C^*-algebras, and The Gelfand-Naimark theorem

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Friday 2nd December 2016

A Brief History

- 1925 – Heisenberg, *Über quantentheoretische...*
  new QM model.
  $$PQ - QP = \frac{h}{2\pi i}.$$
- 1925 – Born & Jordan, *Zur Quantenmechanik*
  developed matrix mechanics.
- 1935-1943 – Murray & von Neumann, *On rings of operators*
  a general framework.

A Brief History

- 1943 – Gelfand & Naimark, *On the embedding of normed rings...*
  abstract C*-algebras.

Aims

In my project

- Background understanding on C^*-algebras, standard results,
- Representation theory, considering GNS construction,
- Commutative and general GN theorems and proofs.

Gelfand-Naimark-Segal construction

Two theorems:

- Existence, for every state ρ of a C*-algebra, of a cyclic $*$-representation π and a unit cyclic vector x, such that

$$\rho(A) = \langle \pi_\rho(A) x_\rho, x_\rho \rangle.$$

- Uniqueness of these representations and vectors.

Gelfand-Naimark theorems

References