Chapter 3 Homework Answers

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Due Wednesday, Sept. 5, 2018

Instructions

The exercises from Chapter 3, below, are not to do for the lab but are homework exercises. You have a choice:

- You can do the exercises out of the book like regular homework and turn them in at the beginning of class on Wednesday, Sept. 5.
- You can do them using RMarkdown and turn them by submitting them electronically on GitHub by the start of class on Wednesday, Sept. 5.

  To submit the homework electronically,

  - Clone a local copy of the file repository from github.
  - In your local repository, answer the exercises in the template `homework_01.Rmd`.
  - When you are done, knit your `.Rmd` file into a PDF or DOCX (Microsoft Word) file.
  - Use git to commit your changes (including the edits to `homework_01.Rmd` and the new PDF or DOCX file) to your local git repository.
  - Push the changes from your local git repository to github.

    The last changes that you push before the due date (start of class on Wednesday, Sept. 5) will be graded.

It is your choice how to do them. Either way is acceptable and will get equal credit.

Exercises from Chapter 3

For the following exercises, use the following numbers:

- $I_{solar} = 1350\ W/m^2$
- $\sigma = 5.67 \times 10^{-8}$
- $\alpha = 0.30$
- $\varepsilon = 1.0$

```
I_solar = 1350
alpha = 0.30
```

```
sigma = 5.67E-8
epsilon = 1
```

Worked Example for One-Layer Atmosphere

A One-Layer Model.

```
make_layer_diagram(1)
```

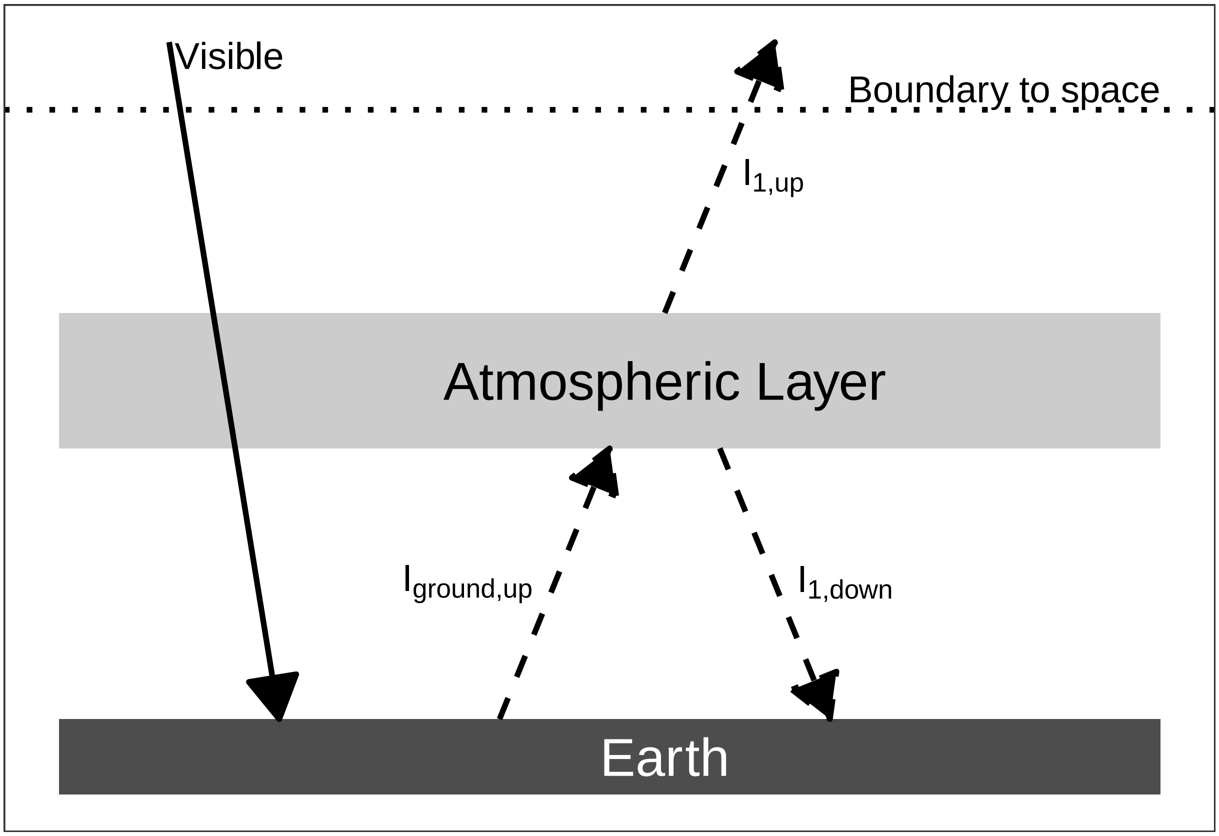

Figure 1: An energy diagram for a planet with one pane of glass for an atmosphere. The intensity of heat from visible light is $(1-\alpha)I_{\text{solar}}/4$.

a) Write the energy budgets for the atmospheric layer, for the ground, and for the Earth as a whole.

Answer: Start at the top, at the boundary to space, and work downward:

- At the boundary to space, $I_{1,\text{up}} = (1-\alpha)I_{\text{solar}}/4$.
- At the atmospheric layer, $I_{1,\text{up}} + I_{1,\text{down}} = I_{\text{ground,up}}$
- At the ground, $(1-\alpha)I_{\text{solar}} + I_{1,\text{down}} = I_{\text{ground,up}}$

We also know that

- $I_{1,\text{up}} = I_{1,\text{down}} = \varepsilon\sigma T_1^4$
- $I_{\text{ground,up}} = \sigma T_{\text{ground}}^4$

b) Manipulate the budget for the Earth as a whole to obtain the temperature T_1 of the atmospheric layer. Does this part of the exercise seem familiar in any way? Does the term ring any bells?

Answer:

$$(1 - \alpha) I_{\text{solar}}/4 = I_{1,\text{up}} = \sigma T_1^4$$

$$(1 - \alpha) I_{\text{solar}}/4\varepsilon\sigma = T_1^4$$

$$T_1 = \sqrt[4]{\frac{(1 - \alpha) I_{\text{solar}}}{4\varepsilon\sigma}}$$

This is familiar, because it's the same as the formula for the bare-rock temperature.

Here is R code to calculate $I_{1,\text{up}}$ and T_1:

```
I_1_up = (1 - alpha) * I_solar / 4
T_1 = (I_1_up / (epsilon * sigma))^0.25
```

From the algebraic solution, we expect T_1 to be 254. K. From the R code above, we get T_1 = 254. K.

c) Now insert the value you found for T_1 into the budget for atmospheric layer 1 to obtain the temperature of the ground, T_{ground}.

Answer:

- $I_{\text{ground}} = I_{1,\text{up}} + I_{1,\text{down}} = 2 \times I_{1,\text{up}}$
- $\varepsilon\sigma T_{\text{ground}}^4 = 2\varepsilon\sigma T_1^4$
- $T_{\text{ground}}^4 = 2T_1^4$
- $T_{\text{ground}} = \sqrt[4]{2} \times T_1$

And here is R code to calculate $I_{1,\text{down}}$, I_{ground}, and T_{ground}:

```
I_1_down = I_1_up
I_ground = I_1_up + I_1_down
T_ground = (I_ground / (epsilon * sigma))^0.25
```

From the algebraic solution, we get T_{ground} = 302. K and from the R code above, we get T_{ground} = 302. K.

Exercise 3.1 (Grad. students only)

> **The moon with no heat transport.** The Layer Model assumes that the temperature of the body in space is all the same. This is not really very accurate, as you know that it is colder at the poles than it is at the equator. For a bare rock with no atmosphere or ocean, like the moon, the situation is even worse because fluids like air and water are how heat is carried around on the planet. So let us make the other extreme assumption, that there is no heat transport on a bare rock like the moon. Assume for comparability that the albedo of this world is 0.30, same as Earth's.
>
> What is the equilibrium temperature of the surface of the moon, on the equator, at local noon, when the sun is directly overhead? (Hint: First figure out the intensity of the local solar radiation I_{solar})

Answer: Since the moon has no heat transport, we can consider I_{in} and I_{out} at the point where the sun is directly overhead, without worrying about averaging over all of the surface area of the moon. This means that instead of

$$I_{\text{in}} = \frac{(1-\alpha)I_{\text{solar}}}{4},$$

we use

$$I_{\text{in}} = (1-\alpha)I_{\text{solar}},$$

so

$$\begin{aligned} I_{\text{out}} &= I_{\text{in}} \\ \varepsilon\sigma T_{\text{moon}}^4 &= (1-\alpha)I_{\text{solar}} \\ T_{\text{moon}}^4 &= \frac{(1-\alpha)I_{\text{solar}}}{\varepsilon\sigma} \\ T_{\text{moon}} &= \sqrt[4]{\frac{(1-\alpha)I_{\text{solar}}}{\varepsilon\sigma}} \end{aligned}$$

```
I_solar = 1350 # W / m^2
albedo = 0.3
emissivity = 1
T_moon = ( (1 - albedo) * I_solar / (emissivity * sigma) )^0.25
```

T_{moon} is 359. Kelvin.

> What is the equilibrium temperature on the dark side of the moon?

Answer:

On the dark side of the moon I_{solar} is zero, so we expect the temperature to be zero.

```
I_solar_dark = 0
T_dark_side = ( (1 - albedo) * I_solar_dark / (emissivity * sigma) )^0.25
```

$T_{dark\text{-}side}$ = 0 Kelvin.

In fact, outer space has a radiation that corresponds to black body with a temperature of around 3 Kelvin. This radiation is heat left over from the big bang, almost 14 billion years ago. Scientists call the the "Cosmic Microwave Background Radiation," and it means that empty deep space behaves as though it has a temperature of 3 K. Thus, in our example we would really expect the dark side of the moon to have a temperature of around 3 K.

The real dark side of the moon is cold, but not this cold.

- First, the moon orbits around the earth, so each side gets sunlight for half of each month and is in darkness for half of each month, so even when it's dark, the surface still has some leftover heat from the last time it was in the sun (it takes a long time to cool all the way from 359. K to 0 K).
- Second, there is a small, but nonzero, flow of heat through the solid moon (this is thermal conduction), so heat does travel from the bright side to the dark side.

These phenomena make the dark side of the moon much warmer than 3 Kelvin, but it is still bitterly cold there.

However, the homework explicitly told you to ignore these details, so for the purposes of this homework exercise, the correct answer is zero Kelvin.

Exercise 3.2

A Two-Layer Model. Insert another atmospheric layer into the model, just like the first one. The layer is transparent to visible light but a blackbody for infrared.

```
make_layer_diagram(2)
```

a) Write the energy budgets for both atmospheric layers, for the ground, and for the Earth as a whole, like we did for the One-Layer Model.

Answer:

First, we balance the flow of heat up and down at the imaginary boundary between the atmosphere and outer space.:

$$I_{2,\text{up}} = (1 - \alpha) I_{\text{solar}}/4$$

The top layer of the atmosphere (layer 2) behaves exactly the way it did in the one-layer model:

$$\begin{aligned} I_{2,\text{in}} &= I_{2,\text{out}} \\ I_{1,\text{up}} &= I_{2,\text{up}} + I_{2,\text{down}} = 2 I_{2,\text{up}} \end{aligned}$$

($I_{2,\text{up}} = I_{2,\text{down}}$ because both are determined by the temperature of layer 2, T_2, and the Stefan-Boltzmann equation.)

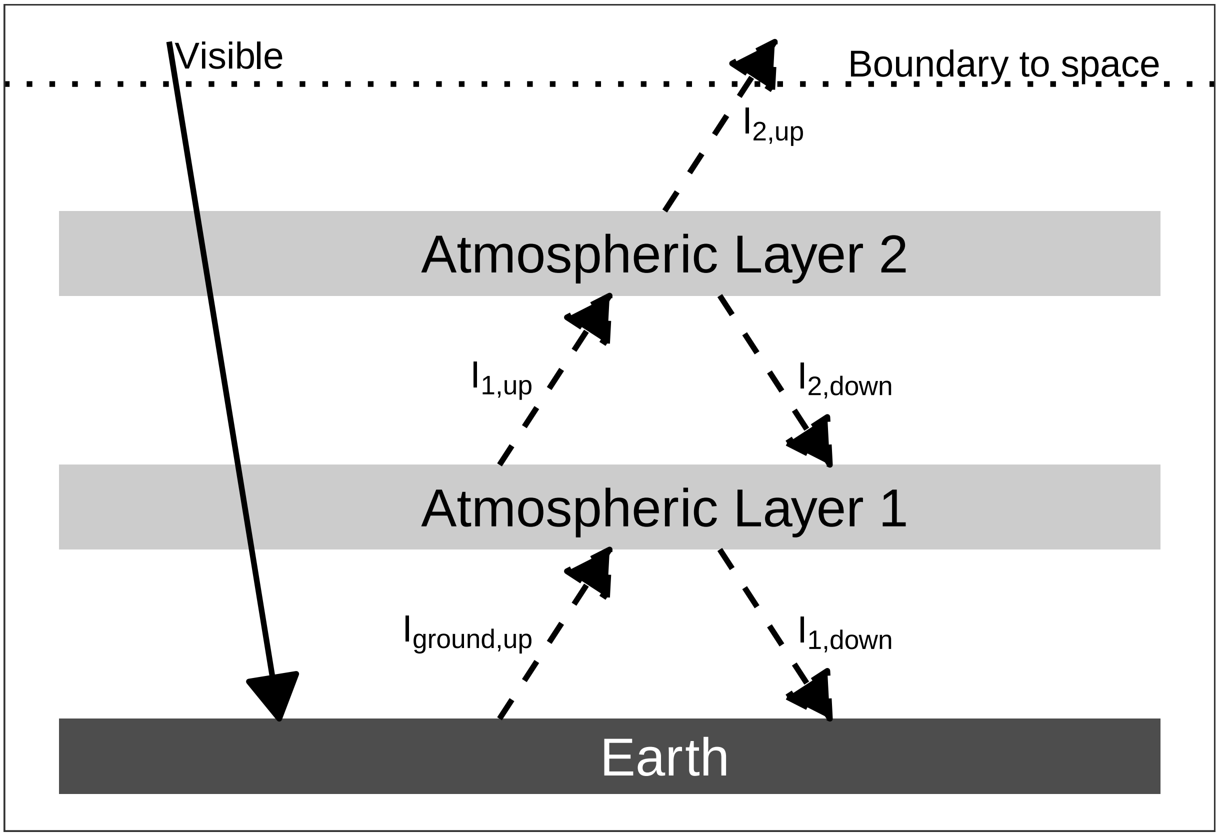

Figure 2: An energy diagram for a planet with two panes of glass for an atmosphere. The intensity of absorbed visible light is $(1-\alpha)I_{\text{solar}}/4$.

This is essentially the same equation we used for the heat balance of the atmosphere in the one-layer model.

The heat-balance for the lower layer of the atmosphere is a little more complicated because there are two sources of heat in:

$$I_{1,\text{in}} = I_{1,\text{out}}$$
$$I_{\text{ground,up}} + I_{2,\text{down}} = I_{1,\text{up}} + I_{1,\text{down}} = 2I_{1,\text{up}}$$

Finally, the heat balance for the ground is

$$I_{\text{ground,in}} = I_{\text{ground,out}}$$
$$\frac{(1-\alpha)I_{\text{solar}}}{4} + I_{1,\text{down}} = I_{2,\text{up}} + I_{2,\text{down}} = 2I_{2,\text{up}}$$

b) Manipulate the budget for the Earth as a whole to obtain the temperature T_2 of the top atmospheric layer, labeled Atmospheric Layer 2 in the figure above. Does this part of the exercise seem familiar in any way? Does the term ring any bells?

Answer:

$$
\begin{aligned}
I_{2,\text{up}} &= \frac{(1-\alpha)I_{\text{solar}}}{4} \\
\sigma T_2^4 &= \frac{(1-\alpha)I_{\text{solar}}}{4} \\
T_2^4 &= \frac{(1-\alpha)I_{\text{solar}}}{4\sigma} \\
T_2 &= \sqrt[4]{\frac{(1-\alpha)I_{\text{solar}}}{4\sigma}}
\end{aligned}
$$

```
T_2 = ( (1 - albedo) * I_solar / (4 * sigma) )^0.25
```

This is just like the one-layer model, and we get the same bare-rock temperature for the top of the atmosphere: $T_2 = 254.$ Kelvin.

c) Insert the value you found for T_2 into the energy budget for layer 2, and solve for the temperature of layer 1 in terms of layer 2. How much bigger is T_1 than T_2?

Answer:

$$
\begin{aligned}
I_{2,\text{in}} &= I_{2,\text{out}} \\
I_{1,\text{up}} &= I_{2,\text{up}} + I_{2,\text{down}} = 2I_{2,\text{up}} \\
\sigma T_1^4 &= 2\sigma T_2^4 \\
T_1^4 &= 2T_2^4 \\
T_1 &= \sqrt[4]{2}\, T_2
\end{aligned}
$$

```
T_1 = 2^0.25 * T_2
```

This gives layer 1 the same temperature that the ground had in the one-layer model: $T_1 = 302.$, which is $\sqrt[4]{2} = 1.19$ times bigger than T_2.

d) Now insert the value you found for T_1 into the budget for atmospheric layer 1 to obtain the temperature of the ground, T_{ground}. Is the greenhouse effect stronger or weaker because of the second layer?

Answer:

This gets a little more complicated:

$$
\begin{aligned}
I_{1,\text{in}} &= I_{1,\text{out}} \\
I_{\text{ground,up}} + I_{2,\text{down}} &= I_{1,\text{up}} + I_{1,\text{down}} = 2I_{1,\text{up}} \\
\sigma T_{\text{ground}}^4 + \sigma T_2^4 &= 2\sigma T_1^4 \\
T_{\text{ground}}^4 + T_2^4 &= 2T_1^4 \\
T_{\text{ground}}^4 &= 2T_1^4 - T_2^4
\end{aligned}
$$

But $T_1^4 = 2T_2^4$, so

$$T_{\text{ground}}^4 = 4T_2^4 - T_2^4 = 3T_2^4$$
$$T_{\text{ground}} = \sqrt[4]{3}\, T_2^4$$

```
T_ground = 3^0.25 * T_2
```

Thus, the ground temperature in the two-layer model is 334. Kelvin. This is 32. Kelvin hotter than the ground was in the one-layer model, so adding another layer made the greenhouse effect stronger.

Exercise 3.3

```
make_nuclear_winter_diagram()
```

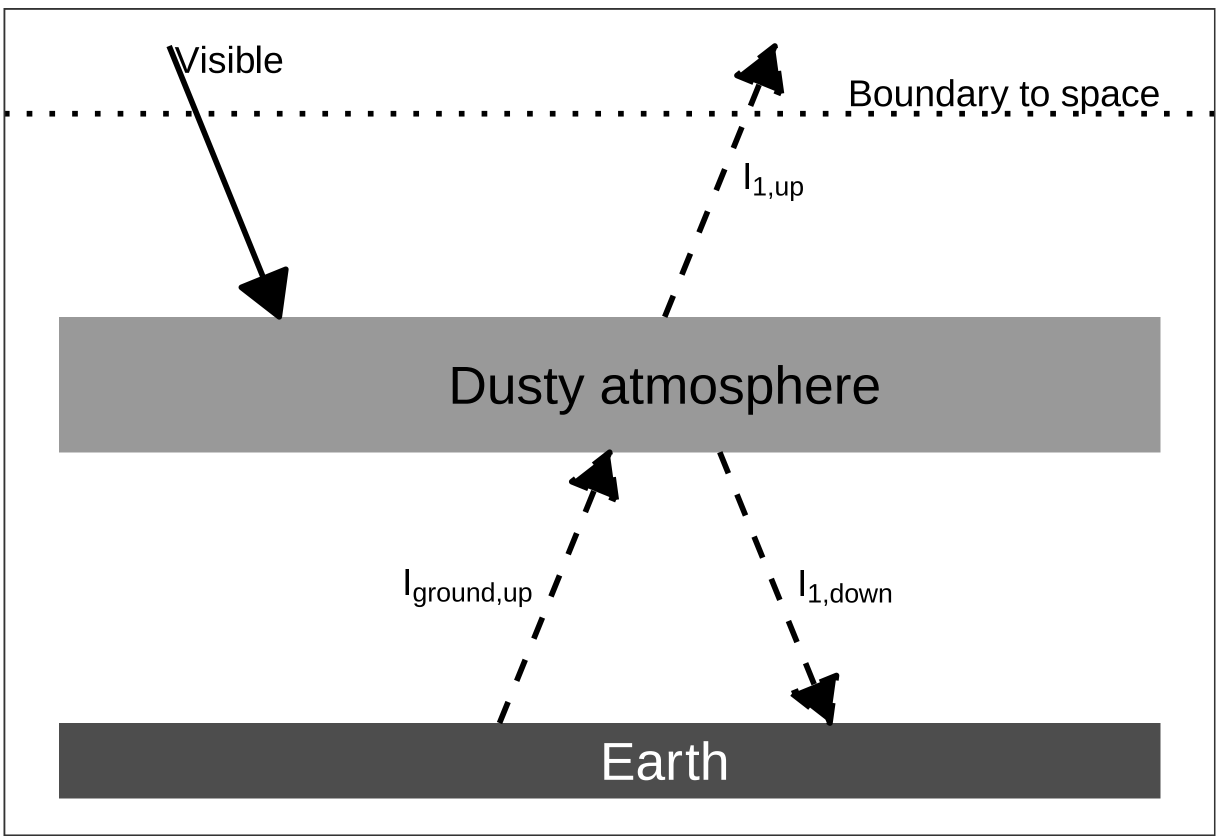

Figure 3: An energy diagram for a planet with an opaque pane of glass for an atmosphere. The intensity of absorbed visible light is $(1 - \alpha)I_{\text{solar}}/4$.

> **Nuclear Winter.** Let us go back to the One-Layer Model but change it so that the atmospheric layer absorbs visible light rather than allowing it to pass through (See the figure above). This could happen if the upper atmosphere were filled with dust. For simplicity, assume that the albedo of the Earth remains the same, even though in the

real world it might change with a dusty atmosphere.> What is the temperature of the ground in this case?

Answer:

We start, as always, by balancing the energy. Just as in all of the other layer models, the temperature of the top layer of the atmosphere (the dusty layer) is the basic bare-rock temperature (because the dust doesn't change the albedo in this problem).

$$
\begin{aligned}
I_{\text{atm,up}} &= \frac{(1-\alpha)I_{\text{solar}}}{4} \\
\sigma T_1^4 &= \frac{(1-\alpha)I_{\text{solar}}}{4} \\
T_1^4 &= \frac{(1-\alpha)I_{\text{solar}}}{4\sigma} \\
T_1 &= \sqrt[4]{\frac{(1-\alpha)I_{\text{solar}}}{4\sigma}}
\end{aligned}
$$

What's new is that when we balance the heat flow in the dusty atmosphere, we have:

$$
\begin{aligned}
I_{\text{atm,in}} &= I_{\text{atm,out}} \\
\frac{((1-\alpha)I_{\text{solar}}}{4} + I_{\text{ground,up}} &= I_{\text{atm,up}} + I_{\text{atm,down}} = 2I_{\text{atm,up}}
\end{aligned}
$$

We could just put numbers in and solve for T_{ground}, but if we remember that

$$
\frac{(1-\alpha)I_{\text{solar}}}{4} = I_{\text{atm,up}}
$$

we can substitute

$$
\begin{aligned}
I_{\text{atm,up}} + I_{\text{ground,up}} &= 2I_{\text{atm,up}} \\
I_{\text{ground,up}} &= I_{\text{atm,up}} \\
\sigma T_{\text{ground}}^4 &= \sigma T_{\text{atm}}^4 \\
T_{\text{ground}} &= T_{\text{atm}}
\end{aligned}
$$

```
T_atm = ( (1 - albedo) * I_solar / (4 * sigma) )^0.25
T_ground = T_atm
```

So the ground temperature will be 254. K, the same as the temperature of the atmosphere, which is the bare-rock temperature, so there is no greenhouse effect.