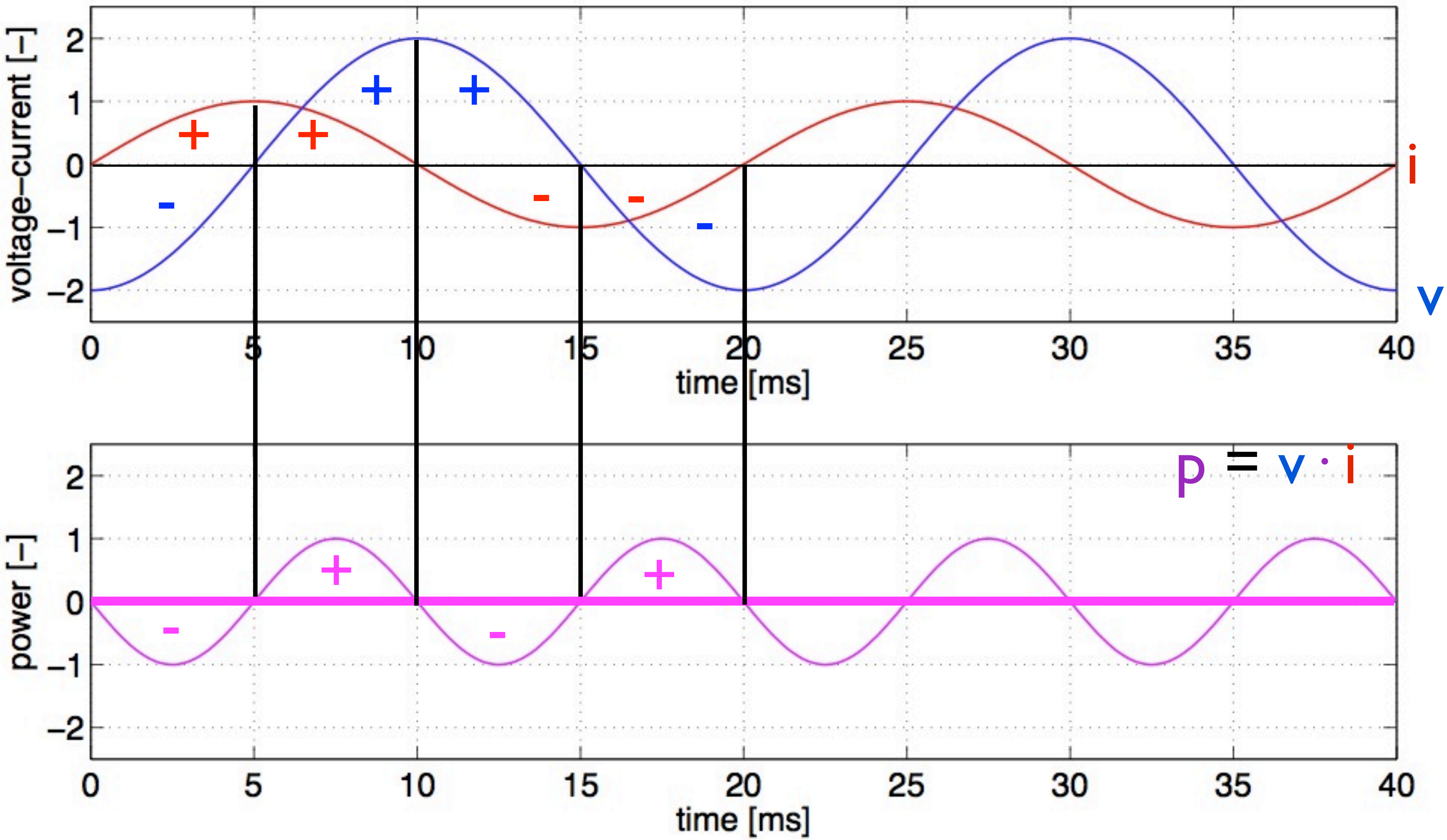
$$P = \frac{1}{T}\int_0^T p \;dt = \frac{1}{T}\int_0^T v \cdot i \, dt$$

If the current is π/2 shifted then the average power is 0

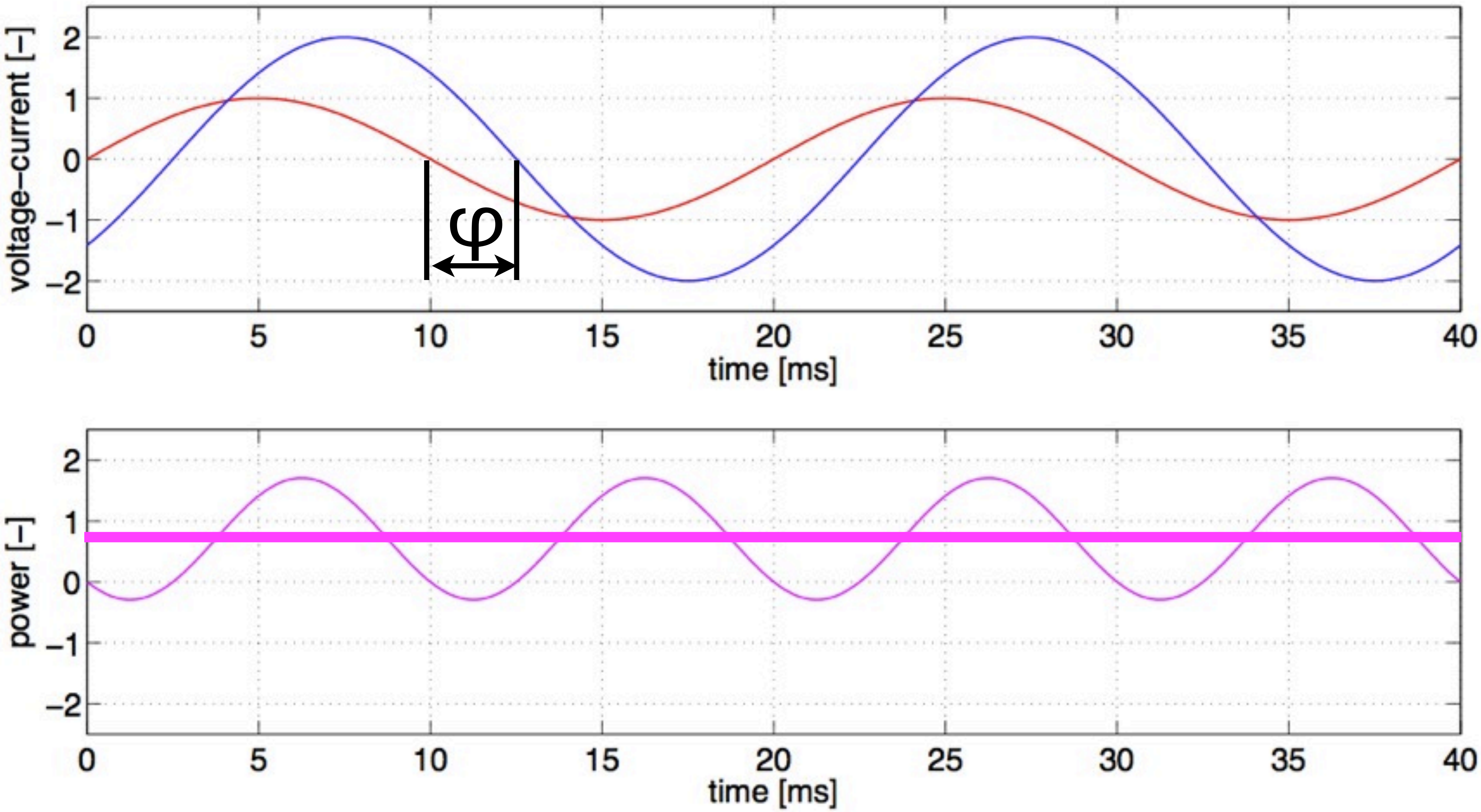




If the current is π/2 shifted then the average power is 0

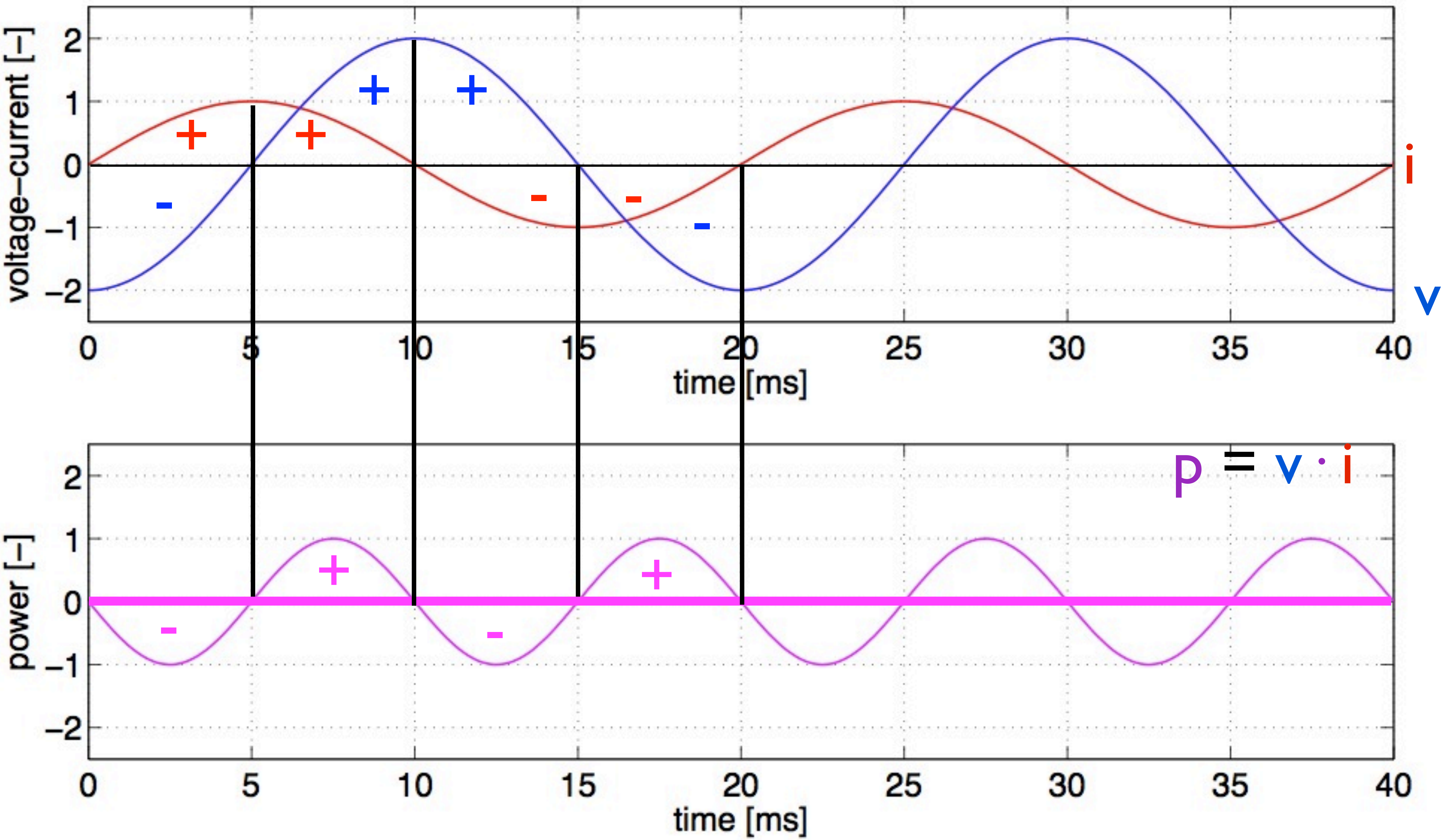


If the phase difference between voltage and current is an angle φ



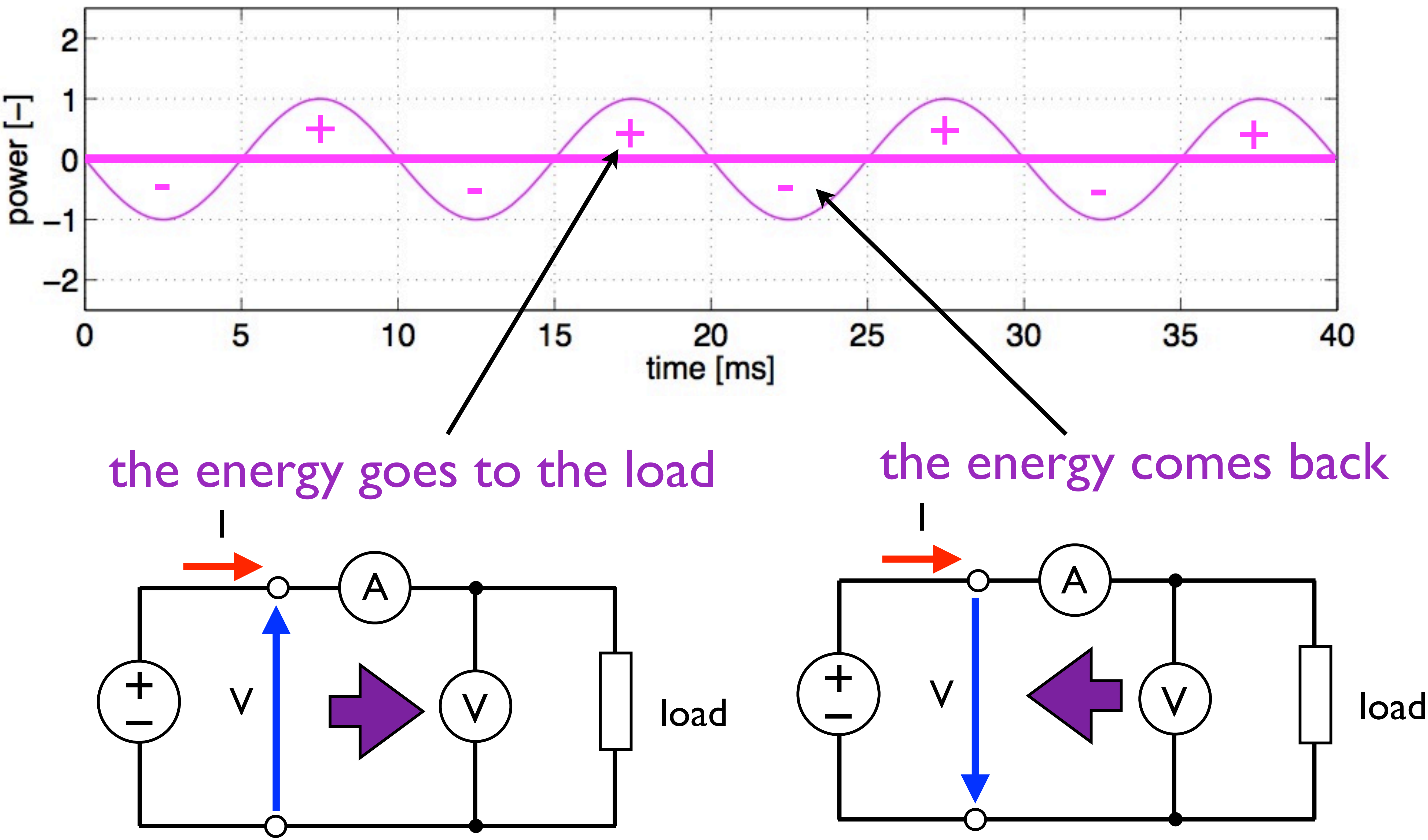
$$P = V_{RMS} \cdot I_{RMS} \cdot \cos\varphi$$

Let's go back to the case when φ =π/2



Let’s go back to the case when $\varphi = \pi/2$

$p = v \cdot i$

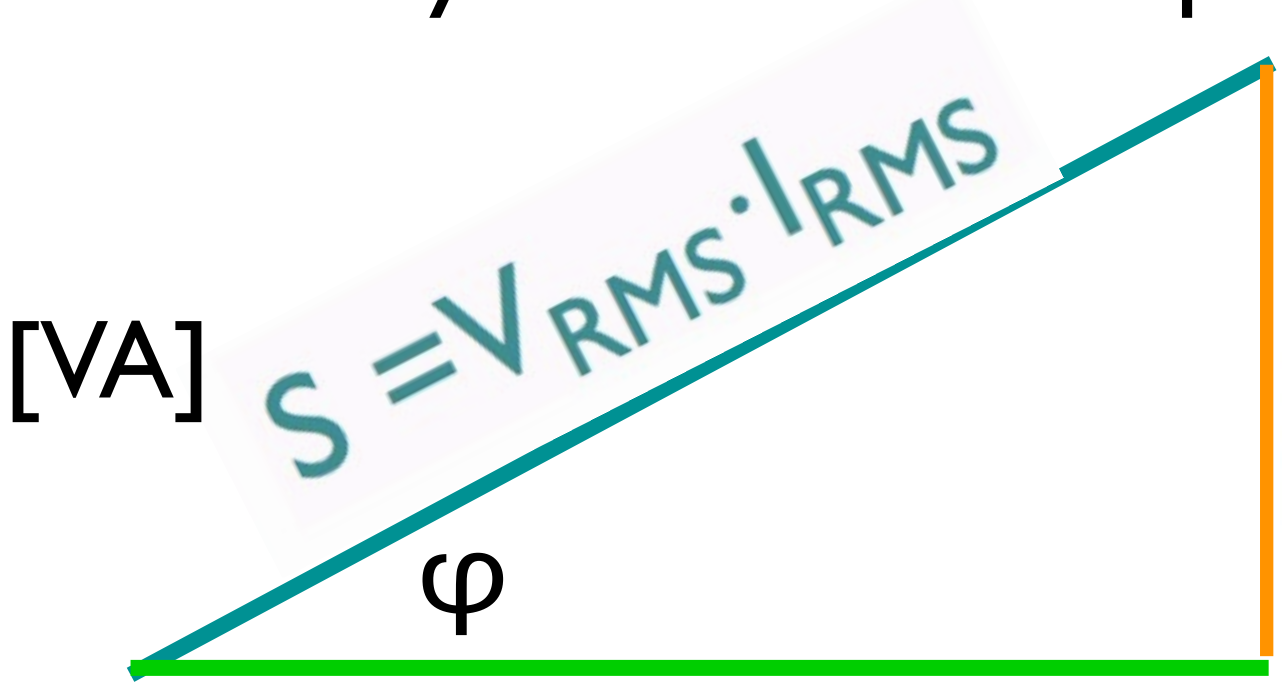


REACTIVE POWER

Represents the energy which oscillates between the generator and the load without producing any transfer on energy

$$Q = V_{RMS} \cdot I_{RMS} \cdot \sin\varphi$$

Finally we have three powers to measure



$$Q = V_{RMS} \cdot I_{RMS} \cdot \sin\varphi$$

[VAR]

$$P = V_{RMS} \cdot I_{RMS} \cdot \cos\varphi$$

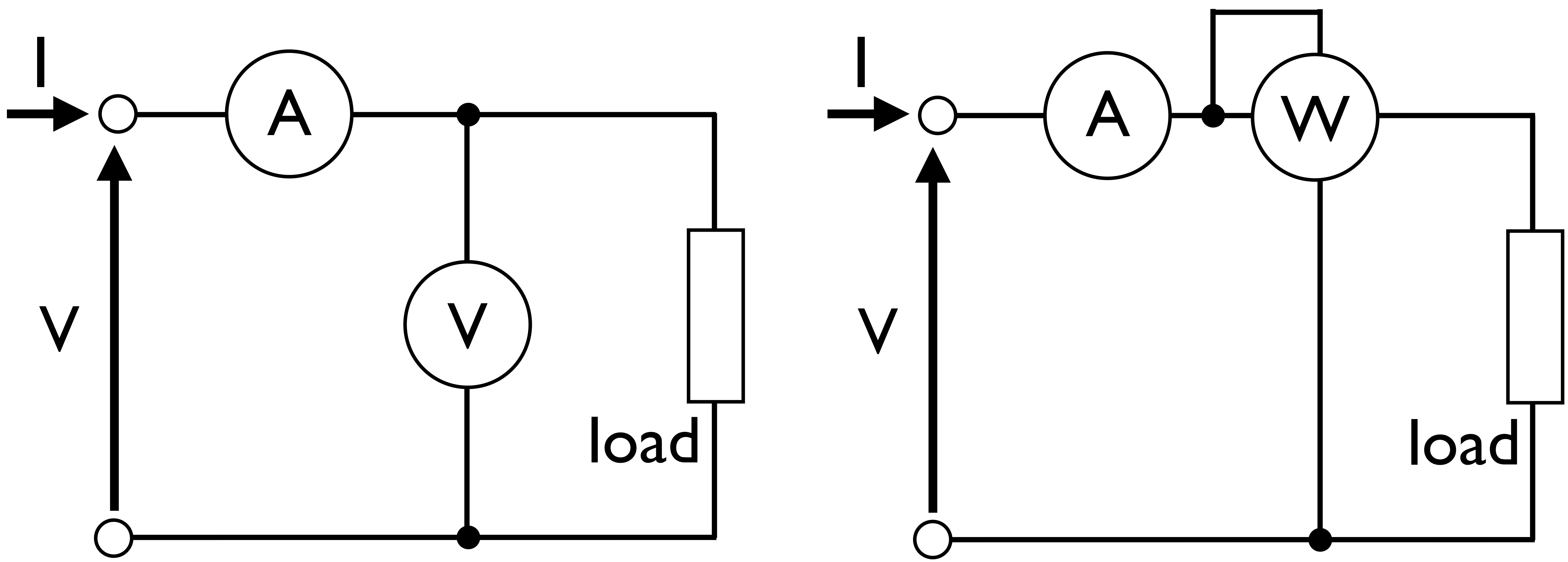
[W]

$$S = \sqrt{P^2 + Q^2}$$

In case of non-harmonic waveforms: $S = \sqrt{P^2 + Q^2 + D^2}$

But why should we measure all of them?

In this case I can't simply use a voltmeter and an ammeter, because besides voltage and current I have to measure also the phase between them



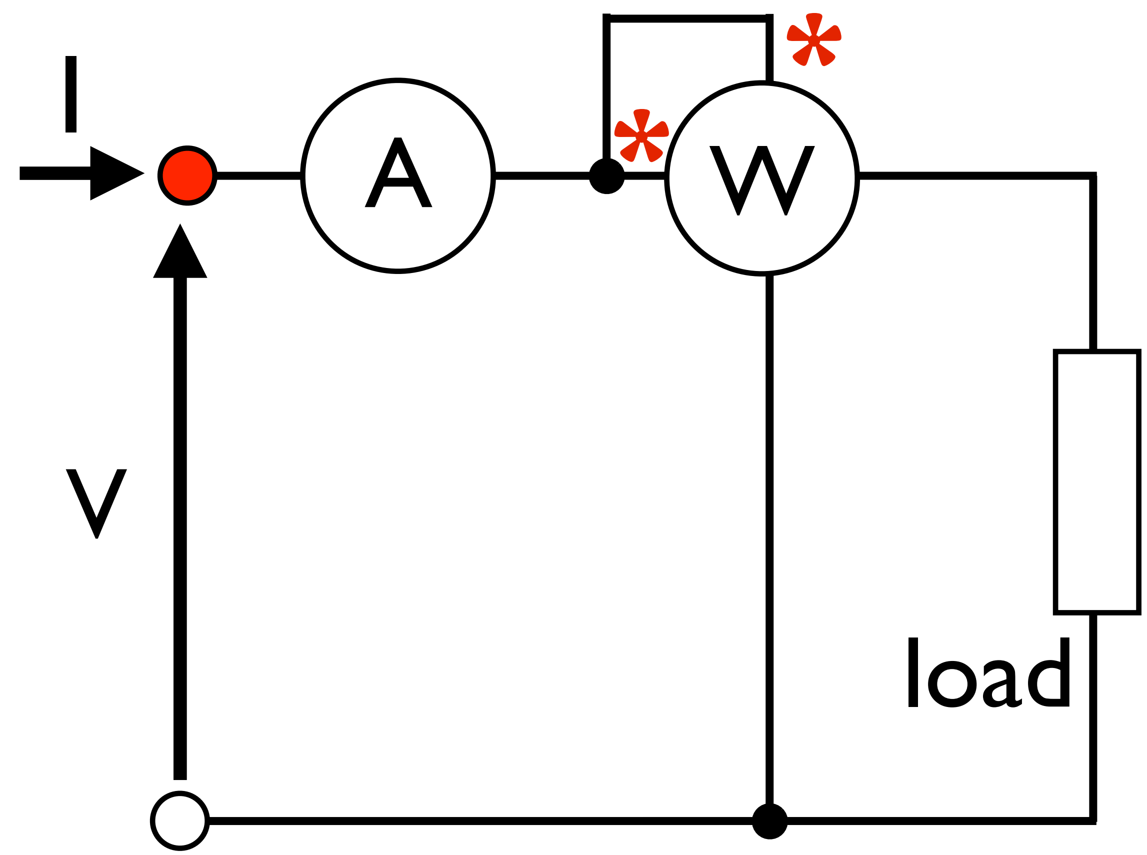
The wattmeter

It has two coils: a voltmeter coil and ammeter coil

BE CAREFUL:

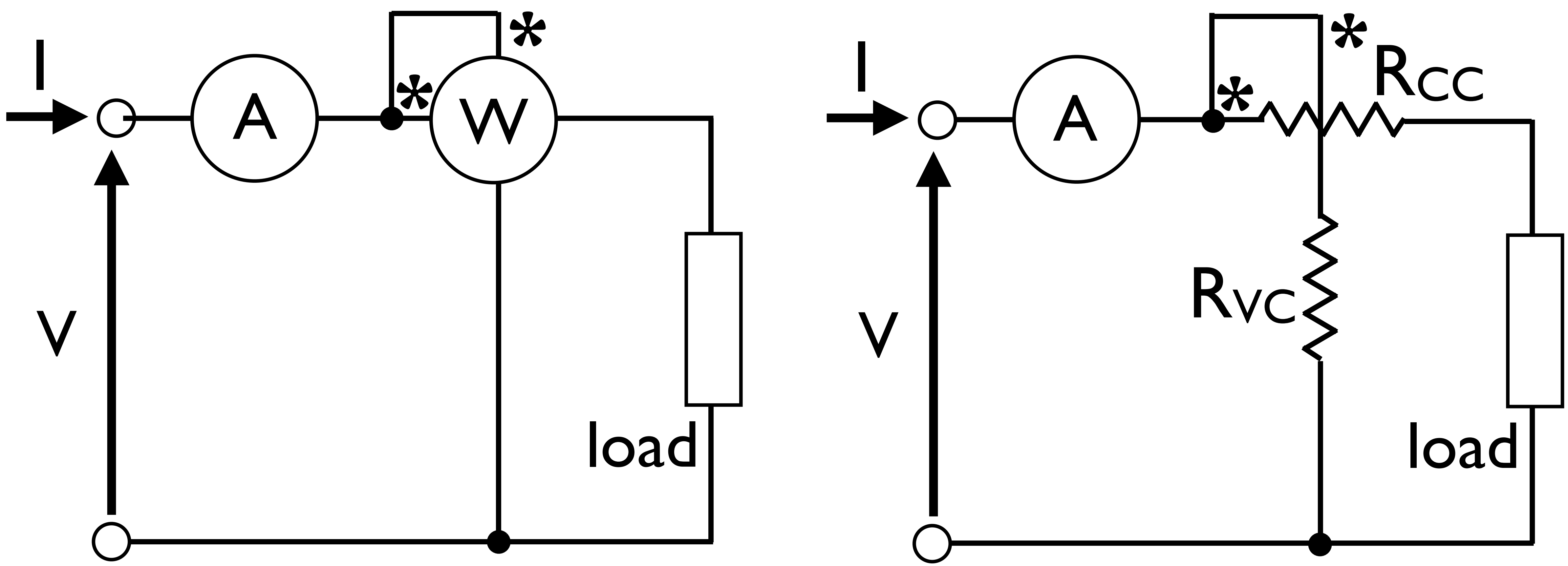
1) both coils have a sign

2) if $I>I_{max}$ the instrument is saturated (bad!). But if simultaneousely $V<<V_{max}$ the total reading will appear ok



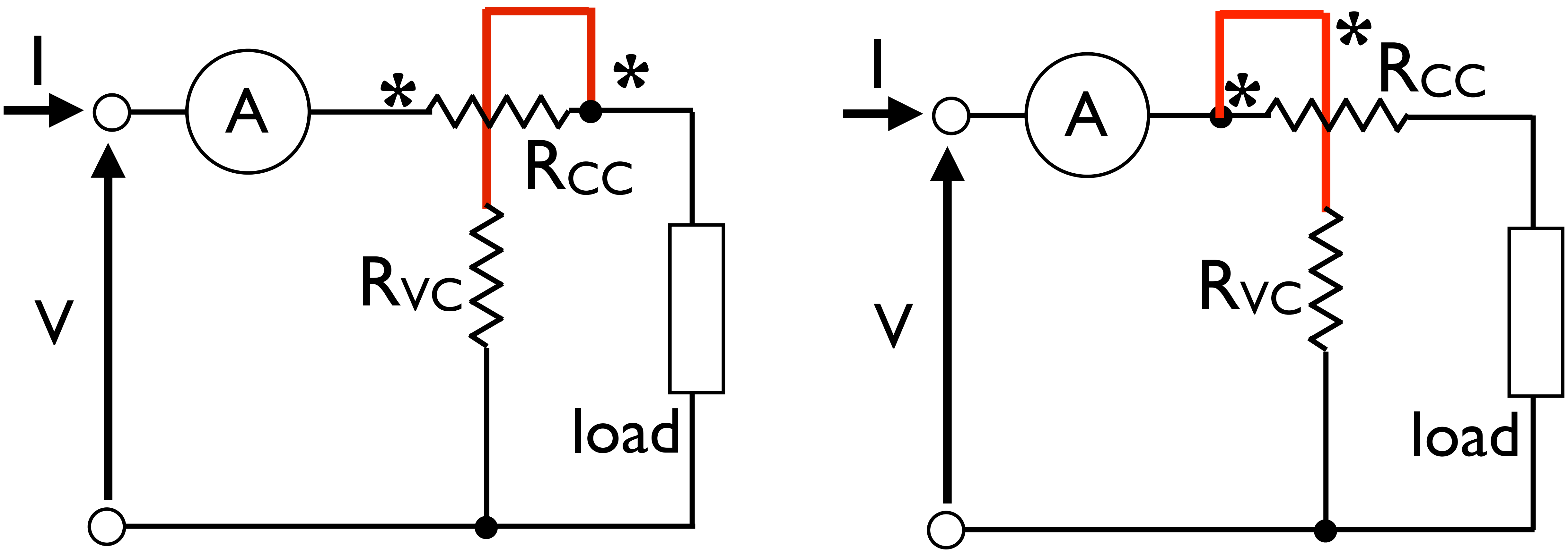
The wattmeter

Still we have to apply correction of methodical error



The wattmeter

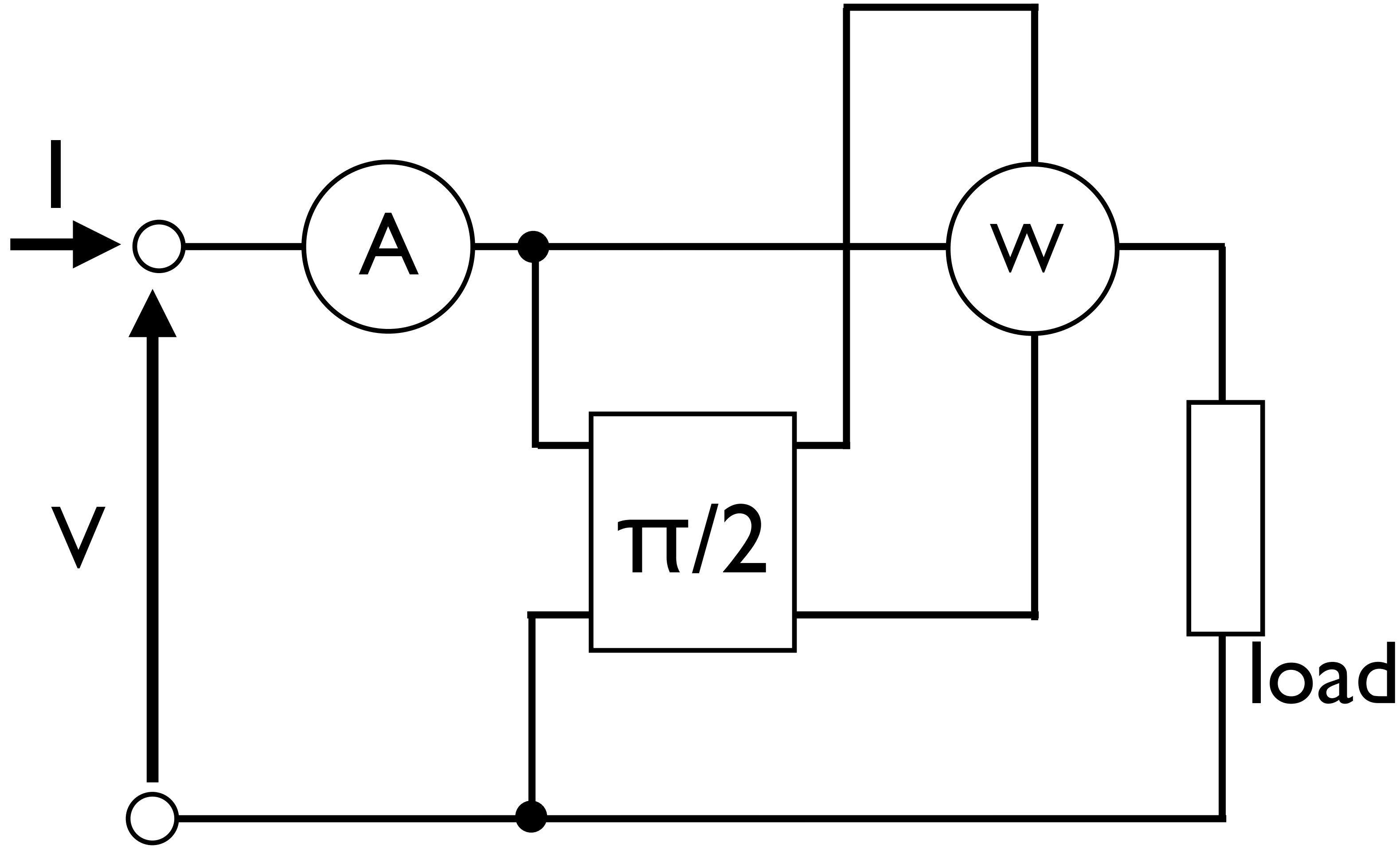
Still we have to apply correction of methodical error



A B

B is better if you don't correct methodical error because $P_{CC}<P_{VC}$

Measurement of reactive power

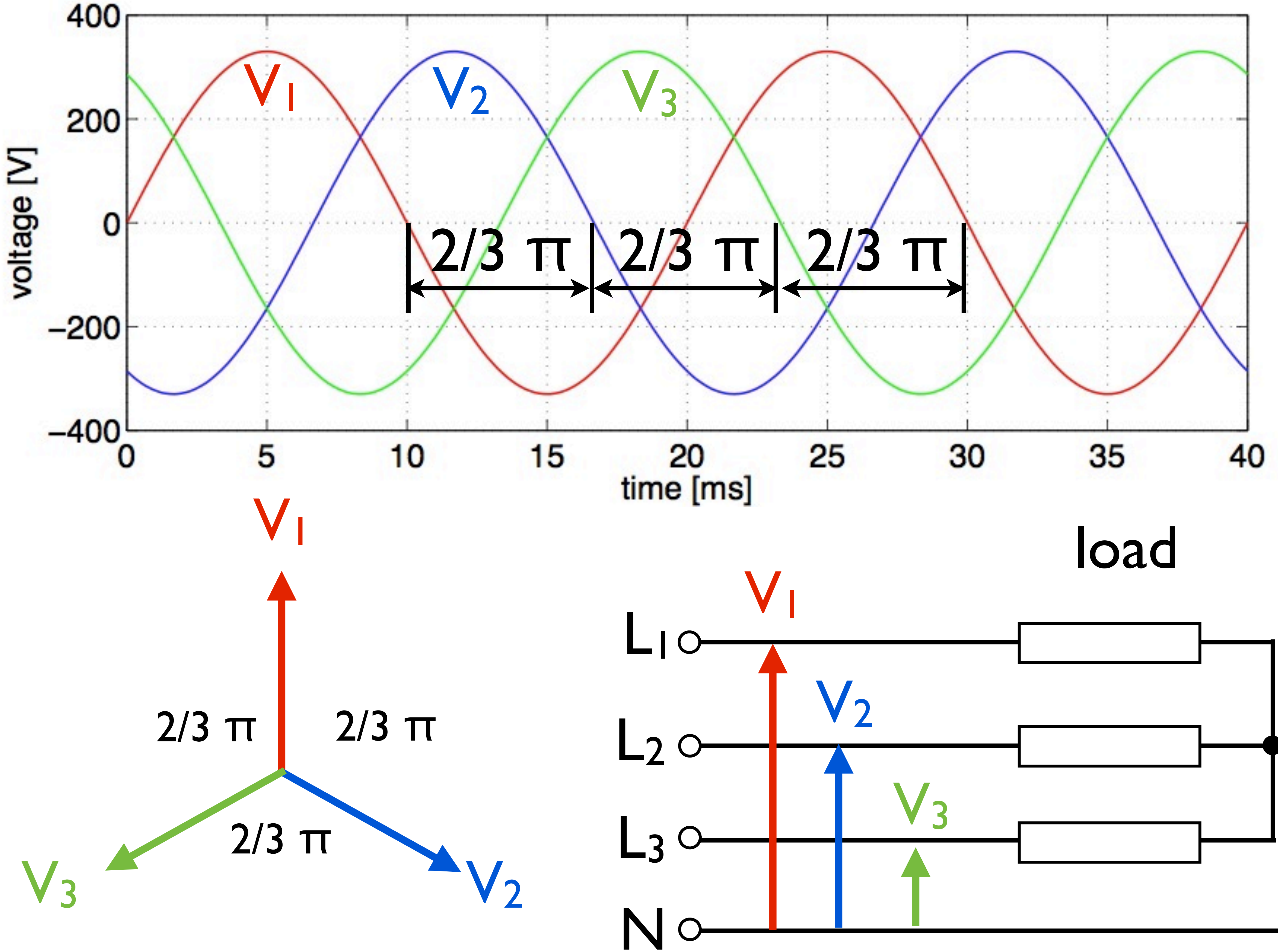


$$P = V_{RMS} \cdot I_{RMS} \cdot \cos\varphi$$

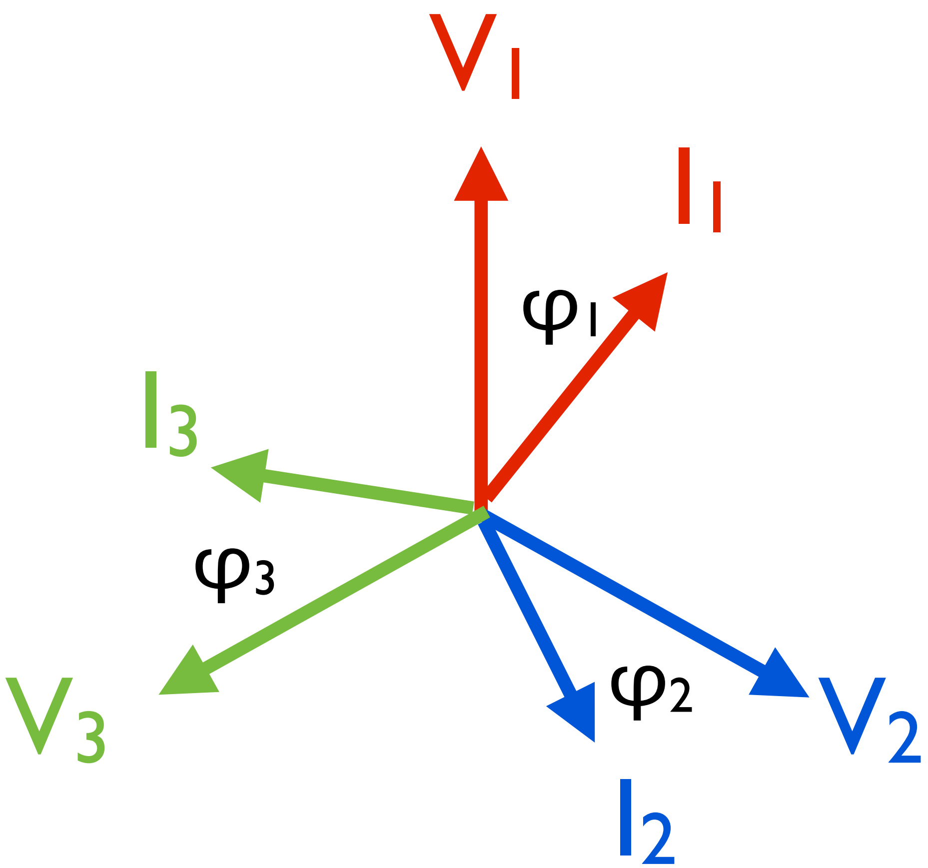
$$Q = V_{RMS} \cdot I_{RMS} \cdot \sin\varphi$$

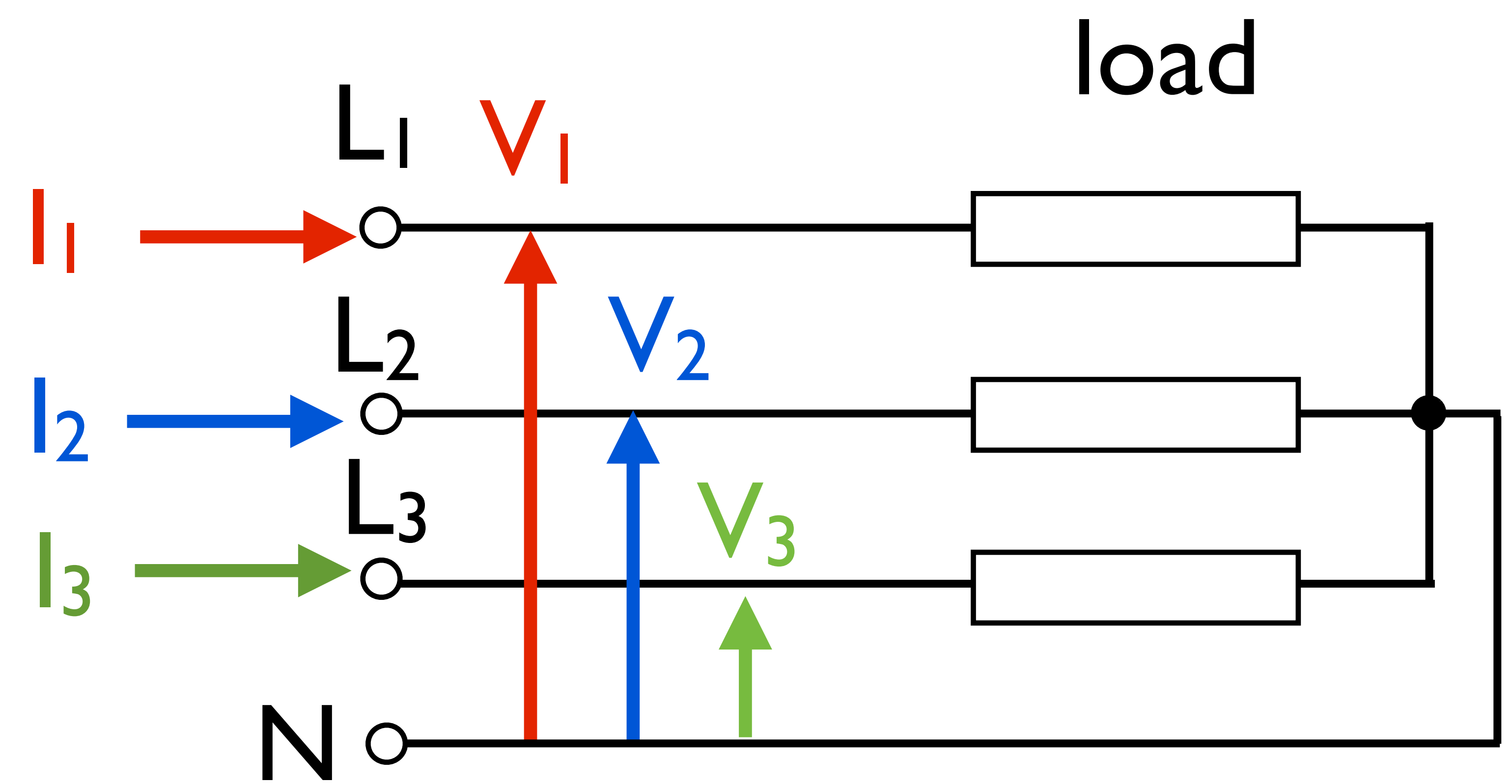
We change from cosφ
to sinφ
shifting the voltage by π/2

Three phase industrial net



CASE A - Symmetrical voltage system - balanced load
ACTIVE POWER





If the load is balanced and all voltages have the same amplitude, all the currents have also the same amplitude

$$|V_1| = |V_2| = |V_3| \qquad |\varphi_1| = |\varphi_2| = |\varphi_3|$$

$$|I_1| = |I_2| = |I_3|$$

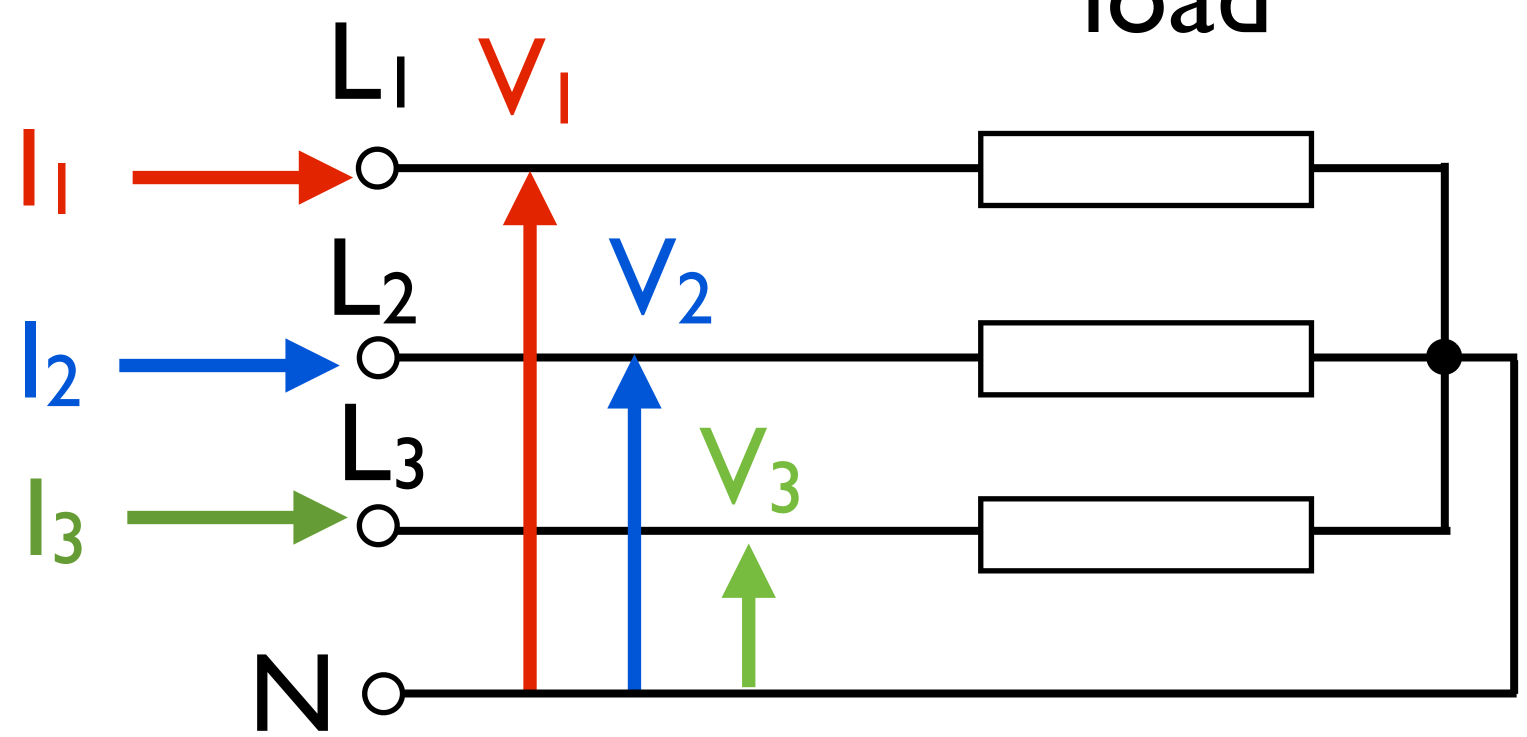
CASE A - Symmetrical voltage system - balanced load
ACTIVE POWER

$|V_1| = |V_2| = |V_3|$

$|I_1| = |I_2| = |I_3|$

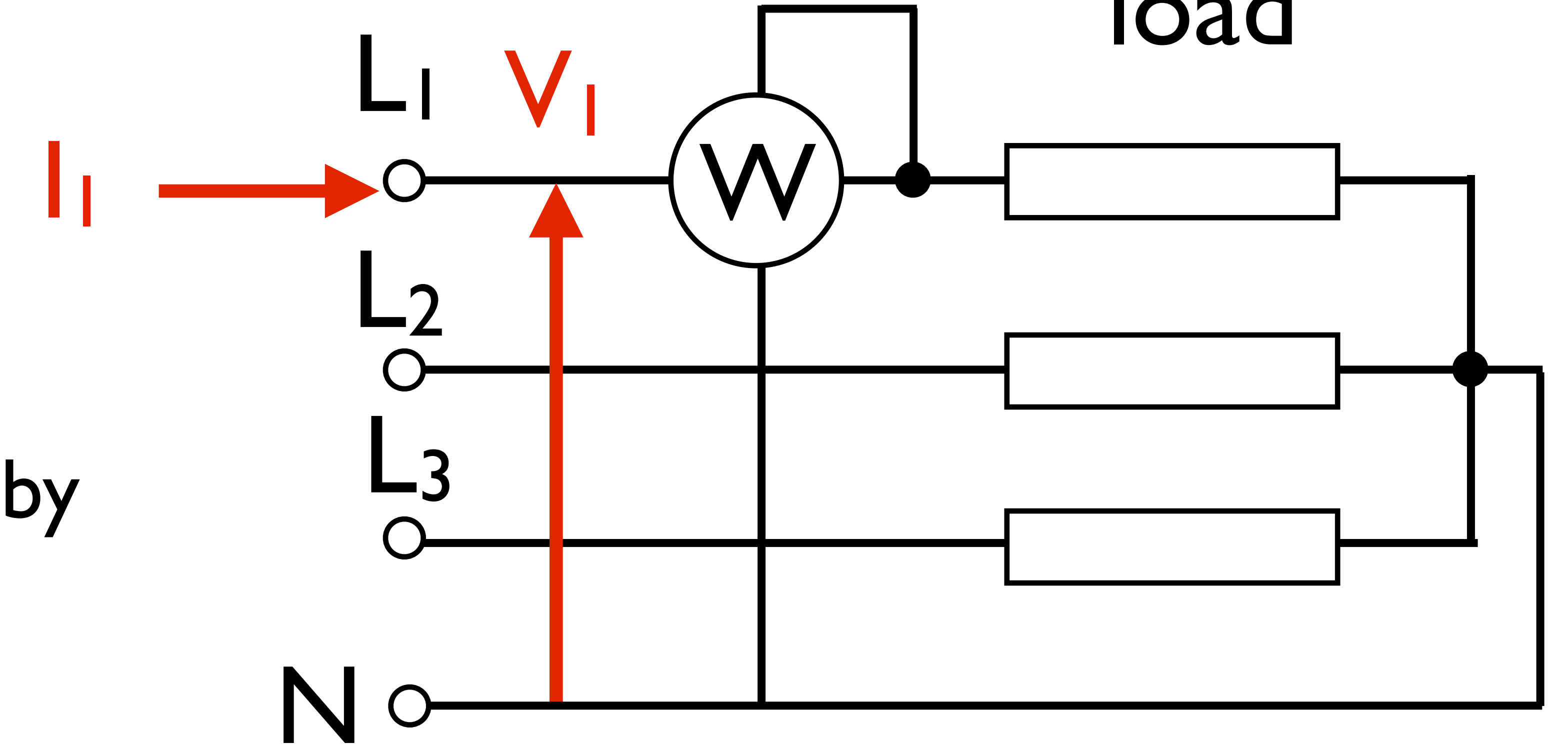
$|\varphi_1| = |\varphi_2| = |\varphi_3|$

Each phase absorbs the same power. It's enough to measure one of them and triple it



$P=3\ P_m$

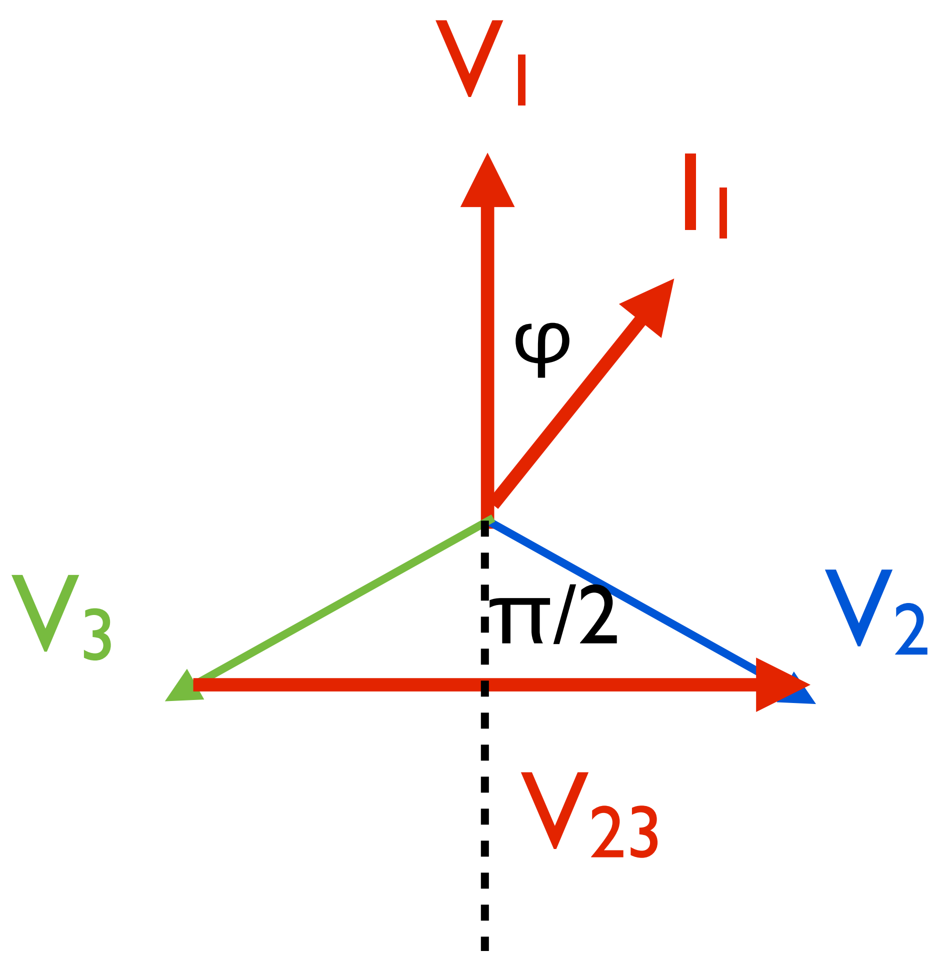
P_m is the power measure by the wattmeter

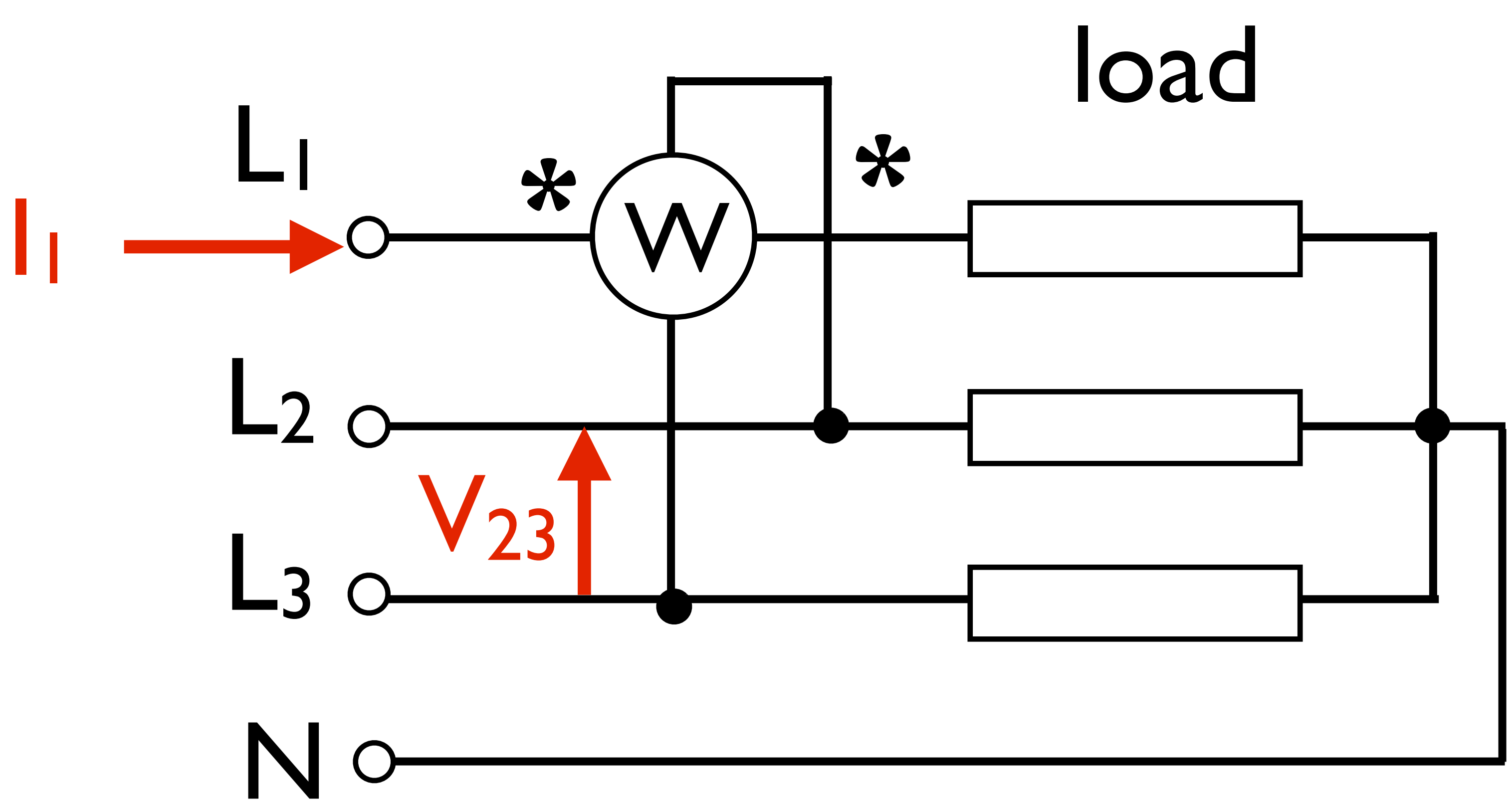


CASE A - Symmetrical voltage system - balanced load
REACTIVE POWER

Again, we can measure one reactive power and then triple it... but how can we obtain π/2 phase shift?

$$Q = V_{RMS} \cdot I_{RMS} \cdot \sin\varphi = V_{RMS} \cdot I_{RMS} \cdot \cos\left(\frac{\pi}{2} - \varphi\right)$$



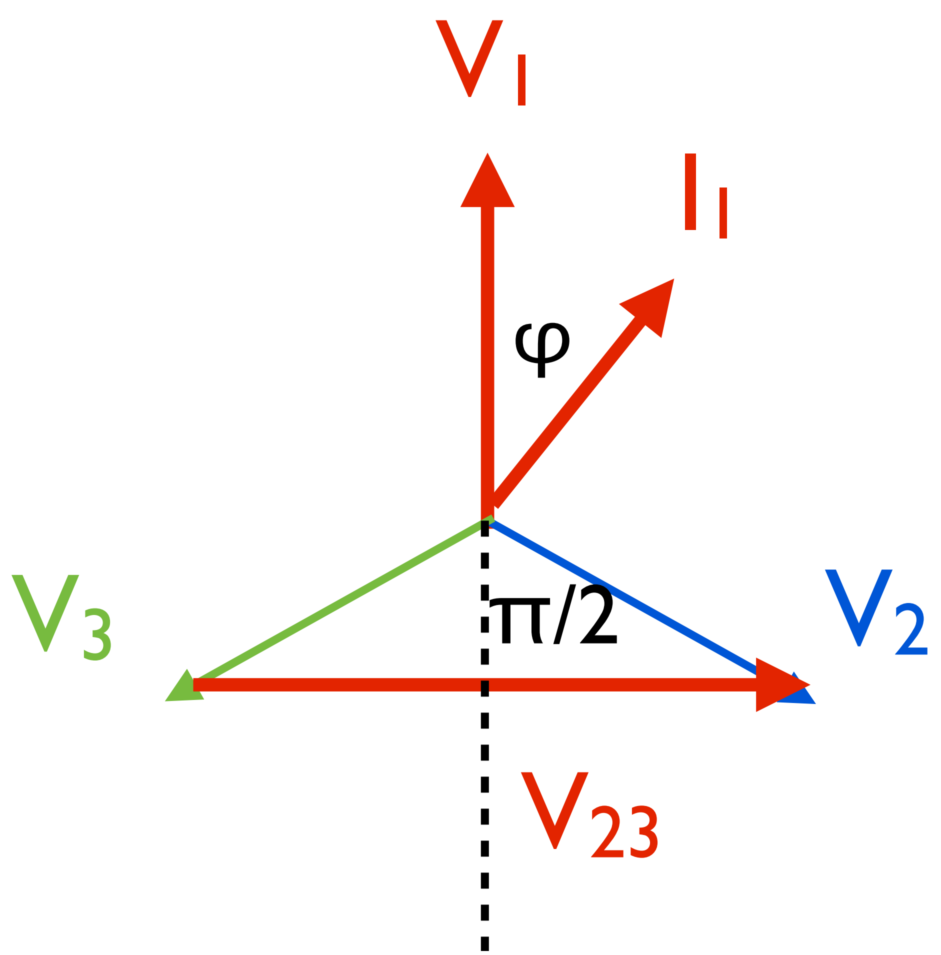


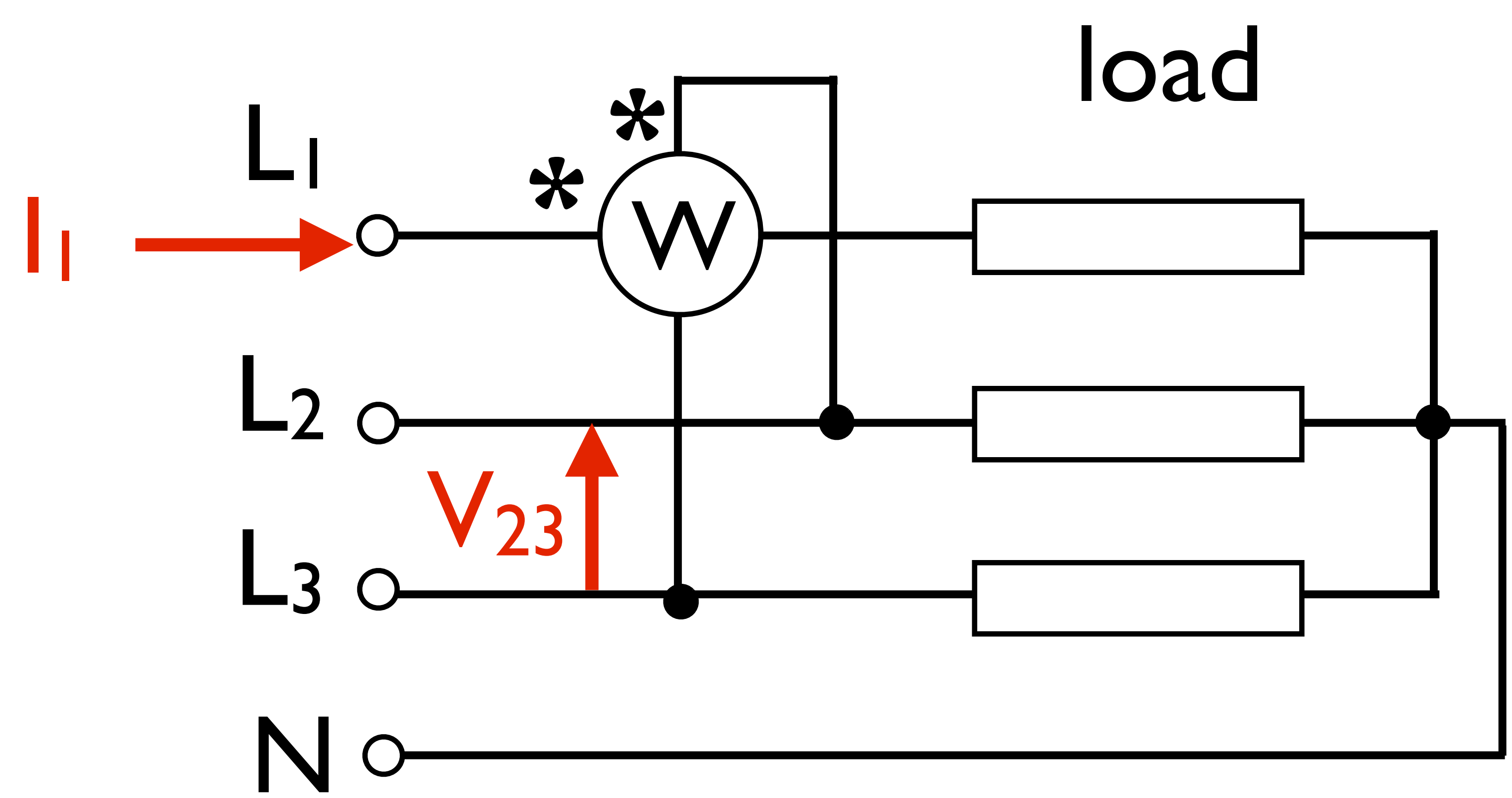
We can still use a wattmeter to measure reactive power but V_{23} is larger than V_1!

$$V_{23} = \sqrt{3}\, V_1$$

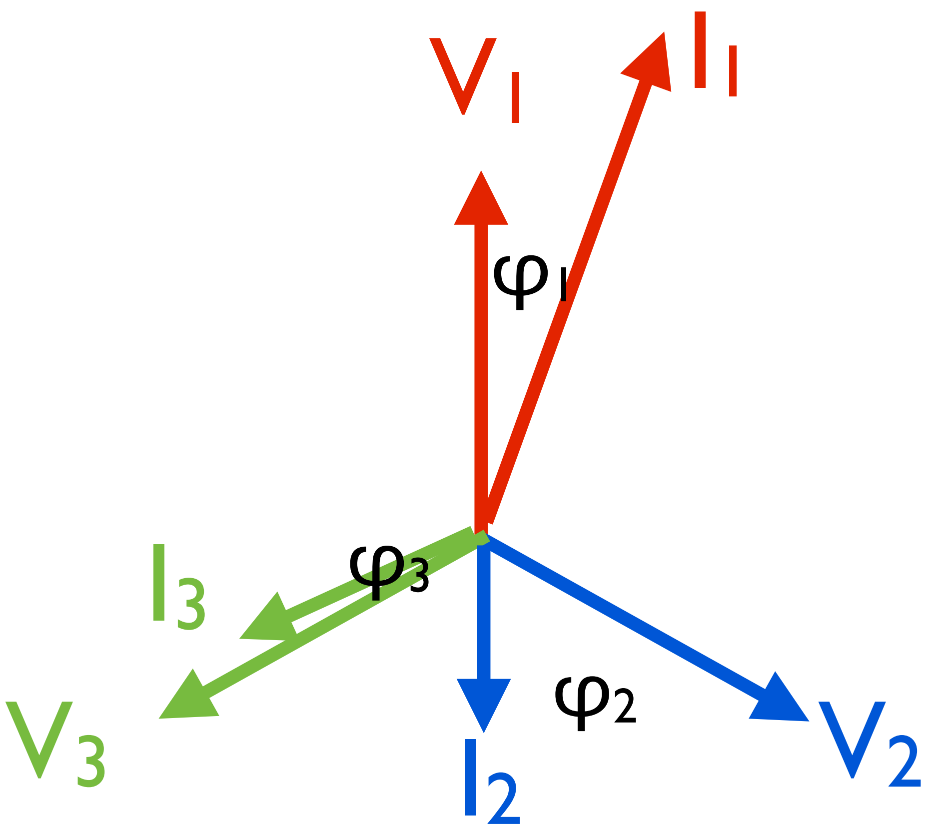
Therefore, we must divide the measured power by $\sqrt{3}$

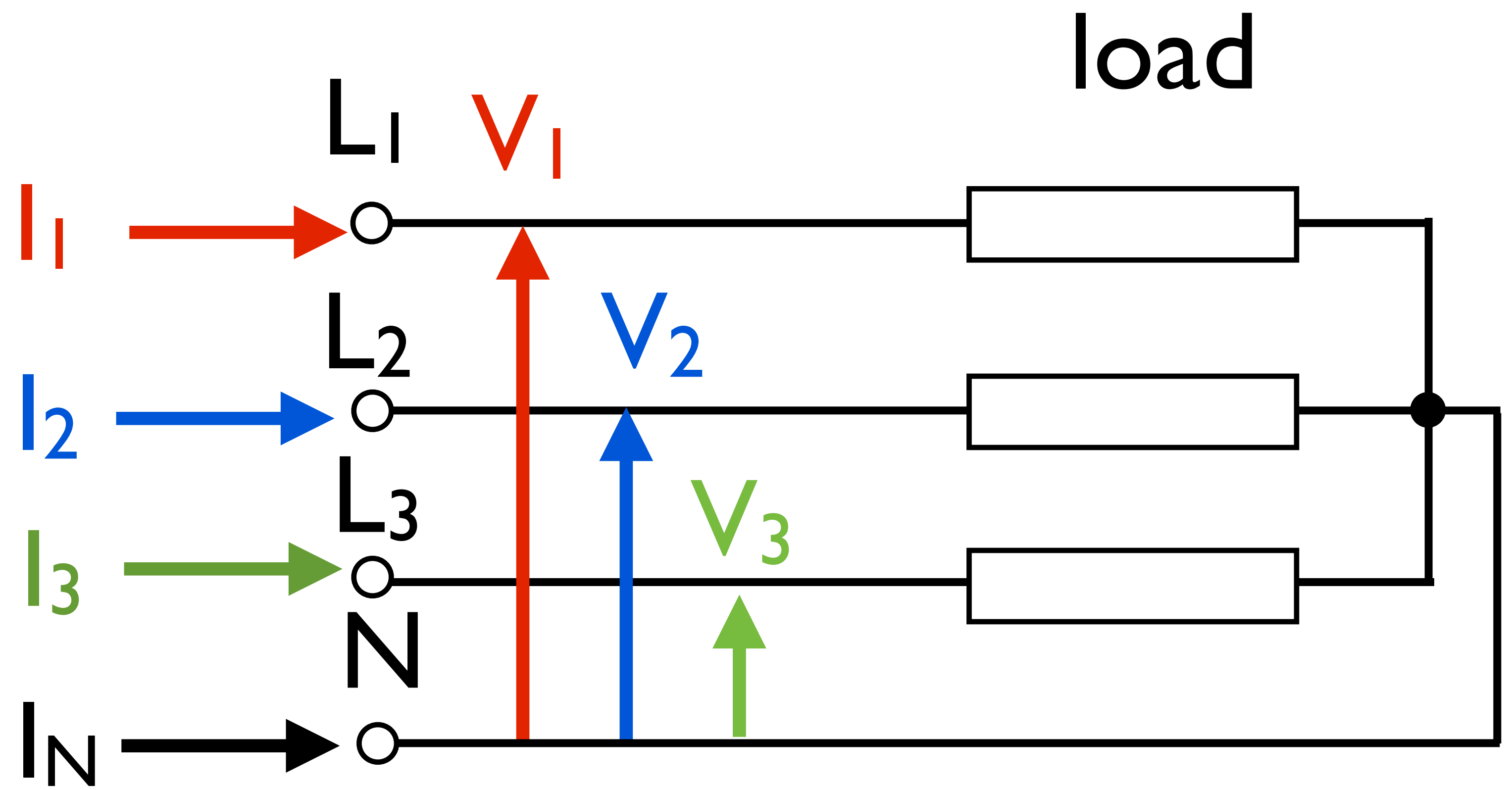
$$Q = 3\, P_m \,/\, \sqrt{3}$$





CASE B - Symmetrical voltage system - UNbalanced load
ACTIVE POWER





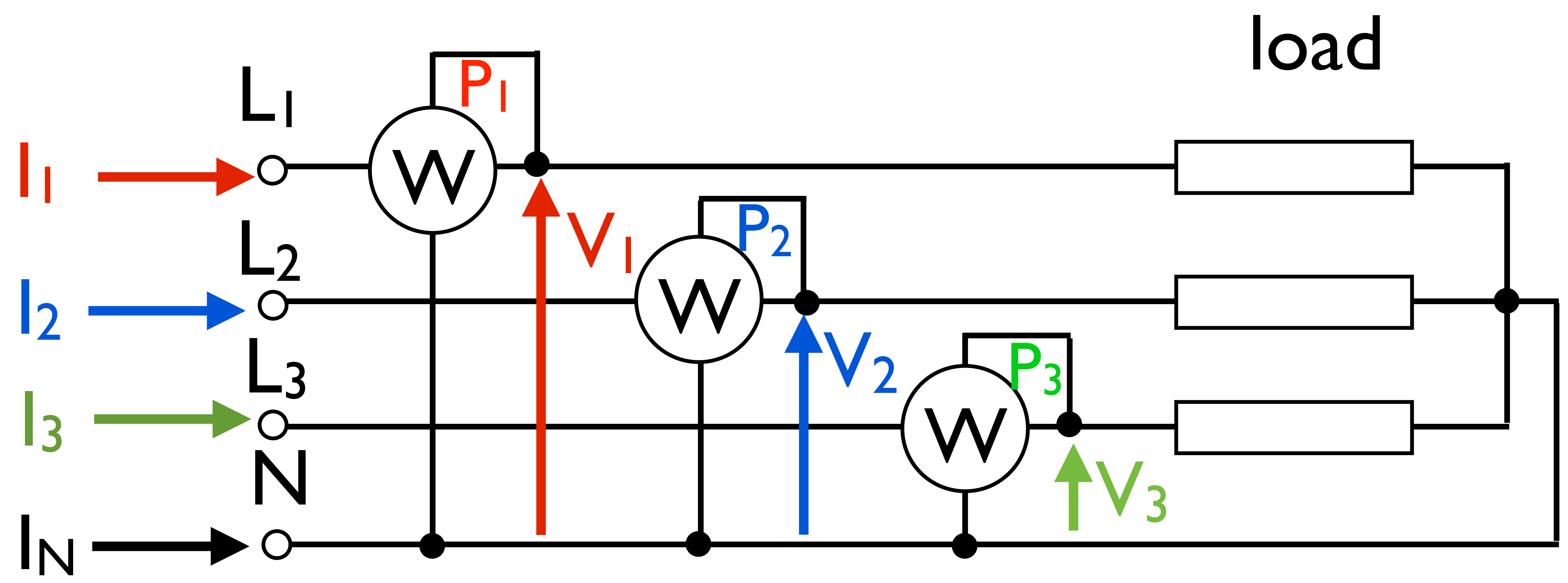
The voltages are still symmetrical, but each line absorbs a different current.

The vectorial sum of the currents is 0.

$$I_1 + I_2 + I_3 + I_N = 0$$

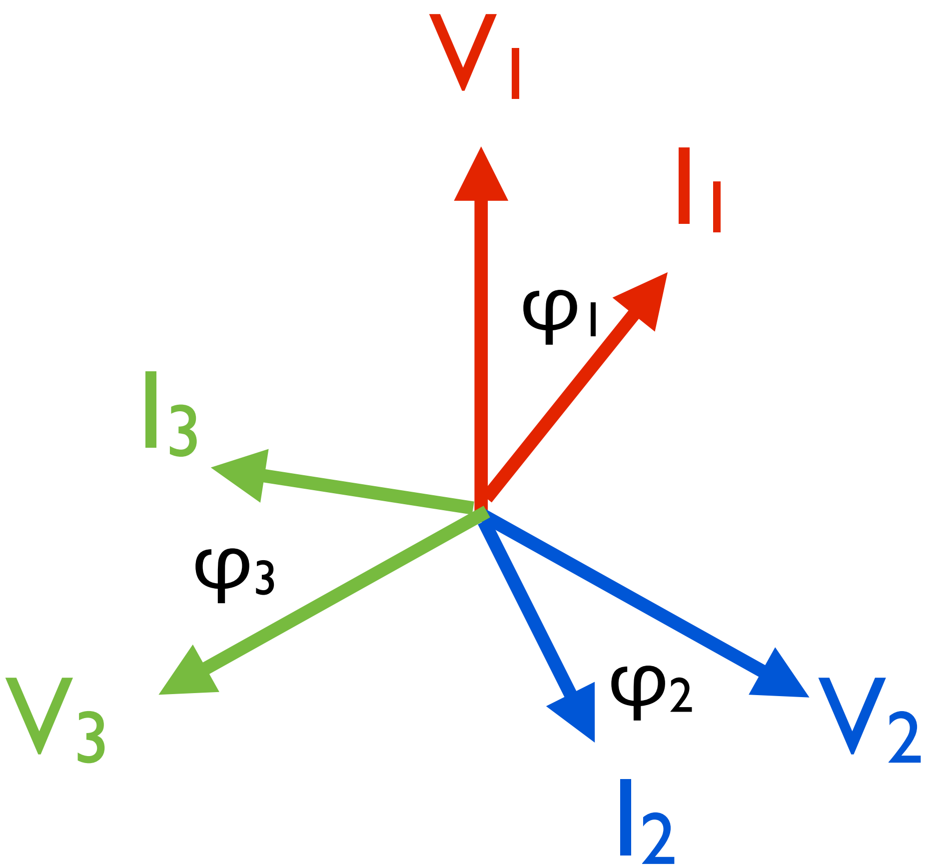
CASE B - Symmetrical voltage system - UNbalanced load

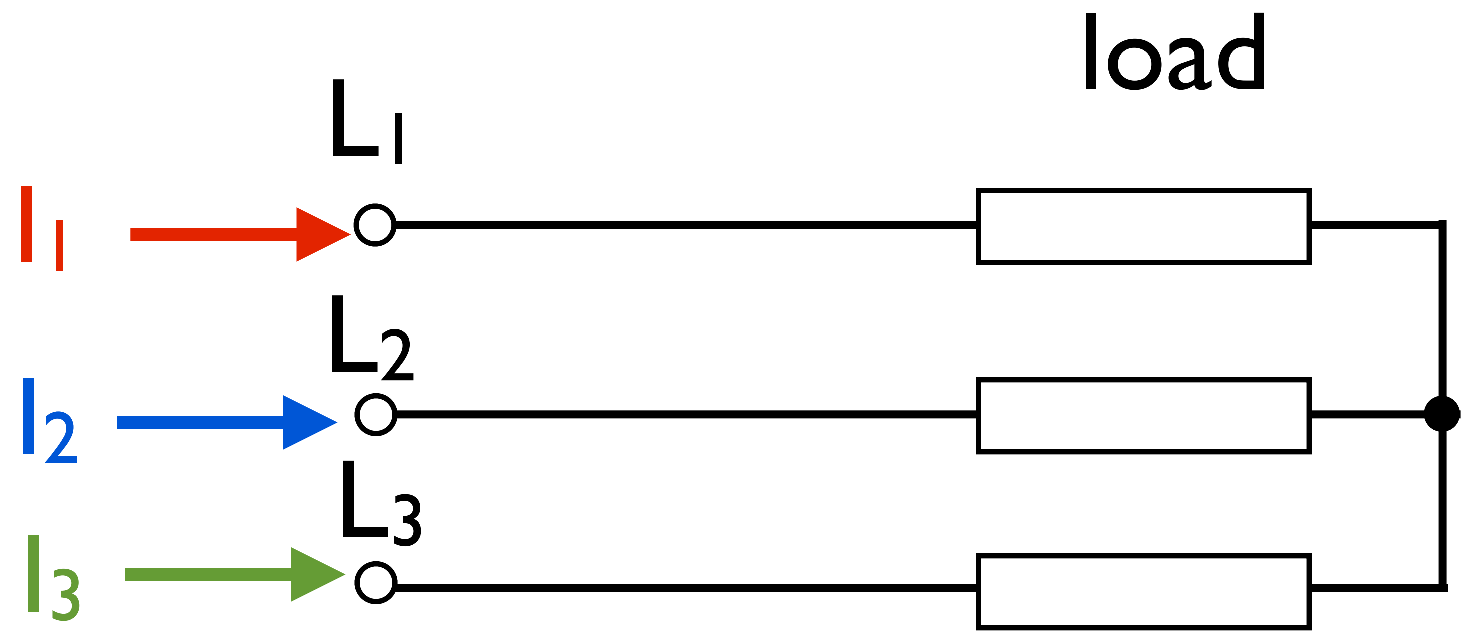
I can't do anything else than measure the power of each line and then sum them up



$$P = P_1 + P_2 + P_3$$

Special case: load with no N connection





$$I_1 + I_2 + I_3 = 0$$

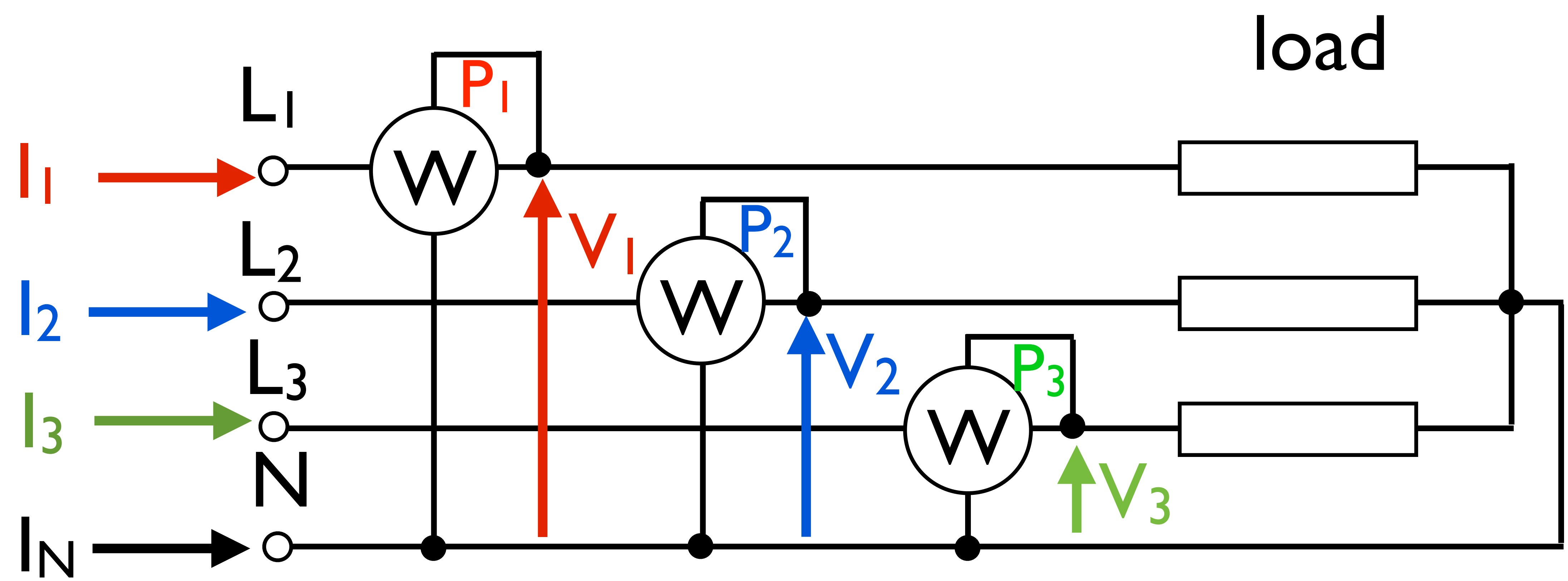
$$I_N = 0$$

ONLY 2 WATTMETERS

are required to measure the total power

CASE B - Symmetrical voltage system - UNbalanced load

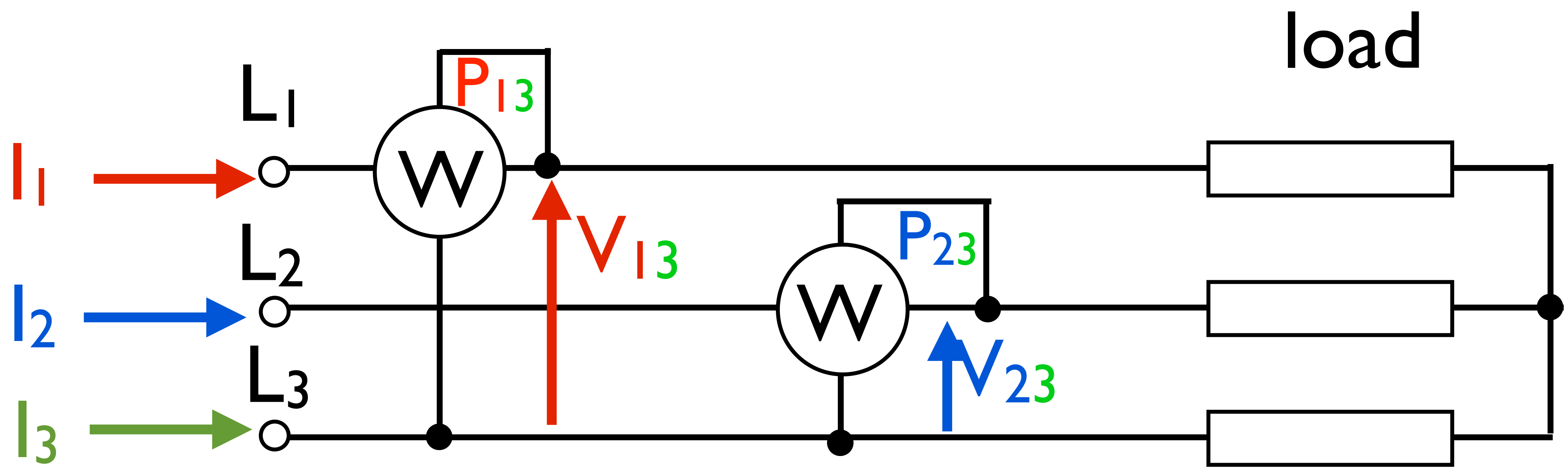
I can't do anything else than measure the power of each line and then sum them up



$$P = P_1 + P_2 + P_3$$

Aaron connection

$$P = P_{13} + P_{23}$$



$$i_1 + i_2 + i_3 = 0$$

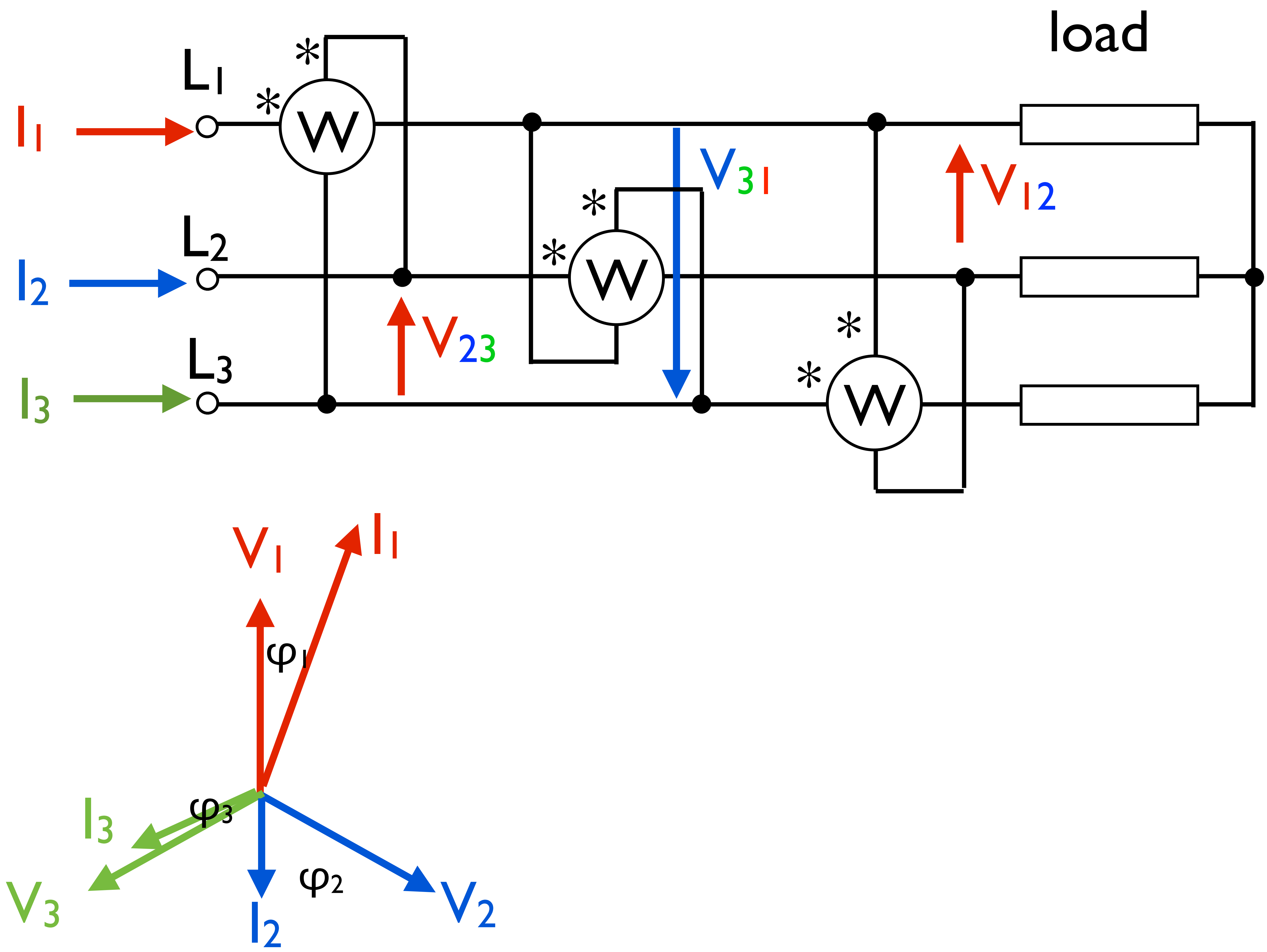
$$P = \frac{1}{T}\int_0^T (v_1 \cdot i_1 + v_2 \cdot i_2 + v_3 \cdot i_3)\, dt$$

$$\left(v_1 \cdot i_1 + v_2 \cdot i_2 - v_3 \cdot (i_1 + i_2)\right)$$

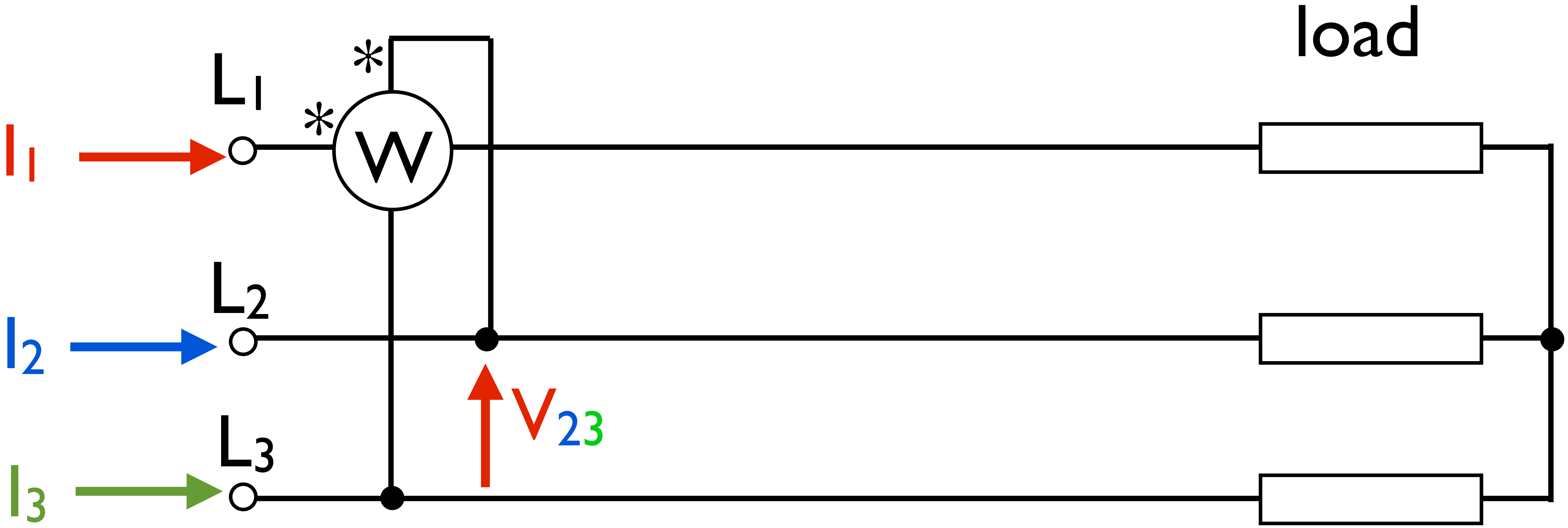
$$\left((v_1 - v_3) \cdot i_1 + (v_2 - v_3) \cdot i_2\right)$$

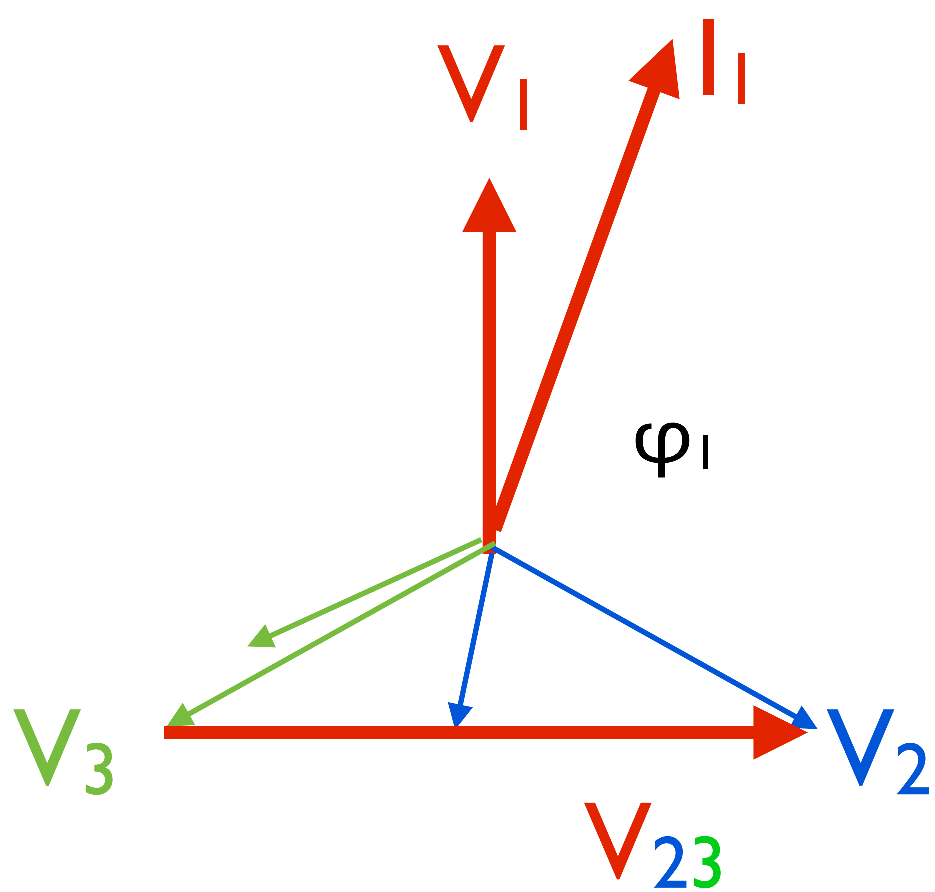
$$(v_{13} \cdot i_1 + v_{23} \cdot i_2)$$

REACTIVE POWER



REACTIVE POWER



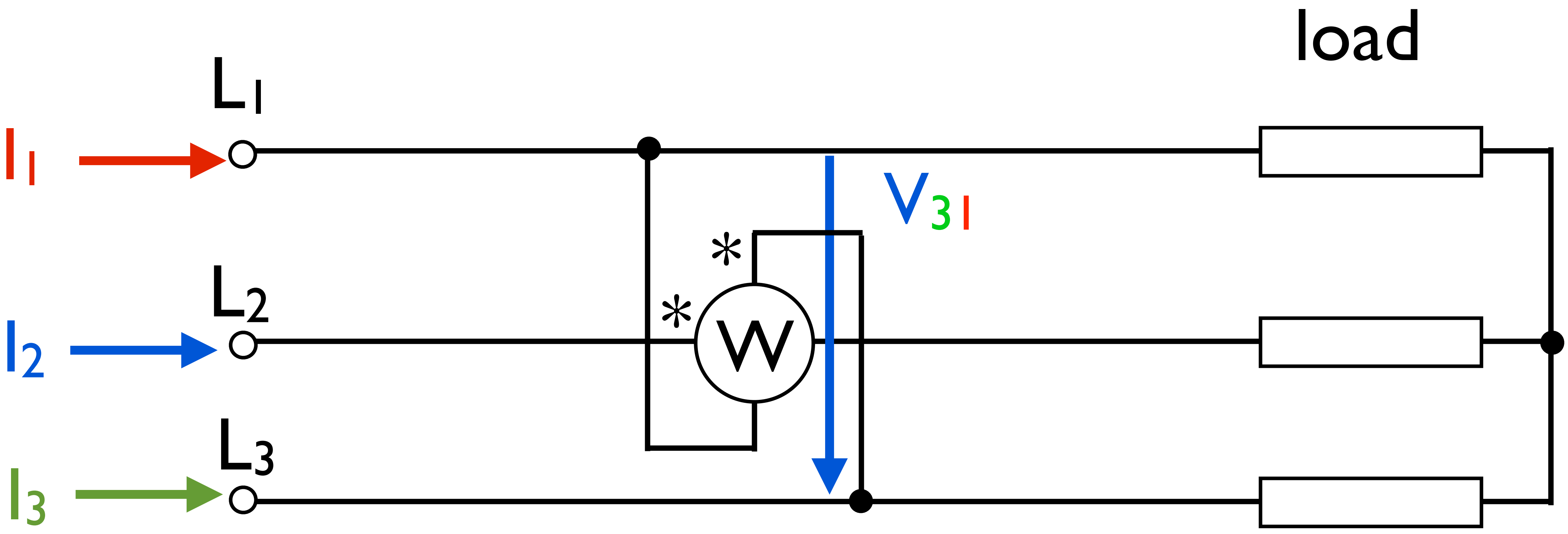


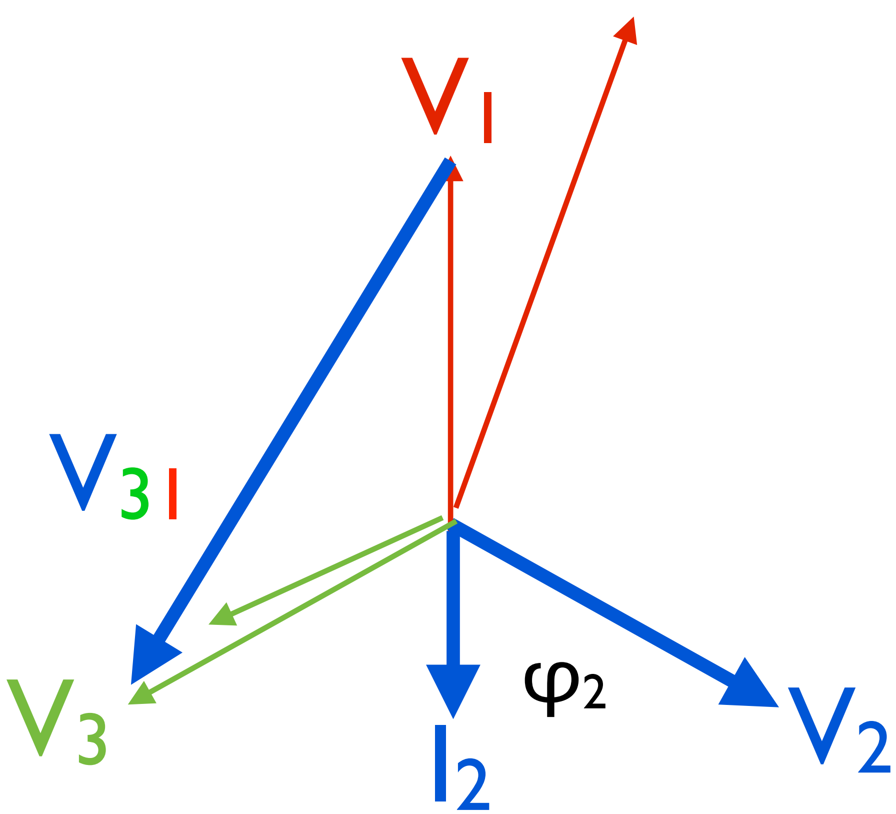
$\pi/2$ between V_1 and V_{23}

$$Q_1 = V_1 \cdot I_1 \cdot \sin\varphi_1$$

$$Q_1 = \frac{V_{23}}{\sqrt{3}} \cdot I_1 \cdot \cos\left(\frac{\pi}{2} - \varphi_1\right)$$

REACTIVE POWER



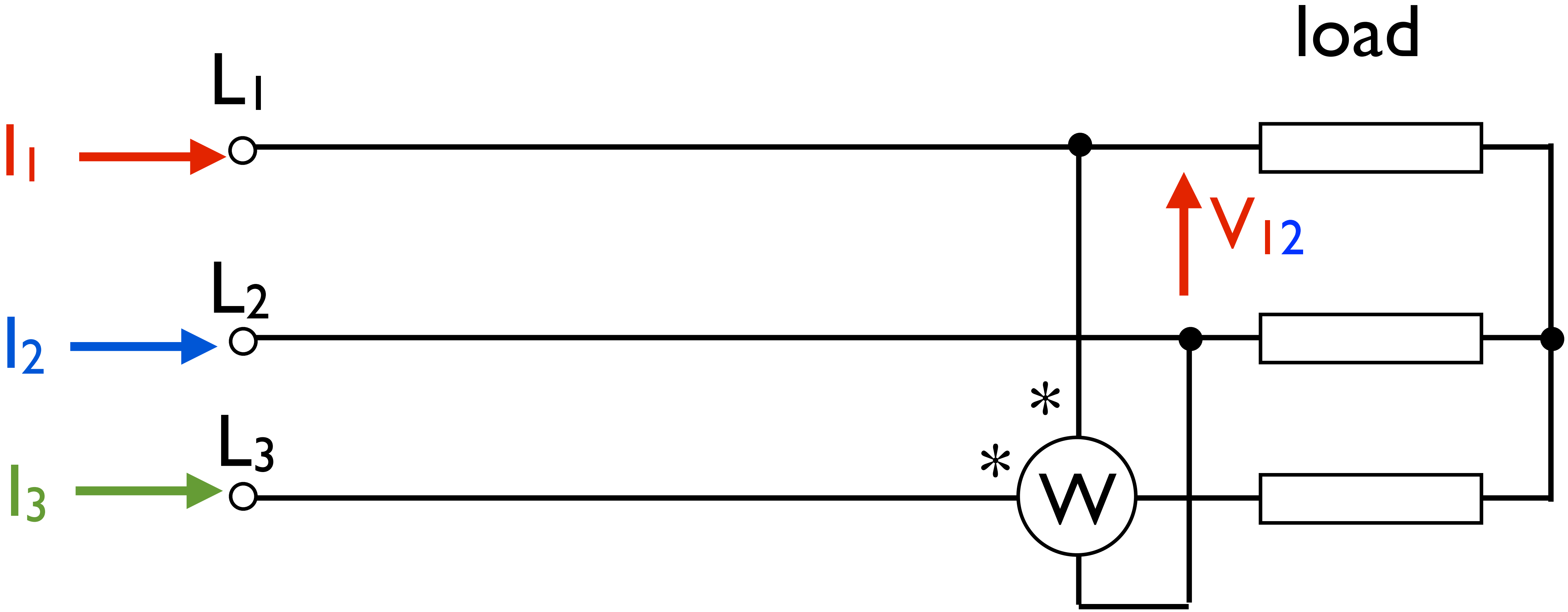


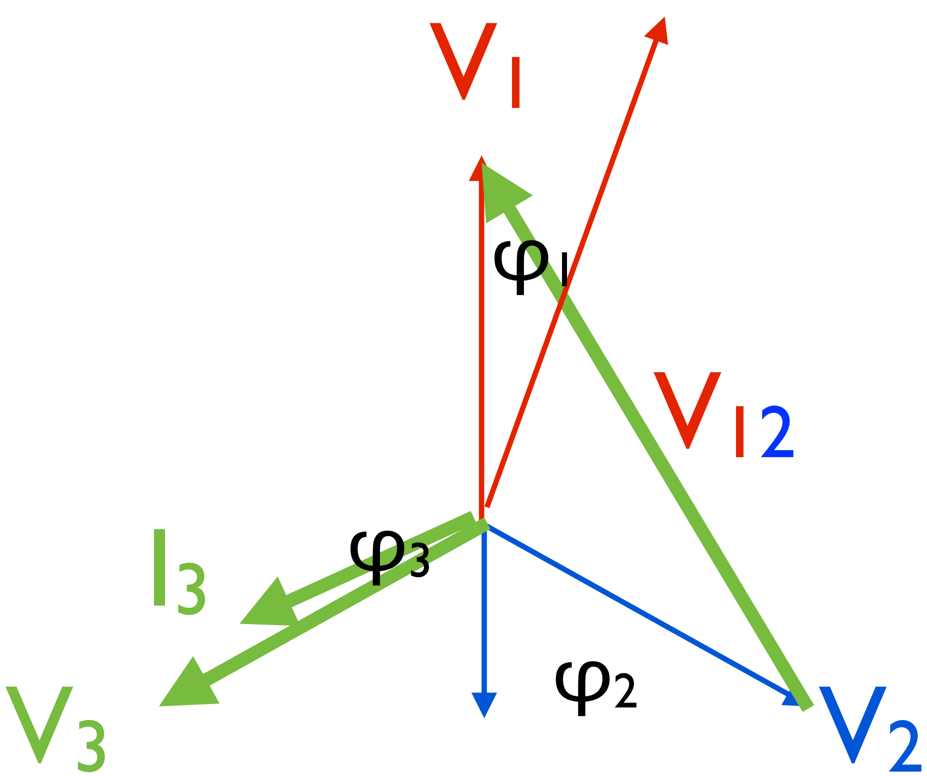
$\pi/2$ between V_2 and V_{13}

$$Q_2 = V_2 \cdot I_2 \cdot \sin\varphi_2$$

$$Q_2 = \frac{V_{31}}{\sqrt{3}} \cdot I_2 \cdot \cos\left(\frac{\pi}{2} - \varphi_2\right)$$

REACTIVE POWER

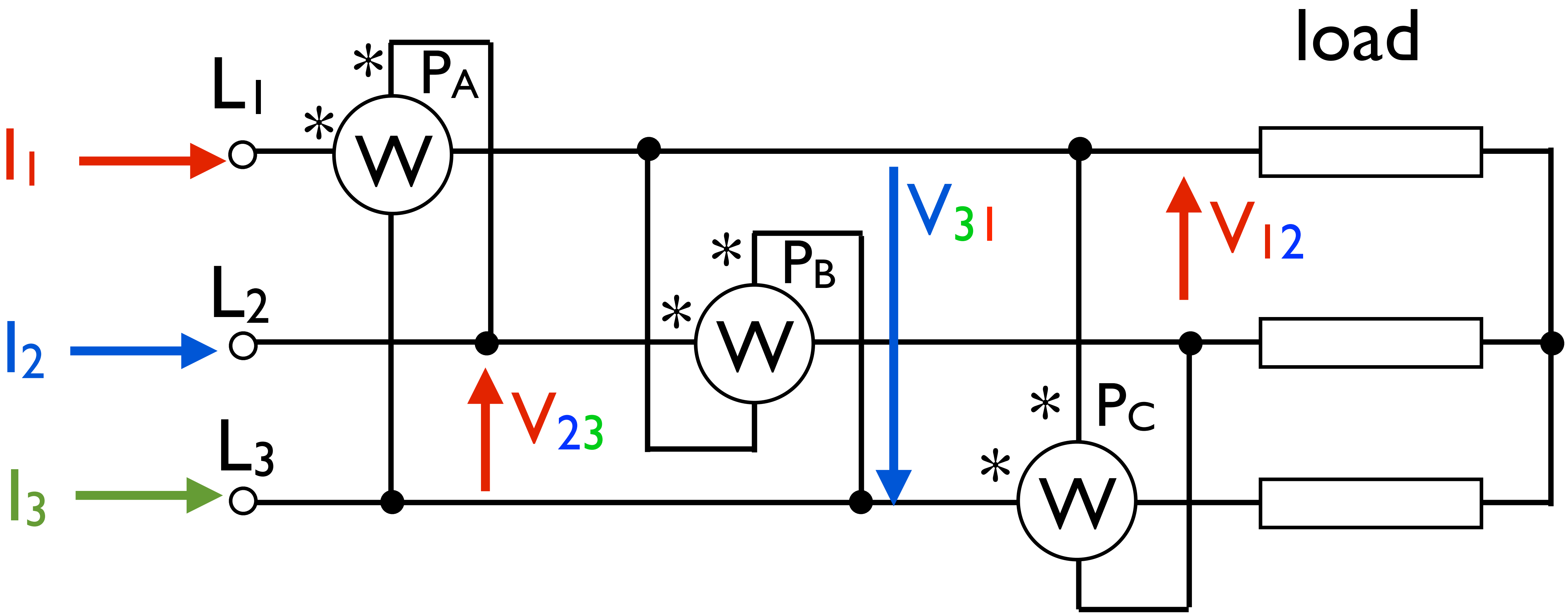


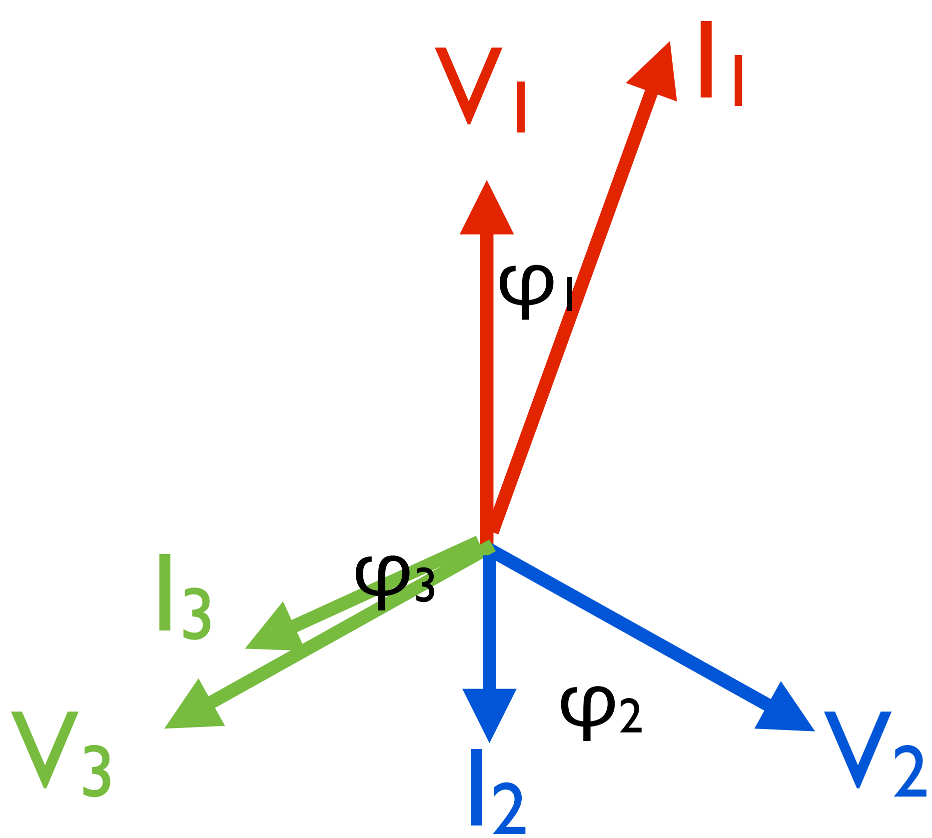


$$Q_3 = V_3 \cdot I_3 \cdot \sin\varphi_3$$

$$Q_3 = \frac{V_{12}}{\sqrt{3}} \cdot I_3 \cdot \cos\left(\frac{\pi}{2} - \varphi_3\right)$$

REACTIVE POWER

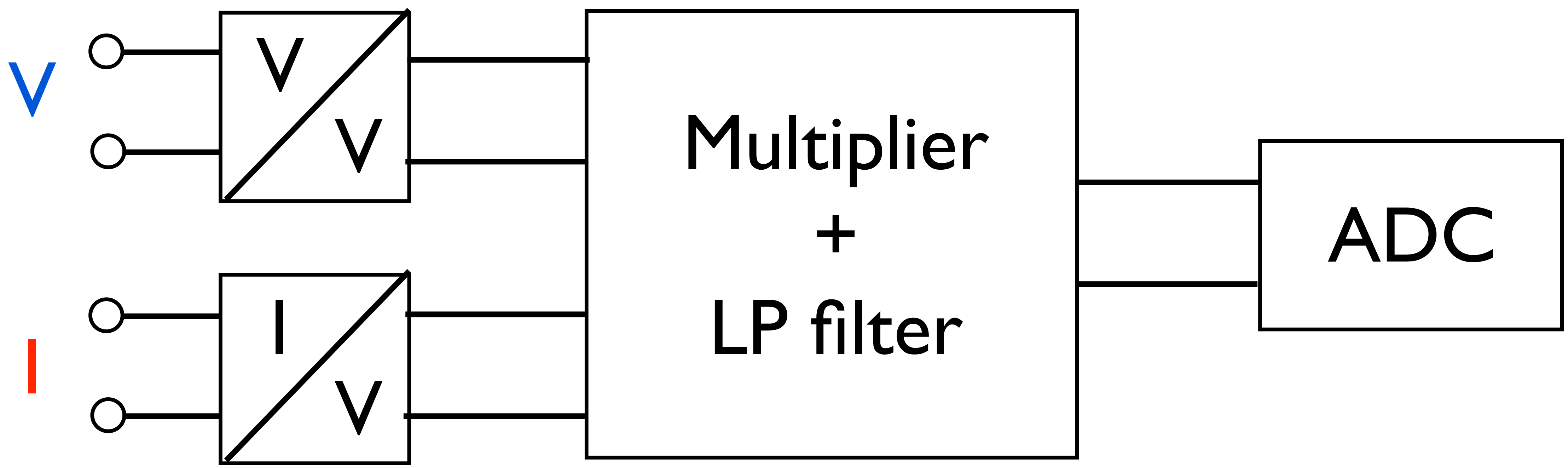




$$Q = \frac{P_A}{\sqrt{3}} + \frac{P_B}{\sqrt{3}} + \frac{P_C}{\sqrt{3}}$$

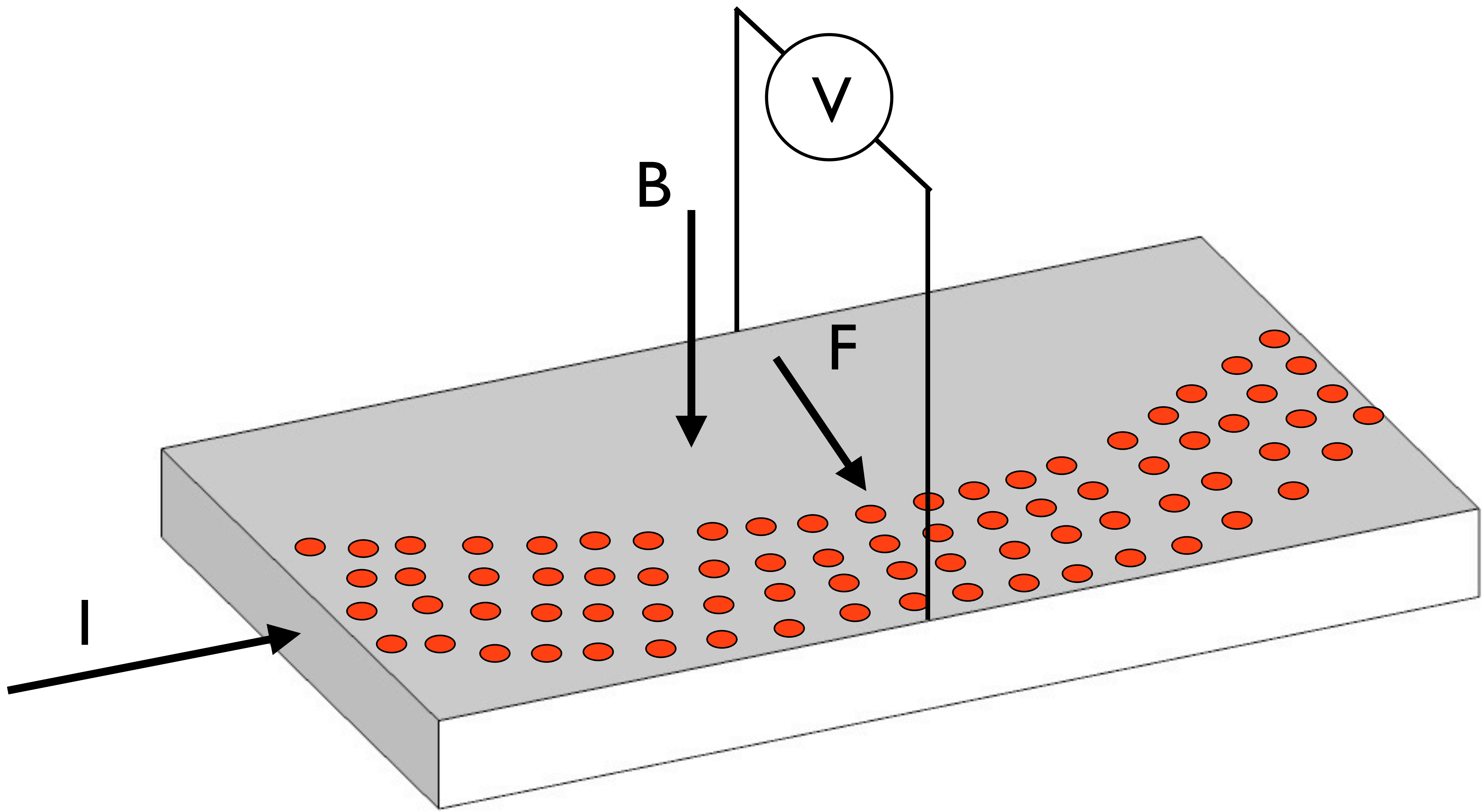
ELECTRONIC WATTMETER

A) Analog multiplier



Analog multiplier can be based on
- Hall effect
- log-antilog
-TDM (Time Division Multiplier)

Hall effect multiplier



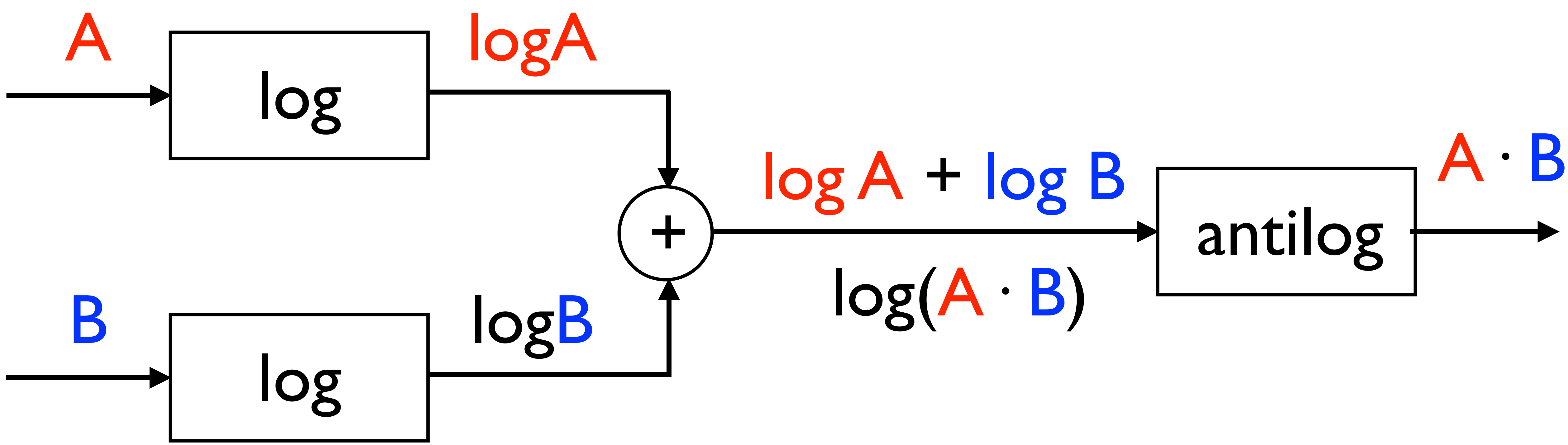
$$V = k \cdot F = k \cdot I \cdot B$$

B is created from a voltage by a solenoid and multiplied by the current

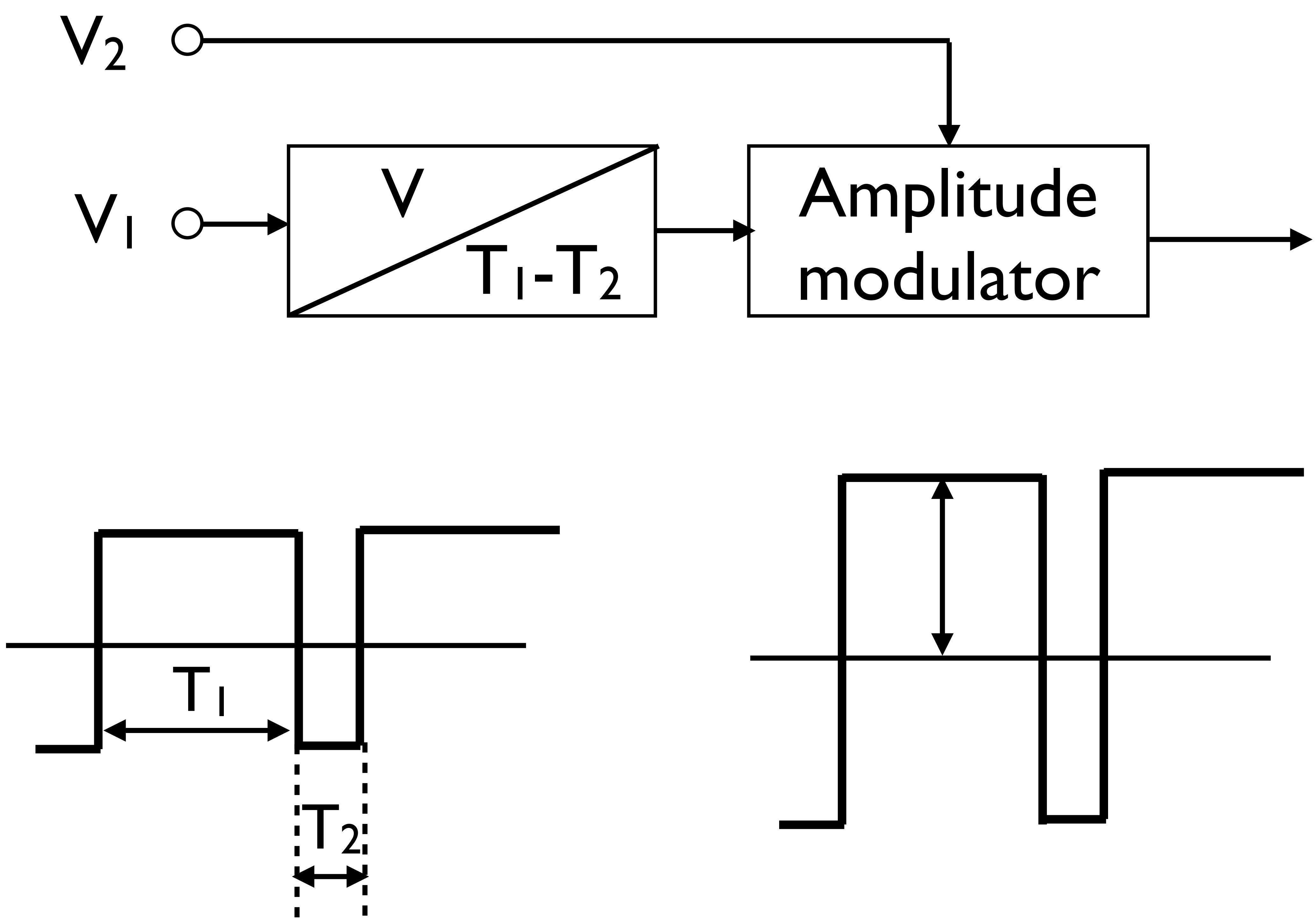
LOG-ANTILOG MULTIPLIER

It's based on the funny property which makes possible to transform a multiplication to an addition in logarithmic world

$$\log(A \cdot B) = \log A + \log B$$

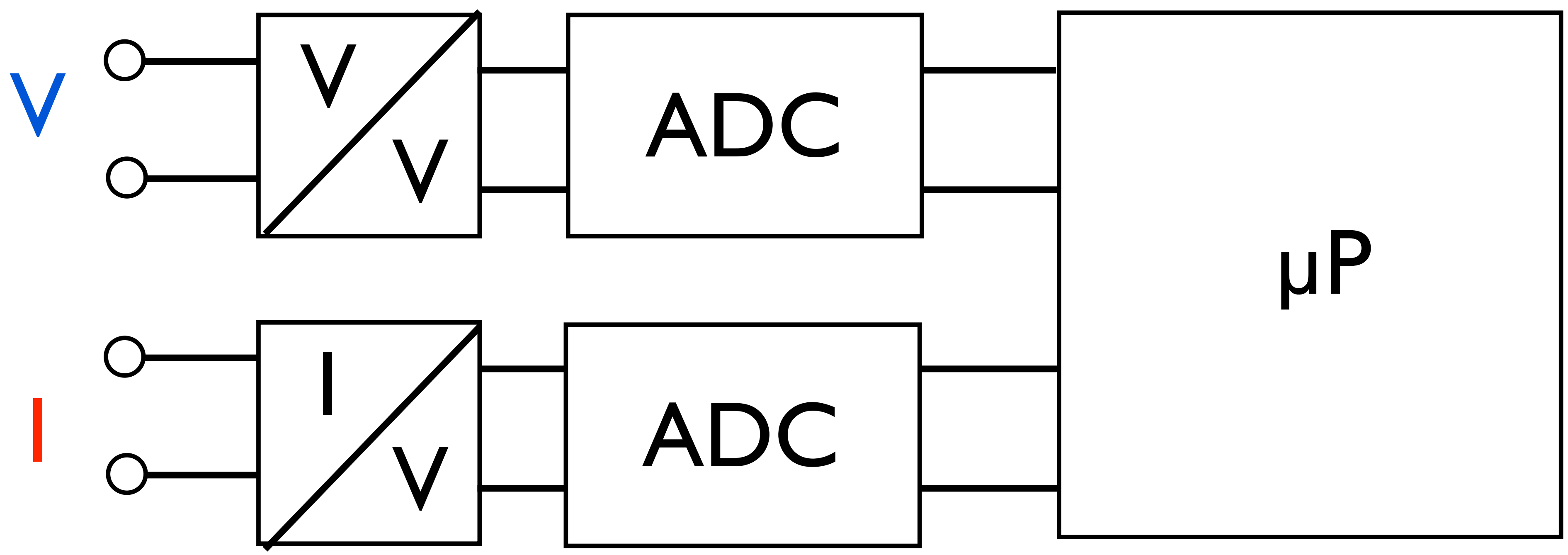


TDM (Time division multiplier)



ELECTRONIC WATTMETER

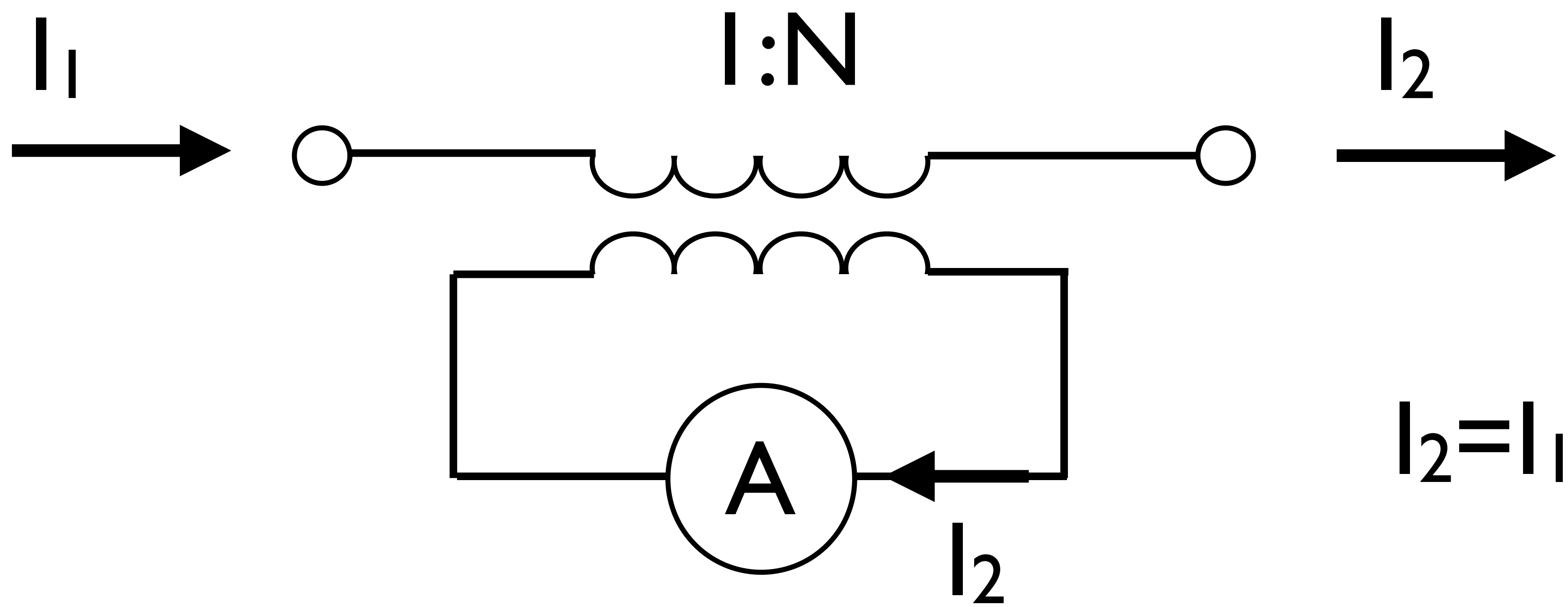
B) Digital multiplier



Both current and voltage are sampled, then the product is calculated numerically

What if the current is too high?

CURRENT TRANSFORMER

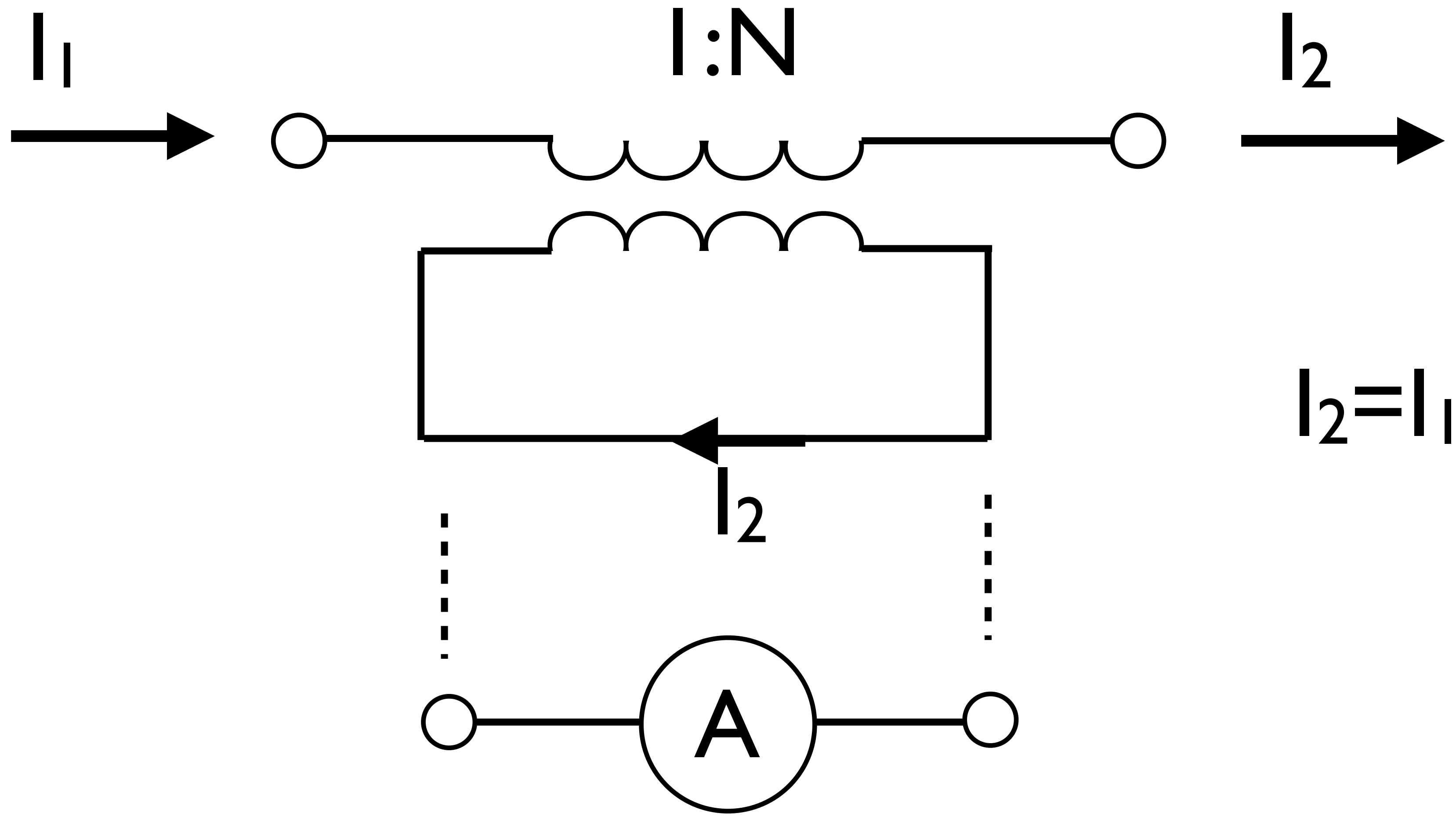


$I_2 = I_1/N$

BE CAREFUL: never open the secondary winding

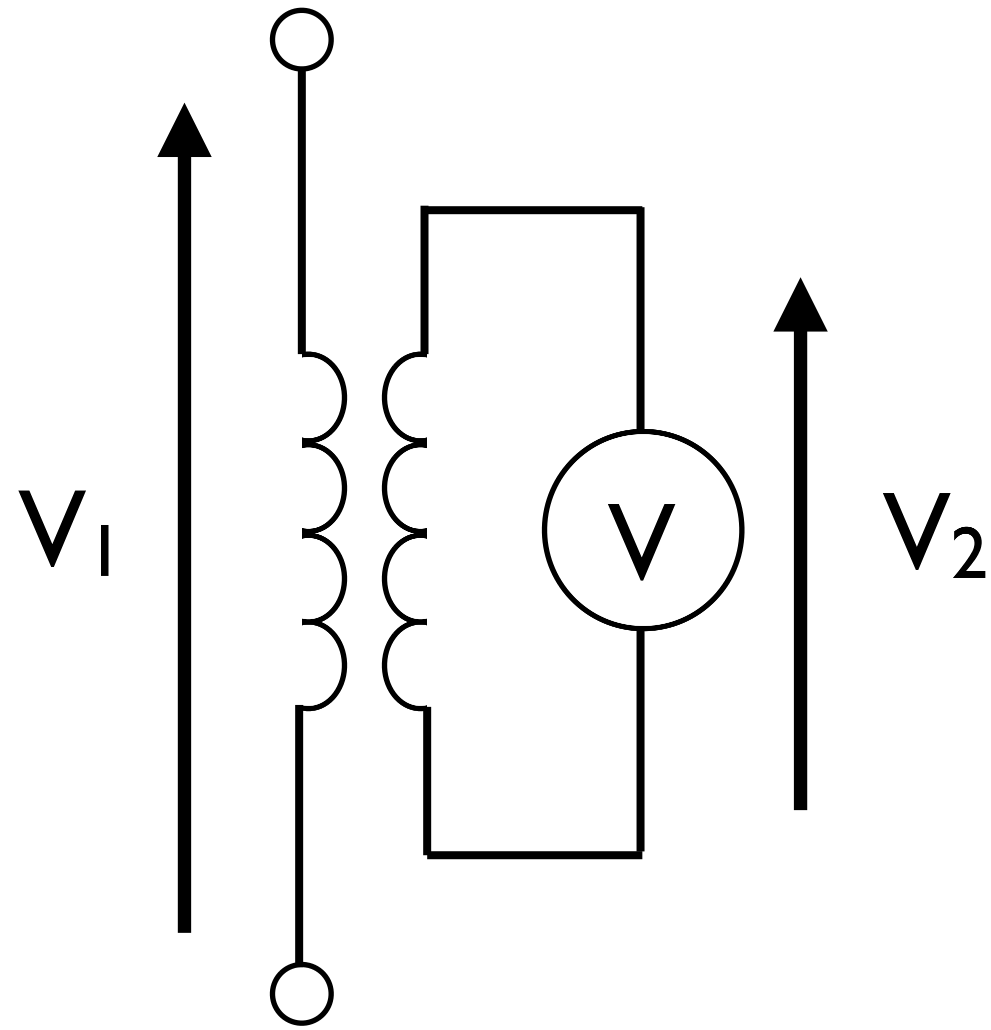
CURRENT TRANSFORMER

BE CAREFUL: if you remove the ammeter short circuit the secondary winding



$I_2 = I_1/N$

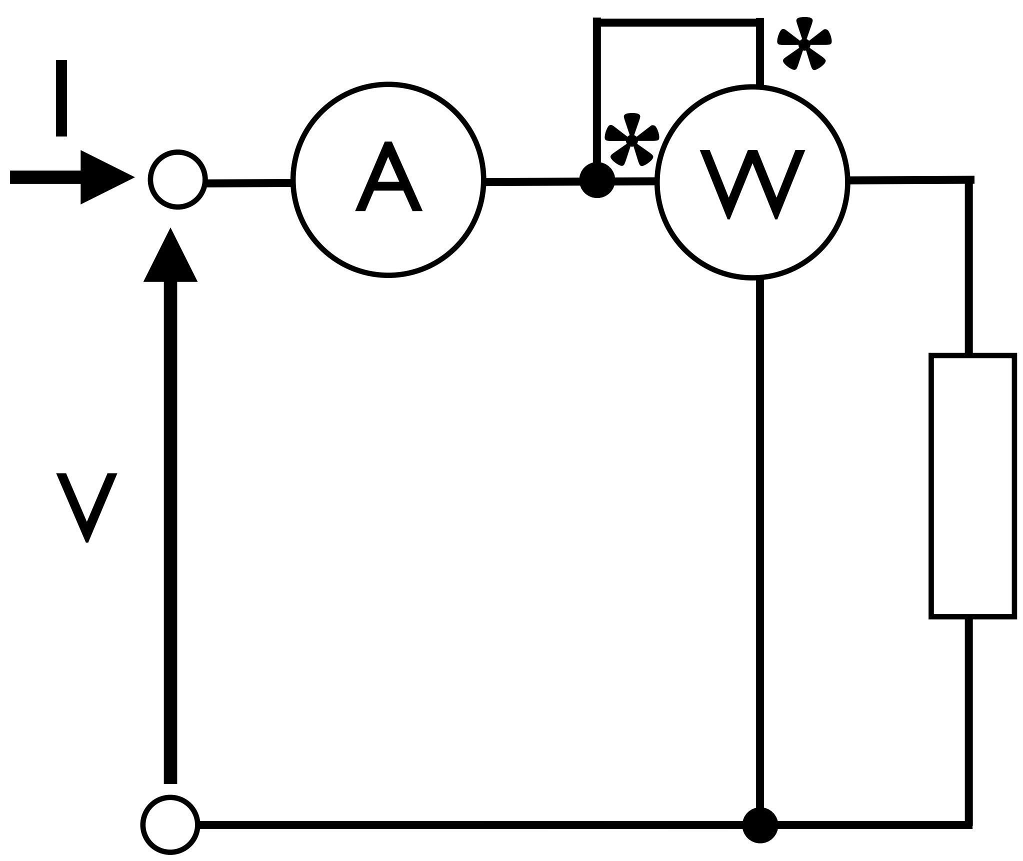
VOLTAGE TRANSFORMER

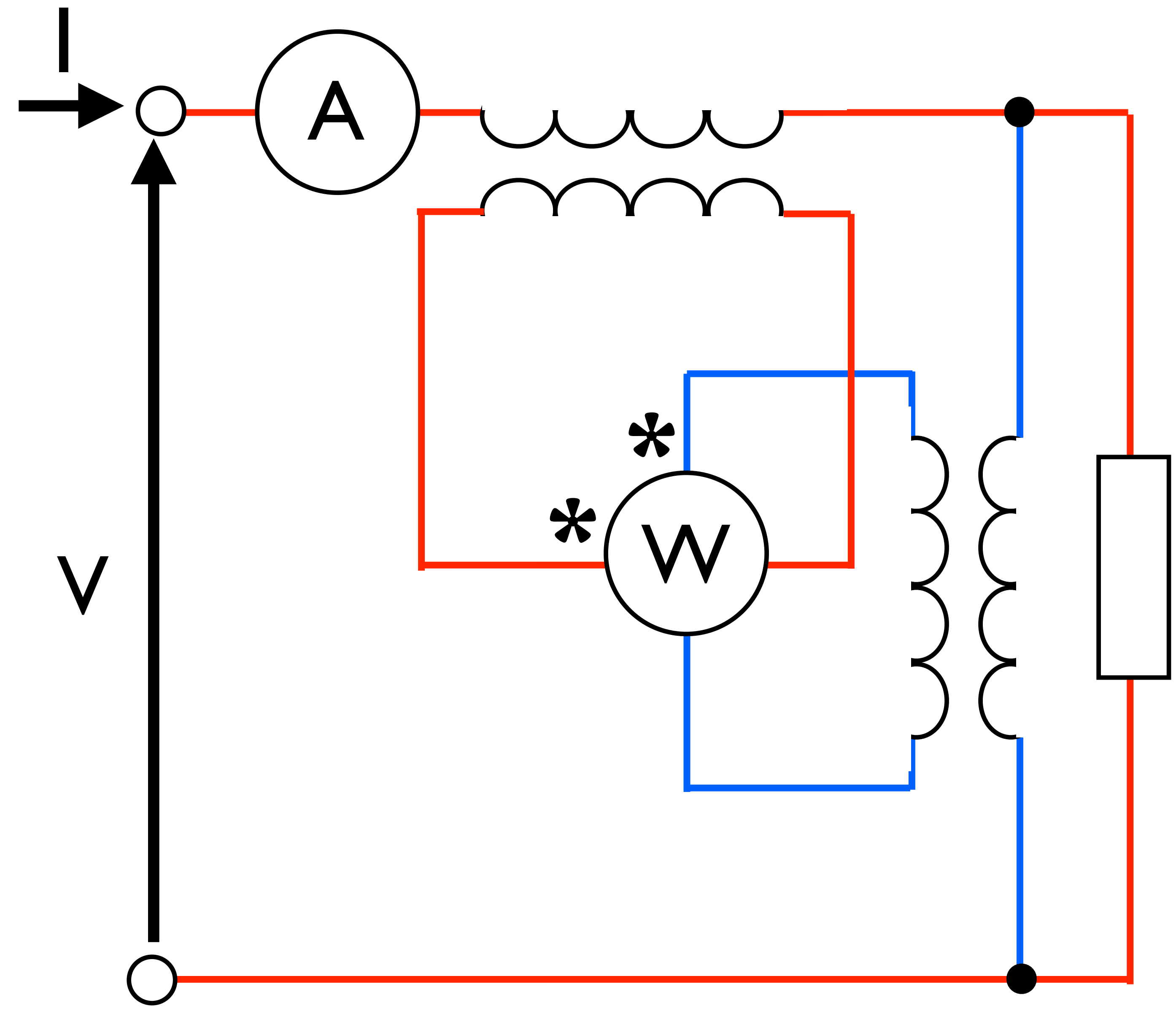


$V_2 = V_1/N$

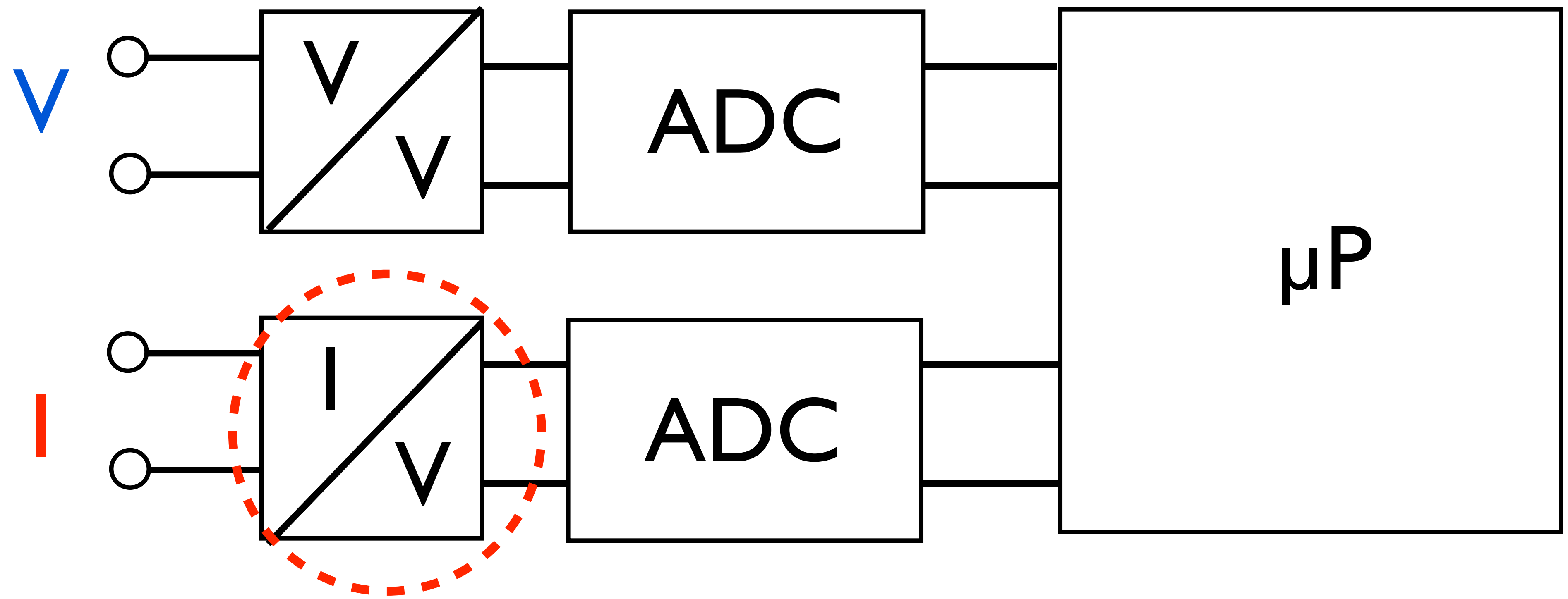
Always leave secondary winding open

Measurement of power with MEASUREMENT TRANSFORMS





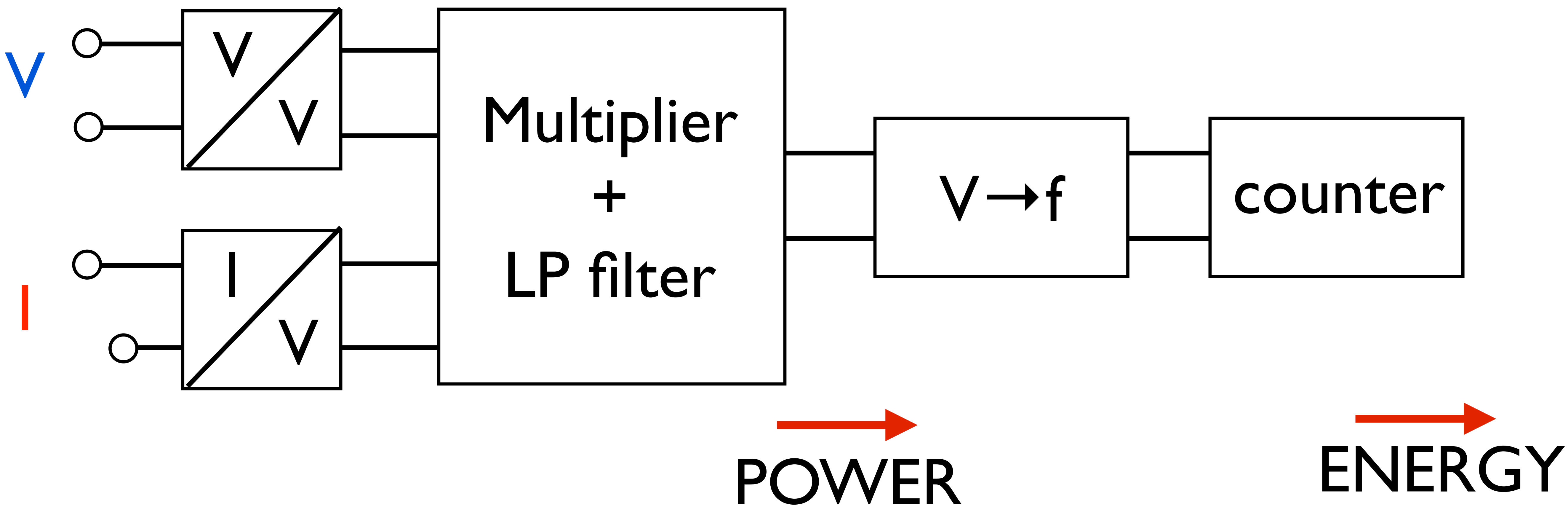
How can I convert a current into a voltage?



- Transformer and current to voltage converter *(with opamp)*
- Coaxial shunt
- Hall effect sensor *(I measure the magnetic field produced by the current)*

MEASUREMENT OF ENERGY

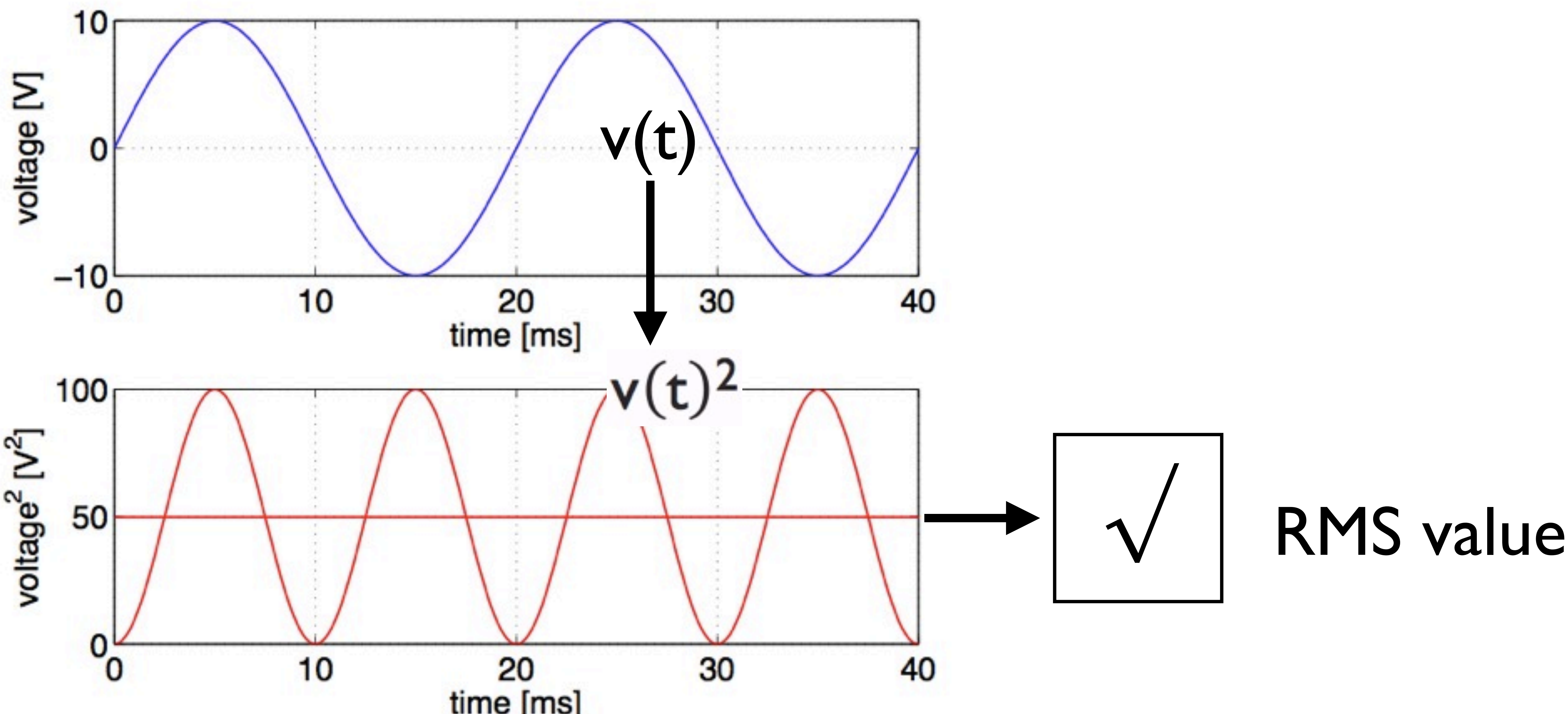
We must integrate the power



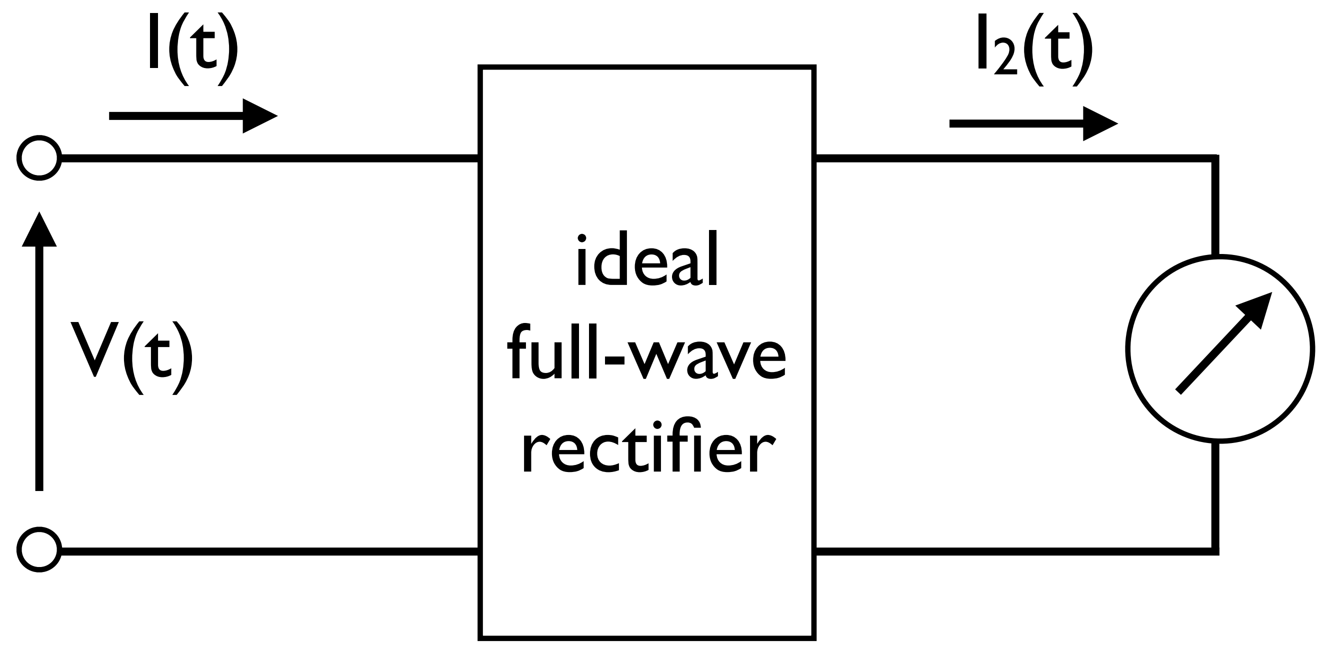
We convert the signal representing the power into frequency and then we count the pulses

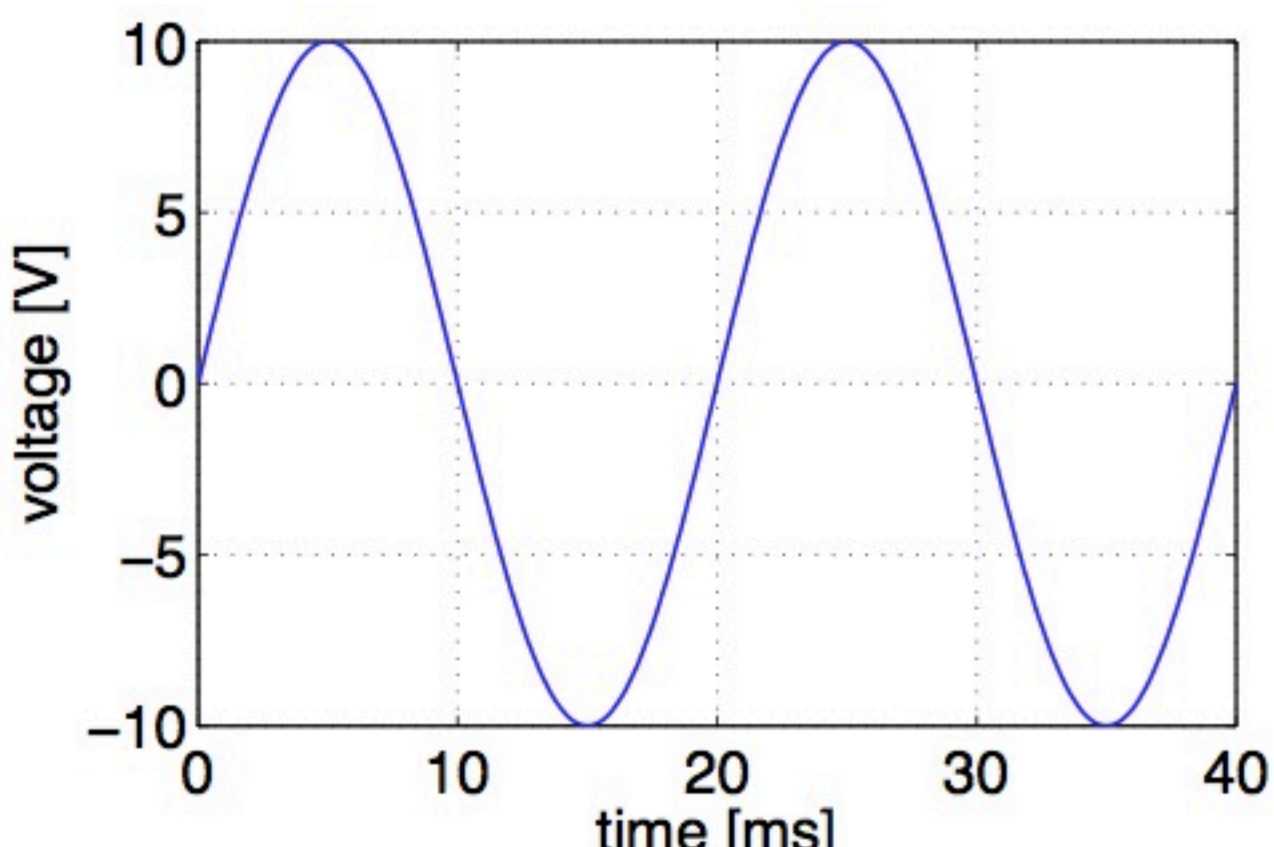
MEASUREMENT OF RMS VALUE

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt}$$

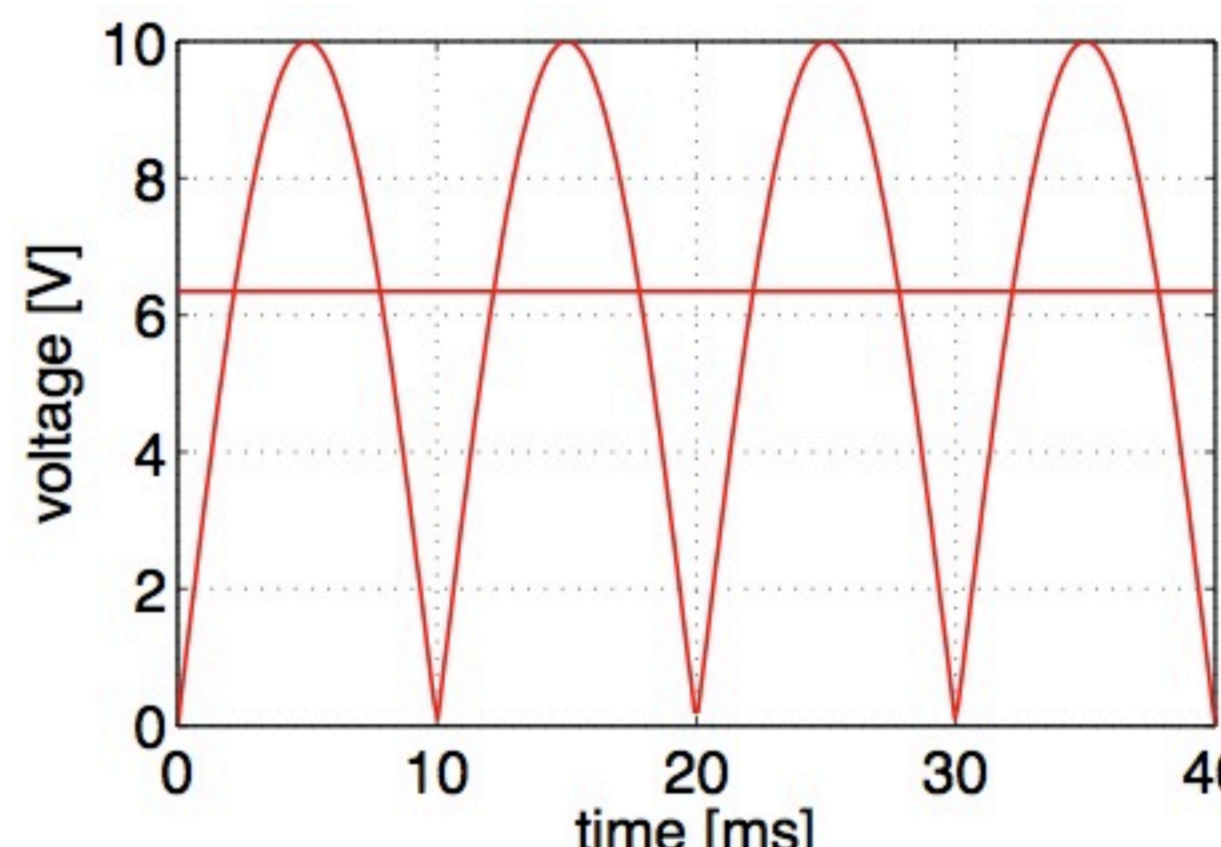


Analog instruments and cheap digital intruments

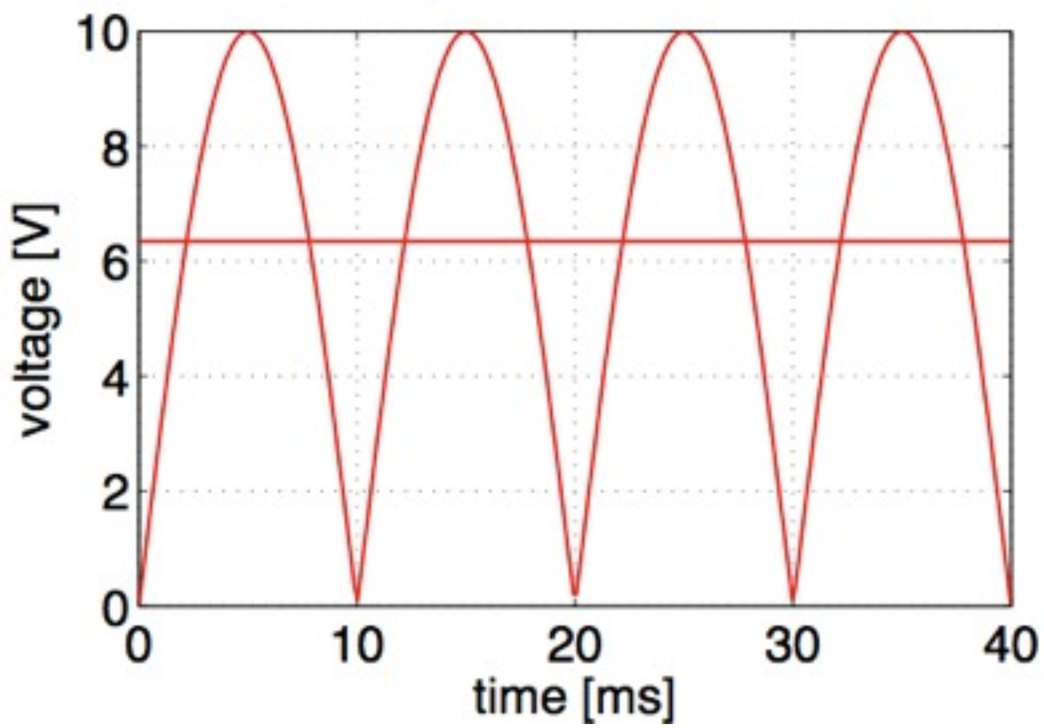




Rectified Mean value (RM)

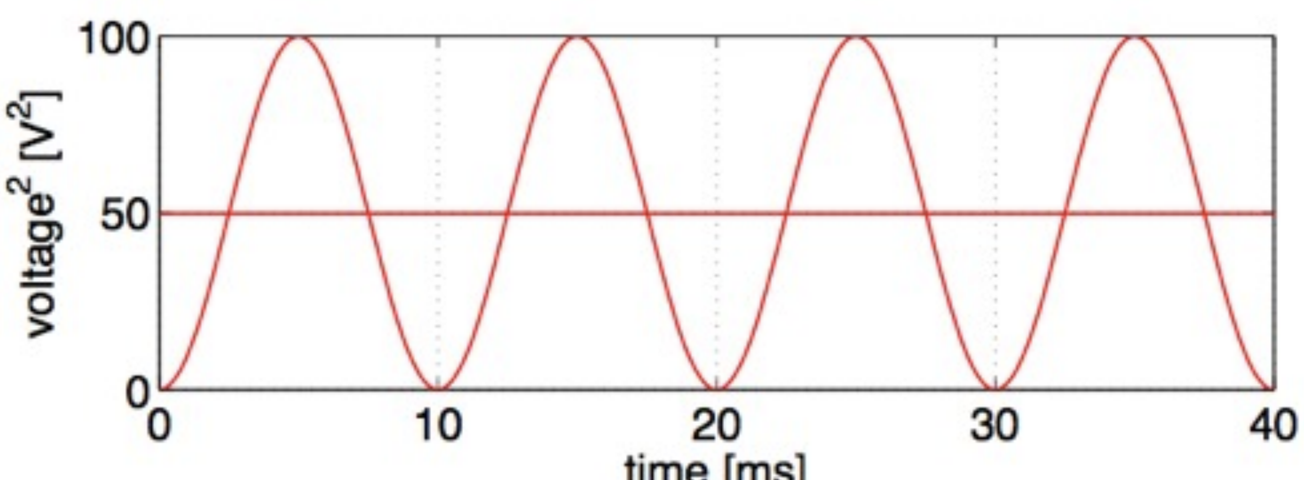


RM value and RMS are different!



Rectified Mean value

$$V_{RM} = \frac{1}{T}\int_0^T |v(t)|dt$$



RMS value

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt}$$

For sinewaves:

$$V_{RMS} = \frac{V_{MAX}}{\sqrt{2}} \qquad V_{RM} = \frac{2}{\pi} V_{MAX}$$

Cheap instruments measure V_{RM} but they multiply the scale 1.11 times to show V_{RMS}

$$V_{RMS} = \frac{V_{RM} \cdot \frac{\pi}{2}}{\sqrt{2}} \approx 1.11 \cdot V_{RM}$$

Wrong measurement if the voltage is NOT sinewave!

SMARTER METHODS TO MEASURE RMS VALUE

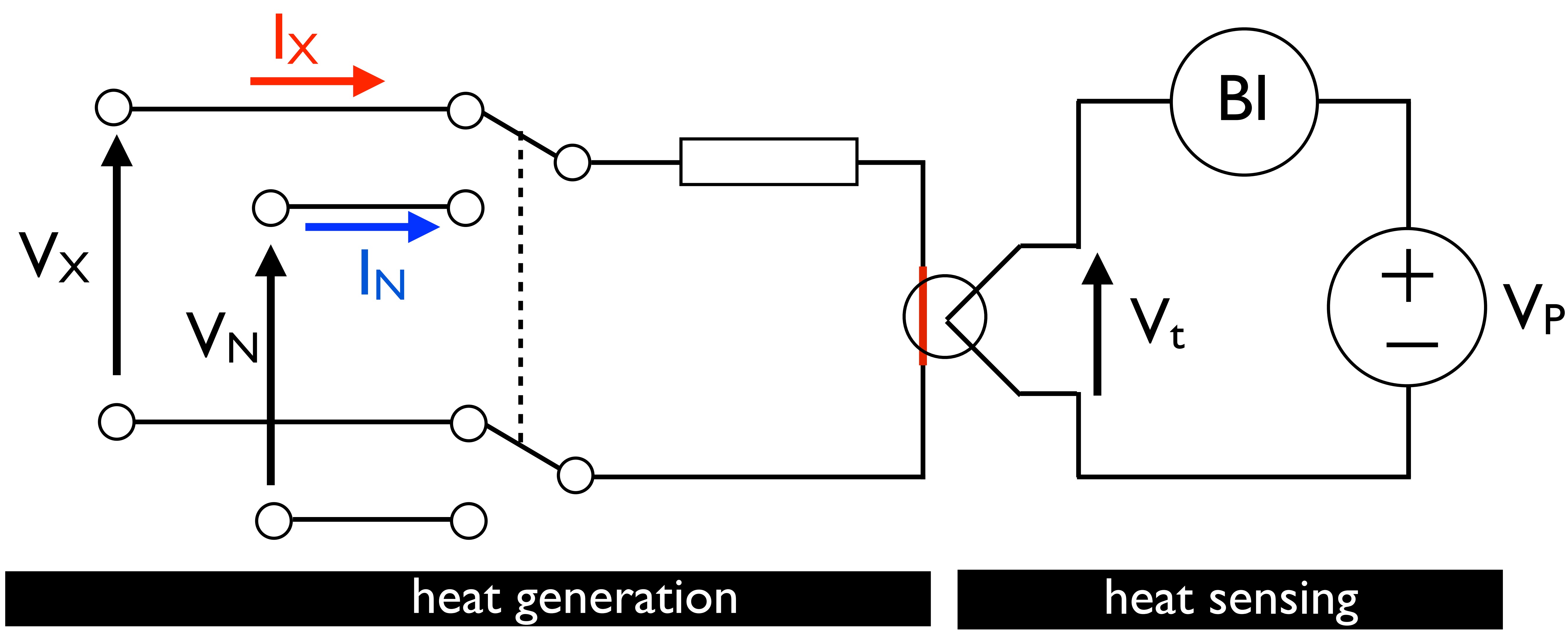
- Sampling the waveform followed by numerical computation of the RMS value

- Multiplication
    - hall effect
    - log-antilog

      log is defined only for positive values!
      You must add dc value to use log-antilog multiplier for negative value

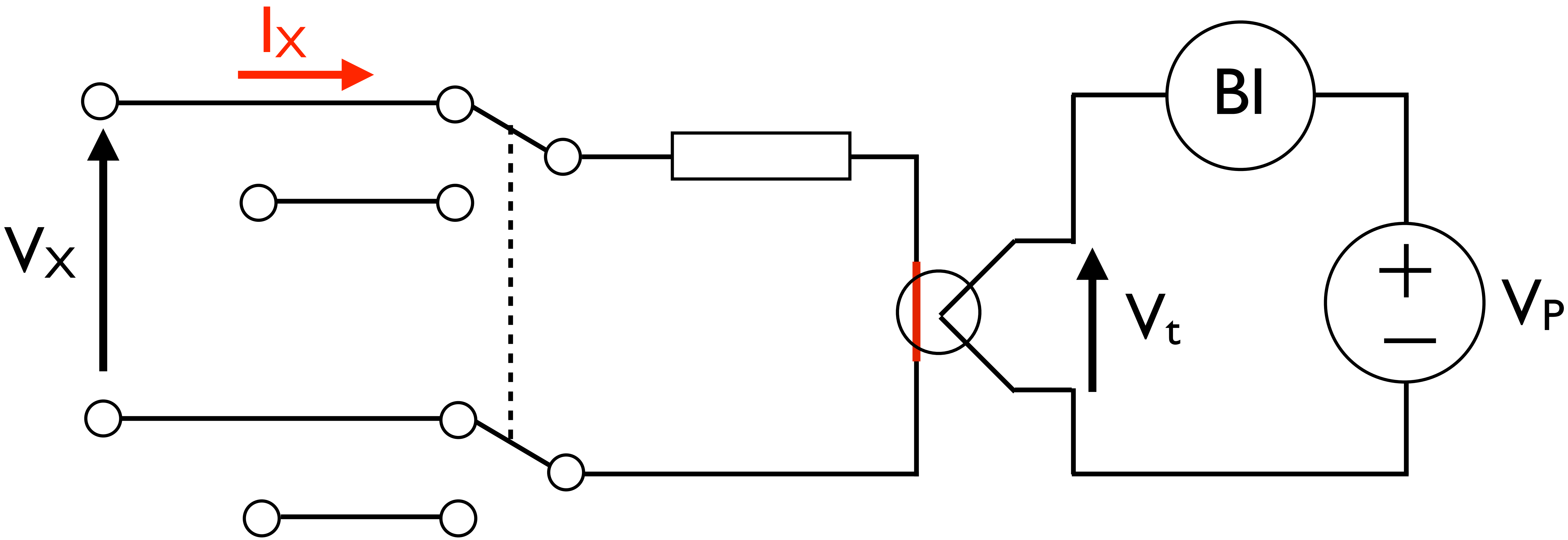
- Thermal RMS to DC converter

THERMAL RMS to DC CONVERTER



It compares the heat produced by I_X (unknown) and the heat produced by I_N (known)

THERMAL RMS to DC CONVERTER

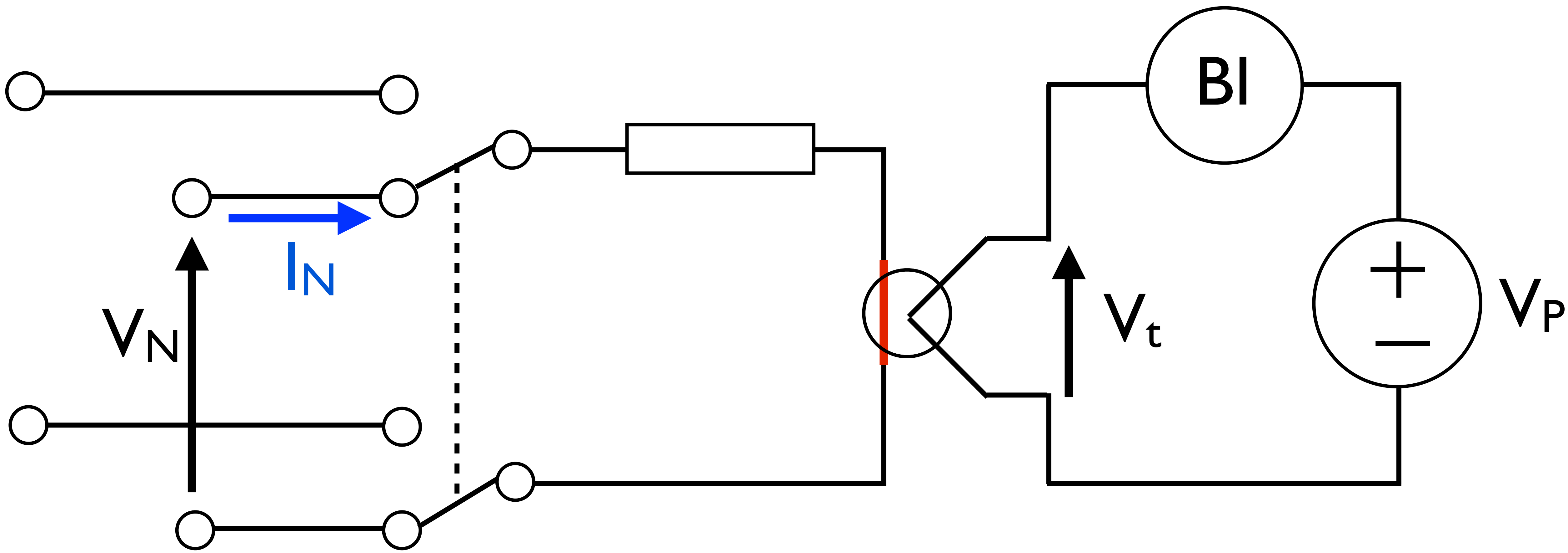


STEP 1

The heat produced by I_X generates heat which generates in turn V_t.

I change V_P so that is equals V_t

THERMAL RMS to DC CONVERTER



STEP 2

I change I_N until the heats it generates gives rise to a voltage V_t equal to V_P

In this condition the two heats are equivalent and

$$I_N = I_{X\ RMS}$$