Feedback Control of Dynamic Systems

Fourth Edition

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To Gertrude, David, Carole
Valerie, Daisy, Annika, Davenport
Malahat, Sheila, Nima

Contents

Preface

In this fourth edition we again had the objectives of retaining the best of the previous editions, to rewrite key sections where we felt it was possible to improve the presentations and enhance the book's pedagogical effectiveness, and to take better advantage of the wide use of computers in control design, especially the toolboxes of MATLAB and Simulink, from The Mathworks, Inc.

The basic structure of the book is unchanged and we continue to combine analysis with design using the three approaches of the root locus, frequency response, and state variable equations. The text continues to include carefully worked out examples, many of them new to this edition, to illustrate the material. As a new feature, to assist the students in verifying that they have learned the material, we provide a set of review questions at the end of each chapter with answers in the back of the book. While modest changes were made throughout the entire book, special attention was given to the introduction of transforms in Chapter 3, to the introduction to feedback in Chapter 4, and to the organization and statements of the problems appearing at the end of each chapter.

In the three central chapters on the design methods, we continue to expect the students to learn how to perform the basic calculations by hand in order to be able to guide a design by understanding (and frequently by a quick sketch) rather than by computer rote. However, more than in previous editions, we de-emphasize the manual work and introduce computer tools early on in recognition of the universal use of these tools in control analysis and design. For example, we no longer mark certain problems as requiring a computer but, rather, expect that the student has access to a computer in every case, as needed.

Furthermore, in recognition of the fact that, increasingly, controllers are implemented in embedded computers, we introduce digital control in Chapter 4 and in a number of cases compare the responses of feedback systems using analog controllers with those having a digital "equivalent" controller. As before, we have prepared a collection of all the MATLAB ".m" files used to produce the figures in the book and these are available at the companion web site for this title:

http://www.prenhall.com/franklin

or at the homepage for SC Solutions, Inc.:

http://scsolutions.com/scsolutions.control.html

As representative applications of control, we again present extensive case studies in Chapter 9. In this edition we have added new studies of the control of the read-write head assembly of a computer hard disk and the temperature control of a silicon wafer in a Rapid Thermal Processor used in the fabrication of integrated circuits.

We feel that this fourth edition presents the material with good pedagogical support, provides strong motivation for the study of control, and represents a solid foundation for meeting the educational challenges of a study of feedback control.

Addressing the Educational Challenges

Some of the educational challenges facing students of feedback control are long-standing; others have emerged in recent years. Some of the challenges remain for students across their entire engineering education; others are unique to this relatively sophisticated course. Whether they are old or new, general or particular, the educational challenges we perceived were critical to the evolution of this text. Here we will state several educational challenges and describe our approaches to each of them.

- CHALLENGE: *Students must master design as well as analysis techniques.*

Design is central to all of engineering and especially to control systems. Students find that design issues, with their corresponding opportunities to tackle practical applications, particularly motivating. But students also find design problems difficult because design problem statements are usually poorly posed and lack unique solutions. Because of both its inherent importance for and its motivational effect on students, design is emphasized throughout this text so that confidence in solving design problems is developed from the start.

The emphasis on design begins in Chapter 4, following the development of modeling and dynamic response. The basic idea of feedback is introduced first, showing its influence on disturbance rejection, tracking accuracy, and robustness to parameter changes. The design orientation continues with uniform treatments of the root locus, frequency response, and state variable feedback techniques. All of the treatments are aimed at providing the knowledge necessary to find a good feedback control design with no more complex mathematical development than is essential to clear understanding.

Throughout the text, examples are used to compare and contrast the design techniques afforded by the different design methods and, in the capstone case studies of Chapter 9, complex real-world design problems are tackled using all of the methods in a unified way.

- CHALLENGE: *New ideas continue to be introduced into control.*

Control is an active field of research and hence there is a steady influx of new concepts, ideas, and techniques. In time, some of these elements develop to the point where they join the list of things every control engineer must know. This text is devoted to supporting students equally in their need to grasp both traditional and more modern topics.

In each of our previous editions we have tried to give equal time to root locus, frequency response, and state variable methods for design. In this edition we have shifted the emphasis from manual design methods augmented with computer tools to an emphasis on computer-aided methods augmented with a solid mastery of the underlying techniques. Included in this re-emphasis is the early introduction of sampling, which enables one to design digital controllers. While this material can be skipped to save time without disruption of the flow of the text, we feel that it is very important for students to recognize that digital control is being used increasingly and that the most basic techniques of digital control are easily mastered.

With regret we acknowledge that we are not able at this time to introduce the important topics of hybrid control or designs based on various optimization methods.

- CHALLENGE: *Students need to manage a great deal of information.*

The vast array of systems to which feedback control is applied and the growing variety of techniques available for the solution of control problems means that today's student of feedback control must learn many new ideas. How do students keep their perspective as they plow through lengthy and complex textual passages? How do they identify highlights and draw conclusions? How do they review for exams? Helping students with these tasks was a criterion for the fourth edition. We outline these features in the accompanying table on page xiv.

- CHALLENGE: *Students of feedback control come from a wide range of disciplines.*

Feedback control is an interdisciplinary field in that control is applied to systems in every conceivable area of engineering. Consequently, some schools have separate introductory courses for control within the standard disciplines and some, such as Stanford University, have a single set of courses taken by students from many disciplines. However, to restrict the examples to one field is to miss much of the range and power of feedback; but to cover the whole range of applications is overwhelming. In this book we develop the interdisciplinary nature of the field and provide review material for several of the most common technologies so that students from many disciplines will be comfortable with the presentation. For electrical engineering students who typically have a good background in transform analysis, we include an introduction to writing equations of motion for mechanical mechanisms in Chapter 2. For mechanical engineers, we include in Chapter 3 a review of the Laplace Transform and dynamic response as needed in control. In addition, we introduce other technologies briefly and, from time to time, we present the equations of motion of a physical system without derivation but with enough physical description to

FEATURE	REFERENCE EXAMPLE
Chapter openers offer perspective and overview. They place the specific chapter topic in the context of the discipline as a whole and they briefly overview the chapter sections.	Chapter 3 opener, pp. 94–95
Margin notes help students scan for chapter highlights. They point to important definitions, equations, and concepts.	pp. 49–50
Boxed highlights identify key concepts within the running text. They also function to summarize important design procedures.	Advantage of feedback, p. 206; compensation design, p. 440
Bulleted chapter summaries help with student review and prioritization. These summaries briefly reiterate the key concepts and conclusions of the chapter.	Chapter 2 summary, pp. 77–78
Synopsis of design aids. Relationships used in design and throughout the book are collected in one place for easy reference.	Inside back cover
The *color blue* is used (1) to highlight useful; pedagogical features; (2) to highlight components under particular scrutiny within block diagrams; (3) to distinguish curves on graphs; and (4) to lend a more realistic look to figures of physical systems.	Fig. 5.43, p. 330 Fig. 2.9, p. 32
Review questions at the end of each chapter with solutions in the back guide the student in self-study.	Chapter 2, p. 78

be understood from a response point of view. Examples of some of the physical systems represented in the text include the read-write head for a computer disk drive, a satellite tracking system, the fuel-air ratio in an automobile engine, and an airplane autopilot system.

Outline of the Book

The contents of the book is organized into nine chapters and seven appendixes. The chapters include some sections of advanced or enrichment material marked with a triangular blue icon that can be omitted without interfering with the flow of the material. Examples and problems based on this material are also marked with these icons. The appendixes include background and reference material such as Laplace transform tables, a review of complex variables, a review of matrix theory, and answers to the end-of-chapter review questions.

In Chapter 1, the essential ideas of feedback and some of the key design issues are introduced. The chapter also contains a brief history of control, from

the ancient beginnings of process control to the contributions of flight control and electronic feedback amplifiers. It is hoped that this brief history will give a context for the field, introduce some of the key figures who contributed to its development, and provide motivation to the student for the studies to come.

Chapter 2 is a short presentation of dynamic modeling and includes mechanical, electrical, electro-mechanical, fluid, and thermodynamic devices. It also discusses the state variable formulation of differential equations. This material can be omitted, used as the basis for review homework to smooth out the usual non-uniform preparation of students, or covered in depth.

Chapter 3 covers dynamic response as used in control. Again, much of this material may have been covered previously, especially by electrical engineering students. For many students, the correlation between pole locations and transient response and the effects of extra zeros and poles on dynamic response is new material, as is the notion of stability of a closed-loop system. This material needs to be covered carefully.

Chapter 4 introduces feedback in the most elementary context, permitting concentration on the essential effects of feedback on tracking accuracy, disturbance rejection, and sensitivity to model errors. The basic equation and transfer functions of feedback are introduced along with the definitions of the sensitivity and complementary sensitivity functions. In the context of a first-order model for speed control, the concepts of proportional, integral, and derivative (PID) control are introduced. In this way, the student gets the idea of what control is all about before the tedious rules of root locus or the Nyquist Stability Criterion are developed. Finally, in this chapter the basic issues of digital control are introduced, along with the idea of a digital equivalent controller. In this approach, the central issues of control design are brought forward and can remain in the foreground during the development of the necessary analysis that goes with construction of sophisticated design tools. The concepts of steady-state tracking error and system type are also treated here.

Following the overview of feedback, the core of the book presents the design methods based on root locus, frequency response, and state variable feedback in Chapters 5, 6, and 7, respectively.

Chapter 8 develops in more detail the tools needed to design feedback control for implementation in a digital computer. However, for a complete treatment of feedback control using digital computers, the reader is referred to the companion text, *Digital Control of Dynamic Systems*, by Franklin, Powell, and Workman (Prentice Hall, 1998).

In Chapter 9, the three primary approaches are integrated in several case studies and a framework for design is described that includes a touch of the real-world context of practical control design.

Course Configurations

The material in this text can be covered flexibly. Most first-course students in controls will have some background in dynamics and Laplace transforms.

Therefore, Chapter 2 and most of Chapter 3 would be a review for those students. In a 10-week quarter, it is possible to review Chapter 3, and cover all of Chapters 1, 4, 5, and 6. Most optional sections noted with a blue triangle should be omitted. In the second quarter, Chapters 7 and 9 can be covered comfortably including these optional sections. Alternatively, some optional sections could be omitted and selected portions of Chapter 8 included. A semester course should comfortably accommodate Chapters 1–7, including the review material of Chapters 2 and 3, if needed. If time remains after this core coverage, selected case studies from Chapter 9 or some introduction of digital control from Chapter 8 may be added.

The entire book can also be used for a three-quarter sequence of courses consisting of modeling and dynamic response (Chapters 2 and 3), classical control (Chapters 4–6), and modern control (Chapters 7–9).

Two basic 10-week courses are offered at Stanford and are taken by seniors and first-year graduate students who have not had a course in control, mostly in the Departments of Aeronautics and Astronautics, Mechanical Engineering, and Electrical Engineering. The first course reviews Chapters 2 and 3 and covers Chapters 4–6. The more advanced course is intended for graduate students and reviews Chapters 4–6 and covers Chapters 7–9. This sequence complements a graduate course in linear systems and is the prerequisite to courses in digital control, optimal control, flight control, and smart product design. Several of the subsequent courses include extensive laboratory experiments. Prerequisites for the course sequence include dynamics or circuit analysis and Laplace transforms.

Prerequisites to this Feedback Control Course

This book is for a first course at the senior level for all engineering majors. For the core topics in Chapters 4–7, prerequisite understanding of modeling and dynamic response is necessary. Many students will come into the course with sufficient background in those concepts from previous courses in physics, circuits, and dynamic response. For those needing review, Chapters 2 and 3 should fill in the gaps.

An elementary understanding of matrix algebra is necessary to understand the state-space material. While all students will have much of this in prerequisite math courses, a review of the basic relations is given in Appendix C and a brief treatment of particular material needed in control is given at the start of Chapter 7. The emphasis is on the relations between linear dynamic systems and linear algebra.

Supplements

An Instructor's Manual with complete solutions to homework problems is available to faculty who adopt the fourth edition. The web sites mentioned above include the .m files used to generate all of the MATLAB figures in the book.

Acknowledgments

Finally, we wish to acknowledge our great debt to all those who have contributed to the development of feedback control into the exciting field it is today and specifically to the considerable help and education we have received from our students and our colleagues. In particular, we have benefited in this effort by many discussions with the following individuals, who have taught introductory control at Stanford: A. E. Bryson, Jr., R. H. Cannon, Jr., D. B. DeBra, S. Rock, C. Tomlin, S. Boyd, and P. Enge. Special thanks go to Prof. Dan DeBra for his careful reading of the manuscript and many useful comments. In addition, our colleagues M. Spong, L. Pao, D. Meyer, K. Pasino, P. Dorato, and M. Saif have provided valuable feedback. We also appreciate the help of D. de Roover, G. van der Linden, J. Ebert, R. Kosut, M. Tao, and A. Rahimi. Furthermore, L. Kobayashi, H-T. Lee, E. Thuriyasena, and M. Matsuoka provided valuable help in proofreading the chapters as well as in the preparation of the solutions manual.

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Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$1/s$	$1(t)$
3	$1/s^2$	t
4	$2!/s^3$	t^2
5	$3!/s^4$	t^3
6	$m!/s^{m+1}$	t^m
7	$1/(s+a)$	e^{-at}
8	$1/(s+a)^2$	te^{-at}
9	$1/(s+a)^3$	$\frac{1}{2!}t^2e^{-at}$
10	$1/(s+a)^m$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bt}-ae^{-at}$
17	$a/(s^2+a^2)$	$\sin at$
18	$s/(s^2+a^2)$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
21	$\frac{a^2+b^2}{s\left[(s+a)^2+b^2\right]}$	$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$

Chronological History of Feedback Control

1624
Drebble, Incubator

1728
Watt, Flyball governor

1868
Maxwell, Flyball stability analysis

1877
Routh, Stability

1890
Liapunov, Nonlinear stability

1910
Sperry, Gyroscope and autopilot

1927
Black, Feedback electronic amplifier: Bush, Differential analyzer

1932
Nyquist, Nyquist stability criterion

1938
Bode, Frequency response methods

1942
Wiener, Optimal filter design
Ziegler-Nichols PID tuning

1947
Hurewicz, Sampled data systems; Nichols, Nichols chart

1948
Evans, Root locus

1950
Kochenberger, Nonlinear analysis

1956
Pontryagin, Maximum principle

1957
Bellman, Dynamic programming

1960
Draper, Inertial navigation; Kalman, Optimal estimation