Sensitivity, Uncertainty, and Robustness Analysis

EES 4760/5760

Agent-Based & Individual-Based Computational Modeling

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Sensitivity, Uncertainty, Robustness

- Does ABM reproduce patterns **robustly**?
  - or are patterns **sensitive** to specific values for parameters?
- How **uncertain** are the results?
  - What can the model tell us about parameters that we can’t measure?
- **Sensitivity Analysis** focuses on small changes in parameters.
- **Robustness Analysis** looks at large changes in parameters.

Is high sensitivity good or bad?

- **Bad:** If model is testing a general theory, but is very sensitive to parameter values, that is evidence *against* the theory.
  - Does model work across the entire range of *observed values* for parameters?
- **Good:** If the model is being used to evaluate parameters we can't measure, higher sensitivity can mean less uncertainty about parameters.

Challenges: Computational Complexity

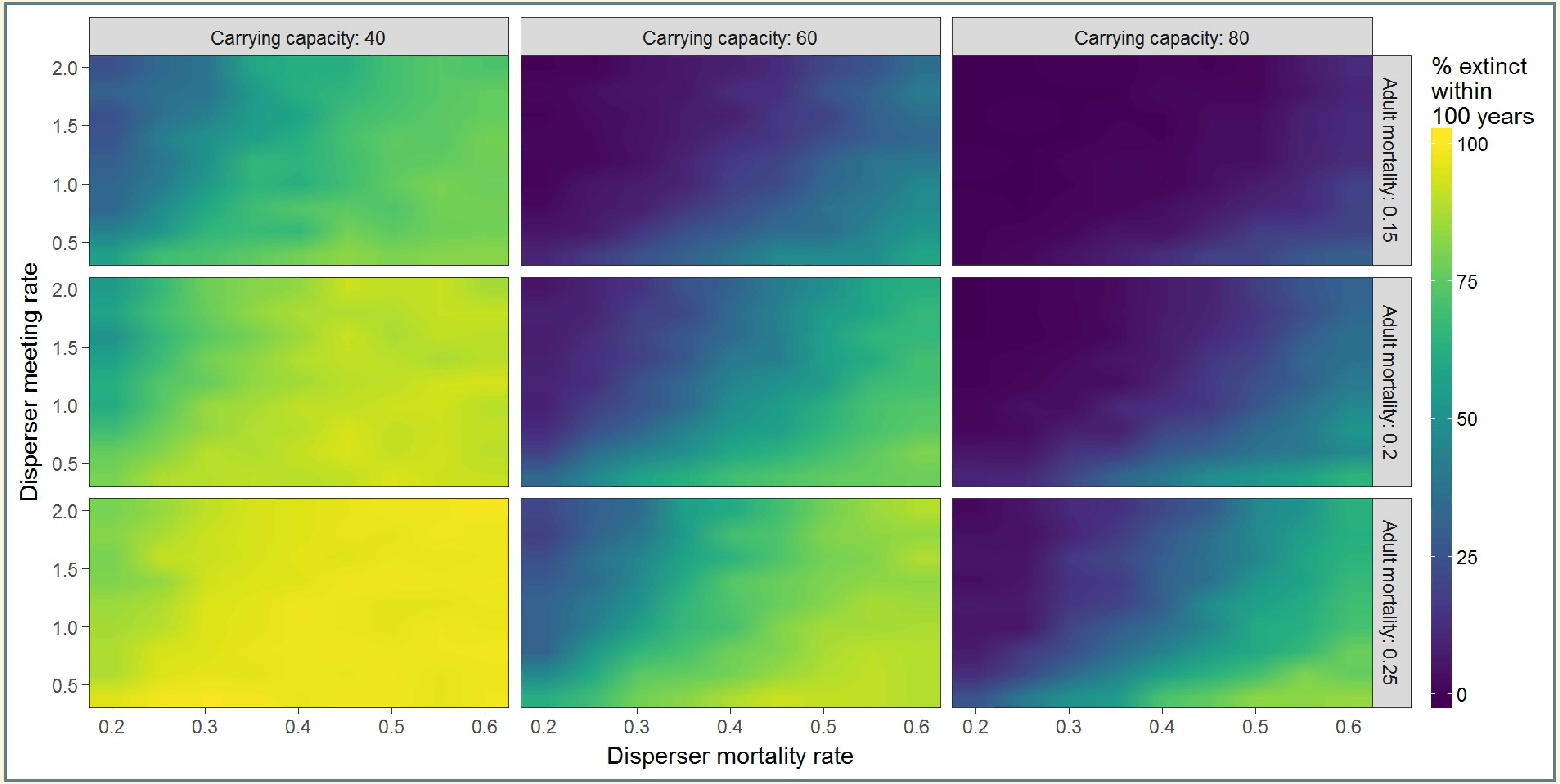
- We would like to do *global sensitivity analysis:*
  - Vary all parameters over their entire ranges, in every combination.
  - Can’t: computationally unfeasible.
  - There are strategies to make global sensitivity analysis feasible, but they are complicated.
- Instead: *local sensitiity analysis:*
  - Small variations around most likely values of parameters.
  - Vary one parameter at a time, or multiple parameters?
    - Interactions
  - Sampling parameter values
  - Random (Monte Carlo)
  - Systematic (e.g., Latin Hypercube)

Example: Wild Dog Model

- Packs of wild dogs in nature preserve.
- Goal: Keep them from going extinct in next 100 years.
- Vary parameters:
  - Mortality rate of adult dogs in pack
  - Mortality rate of dispersers
  - Meeting rate of disperser groups
  - Carrying capacity

Analyzing data:

- Contour plots
- “Small multiple” plots
- Analyze four-dimensional data set using a grid of nine plots.



Carrying capacity: 40
Carrying capacity: 60
Carrying capacity: 80
Adult mortality: 0.15
Adult mortality: 0.2
Adult mortality: 0.25
% extinct within 100 years
100
75
50
25
0
2.0
1.5
1.0
0.5
0.2
0.3
0.4
0.5
0.6
Disperser meeting rate
Disperser mortality rate

Example Research Model

Example Research Model

J.J. Jordan *et al.*, "Third-party punishment as a costly signal of trustworthiness," Nature **530**, 473 (2016). doi:10.1038/nature16981

- Cooperation and Cheating
  - Common situation:
    - Everyone is better off if everyone cooperates than if everyone cheats.
    - Once everyone else has chosen their action, any individual is better off cheating than cooperating.
    - Nash equilibrium: Everyone making the best choice for himself produces the worst outcome for everyone.
    - Opposite of the "invisible hand" in economics.

Prisoner’s Dilemma

	B Cooperates	B Defects
A Cooperates	5, 5	0, 7
A Defects	7, 0	1, 1

- No matter what player A does, player B is better off defecting
- No matter what player B does, player A is better off defecting
- If both players defect, both are worse off than if both cooperated.

Tragedy of the commons

- Ten farmers share a pasture.
- A pasture can support 100 cows.
- If $N_{\text{cows}} \leq 100$, each cow produces $1,000 worth of milk per month.
- If $N_{\text{cows}} > 100$, each cow produces

  $$\$1,000 \times \left(1 - \frac{(N_{\text{cows}} - 100)}{100}\right)$$

  worth of milk per month.
- Each farmer has 10 cows, each farmer earns $10,000 per month.
- One farmer adds 1 cow: total 101.
  - .fragment Each cow produces $\$1000 \times (1 - (101 - 100)/100) = \$990$.
  - .fragment First farmer earns (11 $990 = $10,890),
  - .fragment Everyone else earns $9,900.
- Each farmer adds 1 cow: total 110.
  - Each cow produces $900. Each farmer earns $9,000.

Iterated games

- If only playing once, best strategy is to cheat, because it is rational for everyone else to cheat.
- If playing multiple turns, threat of punishment in future rounds promotes cooperation.
- It is generally costly to punish people.
- If someone cheats against you, it’s often worthwhile to punish them.
- If you see someone cheating against another person and you aren’t affected, is it worth your while to punish the cheater, even if it costs you?
- Does tragedy of commons inhibit people from punishing?

Theory

- Punishment sends a signal:
  - Deters cheaters.
  - Signals that you are trustworthy.

Game

- Player has two roles: Signaler and Chooser
- Signaler can be either Trustworthy or Exploitative.
- Two kinds of signals: Helping or Punishing a third party.
- Two stages:

1. Signalers can pay costs to send signals.
2. Choosers decide whether to accept Signalers as partners.

- Cost of signaling can be either small (s) or large (ℓ)

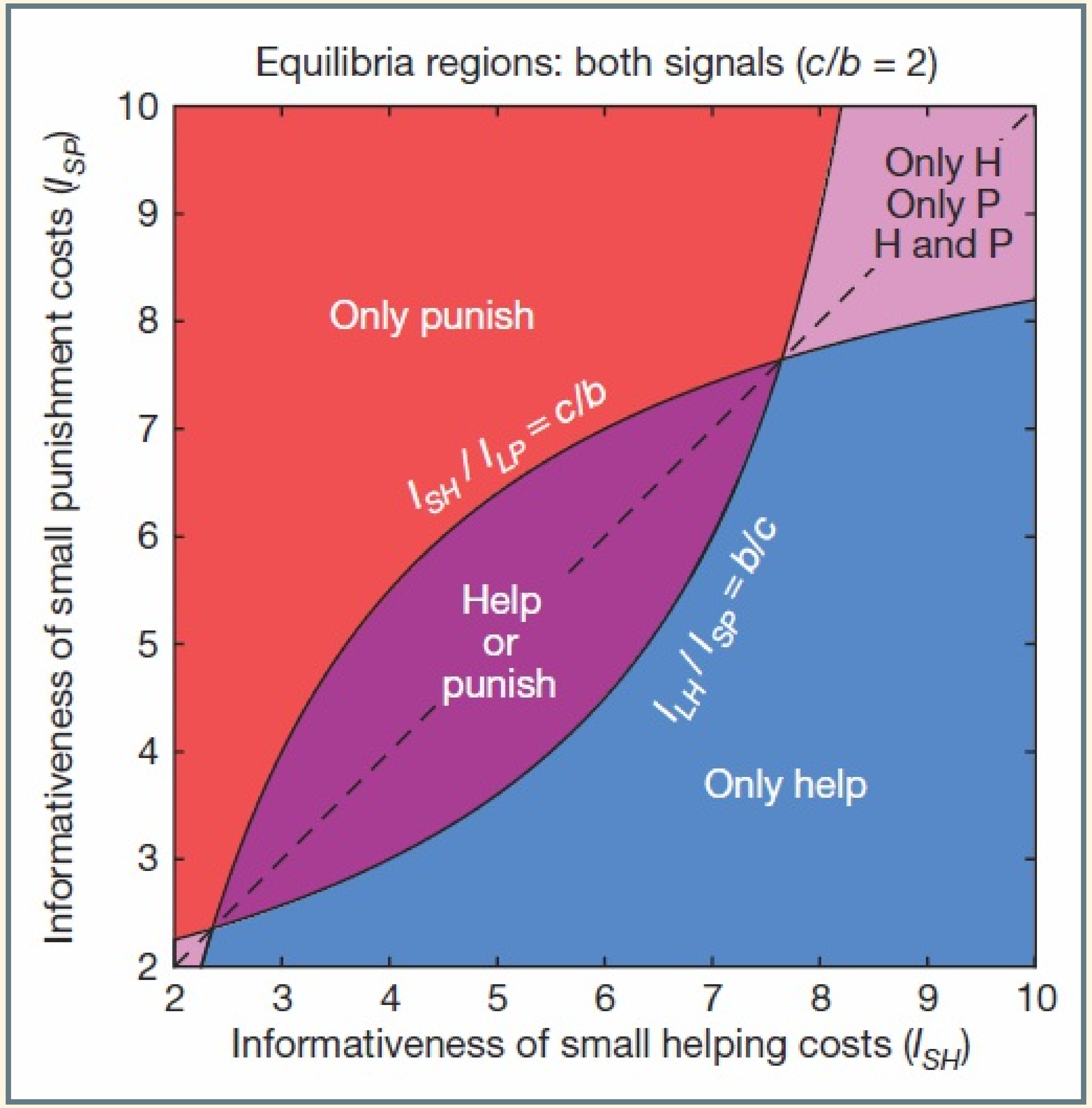
Payoffs

- Payoffs after second stage are:

	Trustworthy Signaller	**Exploitative Signaller**
Chooser Accepts	m, r	$-e$, r
Chooser Rejects	0, 0	0, 0

- m is benefit of mutual cooperation, r is reward for being trustworthy, e is harm from exploitation.

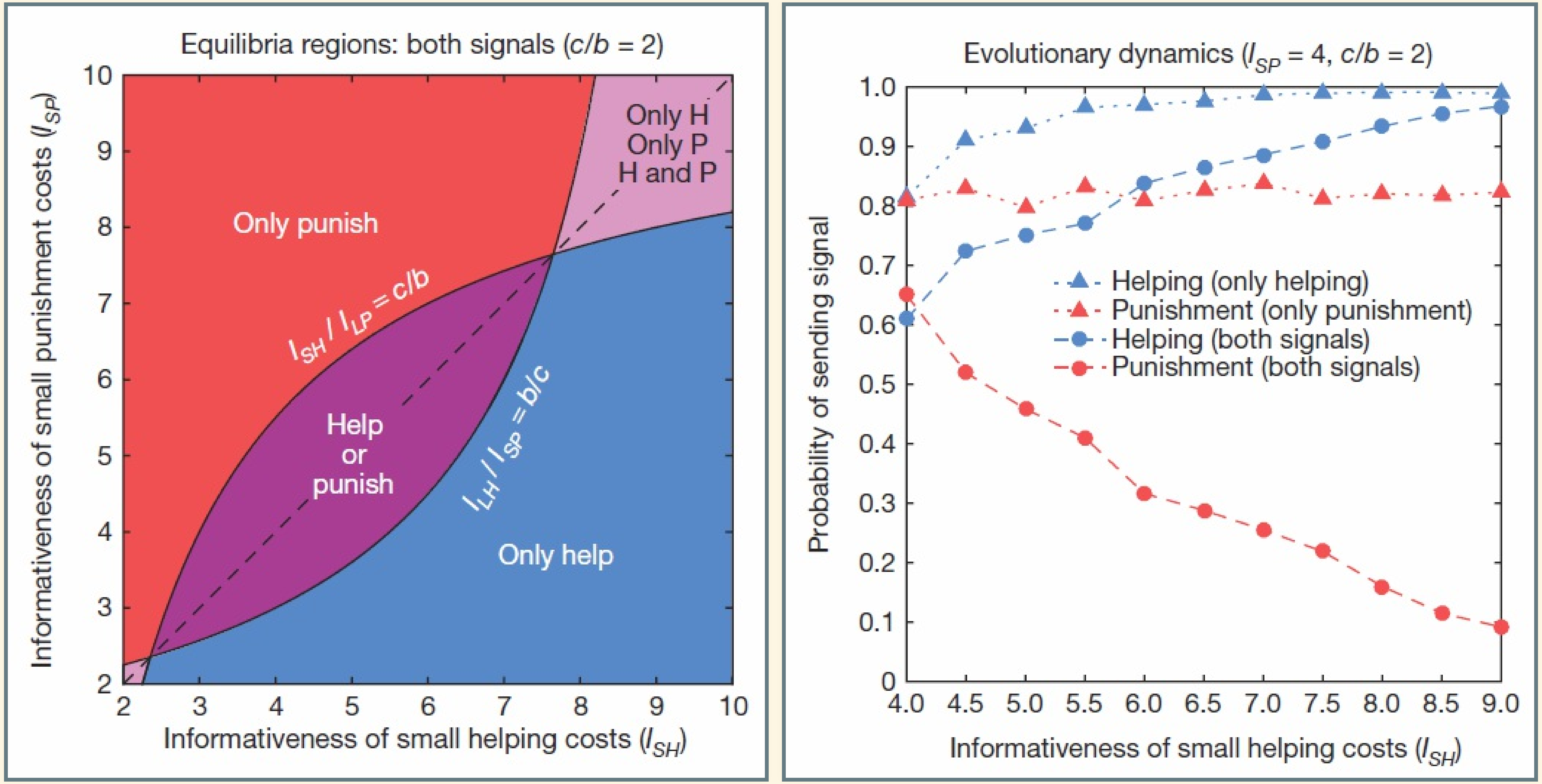
Rational strategies



Agent-based model

- Signaler randomly chosen to be Trustworthy or Exploitative.
- Chooser does not know Signaler type
- Evolution of strategies:
  - Each agent plays a certain number of turns (a generation)
  - Agents have probability of reproducing based on earnings from game.
    - Offspring inherit strategy with some random “mutations”

Outcome of evolution



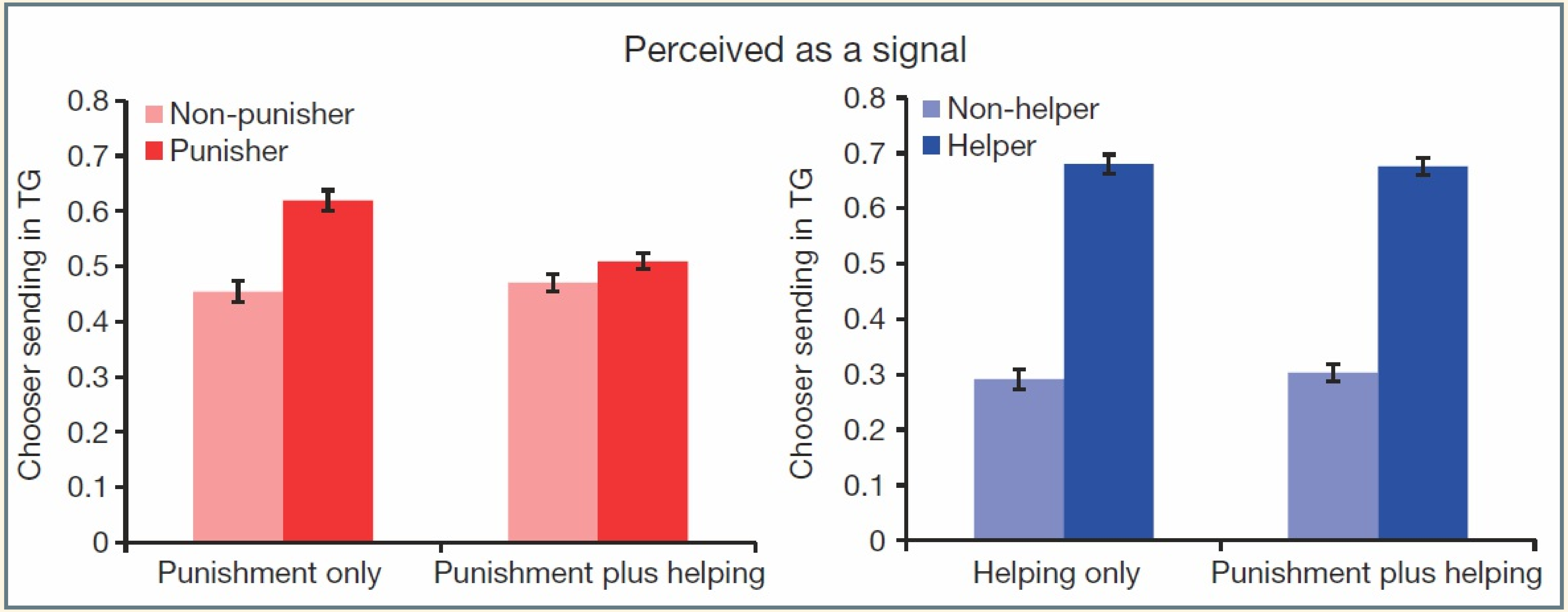
Human Game

- Amazon Mechanical Turk (Internet)
- Human players assigned to one of three games:
  - Signaler can only punish.
  - Signaler can only help.
  - Signaler can help and punish.

Trust Game

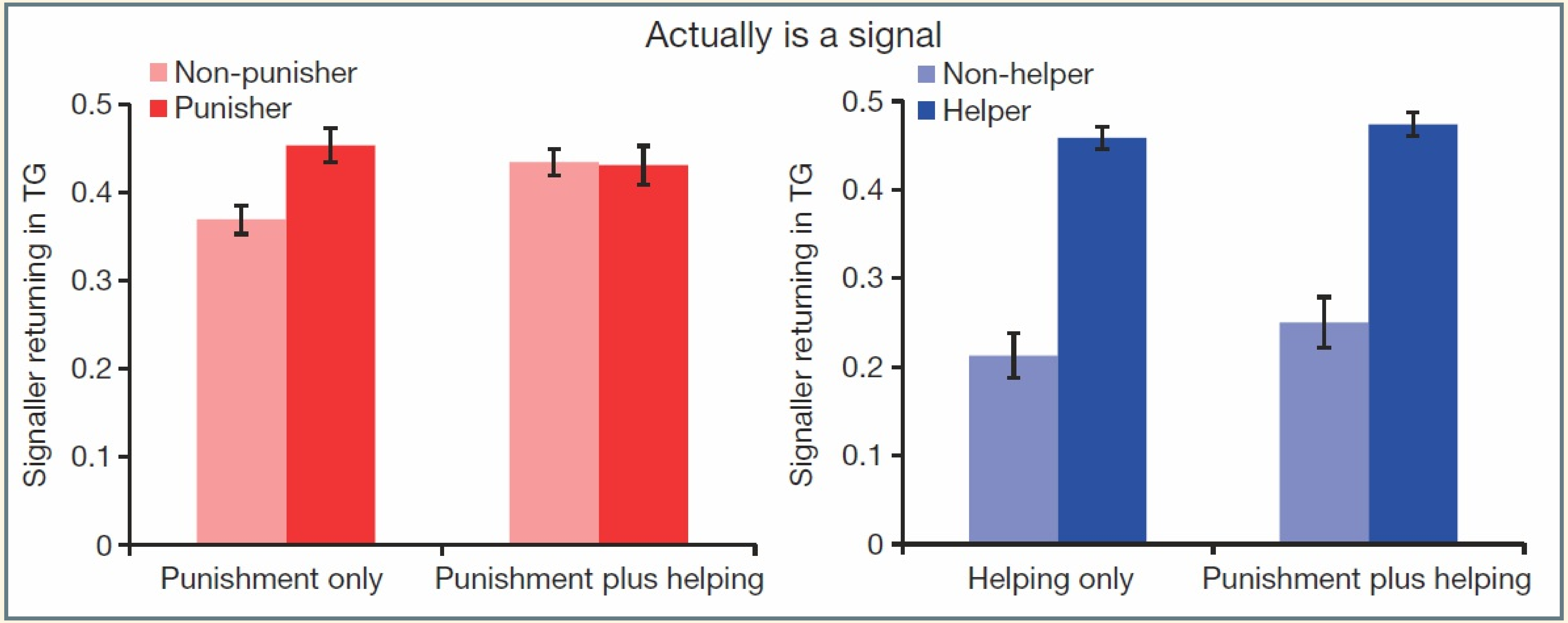
- To check whether signals are interpreted accurately by Chooser agents, run a second game:
  - Chooser gets some money M.
    - Chooses how much to *send* to Signaler (x).
  - Money sent to Signaler is tripled (Signaler get $3x$)
  - Signaler decides how much of the $3x$ to *return* to Chooser.

Perception of signaling:



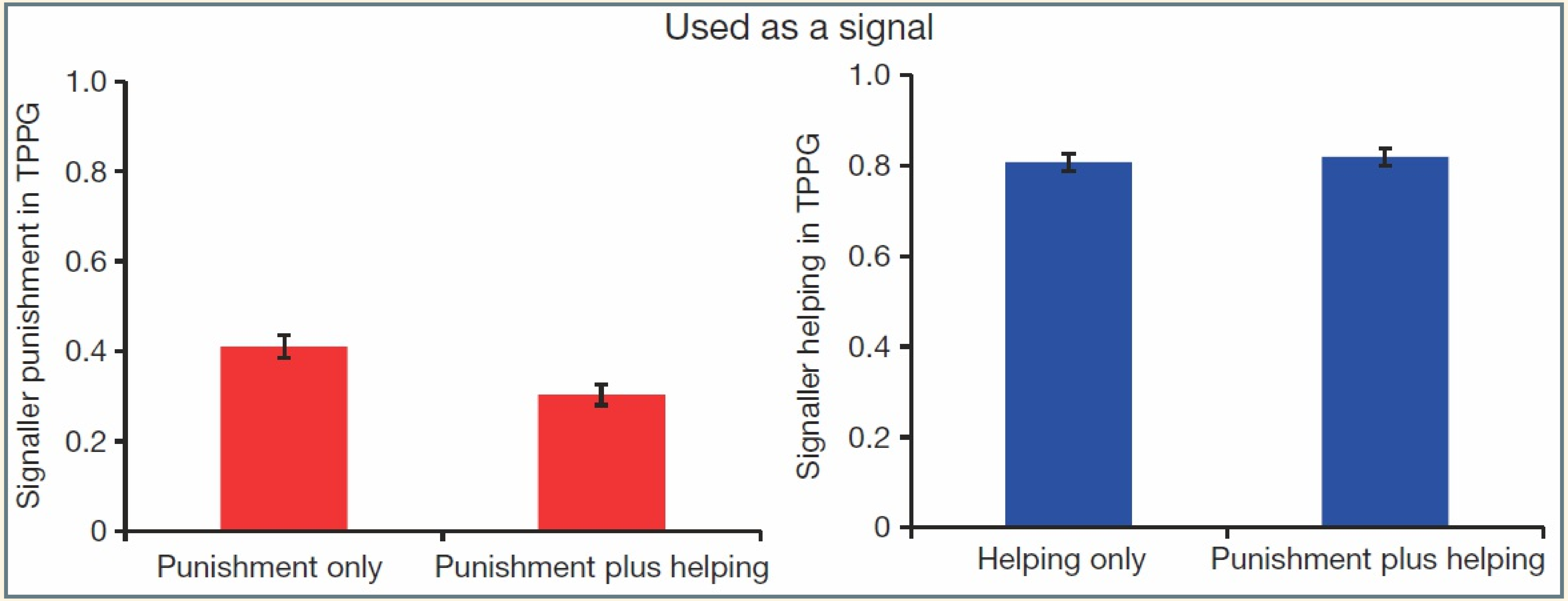
- Chooser shows trust by sending more money to Signalers who punish and who help.
- Helping is a more powerful signal to Chooser than punishing.
- This matches theory of rational behavior.

Actual signaliing



- Signalers who punish are more trustworthy: return more money to Chooser.
- Helping is indeed a more accurate signal of trustworthiness.

Signaler Choice



- Signaler is less likely to punish when helping is an option.

Conclusions

- Evolved strategies of agents match both pure theory (rational strategy) and experimental results.
- It is advantageous for third parties to carry out costly punishments when the punishments can signal trustworthiness to others in the community.
- When there are less costly or more effective ways to signal trustworthiness, third parties are less likely to punish.

General ideas about agent-based modeling

- Model interactions between individuals
  - Direct: individual-individual
  - Indirect: individual-environment, environment-individual
- Focus on emergent properties
  - Patterns or phenomena that were not deliberately programmed in, but arise spontaneously from interactions of agents with each other and with environment.
- Pattern-oriented modeling:
  - Start simple, but aim to build in enough complexity to produce multiple patterns seen in nature, or predicted in theory.
  - As you design model think about what kinds of “currency” you will use to assess its value.