C*-ALGEBRAS, THE GELFAND-NAIMARK THEOREM, AND [OTHER THING]

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1. Introduction

1.1. **History of C*-Algebras.** A brief timeline of the development on the theory. The theory of C*-algebras has its basis in W. Heisenberg's use of so-called "matrix mechanics" in the modelling of algebras of physical observables in quantum mechanical systems. In [1925], Heisenberg

1.2. **C*-Algebras.** A brief overview of what a C*-algebra is.

- banach space
- banach algebra
- *-algebra
- C* algebra
- representations

Definition 1. *A* C*-algebra *is a Banach algebra* $(A, \|\cdot\|)$ *with involution* $*: A \to A$ *with the condition that*

$$\|x^*x\| = \|x\|^2 \text{ for all } x \in A.$$

This condition is known as the *C* axiom*. The study of C*-algebras started with the consideration of matrix mechanics by H

There are many statements of the theorem; this here comes from [ref].

Theorem (Gelfand-Naimark, Commutative)**.** *Every commutative, unital C*-algebra A is isometrically ∗-isomorphic to the algebra of continuous functions on the algebra of characters on A.*

The theorem first appeared in [gelfand-naimark] in a form relating 'normed rings' and a closed subrings of the set of bounded operators on a Hilbert space.

1.3. **Aims.** The aims for my project are, provisionally:

- Take the Gelfand-Naimark theorem, and understand its contents and proof.
- Consider the representation theory of C*-algebras, using the Gelfand-Naimark-Segal construction as a starting point.

- (as 'further reading') Give an overview of areas in which operator algebra theory can be taken (for example, Cuntz algebras and operator K-theory, von Neumann algebras and Factors, abstract harmonic analysis).

References