Stochasticity

EES 4760/5760

Agent-Based and Individual-Based Computational Modeling

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Stochastic Business Investor model

On “downloads” page, go to “18. Stochasticity: Stochastic Business Investor Model”

https://ees4760.jgilligan.org/models/class_18/class_18_models.zip

Stochasticity

Stochasticity:

Why do we use random numbers?

- To “inject ignorance” into a model:
  - We want to represent some kind of variability *but*
  - We do not know the details of what causes the variability

```
ask patches [set profit 1000 + (random 1000)]
ask turtles [ if random-float 1.0 < mortality-prob [die] ]
```

- To simplify complex processes:
  - We know what causes the variability
  - But calculating it in our model would be too complex, slow, etc.

```
ifelse random-float 1.0 > prob-rain
[ set rains-today true]
[ set rains-today false]
```

Common uses of stochasticity

- Initialization

```
set fish-length random-normal 50 10
```

  Pick a random number from a normal distribution with mean 50 and standard deviation 10

- In submodels

```
ifelse random-float 1.0 < q
[ uphill elevation ]
[ move-to one-of neighbors ]
```

Guidance for Stochasticity

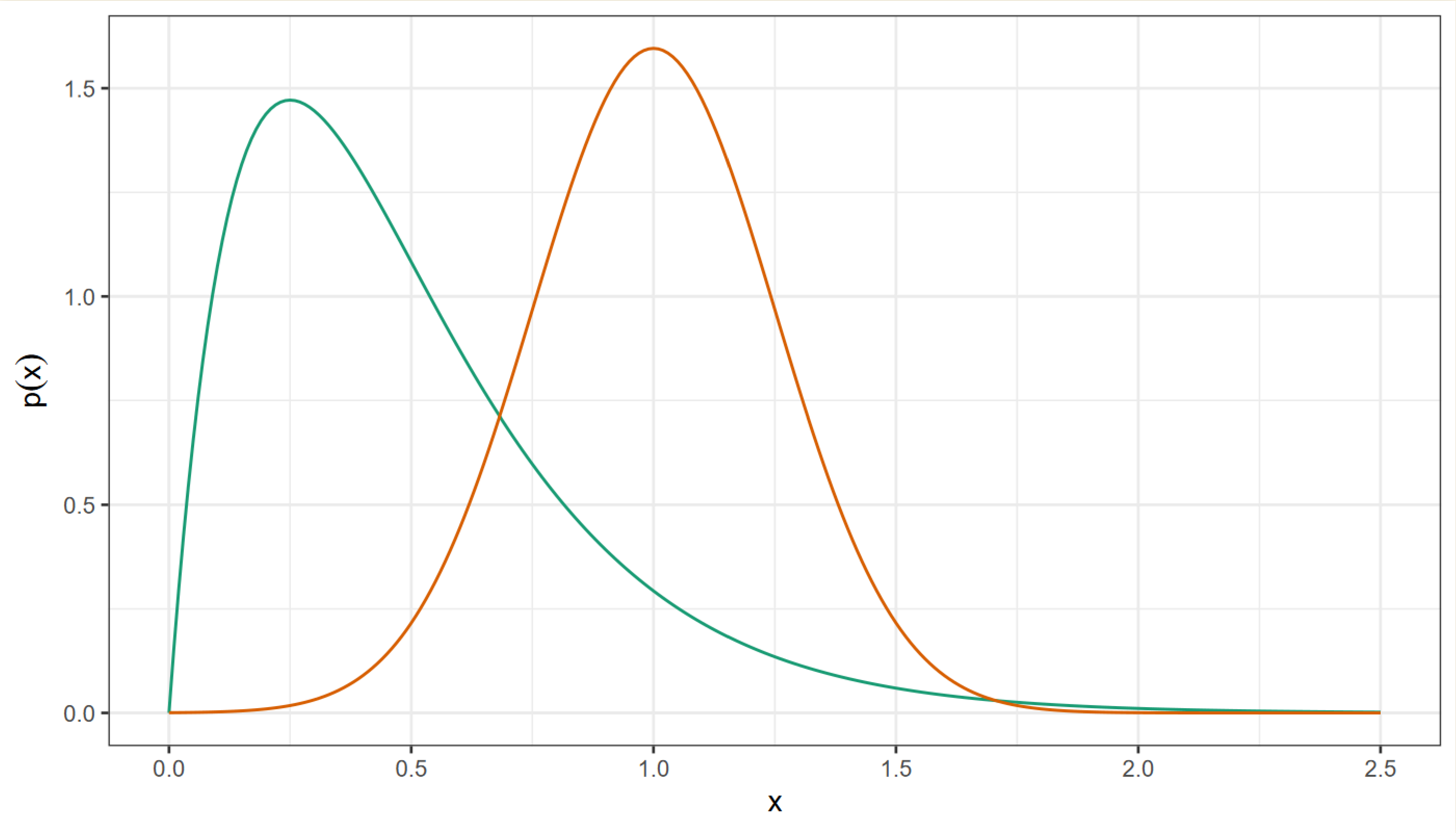
- **Do** use stochasticity to initialize model differently on different runs
  - Makes sure that effects you see are not *artifacts* of a specific initialization
- **Do** use stochasticity to simplify representation of very complex processes
  - If wild dogs live an average of 5 years:
    - instead of a detailed submodel that determines exactly when each dog will die,
    - let dogs die at random with a 20% probability of dying each tick.
- **Don't** use too much stochasticity
  - If you put too many different sources of randomness into your models every run may be *so different* you can't discover any general properties.

Limitations of Stochasticity

- Stochastic treatments often fail to account for *contingency*:
  - The way results depend on other variables
- Stochastic treatments often fail to account for *correlation*:
  - Hotter or wetter times (days, months, years) tend to follow other hotter or wetter times
  - B.B. Mandelbrot & J.R. Wallis, “Noah, Joseph, and Operational Hydrology”, *Water Resources Research*, **4**, 909 (1968)
    - **Noah effect:** 40 days & 40 nights of rain.
      - A few years have floods so intense they change the average for an entire century
    - **Joseph effect:** Seven years of plenty followed by seven years of famine.
      - The wettest decade in a century includes an extraordinary run of consecutive wet years

Distributions

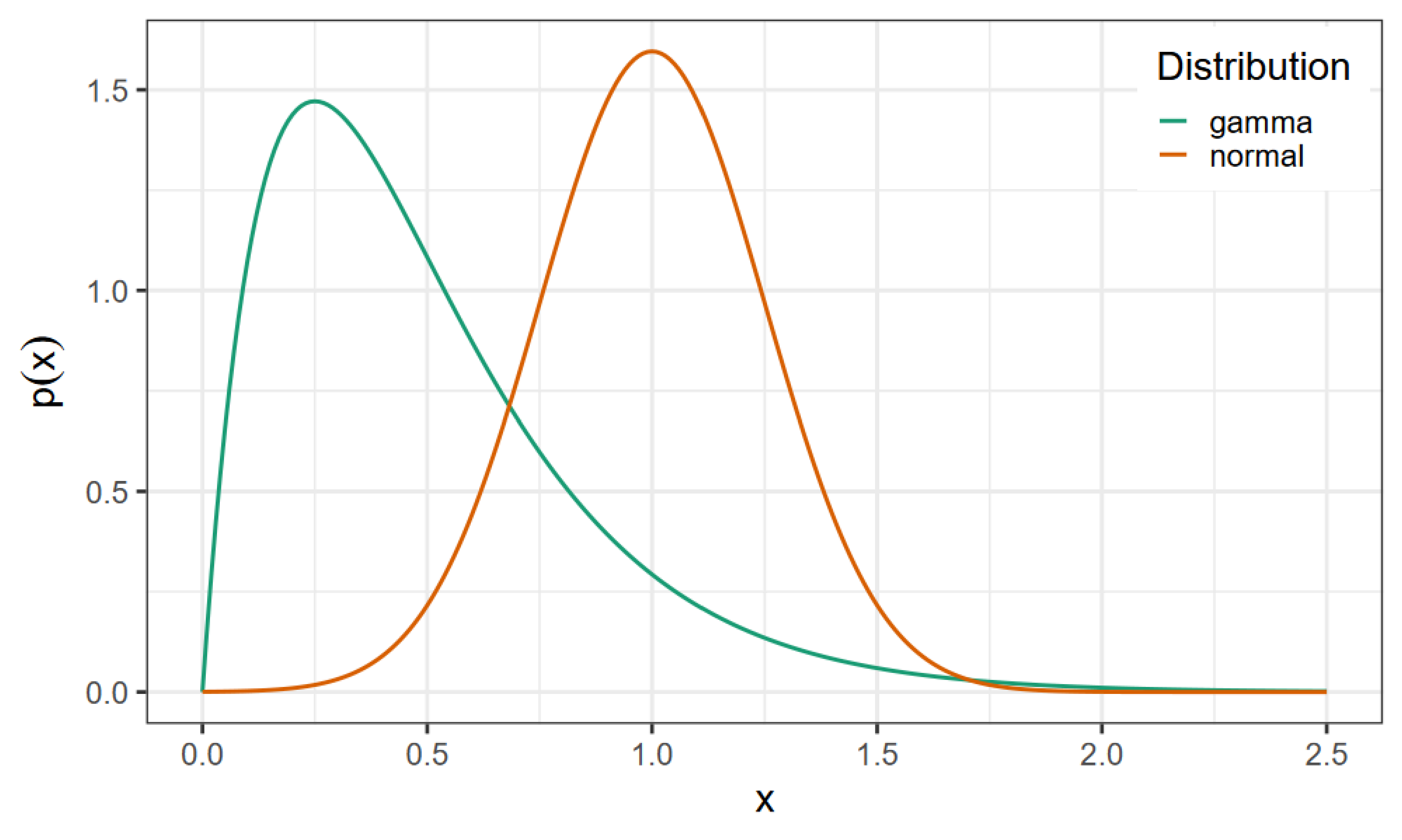
What is a Distribution?



What is a Distribution?

- In simulation programming, an algorithm that produces (pseudo)random numbers that fit a particular statistical distribution.

```
let x1 random-normal 1.0  0.25
let x2 random-gamma  2.0  4.0
```



Distributions in NetLogo

- Continuous (real-number)
  - Uniform: `random-float` *`upper-limit`*
  - Normal: `random-normal` *`mean sd`* (beware of outliers)
  - Also: `random-gamma`, `random-exponential`
- Discrete (integer):
  - Uniform: `random` *`upper-limit`*
    - 0 to *`upper-limit`* `- 1`
  - Poisson: `random-poisson` *`mean`*
    - `mean` = average value
  - Bernoulli (`true` or `false`): `random-float 1.0 < p`
    - `true` with probability `p`
    - See `random-bernoulli` reporter on p. 206 of the textbook.

Controlling randomness

Controlling randomness

- `random-seed` *number*
  - As long as *number* is the same, you get the same sequence of random numbers

```
to setup
    clear-all
    random-seed 12345
    ...
end
```

  - Every time you run the model, it does exactly the same thing.

Controlling randomness

- `with-local-randomness [ commands ]`
  Runs without changing sequence of random numbers in other parts of the model.

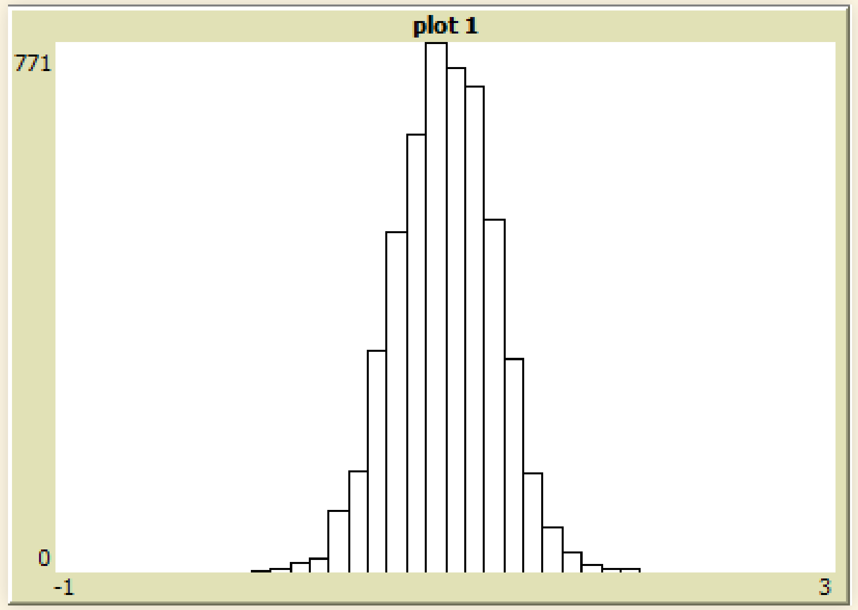
```
to setup
  with-local-randomness
  [
    random-seed 565656
    ...
  ]
end
```

- Examples:
  - You want to have the same random thing happen every time you run the model, but you want the rest of the model to be randomly different every time you run it.
  - You want a part of the model to run the same way every time, even if you edit code in other parts of the model.

How can we see a distribution?

- Histograms

```
to plot-histogram-normal
  clear-all
  set-plot-pen-mode 1 ; bar mode
  set-plot-pen-interval 0.1
  set-plot-x-range -1 3
  let x (list)
  ; fill x with 5000 random numbers from a normal distribution
  repeat 5000 [ set x fput (random-normal 1.0 0.25) x]
  histogram x
end
```



Uniform distributions

- Integer: `random n` gives an integer i: $0 \leq i < n$
  - From 0 to $(n - 1)$
- Continuous: `random-float z` gives a number x: $0 \leq x < z$
  - Should we worry that $x < z$?

```
to test
  let num_draws 10000
  let max-rand 0
  repeat num_draws
  [
    let x random-float 1000
    if x > max-rand [ set max-rand x ]
  ]
  show max-rand
end
```

```
observer> test
observer: 999.9869678378017
```

Poisson distribution

- For countable things that happen at a small rate.
  - On every turn a random number of agents turn red, with an average of 5% of agents

```
ask n-of (random-poisson (0.05 * count turtles)) turtles
[set color red]
```

or

```
let red-count random-poisson (0.05 * count turtles)
ask n-of red-count turtles [set color red]
```

Normal distribution

- For measurable things with an average value

```
set weight random-normal 150 20  ; weight in pounds
set height random-normal 70 2    ; height in inches
```

- Be careful of outliers. There is no limit, so there is a small probability of getting a very large value or a negative value.

```
repeat 5000 [
  let x random-normal 30 10
  if x < 0 [ print precision x 2]
]
```

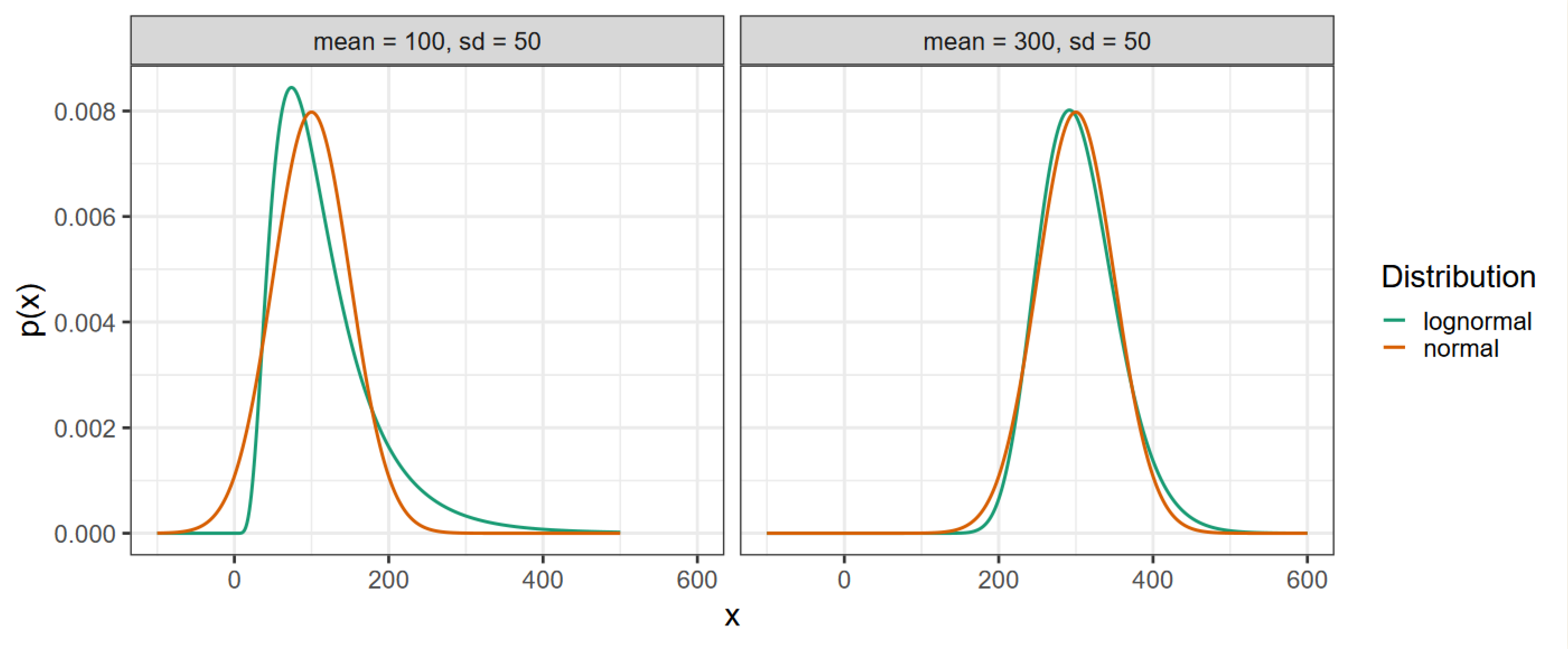
Output:

```
-1.75
-1.6
-1.08
-6.66
-5.89
-10.87
-0.17
```

Lognormal distribution

- If you want something like a normal distribution, but where the result **must** be positive, try a lognormal distribution:

```
to-report random-lognormal [ m s ]
  let mm ln m
  let ss (ln ((m + s) / (m - s))) / 2
  report exp (random-normal mm ss)
end
```



Stochastic Business Investors

Stochastic Business Investors

Model: https://ees4760.jgilligan.org/models/class_18/business_investor_class_18.nlogo

Original model:

- Investors move to neighbor with highest expected utility (including own patch)
- Average over 10,000 runs:
  - Count fraction of the time a choice is made *when it is available*:

Alternative	Frequency
Higher profit, lower risk	78.0%
Higher profit, higher risk	9.3%
Lower profit, lower risk	3.4%
Lower profit, higher risk	0.0%
Don’t move	92.1%

- Mean wealth = $212,434
- Total wealth = $5,310,861

Stochastic Model

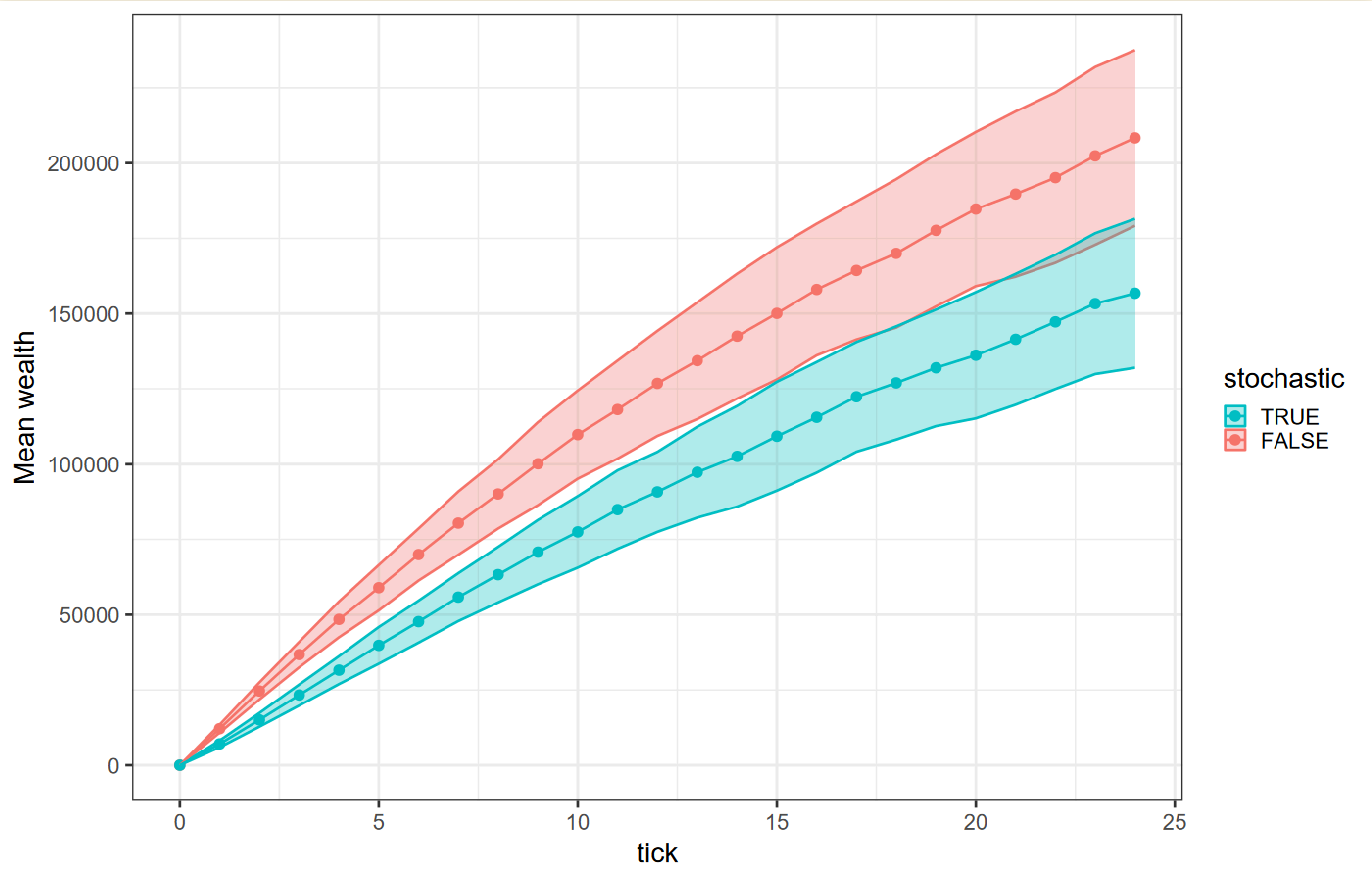
Original model:

Alternative	Frequency
Higher profit, lower risk	78.0%
Higher profit, higher risk	9.3%
Lower profit, lower risk	3.4%
Lower profit, higher risk	0.0%
Don’t move	92.1%

Stochastic model

- If there are neighbors with higher profit and lower risk:
  - 78.0% probability of moving to one of them
- Otherwise, if there are neighbors with higher profit and higher risk:
  - 9.3% probability of moving to one of them
- etc.

Compare models:



What Is Missing?

- The stochastic model moves with the same probabilities as the original model
  - Why are the turtles less wealthy?
  - The stochastic model ignores *contingency*:
    - The probability of moving to a
      - higher-profit, higher-risk patch
      - or a lower-profit, loser-risk patch

    depends on the turtle’s wealth.