C^*-algebras, and the Gelfand-Naimark theorems

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Friday 2nd December 2016

A Brief History

- 1925 – Heisenberg, *Über quantentheoretische...*. New QM model.
$$pq - qp = \frac{h}{2\pi i}.$$
- 1925 – Born & Jordan, *Zur Quantenmechanik*. Developed matrix mechanics.
- 1935-1943 – Murray & von Neumann, *On rings of operators*. A general framework.
- 1943 – Gelfand & Naimark, *On the embedding of normed rings...*. Abstract C*-algebras.

Aims

In my project

- Background understanding of C^*-algebras, standard results,
- Representation theory, considering the Gelfand-Naimark-Segal construction,
- Commutative and general GN theorems and their proofs.

C*-algebras

- A *C^*-algebra* is a Banach algebra $(A, \|\cdot\|)$ with involution map $a \mapsto a^*$, with the condition that

$$\|a^* a\| = \|a\|^2 \text{ for all } a \in A.$$

Gelfand-Naimark theorem

Commutative

- *Every commutative, unital C^*-algebra A is isometrically $*$-isomorphic to the algebra of continuous functions on the algebra of characters on A.*

A relation between commutative C*-algebras, and the space of continuous functions on a compact topological space.
Gives us a way to explore non-commutative analogues to geometry and topology.

Gelfand-Naimark theorem

General

- *Every C^*-algebra A is isometrically $*$-isomorphic to the algebra of bounded operators on a Hilbert space.*

References

- Kadison, R. V. & Ringrose, J. R., *Fundamentals of the theory of operator algebras: Vol. I. Elementary theory.*
- MacKinnon, E., *Heisenberg, Models, and the Rise of Matrix Mechanics.*
- Schroer, B., *Pascual Jordan, Glory and Demise and his legacy in contemporary local quantum physics.*