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CS 331

Written Assignment #3

1. Probability distributions
    a. P(toothache = true) = 0.064 + 0.016 + 0.012 + 0.108 = 0.2
    P(toothache = false) = 1 - P(toothache = true) = 0.8

    b. P(cavity = true) = 0.008 + 0.072 + 0.012 + 0.108 = 0.2
    P(cavity = false) = 1 - P(cavity = true) = 0.8

    c. P(toothache = true | cavity = true) = P(toothache = true A cavity = true)/P(cavity = true) = (0.012 + 0.108)/0.2 = 0.6
    P(toothache = false | cavity = true) = 1 - P(toothache = true | cavity = true) = 0.4
    P(toothache = true | cavity = false) = P(toothache = true A cavity = false)/P(cavity = false) = (0.064 + 0.016)/0.8 = 0.1
    P(toothache = false | cavity = false) = 1 - P(toothache = true | cavity = false) = 0.9

2. P(X,Y) = P(X)P(Y)   (1)
    P(X,Y) = P(X|Y)P(Y)   joint probability definition
    P(X|Y) = P(X)   (2)
    therefore P(X,Y) = P(X)P(Y)
    P(X|Y) = P(Y|X)P(X)/P(Y)   Bayes' Theorem
    P(X,Y) = P(Y|X)P(X)
    P(Y|X) = P(Y)   (3)
    P(X,Y) = P(Y)P(X) = P(X)P(Y)

3. Coin flipping indepence
    a. The flipping of a coin is an event that is independent by nature so successive flips do not depend on each other, however the probability of an outcome of many coin flips does change with each coin flip. For example, the probability of HH = 0.25 with P(H) = P(T) = 0.5, but P(HH | T) = 0. So outcomes are not necessarily independent of each other if outcomes are defined as a series of flips. Otherwise, independent flips are independent.

    b. The answer in (a) still stands whether you know the probability distribution or not of a single coin flip.

4. P(test = true | disease = true) = 0.99
    P(test = false | disease = true) = 1 - P(test = true | disease = true) = 0.01

P(test = false | disease = false) = 0.99
P(test = true | disease = false) = 1 - P(test = false | disease = false) = 0.01

P(disease = true) = 0.0001
P(disease = false) = 1 - P(disease = true) = 0.9999

P(disease = true | test = true) = P(test = true | disease = true)P(disease=true)/P(test = true)
P(test = true) = P(test = true | disease = false)P(disease = false) + P(test = true | disease = true)P(disease = true)
P(disease = true | test = true) = (0.99)(0.0001) / ((0.01)(0.9999) + (0.99)(0.0001)) = 0.0098039216

Which is good news your chances of actually having the disease are still quite low.

5. Taxi colors
   a. P(appears = green | taxi = green) = 0.75
      P(appears = blue | taxi = green) = 1 - P(appears = green | taxi = green) = 0.25
      P(appears = blue | taxi = blue) = 0.75
      P(appears = green | taxi = blue) = 1 - P(appears = blue | taxi = blue) = 0.25

      P(taxi = blue | appears = blue) = P(appears = blue | taxi = blue)P(taxi = blue) / P(appears = blue)

      Which cannot be determined because we don't know and cannot determine P(taxi = blue)

   b. P(taxi = green) = 0.9
      P(taxi = blue) = 1 - P(taxi = green) = 0.1

      P(appears = blue) = P(appears = blue | taxi = blue)P(taxi = blue) + P(appears = blue | taxi = green)P(taxi = green)

      P(taxi = blue | appears = blue) = (0.75)(0.1) / ((0.75)(0.1) + (0.25)(0.9)) = 0.25