C∗-ALGEBRAS, THE GELFAND-NAIMARK THEOREM, AND [OTHER THING]

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1. Introduction

1.1. History of C∗-Algebras. The noncommutative nature of W. Heisenberg's work in [**?**] lead to Born and Jordan (find reference) developing the matrix mechanics required to concisely convey the new quantum mechanical model. From 1930-1943, J. von Neumann, together with F. J. Murray, developed the theory of *rings of operators* acting on a Hilbert space in [**?**], [**?**], [**?**], [**?**], in an attempt to establish a general framework for this matrix mechanics. These rings of operators are now considered part of the theory *von Neumann algebras*, a subsection of C∗-algebra theory. In [**?**], Gelfand and Naimark established an abstract characterisation of C∗-algebras, free from dependence on the operators acting on a Hilbert space. The Gelfand-Naimark, which we will be considering here at length, gives the link between these abstract C∗-algebras and the rings of operators previously studied.

1.2. C∗-Algebras. A brief overview of what a C∗-algebra is.

- banach space
- banach algebra
- *-algebra
- C* algebra
- representations

Definition 1. *A C∗-algebra is a Banach algebra $(A, \|\cdot\|)$ with involution $* : A \to A$ with the condition that*

$$\|x^*x\| = \|x\|^2 \text{ for all } x \in A.$$

This condition is known as the *C∗ axiom*. The study of C∗-algebras started with the consideration of matrix mechanics by H

There are many statements of the theorem; this here comes from [ref].

Theorem (Gelfand-Naimark, Commutative)**.** *Every commutative, unital C*-algebra A is isometrically ∗-isomorphic to the algebra of continuous functions on the algebra of characters on A.*

The theorem first appeared in [gelfand-naimark] in a form relating 'normed rings' and a closed subrings of the set of bounded operators on a Hilbert space.

1.3. **Aims.** The aims for my project are, provisionally:

- Take the Gelfand-Naimark theorem, and understand its contents and proof.
- Consider the representation theory of C*-algebras, using the Gelfand-Naimark-Segal construction as a starting point.
- (as 'further reading') Give an overview of areas in which operator algebra theory can be taken (for example, Cuntz algebras and operator K-theory, von Neumann algebras and Factors, abstract harmonic analysis).

References