Lecture 4

- Measurement of small voltages
- Measurement of resistance

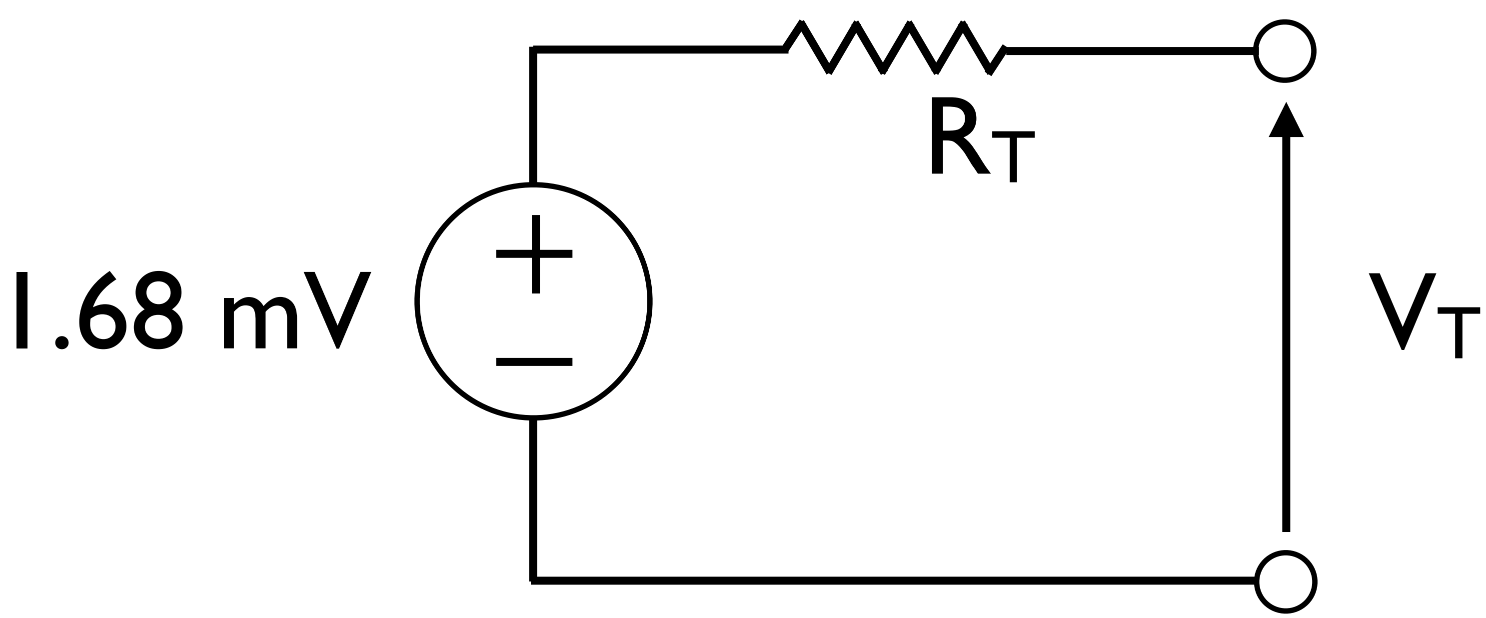
MEASUREMENT OF SMALL VOLTAGES

- *voltage has to be amplified before being measured*
- *input impedance has to be increased in some particular case*

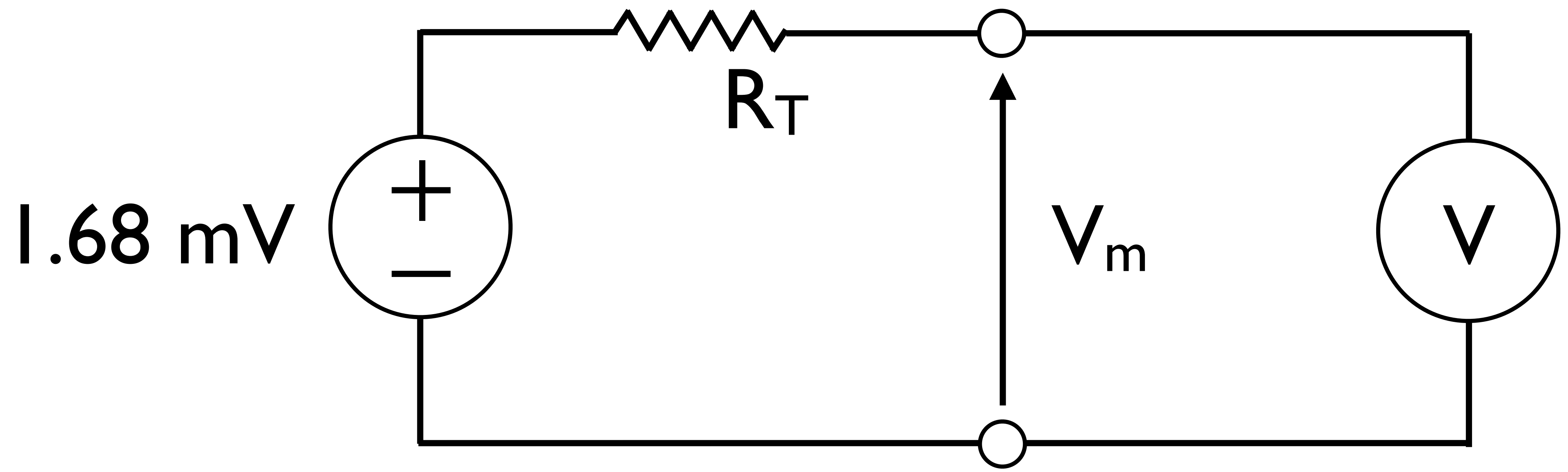
For example

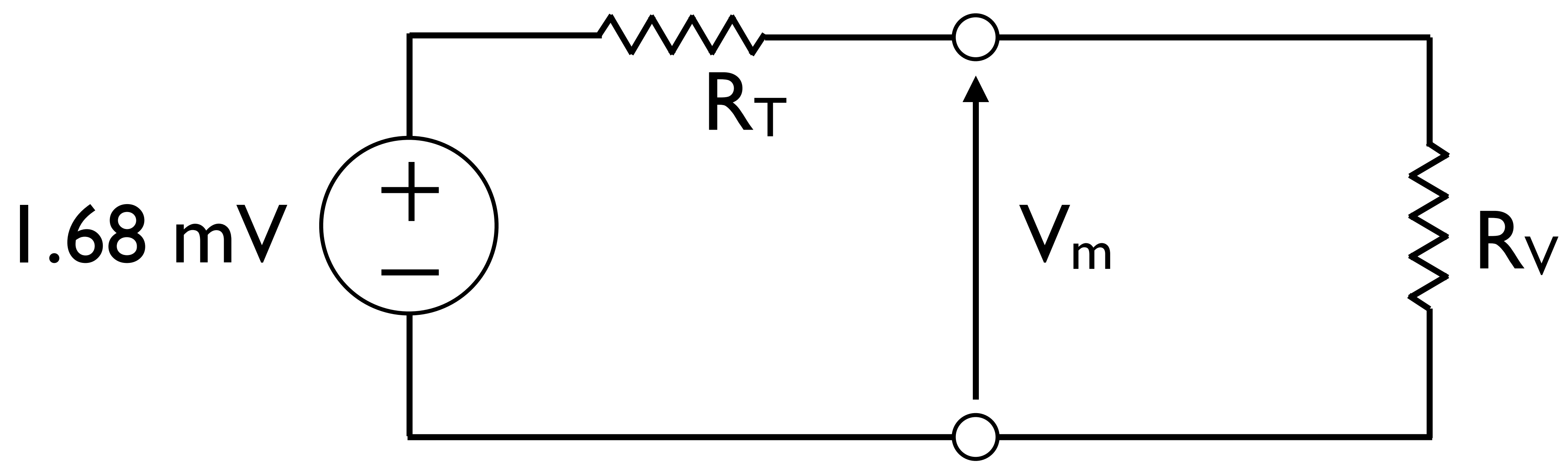
THERMOCOUPLE

$$V_T = k \cdot \Delta T = 42\ \mu V/K \cdot (60-20) = 1.68\ mV$$



Problem n. 1: we MUST amplify the voltage

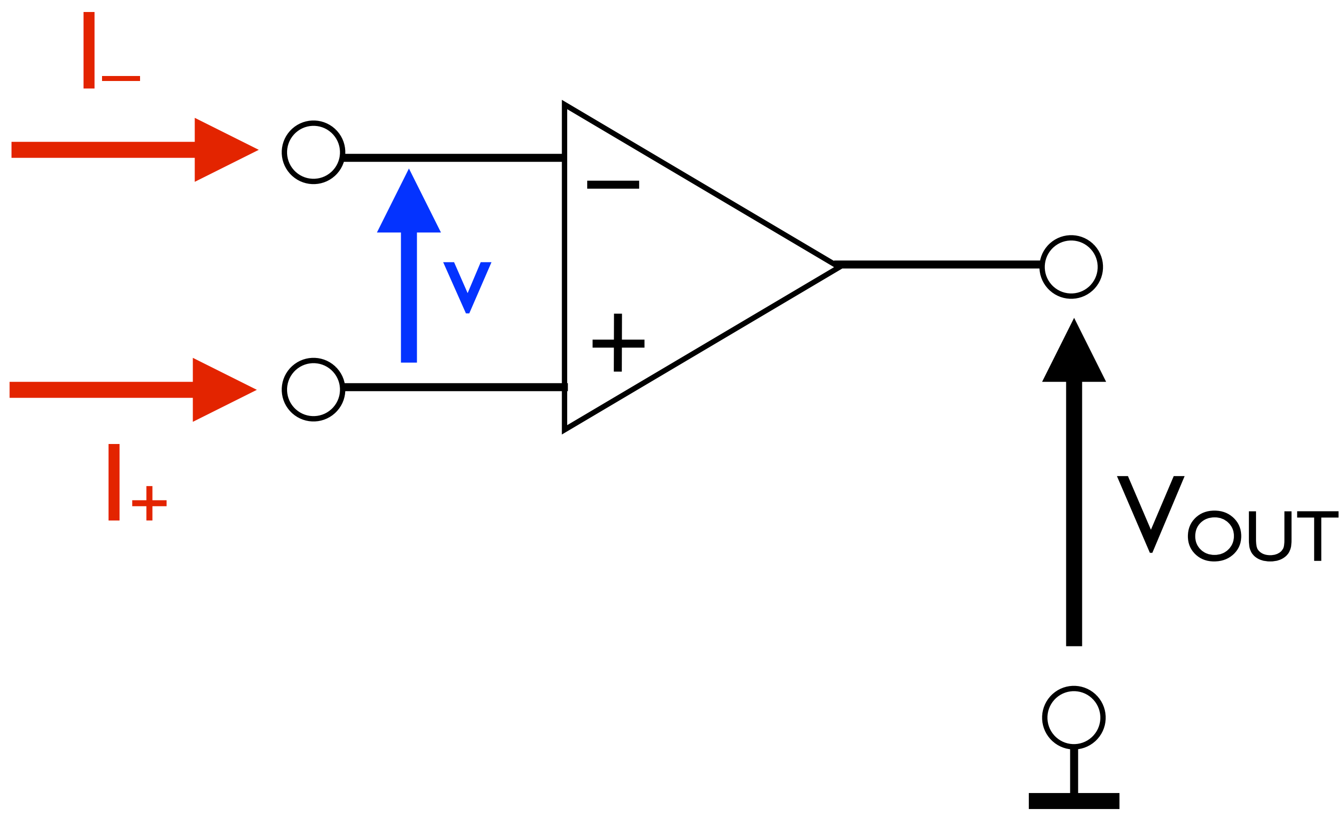




$$V_m = 1.68\ \mathrm{mV} \cdot \frac{R_V}{R_V + R_T}$$

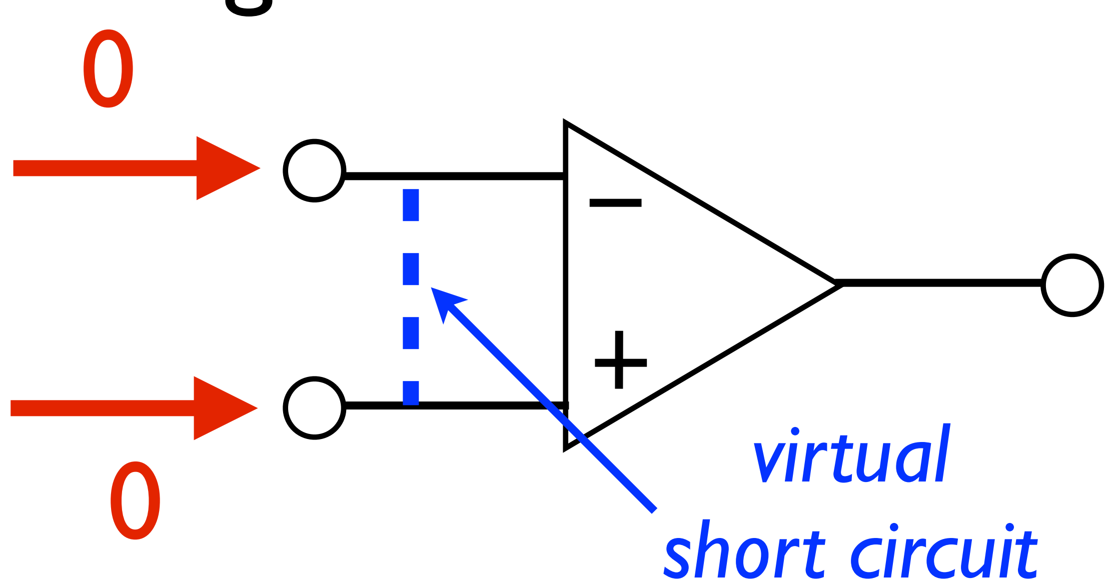
Problem n. 2: we must increase R_V as much as possible

OPERATIONAL AMPLIFIER

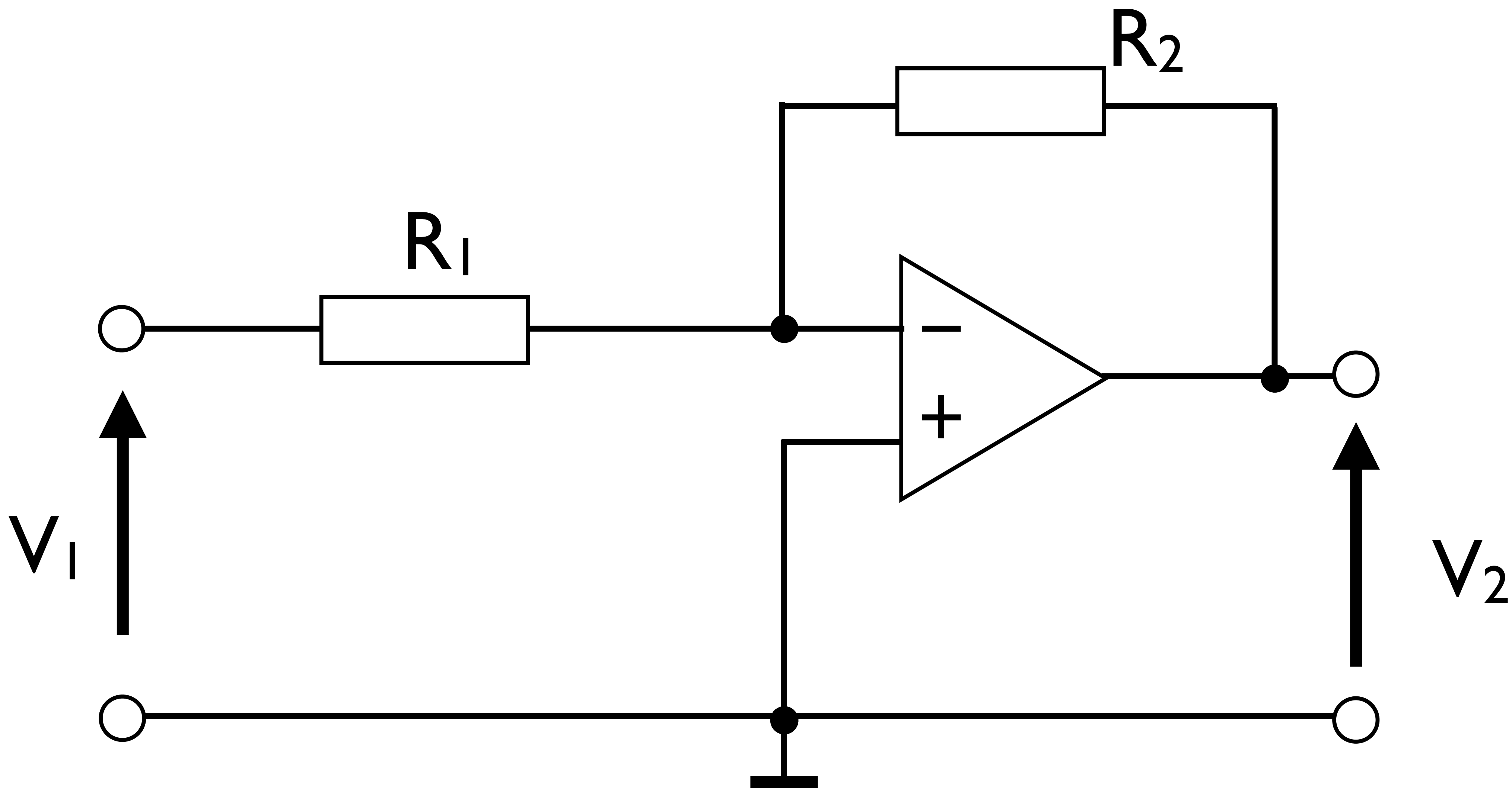


The opamp set the output voltage V_{OUT} to a value which assures the following conditions are fullfilled

1) I_- =0
2) I_+ =0
3) v =0

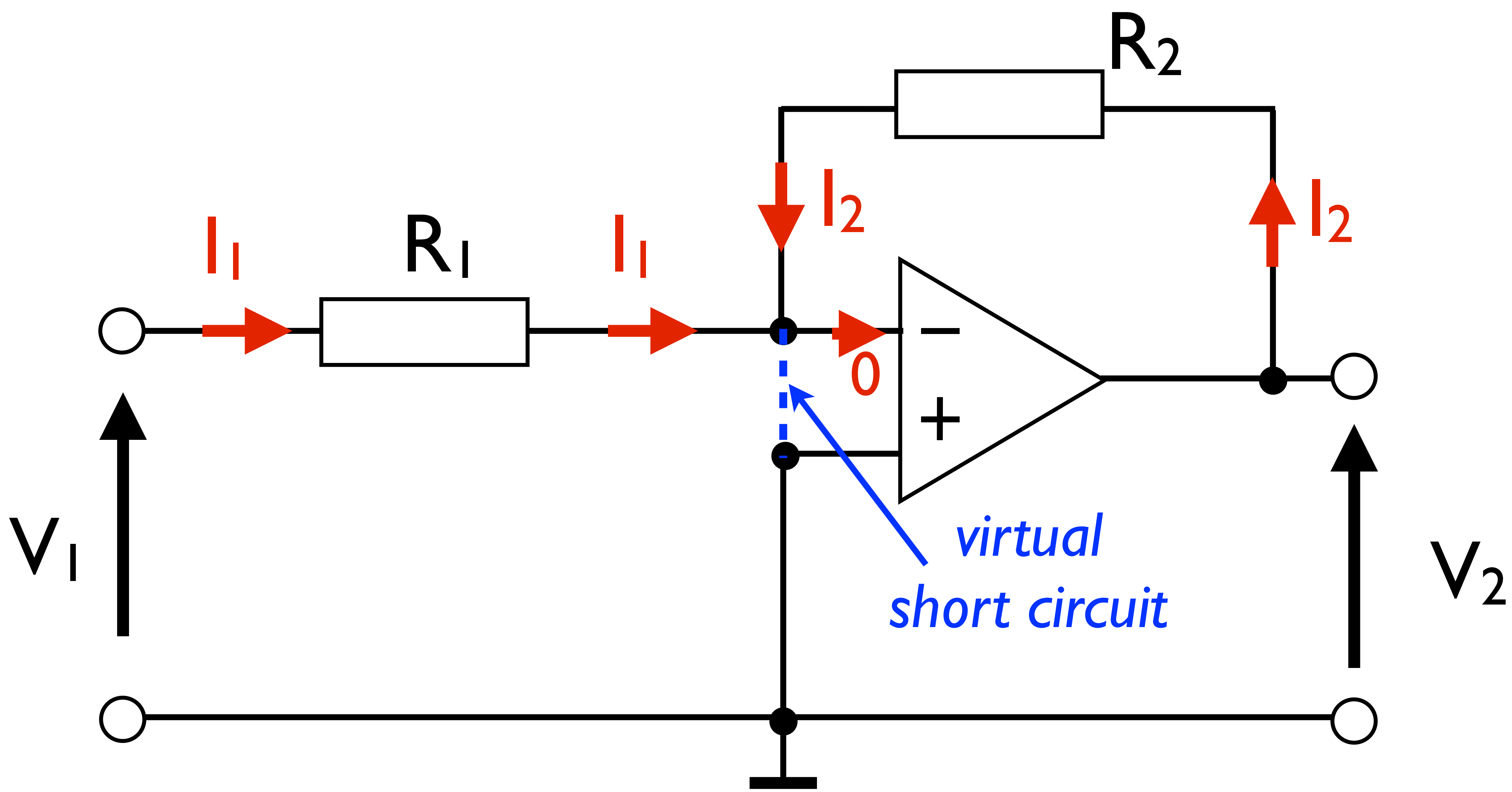


INVERTING AMPLIFIER



$$V_2 = -V_1 \cdot \frac{R_2}{R_1}$$

INVERTING AMPLIFIER



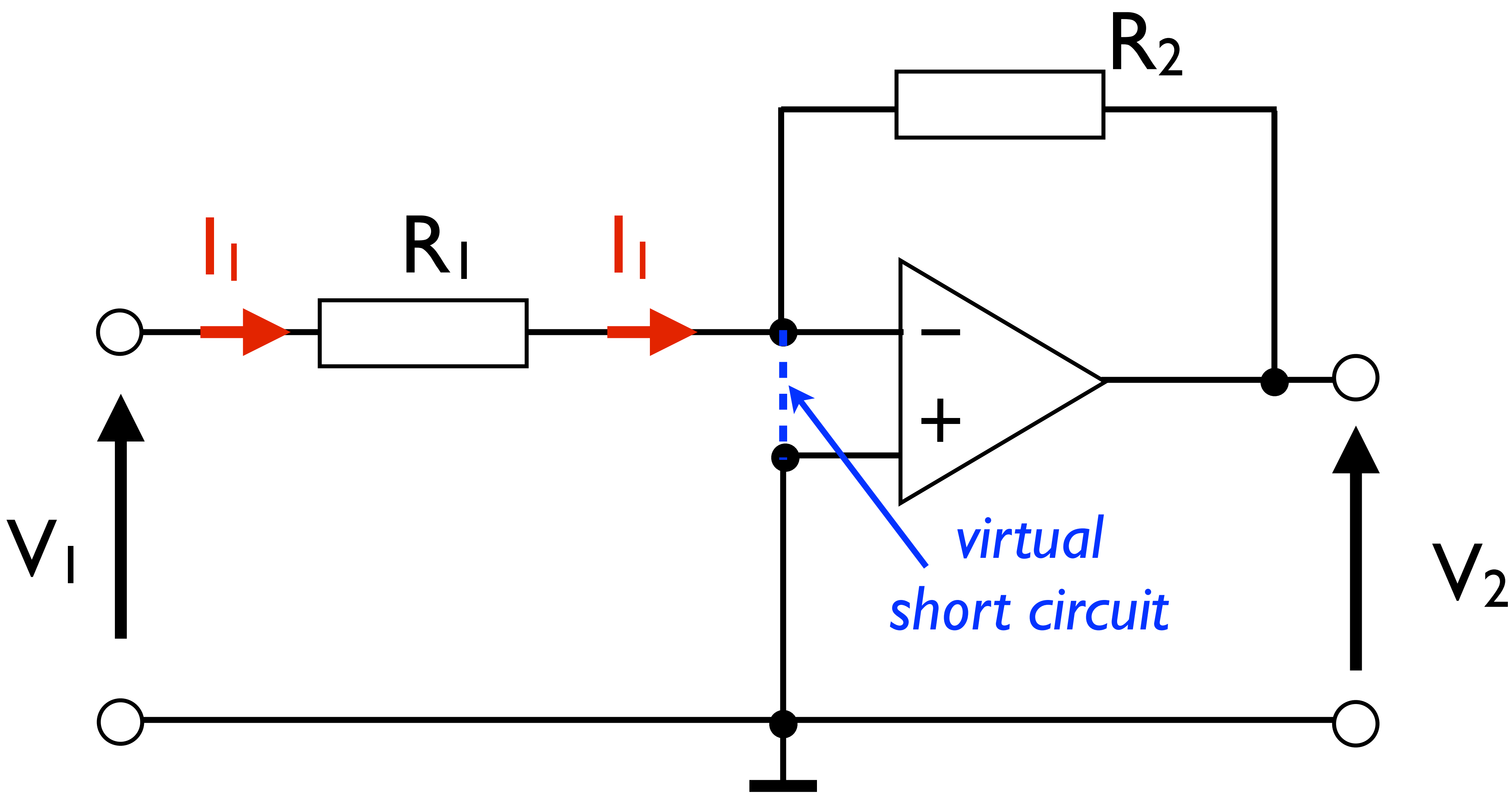
1) $I_1 = {}^{V_1}/_{R_1}$

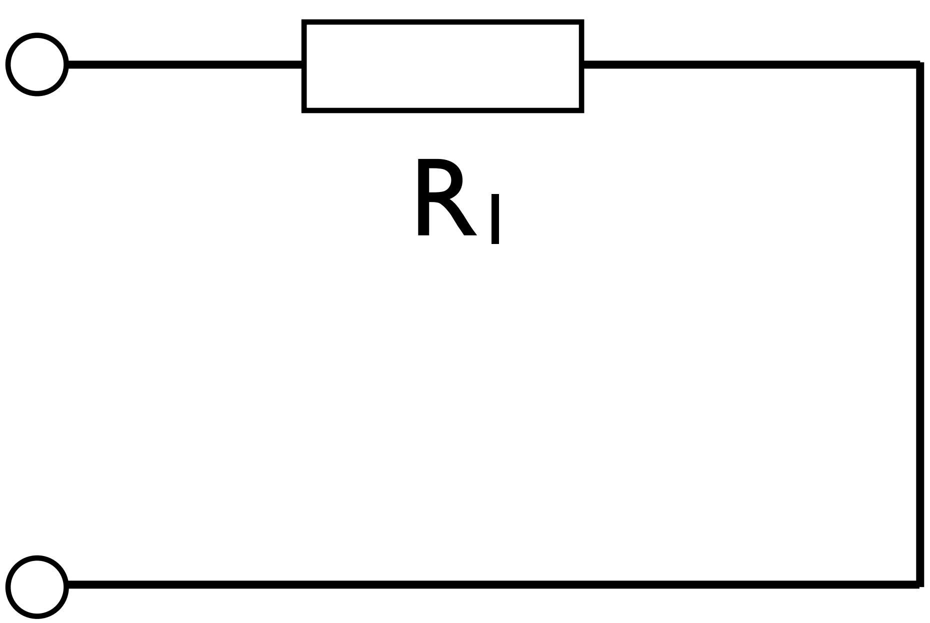
2) $V_2 = R_2 \cdot I_2$

3) $I_2 = -I_1$

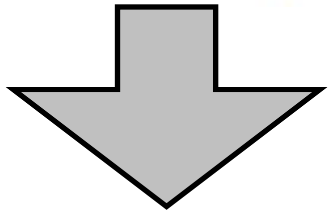
$$V_2 = -V_1 \cdot \frac{R_2}{R_1}$$

INVERTING AMPLIFIER



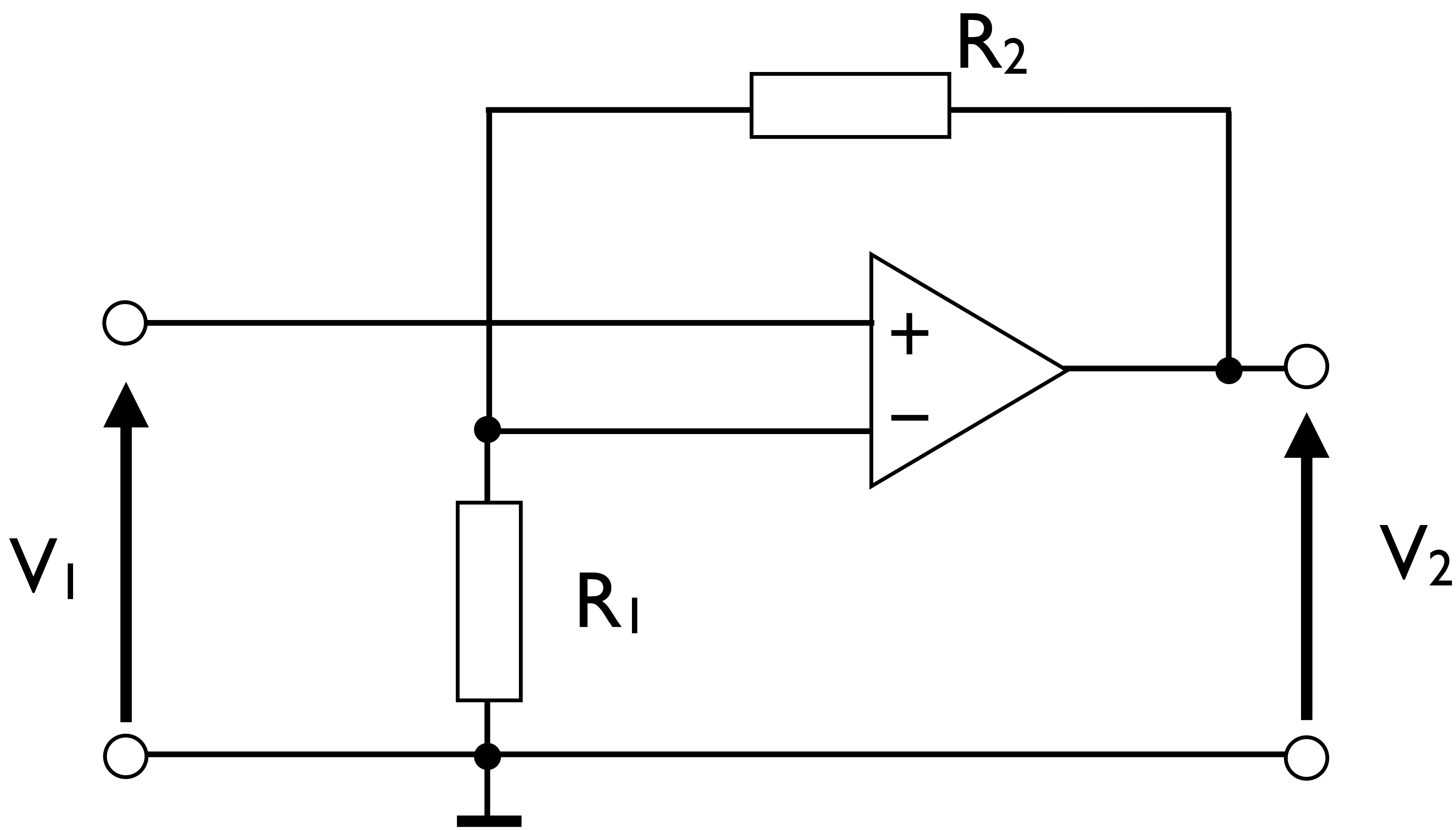


$$I_1 = {}^{V_1}/_{R_1}$$



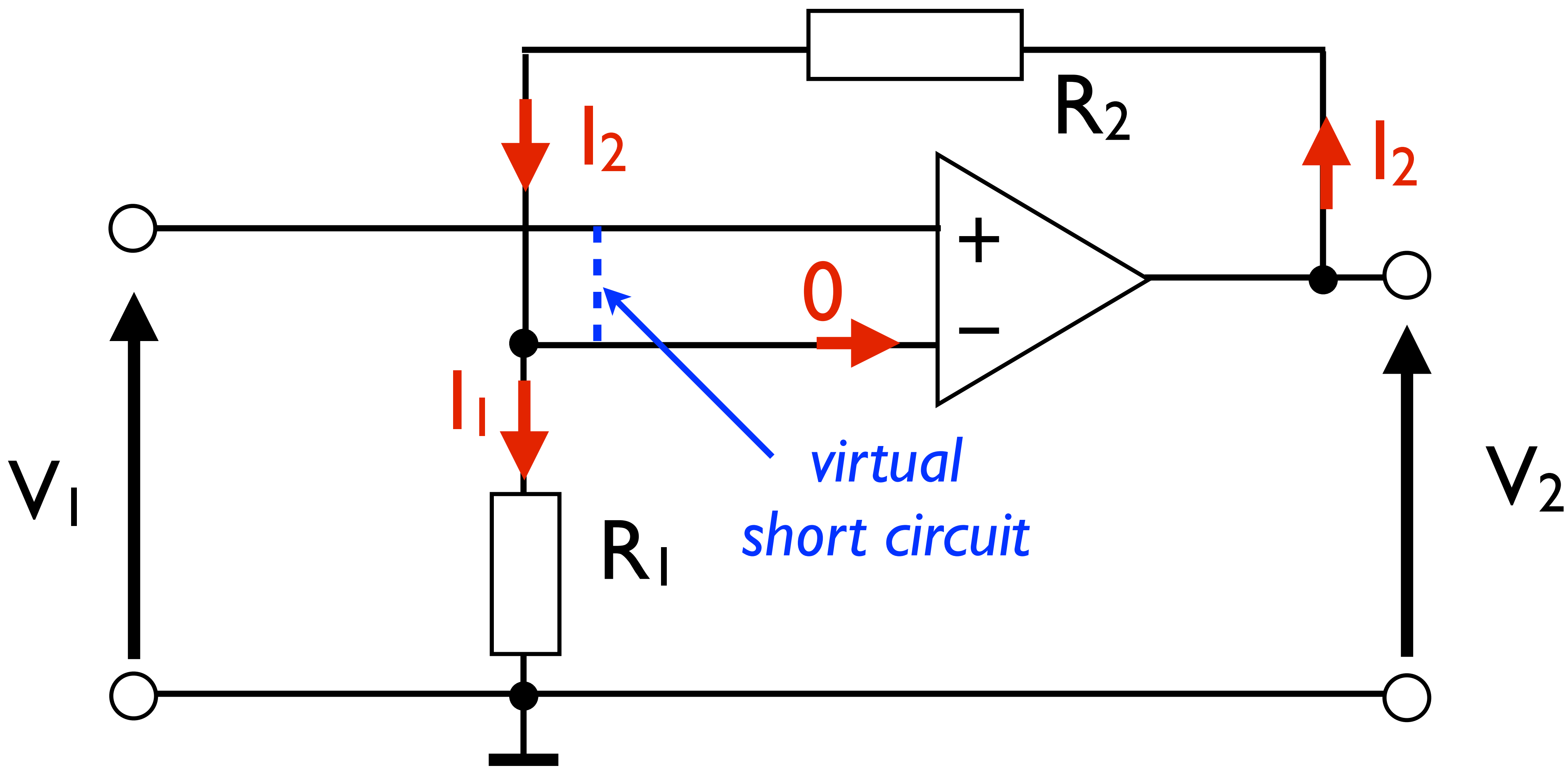
Input resistance R_1

NON-INVERTING AMPLIFIER



$$V_2 = \left(1 + \frac{R_2}{R_1}\right) \cdot V_1$$

NON-INVERTING AMPLIFIER



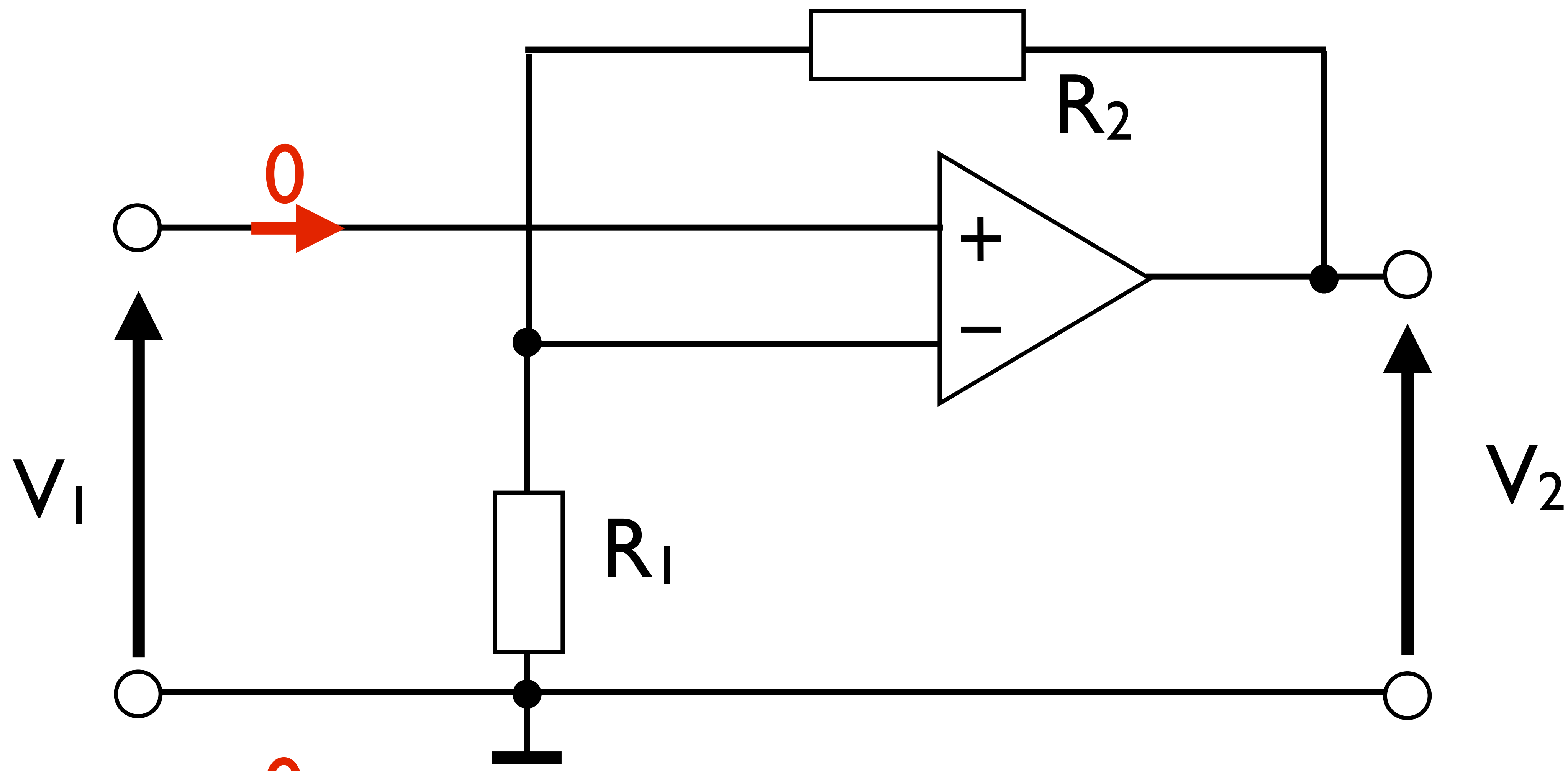
1) $I_1 = {}^{V_1}/_{R_1}$

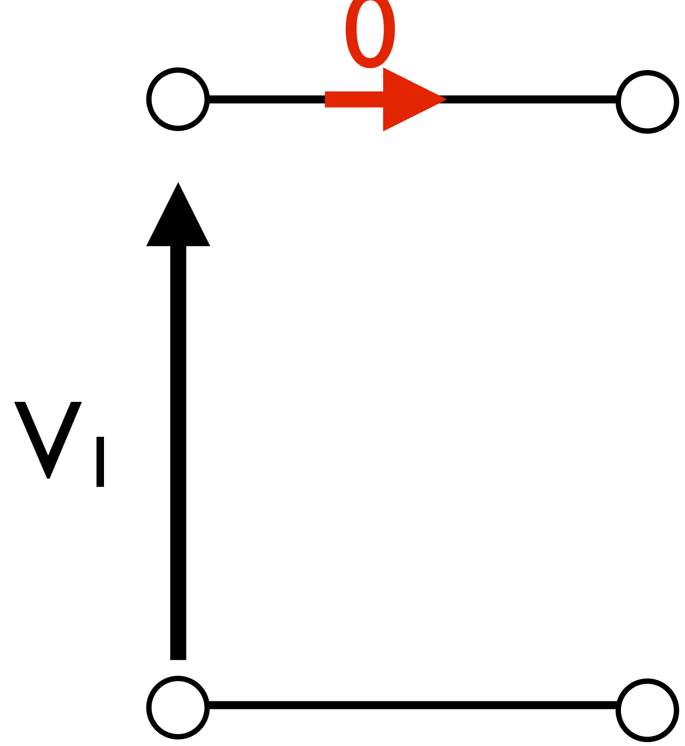
2) $I_2 = I_1$

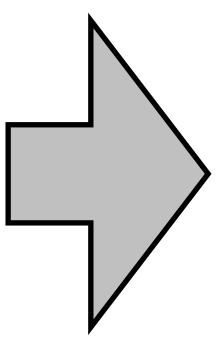
3) $V_2 = R_2 \cdot I_2 + V_1$

$$V_2 = \left(1 + \frac{R_2}{R_1}\right) \cdot V_1$$

NON-INVERTING AMPLIFIER

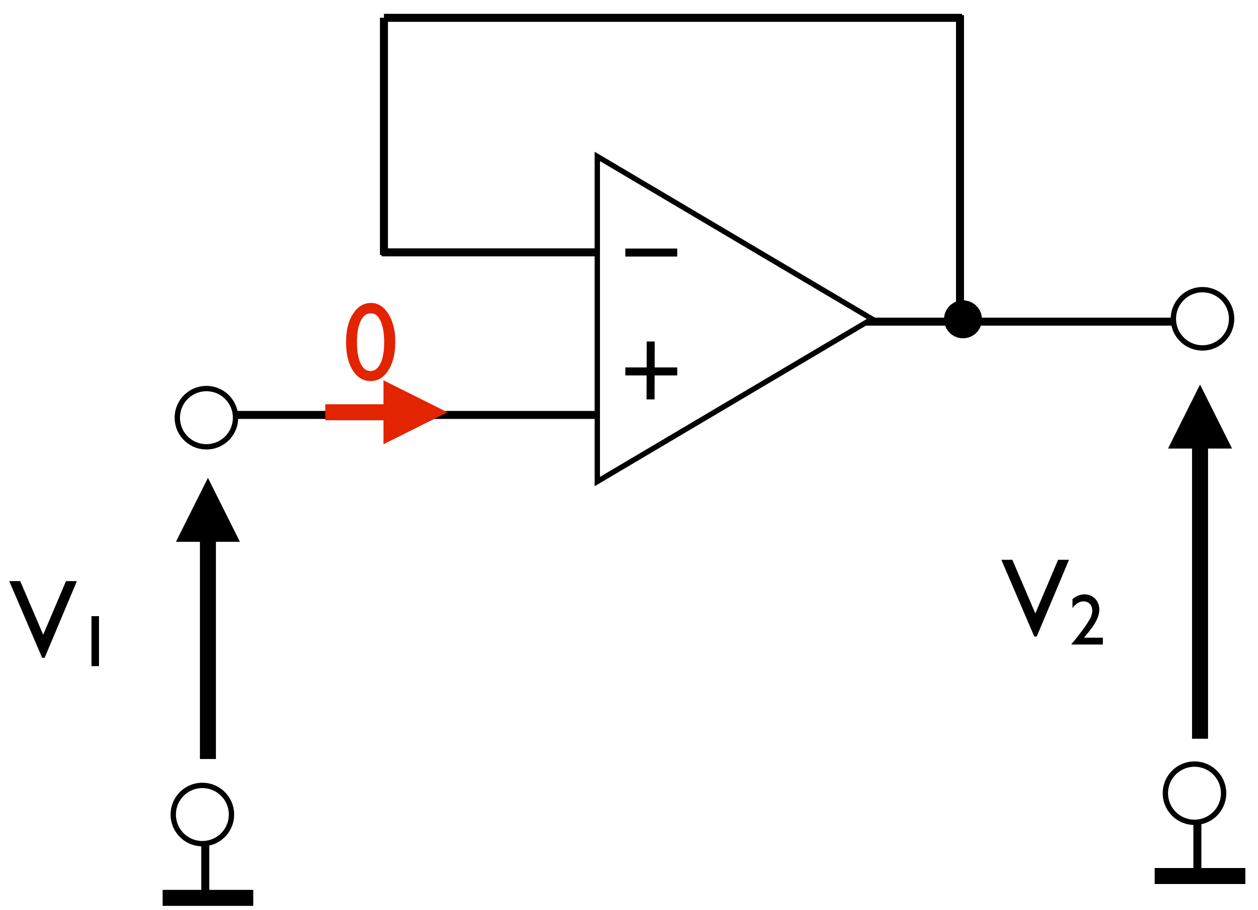






Input resistance ∞

VOLTAGE FOLLOWER

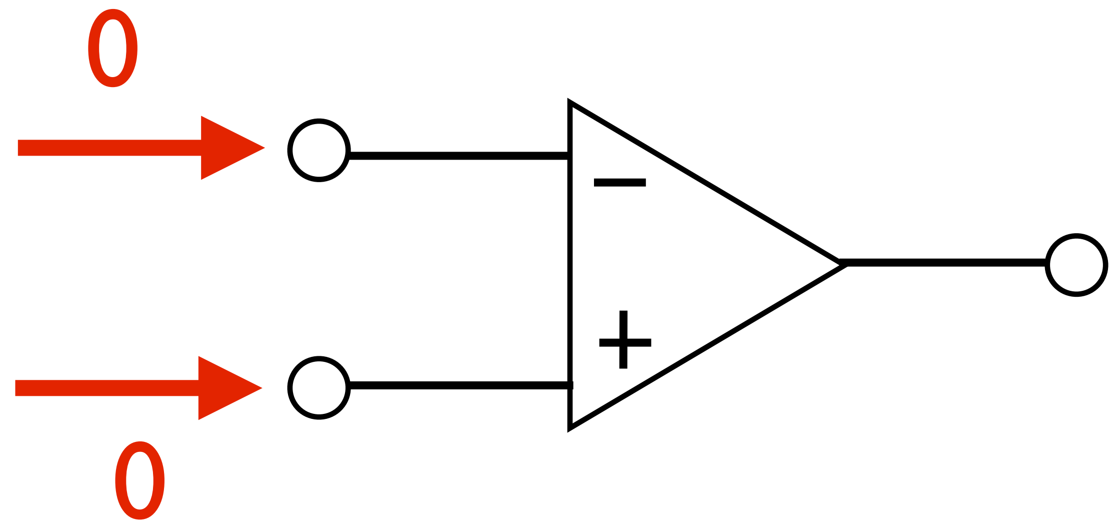


$$V_2 = V_1$$

Input resistance ∞

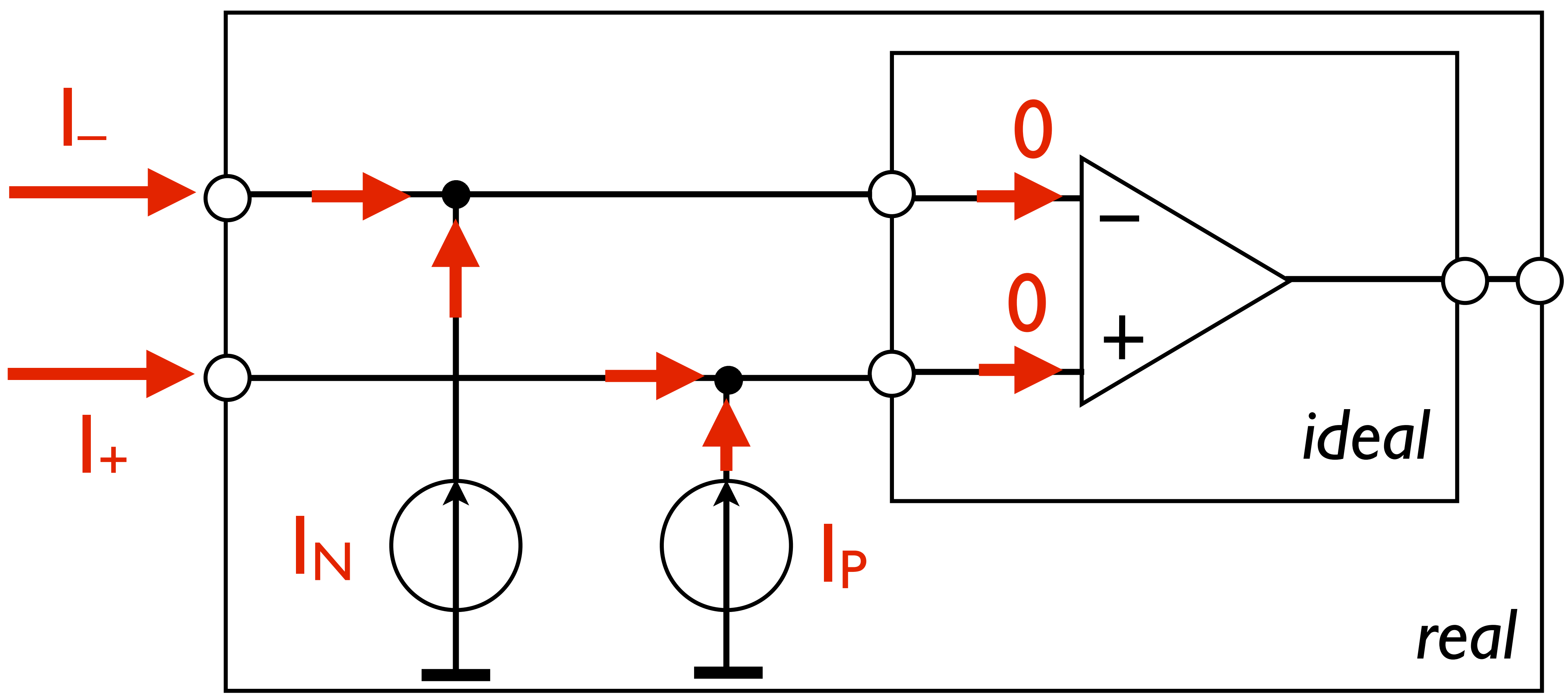
Output resistance 0

IDEAL OPAMP

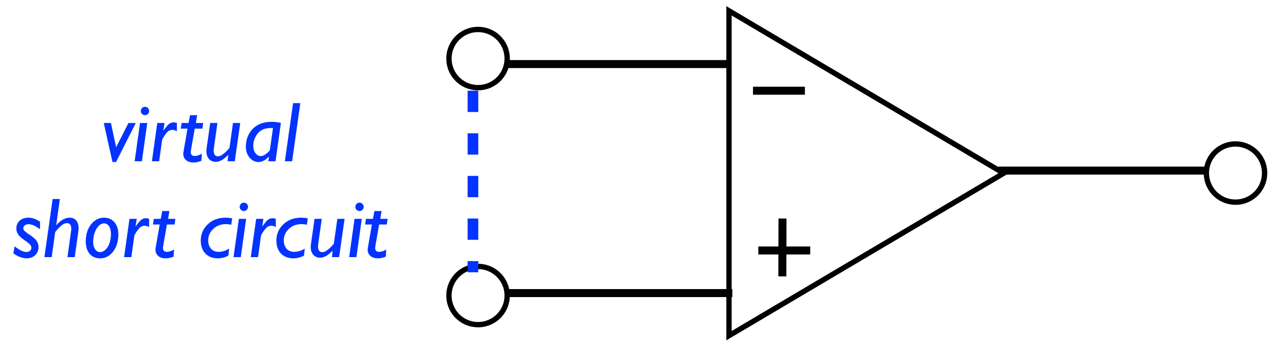


REAL OPAMP

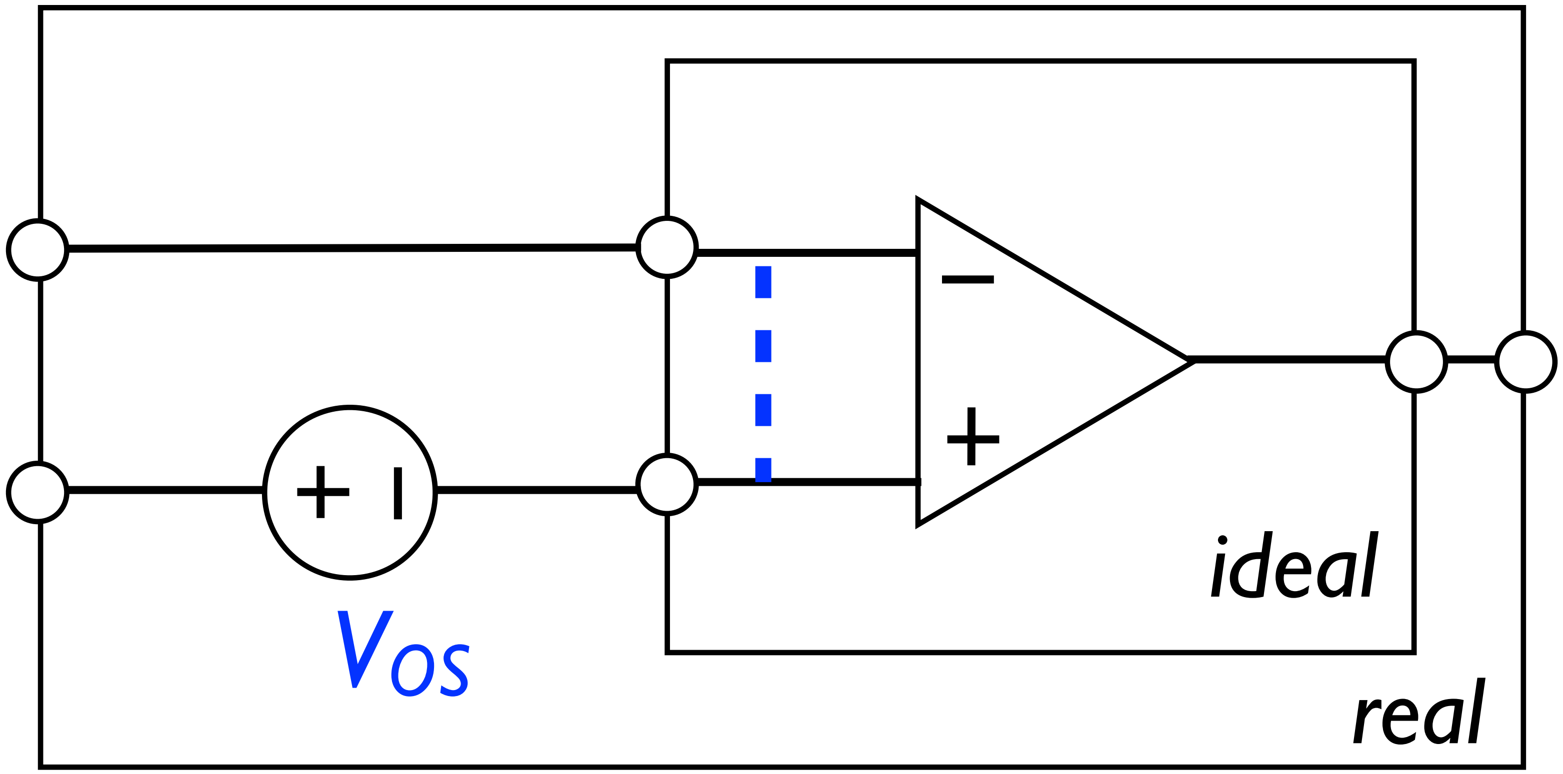
1) Bias currents



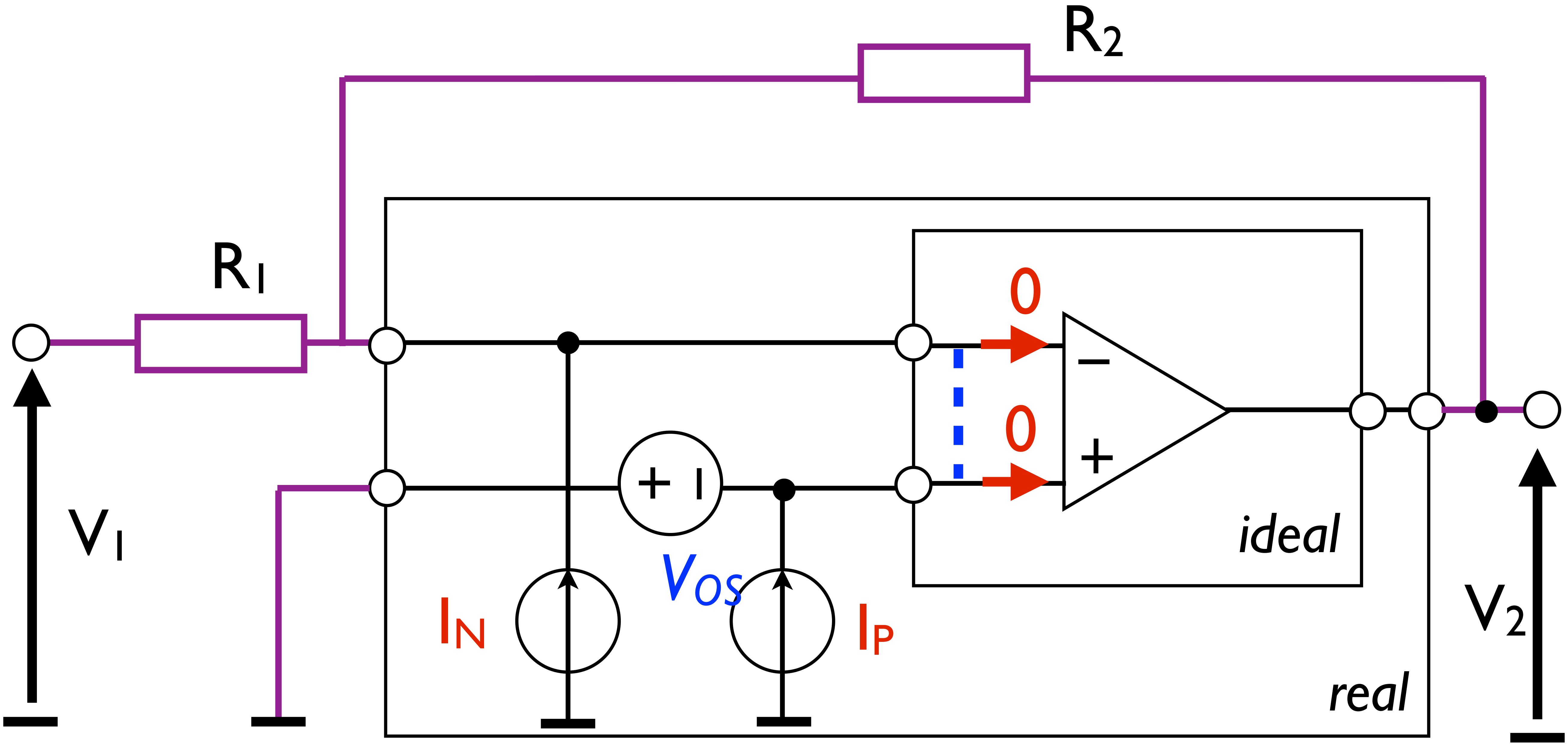
IDEAL OPAMP



REAL OPAMP
2) offset voltage

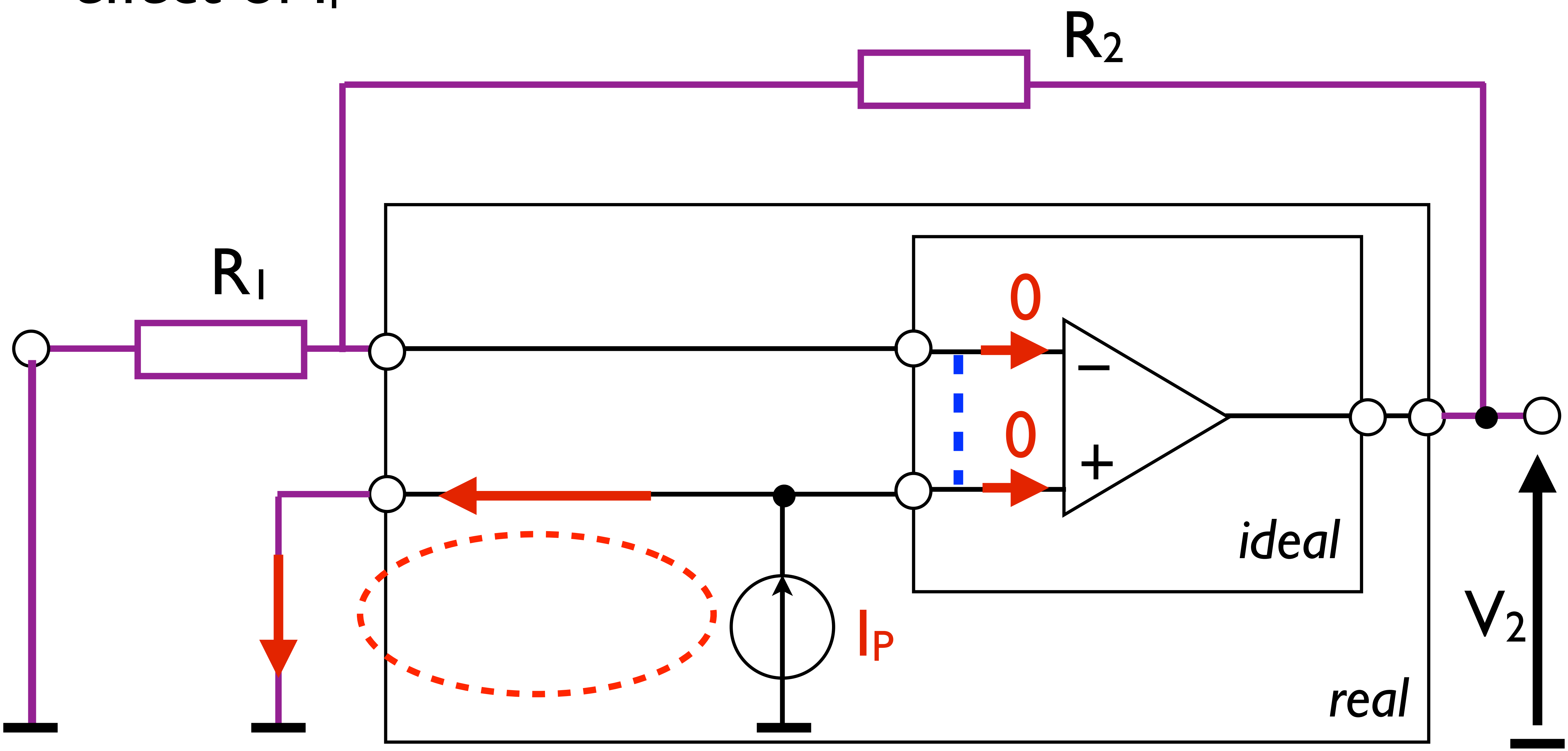


REAL INVERTING AMPLIFIER



REAL INVERTING AMPLIFIER

- effect of I_P

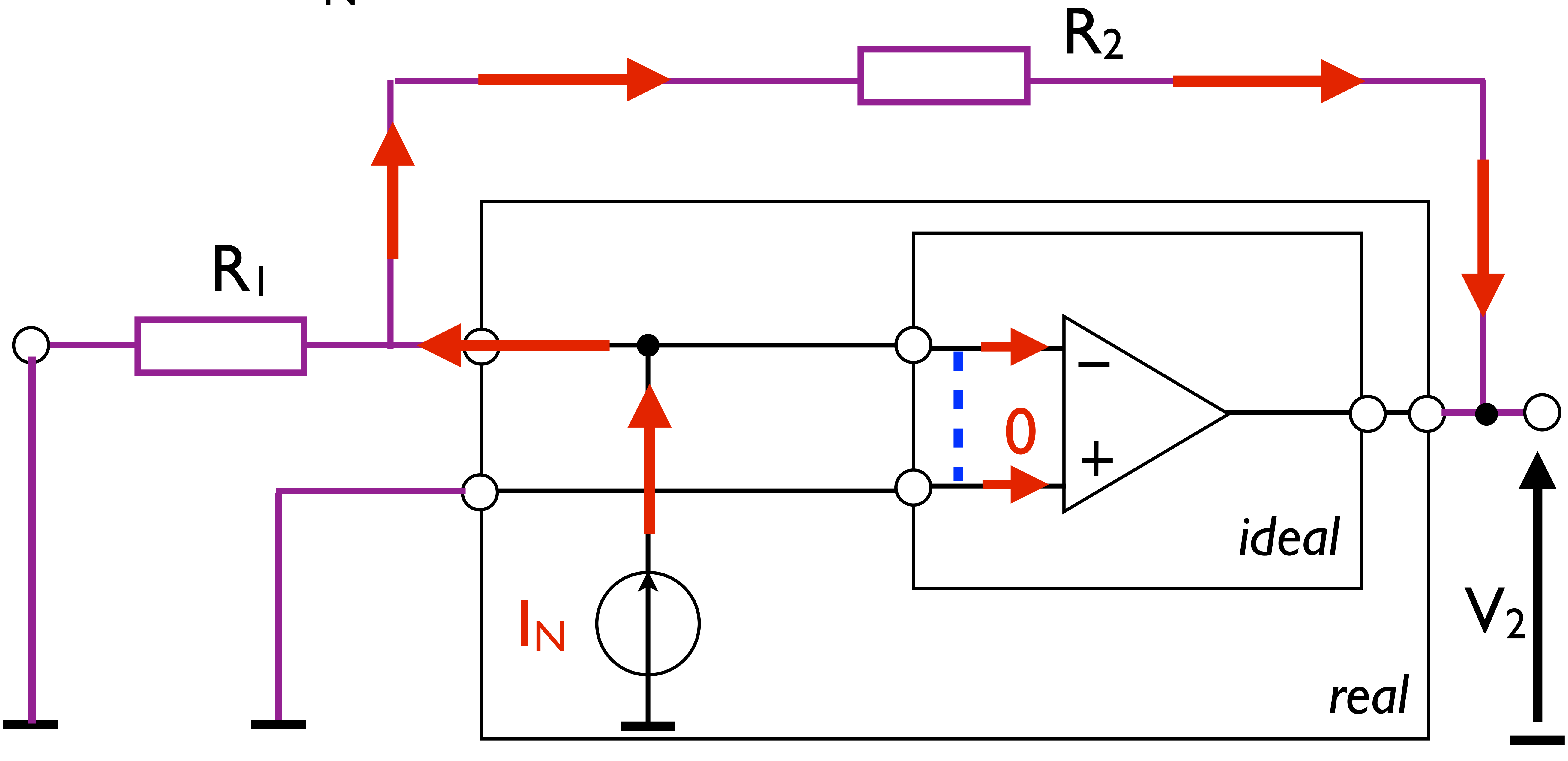


$V_2=0$

I_P has no effect on V_2

REAL INVERTING AMPLIFIER

- effect of I_N

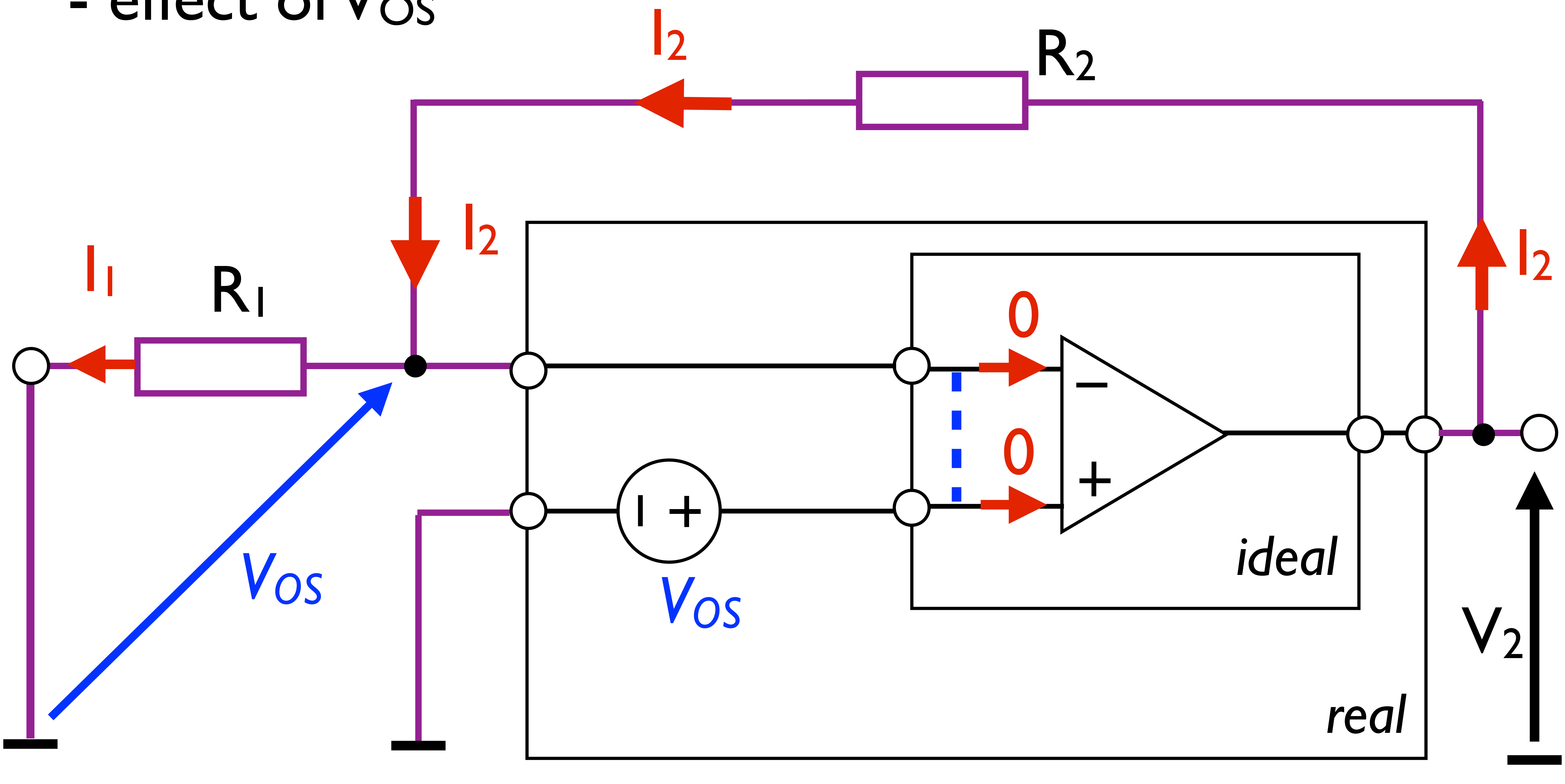


The current in R_1 is 0 because
R_1 is short-circuited

$$V_2 = R_2 \cdot I_N$$

REAL INVERTING AMPLIFIER

- effect of V_{OS}



$I_1 = {}^{V_{OS}}/_{R_1}$; $I_2 = I_1$; $V_2 = \dfrac{V_{OS}}{R_1} \cdot R_2 + V_{OS}$

REAL INVERTING AMPLIFIER

Taking all the contributions into account

$$V_2 = -\frac{R_2}{R_1} \cdot V_1 \pm R_2 \cdot I_N \pm V_{OS}\left(1 + \frac{R_2}{R_1}\right)$$

The input voltage will recalculated as:

$$V_1 = -\frac{R_1}{R_2} \cdot V_2 \mp R_1 \cdot I_N \mp V_{OS}\left(1 + \frac{R_1}{R_2}\right)$$

UNCERTAINTY: IDEAL INVERTING AMPLIFIER

$$V_1 = -\frac{R_1}{R_2} \cdot V_2$$

derivates to all variables

$$\frac{dV_1}{dV_2} = -\frac{R_1}{R_2} \qquad \frac{dV_1}{dR_1} = -\frac{V_2}{R_2} \qquad \frac{dV_1}{dR_2} = \frac{R_1}{R_2{}^2} \cdot V_2$$

$$u_{V_1}\Big|_{id} = \sqrt{\left(-\frac{R_1}{R_2} \cdot u_{V_2}\right)^2 + \left(-\frac{V_2}{R_2} \cdot u_{R_1}\right)^2 + \left(\frac{R_1}{R_2{}^2} \cdot V_2 \cdot u_{R_2}\right)^2}$$

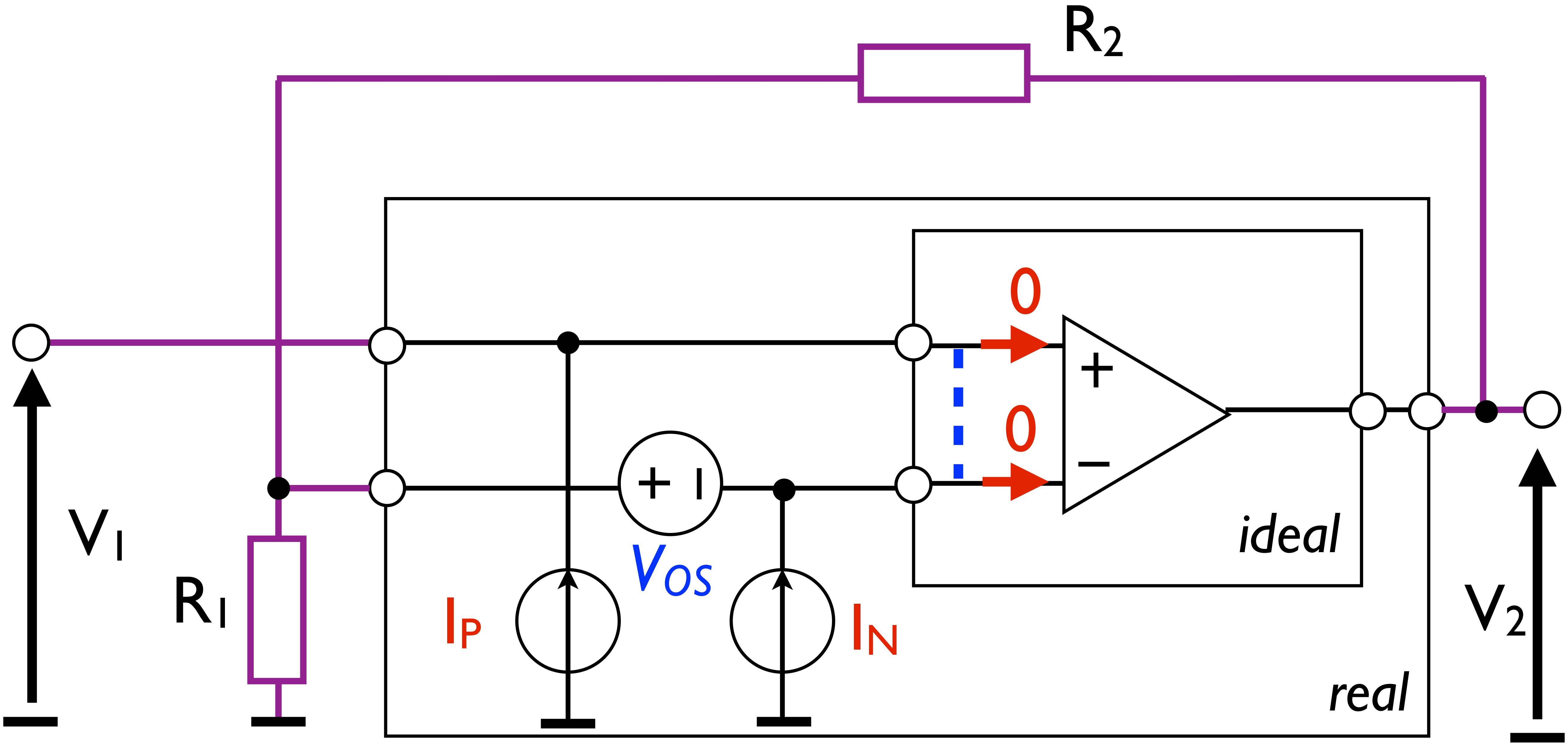
UNCERTAINTY: REAL INVERTING AMPLIFIER

$$V_1 = -\frac{R_1}{R_2} \cdot V_2 \mp R_1 \cdot I_N \mp V_{OS}\left(1 + \frac{R_1}{R_2}\right)$$

ideal | additional sources of uncertainty

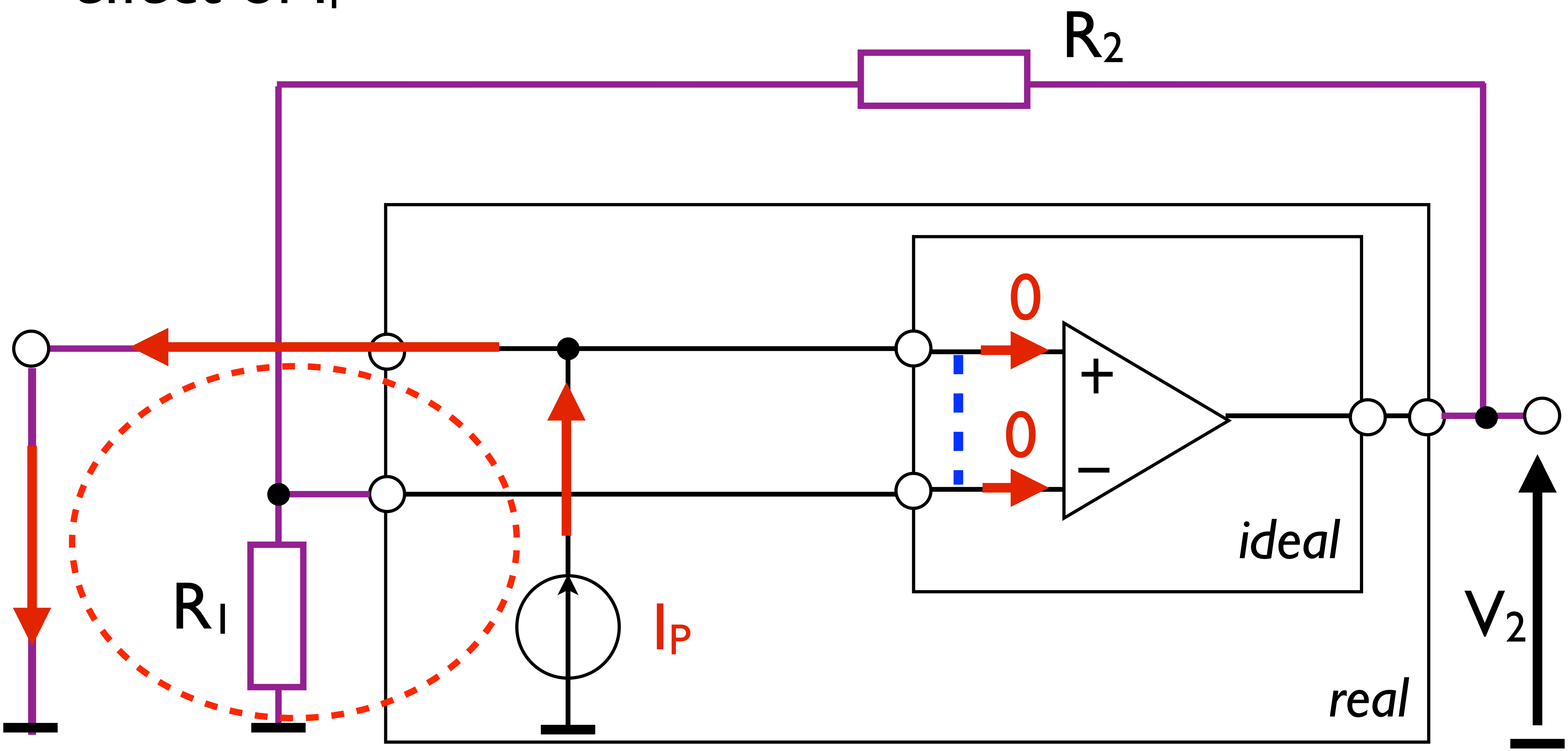
$$u_{V_1}\Big|_{real} = \sqrt{u_{V_1}\Big|_{id}^{2} \mp \left(\frac{R_1 \cdot I_N}{\sqrt{3}}\right)^2 \mp \left(\frac{V_{OS}\left(1 + \frac{R_1}{R_2}\right)}{\sqrt{3}}\right)^2}$$

REAL NON-INVERTING AMPLIFIER



REAL NON-INVERTING AMPLIFIER

- effect of I_P

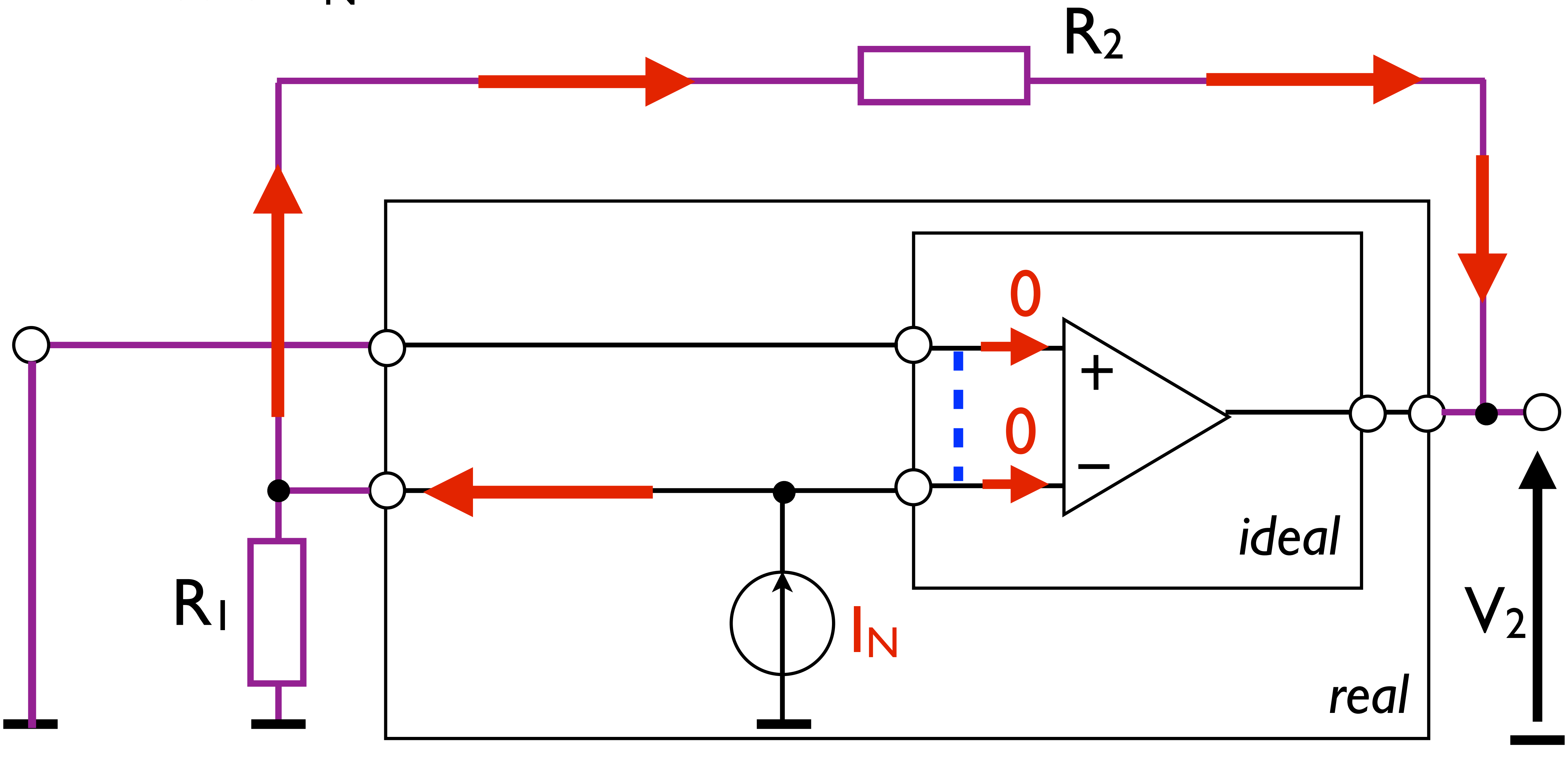


I_P has no effect on V_2

$V_2=0$

REAL NON-INVERTING AMPLIFIER

- effect of I_N

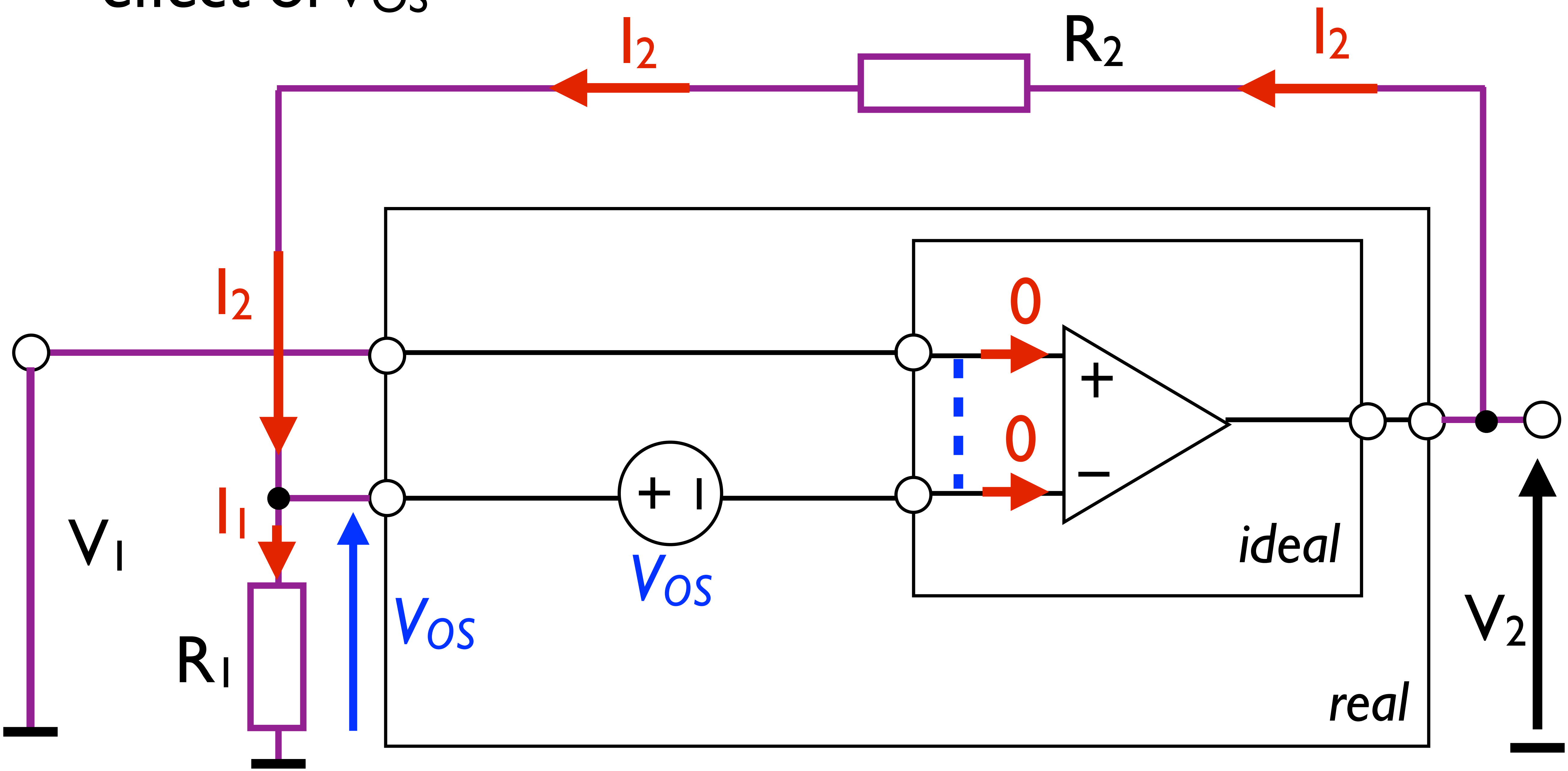


The current in R_1 is 0 because
R_1 is short-circuited

$$V_2 = R_2 \cdot I_N$$

REAL NON-INVERTING AMPLIFIER

- effect of V_{OS}



$$V_2 = \left(1 + \frac{R_2}{R_1}\right) V_{OS}$$

REAL NON-INVERTING AMPLIFIER

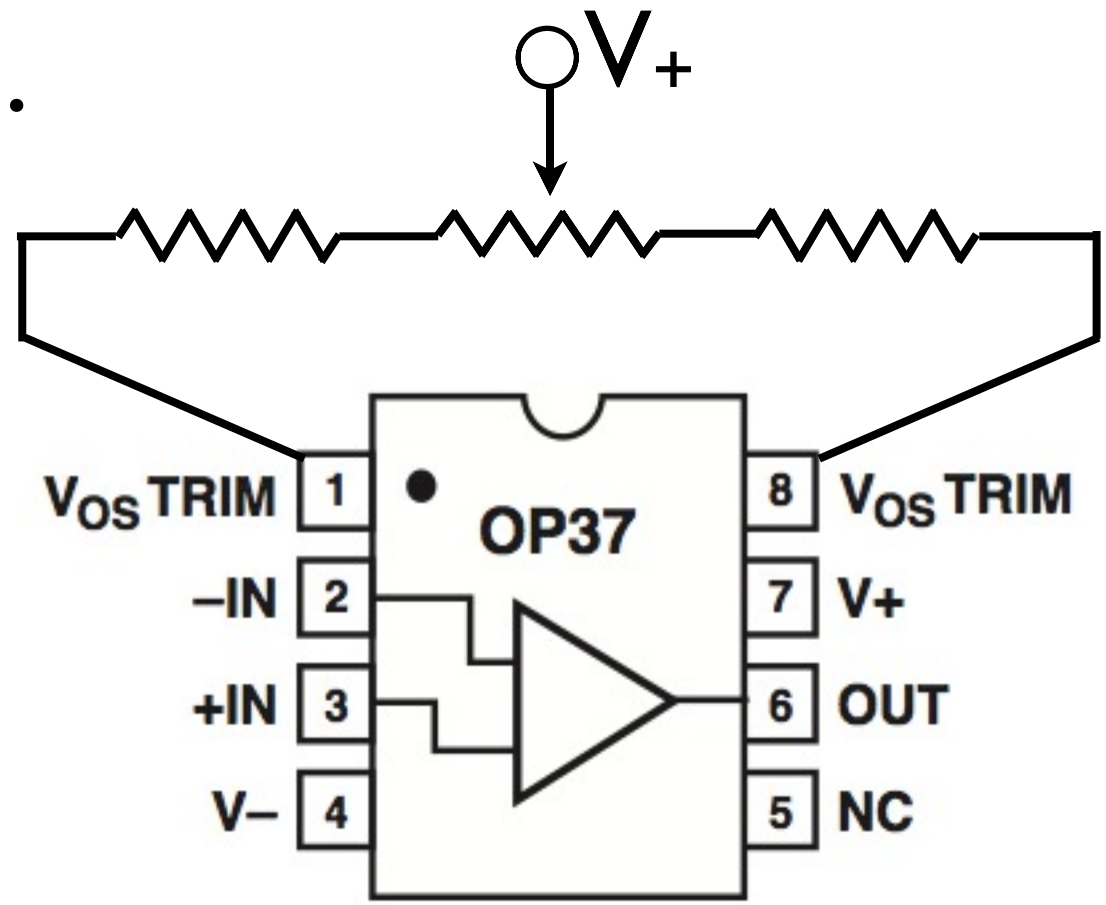
Taking all the contributions into account

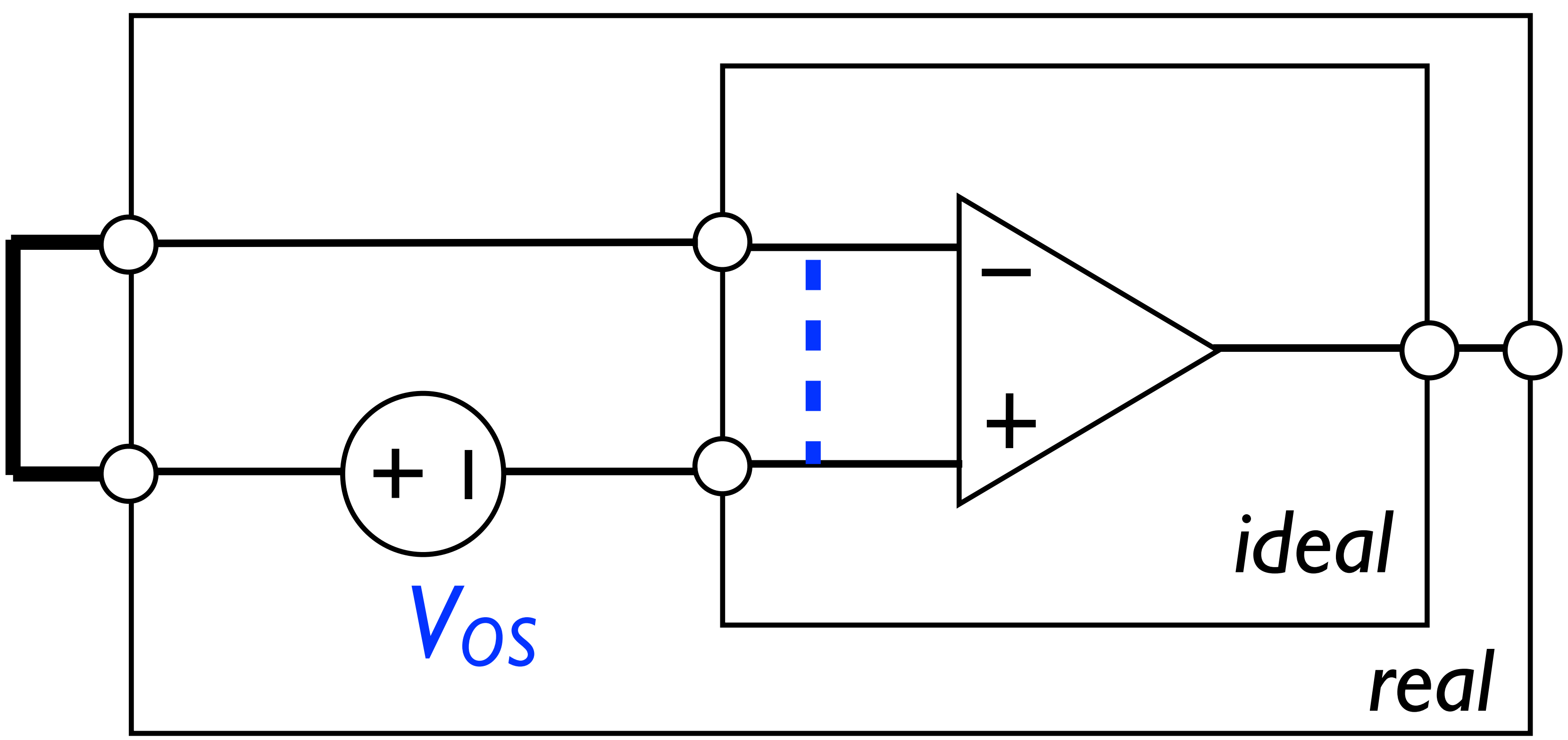
$$V_2 = \left(1 + \frac{R_2}{R_1}\right) V_1 \pm I_N \cdot R_2 \pm \left(1 + \frac{R_2}{R_1}\right) V_{OS}$$

The input voltage will recalculated as:

$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right) V_2 \mp I_N \cdot R_2 \left(\frac{R_1}{R_1 + R_2}\right) \mp V_{OS}$$

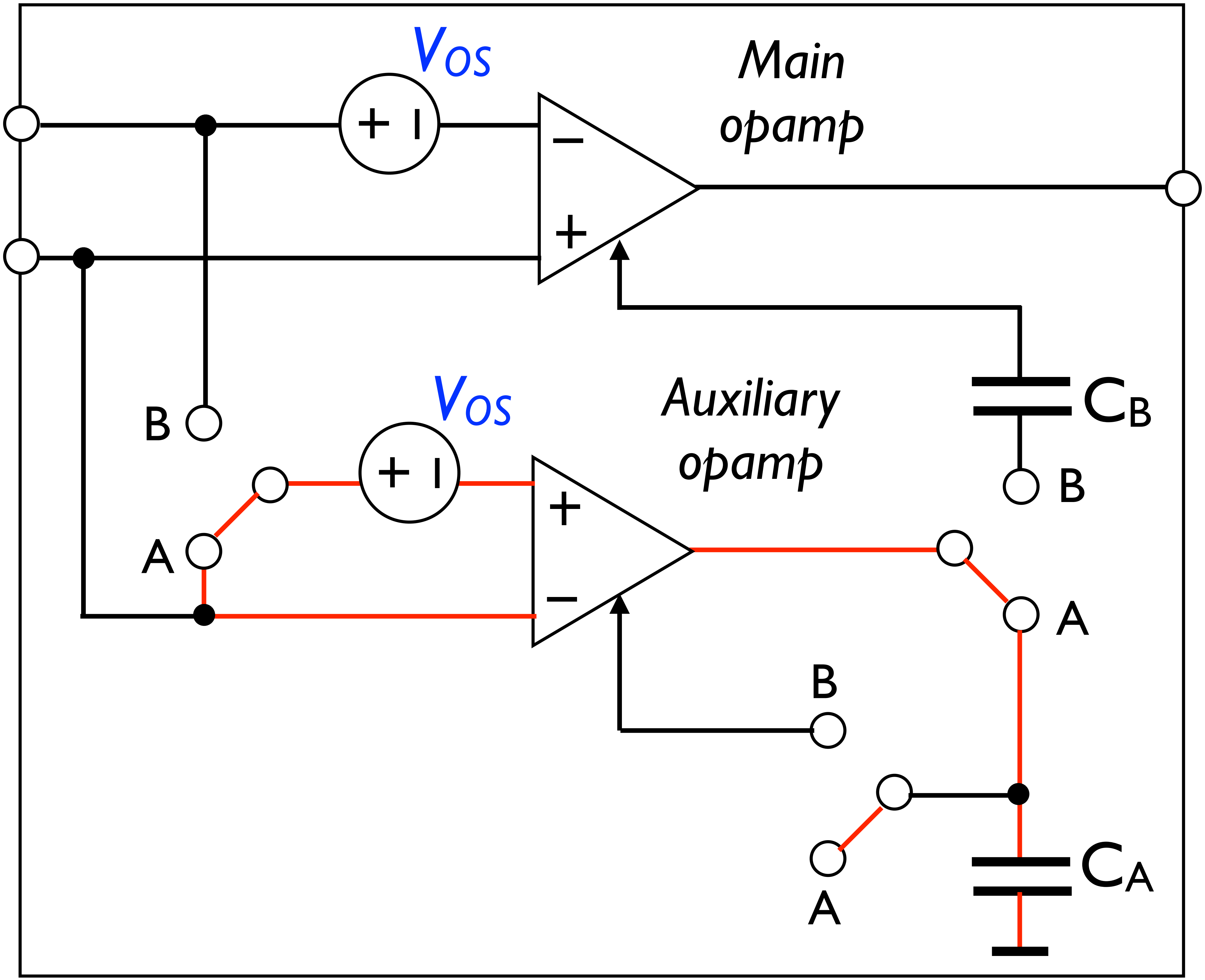
V_{OS} can be nulled...



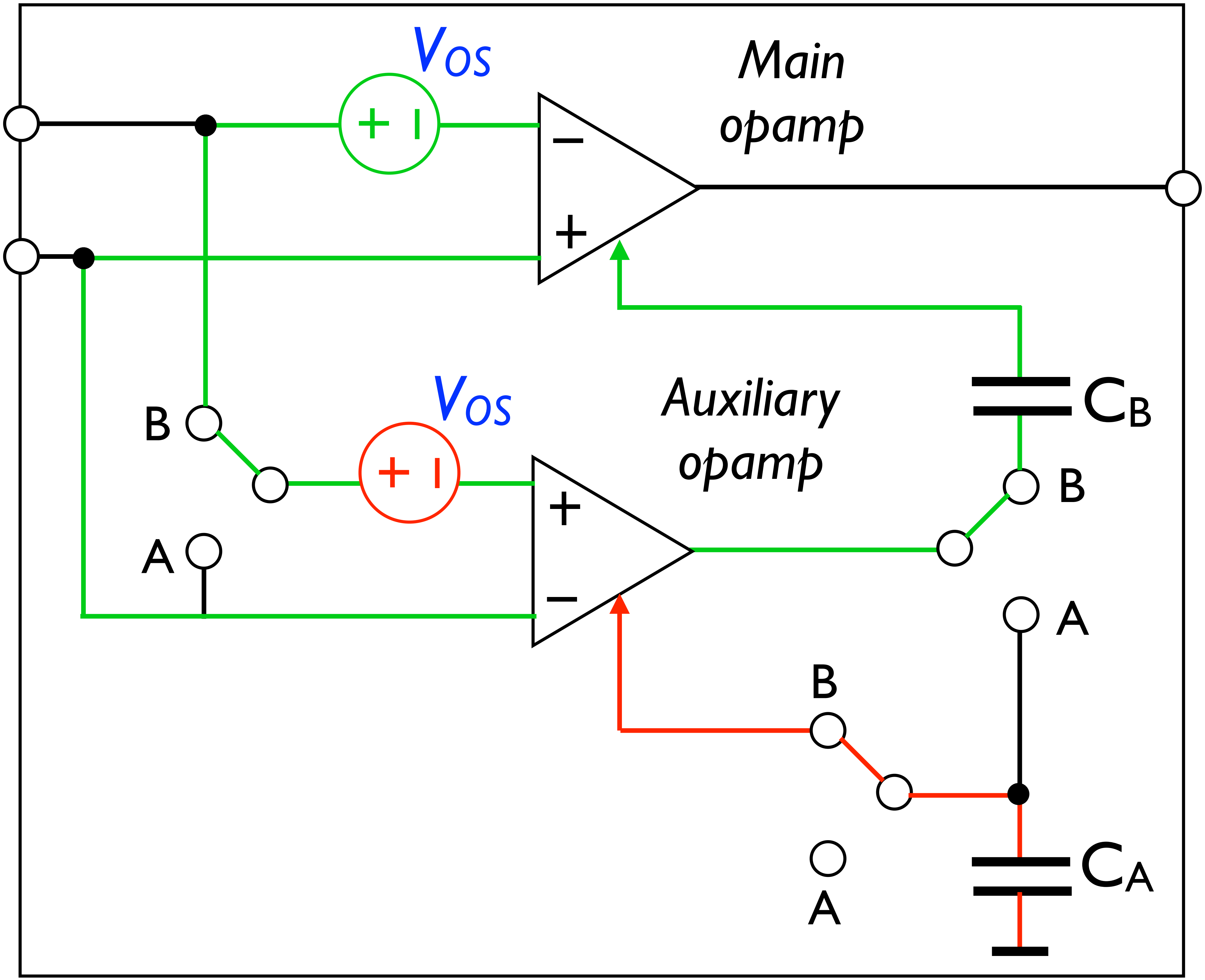


...but it drifts!

AUTOMATICALLY NULLED AMPLIFIER



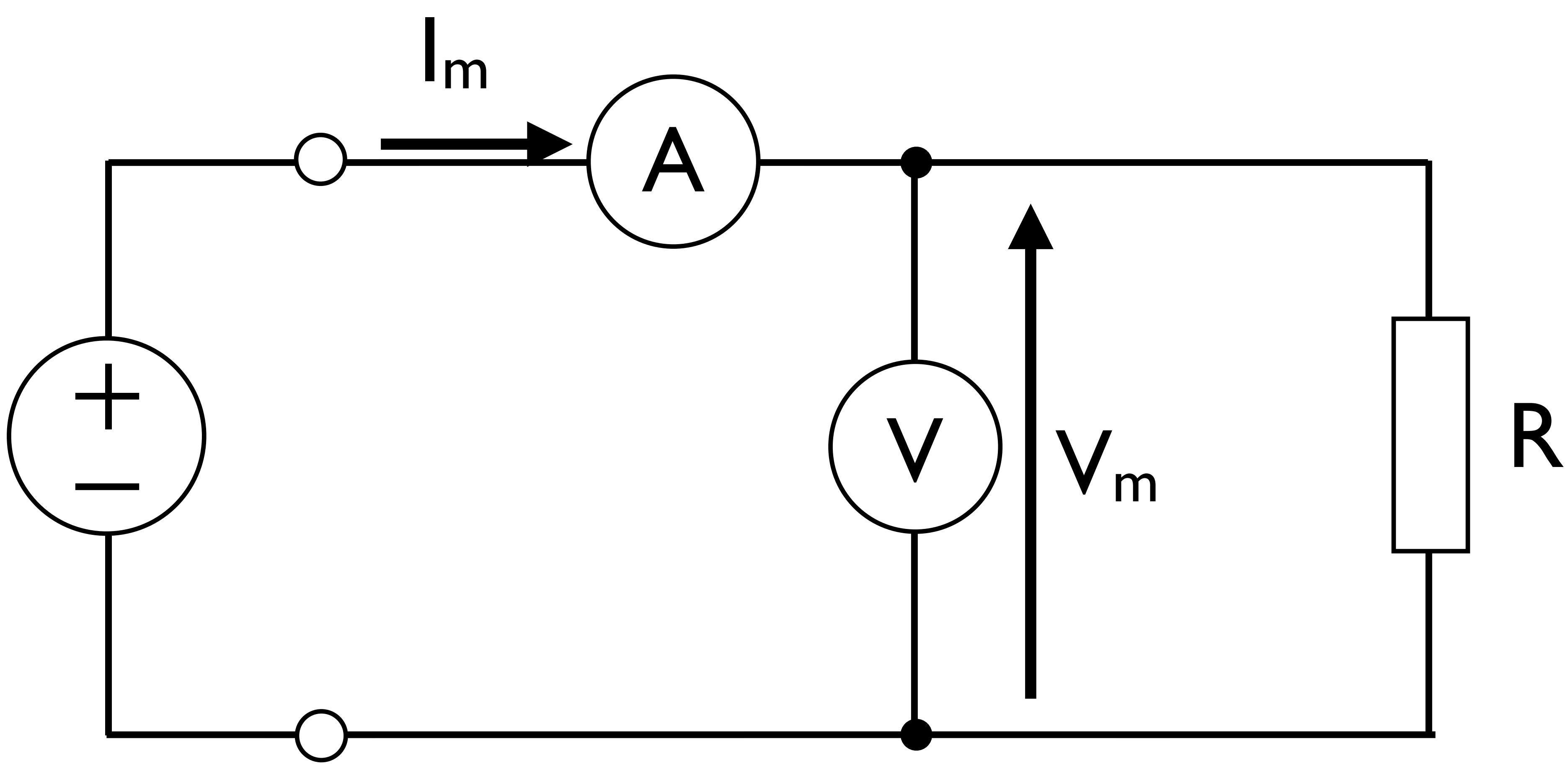
AUTOMATICALLY NULLED AMPLIFIER



MEASUREMENT OF RESISTANCE

V-A method

(aka the simplest way to measure a resistance)



$R = V_m / I_m$

MEASUREMENT OF RESISTANCE

V-A method

Uncertainty

Function to derivate $R(V_m, I_m) = V_m / I_m$

Derivates:

$$\frac{dR}{dV_m} = \frac{1}{I_m}$$

$$\frac{dR}{dI_m} = -\frac{V_m}{{I_m}^2}$$

Uncertainty of Resistance

$$u_R = \sqrt{\left(\frac{dR}{dV_m} \cdot u_{V_m}\right)^2 + \left(\frac{dR}{dI_m} \cdot u_{I_m}\right)^2}$$

$$= \sqrt{\left(\frac{1}{I_m} \cdot u_{V_m}\right)^2 + \left(-\frac{V_m}{{I_m}^2} \cdot u_{I_m}\right)^2}$$

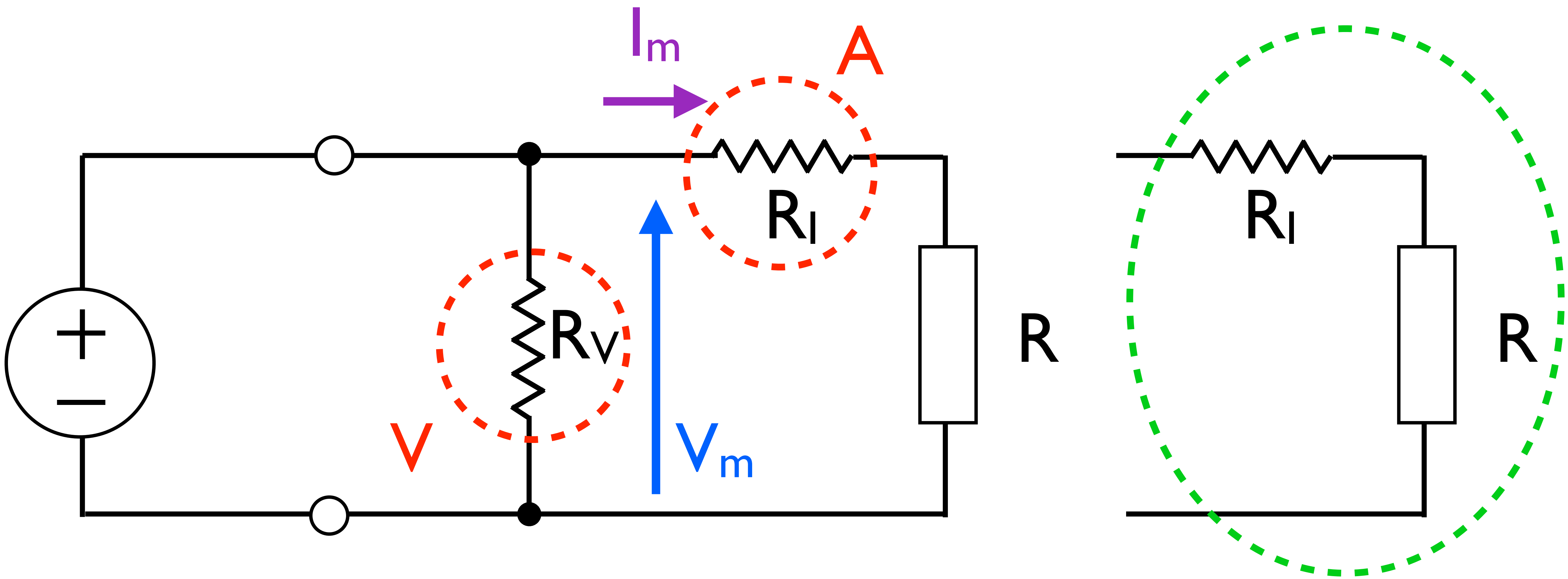
Let’s check if units are correct:

$$u_R = \sqrt{\left(\frac{1}{I_m} \cdot u_{V_m}\right)^2 + \left(-\frac{V_m}{I_m{}^2} \cdot u_{I_m}\right)^2}$$

$$u_R = \sqrt{\left(\frac{1}{[A]} \cdot [V]\right)^2 + \left(-\frac{[V]}{[A]^2} \cdot [A]\right)^2}$$

$$u_R = \sqrt{([\Omega])^2 + ([\Omega])^2} = [\Omega]$$

Problem: non-ideal instruments

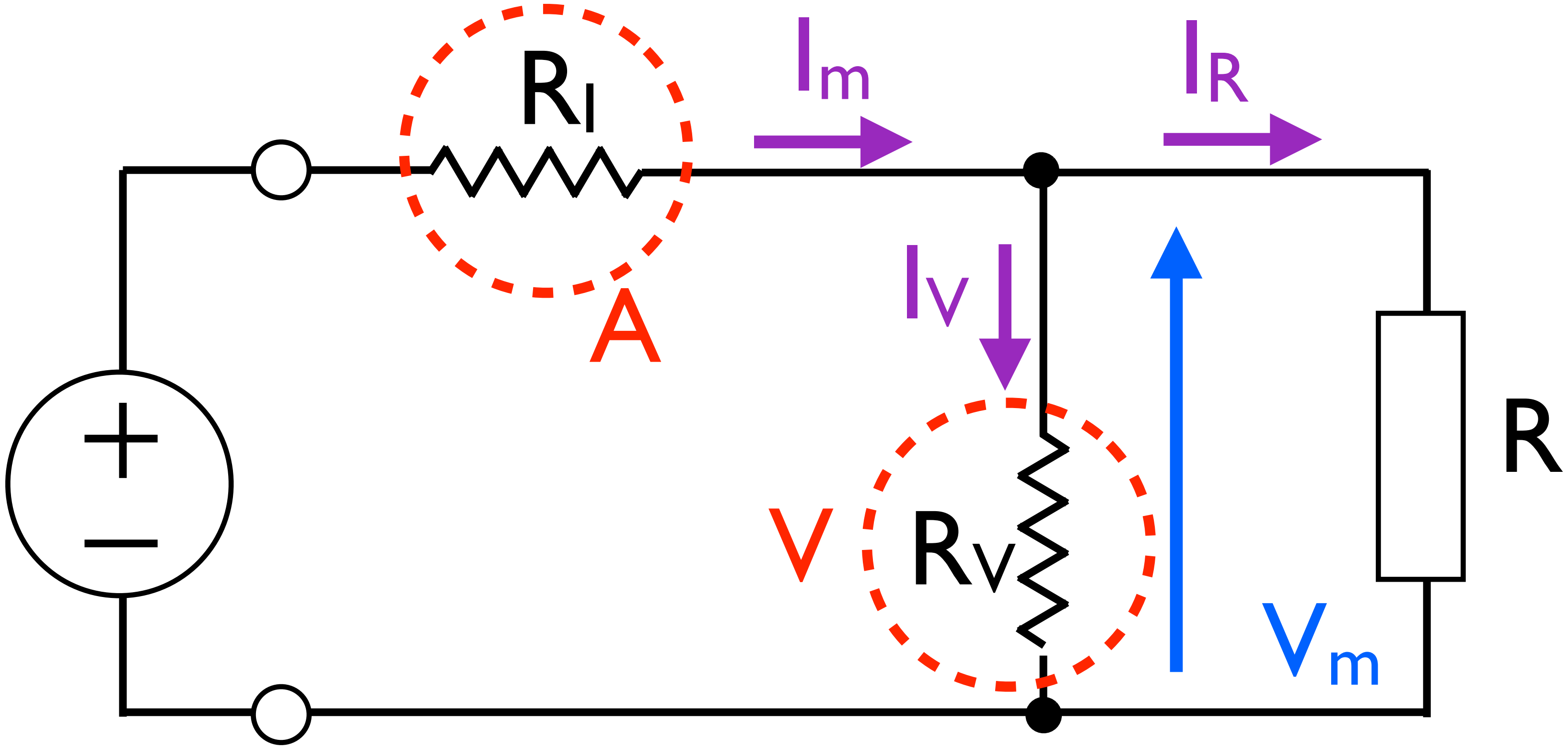


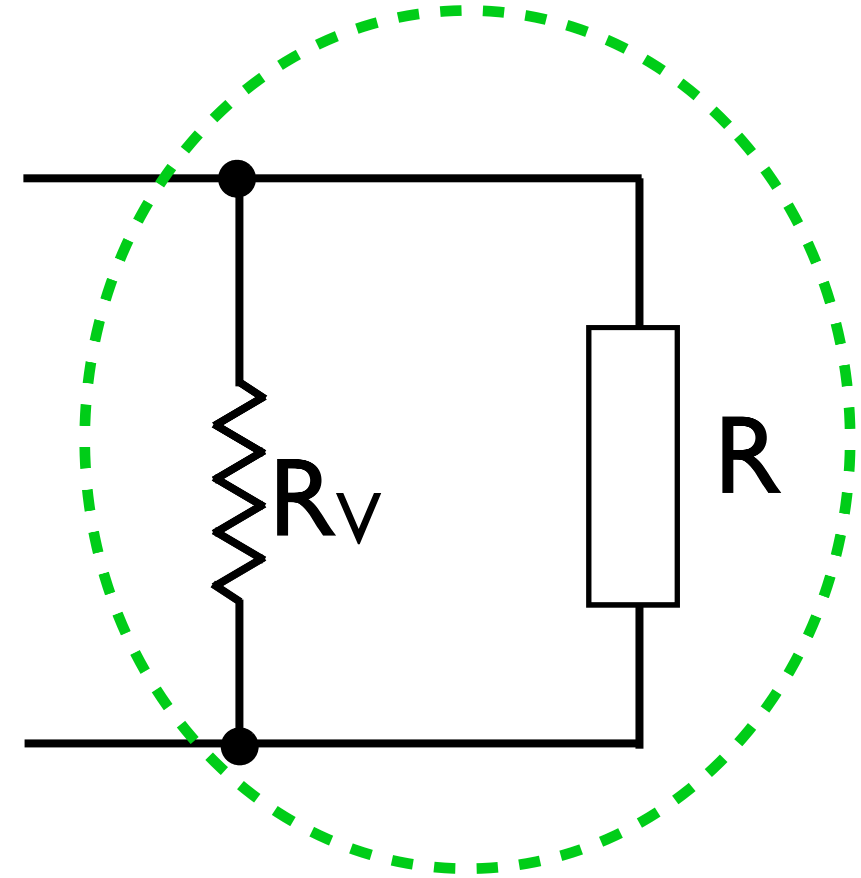
$R_m = V_m / I_m = R_I + R$

Correction of measured value

$$R = \frac{V_m}{I_m} - R_I$$

Problem: non-ideal instruments





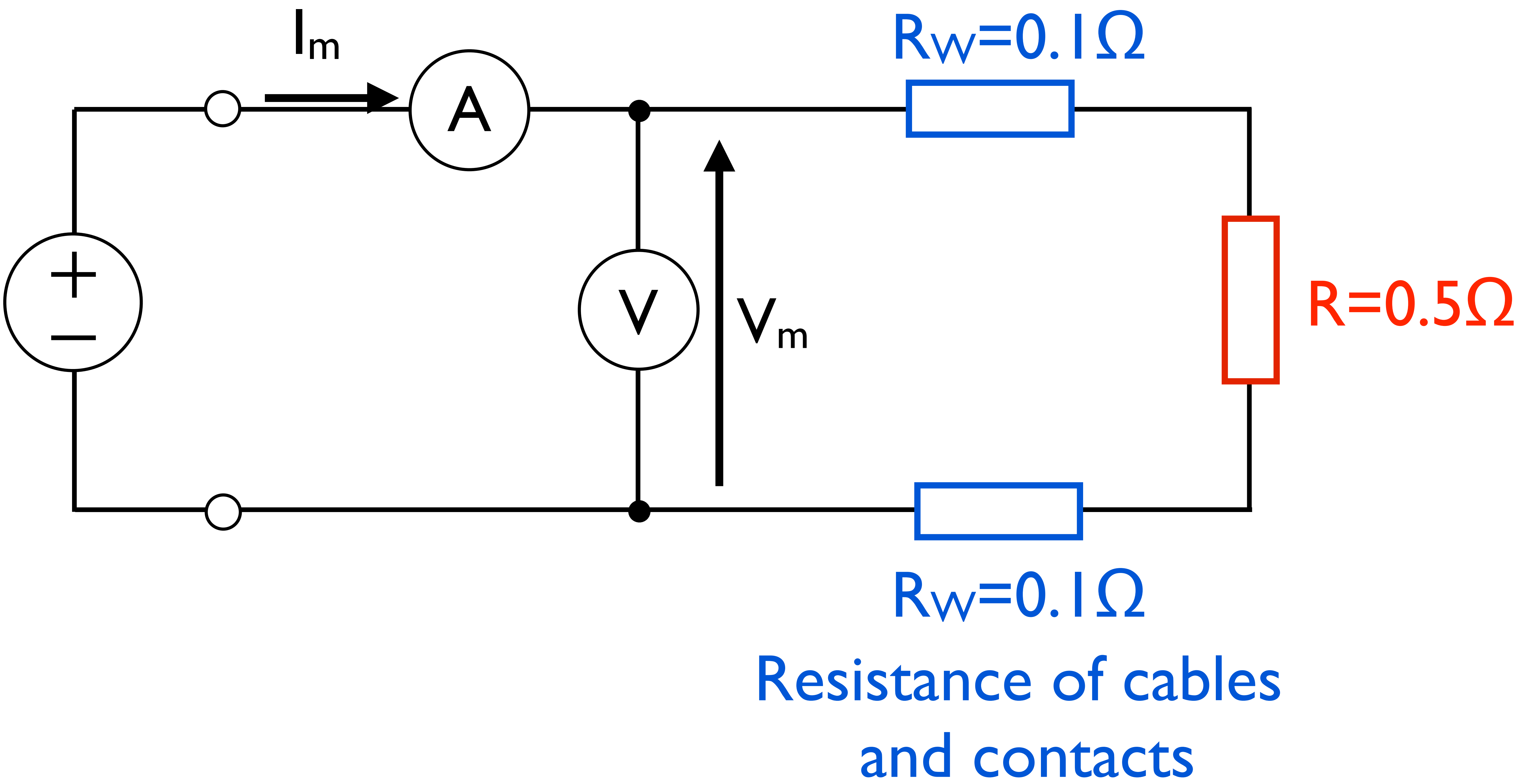
what you measure

$$\frac{V_m}{I_m} = \frac{1}{1/_R - 1/_{R_V}}$$

Correction of measured value

$$R = \frac{V_m}{I_m - I_V} = \frac{V_m}{I_m - V_V/_{R_V}}$$

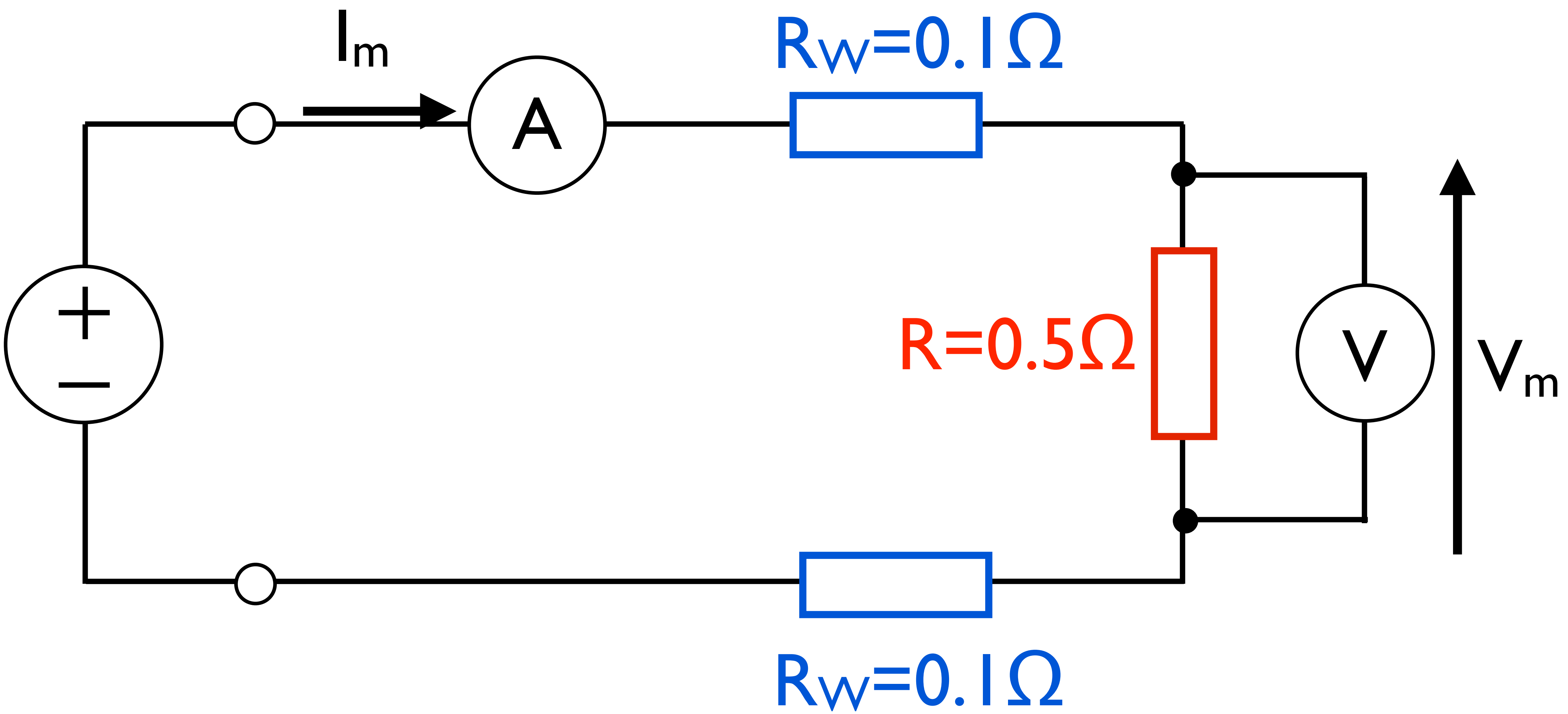
MEASUREMENT OF LOW RESISTANCE



We actually measure $V_m/I_m = R + 2R_W$

MEASUREMENT OF LOW RESISTANCE

Four wire method

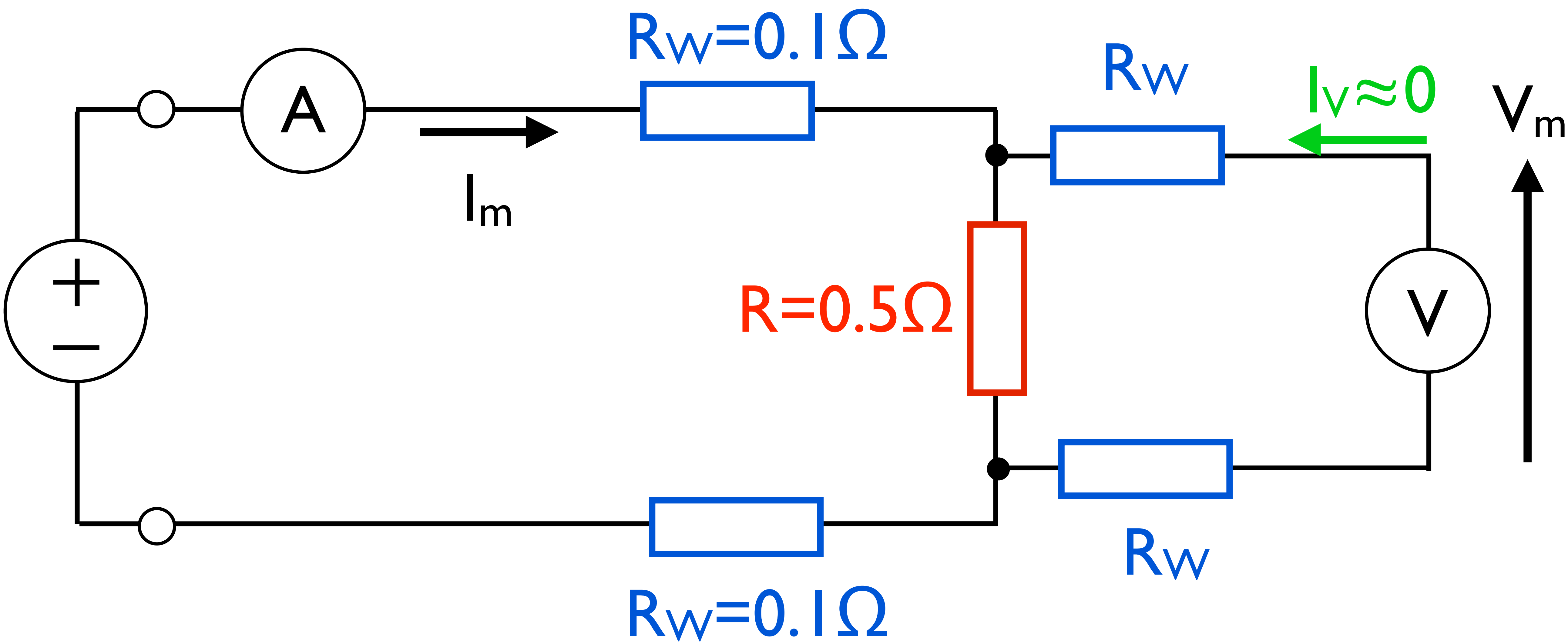


In this case we measure $V_m/I_m = R$

MEASUREMENT OF LOW RESISTANCE

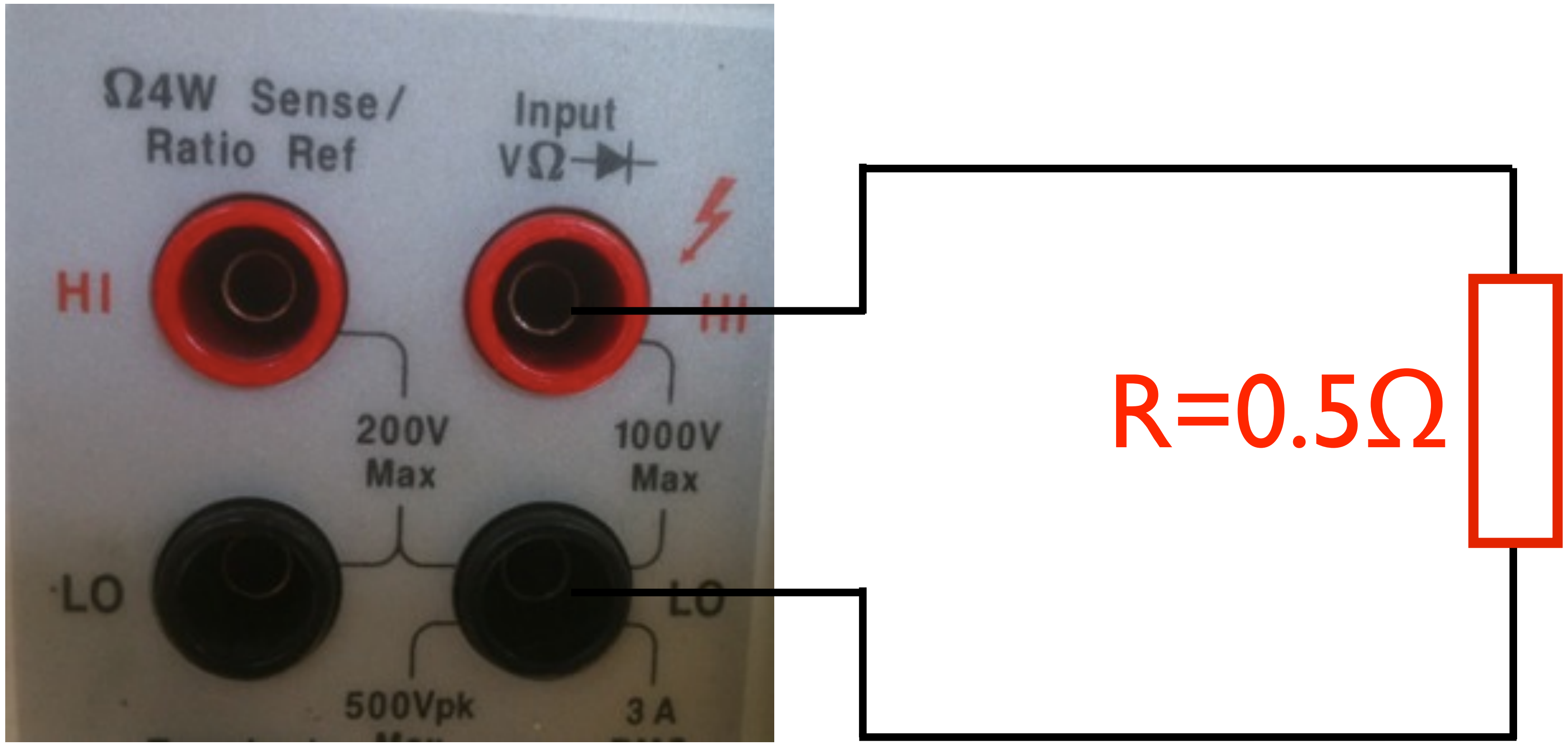
Four wire method

What about the cables of the voltmeter?

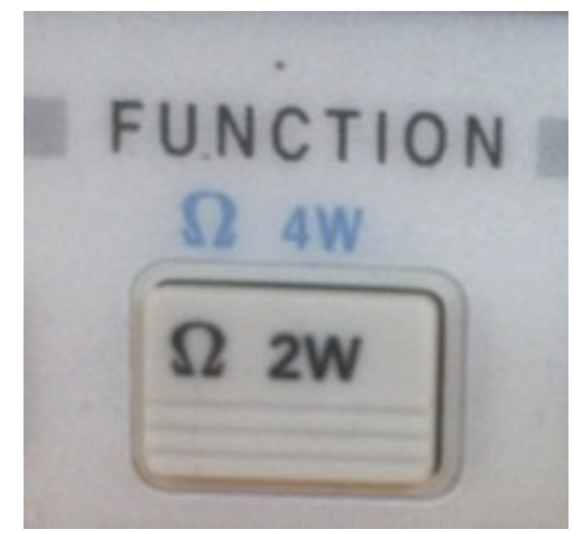


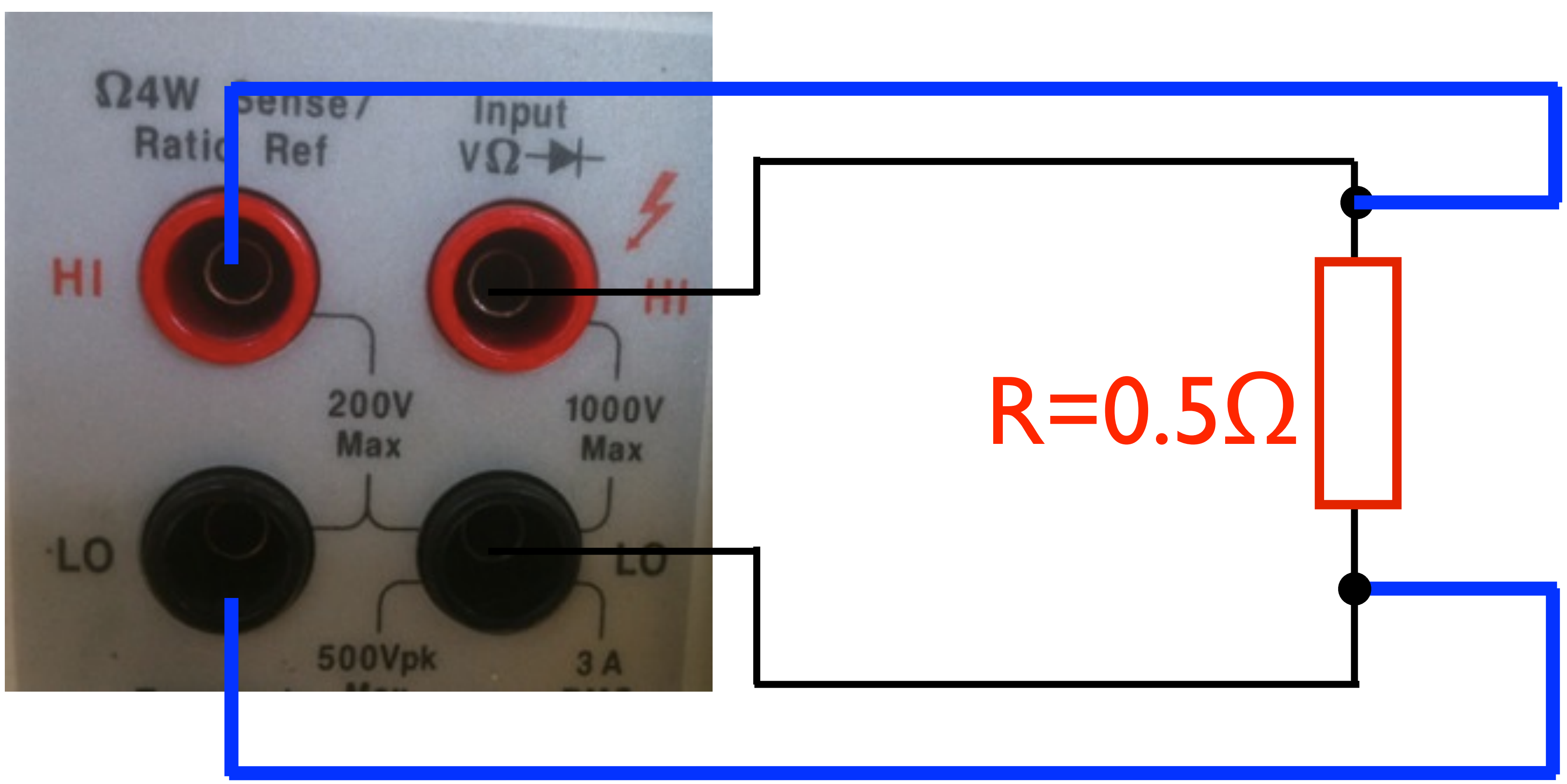
The voltage drop on R_W is negligible because $I_V \approx 0$

MEASUREMENT OF LOW RESISTANCE



Two wire method





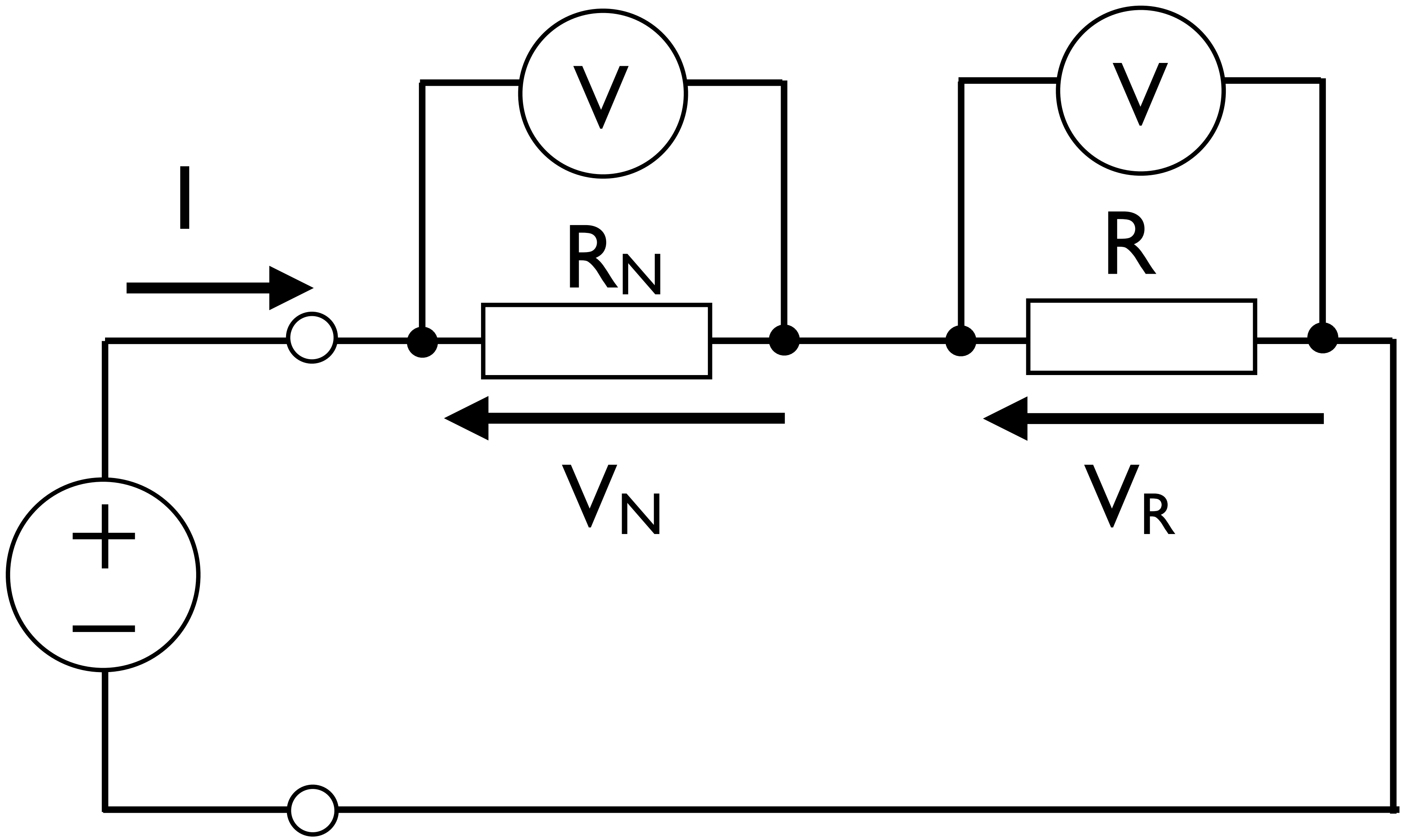
Four wire method

What if the resistance is very, very low?

We should use large current to obtain an acceptable voltage. Often, it's a problem to measure a large current

By using COMPARISON METHOD we do not need to measure the current anymore

COMPARISON METHOD



$V_N = R_N \cdot I$ $V_R = R \cdot I$ both resistor share the same current

$$\frac{V_R}{V_N} = \frac{R \cdot I}{R_N \cdot I} = \frac{R}{R_N}$$

$$R = \frac{V_R}{V_N} \cdot R_N$$

COMPARISON METHOD

$$R = \frac{V_R}{V_N} \cdot R_N$$

Sources of uncertainty are only the measured voltages and the standard resistor

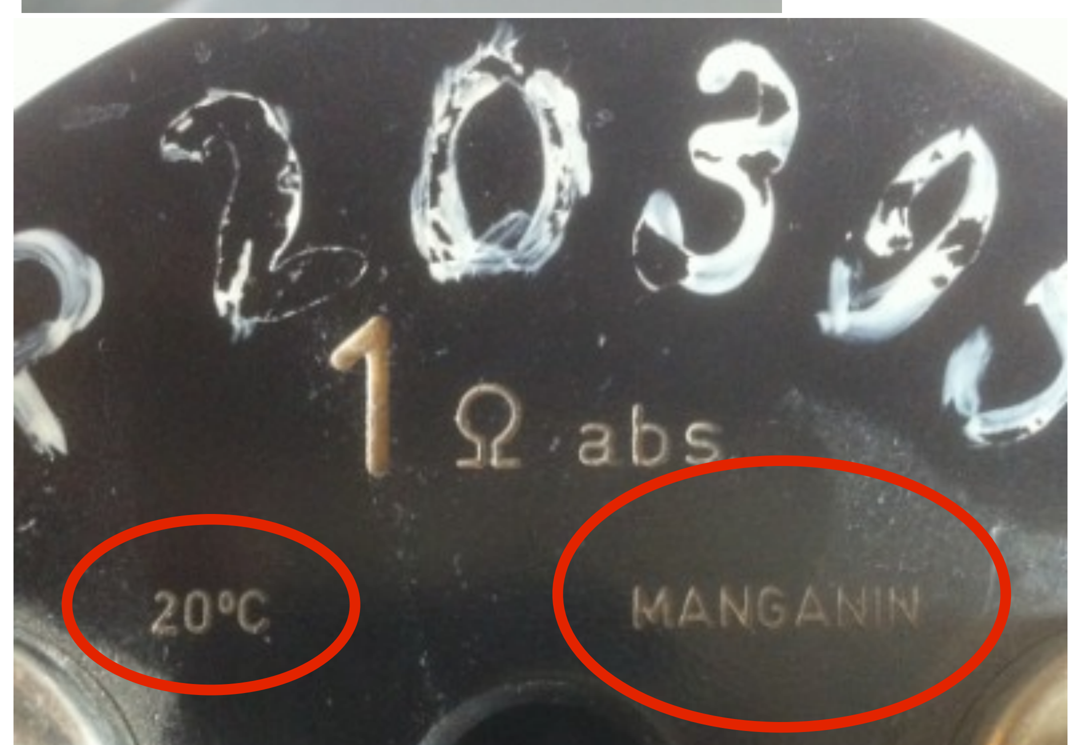
STANDARD RESISTORS

PRIMARY standards	SECONDARY standards
Quantum Hall effect	*Wire (>1Ω) of sheet (<1Ω) of alloy with low temperature coefficient*
	For AC: quadrifilar resistors

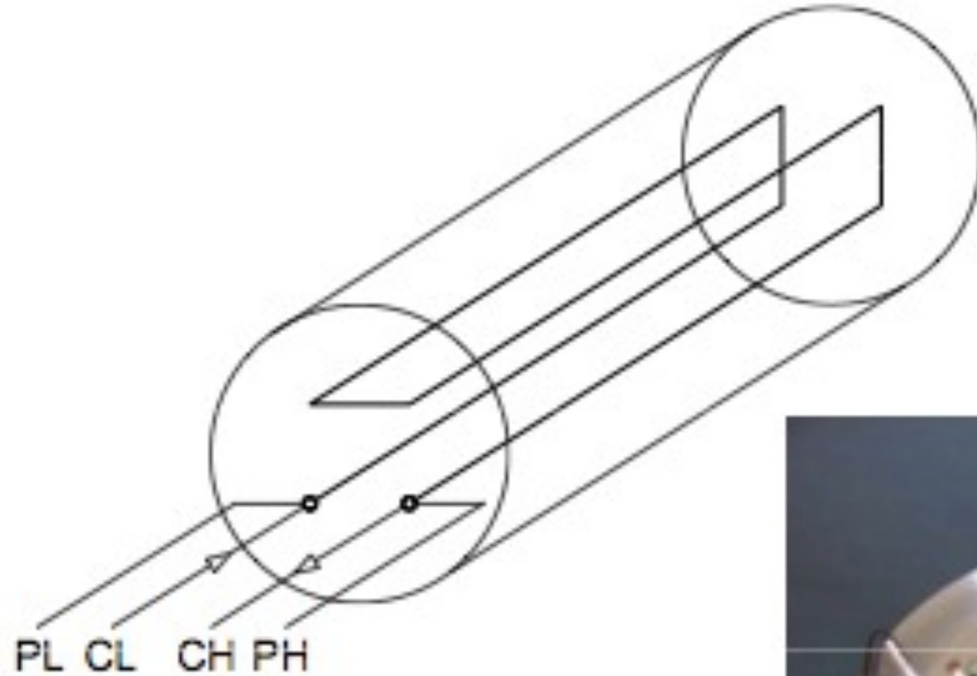
SECONDARY STANDARDS

Accuracy: 0.01%

Two connectors per terminal



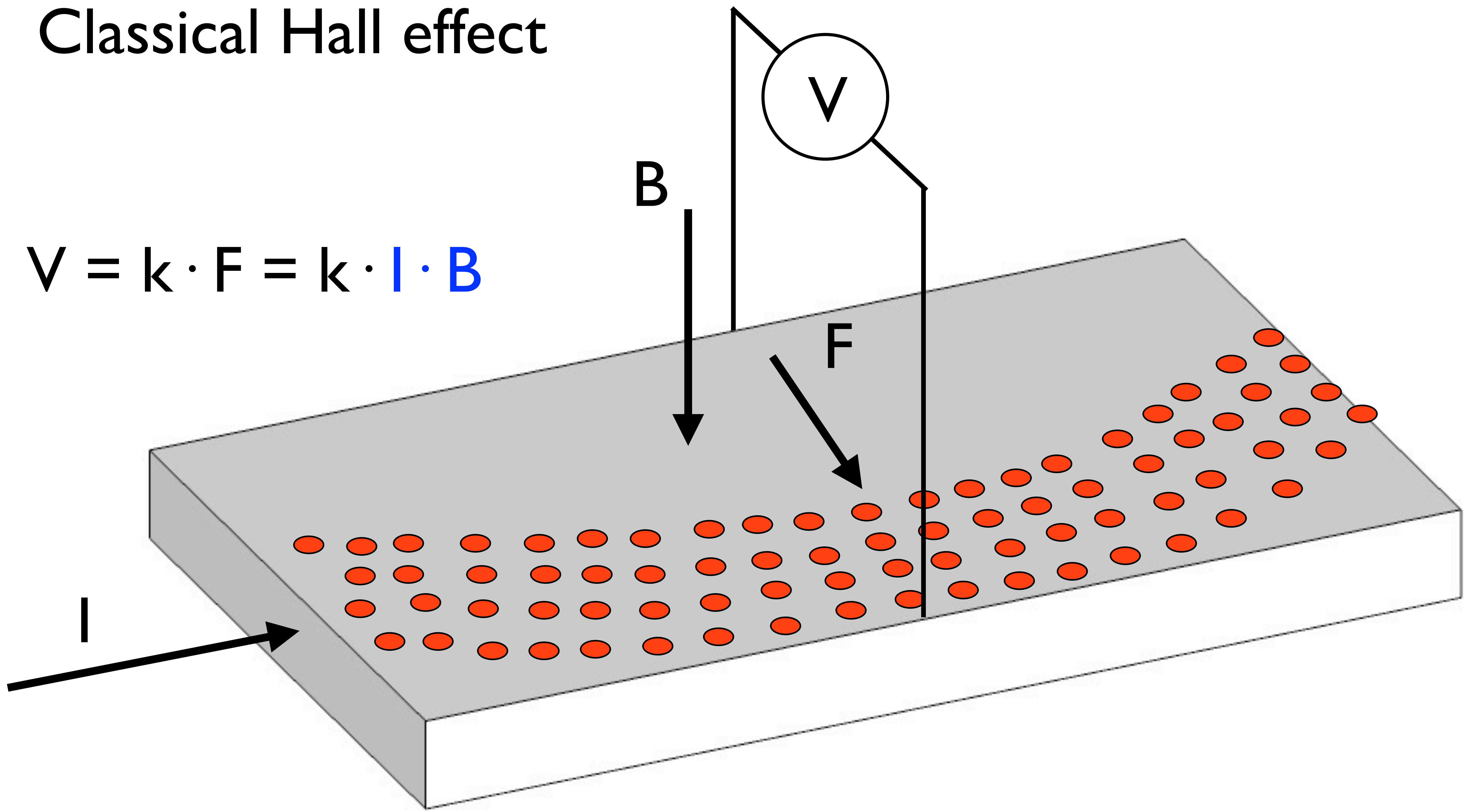
QUADRIFILAR STANDARDS



PRIMARY STANDARDS

Classical Hall effect

$V = k \cdot F = k \cdot I \cdot B$



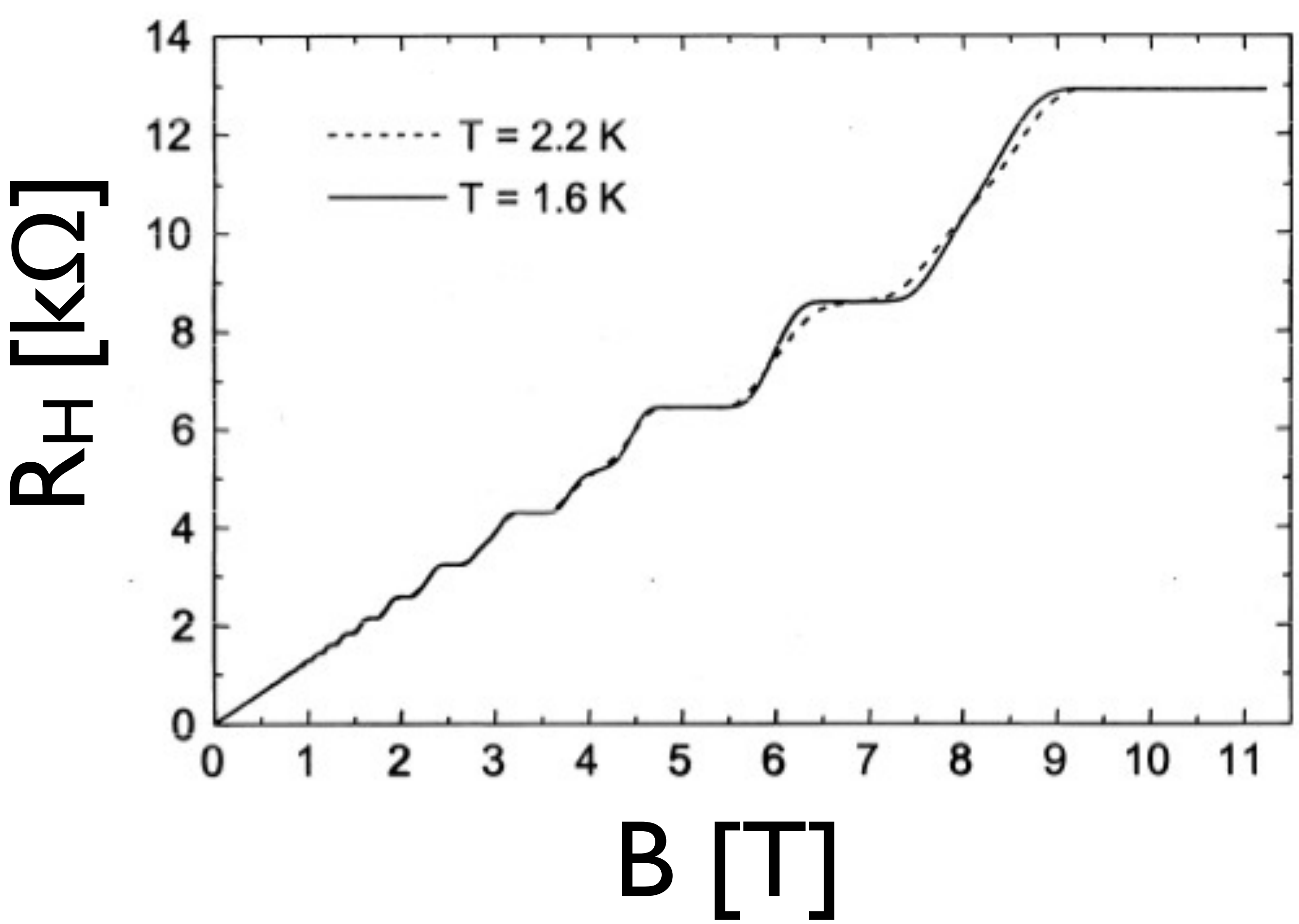
Hall resistance $R_H = V / I = k \cdot B$ Linear to B

PRIMARY STANDARDS

Quantum Hall effect

- very low temperature
- very high magnetic field

The Hall resistance is not linear to B but quantized

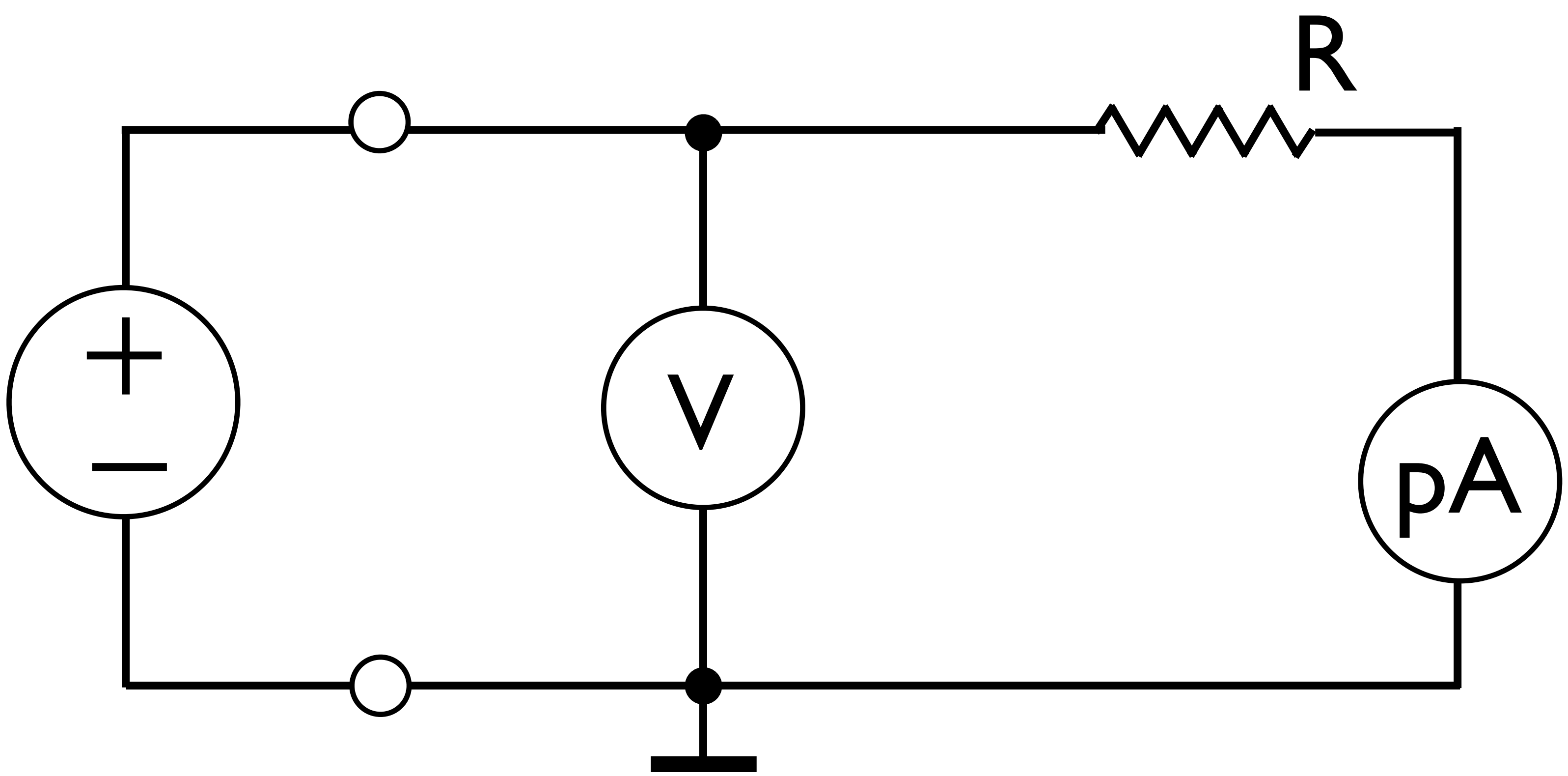


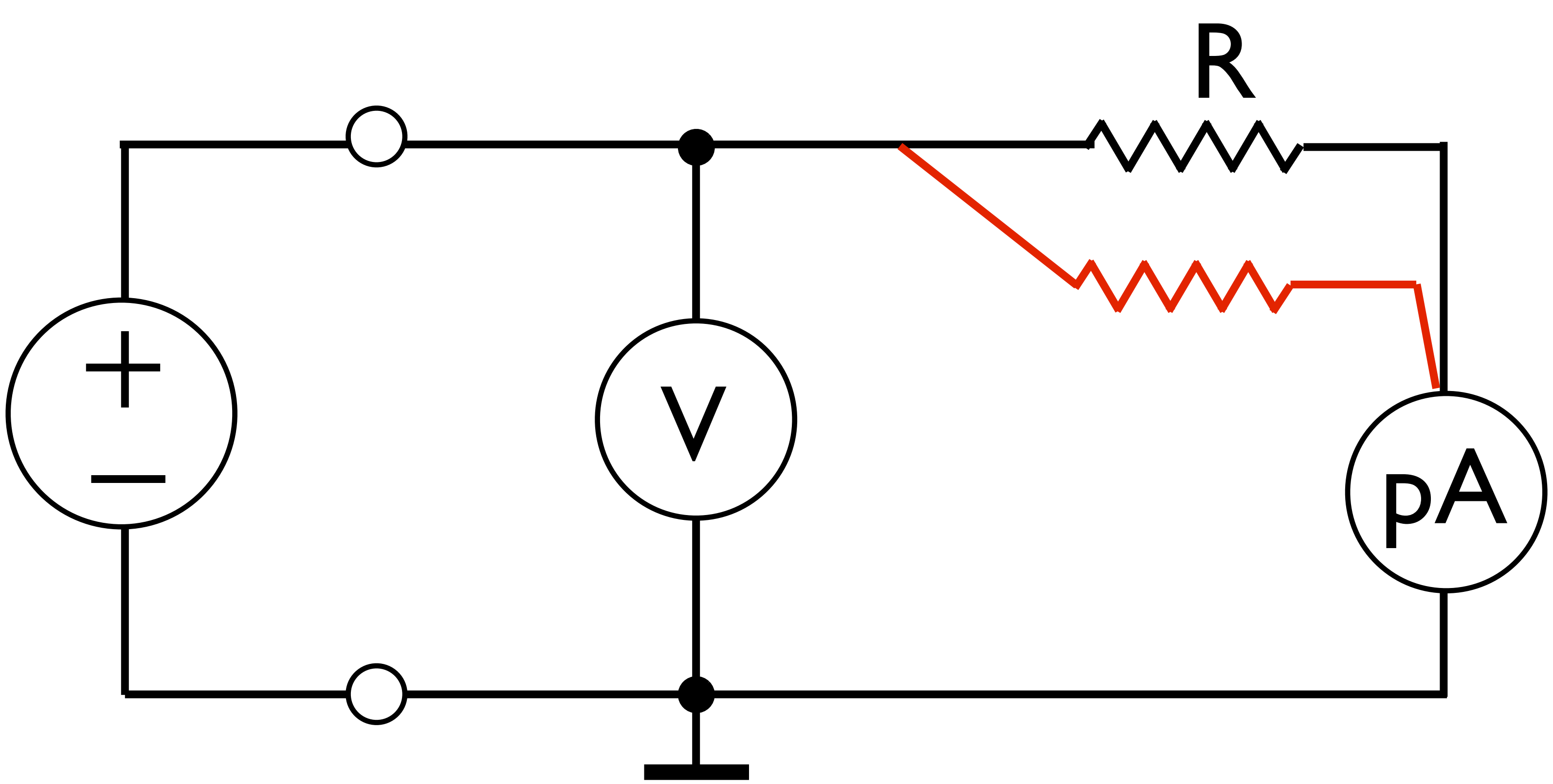
Steps of resistance at:

$$R_H = \frac{1}{i} \cdot \frac{h}{e^2}$$

$$R_H = \frac{1}{i} \cdot 25812.8\Omega$$

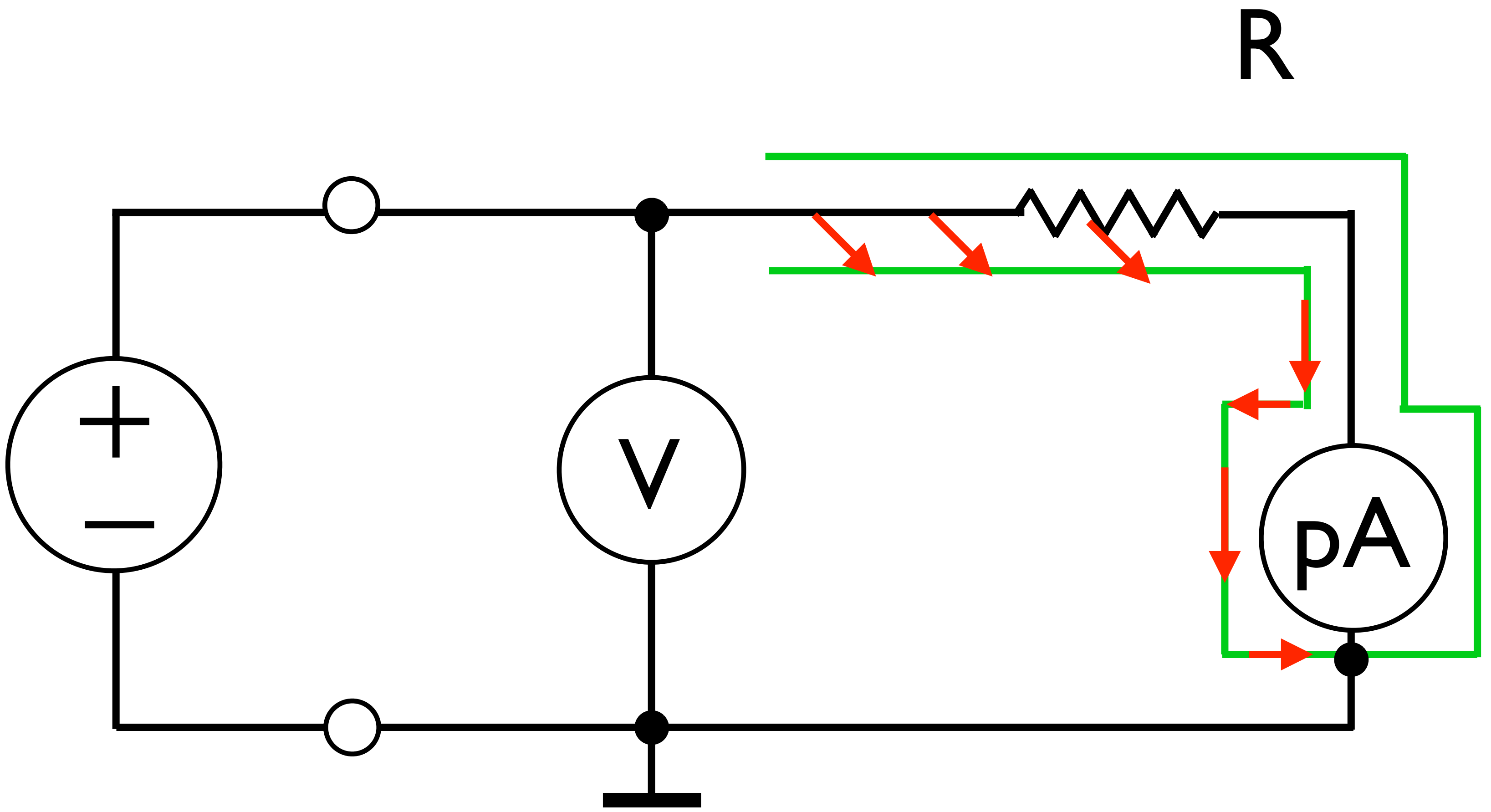
MEASUREMENT OF LARGE RESISTANCE





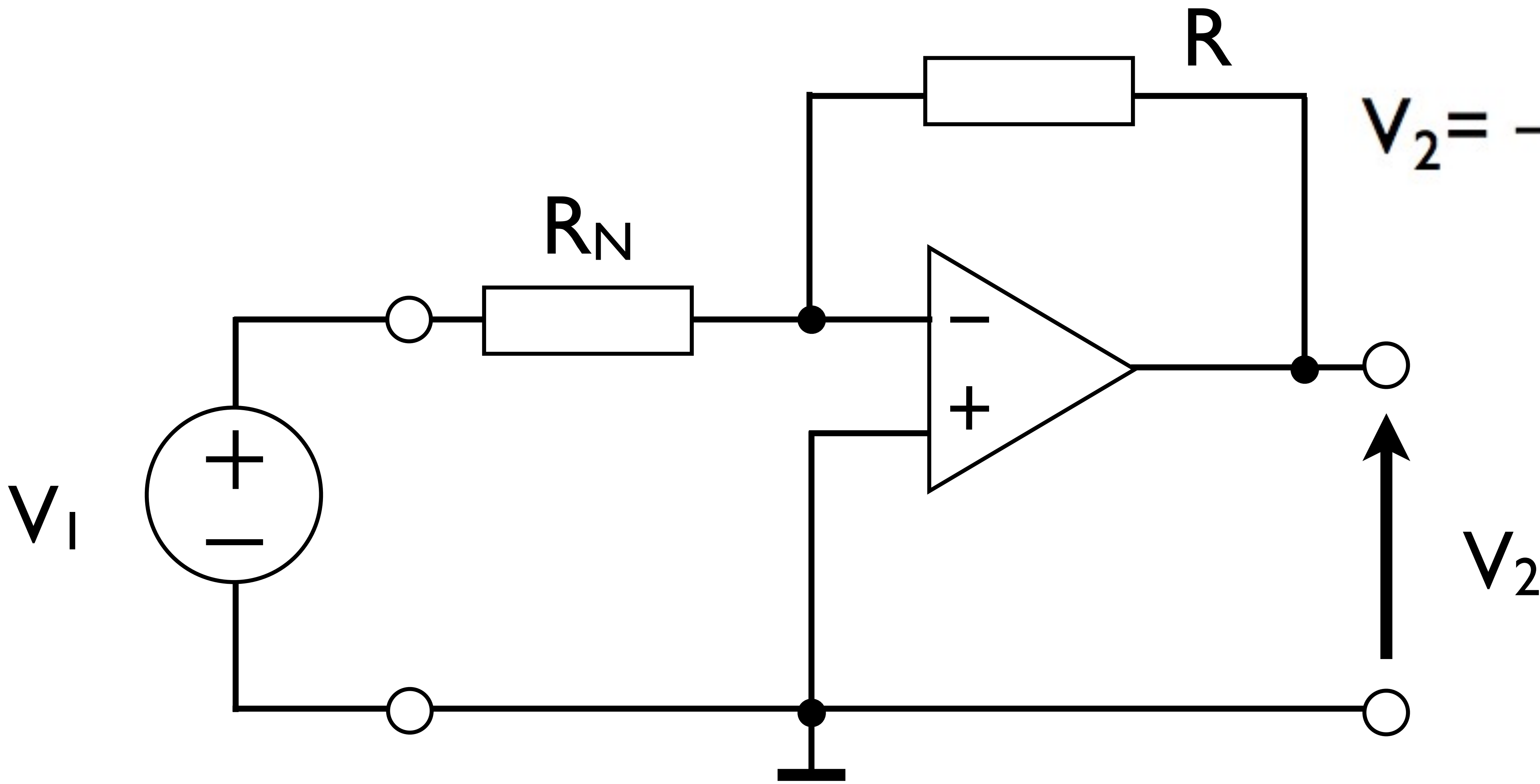
Resistance of insulation might bias pAmmeter

MEASUREMENT OF LARGE RESISTANCE



The current flows through the shield instead of loading the pAmmeter

RESISTANCE TO VOLTAGE CONVERTER



$$V_2 = -V_1 \cdot \frac{R}{R_N}$$

Both V_1 and R_N are constant, so the output V_2 depends only on R

$$R = -\frac{V_2}{V_1} \cdot R_N = k \cdot V_2$$

RESISTANCE TO VOLTAGE CONVERTER

$$R = -\frac{V_2}{V_1} \cdot R_N$$

Sources of uncertainty: V_1, V_2, R_N

Derivates to all variables

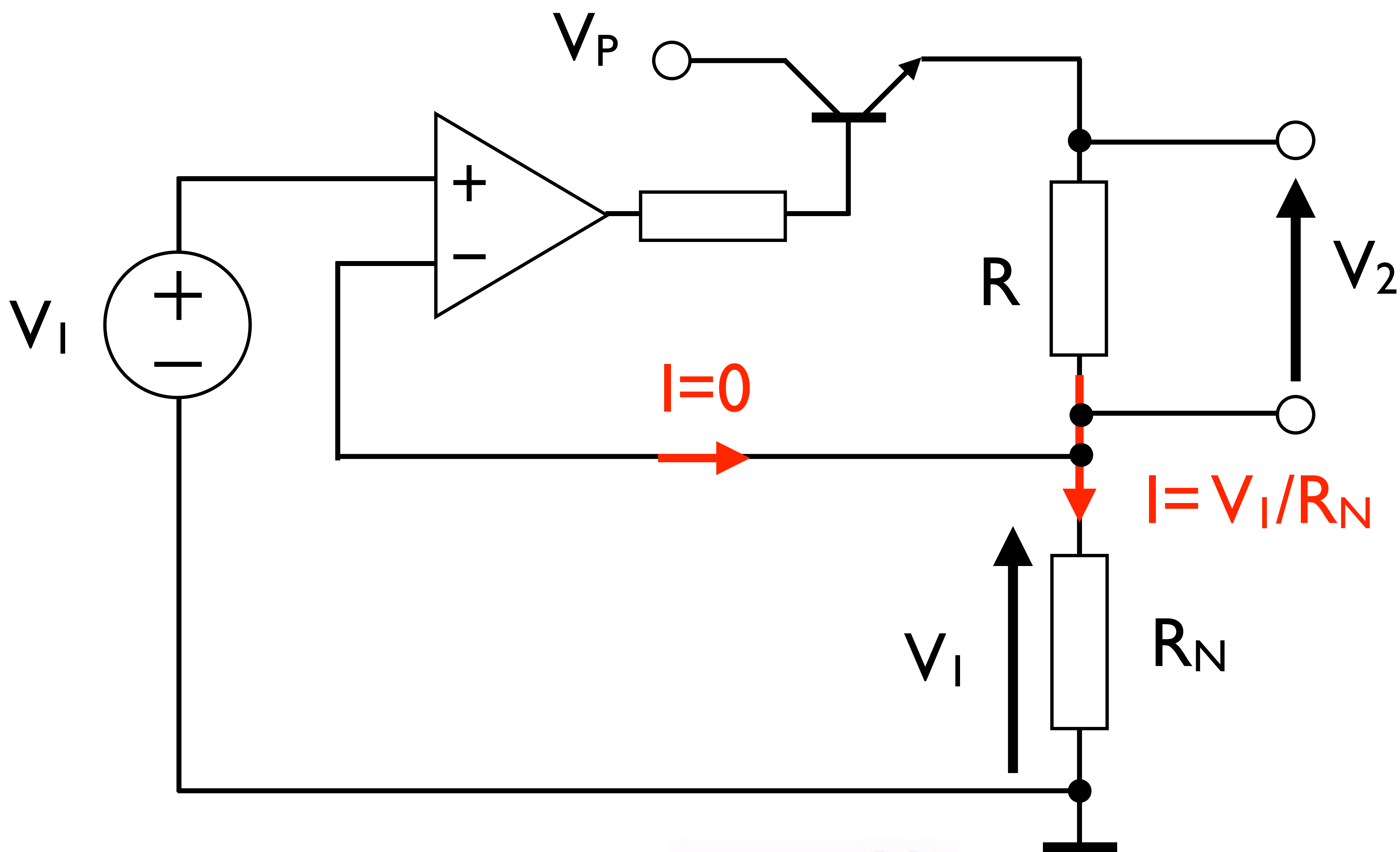
V_1: $$\frac{dR}{dV_1} = -\frac{V_2}{V_1^2} \cdot R_N$$

V_2: $$\frac{dR}{dV_2} = -\frac{1}{V_1} \cdot R_N$$

R_N: $$\frac{dR}{dR_N} = -\frac{V_2}{V_1}$$

$$u_R = \sqrt{\left(-\frac{V_2}{V_1^2} \cdot R_N \cdot u_{V_1}\right)^2 + \left(-\frac{1}{V_1} \cdot R_N \cdot u_{V_2}\right)^2 + \left(-\frac{V_2}{V_1} \cdot u_{R_N}\right)^2}$$

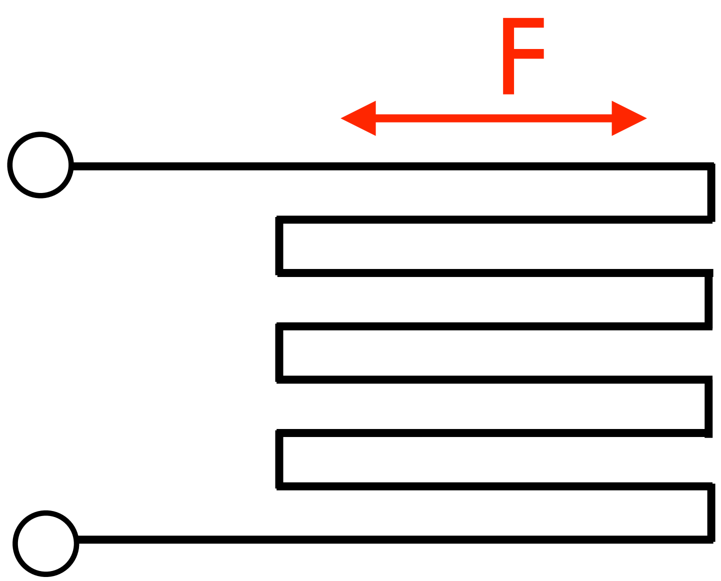
R to V converter for LOW RESISTANCE



$$V_2 = R \cdot \frac{V_I}{R_N} \longrightarrow R = R_N \cdot \frac{V_2}{V_I}$$

MEASUREMENT OF RESISTANCE
with large idle value and little changing value

Strain gage



119.95Ω

120.05Ω

If you use a 5 digit ohmmeter you will have only 1 digit resolution, whereas the first 4 digits are "wasted"

119.95Ω

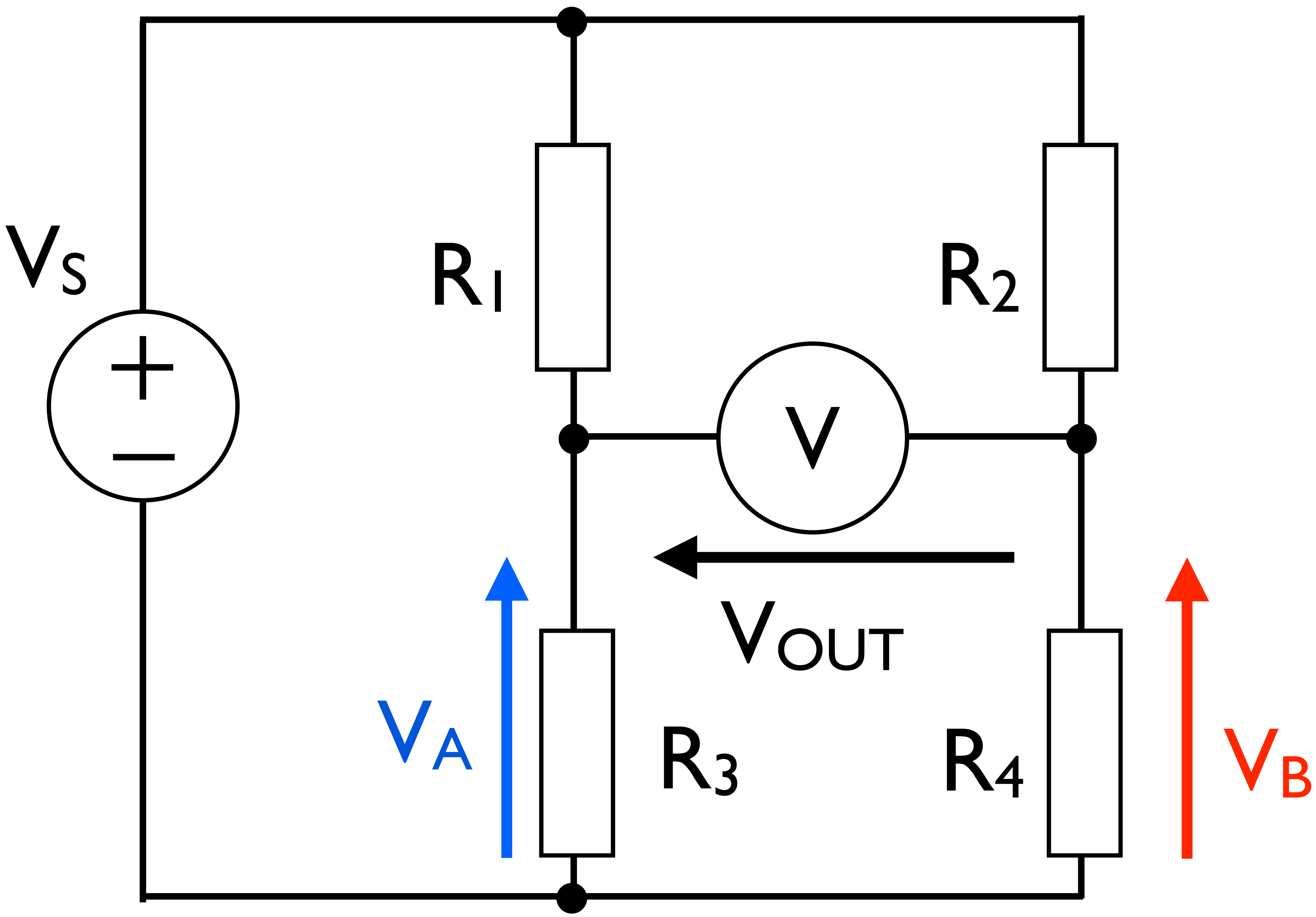
120.05Ω

We must get rid of $120\ \Omega$ and measure only $0.05\ \Omega$

We cannot amplify! Because the big idle value would saturate the instruments. First we must remove $120\ \Omega$, then we can amplify.

Solution:

WHEATSTONE BRIDGE



Balanced if:

$$V_{OUT} = 0$$

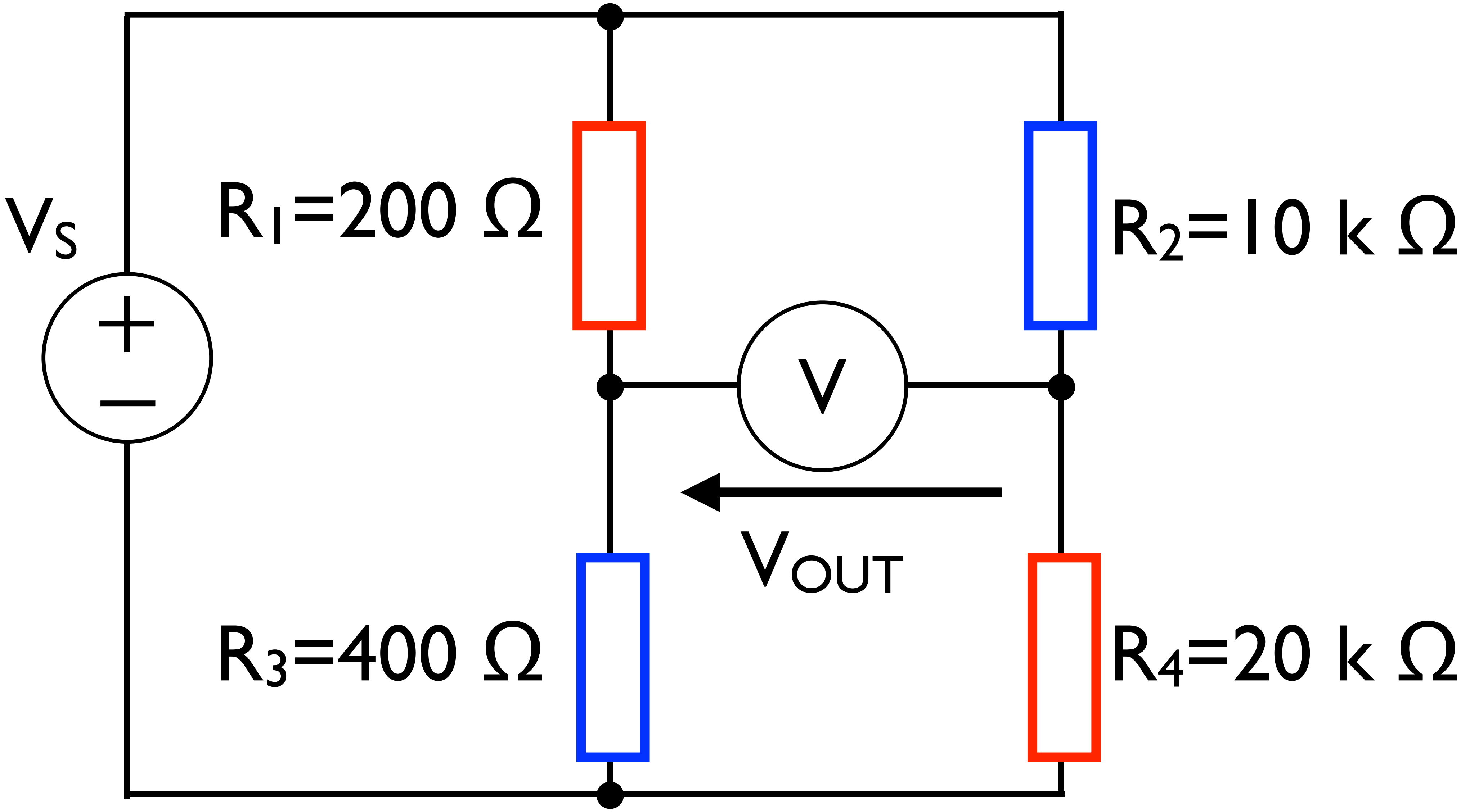
$$V_A = V_B$$

$$\frac{R_3}{R_1+R_3} \cdot V_s = \frac{R_4}{R_2+R_4} \cdot V_s$$

$$\frac{R_3}{R_1+R_3} = \frac{R_4}{R_2+R_4}$$

$$R_2 \cdot R_3 = R_1 \cdot R_4$$

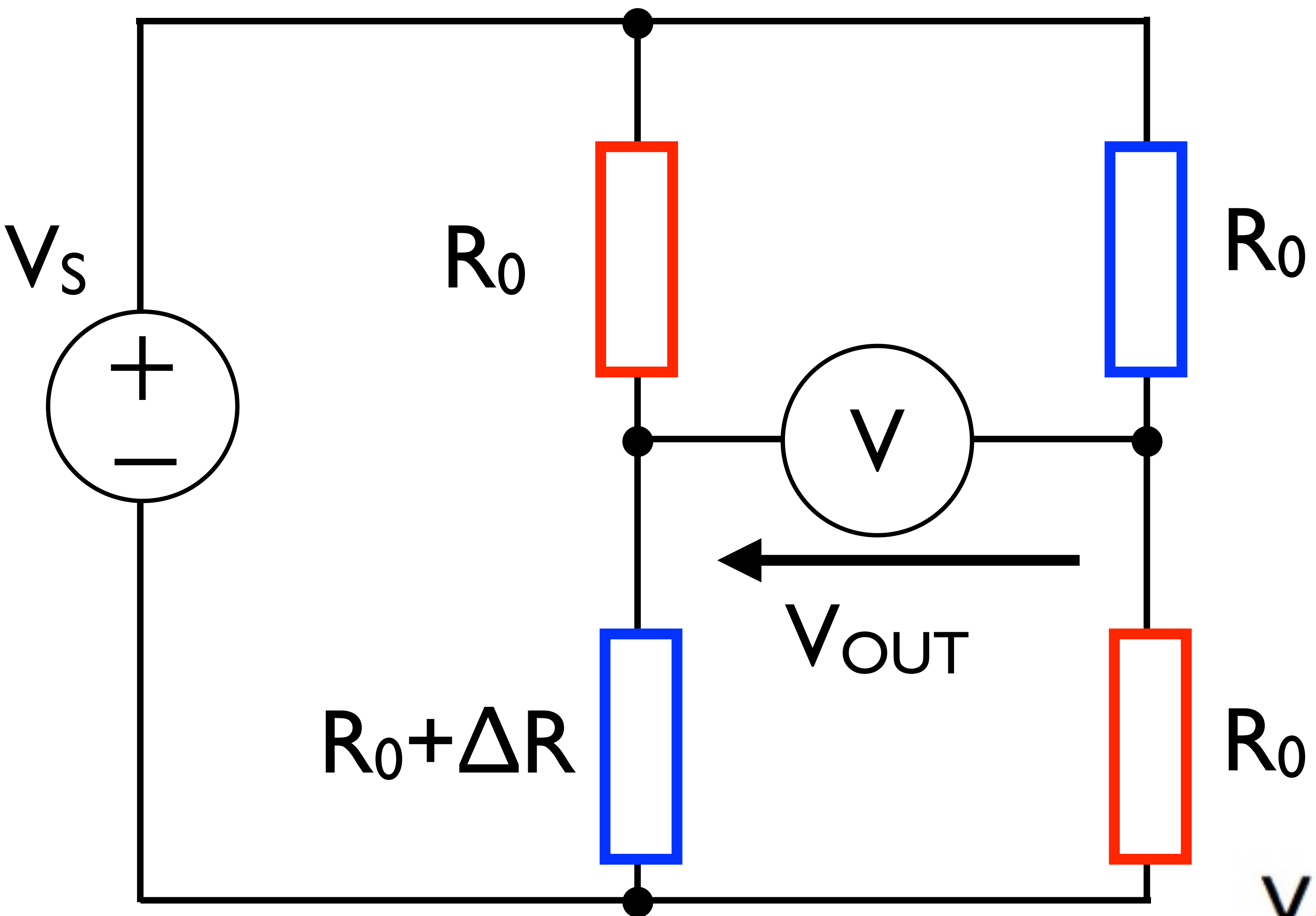
WHEATSTONE BRIDGE



$$R_2 \cdot R_3 = R_1 \cdot R_4$$

$$10{,}000 \cdot 400 = 200 \cdot 20{,}000$$

WHEATSTONE BRIDGE
Unbalanced bridge



Let us assume all resistance to be nominally equal to R_0

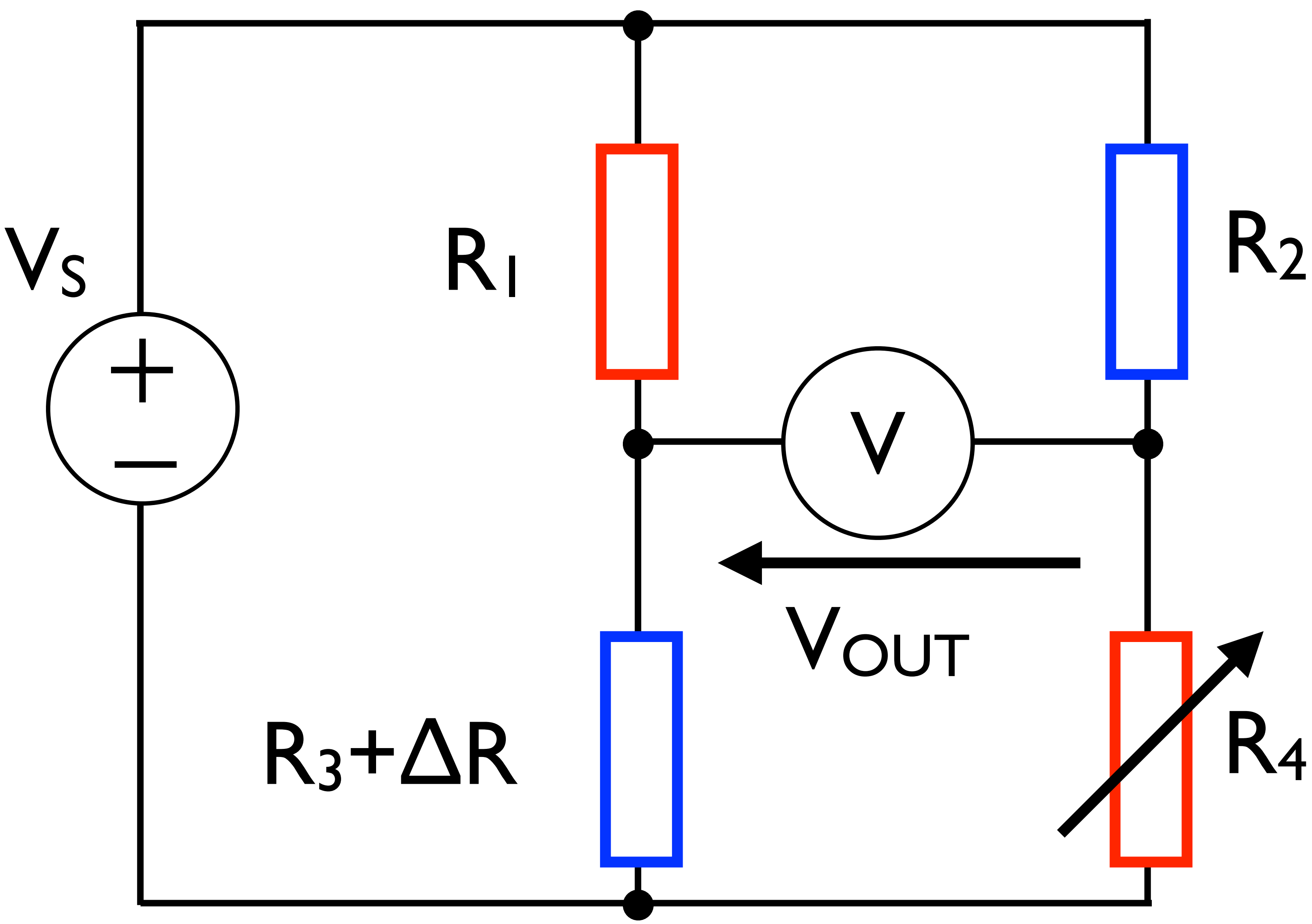
$$V_{OUT} = \left(\frac{R_0+\Delta R}{2\cdot R_0+\Delta R} - \frac{1}{2}\right)\cdot V_S$$

$$V_{OUT} = \frac{\Delta R}{4\cdot R_0\left(1+\frac{\Delta R}{2\cdot R_0}\right)}\cdot V_S$$

Linear only for $\Delta R << R_0$

WHEATSTONE BRIDGE

Controlled bridge



I change the value of R4 until V_{OUT}=0 in order to restore the balanced condition

$$R_2 \cdot R_3 = R_1 \cdot R_4$$

Then, R_4 - which is known - equals $R_3+\Delta R$