Lecture 2

- Uncertainty of measurement
- Digital multimeters
- Analog instruments
- Vectorial voltmeter
- Measurement of frequency and period

Accuracy of measurement

Old style approach

I provide the measurement with two values:

- measured value X_M
- error of measurement Δ_M

$$X_M = X_S + \Delta_M$$

E.g.: I can say I measure 3.23 V with 0.02 V error

X_S is the “true” value

How can we calculate the error?

The error is the difference between the measured value and the true value

$$\Delta_M = X_M - X_S$$

- true value X_S
- measured vale X_M
- error of measurement Δ_M

Problem: we do not know the "true value". If we knew it we would have a perfect measurement and we would not talk about error.

New approach: the UNCERTAINTY

Definition:

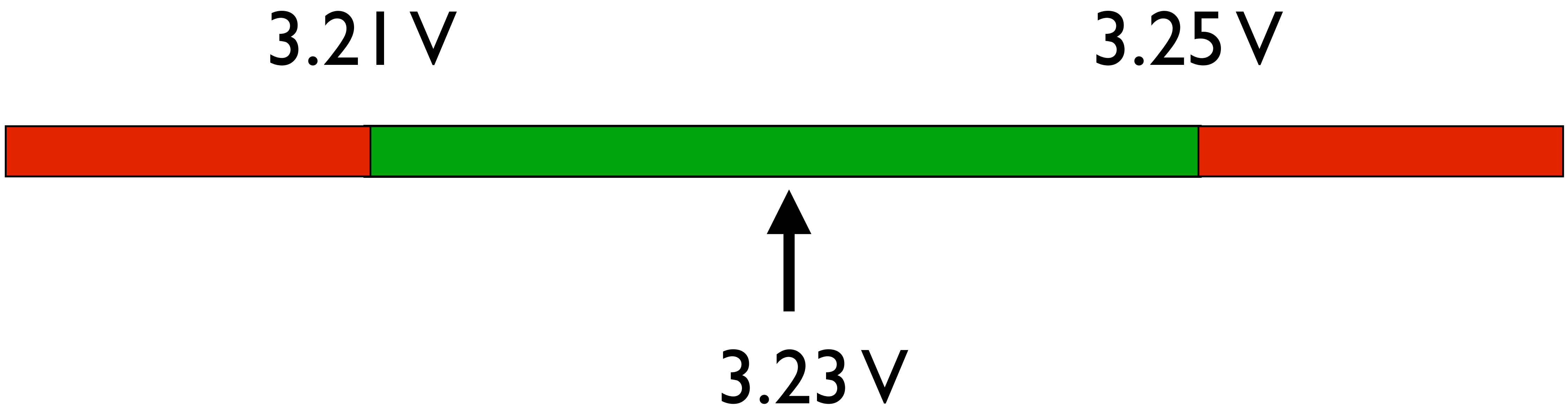
parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand.

Rough translation:

I can't tell you where the true value is, I don't know it.
But at least I can give a range of values where I expect it to be.

1993 – International Standard Organization (ISO)

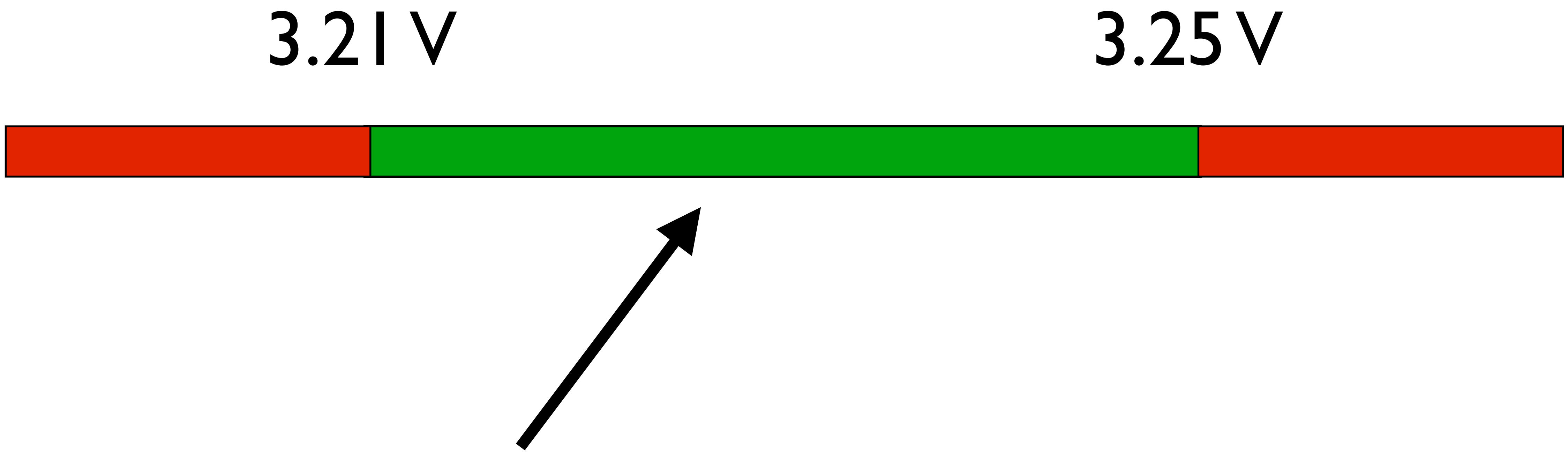
Guide to the Expression of Uncertainty of Measurements



I give a measurement (3.23V) and an uncertainty (0.2V).
It means: I expect the real value to be between 3.21 V and 3.25 V.

Question n. 1

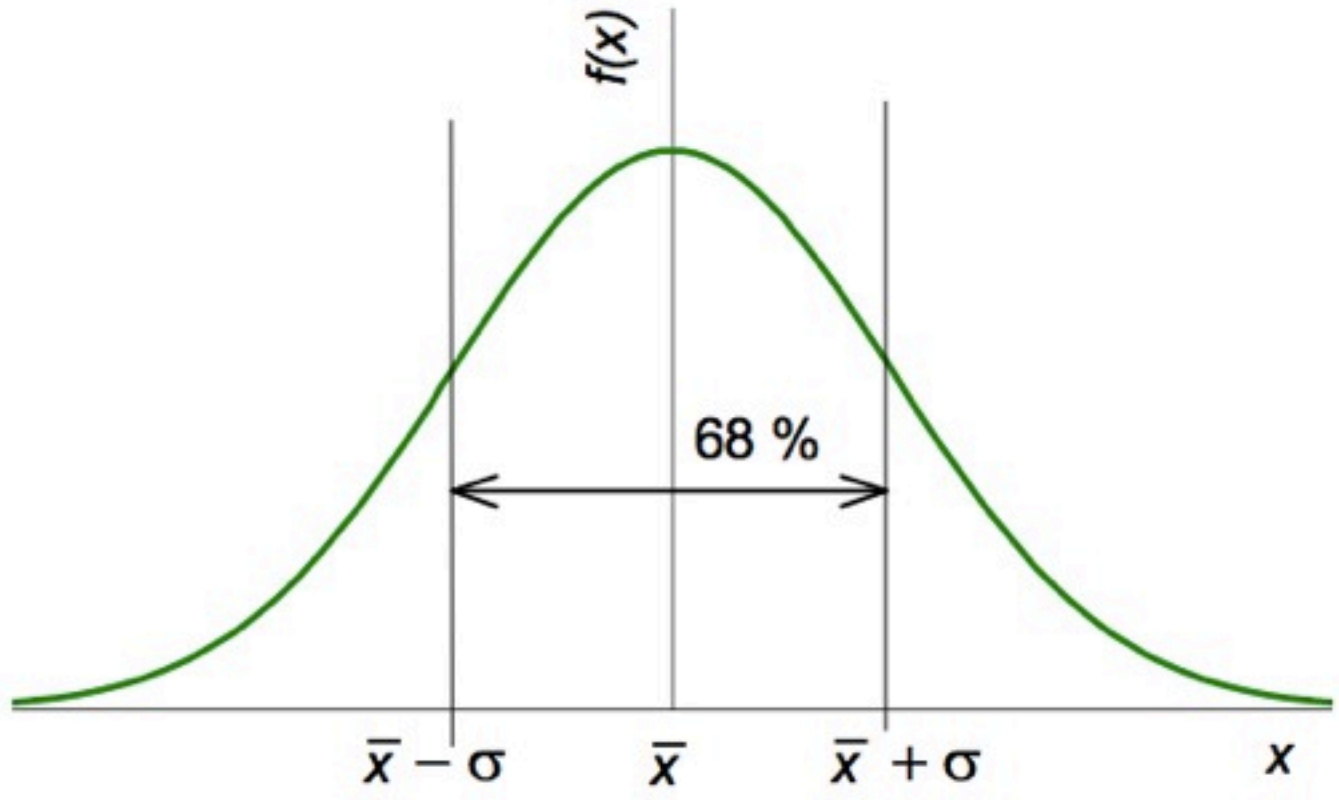
Does this mean that the measurement is for sure between 3.21 and 3.25?



You can say the measurement is somewhere here with some confidence

We must be more precise and agree on what we mean by "some confidence"

Normal (gaussian) distribution



Probability density:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(x-\bar{x})^2}{2\sigma^2}}$$

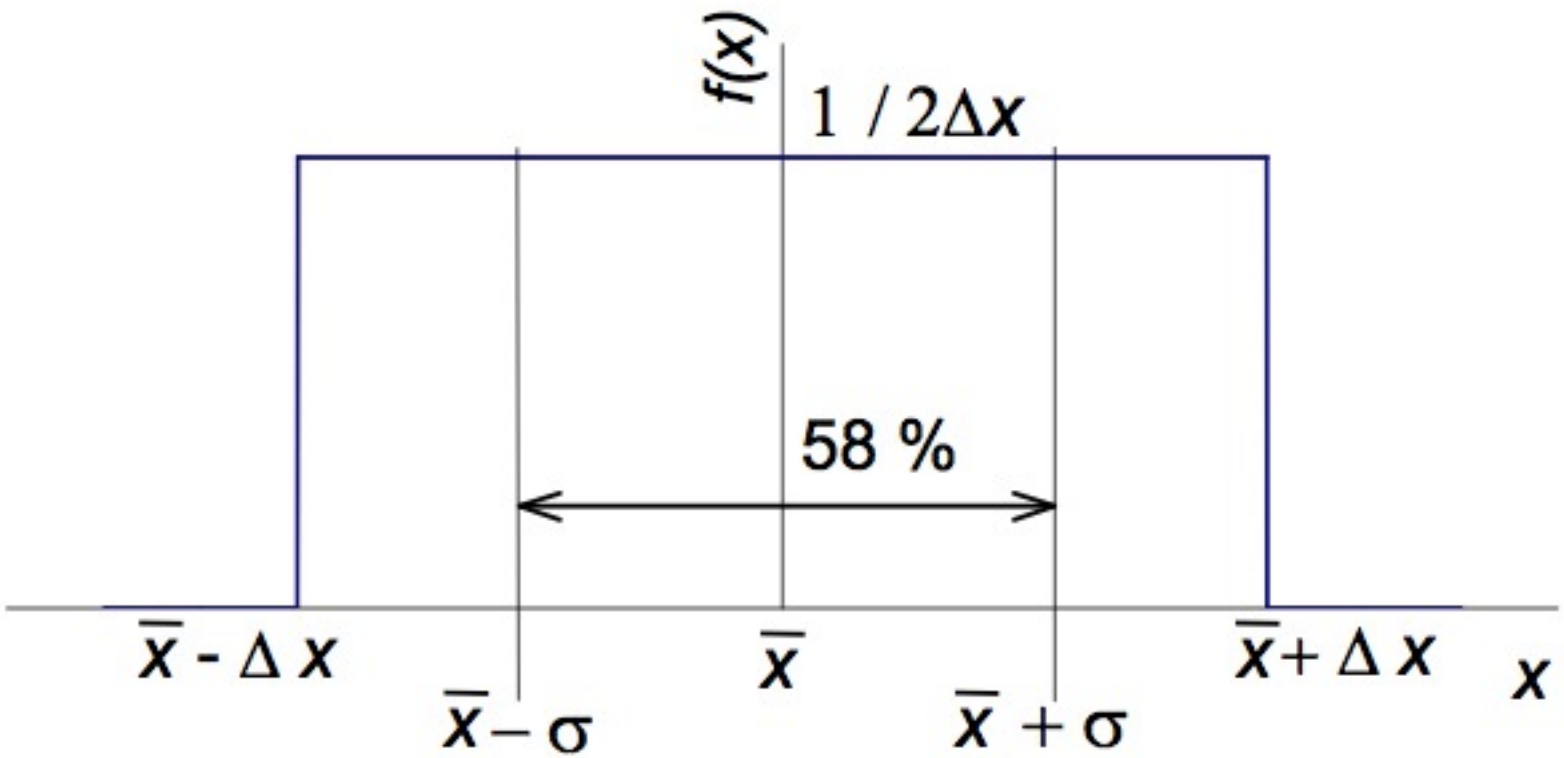
The standard uncertainty is given by the standard deviation. It covers 68% of probability...

Question n. 2

Does this mean that the measured voltage is between 3.21 and 3.25 with 68% probability?

In some case we don't know the probability distribution, but we know that the measured value will be *for sure* within a specific range $\pm\Delta x$

We assume uniform distribution



In this case the standard deviation is

$$\sigma = \frac{\Delta x}{\sqrt{3}}$$

When we state an uncertainty we must always specify what kind of distribution we assume

Then, we can give the standard uncertainty $u(x)$

or the expanded uncertainty $U(x)$

$$U(x) = k\,u(x)$$

k is the coverage factor (k=2,3...)

2 TYPES OF UNCERTAINTY

Type A

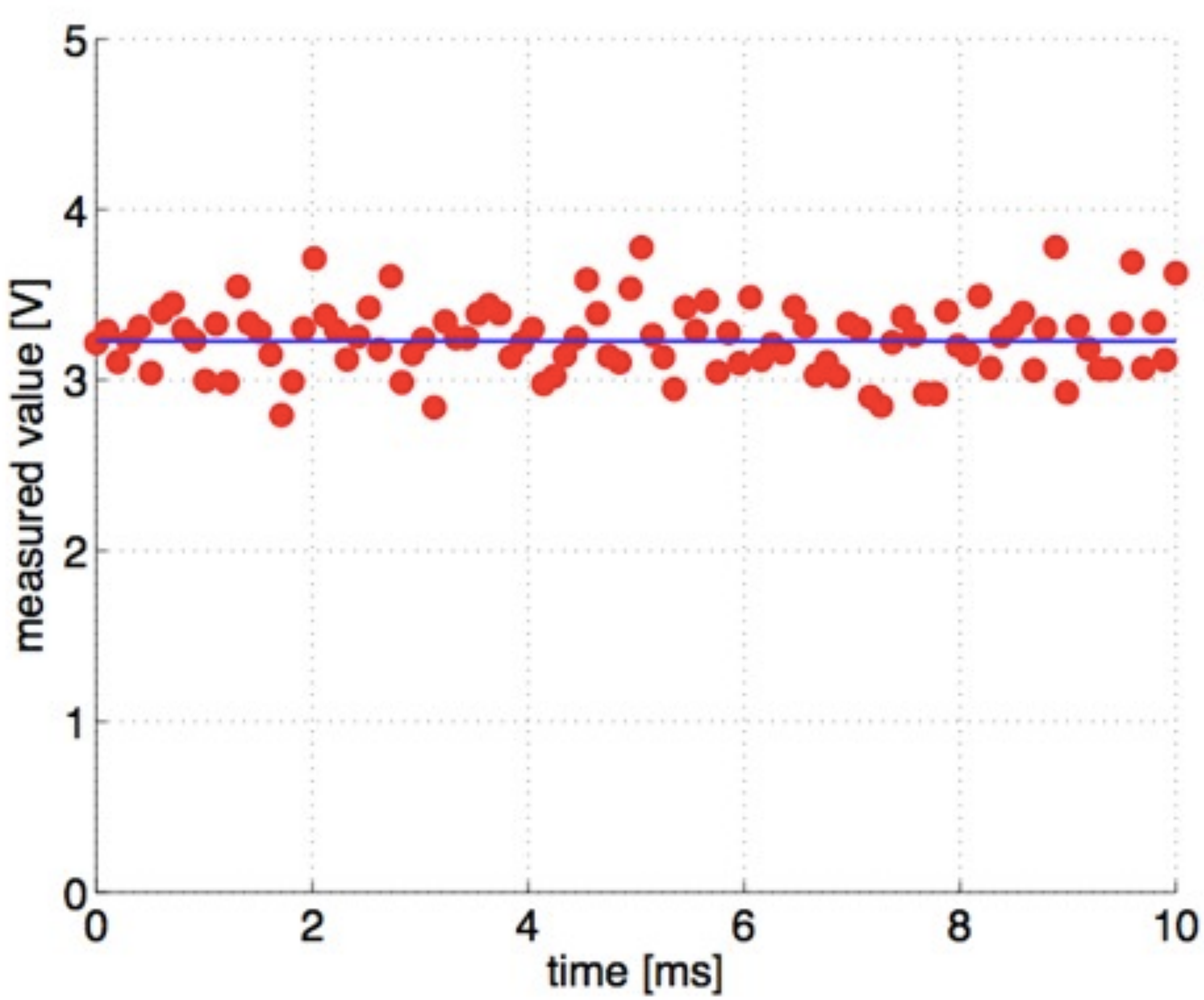
Definition:

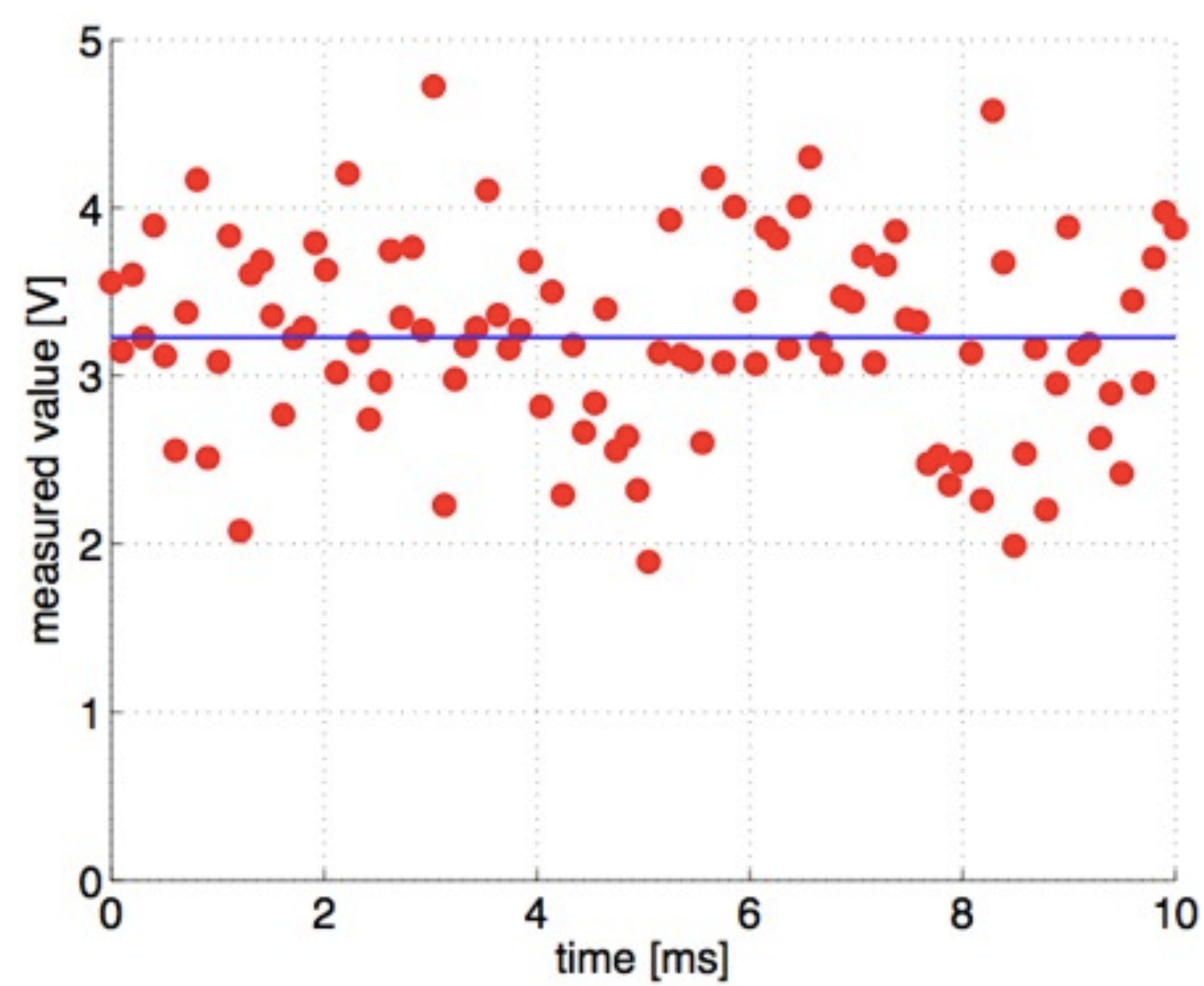
uncertainty by the statistical analysis of series of observations.

Rough translation:

I repeat the measurement several times and every time and every time I measure a different value.
The more the measured values differ the higher is the uncertainty.

Example: in both cases the average value is 3.23 V but the standard deviation is different

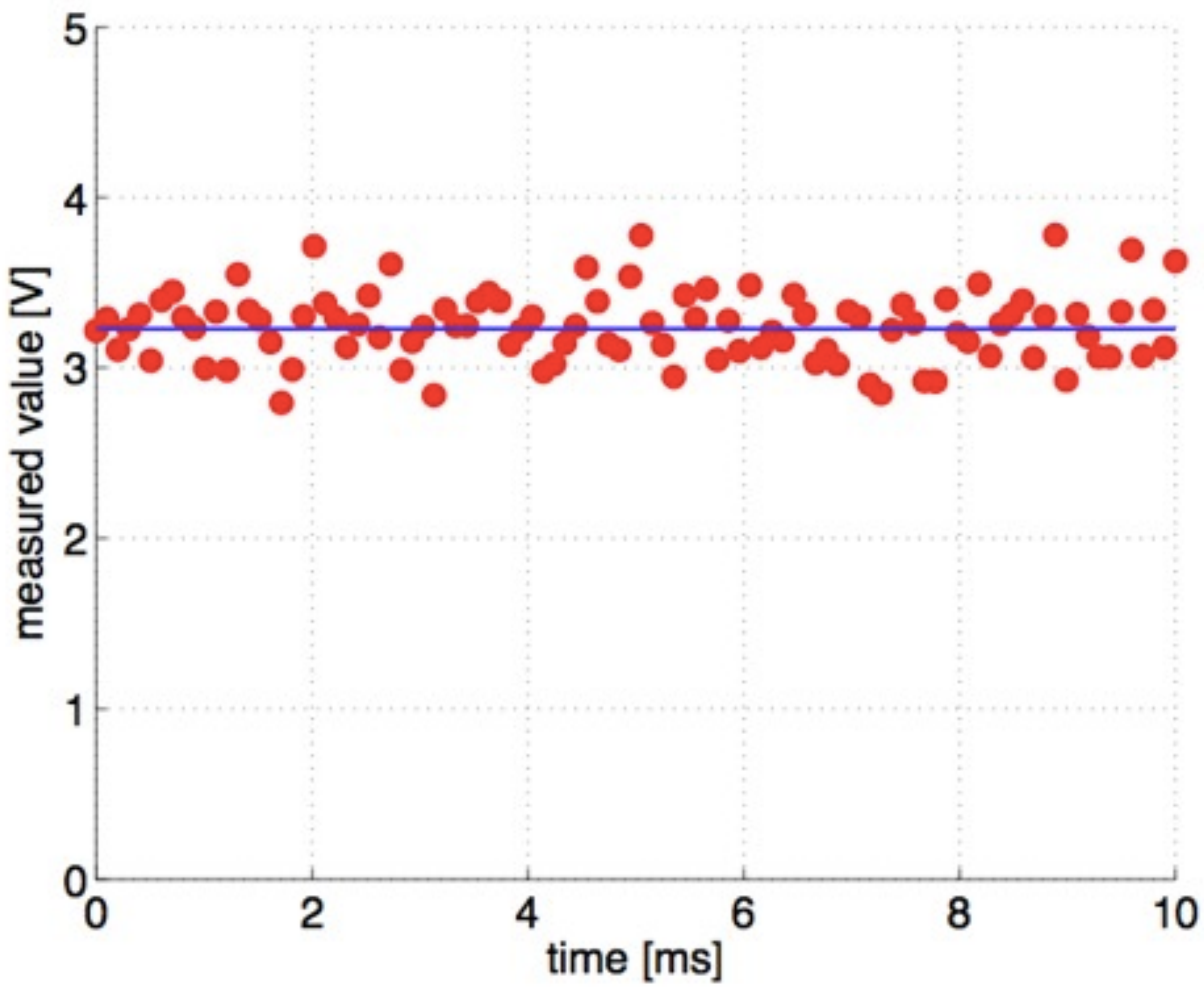




$\sigma = 0.2\,\mathrm{V}$

$\sigma = 0.6\,\mathrm{V}$

TYPE A UNCERTAINTY



$$u_A(x) = \hat{\sigma}(\bar{X}) = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

2 TYPES OF UNCERTAINTY

Type B

Definition:

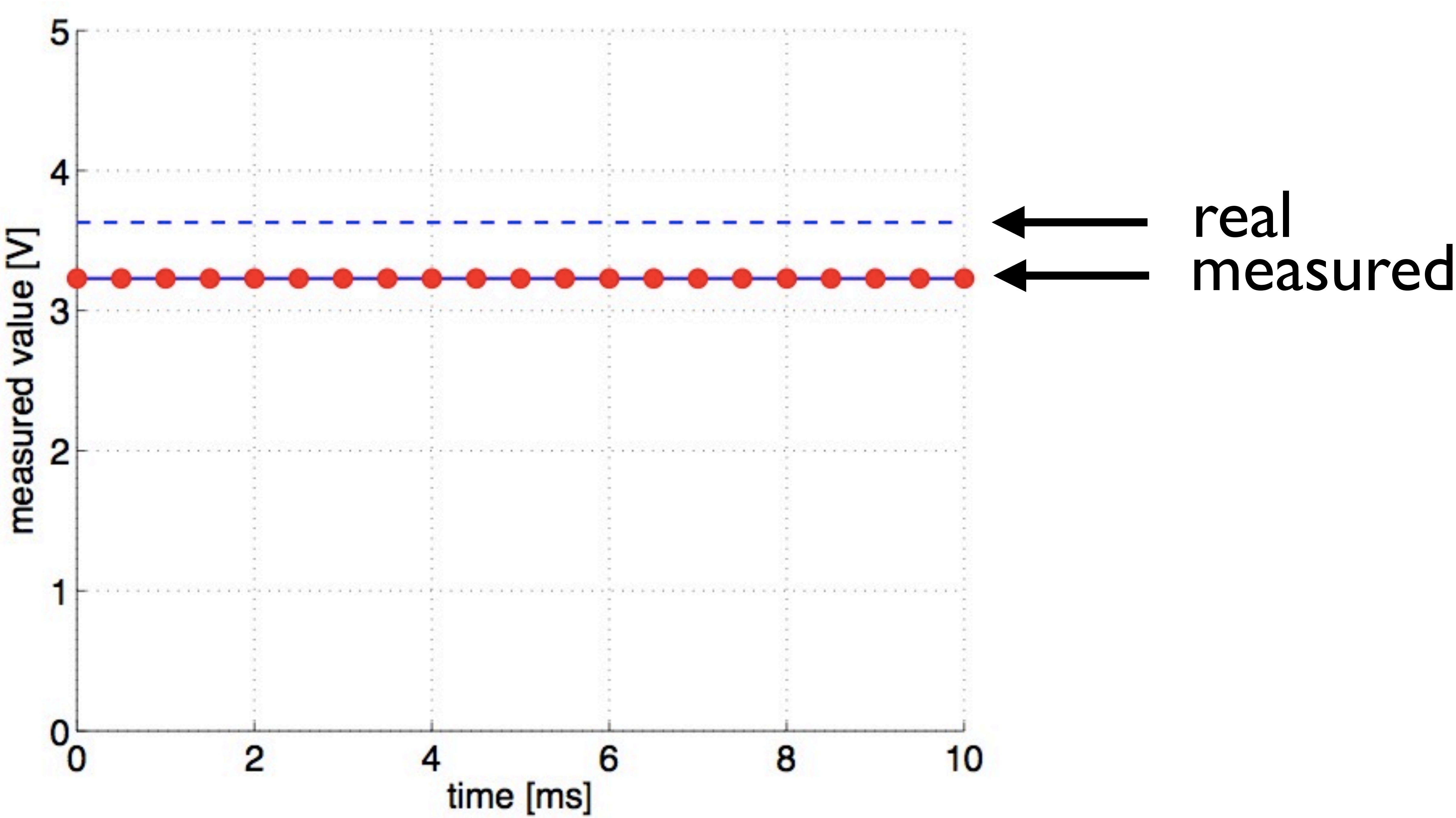
uncertainty by means other than the statistical analysis of series of observations.

Rough translation:

The measured value does not change with time. I repeat the measurement 1000 times and I always get the same value. But I can't say I am doing a "perfect" measurement. Maybe the instrument is simply giving a very constant but also "very wrong" value.

2 TYPES OF UNCERTAINTY

Type B



2 TYPES OF UNCERTAINTY

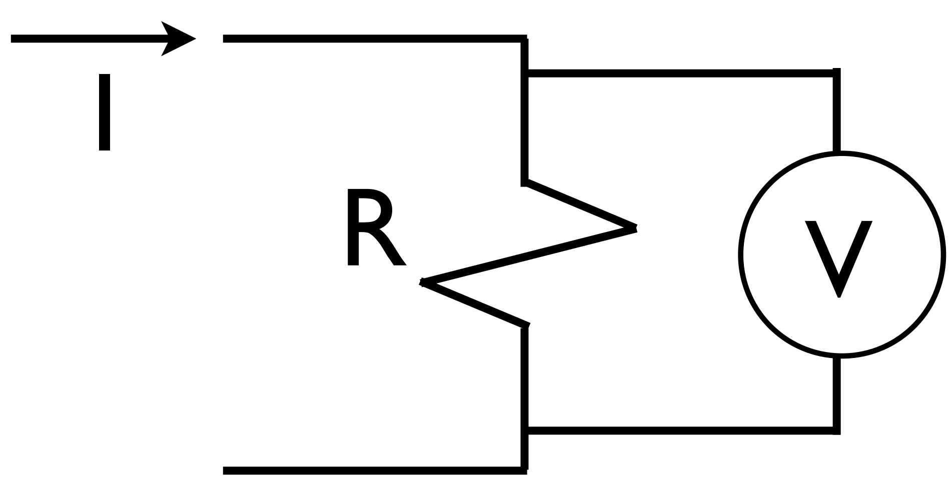
Type B

Sources:

- the instrument itself
(high quality instruments provide more accurate measurement, thus lower uncertainty)

- the measurement set-up

Ex. measurement of current



I=V/R

but... how precisely do I know the value or R?

2 TYPES OF UNCERTAINTY

Type B

Manufacturers provide information about the uncertainty of the instrument

E.g.: Digital multimeter

% of reading, % of range

Accuracy $$\Delta_X = \frac{\delta_1}{100} X + \frac{\delta_2}{100} M$$

■ DC Characteristics

Accuracy Specifications ± (% of reading + % of range) [1]

Function	Range [3]	Test Current or Burden Voltage	24 Hour [2] 23°C ± 1°C	90 Day 23°C ± 5°C	1 Year 23°C ± 5°C	Temperature Coefficient /°C 0°C – 18°C 28°C – 55°C
DC Voltage	100.0000 mV		0.0030 + 0.0030	0.0040 + 0.0035	0.0050 + 0.0035	0.0005 + 0.0005
	1.000000 V		0.0020 + 0.0006	0.0030 + 0.0007	0.0040 + 0.0007	0.0005 + 0.0001
	10.00000 V		0.0015 + 0.0004	0.0020 + 0.0005	0.0035 + 0.0005	0.0005 + 0.0001
	100.0000 V		0.0020 + 0.0006	0.0035 + 0.0006	0.0045 + 0.0006	0.0005 + 0.0001
	1000.000 V		0.0020 + 0.0006	0.0035 + 0.0010	0.0045 + 0.0010	0.0005 + 0.0001

2 TYPES OF UNCERTAINTY

Type B

Manufacturers provide information about the uncertainty of the instrument

E.g.: Digital multimeter
% of reading, % of range

Accuracy $$\Delta_X = \frac{\delta_1}{100} X + \frac{\delta_2}{100} M$$

Assuming uniform distribution the uncertainty is $$u_B = \sigma = \frac{\Delta z_{max}}{\sqrt{3}} = \frac{\frac{\delta_1}{100} X + \frac{\delta_2}{100} M}{\sqrt{3}}$$

2 TYPES OF UNCERTAINTY

Type B

Manufacturers provide information about the uncertainty of the instrument

E.g.: Analog (pointer) voltmeter

ACCURACY CLASS (AC)

The maximum possible deviation of the measured value from the value of the measured quantity

Typically given in % of full scale

The accuracy is then $\Delta_P = \frac{AC}{100} M$ M=full scale range

and assuming uniform distribution $u_B = \sigma = \frac{AC/100}{\sqrt{3}} M$

Joining Type A and Type B uncertainty

$$u(x)=\sqrt{u_A^2(x)+u_B^2(x)}$$

1. Example of estimation of uncertainty

Digital multimeter

Measuring range M=10 V

Accuracy= ±0.01% range ±0.005% of range

Type A uncertainty

I measure 10 values

5.0009
5.0019
4.9992
4.9998
5.0011
4.9989
5.0007
5.0003
4.9995
5.0014
(V)

Average=5.0004 V

$u_A = \sigma = 0.00032\,V$

1. Example of estimation of uncertainty

Digital multimeter

Measuring range M=10V

Accuracy= ±0.01% range ±0.005% of range

Type B uncertainty

$$u_B = \frac{\frac{\delta_1}{100} X + \frac{\delta_2}{100} M}{\sqrt{3}} = \frac{\frac{0.01}{100} 5.0004 + \frac{0.005}{100} 10}{\sqrt{3}} = 0.00058 \text{ V}$$

1. Example of estimation of uncertainty

Digital multimeter

Measuring range M=10V

Accuracy= ±0.01% range ±0.005% of range

Type A and Type B together

$$u(x)=\sqrt{u_A^2(x)+u_B^2(x)}=\sqrt{0.00032^2+0.00058^2}=0.00066\ V$$

So, I can finally say that:

The measured value is V=5.0004V
with expanded uncertainty u=0.0013 V (k=2)

2. Example of estimation of uncertainty

Electromechanical voltmeter

Accuracy class AC= 0.5

Measurement range M=130 V

Measured value V=71.1 V

Type A the reading does not change, I neglect it

Type B

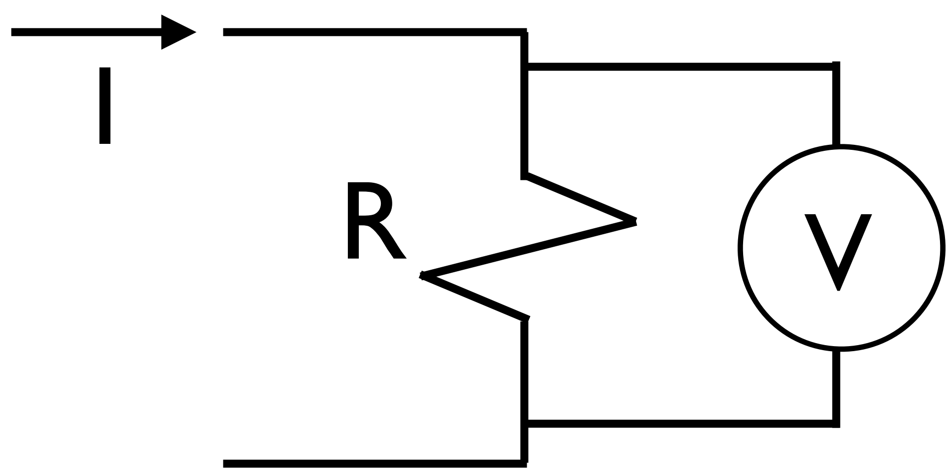
$$u_B = \frac{AC/100}{\sqrt{3}} M = \frac{0.5/100}{\sqrt{3}} 130 = 0.375 \text{ V}$$

Finally I can say the measurment is V=71.1 V with expanded uncertainty 0.75 V (k=2)

UNCERTAINTY OF DERIVED MEASUREMENT

Now, what if I measure one or more physical quantities and then I calculate another quantity from them?

E.g.



$I = V/R$

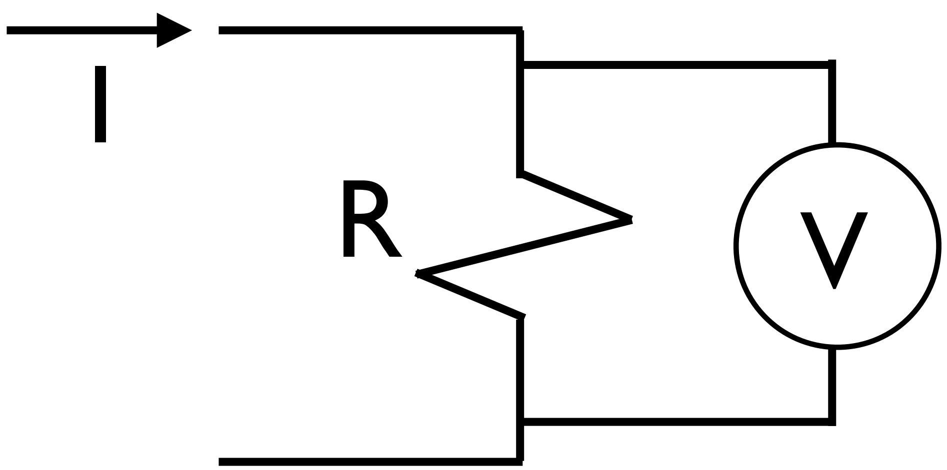
I directly measure the voltage V

I indirectly measure the current I

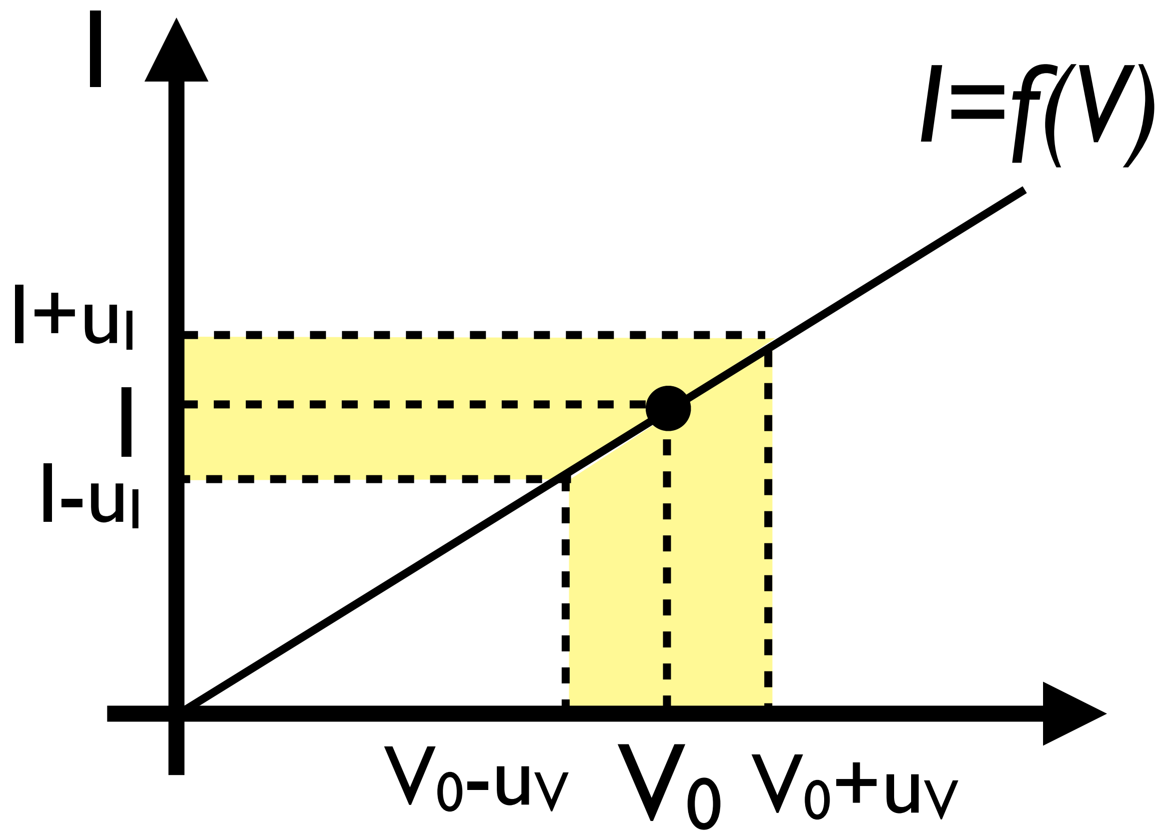
I know how to calculate u_V, the uncertainty of V.

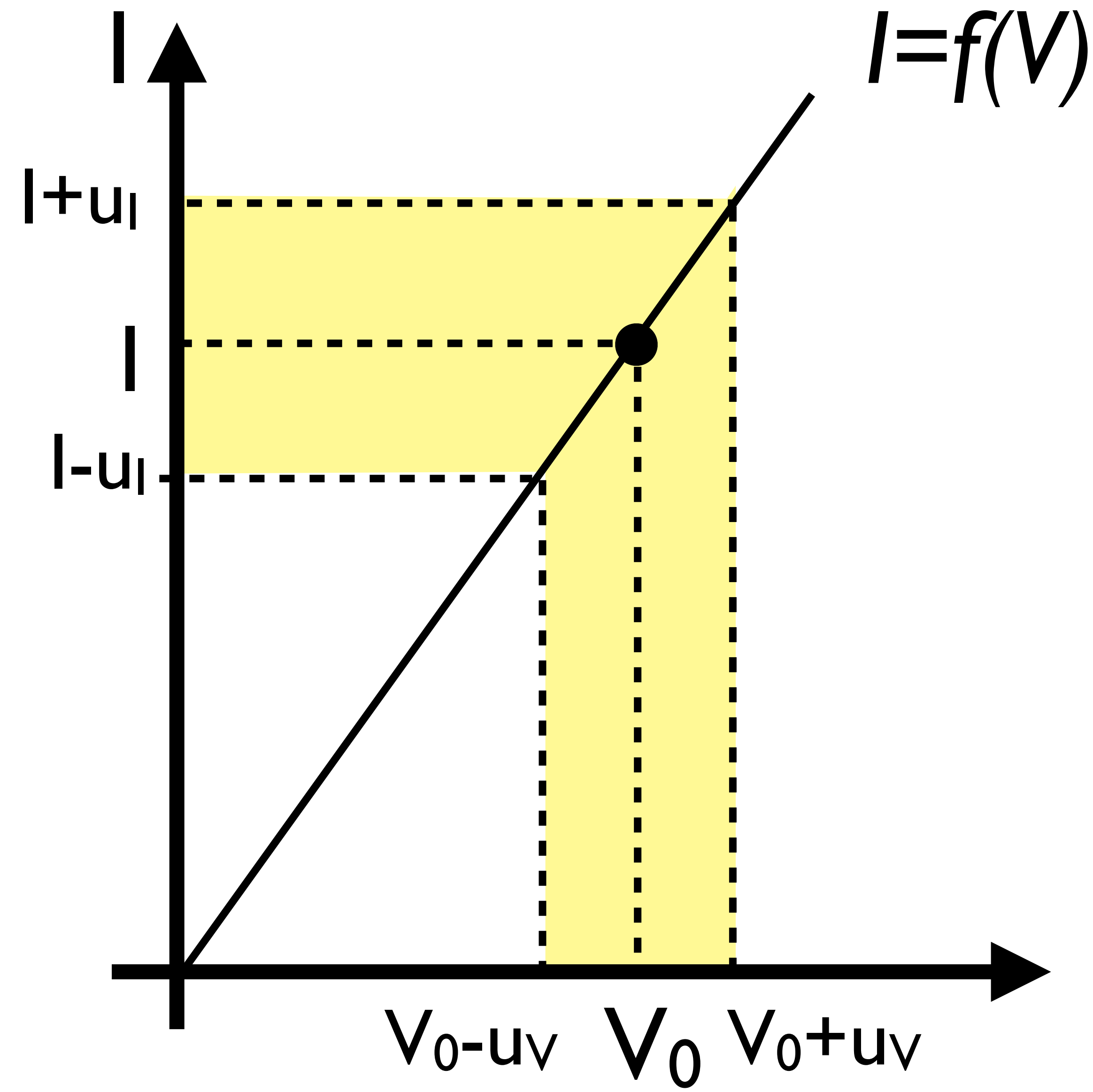
How can I calculate u_I?

UNCERTAINTY OF DERIVED MEASUREMENT

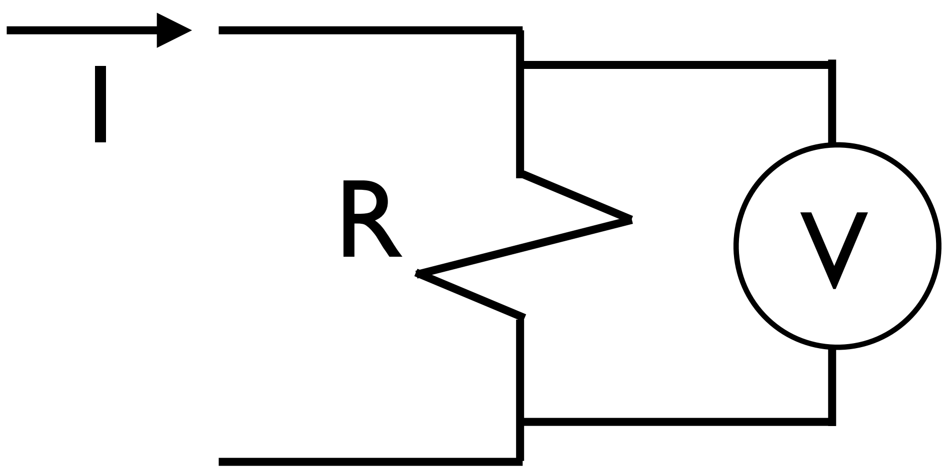


I=V/R

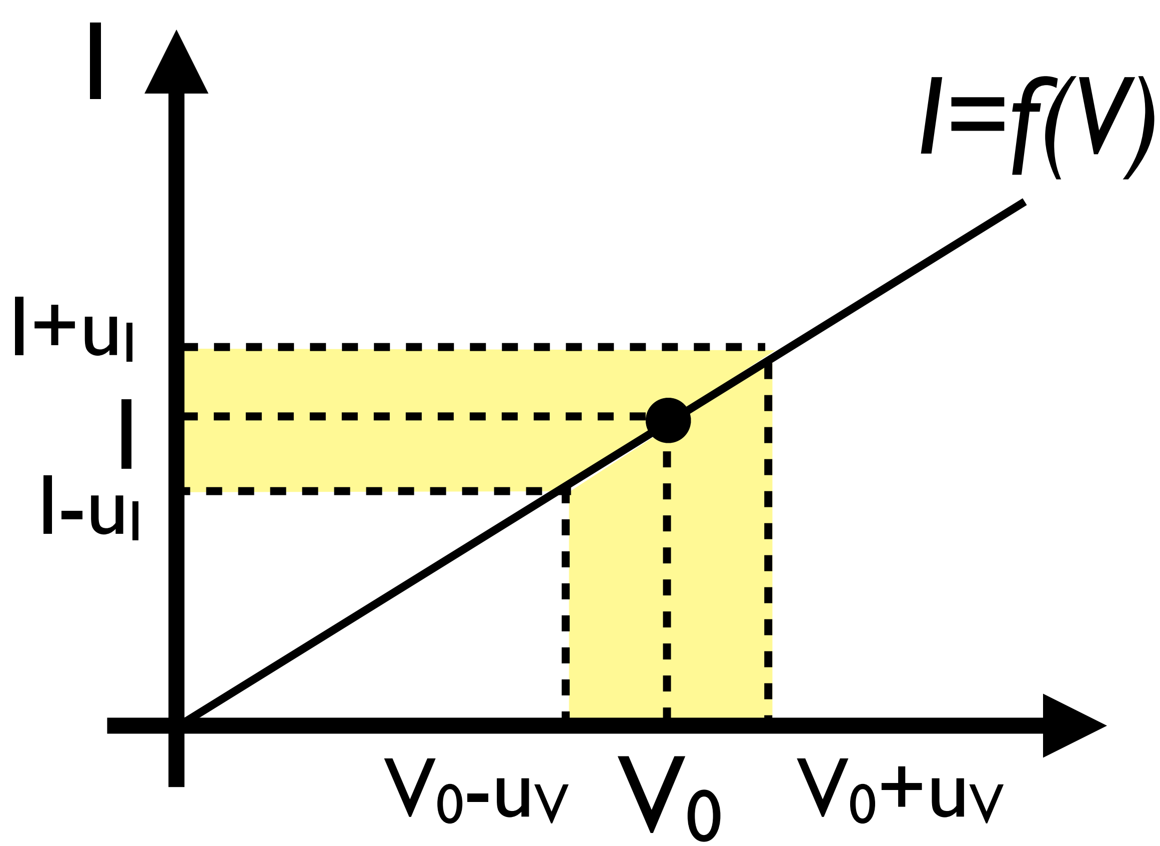




UNCERTAINTY OF DERIVED MEASUREMENT

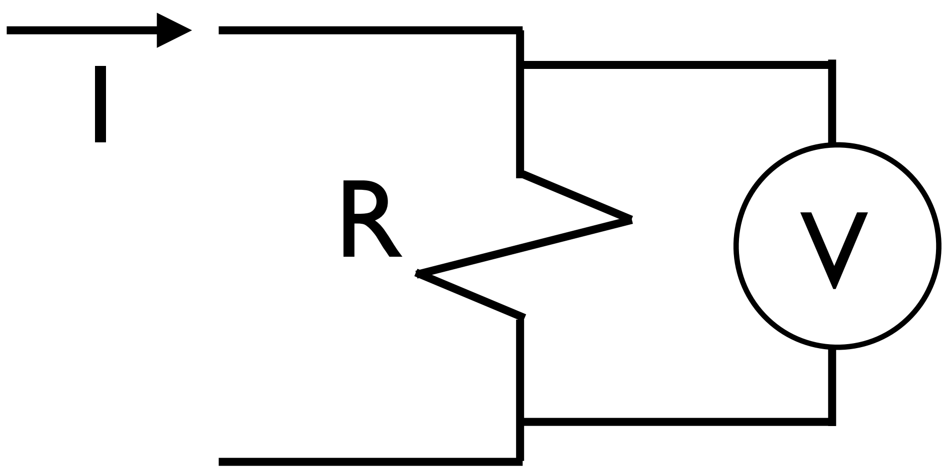


$I = V/R$



$$u_I = \left.\frac{\partial f}{\partial V}\right|_{V_0} \cdot u_V = \frac{1}{R} \cdot u_V$$

UNCERTAINTY OF DERIVED MEASUREMENT

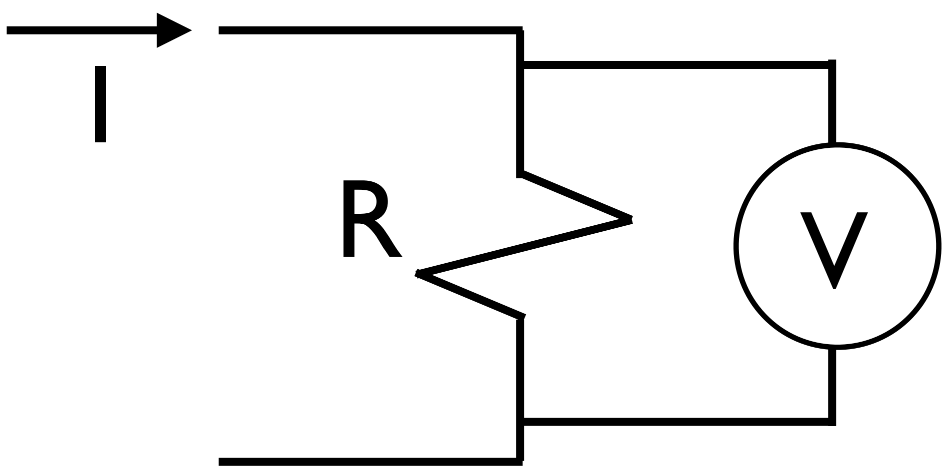


I=V/R

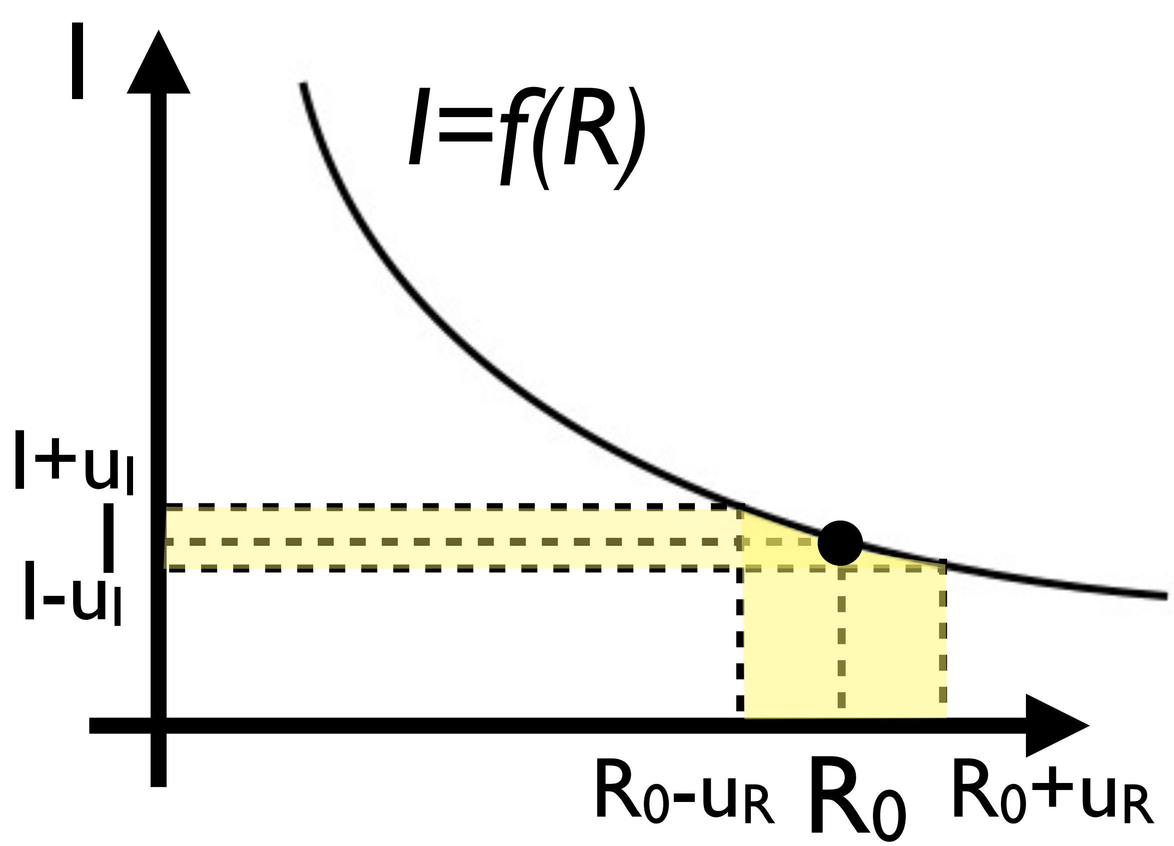
$$u_I = \left.\frac{\partial f}{\partial V}\right|_{V_0} \cdot u_V = \frac{1}{R} \cdot u_V$$

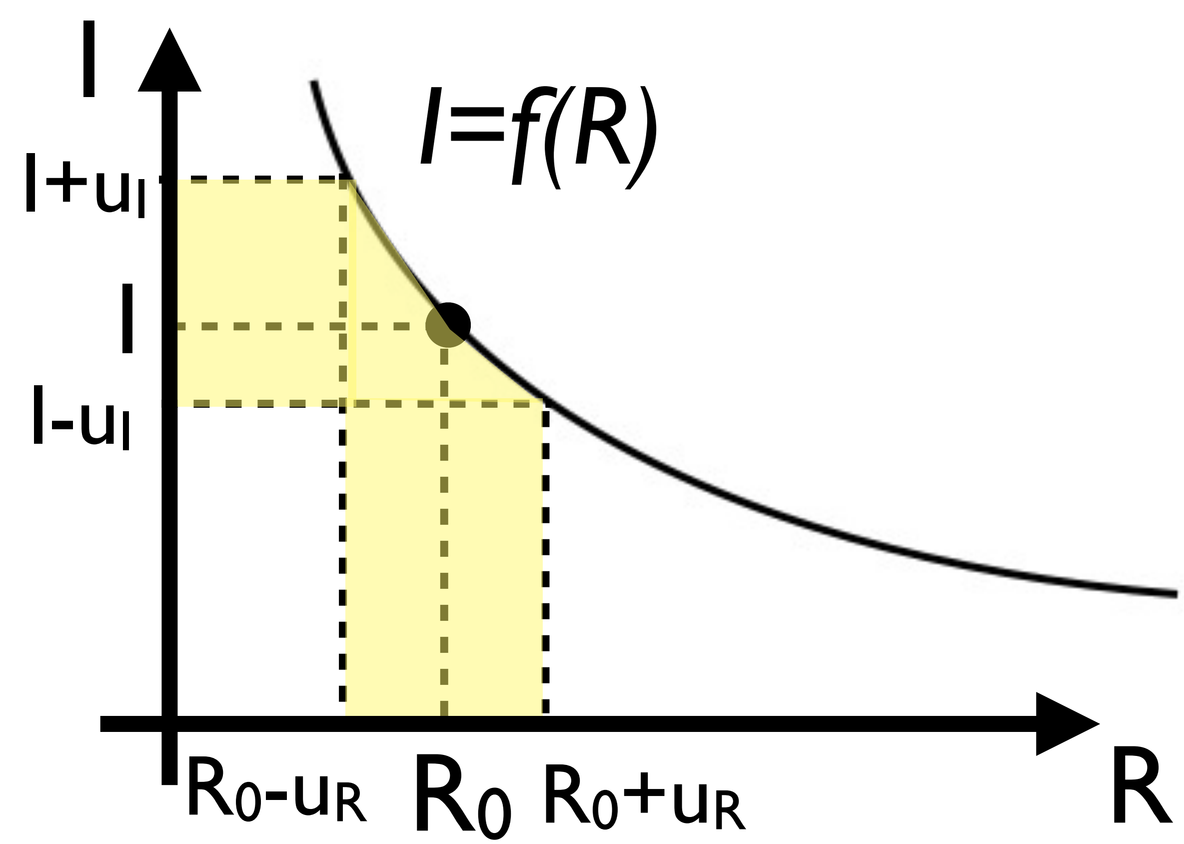
This equation considers only V as source of uncertainty whereas R is considered "perfect". Now we should also consider the uncertainty of R

UNCERTAINTY OF DERIVED MEASUREMENT

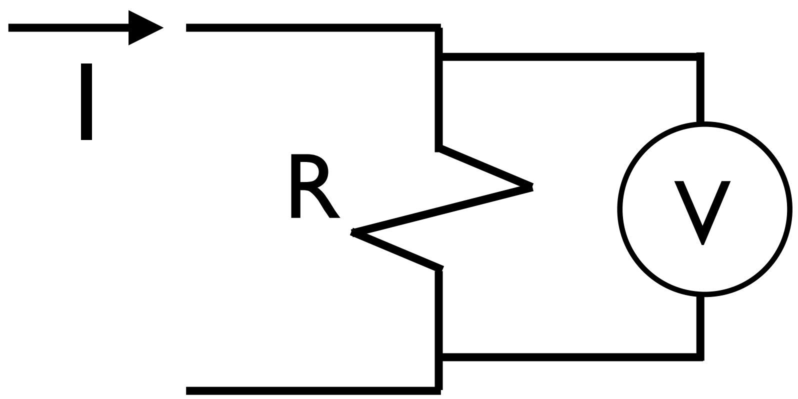


$I=V/R$

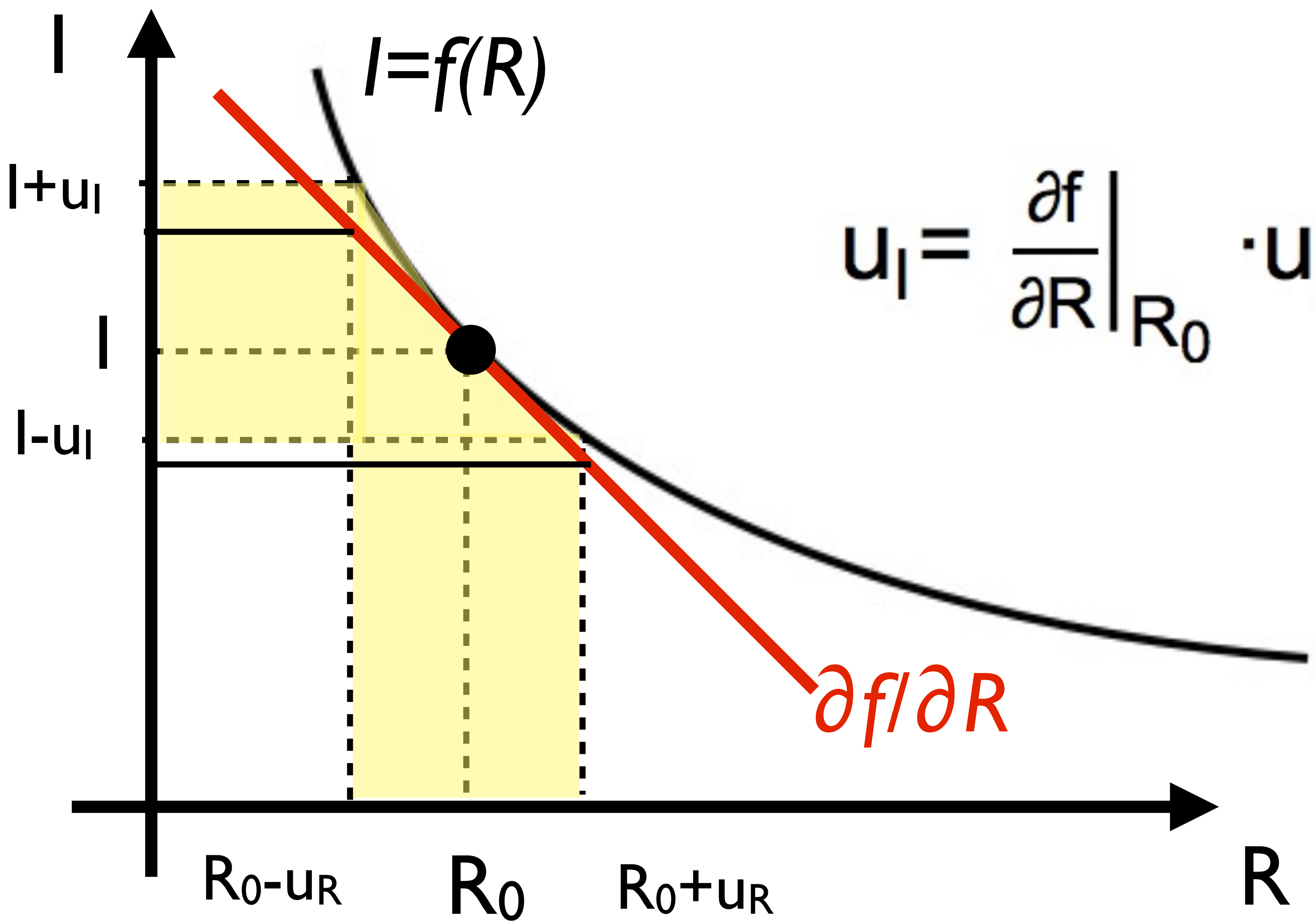




UNCERTAINTY OF DERIVED MEASUREMENT

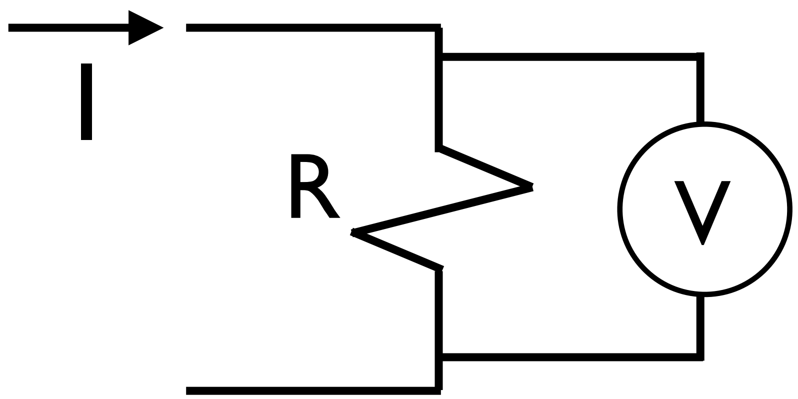


$I = V/R$



$$u_I = \left.\frac{\partial f}{\partial R}\right|_{R_0} \cdot u_R = -\frac{1}{R_0^2} \cdot u_V$$

UNCERTAINTY OF DERIVED MEASUREMENT



I=V/R

I have two contributions to the uncertainty of I

u_V u_R

u_I

$$u_I = \sqrt[2]{\left(\left.\frac{\partial f}{\partial V}\right|_{V_0} \cdot u_V\right)^2 + \left(\left.\frac{\partial f}{\partial R}\right|_{R_0} \cdot u_R\right)^2}$$

UNCERTAINTY OF DERIVED MEASUREMENT

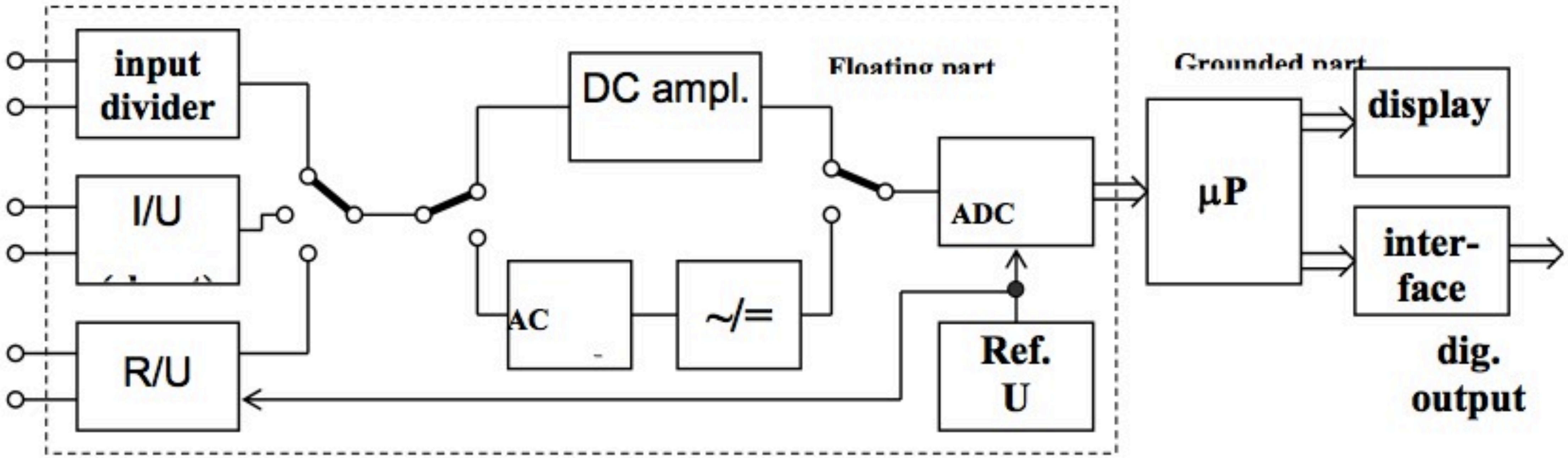
Generally if you indirectly measure a quantity Y as function f of N variables $X_1, X_2, ... X_N$

$$Y=f(X_1, X_2, ... X_N)$$

the uncertainty of Y is

$$u_Y = \sqrt[2]{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial X}\bigg|_{X_{i_0}} \cdot u_{X_i}\right)^2}$$

Digital multimeters



Multimeters of higher class:

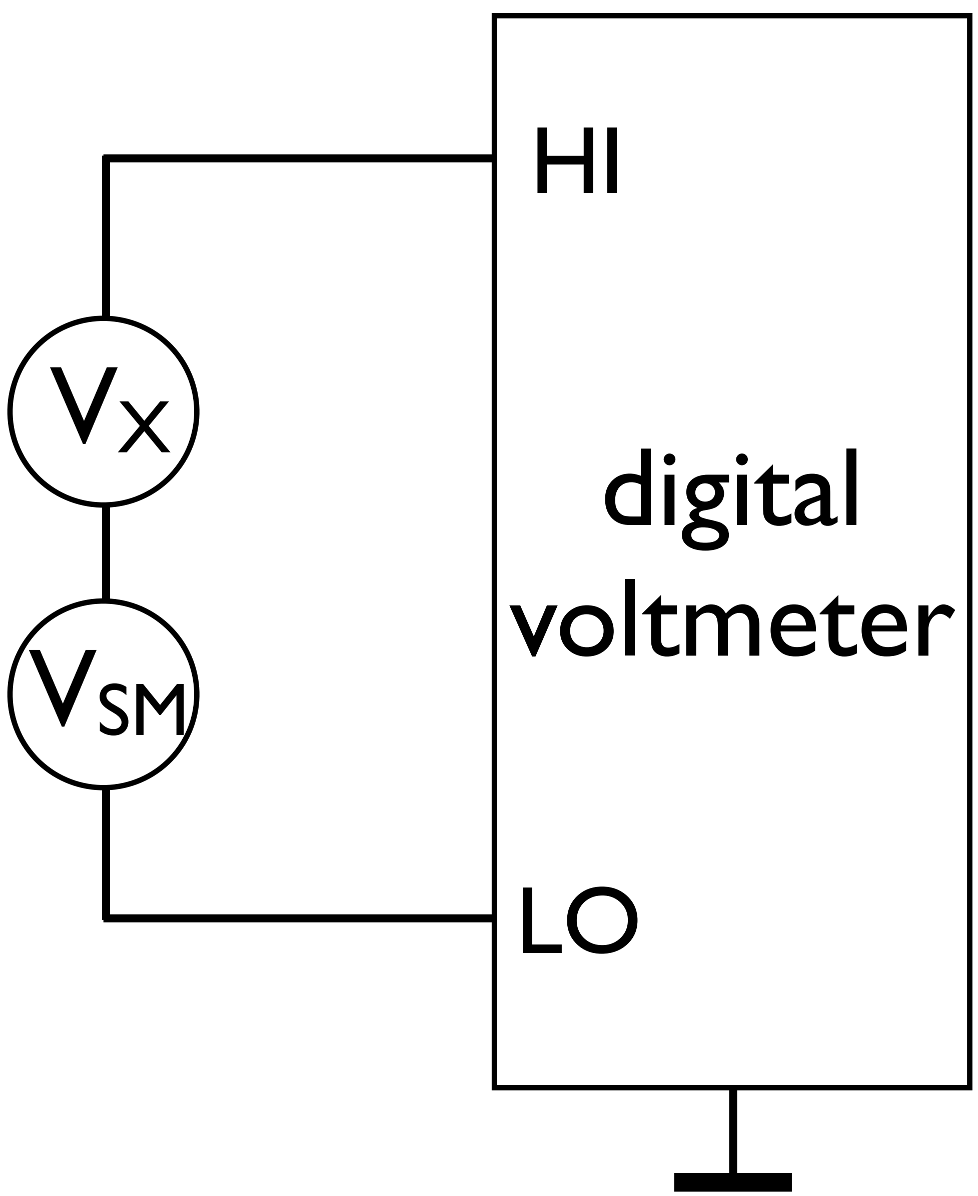
a) have floating screen - guard (see further);

b) allow for 4-ternimal connection by resistance measurement;

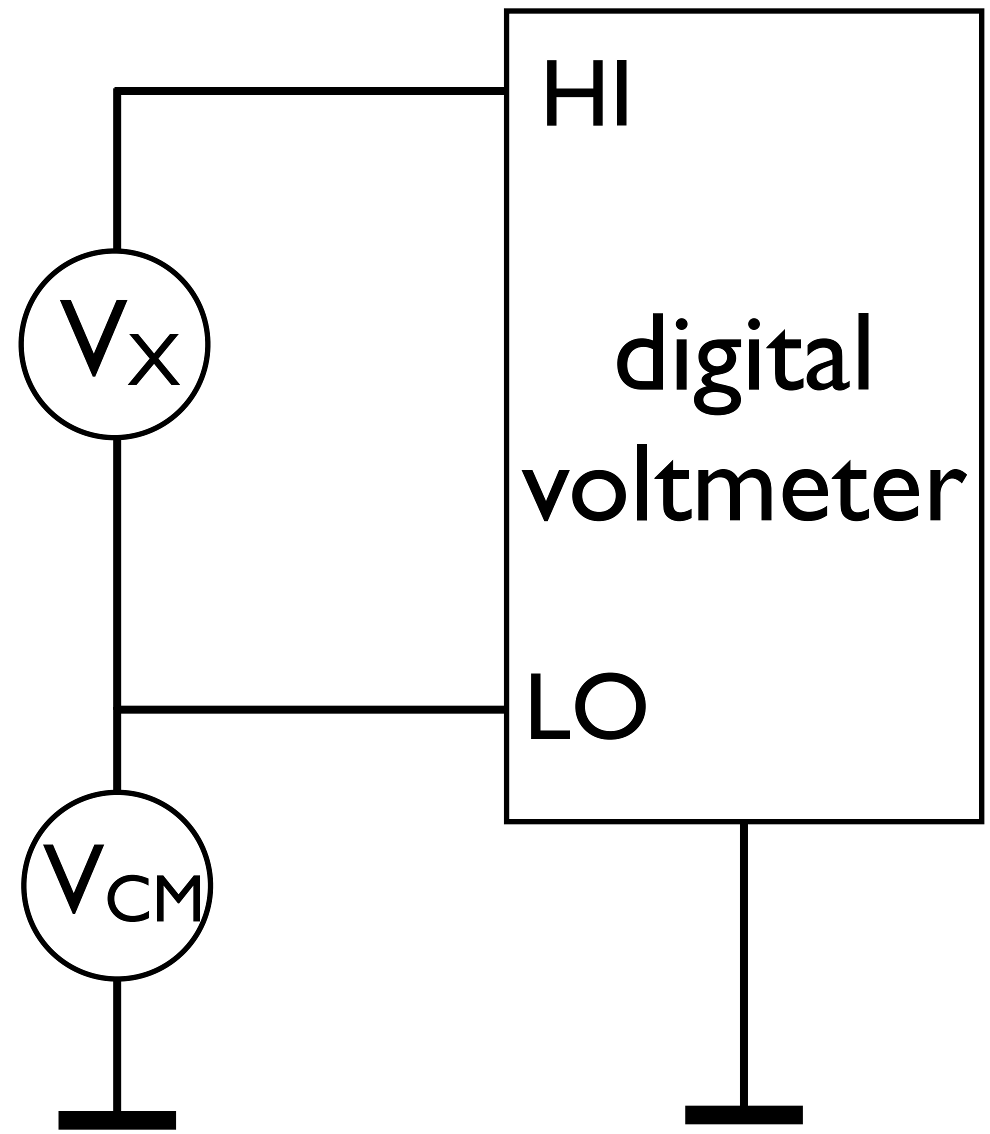
c) built-in processor allows auto-calibration and software calibration (see lecture 13);

d) allow connection to a computer using standard interface (RS-232 / IEEE 488 / USB).

INTERFERENCE TYPES

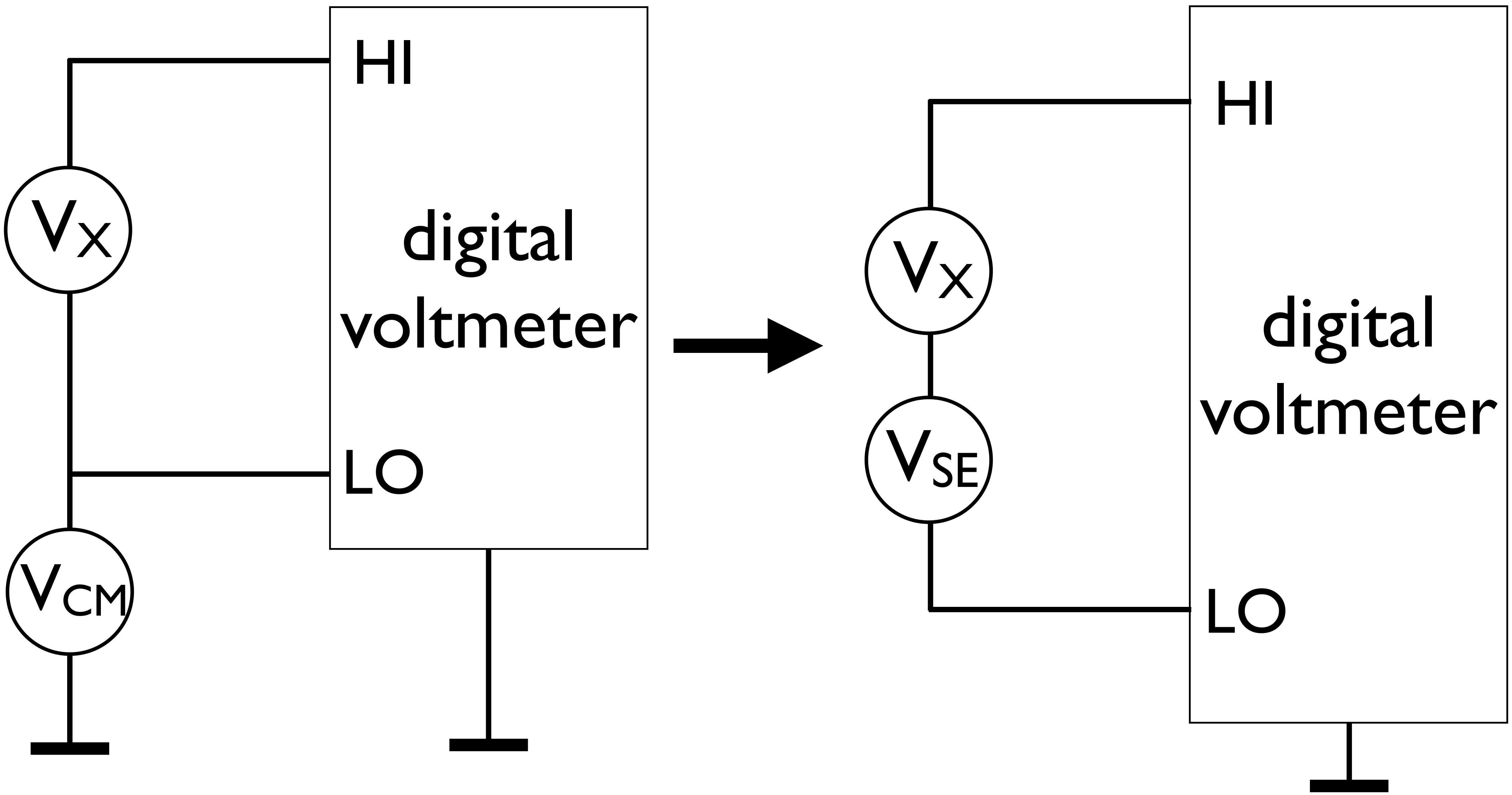


serial mode



common mode

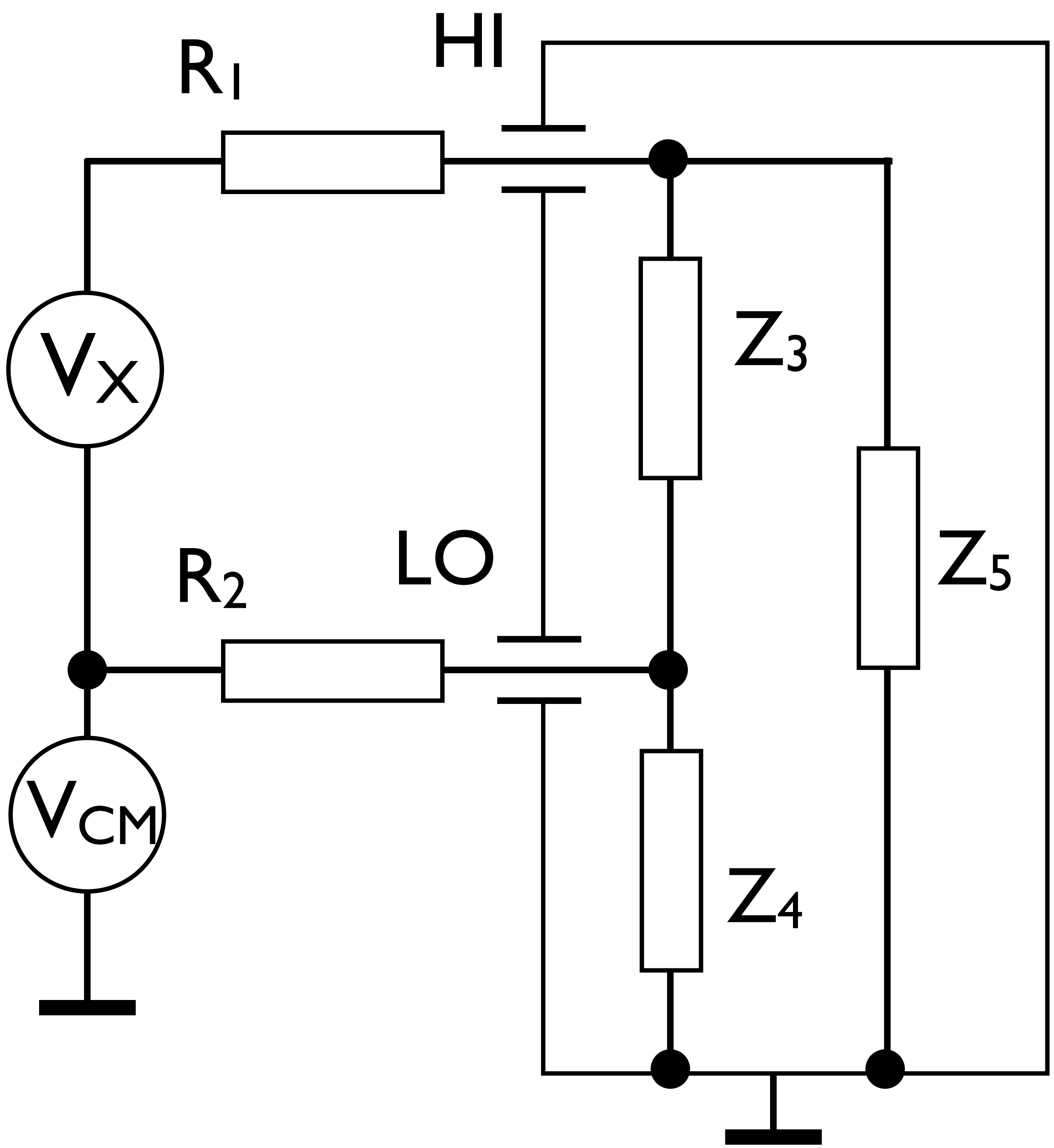
COMMON MODE



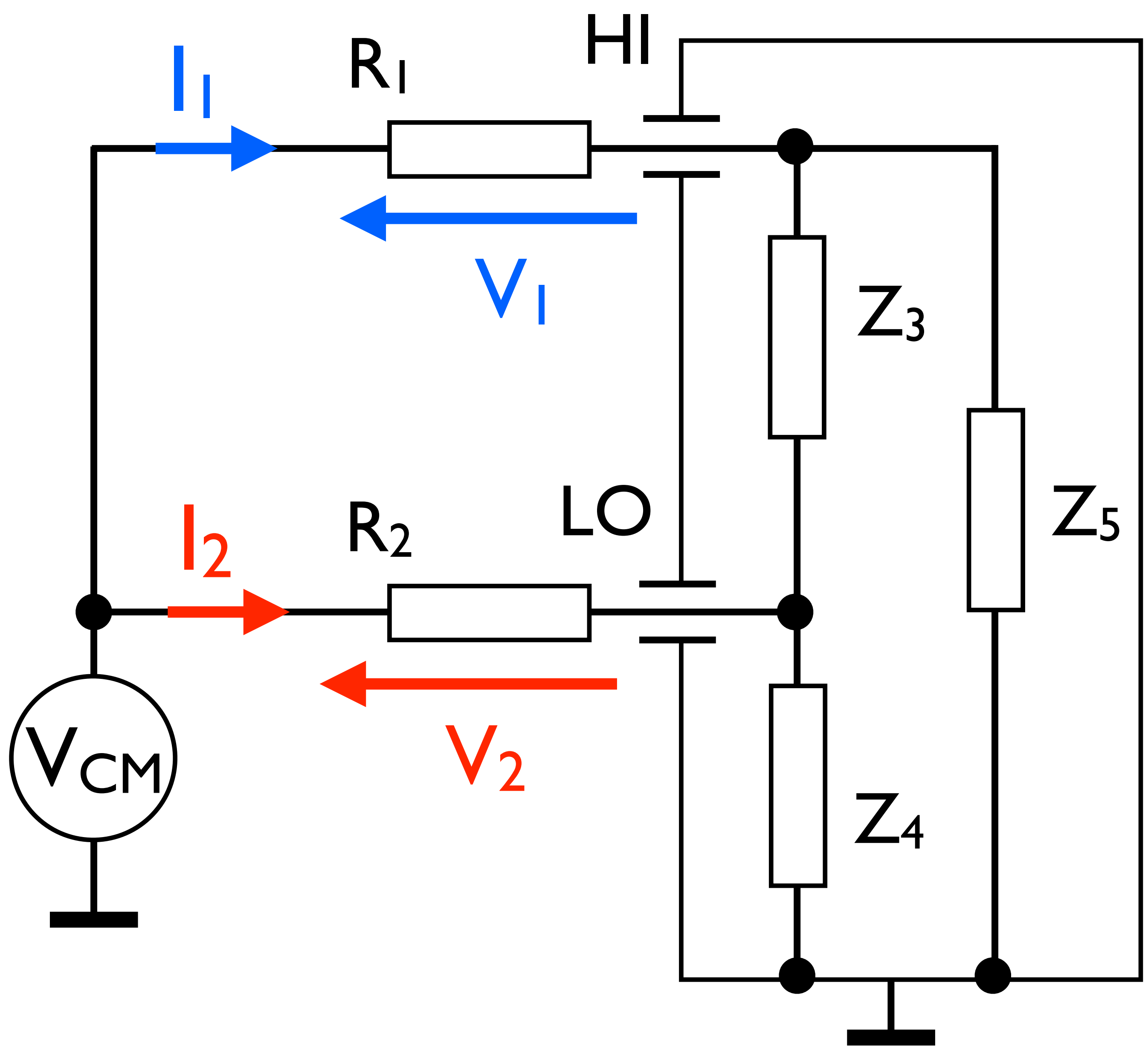
common mode

equivalent serial mode

COMMON MODE



COMMON MODE



If $I_1 = I_2$
then $V_1 + V_2 = 0$

unfortunately

$|Z_5| >> |Z_4|$

therefore

$|I_1| << |I_2|$

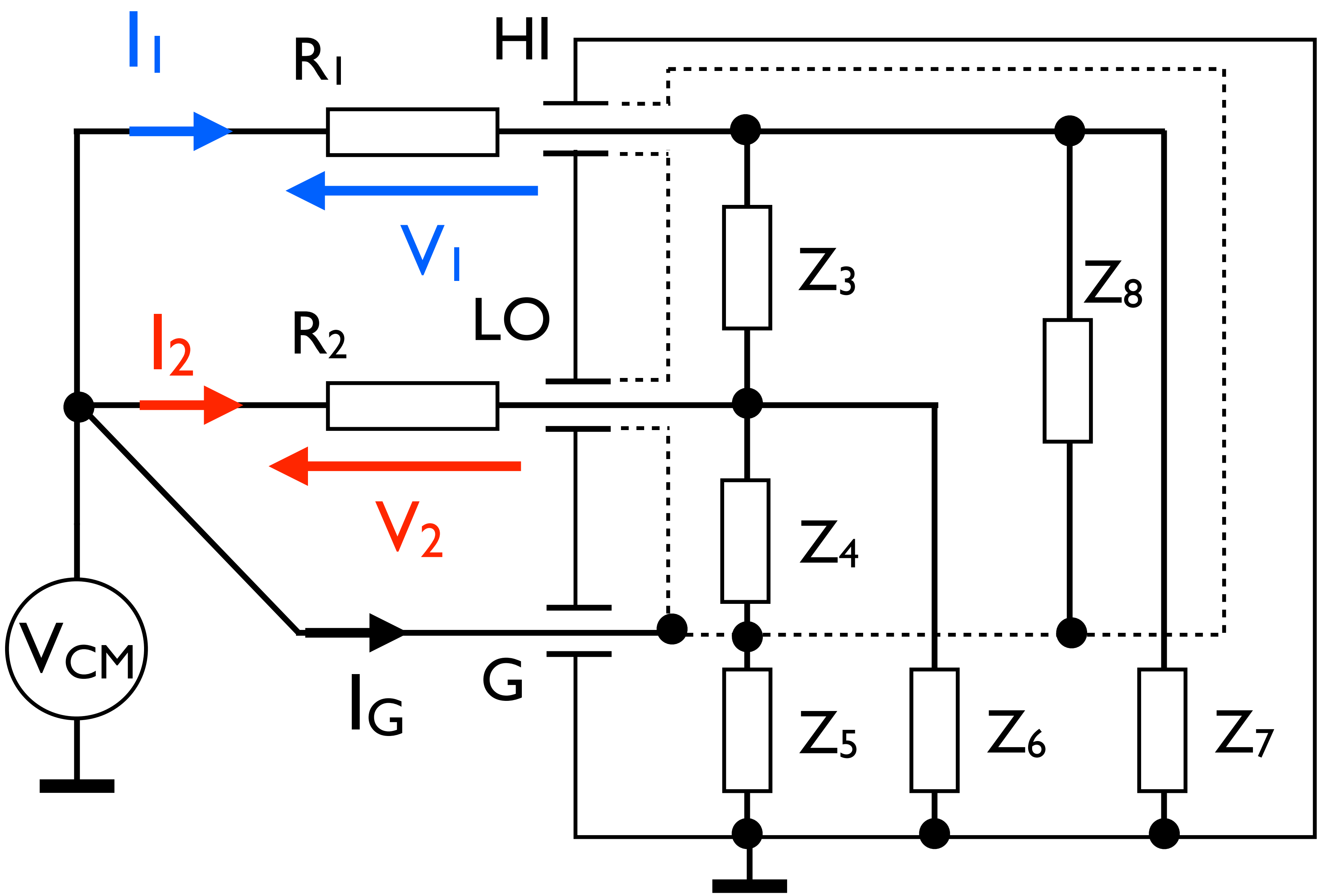
and

$|V_1| << |V_2|$

$$|V_{SME}| = |V_1 - V_2| \cong |V_2| = |R_2 \cdot I_2| \cong \left| R_2 \cdot \frac{V_{CM}}{|Z_4|} \right|$$

$$CMR = 20\log\left(\frac{V_{CM}}{V_{SME}}\right) \cong 20\log\left(\frac{|Z_4|}{R_2}\right)$$

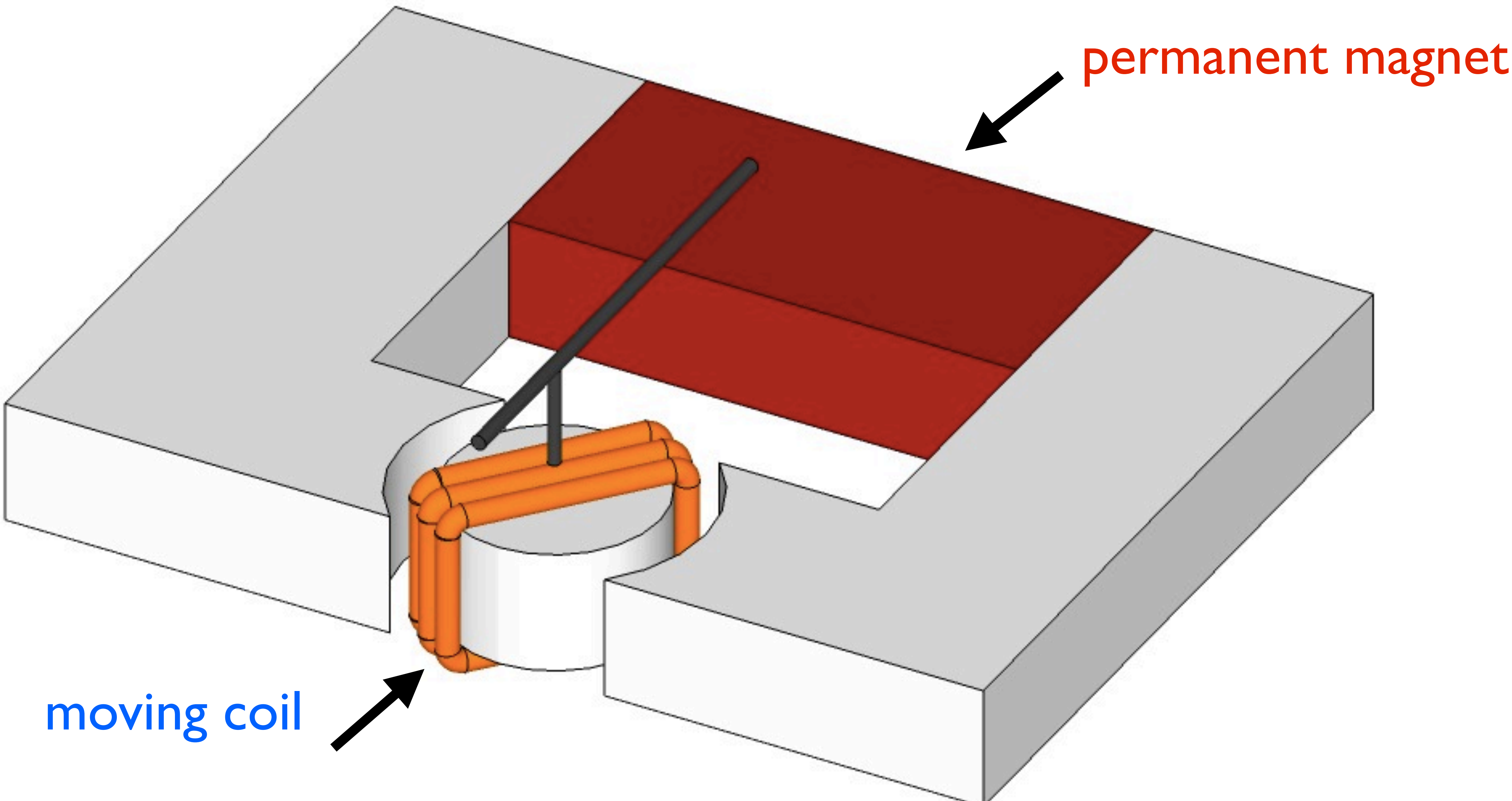
COMMON MODE
with guard



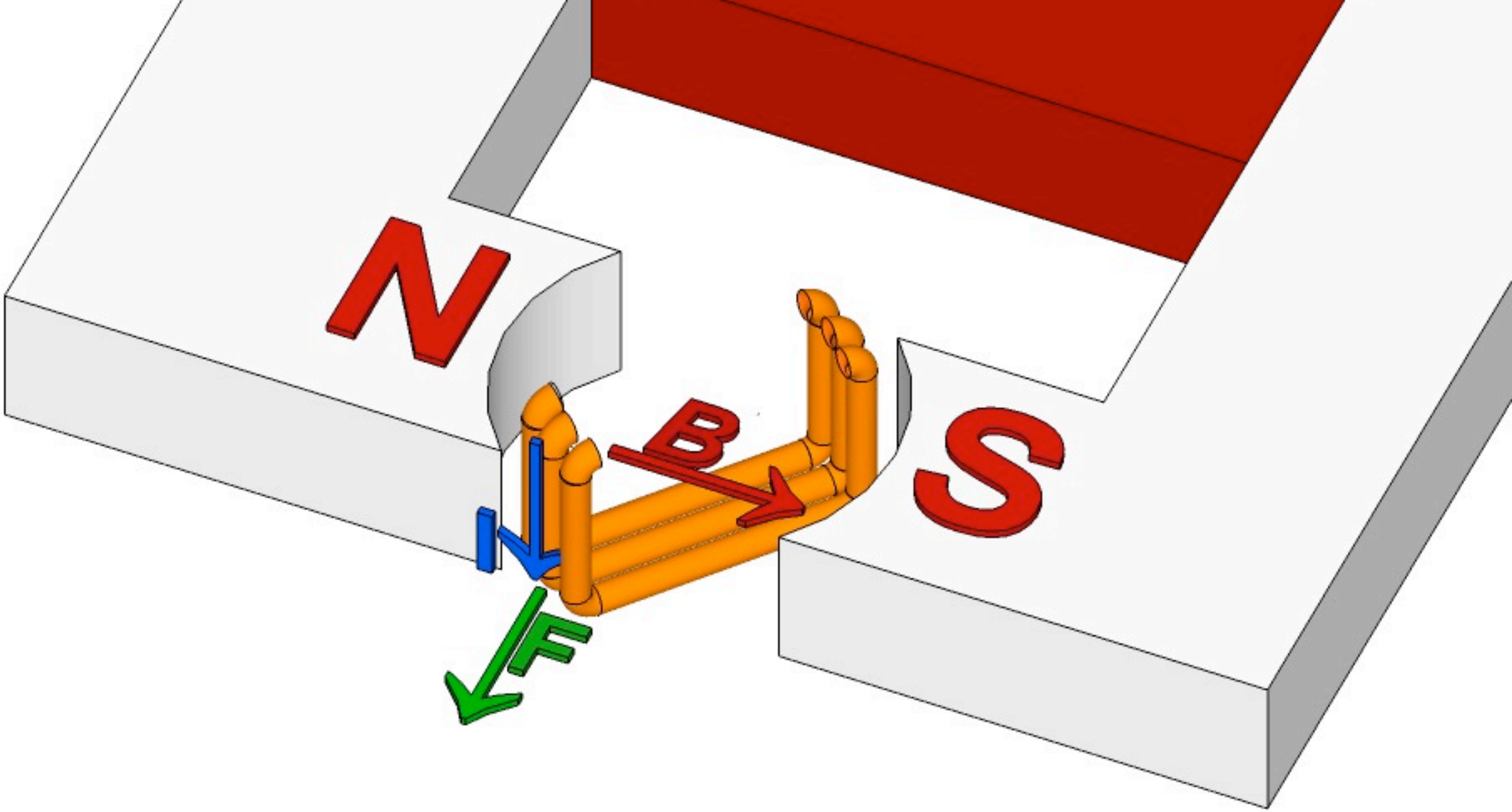
Now the voltage between LO and G is almost 0, thus Z_4 is basically shortcircuited. I_G does not affect measurement

PMMC METER

permanent magnet moving coil meter



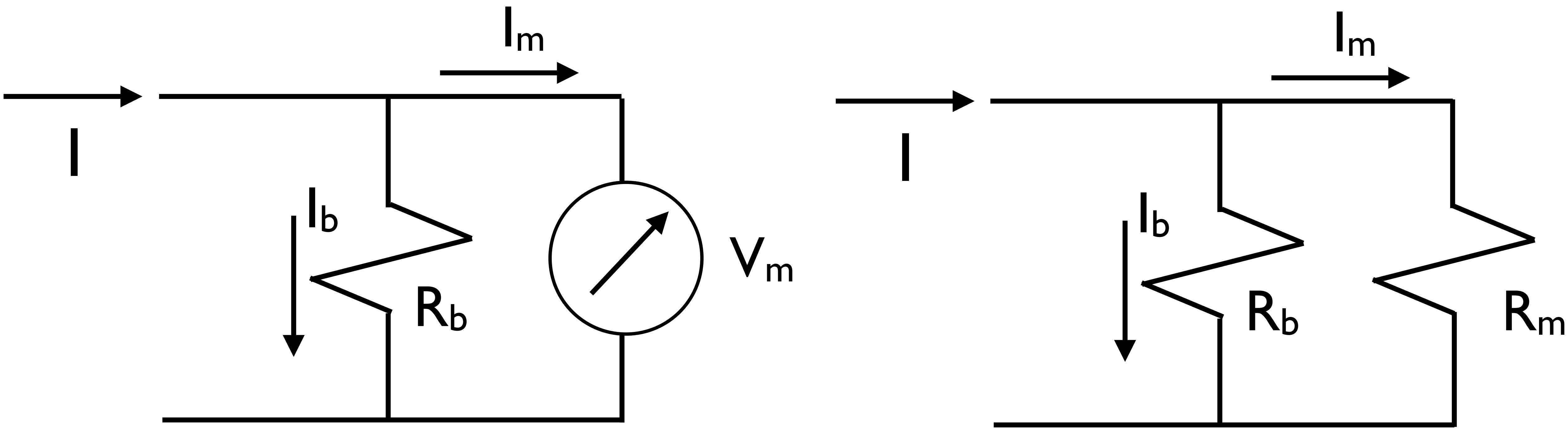
DC ammeters and voltmeters



The coil rotates as an effect of a force $F = I \cdot L \cdot B$
which creates a torque $T = 2 \cdot N \cdot (I \cdot L \cdot B) \cdot r = c_d \cdot I$

I=current in the coil; L=length of the wire; N=number of turns; r=radius of coil

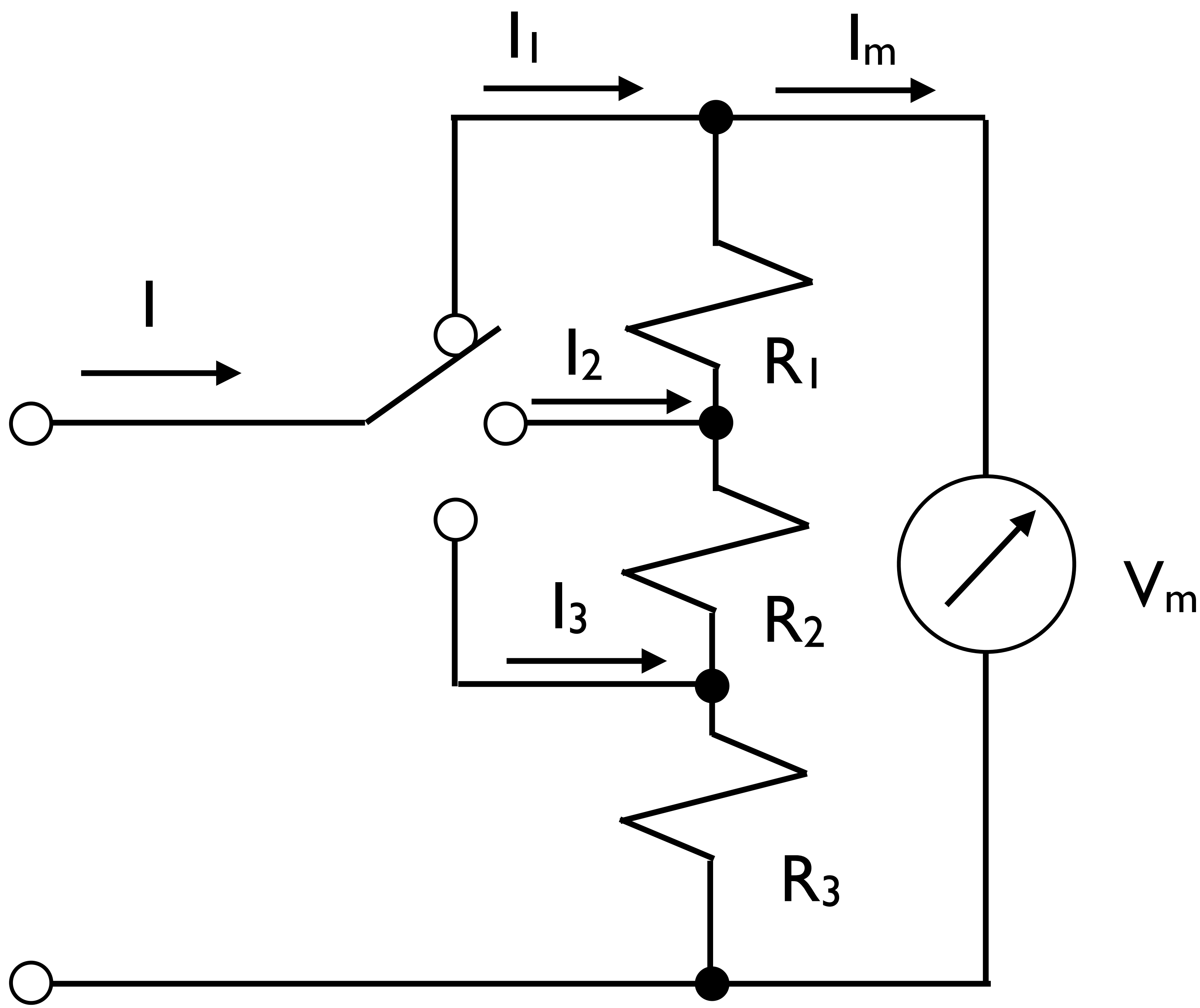
PMMC DC Ammeters



R_m= resistance of the moving coil
R_b= bias resistance

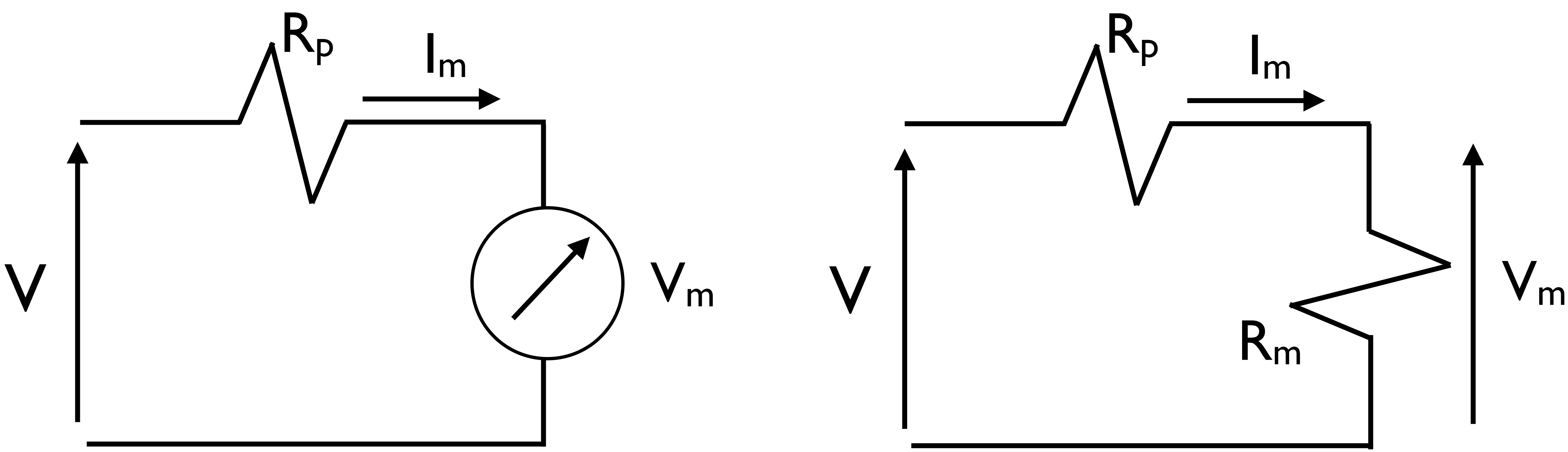
If we want to increase the range by a factor N we must choose bias resistance $R_b = R_b/(N-1)$

PMMC DC Ammeters with multiple ranges



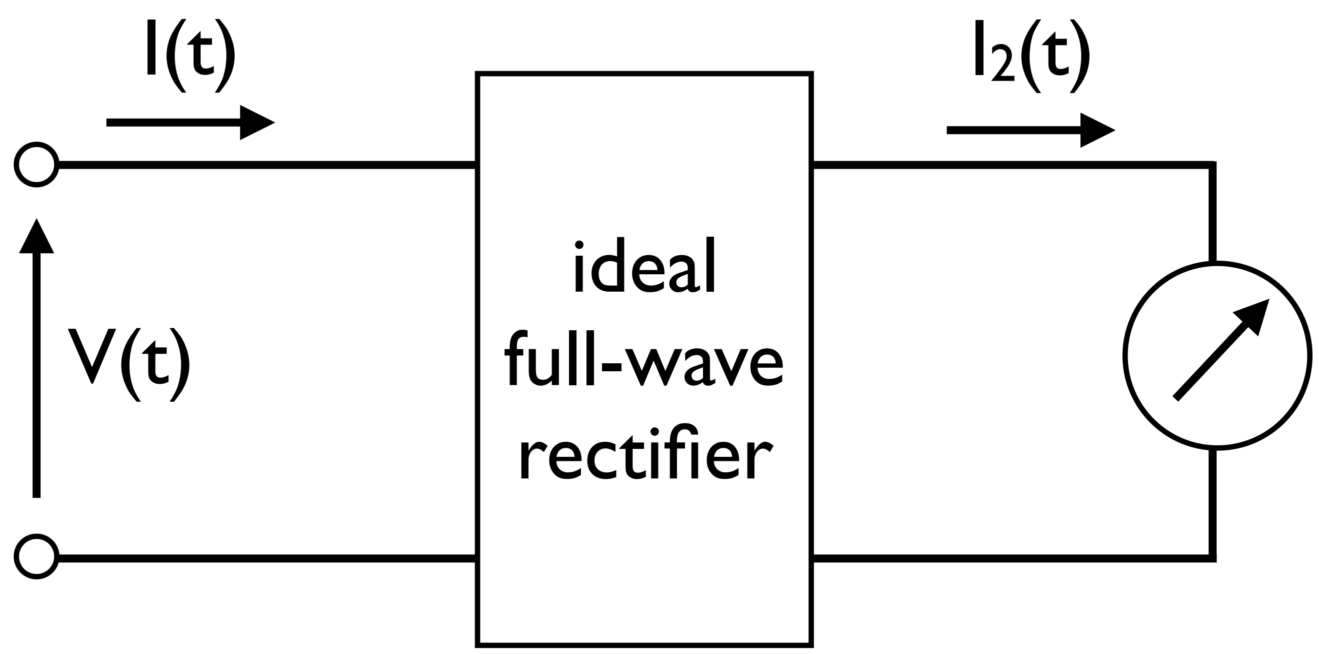
If we want to increase the range by a factor N we must choose bias resistance $R_b= R_b/(N-1)$

PMMC DC Voltmeters



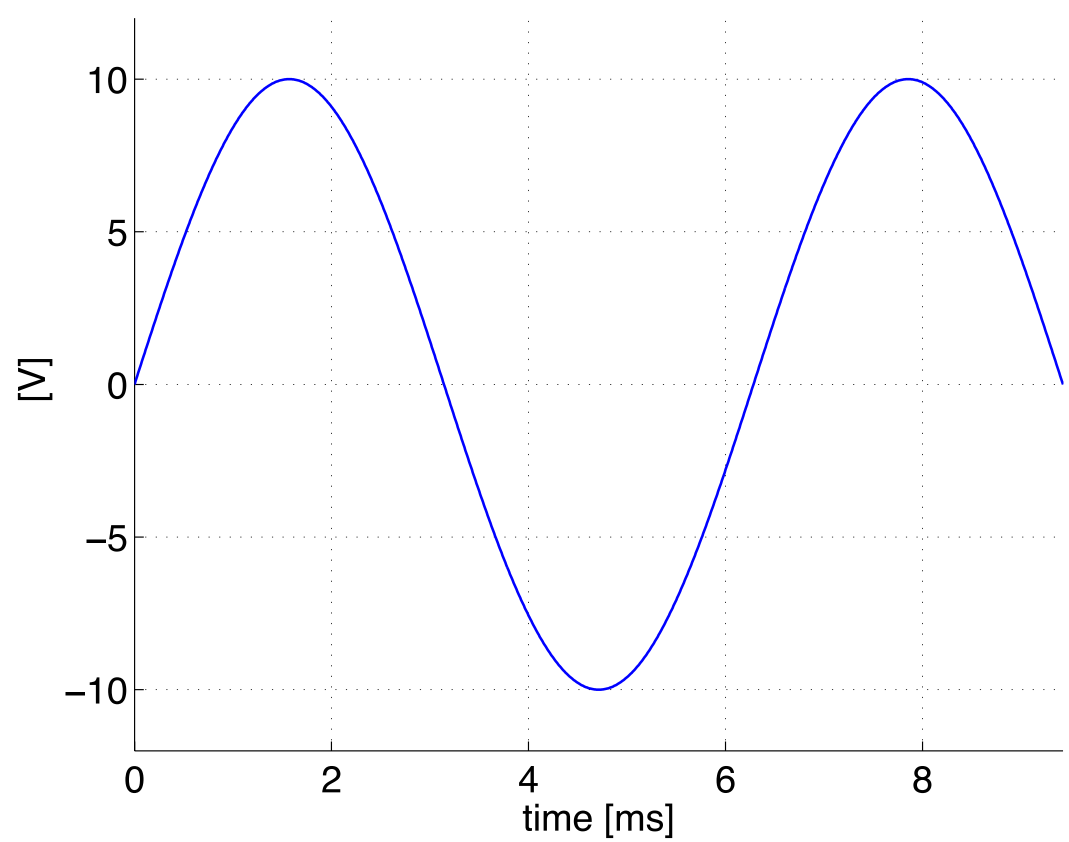
The meter measures a voltage $V_m = R_m \cdot I_m$.
We can increase the voltage range by a factor N with a voltage divider using a resistor
$R_p=(N-1)R_m$

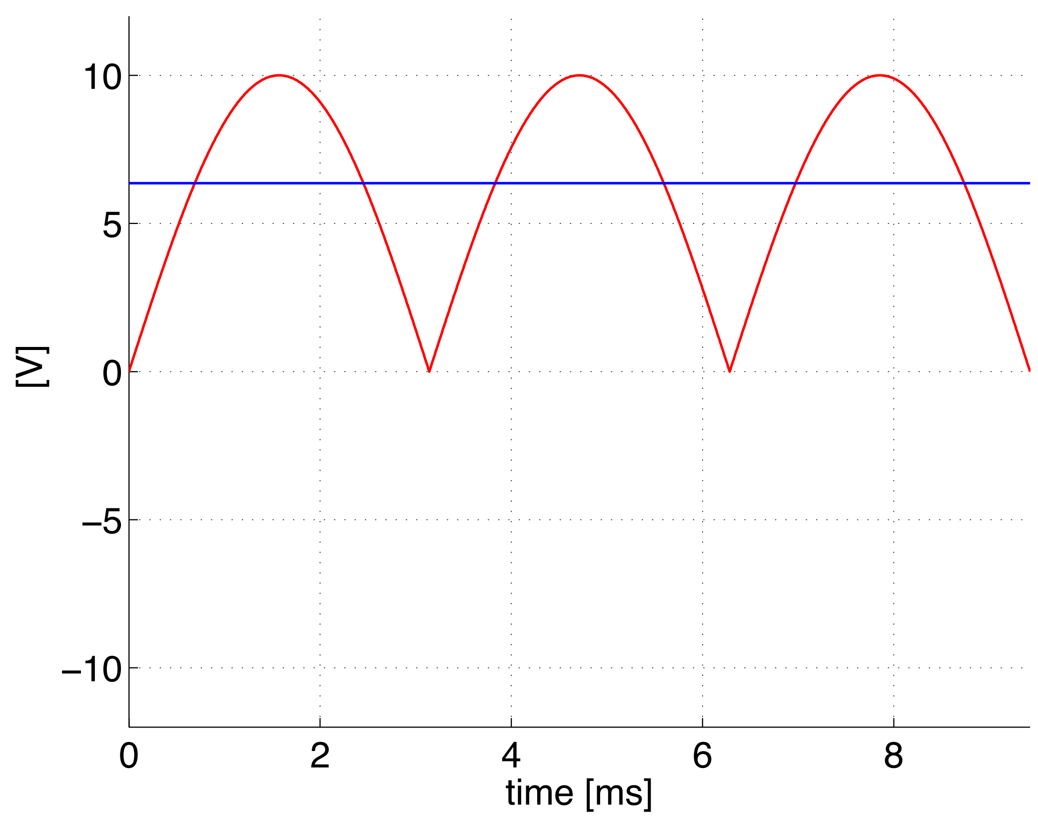
What if we want to measure AC current or voltage?



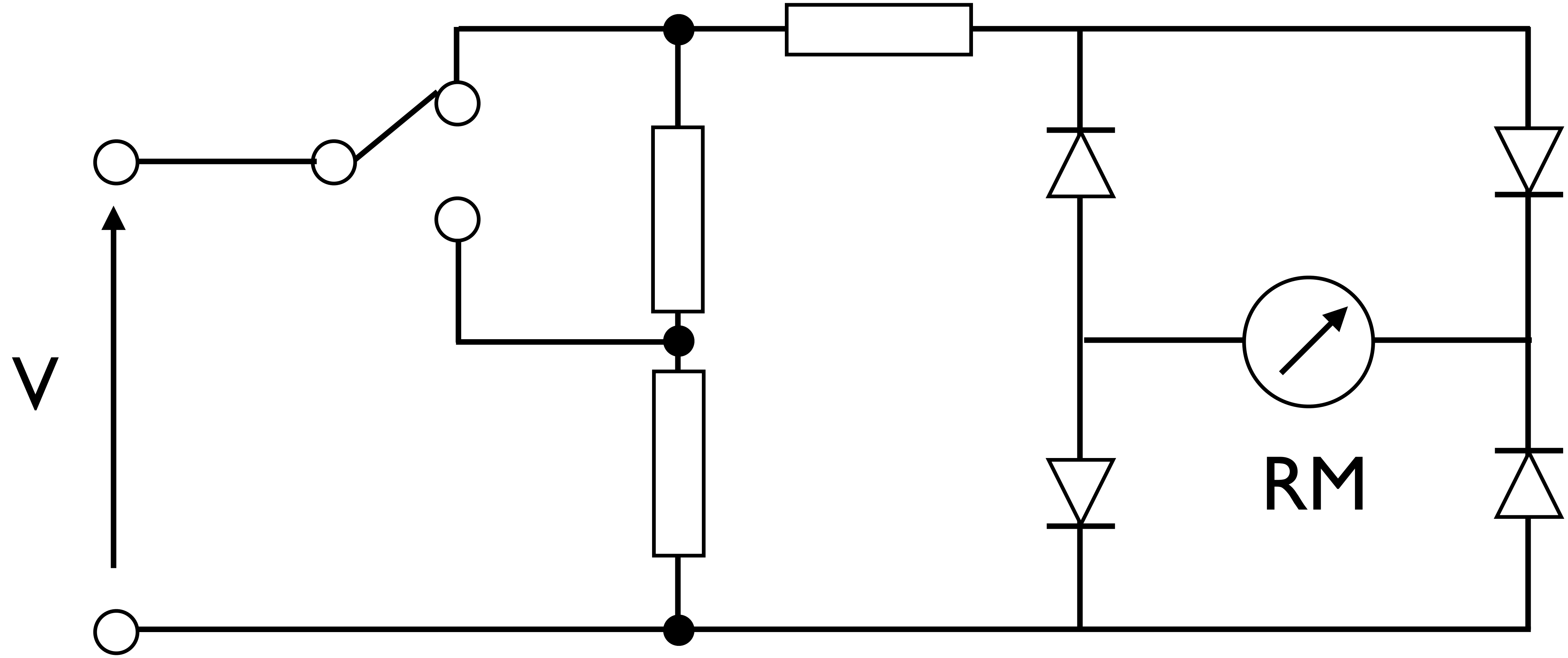
Rectified mean value is NOT RMS value

RMS value = 1.11 RM value (for sinewave)



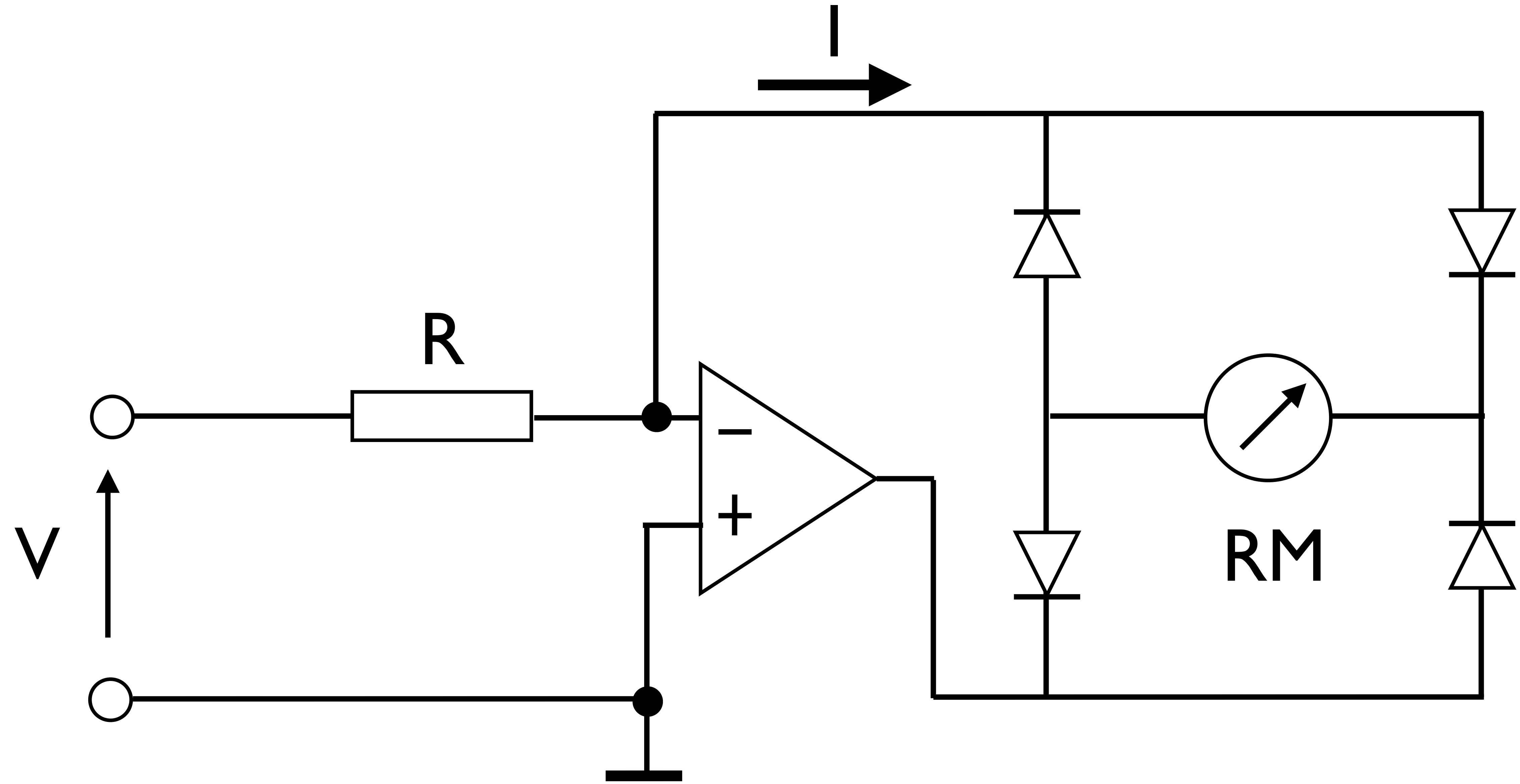


Real rectifier



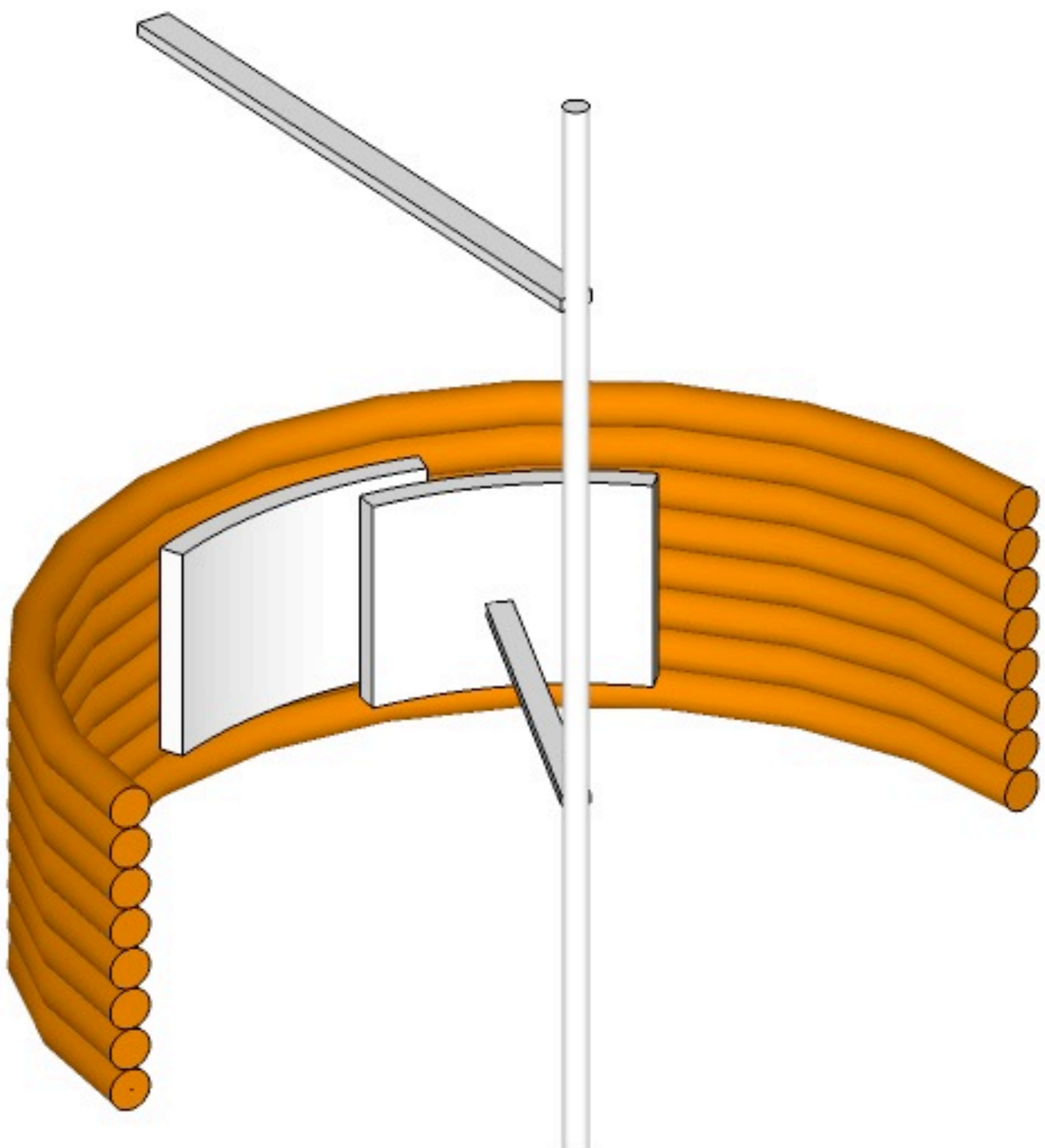
Problem: the diodes are
NOT linear.
At low voltages non-linearity
must be compensated

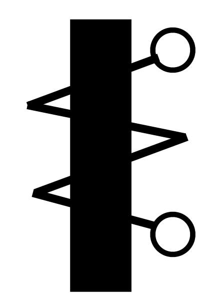
For very low voltage (<1 V) active compensation of rectifier non-linearity must be used

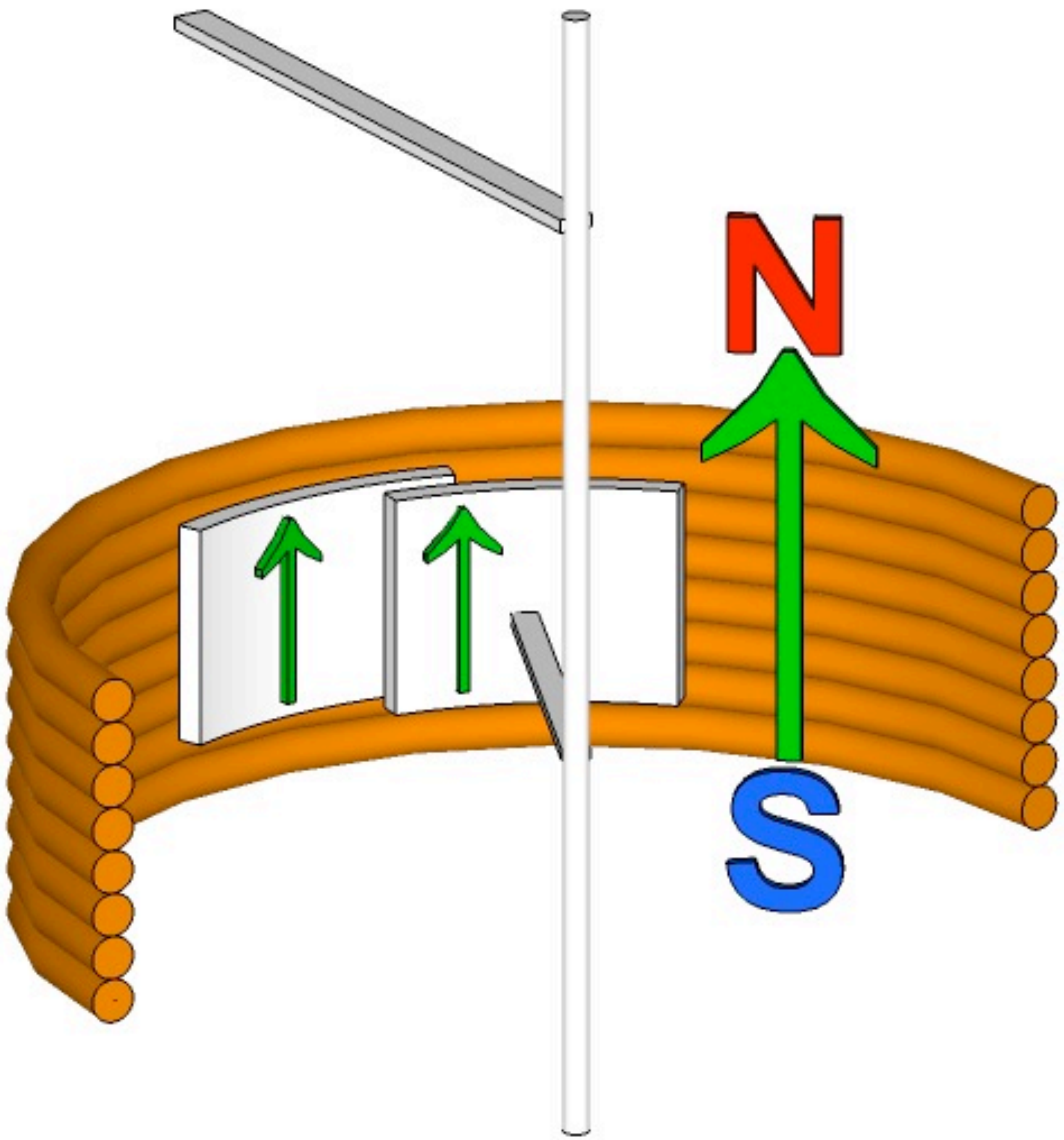


The current I does not depend on rectifier, it always results I=V/R.

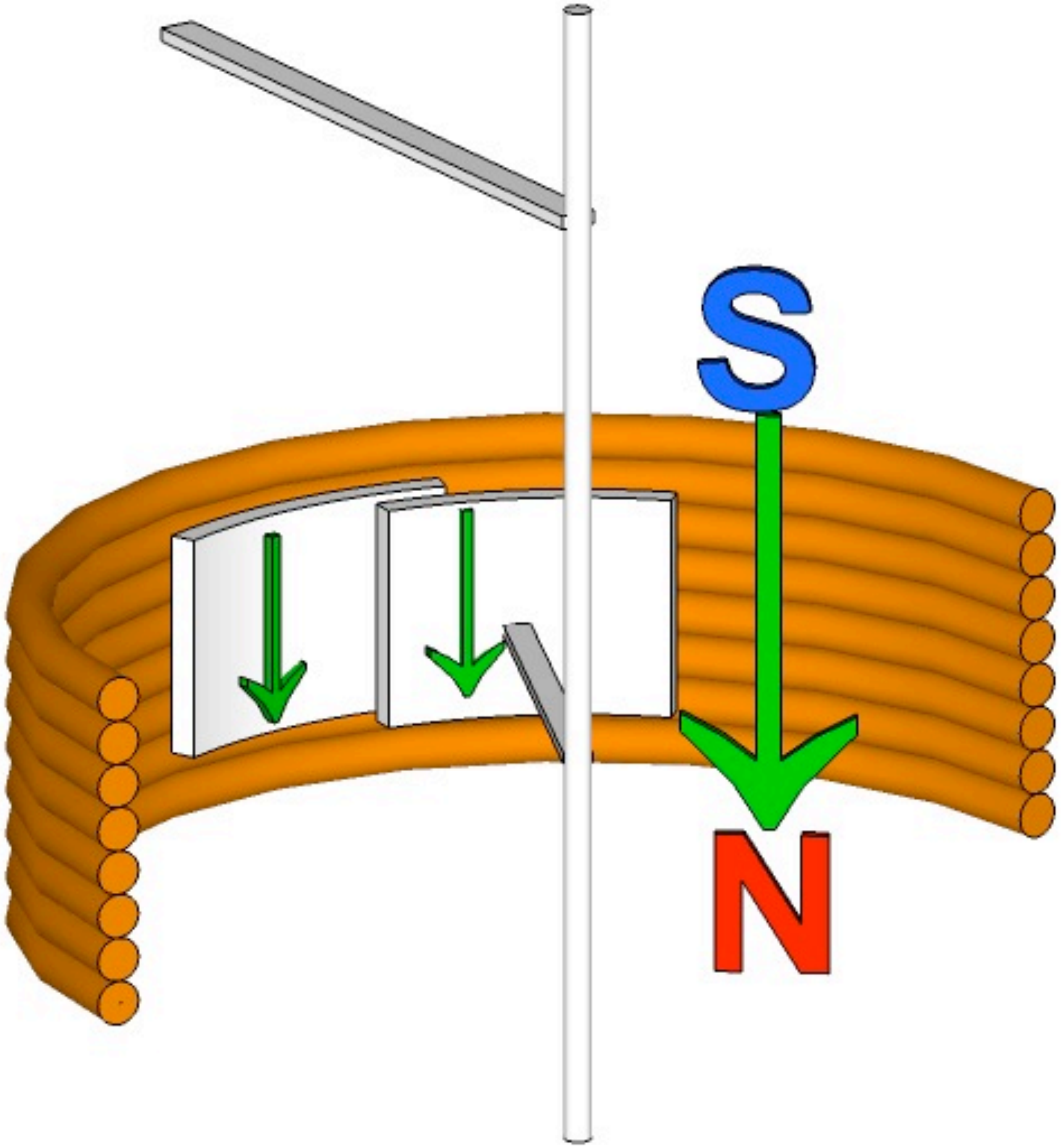
Moving Iron Instruments
(Iron Vane Meters)







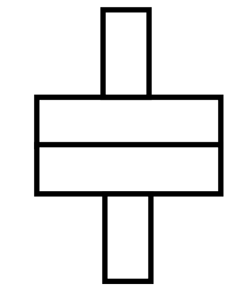
The current flowing in the coil magnetizes both iron segments in the same direction. Thus, they repel and rotation occurs.

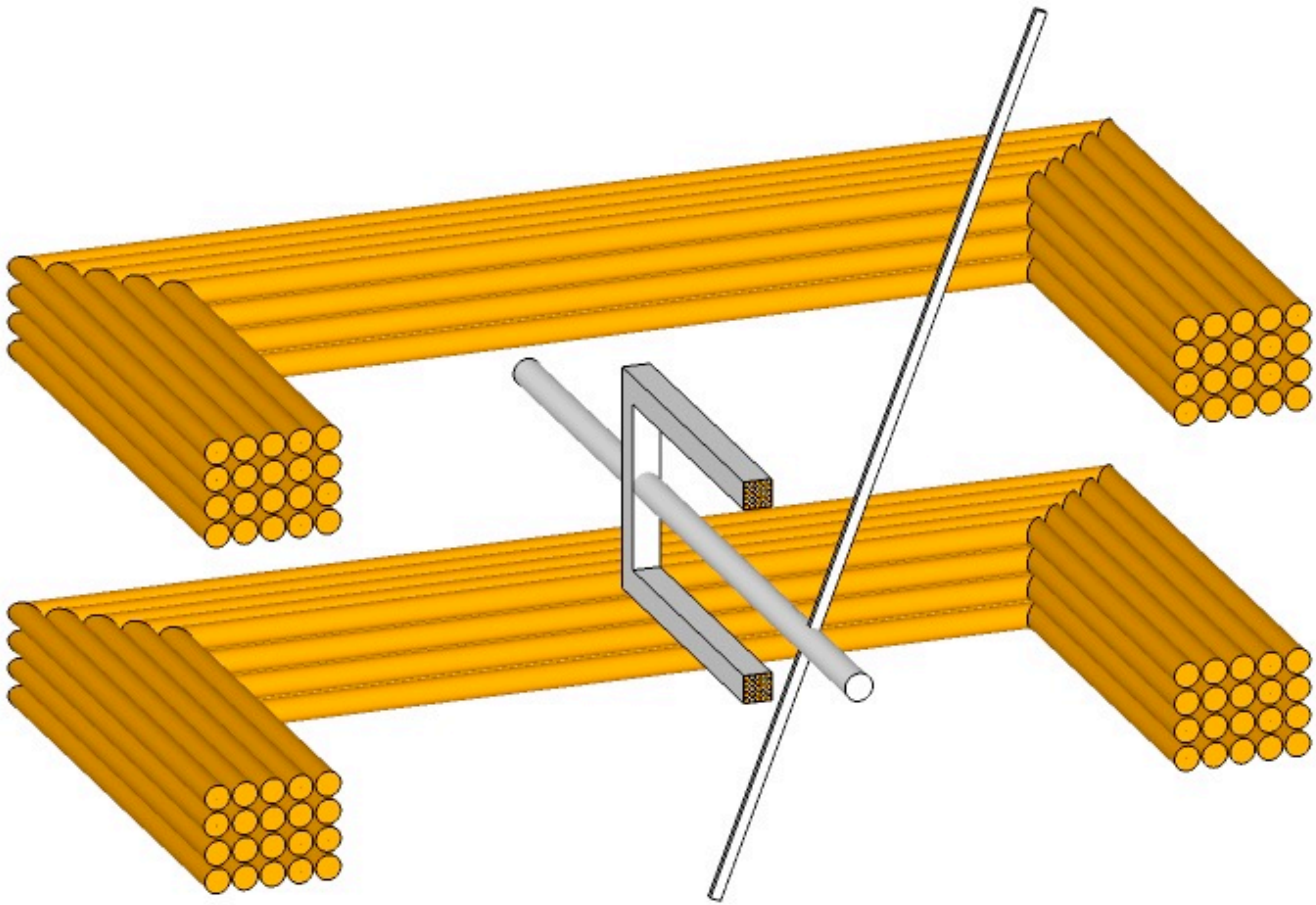


Repulsion direction does not depend on the direction of magnetization, because the iron segments repels in the same way either magnetized in one direction or in the opposite direction.

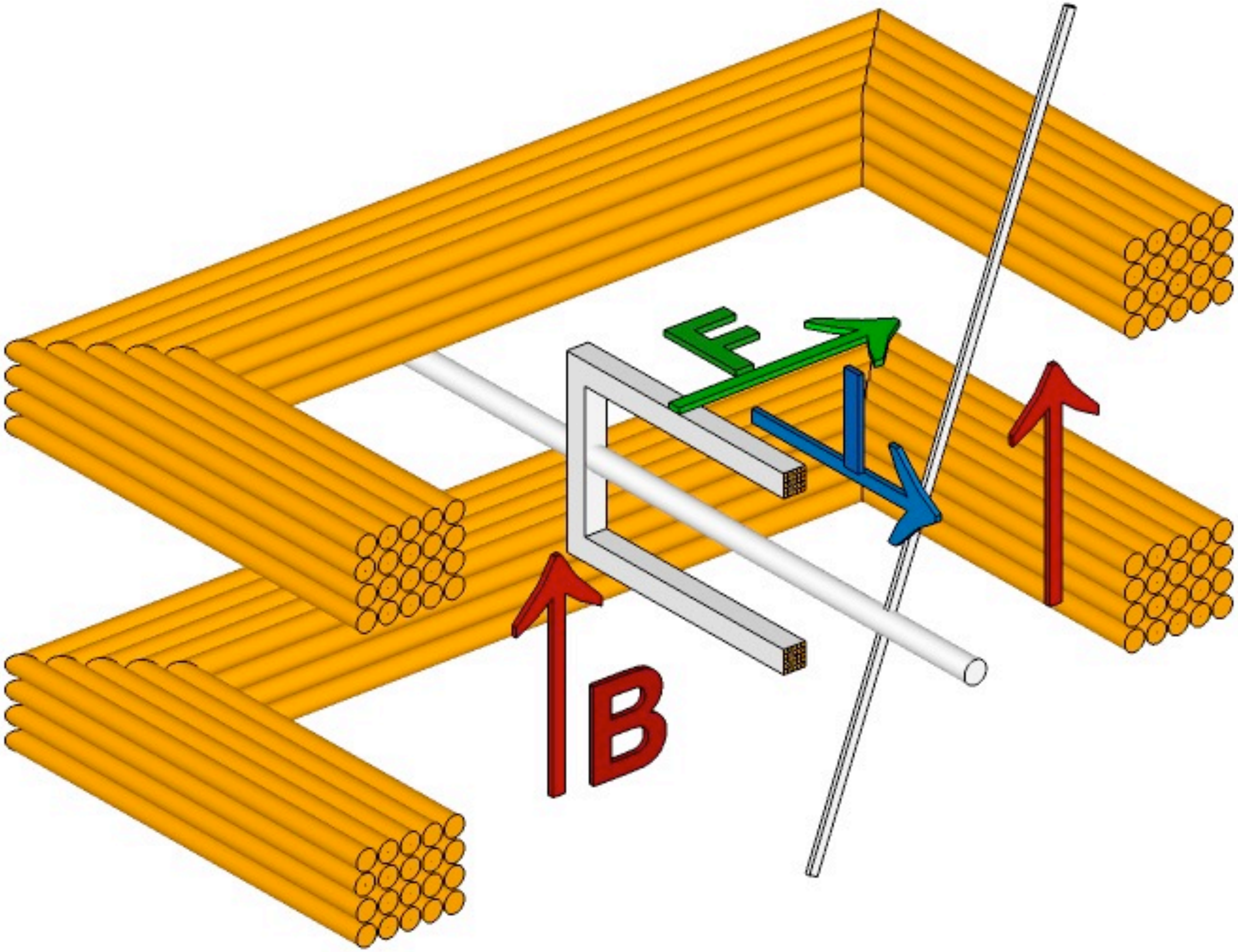
AC METERS

ELECTRODYNAMIC INSTRUMENTS



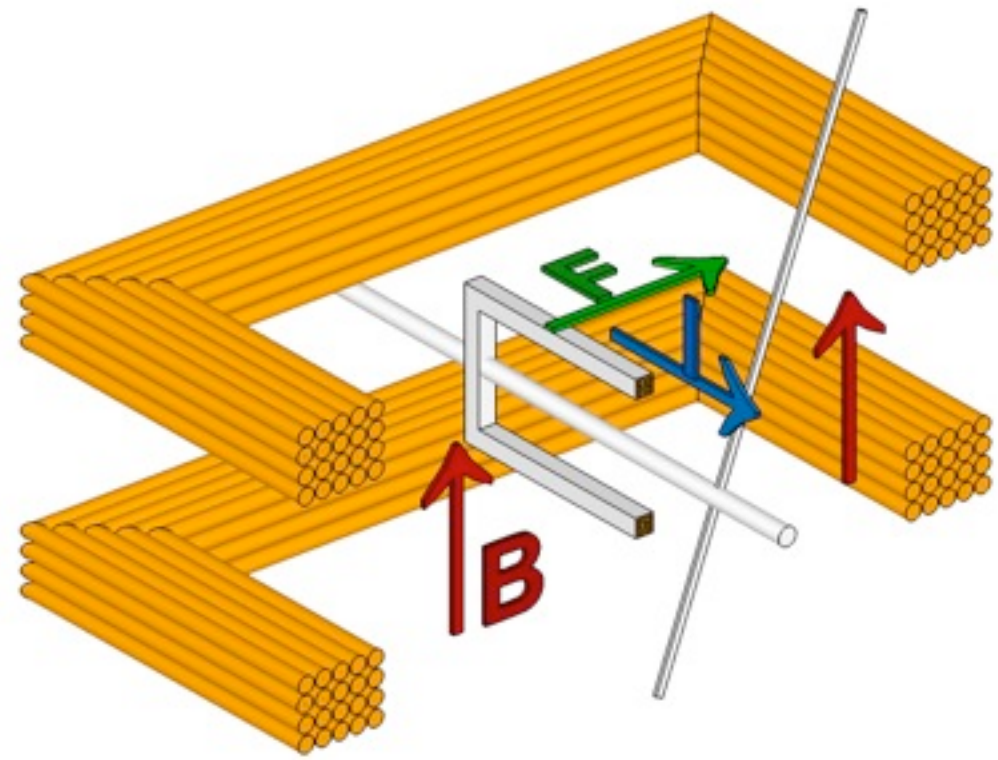


Two coils: one fixed, one moving. Two currents.



The torque depends of both currents $T_d = k_d \cdot I_1 \cdot I_2$

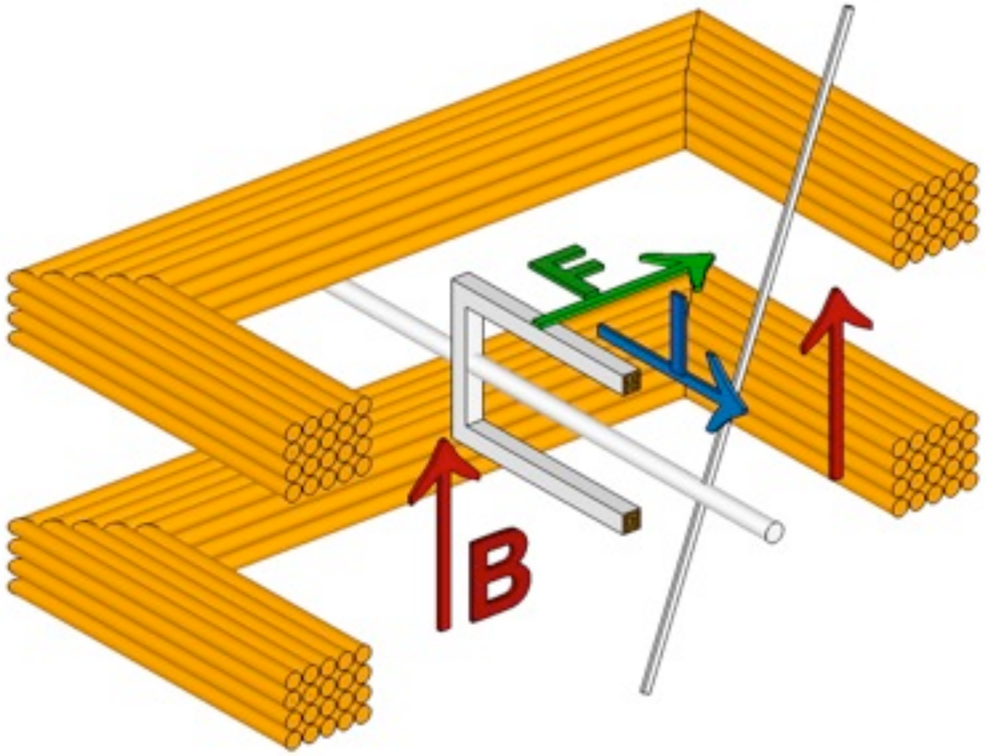
We can use it as wattmeter, as long as one of the two currents is proportional to a voltage



AC wattmeter

$$T_d = \frac{1}{T}\int_0^T t_d \mathrm{d}t = k_d \frac{1}{T}\int_0^T i_1 i_2 \mathrm{d}t = k_d \frac{1}{R_{VC}} \frac{1}{T}\int_0^T i_{CC} u_{VC} \mathrm{d}t = k_d \frac{P}{R_{VC}}$$

Voltage coil: like a voltmeter (series resistor to change range)
Ammeter coil: like an ammeter (parallel resistor to change range)



Be careful to the voltmeter coil

The current is not simply given by the voltage divided by the coil's resistance. If the frequency is high enough, you should consider also the reactance. Thus, for a given voltage the current flowing in the coil will depend on the frequency

For sinewaves:

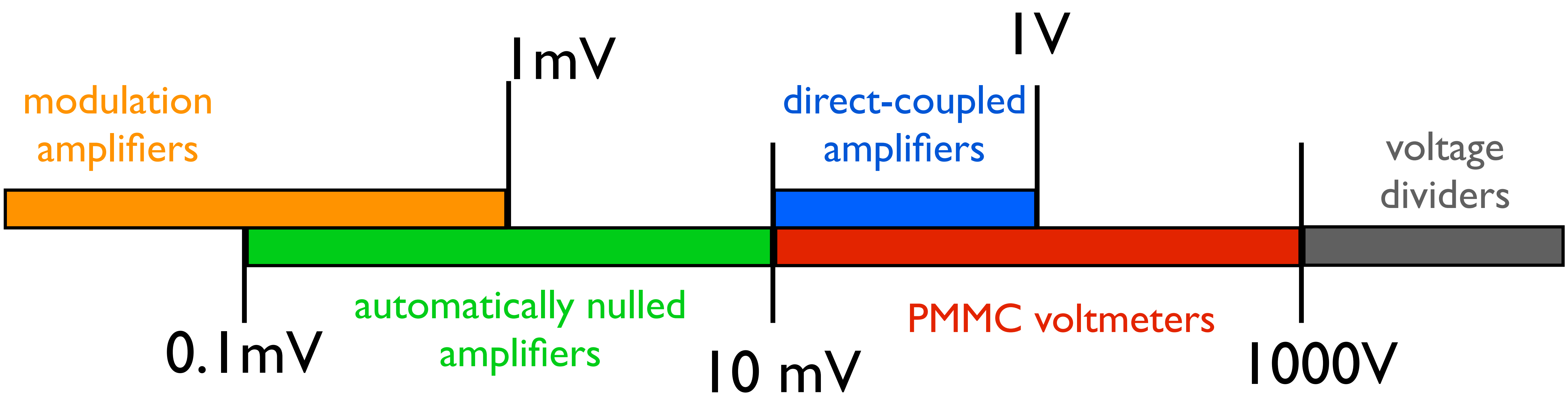
$$T_{\mathrm{d}} = k_{\mathrm{d}} \frac{U_{\mathrm{VC}}}{\sqrt{R_{\mathrm{VC}}^2 + \omega^2 L_{\mathrm{VC}}^2}} I_1 \cos(\varphi + \Delta\varphi)$$

How to measure:

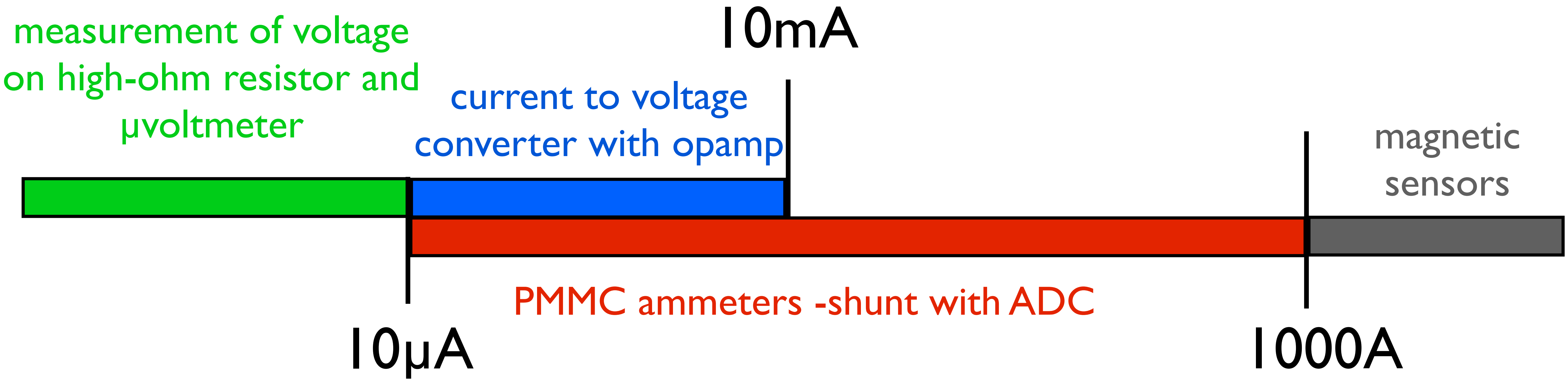
DC VOLTAGE — DC CURRENT

AC VOLTAGE — AC CURRENT

DC VOLTAGE



DC CURRENT


measurement of voltage on high-ohm resistor and μvoltmeter
10mA
current to voltage converter with opamp
magnetic sensors
PMMC ammeters -shunt with ADC
10μA
1000A

AC VOLTAGE

1. Rectified mean value (RM)

    - PMMC with rectifies

        2 ÷ 1000 V and 50 Hz ÷ 5 kHz

    - Digital Multimeters

        up to 100 kHz

    - For very low voltage lock-in amplifier

2. RMS value

    - Iron vane, electrodynamic instruments, PMMC with thermocouples (low frequency)
    - RMS to DC converters
    - Sampling and computation

AC CURRENT

- RMS value directly - iron vane ammeter
(1 mA ÷ 10A, up tp 1 kHz)

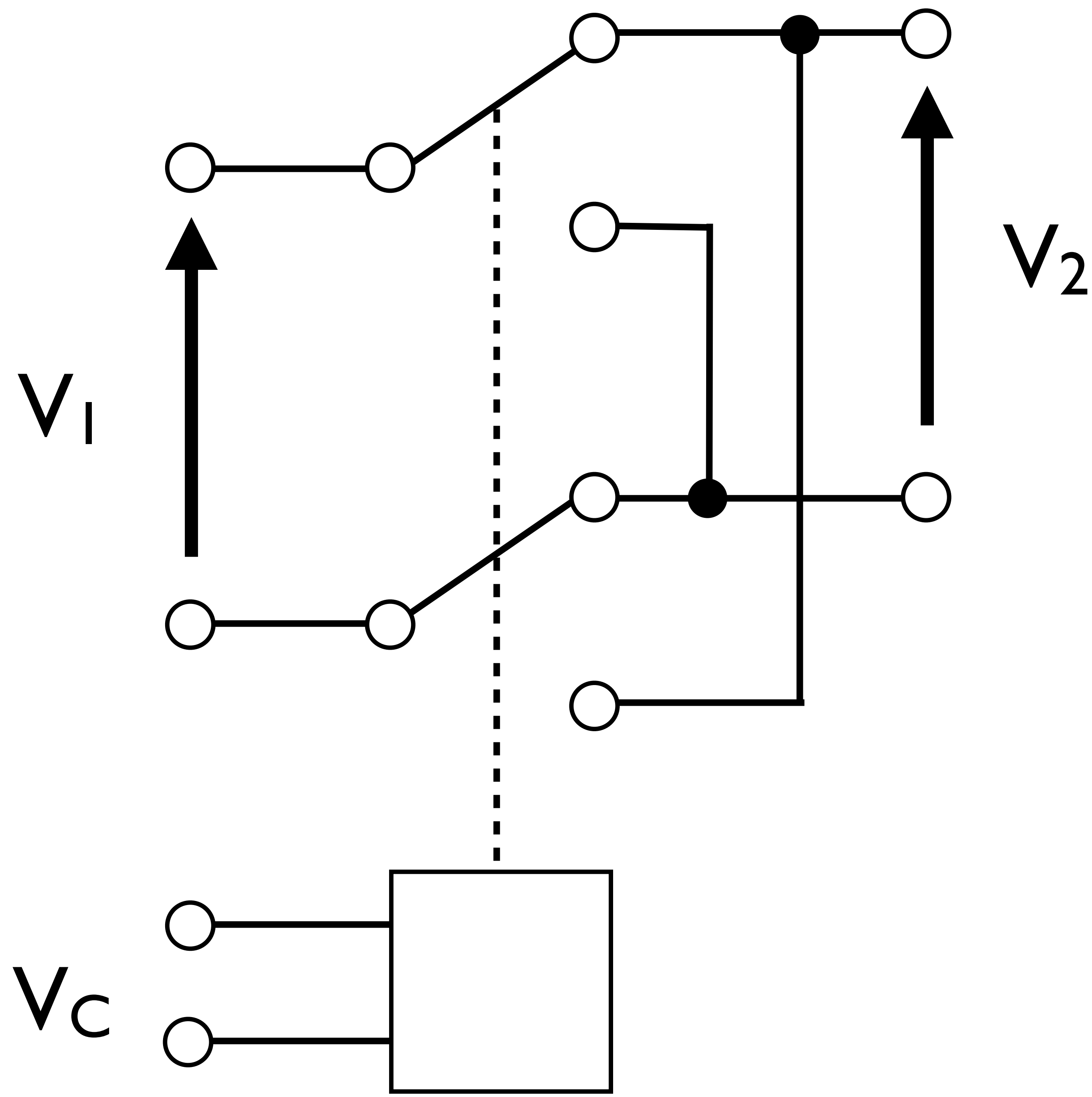
- PMMC with rectifier for sinewaves

- Digital multimetes (up to hunderds kHz)

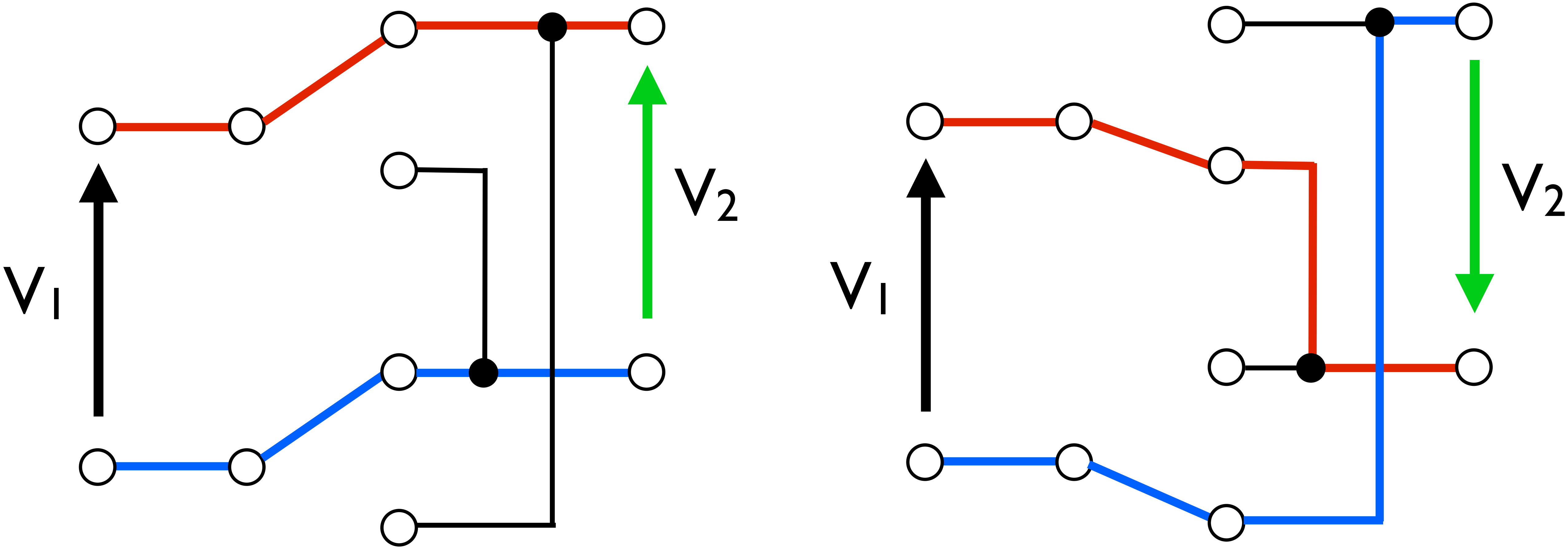
- currents probes (galvanic insolation)

CONTROLLED PHASE SENSITIVE RECTIFIERS

Principle

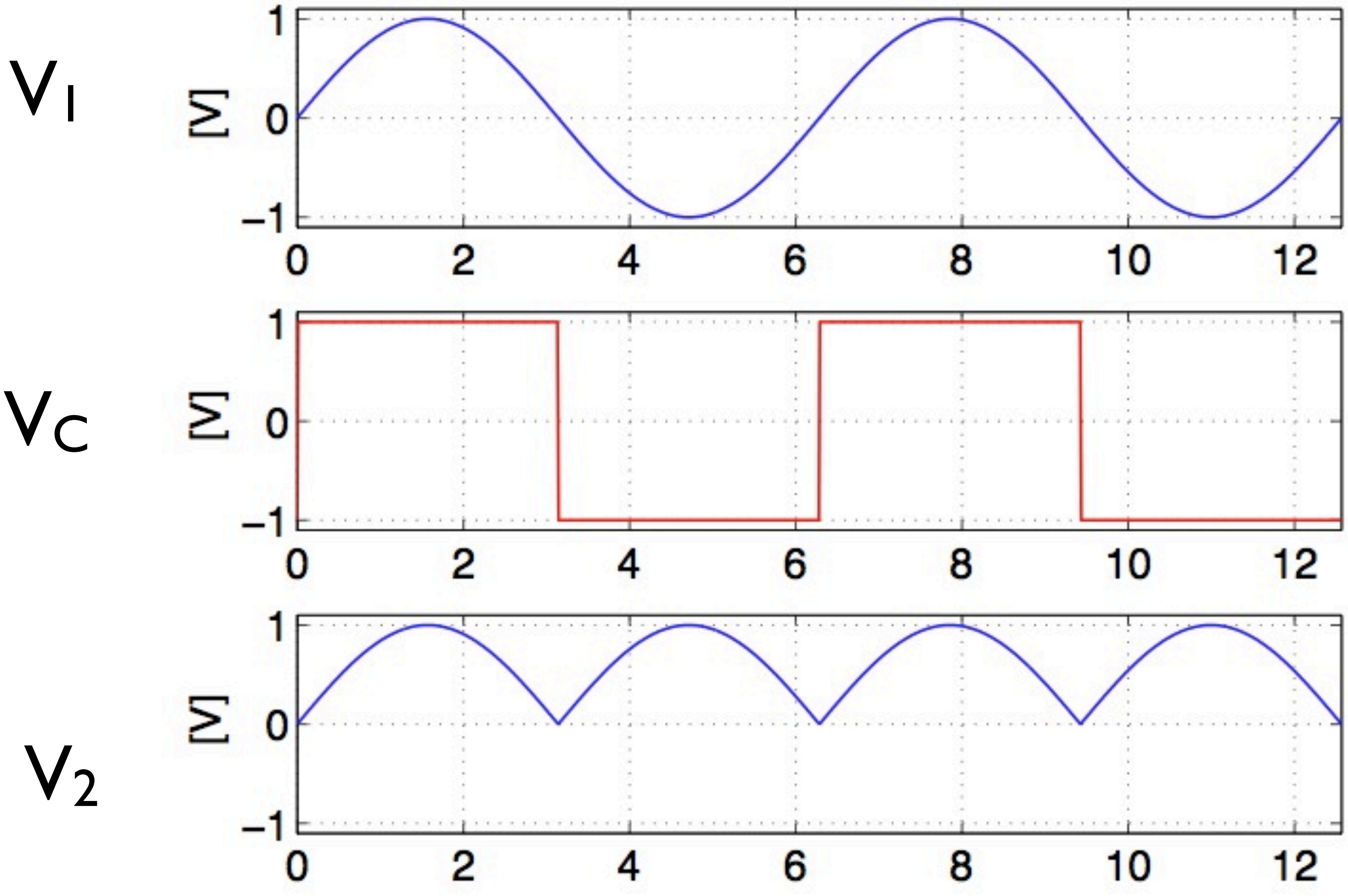


CONTROLLED PHASE SENSITIVE RECTIFIERS

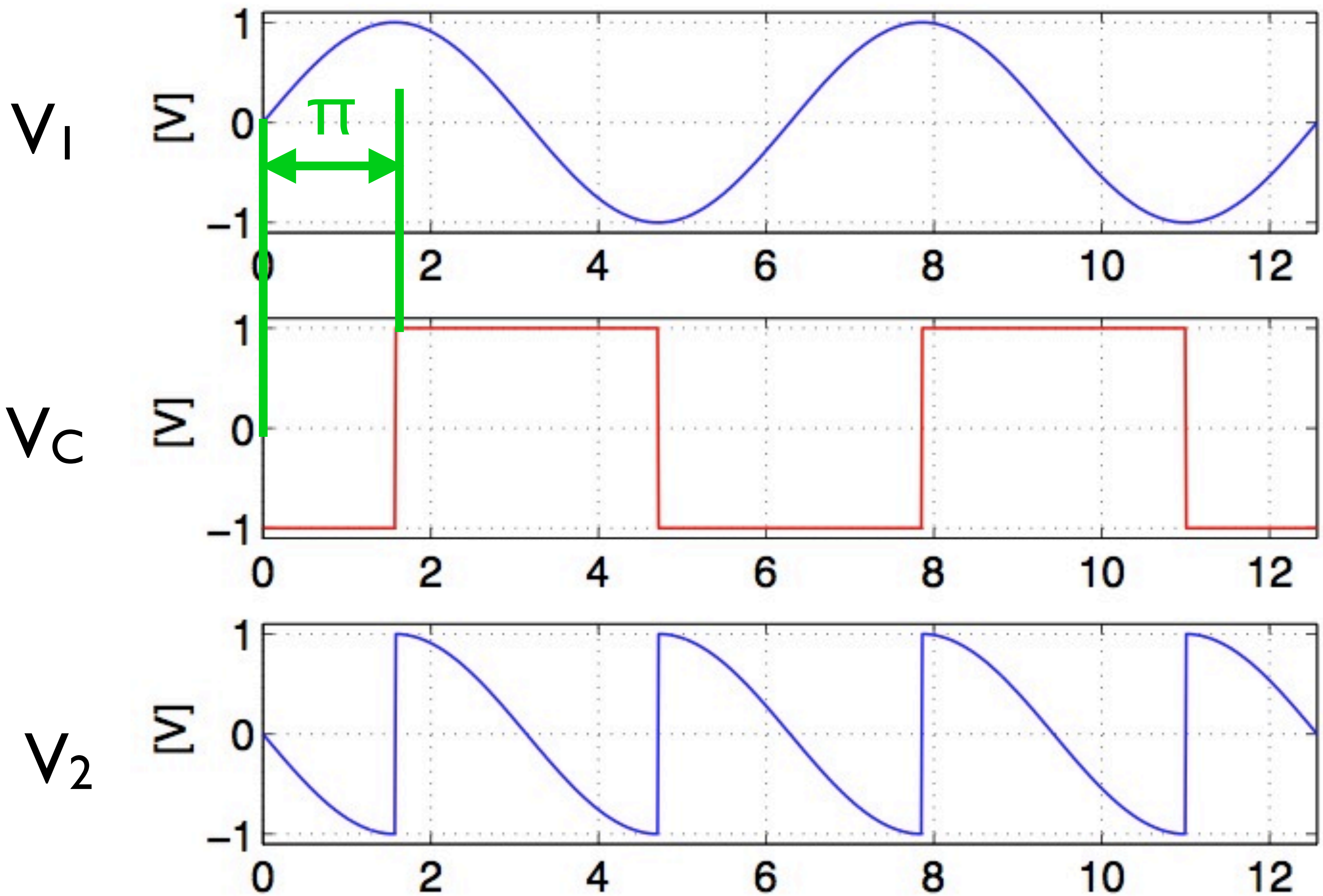


Inverting the state of the switches you invert the polarity of the output voltage

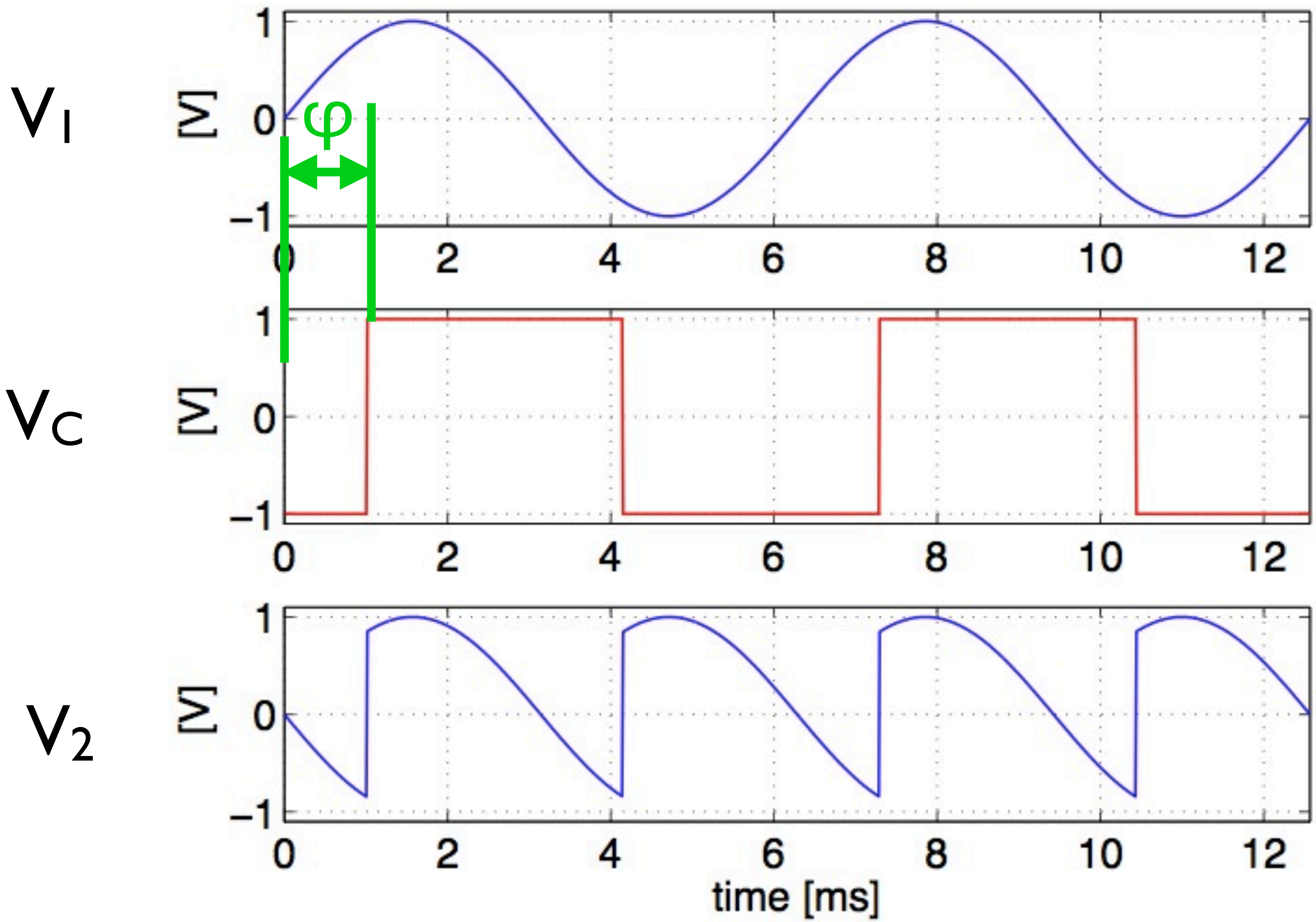
The output voltage V_2 inverts its polarity according to the control voltage V_C



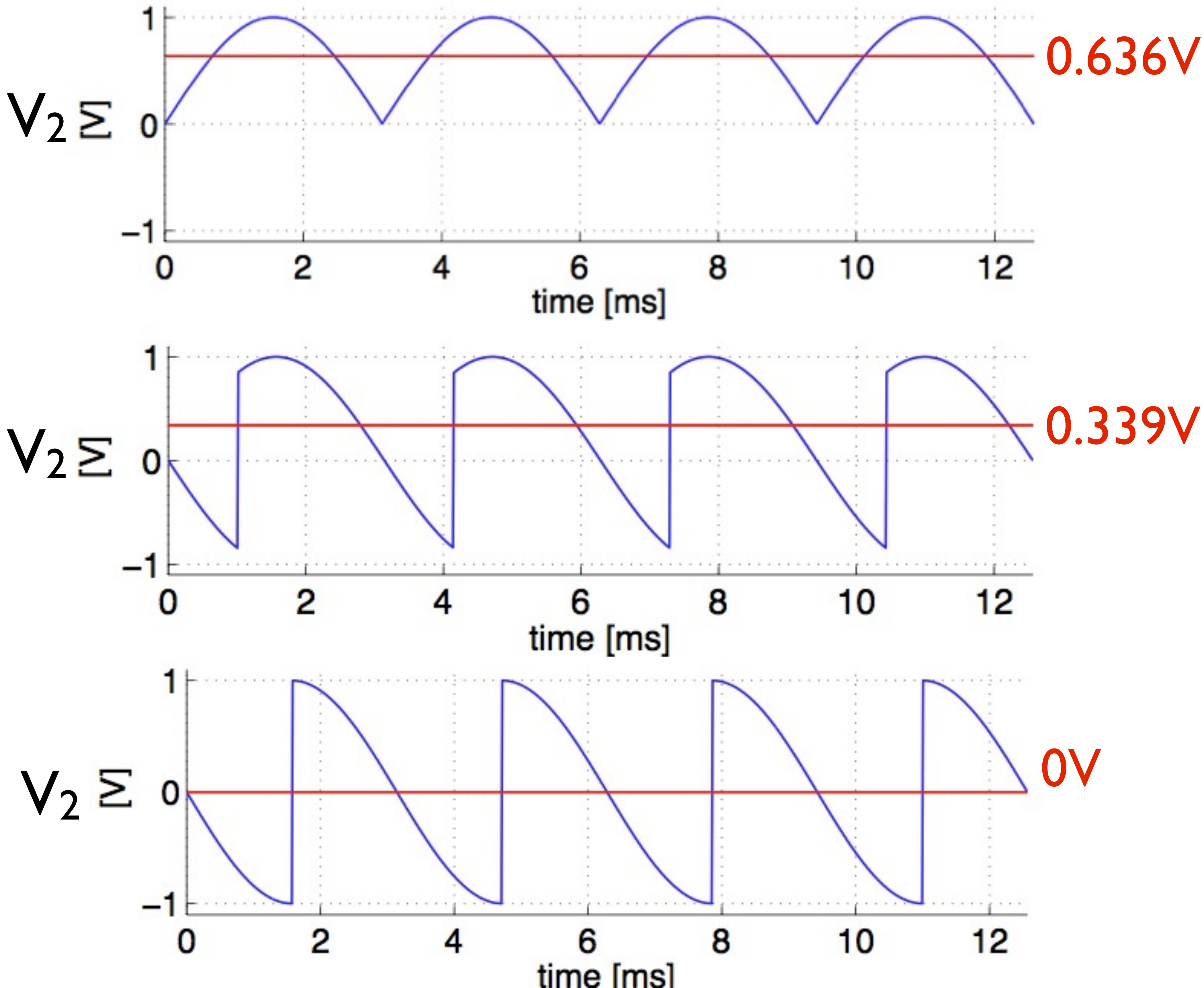
If we shift the phase φ of V_C with the respect V_1 by π



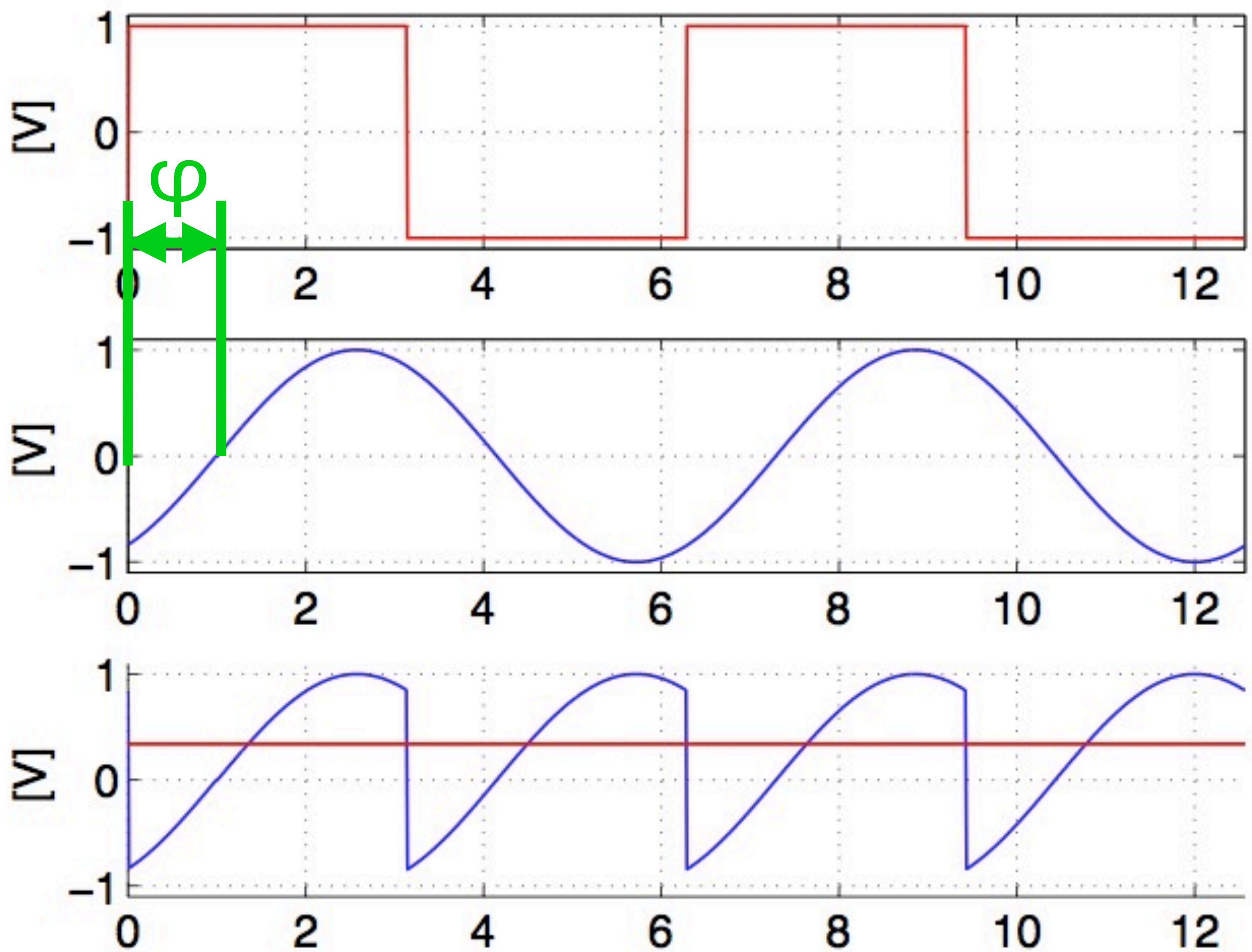
If we change the phase φ of V_C with the respect V_1 of an arbitrary value



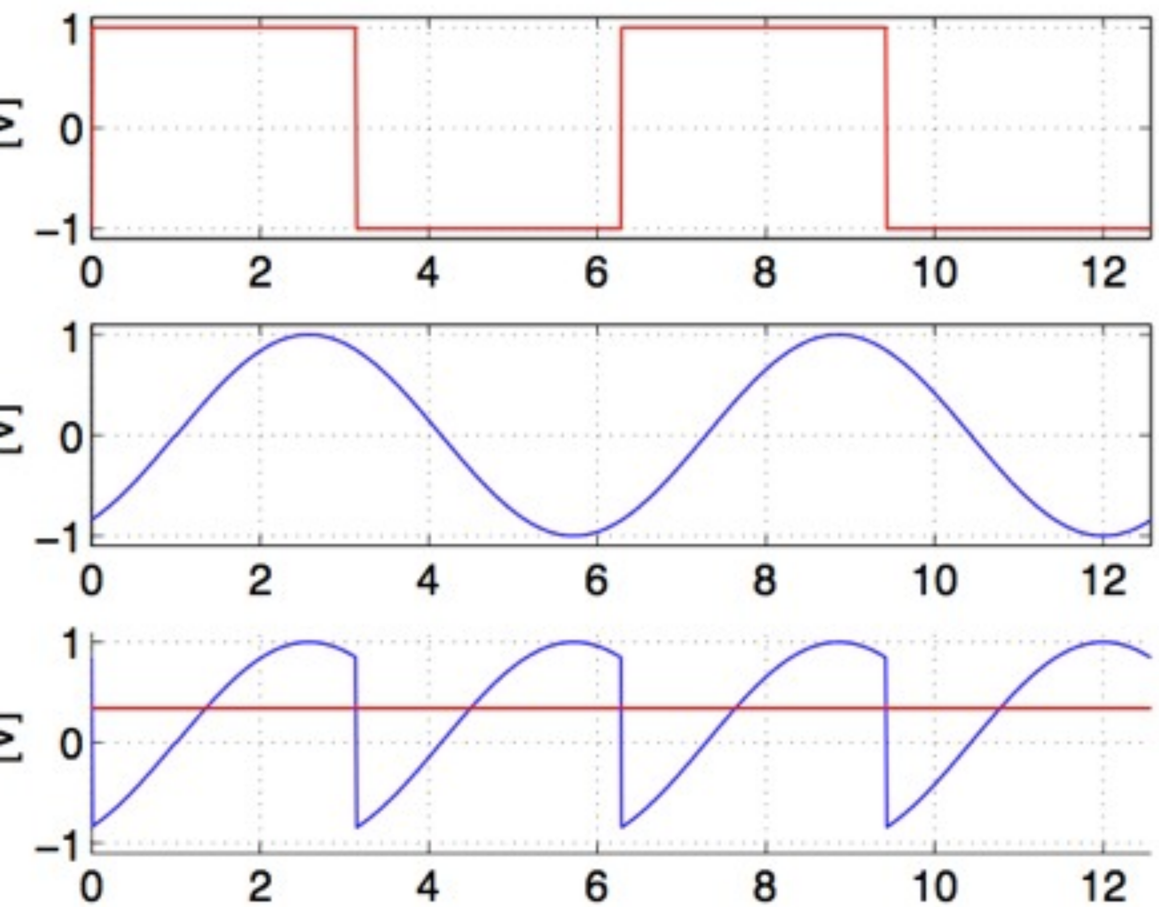
The average value of V_2 tells us something about the phase



In fact you consider the control voltage V_C as a reference and I obtain the real part of the voltage



In fact you consider the control voltage V_C as a reference and I obtain the real part of the voltage

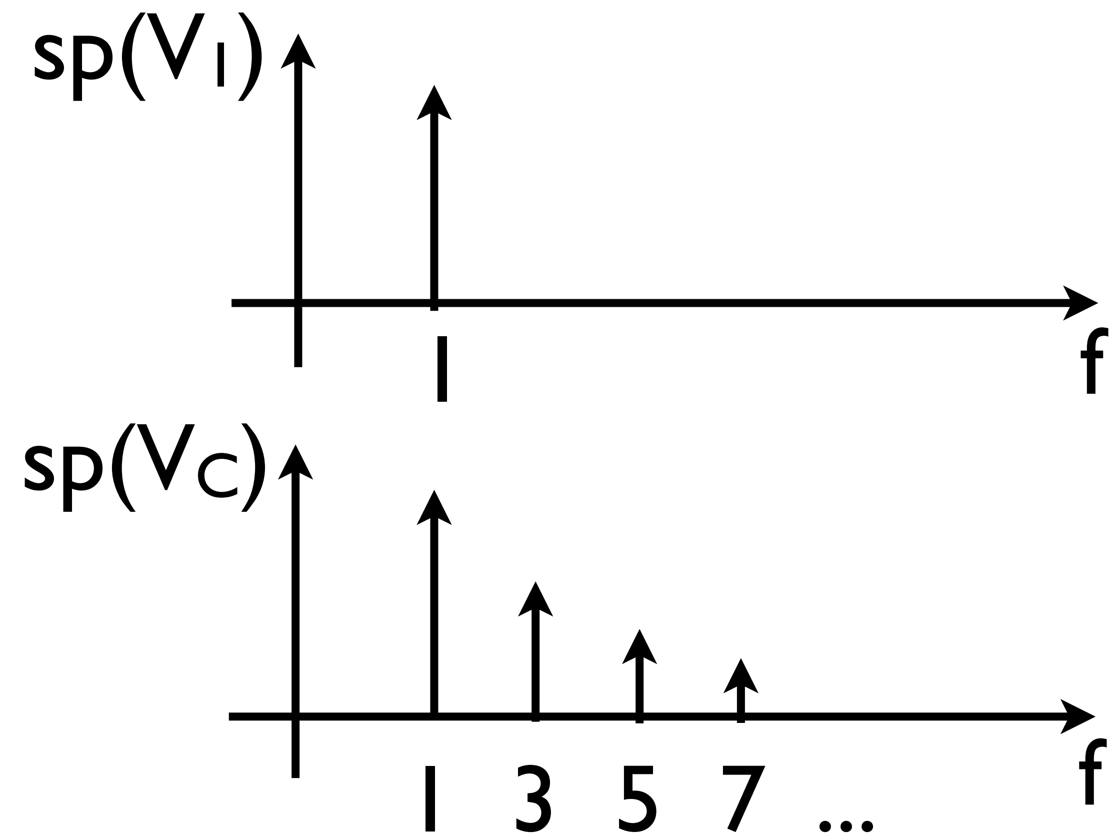


V_C

V_1

V_2

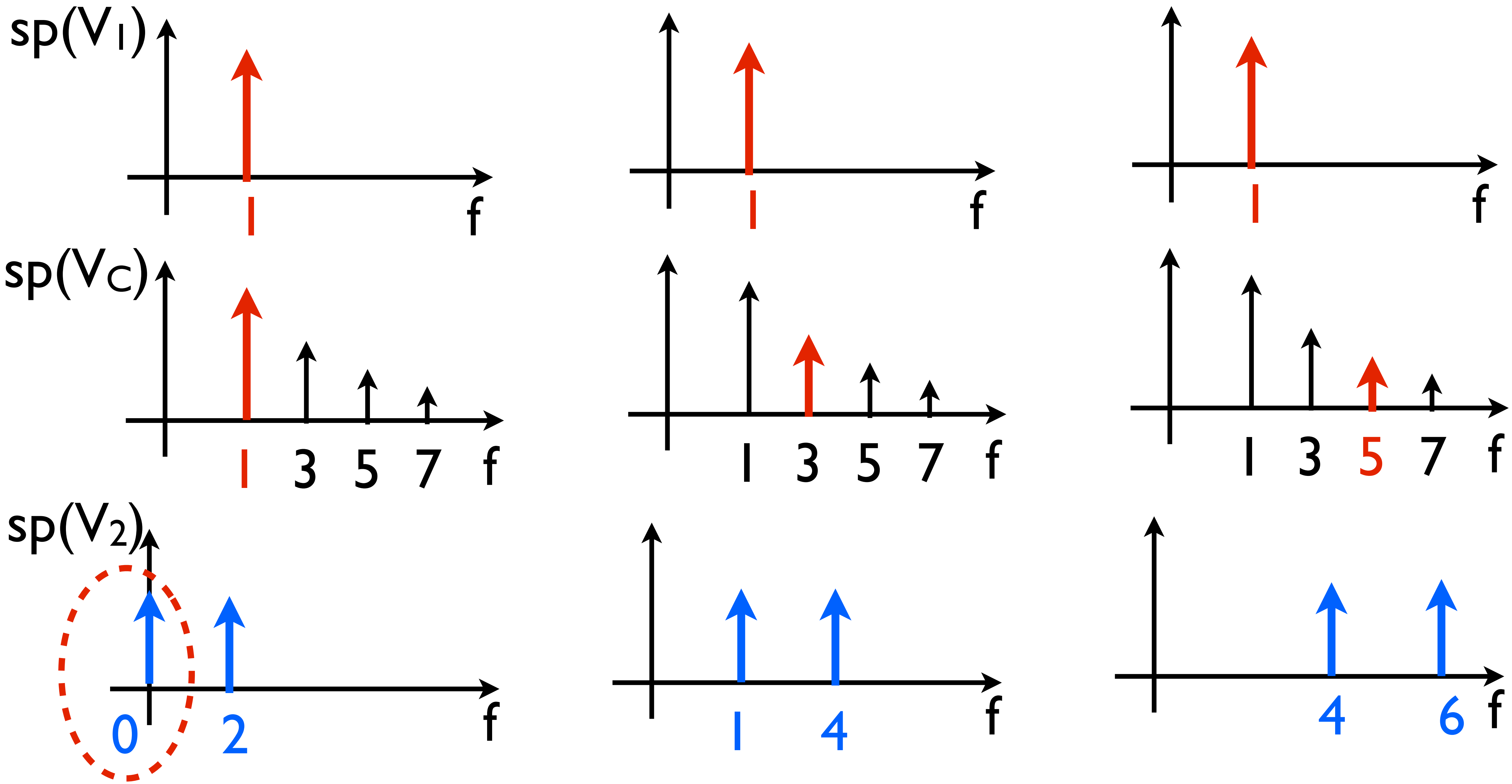
$$V_2 = V_1 \cdot V_C$$



$$\sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

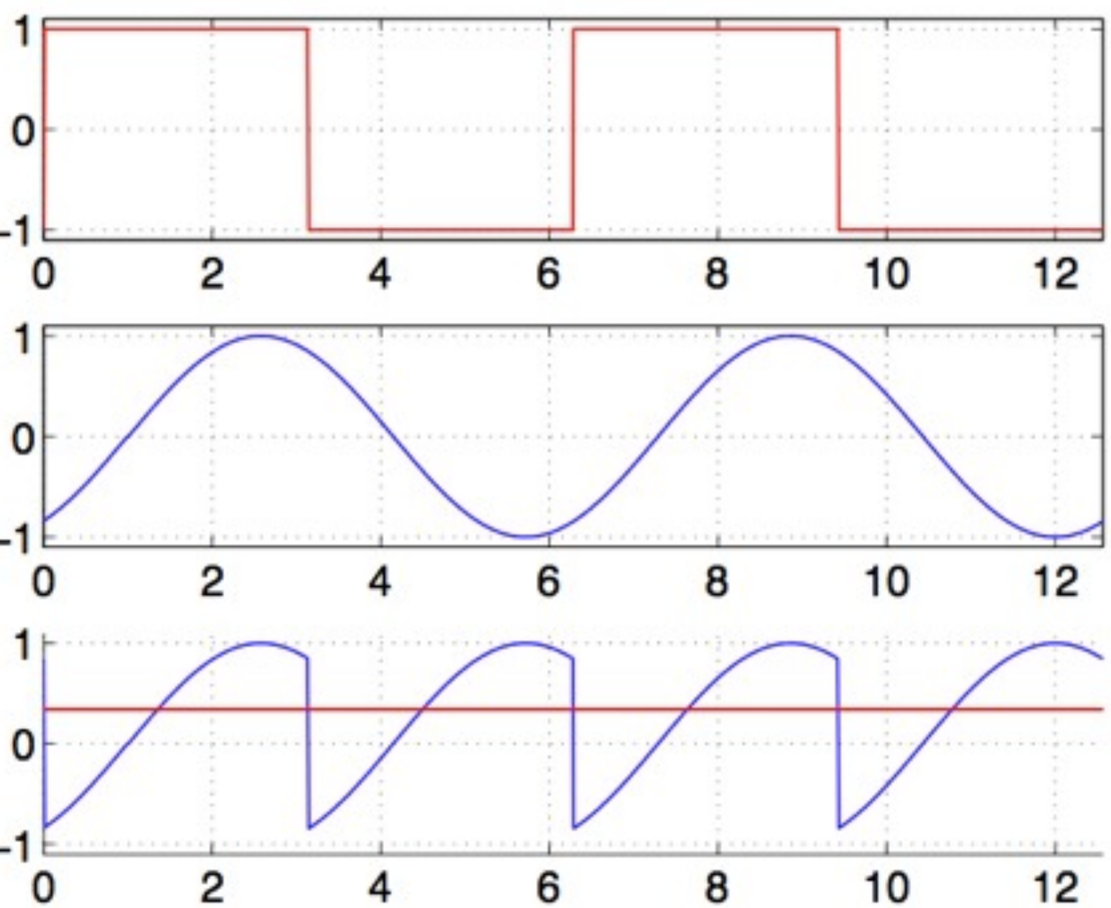
$$\sin(\alpha)\sin(\beta)=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$

$$V_2 = V_1 \cdot V_C$$



When we consider the DC value of V_2 we get the product of V_1 and first harmonic of V_C

In fact you consider the control voltage V_C as a reference and I obtain the real part of the voltage



V_C

V_1

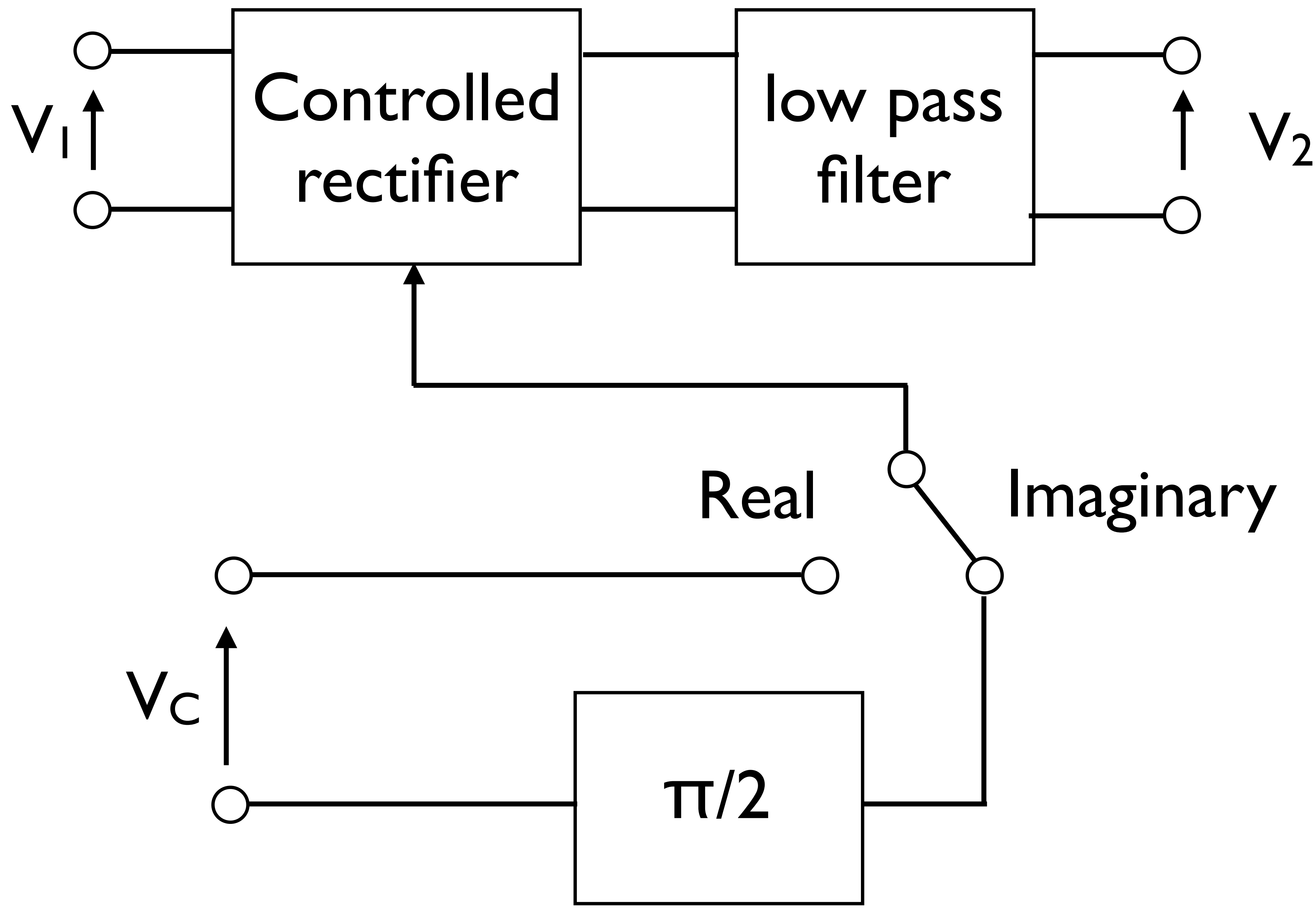
V_2

$$V_2 = V_1 \cdot V_C$$

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\mathrm{mean}(V_2) = \frac{1}{2} V_1 \frac{4}{\pi} \cos(\omega t - \varphi - \omega t) = \frac{2}{\pi} V_1 \cdot \cos\varphi$$

Shifting the control voltage by π/2 we measure the imaginary part instead of the real part of the voltage

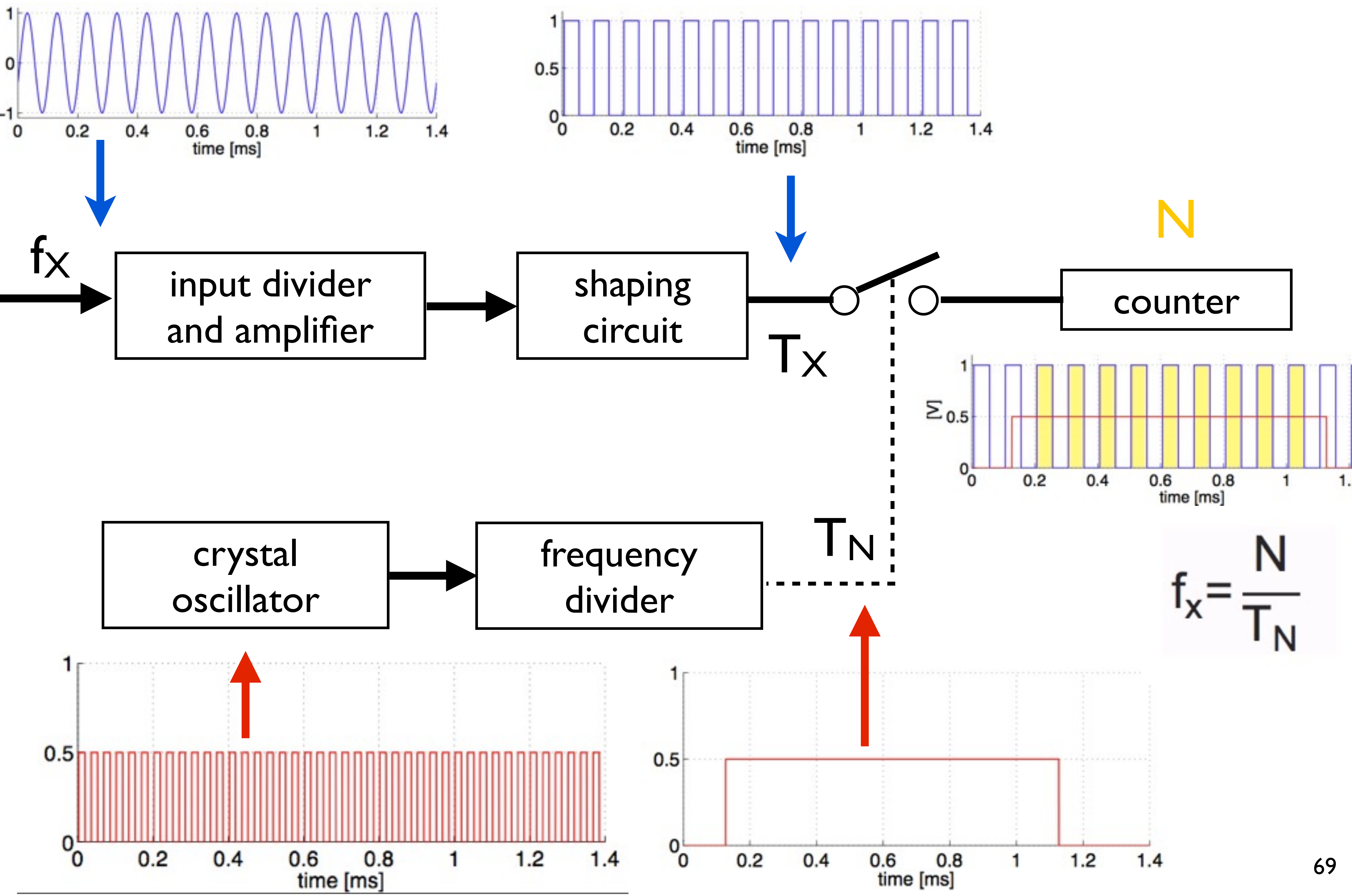


Measurement of frequency and period

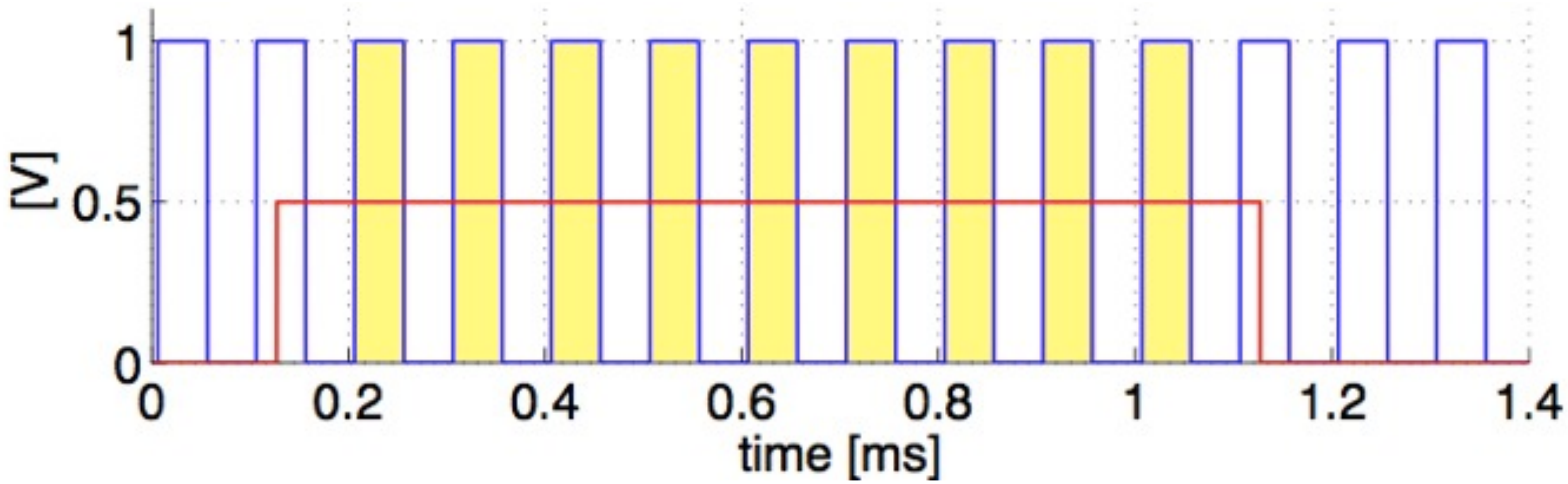
- Direct measurement with oscilloscope
  (poor resolution)

- Measurement with counter
  $T=1/f$ ... but measuring the period is different than measuring the frequency

Measurement of FREQUENCY



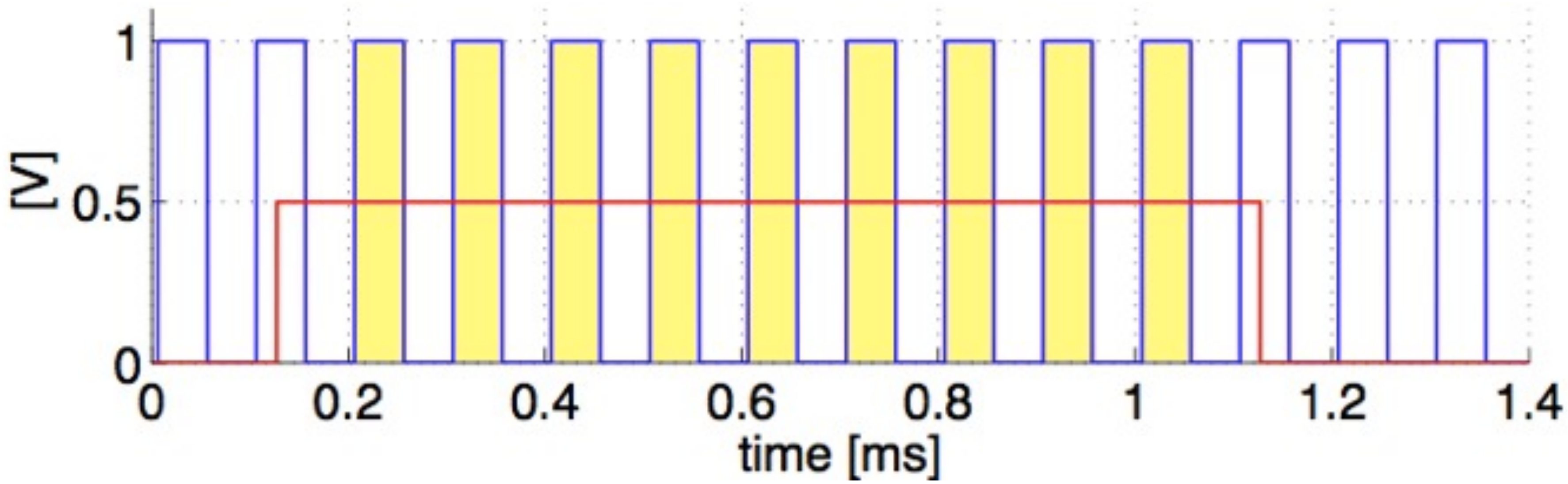
Measurement of FREQUENCY



If the gate is open for	T_N=1s	T_N=10s	T_N=100s
and I count N pulses	N=9	N=93	N=934
the frequency f_X is	9 Hz	9.3 Hz	9.34 Hz

The accuracy is $\Delta f_x' = \dfrac{1}{T_N}$

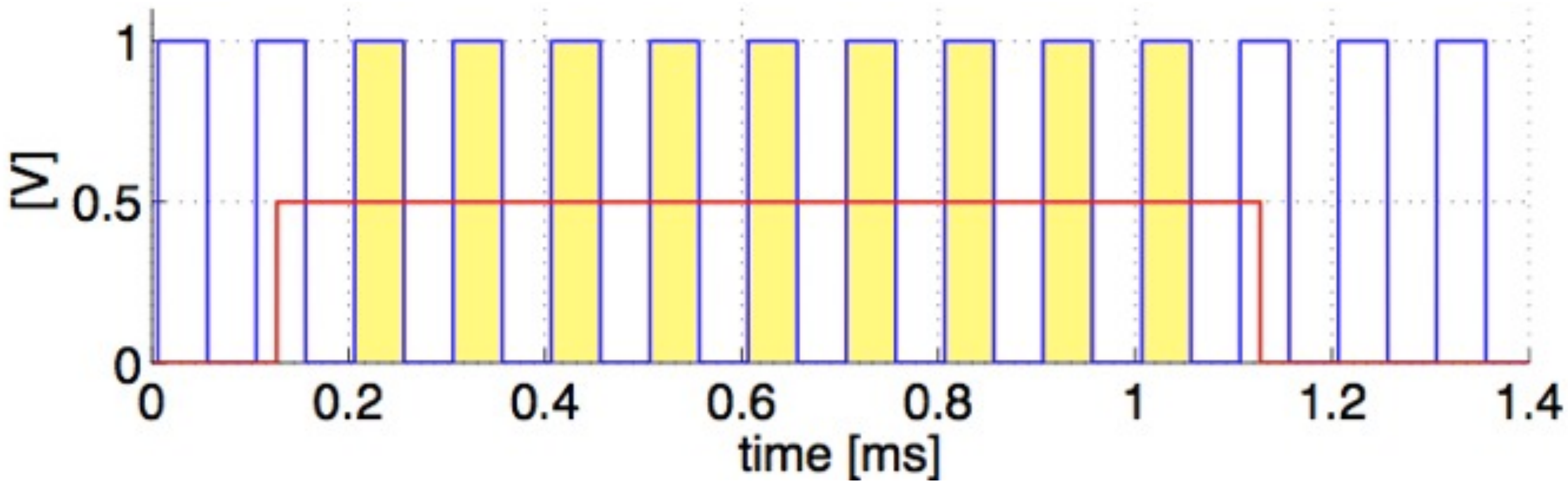
Measurement of FREQUENCY



The quartz has an instability δf_0 which determines an opening time of the gate longer or shorter than supposed

$$\Delta f_x = \frac{\delta f_0}{100} \frac{N}{T_N} = \frac{\delta f_0}{100} \cdot f_x$$

Measurement of FREQUENCY

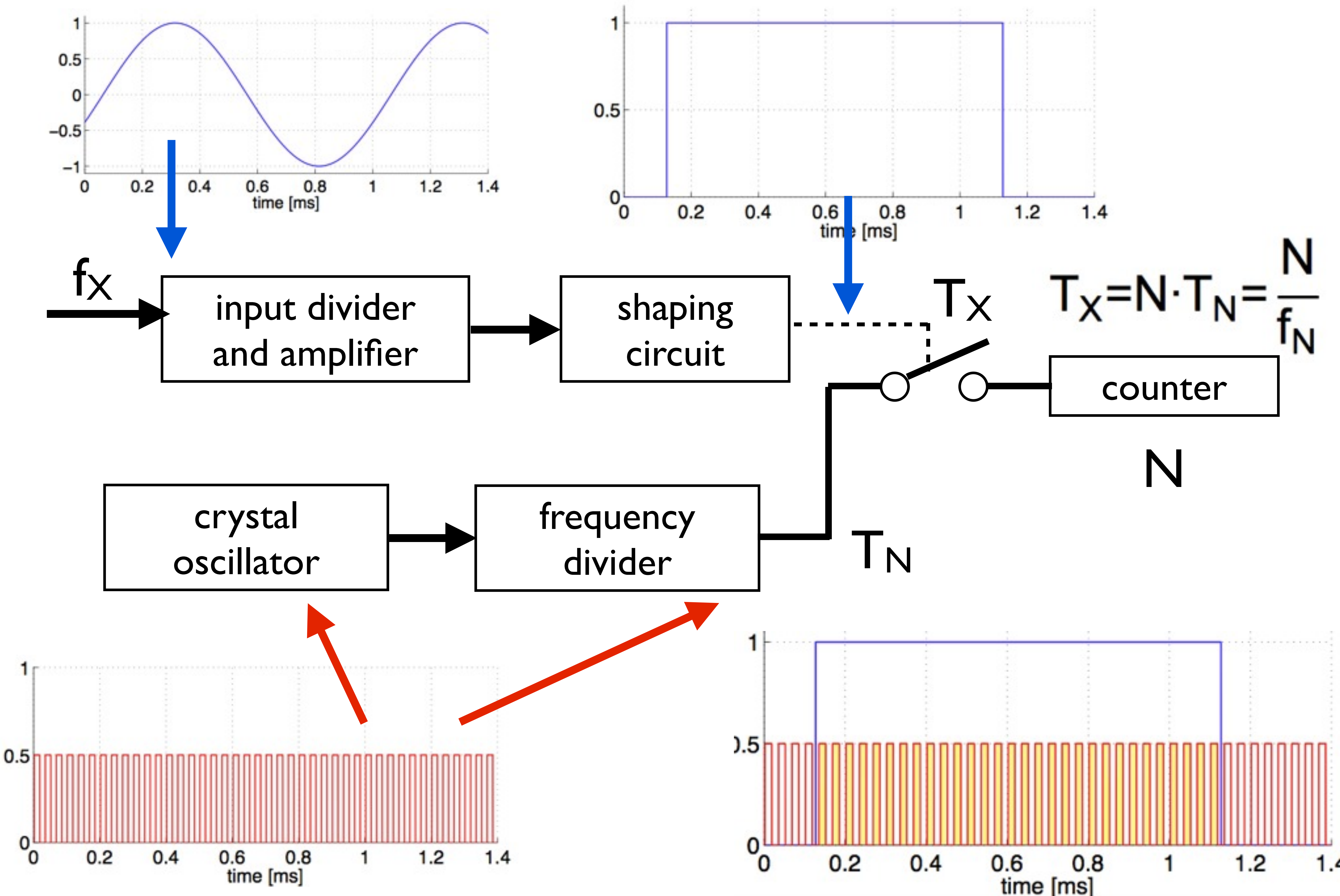


Intrinsic accuracy $$\Delta f_x' = \frac{1}{T_N}$$

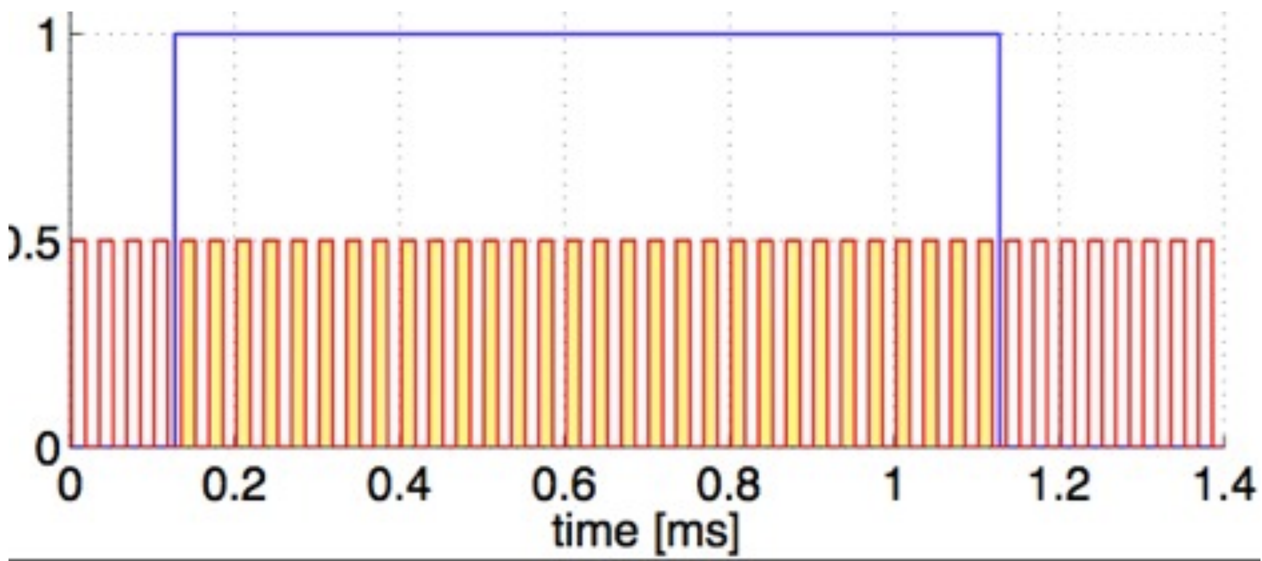
Quartz instability accuracy $$\Delta f_x = \frac{\delta f_0}{100}\frac{N}{T_N} = \frac{\delta f_0}{100}\cdot f_x$$

Uncertainty $$u_{f_x} = \sqrt{\left(\frac{\Delta f_x'}{\sqrt{3}}\right)^2 + \left(\frac{\Delta f_x}{\sqrt{3}}\right)^2}$$

Measurement of PERIOD



Measurement of PERIOD



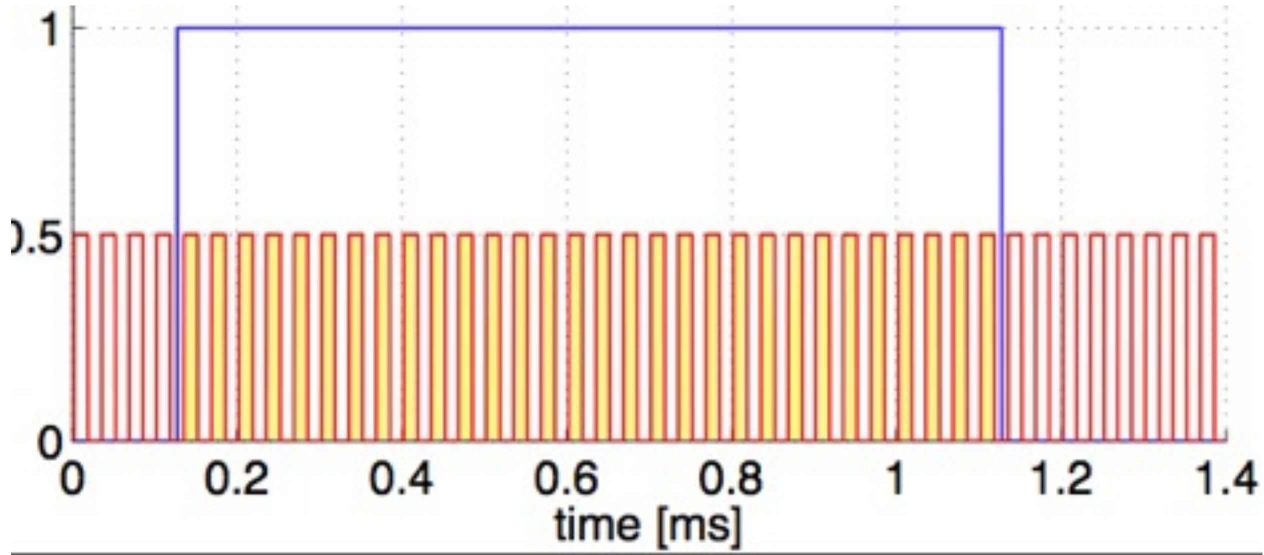
$T_X = N \cdot T_N$

For a given T_X=1.1324 s to be measures

If the quartz period	T_N=0.1s	T_N=0.01s	T_N=0.001s
and I count N pulses	N=11	N=113	N=1132
The measured period T_X is	1.1 s	1.13 s	1.1132 Hz

The accuracy is $\Delta T_X' = T_N = \frac{1}{f_N}$

Measurement of PERIOD



$$T_X = N \cdot T_N$$

Again, we should consider the instability δf_0 of the quartz.
If T_N is longer than suppose
I will count lower N and vice versa

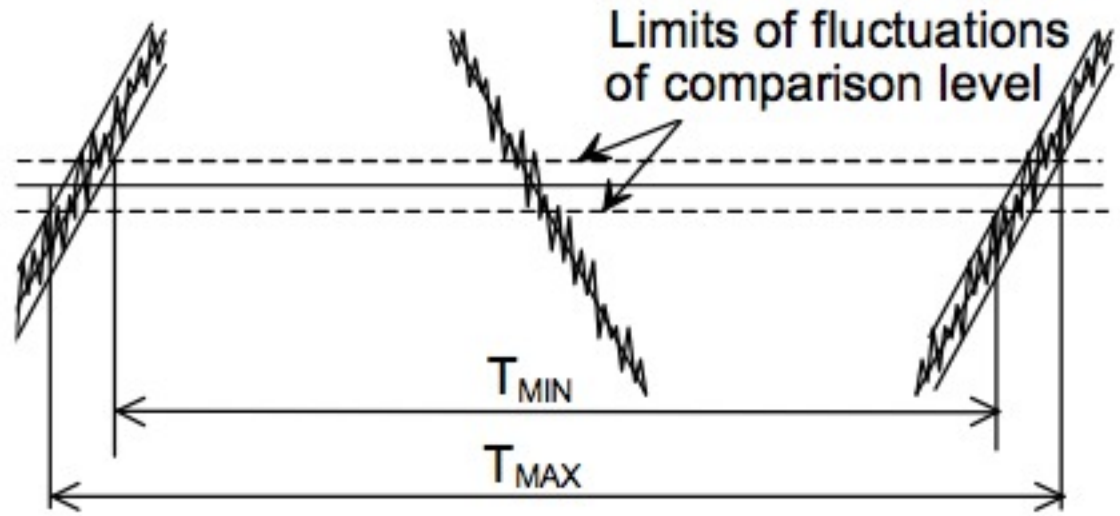
The accuracy is $\Delta T_X = \frac{\delta f_0}{100} T_N \cdot N$

Measurement of PERIOD

Intrinsic instability $$\Delta T_X' = T_N = \frac{1}{f_N}$$

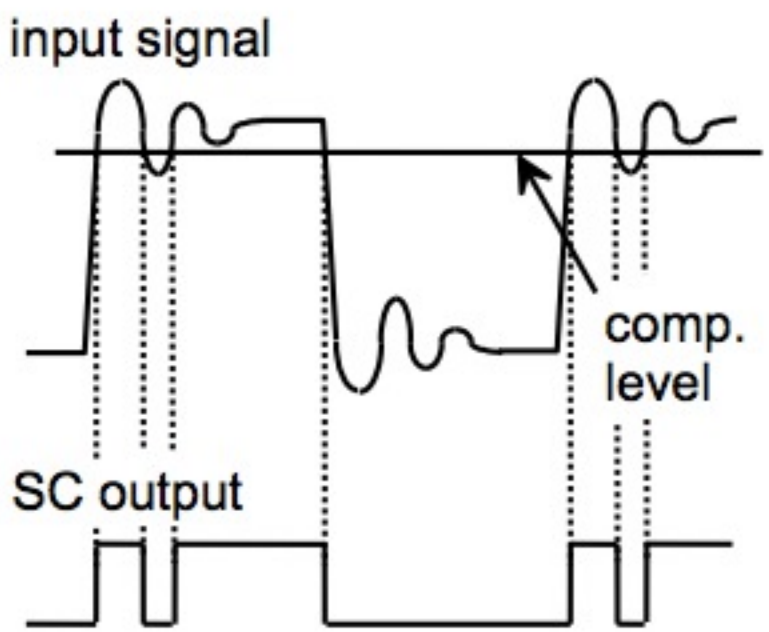
Quartz instability $$\Delta T_X = \frac{\delta f_0}{100} T_N \cdot N$$

Noise of threshold



$$u_{T_x} = \sqrt{\left(\frac{\Delta T_x'}{\sqrt{3}}\right)^2 + \left(\frac{\Delta T_x}{\sqrt{3}}\right)^2 + 2u_C}$$

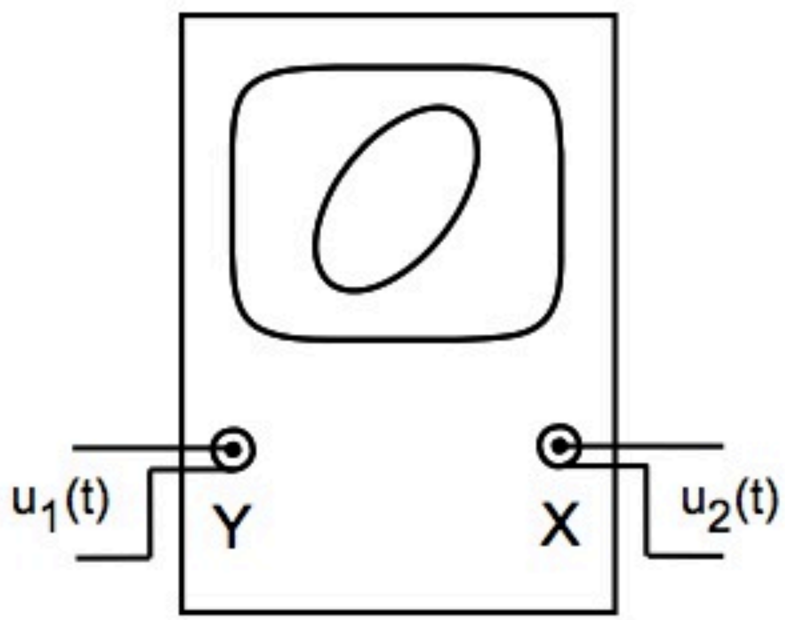
Be careful to proper setting in the shaping circuit

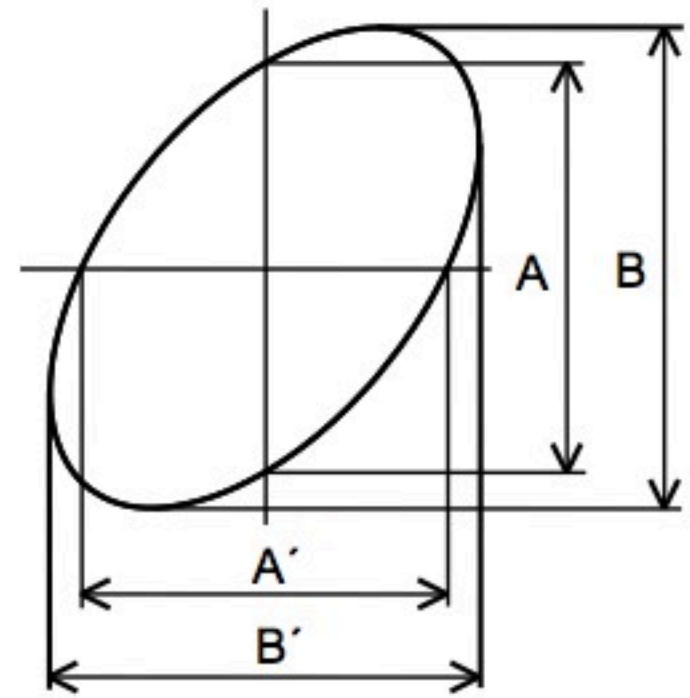


Ripple in the signal might cause false output

Measurement of PHASE DIFFERENCE

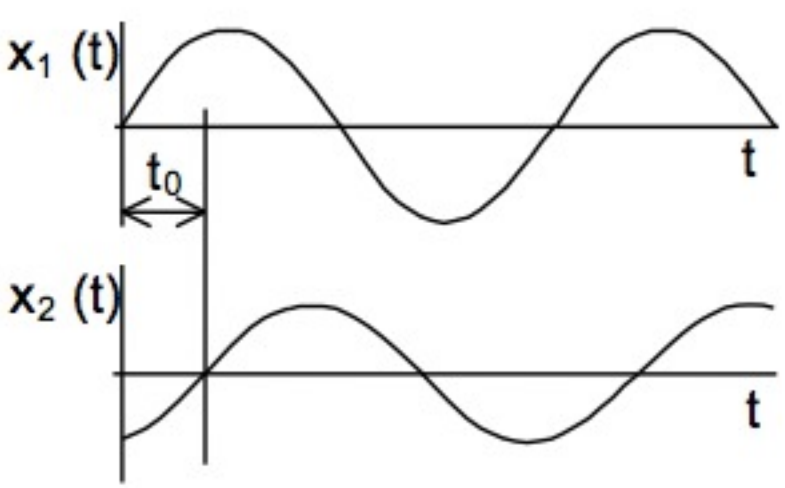
Oscilloscope in XY mode

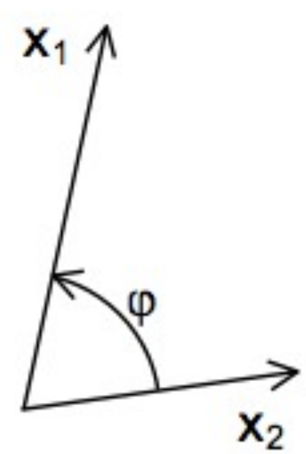




$$\varphi = \arcsin\frac{A}{B} = \arcsin\frac{A'}{B'}$$

Oscilloscope in time domain





$$\varphi = \omega t_0 = 2\pi f t_0 = \frac{2\pi t_0}{T} \quad (rad)$$

$$\varphi = \frac{360 t_0}{T} \quad (^0)$$