# Variation of the speed of sound in air with humidity and temperature

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(Received 7 December 1984; accepted for publication 7 January 1985)

This paper describes the influence of humidity on the speed of sound in standard atmospheric air at various temperatures. The prediction is based on theoretical and experimental data obtained for the variation of the ratio of specific heats, $\gamma$, in humid air. Over a temperature range from 0°–30° Celsius, the maximum uncertainty in the sound speed ratio $c_h/c_0$, is estimated to be about 400 ppm. Comparison is made with known experimental measurements and theoretical data.

PACS numbers: 43.28.Fp, 51.40. + p

## LIST OF SYMBOLS

| | |
|---|---|
| $A_t$ | a function of $t$ |
| c | speed of sound |
| $c_0$,$c_h$ | speed of sound in dry and humid air, respectively |
| $h$ | relative humidity, dimensionless |
| $M$ | molar mass |
| $M_0$,$M_h$ | molar mass of dry and humid air, respectively |
| $P$ | atmospheric pressure |
| $R$ | universal gas constant |
| $t$ | temperature in degrees Celsius |
| $T_0$ | absolute temperature in kelvins |
| $\gamma$ | ratio of specific heats in air, dimensionless |

## INTRODUCTION

Theories of the speed of sound in gases are well known, and experiments to determine the sound speed in atmospheric air have been performed by many authors. Lenihan[1] and Hardy *et al.*[2] gave details which contained the pertinent data on such experiments which took place over a period of more than 300 years. As pointed out by Bancroft,[3] the measurement of the speed of sound could lead to the determination of other thermodynamic properties of gases, such as the departure of a real gas from the ideal gas law, and the specific heat ratio, if other parameters of the equation of state were to be known. In a relatively recent publication, Harris[4] investigated experimentally the variation of the speed of sound in air with humidity at 20° Celsius; and the uncertainty of his experimental method, which was based on the accurate assessment of the frequency of the decaying normal modes in a spherical chamber, was estimated to be within $\pm$ 150 ppm. Morfey and Howell[5] gave theoretical sound speed predictions which were in good agreement with the measurements obtained by Harris.[4]

It is the aim here to show that the information derived from our theoretical and experimental investigations[6,7] of the specific heat ratio, can enable one to determine the variation on the speed of sound in air with humidity at various temperatures.

## I. THEORY

The speed of sound[8] in an ideal gas at a constant pressure is

$$c = [(\gamma/M)R\, T_0]^{1/2}, \tag{1}$$

where $\gamma$, $M$, $R$, and $T_0$ are the specific heat ratio, molar mass, universal gas constant, and the absolute temperature (in kelvins), respectively. As can be seen from Eq. (1), the effects of temperature on the sound speed are relatively large. However, at a constant temperature the sound speed ratio in air is given by

$$c_h/c_0 = [(\gamma_h/M_h)/(\gamma_0/M_0)]^{1/2}, \tag{2}$$

where $c_h$,$\gamma_h$,$M_h$ and $c_0$,$\gamma_0$,$M_0$ are the sound speed, specific heat ratio, and the molar mass of air at two different concentrations of humidity. In the following, the atmospheric pressure is assumed to be equal to 101.325 kPa in all cases.

Based on theoretical and experimental thermodynamic data on the constituents of standard air[9] and humidity, the variation of $\gamma/M$ with humidity has been calculated,[6] and one is able to deduce the following approximate equation:

$$\gamma_h/M_h = 0.04833 + (h - 0.023)A_t, \tag{3}$$

where

$$A_t = 9.2 \times 10^{-5} + 5.5 \times 10^{-6}\, t + 4.25 \times 10^{-7}\, t^2. \tag{4}$$

Over the range for relative humidity $h$ from 0–1.0, and temperature $t$ from 0°–30° Celsius, the difference between the value of $\gamma/M$ obtained with Eq. (3) and the calculated values (shown in Fig. 2 of Ref. 6) is within $\pm$ 1.5 digits in the fifth decimal place. From Eqs. (2) and (3), the variation of the sound speed with humidity and temperature can be deduced.

## II. DATA PRESENTATION

The variation of the sound speed ratio with humidity for temperatures from 0°–30° Celsius in steps of 5° Celsius is shown in Fig. 1. It can be seen that $c_h/c_0$ increases with humdity and temperature. With the computed data, one is able to obtain the following approximate equation:

$$c_h/c_0 = 1 + h(9.66 \times 10^{-4} + 7.2 \times 10^{-5}\, t + 1.8 \times 10^{-6}\, t^2 + 7.2 \times 10^{-8}\, t^3 + 6.5 \times 10^{-11}\, t^4). \tag{5}$$

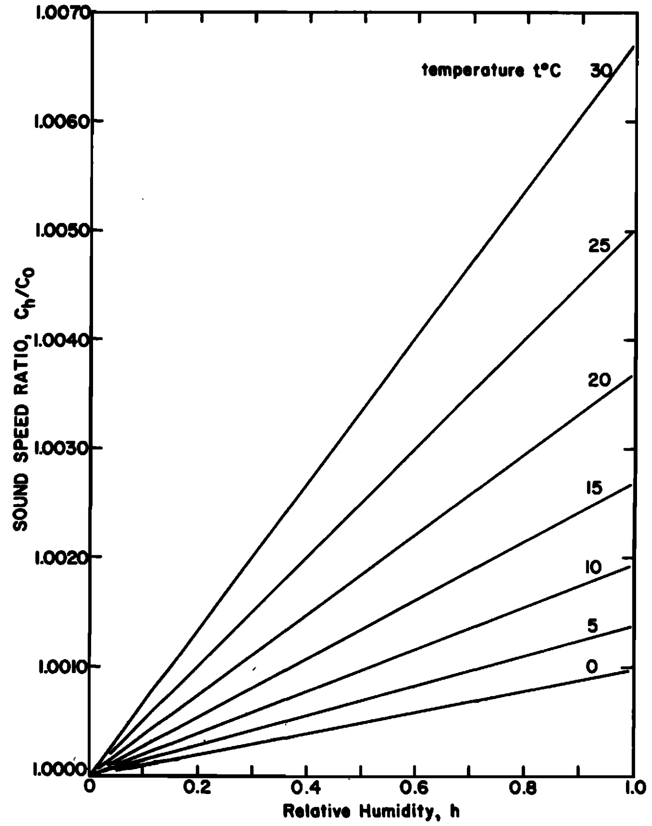


FIG. 1. The effects of humidity on sound speed in air at various temperatures, at a pressure of 101.325 kPa.

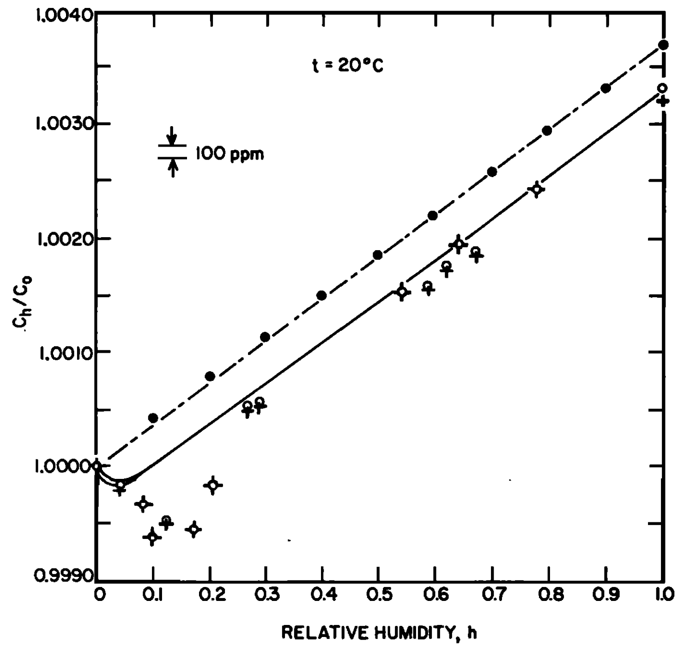


FIG. 2. The variation of sound speed in air with humidity at a pressure of 101.325 kPa. The sound speed is expressed in the ratio of the speed at a given humidity ($c_h$) to the speed for dry air ($c_0$). Experimental data obtained by Harris[4]; circles, 293 Hz; crosses, 503 Hz. The full lines indicate the prediction by Morfey and Howell.[5] The chain–dot line is derived from $\gamma/M$ data predicted by Wong and Embleton.[6] When a correction is applied to the prediction by Morfey *et al.* (dots, see text), both predictions are in agreement to within 50 ppm for relative humidity at and above 0.1.

The difference between the value of $c_h/c_0$ obtained with Eq. (5) and the computed values shown in Fig. 1 is within $\pm 2$ digits in the fifth decimal place.

## III. UNCERTAINTIES

The experimental data on the influence of humidity on sound speed in air at 20° Celsius, and at 1 atm, obtained by Harris[4] (crosses and circles), together with our Fig. 1 prediction (chain–dot line), are shown in Fig. 2. The full line curves shown are theoretical predictions, based on vibrational relaxation effects, obtained by Morfey and Howell.[5] For the range of $h$ from 0.1–1, the maximum deviation between our calculations (chain–dot line) and their theoretical curves is approximately 400 ppm.

In order to obtain good agreement with Harris' experimental measurement, Morfey and Howell used a dry air value of $\gamma_0 = 1.4007$ (see p. 1526, Ref. 5) which is relatively larger than our predicted dry air value of 1.3996 for $\gamma$ at 20° Celsius (see Table II C, Ref. 6). In view of the fact that the sound speed ratio is proportional to $(1/\gamma_0)^{1/2}$, a correction factor equals to $(1.4007/1.3996)^{1/2}$ can be applied to the full line theoretical curves. Over the range of $h$ from 0.1–1, the maximum deviation between our prediction (chain–dot line) and the corrected values (dots) of Morfey and Howell curves, shown in Fig. 2, is within 50 ppm.

Our data (Fig. 1) on the variation of $c_h/c_0$ with humidity and temperature are derived from the theoretical prediction obtained for the ratio of specific heats in air, $\gamma$. The uncertainty of the latter was estimated to be of the order of 400 ppm (see p. 558, Ref. 6). In a recent publication,[7] the authors confirmed the predicted changes in $\gamma$ by measurement, using a precision direct acoustical method; and the measured data were within 100–250 ppm of the theoretical prediction. It may be reasonable to assume that the maximum uncertainty of our present prediction of $c_h/c_0$ is also about 400 ppm.

## IV. CONCLUSIONS

The effects of humidity on the speed of sound in air at various temperatures can be calculated from Eq. (5). The maximum uncertainty is estimated to be about 400 ppm which is sufficient to encompass the deviations from Morfey and Howell's predictions. When compared with Harris' measurements, we are unable to explain the discrepancy at relative humidity below approximately 0.2. However, it must be realized that unlike Harris' experimental measurements (performed at 20° Celsius), our predictions are applicable to temperatures from 0°–30° Celsius.

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