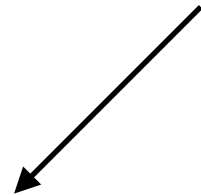


# Sensors of velocity, acceleration and vibration

AE3B38SME - Sensors and Measurement

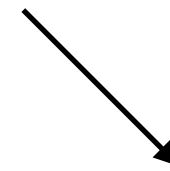
# Measurement of velocity



## Indirect

I measure position  $s$  and I  
derivate in time

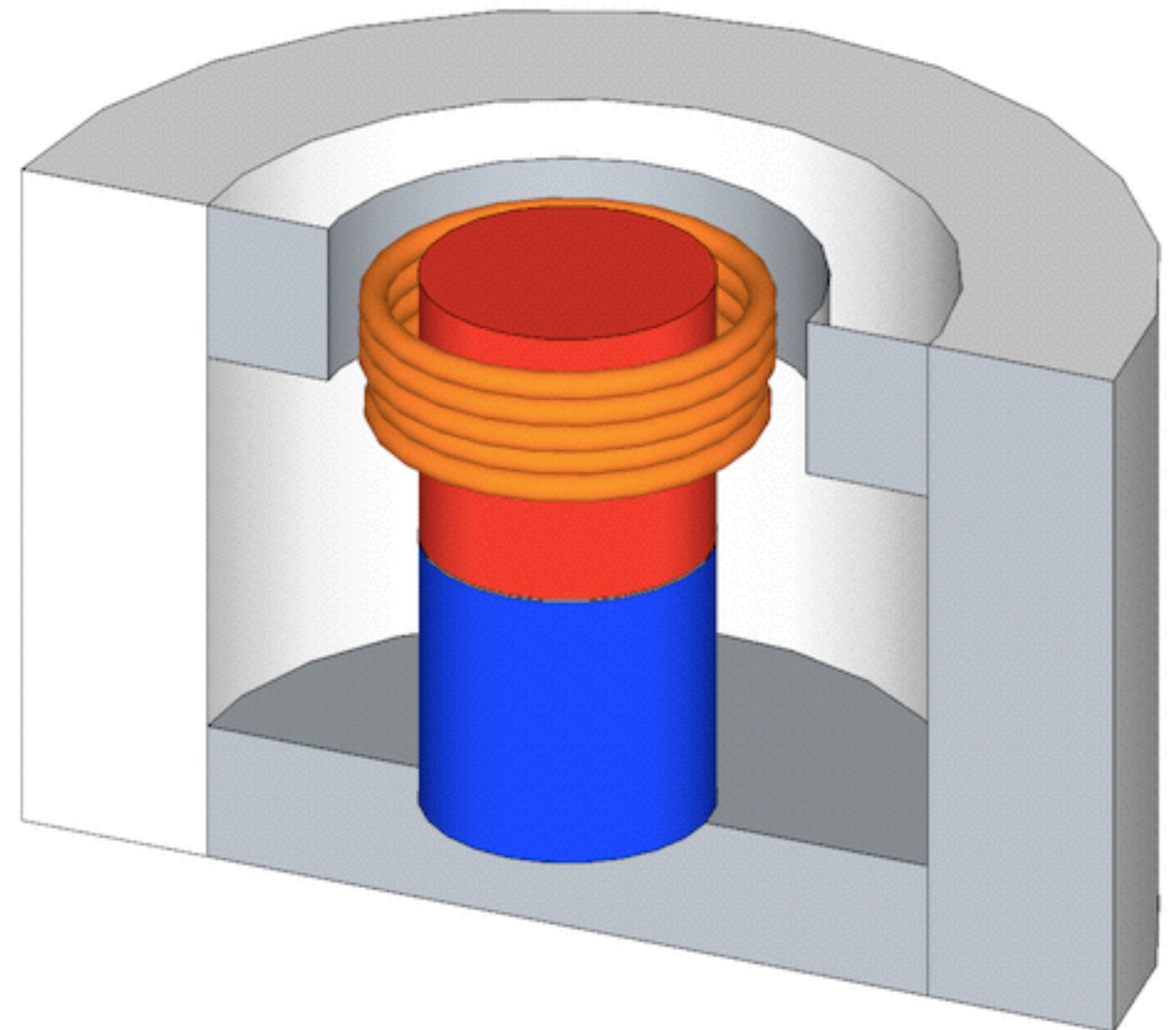
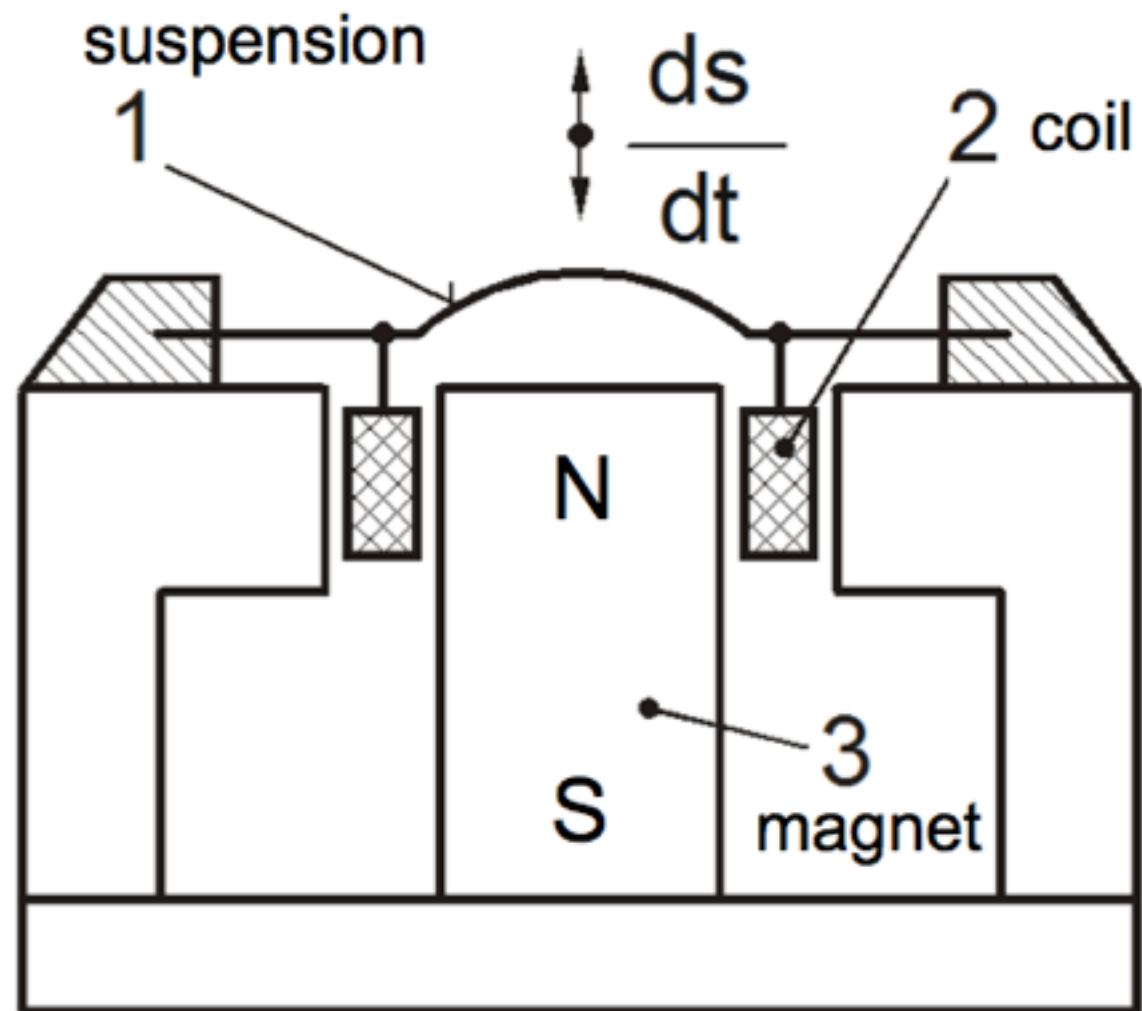
$$V = ds/dt$$



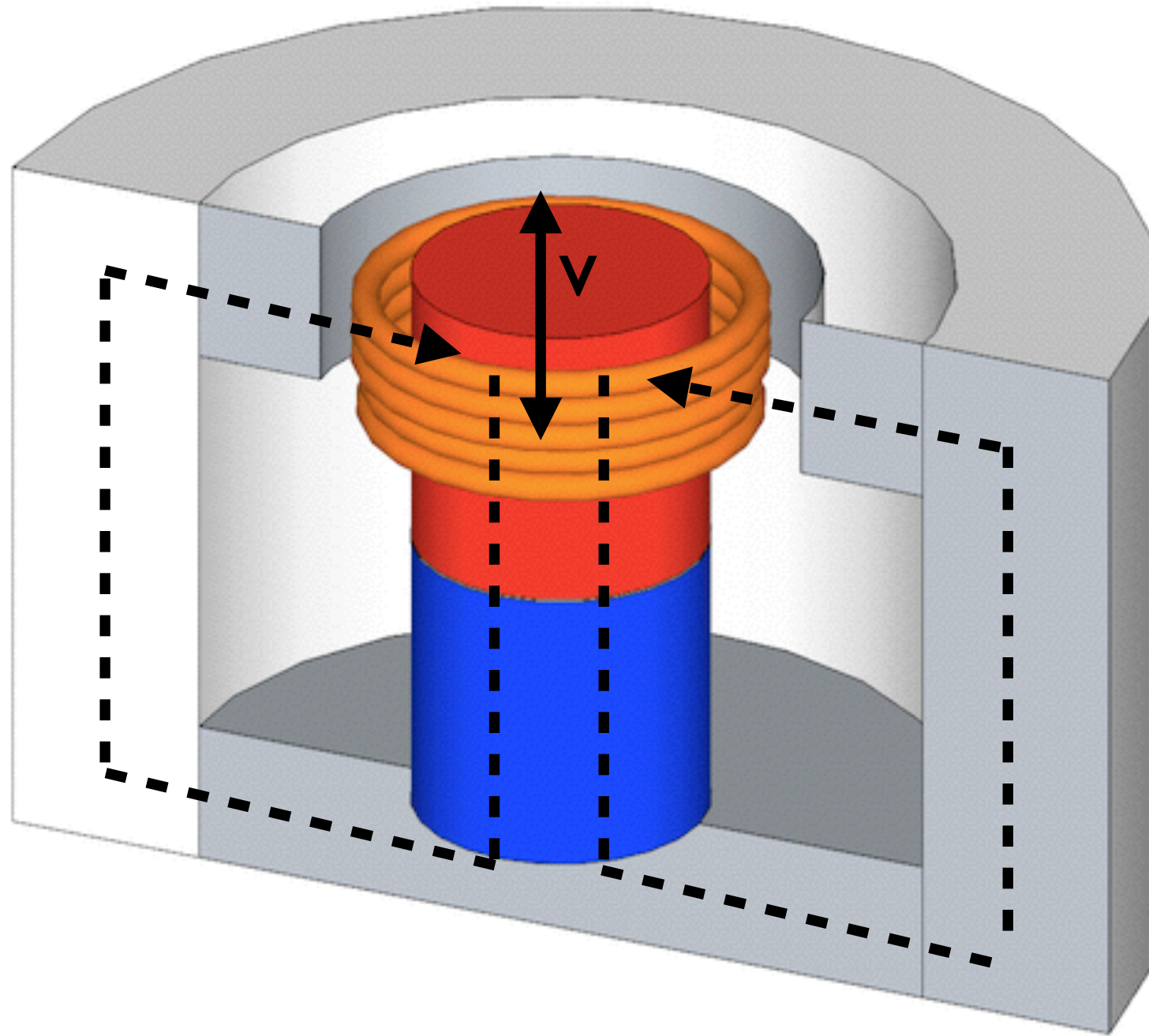
## Direct

I measure a physical quantity  
proportional to velocity

# Electrodynamic sensor of velocity

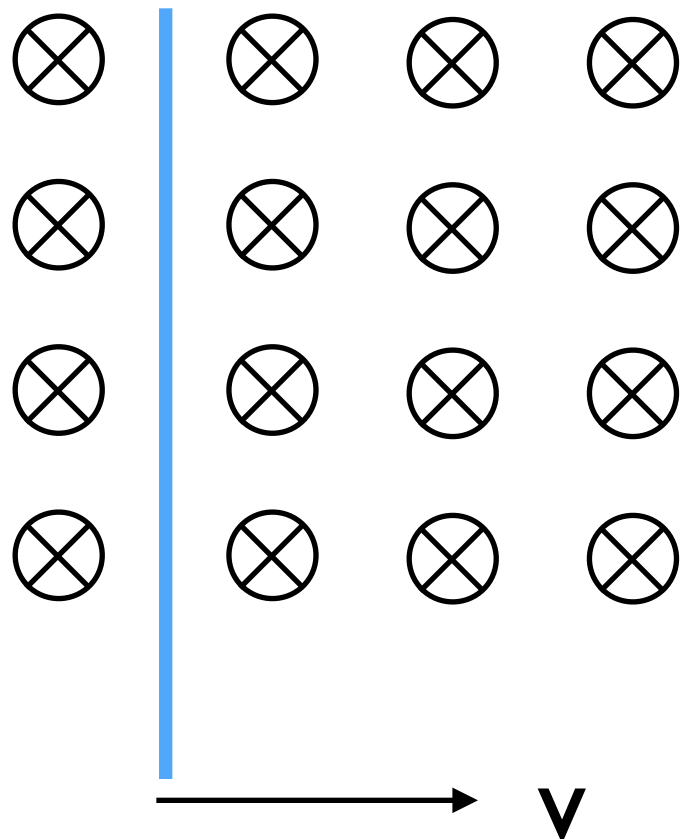


# The magnetic field in the yoke

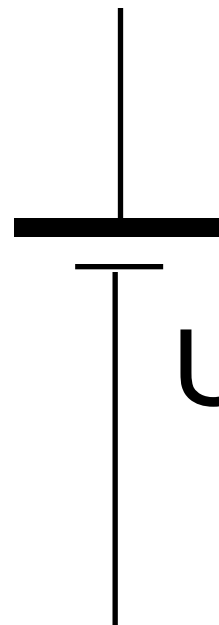


# A conductor moving in a magnetic field

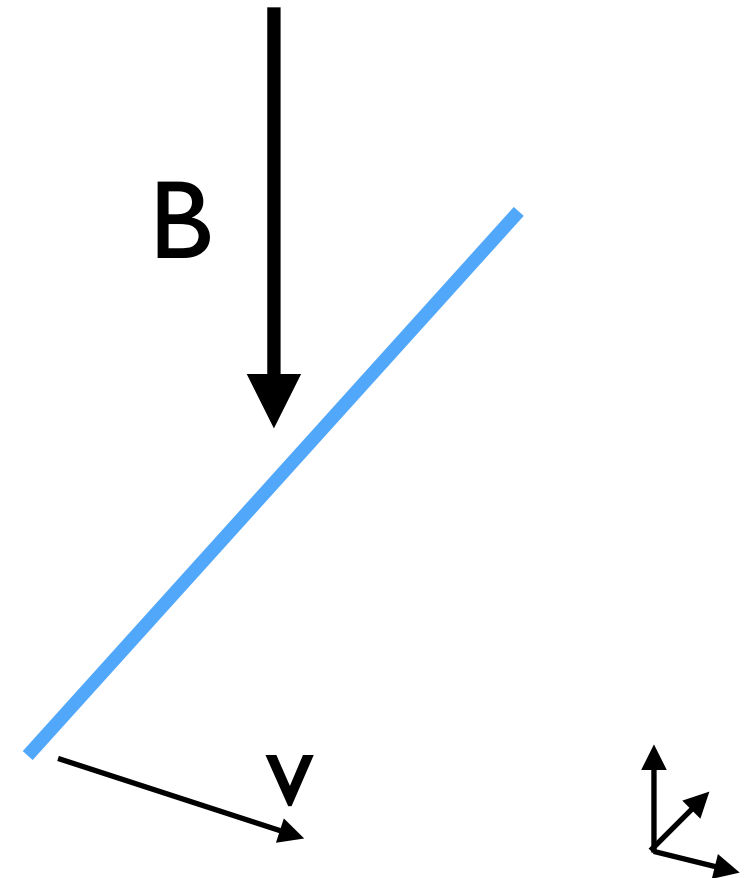
top view



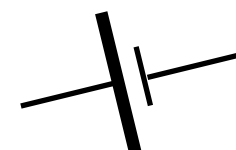
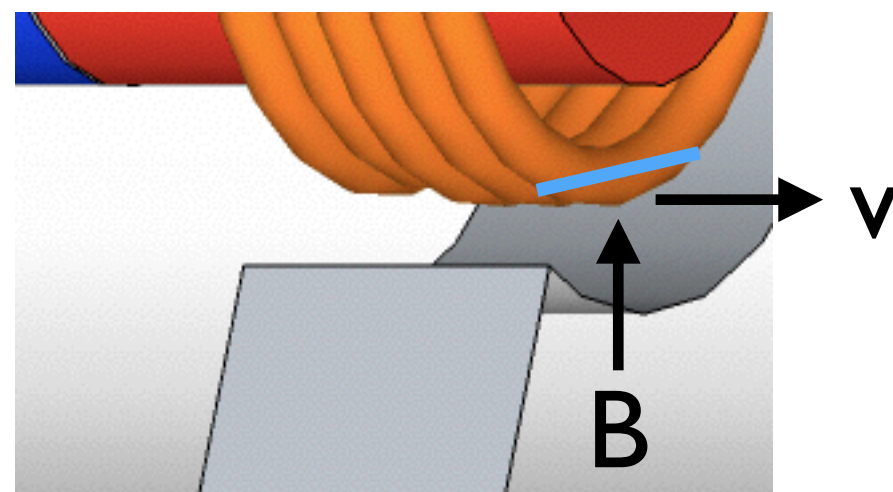
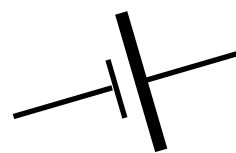
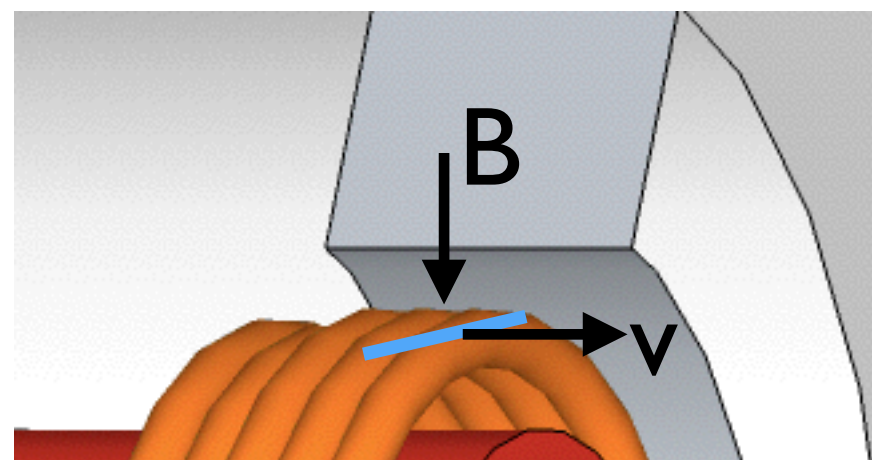
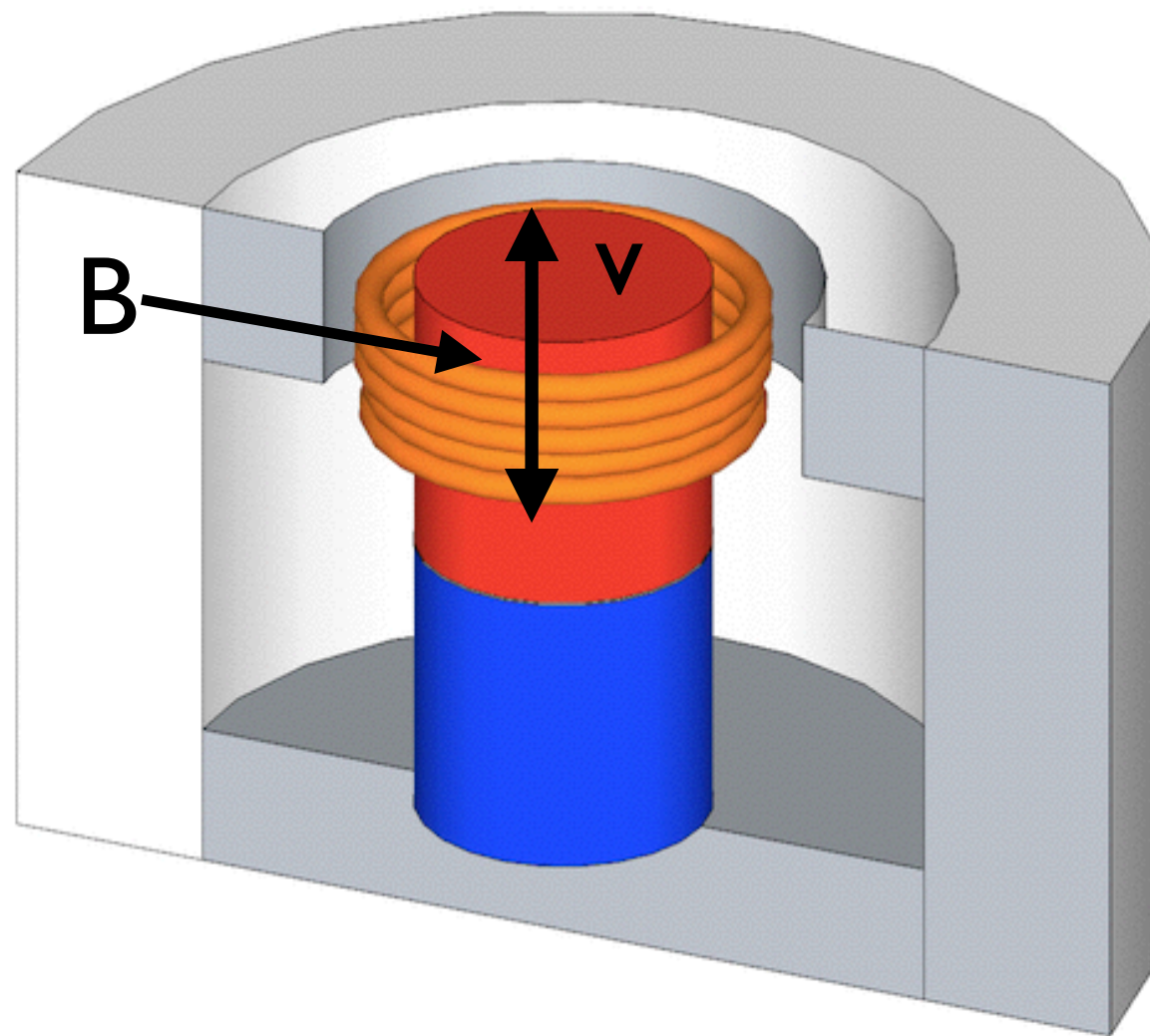
=



$$U = Bdv$$

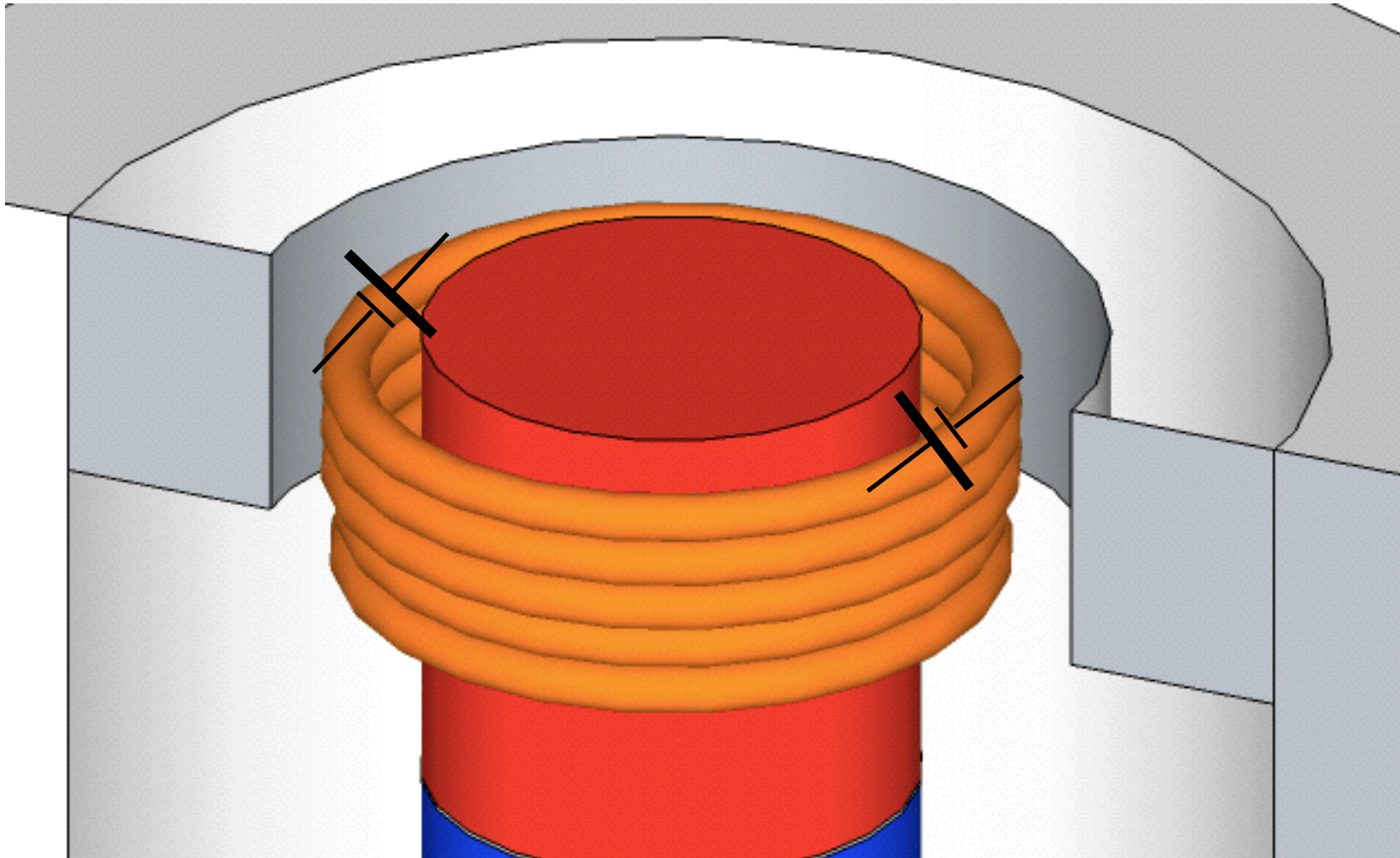


$d$  = length of the wire

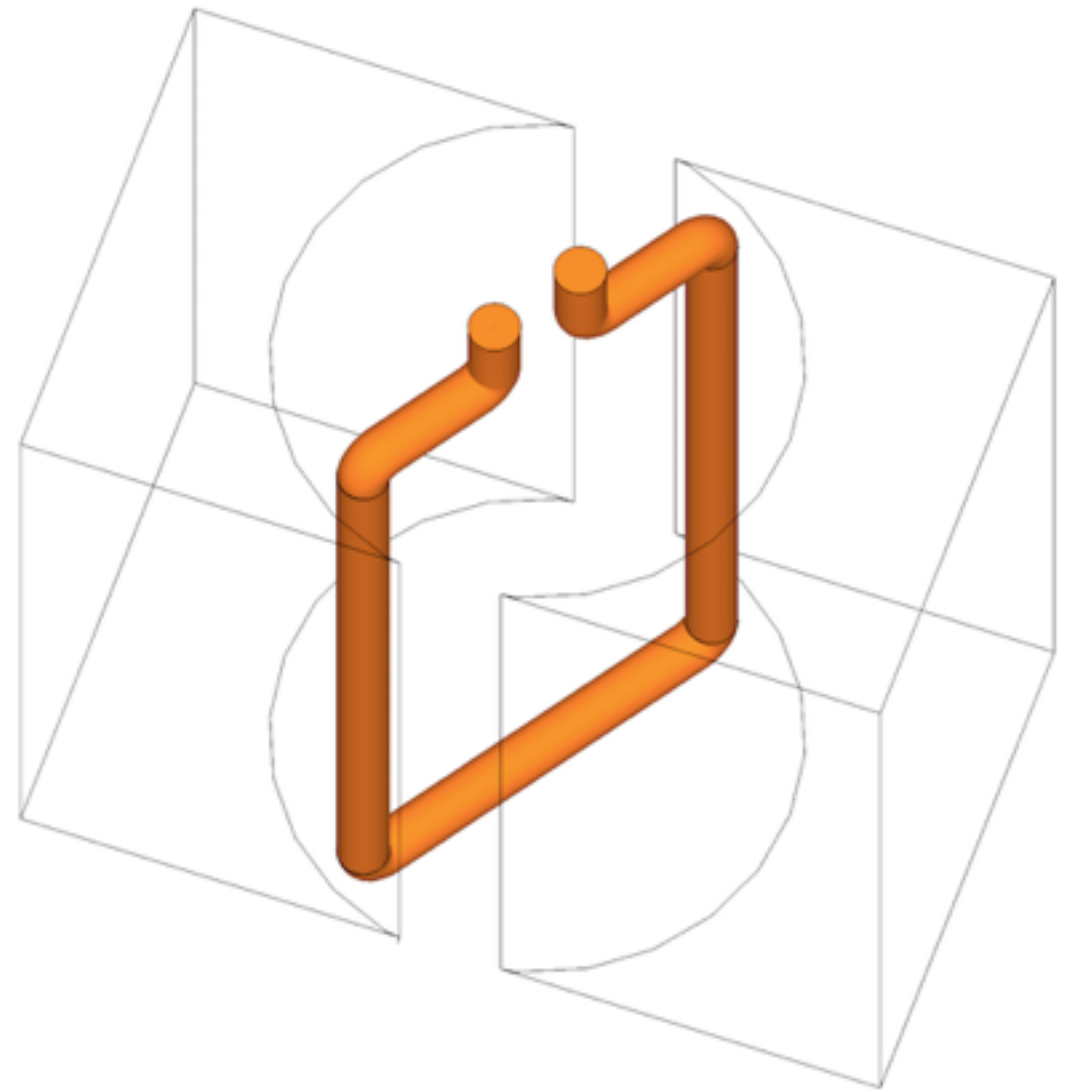




A voltage is induced in the coil. This voltage is proportional to the velocity.

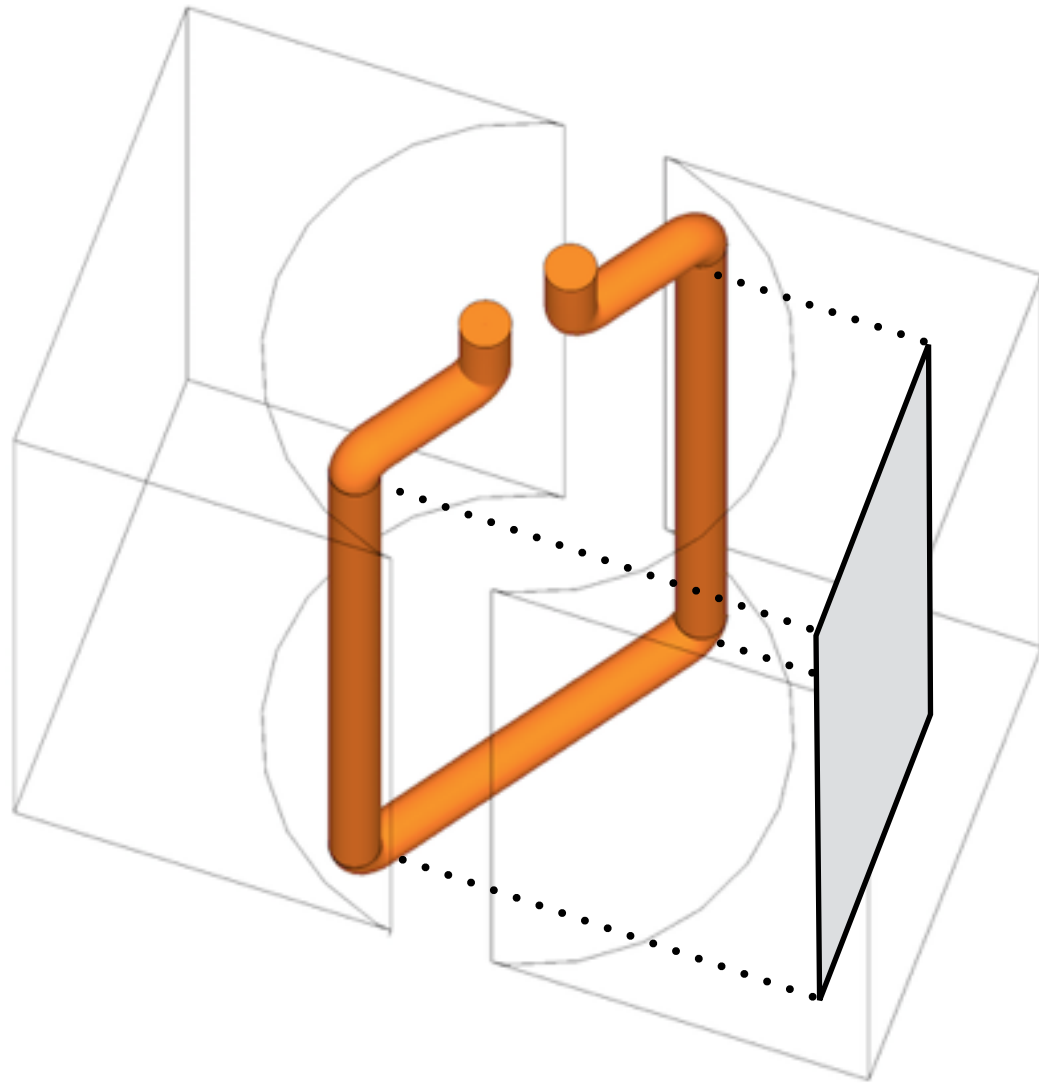


# Tachometer based on DC generator



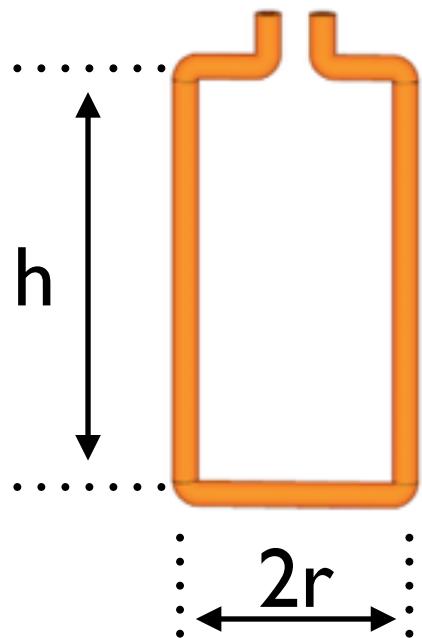
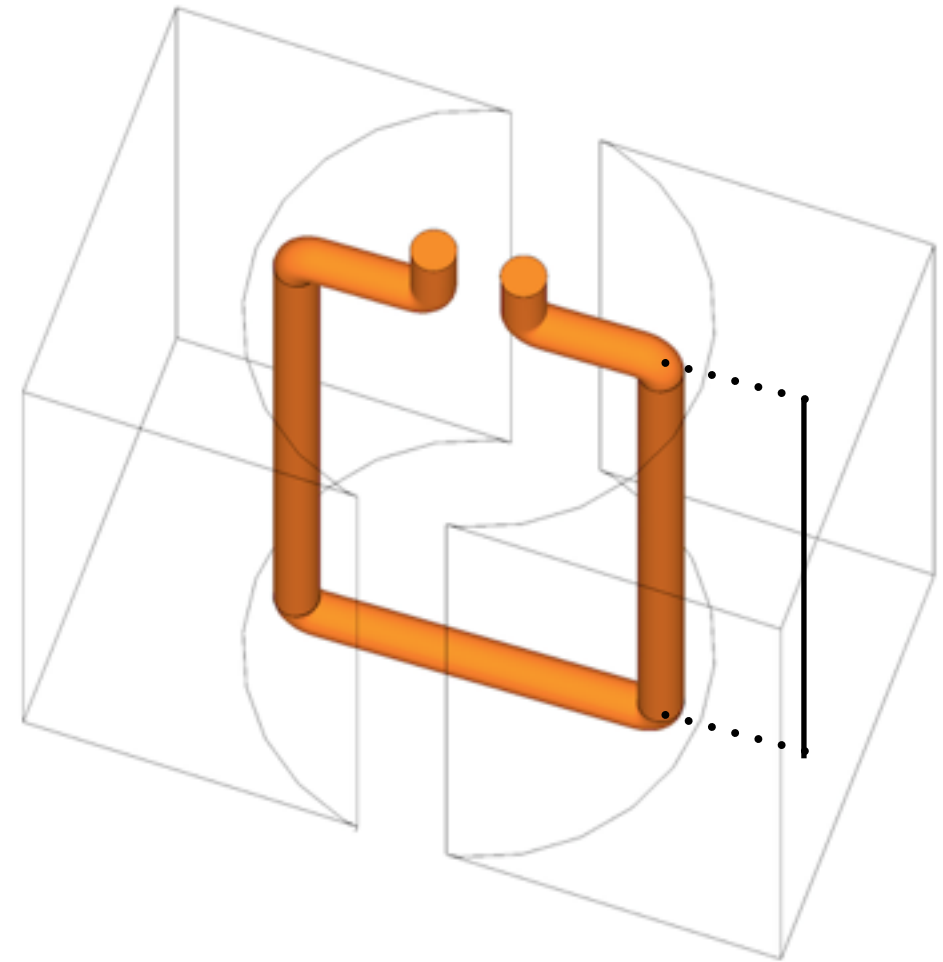
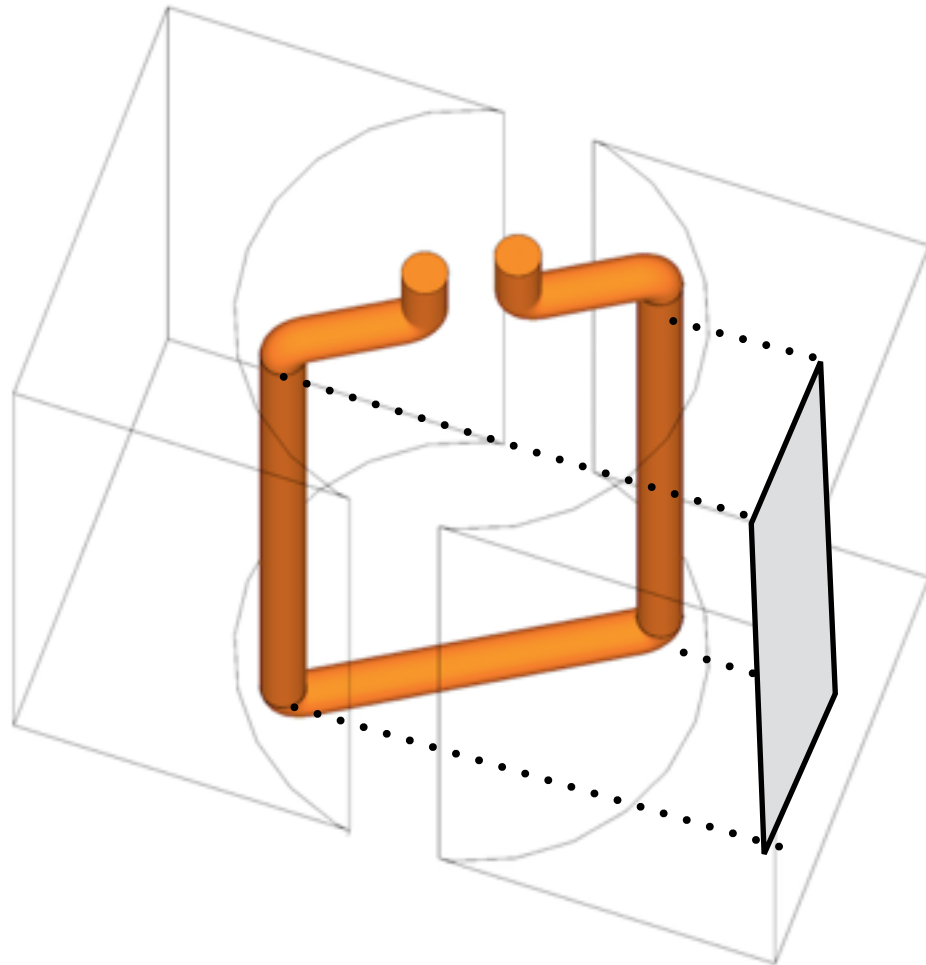


# Tachometer based on DC generator



the effective area of the coil depends on the relative angle to the magnetic field

# Tachometer based on DC generator



$$V = (2rhN)\omega$$

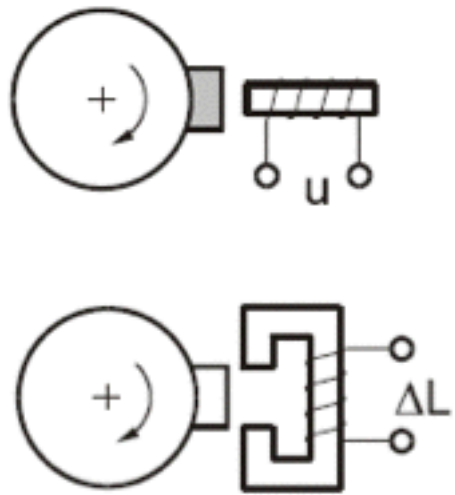
N=number of turns

ideally: no load - to avoid effect of inductance at high  $\omega$

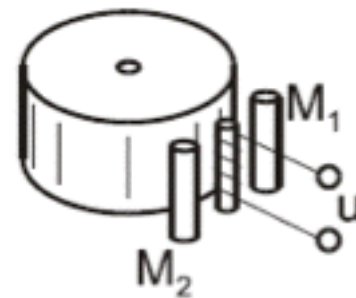
The induced voltage depends on the rotational speed

# Sensors of velocity with pulse output

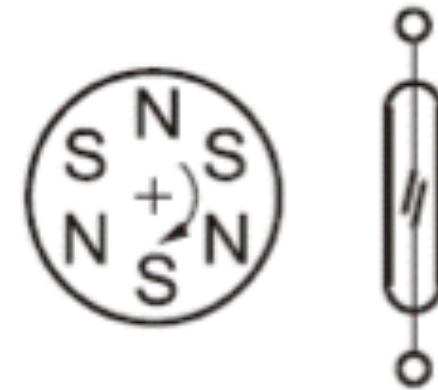
induction



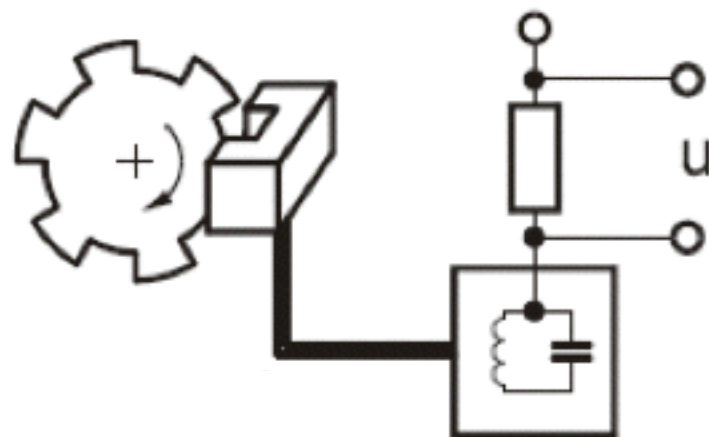
wiegand sensor



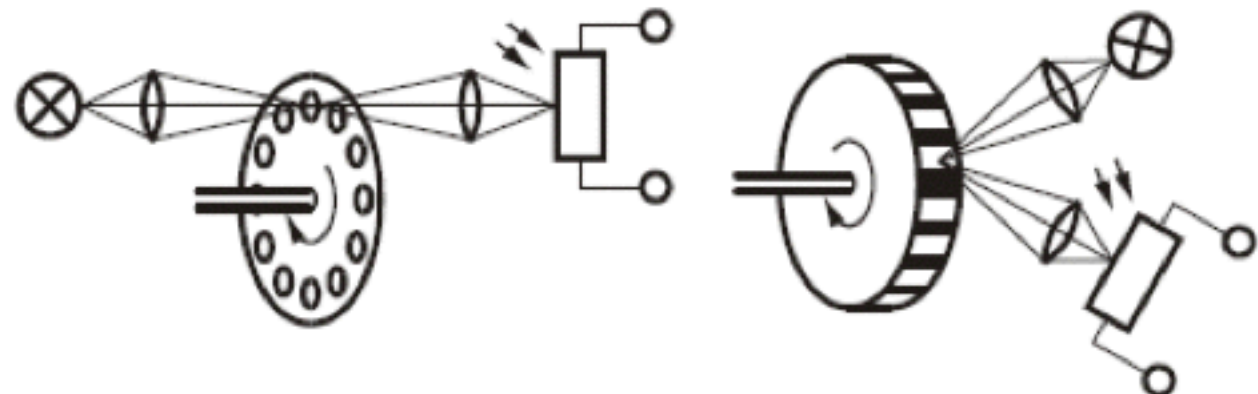
reed sensor



eddy current

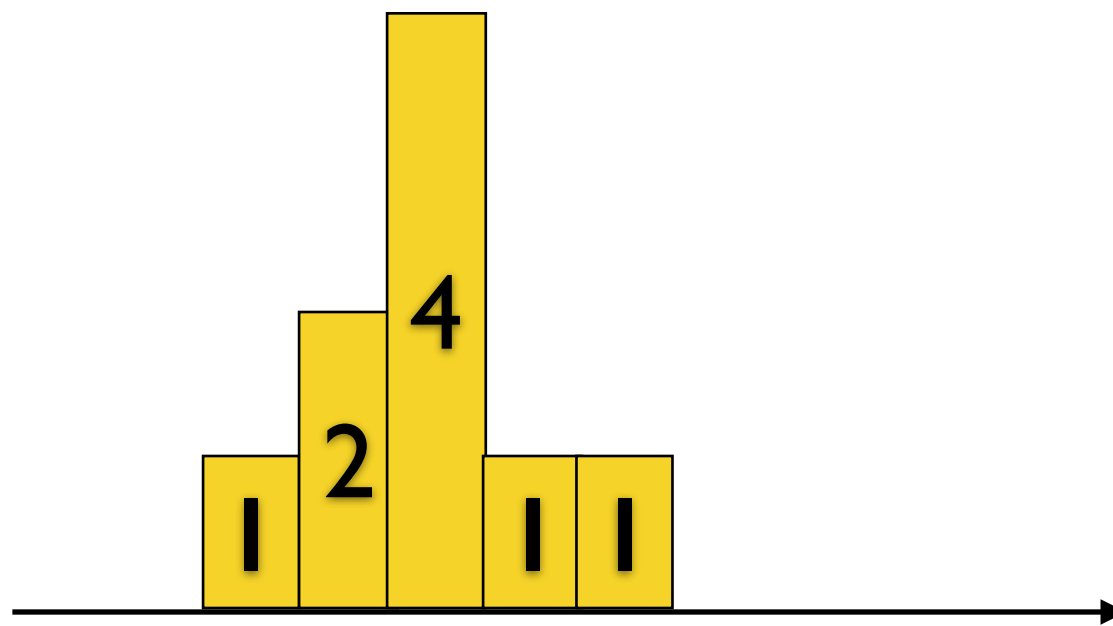


optoelectronics

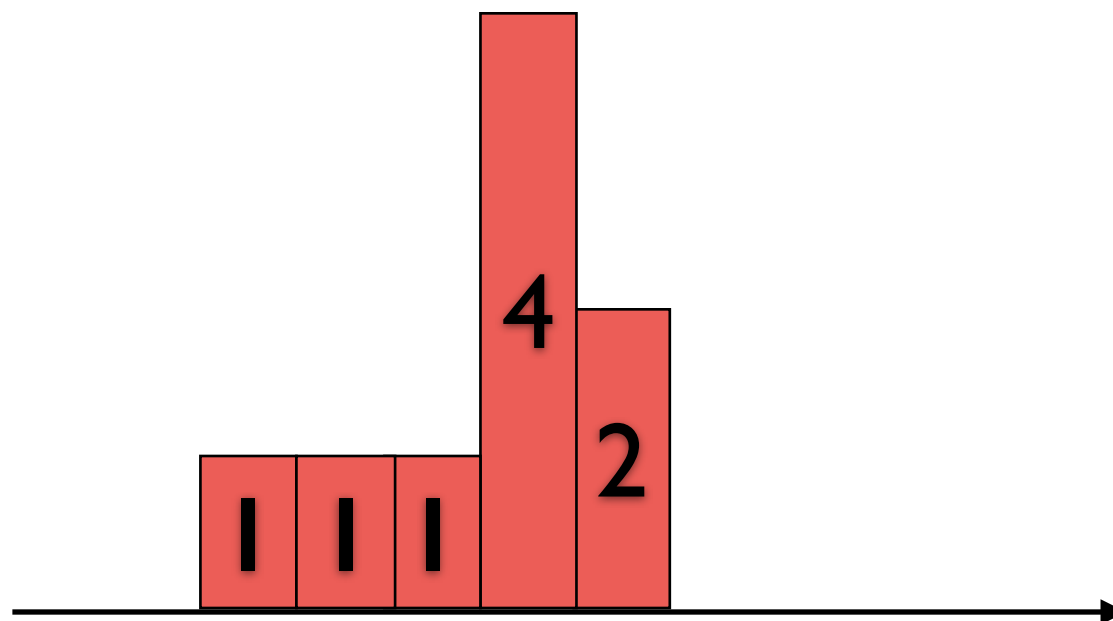


# Measurement by means of correlation

what is correlation?

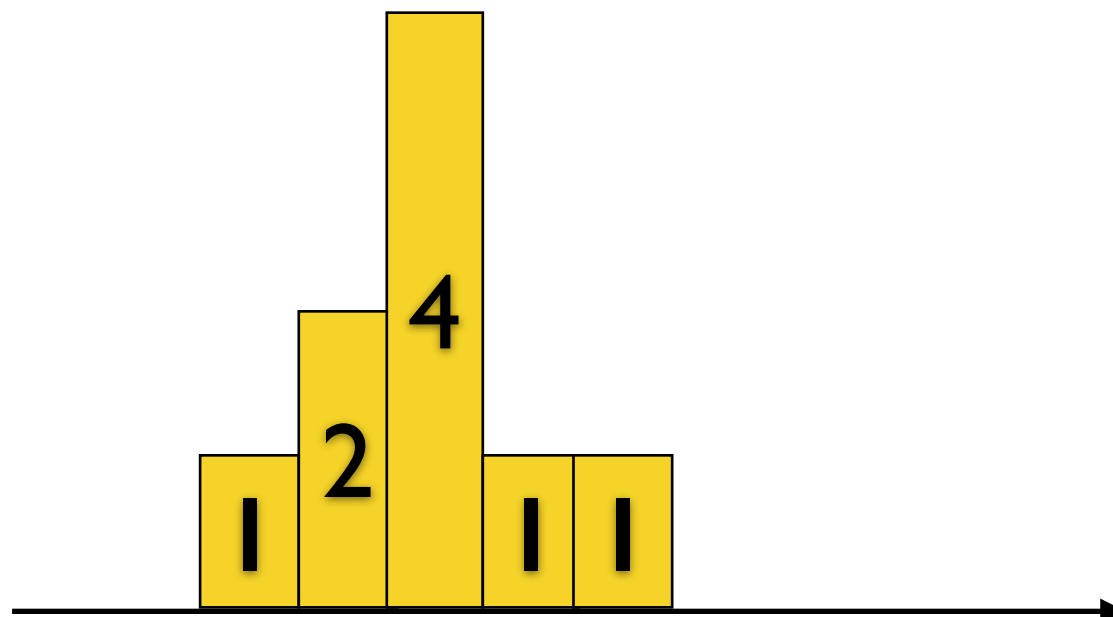


sequence A

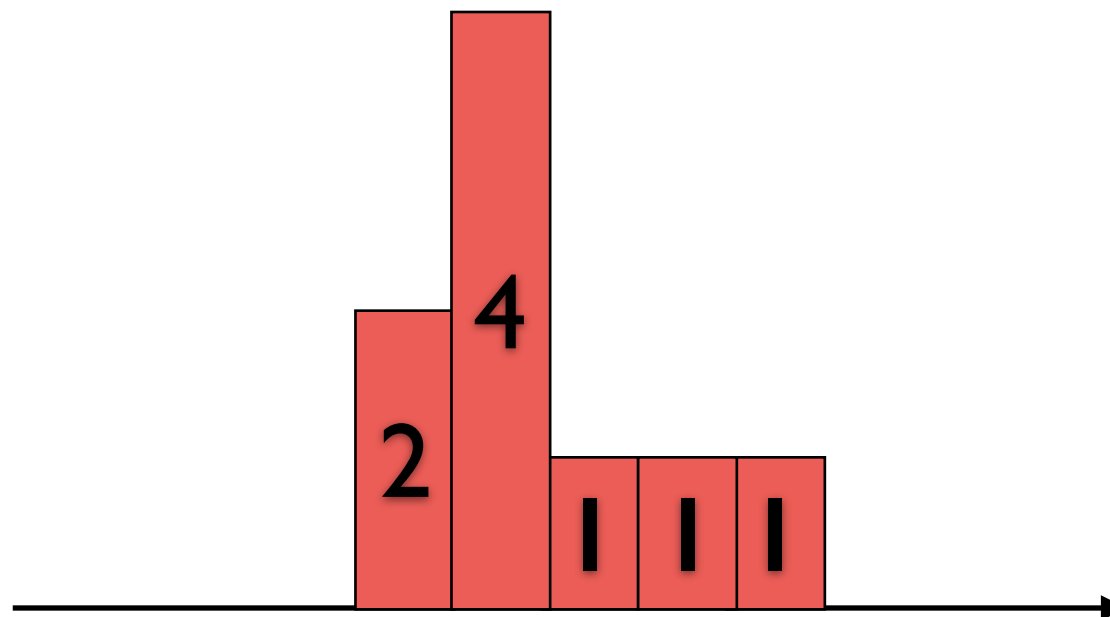


sequence B

A

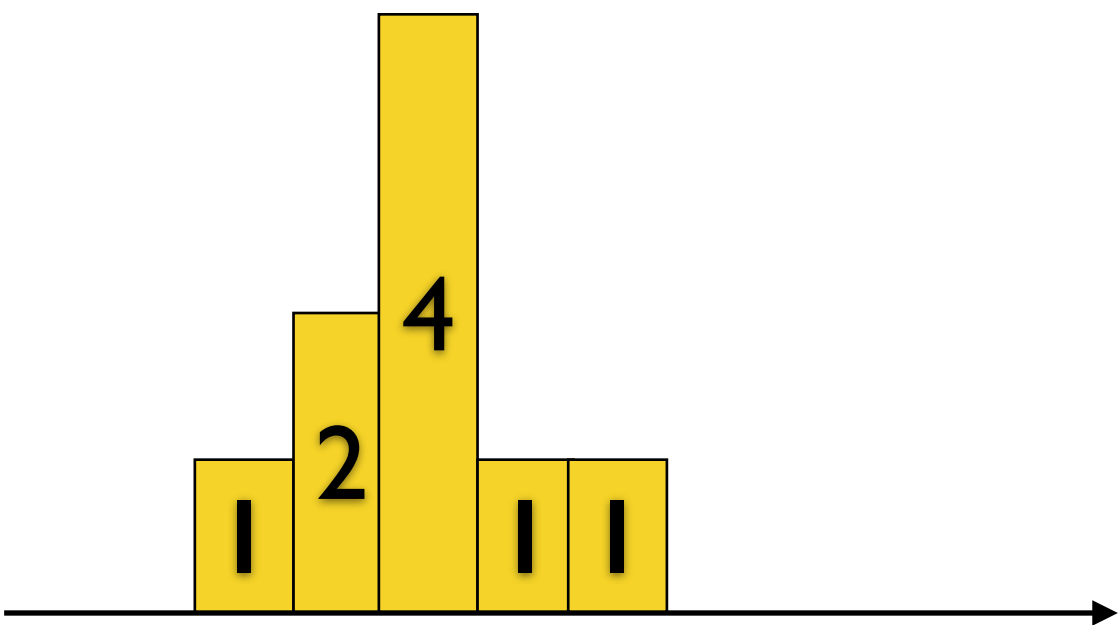


flipped B

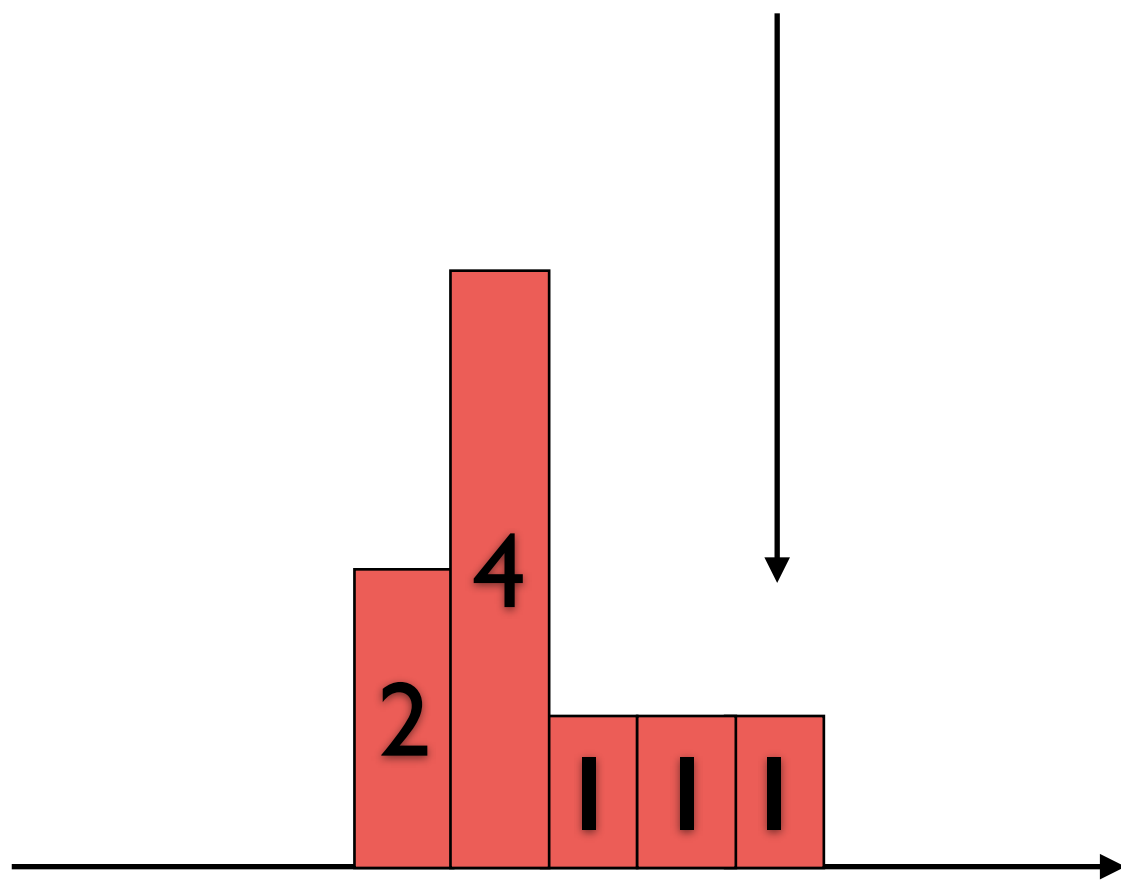


step 1    corr= [ 1

A



flipped B

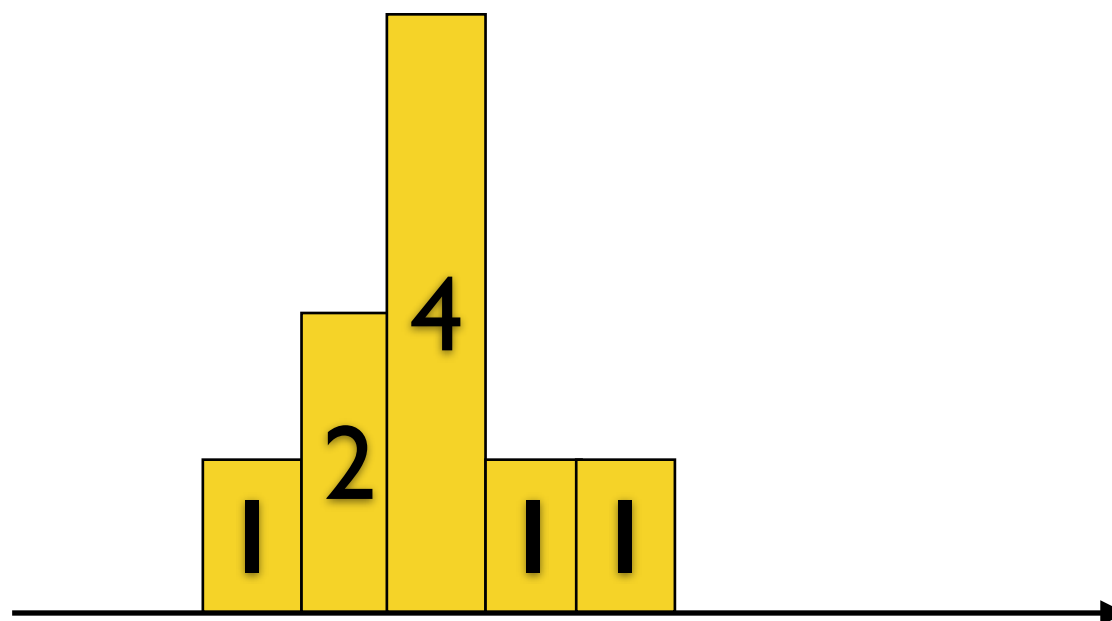


$| \times | = |$

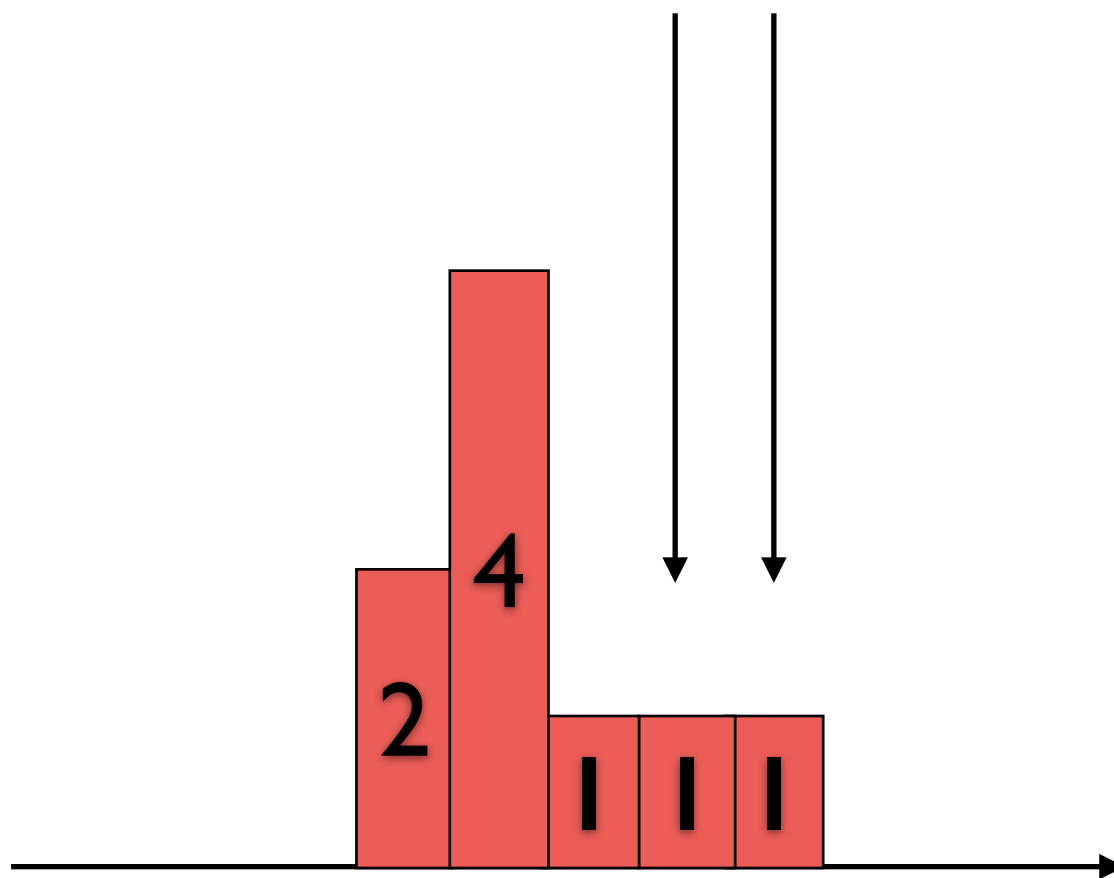


step 2 corr= [ 1; 3

A



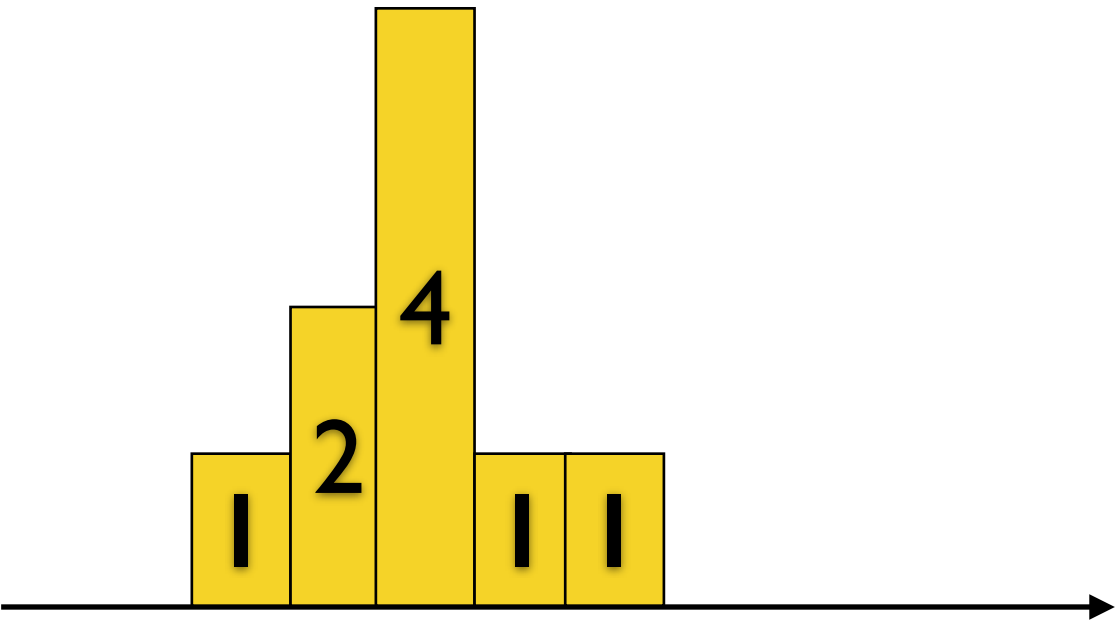
flipped B



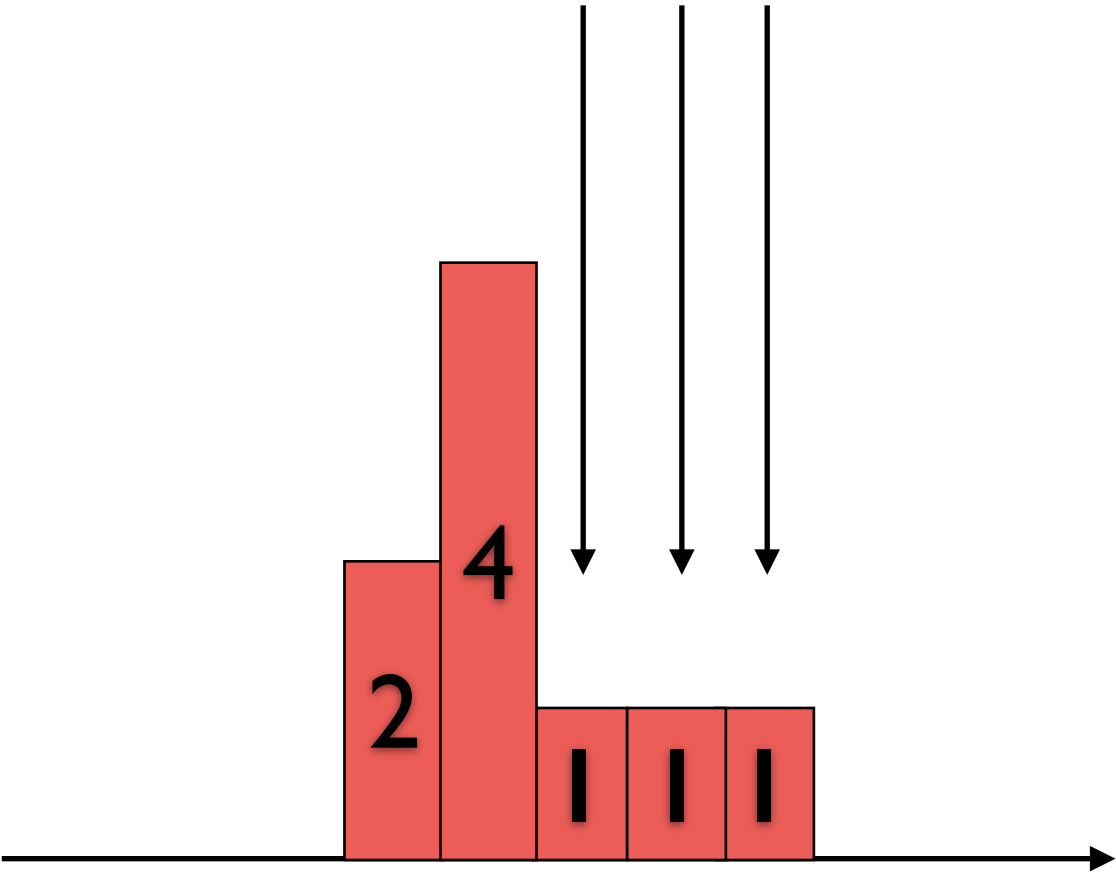
$$1 \times 1 + 2 \times 1 = 3$$

step 2    corr= [ 1; 3; 7

A



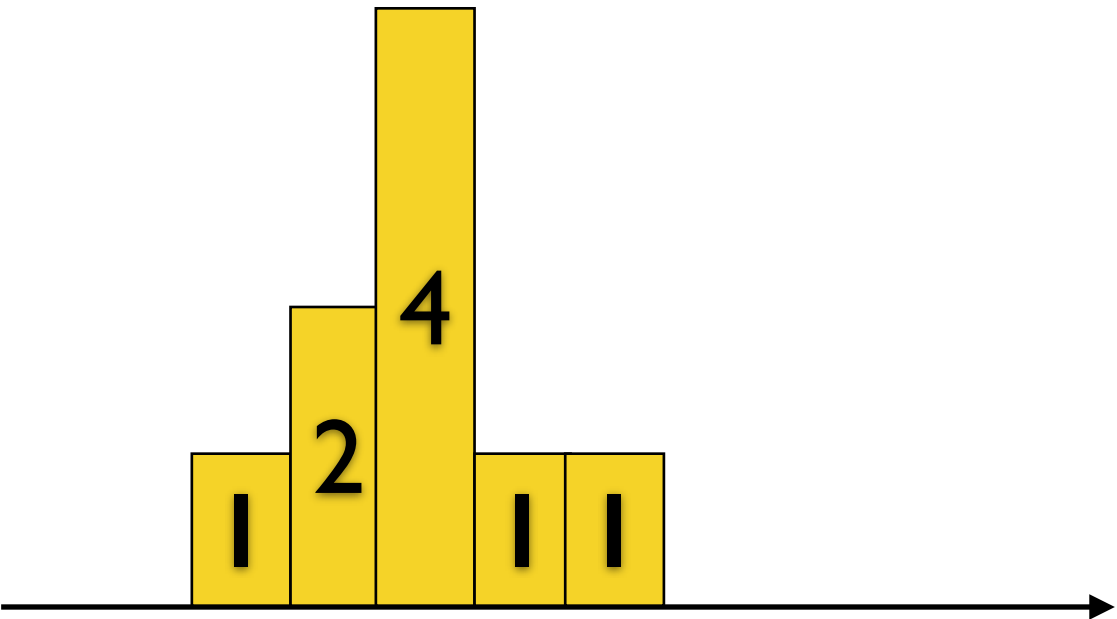
flipped B



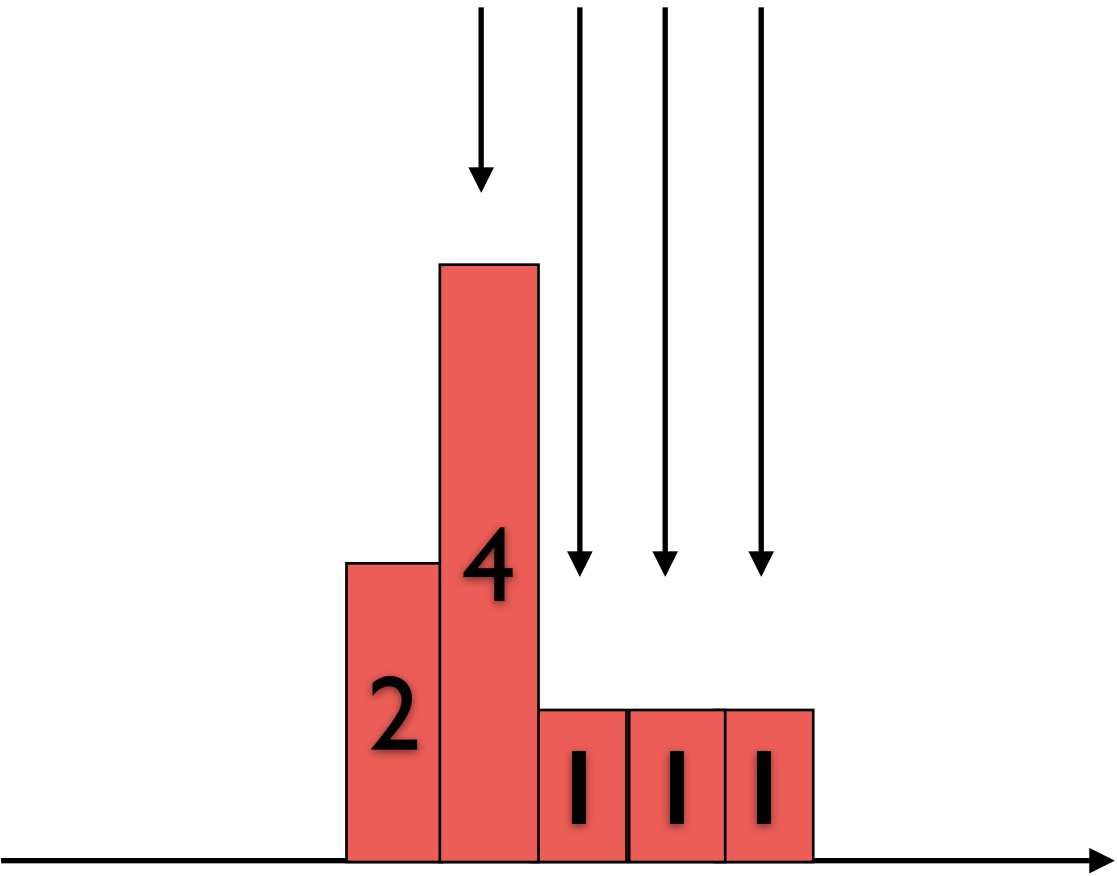
$$1 \times 1 + 2 \times 1 + 4 \times 1 = 7$$

step 2 corr= [ 1; 3; 7; 1 1

A



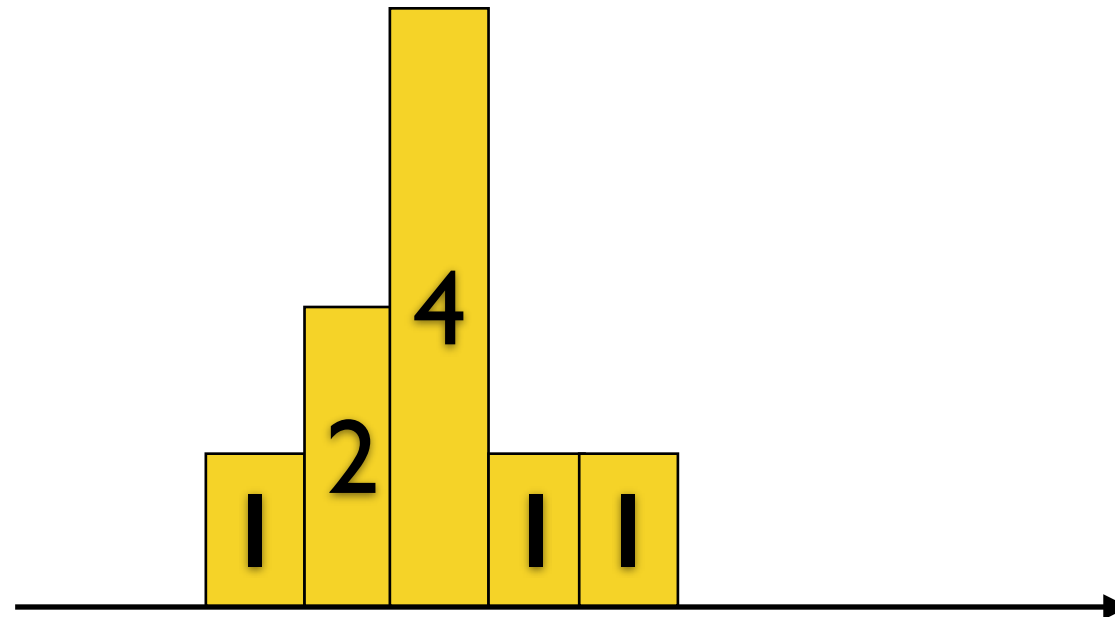
flipped B



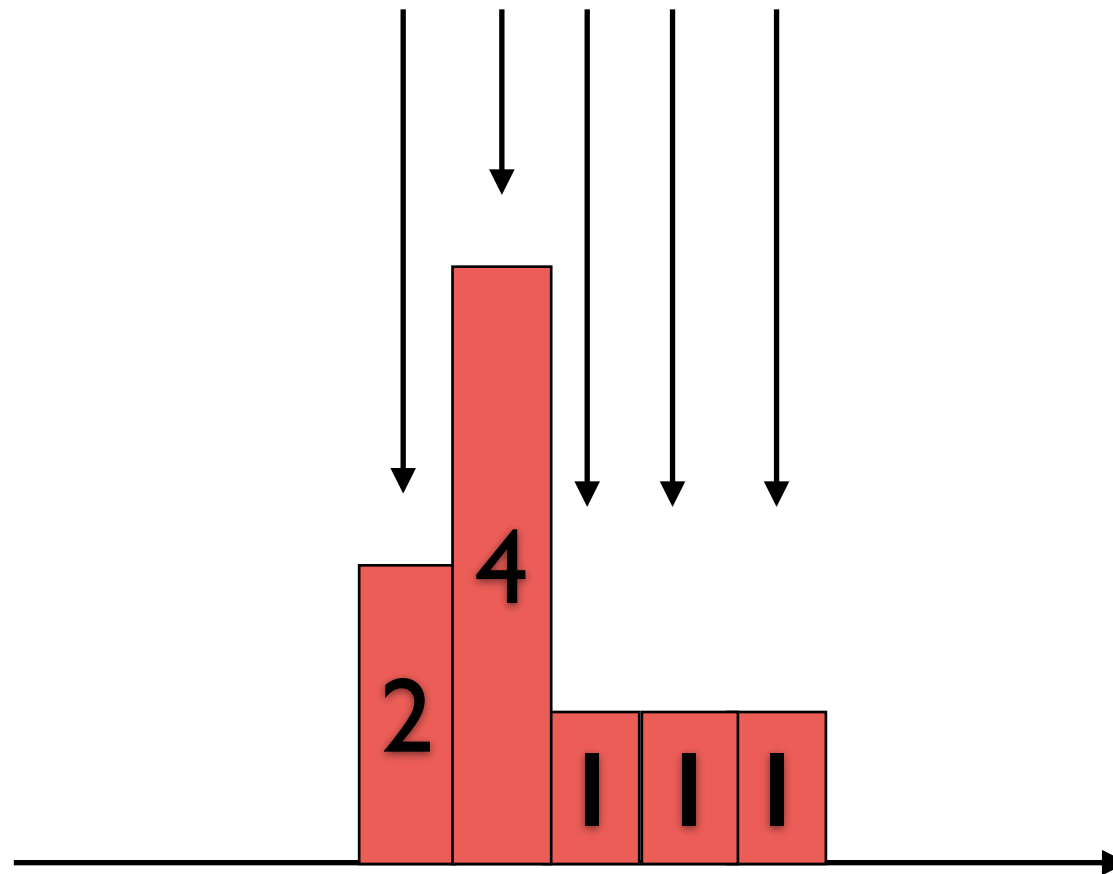
$$4 \times 1 + 2 \times 1 + 4 \times 1 + 1 \times 1 = 11$$

step 2 corr= [ 1; 3; 7; 11; 16

A



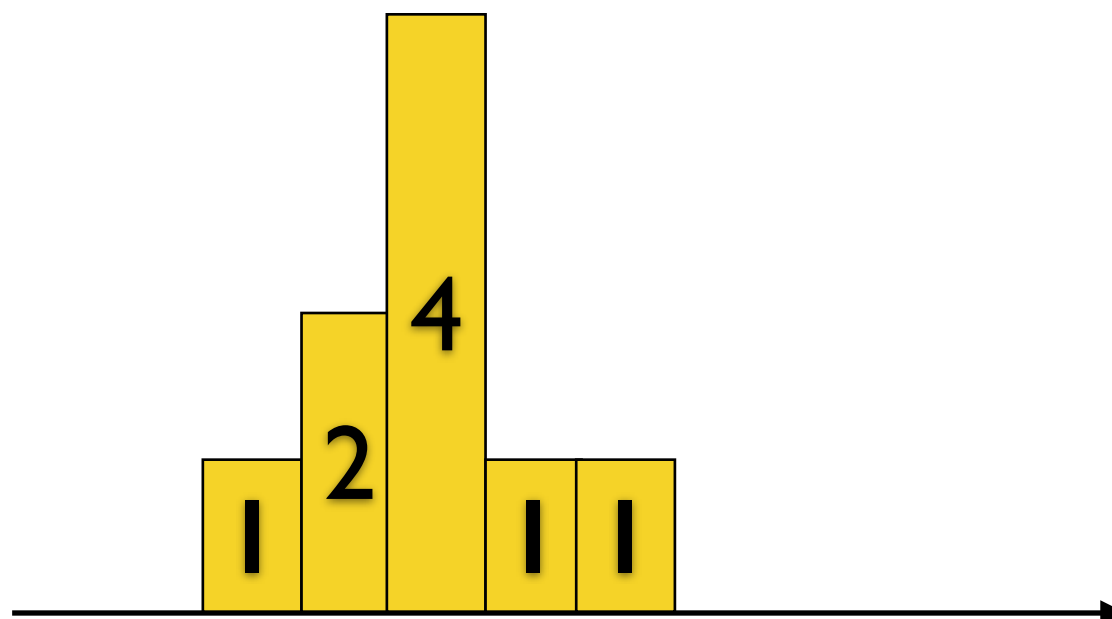
flipped B



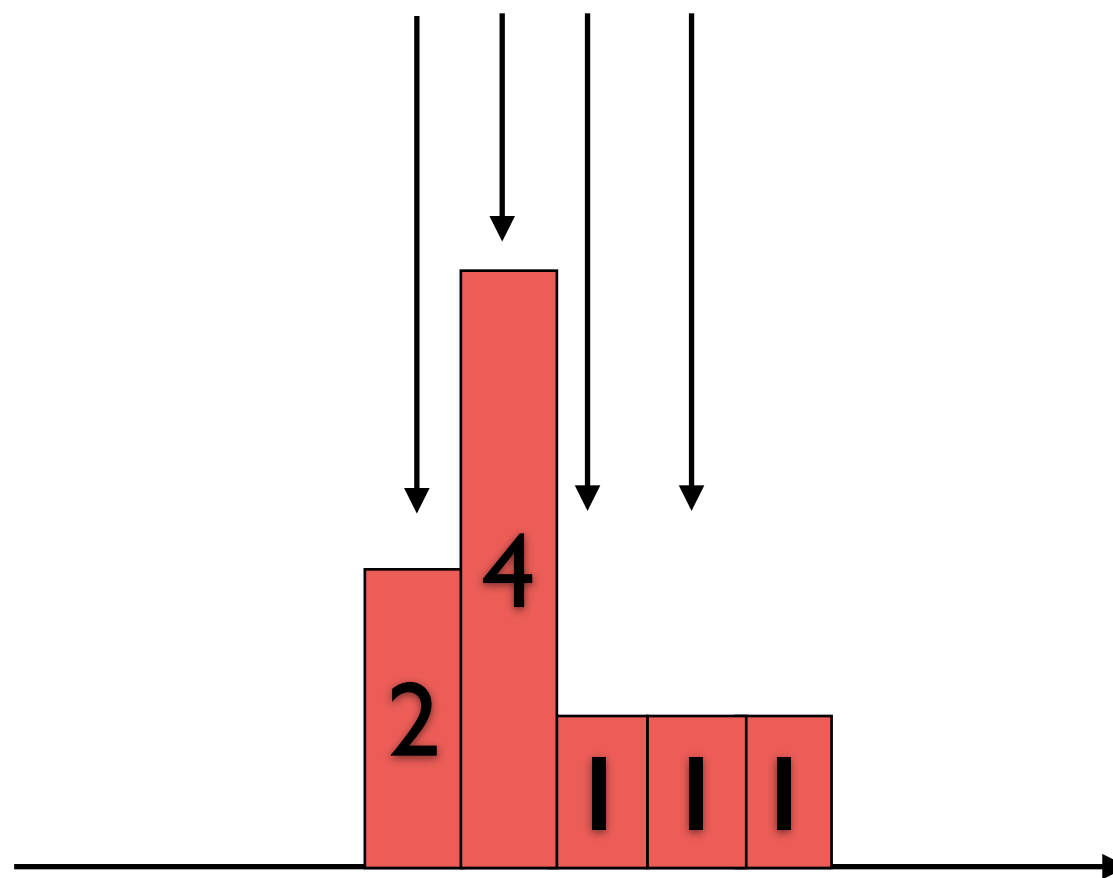
$$2 \times 1 + 2 \times 4 + 4 \times 1 + 1 \times 1 + 1 \times 1 = 16$$

step 2 corr= [ 1; 3; 7; 11; 16; 22

A



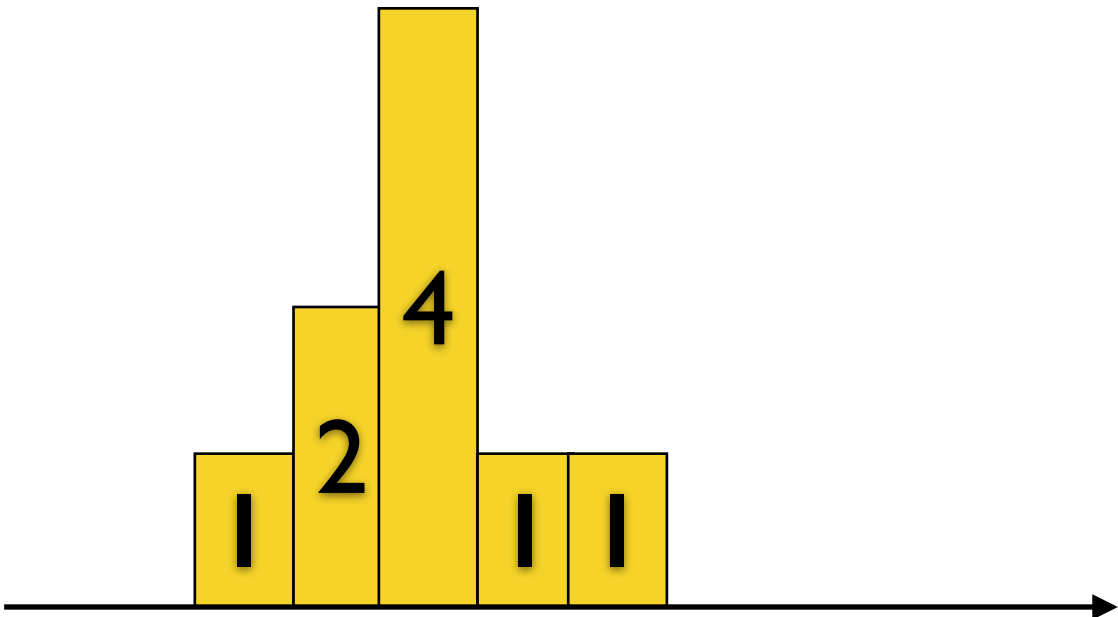
flipped B



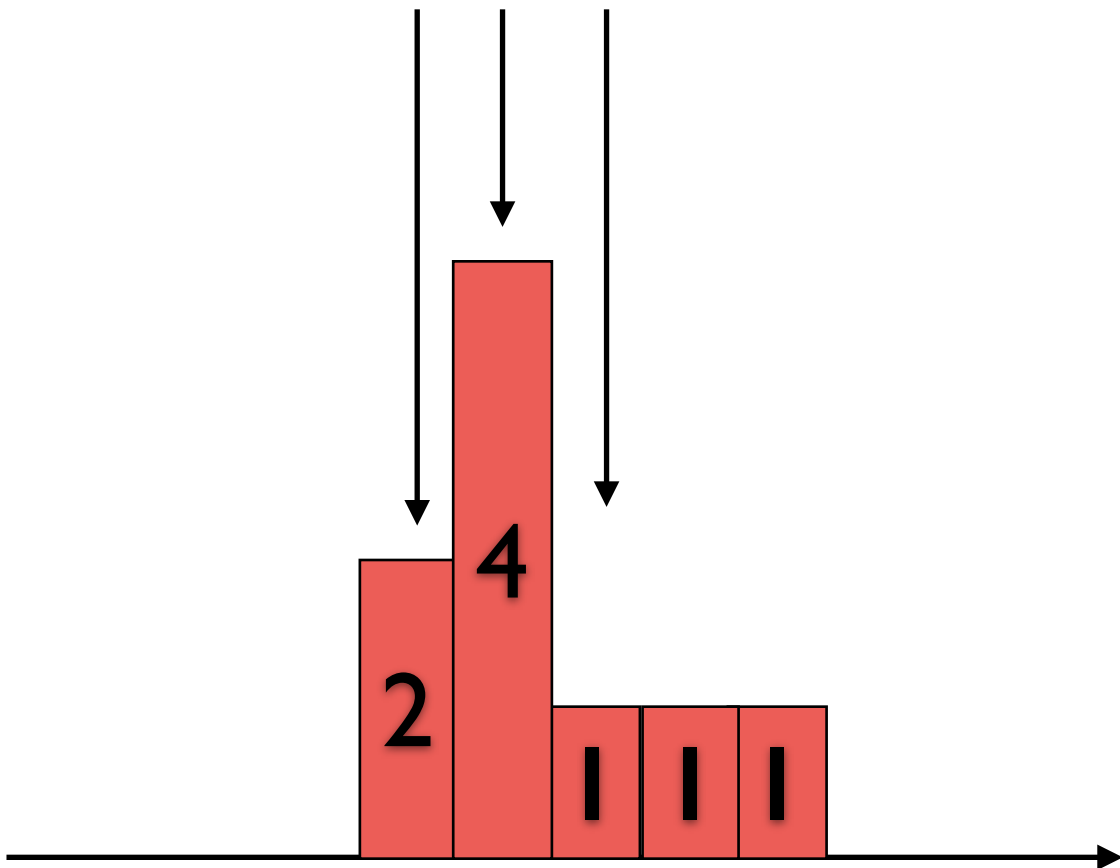
$$2 \times 2 + 4 \times 4 + 1 \times 1 + 1 \times 1 = 22$$

step 2 corr= [ 1; 3; 7; 11; 16; 22; 13

A



flipped B

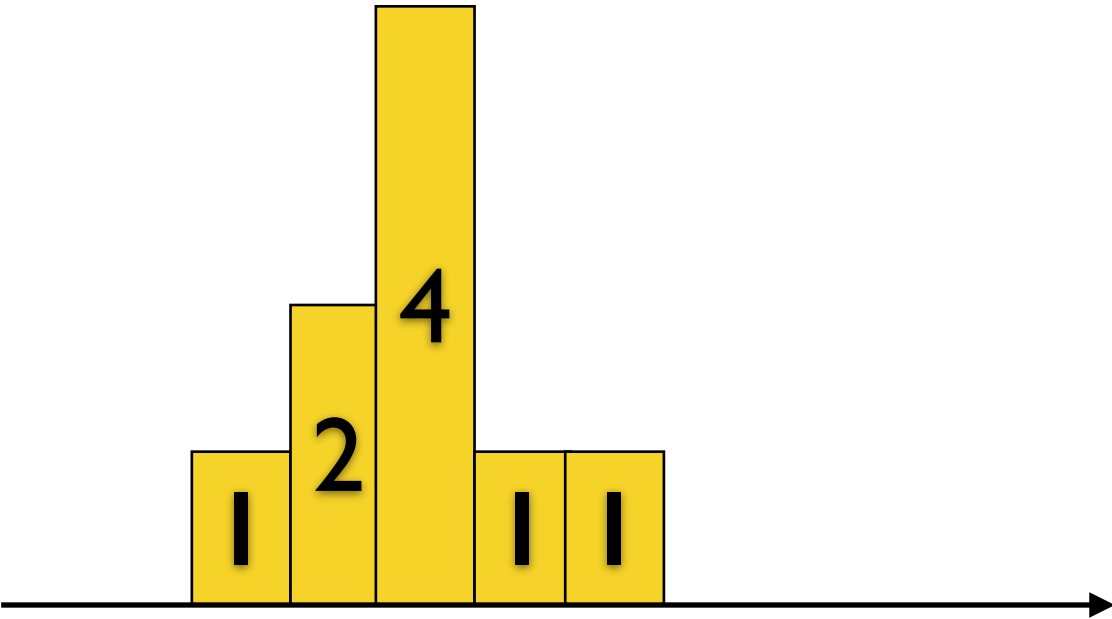


$$4 \times 2 + 4 \times 1 + 1 \times 1 = 13$$

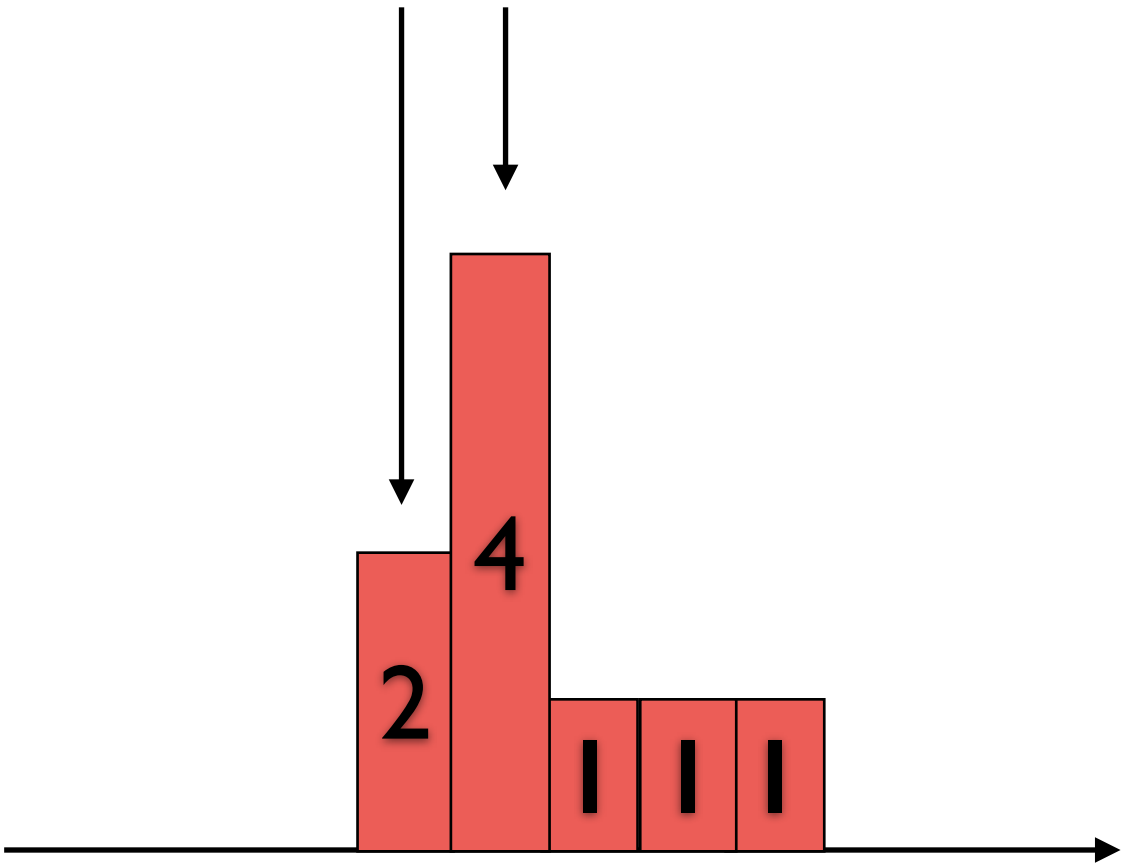


step 2 corr= [ 1; 3; 7; 11; 16; 22; 13; 6

A



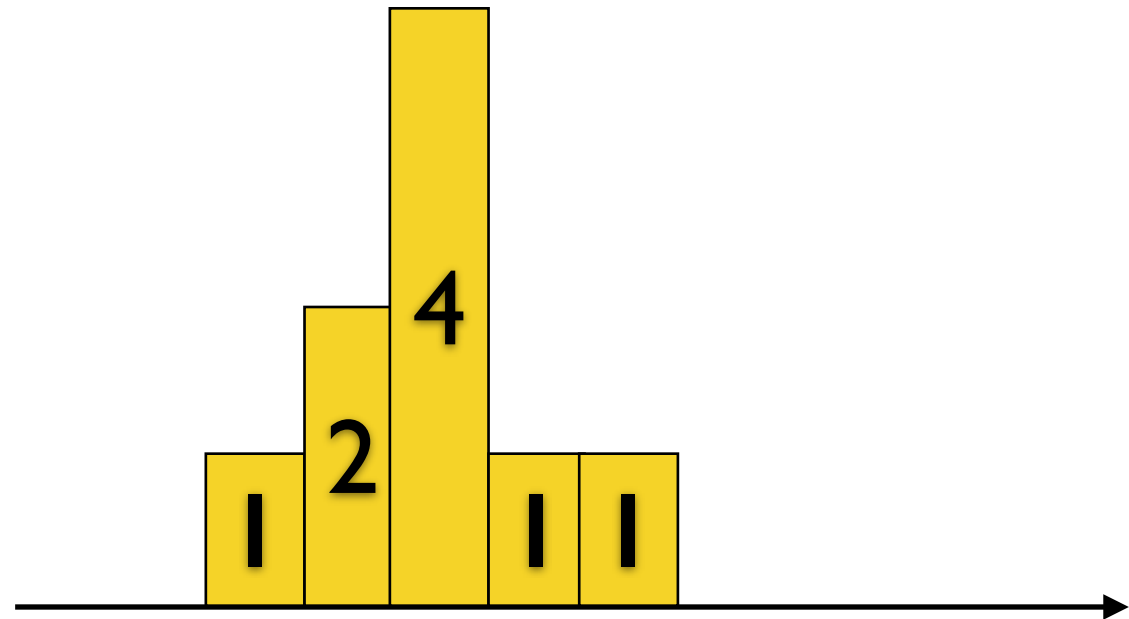
flipped B



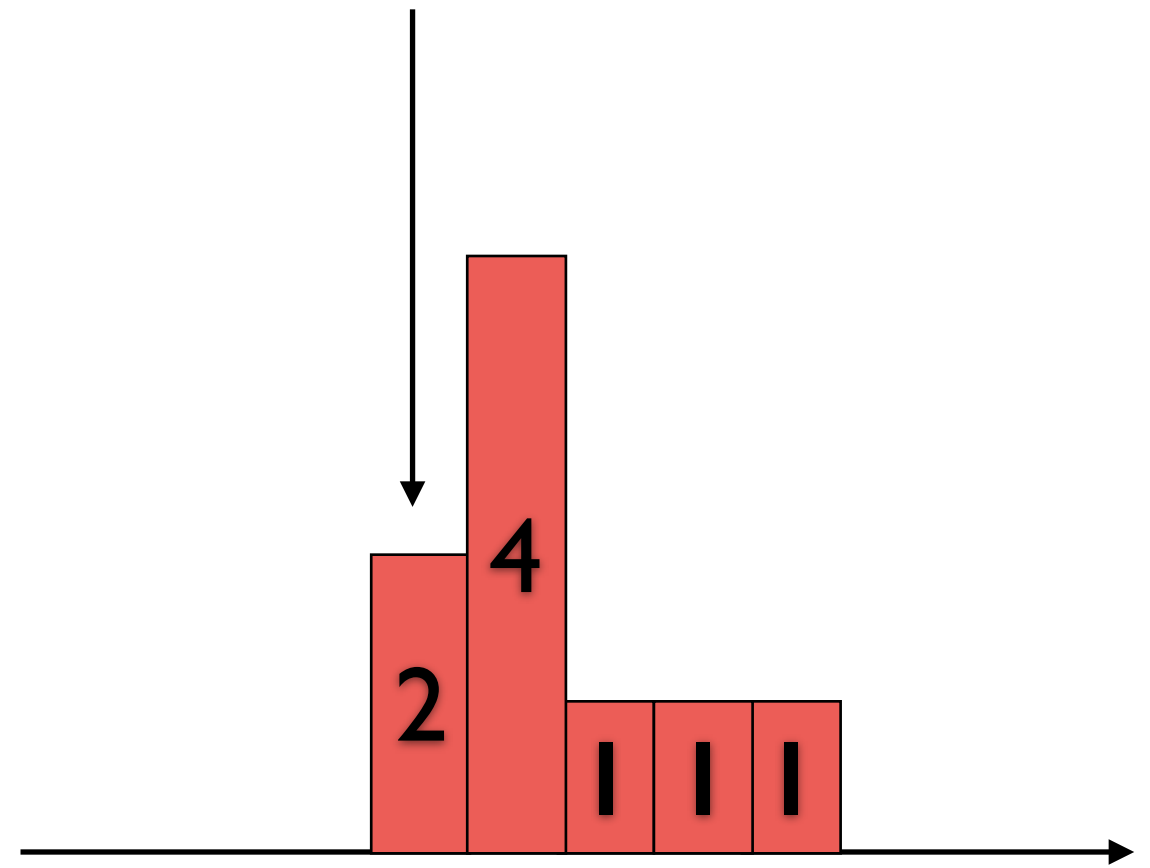
$$2 \times 1 + 4 \times 1 = 6$$

step 2 corr= [ 1; 3; 7; 11; 16; 22; 13; 6; 2]

A

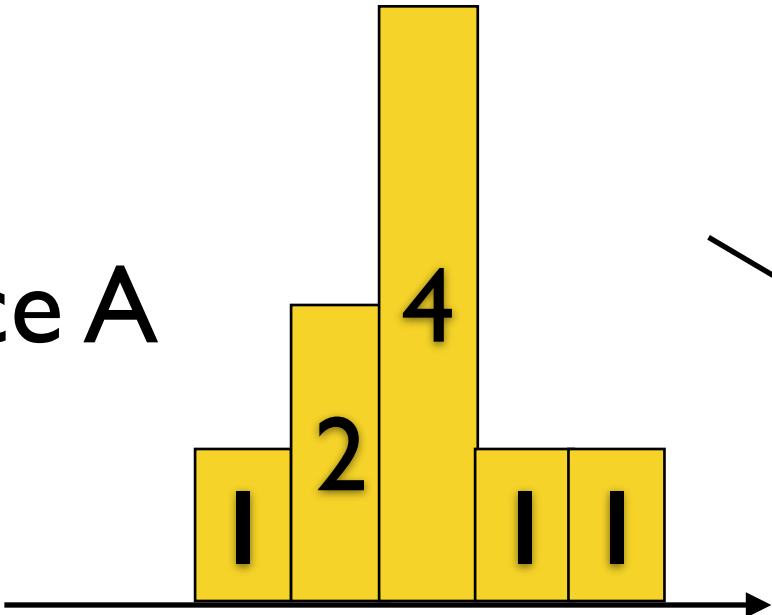


flipped B

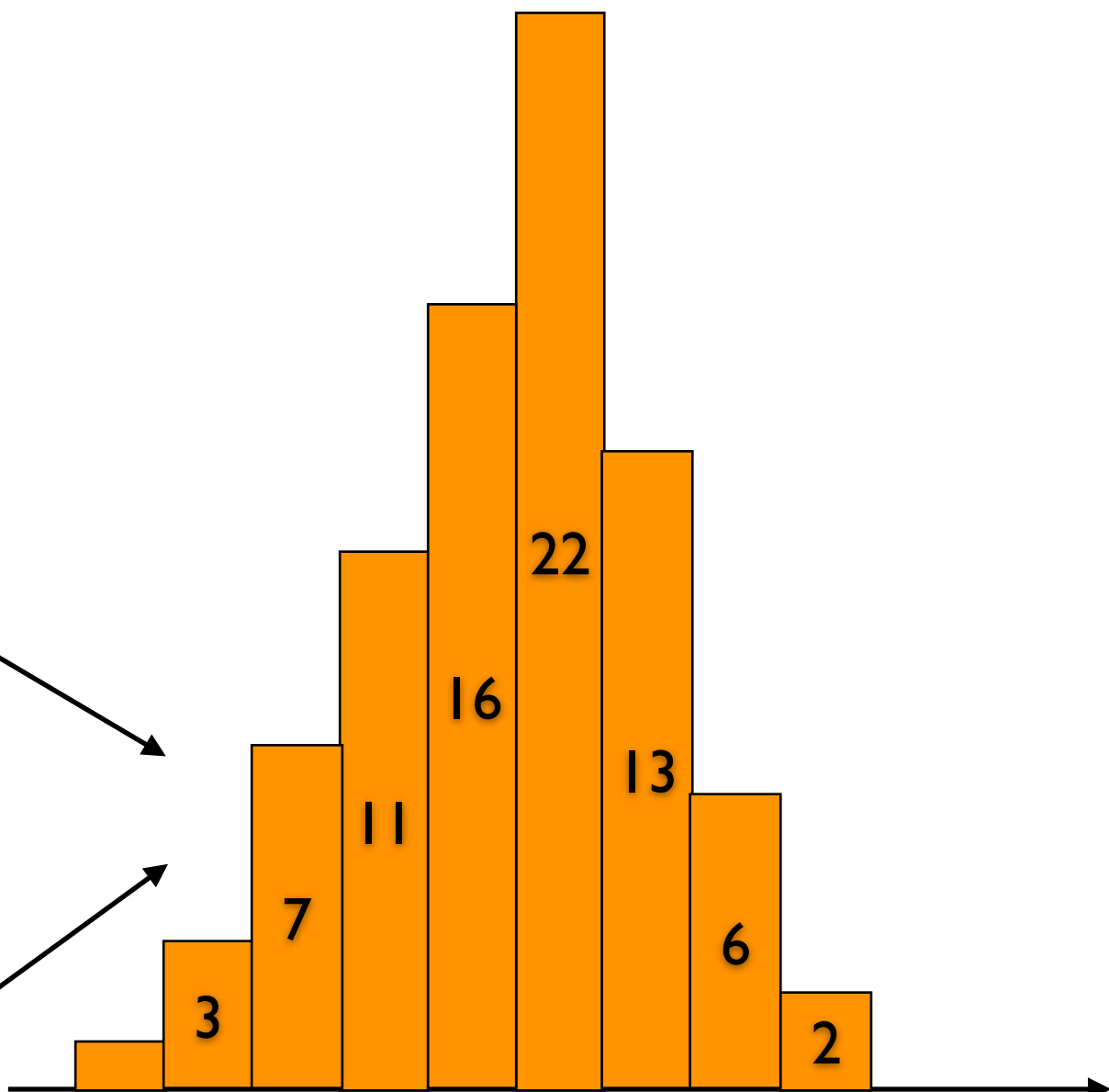
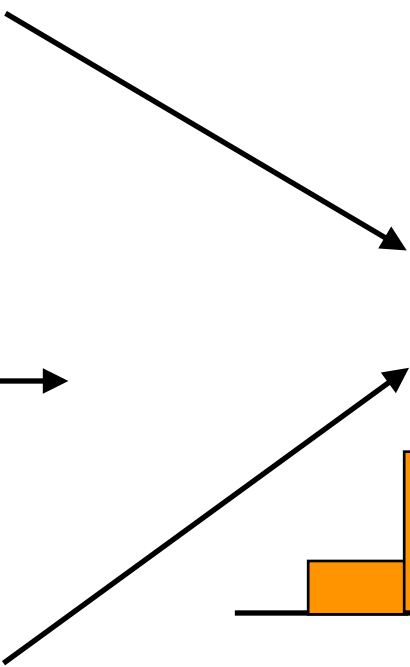
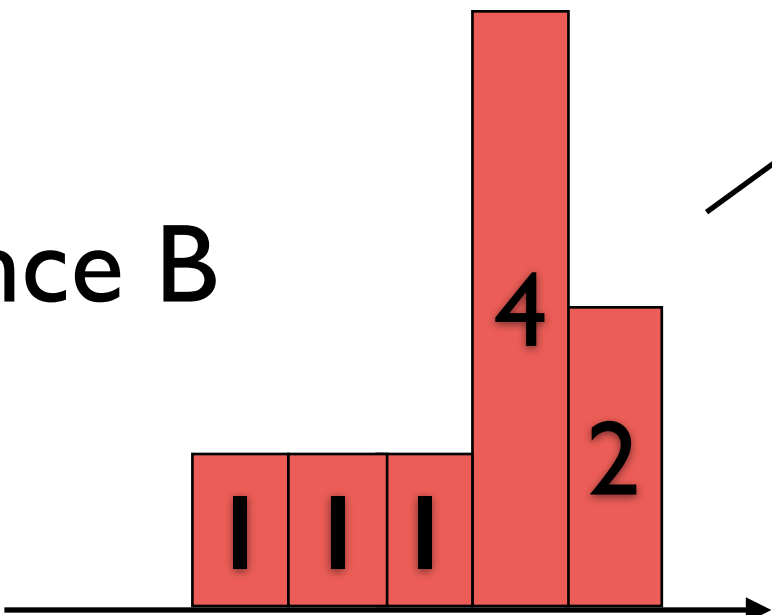


$$2 \times 1 = 2$$

sequence A

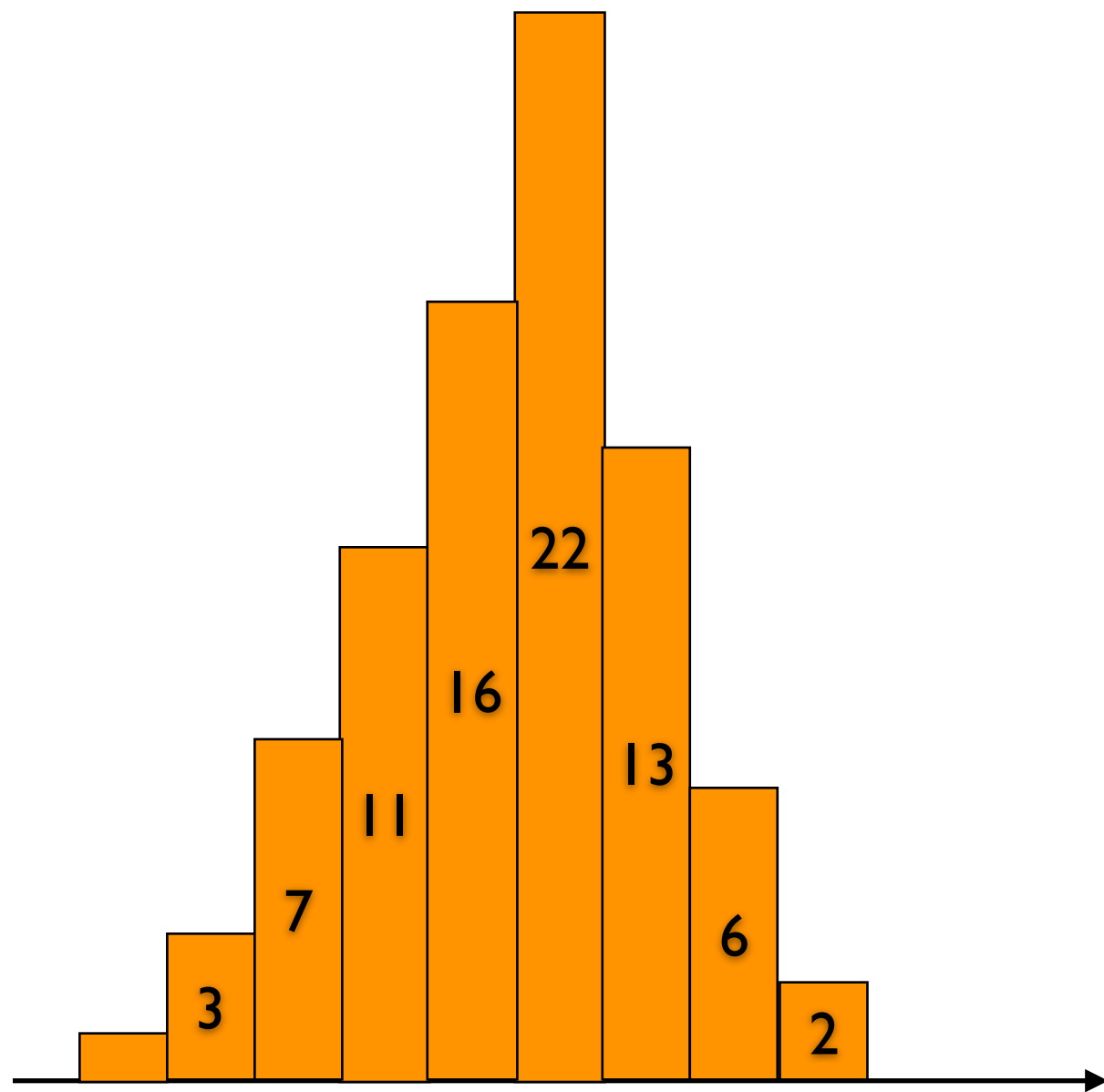


sequence B

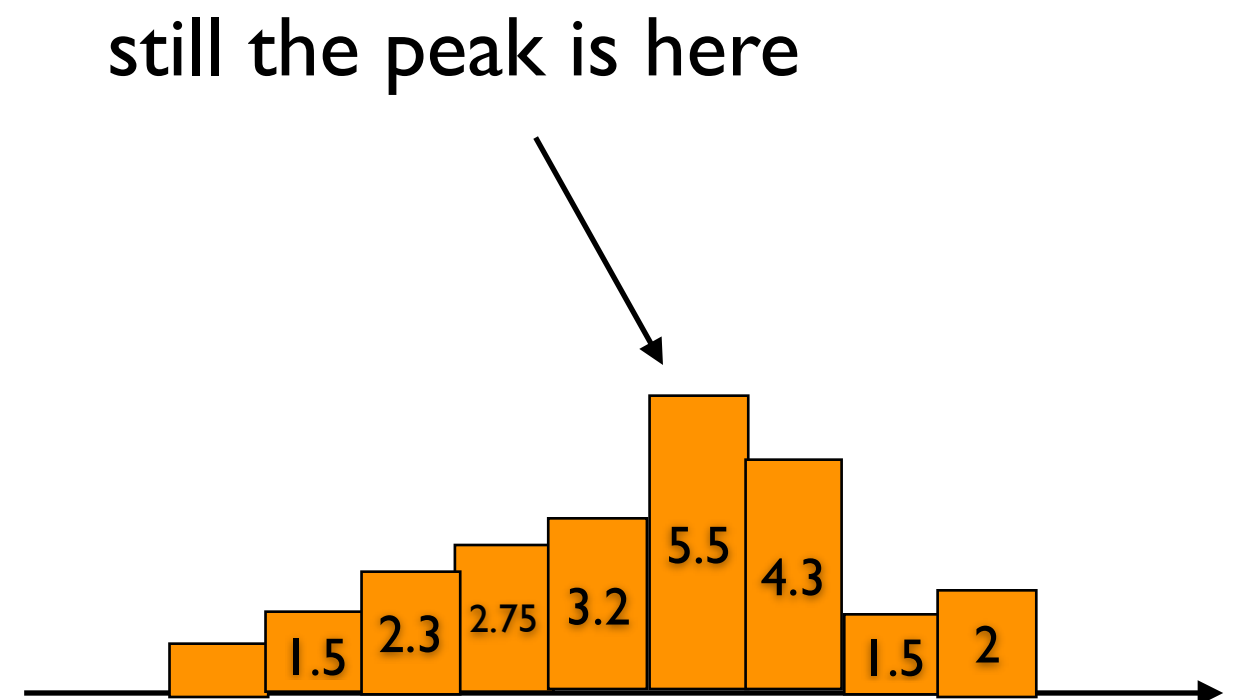


correlation

Let us divide by the number of products we summed

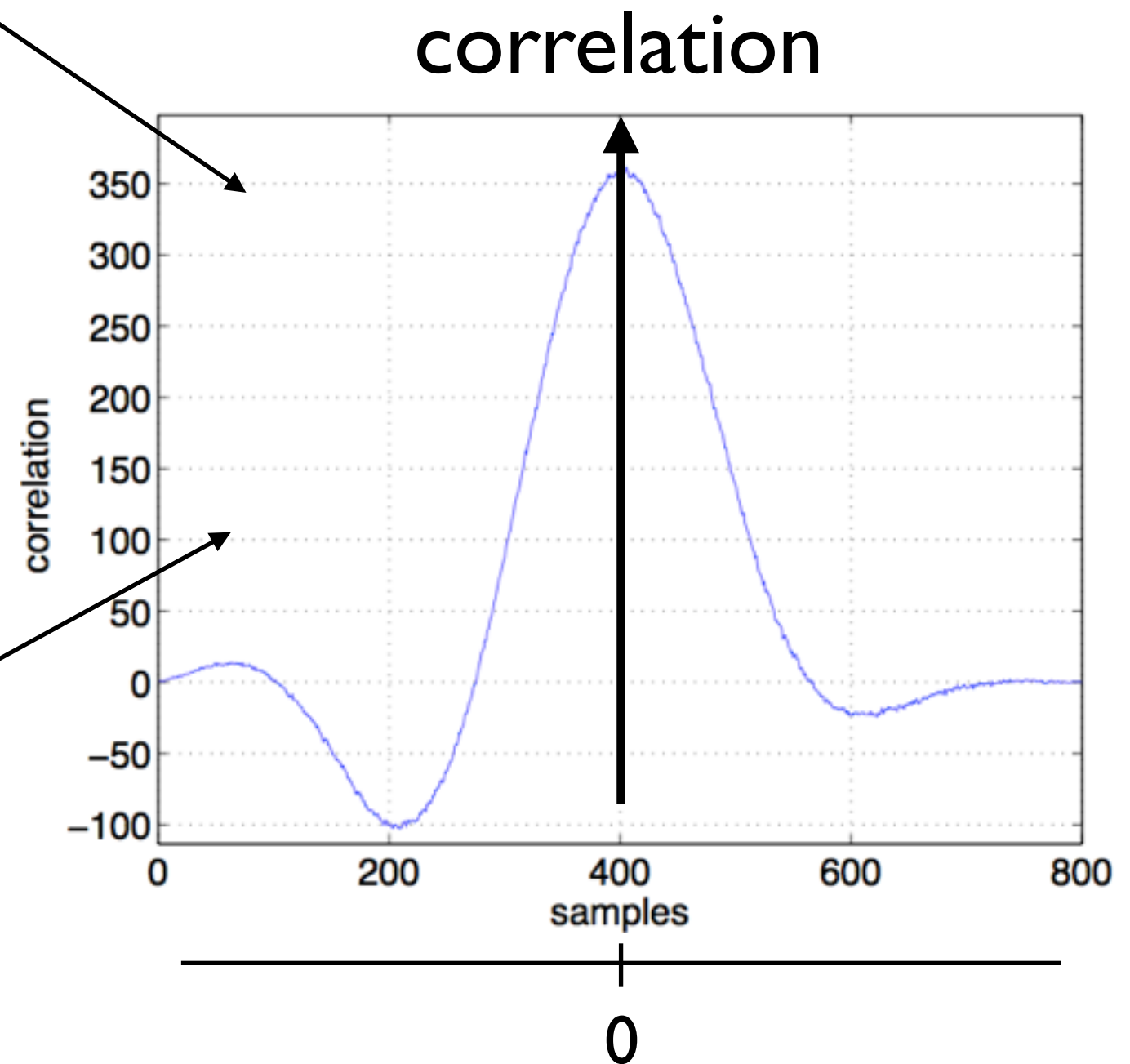
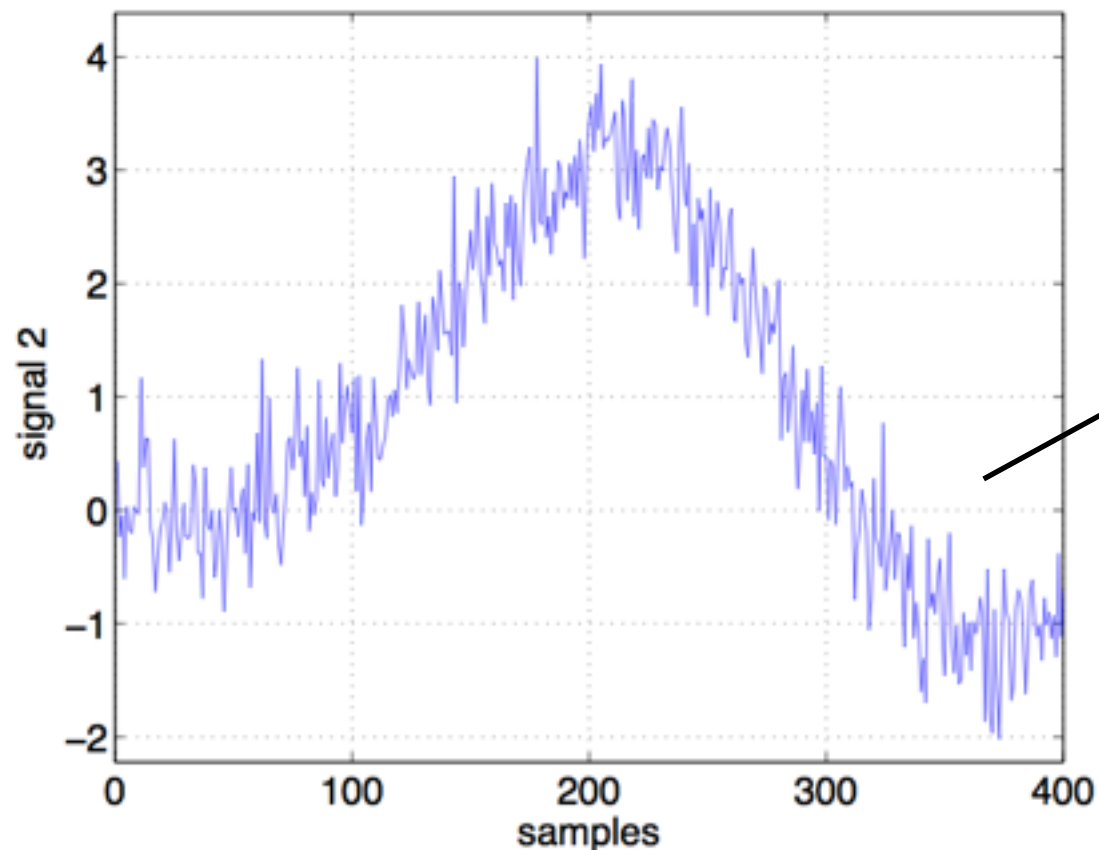
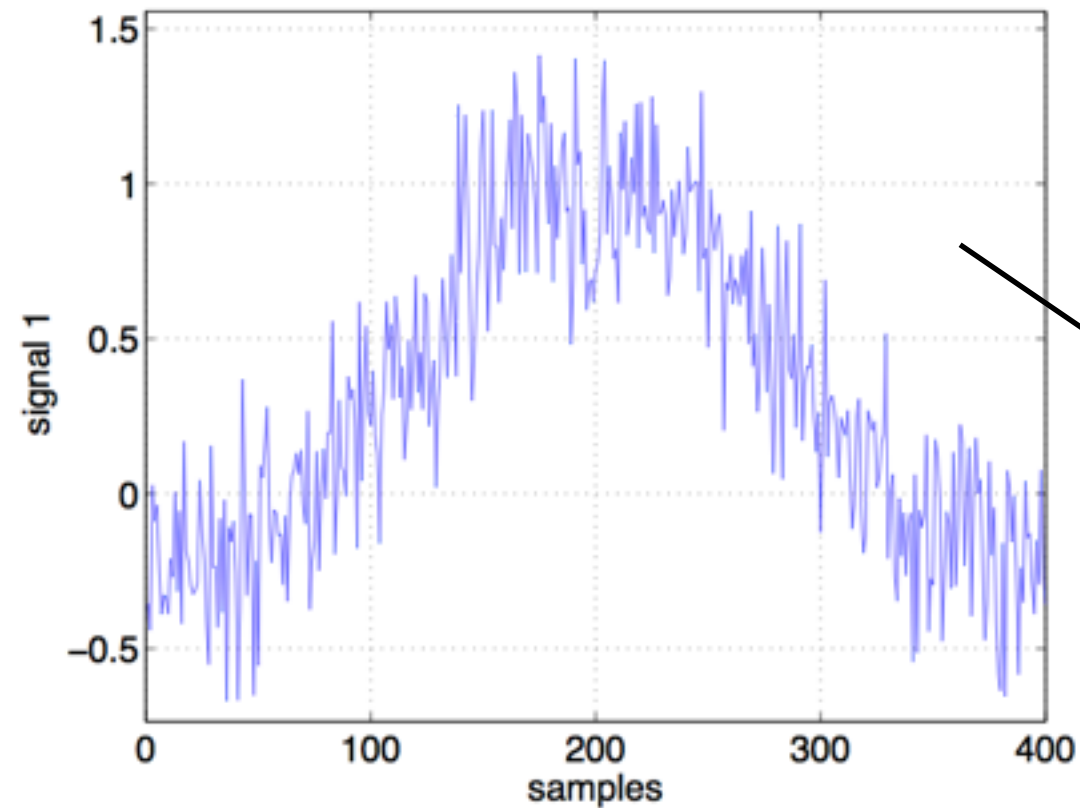


correlation

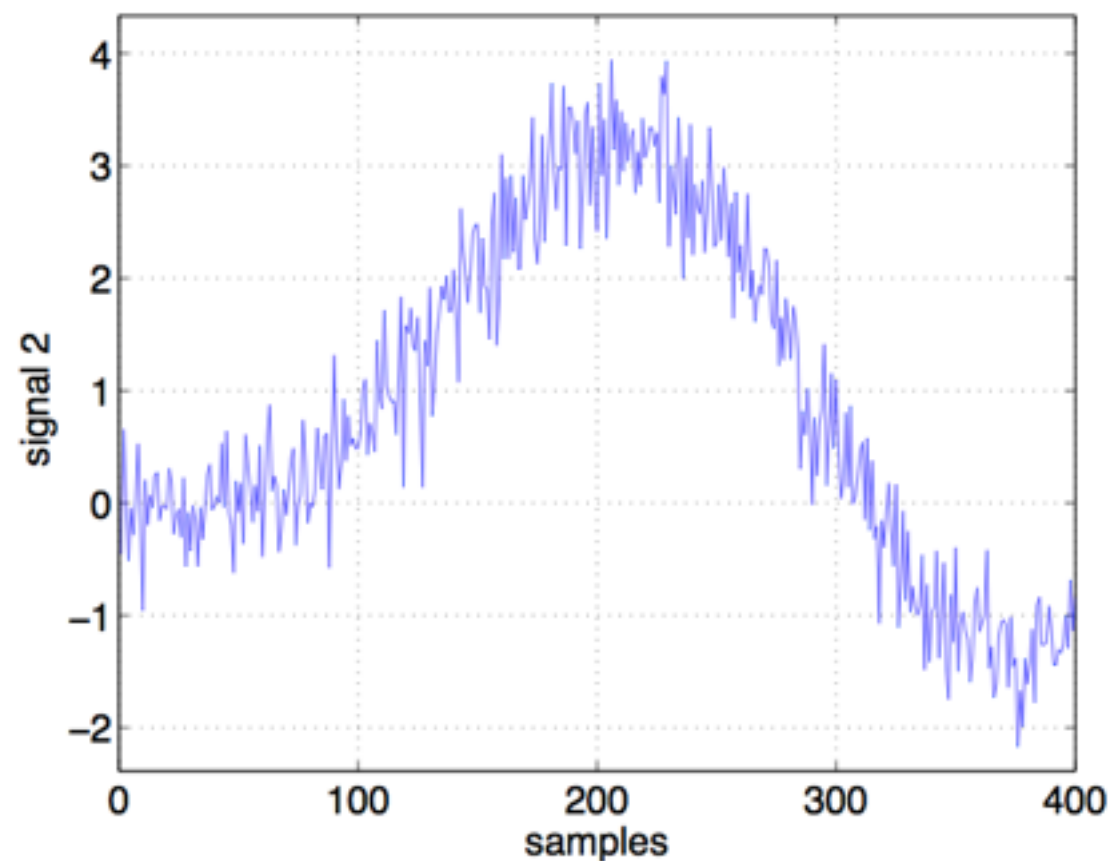
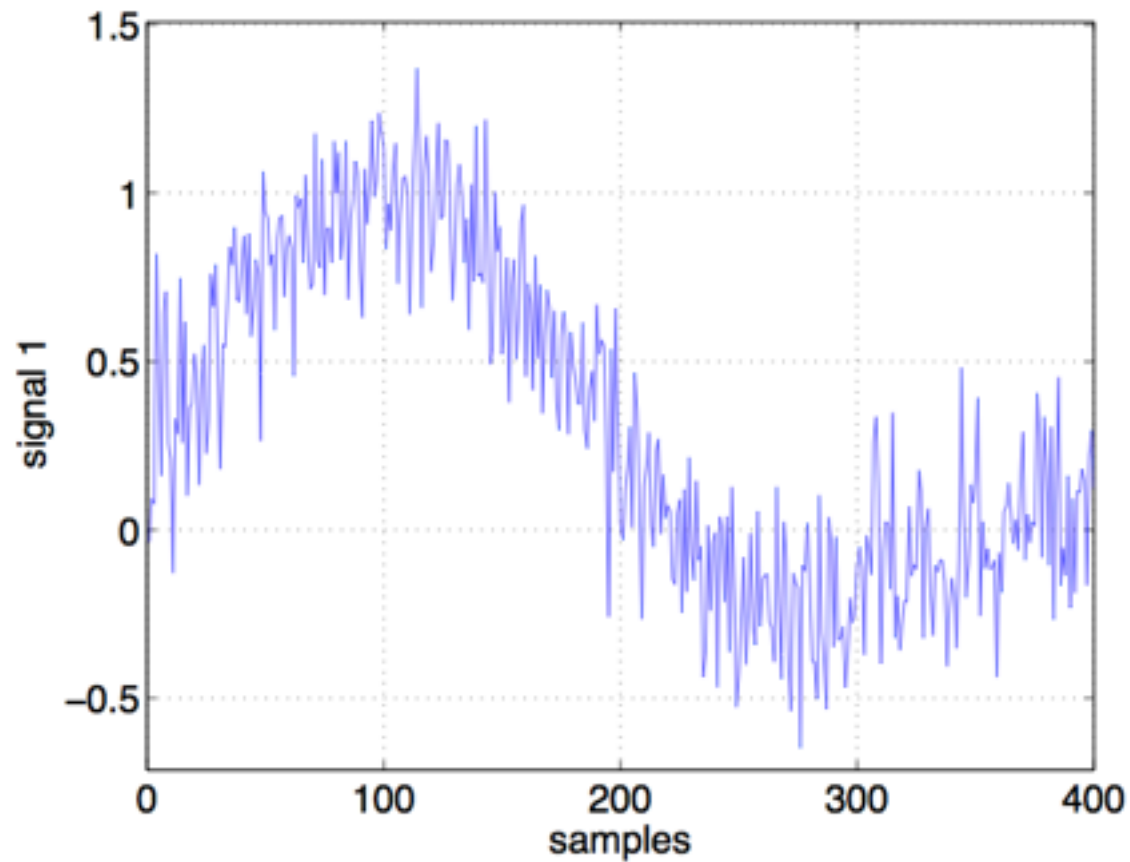


correlation

The peak in the correlation tells us where the signals are mostly similar

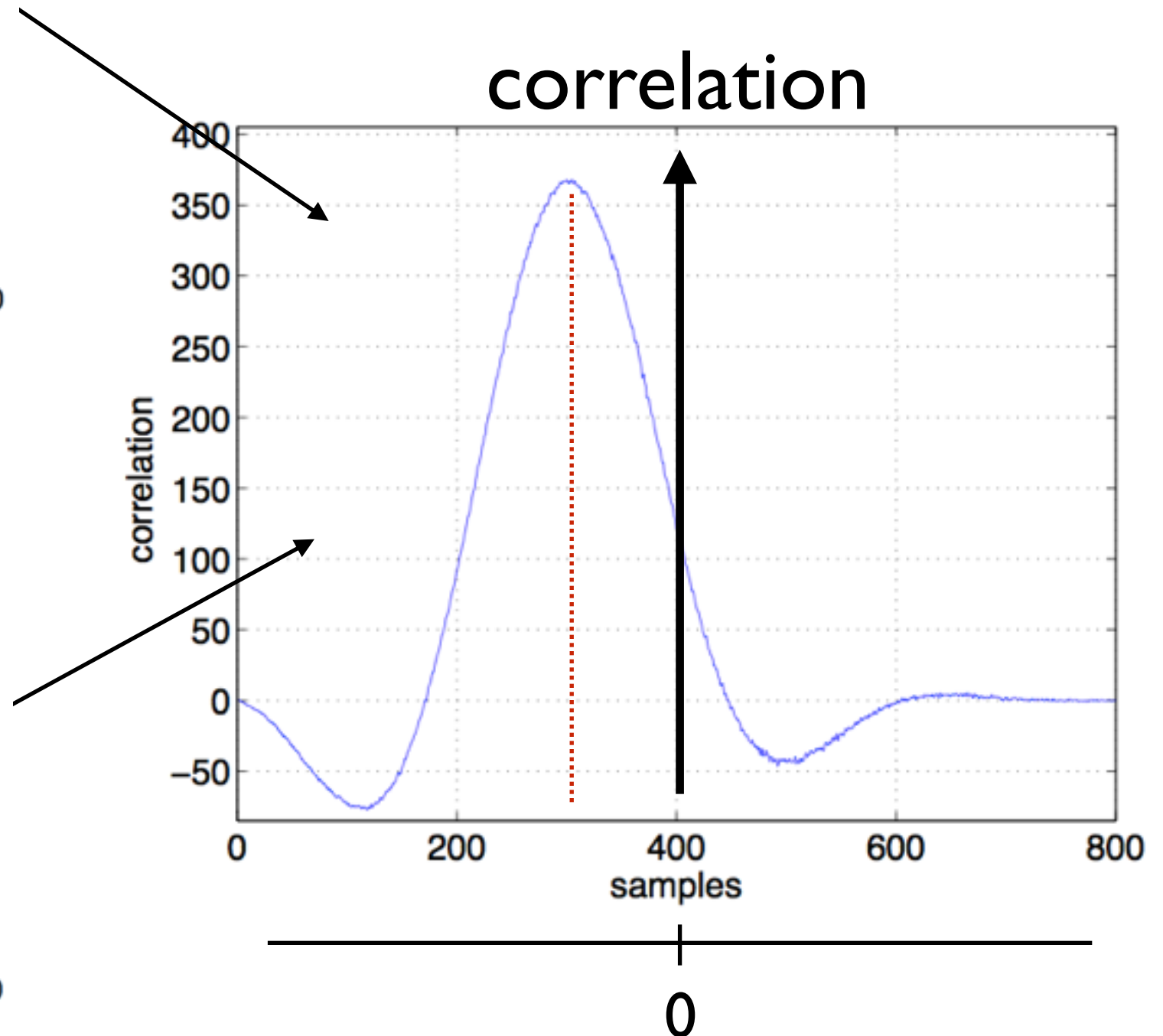


if the first signal moves “to the left”...

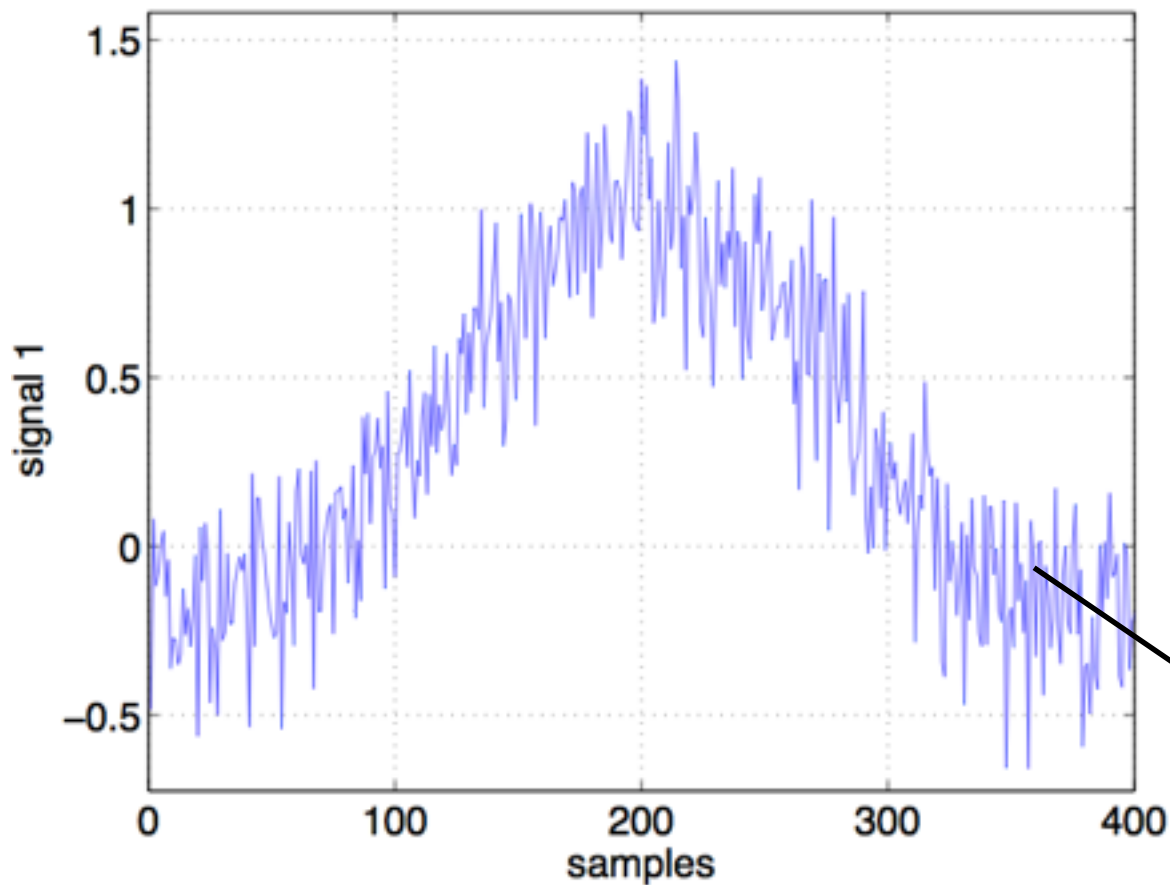


...the condition when the signal will be mostly similar will occur earlier

correlation

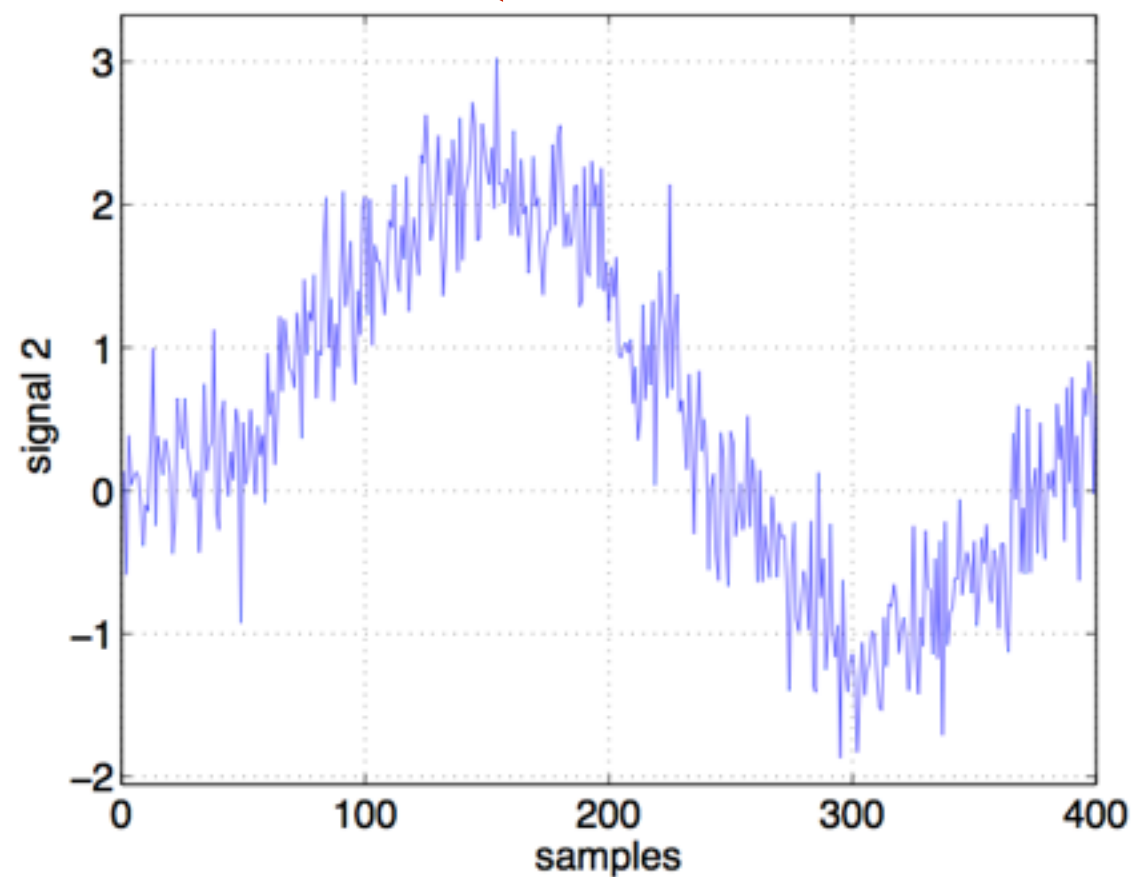




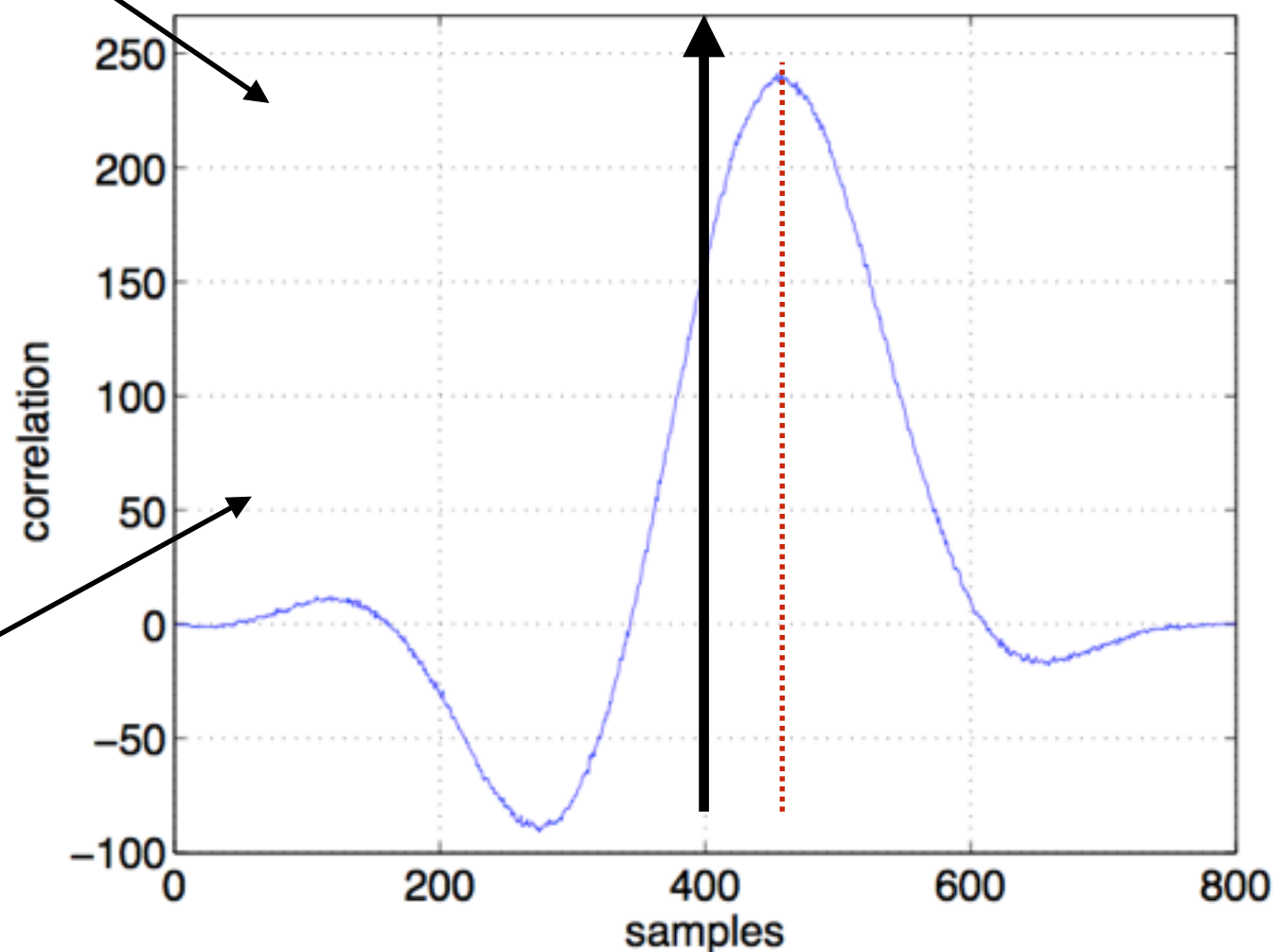


...the condition when the signal will be mostly similar will occur later

if the second signal moves “to the left”...

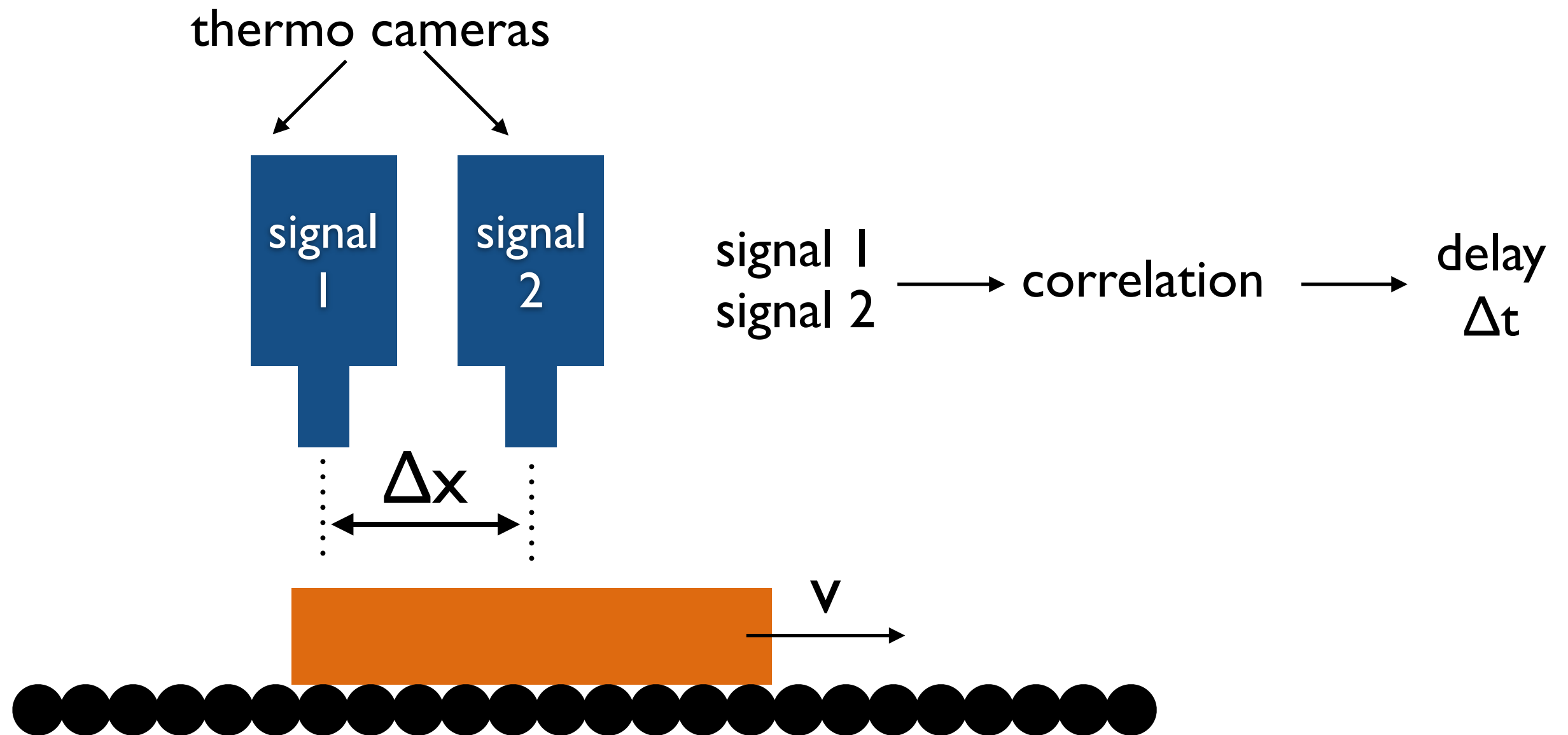


correlation



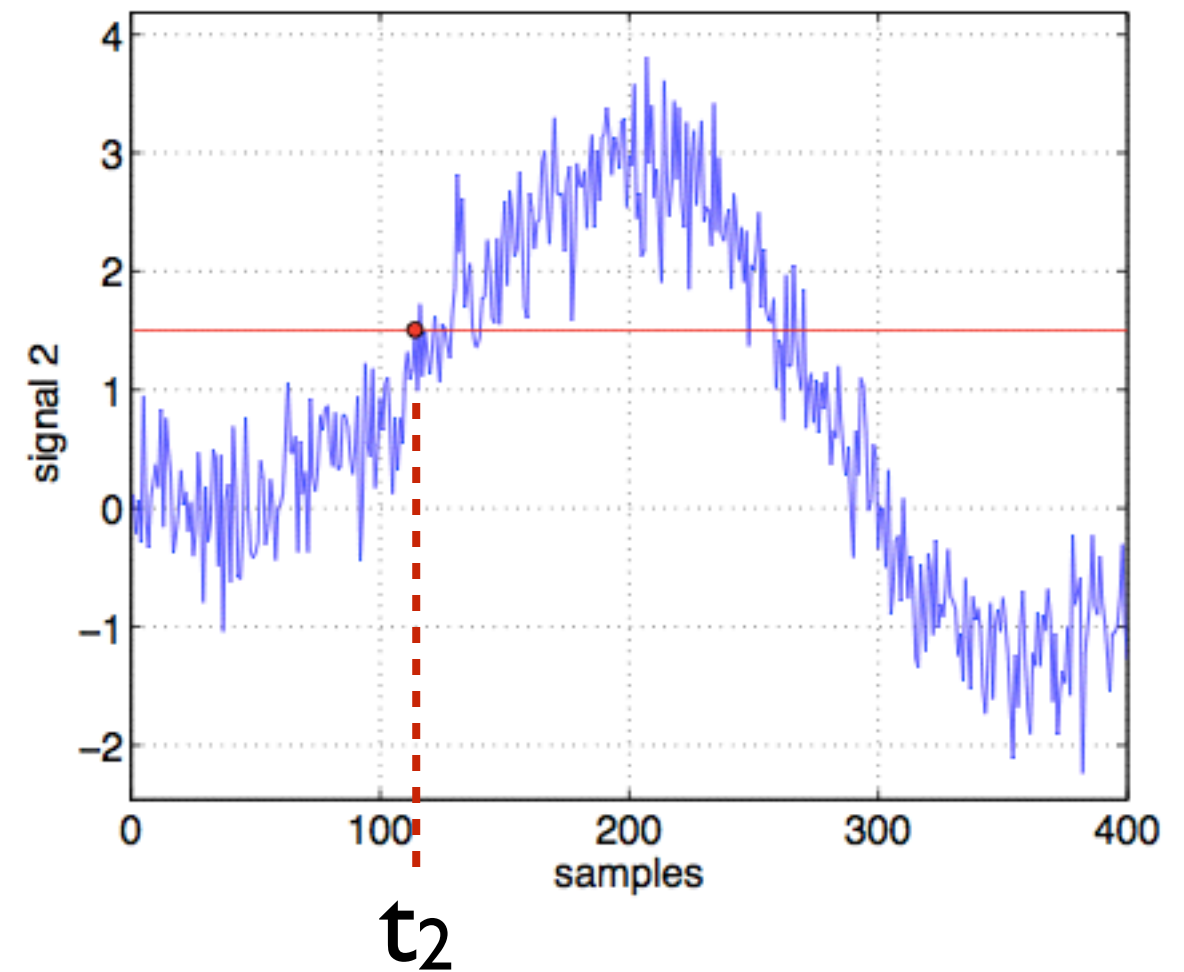
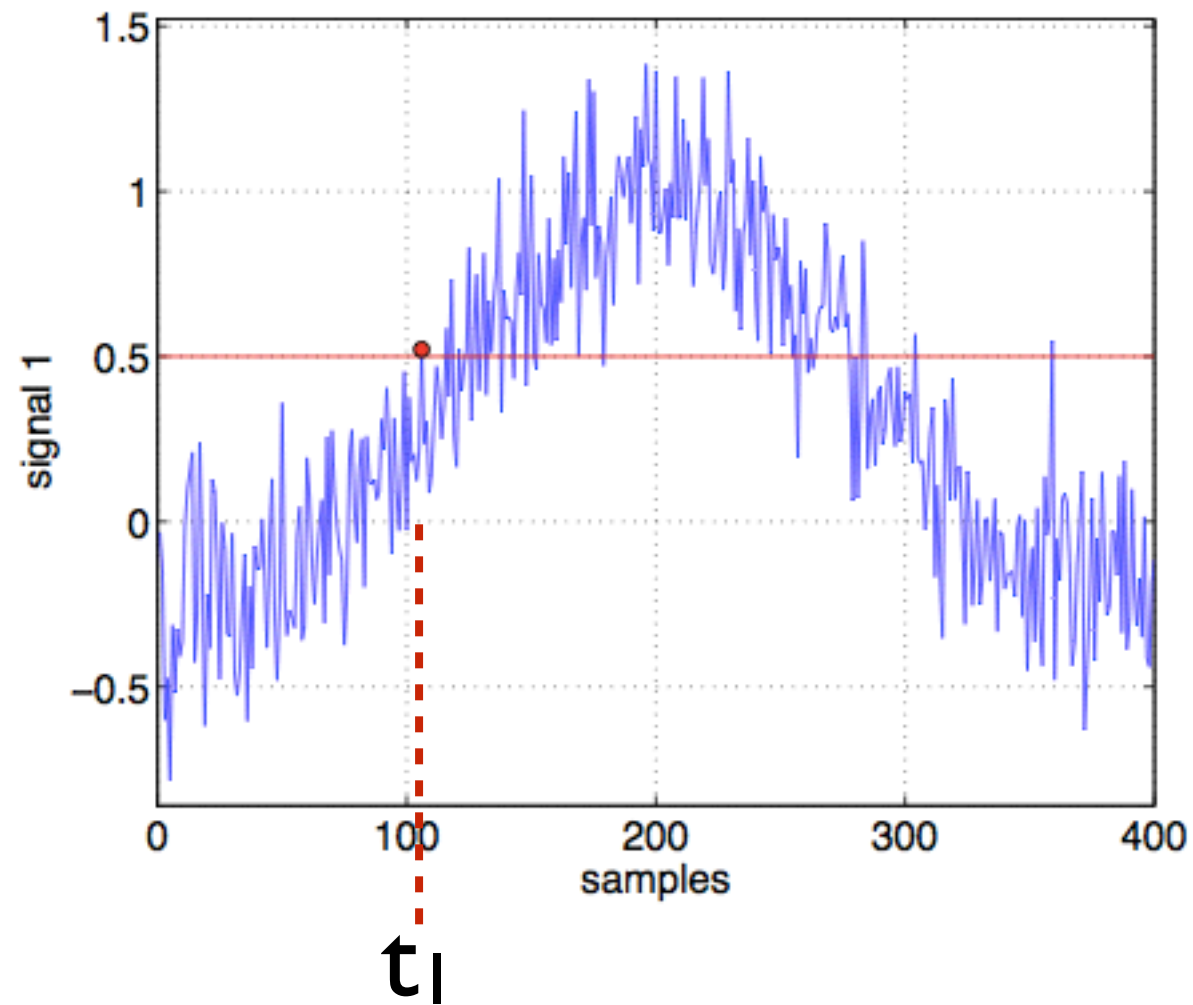
0

# How to use this principle to measure velocity



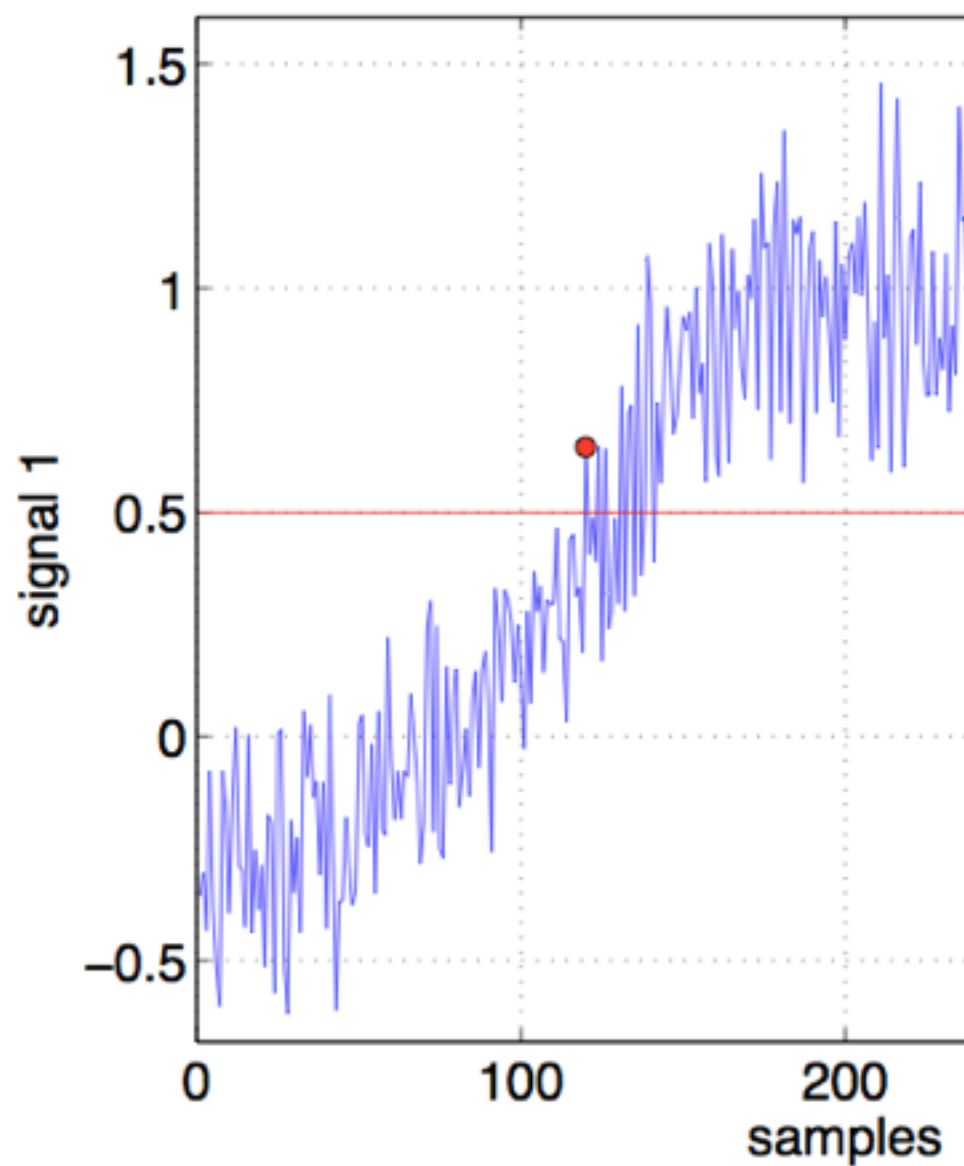
$$v = \Delta x / \Delta t$$

why not using a threshold?

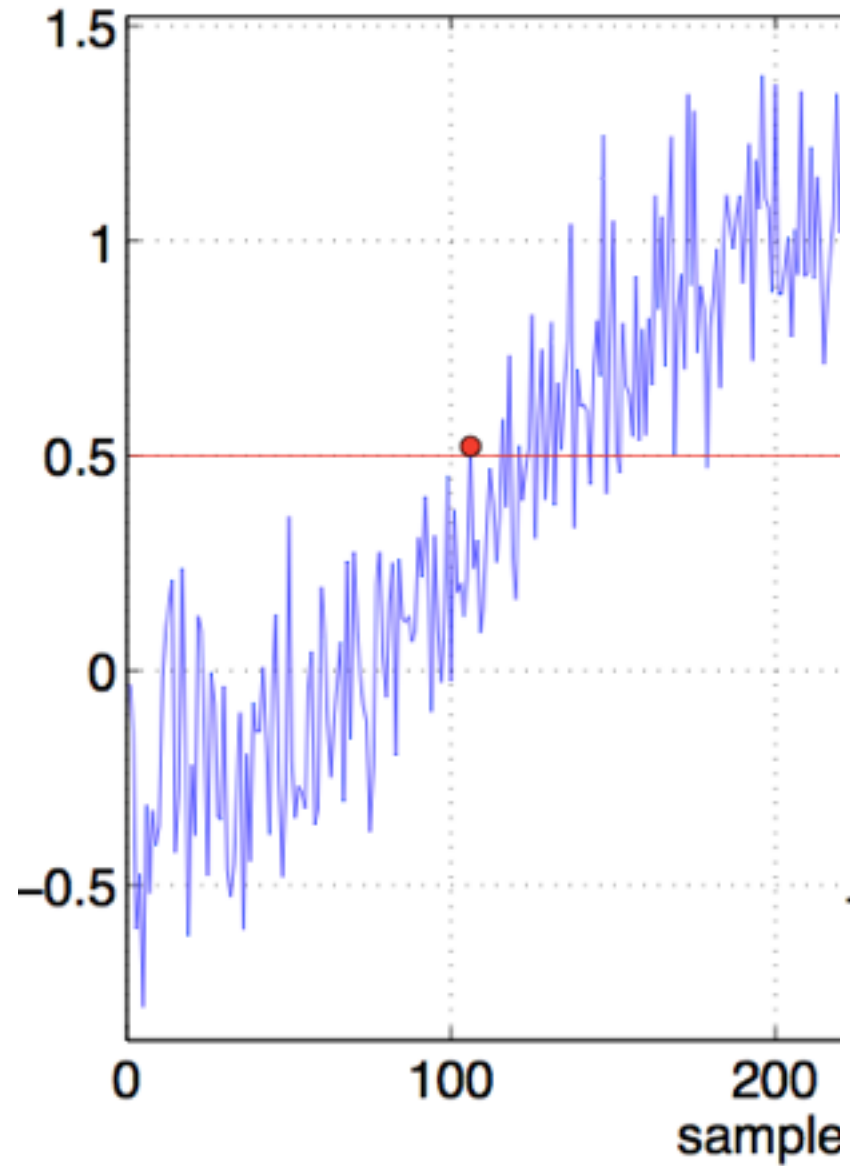


$$\Delta t = t_1 - t_2$$

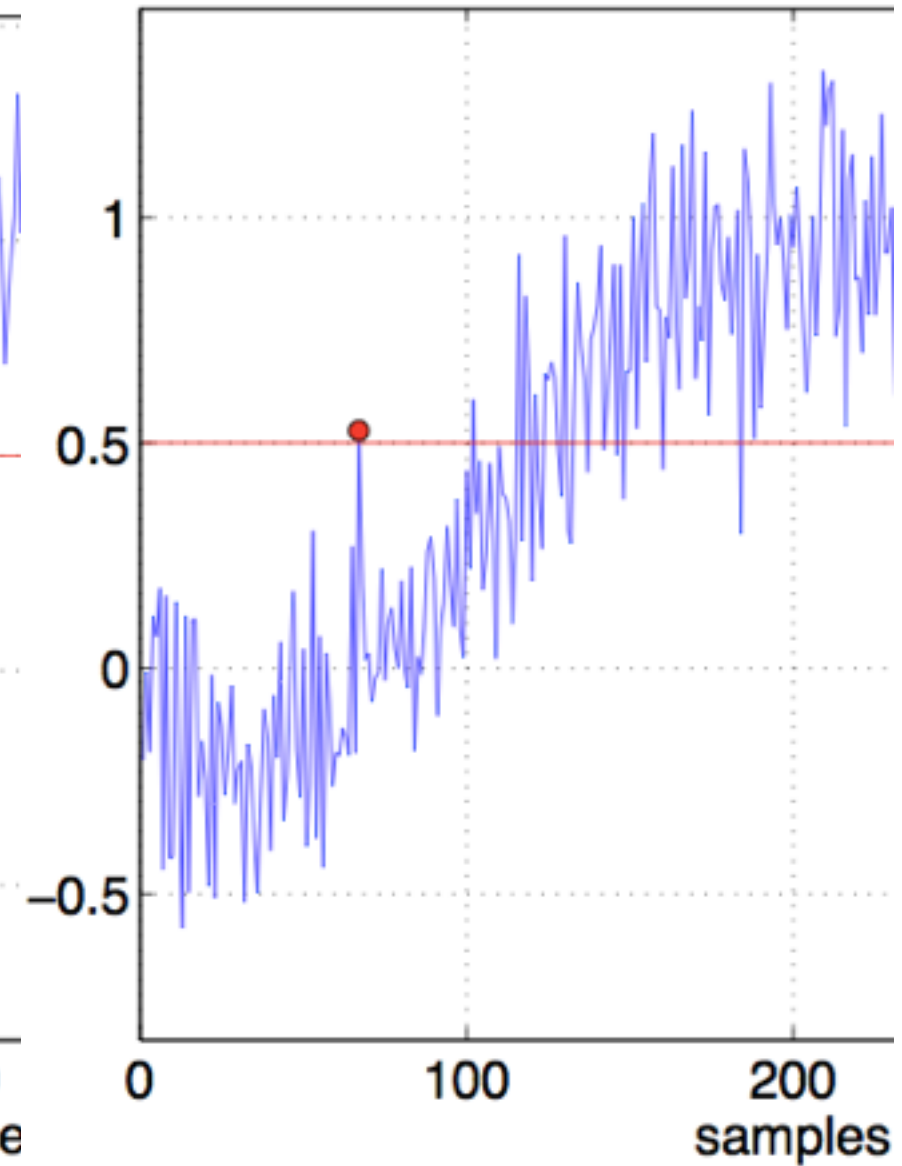
The noise makes this system highly unreliable



120

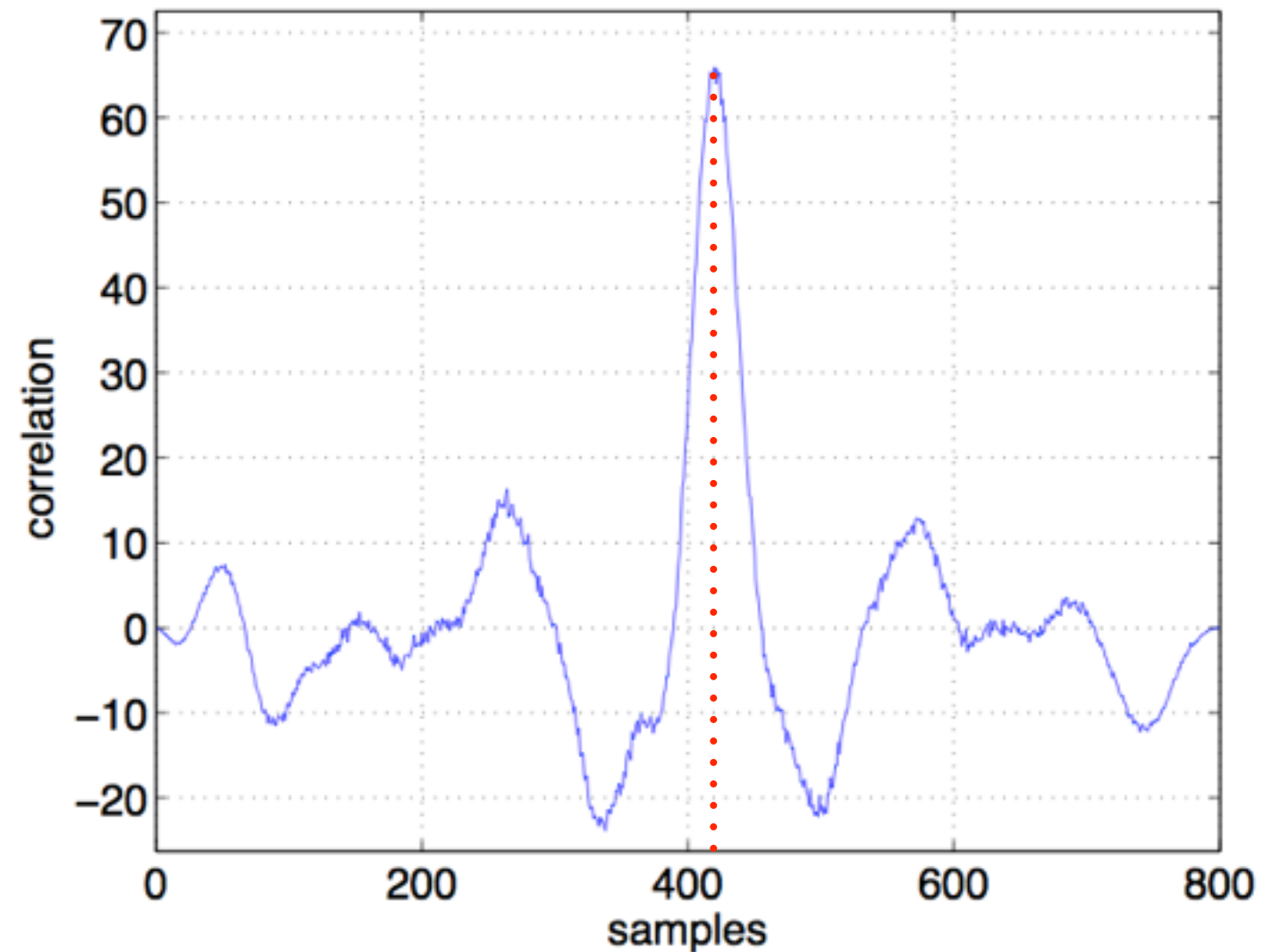
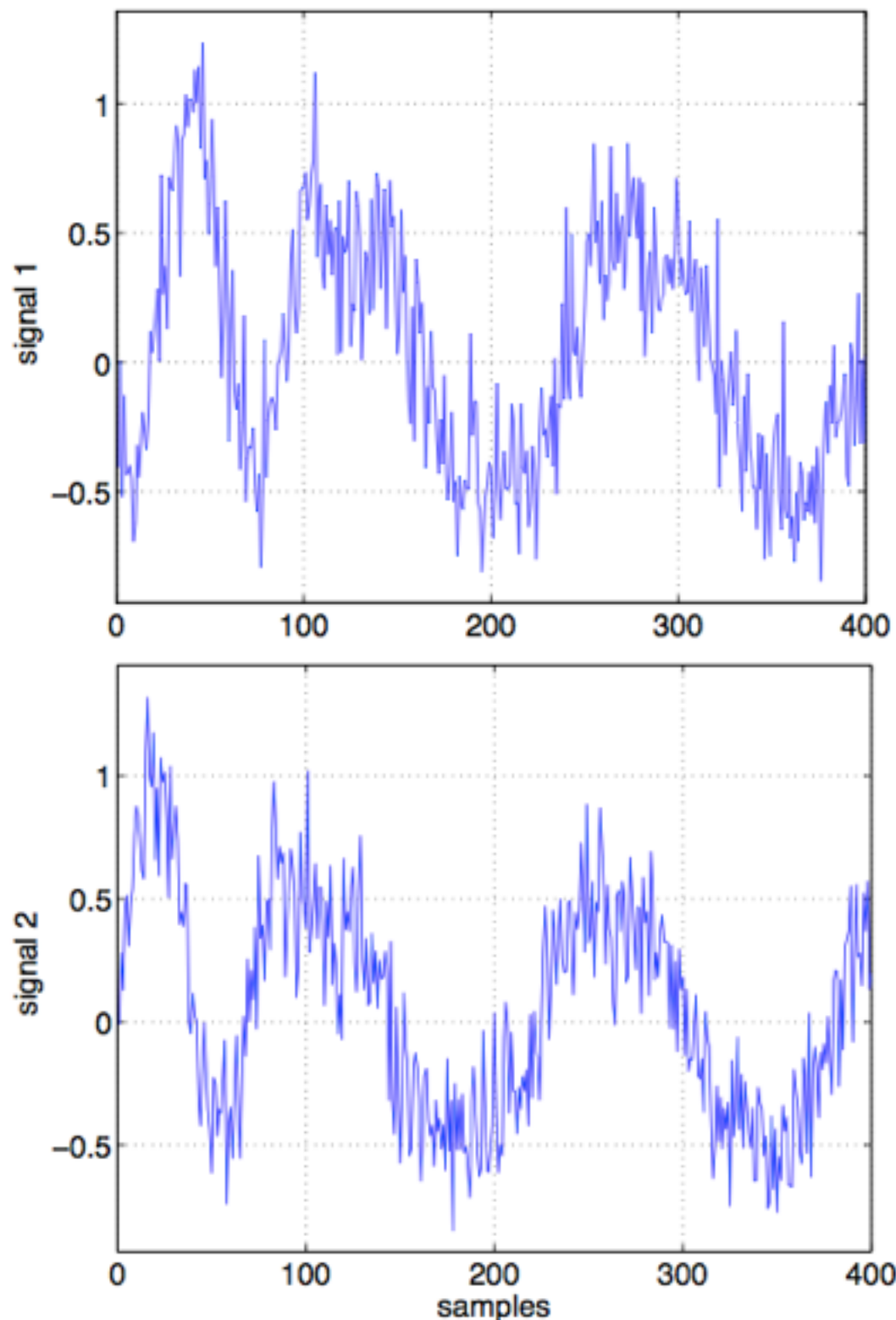


108



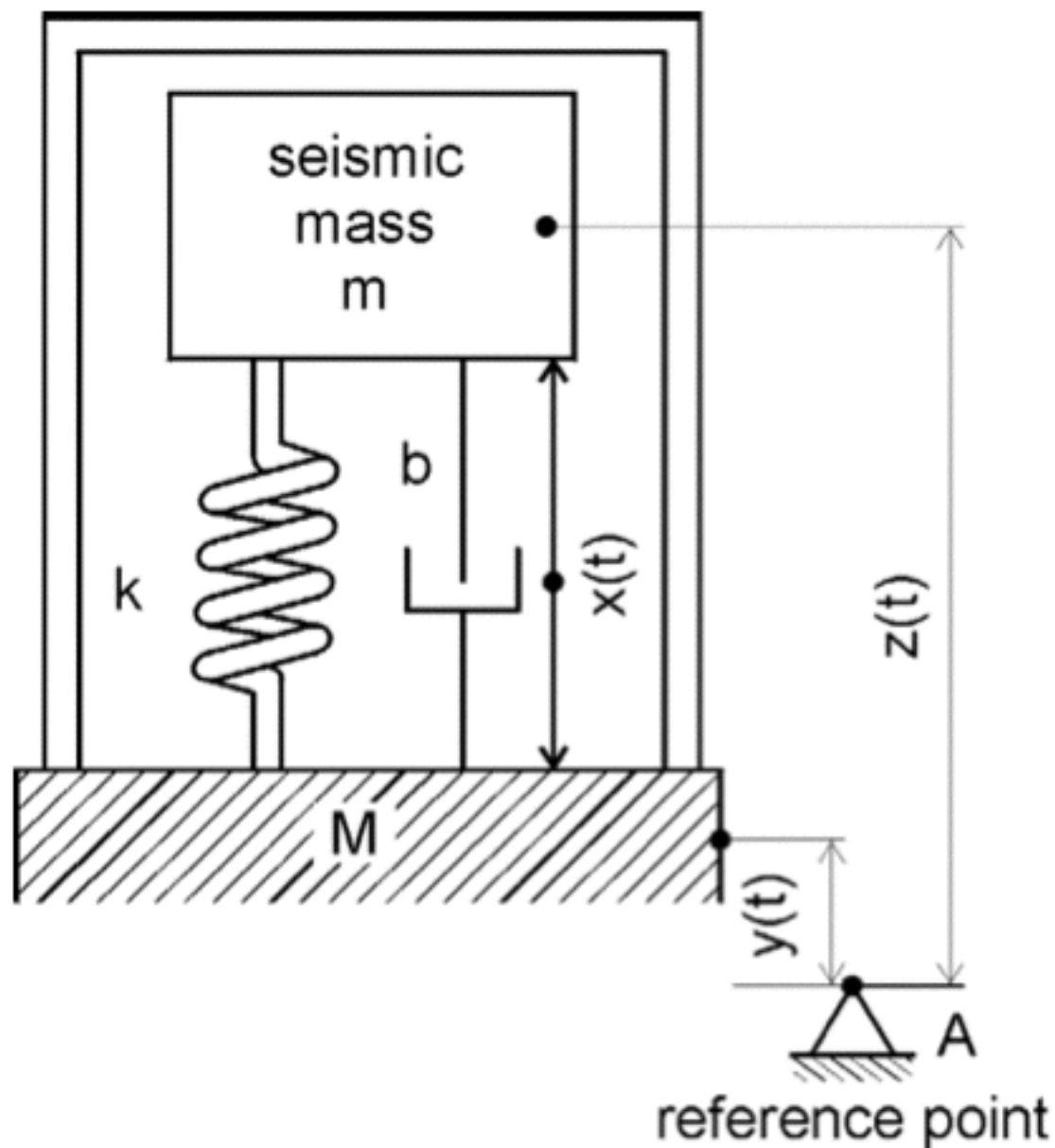
64

Signals do not necessarily need to have a peak.  
Correlation tells us when they are mostly similar  
whatever are the signals.





# Absolute sensors of vibrations



$m$  - mass

$k$  - stiffness (spring constant)

$b$  - viscous friction coefficient

Equation of motion of mass-spring system

$$m \frac{d^2 z}{dt^2} + b \frac{dx}{dt} + kx = 0$$

$$z(t) = x(t) + y(t)$$

$$m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = -m \frac{d^2 y}{dt^2}$$

inertial  
force

spring force

damping

presumption:

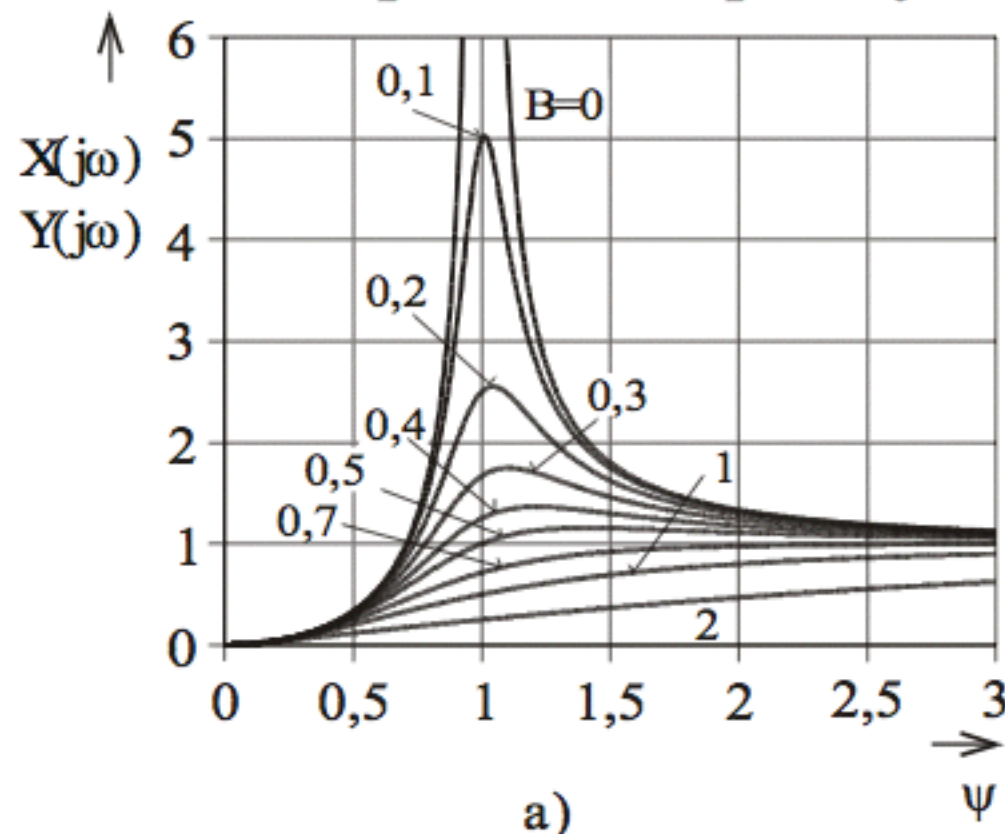
$$y(t) = Y(j\omega)e^{j\omega t}$$

Solution:

$$x(t) = X(j\omega)e^{j(\omega t - \varphi)}$$

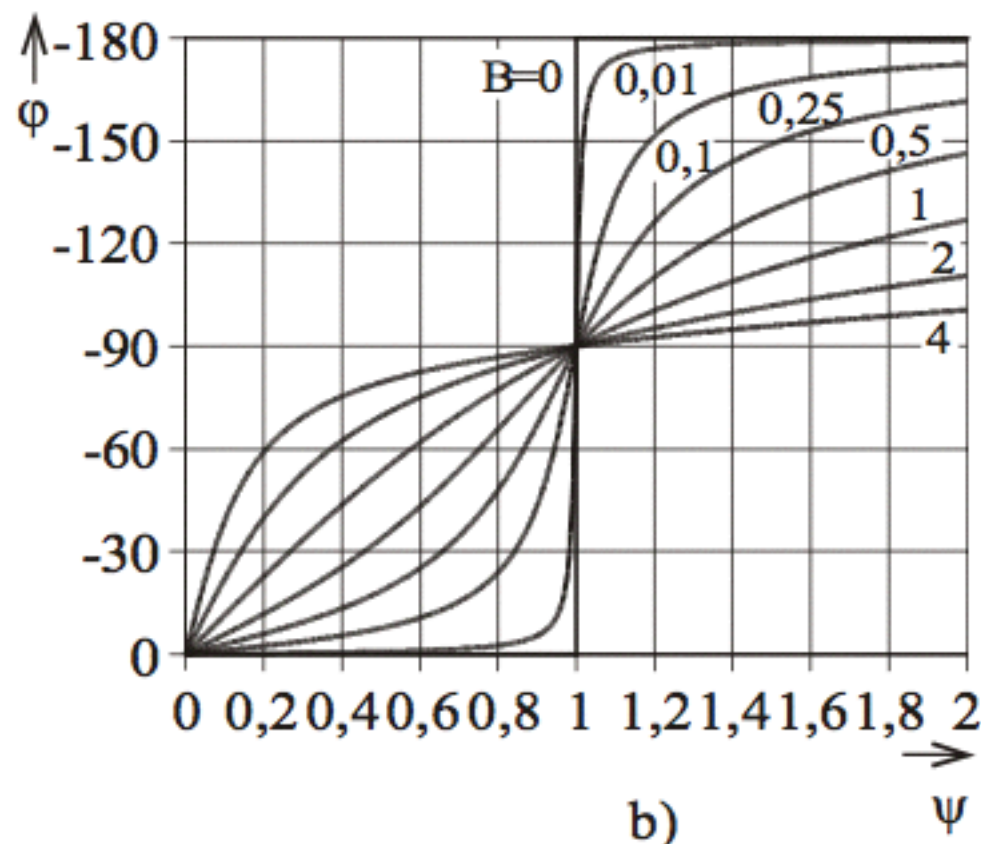


## Amplitude frequency response:



$$\left| \frac{X(j\omega)}{Y(j\omega)} \right| = \frac{\Psi^2}{\sqrt{(1 - \Psi^2)^2 + (2B\Psi)^2}}$$

## Phase frequency response



$$\varphi = \text{arctg} \frac{-2B\Psi}{1 - \Psi^2}$$

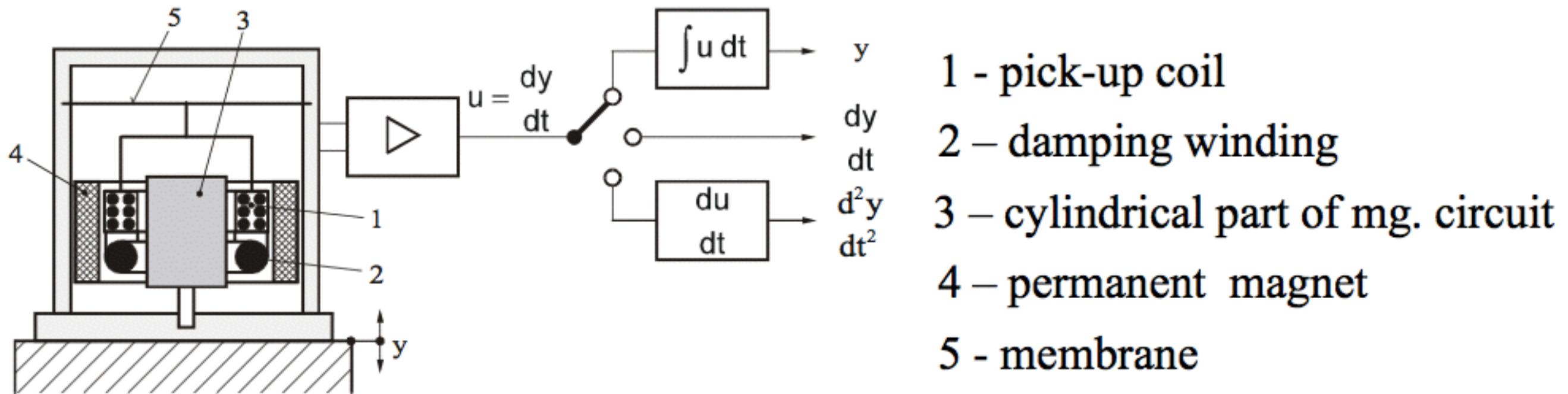
where:  $\Psi = \frac{\omega}{\omega_0}$  - normalised frequency  
(referenced to resonance freq.)

$B = \frac{b}{b_{kr}}$  - damping ratio

$b_{kr} = 2m\omega_0$  - critical damping ratio

$\omega_0 = \sqrt{\frac{k}{m}}$  - resonant frequency

# Electrodynamic sensor of vibration



- seismic mass = mass of coil 1 + mass of winding 2
- viscous damping – due to currents induced in winding 2
- induced voltage  $u = \frac{d\Phi}{dt}$  proportional to the *velocity* of coil

☺ - universality

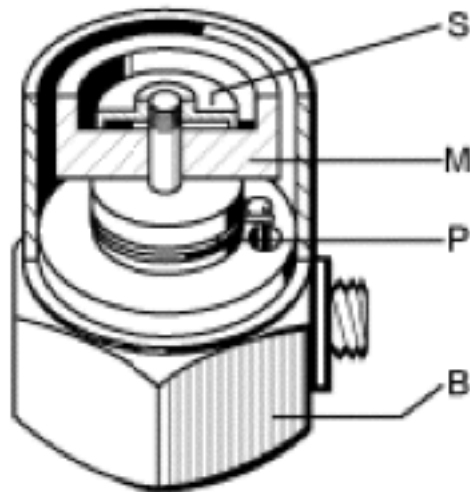
## GEOPHONE

- application: vibrations of machines, buildings, occupancy detection

☺ - cheap  $f_r = 1 \dots 100 \text{ Hz}$ ,  $m = 20\text{g} \dots 5 \text{ kg}$

# Implementation

## Classical (compression, centre mounted)



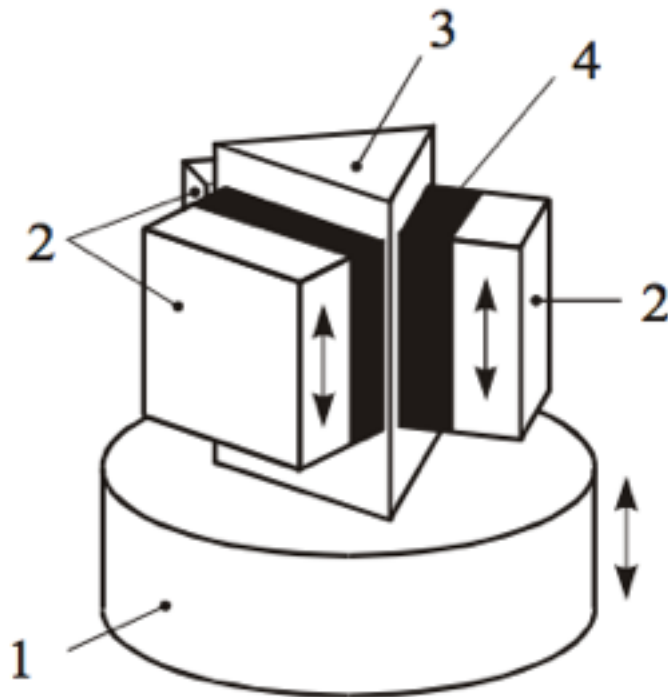
M = seismic mass

P = piezoelectric element

B = base

S = spring

## Shear deformation



1 = base

2 = mass

3 = central stick

4 = piezoelectric element

Bruel -Kjaer

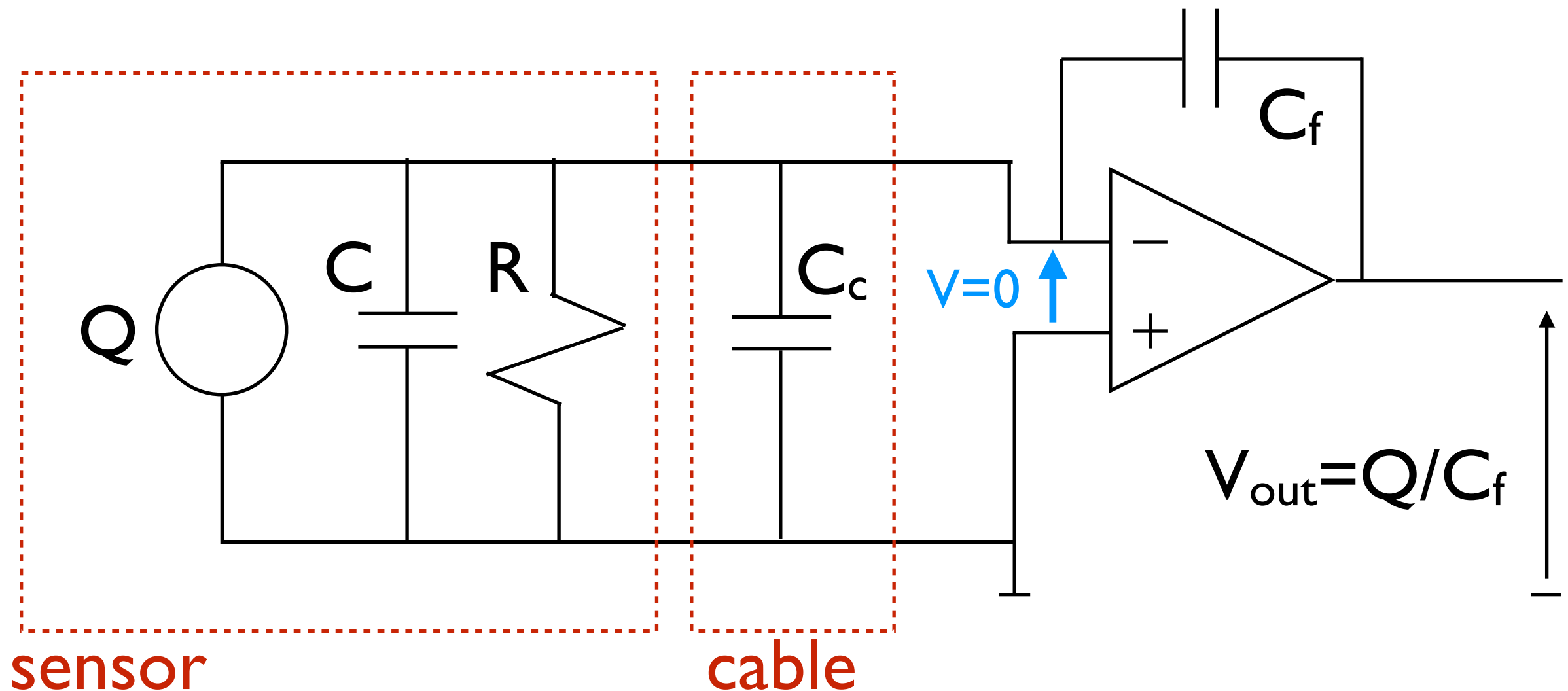


less sensitive to deformation of  
bottom of sensor case, thermal  
dilatation, acoustic effects

# Signal conditioning of piezoelectric sensors

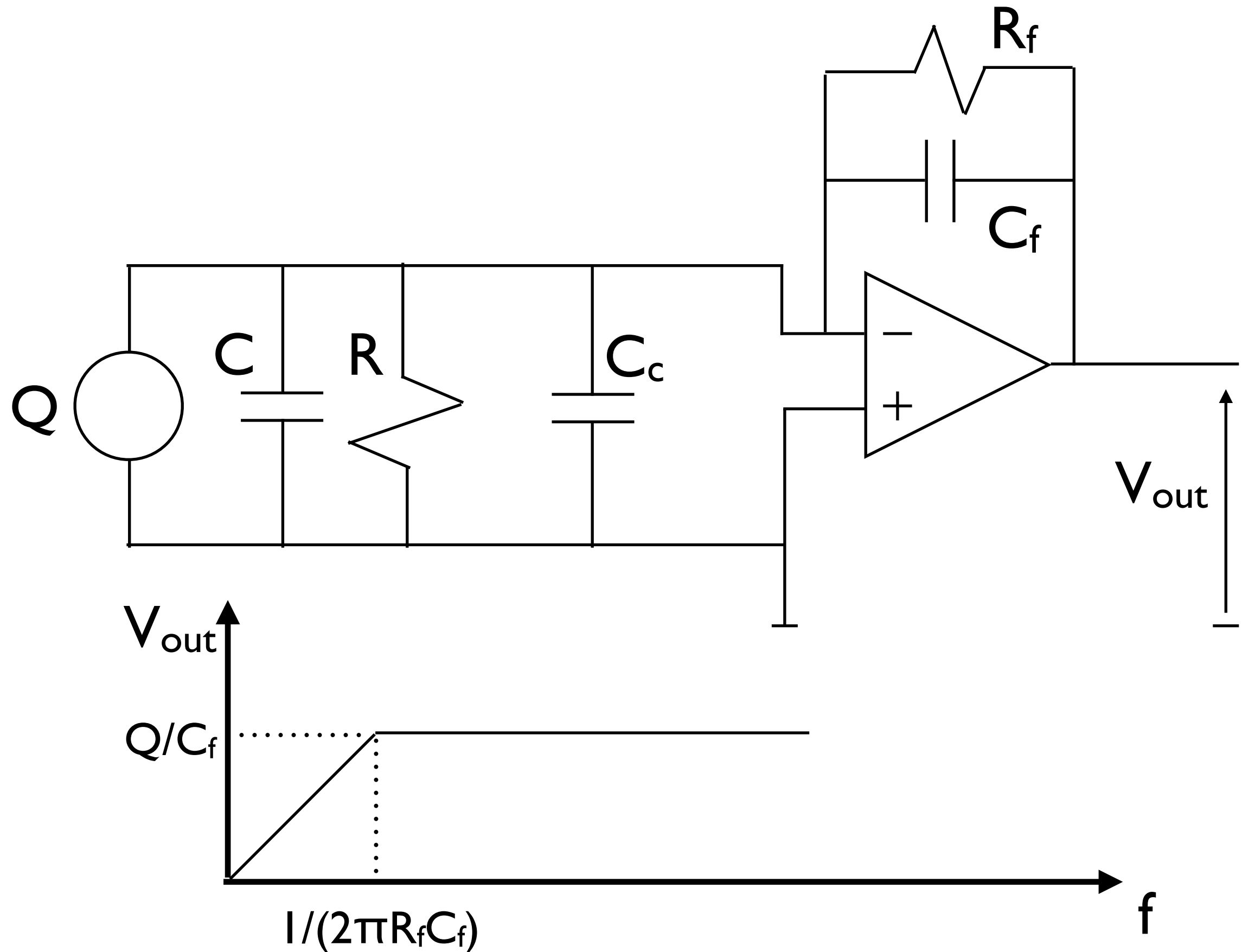
- capacitance of cable affect the voltage if sensor is used in voltage mode amplifier

## CHARGE AMPLIFIER

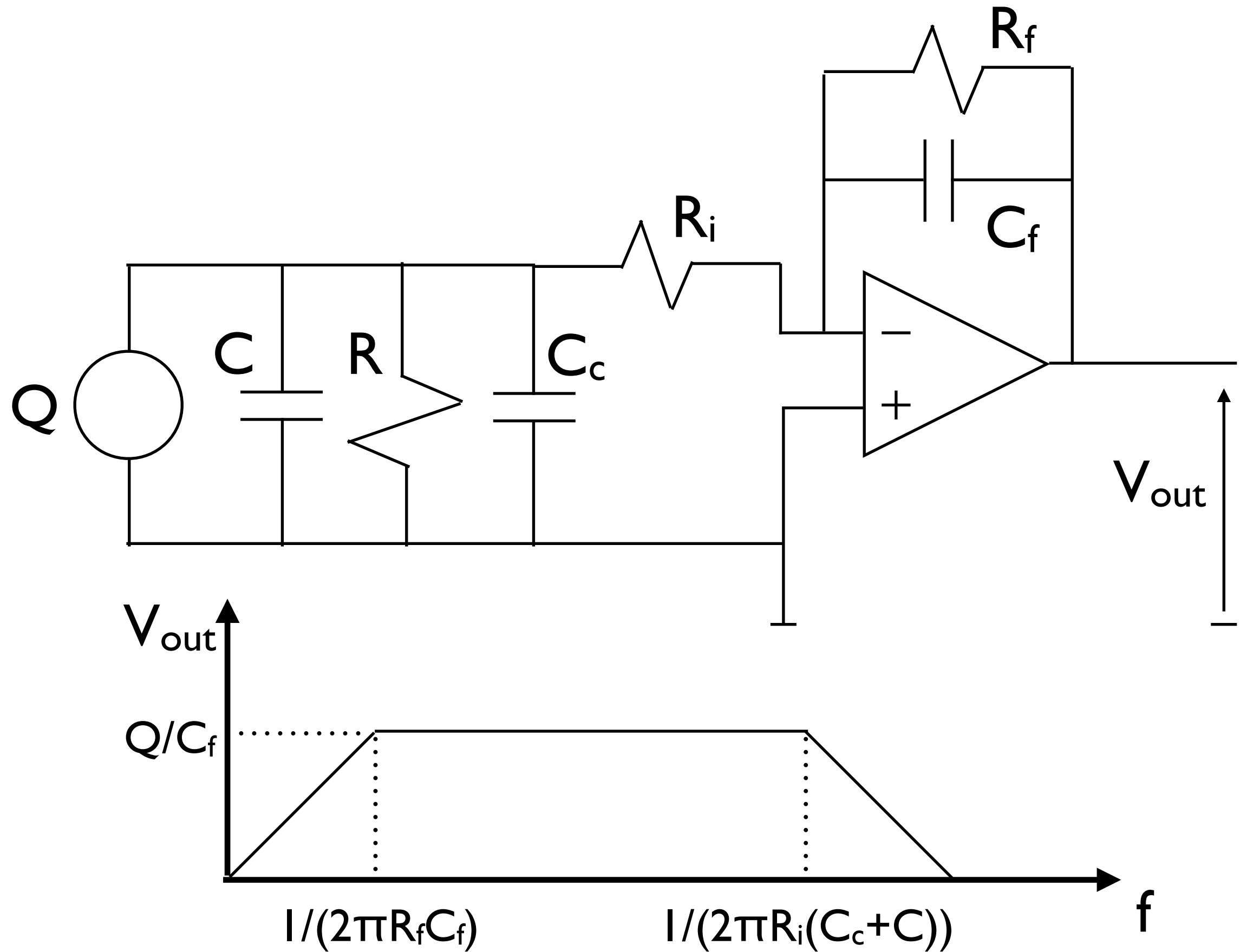


The virtual short circuit grounds the capacities, so that any change in cable's capacity does not affect the output voltage, which depends only on  $Q$  and  $C_f$

To avoid saturation of the amplifier  $R_f$  is required



To limit bandwidth (and avoid resonance)  $R_i$  is required

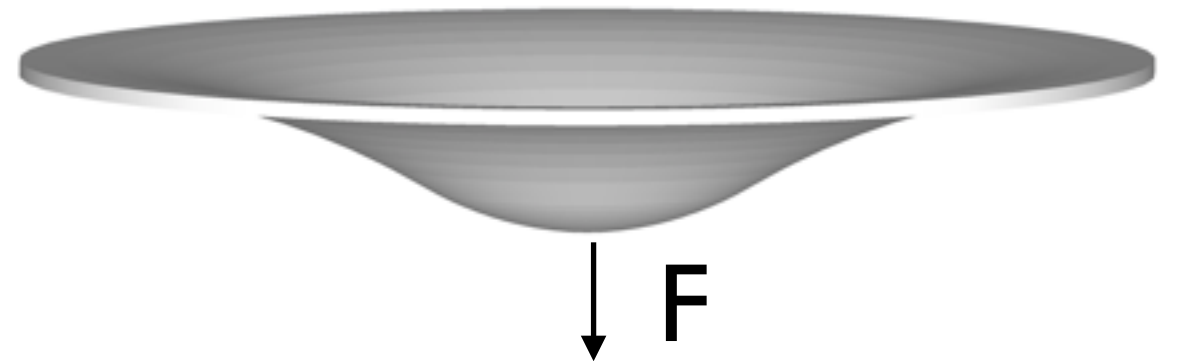
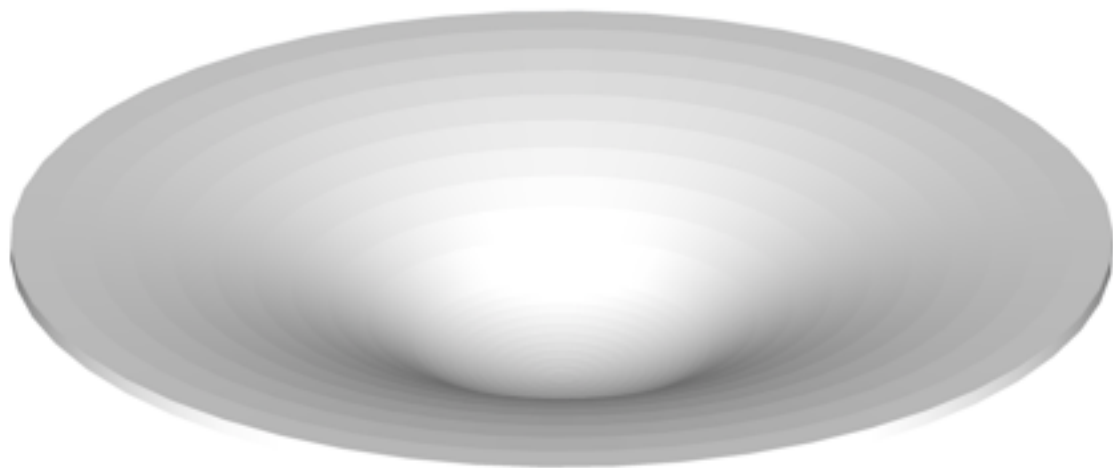




# Capacitive accelerometers

Principle: we measure the force produced by acceleration by means of the deformation produced on a capacitor

## Basic idea



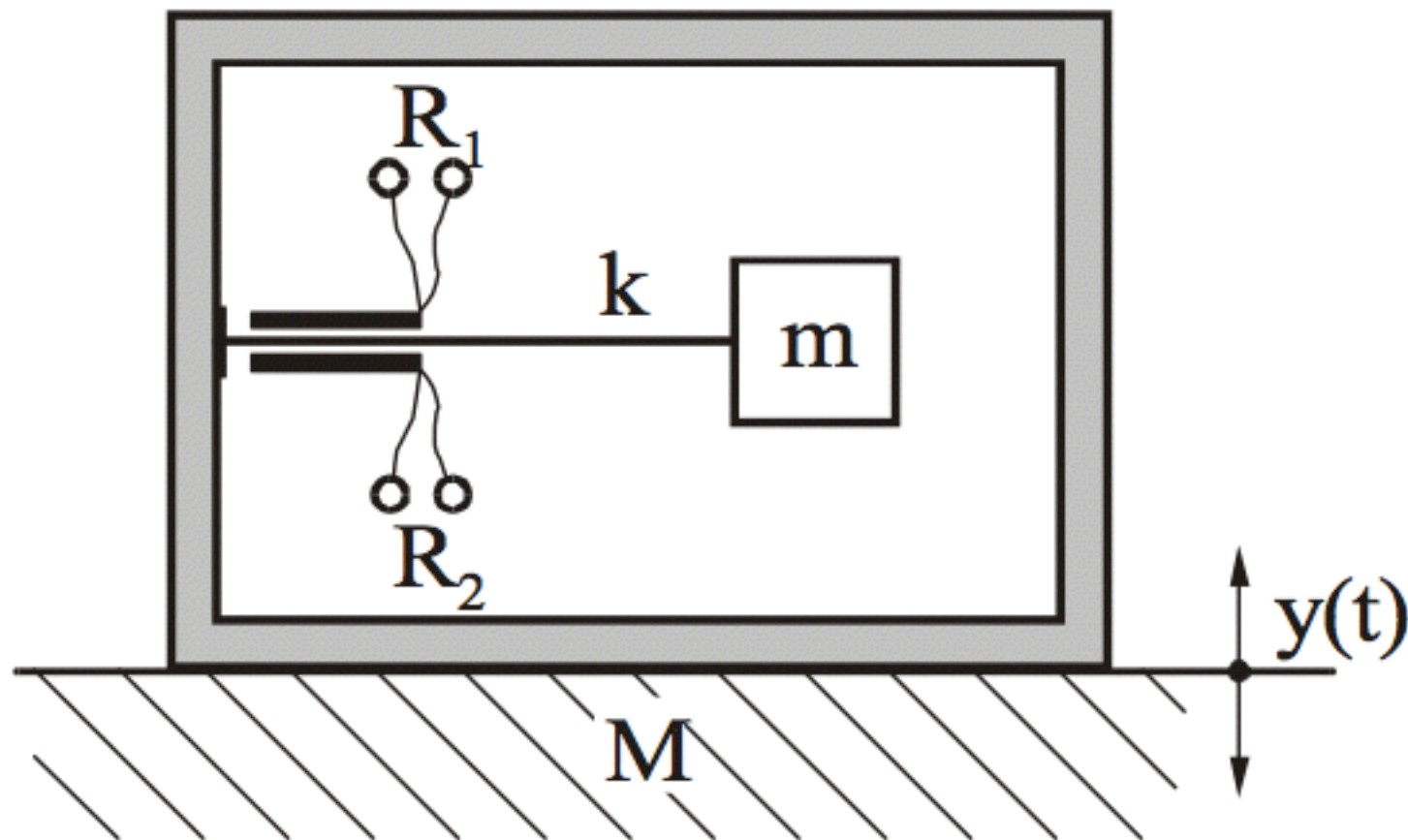
## Actual implementation

thin membrane with  
helical slits and openings

fr= 20 to 15 kHz



# Accelerometer with vibrating beam and strain gages



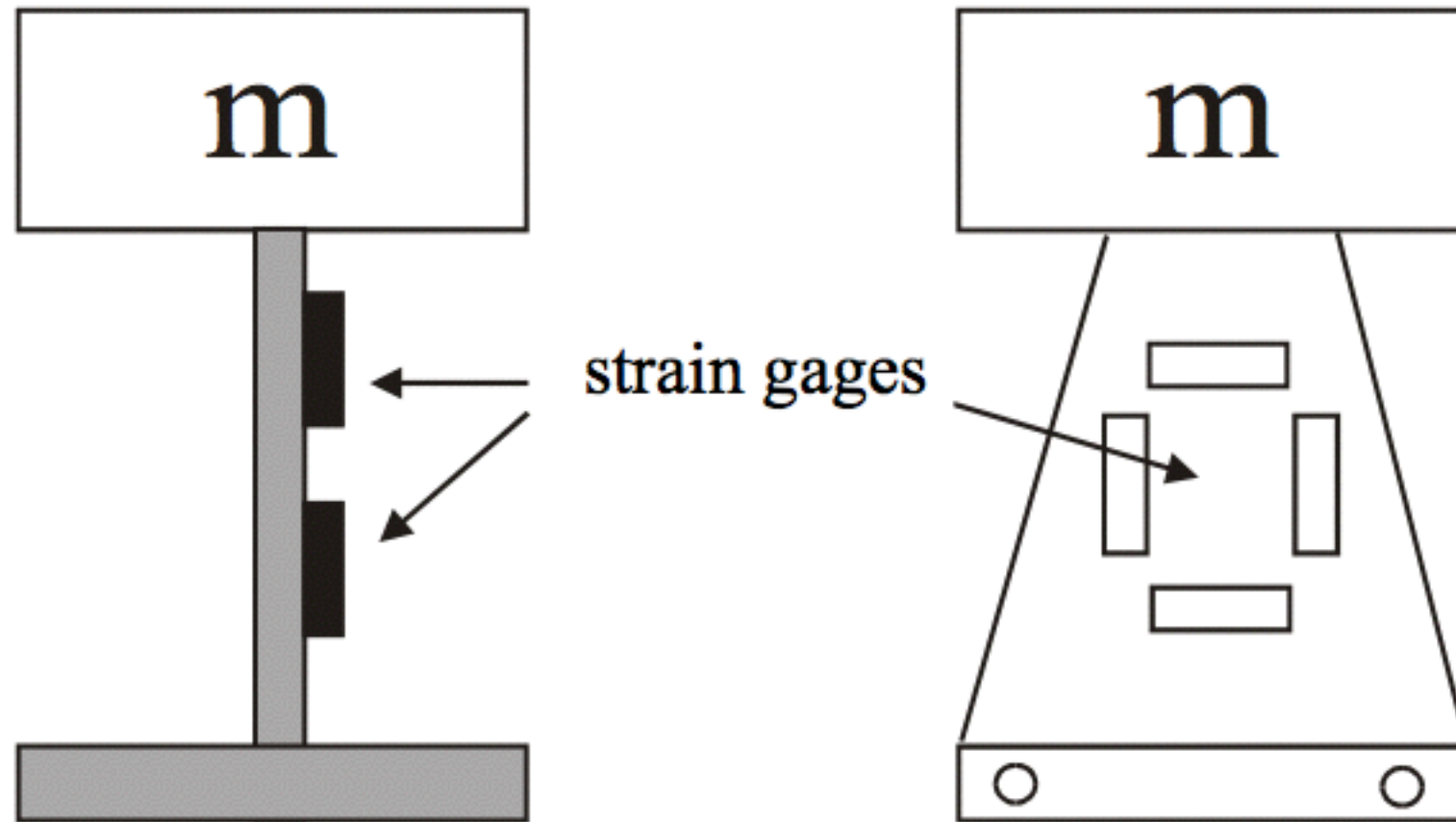
the deformation of the beam is detected by means of strain gages (see more on the lecture about force sensors)

suitable for MEMS



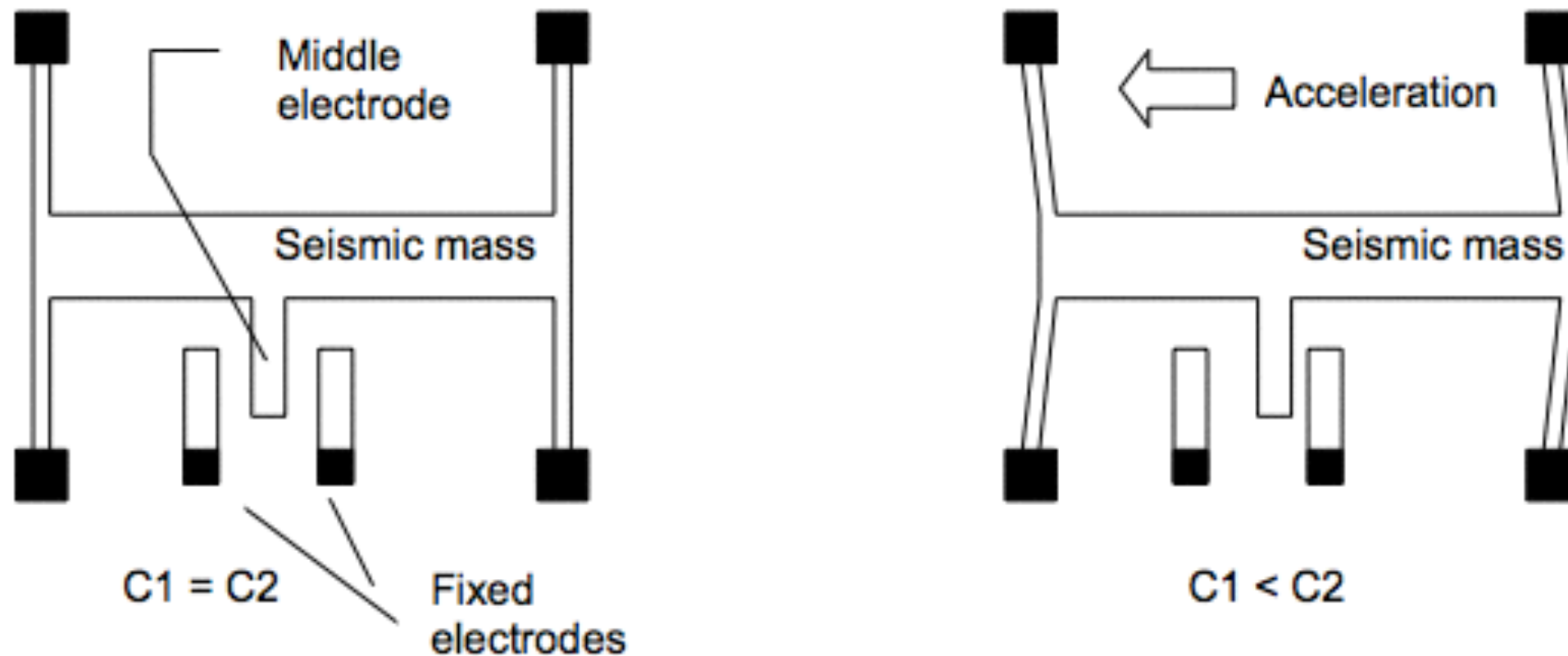
# Alternative configuration

e.g. in airbags

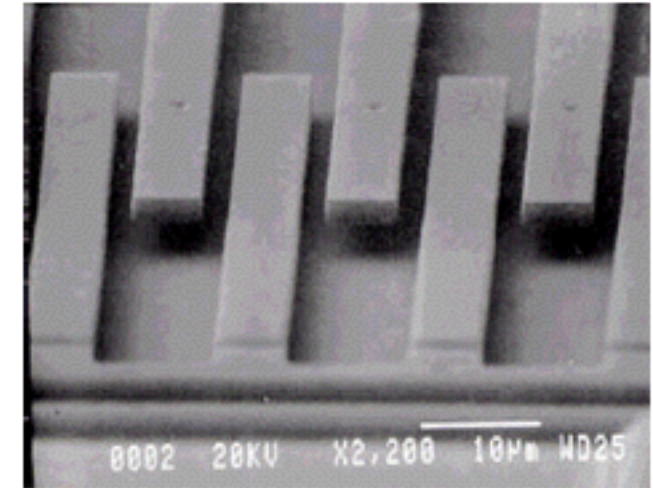
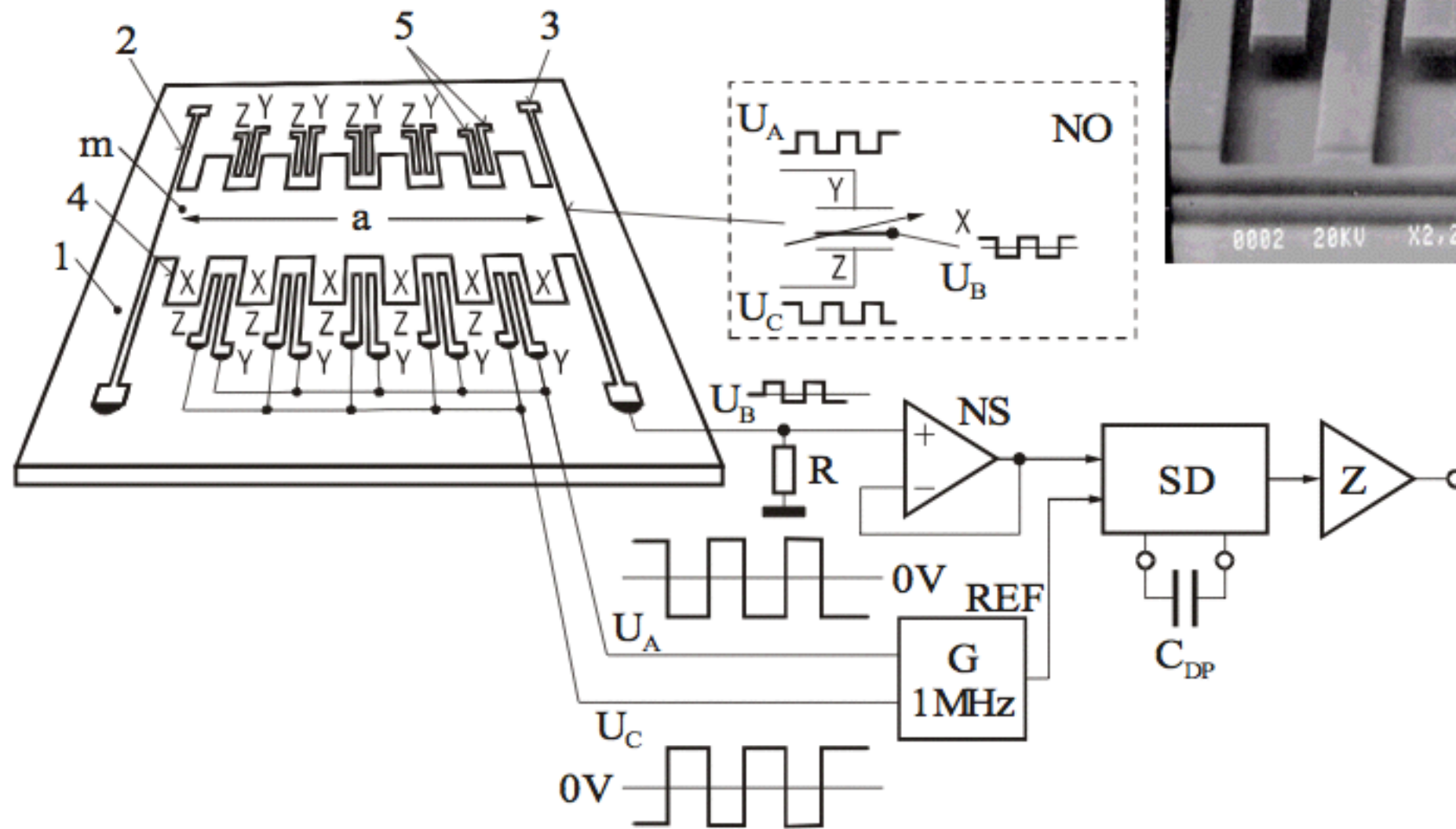


# MEMS accelerometers

## Basic principle

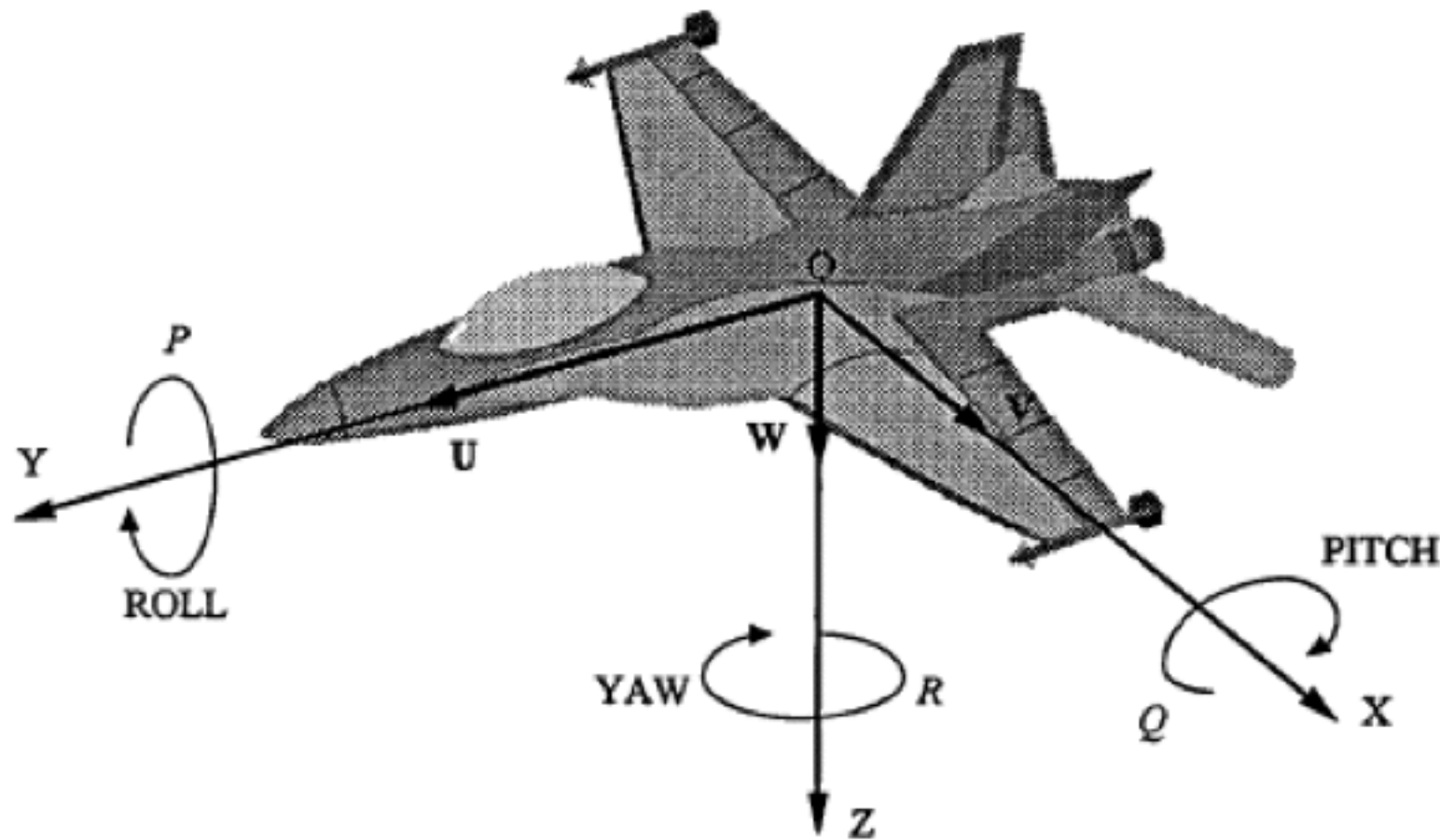


- based on dependence of capacity on distance of electrodes
- differential configuration to improve linearity



- 1 - platform made from polycrystalline Si
- 2 - bow strings
- 3 - anchors on the platform
- 4 - seismic mass (movable central electrodes)
- 5 - fixed electrodes

# Inertial navigation system



3 sensors of acceleration:  
integrating 2 times we obtain position

3 sensor of angular position:  
to understand in which direction the plane is moving and then  
integrate the accelerometers

# Actual gyroscopes

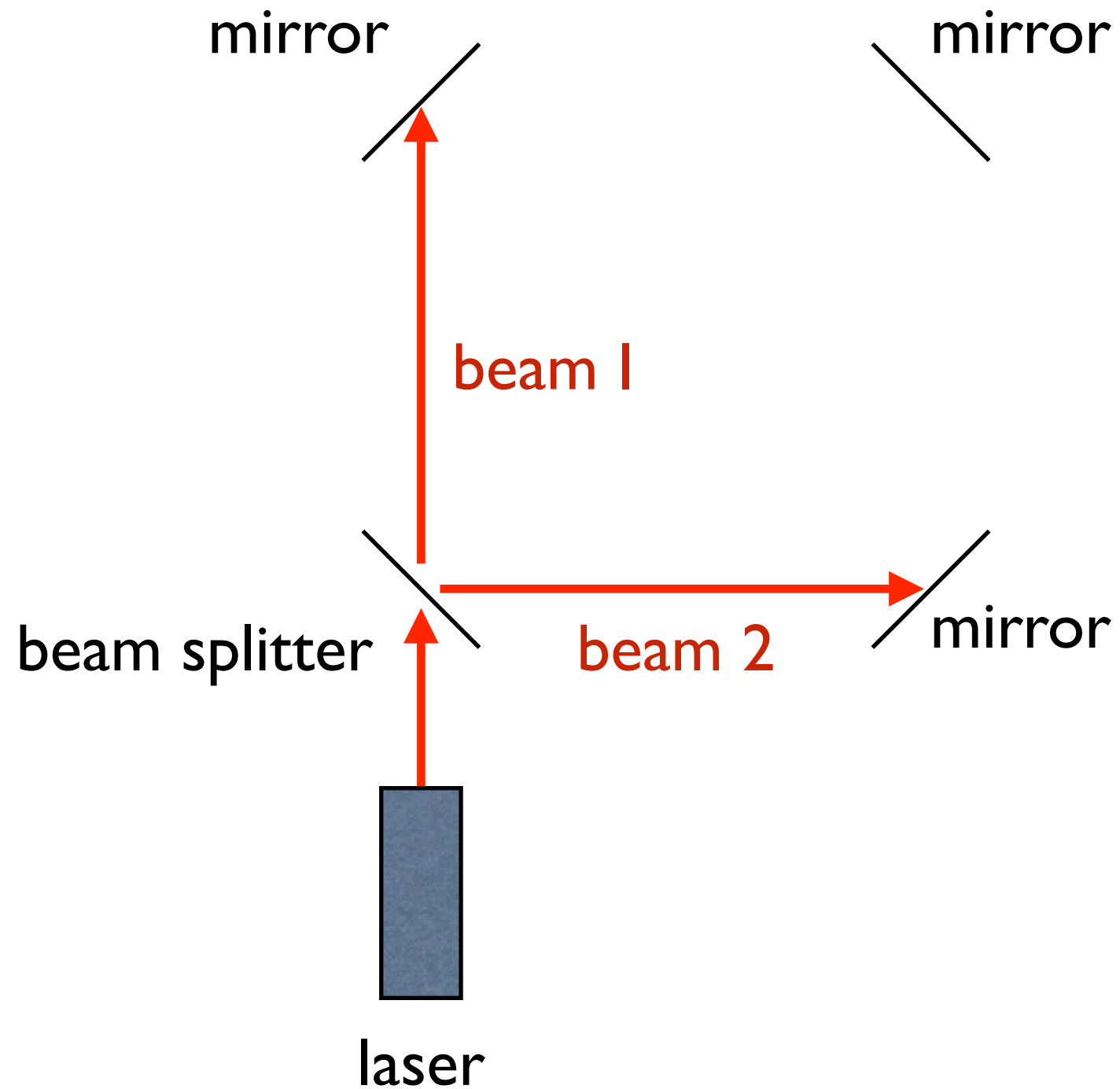


A mass is free spinning in a gimbal

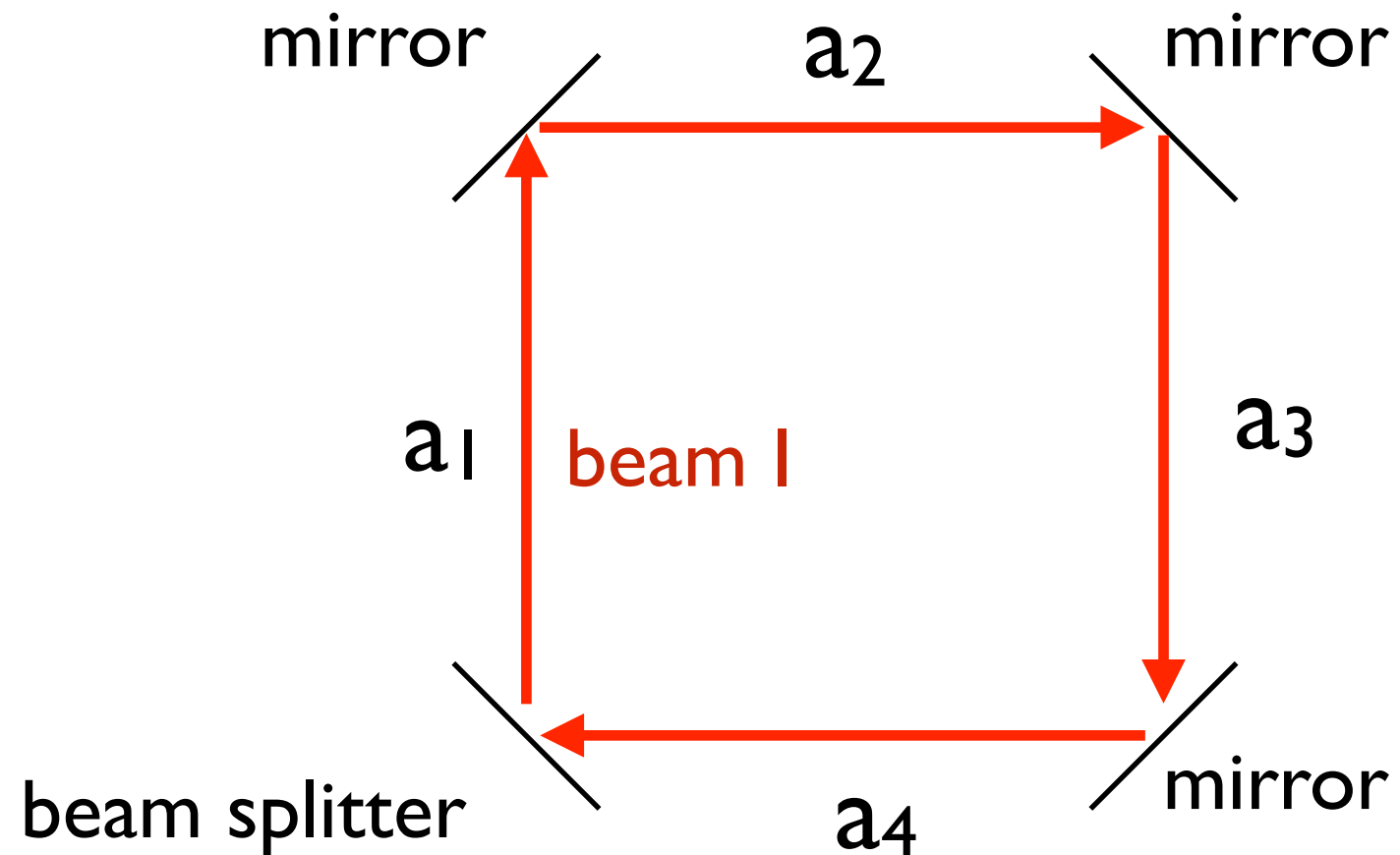
The angular momentum of the spinning mass makes the stay in the same direction

It is used to have a reference of position.

# Ring Laser Gyroscopes



# Ring Laser Gyroscopes

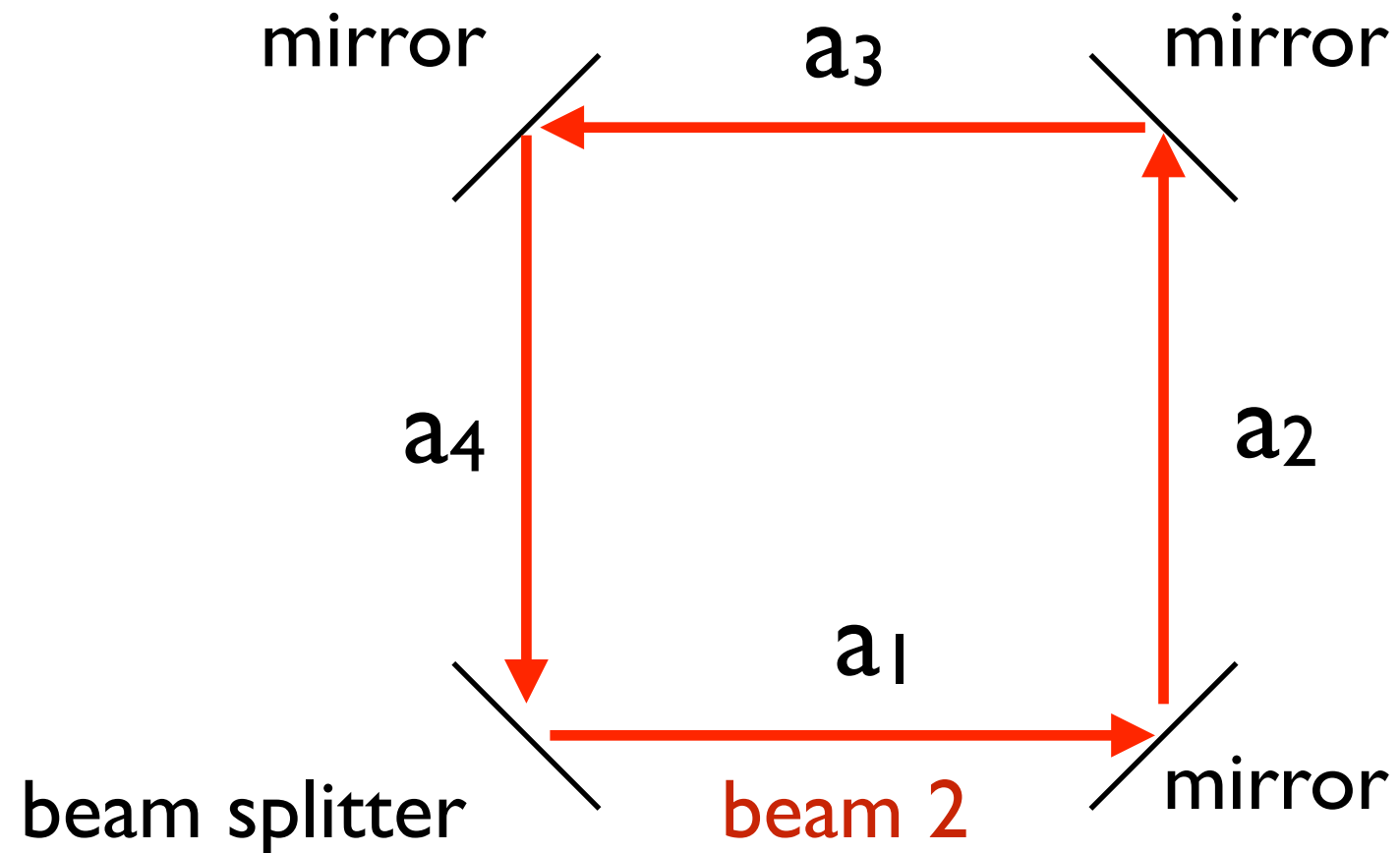


laser

Total length of the path

$$L_1 = a_1 + a_2 + a_3 + a_4$$

# Ring Laser Gyroscopes



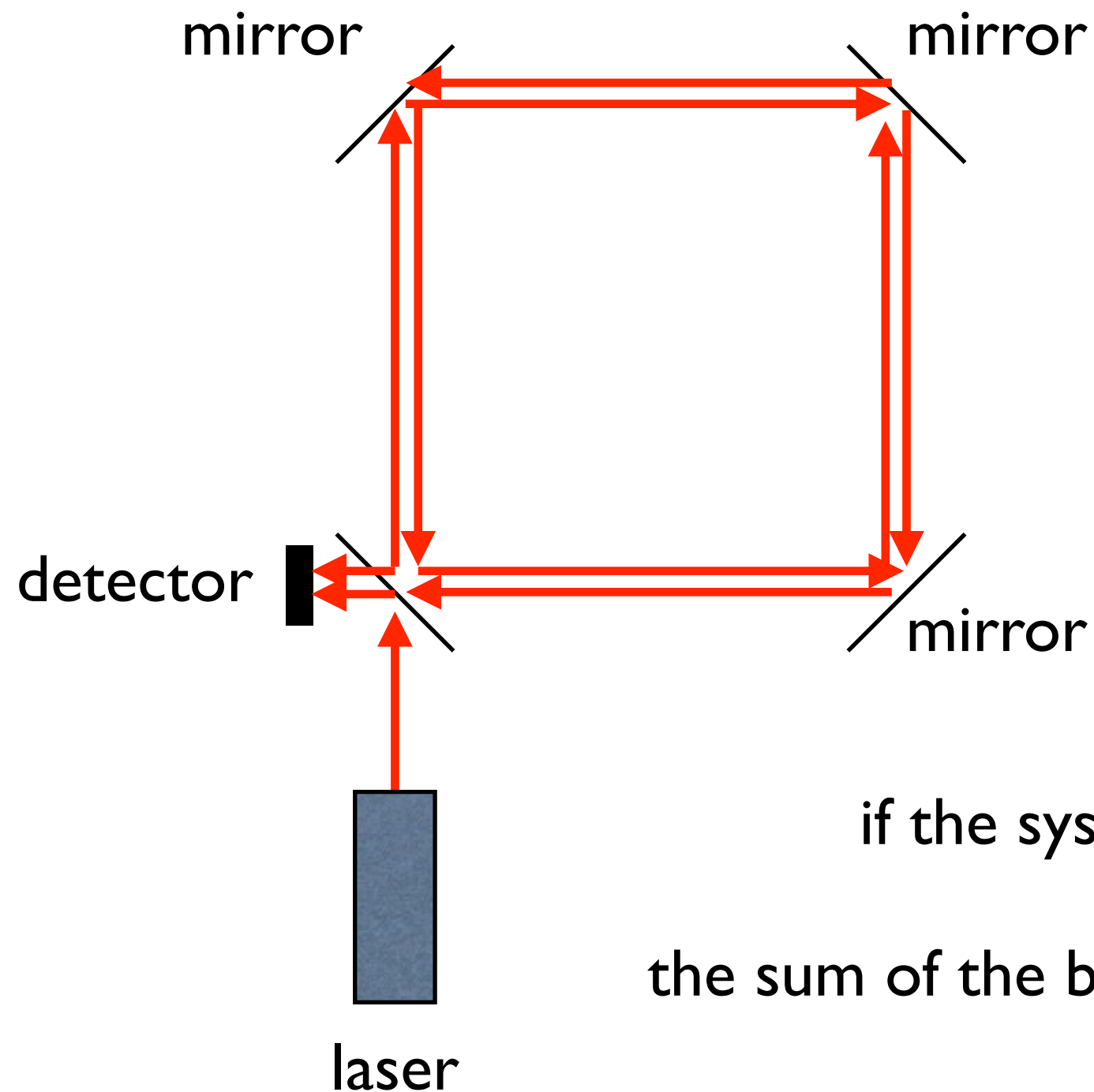
laser

Total length of the path

$$L_2 = a_1 + a_2 + a_3 + a_4$$



# Ring Laser Gyroscopes (RLG)

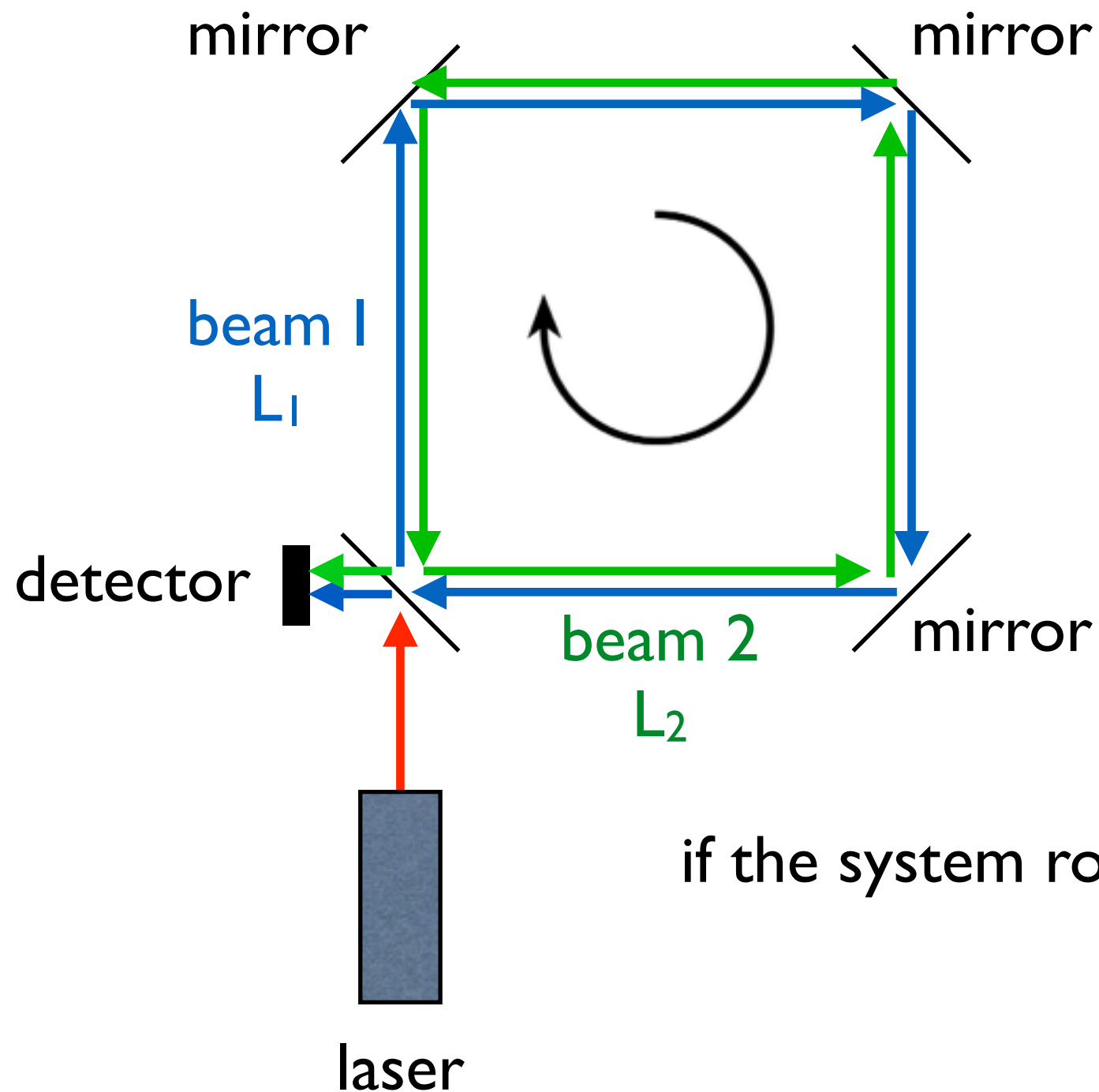


if the system does not move

$$L_1 = L_2$$

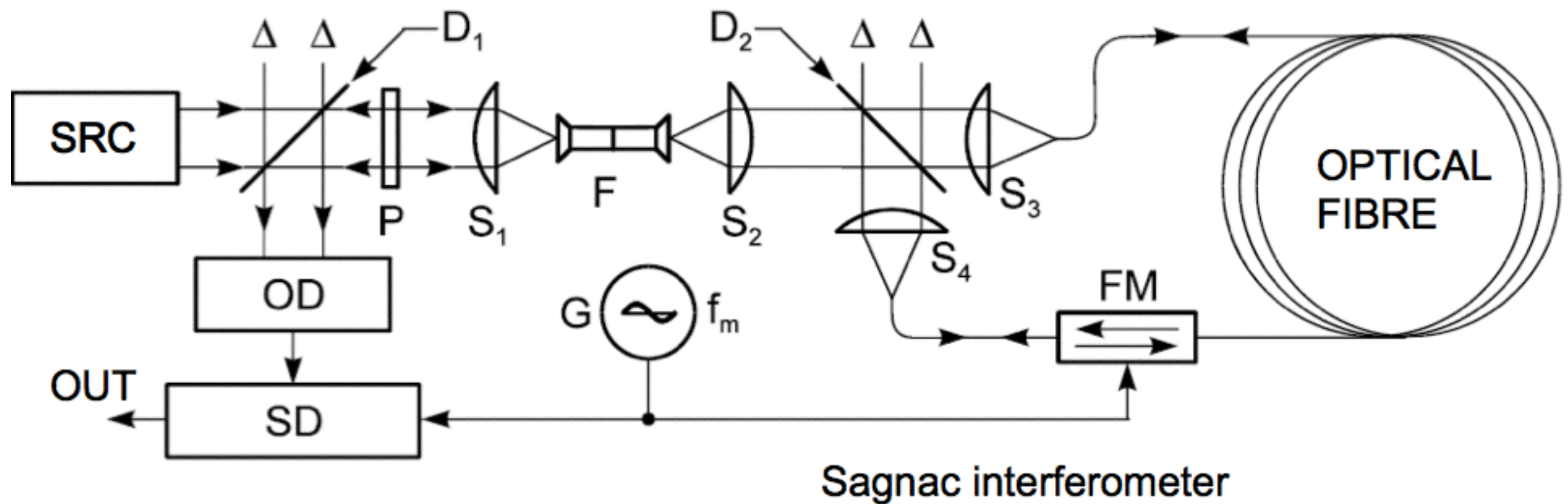
the sum of the beams at the detector is null

# Ring Laser Gyroscopes (RLG)



if the system rotates clockwise apparently  
 $L_1 < L_2$

# Optical fibre gyroscopes

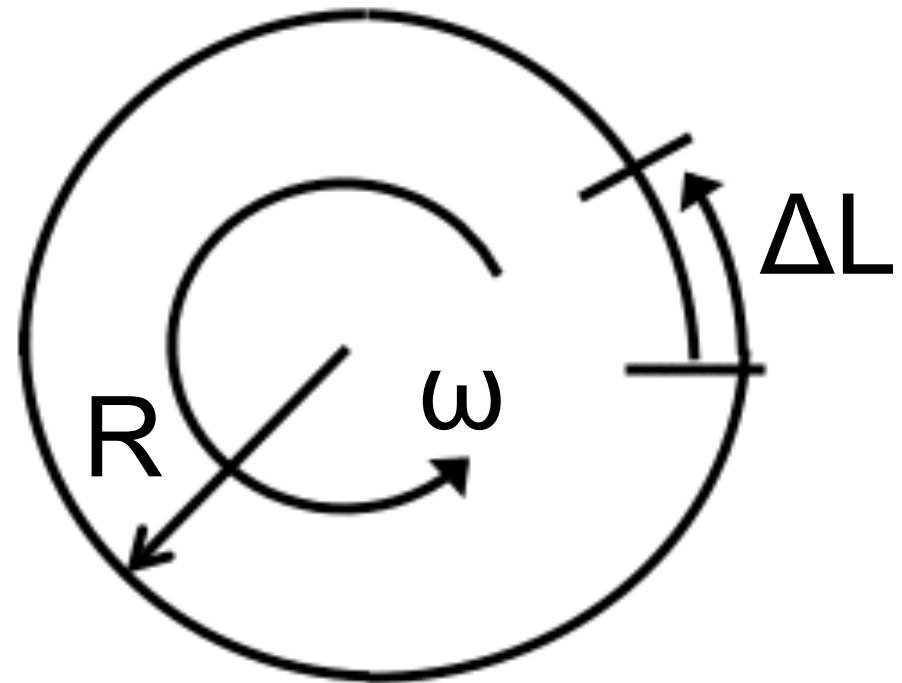


counterclockwise

$$t_1 = \frac{2\pi R + \Delta L}{c}$$

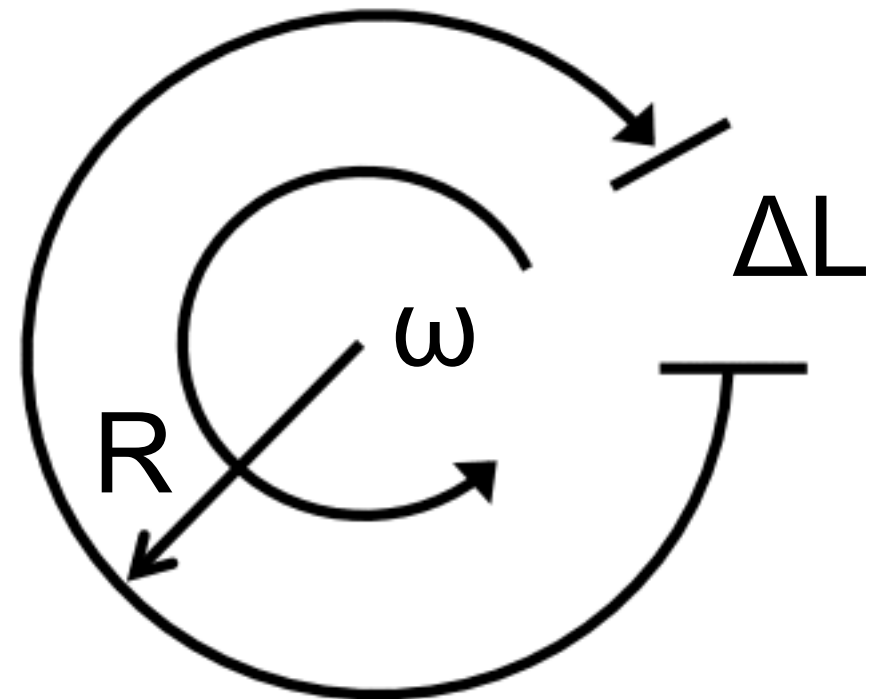
$$\Delta L = R\omega t_1$$

$$t_1 = \frac{2\pi R}{c - R\omega}$$



clockwise

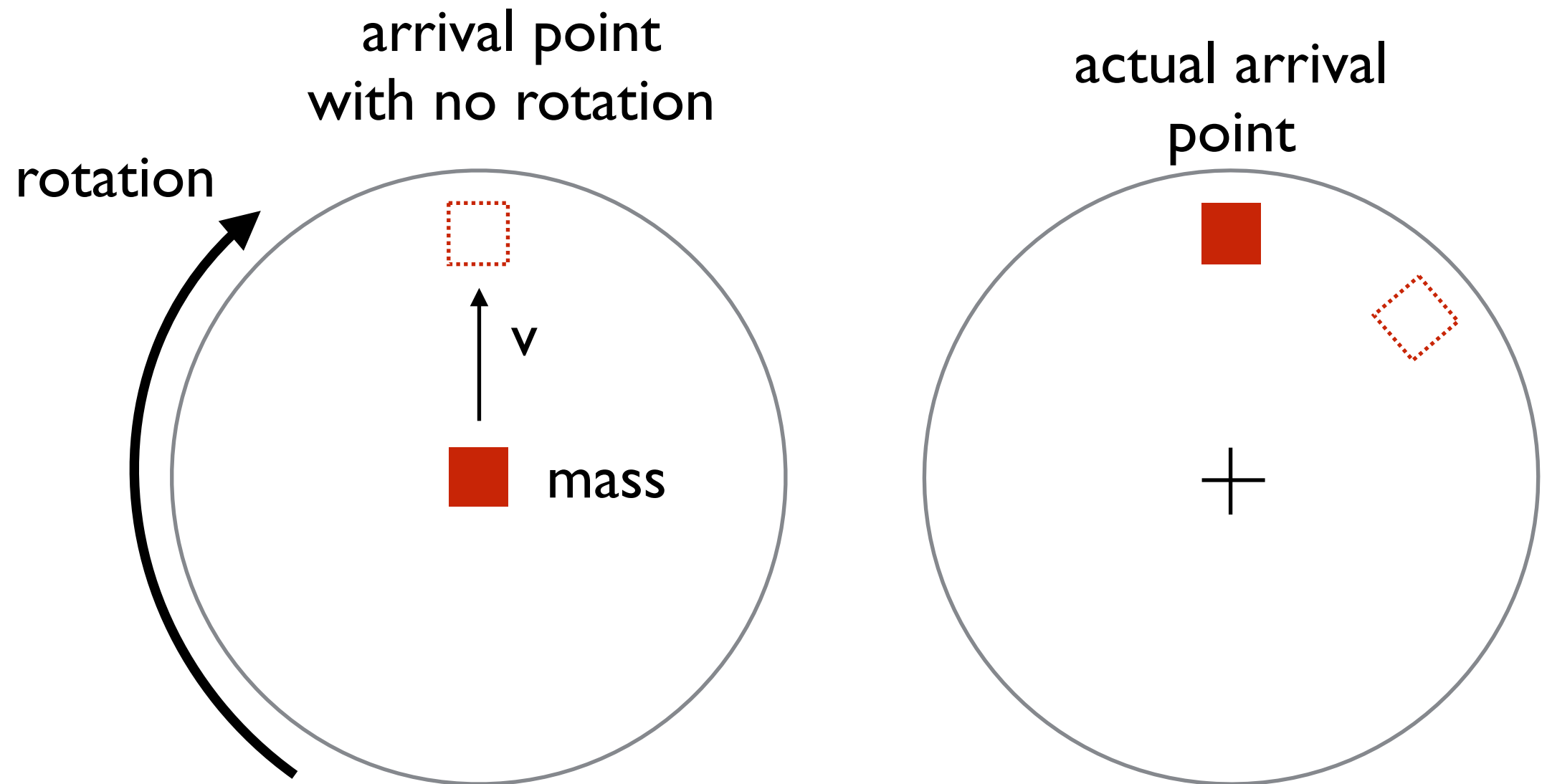
$$t_2 = \frac{2\pi R}{c + R\omega}$$



$$\Delta t = t_1 - t_2 = \frac{4\pi R^2 \omega}{c^2 - R^2 \omega^2} \approx \frac{4\pi R^2 \omega}{c^2}$$

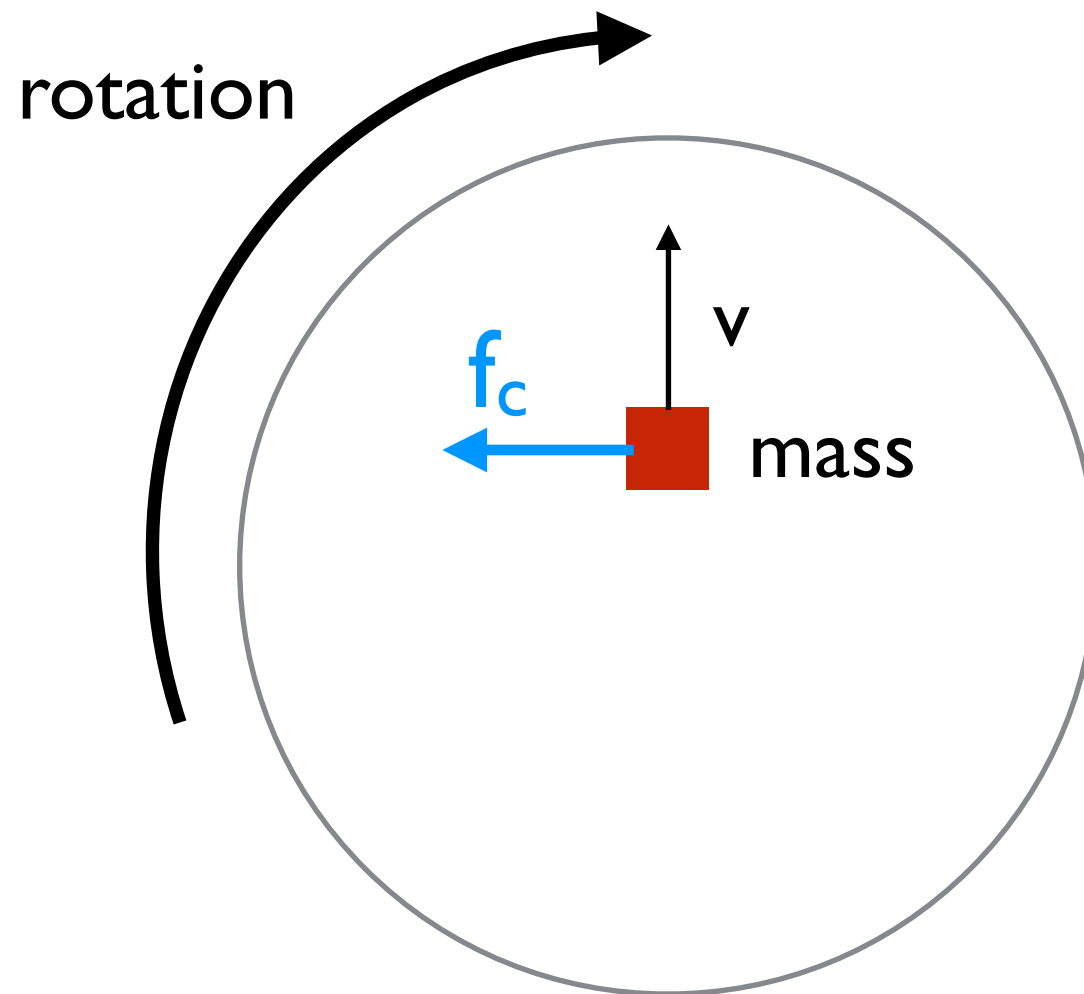
# Angular rate sensors based on Coriolis effect

## iMEMS<sup>®</sup> from Analog



# Angular rate sensors based on Coriolis effect

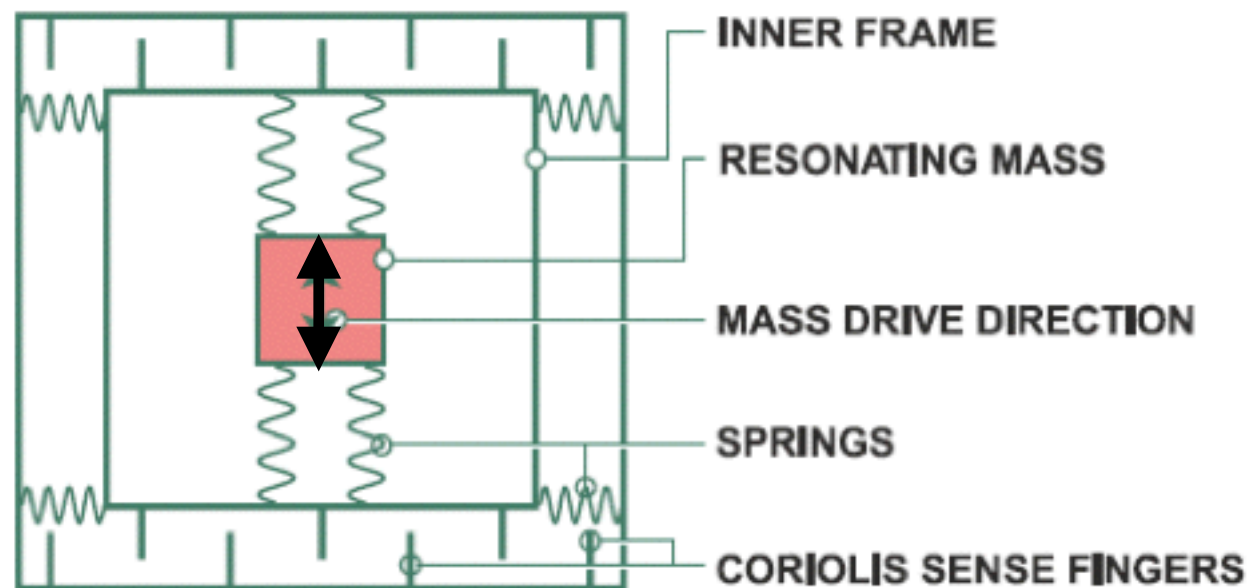
iMEMS<sup>®</sup> from Analog



# Angular rate sensors based on Coriolis effect

## iMEMS<sup>®</sup> from Analog

How to use the Coriolis force to detect rotation?



A mass oscillates vertically.  
So, the velocity is periodically

↑  $v$

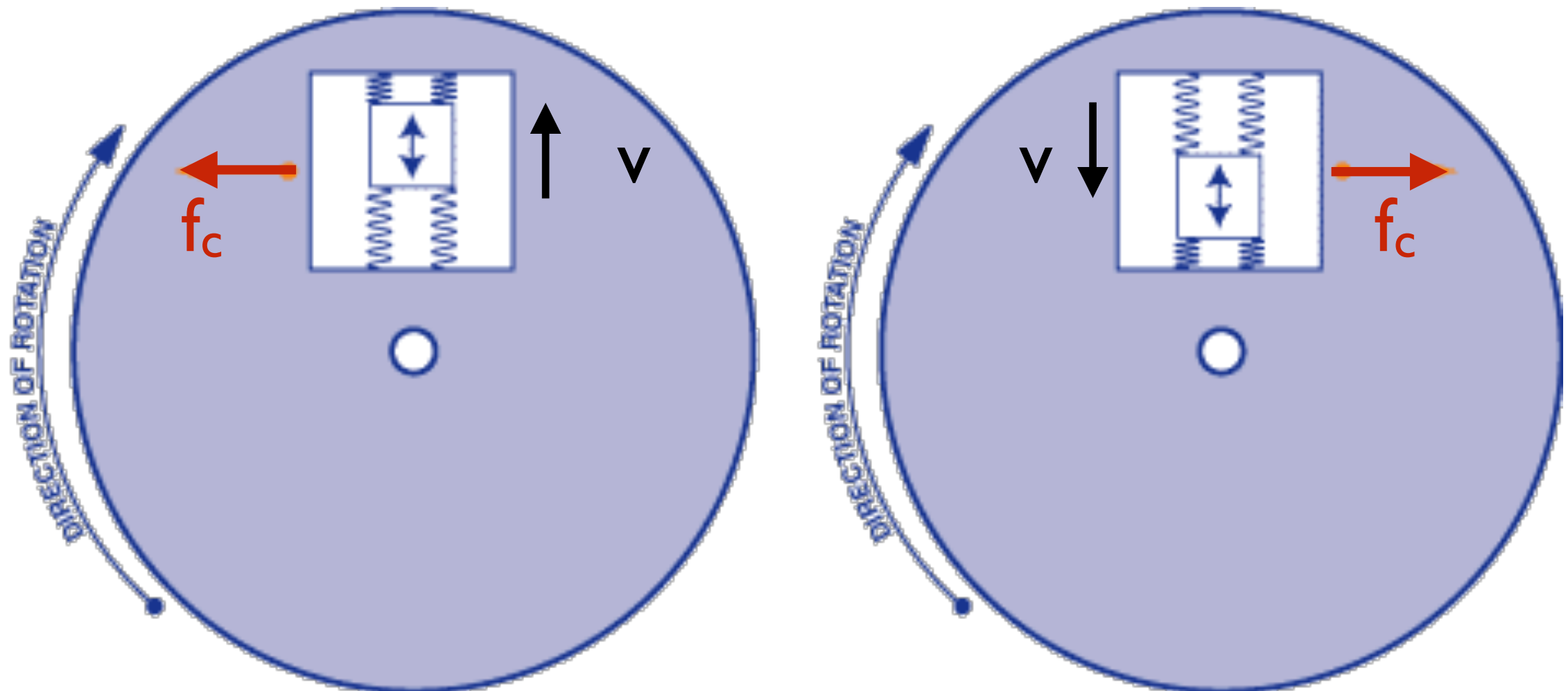
and

↓  $v$

# Angular rate sensors based on Coriolis effect

## iMEMS<sup>®</sup> from Analog

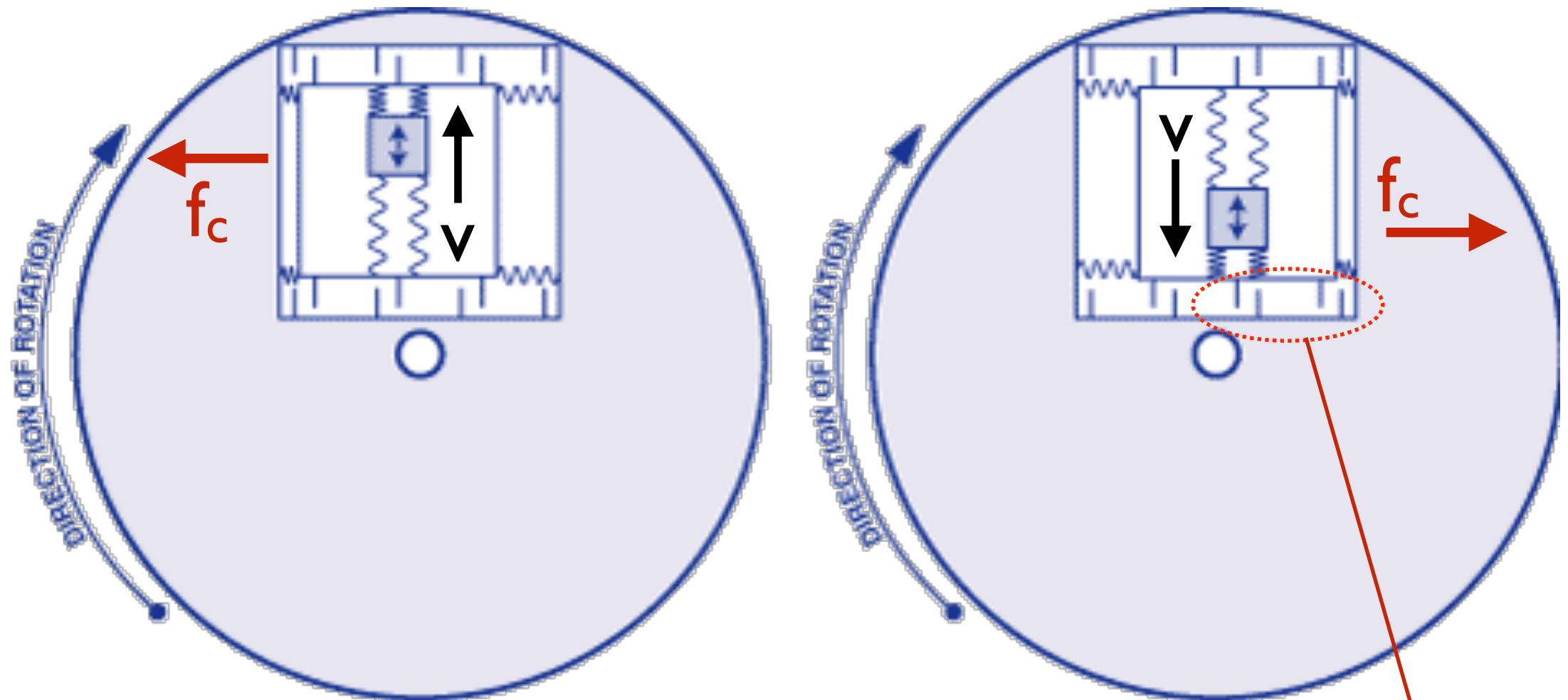
According to the direction of  $v$  the Coriolis force changes direction





# Angular rate sensors based on Coriolis effect

## iMEMS<sup>®</sup> from Analog



horizontal displacement is detected  
by capacitive sensor technique