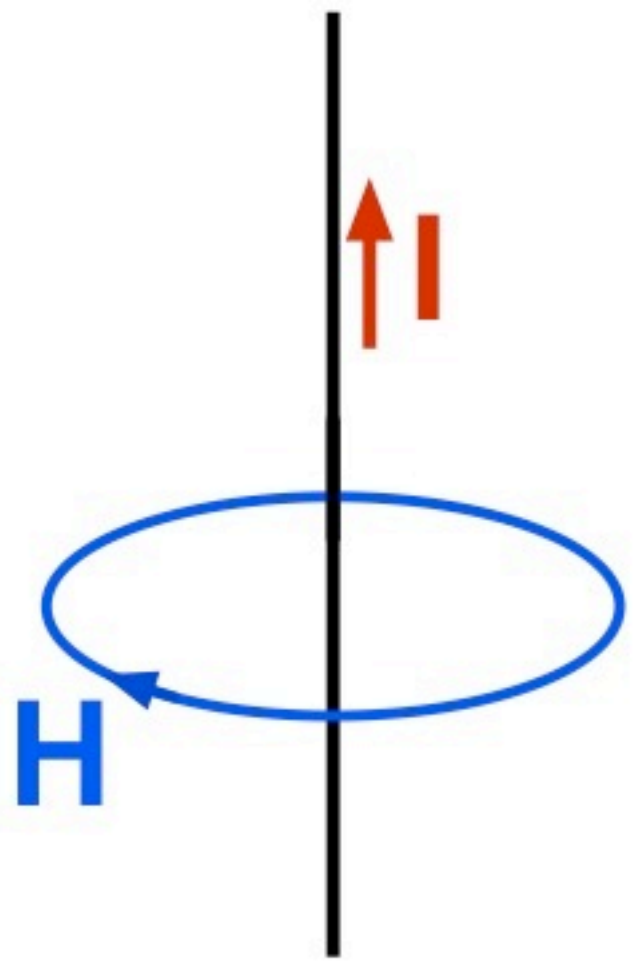
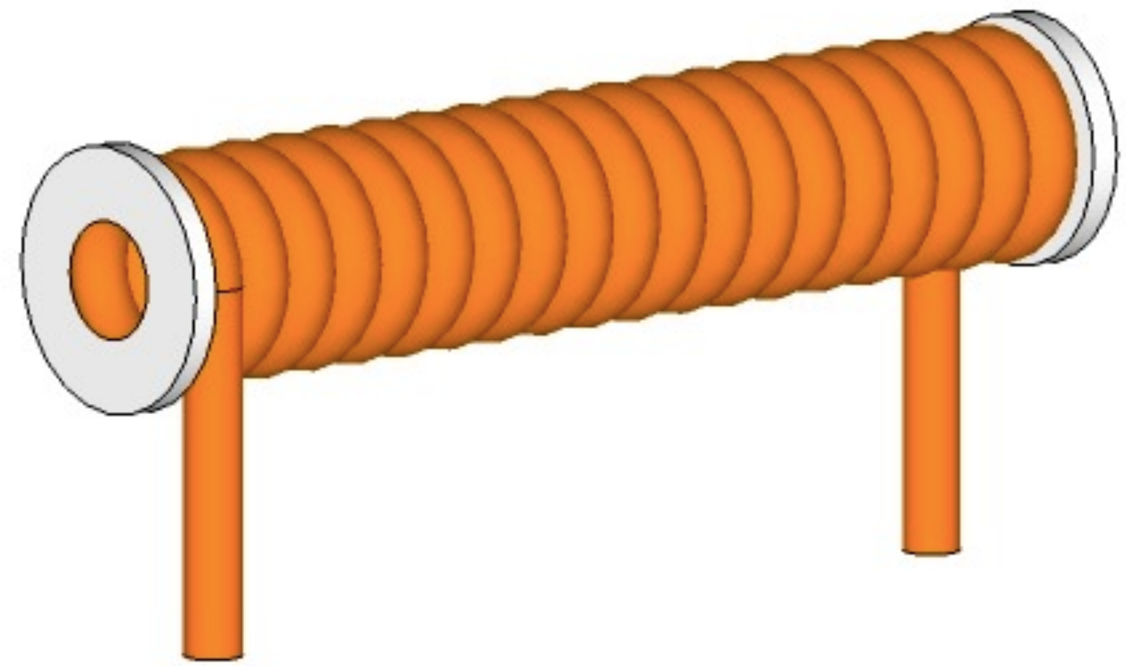
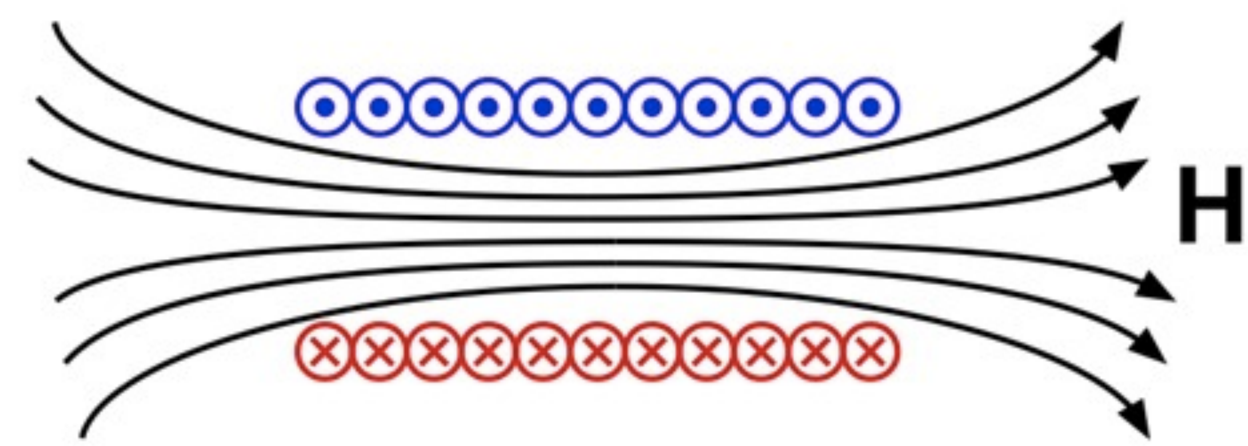
Lecture 5

- Magnetic measurement
- Impedance measurement

MAGNETIC FIELD







$$\oint \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{S}$$

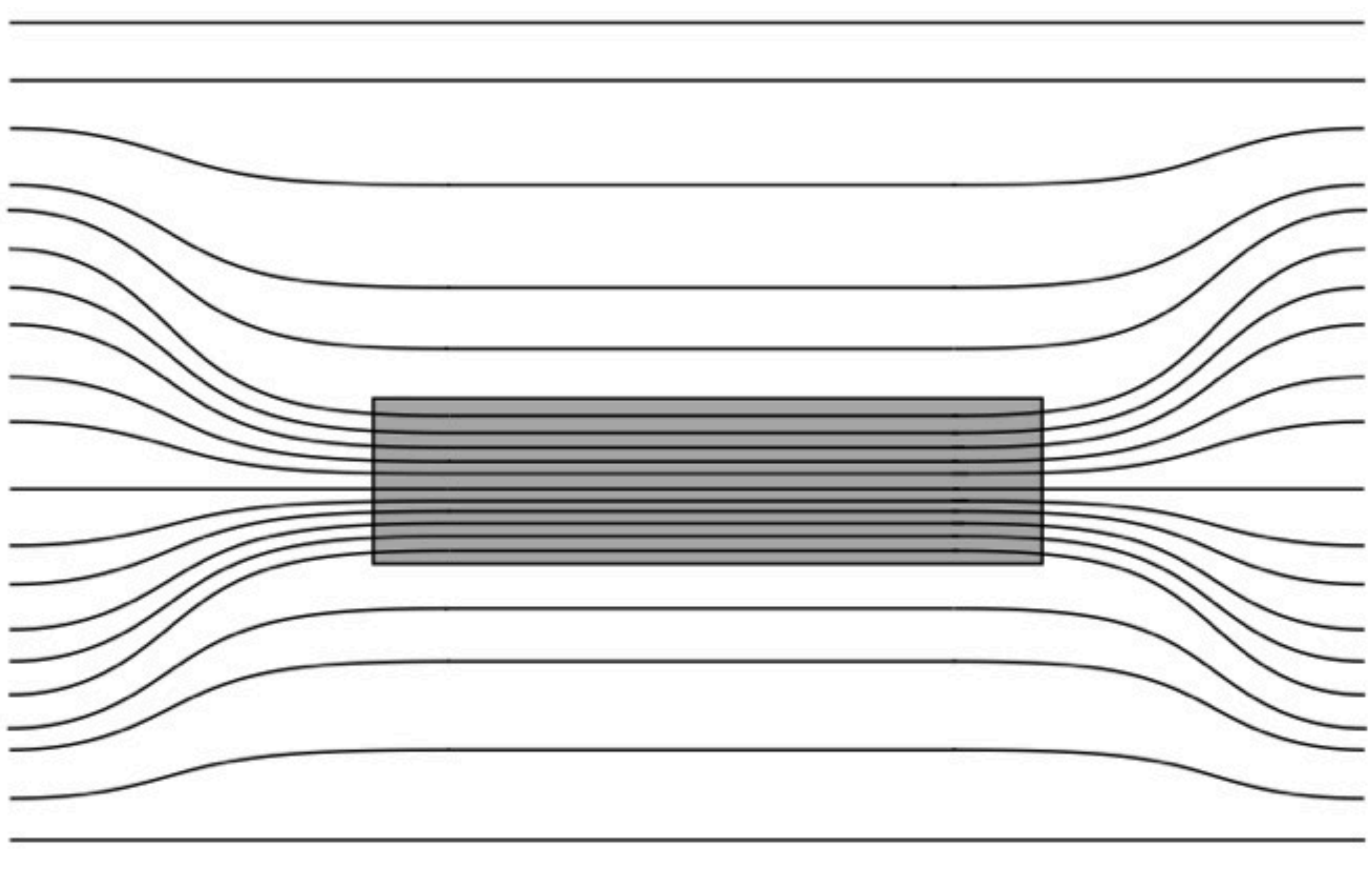
H [A/m]

J current density
S surface

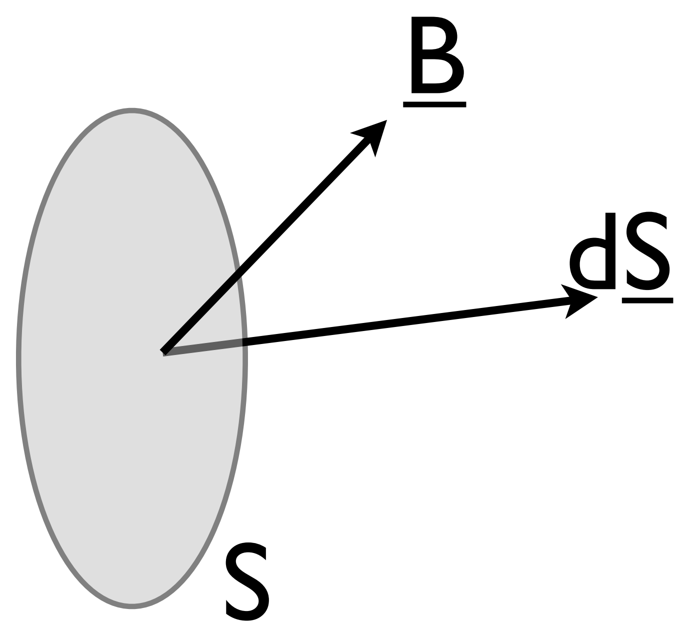
(ANOTHER) MAGNETIC FIELD

$$\underline{B}=\mu\underline{H}$$

μ = Permeability
that is "how much a material can be permeated by H"



B is the concentration of the MAGNETIC FLUX over an area

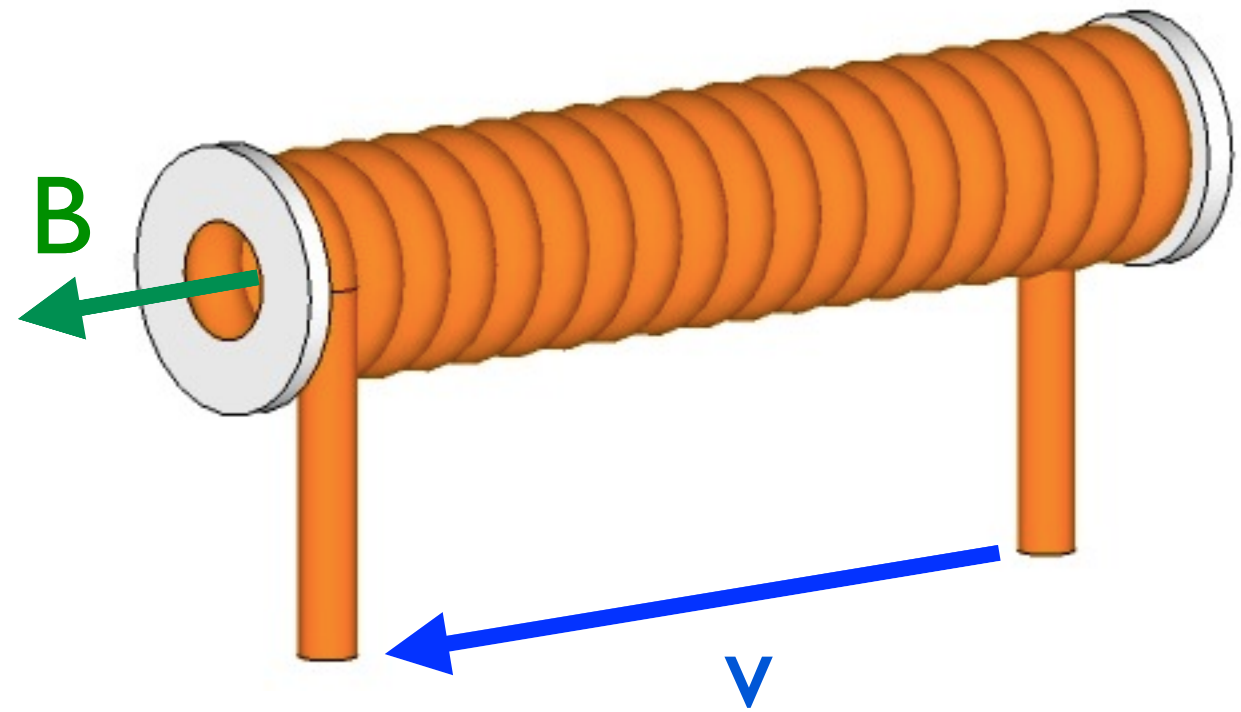


$$\Phi = \int_S \underline{B} \cdot d\underline{S}$$

An interesting property of the magnetic flux

$$v = -\frac{d\phi}{dt}$$

don't forget the minus!



$$v = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S}$$

If you want to measure B you can simply integrate the voltage induced in a search coil, then divide by the area of the coil

$$v = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S}$$

We assume B uniform, so $\phi = B \cdot S$

$$v = -\frac{d}{dt} B \cdot S$$

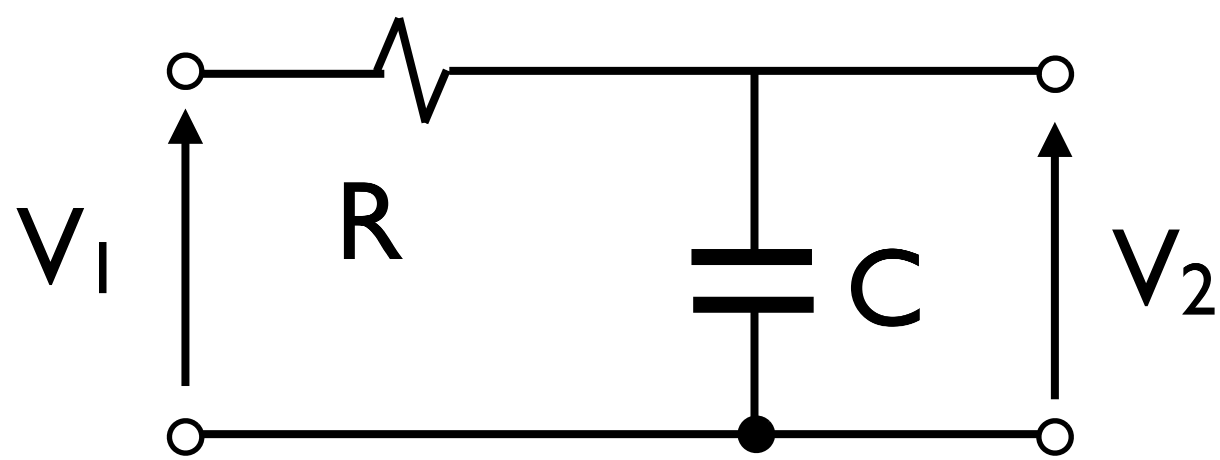
The amplitude of B is obtained by

$$B = -\frac{1}{S}\int_{t_1}^{t_2} v \cdot dt$$

What about the direction?

HOW TO INTEGRATE A VOLTAGE

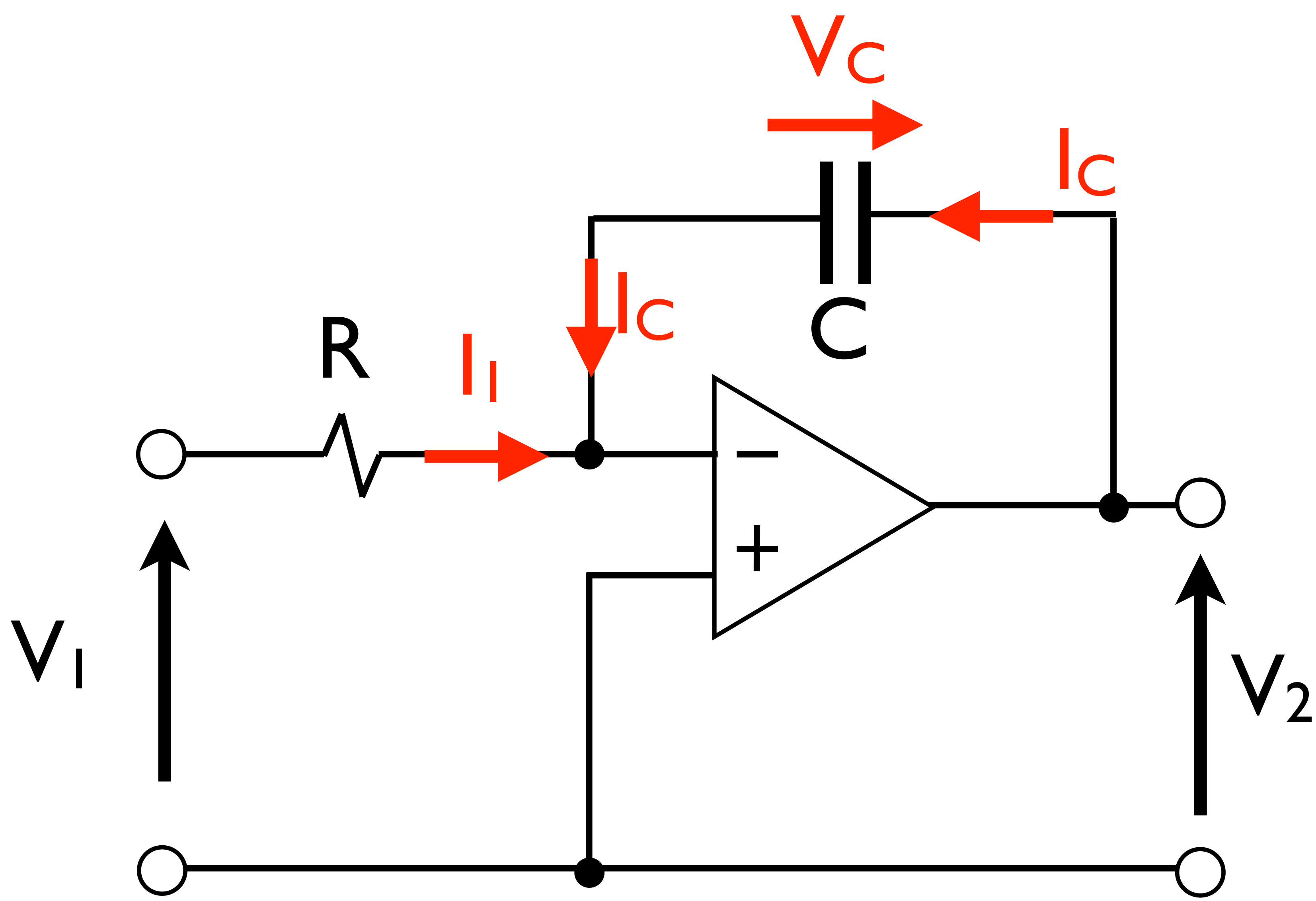
1 - Passive integrator



$$V_2 = \frac{1}{1 + j\omega CR} V_1$$

HOW TO INTEGRATE A VOLTAGE

2 - Active integrator

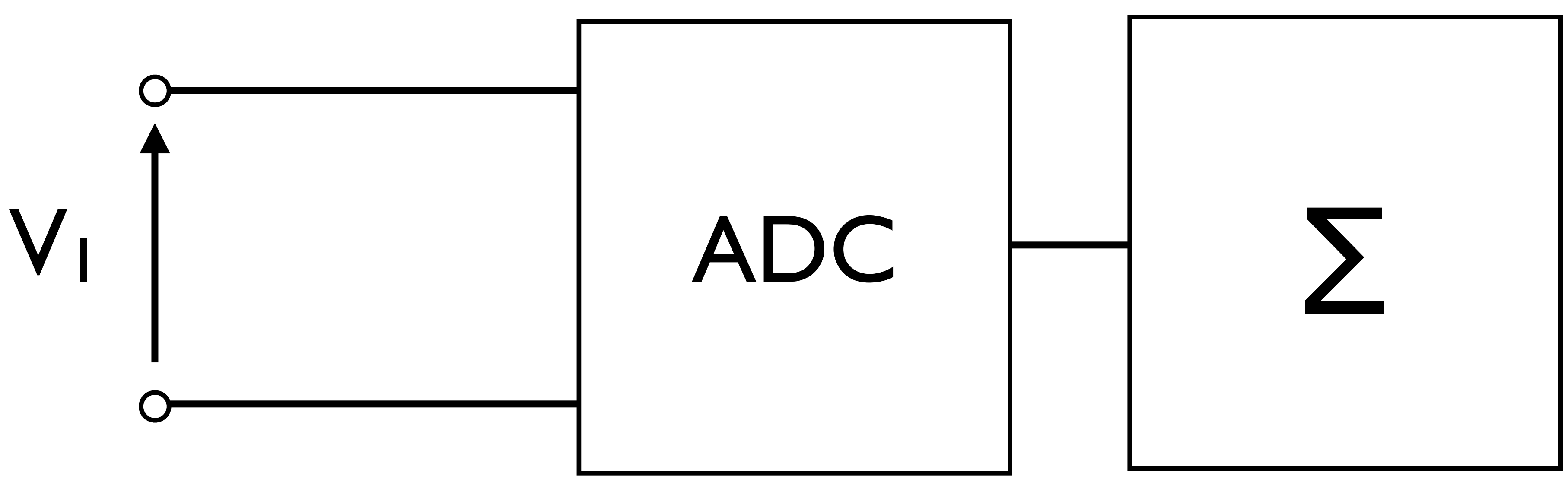


$$V_2 = \frac{1}{C}\int_{t_1}^{t_2} i_c\, dt = \frac{1}{C}\int_{t_1}^{t_2} -\frac{V_1}{R} dt$$

HOW TO INTEGRATE A VOLTAGE

3 - Numerically

You digitize the waveform of the voltage and then you sum up the samples multiplied time the sampling time and you obtain the integral of the voltage.



$$B = -\frac{1}{S}\int_{t_1}^{t_2} v \cdot dt$$

Problem 1.

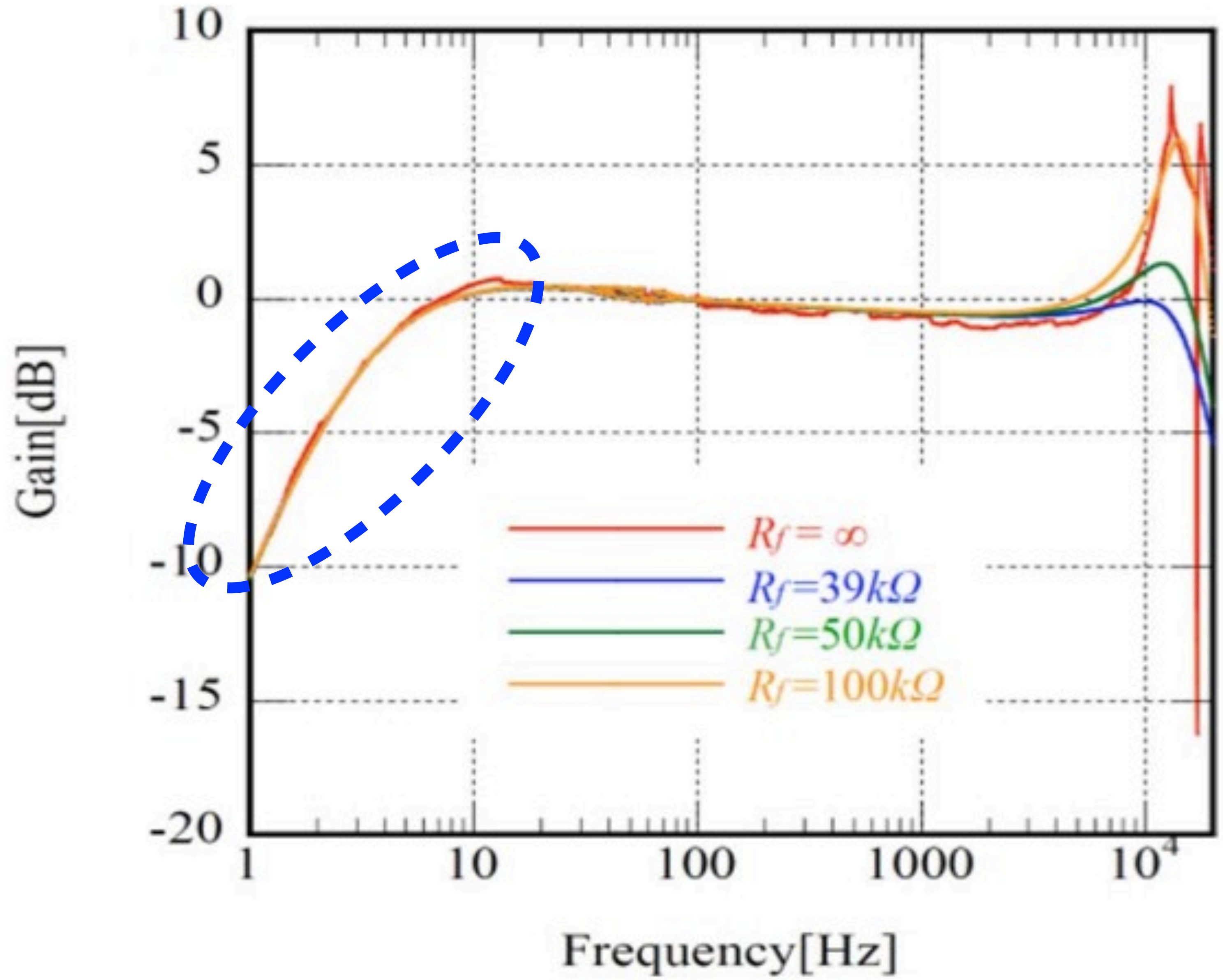
Do I really measure B?

What is missing in this equation?

Problem 2. →?

A search coil is NOT suitable to measure low frequency field

Response of Search Coil Magnetometer

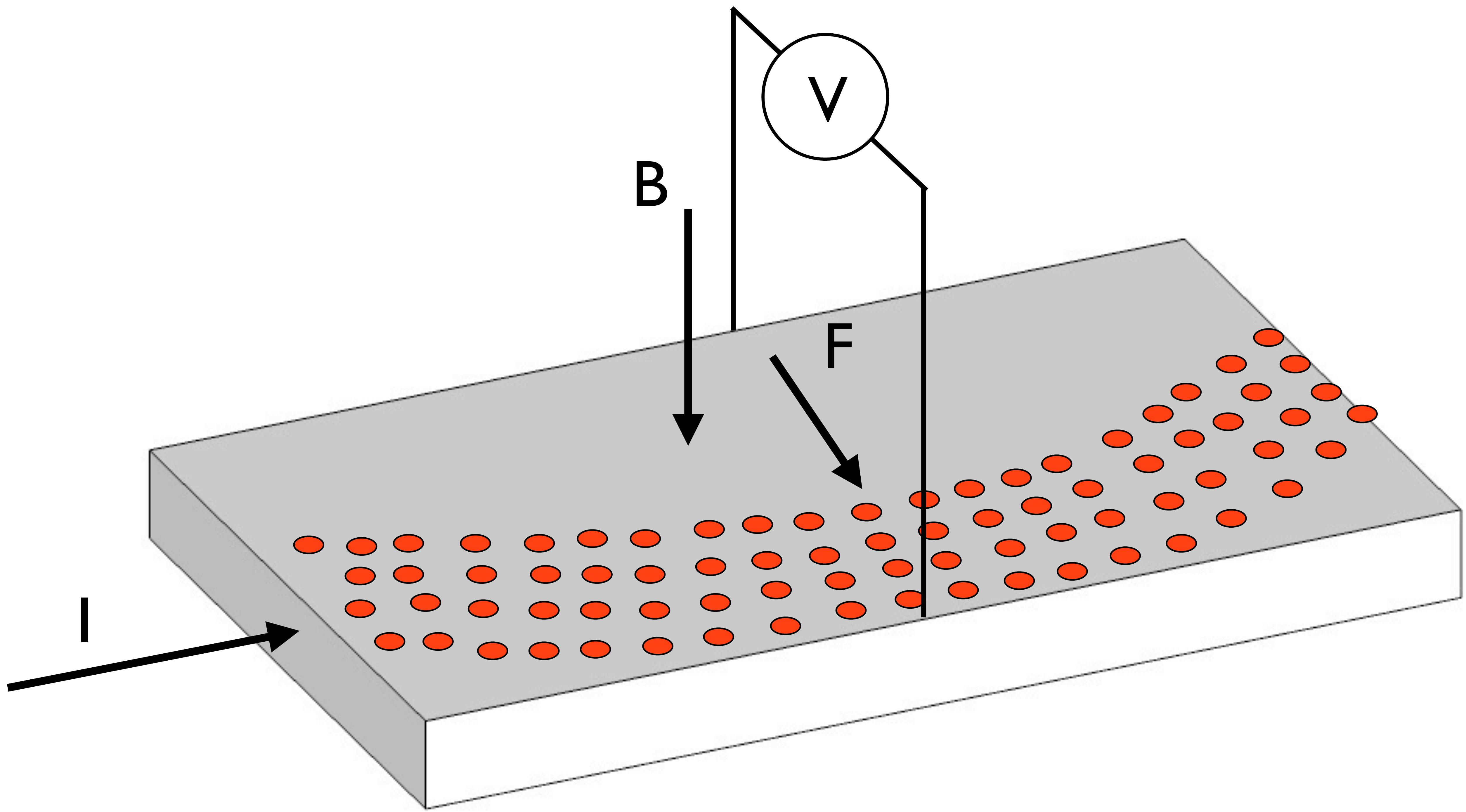


from Feng Han et al., Fluxgate and Search Coil Hybrid: A Low-Noise Wide-Band Magnetometer

Measurement of DC magnetic field

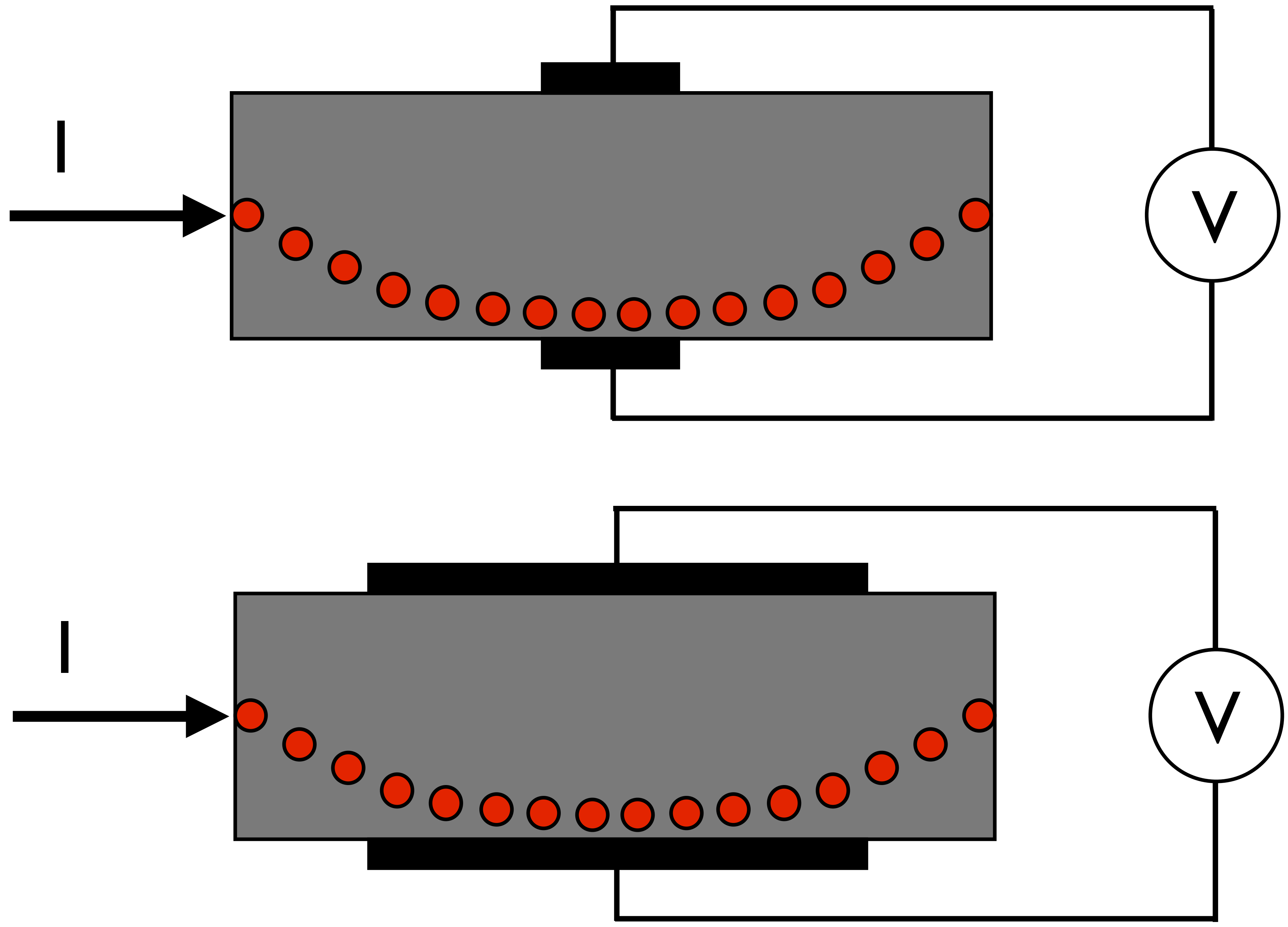
1- Hall effect sensor
2 - Anisotropic MagnetoResistor (AMR)
3 - Fluxgate

HALL EFFECT SENSOR

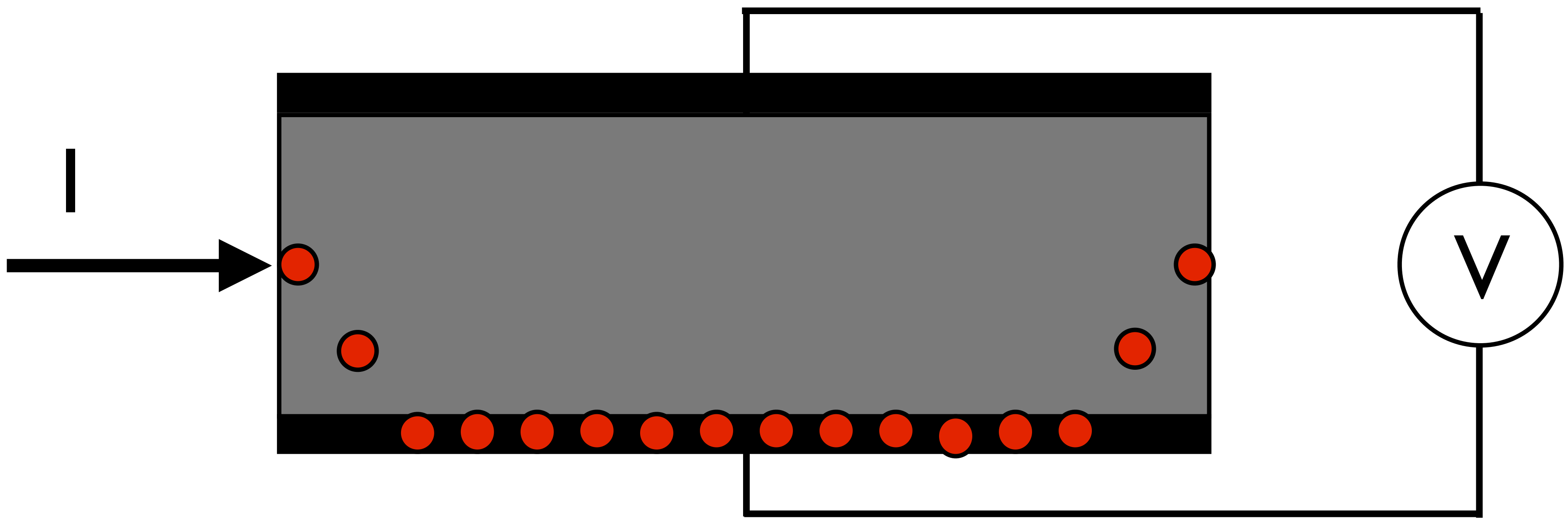


$V = k \cdot F = k \cdot I \cdot B$

Problem 1. How can I pick up the voltage?

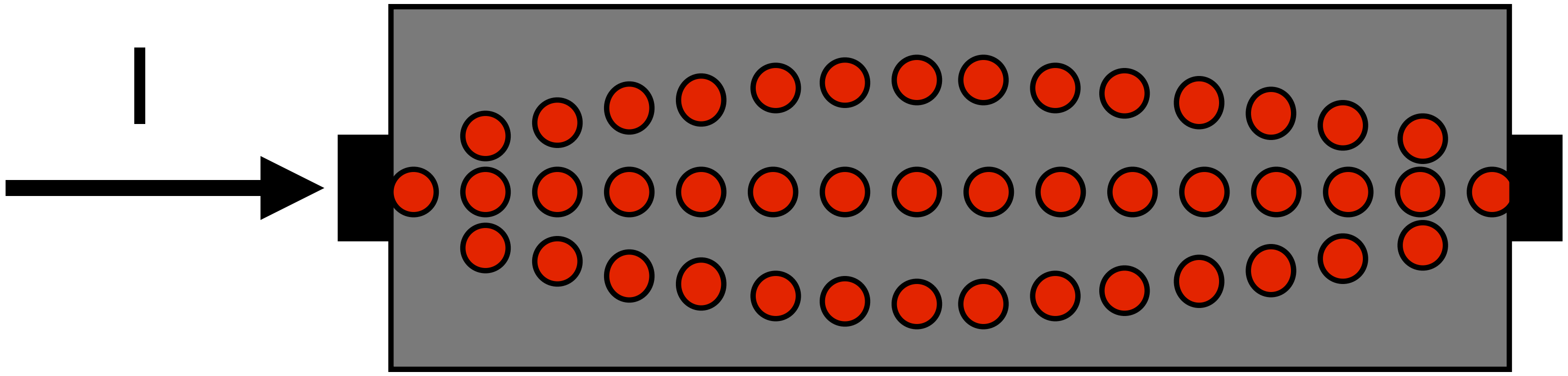


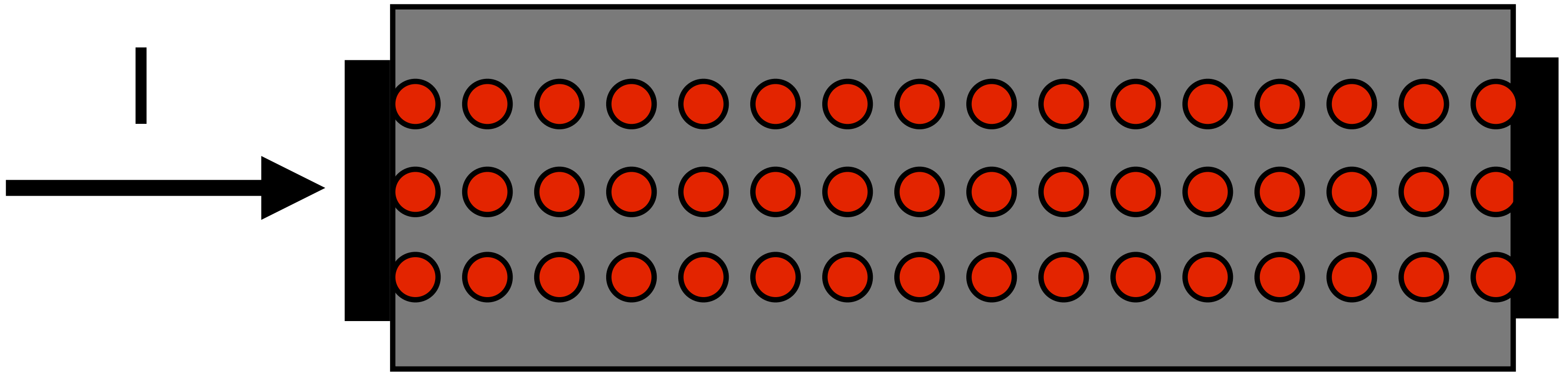
I would like to have the contacts as large as possible



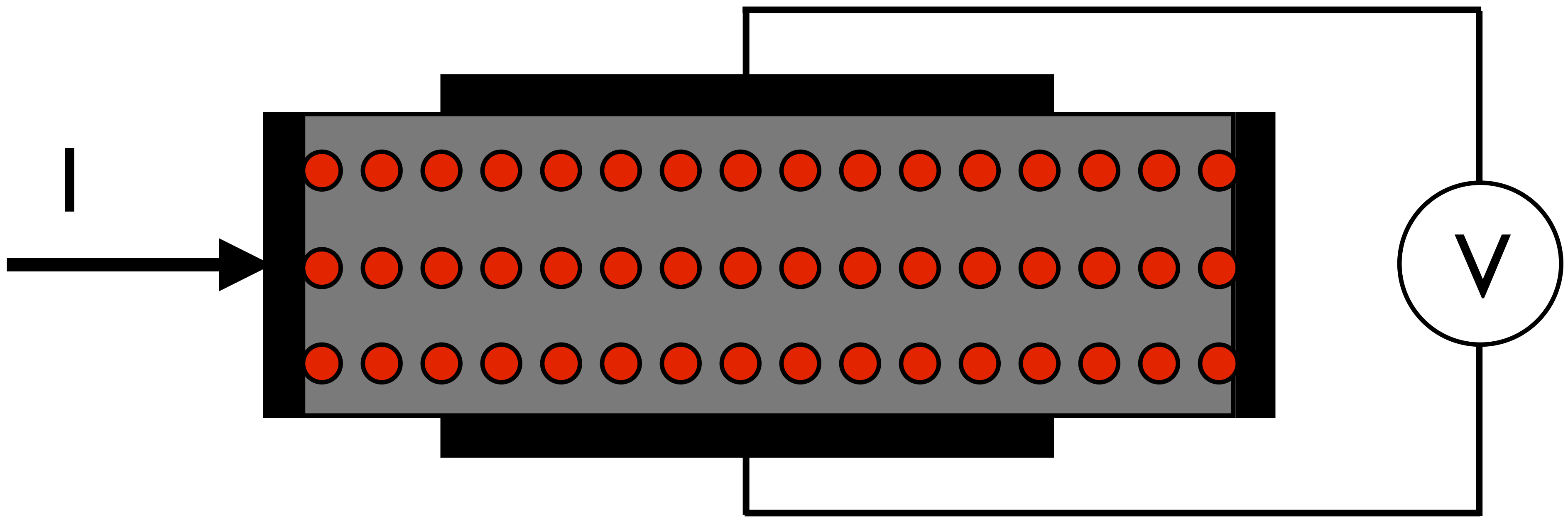
but if the contacts are too large
they short circuit the current

Problem 2. How can I obtain uniform current?



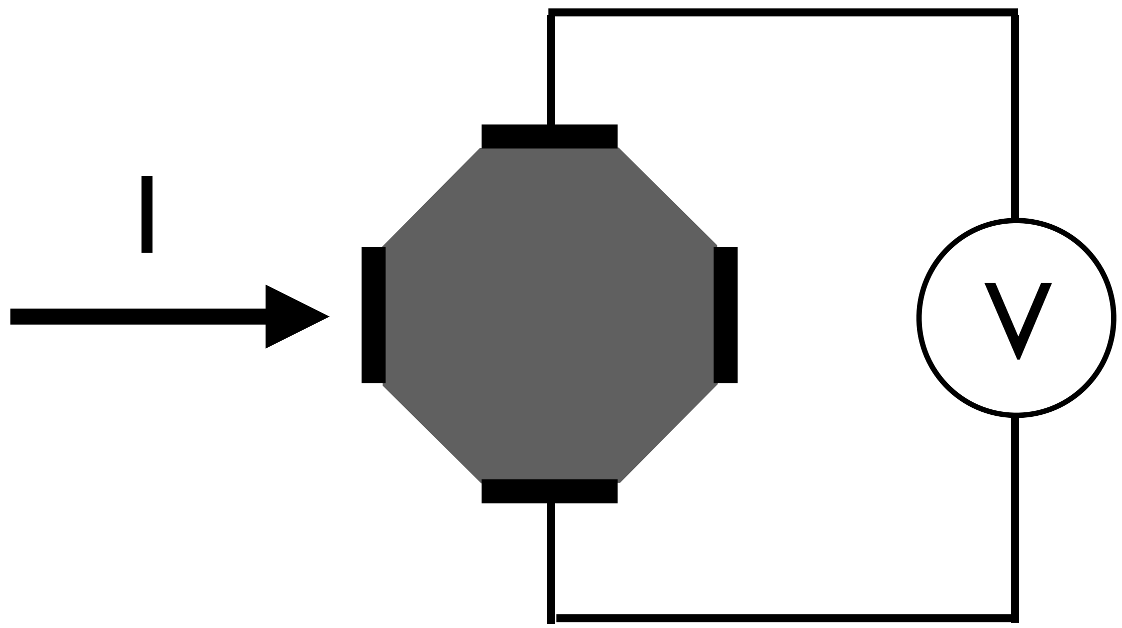


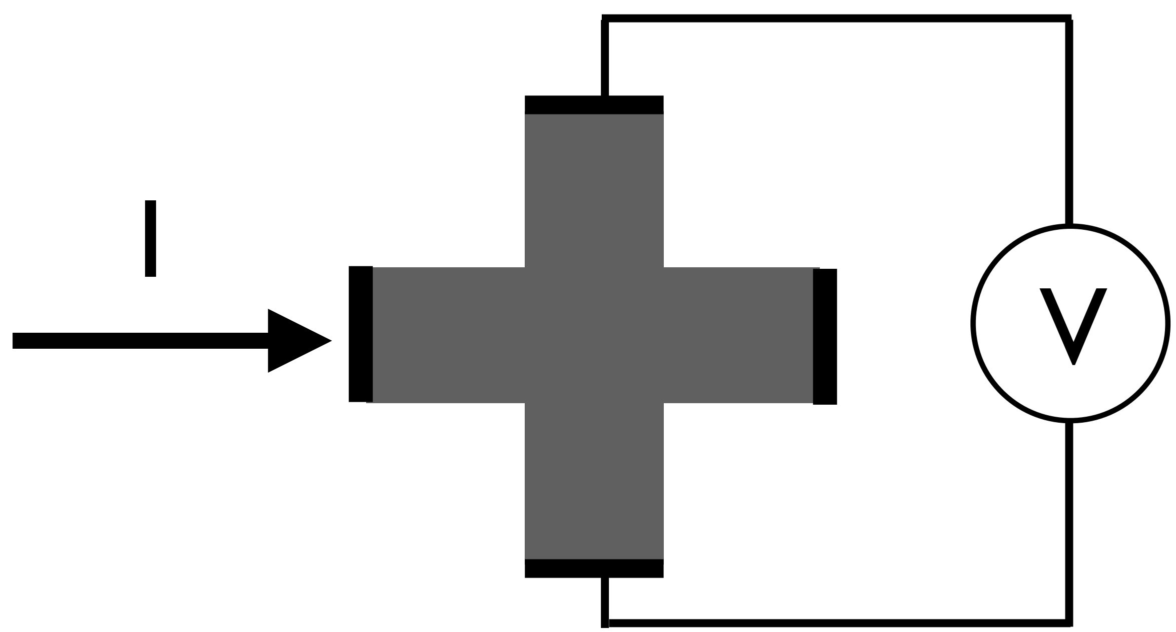
I would like to have contacts as long as possible



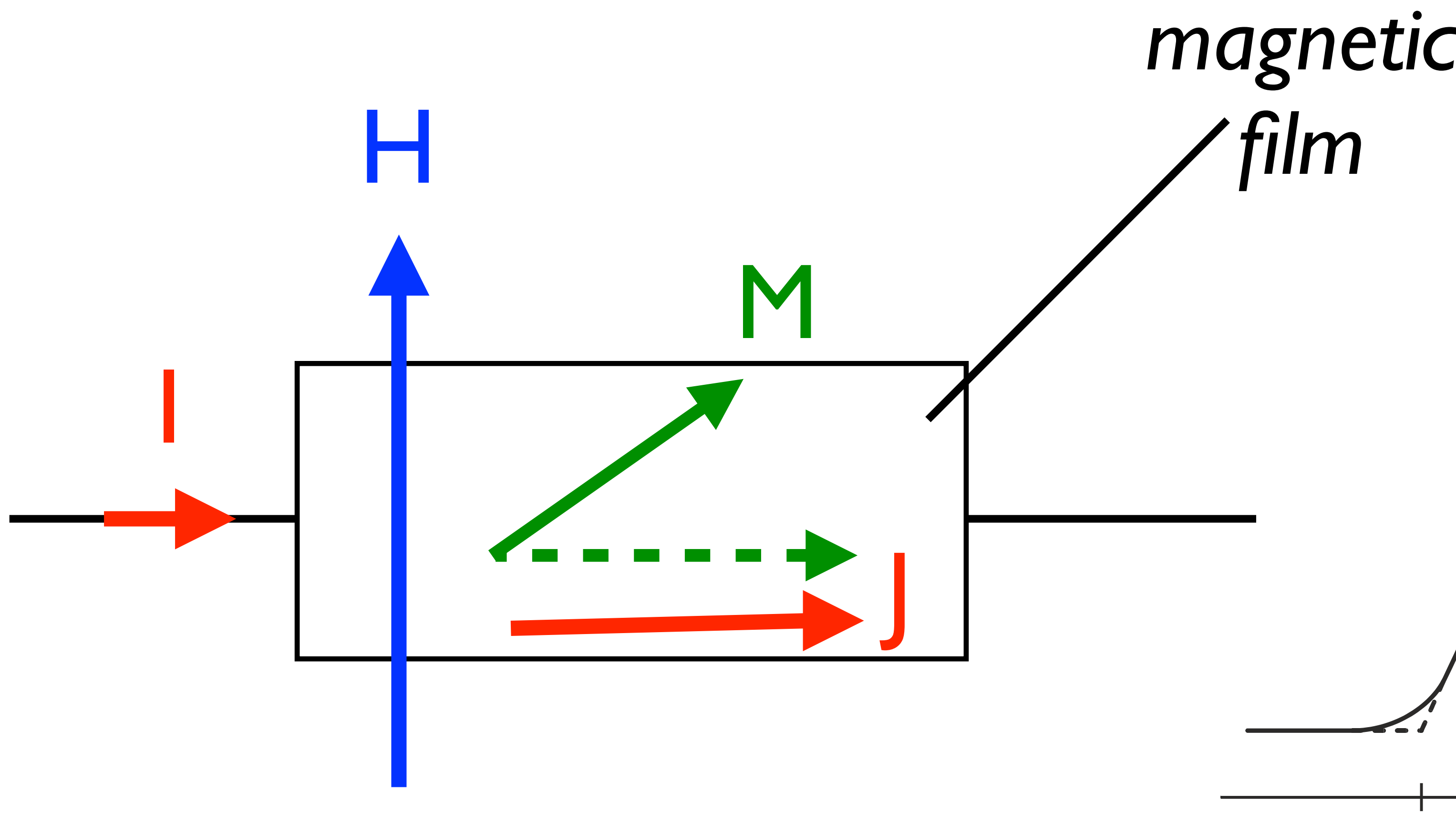
but this would short circuit the voltmeter.

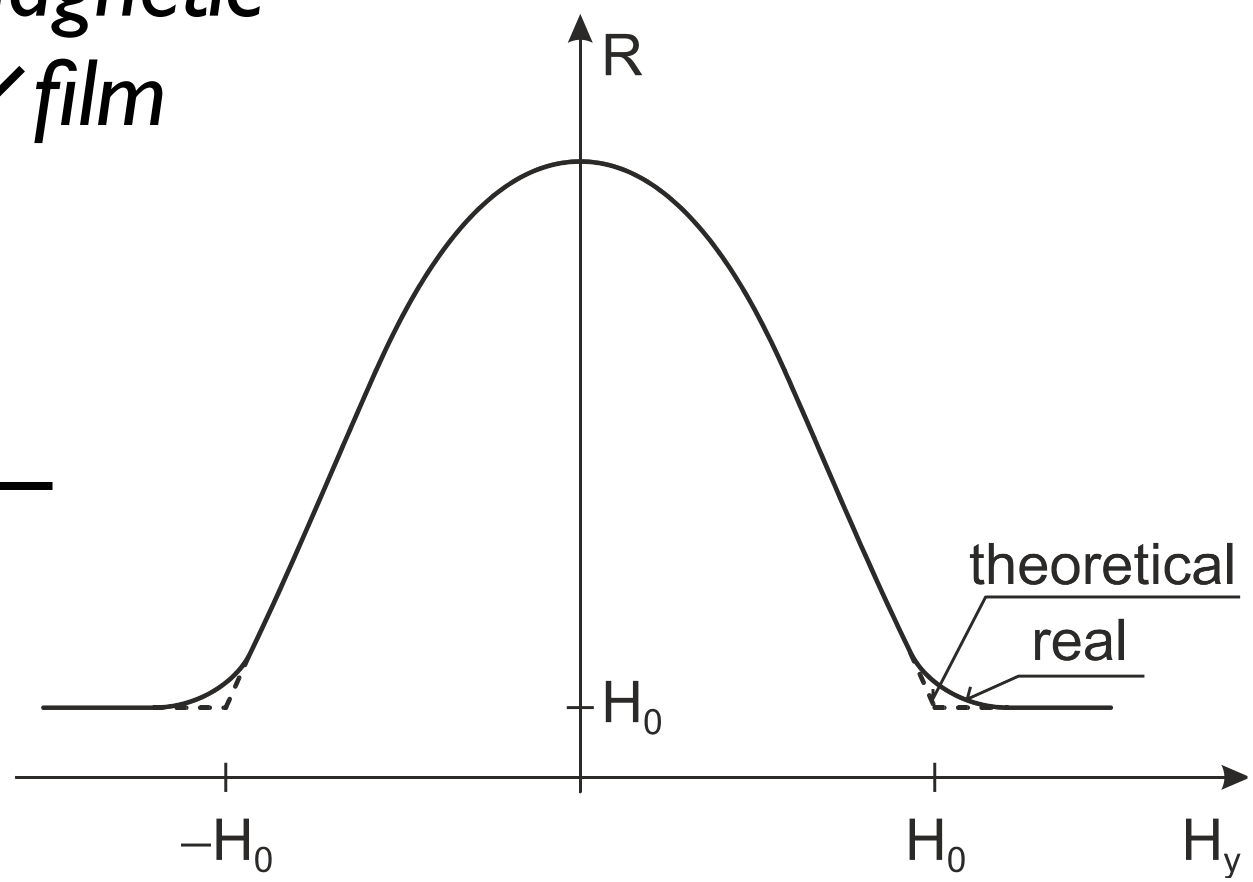
Several shapes provide a compromise





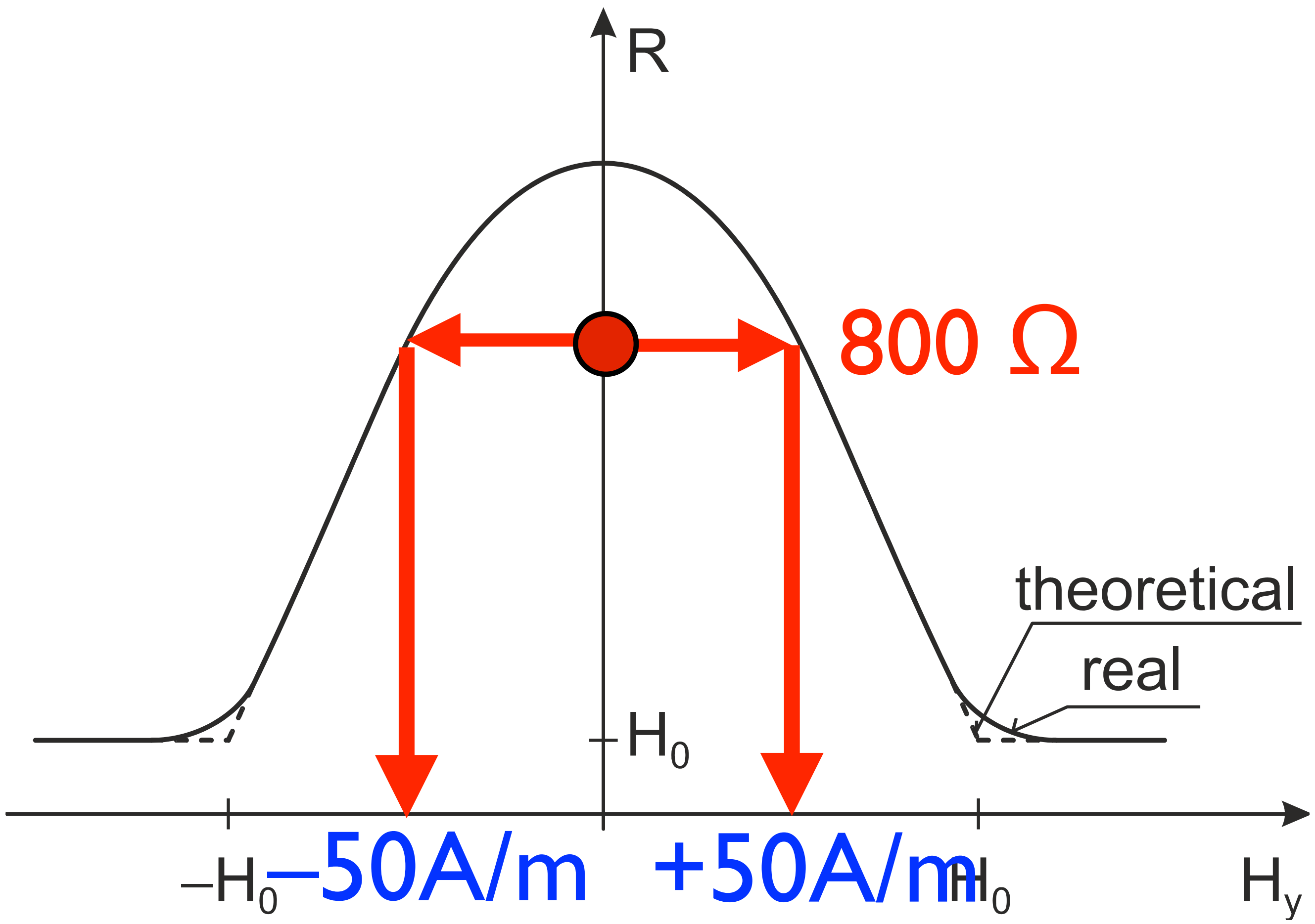
Anisotropic MagnetoResistor (AMR)





The resistance (slightly) changes when a magnetic field deflects its magnetization M

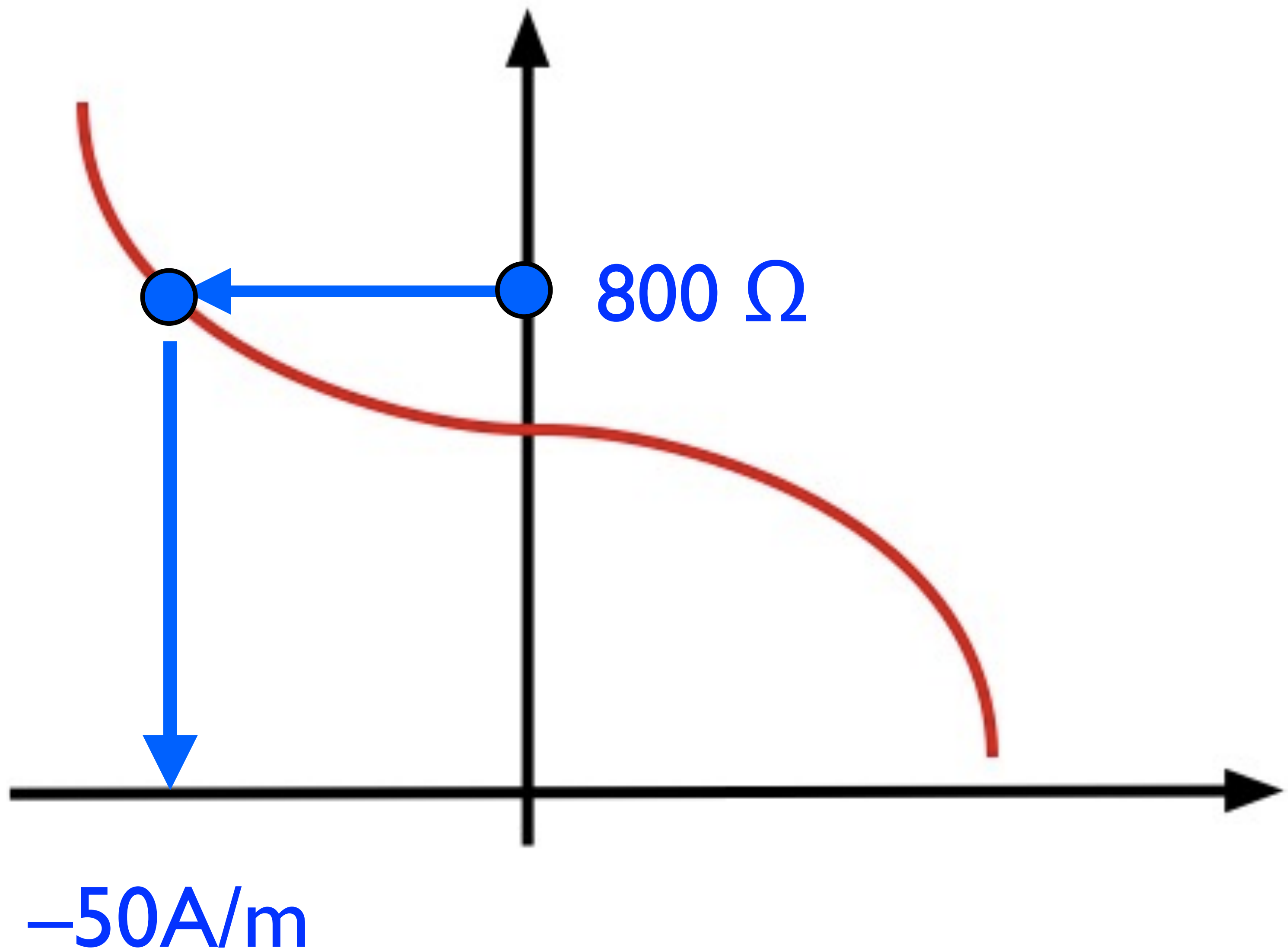
Problem: the characteristic is NOT linear



If the resistance is 800 Ω how can I know if the field is - 50A/m or + 50 A/m?

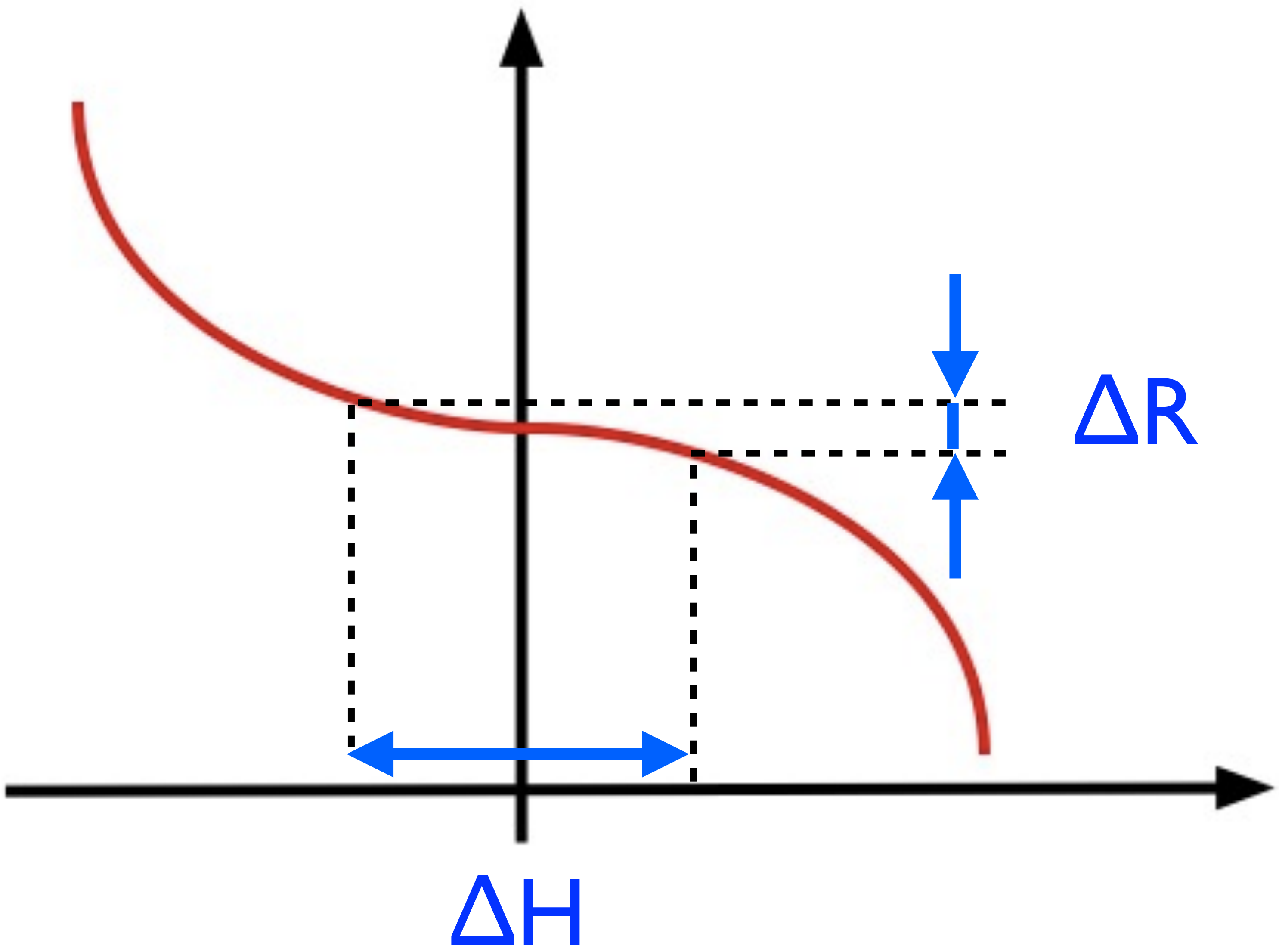
Both of them cause the same drop of resistance!

If the characteristic was at least monotonic I could distinguish positive and negative magnetic field



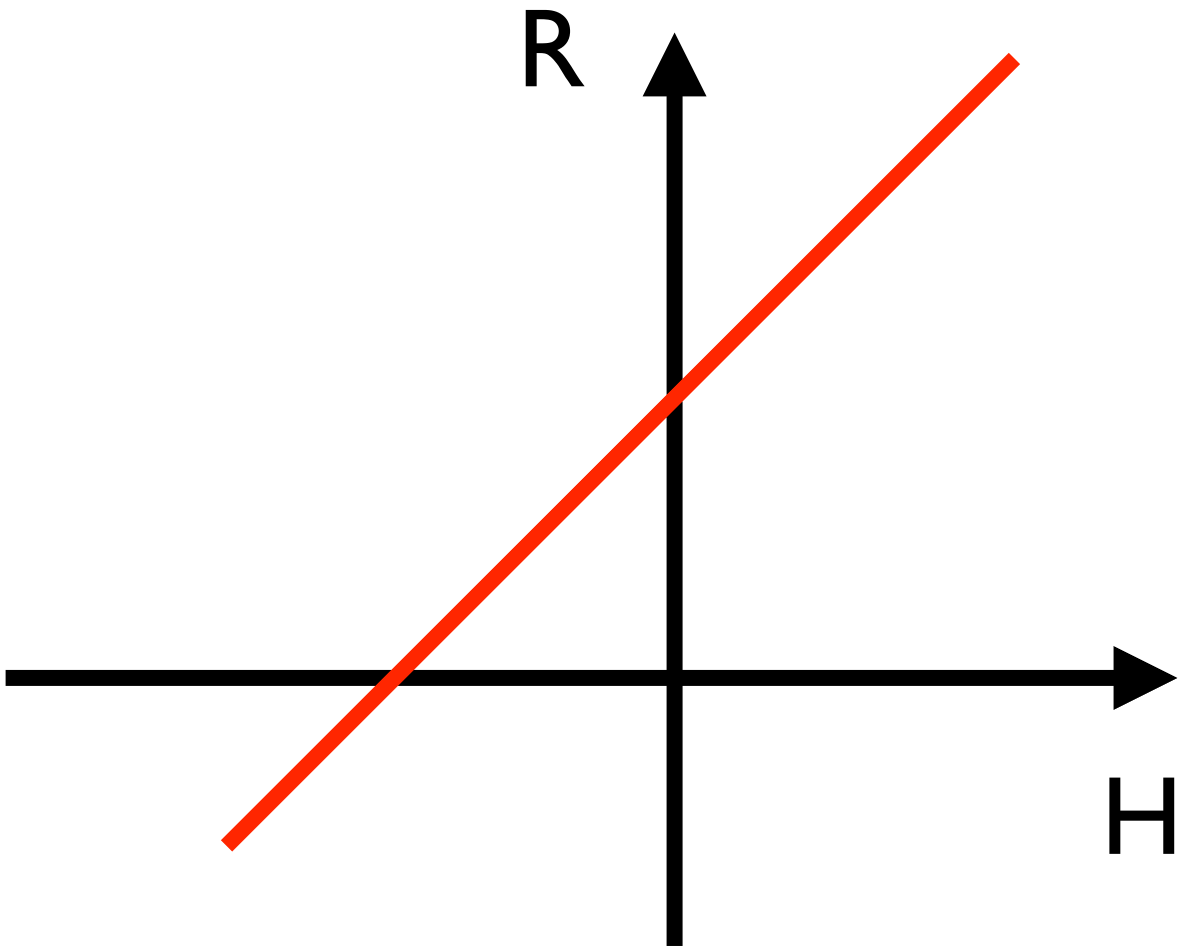
If I measure 800 Ω, for sure the field is - 50A/m because no other field causes 800 Ω

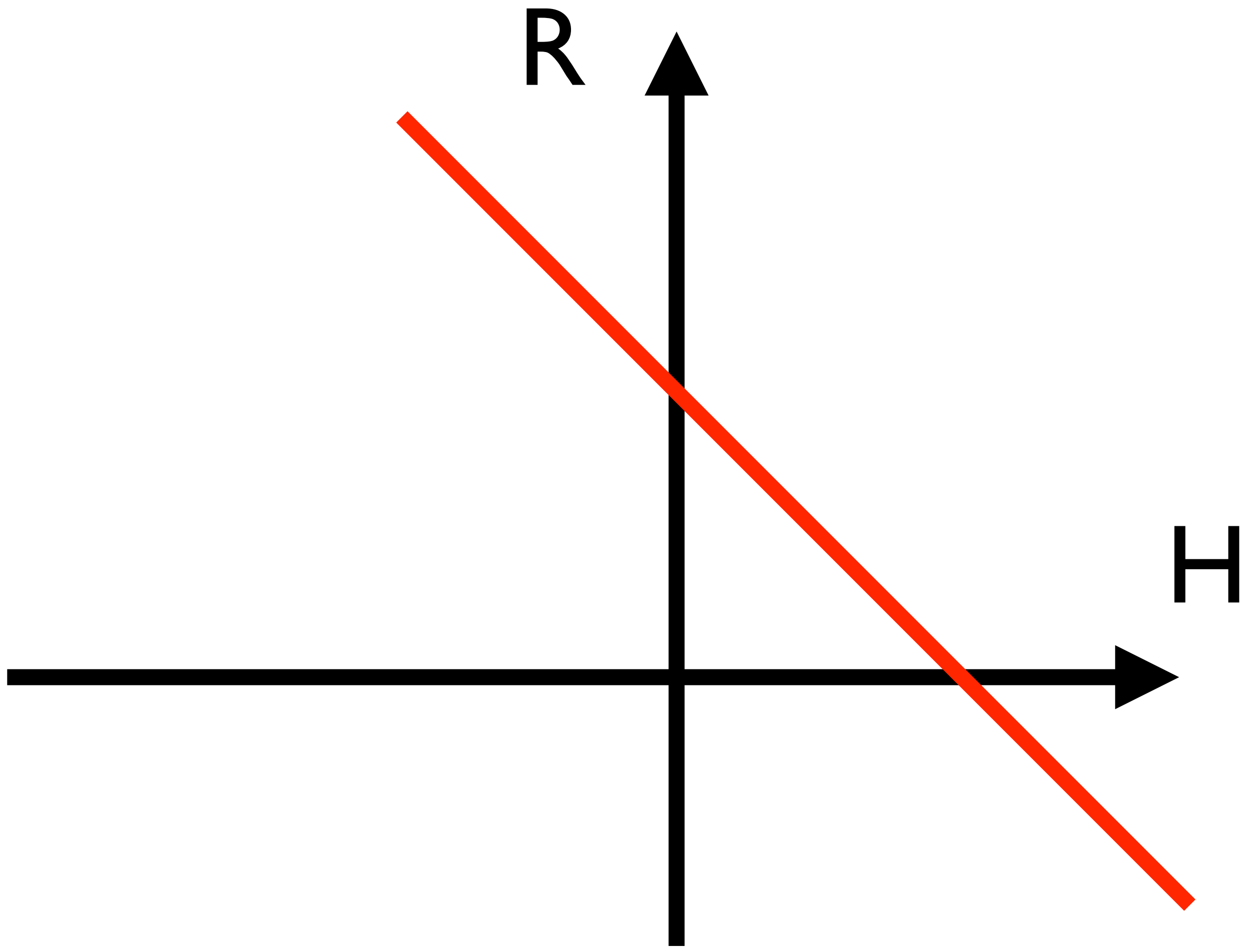
Monotonic function is not enough.



Large variation of H cause only little variation of R

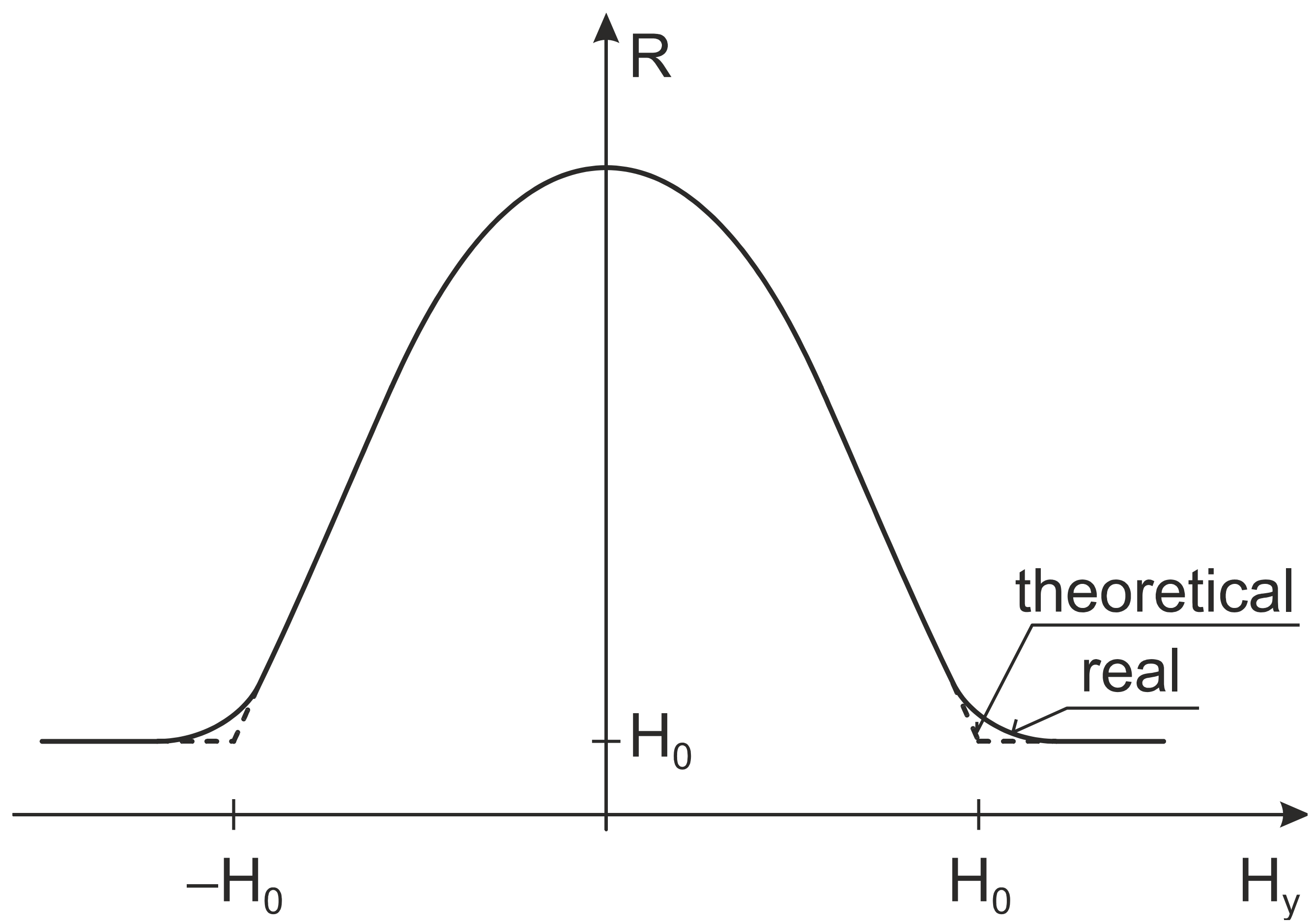
We need a LINEAR charateristic





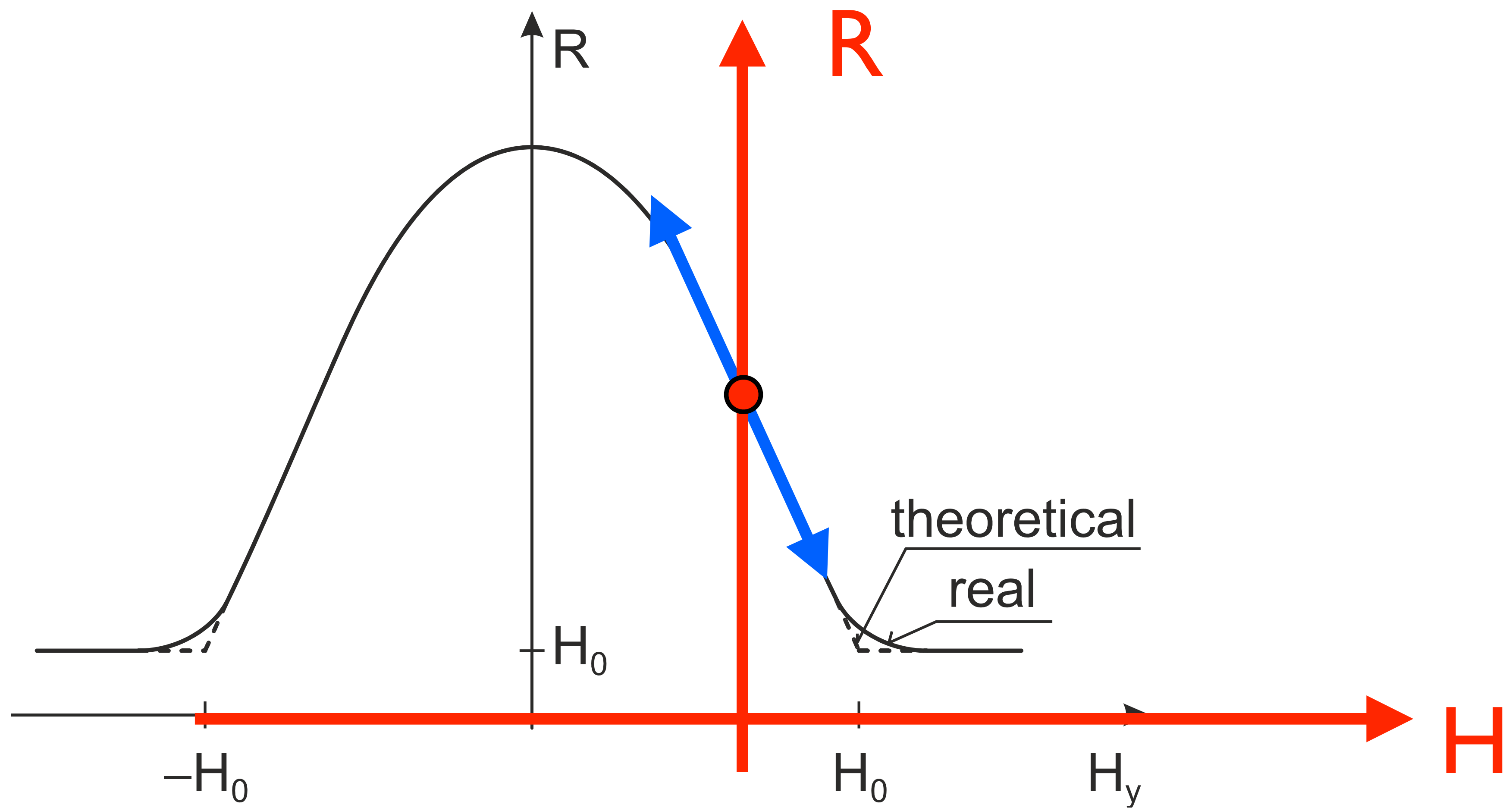
- it's monotonic
- the slope (sensitivity) is uniform everywhere

Unfortunately the resistance behave non linearly



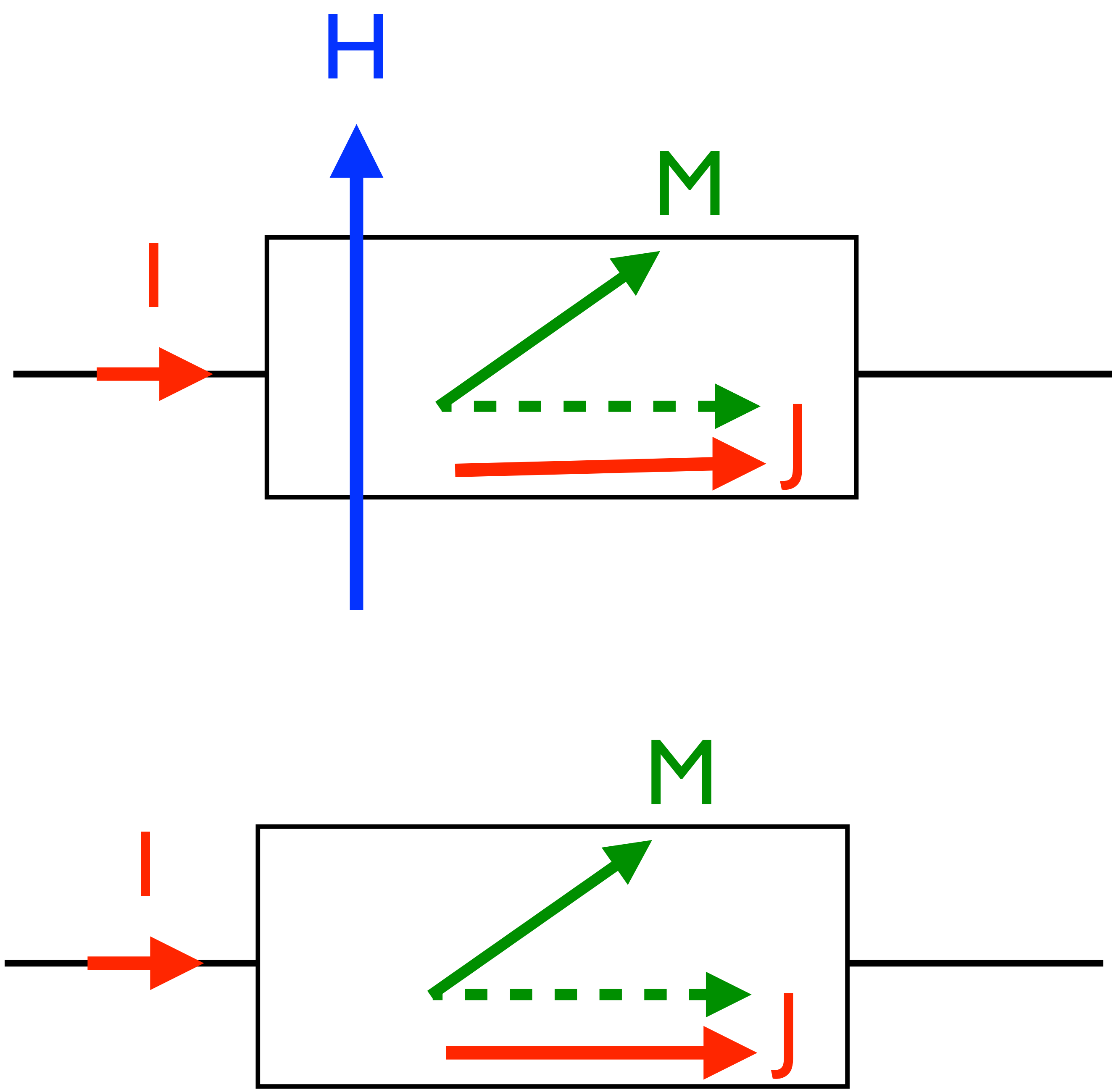
How can we get something linear out of this?

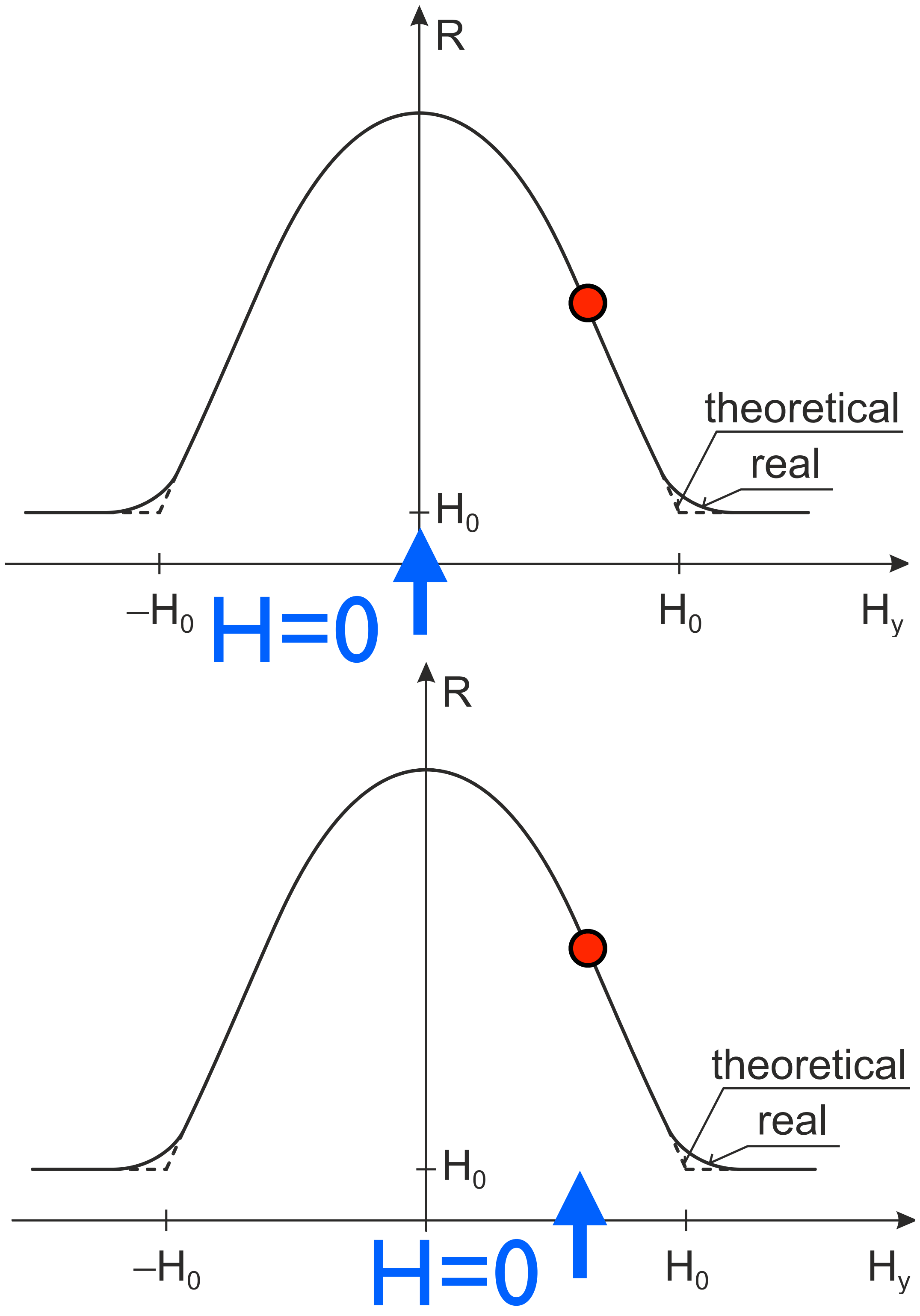
I can use only this part of the characteristic:



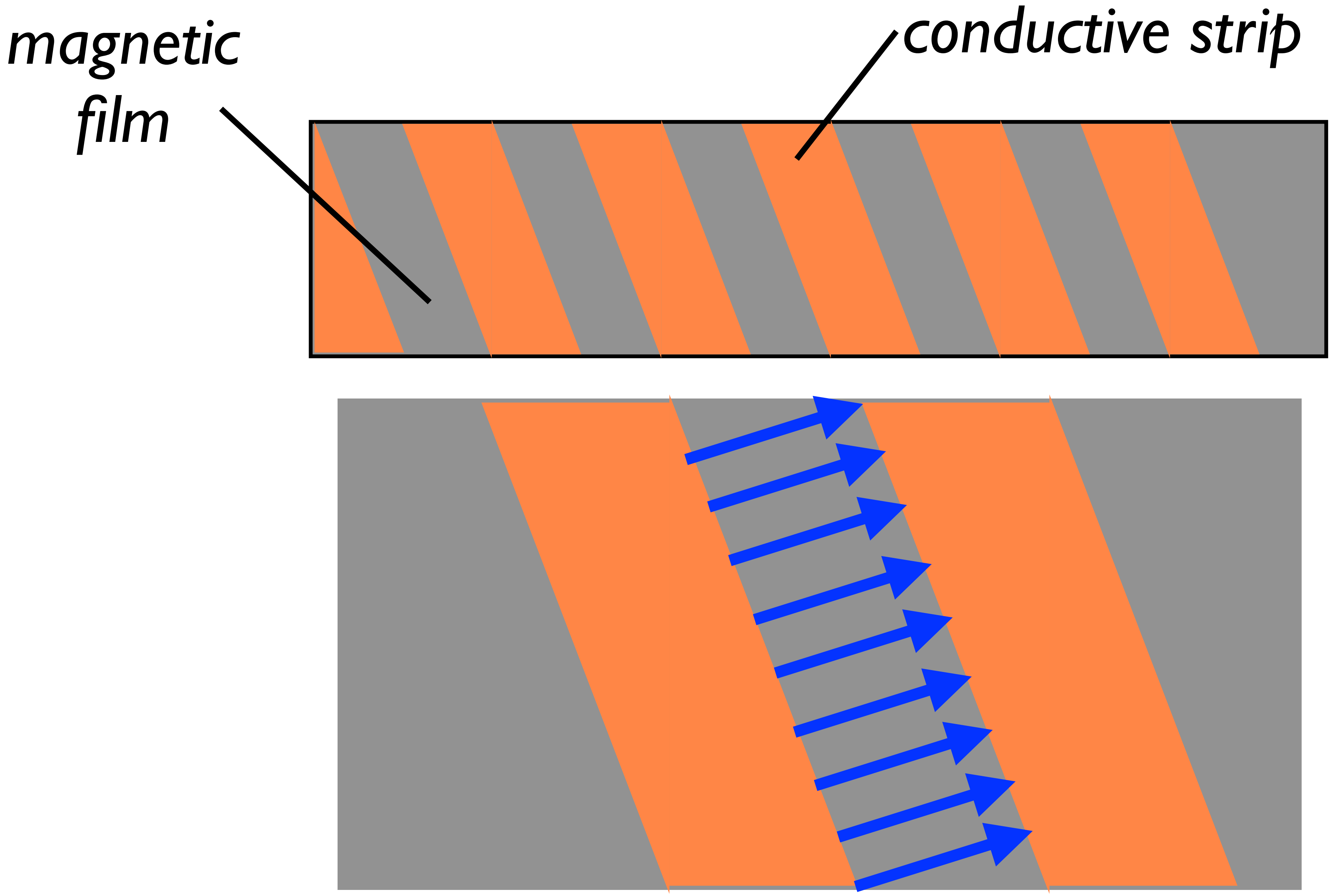
I shift the characteristic so that it is centered in the linear part

In order to do so, I should rotate the Magnetization M of 45° for no magnetic field applied



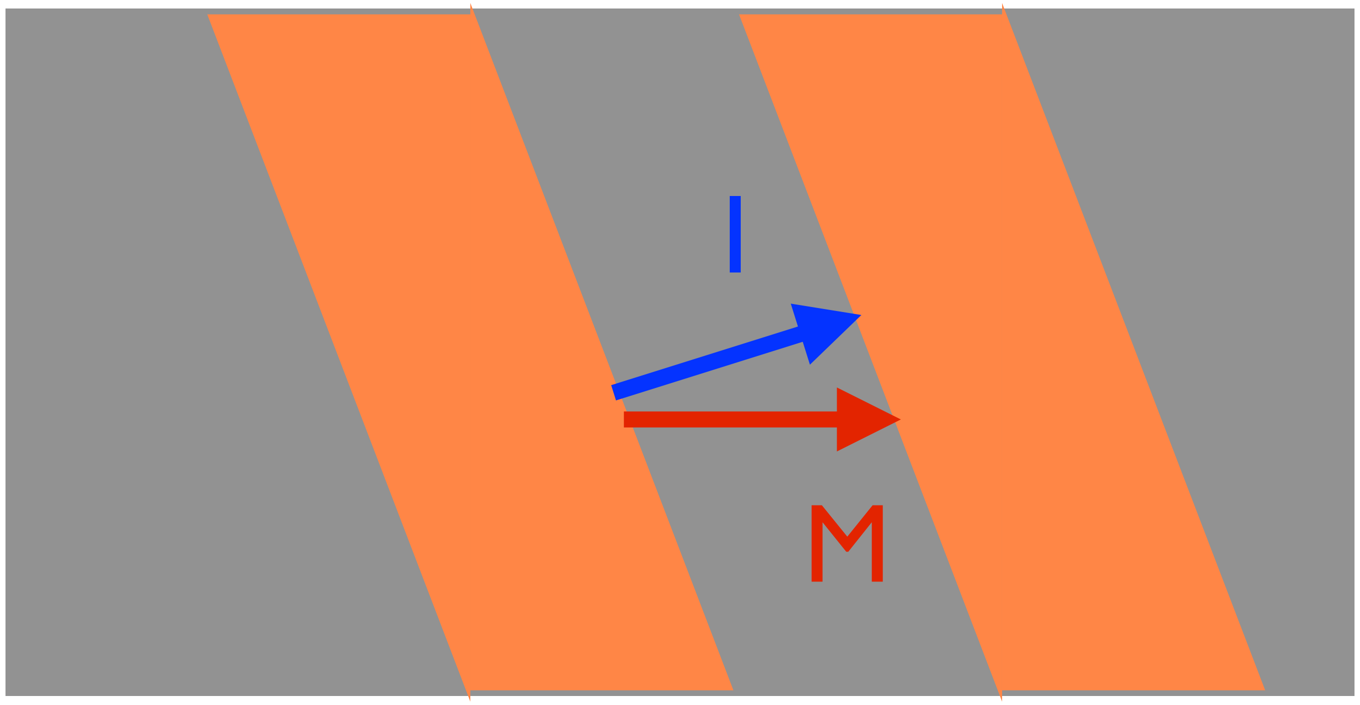


We use barber poles to shift the angle between Magnetization and Current



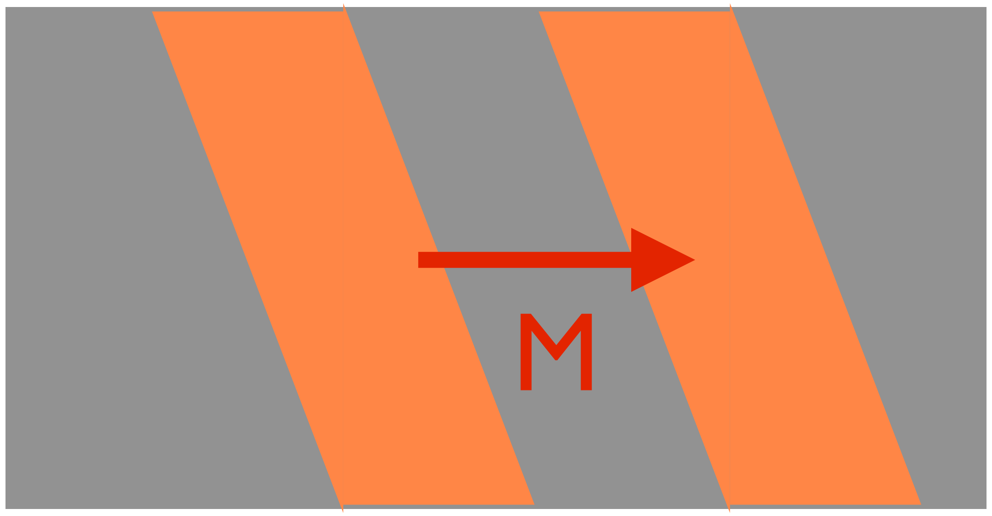
The current flows from a conductive strip to the following conductive strip following the closest distance

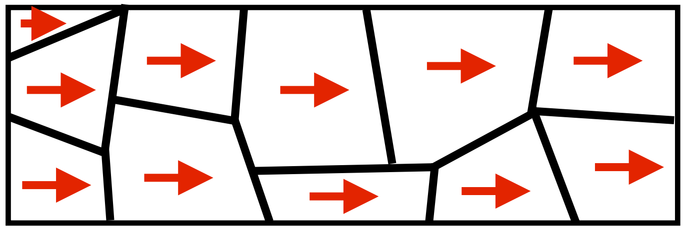
In fact, we do not rotate M, we rotate I



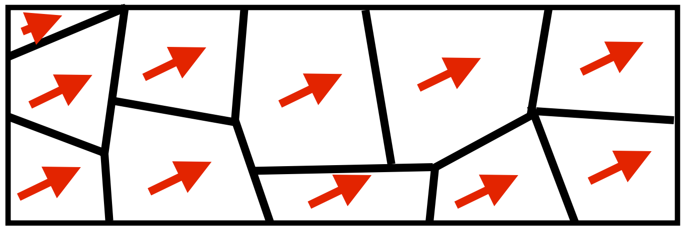
Problem: M can "deteriorate" if we expose it to very large magnetic field

H=0

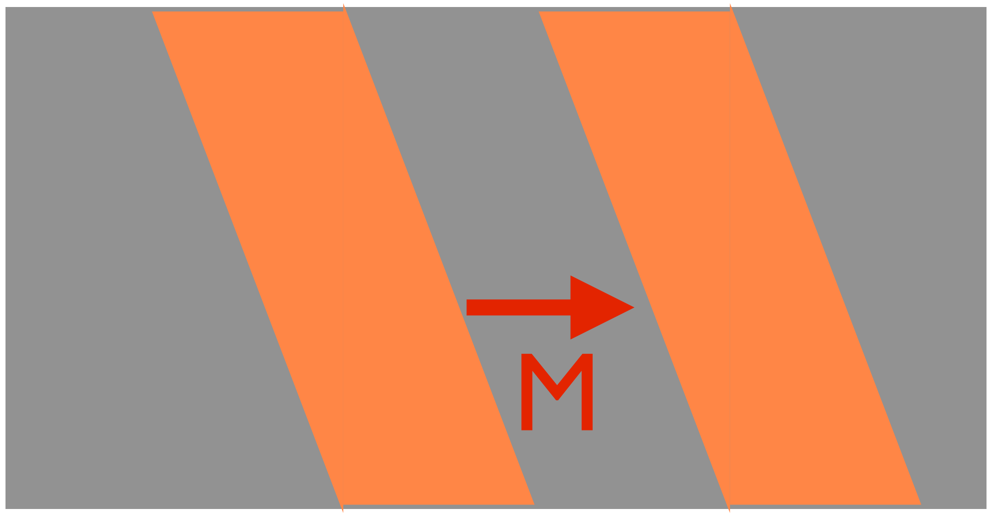


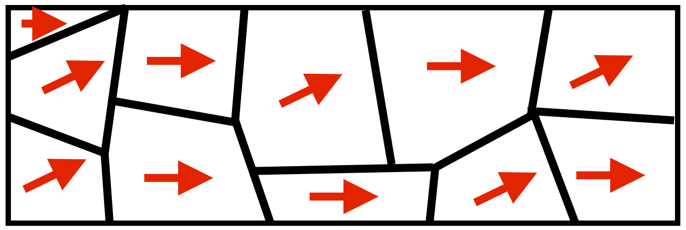


H=100 A/m

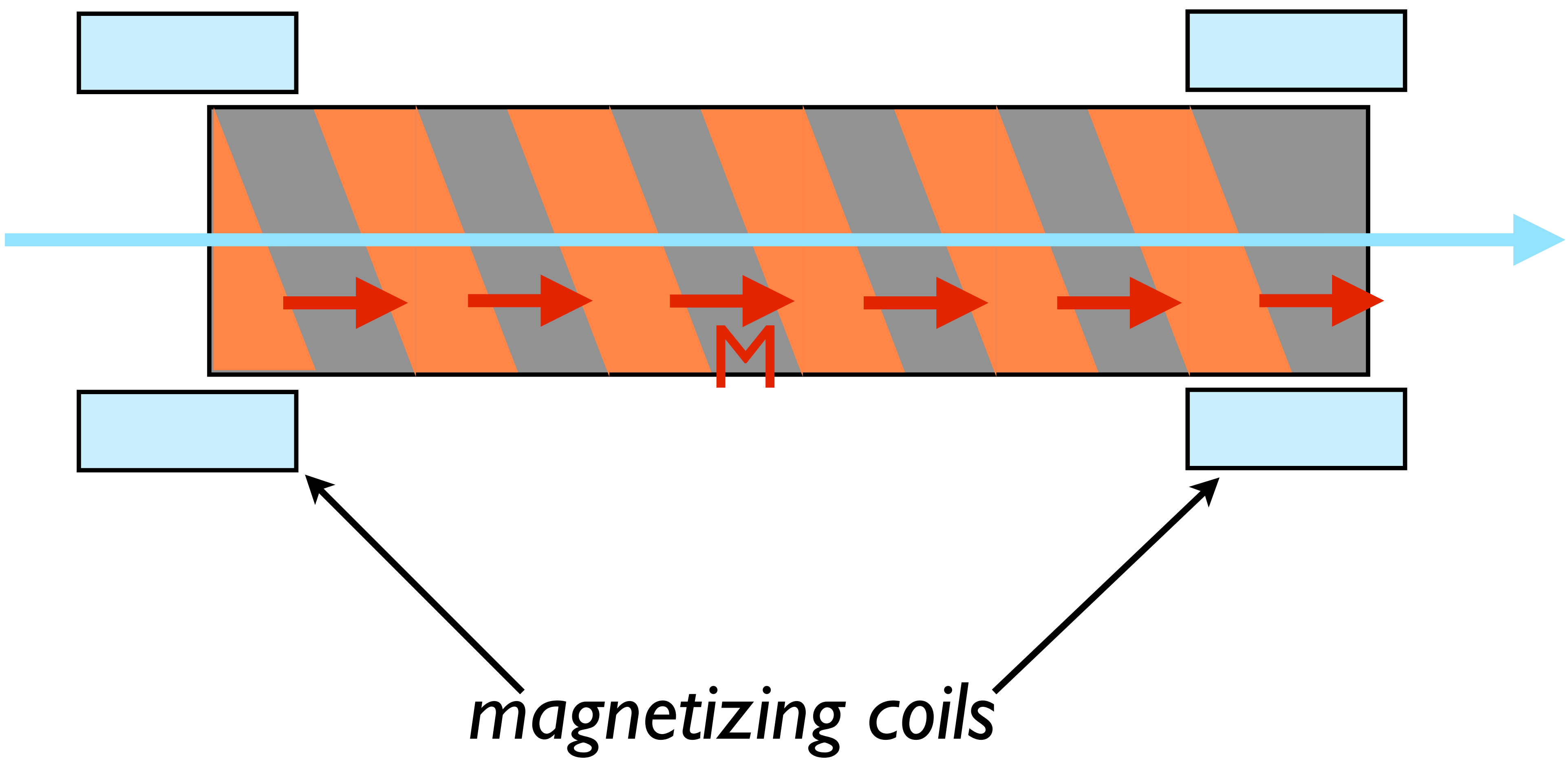


H=0





AMR sensors must be periodically magnetized to restore full magnetization



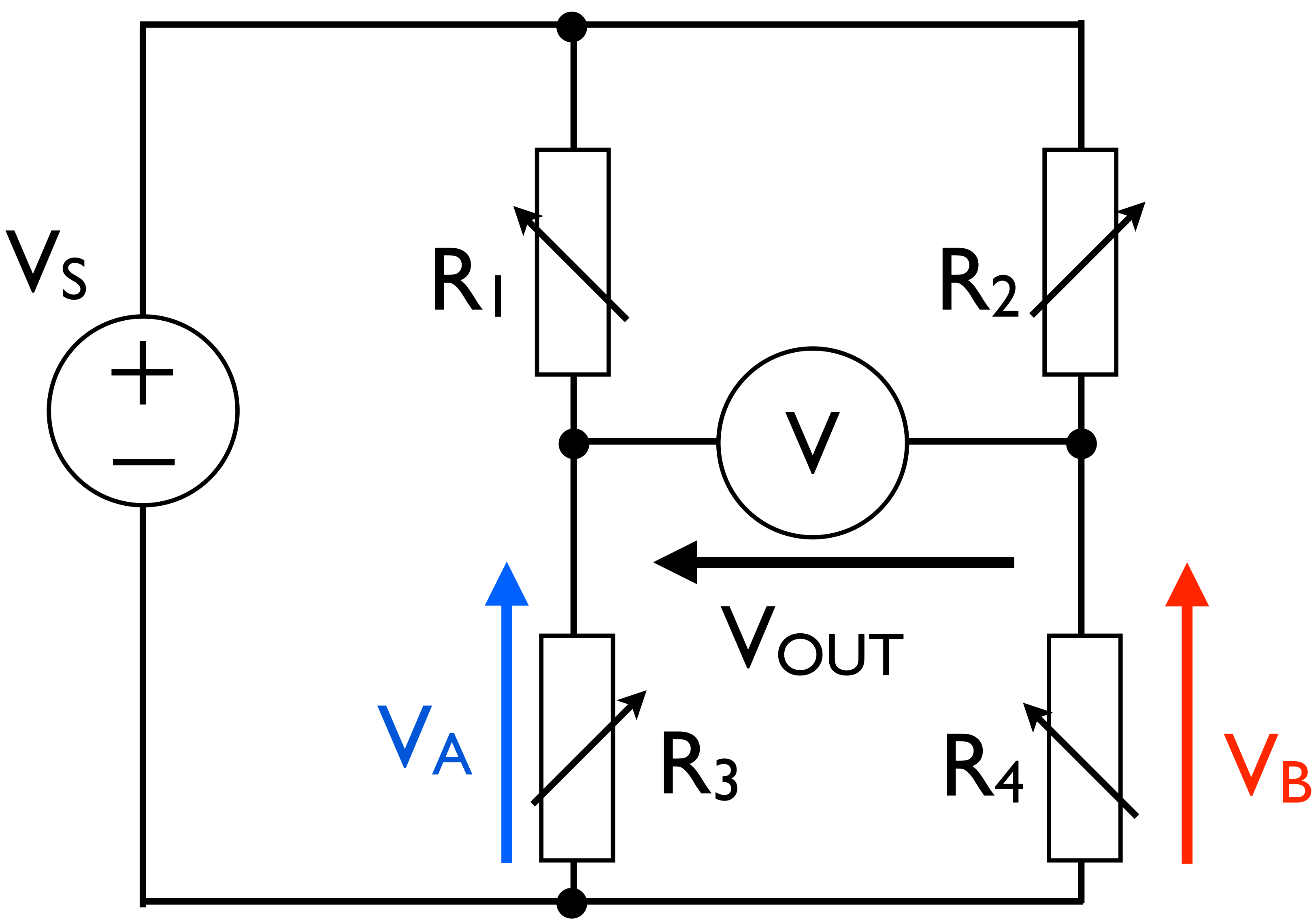
Additional problem: the magnetoresistive effect is low!

The resistance changes only a few % of its value at H=0

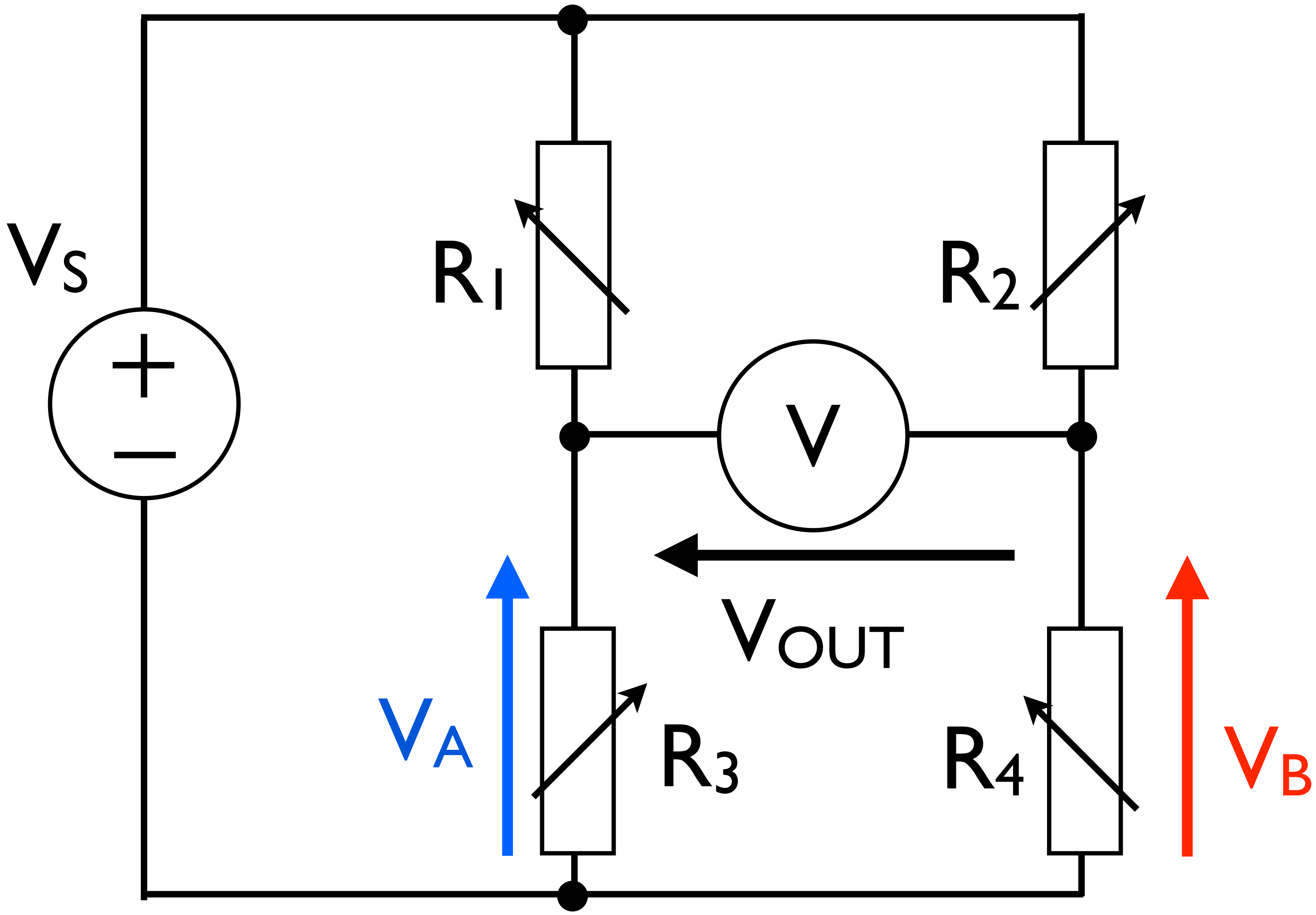
How can we measure such a resistance?

Back to lecture n. 4., we can use a

WHEATSTONE BRIDGE



We use 4 magnetoresistors, connected with opposite sensitivity direction



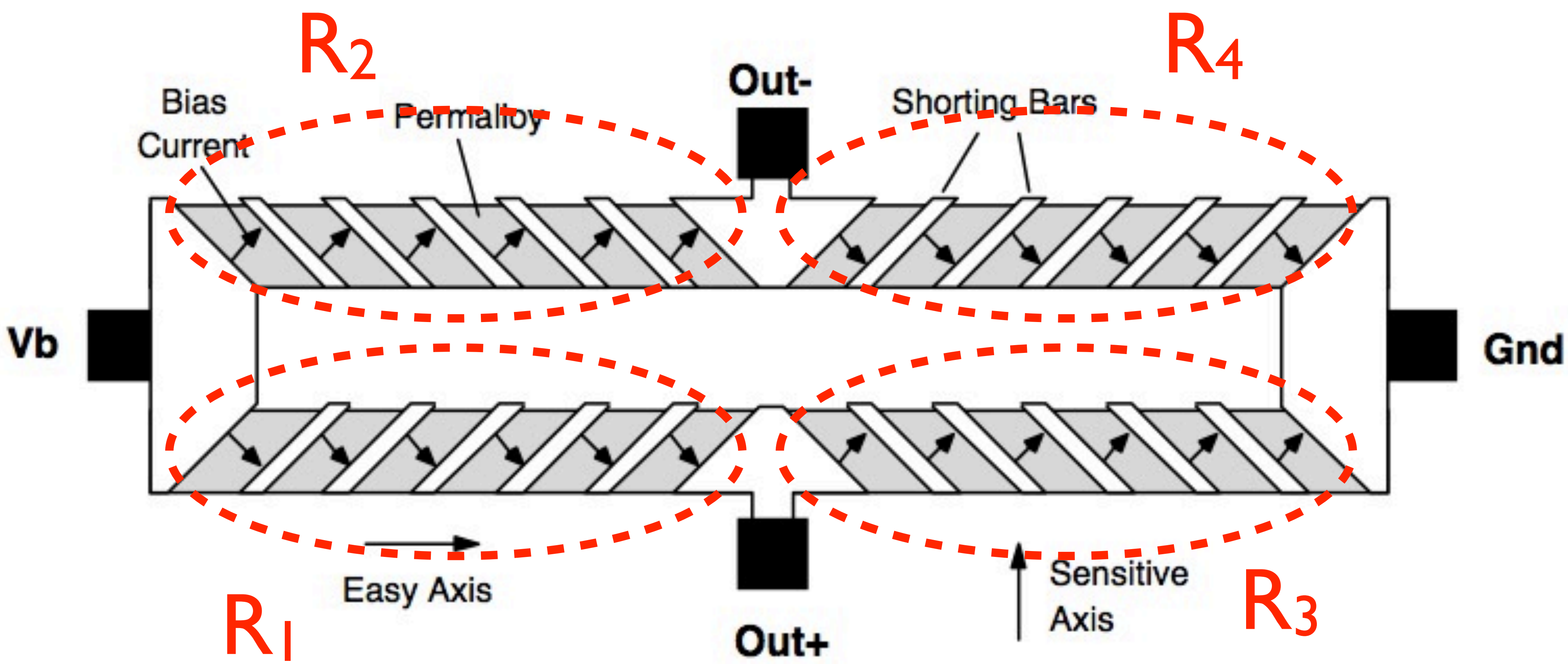
If H increases

R_3 (and R_2) increases → V_A increases

R_4 (and R_1) decreases → V_B decreases

Finally $V_{OUT} = V_A - V_B$ increases

If they had the same sensitive direction V_{OUT} would not change!



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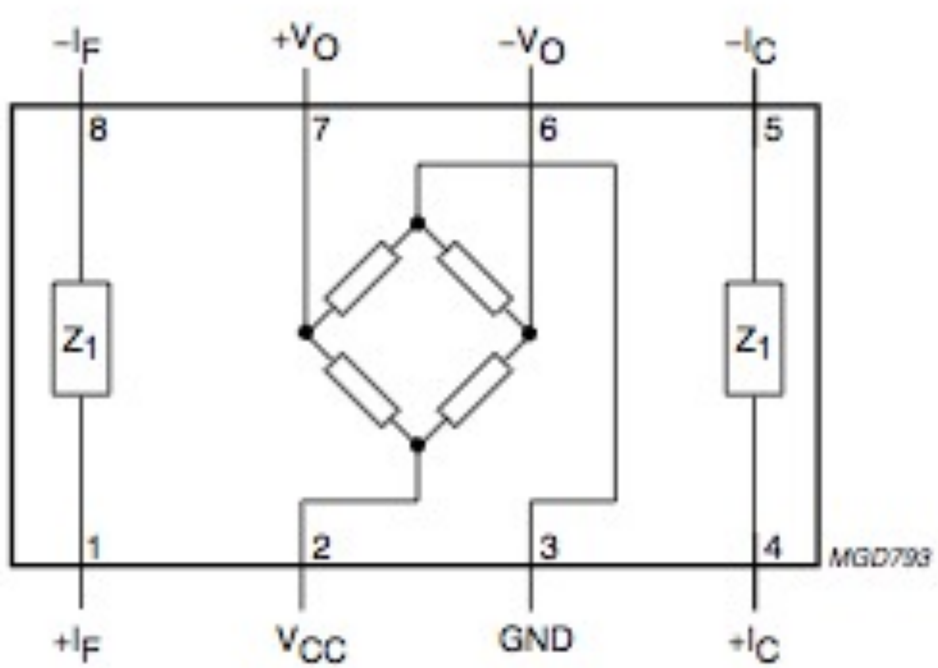

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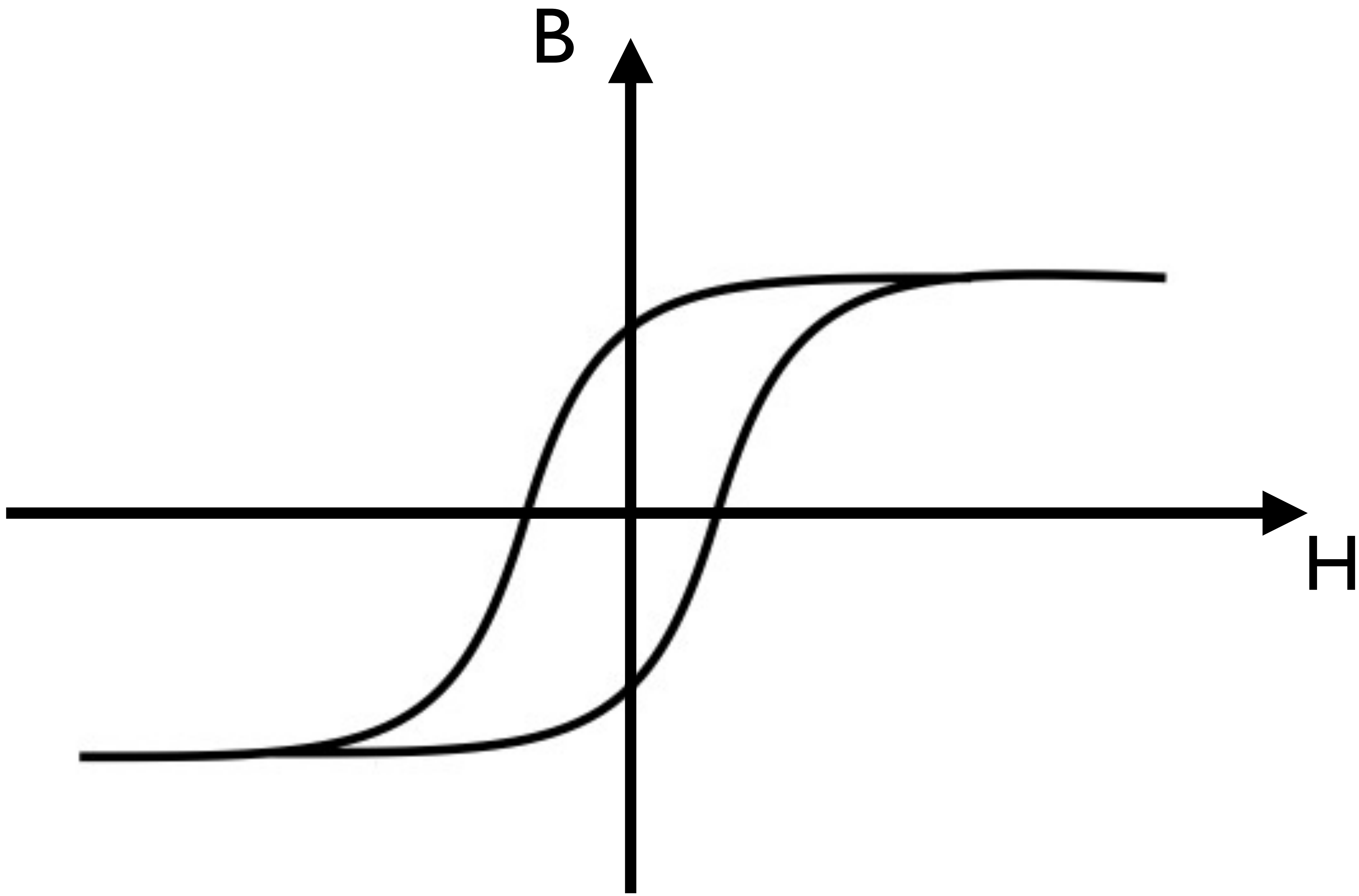
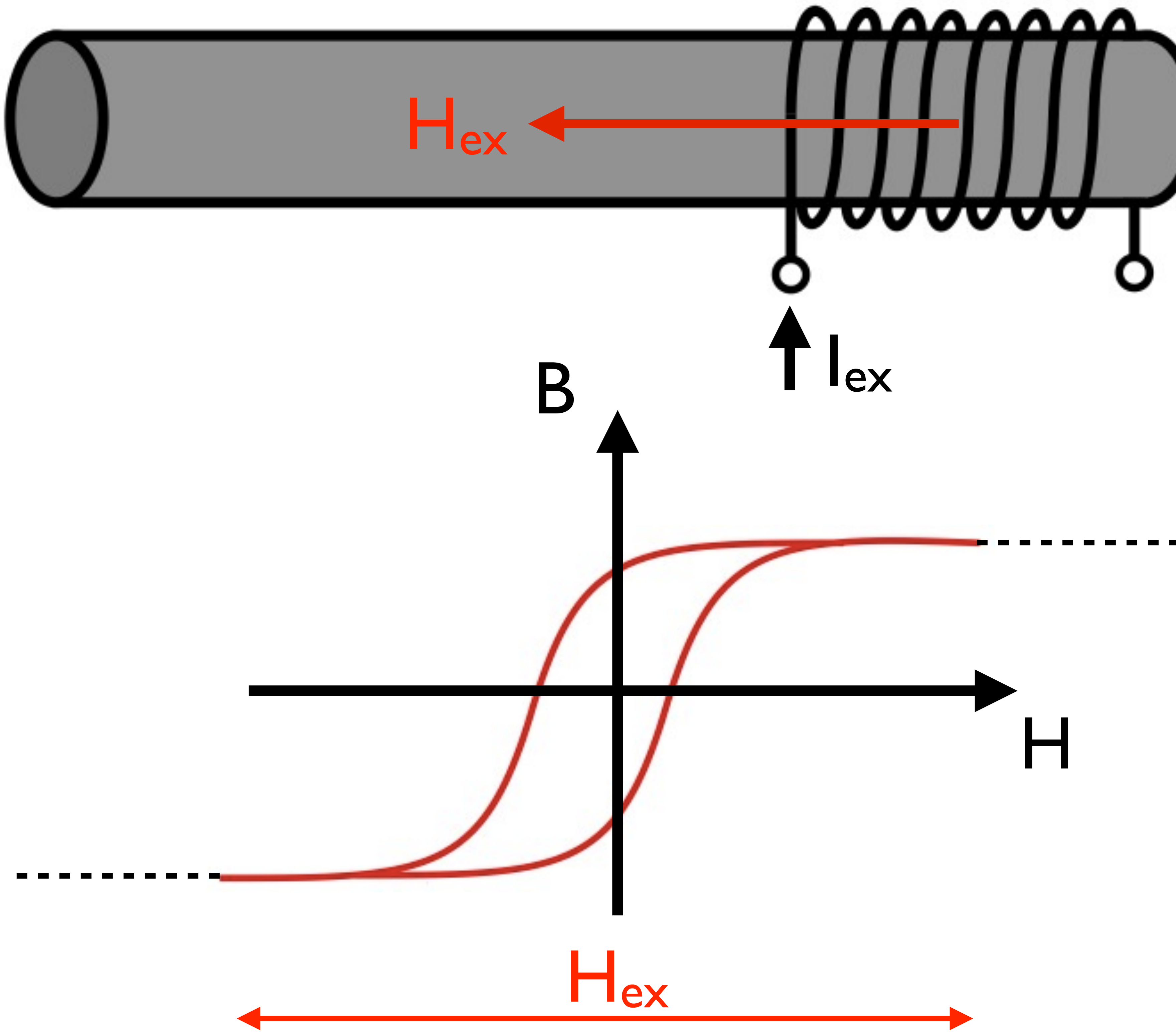
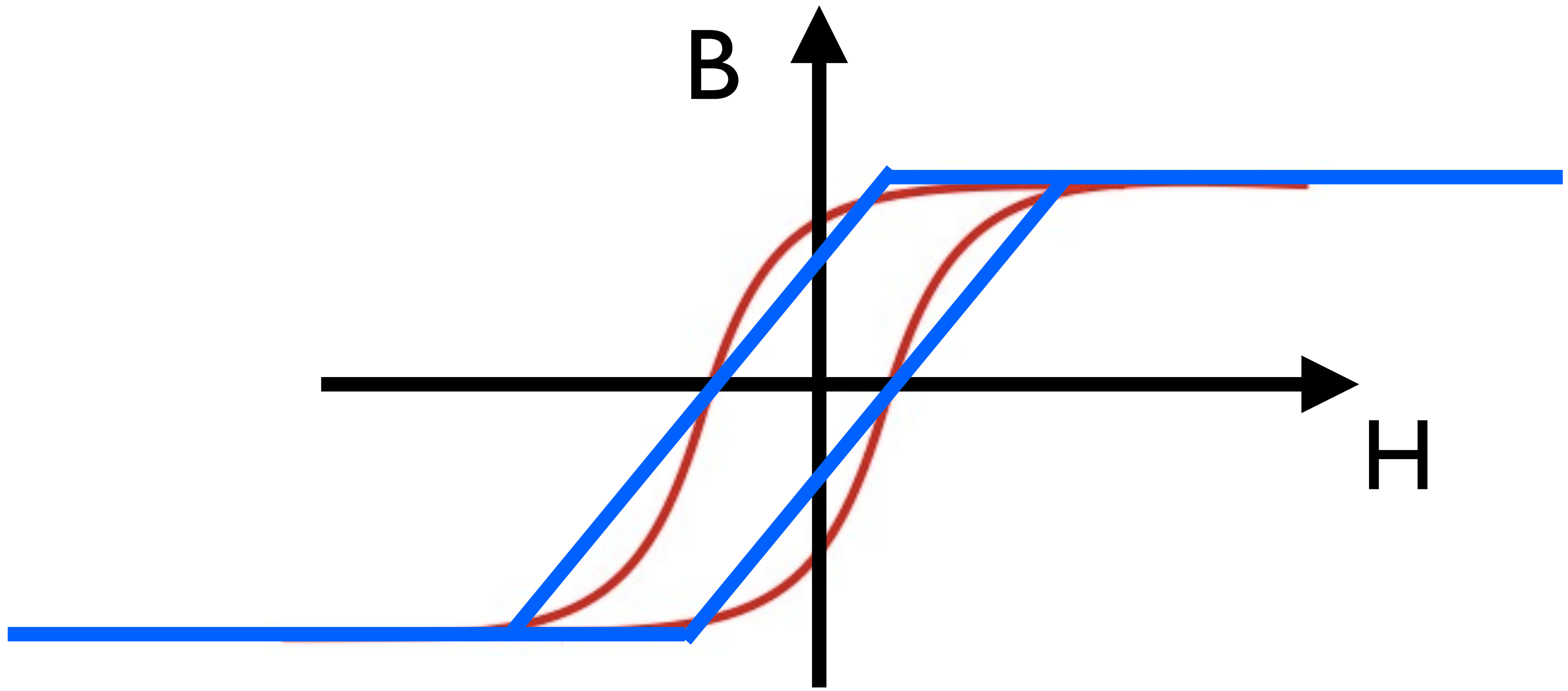
3 - Fluxgate

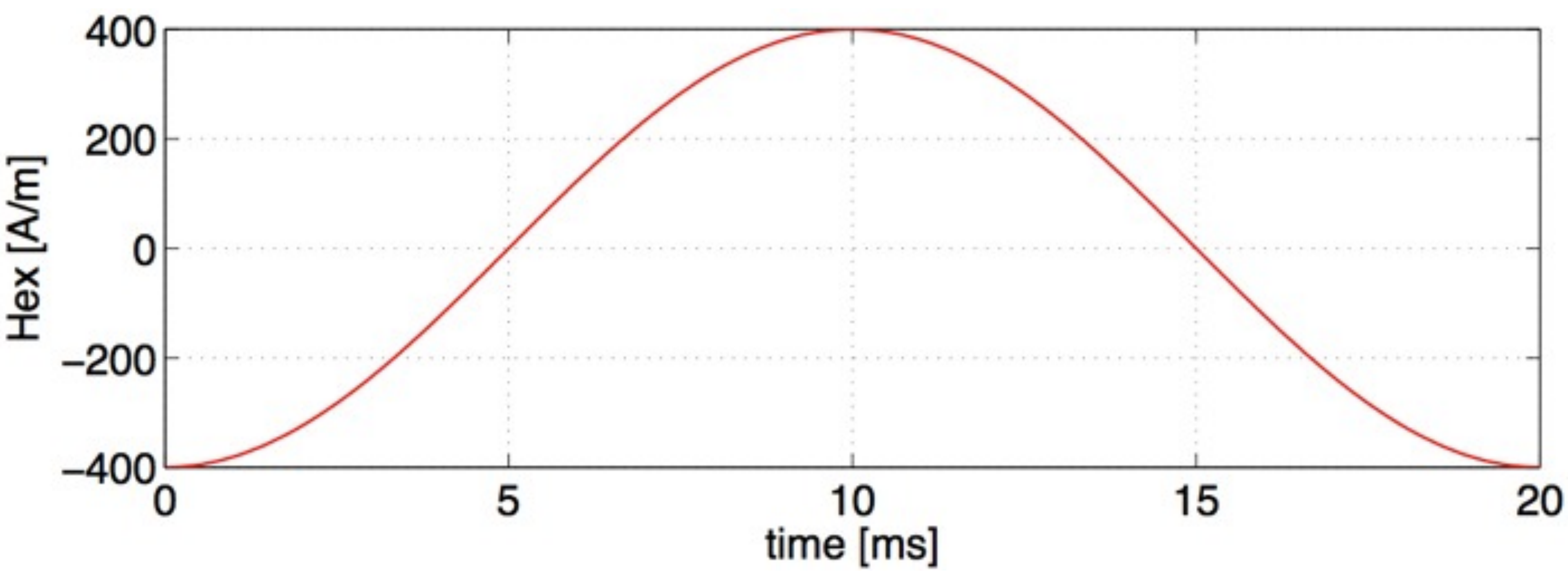


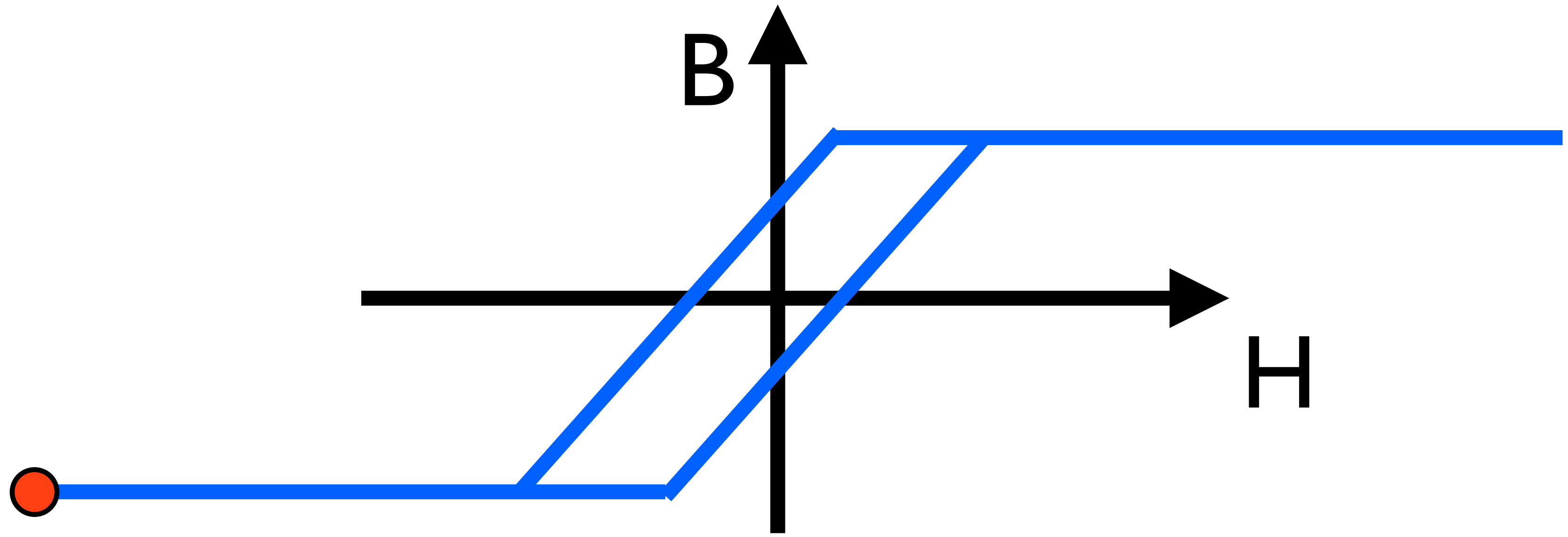
I generate a large excitation field H_{ex} using an excitation coil on a magnetic core

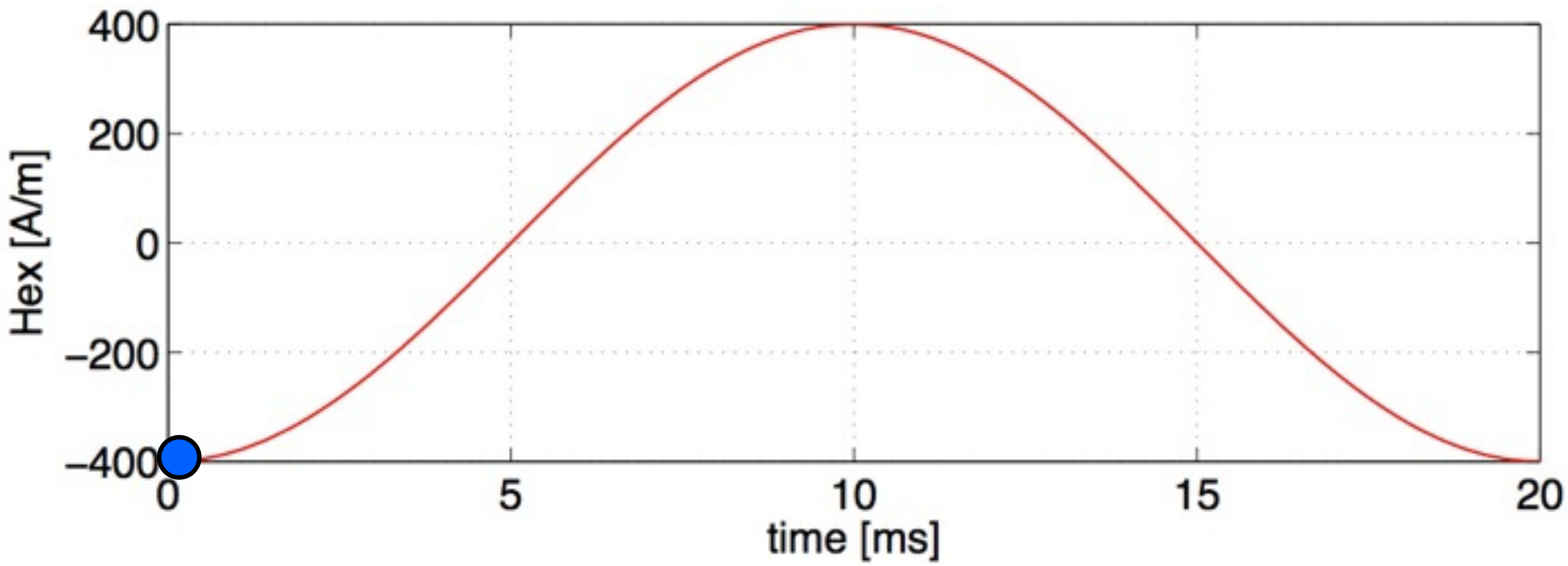


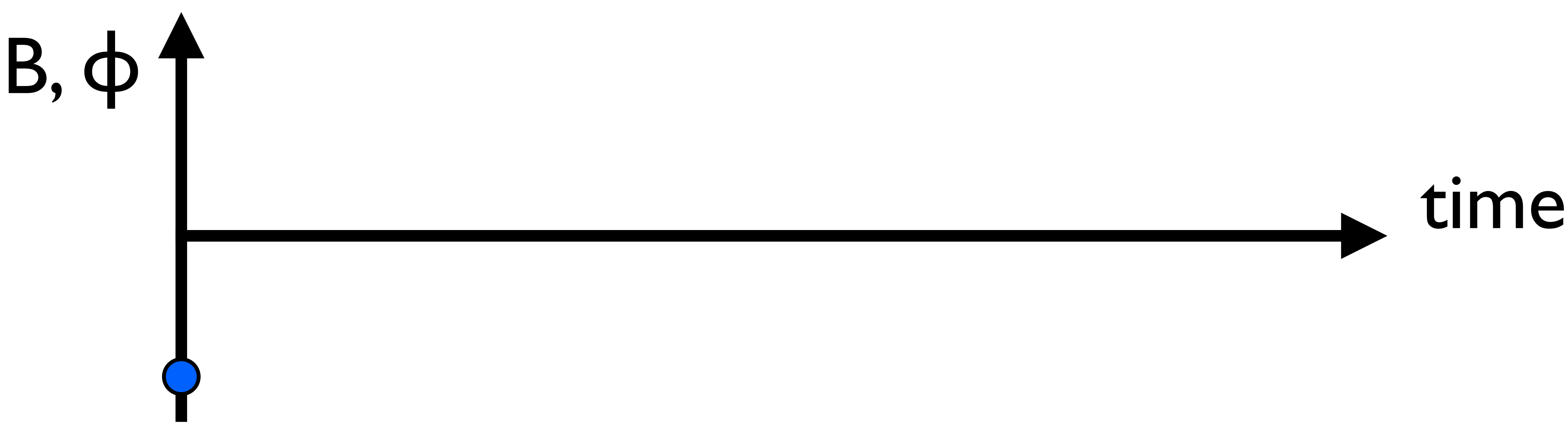
Let us assume a sinusoidal Hex and simplified hysteresis curve

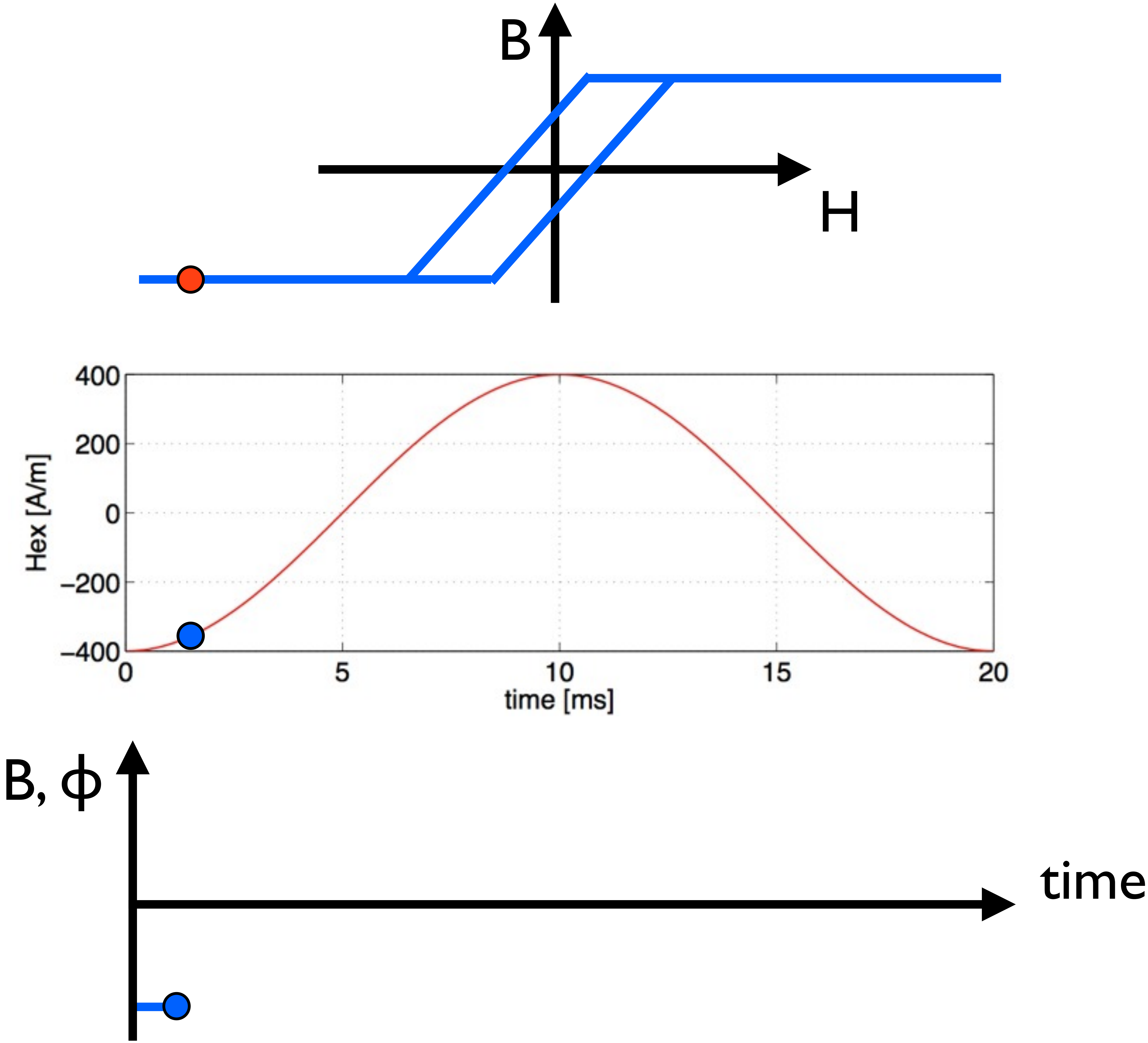





B
H


400
200
0
−200
−400
Hex [A/m]
0
5
10
15
20
time [ms]


B, ϕ
time



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B, ϕ
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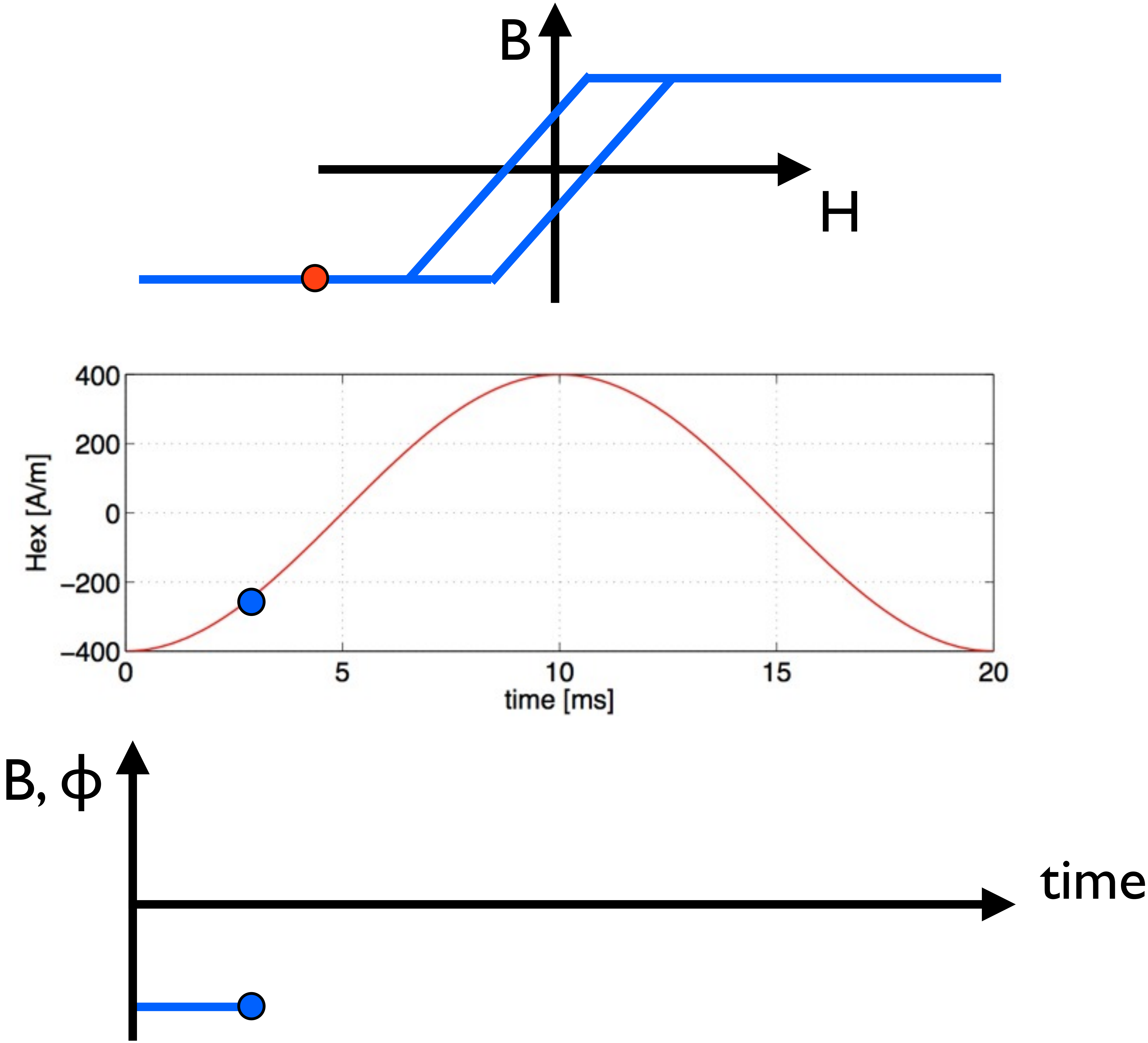
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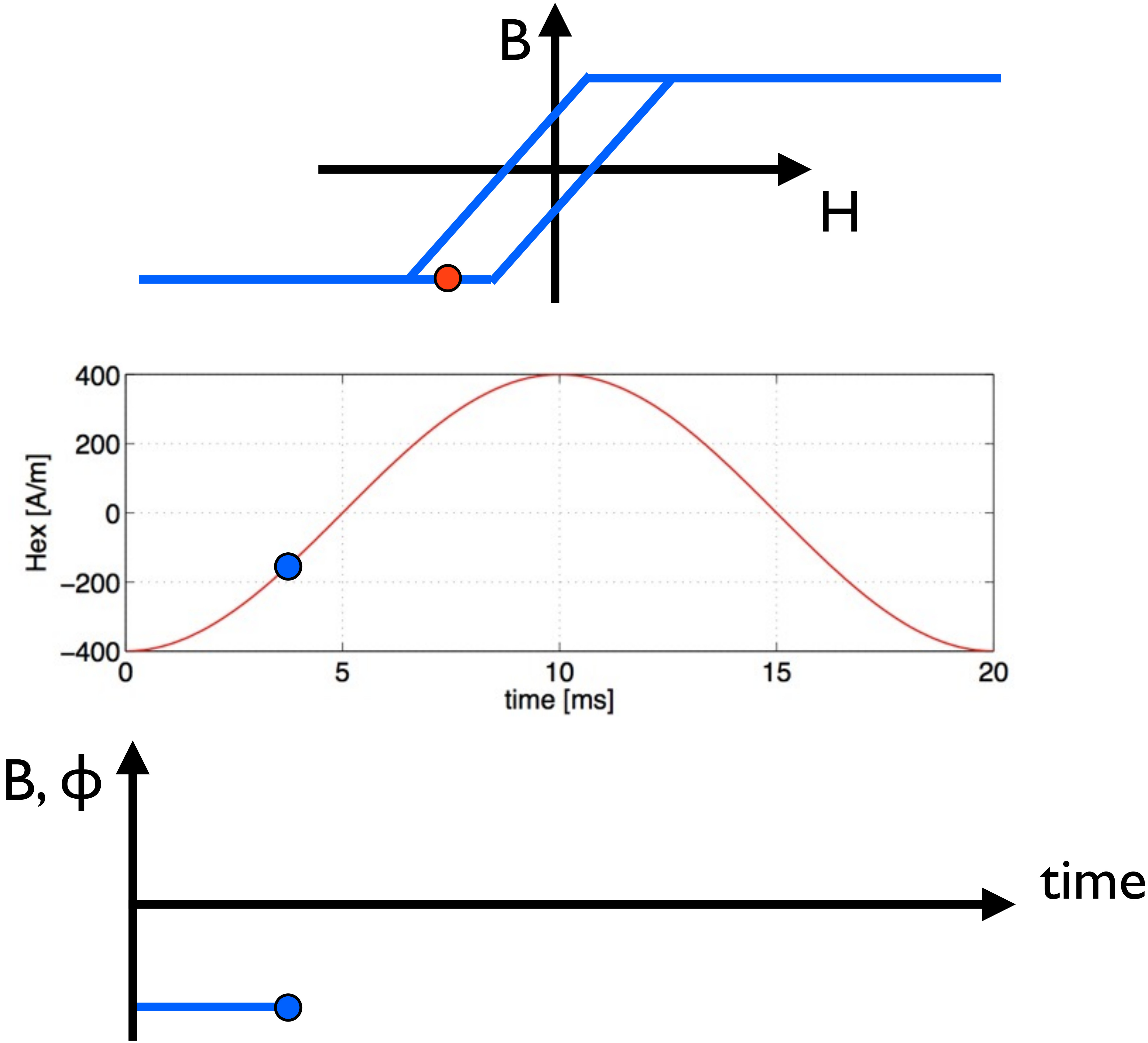

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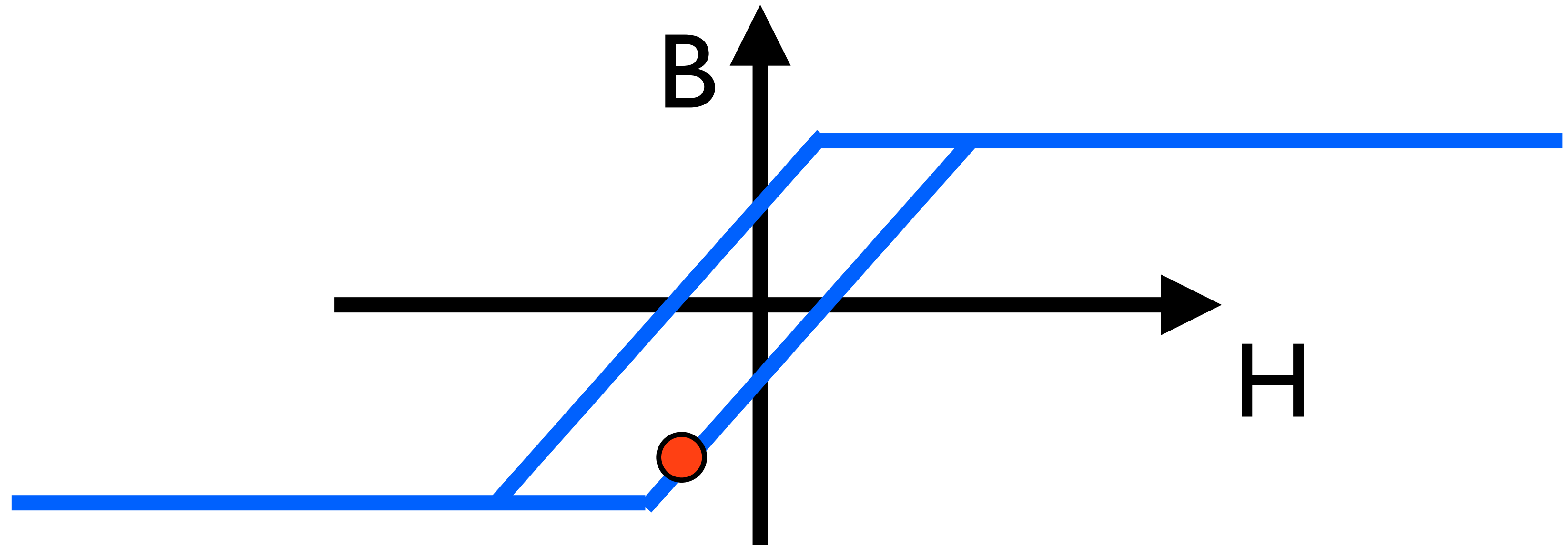
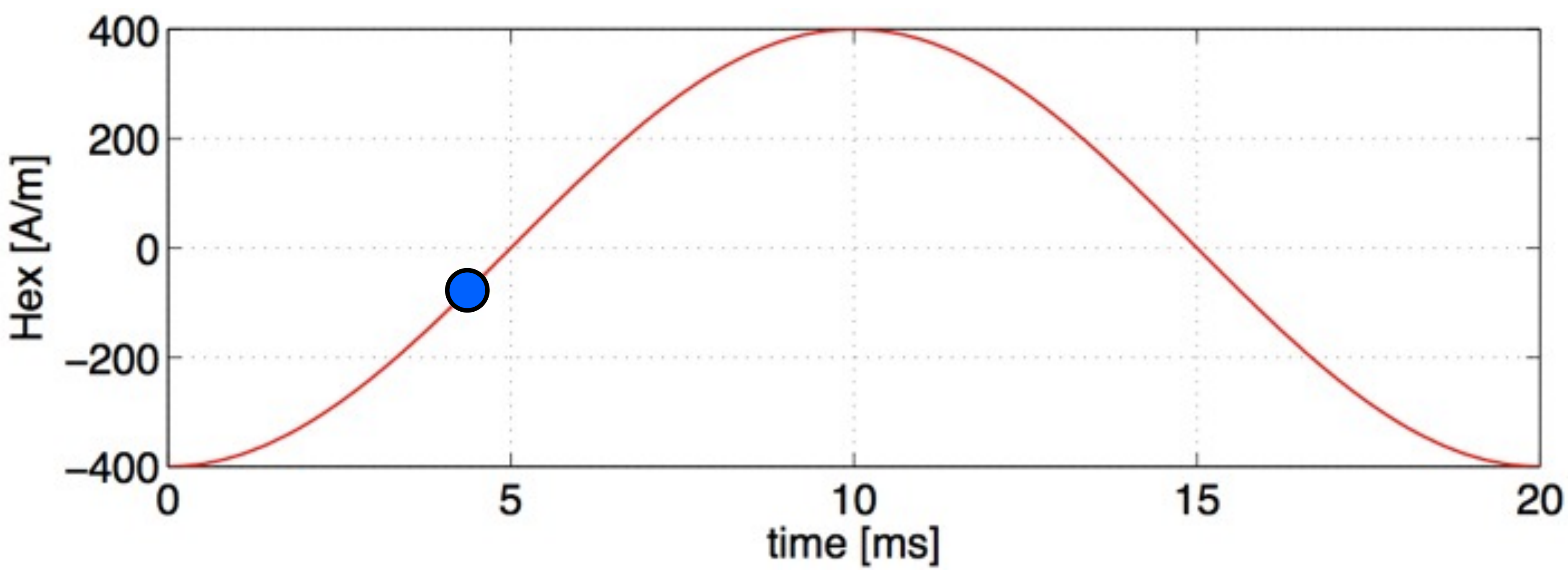
Hex [A/m]
time [ms]
B, ϕ
time


B
H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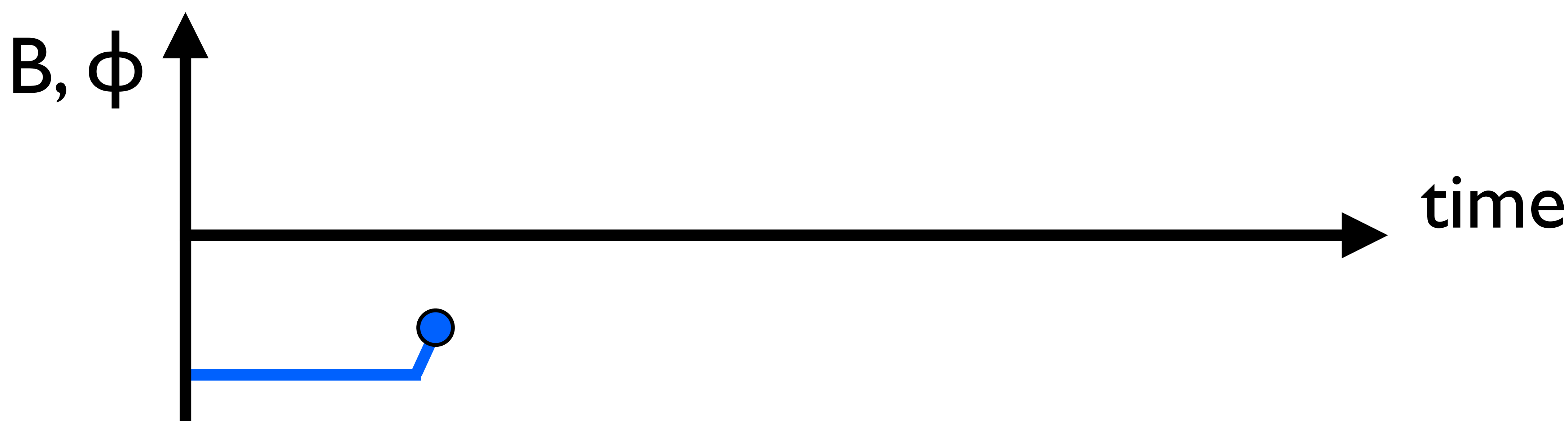
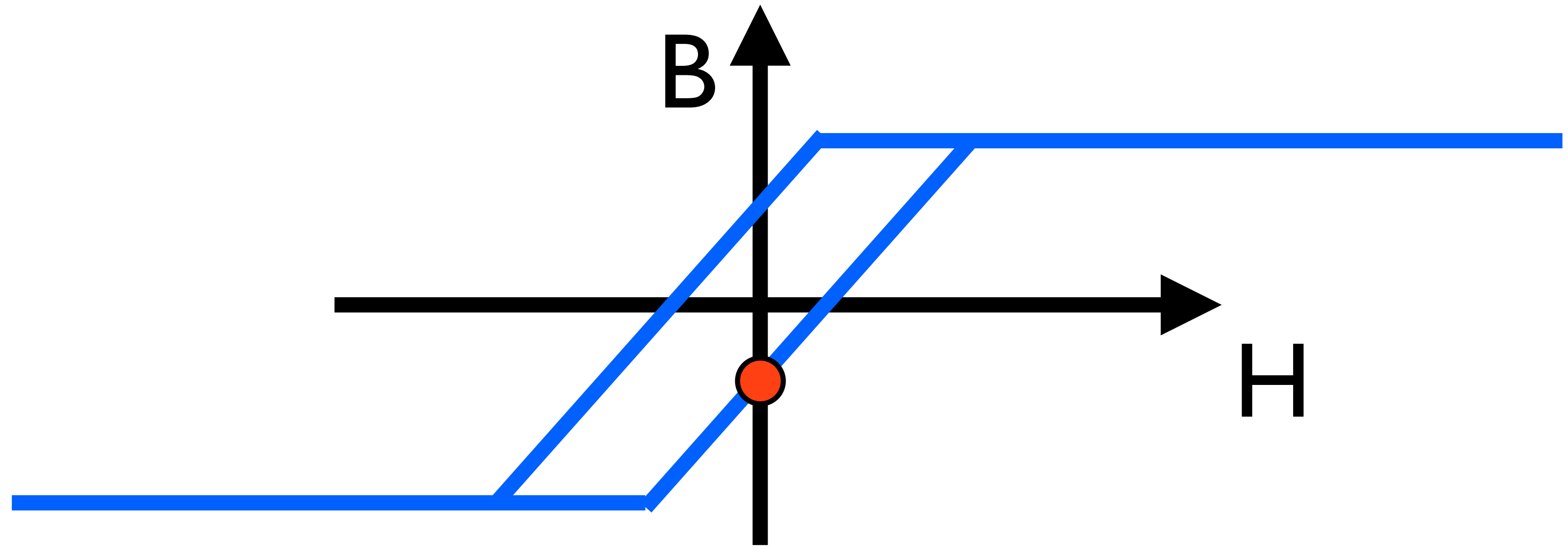
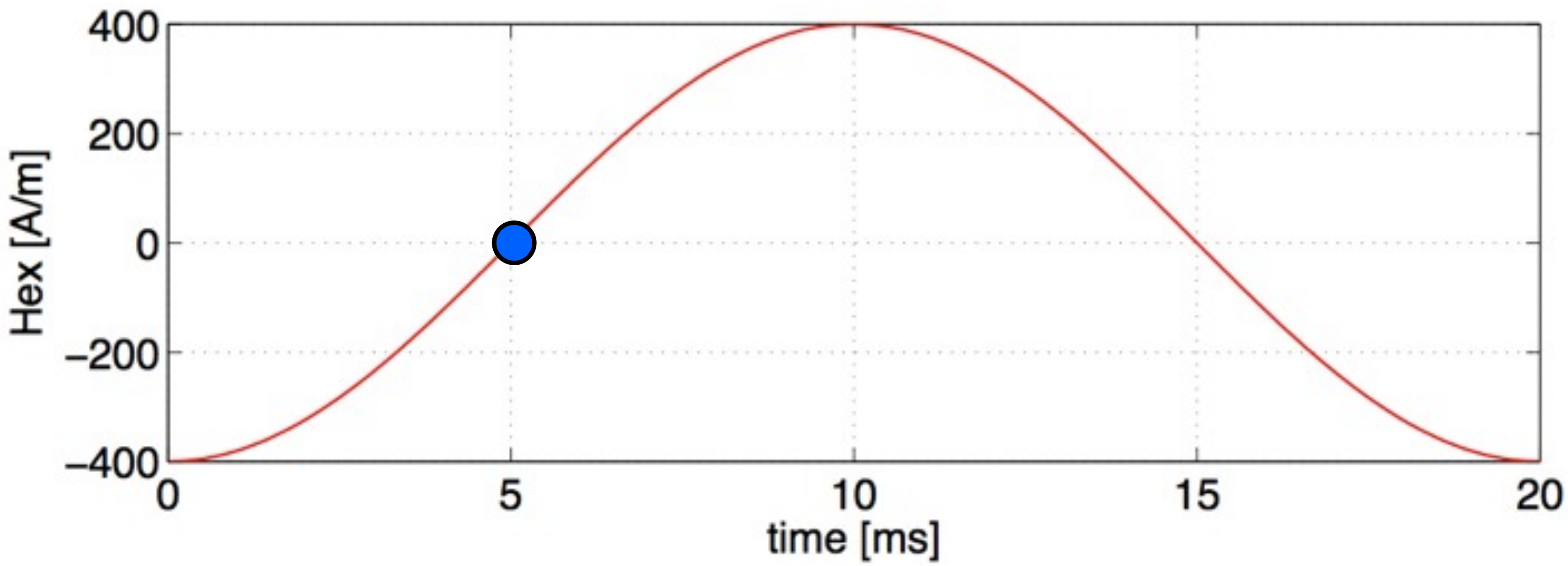
time [ms]


B, ϕ
time


B
H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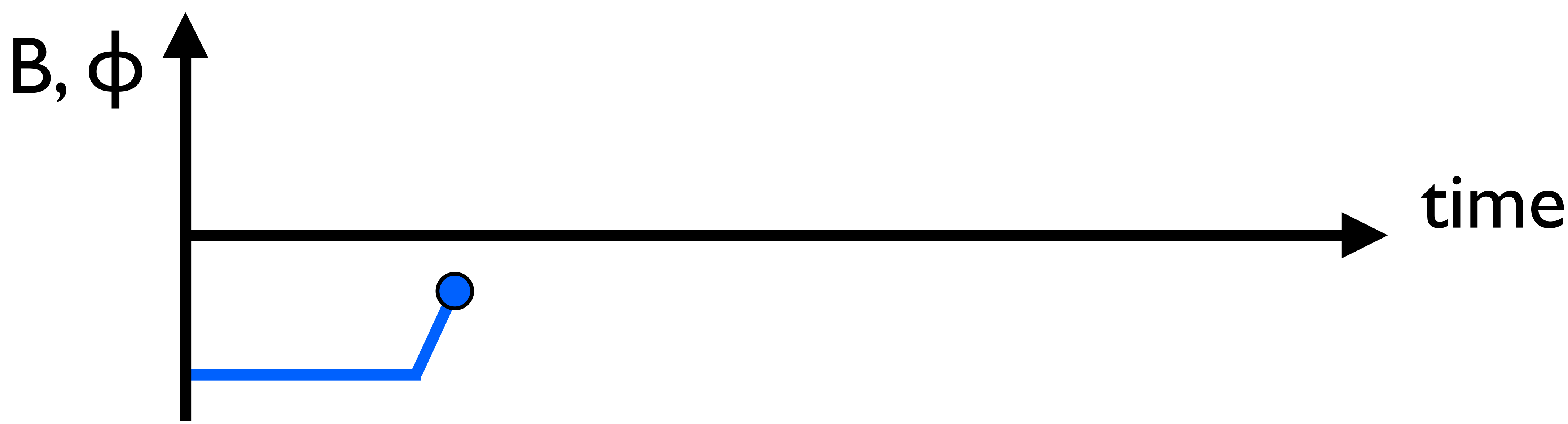
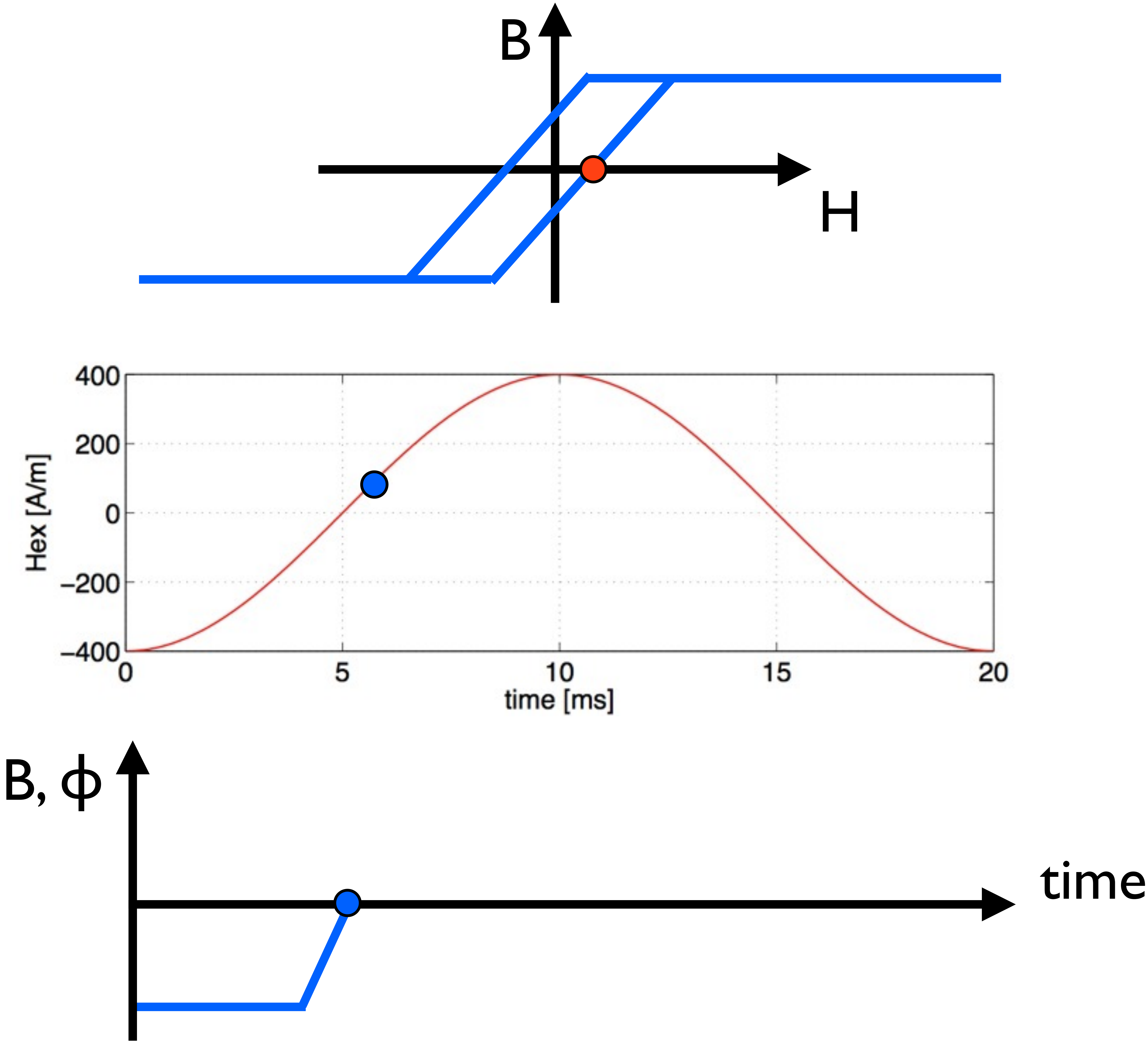

B, ϕ
time


B
H
400
200
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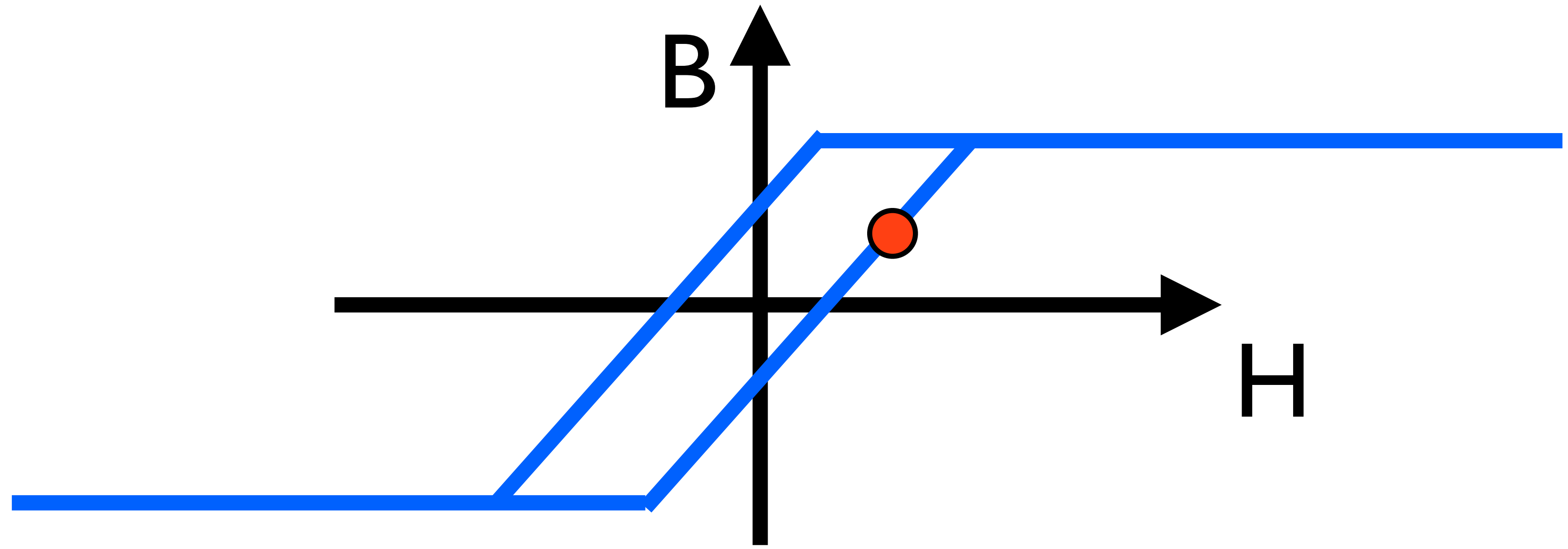
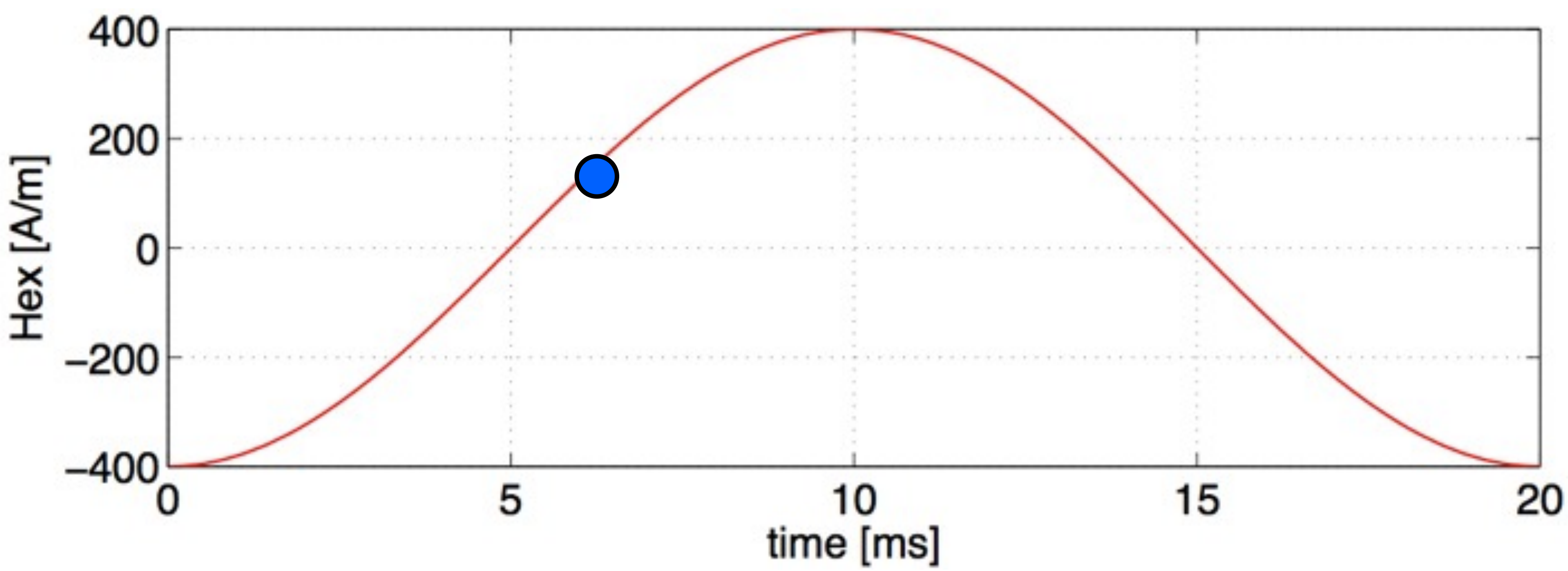
time [ms]
Hex [A/m]
B, ϕ
time


B
H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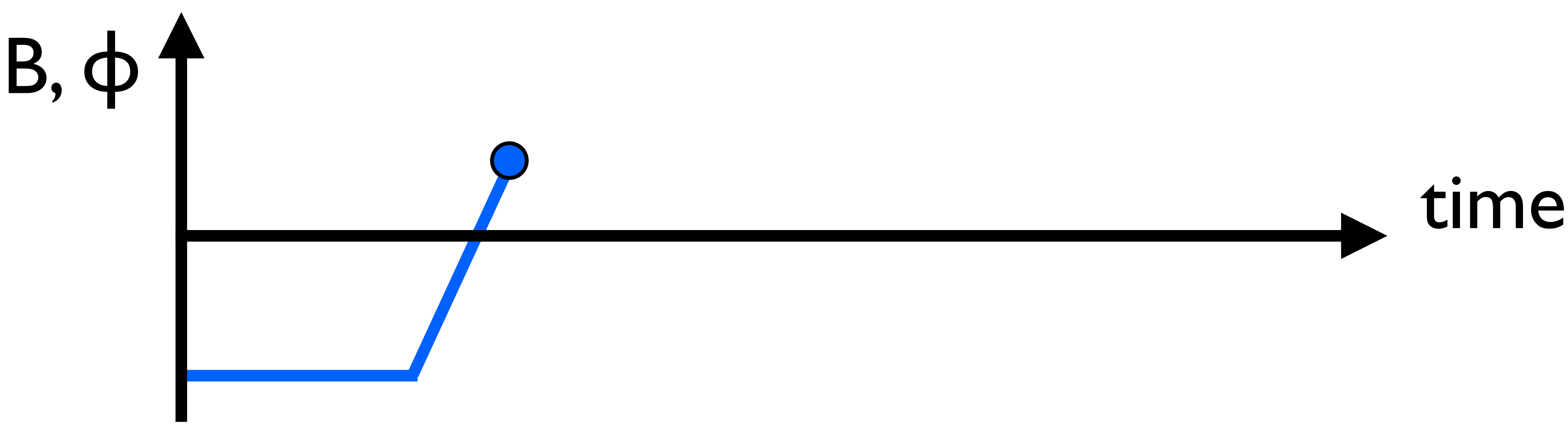
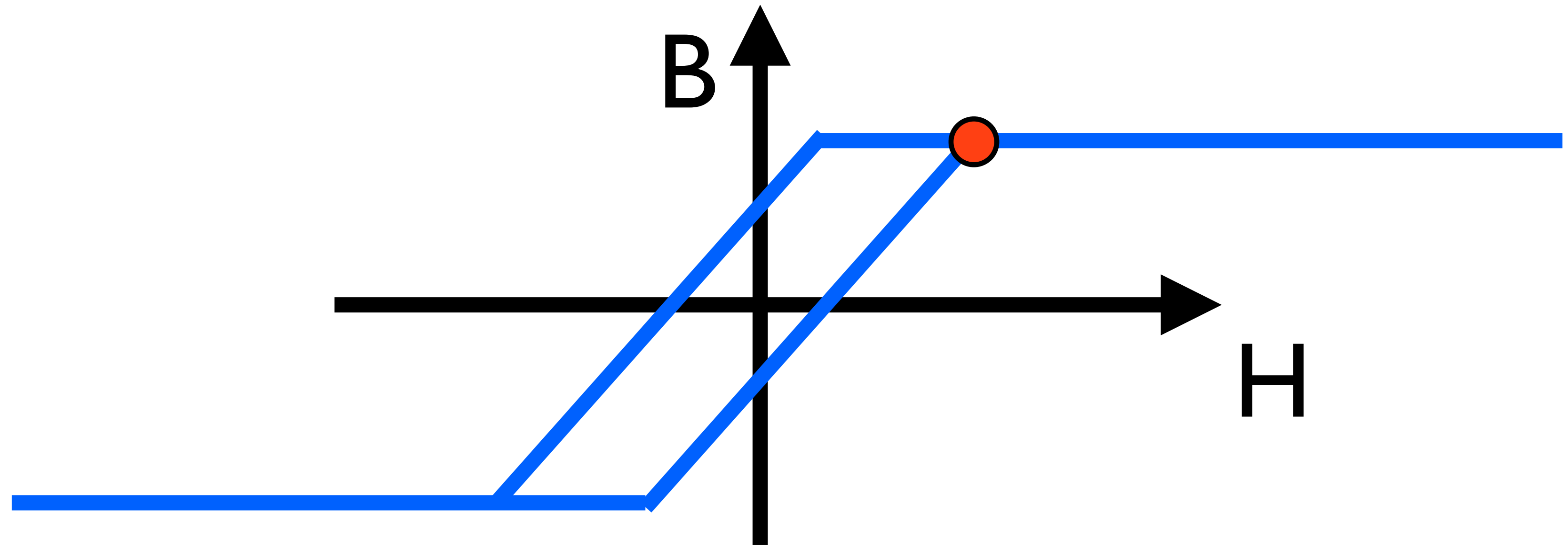
Hex [A/m]
time [ms]


B, ϕ
time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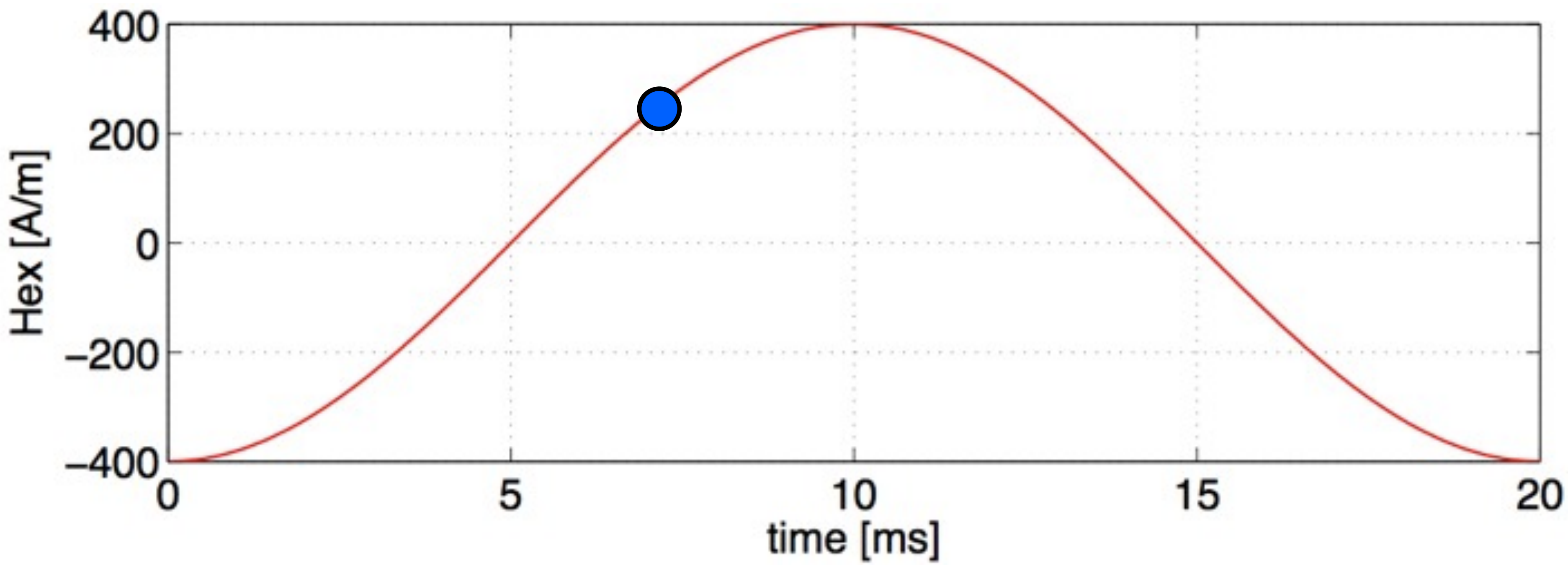
H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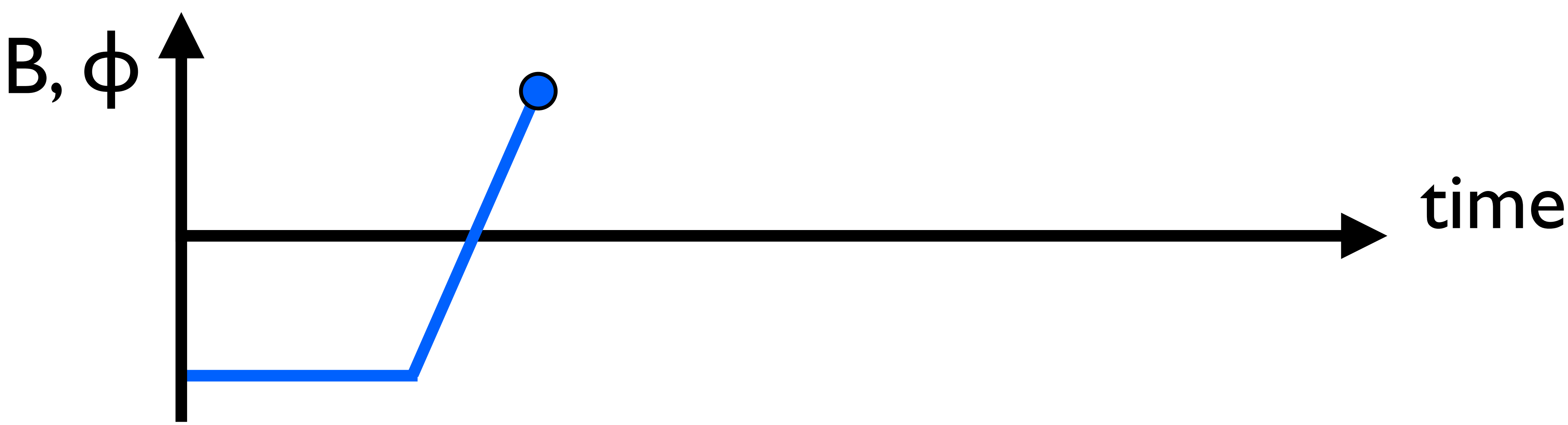
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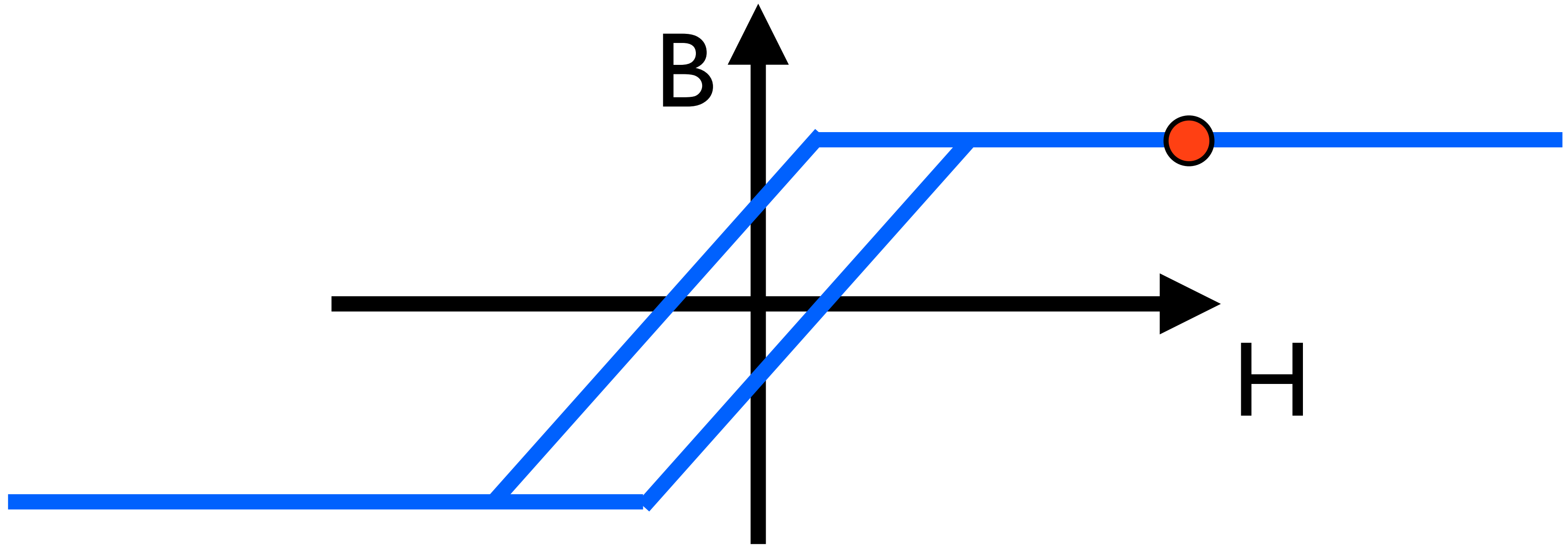
time


B
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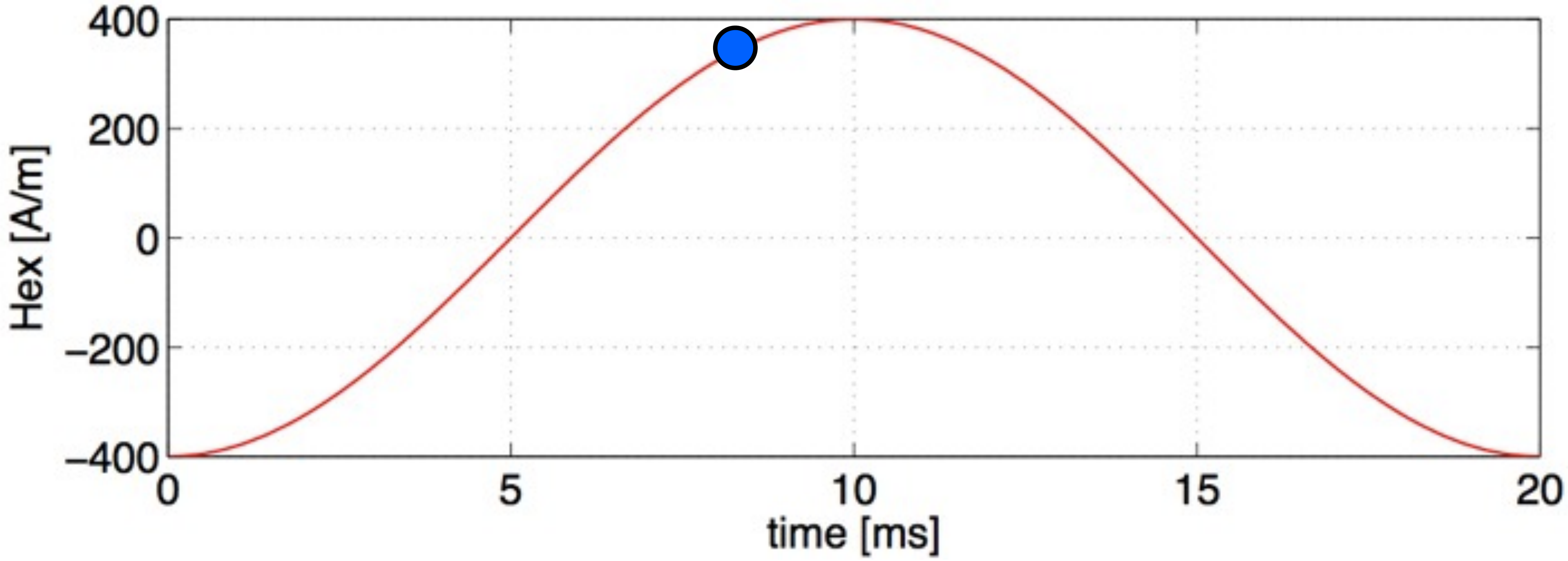
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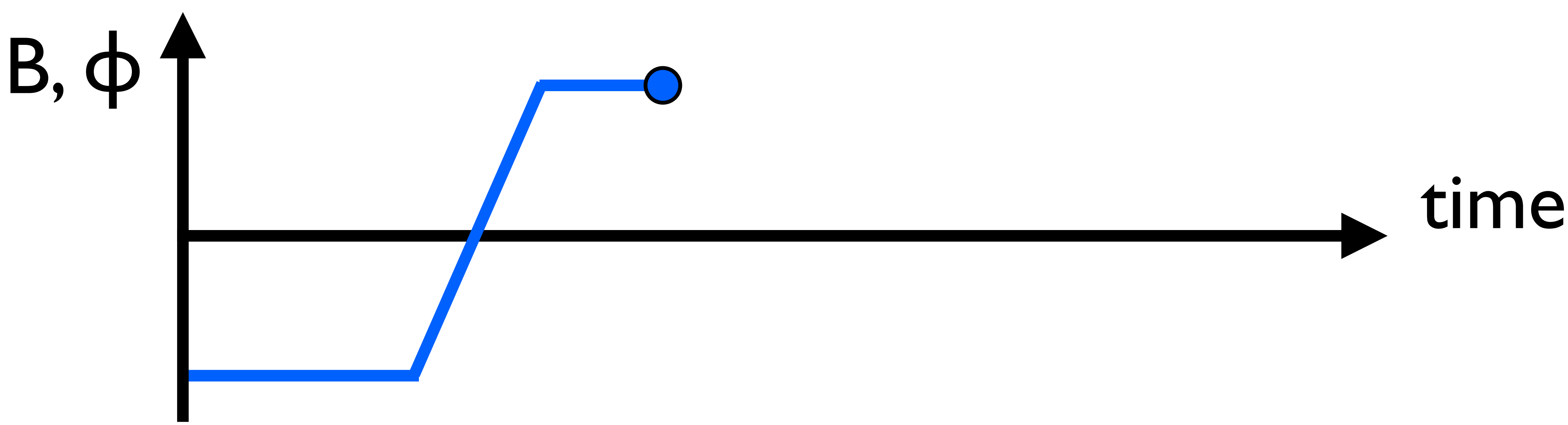
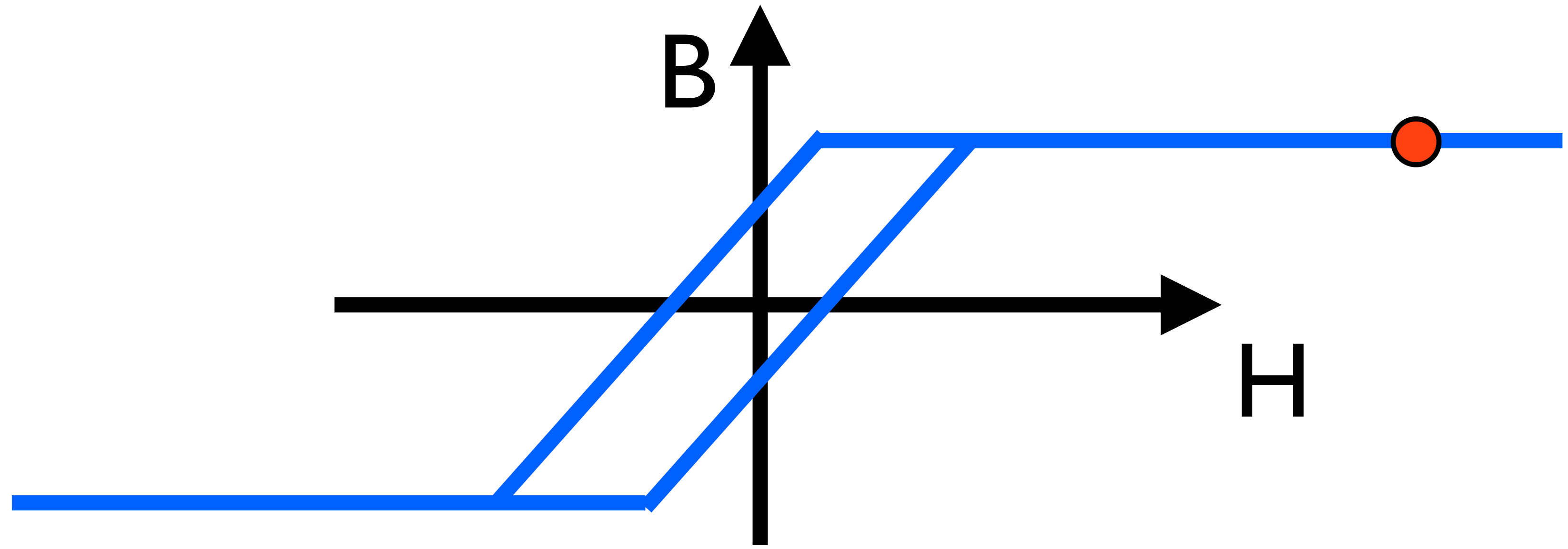

B, ϕ
time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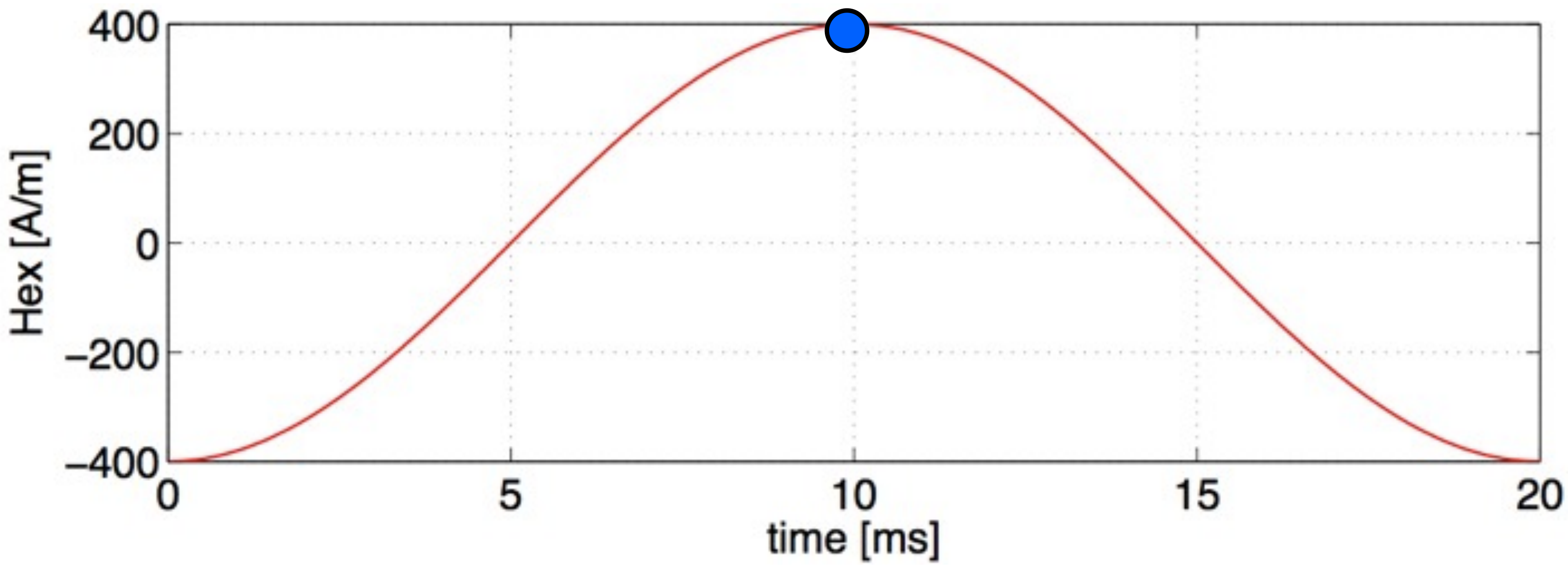
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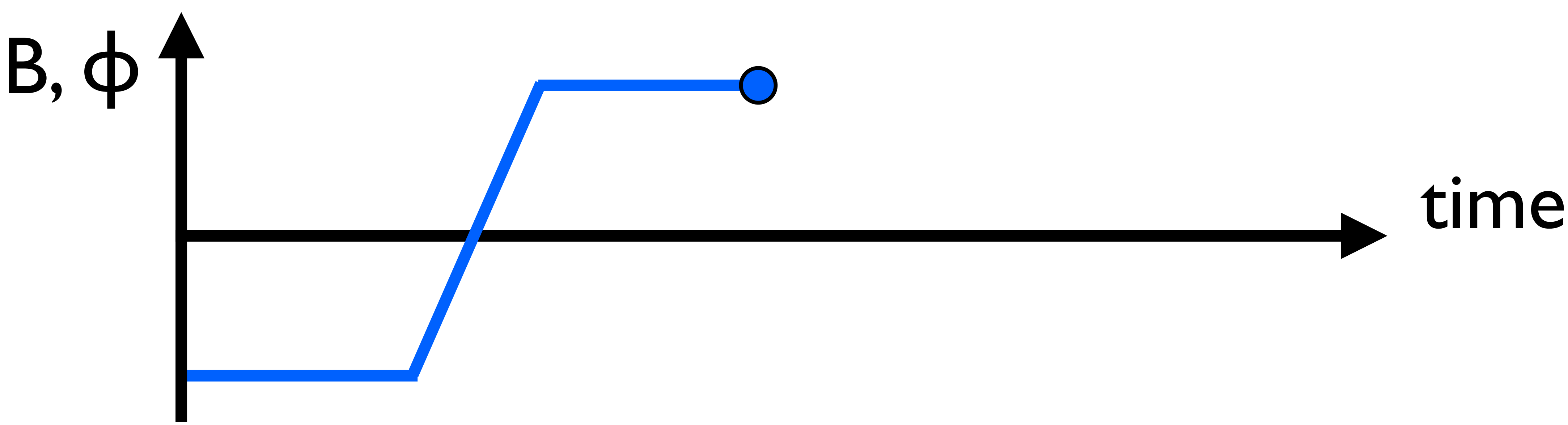
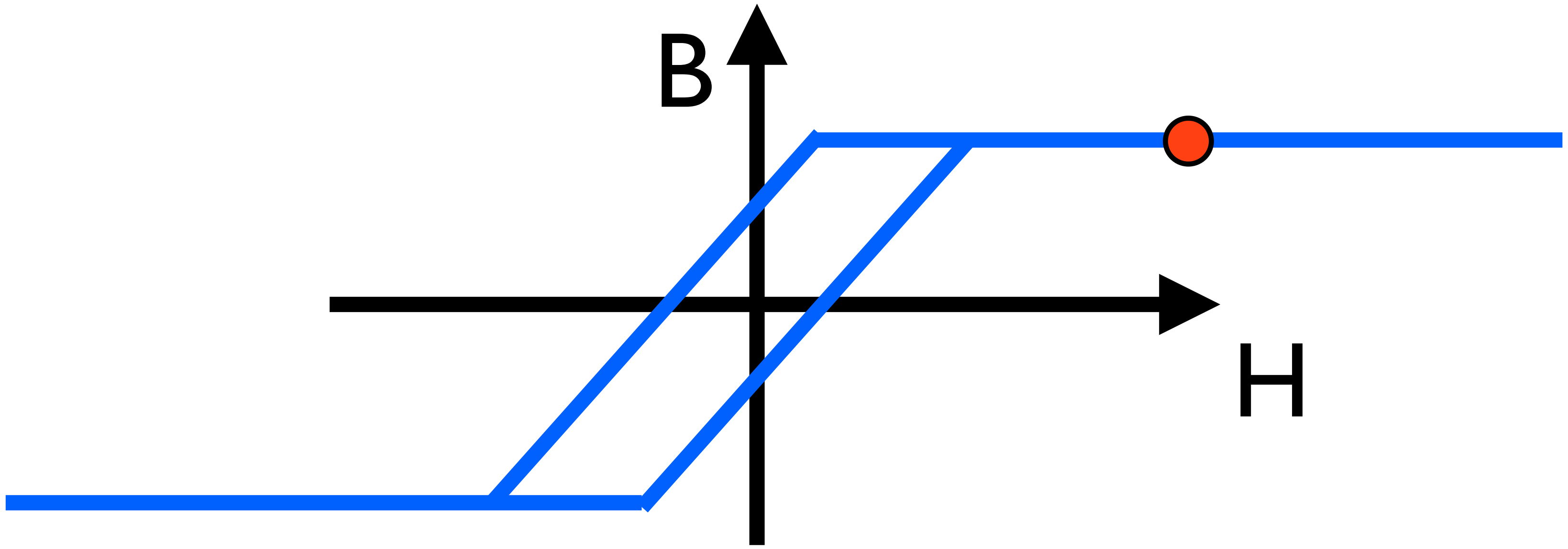

B, ϕ
time


B
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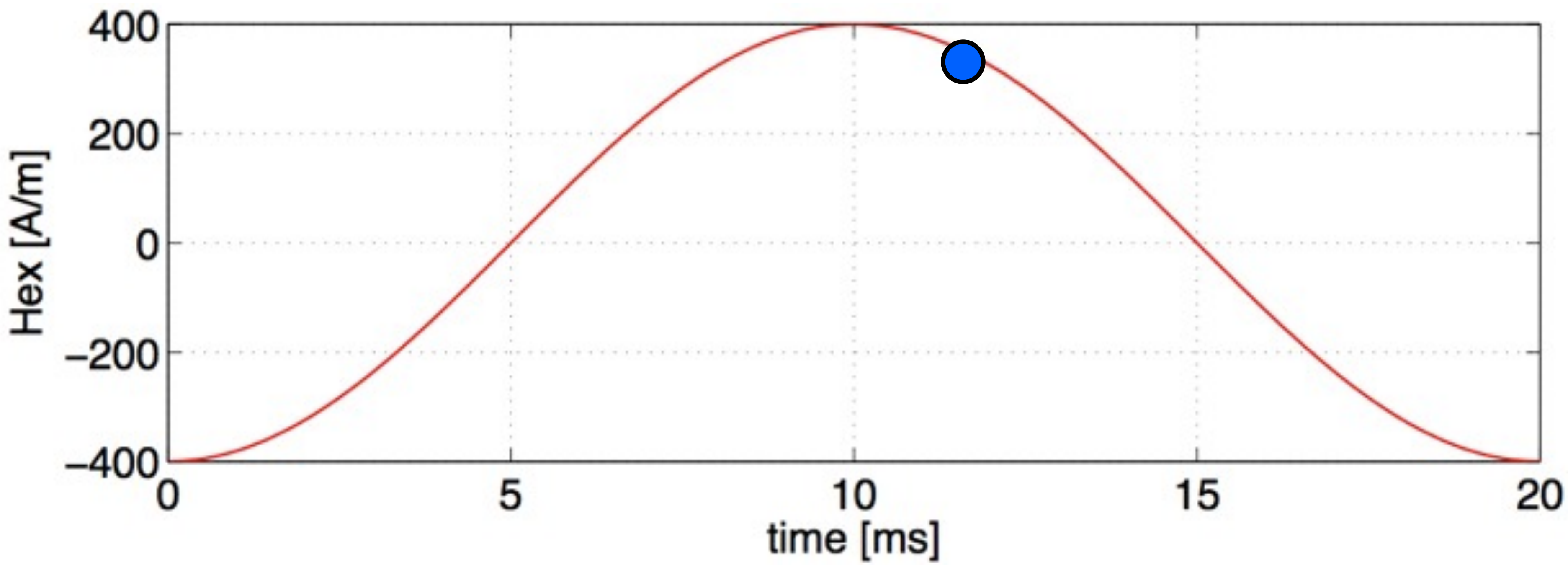
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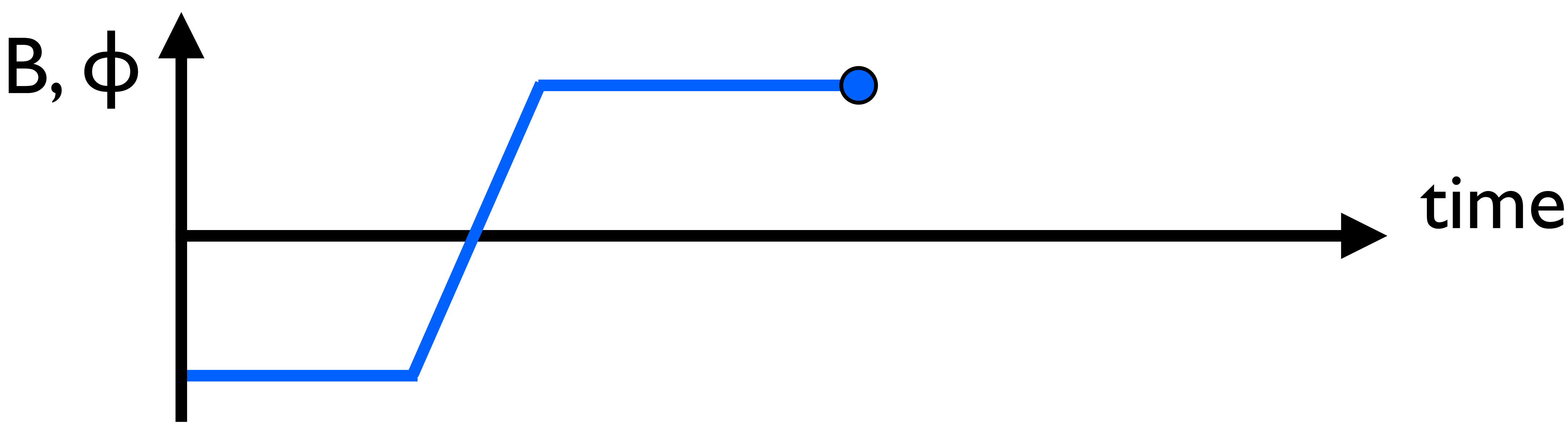
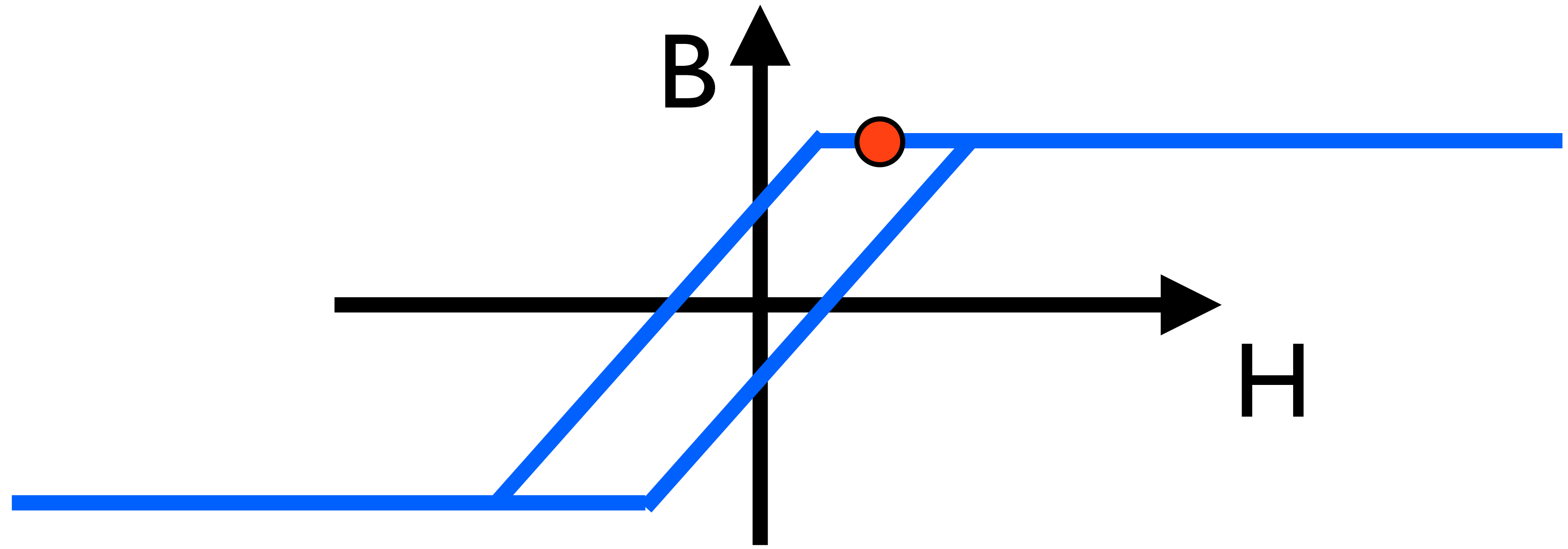

B, ϕ
time


B
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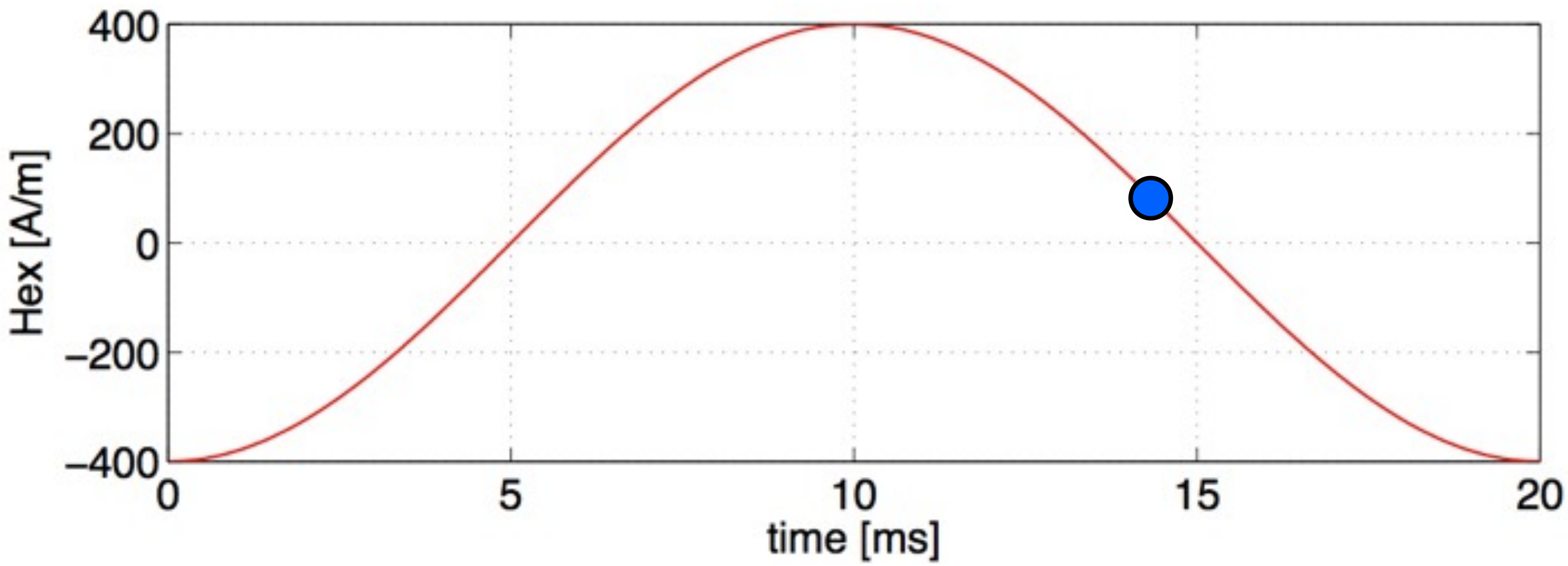
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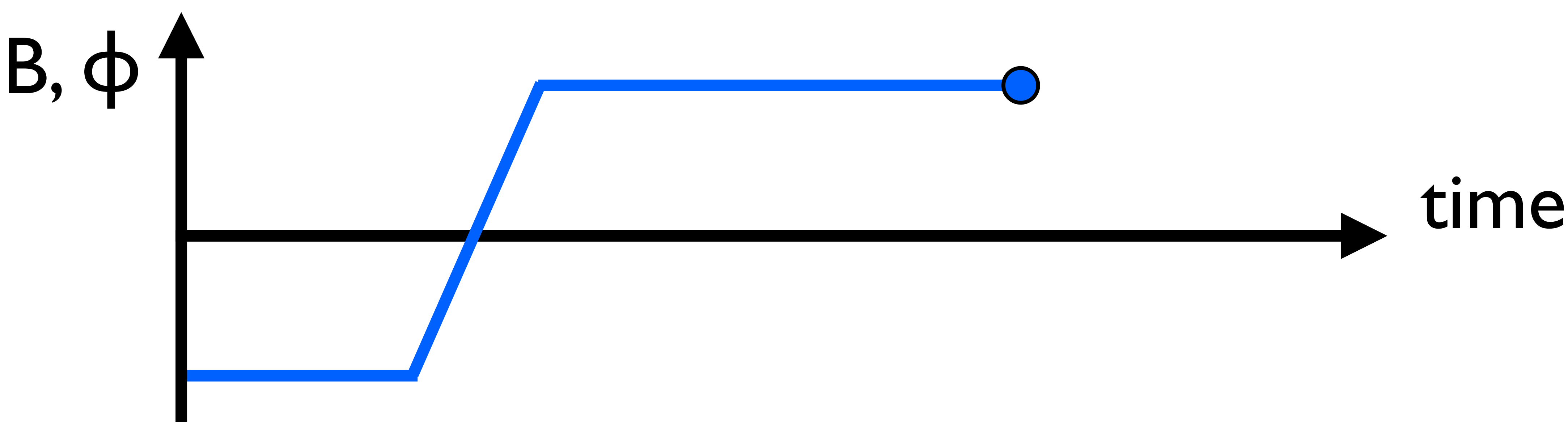
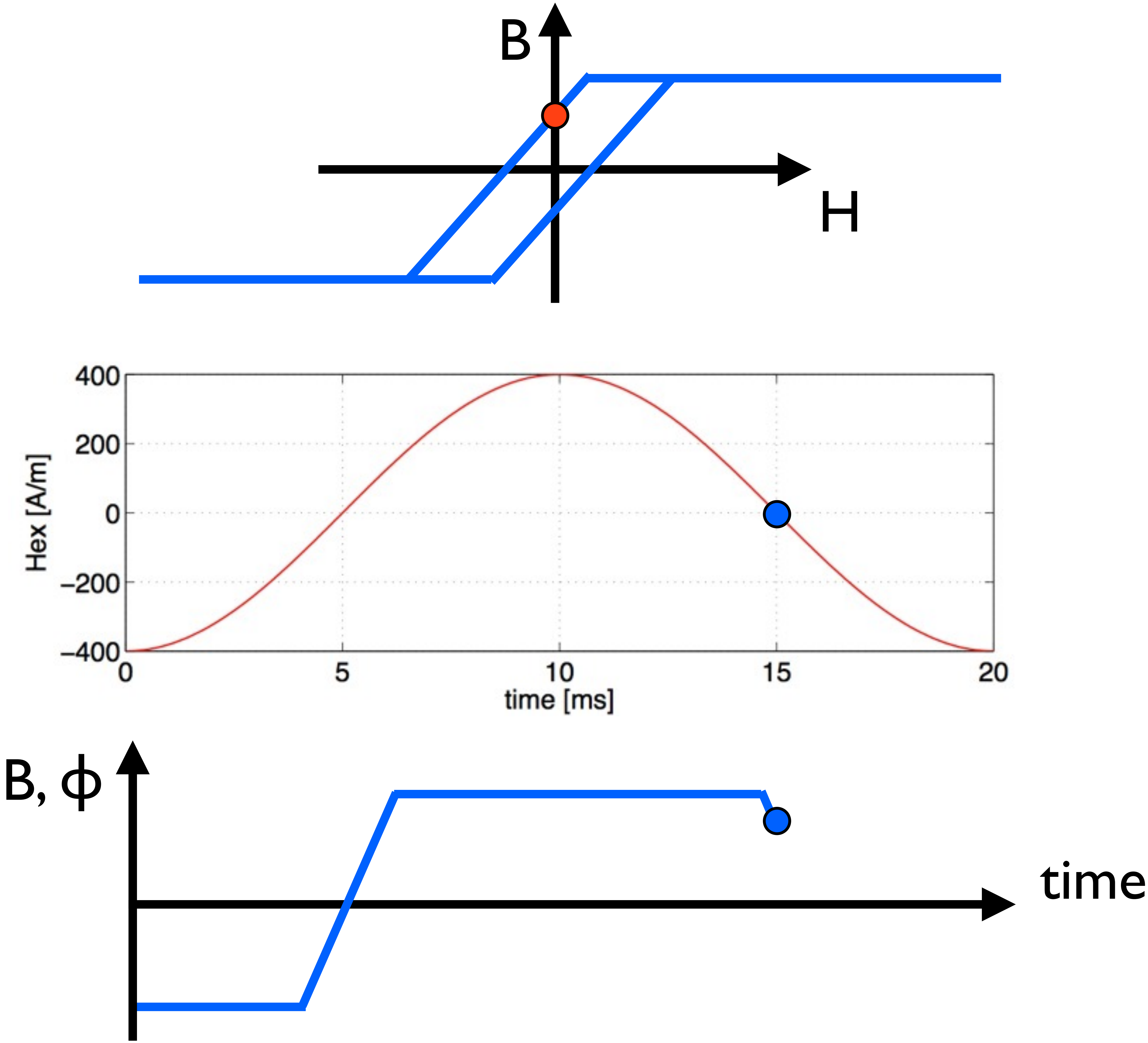

B, ϕ
time


B
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Hex [A/m]
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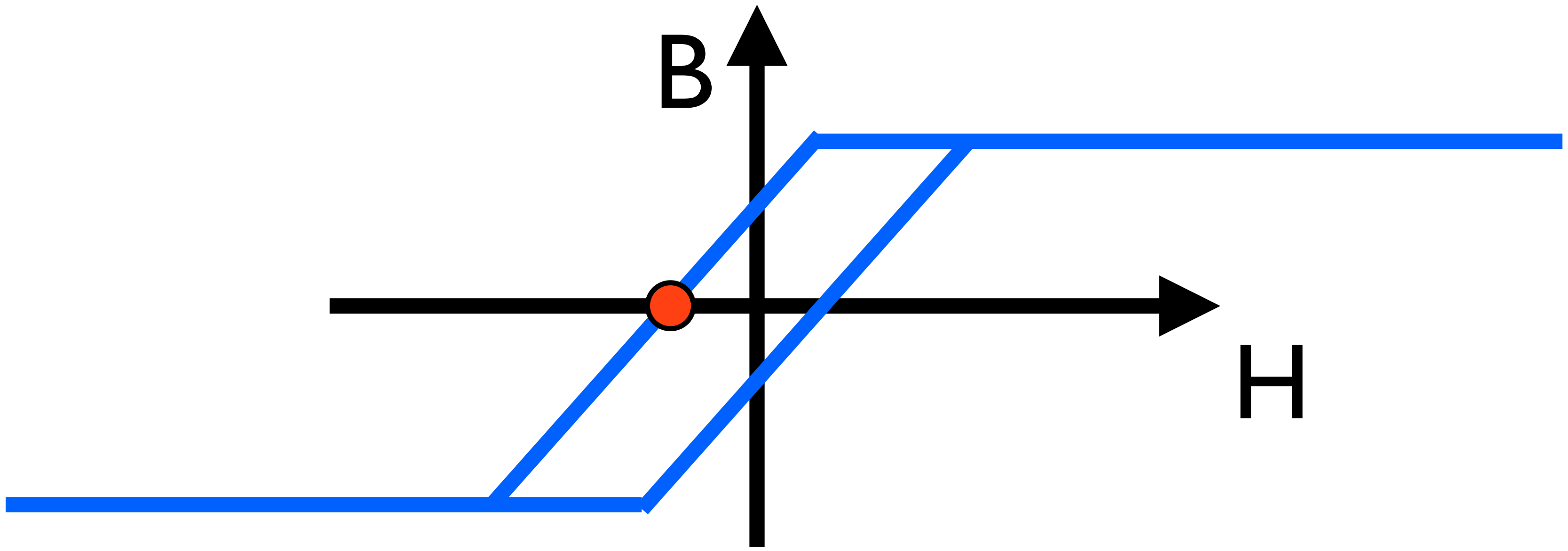
time [ms]
B, ϕ
time


B
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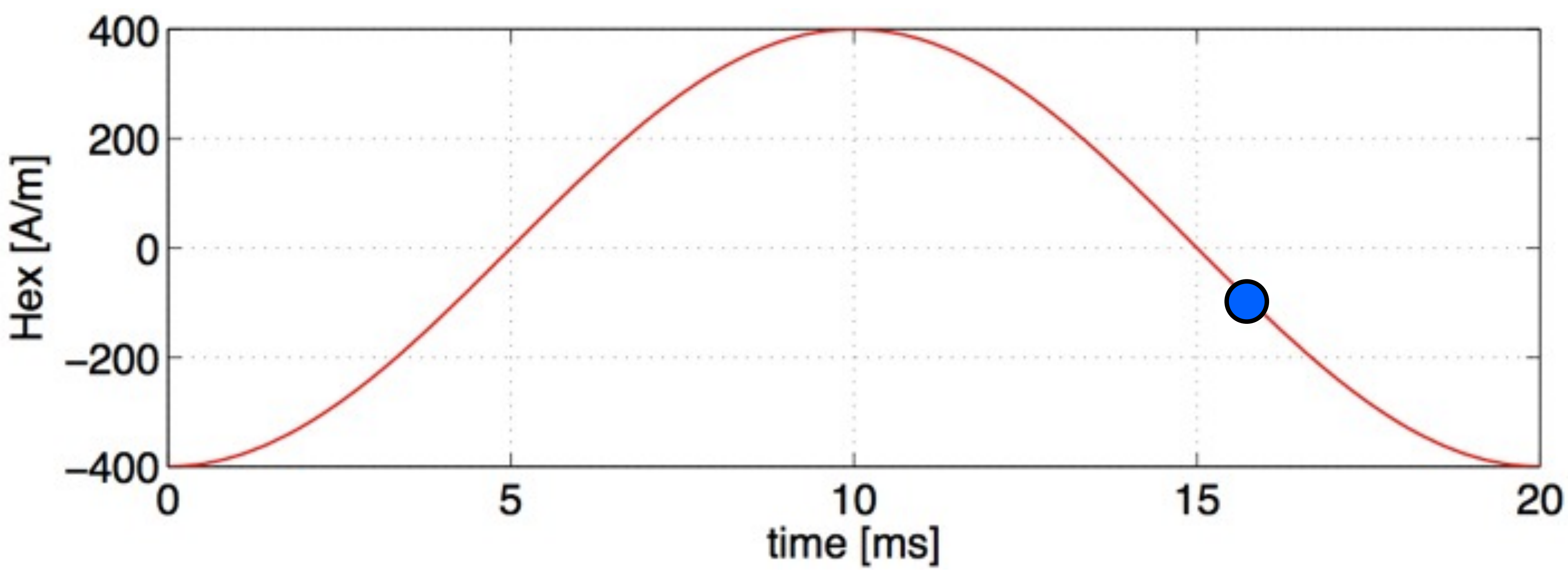
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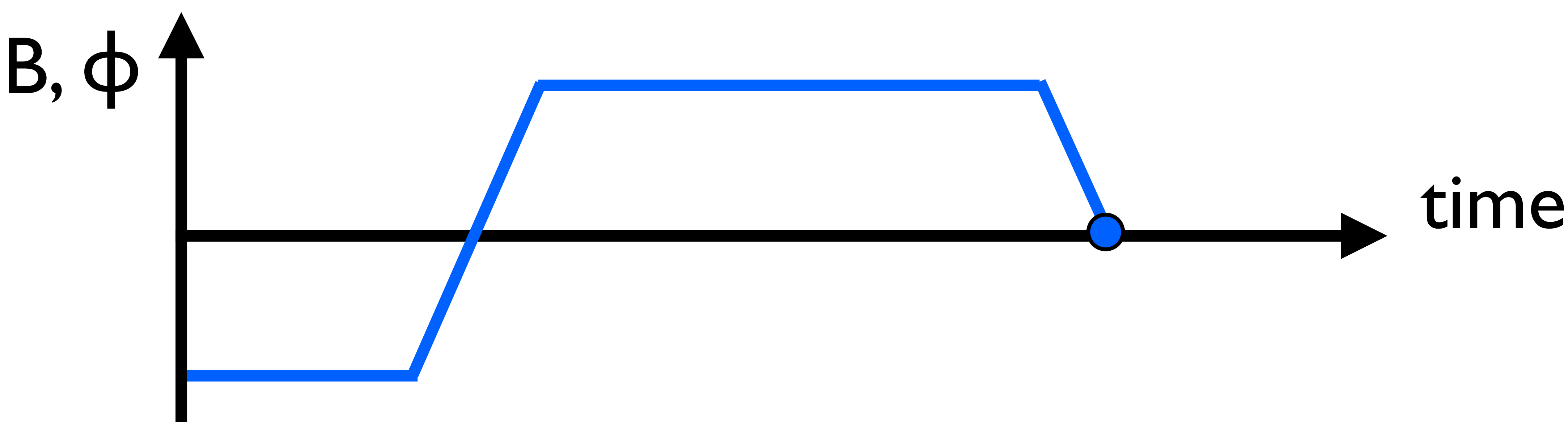
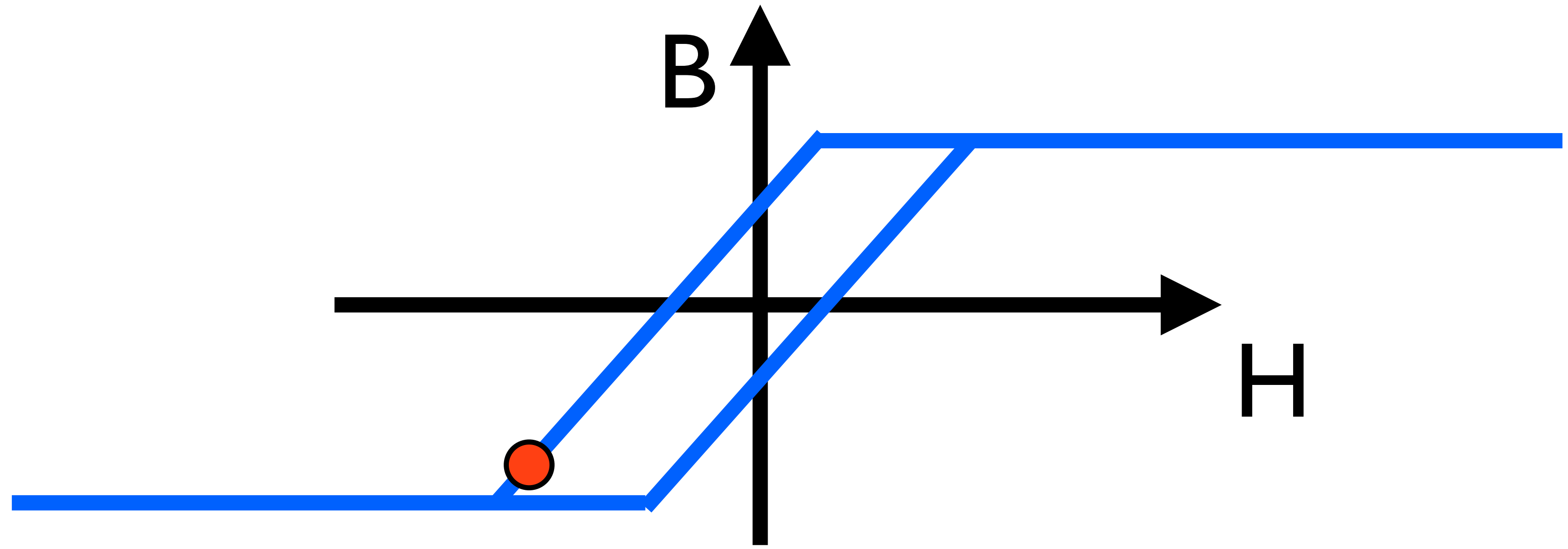
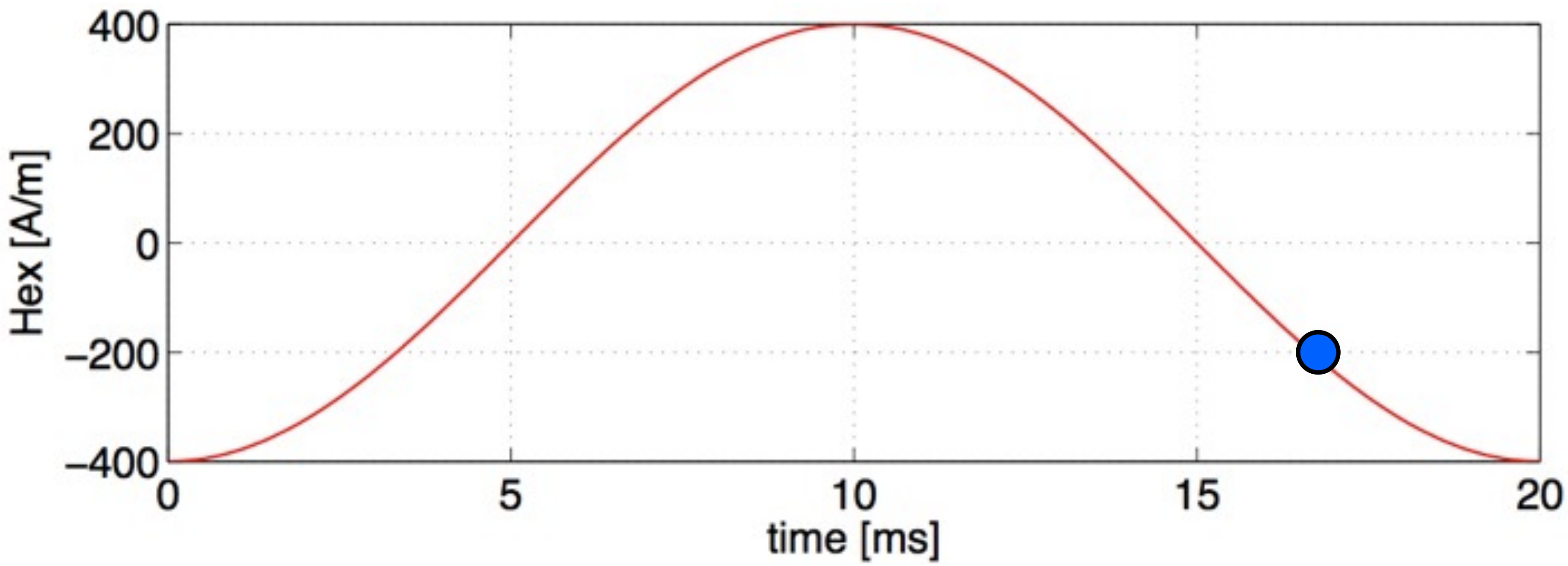

B, ϕ
time



B
H



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Hex [A/m]
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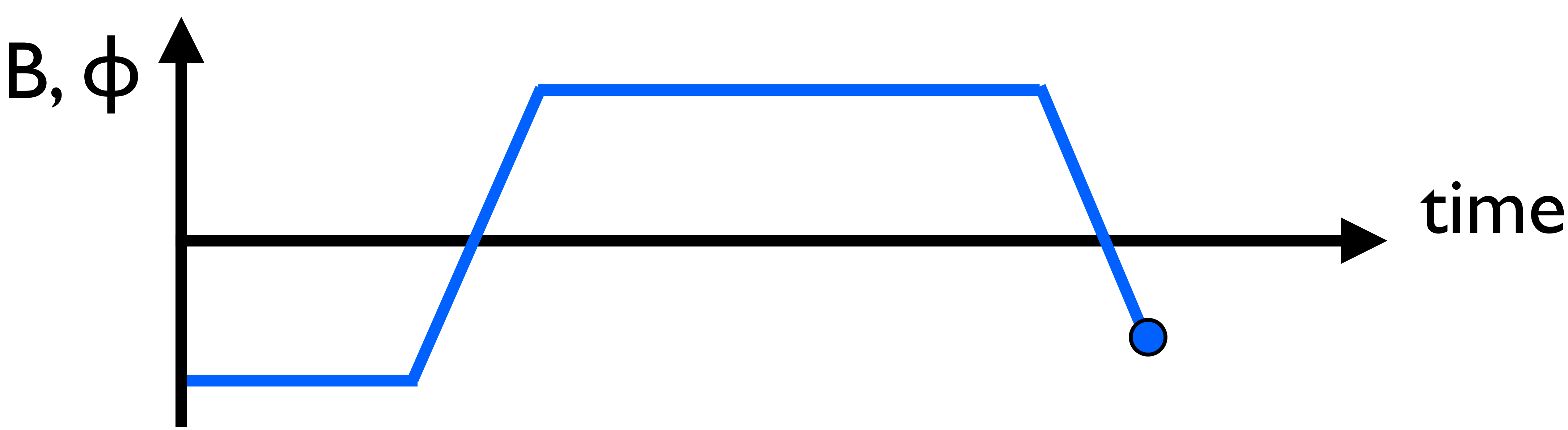
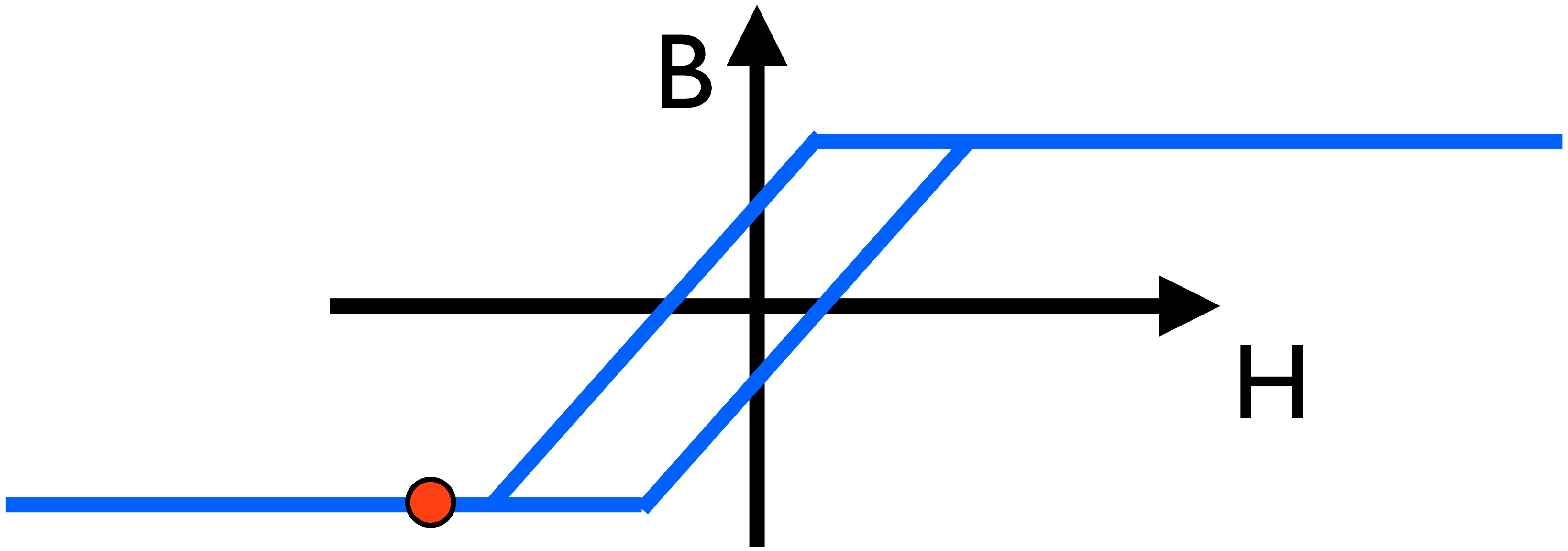
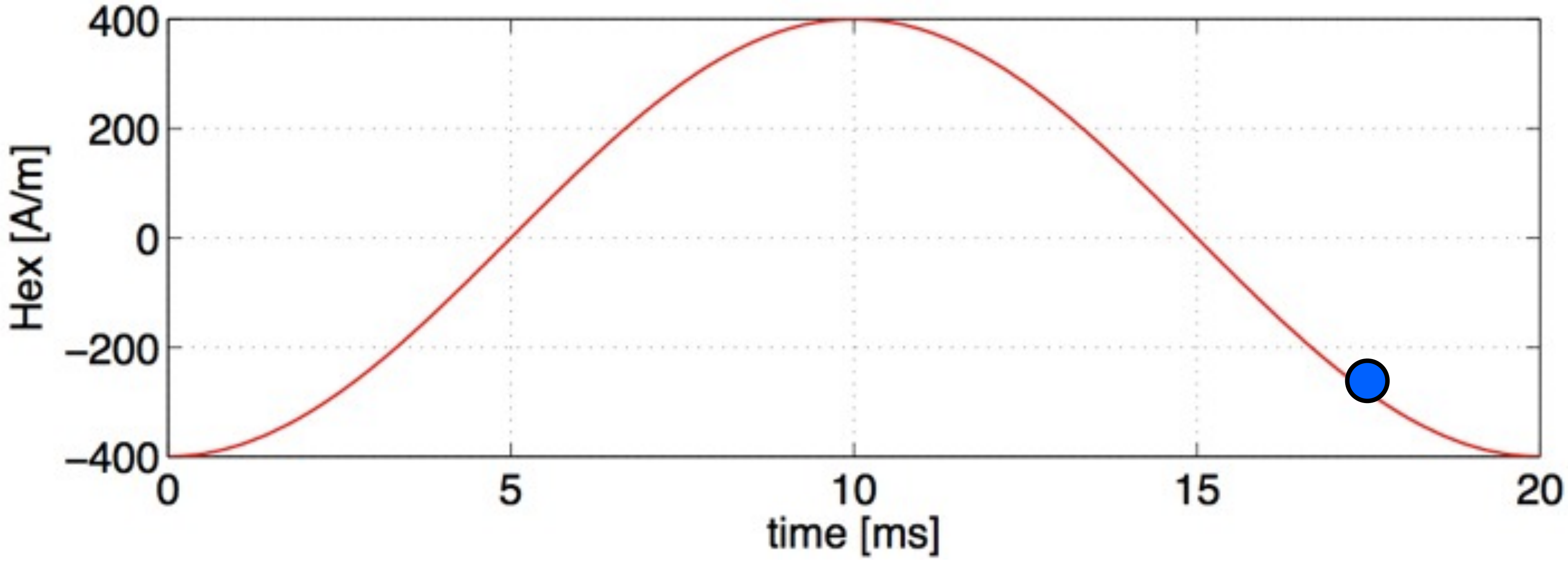


B, ϕ
time


B
H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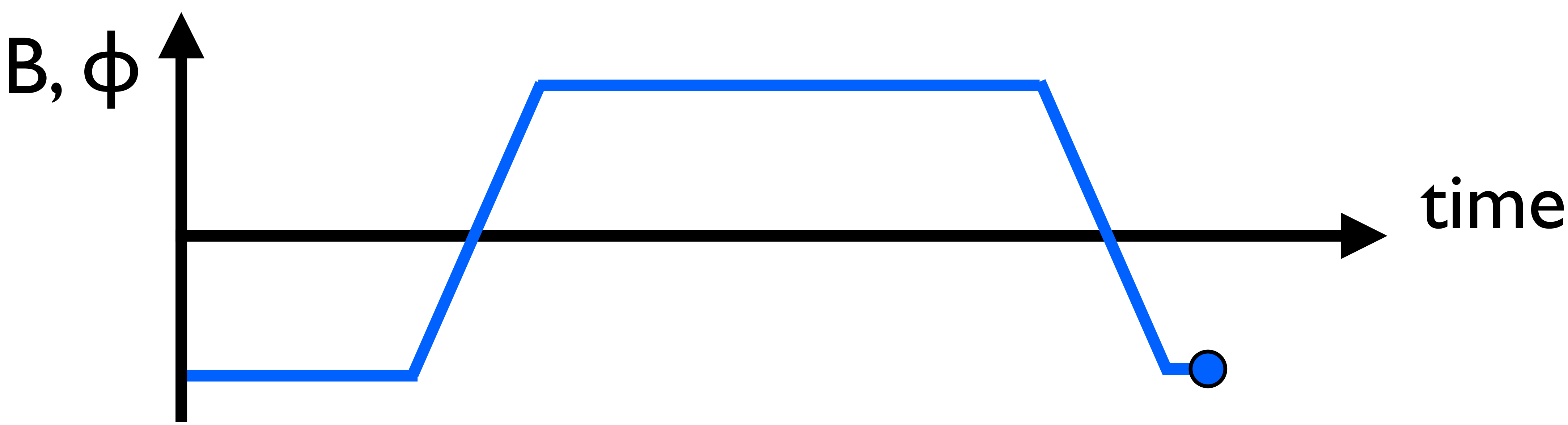
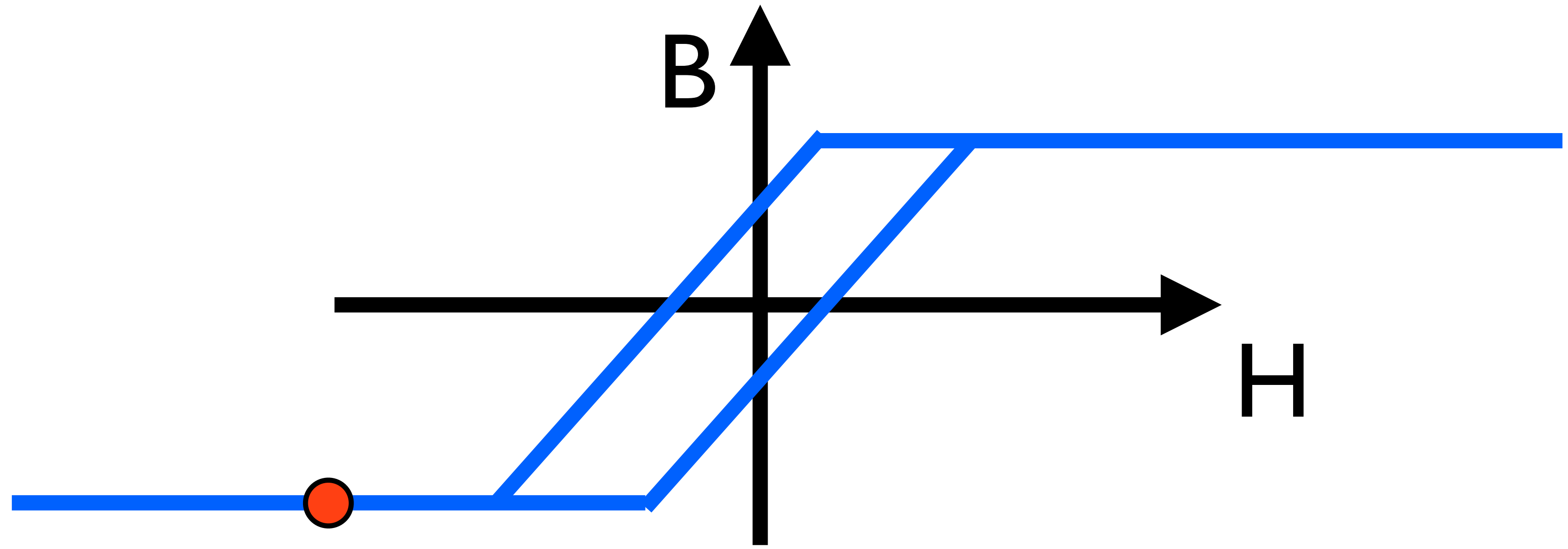
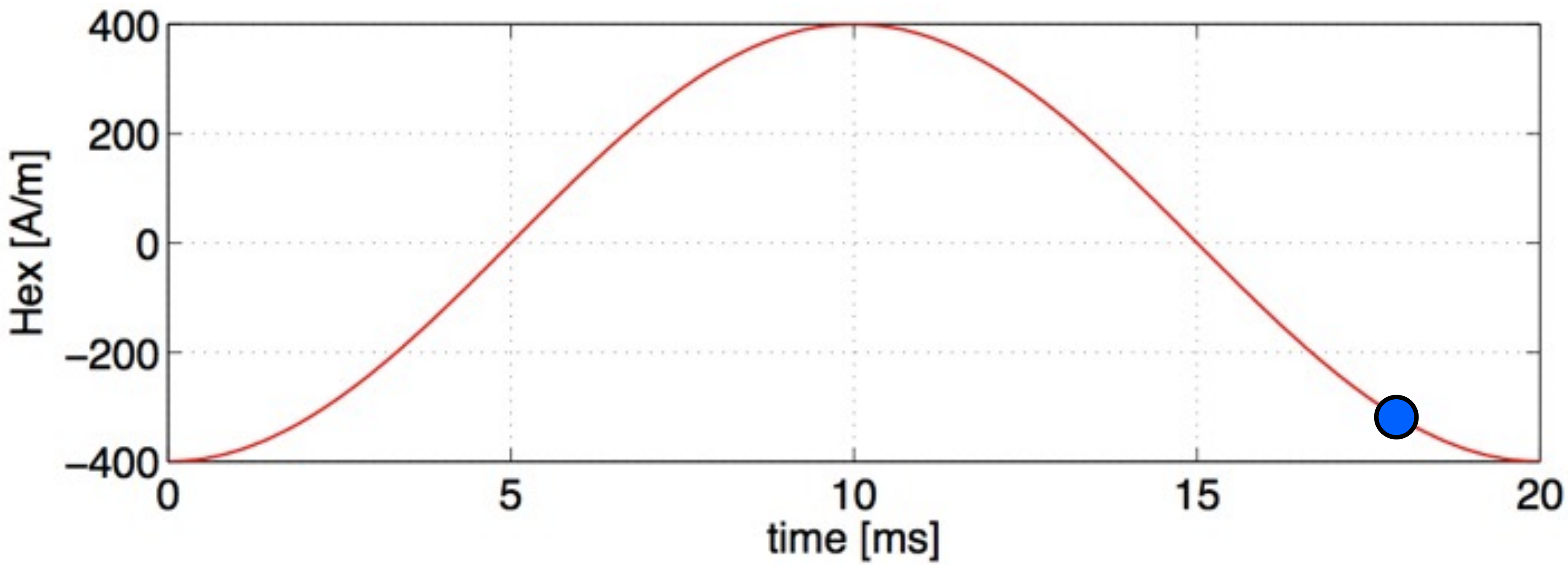
time [ms]


B, ϕ
time


B
H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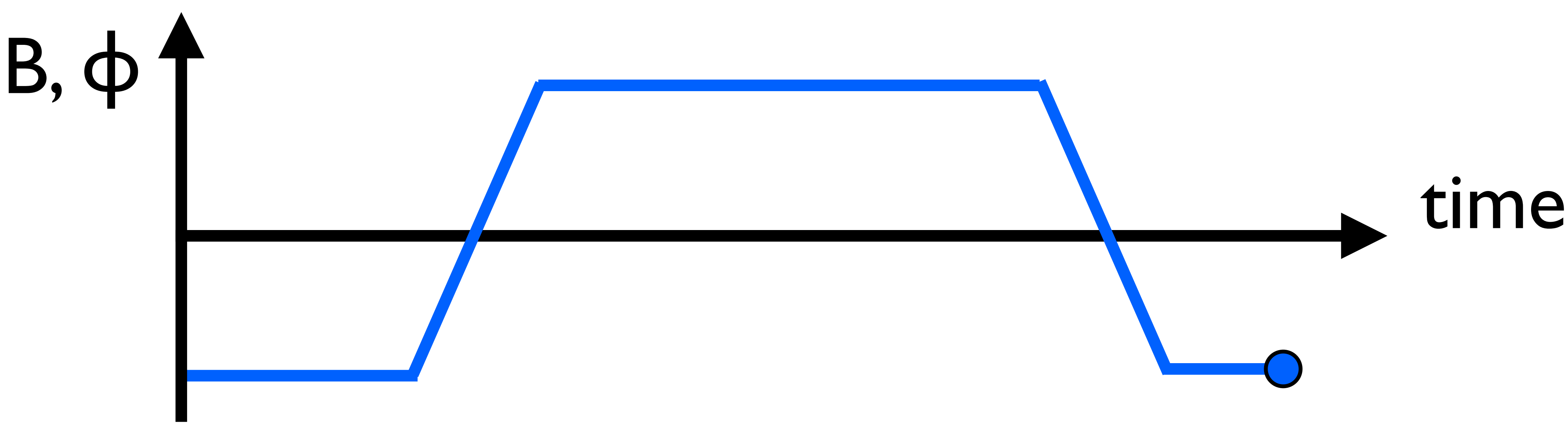
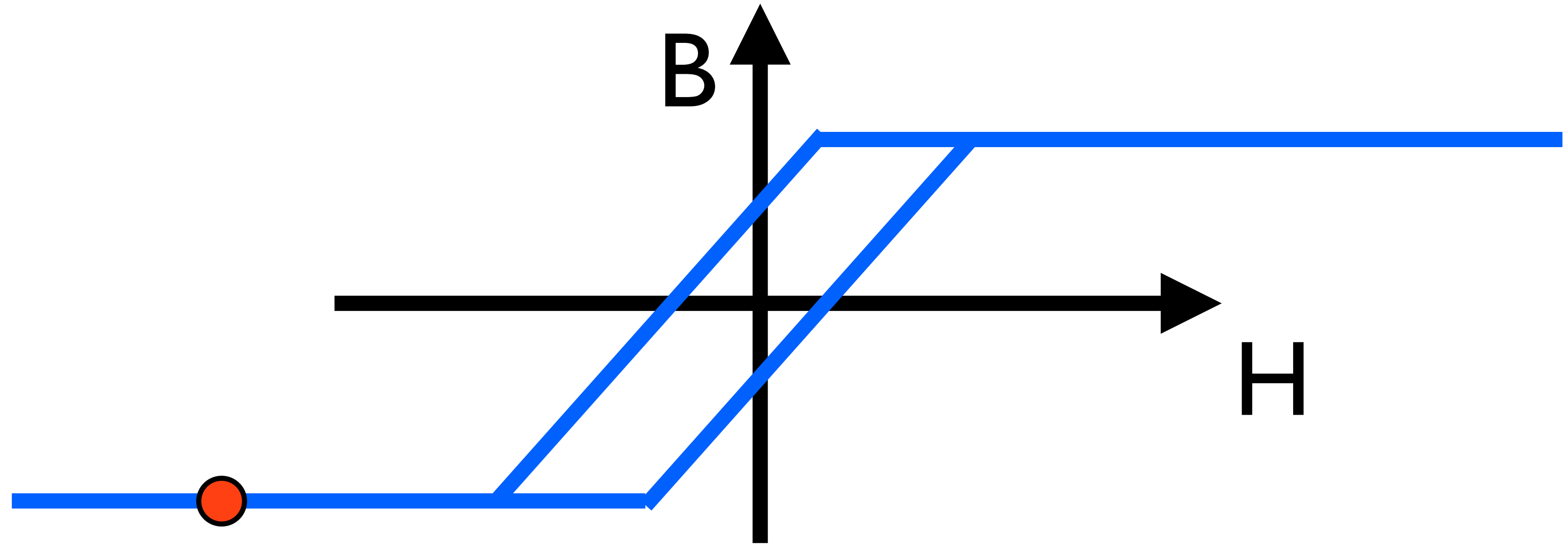
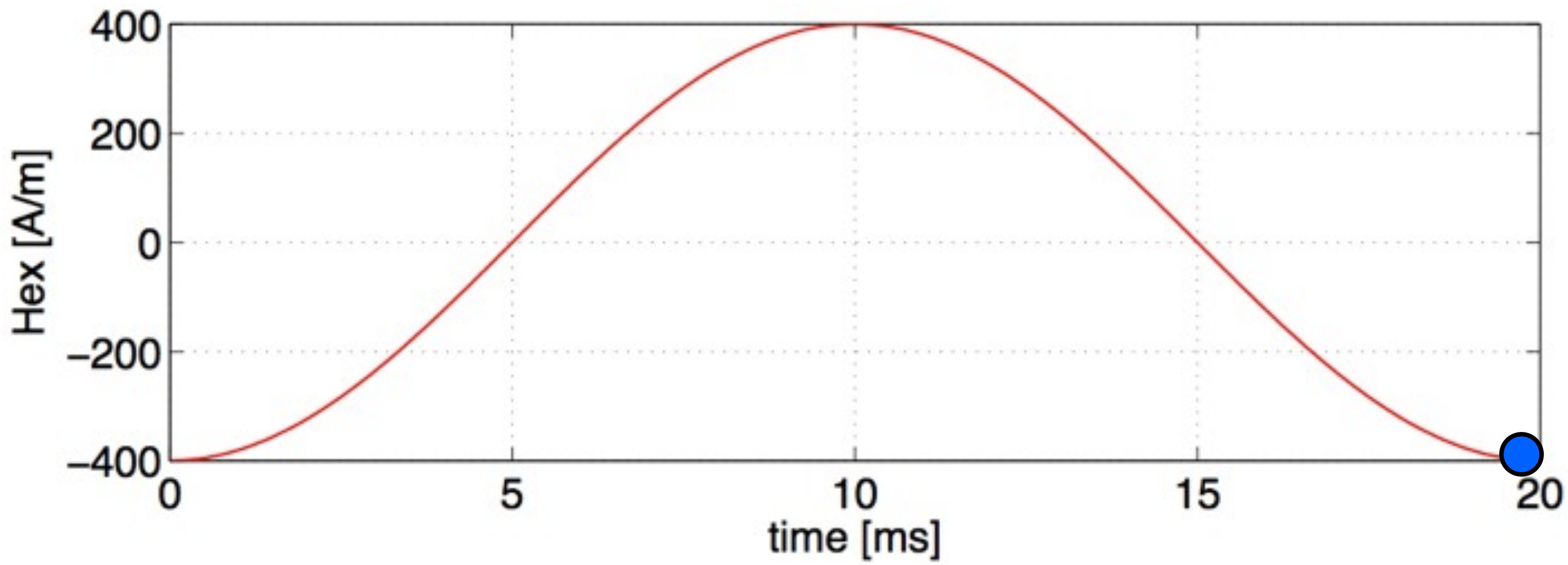
time [ms]


B, ϕ
time


B
H


400
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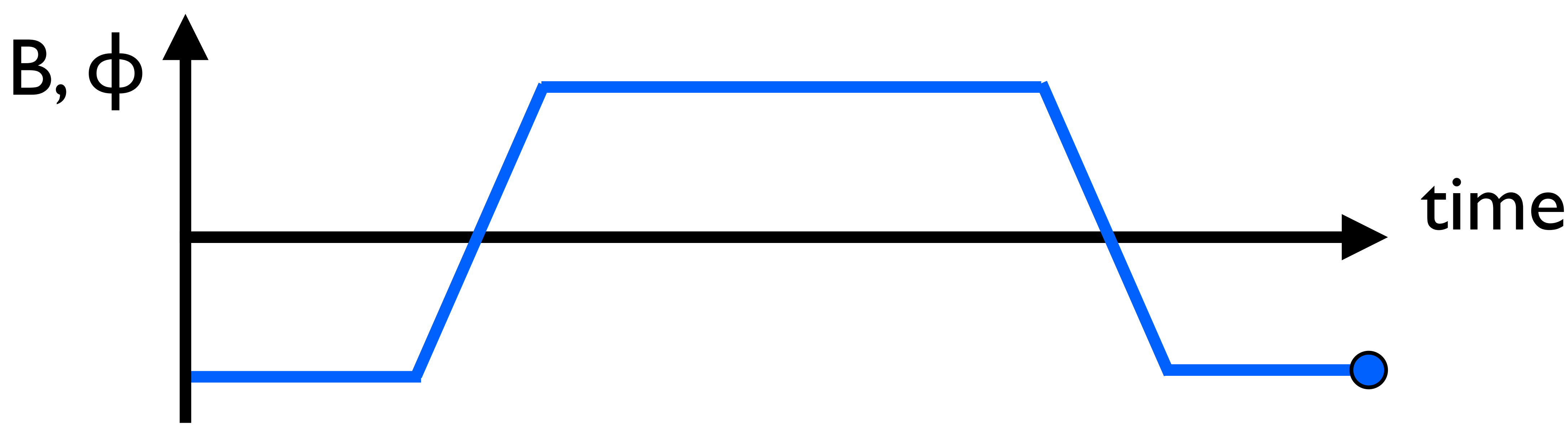
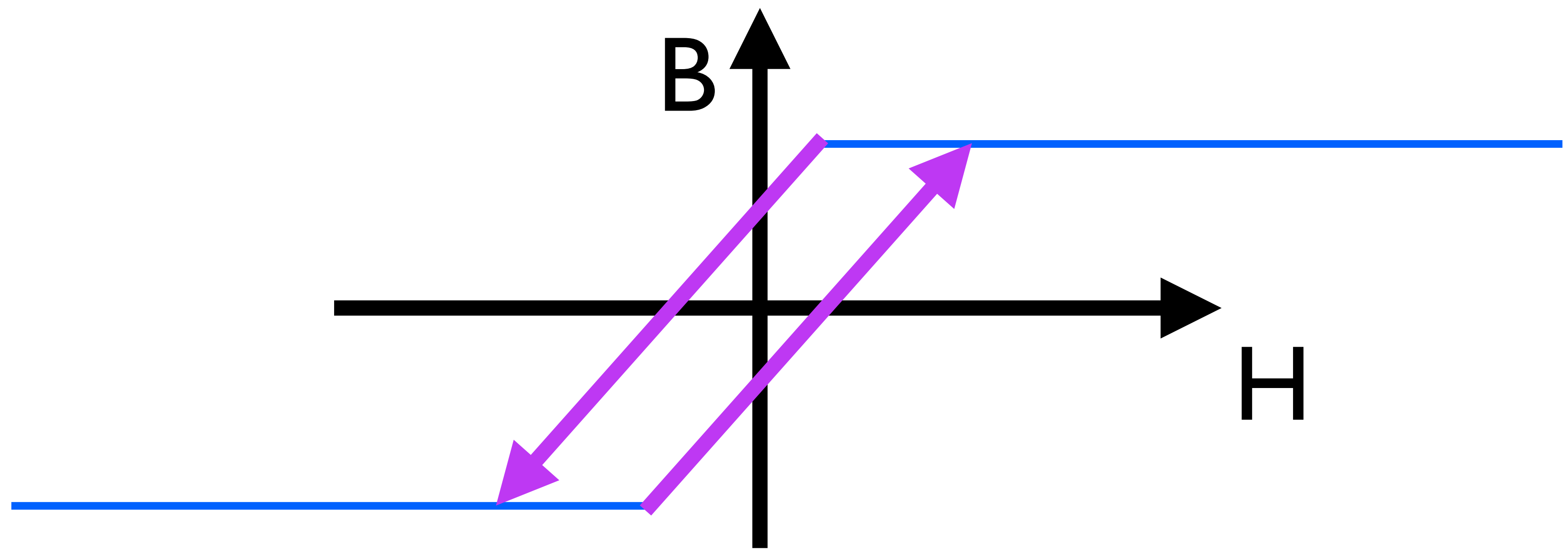
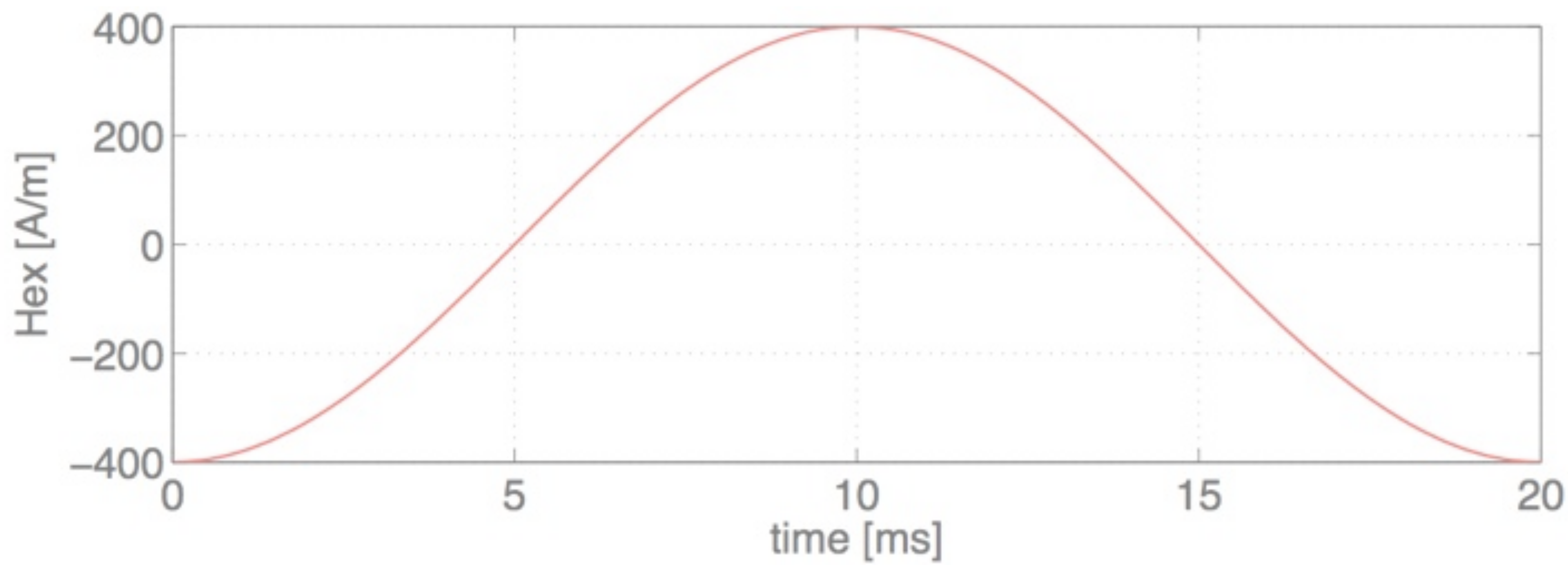
time [ms]
Hex [A/m]


B, ϕ
time


B
H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Hex [A/m]
0
5
10
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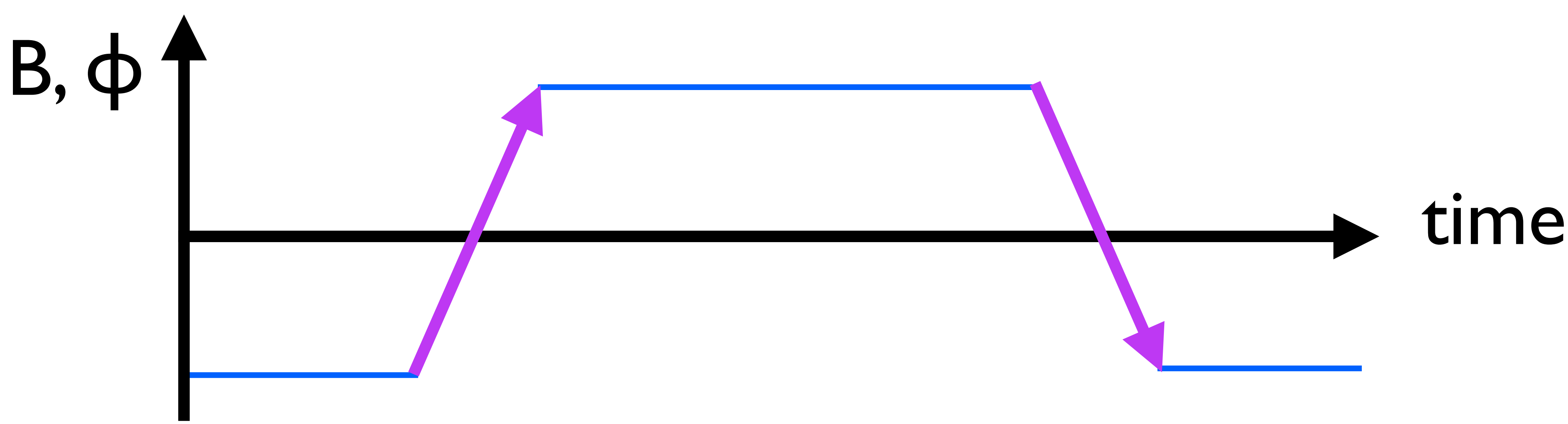
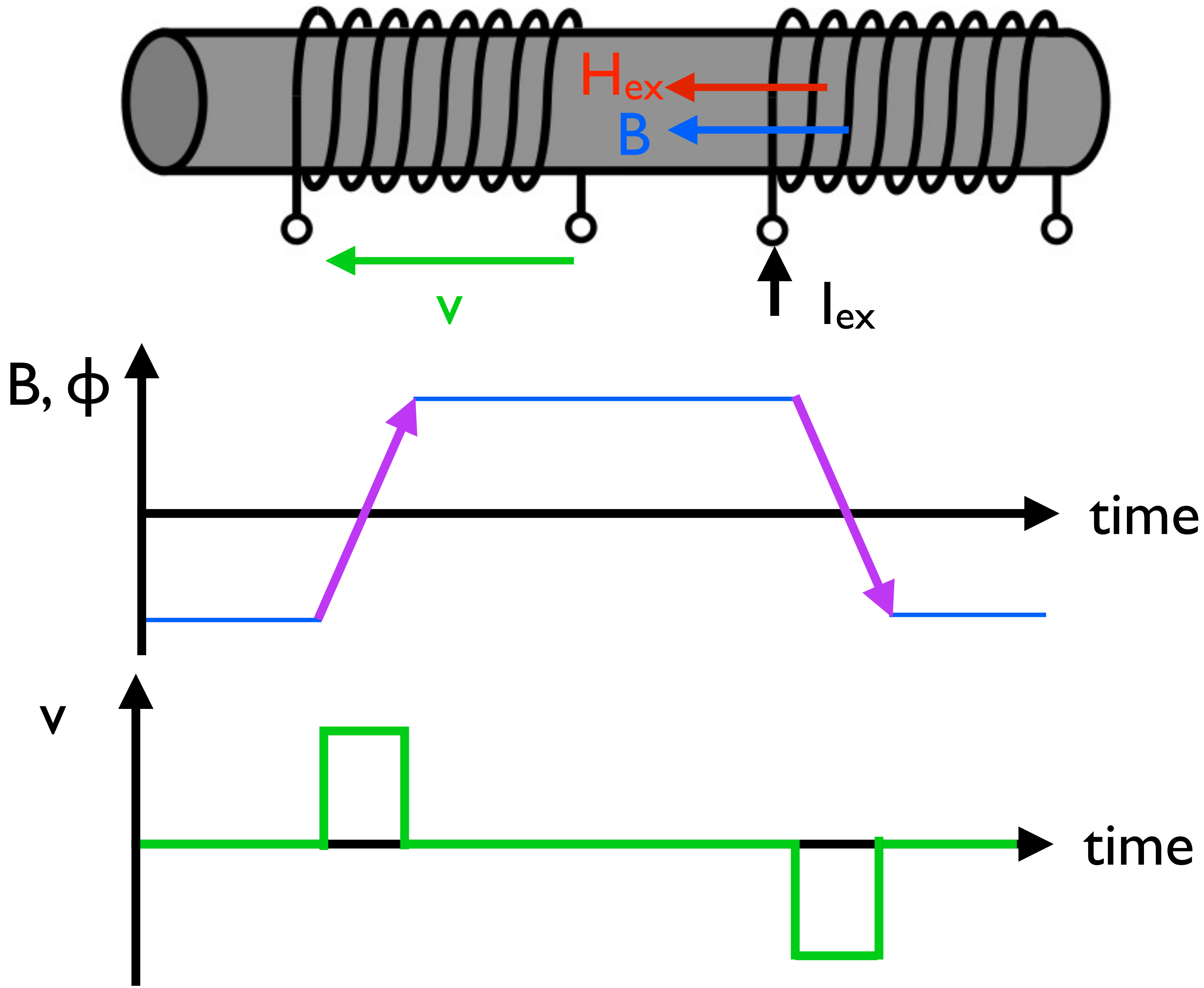
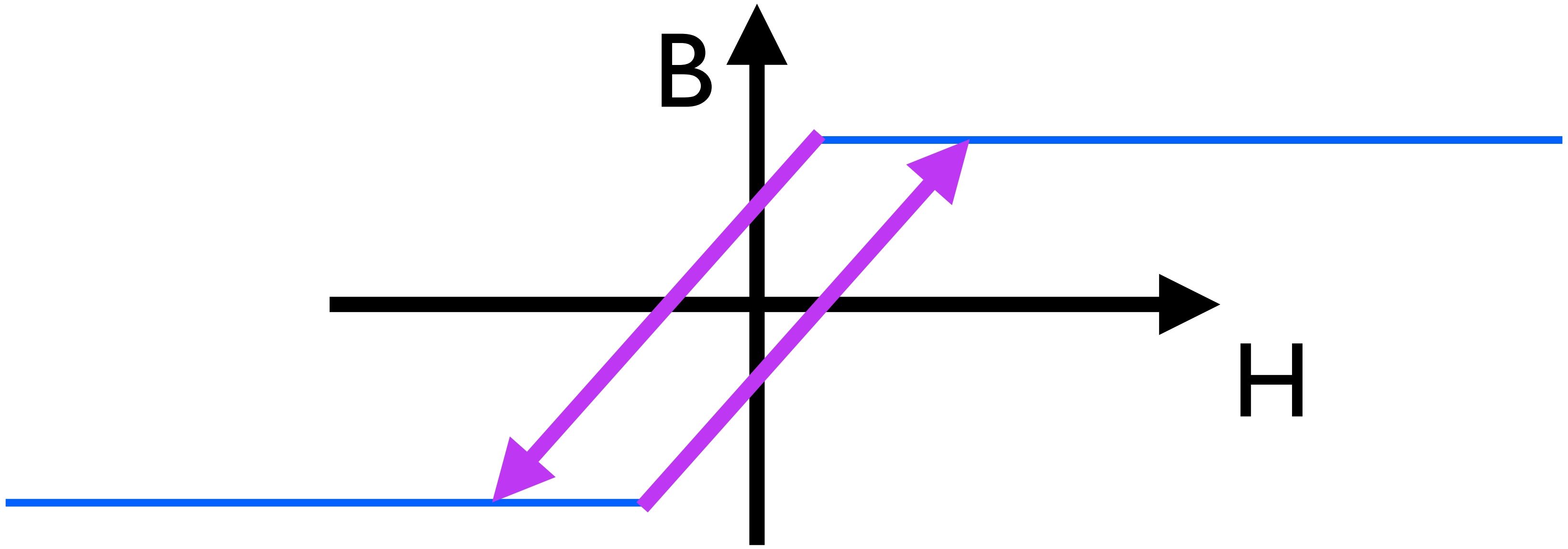
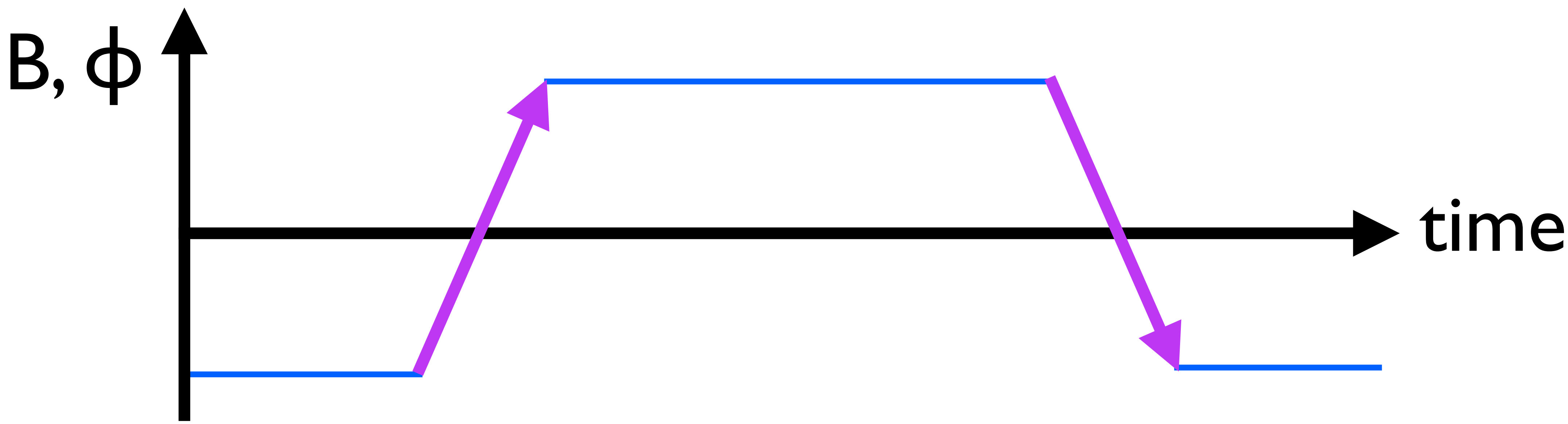
time [ms]

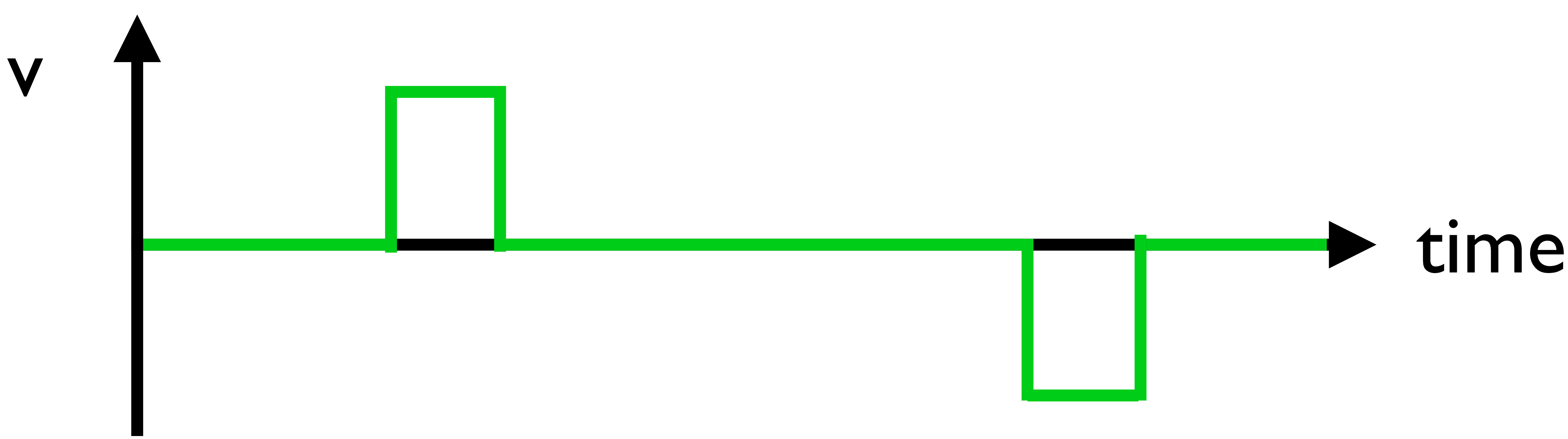

B, ϕ
time

we wind a pick-up coil to measure the variation of flux

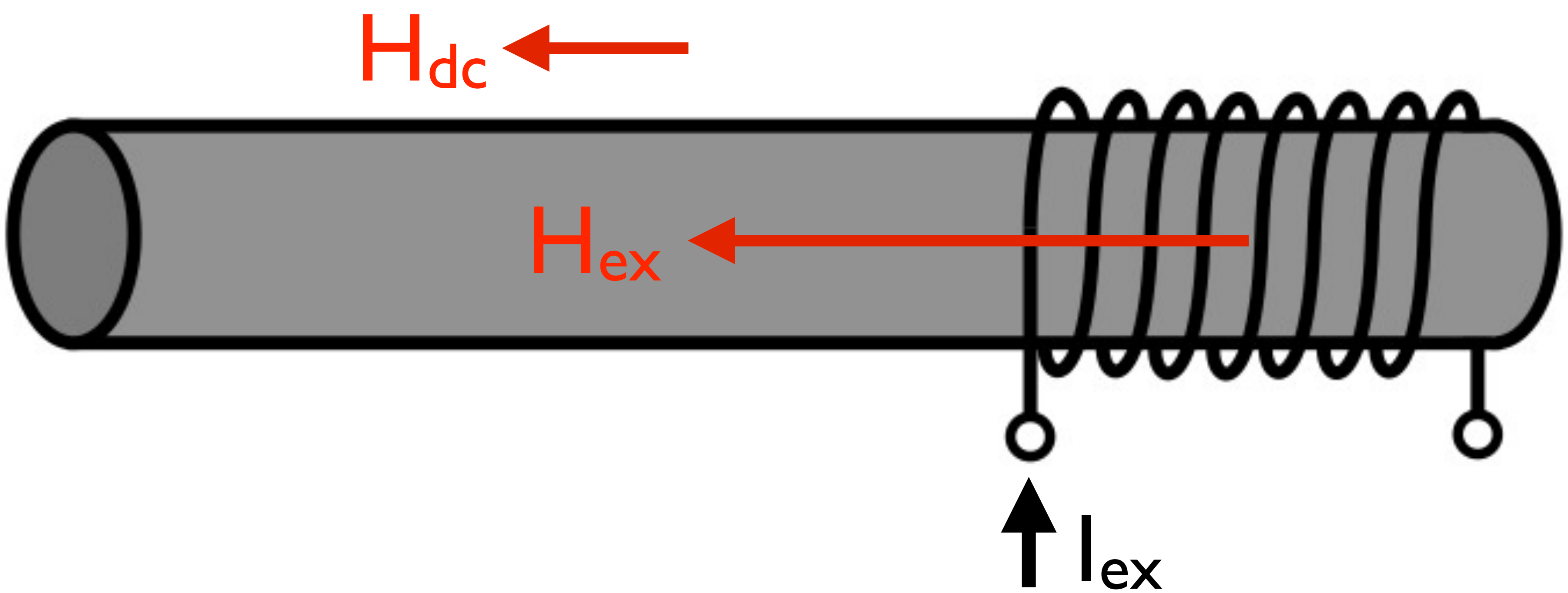



B
H

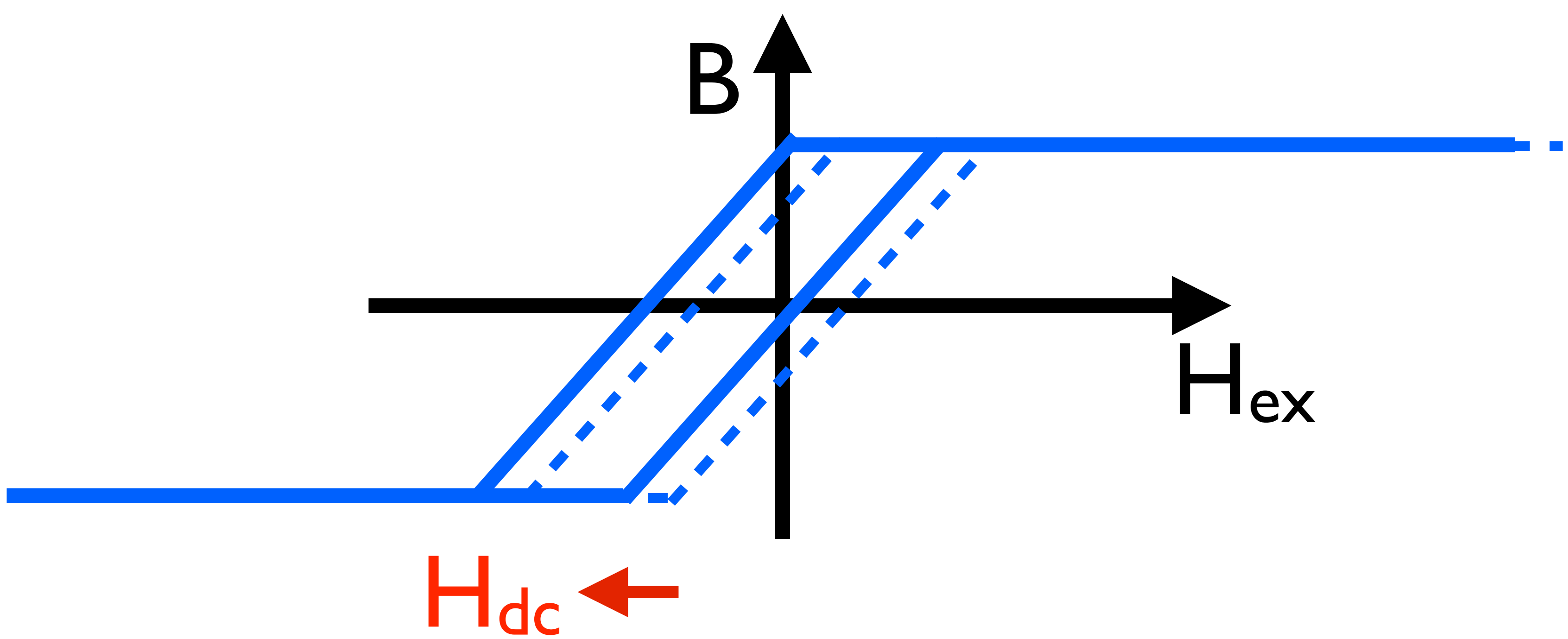

B, ϕ
time


v
time

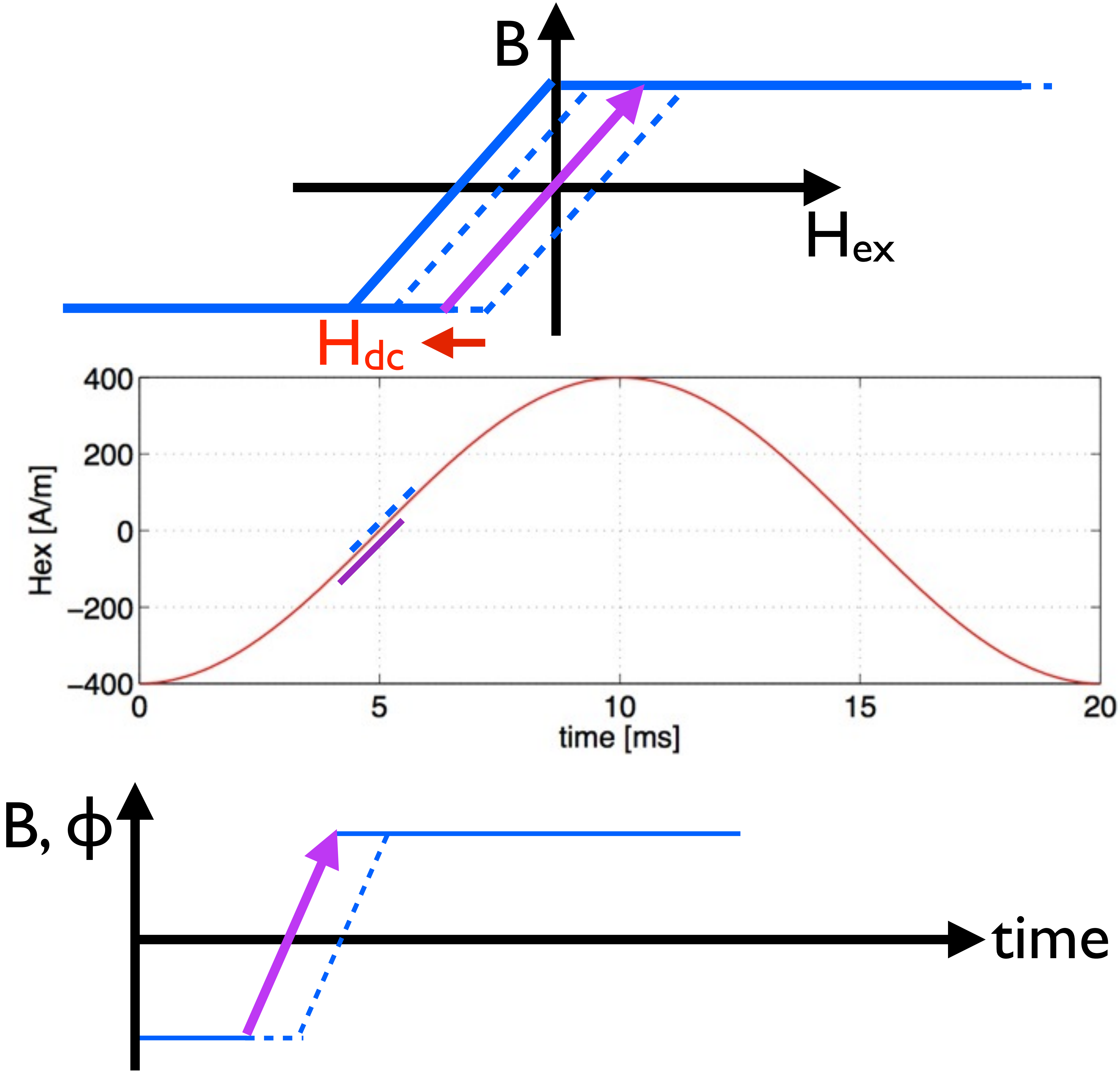
What happens if I add an additional DC field?



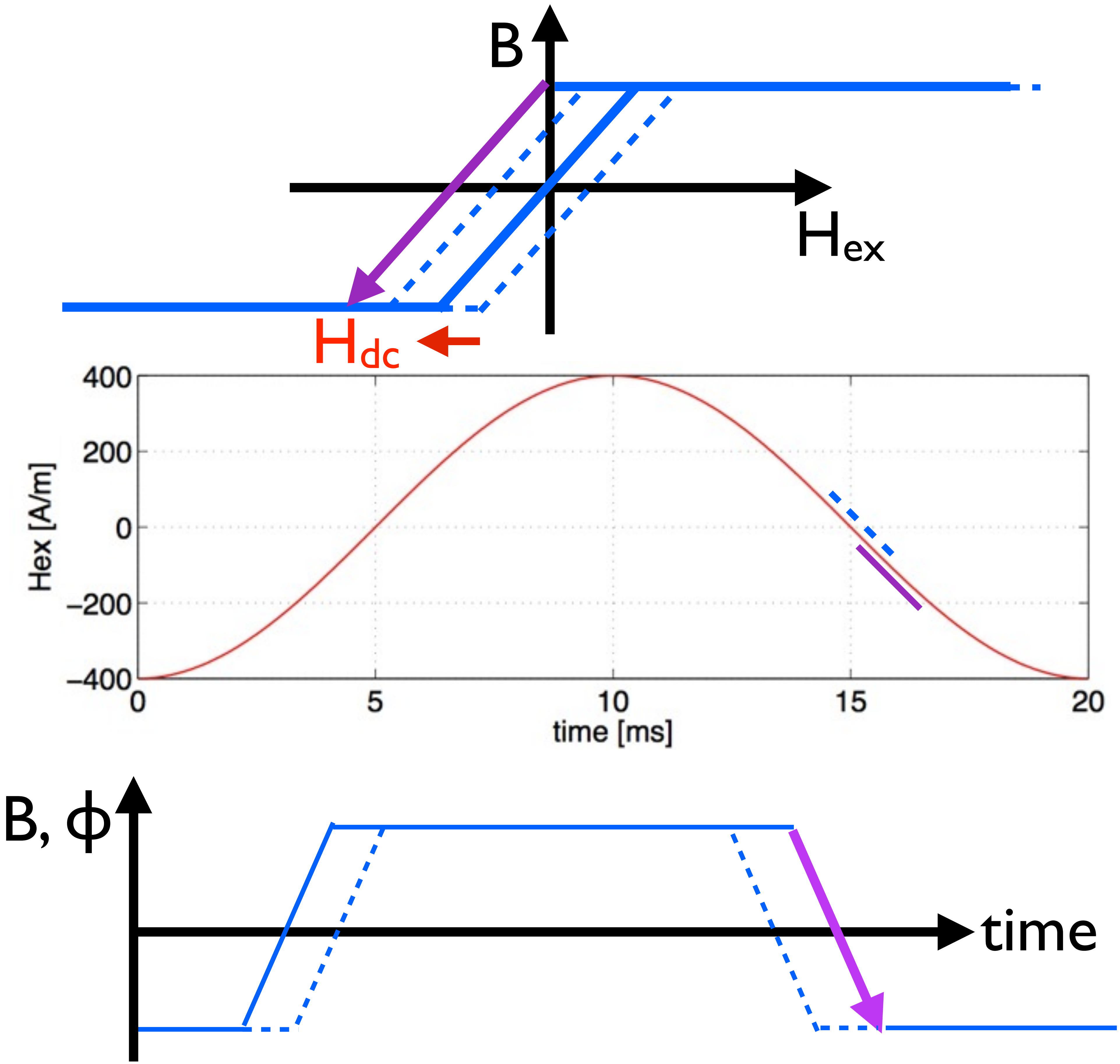
The effective BH loop is shifted by H_{dc}



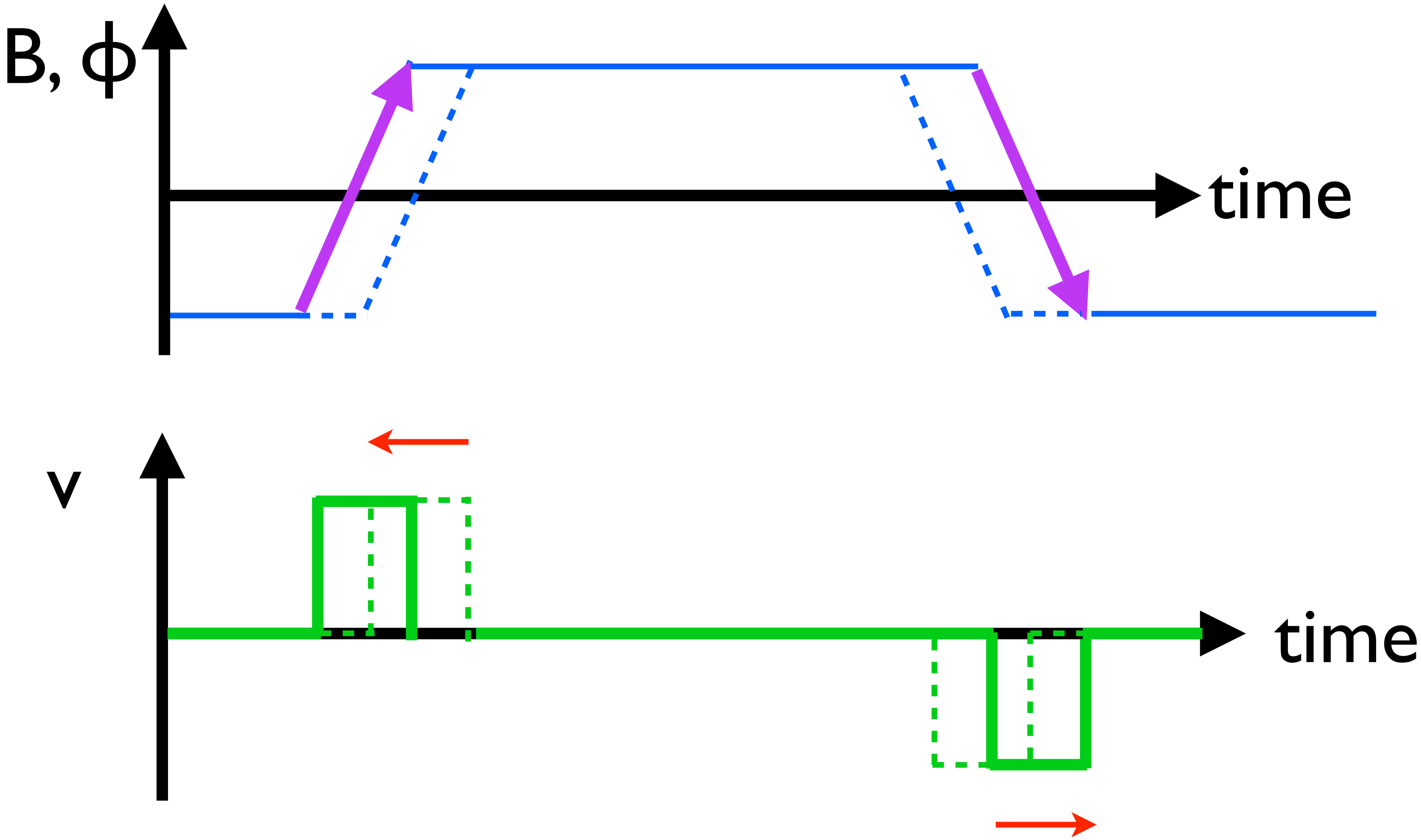
The first transition occurs earlier



The second transition occurs later

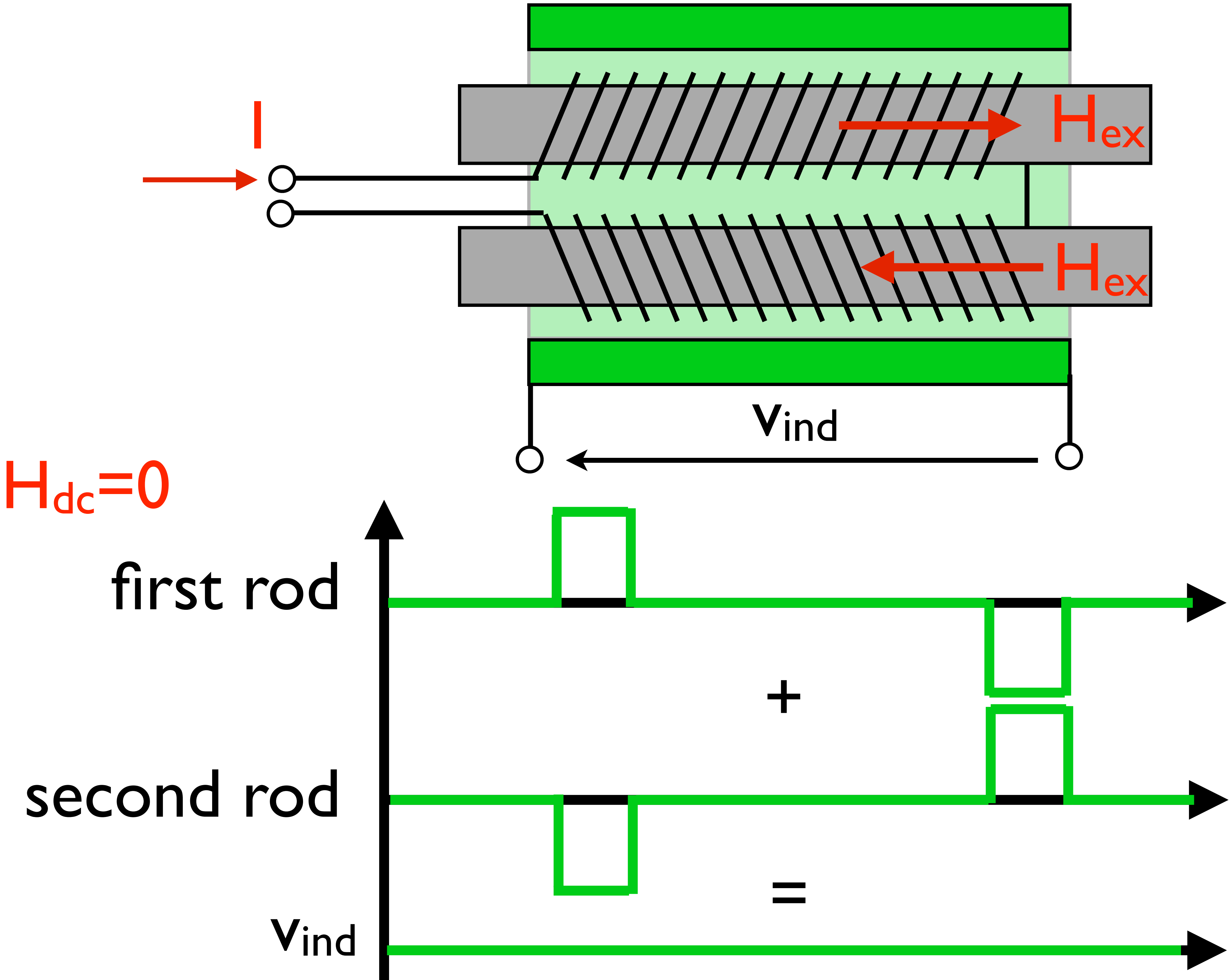


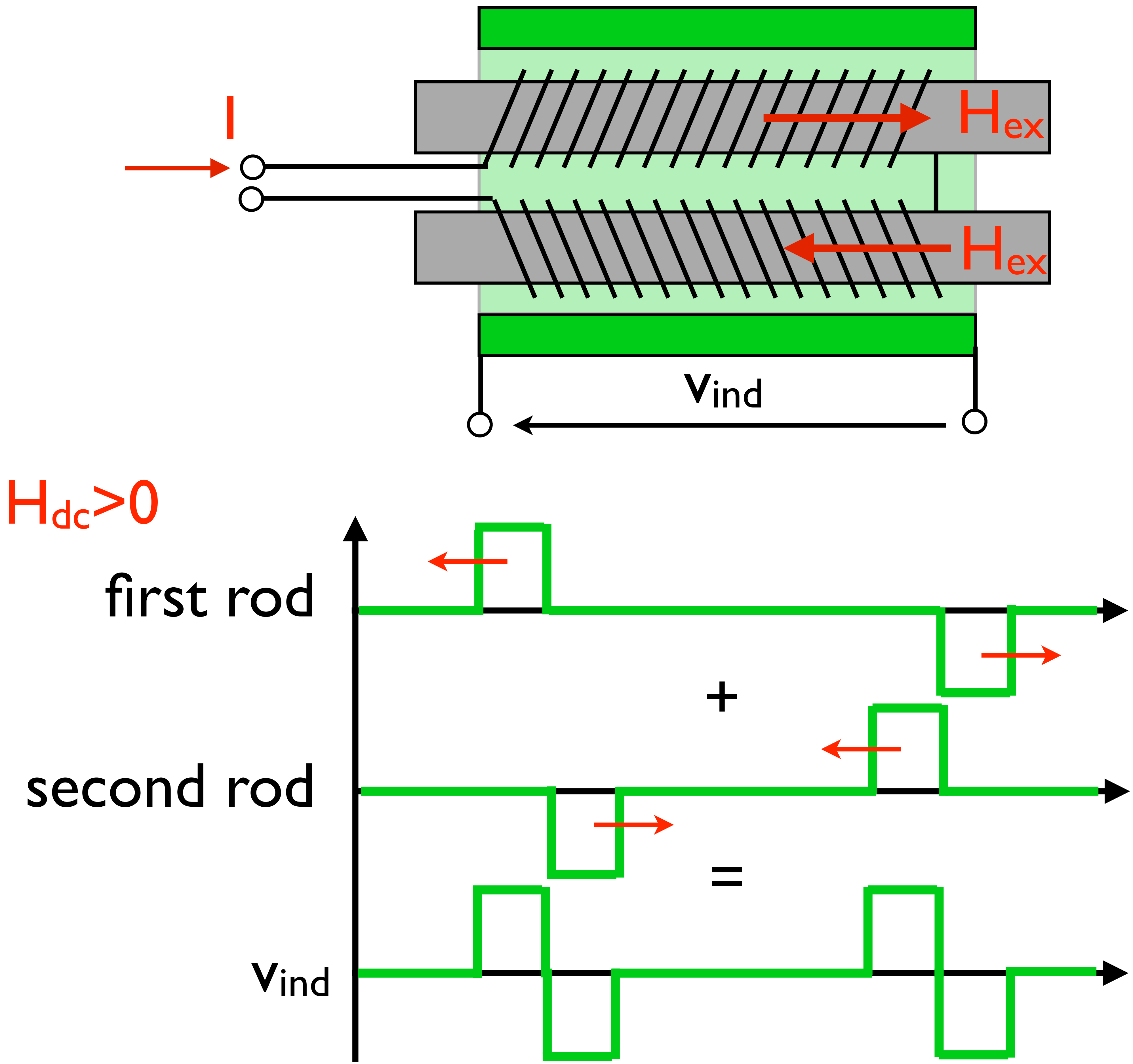
The voltage peaks shift in opposite direction



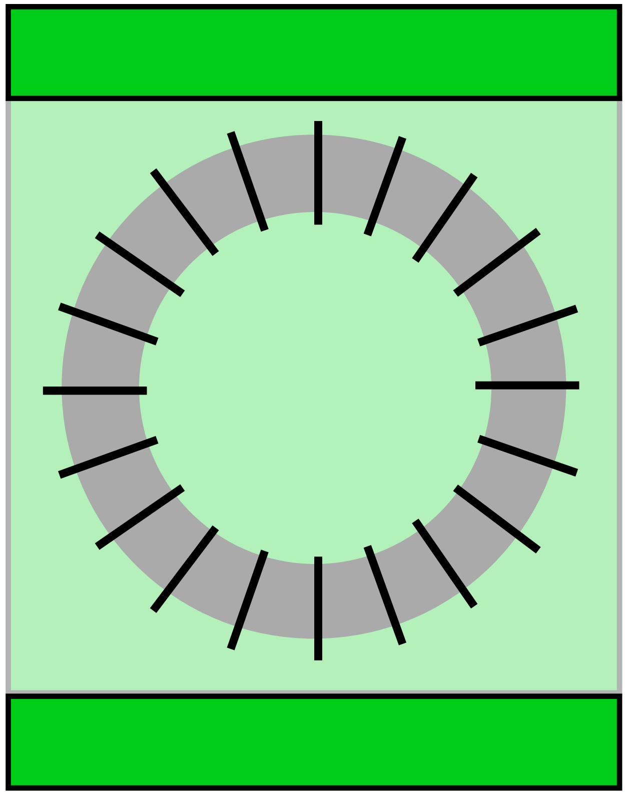
This shift is typically detected by extracting the 2nd harmonic

In fact, the core is not single a rod.
To suppress first harmonic two rods excited in opposite direction are used



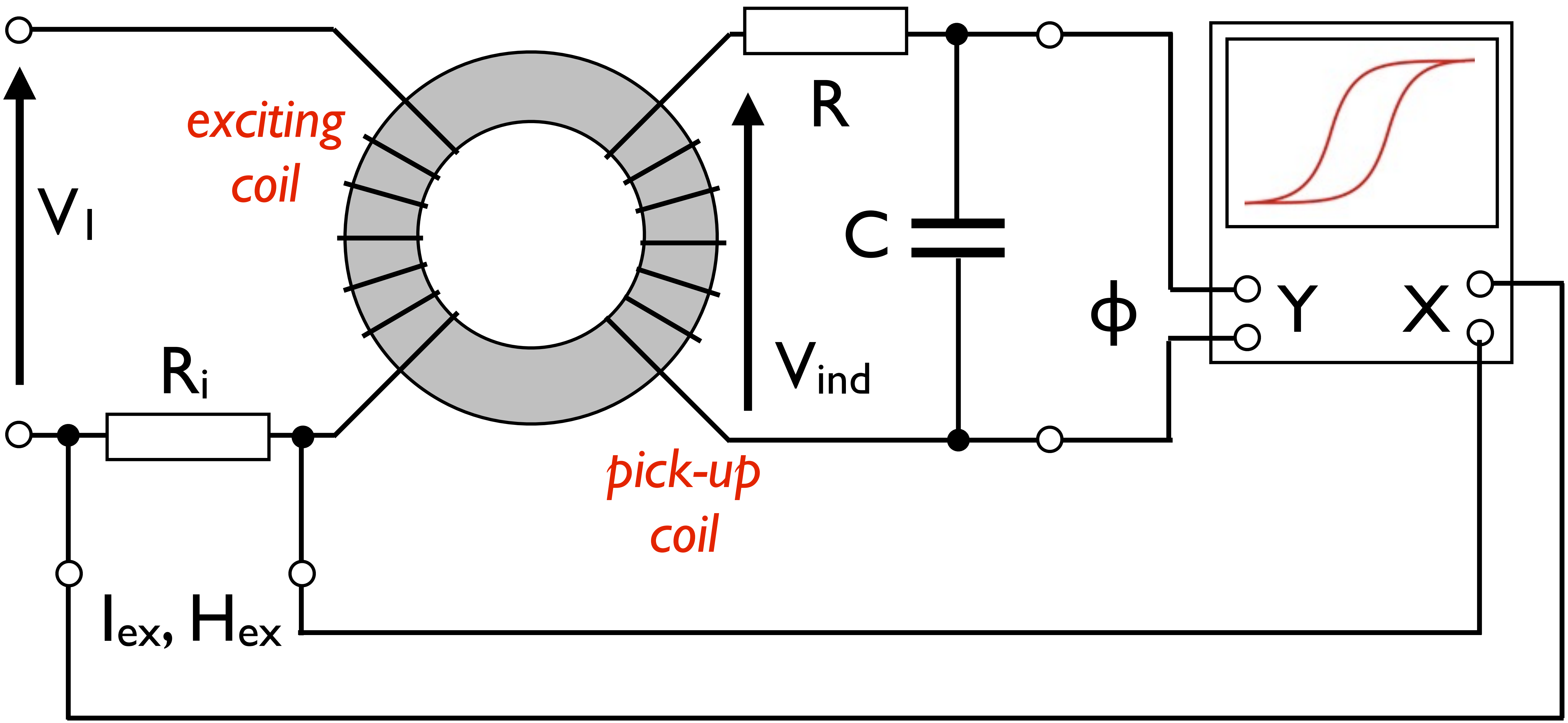

I
Hex
Hex
vind
Hdc>0
first rod
+
second rod
=
vind

Evolution of double rod to ring core
(to reduce demagnetizing factor)



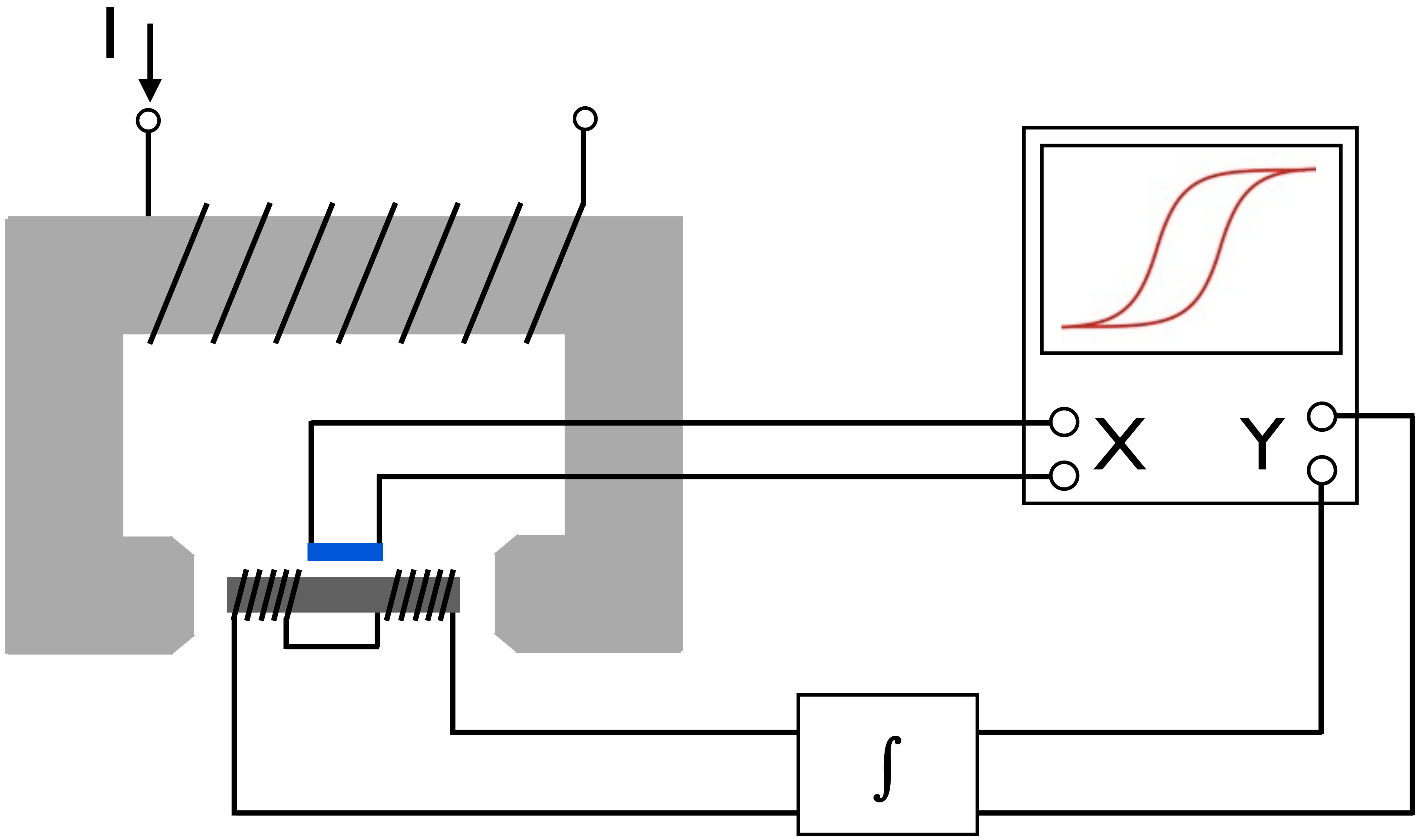
Measurement of BH loop

a) closed (and uniform) specimen



Measurement of BH loop

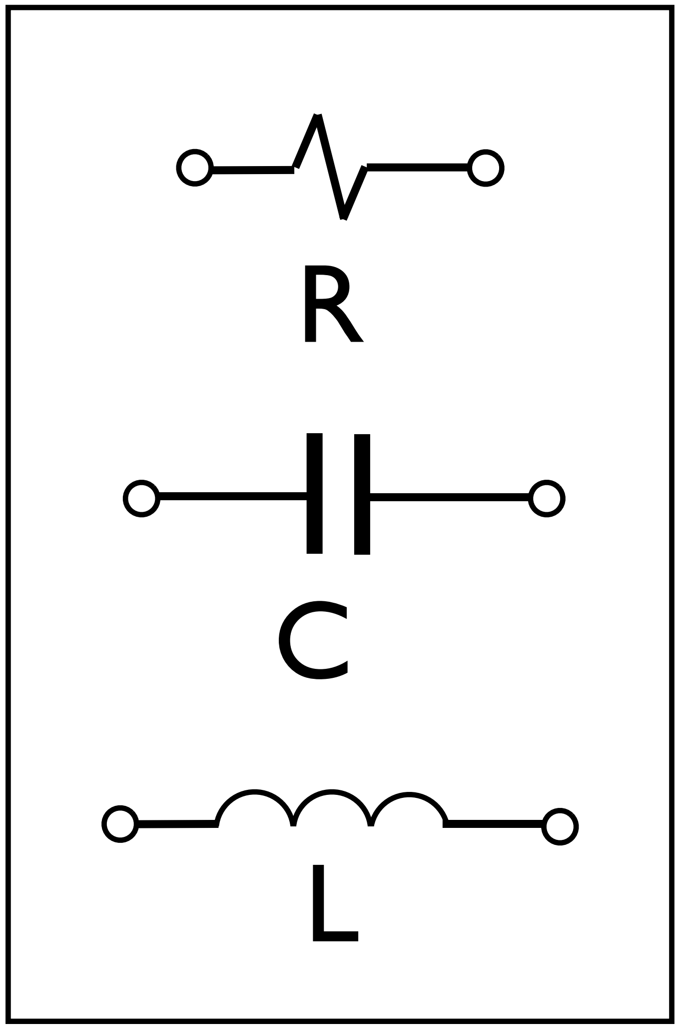
b) open specimen



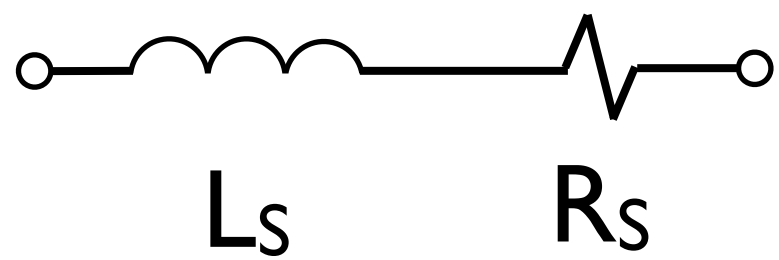
H inside the yoke is directly measured in the air gap of the yoke, for example using Hall sensor

MEASUREMENT OF IMPEDANCE

First of all, let's give a model of an impedance



$R_S + j\omega L_S$

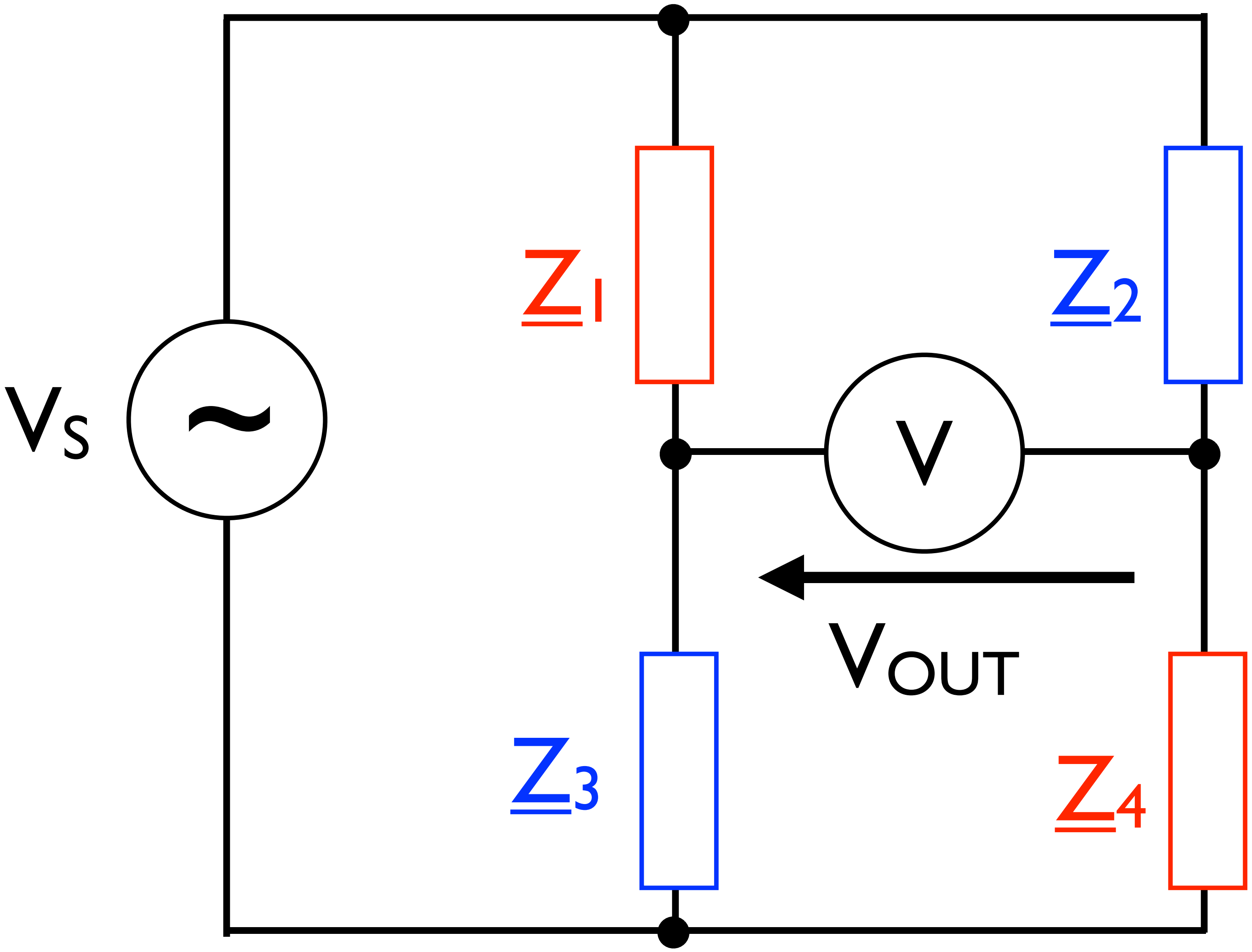


$Q=\omega L_S/R_S$

$R_S + 1/j\omega C_S$

$tg\delta=\omega C_S R_S$

AC WHEATSTONE BRIDGE



The condition for balanced bridge is still the same

$$\underline{Z}_1 \cdot \underline{Z}_4 = \underline{Z}_2 \cdot \underline{Z}_3$$

to obtain $V_{OUT} = 0$

... but it's a complex equation!

$$\underline{\underline{Z}}_1 \cdot \underline{\underline{Z}}_4 = \underline{\underline{Z}}_2 \cdot \underline{\underline{Z}}_3$$

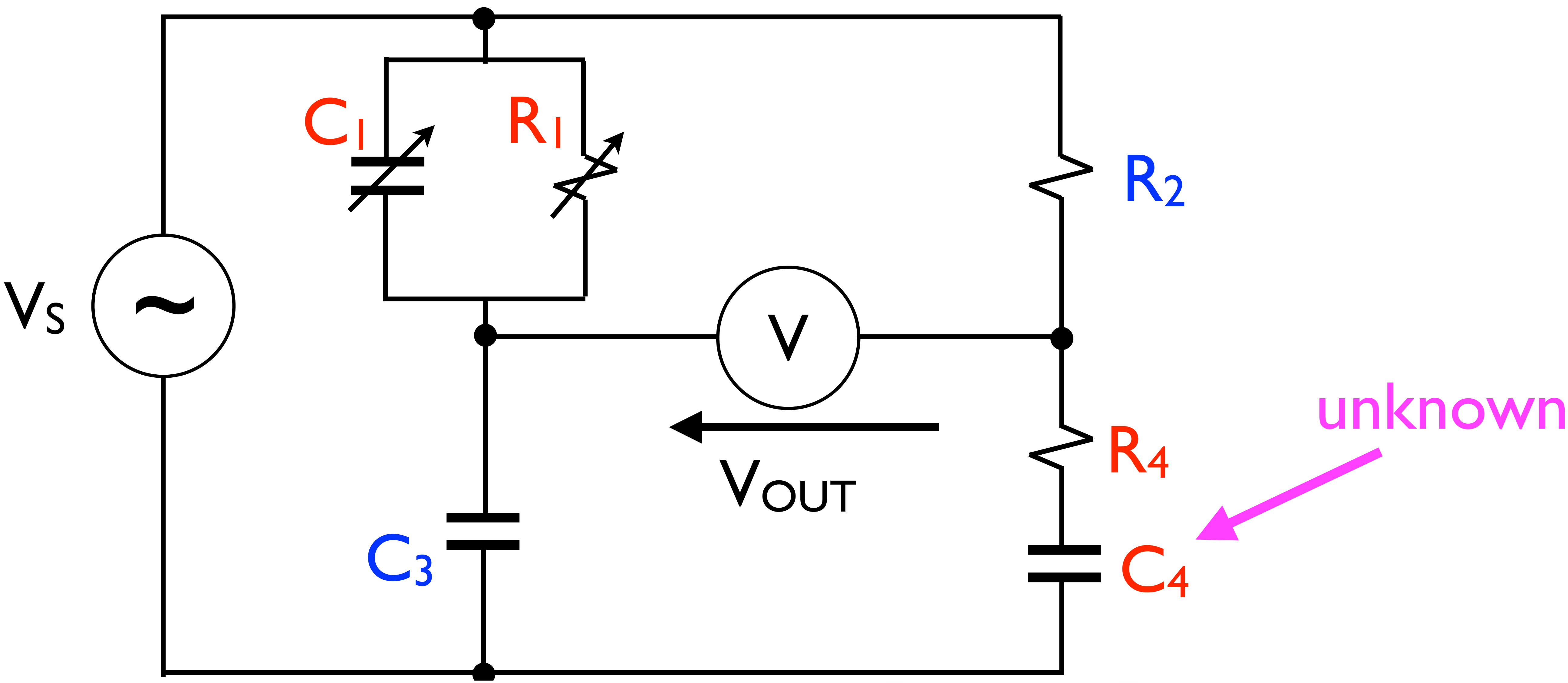
1 complex equation $\rightarrow$ 2 scalar equations

Real $\quad \mathrm{Re}[\underline{\underline{Z}}_1 \cdot \underline{\underline{Z}}_4] = \mathrm{Re}[\underline{\underline{Z}}_2 \cdot \underline{\underline{Z}}_3]$

Imaginary $\quad \mathrm{Im}[\underline{\underline{Z}}_1 \cdot \underline{\underline{Z}}_4] = \mathrm{Im}[\underline{\underline{Z}}_2 \cdot \underline{\underline{Z}}_3]$

Measurement of Capacitance

Schering bridge



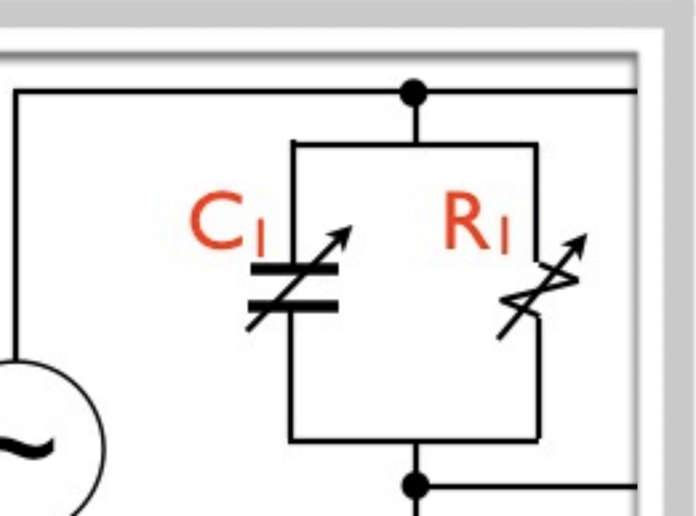
$$R_2 \cdot \frac{1}{j\omega C_3} = \left(R_4 + \frac{1}{j\omega C_4}\right) \cdot \left(\frac{1}{\frac{1}{R_1} + j\omega C_1}\right)$$

$$R_2 \cdot \frac{1}{j\omega C_3} = \left(R_4 + \frac{1}{j\omega C_4}\right) \cdot \left(\frac{1}{\frac{1}{R_1} + j\omega C_1}\right)$$

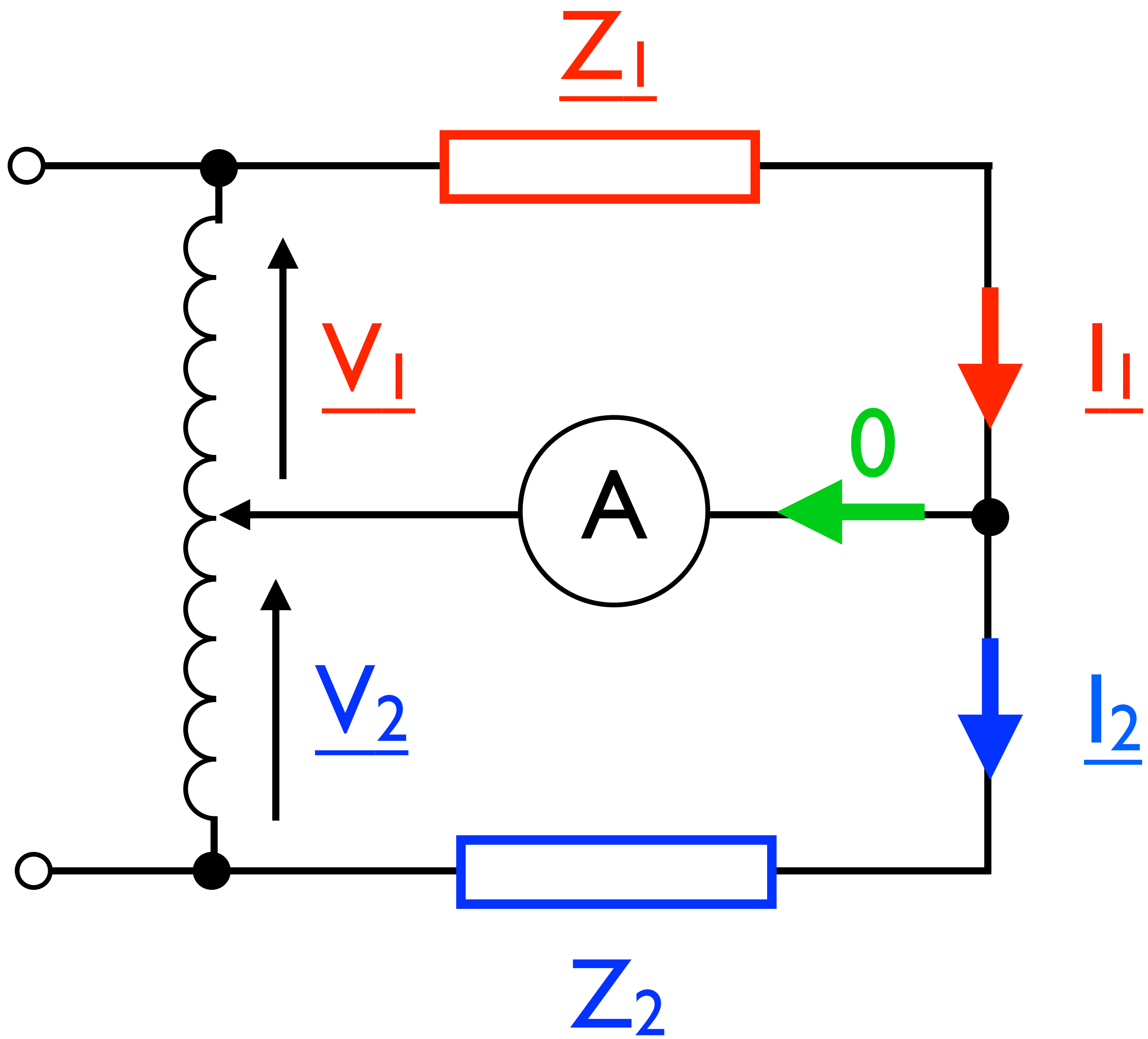
Solving for real and imaginary part

$$R_4 = \frac{R_2 \cdot C_1}{C_3} \qquad C_4 = \frac{C_3 \cdot R_1}{R_2}$$

C_1 is variable $\qquad$ R_1 is variable



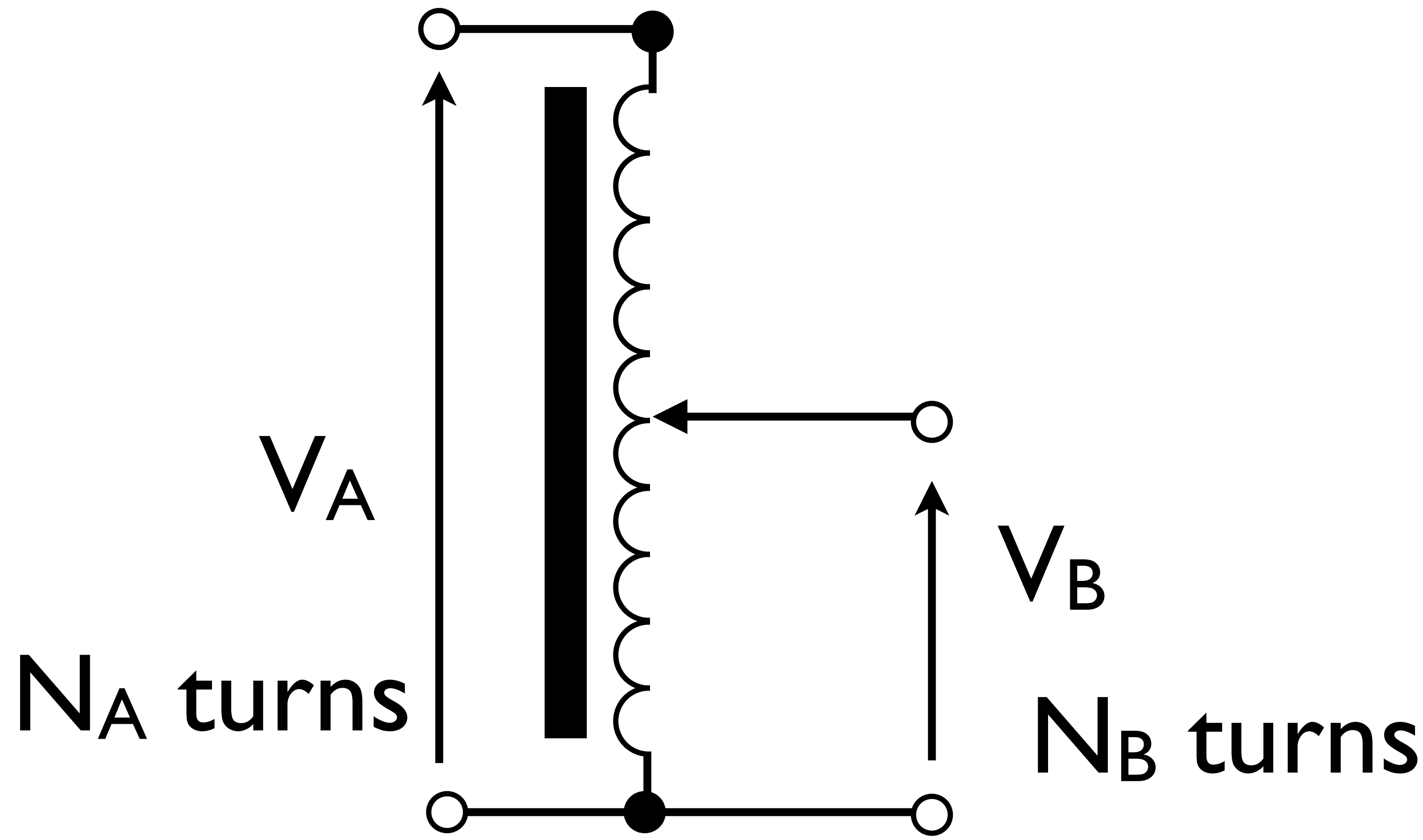
TRANSFORMER BRIDGE



The bridge is balanced if no current flows through the ammeter, that is if the $I_1 = I_2$ or

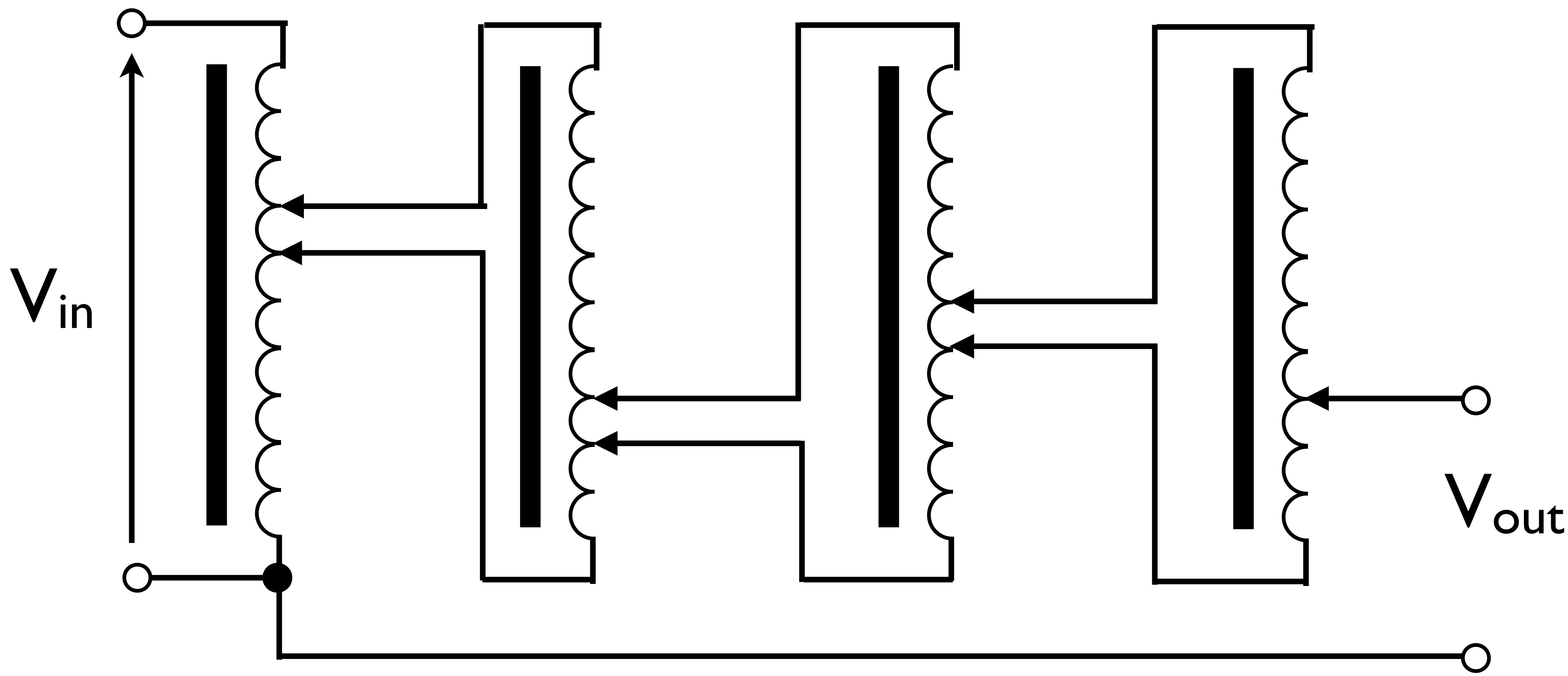
$V_1 / Z_1 = V_2 / Z_2$

AUTOTRANSFORMER



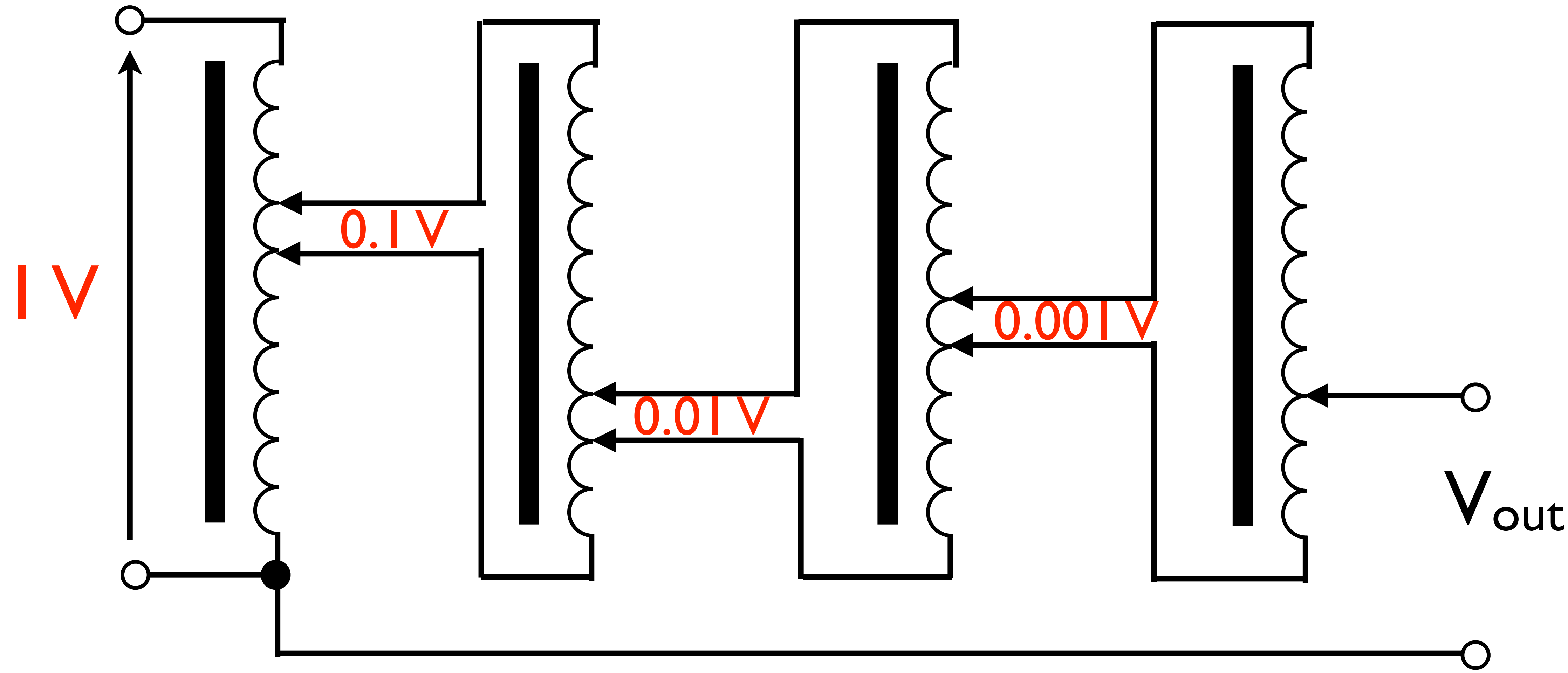
$$V_B = V_A \cdot N_B / N_A$$

The voltage can be very finely regulated by using cascade dividers

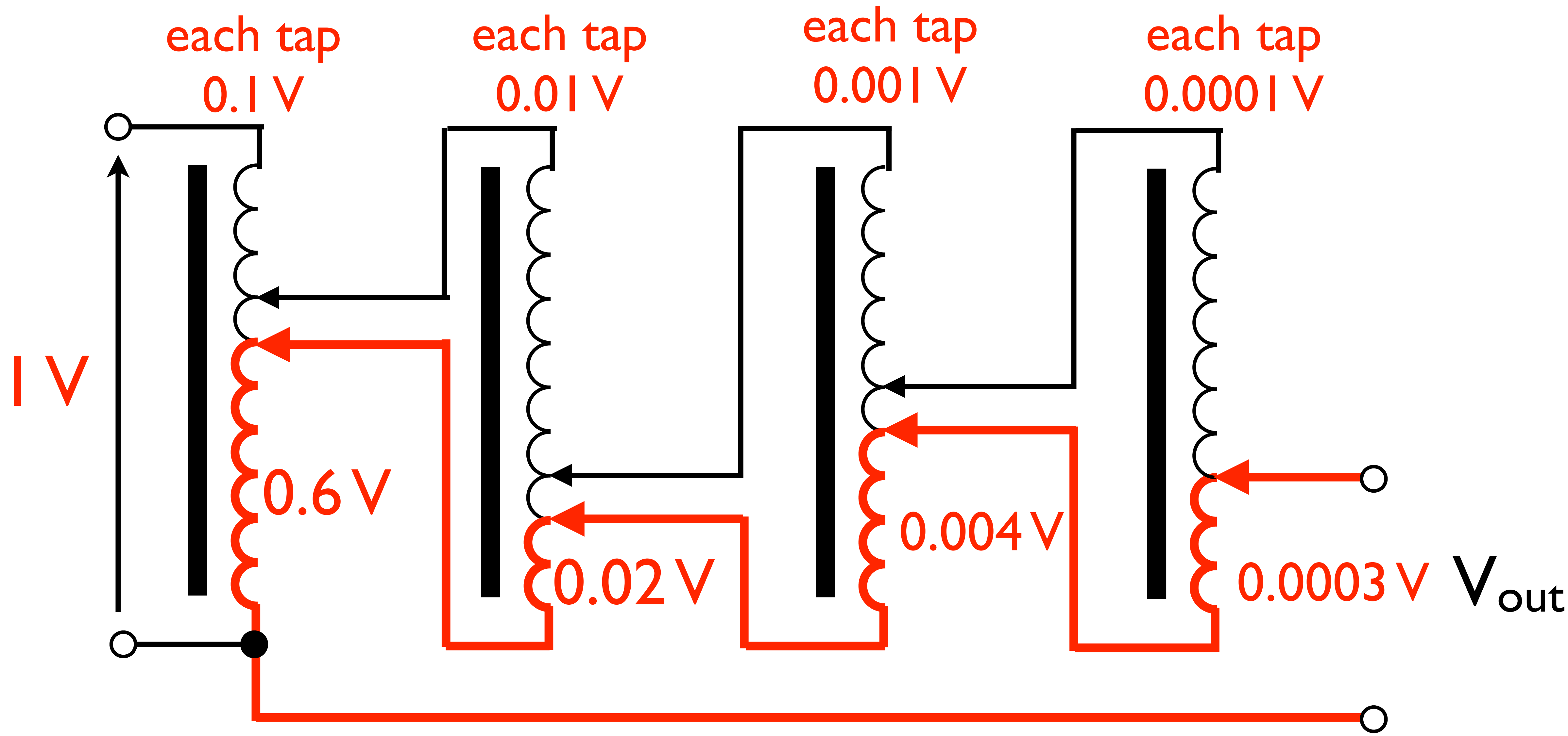


How does it work?

Each stage is powered by a voltage
10 times lower than the previous stage

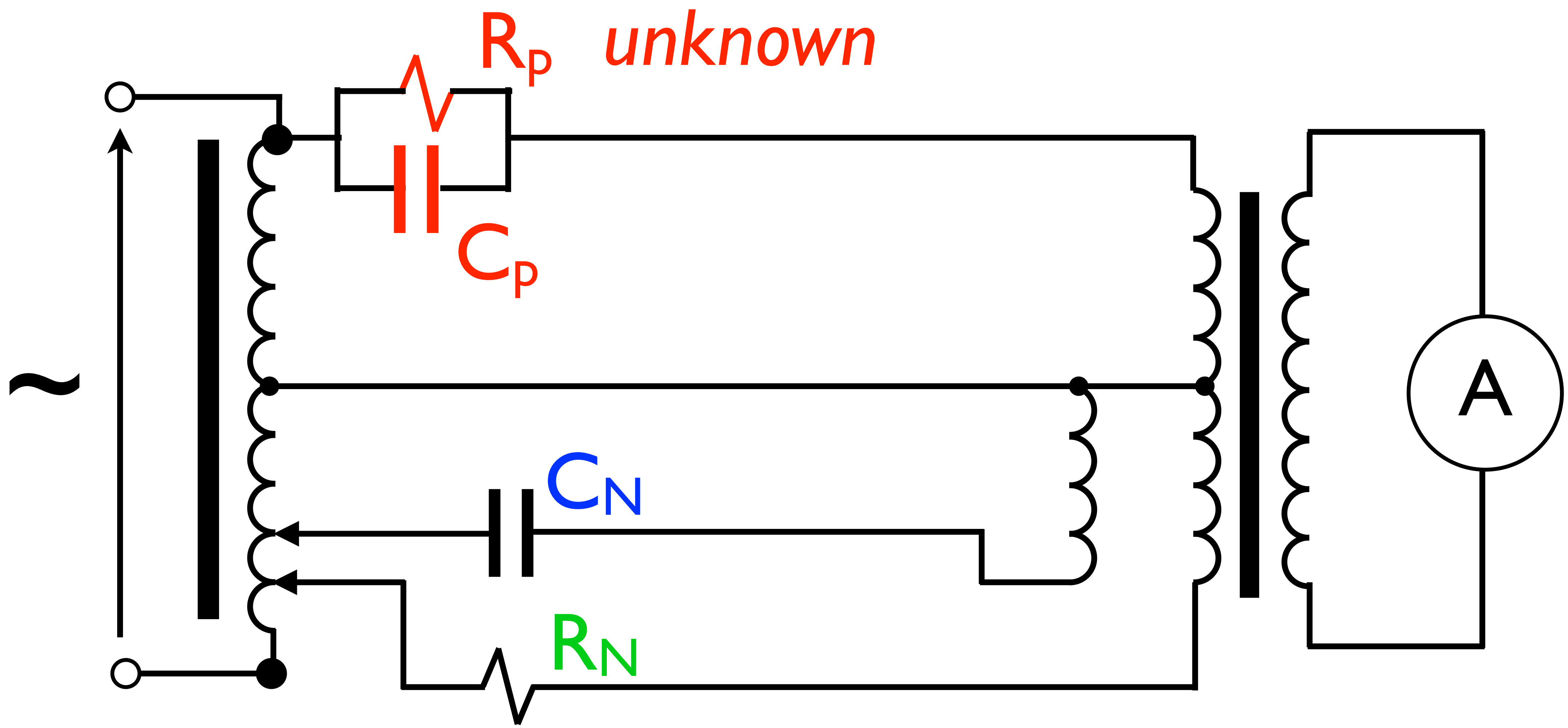


At each stage we can select the value of each decimal number



$$V_{out}=0.6+0.02+0.004+0.0003\,V = 0.6243\,V$$

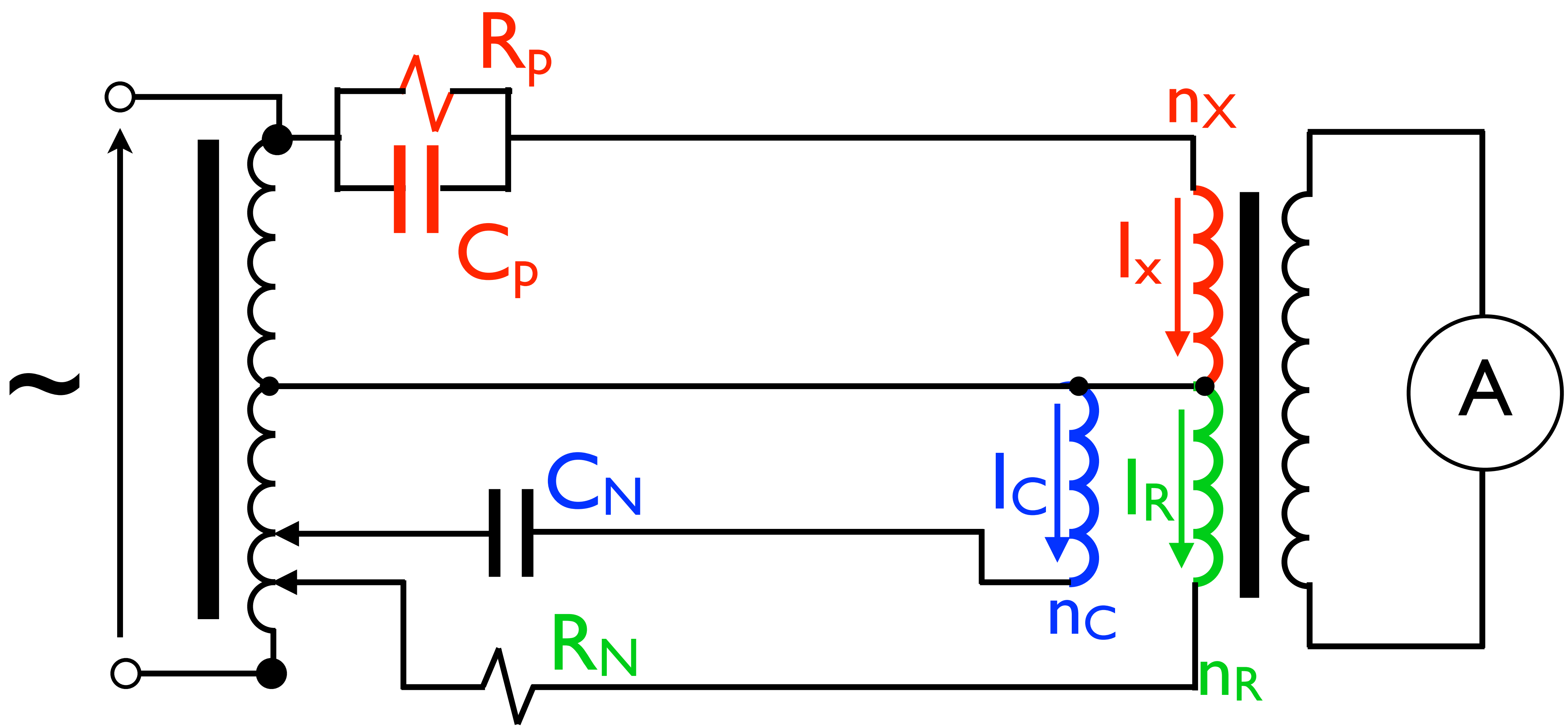
The actual configuration



C_N, R_N: reference but not variable

We change the current acting on the voltage

The actual configuration

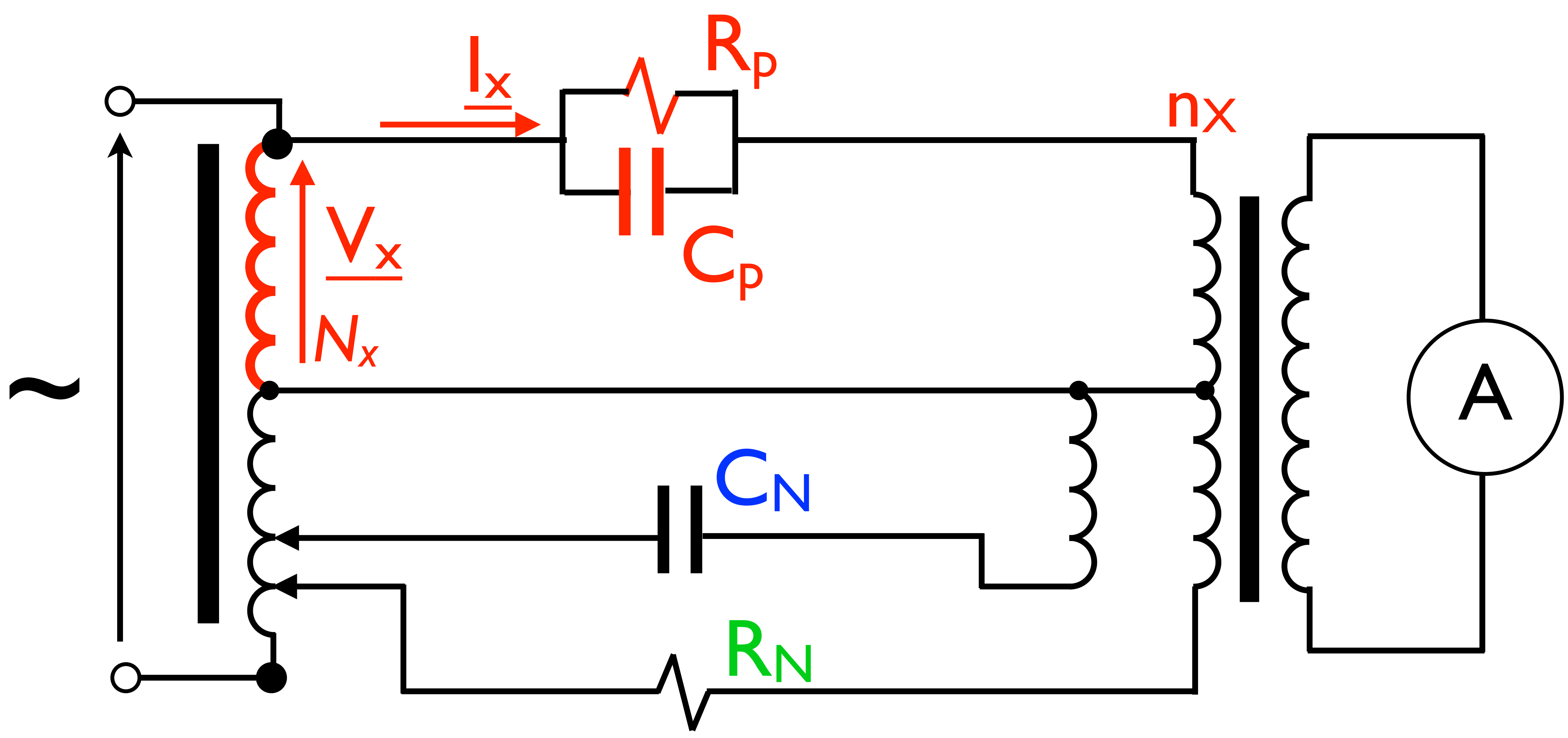


n_X, n_C, n_R: number of turns of the windings

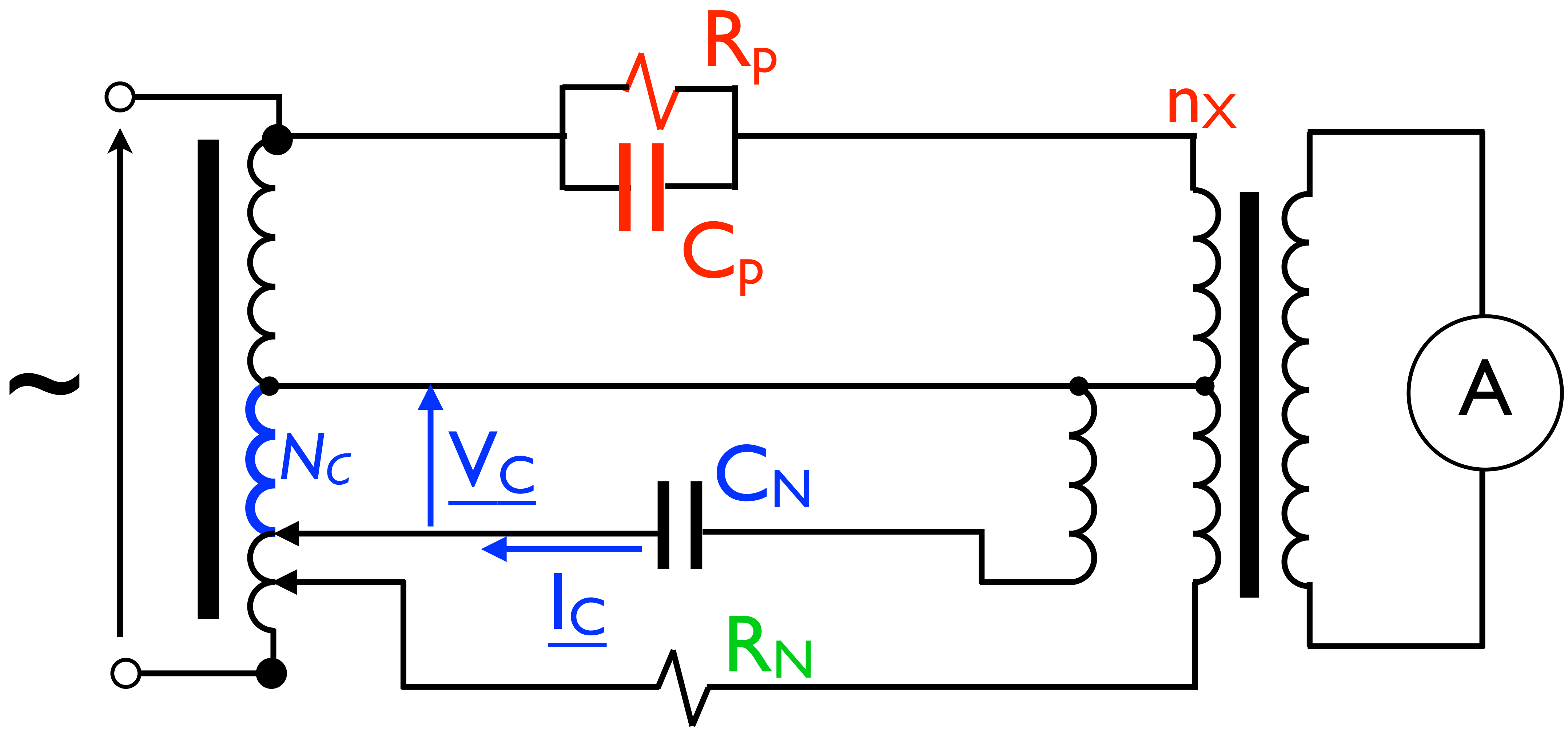
The bridge is balanced if

$$n_X \cdot \underline{I_x} = n_C \cdot \underline{I_C} + n_R \cdot \underline{I_R}$$

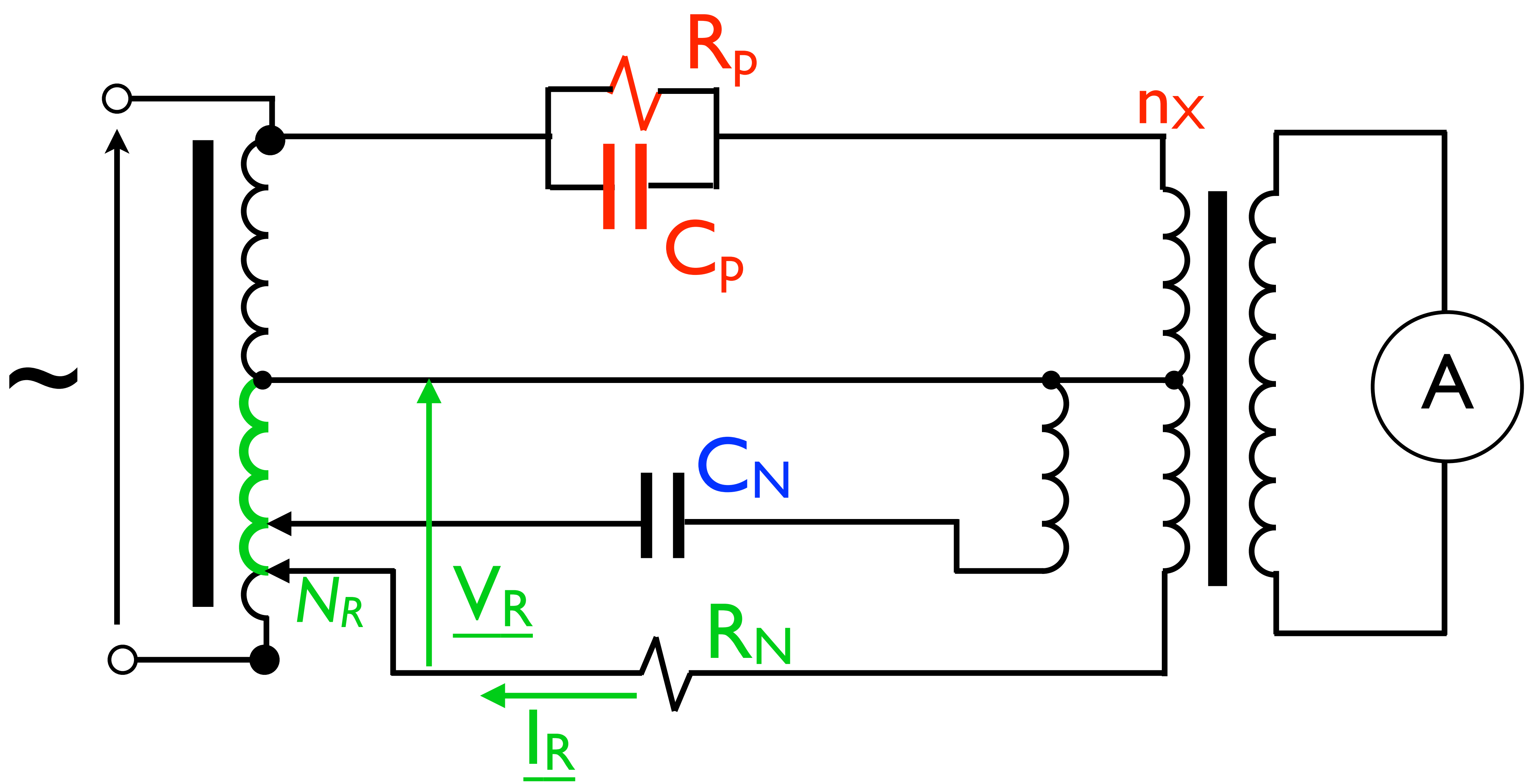
The actual configuration



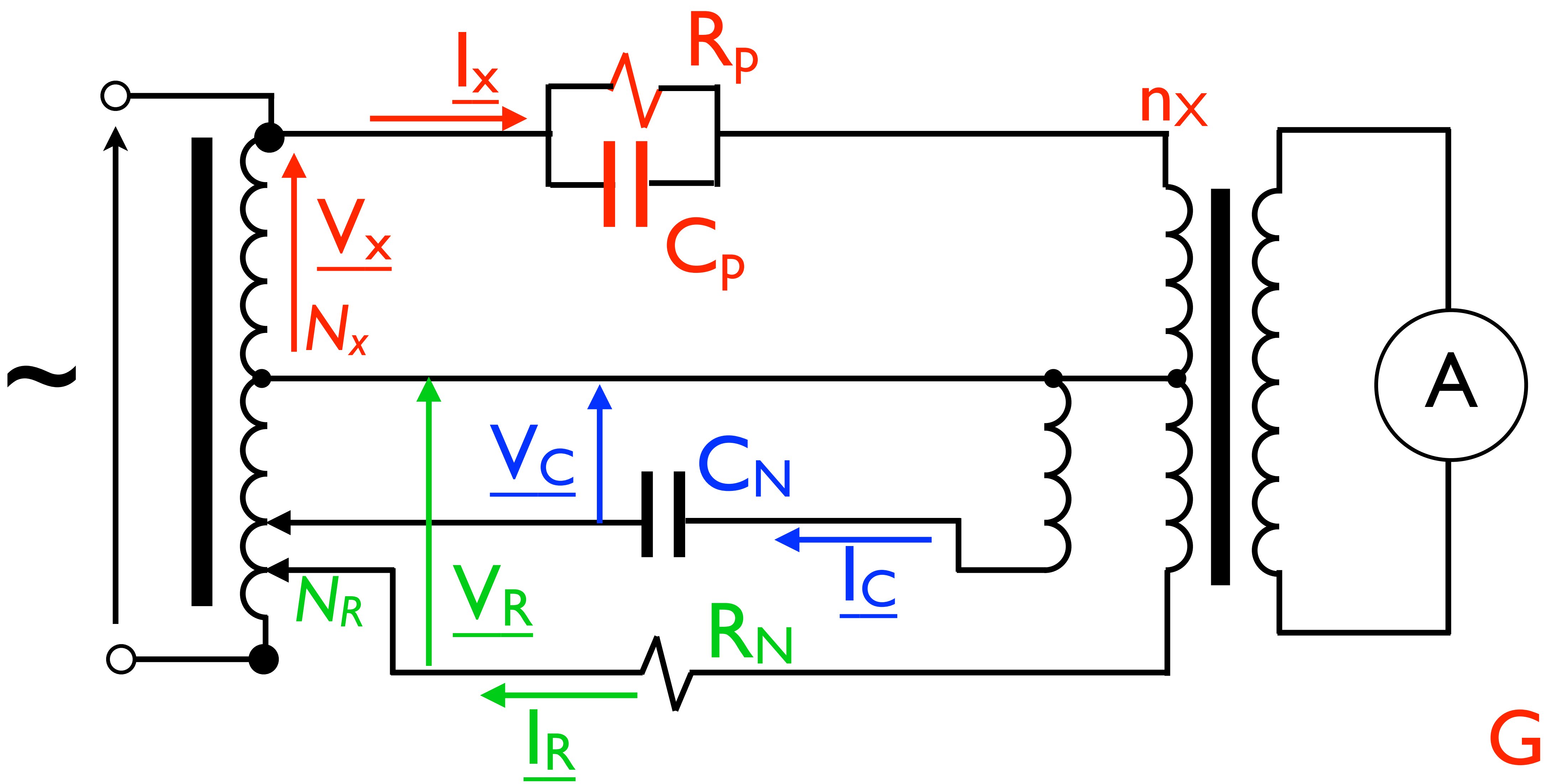
The actual configuration



The actual configuration



The actual configuration



$G_p = 1/R_p$

$G_N = 1/R_N$

$$n_X \cdot \underline{I_x} = n_C \cdot \underline{I_C} + n_R \cdot \underline{I_R}$$

becomes

$$n_X \cdot \underline{V_x} \cdot (G_p + j\omega C_p) = n_C \cdot \underline{V_C} \cdot j\omega C_N + n_R \cdot \underline{V_R} \cdot G_N$$

$$n_X \cdot N_x \cdot (G_p + j\omega C_p) = n_C \cdot N_C \cdot j\omega C_N + n_R \cdot N_R \cdot G_N$$

$$n_X \cdot N_X \cdot (G_p + j\omega C_p) = n_C \cdot N_C \cdot j\omega C_N + n_R \cdot N_R \cdot G_N$$

Real

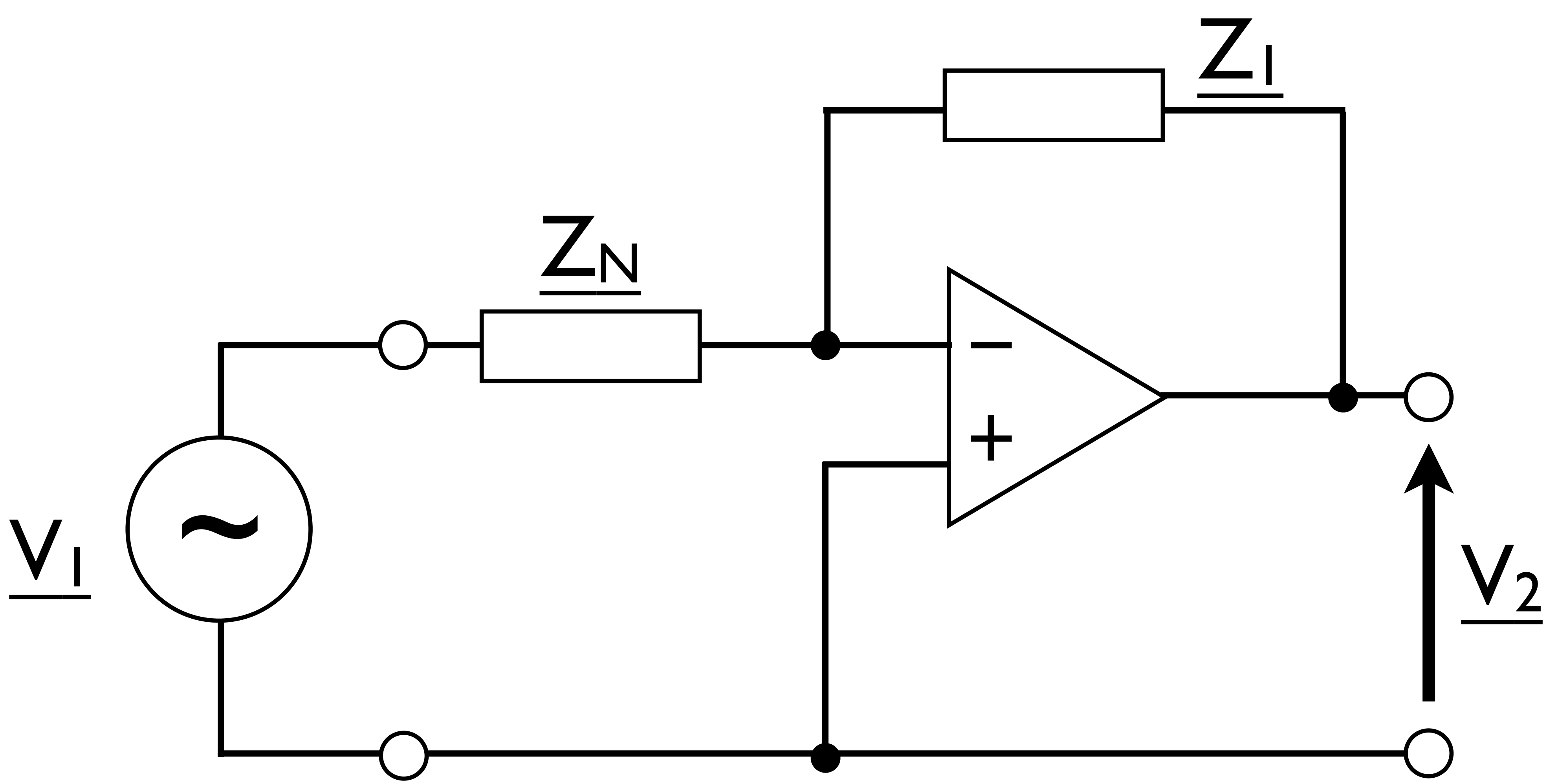
Imaginary

$$G_p = \frac{n_R}{n_X} \cdot \frac{N_R}{N_X} \cdot G_N$$

$$C_p = \frac{n_C}{n_X} \cdot \frac{N_C}{N_X} \cdot C_N$$

The unknown resistance and capacitance depend only on the ratio of the transformers

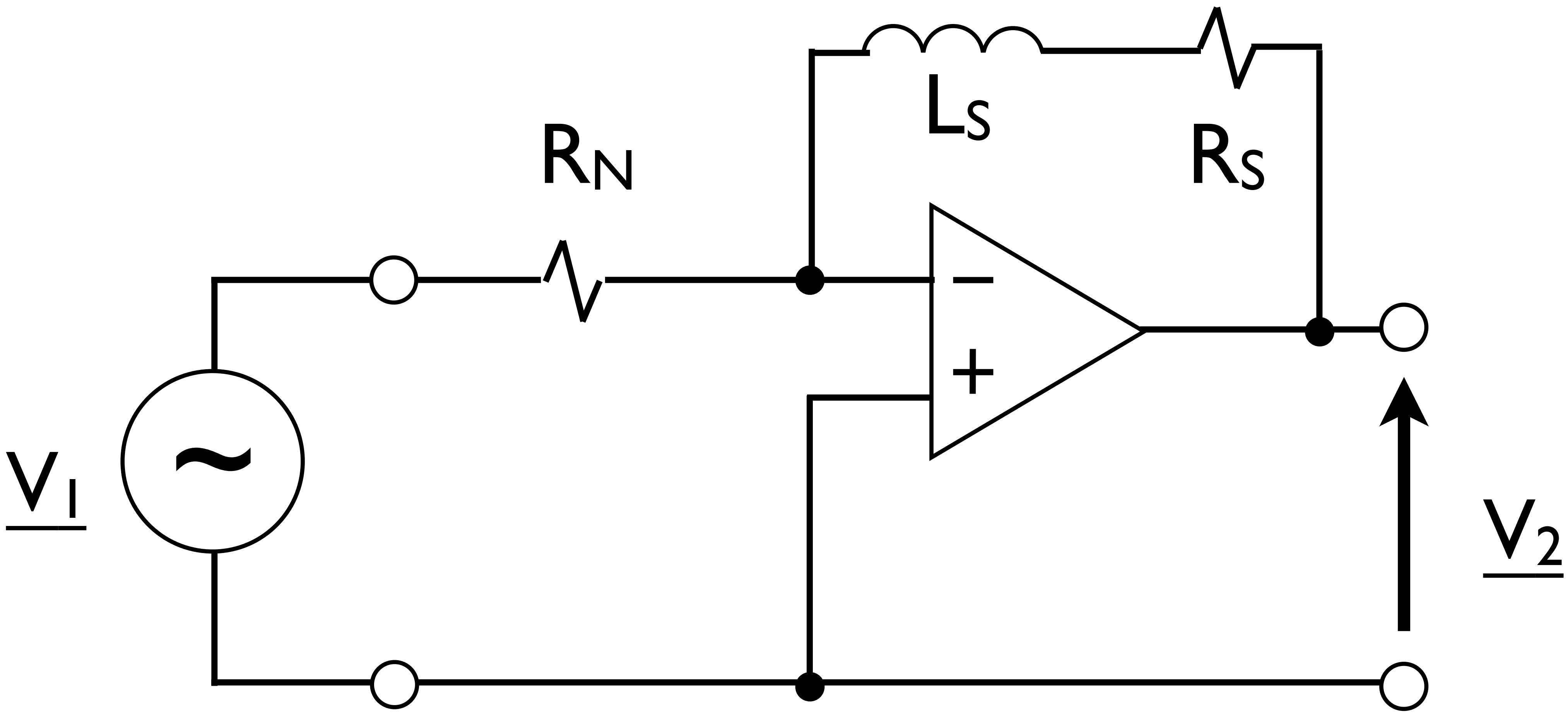
IMPEDANCE TO VOLTAGE CONVERTER



$$\underline{V}_2 = -\frac{\underline{Z}_1}{\underline{Z}_N} \cdot \underline{V}_1$$

The very same thing as Resistance to Voltage converter... but complex!

IMPEDANCE TO VOLTAGE CONVERTER



$$\underline{V_2} = -\frac{R_S + j\omega L_S}{R_N} \cdot \underline{V_1}$$

IMPEDANCE TO VOLTAGE CONVERTER

$$\underline{V_2} = -\frac{R_S + j\omega L_S}{R_N} \cdot V_1$$

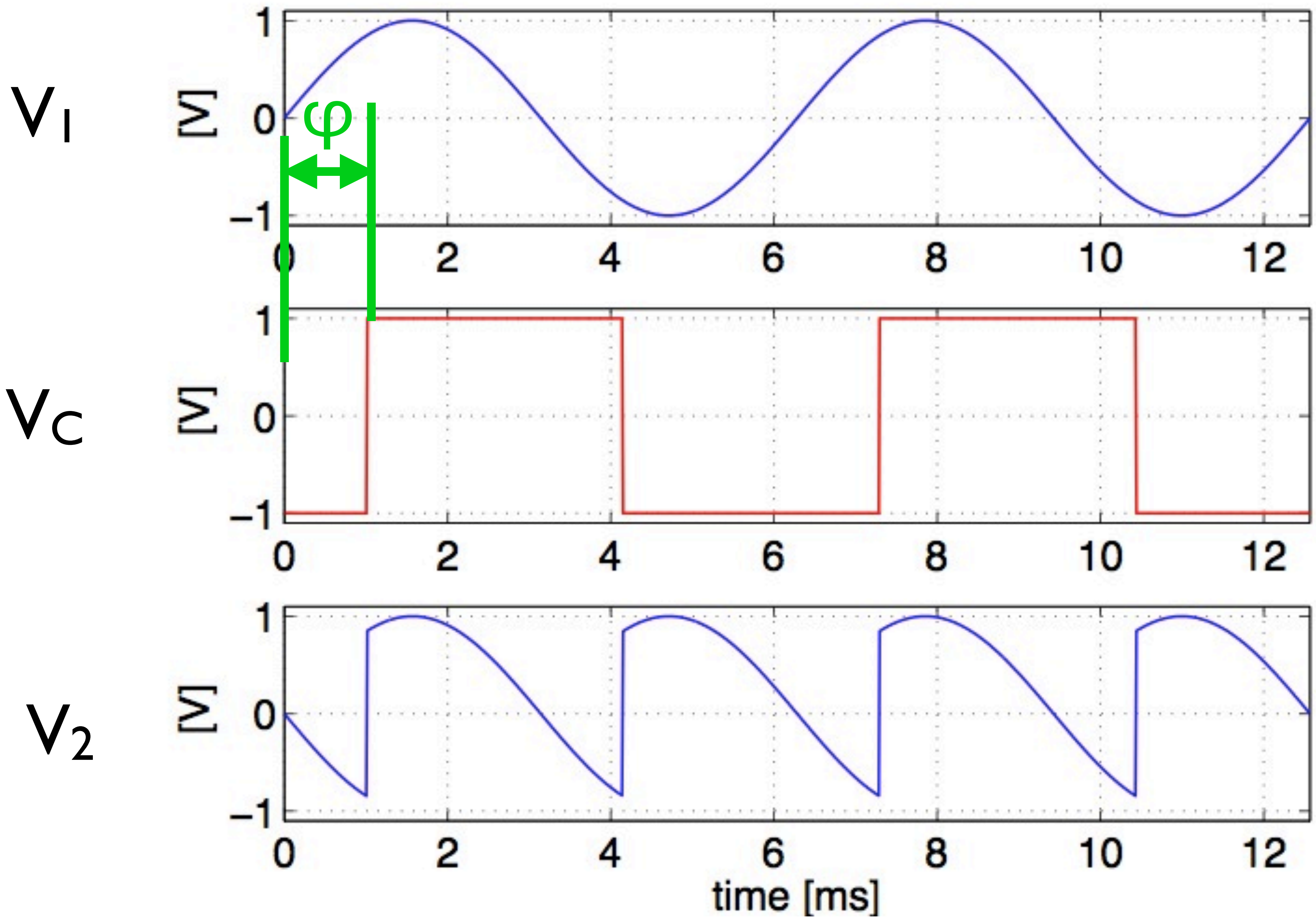
$$R_S + j\omega L_S = -R_N \cdot \frac{\underline{V_2}}{\underline{V_1}}$$

I consider V_1 at phase=0, so Real

$$R_S + j\omega L_S = -R_N \cdot \frac{\underline{V_2}}{V_1}$$

$$R_S = -R_N \cdot \frac{\mathrm{Re}\left(\underline{V_2}\right)}{V_1} \qquad L_S = -R_N \cdot \frac{\mathrm{Im}\left(\underline{V_2}\right)}{\omega V_1}$$

Back to lesson 2 - Controlled rectifier



$$\text{mean}(V_2) = \frac{1}{2} V_1 \frac{4}{\pi} \cos(\omega t - \varphi - \omega t) = \frac{2}{\pi} V_1 \cdot \cos\varphi$$

Shifting the control voltage by π/2 we measure the imaginary part instead of the real part of the voltage