Estimating the Demand for Risk in Financial Assets: A Welfare and Demographic Analysis

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1 Introduction

1.1 Motivation

In recent years, the widespread availability of new investing technologies has permanently changed financial markets, enabling a rise in market participation among non-institutional investors. With digital platforms, individuals can access a wide range of financial instruments from their personal devices, altering the demographics of investors and lowering entry barriers across financial markets. This shift has increased the diversity of investors and presented new opportunities to change how to manage client risk profiles. Traditionally, banks have relied on questionnaires to gauge clients' risk tolerance. However, integrating detailed demographic data may offer a closer estimation of clients' true risk preferences, allowing for more personalized and welfare-maximizing investment strategies.

1.2 Research Question

How can integrating demographic and characteristics data improve clients' risk tolerance and time preference estimation for different financial assets across various demographics? This question explores whether additional demographic factors can provide deeper insights into investors' investment choices, getting closer to their utility-maximizing investment decisions.

1.3 Method

My analysis will explore different risk aversion measures and gauge their effectiveness in approximating a utility-maximizing solution for risk demand. The first measure is derived from private information, incorporating income, historical portfolio holdings and transactions to reflect an individual's true risk aversion based on past financial decisions. The second is an estimate of risk aversion as advised by the bank, deduced from current portfolio allocations. These are presumed to be bank-recommended. By examining these measures, I aim to determine whether a potentially more precise measurement of risk aversion and incorporating time preference can lead us toward a utility-maximizing solution, ultimately enhancing financial welfare.

To address this research question, I propose a structural model that will econometrically estimate individuals' time preferences and risk aversion based on an expanded set of demographic characteristics. This model will incorporate traditional demographic factors such as Age, Income, Education, Employment Status, Race, Gender, and Marital Status, along with other relevant variables that might influence risk preference and time preference. The model will be calibrated using a dataset provided by the Canada Revenue Agency (CRA) and other publicly available investment and macroeconomic data. This method examines how these characteristics influence preferences, potentially leading to improved portfolio management strategies and enhancing aggregate welfare.

2 Literature Review

This literature review looks at findings from various studies, providing an understanding of how demographic influences affect financial behaviour. By integrating economic theory, this section offers insights into evolving investment behaviours.

Sydney C. Ludvigson's chapter in the "Handbook of the Economics of Finance" discusses advances in consumption-based asset pricing and their empirical tests, emphasizing the role of consumption data in explaining stock market behaviours. This research aligns closely with my proposal to integrate consumption data into asset pricing models. Ludvigson advocates for method-of-moments estimators, which supports my approach of using the Generalized Method of Moments (GMM) for dynamic model estimation of risk preferences [1].

Douglas T. Breeden, Robert H. Litzenberger, and Tingyan Jia discuss the role of the consumption capital asset pricing model (CAPM) and how it has evolved to include heterogeneous agents and limited market participation, which is needed for understanding different investment behaviours across demographic groups. Their findings support my model's premise that different demographic characteristics can lead to varied investment preferences and risk tolerances. They provide some theoretical work that supports my analysis of demographic impacts on asset pricing [2].

Centreville (2014) analyzes risk aversion and insurance demand, discussing how demographic characteristics influence individual decisions in insurance markets. This is relevant to my research as it explains the importance of age, gender, and economic status in shaping risk behaviours [3].

Bernard Caillaud's exploration of behavioural models introduces the concept of time-inconsistent preferences, which highlights how varying risk preferences over time can impact financial decisions and market dynamics. This supports my methodological approach to incorporate time-varying risk preferences in modelling financial behaviours [4].

Douglas W. Blackburn and Andrey D. Ukhov's research provides insight into investor utility functions. Their study utilizes the returns on stocks and the prices of call options to deduce the utility of wealth functions of marginal investors, revealing strong support for non-concave utility functions. These functions, which include segments that are both risk-averse and risk-seeking, reflect a bigger picture of investor behaviour than the traditional models assume. This evidence suggests that individual behaviours significantly influence market dynamics [5].

The study by Johannes G. Jaspersen et al. on predicting insurance demand from risk attitudes uses experimental data to parameterize structural models of risk preference. This empirical approach provides a foundation for using similar models to estimate risk aversion and demographic influences more accurately [6].

Carmona and Delarue (2013) leverage mean-field game theory to explore market behaviours. This aids in understanding how aggregated demographic characteristics can impact collective risk-taking behaviours in financial markets. [7].

Keys et al. (2016) investigate the impact of economic shocks on various demographic groups, illuminating how differences in education and income levels influence financial decision-making. This research supports my goal of integrating such demographic data to more accurately predict and understand risk preferences and investment decisions across diverse populations, especially under varying economic conditions [8].

Conlon and Gortmaker (2020) analyze the demand for differentiated products concerning consumer characteristics, drawing parallels to the varied financial instruments in my study. Their insights into how different demographic traits affect product choices show how similar factors may influence preferences for different types of financial assets, aiding in developing more tailored investment strategies [9].

3 Data Description

I have secured a dataset from the Canada Revenue Agency (CRA). This dataset encompasses an array of individual-level data cleaned and ready for modelling financial behaviour and risk preferences.

In reality, the data obtained from the (CRA) is simulated for the purposes of this analysis. The simulation is designed to replicate hyper-realistic demographic and financial behaviour based on real-world statistics.

Demographic and Financial Data

My dataset includes the following demographic information:

- Age, Gender, Income, Education Level, Marital Status, Employment Status and Race.
- Types of Assets Held, Portfolio Size, Historical Asset Holdings Data, Transaction History, Participation in Retirement Accounts, Credit Score, and Existing Debts.
- Measure of risk aversion and the bank's estimate of it.

Formula for Risk Aversion Coefficient Calculation

Asset Score Calculation:

$$\text{Asset\_Score} = \text{Stocks\_Held} + \text{Crypto\_Held} - \text{Bonds\_Held} - \text{Real\_Estate\_Held}$$

Transaction Score Calculation:

$$\text{Transaction\_Score} = \text{Total\_Transactions} \times \text{Avg\_Transaction\_Amount}$$

Safe Savings Handling:

$$\text{Savings\_Safe} = \text{replace}(\text{Savings}, 0, 0.01)$$

Debt to Savings Ratio Calculation:

$$\text{Debt\_to\_Savings\_Ratio} = \frac{\text{Total\_Debt}}{\text{Savings\_Safe}}$$

Normalization of Fields:
Normalized Fields = MinMaxScaler(Income, Credit Score, Asset Score, Transaction Score, Debt to savings Ratio)

Weights Definition: Weights = { Income: 0.2, Credit_Score: 0.2, Asset_Score: 0.2, Transaction_Score: 0.2, Debt_to_Savings_Ratio: -0.2 }

Risk Aversion Coefficient Calculation:

$$\text{Risk\_Aversion\_Coefficient} = \text{Normalized Fields} \cdot \text{Weights}$$

Calculation of Risk Scores and Bank's Risk Aversion Estimate

Define Risk Scores for Assets

$$\begin{aligned}\text{Risk scores} = \{ \\ &\text{Crypto\_Held} : 3, \quad \text{(Most risk-loving)} \\ &\text{Stocks\_Held} : 1, \\ &\text{Real\_Estate\_Held} : -1, \\ &\text{Bonds\_Held} : -2 \quad \text{(Most risk-averse)} \\ \}\end{aligned}$$

Calculate Weighted Risk Score for Assets
Weighted Risk Score = Crypto Held (3) + Stocks Held (1) + Real Estate Held (-1) + Bonds Held (-2)

Normalize the Weighted Risk Score Normalized Fields = MinMaxScaler(Weighted Risk Score)

Calculate Diversification Score Based on Portfolio Size

$$\text{Diversification\_Score} = \begin{cases} (\log(1 + \text{Portfolio\_Size}) + 1)^{0.5} & \text{if Portfolio\_Size} = 0 \\ (\log_{10}(\text{Portfolio\_Size}))^{0.5} & \text{otherwise} \end{cases}$$

Normalize the Diversification Score Normalized Fields = MinMaxScaler(Diversification Score)

Calculate the Bank's Risk Aversion Estimate

Banks_Risk_Aversion_Estimate = 1−Normalized_Risk_Score−Normalized_Diversification_Score

3.1 Justification

The Risk Aversion Coefficient is calculated using actual financial actions and personal data to reflect an individual's risk tolerance. It incorporates real behaviours like asset choices and transaction patterns.

On the other hand, the Bank's Risk Aversion Estimate assumes that the bank recommended the assets held. It represents how the bank perceives the individual's risk tolerance based on their visible asset portfolio. This method shows how financial institutions interpret client data to assess risk.

4 Model Specification

4.1 Consumer Utility Function

Consumers are modelled to maximize expected utility over consumption subject to their budget constraint that includes asset choices, accounting for risk aversion and demographic influences: [10] [11]

$$\max_{\{c_{ti}, x_{ti}\}_{t=0}^{\infty}} E\left[\sum_{t=0}^{\infty} \beta_i^t \left(\frac{c_{ti}^{1-\rho_i}}{1-\rho_i}\right) |\mathcal{I}_0\right],$$

Subject to the budget constraint:

$$c_{ti} + q_t' x_{ti} \leq q_t' x_{t-1,i} + y_{ti}, \quad x_{ti} \geq 0,$$

4.2 Euler Equations for Optimal Asset Holdings

The first-order conditions from the utility maximization problem give us the consumption assets pricing model (CAPM) Euler equation for optimal asset holdings. For each asset j that an individual holds ($x_{ti}^j > 0$), I have: [12]

$$\beta_i^t c_{ti}^{-\rho_i} q_t^j = E\left[\beta_i^{t+1} c_{t+1,i}^{-\rho_i} q_{t+1}^j \mid \mathcal{I}_{ti}\right],$$

and for assets k not held ($x_{ti}^k = 0$), I have the following inequality:

$$\beta_i^t c_{ti}^{-\rho_i} q_t^k < E\left[\beta_i^{t+1} c_{t+1,i}^{-\rho_i} q_{t+1}^k \mid \mathcal{I}_{ti}\right].$$

Where:

- c_{ti} is the consumption at time t for individual i, which can depend on current and past income and asset prices.

- x_{ti} denotes the vector of asset holdings at time t for individual i, also state-dependent.
- β_i is the subjective discount factor, varying across for individual i and states.
- ρ_i represents the coefficient of relative risk aversion, which varies with individual is demographics and other characteristics.
- $\mathcal{I}_{ti}$ represents the information set at time t for individual i, including past consumption, income, asset holdings, and prices, as well as individual characteristics.
- y_{ti} is the income at time t for individual i, which can be a random variable reflecting the stochastic nature of income and wages over time.
- q_t is the price vector of assets at time t, reflecting some randomness in asset prices.
- j is an index used to represent an asset currently included in the individual's portfolio.
- k is an index used to represent an asset not currently included in the individual's portfolio.

4.3 Incorporating Demographic Factors into Risk Aversion and Time Preferences

I extend the model to allow for the coefficient of relative risk aversion ρ_i and the discount factor β_i to be functions of demographic characteristics: [13]

$$\rho_i = f(\mathbf{d}_i; \gamma), \quad \beta_i = g(\mathbf{d}_i; \delta),$$

Where $\mathbf{d}_i$ includes demographic characteristics for individual i, and γ, δ are parameters that capture the relationship between demographics and the parameters of risk aversion and time preferences, respectively.[14]

4.4 Modeling Demand for Risk

Demand for assets x_{ti} is a function of the expected utility maximization, subject to individual budget constraints. The demand reflects portfolio choices across various risk categories and evolves over time, influenced by changes in both market conditions and personal demographic factors.

5 Estimation Method

The structural model designed to estimate the demand for risk in financial assets will employ the Generalized Method of Moments (GMM) for parameter estimation.

5.1 Generalized Method of Moments (GMM)

GMM will estimate the risk aversion coefficient ρ_i and the subjective discount factor β_i. The moment conditions will be derived from the Euler equations that represent the optimal conditions for consumption and investment decisions under uncertainty.

$$E\left[(1-\beta_i\frac{c_{ti}^{-\rho_i}}{c_{i,t+1}^{-\rho_i}}\frac{q_{t+1}^j}{q_t^j}z_{ti})\right]=0,$$

$$\forall z_{ti}\in\mathcal{I}_i$$

I can now estimate β_i and ρ_i by minimizing [15]

$$\min_{\beta_i,\rho_i}\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{T}\sum_{t=1}^{T}\left(1-\beta_i\frac{c_{i,t+1}^{-\rho_i}q_{t+1}^j}{c_{i,t}^{-\rho_i}q_t^j}\right)z_{ti}\right)'W\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{T}\sum_{t=1}^{T}\left(1-\beta_i\frac{c_{i,t+1}^{-\rho_i}q_{t+1}^j}{c_{i,t}^{-\rho_i}q_t^j}\right)z_{ti}\right)$$

5.2 Incorporating Demographic Factors

Demographic factors are integrated into the estimation process by allowing ρ_i and β_i to vary across individuals based on their demographic characteristics. The estimation will consider how these personal factors, represented in the vector $\mathbf{d}_i$, influence the individual's risk aversion and time preferences.

$$\hat{\rho}_{ii}=f(\mathbf{d}_i;\hat{\gamma}),\quad\hat{\beta}_{ii}=g(\mathbf{d}_i;\hat{\delta}),$$

The functions f and g will be specified to capture potential non-linear effects of demographics on the risk aversion and discount factor, respectively. [16]

5.3 Estimation Procedure

The GMM estimator will be implemented as follows:

1. Specify the moment conditions based on the model's first-order conditions.
2. Construct an initial weighting matrix, often starting with the identity matrix or the inverse of the sample covariance matrix of the moment conditions.
3. Solve the GMM minimization problem to obtain consistent estimates of ρ_i and β_i.
4. Update the weighting matrix using the estimates from the previous step and re-estimate the parameters until convergence is achieved.
5. Assess the quality of the estimates using Hansen's J-test for overidentifying restrictions, checking the validity of the instruments and the overall model specification. [17]

5.4 Model Validation

Model validation will involve cross-validation techniques to ensure robustness and prevent overfitting. Additionally, a series of diagnostic tests, including tests for autocorrelation, heteroskedasticity, and model specification, will be conducted to ensure the validity of the GMM estimates.

5.5 Planned Research Activities

To directly assess how changes in model parameters affect utility, I will simulate the utility functions' moment-generating conditions and solve the minimization problem under various scenarios. To validate the practical applicability of the models, I will introduce budget constraints into the simulations. This will help assess whether more accurate risk aversion estimates lead to improved utility maximization, aligning theoretical models more closely with real-world financial decision-making.

The research will be conducted in several phases, each designed to build upon the insights gathered from the previous one. The following are the specific steps elaborating on the above:

1. **Data Preparation and Preliminary Analysis:** Initial activities will involve examining the dataset from the Canada Revenue Agency (CRA). This includes data cleaning, normalization of variables, and preliminary statistical analysis to understand the underlying distributions and summary statistics of demographic and financial variables.

2. **Structural Model Development:** The structural econometric model could be improved to estimate individual time preferences and risk aversion.

3. **Parameter Estimation:** Risk aversion coefficients and time preferences will be estimated using the generalized method of moments (GMM). This phase involves setting up the moment conditions derived from the model and solving the equations using the steps in 5.3.

4. **Model Validation and Refinement:** Once the initial estimates are obtained, the model will undergo several validation checks, including out-of-sample testing and cross-validation. Based on the outcomes of these tests, adjustments/refinements will be made.

5. **Policy Simulation and Welfare Analysis:** Simulations to test how different policy interventions that cause changes in market conditions might affect the welfare of different demographic groups.

6. **Longitudinal Study Setup:** If initial results are promising, plans will be made to set up a longitudinal study that tracks the changes in risk aversion and time preferences over time, providing insights into how these preferences evolve.

Anticipated Challenges and Mitigation Strategies: Challenges such as data inconsistencies, model overfitting, and potential biases in parameter estimation are anticipated. These will be addressed through data validation and model evaluations. Collaborations with industry experts and feedback from the academic community will also play a crucial role.

6 Preliminary Results

- **Findings:** The preliminary analysis conducted using the Ordinary Least Squares (OLS) model demonstrates a significant improvement in estimating the Risk Aversion Coefficient compared to the bank's estimates. The mean squared errors (MSE) for the models were as follows:

  - OLS MSE: 0.0075,
  - Bank Estimate MSE: 0.0697.

- **Implications:** These results imply that the current methodologies employed by banks to assess risk aversion could be substantially improved by integrating this model. Adopting it could lead to more personalized, accurate financial advice and improved financial planning. This improvement in risk assessment accuracy could potentially increase the welfare of individuals by aligning their investment strategies more closely with their actual risk preferences.

7 Conclusion

This proposal outlines an approach to explore whether a model that includes demographics can improve the estimation of clients' risk aversion and time preference compared to traditional methods banks use.

7.1 Implications of the Proposal

Financial institutions can improve their assessments by using these modelling techniques. This improvement can lead to more personalized financial advice, matching investment strategies more closely with individual risk preferences, thereby bettering client utility and welfare.

7.2 Anticipated Outcomes

I expect that the models will show how these decisions are influenced by their demographic backgrounds. The goal is to demonstrate that closer approximations to actual risk aversion and time preference can significantly improve the welfare of individuals by providing investment recommendations that better match their risk profiles.

7.3 Future Research Directions

Following the initial study, future research could explore how additional demographic and psychological variables can enhance accuracy. Another is how longitudinal changes in risk aversion and time preference over time.

References

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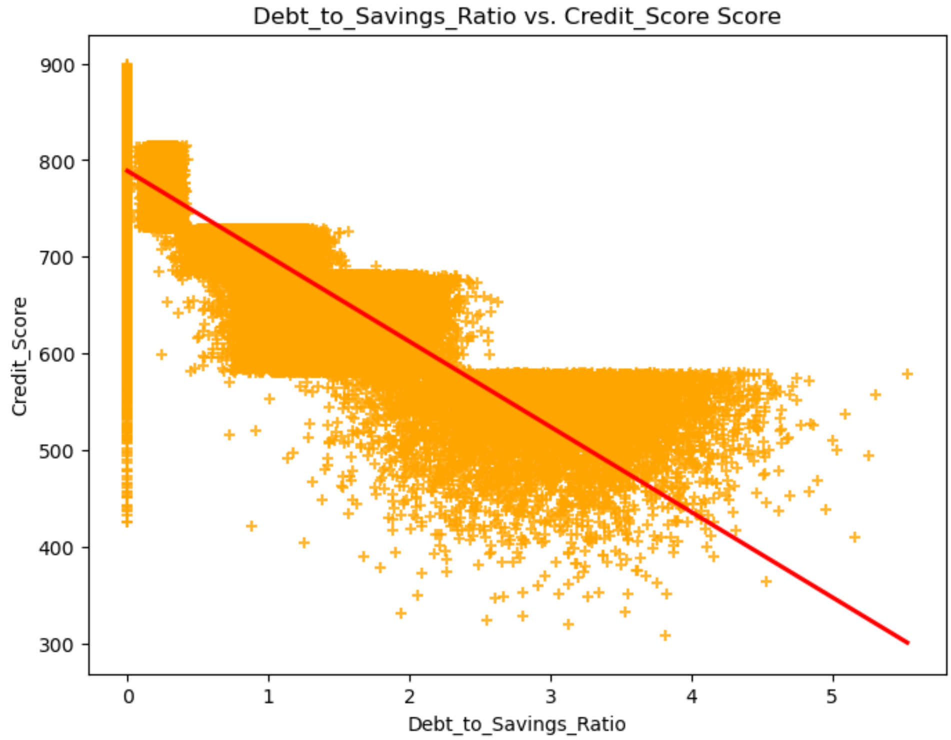
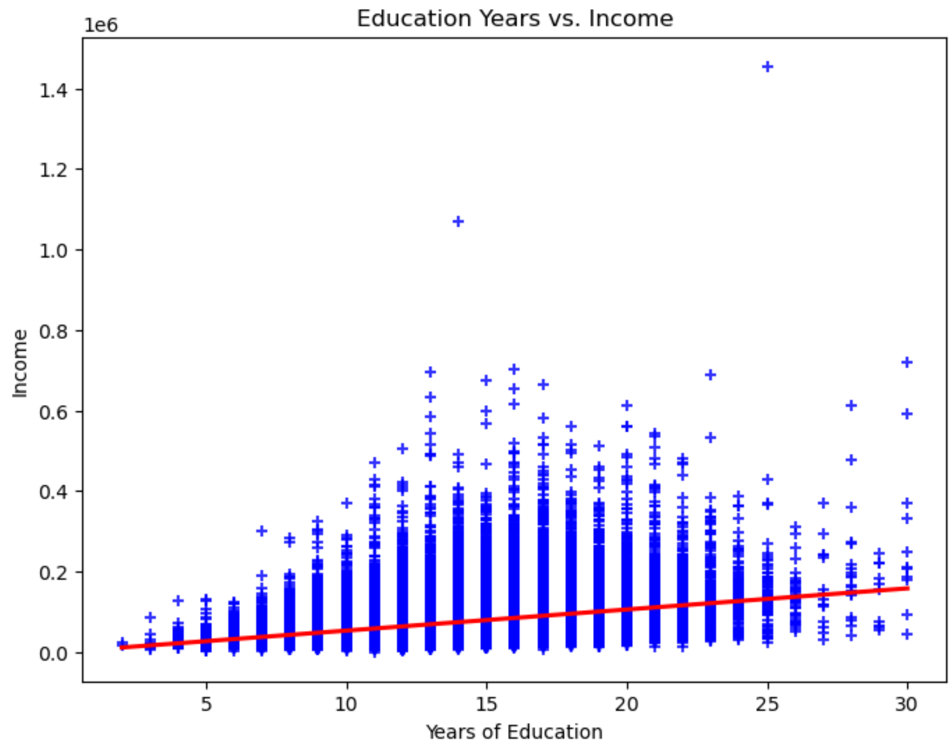
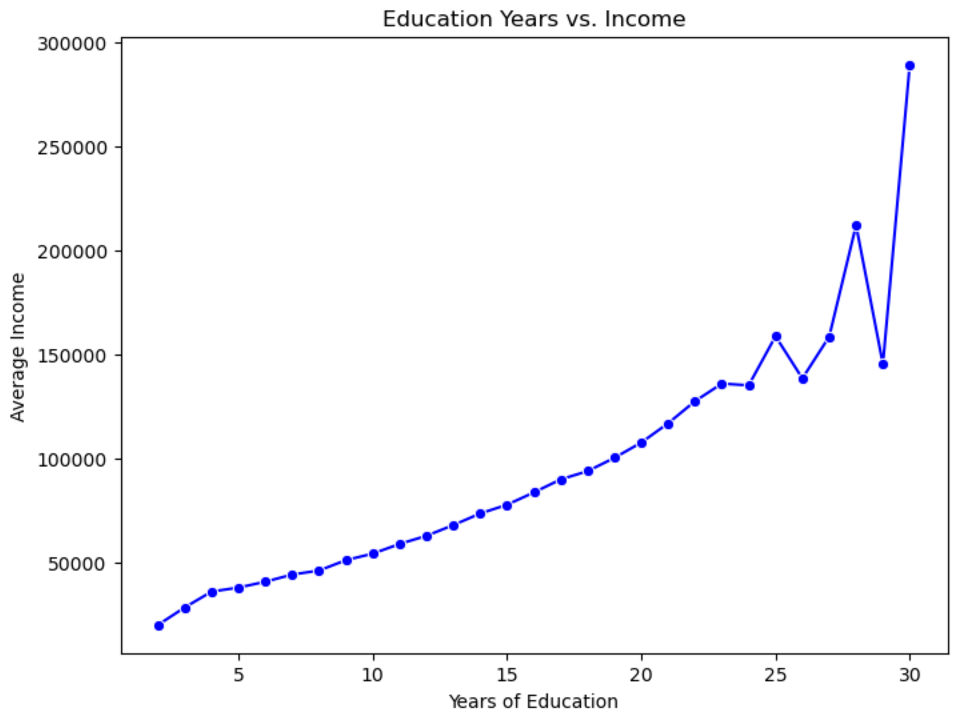
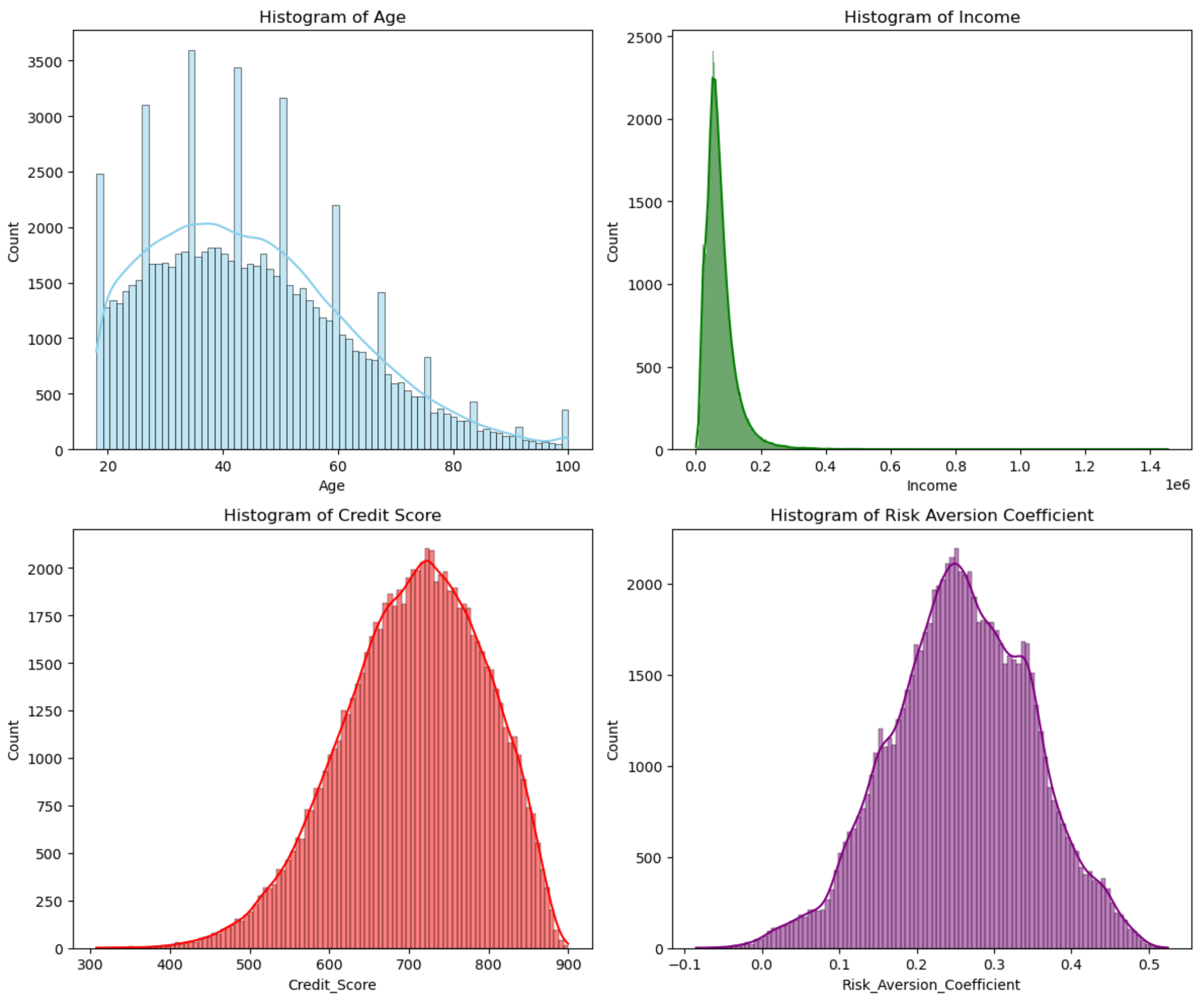
8 Appendix

Here, I'll put down some descriptive graphs and charts about my population sample. (It took a long time to simulate it so that it looked accurate. And I want to use it for something)






Education Years vs. Income
Average Income
300000
250000
200000
150000
100000
50000
5
10
15
20
25
30
Years of Education


Histogram of Age
Count
Age
Histogram of Income
Count
Income
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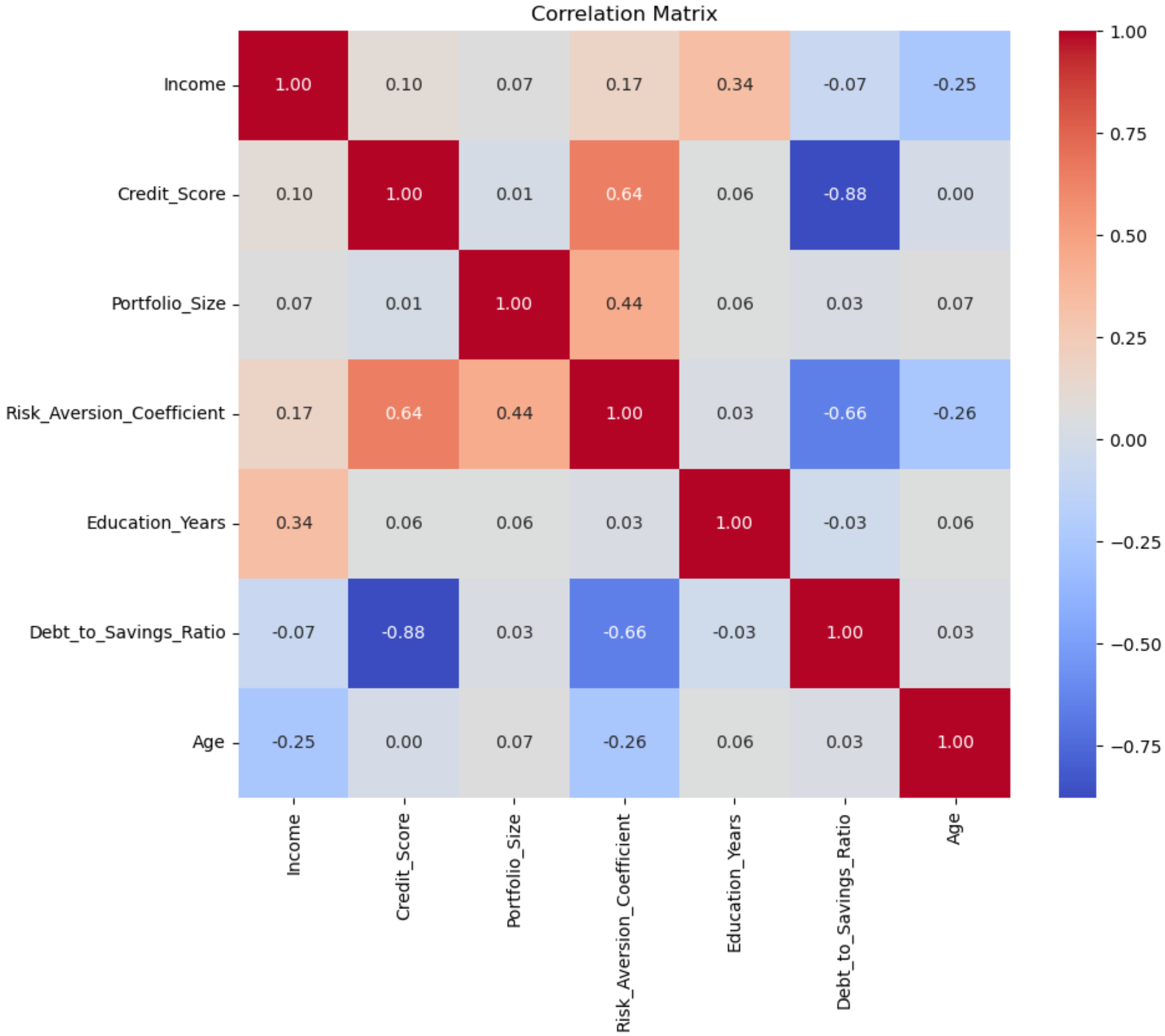
Histogram of Credit Score
Count
Credit_Score
Histogram of Risk Aversion Coefficient
Count
Risk_Aversion_Coefficient


Correlation Matrix
Income
Credit_Score
Portfolio_Size
Risk_Aversion_Coefficient
Education_Years
Debt_to_Savings_Ratio
Age
1.00 0.10 0.07 0.17 0.34 -0.07 -0.25
0.10 1.00 0.01 0.64 0.06 -0.88 0.00
0.07 0.01 1.00 0.44 0.06 0.03 0.07
0.17 0.64 0.44 1.00 0.03 -0.66 -0.26
0.34 0.06 0.06 0.03 1.00 -0.03 0.06
-0.07 -0.88 0.03 -0.66 -0.03 1.00 0.03
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1.00
0.75
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