

lineaire tijdsafhankelijke

Systemen en hun

mathematisch model

Samenvatting

Gilles Collebaut

Hoofdstuk 1 : lineaire tydsongh. systemen en hun mathematisch model

Het mathematische model

Voor een LTO: $B_n \frac{d^n y}{dt^n} + \dots + B_0 y = A_m \frac{d^m u}{dt^m} + \dots + A_0 u$

lineariteit:

↳ voldoen aan superpositie-principe

$$+ \begin{aligned} u_1(t) &\rightarrow y_1(t) \\ u_2(t) &\rightarrow y_2(t) \end{aligned}$$

$$u_1 + u_2 \rightarrow y_1 + y_2$$

Tydsafhankelik:

dan
↳ $u(t) \rightarrow y(t)$
 $u(t-z) \rightarrow y(t-z)$

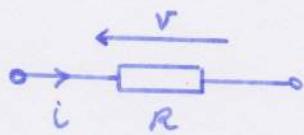
causaliteit:

op een bepaald tydstip t
↳ uitgangssignaal enkel afhanklik van op dat moment ingangssignaal

↑
kan de toekomst niet voorzien

Eenvoudige elektrische L.T.O.-Systemen

Weerstand

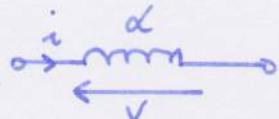


$$v = Ri$$

$$i = \frac{v}{R}$$

$$dW = v dq = Ri^2 dt \quad \leftarrow \text{warmte door joule-effect. (niet-reversibel)}$$

Spoel



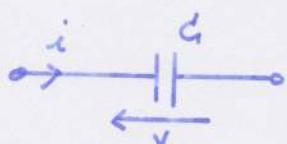
$$v = \alpha \frac{di}{dt}$$

$$i = \frac{1}{\alpha} \int_0^t v dt + i_0$$

$$dW = v dq = \alpha \frac{di}{dt} dq$$

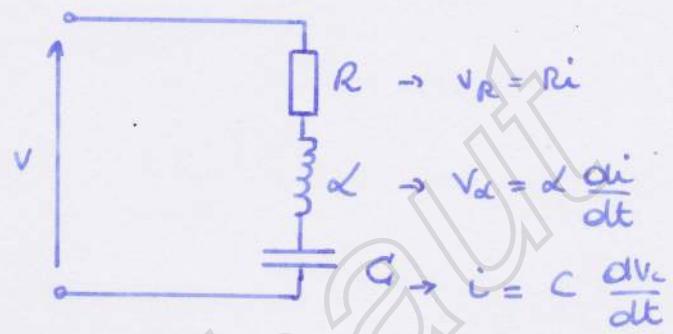
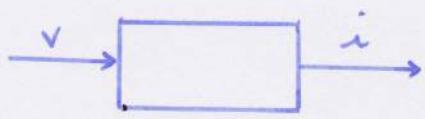
$$= \alpha d\left(\frac{i^2}{2}\right) \quad \leftarrow \text{magn. veld (reversibel)}$$

Condensator



$$v = v_0 + \frac{1}{C} \int_0^t i dt \quad \leftarrow \quad i = C \frac{dv}{dt}$$

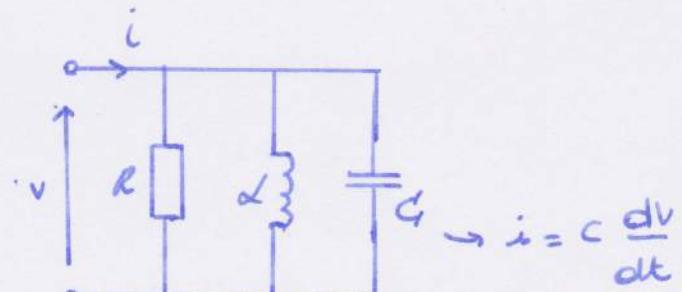
$$dW = v dq = \frac{1}{C} dq^2 = \alpha \left(\frac{q^2}{2C} \right) \quad \leftarrow \text{elektr. veld (reversibel)}$$



$$L \frac{di^2}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i = \frac{dv}{dt}$$

$$i = \frac{dq}{dt}$$

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = V$$

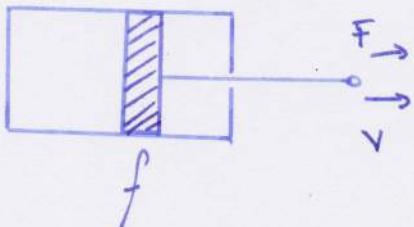


$$i_R = \frac{V}{R} \quad V = \alpha \frac{di}{dt}$$

$$C \frac{d^2v}{dt^2} + \frac{1}{R} \frac{dv}{dt} + \frac{1}{L} v = \frac{di}{dt}$$

Eenvoudige mechanische LTO.-systemen

wrijvingsweerstand

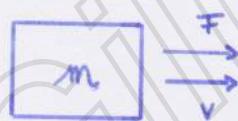


$$v = \frac{1}{f} F \quad F = fv$$

$$\begin{aligned} dW &= F dx \\ &= f v dx \\ &= f v^2 dt \end{aligned}$$

← wrijvingsweerstand (wrijte)

Massa

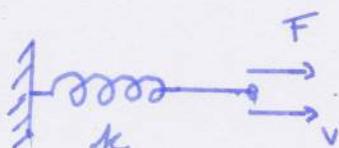


$$v(t) = v(0) + \int_0^t F dt \quad F = m \frac{dv}{dt}$$

$$dW = m \frac{dv}{dt} dx$$

$$= d\left(\frac{mv^2}{2}\right) \quad \leftarrow \text{kinetische energie}$$

veer



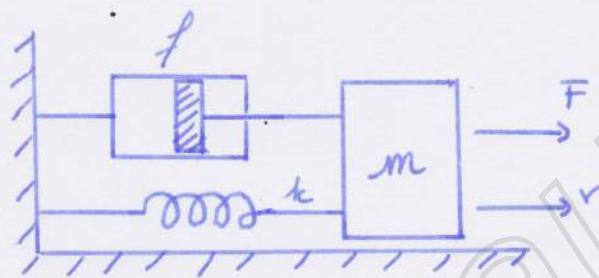
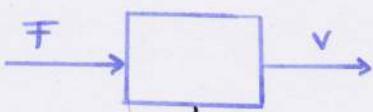
$$v = \frac{1}{k} \frac{dF}{dt}$$

$$F = kn$$

$$F(t) = F_0 + k \int_0^t v dt$$

$$dW = kn dx$$

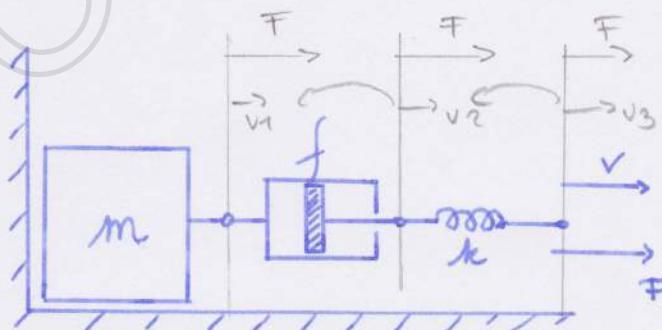
$$= d\left(k \frac{x^2}{2}\right) \quad \leftarrow \text{potentiële energie}$$



$$ma = \sum F \Leftrightarrow m \frac{dv}{dt} = F - fv - kn$$

$$m \frac{dv}{dt} + fv + kn = F \quad \downarrow v = \frac{dx}{dt}$$

$$m \frac{d^2x}{dt^2} + f \frac{dx}{dt} + kn = F$$



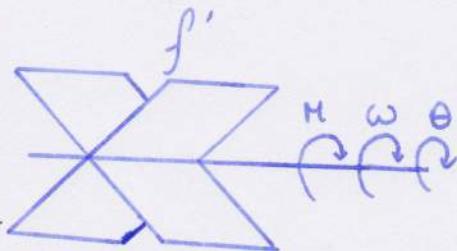
$$v = v_1 + v_2 + v_3 \quad \rightarrow \frac{dv_1}{dt} = \frac{F}{m}$$

$$\rightarrow v_2 = \frac{F}{f}$$

$$\rightarrow v_3 = \frac{1}{k} \frac{dF}{dt}$$

$$\rightarrow \frac{1}{k} \frac{d^2F}{dt^2} + \frac{1}{f} \frac{dF}{dt} + \frac{1}{m} F = \frac{dv}{dt}$$

Wrijingsweerstand



Rotatie

$$\omega = \frac{1}{f'} M$$

$$M = f' \omega$$

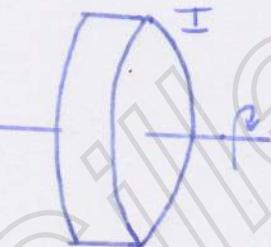
$$d\omega = M d\theta$$

$$= f' \omega d\theta$$

$$= f' \omega^2 dt$$

← wrijingsweerstand (waarde)

Traagheidsmoment

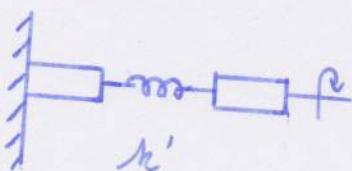


$$\omega(t) = \omega_0 + \frac{1}{I} \int_0^t M dt \quad \leftarrow \quad M = I \frac{d\omega}{dt}$$

$$d\omega = d\left(\frac{I\omega^2}{2}\right)$$

← kin. en.

Torsie - veer

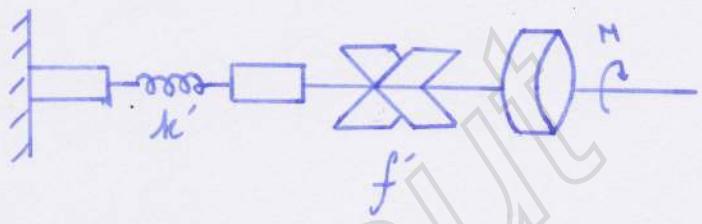
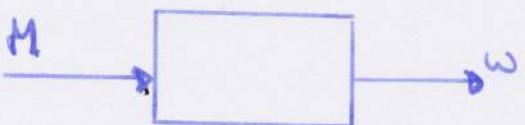


$$\omega = \frac{1}{k'} \frac{dt}{dt}$$

$$M = k' \theta$$

$$M(t) = M_0 + k' \int_0^t \omega dt$$

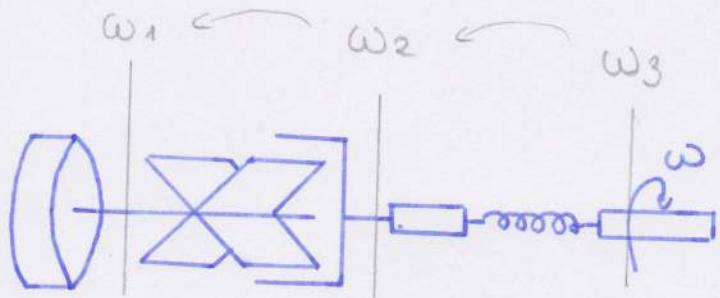
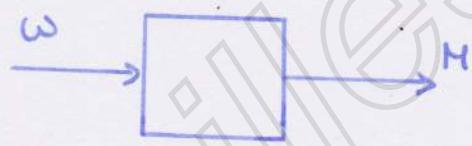
$$d\omega = d\left(\frac{k'\theta^2}{2}\right) \quad \leftarrow \text{pot. en.}$$



$$I \frac{d\omega}{dt} = \sum M = -k'\theta - f'\omega + M$$

$$\rightarrow I \frac{d^2\omega}{dt^2} + f' \frac{d\omega}{dt} + k' \omega = \frac{dM}{dt}$$

$$I \frac{d^2\theta}{dt^2} + f' \frac{d\theta}{dt} + k' \theta = M$$



$$\omega = \omega_1 + \omega_2 + \omega_3$$

$$M = I \frac{d\omega_1}{dt} \quad \omega_2 = \frac{1}{f'} M$$

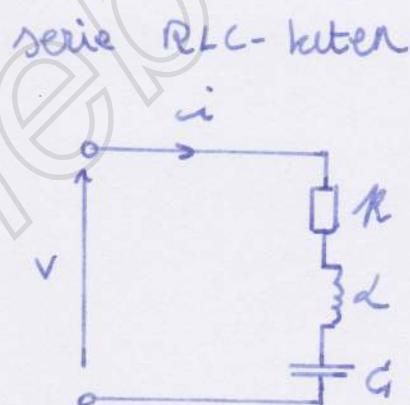
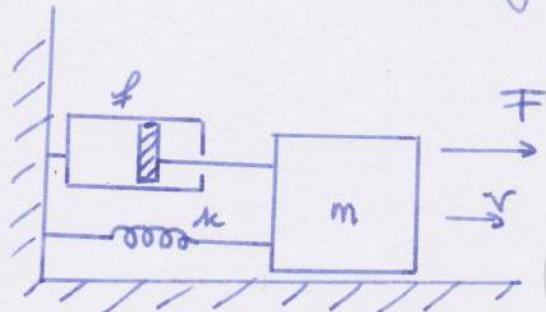
$$\omega_3 = \frac{1}{k'} \frac{dM}{dt}$$

$$\rightarrow \frac{1}{k'} \frac{d^2M}{dt^2} + \frac{1}{f'} \frac{dM}{dt} + \frac{1}{I} M = \frac{d\omega}{dt}$$

Analogie tussen systemen

\leftrightarrow + omrekeningsfactor

massa - veer - dumper systeem:



$$m \frac{d^2v}{dt^2} + f \frac{dv}{dt} + kv = \frac{df}{dt}$$

$$L \frac{d^2i}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i = \frac{dv}{dt}$$



$$\alpha = K_1 m$$

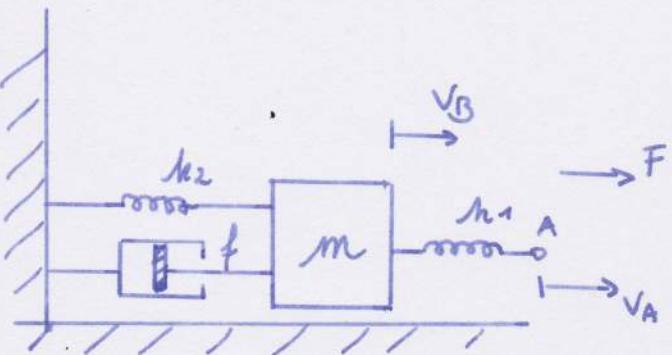
$$R = K_1 f$$

$$\frac{1}{C} = K_1 k$$

$$V(t) = K_2 F(t)$$

$$\rightarrow m \frac{d^2 \left[\frac{K_1}{K_2} i \right]}{dt^2} + f \frac{d \left[\frac{K_1}{K_2} i \right]}{dt} + k \left[\frac{K_1}{K_2} i \right] = \frac{df}{dt}$$

$$\rightarrow V(t) = \frac{K_1}{K_2} i(t)$$



$$\text{Veer 1: } V_A - V_B = \frac{1}{k_1} \frac{dF}{dt}$$

$$\text{massa: } m \frac{dV_B}{dt} + f V_0 + k_2 \int_0^t V_B dt = F$$

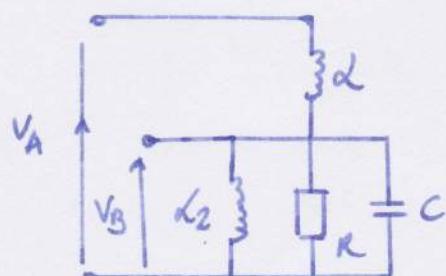
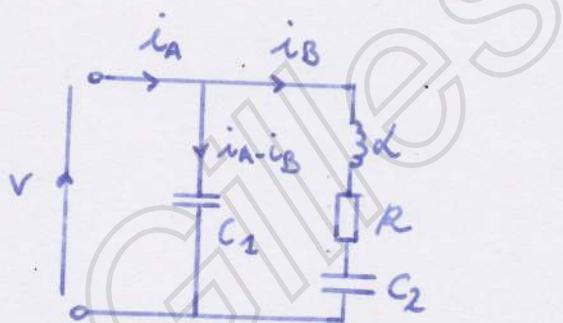
Analogie

$$L \frac{di_B}{dt} + R i_B + \frac{1}{C_2} \int_0^t i_B dt = v$$

$$C \frac{dV_B}{dt} + \frac{V_B}{R} + \frac{1}{L} \int_0^t V_B dt = i$$

$$i_A - i_B = C_1 \frac{dv}{dt}$$

$$V_A - V_B = L_1 \frac{di}{dt}$$



$$V_A(t) = \frac{k_1}{k_2} i_A(t)$$

$$V_A(t) = \frac{k_1}{k_2} V_A(t)$$

Snelheid

$$V_B(t) = \frac{k_1}{k_2} i_B(t)$$

$$V_B(t) = \frac{k_1}{k_2} V_B(t)$$

Snelheid

$$\frac{d}{dt} = \frac{1}{a} \frac{d}{dt} \quad \begin{cases} a < 1: \text{ versneld} \\ a > 1: \text{ vertraagd} \end{cases}$$

linearisatie

niet-lineair verg

$\xrightarrow{\text{Taylor}}$ als
L.T.O. systemen
bestuderen

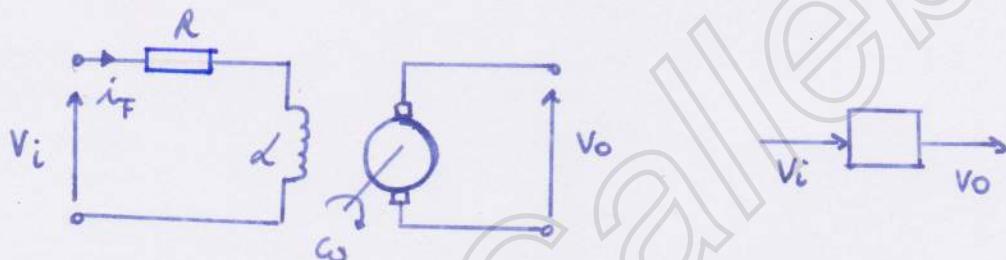
$f(n)$ berekening dicht bij n_0

$$\rightarrow f(n) = f(n_0) + \frac{df}{dn} \Big|_{n_0} (n - n_0) + \dots$$

als $(n - n_0)$ voldoende klein.

Eenvoudige elektromagnetische systemen

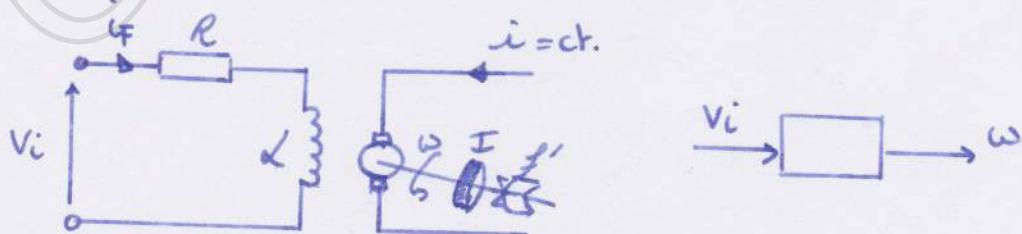
De Shunt-generator



$$\left. \begin{aligned} V_i &= R i_F + \alpha \frac{d i_F}{dt} \\ v_o &= K i_F \end{aligned} \right\} \rightarrow \frac{\alpha}{K} \frac{dv_o}{dt} + \frac{R}{K} v_o = V_i$$

eerste orde
systeem

Een belaste dc-motor



$$\alpha \frac{di_F}{dt} + R i_F = V_i$$

$$M = K i_F$$

$$M = f' \omega + I \frac{d\omega}{dt}$$

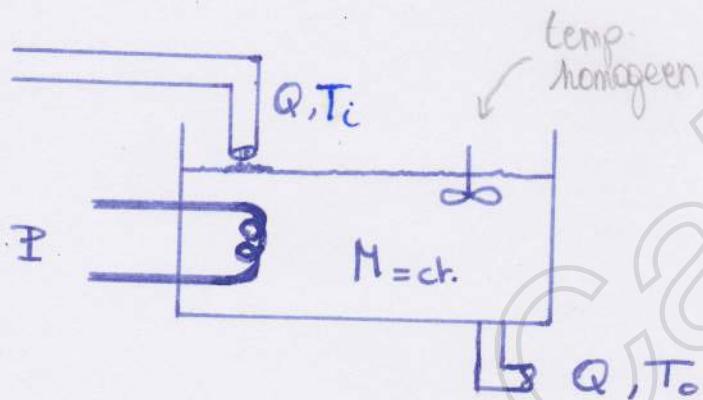
↓

tweede orde
systeem

$$\frac{d^2\omega}{dt^2} + \left(\frac{L'}{I} + \frac{R}{\alpha} \right) \frac{d\omega}{dt} + \frac{R f'}{I} \omega = \frac{K}{\alpha I} V_i$$

Eenvoudige L.T.O. processen

Een verwarmingsketel



bij evenwicht:

$\frac{P}{Q}$
tauw.
warmte

[J/s]

$$= C_h \cdot Q \cdot (T_o - T_i)$$

$\underbrace{ }$ $\underbrace{ }$ $\underbrace{ }$

zoektyke- dubiet temp.
warmte warmte verschil

[J/kg.k] [kg/s] [K]

$$\rightarrow T_{oe} = T_{ie} + \frac{P}{C_h Q}$$

energibalans in dt:

$$\rightarrow T_i = T_{ie} + t_i$$

$$T_o = T_{oe} + t_o$$

$$P dt = \underbrace{C_h Q (T_o - T_i) dt}_{\text{opwarmen instromende vloeistof}} + \underbrace{C_h N dt}_{\text{gewarmen rd ketel in zijn geheel}}$$

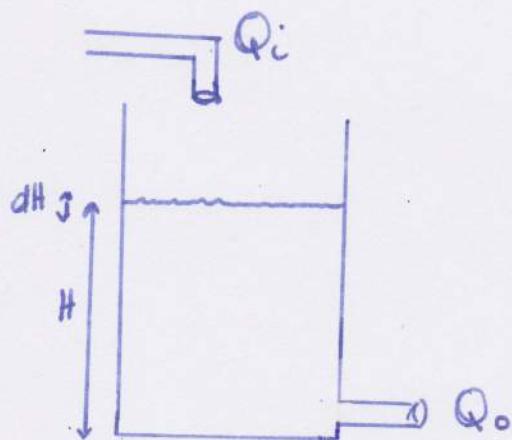
opwarmen
instromende
vloeistof

gewarmen
rd ketel
in zijn geheel

$$\Rightarrow P dt = C_h Q [(T_{oe} + t_o) - (T_{ie} + t_i)] dt + C_h N d (T_{oe} + t_o)$$

$$\Rightarrow \frac{M}{Q} \frac{dt_o}{dt} + t_o = t_i$$

Een vloeistoftank



$$Q_o = C \sqrt{H}$$

$$\text{evenwicht: } Q_{ie} = Q_{oe} = C \sqrt{H_e}$$

over kleine dt:

$$Q_i \, dt = Q_o \, dt + A \, dH$$

⌊ ⌋ ⌊ ⌋
 instroomdubiet uitstroomdubiet aangrenzende haardheid in tank

Variaties over evenw.: $Q_i = Q_{ie} + q_i$

$$Q_o = Q_{oe} + q_o$$

$$H = H_e + h$$

$$\Rightarrow A \frac{dH}{dt} + q_o = q_i \quad (\alpha)$$

verband $h \propto q_o$

nt. lineair: $Q_{oe} + q_o = C \sqrt{H_e + h}$

Taylor

$$Q_o(H) = Q_o(H_e) + \left. \frac{dQ_o}{dH} \right|_{H_e} (H - H_e)$$

$$\underbrace{Q_o(H) - Q_o(H_e)}_{Q_{oe}} = \left. \frac{dQ_o}{dH} \right|_{H_e} (H - H_e)$$

$$q_{io} = \left. \frac{dQ_o}{dH} \right|_{H_e} \cdot h$$

$$q_0 = \left. \frac{dQ_0}{dt} \right|_{He} \cdot h$$

$$\rightarrow q_0 = \left. \frac{d(C\sqrt{H})}{dt} \right|_{He} \cdot h$$

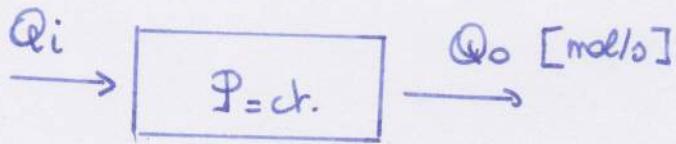
$$q_0 = \frac{C}{L\sqrt{He}} \cdot h = \frac{C\sqrt{He}}{2He} \xrightarrow{\text{Qoe}} Q_{oe}$$

$$q_0 = \frac{Q_{oe}}{2He} \cdot h \quad \text{in (a)}$$

$$\Rightarrow \frac{\lambda A He}{Q_{oe}} \frac{dq_0}{dt} + q_0 = q_i$$

← eerste orde
systrem

Een gasreservoir



$$Q_o = C \sqrt{P}$$

$$Q_{ic} = Q_{oc} = C \sqrt{P_e}$$

algemene
gaswet: $M_e = \frac{P_e V}{R T}$

massabalans: $Q_i dt = Q_o dt + \underbrace{dm}_{\text{hoeveelheid gas in reservoir}}$

evenw.: $Q_i = Q_{ic} + q_i$

$$Q_o = Q_{oc} + q_o$$

$$P = P_e + p$$

$$M = M_e + m$$

Verband p en m : $M_e + m = \frac{(P_e + p)V}{R T}$

$$M_e = \frac{P_e V}{R T}$$

$$m = \frac{M_e}{P_e} p$$

$$\frac{M_e}{P_e} \frac{dp}{dt} + q_o = q_i$$

Verband per q_o : $Q_{oc} + q_o = C \sqrt{P_e + p}$

Taylor {

$$p = \frac{2P_e}{Q_{oc}} q_o$$

eerste orde
systeem

$$\frac{2M_e}{Q_{oc}} \frac{dq_o}{dt} + q_o = q_i$$

Besluit

eerste - orde systeem:

$$\frac{dy(t)}{dt} + y(t) = K u(t)$$

↑
statische
verstärkungsfaktor

tweede- orde systeem:

dempingsgraad

$$\frac{1}{\omega_n^2} \frac{d^2 y(t)}{dt^2} + \frac{2z}{\omega_n} \frac{dy(t)}{dt} + y(t) = K u(t)$$

eigen freq. $\left. \begin{array}{c} \\ \end{array} \right\}$ statische
versteckungsfaktor

Hoofdstuk 2

Studie van L.T.O systemen
in het
tijdsdomein

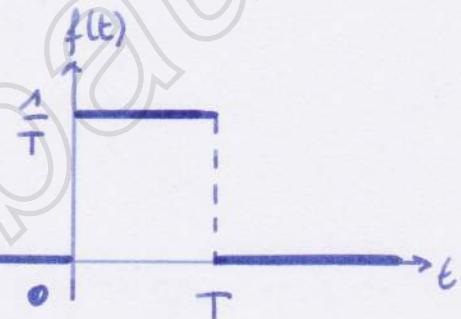
Samenvatting

Gilles Callebaut

Impulsresponsie

Impulsfunctie

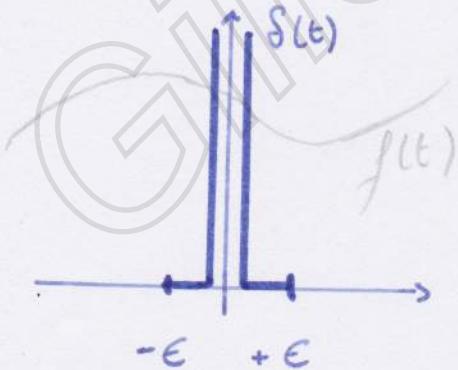
Diracfunctie: $\delta(t) = \lim_{T \rightarrow 0} f(t)$



↓

$$\int_{-\epsilon}^{+\epsilon} \delta(t) dt = 1$$

↑ impulsfunctie met intensiteit 1



$$\begin{cases} f(t) = 0 & t < 0 \\ f(t) = \frac{1}{T} & 0 \leq t \leq T \\ f(t) = 0 & t > T \end{cases}$$

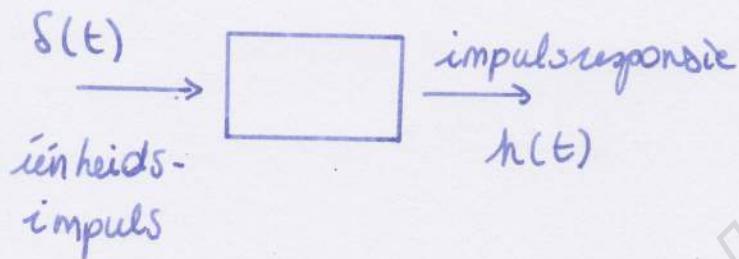
eigenschap: $\int_{-\epsilon}^{+\epsilon} f(t) \cdot \delta(t) dt = f(0)$

Dirac op tydstip z:

$$\int_{z-\epsilon}^{z+\epsilon} \delta(t-z) dt = 1$$

$$\int_{z-\epsilon}^{z+\epsilon} f(t) \delta(t-z) dt = f(z)$$

De Impulsresponsie



eerste-order systeem:

$$\zeta \frac{dy}{dt} + y(t) = Ku(t)$$

$u(t) = \delta(t)$

impulsresponsie:

$$h(t) = 0 \quad t < 0$$

$$\zeta \frac{dh}{dt} + h = 0 \quad t > 0$$

$$\zeta [h(t)]_{-\epsilon}^{+\epsilon} + \int_{-\epsilon}^{+\epsilon} h(t) dt = K \int_{-\epsilon}^{+\epsilon} \delta(t) dt \quad \text{interval } [-\epsilon, +\epsilon]$$

$$\rightarrow \zeta [h(+\epsilon) - h(-\epsilon)] + \underbrace{\int_{-\epsilon}^{+\epsilon} h(t) dt}_{=0 \atop \text{want g'n responsie}} = K \int_{-\epsilon}^{+\epsilon} \delta(t) dt$$

$\underset{\epsilon \rightarrow 0}{\lim}$

$\underset{\epsilon \rightarrow 0}{\lim}$

$$\rightarrow h(0^+) = \frac{K}{\zeta} \quad (\text{begin voorwaarde})$$

We weten:

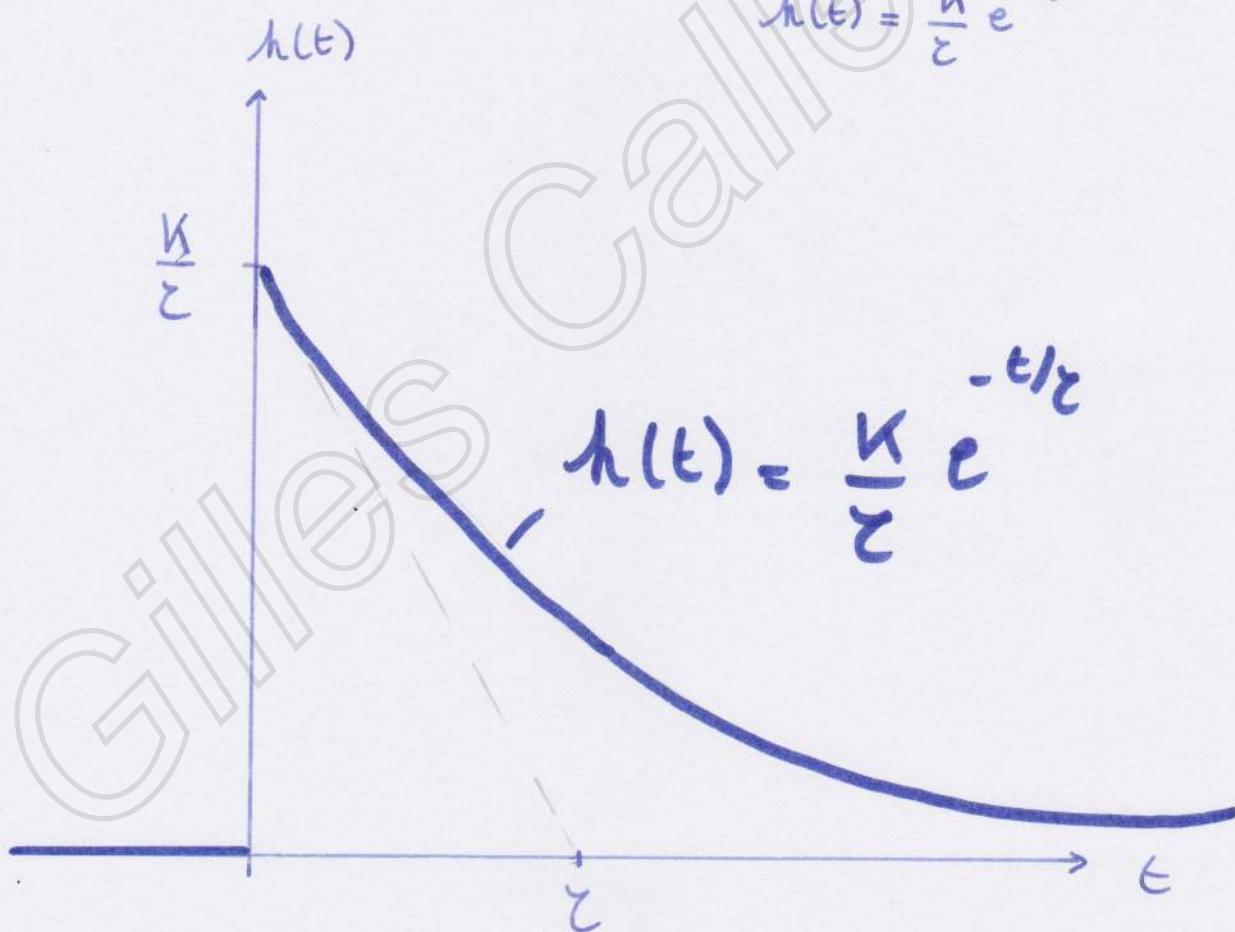
$$\zeta \frac{dh}{dt} + h = 0$$

met beginVW: $h(0^+) = \frac{K}{\zeta}$

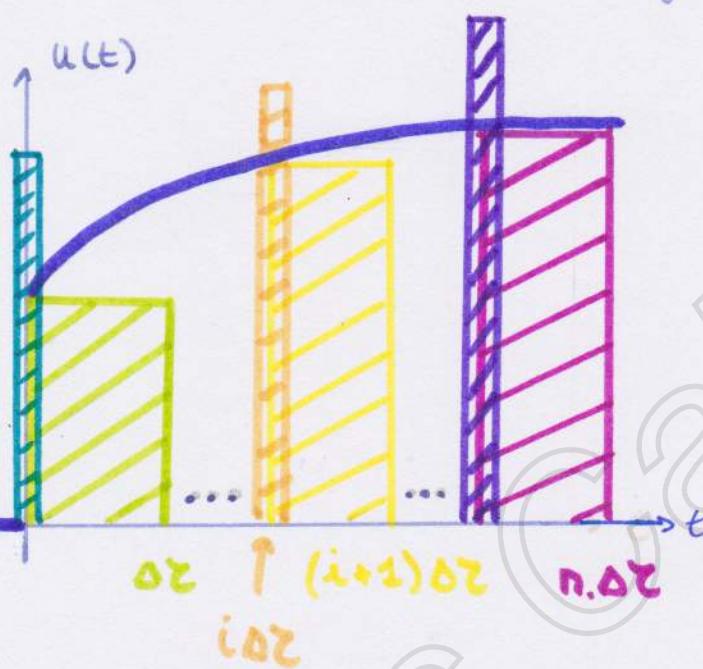
$$\rightarrow \zeta \lambda + 1 = 0$$

$$\hookrightarrow \lambda = -\frac{1}{\zeta} \rightarrow h(t) = A e^{-\frac{t}{\zeta}} \text{ met BVW}$$

$$h(t) = \frac{K}{\zeta} e^{-\frac{t}{\zeta}}$$



De responsie op een willekeurige ingang



$u(t)$ beschouwen als som van impulsfuncties.

$y(t)$ som v. alle responsies op de impulsfuncties.

Opp interval: $u(i\Delta t) \cdot \Delta t \xrightarrow{\text{begin}} u(i\Delta t) \Delta t \delta(t - i\Delta t)$
eigensch. dirac.

impulsrespons:

$$u(i\Delta t) \Delta t h(t - i\Delta t)$$

ingang: $\sum_{i=0}^{n-1} u(i\Delta t) \Delta t \cdot \delta(t - i\Delta t)$

uitgang: $\sum_{i=0}^{n-1} u(i\Delta t) \Delta t \cdot h(t - i\Delta t)$

$$\left(t' = i\Delta t \quad \Delta t' = \Delta t \right)$$

$$\sum_{t'=0}^{t-\Delta t} u(t') h(t-t') \Delta t'$$

$$\left(\Delta t' \rightarrow dt' \right)$$

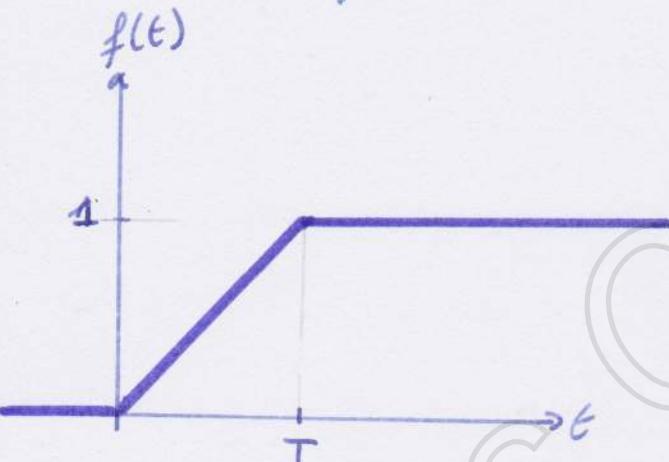
van de eerste noot

convolutie-integraal

$$\rightarrow y(t) = \int_0^t u(t') h(t-t') dt'$$

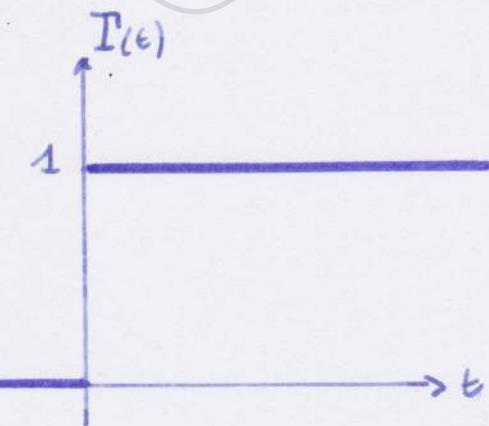
Stapresponsie

De Stafunctie



Heavyside:

$$I(t) = \lim_{T \rightarrow 0} f(t)$$



$$\begin{cases} f(t) = 0 & \text{voor } t < 0 \\ f(t) = \frac{t}{T} & \text{voor } 0 \leq t \leq T \\ f(t) = 1 & \text{voor } t > T \end{cases}$$

$$\rightarrow \begin{cases} \frac{df}{dt} = 0 & \text{voor } t < 0 \\ \frac{df}{dt} = \frac{1}{T} & \text{voor } 0 \leq t < T \\ \frac{df}{dt} = 0 & \text{voor } t > T \end{cases}$$

bij limietovergang:
 $f(t) \rightarrow \delta(t)$

$$\delta(t) = \frac{dI(t)}{dt}$$

$$I(t) = \int_{-\infty}^t \delta(t) dt$$

op tydstip τ : $I(t-\tau) = 0$ voor $t < \tau$

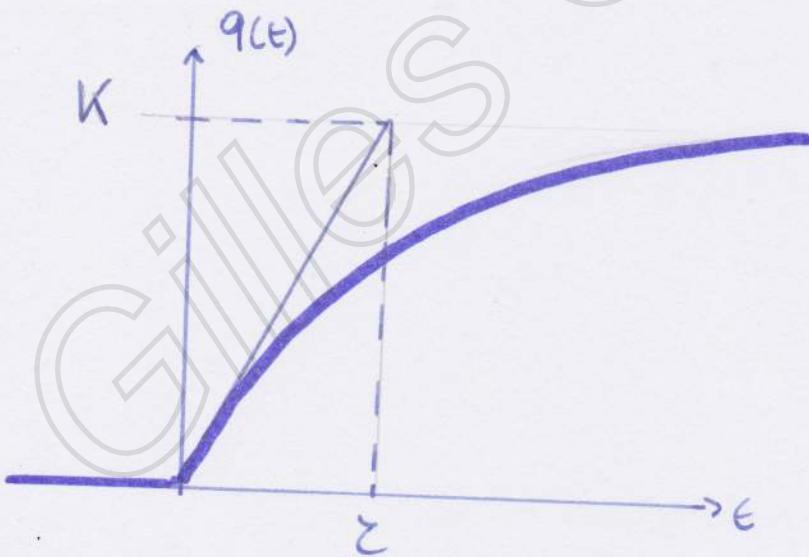
$$I(t-\tau) = 1 \quad \text{voor } t \geq \tau$$

De stapresponsie

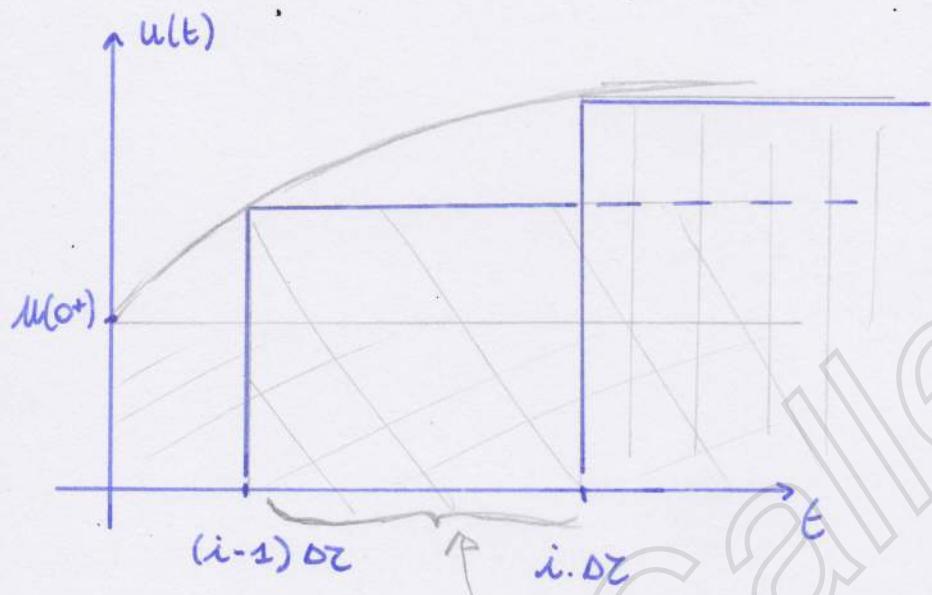
convolutie-integraal : $\Gamma(t) * h(t)$

$$\rightarrow q(t) = \int_0^t 1 \cdot \frac{K}{\zeta} e^{-t'/\zeta} dt' = \left(\frac{K}{\zeta} - \zeta \right) \left[e^{-t'/\zeta} \right]_0^t$$

$$q(t) = K \left(1 - e^{-t/\zeta} \right)$$



de convolutie-integraal van tweede soort



stapfunctie:
in het interval

$$[u(i\Delta t) - u((i-1)\Delta t)] \cdot I(t - i\Delta t)$$

↓
responsie:

$$[u(i\Delta t) - u((i-1)\Delta t)] \cdot q(t - i\Delta t)$$

ingang $u(t)$: $u(0+) I(t) + \sum_{i=0}^n [u(i\Delta t) - u((i-1)\Delta t)] I(t - i\Delta t)$

responsie: $u(0+) q(t) + \sum_{i=0}^n [u(i\Delta t) - u((i-1)\Delta t)] q(t - i\Delta t)$

met $t' = i\Delta t$ is $\Delta t' = \Delta t$
en $\Delta t \rightarrow dt'$

$$\Rightarrow y(t) = u(0+) \cdot q(t) + \int_0^t \frac{du(t')}{dt'} q(t-t') dt'$$

Harmonische response

Harmonische functies

$$F \cos(wt + \varphi)$$

↑ pulsatie [rad/s]

↑ amplitude

↑ fase [rad]

C.N.

$$\tilde{F} = F e^{j\varphi}$$

↑ argument

modulus

afleiden:

$$\tilde{F} \cdot j\omega$$

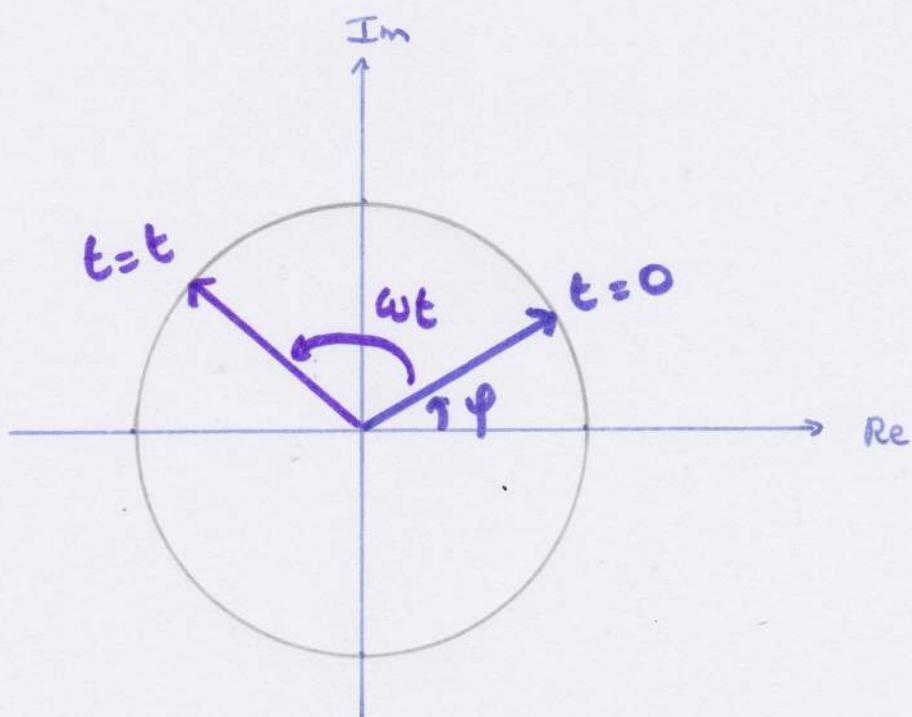
integrieren:

$$\tilde{F} \cdot \frac{1}{j\omega}$$

$$\tilde{f}(t) = \tilde{F} \cos(wt + \varphi) + j \tilde{F} \sin(wt + \varphi)$$

$$= \tilde{F} e^{j(wt + \varphi)}$$

$$= \tilde{F} e^{j\varphi} e^{jwt} = \tilde{F} e^{j\varphi} e^{jwt}$$



een complexe harmonische functie $\tilde{f}(t)$ in complex vlak

De harmonische responsie

$$u(t) = U \cos(\omega t + \alpha) \rightarrow y(t) = Y \cos(\omega t + \beta)$$

$$\bar{U} = U e^{j\alpha}$$

$$\bar{Y} = Y e^{j\beta}$$

harmonische responsie = P.O. vd diff. verg:

$$B_n \frac{d^2y}{dt^n} + \dots + B_0 y = A_m \frac{d^m u}{dt^m} + \dots + A_0 u$$

↓ C.V.

$$B_n (j\omega)^n \bar{Y} + \dots + B_0 \bar{Y} = A_m (j\omega)^m \bar{U} + \dots + A_0 \bar{U}$$

$$\Rightarrow \bar{Y} = H(j\omega) \bar{U} \quad \text{met } H(j\omega) = \frac{A_m (j\omega)^m + \dots + A_0}{B_n (j\omega)^n + \dots + B_0}$$

complexe overdrachtsfunctie
of
complexe transferfunctie

$$\left\{ \begin{array}{l} Y = |H(j\omega)| \bar{U} \\ \arg \bar{Y} = \arg H(j\omega) + \arg \bar{U} \\ \beta = \arg H(j\omega) + \alpha \end{array} \right.$$

fasevoerings
van
uitgang op ingang

Complexe overdracht functie

1^{ste} orde:

$$\zeta \frac{dy}{dt} + y = K.u$$

$$\rightarrow H(j\omega) = \frac{K}{1 + j\omega\zeta}$$

2^{de} orde:

$$\frac{1}{\omega_n^2} \frac{d^2y}{dt^2} + \frac{2\zeta}{\omega_n} \frac{dy}{dt} + y = K.u$$

$$\rightarrow H(j\omega) = \frac{K}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta \frac{j\omega}{\omega_n} + 1}$$

De responsie op een willekeurig ingangssignaal

periodiek ingangssignaal \rightarrow Fourierreeksontwikkeling

$$u(t) = u_0 + \sum_{h=1}^{\infty} U_h \cos(\omega_h t + \alpha_h)$$

$$\rightarrow \text{responsie: } \bar{Y}_h = H(j\omega_h) \bar{U}_h$$

$$Y_h e^{j\phi_h} \rightarrow y(t) = y_0 + \sum_{h=1}^{\infty} Y_h \cos(\omega_h t + \phi_h)$$

niet-periodiek ingangssignaal

$$\rightarrow u(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} V(j\omega) e^{j\omega t} d\omega$$

$$\text{met } V(j\omega) = \int_{-\infty}^{+\infty} u(t) e^{-j\omega t} dt$$

\rightarrow responsie:

$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(j\omega) V(j\omega) e^{j\omega t} d\omega$$

$$Y(j\omega) = H(j\omega) \bar{V}(j\omega)$$

De grafische voorstelling van de complexe overdrachtfunctie

De Bode-diagramma's

Amplitude karakteristiek

$$|H(j\omega)|_{dB} = 20 \log |H(j\omega)|$$

1ste orde:

$$H(j\omega) = \frac{K}{1 + j\omega\tau}$$

modulus: $|H(j\omega)|_{dB} = 20 \log K - 20 \log \sqrt{1 + \omega^2\tau^2}$

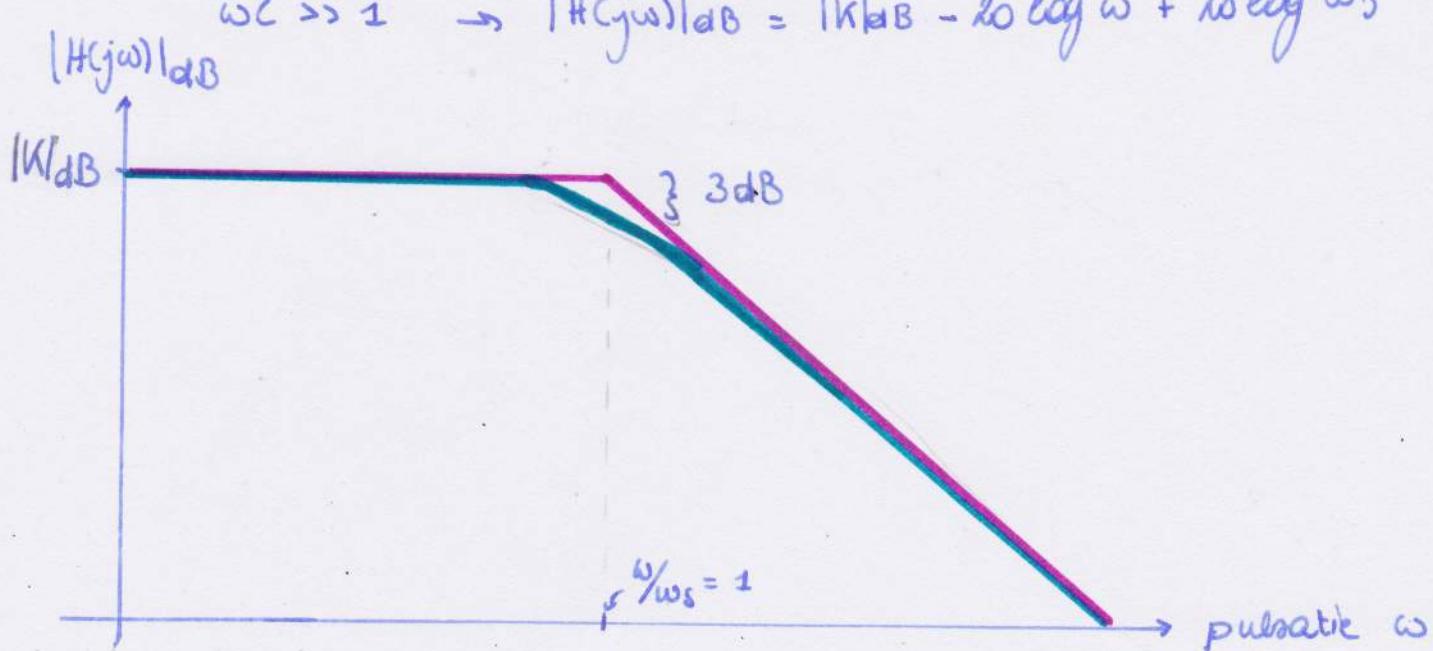
($\omega_s = \frac{1}{\tau}$ kritische freq.)

$$= 20 \log K - 20 \log \sqrt{1 + \frac{\omega^2}{\omega_s^2}}$$

$$\omega\tau \ll 1 \rightarrow |H(j\omega)|_{dB} = |K|_{dB}$$

$$\omega\tau = 1 \rightarrow |H(j\omega)|_{dB} = |K|_{dB} - 3$$

$$\omega\tau \gg 1 \rightarrow |H(j\omega)|_{dB} = |K|_{dB} - 20 \log \omega + 20 \log \omega_s$$



fase karakteristick:

1ste orde: $\frac{K}{1+j\omega z}$

argument: $\arg H(j\omega) = -\arg(1+j\omega z)$

met

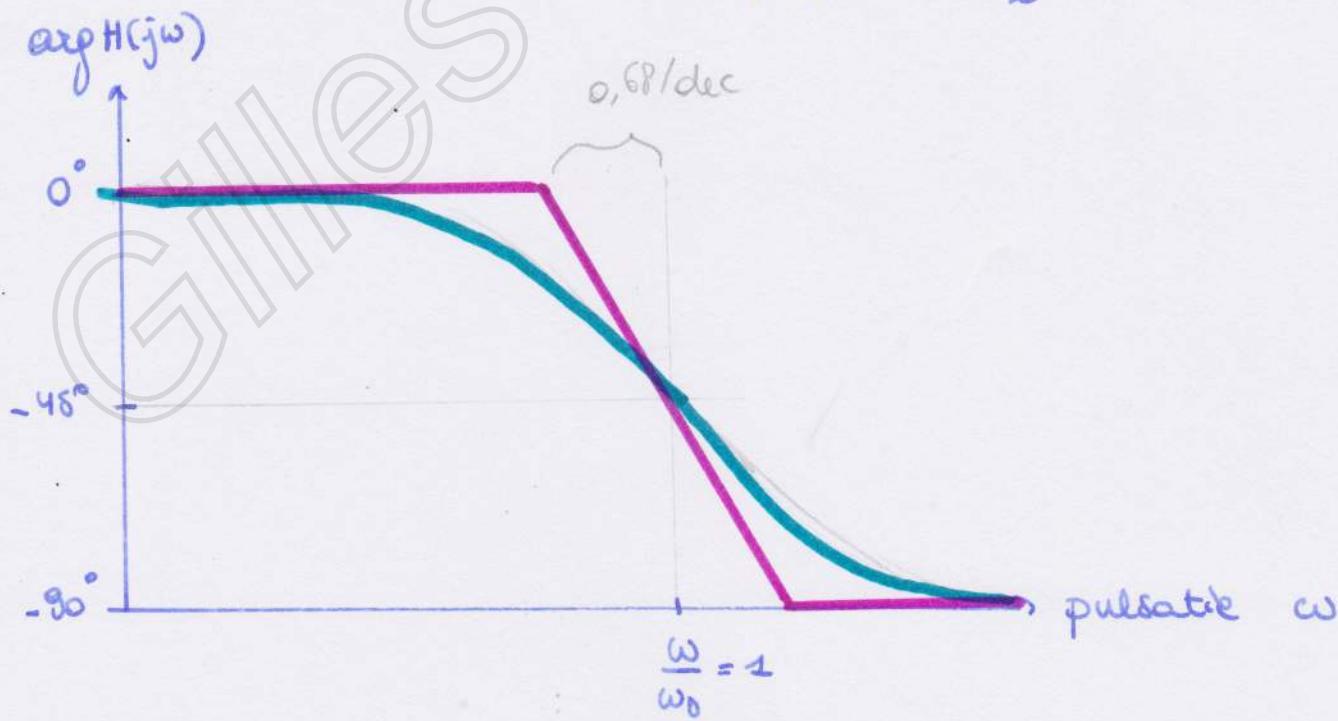
$$K > 0 : \arg = 0$$

$$K < 0 : \arg = \pi$$

$$\omega z \ll 1 : \arg H = 0$$

$$\omega z = 1 : \arg H = -\arg(1+j) = -\frac{\pi}{4} (-45^\circ)$$

$$\omega z \gg 1 : \arg H = -\arg(j\omega z) = -\frac{\pi}{2} (-90^\circ)$$



voor een 2de orde systeem:

$$H(j\omega) = \frac{K}{(j\frac{\omega}{\omega_n})^2 + 2z\frac{j\omega}{\omega_n} + 1} = \frac{K}{[1 - (\frac{\omega}{\omega_n})^2] + j(2z\frac{\omega}{\omega_n})}$$

$$|H(j\omega)|_{dB} = 20 \log K - 20 \log \sqrt{[1 - (\frac{\omega}{\omega_n})^2]^2 + (2z\frac{\omega}{\omega_n})^2}$$

$$\frac{\omega}{\omega_n} \ll 1 \rightarrow |H(j\omega)|_{dB} = 0 \quad \text{met } K=1$$

$$\frac{\omega}{\omega_n} \gg 1 \rightarrow |H(j\omega)|_{dB} = -20 \log \left(\left(\frac{\omega}{\omega_n}\right)^4\right) = -40 \log\left(\frac{\omega}{\omega_n}\right)$$

maximum bij: $\frac{d}{du} |H(j\omega)| = 0$

$$2(1-u^2)(-2u) + (2z)^2 \cdot 2u = 0$$

$$\hookrightarrow u[u^2 + 2z^2 - 1] = 0$$

$$\begin{array}{l} u=0 \\ \Rightarrow u = \sqrt{1-2z^2} \end{array}$$

$$\text{met VW: } 1-2z^2 > 0$$

$$z < \frac{1}{\sqrt{2}} \approx 0,707$$

$$\rightarrow \omega_R = \omega_n \sqrt{1-2z^2}$$

$$\rightarrow Q = \frac{1}{2z\sqrt{1-z^2}}$$

maximale amplitudeverhouding

$$Q = |H(j\omega)|_{\omega=\omega_R}$$

fig 2.21
p51

$$\arg H(j\omega) = -\arg [(1-u^2) + j(2zu)]$$

$$u=1 : \quad \arg H = -\arg (j2z) = -\frac{\pi}{2}$$

$$u \ll 1 : \quad \arg H = -\arg (1) = 0$$

$$u \gg 1 : \quad \arg H = -\arg [(1-u^2) + j(2zu)] = -\pi$$

bepaald door u^2

fig 2.22 en fig 2.23 p52

Algemeen

$$H(j\omega) = \frac{A_m(j\omega)^m + \dots + A_0}{B_n(j\omega)^n + \dots + B_0}$$

voor lage frequenties:

- $A_0 \neq 0$
- $B_0 \neq 0$

$$\left. \begin{array}{l} \\ \end{array} \right\} \rightarrow H(j\omega) = \frac{A_0}{B_0}$$

- $B_0 \neq 0$
- $A_{k-1}, \dots, A_0 = 0$

$$\left. \begin{array}{l} \\ \end{array} \right\} \rightarrow H(j\omega) = \frac{A_k}{B_0} (j\omega)^k$$

Modulus: $|H(j\omega)| \rightarrow 0$

ampl. ker. $k(+20) \text{ dB/dec}$

Argument: $\arg H(j\omega) = \arg \frac{A_k}{B_0} + k \frac{\pi}{2}$

- $A_0 \neq 0$
- $B_{k-1}, \dots, B_0 = 0$

$$\left. \begin{array}{l} \\ \end{array} \right\} \rightarrow H(j\omega) = \frac{A_0}{B_k} \frac{1}{(j\omega)^k}$$

Modulus: $|H(j\omega)|$ varmt ω

ampl. ker. $k(-20) \text{ dB/dec}$

Argument: $\arg H(j\omega) = \arg \frac{A_0}{B_k} + k \left(-\frac{\pi}{2} \right)$

voor hoge frequenties:

$$H(j\omega) = \frac{A_m}{B_n} \frac{1}{(j\omega)^{n-m}}$$

- $n > m \rightarrow |H(j\omega)| \rightarrow 0$

hellings $(n-m)(-20 \text{ dB/dec})$

$$\arg H(j\omega) \approx (n-m)(-\frac{\pi}{2})$$

- $n = m \rightarrow H(j\omega) = \frac{A_m}{B_n}$

- $n < m \rightarrow$ fysische werkelijkheid

↳ ingang steeds sneller

↳ uitgang kan niet meer volgen.

Opsplitsen transferfuncties:

$$H(j\omega) = H_1(j\omega) \cdot H_2(j\omega)$$



$$|H(j\omega)|_{dB} = |H_1(j\omega)|_{dB} + |H_2(j\omega)|_{dB}$$

$$\arg H = \arg H_1 + \arg H_2$$

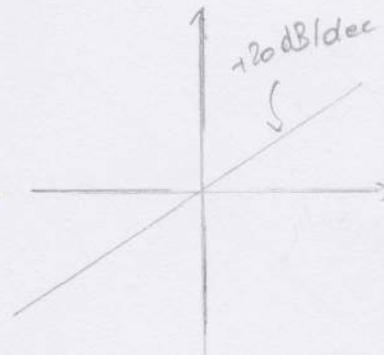
zoeken vd vorm van

$$1 + j\omega\tau$$

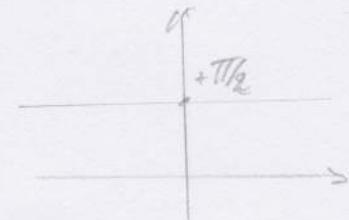
$$1 + \frac{ez}{\omega_n}(j\omega) + \frac{1}{\omega_n^2}(j\omega)^2$$

factoren tegenkomen.

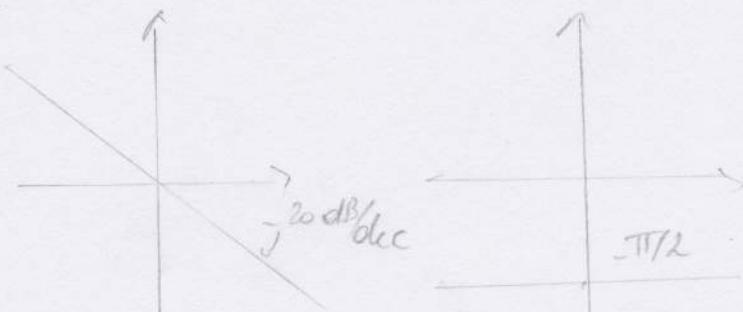
- $\frac{1}{j\omega} \rightarrow$ integratiefactor



- $j\omega \rightarrow$ differentiatiefactor



- $|K|_{dB} \rightarrow$ verschuiving amplitude grafiek



- $K < 0 \rightarrow$ faseverschuiving π

Het Nyquist-diagramma

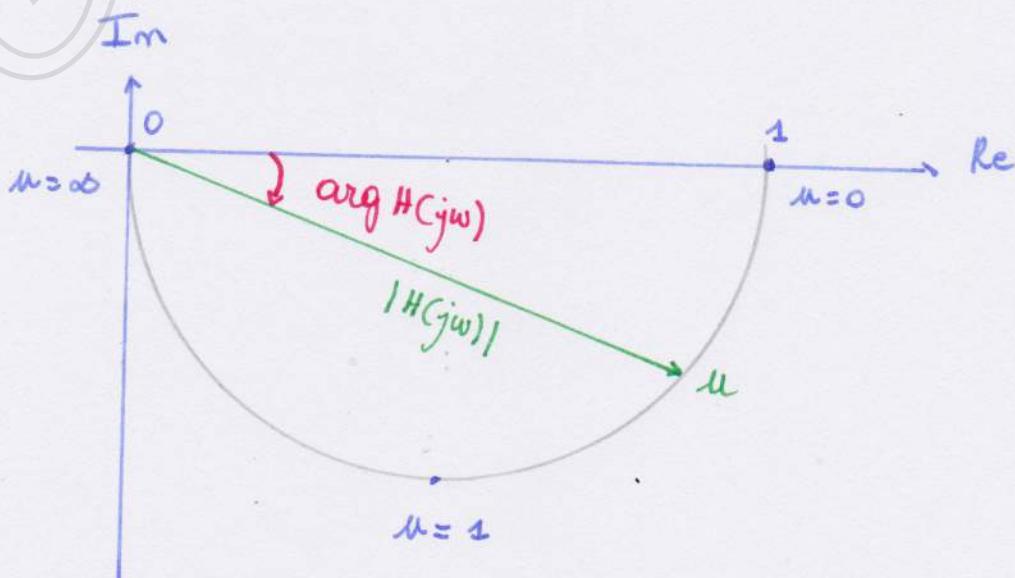
↪ $H(j\omega)$ inzetten $\stackrel{\text{Im}}{\uparrow} \rightarrow \text{Re}$ met $\omega \rightarrow 0 \rightarrow \infty$
 complex vlak

1^{ste} orde

$$H(j\omega) = \frac{1}{1 + j\omega} \quad \begin{aligned} \text{Re} &= \frac{1}{1 + \omega^2} \\ \text{Im} &= \frac{-\omega}{1 + \omega^2} \end{aligned}$$

$$\text{Re}^2 + \text{Im}^2 = \left(\frac{1}{\sqrt{1 + \omega^2}} \right)^2 \rightarrow \frac{1}{1 + \omega^2} = \text{Re}^2 + \text{Im}^2$$

$$\Rightarrow \left(\text{Re} - \frac{1}{2} \right)^2 + \text{Im}^2 = \left(\frac{1}{2} \right)^2$$



2de orde:

$$H(j\omega) = \frac{1}{1 + 2j\omega + (\omega^2)^2} = \frac{1}{(\omega^2 - 1)^2 + j2\omega}$$

$$R_c = \frac{1 - \omega^2}{(\omega^2 - 1)^2 + 4\omega^2}$$

$$Z_m = \frac{-2\omega}{(\omega^2 - 1)^2 + 4\omega^2}$$

$$\omega = 0 \rightarrow R_c = 1 \quad Z_m = 0$$

$$\omega = 1 \rightarrow R_c = 1 \quad Z_m = -\frac{1}{2j}$$

$$\omega = \infty \rightarrow R_c = 0 \quad Z_m = 0$$

- De Nyquist-krommen voor $\omega > 0,7$

↳ binnen $|H(j\omega)| = 1$

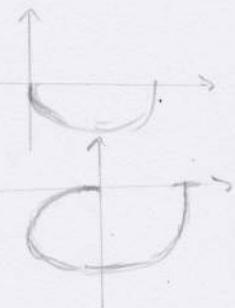
voor $\omega < 0,7$

↳ resonantie

→ kleine freq. → loaduct op Re-0.

→ hoge freq. → 1ste orde: raken neg. Im-as

→ 2de orde: raken neg. Re-as



Algemeen

$$H(j\omega) = \frac{A_m(j\omega)^m + \dots + A_0}{B_n(j\omega)^n + \dots + B_0}$$

hoge frequenties:

$$\rightarrow H(j\omega) = \frac{A_m}{B_n} \frac{1}{(j\omega)^{n-m}}$$

stel $n > m$

$$|H(j\omega)| \rightarrow 0$$

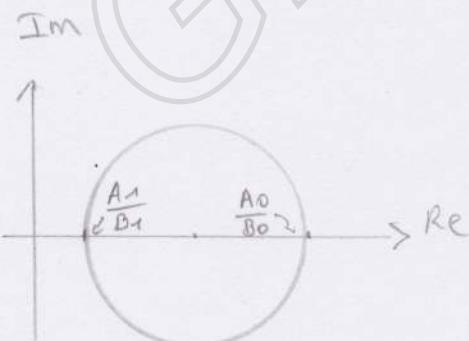
$$\arg H(j\omega) = (n-m)(-\frac{\pi}{2})$$

lage frequenties:

$$\begin{array}{l} A_0 \neq 0 \\ B_0 \neq 0 \end{array} \quad \left. \right\} \rightarrow$$

$$H(j\omega) = \frac{A_0}{B_0} = K$$

\hookrightarrow leedrucht vertrekken in K
op reële as



zelfdake verloop als

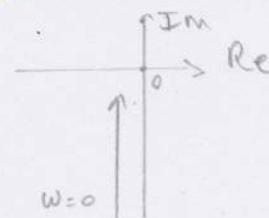
$$H(j\omega) = \frac{A_1(j\omega) + A_0}{B_1(j\omega) + B_0} \rightarrow \begin{aligned} M &= \left(\frac{1}{2} \left(\frac{A_0}{B_0} + \frac{A_1}{B_1} \right); 0 \right) \\ R &= \frac{1}{2} \left| \frac{A_0}{B_0} + \frac{A_1}{B_1} \right| \end{aligned}$$

- noemer: $j\omega$ voorlop (int.)
- teller: $j\omega$ voorlop (diff.)

$$H(j\omega) = -j \frac{A_0}{B_0} \lim_{\omega \rightarrow 0} \frac{1}{\omega}$$

$$|H| \rightarrow \infty$$

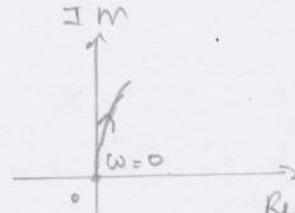
$$\arg H \rightarrow -\frac{\pi}{2}$$



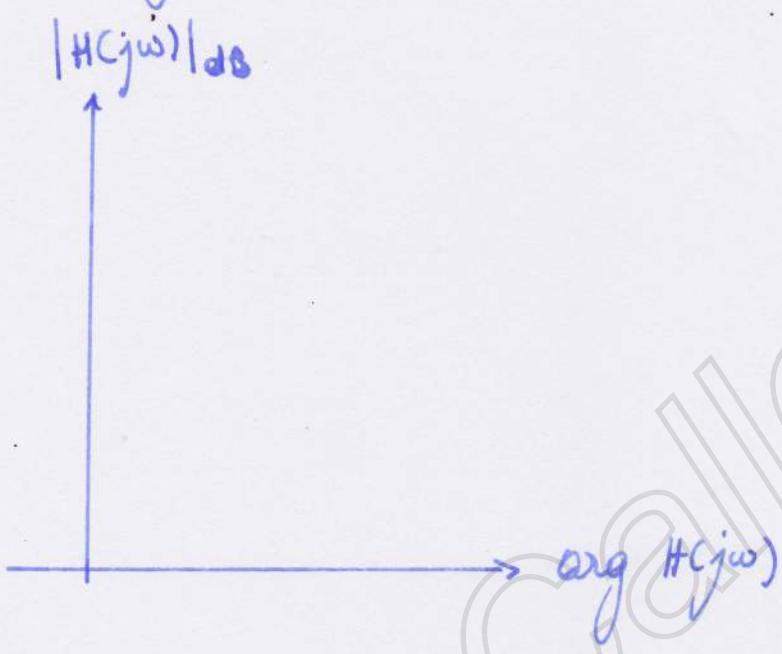
$$H(j\omega) = j \frac{A_0}{B_0} \lim_{\omega \rightarrow 0} \omega$$

$$|H| \rightarrow 0$$

$$\arg H \rightarrow \frac{\pi}{2}$$



Het diagramma van Black



met $\omega \rightarrow \infty$

Hoofdstuk 3

De Laplace - transformatie

gilles callebaut

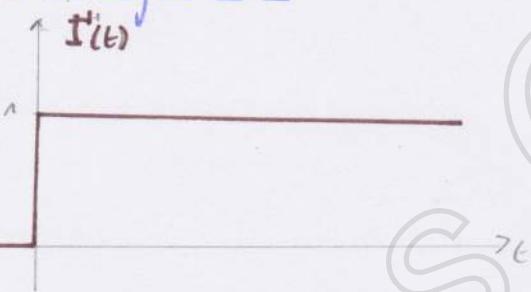
Definitie

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

$$F(s) = \alpha \{ f(t) \}$$

laplace - getransformeerden van enkele vereenvoudigde functies

de Stappfunctie



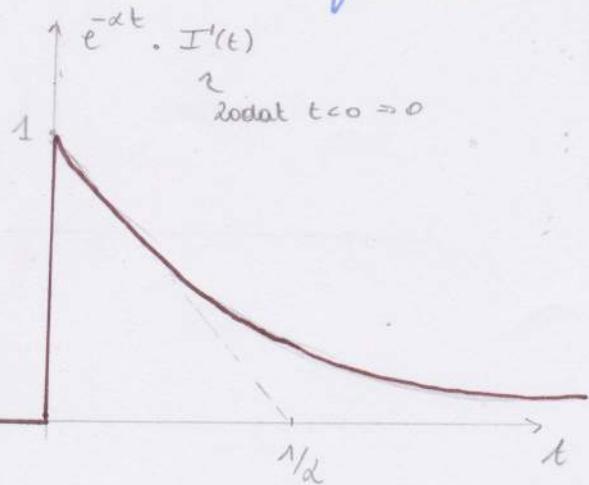
$$\alpha \{ I'(t) \} = \int_0^{\infty} 1 \cdot e^{-st} dt = \frac{e^{-st}}{-s} \Big|_0^{\infty}$$

$$\alpha \{ I'(t) \} = \frac{1}{s}$$

de impulsfunctie

$$\alpha \{ \delta(t) \} = 1$$

de exponentiële functie



$$\begin{aligned} \alpha \{ e^{-at} \} &= \int_0^{\infty} e^{-at} \cdot e^{-st} dt \\ &= \int_0^{\infty} e^{-(s+a)t} dt \\ &= \frac{e^{-(s+a)t}}{-(s+a)} \Big|_0^{\infty} \end{aligned}$$

$$\alpha \{ e^{-at} \} = \frac{1}{s+a}$$

Eigenschaften

linearität

Indien $\mathcal{F}(s)_i \leftrightarrow f(t)_i$ $\xrightarrow{k\mathcal{F}(s) \leftrightarrow kf(t)}$
 $\mathcal{F}_1(s) + \mathcal{F}_2(s) \leftrightarrow f_1(t) + f_2(t)$

$$\Rightarrow \mathcal{L}\{1 - e^{-at}\} = \frac{1}{s} - \frac{1}{s+a}$$

$$\mathcal{L}\{1 - e^{-at}\} = \frac{a}{s(s+a)}$$

$$\mathcal{L}\{1 - e^{-\frac{t}{2}}\} = \frac{1}{s} - \frac{1}{s+\frac{1}{2}}$$

$$\mathcal{L}\{1 - e^{-\frac{t}{2}}\} = \frac{1}{s \cdot (s+\frac{1}{2})}$$

$$\mathcal{L}\{\sin wt\} = \mathcal{L}\left\{\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right\} \rightarrow$$

$$\mathcal{L}\{\sin wt\} = \frac{\omega}{s^2 + \omega^2}$$

$$\mathcal{L}\{\cos wt\} = \mathcal{L}\left\{\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right\} \rightarrow$$

$$\mathcal{L}\{\cos wt\} = \frac{s}{s^2 + \omega^2}$$

$$\mathcal{L}\{\sin wt\} = \frac{1}{2j} \left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega} \right] = \frac{\omega}{s^2 + \omega^2}$$

du - Afgeleide

$$d \left\{ \frac{df(t)}{dt} \right\} = s^2 F(s) - f(0^+)$$

$$d \left\{ \frac{df(t)}{dt} \right\} = \int_0^\infty \frac{df(t)}{dt} e^{-st} dt$$

$$= \int_0^\infty e^{-st} \cdot df(t)$$

$$= f(t) e^{-st} \Big|_0^\infty - \int_0^\infty f(t) \cdot \frac{d(e^{-st})}{dt}$$

$$= -f(0) + s \int_0^\infty f(t) e^{-st} dt$$

$F(s)$

$$d \{ f'' \} = d \{ f' \} - f'(0^+)$$

$$\frac{d^2 f(t)}{dt^2} \leftrightarrow s^2 F(s) - s f(0^+) - \frac{df(0^+)}{dt}$$

want

du Integral

$$d \left\{ \int_0^t f(t) dt \right\} = \frac{d \{ f(t) \}}{s}$$

$$d \left\{ \int_0^t f(t) dt \right\} = \int_0^\infty \int_0^t f(t) dt \cdot e^{-st} dt$$

$$= \int_0^\infty \int_0^t f(t) dt \cdot \frac{d(e^{-st})}{-s}$$

↓ P.I.



$$= \int_0^t f(t) dt \cdot \frac{e^{-st}}{-s} \Big|_0^\infty + \int_0^\infty \frac{e^{-st}}{s} \cdot f(t) dt$$

$\underbrace{-\frac{1}{s} [0-0]}$

$\frac{1}{s} d \{ f(t) \}$

via de integraal eigenschap:

$$t = \int_0^t I'(t) dt \Rightarrow d \left\{ t \cdot I(t) \right\} = \frac{1}{s} \left(\frac{1}{s} \right)$$

$$t = \frac{1}{2} \frac{d(t^2)}{dt} \Rightarrow d \left\{ 2 \int_0^t \int_0^t I'(t) dt \right\} = d \left\{ 2 \int_0^t t \cdot I'(t) dt \right\} \\ = 2 \frac{1}{s} \frac{1}{s^2}$$

=> algemeen:

$$d \left\{ t^n I'(t) \right\} = \frac{n!}{s^{n+1}}$$

beginwaardestelling

$$F(s) \sqsubset f(t) \rightarrow f(0^+) = \lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} s F(s)^*$$

eindwaardestelling

$$F(s) \sqsubset f(t) \rightarrow f(\infty) = \lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s F(s)^{**}$$

$$*\lim_{s \rightarrow \infty} \left[\int_0^\infty \frac{df}{dt} e^{-st} dt \right] = \lim_{s \rightarrow \infty} (s F(s) - f(0^+)) = 0$$

$$** \lim_{s \rightarrow 0} \left[\int_0^\infty \frac{df}{dt} e^{-st} dt \right] = \lim_{s \rightarrow 0} \int_0^\infty df(t) = \lim_{s \rightarrow 0} (s F(s) - f(\infty)) \\ f(\infty) - f(0^+) = ?$$

Translatie in het s - domein

$$f(s) [f(t)] \rightarrow F(s+\alpha) [e^{-\alpha t} f(t)]$$

$$\begin{aligned} d\{e^{-\alpha t} f(t)\} &= \int_0^\infty e^{-\alpha t} f(t) \cdot e^{st} dt \\ &= \int_0^\infty f(t) e^{(s-\alpha)t} dt \\ &= F(s') \end{aligned}$$

$$d\{e^{-\alpha t} f(t)\} = F(s+\alpha)$$

Vermenigvuldigen met t

$$\begin{aligned} d\{t \cdot f(t)\} &= \int_0^\infty t \cdot f(t) \cdot e^{st} dt \\ &= \int_0^\infty f(t) \cdot \left(-\frac{d(e^{st})}{ds}\right) dt \\ &= -\frac{d}{ds} \left[\int_0^\infty f(t) \cdot e^{st} dt \right] \end{aligned}$$

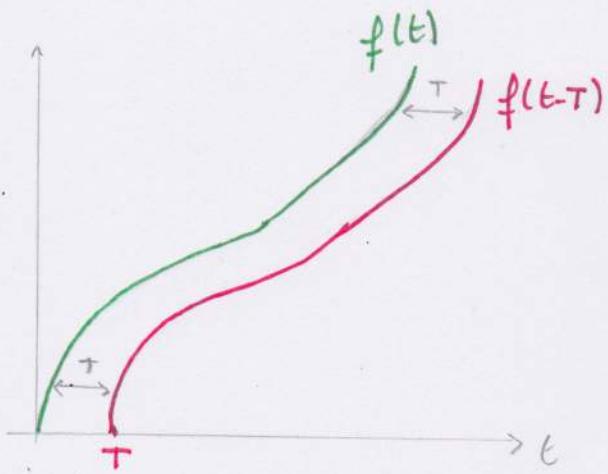
$$d\{t \cdot f(t)\} = -\frac{d}{ds} F(s)$$

Gelykwaardigheidswet

$$\begin{aligned} d\left\{f\left(\frac{t}{k}\right)\right\} &= \int_0^\infty f\left(\frac{t}{k}\right) \cdot e^{-s \frac{t}{k}} \cdot \frac{1}{k} dt \\ &= k \cdot \int_0^\infty f(t') \cdot e^{-s' t'} dt' = k F(s') \end{aligned}$$

$$d\left\{f\left(\frac{t}{k}\right)\right\} = k \cdot F(k \cdot s)$$

Verschuivingswet (in tijdsdomein)



$$\mathcal{F}(s) [f(t) \cdot I'(t)]$$

$$\boxed{\mathcal{L} \{ f(t-T) \} = e^{-sT} F(s)}$$

$$\begin{aligned} \mathcal{L} \{ f(t-T) I'(t-T) \} &= \int_0^\infty f(t') I'(t') e^{-s(t'+T)} dt' \\ &= e^{-sT} \cdot \int_0^\infty f(t') e^{-st'} dt' \end{aligned}$$

$$\begin{aligned} t' &= t - T \\ dt' &= dt \\ t' = 0 &\Rightarrow t = T \\ t' = \infty &\Rightarrow t = \infty \end{aligned}$$

Convolutiestelling

$$y(t) = f(t) * g(t) \quad \leftarrow \text{convolutie}$$

convolutie
stelling

$$\Rightarrow y(t) = \int_0^t f(z) \cdot g(t-z) dz = \int_0^t f(t-z) g(z) dz$$

$$\begin{aligned} G(s) \cdot F(s) &= G(s) \cdot \int_0^\infty f(z) e^{-sz} dz \\ &= \int_0^\infty f(z) \underbrace{G(s) e^{-sz}}_{\left[\int_0^\infty g(t-z) e^{st} dt \right]} dz \quad \leftarrow \text{zie verschuivingswet} \\ &= \int_0^\infty \int_0^\infty f(z) g(t-z) dz \cdot e^{st} dt \\ &\quad \boxed{y(t)} \end{aligned}$$

$$\boxed{\mathcal{L} \{ g(t) * f(t) \} = G(s) \cdot F(s)}$$

De Inverse Laplace-transformatie ↪ Def. !

vd vorm $F(s) = \frac{T(s)}{N(s)}$

→ splitsen in partieelbreuken:

vd vorm

$$\frac{k_1}{s-p} + \frac{k_2}{(s-p)^2} + \frac{k_3}{(s-p)^3} + \dots$$

voor
meervoudige
wortel

algemeen:

$$\frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \frac{k_3}{s-p_3} + \dots$$

↑ polen

v.b. zie p 71.

andere manier:

enkel voor
enkelvoudige
polen!

→ bepalen vd residu's

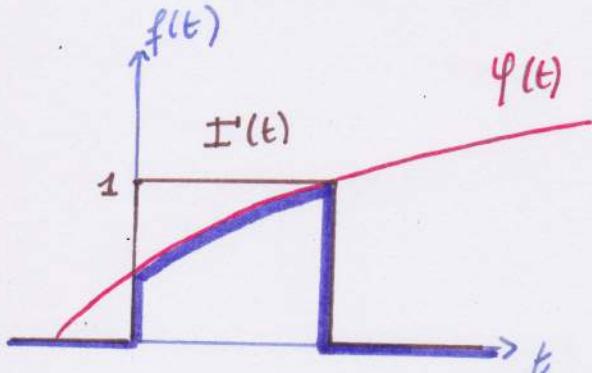
$$k = F(s)(s-p) \Big|_{s=p}$$

opmerking:

$$\frac{k_1 \cdot s + k_2}{(\dots s^2 + \dots s + \dots)} \quad \begin{array}{l} \checkmark \text{ eerste graad} \\ \subset \text{ 2de graad} \end{array}$$

Toepassingen

Toepassingen op de verschuivingswet



$$f \neq 0 \quad (0, T)$$

de rest $f(t) = 0$

$$f(t) = \varphi(t) [I'(t) - I'(t-T)]$$

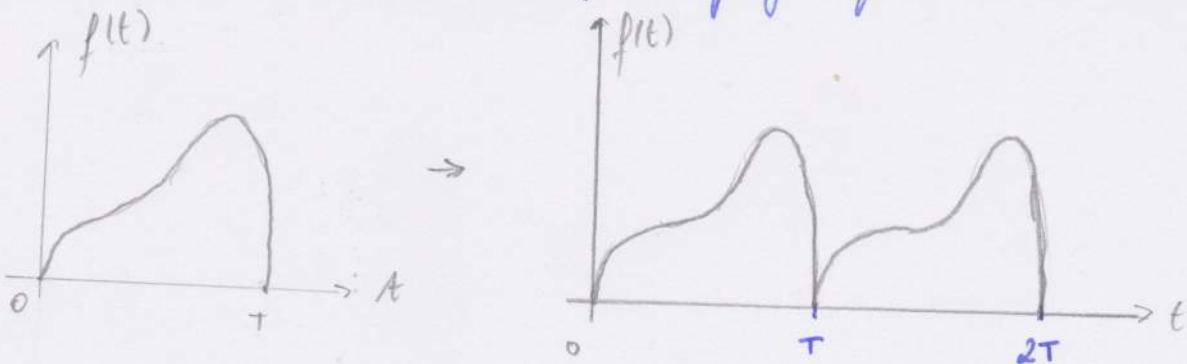
$$f(t) = \varphi(t) I'(t) - \varphi(t-T) I'(t-T)$$

als we $\varphi(t)$ schrijven als $\varphi[(t-T)+T]$

$$\Rightarrow f(t) = \varphi(t) I'(t) - \varphi(t-T) I'(t-T) - \varphi(T) I'(t-T)$$

- De Laplace-grensfunctie van periodische functies

$f(t)$ die nul is buiten $[0, T]$ + opeenvolgingzelfde verloop



$$f(t) = f(t) \cdot I'(t) + \dots + f(t-iT) I'(t-iT)$$

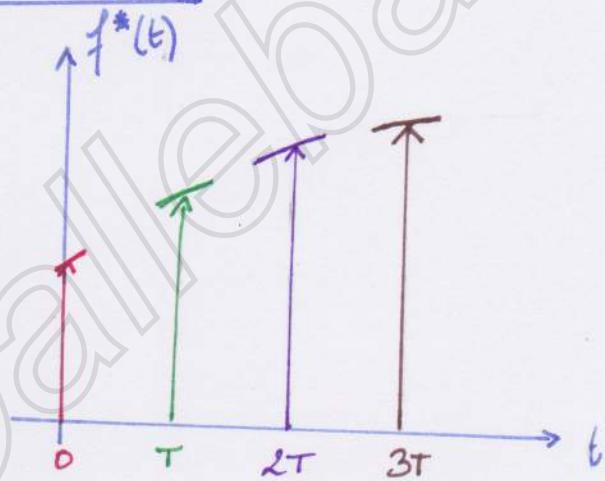
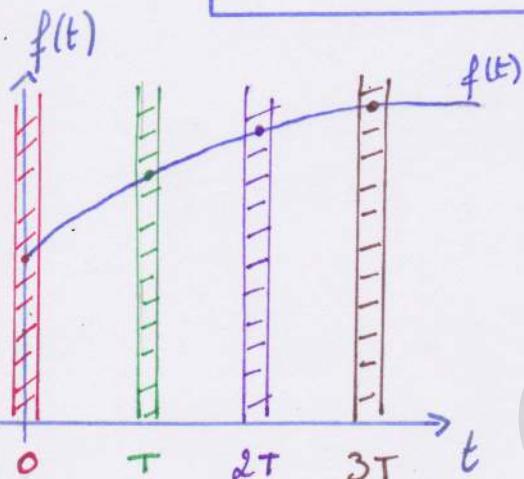
$$f_T(t) = \sum_{i=0}^{\infty} f(t-iT) I'(t-iT)$$

$$\mathcal{F}_T(s) = \mathcal{F}(s) \sum_{i=0}^{\infty} e^{-iT s}$$

$$\sum_{i=0}^{\infty} e^{-iT s} = \frac{1}{1 - e^{-iT s}}$$

- De Laplace - getransformeerde van 'ideaal getoesteide' functies

$$f^*(t) = \sum_{i=0}^{\infty} f(iT) \delta(t-iT)$$



$\delta(t)$ $\delta(t-T)$...

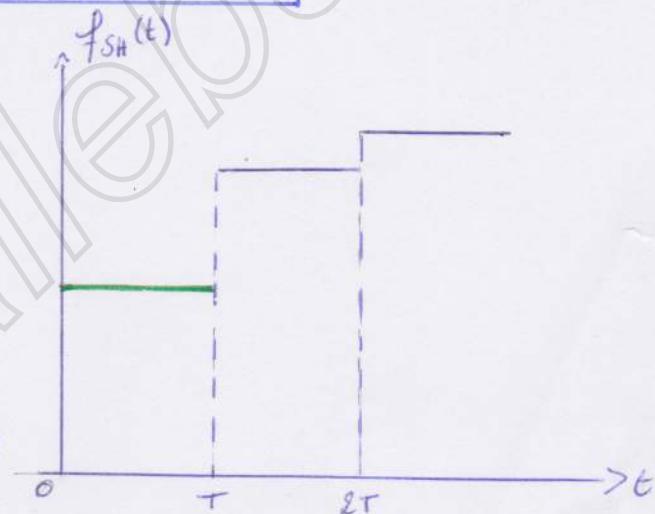
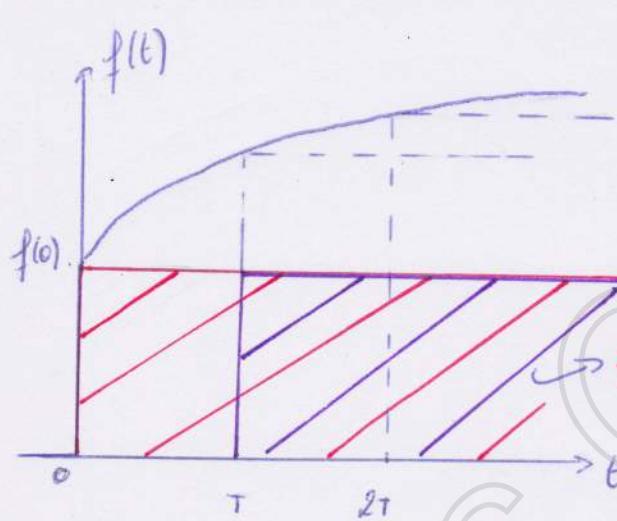
$$F^*(s) = \sum_{i=0}^{\infty} f(iT) e^{-iT s}$$

$$\left\{ \begin{array}{l} z = e^{Ts} \\ e^{-iT s} = (e^{Ts})^{-i} \end{array} \right.$$

$$F(z) = \sum_{i=0}^{\infty} f(iT) z^{-i}$$

- De Laplace - getransformeerde van 'sample and hold' - functies

$$f_{SH}(t) = \sum_{i=0}^{\infty} f(iT) [I(t-iT) - I(t-(i+1)T)]$$



$$F_{SH}(s) = \sum_{i=0}^{\infty} f(iT) \left[\frac{e^{-iTs}}{s} - \frac{e^{-(i+1)Ts}}{s} \right]$$

$$F_{SH}(s) = \frac{1 - e^{-Ts}}{s} \sum_{i=0}^{\infty} f(iT) e^{-its}$$

$$F_{SH}(s) = \frac{1 - e^{-Ts}}{s} F^*(s)$$

Loplosser van lineaire differentiaalverg.

2de ord:

$$B_2 \frac{d^2y(t)}{dt^2} + B_1 \frac{dy(t)}{dt} + B_0 y(t) = A_0 u(t)$$

$$\hookrightarrow B_2 [\zeta^2 Y(s) - \zeta y(0) - y'(0)] + B_1 [\zeta Y(s) - y(0^+)] + B_0 Y(s) = A_0 U(s)$$

$$\rightarrow Y(s) [\zeta^2 B_2 + \zeta B_1 + B_0] = A_0 U(s) + (B_2 \zeta + B_1) y(0^+) + B_2 y'(0^+)$$

$$Y(s) = \frac{A_0 U(s)}{\zeta^2 B_2 + \zeta B_1 + B_0} + \frac{B_2 \zeta + B_1}{\zeta^2 B_2 + \zeta B_1 + B_0} y(0^+) + \frac{B_2}{\zeta^2 B_2 + \zeta B_1 + B_0} y'(0^+)$$

Hoofdstuk 4

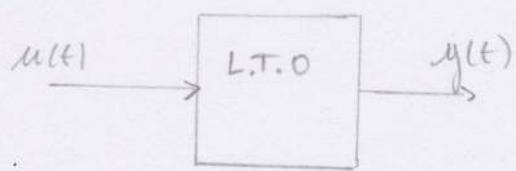
Studie van L.T.O. systemen

met hun

transferfunctie

Gilles Callebaut

De transferfunctie van een L.T.O. systeem



$$y(t) = h(t) * u(t)$$

↓

$$Y(s) = H(s) \cdot U(s)$$

1^{ste} orde:

$$H(s) = \frac{K}{2s+1}$$

$$\zeta \frac{dy(t)}{dt} + y(t) = K \cdot u(t)$$

$$\zeta(s Y(s)) + Y(s) = K \cdot U(s)$$

mits $y(0^+) = 0$

2^{de} orde:

$$H(s) = \frac{K}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1}$$

$$\frac{1}{\omega_n^2} \frac{d^2y(t)}{dt^2} + \frac{2\zeta}{\omega_n} \frac{dy(t)}{dt} + y(t) = K \cdot u(t)$$

$$\frac{1}{\omega_n^2} (s^2 Y(s)) + \frac{2\zeta}{\omega_n} (s Y(s)) + Y(s) = K \cdot U(s)$$

mits $y(0^+) = y'(0^+) = 0$

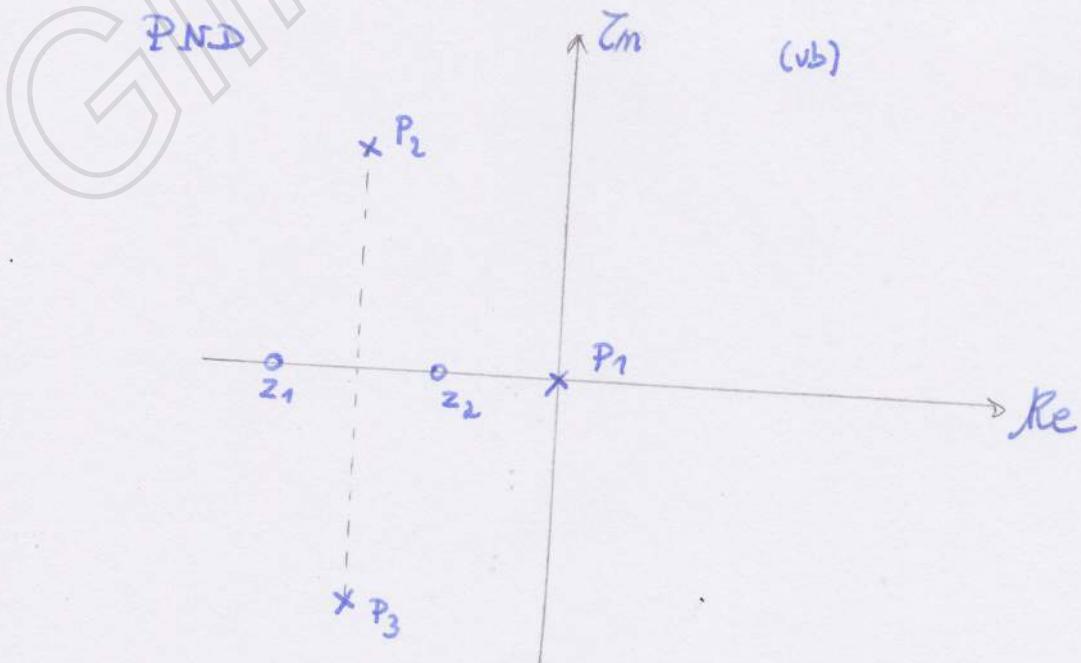
Het polen-nullen-diagramma van de transferfunctie (PND)

- Algemeen -

$$H(s) = \frac{A_m s^m + \dots + A_1 s + A_0}{B_n s^n + \dots + B_1 s + B_0}$$
$$= \frac{A_m}{B_n} \cdot \frac{s^m + \dots + a_0}{s^n + \dots + b_0} = K \cdot \frac{T(s)}{N(s)}$$

$$= K \cdot \frac{(s-z_1)(s-z_2)\dots(s-z_m)}{(s-p_1)(s-p_2)\dots(s-p_n)}$$

→ m nullen
→ n polen



Het PND van een eerste-orde systeem

$$H(s) = \frac{\kappa}{s+1} \rightarrow \frac{\kappa}{2} \cdot \frac{1}{s + \frac{1}{2}} \quad p_1 = -\frac{1}{2}$$

Het PND van een tweede-orde systeem

$$H(s) = \frac{\kappa}{\left(\frac{s}{\omega_n}\right)^2 + \left(\frac{s}{\omega_n}\right)2z + 1} \rightarrow \frac{\kappa \omega_n^2}{s^2 + 2z\omega_n s + \omega_n^2} \rightarrow \frac{\kappa \omega_n^2}{(s+z\omega_n)^2 + \omega_n^2(1-z^2)}$$

wortels: $s^2 + 2z\omega_n s + \omega_n^2 = 0$

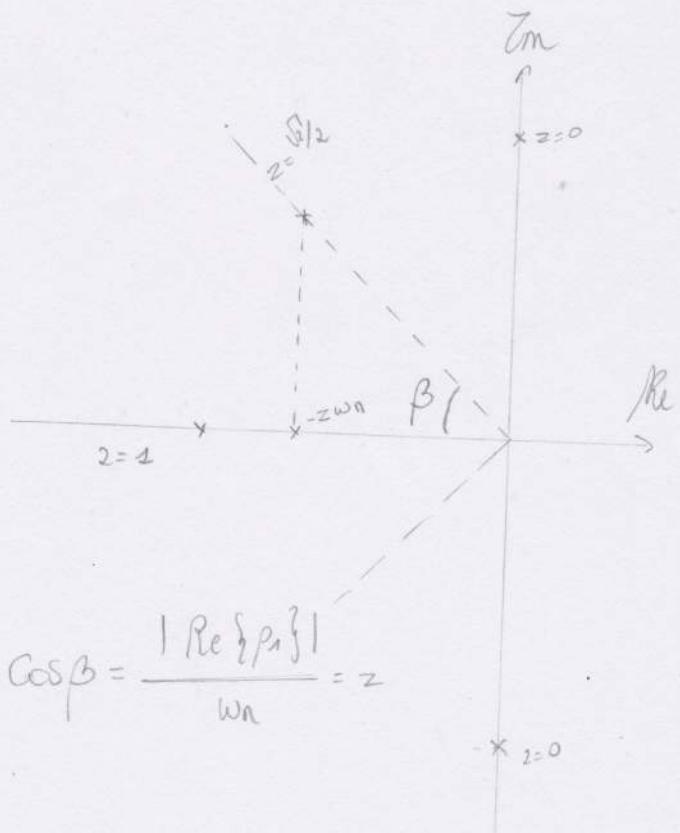
- $z > 1 \rightarrow p_1 = -\omega_n z + \omega_n \sqrt{z^2-1}$

$$\rightarrow p_2 = -\omega_n z - \omega_n \sqrt{z^2-1}$$

- $z = 1 \rightarrow p_1 = p_2 = -\omega_n$

- $z < 1 \rightarrow p_1 = -\omega_n z \pm j\omega_n \sqrt{1-z^2}$

- $z = 0 \rightarrow p_1 = \pm j\omega_n$

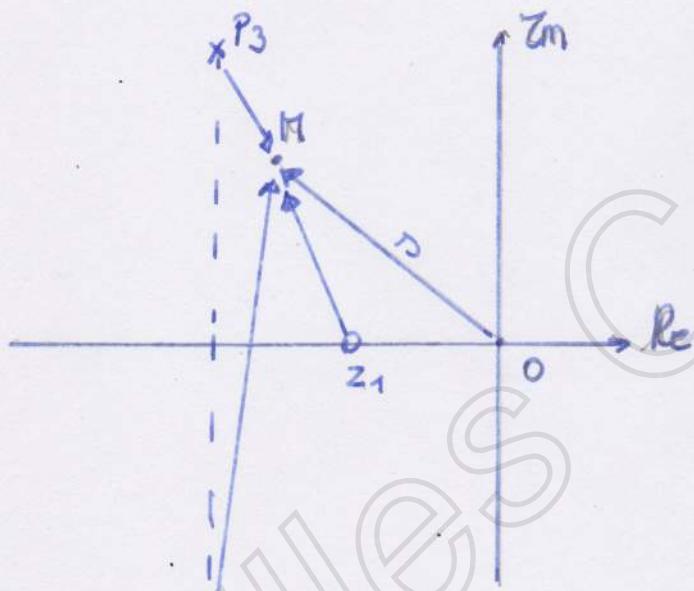


Grafische interpretatie van de transferfunctie i.h. PND

$$H(s) = \frac{(s-z_1) \dots (s-z_m)}{(s-p_1) \dots (s-p_n)} \cdot K$$

M ↴ vechten vanuit
de nullen naar M

K ↴ vechten vanuit
de polen naar M



modulus \Rightarrow lengte
argument \Rightarrow hoek

$$|H(s)| = \frac{|z_1 - M| \dots |z_m - M|}{|p_1 - M| \dots |p_n - M|} K$$

$$\arg H = (\arg z_1 - M + \dots + \arg z_m - M) - (\arg p_1 - M + \dots + \arg p_n - M)$$

$k > 0$

De impulsresponsie van een L.T.O. systeem

$$h(t) = \mathcal{L}^{-1}\{H(s)\}$$

polen bepalen de vorm
splitten in partieelbrakken

inverse Laplace-
transformatie

$$\frac{A}{s-p_i}$$

dubbele reële polen

$$\frac{As+B}{(s-p_i)(s-p_j)} = \frac{As+B}{(s-\alpha)^2 + \omega^2}$$

$$p_i = \alpha + j\omega \quad p_j = \alpha - j\omega$$

$$A e^{p_i t}$$

$$A t e^{p_i t}$$

$$C e^{\alpha t} \sin(\omega t + \phi)$$

$$\frac{As+B}{s^2 + \omega^2}$$

$$C \sin(\omega t + \phi)$$

$$\frac{As+B}{(s-p_i)^2 (s-p_j)^2} = \frac{As+B}{((s-\alpha)^2 + \omega^2)^2}$$

dubbel complexe toep. polen

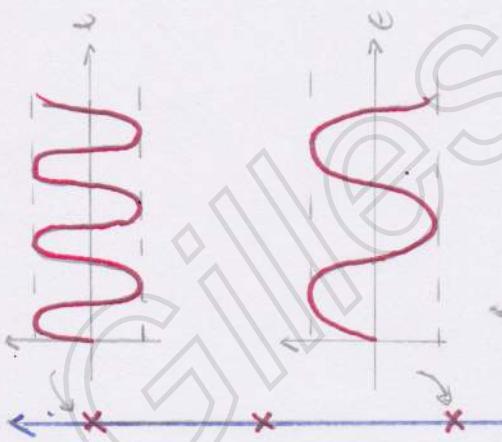
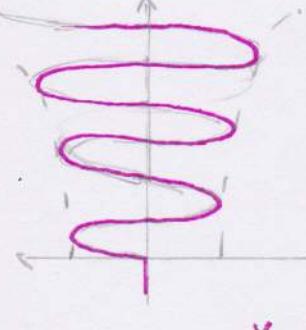
$$e^{\alpha t} [C \sin(\omega t + \phi) + C' t \sin(\omega t + \phi')]$$

$$\frac{As+B}{(s^2 + \omega^2)^2}$$

\tilde{s} dubbele imag.
polen

$$C \sin(\omega t + \phi) + C' t \sin(\omega t + \phi')$$

$$h(t) = \sum \text{moden}$$

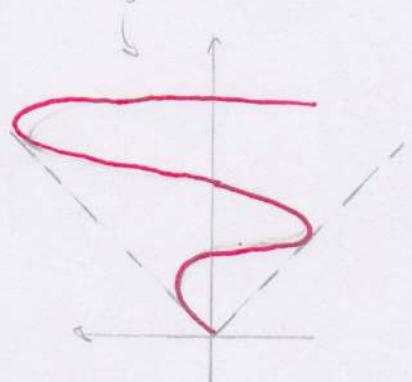
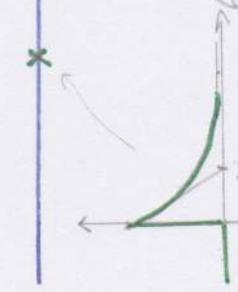
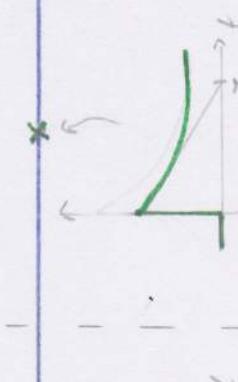
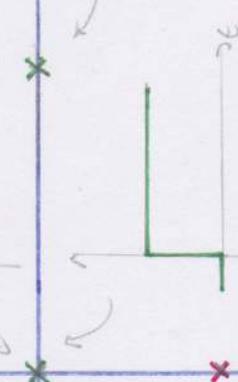


oscillationsmoden
 $\omega > 0 \rightarrow$ mitstreifen
 $\omega > 0 \rightarrow$ grorieren



doppelte
pade

$\rightarrow \epsilon$



doppelte pad

$\rightarrow \epsilon$

harmonische oscillatior

cke A. $\propto \omega$

exp. moden mit

$$\zeta = \frac{1}{|\beta_1|}$$

neg. $\omega \rightarrow$ uitsterven
 pos. $\omega \rightarrow$ groeien

De stabilitéit van een L.T.O. systeem

$h(t) \rightarrow$ uitsterven

\Rightarrow Omdat een L.T.O. systeem stabiel zou zijn, is het nodig en voldoende dat alle polen van zijn transferfunctie $H(s)$ i.h. linkerhalfvlak v.h. PND liggen.

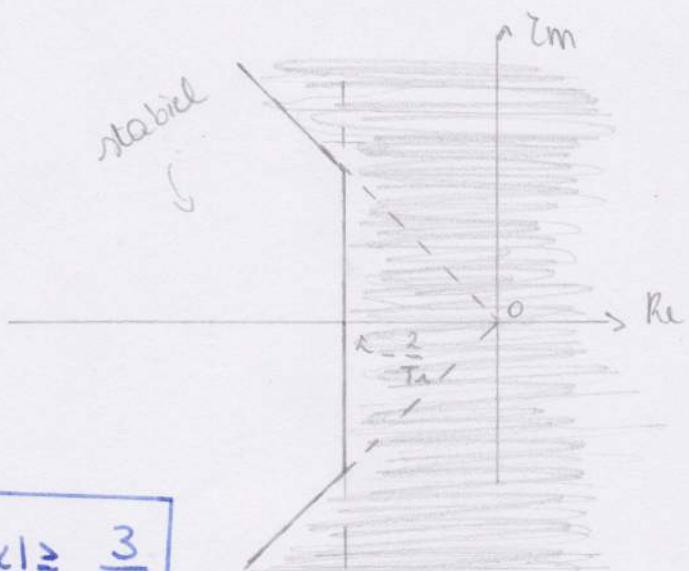
\leadsto na responsietijd $T_r \rightarrow$ verwaarloosbaar t.o.v begin KW:

$$c = \frac{t}{\zeta} \Big|_{t=T_r} \leq 0,05$$

$$\frac{T_r}{\zeta} \geq 3$$

$$\frac{1}{\zeta} \geq \frac{3}{T_r} \rightarrow$$

$$|\alpha| \geq \frac{3}{T_r}$$



\curvearrowleft polen moeten links liggen
tot verticale rechte $- \frac{3}{T_r}$

\leadsto na halve periode gedempt

$T \approx 3$

$$\hookrightarrow \frac{1}{\zeta} = \frac{6}{T} \rightarrow |\alpha| \geq 6$$

\curvearrowleft alle complexen polen tussen de
bissectrices v.h. tweede en
3de kwadrant (45°)

De Mapresponsie v.e. d.T.O. systeem

Algemeen

$$q(t) = \alpha^{-1} \left\{ \frac{H(s)}{s} \right\}$$

$$\alpha \left\{ I^t(t) \right\} = \frac{1}{s}$$



eerste - orde - systeem

$$H(s) = \frac{1}{1+zs} \rightarrow q(t) = \alpha^{-1} \left\{ \frac{1}{s(1+zs)} \right\} = \alpha^{-1} \left\{ \frac{\frac{-z}{s}}{s + \frac{1}{z}} \right\}$$

$$q(t) = [1 - e^{-t/z}] I^t(t)$$

tweede - orde - systeem

$$H(s) = \frac{\omega_n^2}{s^2 + 2zw_n s + \omega_n^2} = \frac{\omega_n^2}{(s + zw_n)^2 + \omega_n^2(1-z^2)}$$

$$\lambda_{1,2} = -zw_n \pm \sqrt{z^2\omega_n^2 - \omega_n^2} = -zw_n \pm \omega_n \sqrt{z^2 - 1}$$

$$\rightarrow q(t) = \alpha^{-1} \left\{ \frac{\omega_n^2}{s[(s + zw_n)^2 + \omega_n^2(1-z^2)]} \right\}$$

- $\lambda = 2$ versch. wortels
- $\lambda = 0$
- $\lambda = 2$ complexe wortels
- $\lambda = 2$ reële gelijke wortels

$z = 1$, kritische dämpfung ← 2 reelle Wurzeln

$$\frac{H(s)}{s} = \frac{\omega_n^2}{s(s + \omega_n)^2} = \frac{A}{s} + \frac{B}{s + \omega_n} + \frac{C}{(s + \omega_n)^2}$$

$$q(t) = \pm -e^{-\omega_n t} (1 + \omega_n t)$$

$z > 1$, kriechende Dämpfung ← 2 reelle Wurzeln (verschieden)
overdampft

$$\lambda_{1,2} = -z\omega_n \pm \sqrt{z^2 - 1} \omega_n \quad \begin{matrix} > 0 \\ \omega_n \end{matrix} \quad \begin{aligned} \lambda_1 &= -\omega_n \underbrace{[z - \sqrt{z^2 - 1}]}_{> 0} = -\frac{1}{\zeta_1} \\ \lambda_2 &= -\omega_n \underbrace{[z + \sqrt{z^2 - 1}]}_{> 0} = -\frac{1}{\zeta_2} \end{aligned}$$

$$\frac{H(s)}{s} = \frac{A}{s} + \frac{B}{s + \frac{1}{\zeta_1}} + \frac{C}{s + \frac{1}{\zeta_2}}$$

$$x. A \Big|_{s=0} : A = \frac{\omega_n^2}{\frac{1}{\zeta_1} \cdot \frac{1}{\zeta_2}} = \pm$$

$$x. (s + \frac{1}{\zeta_1}) \Big|_{s=-\frac{1}{\zeta_1}} : B = \frac{\omega_n^2}{-\frac{1}{\zeta_1} \left(-\frac{1}{\zeta_1} + \frac{1}{\zeta_2}\right)} = \frac{\omega_n^2}{-\frac{1}{\zeta_1} \frac{(-\zeta_2 + \zeta_1)}{\zeta_1 \zeta_2}} \quad \text{"}\omega_n^2\text{"}$$

$$C = \frac{\zeta_2}{\zeta_1 - \zeta_2}$$

$z < 1$, oscillatoire demping \leftarrow 2 complex toegevoegde wortels

$$\begin{aligned} \frac{H(s)}{s} &= \frac{\omega_n^2}{s^2 + 2z\omega_n s + \omega_n^2} \cdot \frac{1}{s} \\ &= \frac{A}{s} + \frac{Bs + C}{s^2 + 2z\omega_n s + \omega_n^2} = \frac{As^2 + 2z\omega_n s A + A\omega_n^2 + Bs^2 + Cs}{s(s^2 + 2z\omega_n s + \omega_n^2)} \\ &= \frac{(A+B)s^2 + (2z\omega_n A + C)s + A\omega_n^2}{s(s^2 + 2z\omega_n s + \omega_n^2)} \end{aligned}$$

$$\rightarrow \begin{cases} A\omega_n^2 = \omega_n^2 \\ 2z\omega_n A + C = 0 \\ A + B = 0 \end{cases} \Rightarrow \begin{cases} A = 1 \\ C = -2z\omega_n \\ B = -1 \end{cases}$$

$$\frac{H(s)}{s} = \frac{1}{s} - \frac{s + 2z\omega_n}{s^2 + 2z\omega_n s + \omega_n^2} = \frac{1}{s} - \frac{(s + z\omega_n) + z\omega_n}{(s + z\omega_n)^2 + (\omega_n \sqrt{1-z^2})^2} \xrightarrow{\omega_p} \frac{\sqrt{1-z^2}}{\sqrt{1-z^2}}$$

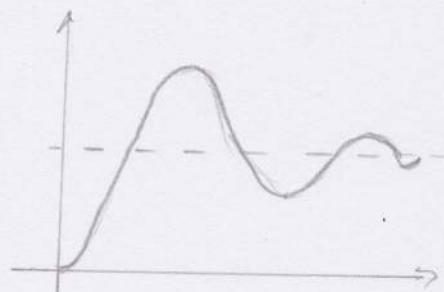
ω_p
eigenpulsatie

$$q(t) = 1 - e^{-z\omega_n t} \cdot \frac{1}{\sqrt{1-z^2}} \cdot \sin(\underbrace{\omega_n \sqrt{1-z^2} t}_{\omega_p} + \Phi)$$

$$\operatorname{tg} \Phi = \frac{\omega}{k} \quad \text{voor de norm} \quad \frac{s+k}{s^2 + \omega^2}$$

$$\operatorname{tg} \Phi = \frac{1-z^2}{z}$$

zie tabel p.84



$$\frac{H(s)}{s} = \frac{1}{s} - \frac{\tau_1}{\tau_1 - \tau_2} \cdot \frac{1}{s + \frac{1}{\tau_1}} + \frac{\tau_2}{\tau_1 - \tau_2} \cdot \frac{1}{s + \frac{1}{\tau_2}}$$

$$q(t) = 1 - \frac{\tau_1}{\tau_1 - \tau_2} \cdot e^{-t/\tau_1} + \frac{\tau_2}{\tau_1 - \tau_2} \cdot e^{-t/\tau_2}$$

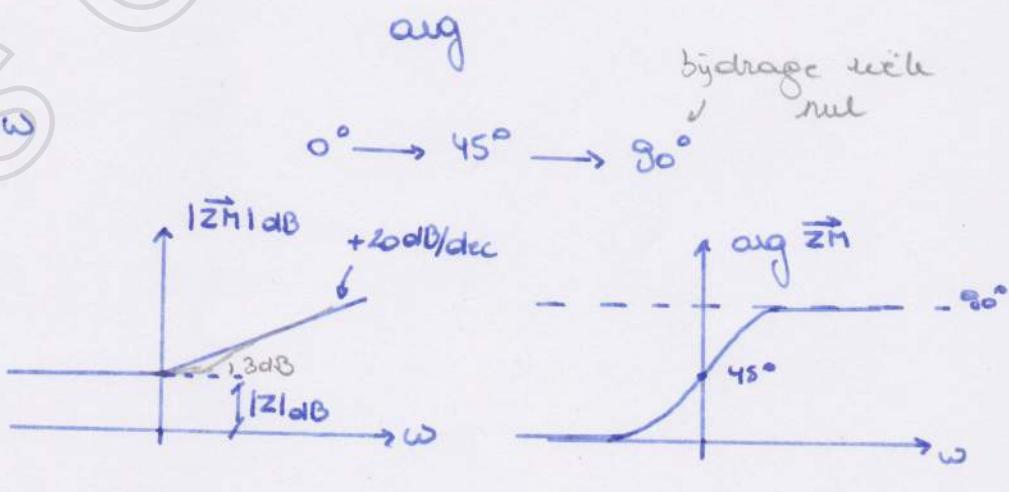
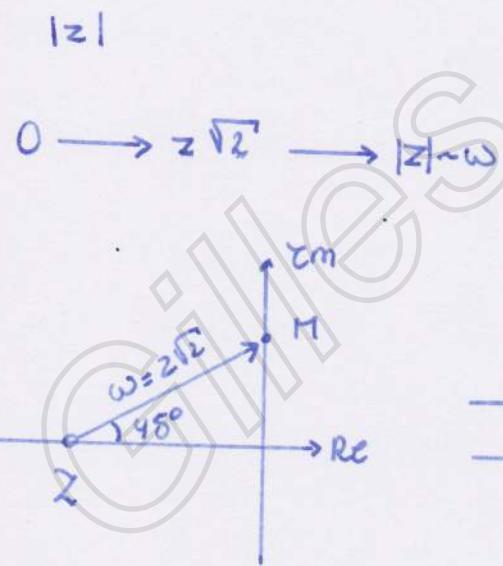
$$\begin{aligned} \tau_1 &= 3\tau_2 \\ &= 3 \frac{z + \sqrt{z^2 - 1}}{\omega_n} \end{aligned}$$

$$z \text{ groot} \rightarrow \begin{cases} \tau_1 \rightarrow \frac{2z}{\omega_n} \\ \tau_2 \rightarrow 0 \end{cases} \quad \left. \begin{array}{l} \text{ } \\ \text{ } \end{array} \right\} \rightarrow \text{lykken op eerste orde}$$

De harmonische responsie v.e. L.T.O. systeem

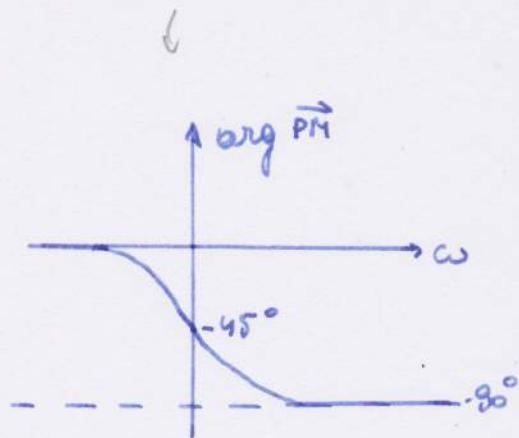
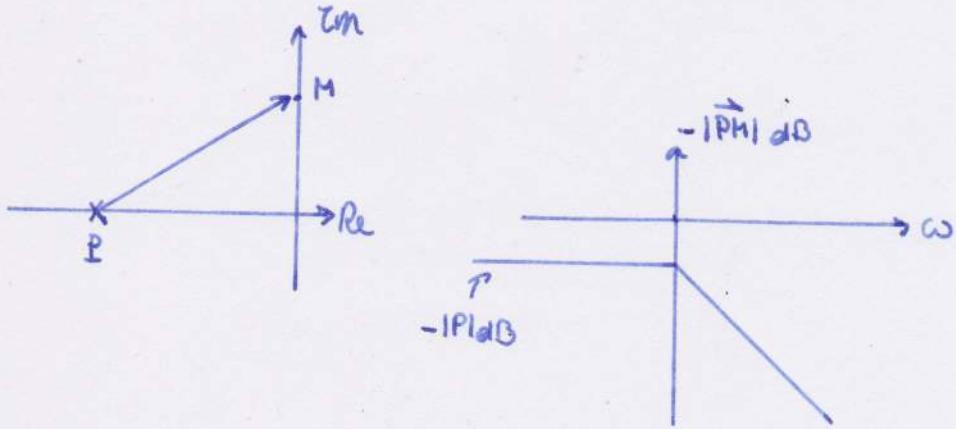
$$\left\{ \begin{array}{l} \bar{Y} = H(j\omega) \bar{U} \\ H(j\omega) = H(s) \Big|_{s=j\omega} \end{array} \right. \rightarrow H(j\omega) = \text{de } \frac{(j\omega - z_1) \dots (j\omega - z_m)}{(j\omega - p_1) \dots (j\omega - p_m)}$$

• reële nul

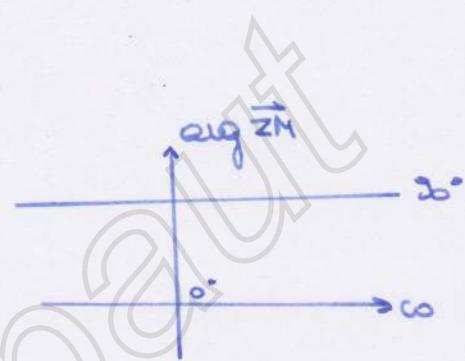
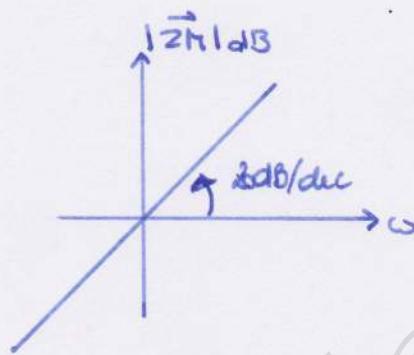
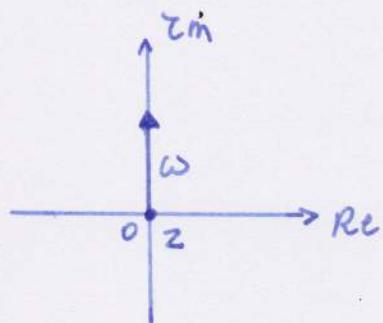


• reële pool

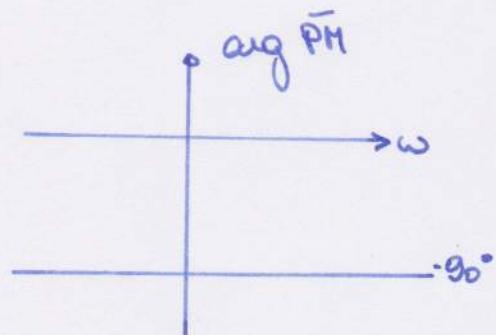
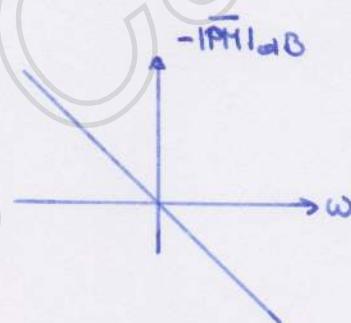
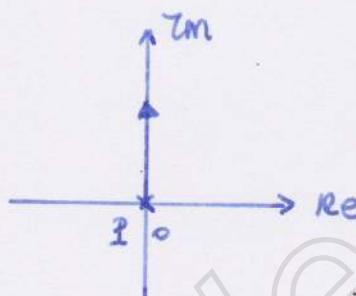
↳ gelijkwaardige vector als reële nul \Rightarrow noemer \Rightarrow min-teken



- nul i.d. oorsprong ← differentiatiefactor

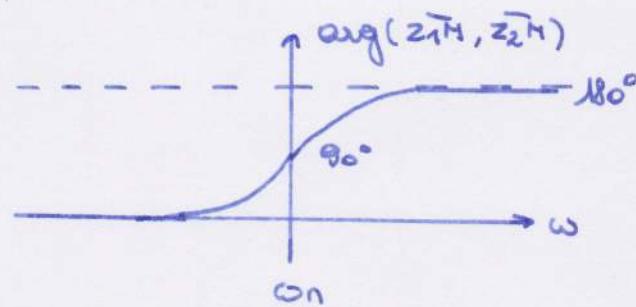
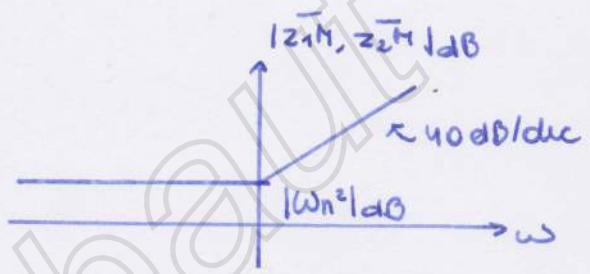
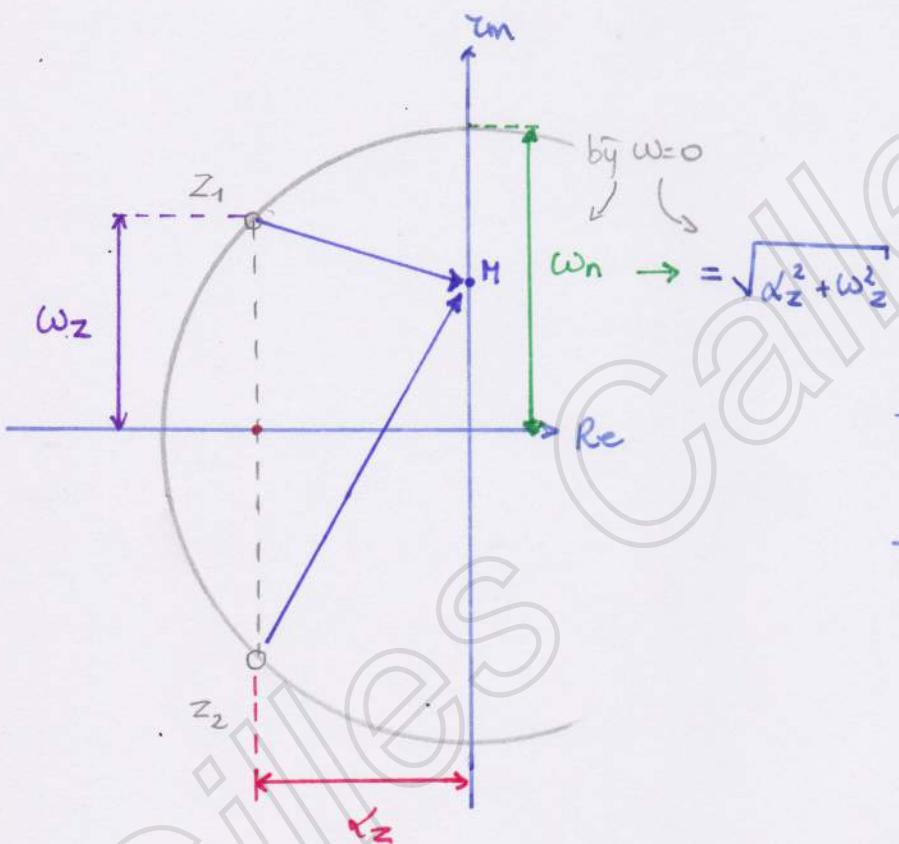


- pool i.d. oorsprong ← integratiefactor



- een paar complex toegevoegde nullen

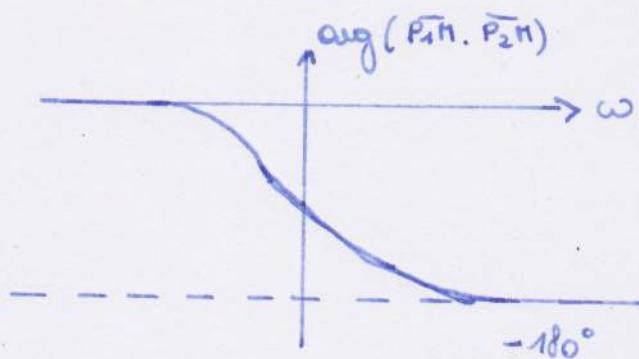
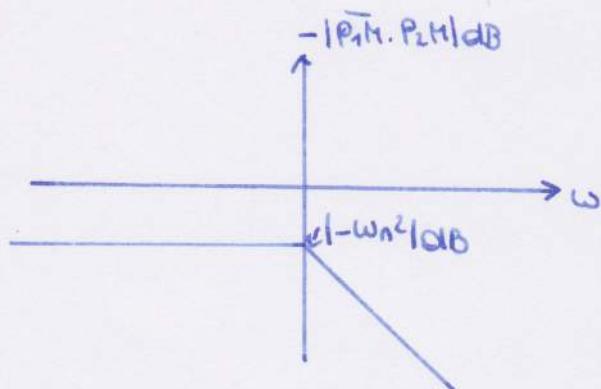
$$z_1 = \alpha_z \pm j\omega_z$$



- een paar complex toegevoegde polen

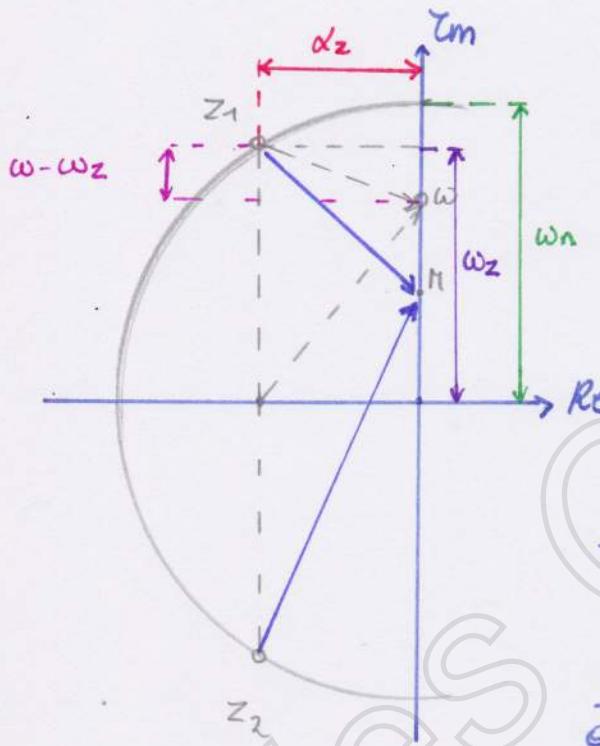
$$P_1 = \alpha_p \pm j\omega_p$$

gelijkaardige vectoren
(zie hierboven)



bijdrage tot amplitude kan.

↳ max: nullen
 ↳ min: polen } tussen bissectrices van
 ↳ d & 3de kwadrant



$$|z_1 \vec{M}| = \sqrt{(\omega_z - \omega)^2 + \alpha_z^2}$$

$$|z_2 \vec{M}| = \sqrt{(\omega_z + \omega)^2 + \alpha_z^2}$$

minimum bepalen:

$$\min \{ |z_1 \vec{M}|, |z_2 \vec{M}| \}$$

$$\frac{d}{d\omega^2} (2(\omega_z^2 + \omega^2 + \alpha_z^2) - 4\omega_z^2) = 0$$

$$\omega^2 = \omega_z^2 - \alpha_z^2$$

$$\rightarrow \boxed{\omega_{\text{Res}} = \sqrt{\omega_z^2 - \alpha_z^2}} \quad) \text{ als } \omega_z > \alpha_z$$

complex toegev. nullen

$$z_1 = \alpha_z \pm j\omega_z \rightarrow [j\omega - (\alpha_z + j\omega_z)][j\omega - (\alpha_z - j\omega_z)] \\ = (j\omega)^2 + \alpha_z \omega_n j\omega + \omega_n^2$$

waarbij $\left\{ \begin{array}{l} \omega_n = \sqrt{\alpha_z^2 + \omega_z^2} \\ z = \frac{j\alpha_z}{\omega_n} \end{array} \right.$

Hoofdstuk 5

De Fourier analyse

+

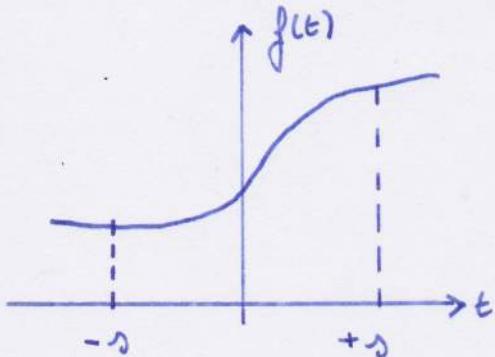
Appendix A

Gilles Callebaut

Hoofdstuk 5

De Fourieranalyse

De Fourierreeksontwikkeling



$$f(t) = \frac{b_0}{2} + \sum_{h=1}^{\infty} \left(b_h \cos \frac{h\pi t}{T} + a_h \sin \frac{h\pi t}{T} \right)$$
$$\left\{ \begin{array}{l} b_0 = \frac{1}{T} \int_{-T}^{+T} f(t) dt \\ b_h = \frac{1}{T} \int_{-T}^{+T} f(t) \cos \frac{h\pi t}{T} dt \\ a_h = \frac{1}{T} \int_{-T}^{+T} f(t) \sin \frac{h\pi t}{T} dt \end{array} \right.$$

som sinus & cosinus herleiden:

$$f(t) = \frac{b_0}{2} + \sum_{h=1}^{\infty} g_h \sin \left(\frac{h\pi t}{T} + \theta_h \right)$$

$$f(t) = \frac{b_0}{2} + \sum_{h=1}^{\infty} g_h \cos \left(\frac{h\pi t}{T} + \theta'_h \right)$$

met $\left\{ \begin{array}{l} g_h = \sqrt{a_h^2 + b_h^2} \\ \theta_h = \operatorname{Bgtg} \frac{b_h}{a_h} \\ \theta'_h = -\operatorname{Bgtg} \frac{a_h}{b_h} \end{array} \right.$

voor periodieke functie \rightarrow interval $(-\frac{T}{2}, +\frac{T}{2})$

Fourieranalyse van een periodieke tijdsfunctie

$$\rightarrow \Delta = \frac{T}{2} \quad \text{met} \quad \omega = \frac{2\pi}{T}$$

$$f(t) = \frac{b_0}{2} + \sum_{h=1}^{\infty} (b_h \cosh ht + a_h \sinh ht)$$

$\frac{b_0}{2}$ = gemiddelde waarde vd periodieke fct

$$\text{met } \left\{ \begin{array}{l} b_0 = \frac{2}{T} \int_{-T}^T f(t) dt \end{array} \right.$$

$$b_h = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos ht dt$$

$$a_h = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin ht dt$$

- even functie ($f(t) = f(-t)$)

$\rightarrow a_h = 0$ voor alle h-waarden \rightarrow enkel cos termen

- oneven functie ($f(t) = -f(-t)$)

$\rightarrow b_0 = 0$ & $b_h = 0$ voor alle h -waarden \rightarrow enkel sin termen

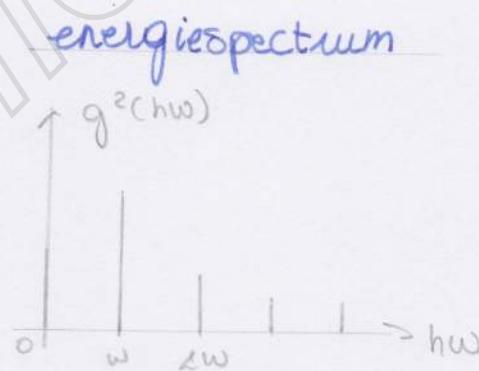
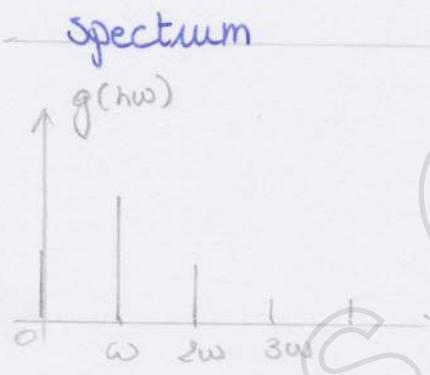
- halve-golfsymm. ($f(t) \equiv -f(t + \frac{T}{2})$) of ($f(t) \equiv f(t + \frac{T}{2})$)
 - enkel oneven harm.
 - enkel even harm.

Het spectrum van een periodieke functie

$$\Rightarrow f(t) = \frac{b_0}{2} + \sum_{h=1}^{\infty} g_h \cos(h\omega t + \theta'_h)$$

amplitude
bij $\omega=0$

$$\sqrt{a_h^2 + b_h^2}$$



De effectieve waarde van een periodieke tijdsfunctie

$$\Rightarrow v(t) = V_0 + \sum_{h=1}^{\infty} V_h \sqrt{2} \cos(h\omega t + \beta_h)$$

gelijkspannings-
componente effectieve
waarde met freq. hω

$$\rightarrow v^2(t) = V_0^2 + V_0 \sum_{h=1}^{\infty} V_h \sqrt{2} \cos(h\omega t + \beta_h) + V_0 \sum_{k=1}^{\infty} V_k \sqrt{2} \cos(k\omega t + \beta_k)$$

$$+ \left[\sum_{h=1}^{\infty} V_h \sqrt{2} \cos(h\omega t + \beta_h) \right] \left[\sum_{k=1}^{\infty} V_k \sqrt{2} \cos(k\omega t + \beta_k) \right]$$

$$\int_T v^2(t) = V_0^2 T + V_0 \sum_{h=1}^{\infty} V_h \sqrt{2} \int_T \cos(hwt + \beta_h) dt +$$

$$V_0 \sum_{k=1}^{\infty} V_k \sqrt{2} \int_T \cos(kwt + \beta_k) dt +$$

$$2 \sum_{h=1}^{\infty} \sum_{k=1}^{\infty} V_h V_k \int_T \cos(hwt + \beta_h) \cos(kwt + \beta_k) dt$$

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$$

$$\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos\alpha \cos\beta$$

$$= V_0^2 T + 2 \sum_{h=1}^{\infty} \sum_{k=1}^{\infty} V_h V_k \int_T \frac{1}{2} [\cos((h+k)wt + \beta_h + \beta_k) + \cos((h-k)wt + \beta_h - \beta_k)] dt$$

$$= 0^* \text{ behalve als } h = k$$

$$\int_T \cos nwt dt = 0 \text{ voor } n=1,2,3,\dots$$

T voor n=0

$$\boxed{\int_T v^2(t) = V_0^2 T + \sum_{h=1}^{\infty} V_h^2 T}$$

Het vermogen geleerd door een periodieke spanning

$$p(t) = v(t) \cdot i(t)$$

&

$$\bar{P} = \frac{1}{T} \int p(t) dt$$

$$\begin{aligned}\bar{P} &= \frac{1}{T} \left[V_0 I_0 T + I_0 \sum_{h=1}^{\infty} V_h \sqrt{2} \int_T \cos(hwt + \beta_h) dt \right. \\ &\quad \left. + V_0 \sum_{k=1}^{\infty} I_k \sqrt{2} \int_T \cos(kwt + \alpha_k) dt \right] \\ &\quad + 2 \sum_{h=1}^{\infty} \sum_{k=1}^{\infty} V_h I_k \int_T \cos(hwt + \beta_h) \cos(kwt + \alpha_k) dt \\ &= V_0 I_0 + \frac{1}{T} \sum_{h=1}^{\infty} \sum_{k=1}^{\infty} V_h I_k \int_T \cos((h+k)wt + \beta_h + \alpha_k) \\ &\quad + \cos((h-k)wt + \beta_h - \alpha_k) dt\end{aligned}$$

$$\boxed{\bar{P} = V_0 I_0 + \sum_{h=1}^{\infty} V_h I_h \cos \phi_h}$$

$$\phi = \beta_h - \alpha_h$$

De exponentiële vorm van de Fourierreeks voor f(t)

$$f(t) = \frac{b_0}{2} + \sum_{k=1}^{\infty} \left(b_k \cos k\omega t + a_k \sin k\omega t \right)$$

$\frac{e^{jk\omega t} + e^{-jk\omega t}}{2}$ $\frac{e^{jk\omega t} - e^{-jk\omega t}}{2j}$

$$f(t) = \frac{b_0}{2} + \sum_{k=1}^{\infty} \left(\frac{b_k - j a_k}{2} e^{jk\omega t} + \frac{b_k + j a_k}{2} e^{-jk\omega t} \right)$$

$$\frac{b_k - j a_k}{2} = \frac{1}{T} \int_{-T/2}^{+T/2} f(t) e^{-jk\omega t} dt \quad (1)$$

$$\frac{b_k + j a_k}{2} = \frac{1}{T} \int_{-T/2}^{+T/2} f(t) e^{+jk\omega t} dt \quad (2)$$

⇒

$$c_n = \frac{1}{T} \int_{-T/2}^{+T/2} f(t) e^{-j n \omega t} dt$$

→ voor $n = k \rightarrow (1)$

→ voor $n = 0 \rightarrow c_n = \frac{b_0}{2}$

→ voor $n = -k \rightarrow (2)$

⇒

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j n \omega t}$$

Responsie van een L.T.O. systeem op een periodiek signaal

- periodiek ingangssignaal \rightarrow goniometr. Fourierreeks

$$u(t) = \sum_{h=0}^{\infty} U_h \cos(h\omega t + \alpha_h)$$

waarbij $\tilde{Y}_h = H(j\omega) \tilde{U}_h$
 $y_h e^{j\beta_h}$ $U_h e^{j\alpha_h}$

$$y(t) = \sum_{h=0}^{\infty} Y_h \cos(h\omega t + \beta_h)$$

- per. ingangssign. \rightarrow exponentiële Fourierreeks

$$u(t) = \sum_{h=-\infty}^{+\infty} \tilde{u}_h(t) = \sum_{h=-\infty}^{+\infty} u_h e^{jh\omega t}$$

waarbij: $\tilde{y}_h(t) = H(j\omega) u_h e^{jh\omega t}$

$$y(t) = \sum_{h=-\infty}^{+\infty} \tilde{y}_h(t)$$

De integraal van Fourier

De exponentiële vorm

$$f(t) = \sum_{h=-\infty}^{+\infty} c_h e^{jhw t} \longrightarrow f(t) = \sum_{h=-\infty}^{+\infty} \frac{\omega}{2\pi} \int_{-\infty}^{+\infty} f(\lambda) e^{-jhw\lambda} d\lambda e^{jhw t}$$

met $c_h = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(\lambda) e^{-jhw\lambda} d\lambda$

Als $s \rightarrow \infty$; $\omega \rightarrow -\infty \rightarrow +\infty$

Stellen we $hw = z$ dan $\omega = \Delta z$ $s \rightarrow \infty$ dan $z = \text{continue variabele}$
& $\Delta z = dz$

$$\Rightarrow f(t) = \frac{1}{2\pi} \lim_{s \rightarrow \infty} \sum_{h=-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} f(\lambda) e^{-jz\lambda} d\lambda \right] e^{jzt} \Delta z$$

λ → t
z → ω

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} f(t) e^{-jwt} dt \right] e^{+jwt} dw$$

↑
Φ(jω) zie volgende pagina

De Fouriertransformatie

- Definitie -

$$\Phi(j\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

↓
Fourier-
transformeerde

$$\rightarrow f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Phi(j\omega) e^{j\omega t} d\omega$$

$$\Phi(j\omega) = \mathcal{F}\{f(t)\}$$

waarbij :

$|\Phi(j\omega)|$ (amplitude)-spectrum

$|\Phi(j\omega)|^2$ energiespectrum

Belangrijke Fouriergetransformeerden

- Als $f(t) = 0$ voor $t < 0 \rightarrow \Phi(j\omega) = F(s) \Big|_{s=j\omega}$

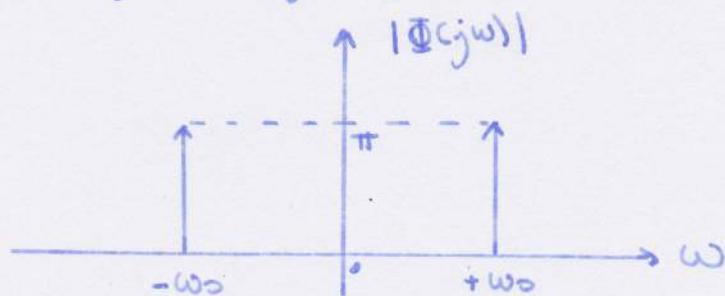
- $\mathcal{F}\{\cos \omega_0 t\}$; $\mathcal{F}\{\sin \omega_0 t\}$

$$\begin{aligned} \cos \omega_0 t &= \frac{1}{2j} \left[e^{j\omega_0 t} + e^{-j\omega_0 t} \right] \\ \sin \omega_0 t &= \frac{1}{2j} \left[e^{j\omega_0 t} - e^{-j\omega_0 t} \right] \end{aligned}$$

dirac: $\int_{-\infty}^{+\infty} f(t) \cdot \delta(t-t_0) dt = f(t_0)$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} -j\pi (\delta(\omega-\omega_0) \pm \delta(\omega+\omega_0)) \cdot e^{j\omega t} d\omega$$

$$\mathcal{F}\{\cos \omega_0 t\} = \pi(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)) \quad \mathcal{F}\{\sin \omega_0 t\} = j\pi(\delta(\omega+\omega_0) - \delta(\omega-\omega_0))$$



← selfde amplitude-spectrum voor sin & cos

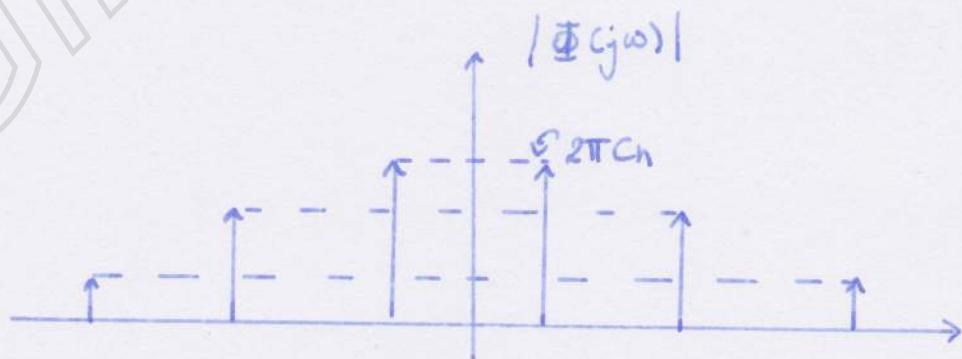
- Fouriergetallen formeerde van periodieke fct

$$f(t) = \sum_{h=-\infty}^{+\infty} c_h e^{j h \omega_0 t}$$

$$\rightarrow = \int_{-\infty}^{+\infty} \sum_{h=-\infty}^{+\infty} c_h \delta(\omega - h\omega_0) e^{j\omega t} d\omega$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[\sum_{h=-\infty}^{+\infty} 2\pi c_h \delta(\omega - h\omega_0) \right] e^{j\omega t} d\omega$$

$$\Phi(j\omega) / \mathcal{F}\{f(t)\}$$



Eigenschappen van de Fouriertransformatie

- lineairiteit

- $\Phi(j\omega) = \Phi^*(-j\omega)$ want $\bar{A} = a+bj = |A|e^{j\theta}$
 $\bar{A}^* = a-bj = |A|e^{-j\theta}$

daaruit volgt:

$$|\Phi(j\omega)| = |\Phi(-j\omega)|$$

$$\arg \Phi(j\omega) = -\arg \Phi(-j\omega)$$

→ spectrum is dus symm. t.o.v. de $|A|$ -as

- Als $f(t)$ even $\rightarrow f(-t) = f(t) \rightarrow \Phi(j\omega) = \Phi^*(j\omega)$

↳ dan is Φ een reële functie

- Als $f(t)$ oneven $\rightarrow f(-t) = -f(t) \rightarrow \Phi(j\omega) = -\Phi^*(j\omega)$

↳ dan is Φ een zuiver imaginair \uparrow functie

- Symmetrie: $\Phi(jt) = \mathcal{F}^{-1}\{2\pi f(-\omega)\}$

- verschuiving tijdsschaal: $\Phi(j\omega)e^{-j\omega t_0} = \mathcal{F}\{f(t-t_0)\}$

- verandering tijdsschaal: $\frac{1}{a}\Phi\left(\frac{j\omega}{a}\right) = \mathcal{F}\{f(at)\}$

- verschuiving freq. domain: $\Phi(j(\omega-\omega_0)) = \mathcal{F}\{e^{j\omega_0 t} f(t)\}$

- Convolutie in de tijd: $\Phi_1(j\omega) \cdot \Phi_2(j\omega) = \mathcal{F}\{f_1(t) * f_2(t)\}$
- convolutie i.h. freq. domain: $\frac{1}{2\pi} [\Phi_1(j\omega) * \Phi_2(j\omega)] = \mathcal{F}\{f_1(t) \cdot f_2(t)\}$

$$= \int_{-\infty}^{+\infty} \Phi_1(j\omega) \Phi_2(j\omega - j\Omega) d\omega$$

- Formule van Parseval

$$\int_{-\infty}^{+\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |\Phi(j\omega)|^2 d\omega$$

gedissipeerd
energie over 1-2

↑
energiedichtheid / brandbreedte
= spectrale energiedichth.

- Sommatische formule van Poisson

$$f(t) \text{ willekeurig} \rightarrow \sum_{n=-\infty}^{+\infty} f(nT) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} \Phi(j_n w_s) \text{ met } w_s = \frac{2\pi}{T}$$

$$f(t) = 0 \text{ voor } t < 0$$

$$\rightarrow \sum_{n=0}^{+\infty} f(nT) = \frac{f(0^+)}{2} + \frac{1}{T} \sum_{n=-\infty}^{+\infty} \Phi(j_n w_s)$$

)

correctterm.

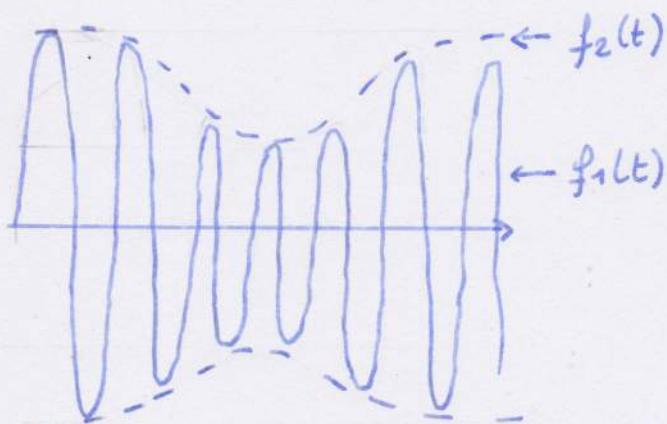
De gemiddelde waarde van een analog signaal

$$V_{\text{gem}} = \frac{1}{T} \int_0^T v(t) dt \quad \& \quad V(j\omega) = \int_0^T v(t) e^{-j\omega t} dt$$
$$\Rightarrow V_{\text{gem}} = \frac{V(j\omega)}{T}$$

De effectieve waarde van een analog signaal

$$V_{\text{eff}}^2 = \frac{1}{T} \int_0^T v^2(t) dt \quad \xrightarrow{\text{Parserval}} \quad V_{\text{eff}}^2 = \frac{1}{2\pi T} \int_{-\infty}^{+\infty} |V(j\omega)|^2 d\omega$$

Het spectrum van een amplitude-gemoduleerd signaal



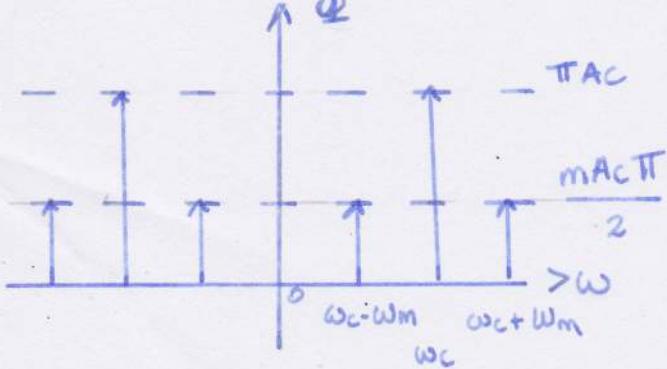
$$\text{met } \begin{cases} f_1(t) = A_c \cos \omega_c t \\ f_2(t) = 1 + m \cos \omega_m t \end{cases} \quad \text{modulatiediepte}$$

$$\omega_c \gg \omega_m$$

$$\rightarrow f_1(t) \cdot f_2(t)$$

$$\rightarrow \mathcal{F} \{ f_1(t) \cdot f_2(t) \}$$

$$= \frac{1}{2\pi} [\Phi_1(j\omega) * \Phi_2(j\omega)]$$



De responsie van een L.T.O. systeem

$$y(t) = \int_0^t u(z) h(t-z) dz = \int_{-\infty}^{+\infty} u(z) h(t-z) dz$$

$$\Rightarrow Y(j\omega) = H(j\omega) U(j\omega)$$

$\stackrel{?}{=}$ $\stackrel{?}{=}$ $\stackrel{?}{=}$

$$\mathcal{F}\{y(t)\} \quad \mathcal{F}\{h(t)\} \quad \mathcal{F}\{u(t)\}$$

$$h(t) = 0 \text{ voor } t < 0$$

$$\Rightarrow H(j\omega) = H(s) \Big|_{s=j\omega}$$

↑

Fourier-
getransf.

↑

Laplace-
getransf.

Appendix A:

Stabiliteitscriterium van Routh - Hurwitz

stabiel als karakter. verg. geen pos. reële wortels
↳ alle coëff. hetzelfde teken.

R-H tabel

s^n	1	b_{n-2}	$b_{n-4} \dots$
s^{n-1}	b_{n-1}	b_{n-3}	$b_{n-5} \dots$
s^{n-2}	c_1	c_2	$c_3 \dots$
s^{n-3}	d_1	$d_2 \dots$	
\dots			
s^0	m_1		

$$C_1 = \frac{\begin{vmatrix} 1 & b_{n-2} \\ b_{n-1} & b_{n-3} \end{vmatrix}}{b_{n-1}}$$

$$C_2 = \frac{\begin{vmatrix} 1 & b_{n-4} \\ b_{n-2} & b_{n-5} \end{vmatrix}}{b_{n-1}}$$

$$d_1 = \frac{\begin{vmatrix} b_{n-1} & b_{n-3} \\ c_1 & c_2 \end{vmatrix}}{c_2} \dots$$

↑
tekenomwisseling

=
wortels met pos. Re. del

Problemen

- eerste element rij = 0 \rightarrow vervangen door E' zeer klein getal
- nul rij \rightarrow rij erboven afleiden \rightarrow invullen i.v.m. nullij.