Conformal Field Theory

Lecture notes

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[Lectures] are fantastically good for learning physics. The lecturer learns a lot of physics. After my first few studies, just about everything I learned about physics came from teaching it. I don't know if the students learned a lot, but I certainly did. So I consider teaching physics very important. — Leonard Susskind

Contents

1 Introduction

1.1 What is conformal field theory?

Introduction: What is CFT? Why is it useful. Examples of CFT. Where does it fit into modern theoretical physics.

in the last few years, CFT has been dominated by
string theory: 2d CFT
holography: geometric approach
condensed matter physics: Euclidean
here focus on the "old" quantum field theory approach
links with: lattice, perturbation theory, etc.
fun fact: the conformal bootstrap was invented by particle physicists
strongly-coupled QFT
no need for action principle

1.2 Examples of conformal field theories

perturbative examples: ϕ^n theory in non-integer d
Caswell-Banks-Zaks
superconformal: $\mathcal{N} = 4$

1.3 Outline

originally covered in 14 periods of 45 minutes each
split into 7 chapters?

2 Classical conformal symmetry

Poincaré symmetry: physics is the same in every coordinate frame
linear transformation of the coordinates

$$x'^{\mu} = x^{\mu} + \omega^{\mu}{}_{\nu} x^{\nu} + a^{\mu} \tag{2.1}$$

infinitesimal line element $dx^2 = \eta_{\mu\nu} dx^{\mu} dx^{\nu}$ is invariant:

$$dx'^2 = dx^2 \tag{2.2}$$

scale symmetry:

$$x'^{\mu} = \lambda x^{\mu} \tag{2.3}$$

special conformal symmetry:

conformal Killing vectors: (see Osborn's notes) infinitesimal transformation

$$x'^{\mu} = x^{\mu} + v^{\mu}(x) \tag{2.4}$$

$$\partial^{\mu} v^{\nu} + \partial^{\nu} v^{\mu} = 2\sigma \eta^{\mu\nu} \tag{2.5}$$

3 Non-perturbative quantum field theory

Wightman axioms

operators are not necessarily invariant under conformal symmetry, but they transform unitarily

unitary representations on Hilbert space

(e.g. pair of point-like particle

no action principle (i.e. no need for "quantization")

only deal with **local** operators

3.1 Spectral representation

operator vs. field

3.2 Scale symmetry

comment on scale vs. conformal invariance

correlation functions:

no need for action principle

3.3 Special conformal symmetry

Mack's classification in 4d

3.4 Exercises

3.5 Bibliography

itzikson zuber

4 Correlation functions

T-products vs. Wightman functions

use retarded products and micro-causality

4.1 Spectral representation for spinning operators

unitary bounds

4.2 From momentum to position

taking fourier transforms

4.3 From Minkowski to Eulidean space

Osterwalder-Schrader

analytic continuation of Wightman functions vs. Wick rotation of T-products

reflection positivity

note that this is usually done in the other direction!

4.4 Embedding-space formalism

in position space

2- and 3-point functions

invariant cross-ratios for 4-point functions

5 State-operator correspondence and OPE

important remark on OPE convergence in Euclidean space!

6 The conformal bootstrap

6.1 Conformal blocks

6.2 The numerical boostrap

generalized free fields/Gaussian

6.3 Results

7 Selected advanced topics

light-cone limit and ?

Virasoro symmetry in 2d: