Authors
CS 109

Handout #1
March 13–20, 2021

Take-Home Quiz

Exam # 00000

Take-Home Quiz information

Here are directions for how to use this LaTeX-generated document:

- Some information here

Name (typed or written): ____________________

Student ID: ____________________

1 Video Views: Binomial Approximation [10 points]

We are studying a video streaming platform. We define a viewer for a particular video as a user who watches the video in entirety. Let the number of viewers for each video be distributed as a Binomial random variable $\mathrm{Bin}(n, p)$, where n is the initial number of viewers and p is the probability that each initial viewer watches the entire video.

Suppose that the number of viewers for a popular video A is distributed as $\mathrm{Bin}(200, 0.4)$. Furthermore, suppose that the number of viewers for a not-so-popular video B is distributed as $\mathrm{Bin}(100, 0.15)$. What is the **approximate** probability that the number of viewers for video A is **more than twice** the number of viewers for video B?

Answer. Let $X \sim \mathrm{Bin}(n_1 = 200, p_1 = 0.4)$ and $Y \sim \mathrm{Bin}(n_2 = 100, p_2 = 0.15)$ be the number of viewers who watch videos A and B, respectively. We would like to compute $P(X > 2Y) = P(X - 2Y > 0)$.

Recall that we can use the Normal approximation if n is large (i.e., $n > 20$) and variance is large (i.e., $np(1 - p) > 10$); otherwise, if $n > 100$ and $p < 0.05$, we can use the Poisson approximation. We first note that X and Y can be approximated as independent Normal random variables, where $X \approx N_X \sim \mathcal{N}(\mu_X = 80, \sigma_X^2 = 48)$ and $Y \approx N_Y \sim \mathcal{N}(\mu_Y = 15, \sigma_Y^2 = 12.75)$. X and Y (and subsequently N_X and N_Y) are independent because their underlying Bernoulli trials are independent.

Next, we note that linear transformations of independent Normal random variables are Normal, and therefore $D = X - 2Y \sim \mathcal{N}(\mu_D = \mu_x - 2\mu_y = 50, \sigma_D^2 = \sigma_X^2 + (-2)^2\sigma_Y^2 = 99)$. Using a continuity correction to account for approximating a discrete difference in viewers with continuous random variables, we obtain $P(X - 2Y > 0) \approx P(N_x - 2N_Y > 0.5) = 1 - P(D \leq 0.5) = 1 - \Phi\left(\frac{0.5 - \mu_D}{\sigma_D}\right)$, where Φ is the Standard Normal CDF.

We compute this numerically using a Standard Normal Table and obtain $1 - \Phi(-4.975) \approx 1$.

2 Jointly Continuous Random Variables [10 points]

X and Y are jointly continuous random variables with the following joint PDF:

$$f_{X,Y}(x,y) = c(2x^2 + 2y) \qquad 0 \le x \le 1 \text{ and } 0 \le y \le 1 \tag{1}$$

Note that $f_{X,Y}(x,y)$ is a valid PDF if the constant $c = 3/5$. What is $E[Y]$? Provide a numeric answer (fractions are fine).

Answer.

$$\begin{aligned}
E[Y] &= \int\int y f_{X,Y}(x,y)\,dx\,dy = \int_0^1 \int_0^1 (3/5)y\left(2x^2 + 2y\right)\,dx\,dy \\
&= \int_0^1 (3/5)\left[\frac{2}{3}yx^3 + 2xy^2\right]_0^1 dy = \int_0^1 (3/5)\left(\frac{2(1)^3 y}{3} + 2(1)y^2\right) dy \\
&= (3/5)\left[\frac{2(1)^3}{6}y^2 + \frac{2(1)}{3}y^3\right]_0^1 = \frac{3}{5}
\end{aligned}$$

3 Mystery Code [10 points]

```
unsigned char mystery(unsigned char n) {
    n |= n >> 1;
    n |= n >> 1;
    n |= n >> 2;
    n++;
    return (n >> 1);
}
```

What does the following code print out?

```
printf("%u", mystery(88));
```

Answer. `64`

This problem can be solved programmatically by implementing the given algorithm.

That's the end of this quiz!