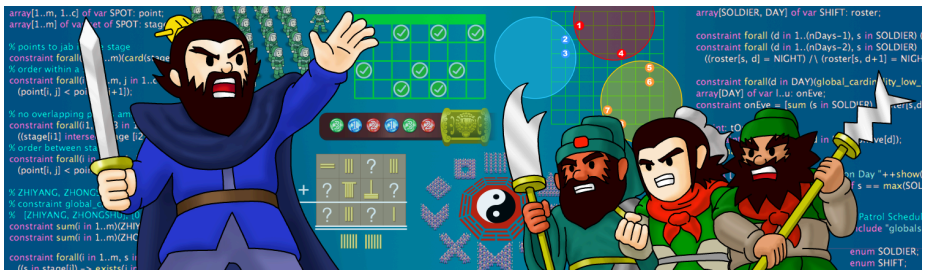
Escape to Jing Province

Advanced Modeling for Discrete Optimization: Assignment 1



Problem Statement

Liu Bei sees the Yuan Shao will certainly be defeated by Cao Cao regardless of his actions. He tells Yuan Shao that he will head to Jing province to bring allies back to help him, but in reality he is plotting to escape to Jing province.

He needs to get to Jing province over a complicated terrain, represented by a grid, where Cao Cao's soldiers are patrolling. Terrain elements in the grid are Plains, Mountain, Forest, City and River. He needs to find a path from where he is now to Jing province such that he abides by the following rules.

- He doesn't enter any mountains, since it's too cold now to travel over them.
- Furthermore, he can enter at most one city. Once he enters a city, he is bound to be recognized and then if he enters another city he will be captured by Cao Cao's soldiers.
- He makes the journey quickly enough. Otherwise, the bulk of Cao Cao's army will arrive, and he will certainly be captured.
- The journey does not involve too many steps, or he will get lost.

His aim is to minimize the number of soldiers he will meet along the route, since that will maximize the chance of a successful escape.

Data Format Specification

The input form for the Escape to Jing Province is files named `data/to_jing_*.dzn`, where

- `nrow` is the number of rows in the grid,
- `ncol` is the number of columns in the grid,
- `start_row` is the row where he starts,
- `start_col` is the column where he starts,
- `delay` is an array mapping terrain to the time in days for traversing a grid square of that terrain type,

- `timelimit` is the number of days in which he must complete his journey,
- `terrain` is a 2D array mapping each grid position to its terrain,
- `Jing` is a 2D array of Booleans saying for each grid position whether it is part of Jing province,
- `soldier` is a 2D array mapping each grid position to the number of squadrons of soldier stationed in that grid position, and
- `maxstep` is the maximum number of steps in the path.

Liu Bei has hurriedly constructed a model, and it seems to work for small data, but does not seem good enough to solve the actual problem data. The model he constructed (in file `to_jing.mzn`) is

```
int: nrow;
set of int: ROW = 1..nrow;
int: ncol;
set of int: COL = 1..ncol;

% Plains, Mountain, Forest, City, River
enum TERRAIN = { P, M, F, C, R };
array[TERRAIN] of int: delay;
int: timelimit;

array[ROW,COL] of TERRAIN: terrain;
array[ROW,COL] of int: soldier;
array[ROW,COL] of bool: Jing;

int: start_row;
int: start_col;

int: maxstep;
set of int: STEP = 1..maxstep;
set of int: STEP0 = 0..maxstep;

var STEP: steps;
array[ROW,COL] of var STEP0: visit;

% start at start position
constraint visit[start_row,start_col] = 1;

% only use steps moves
constraint sum(r in ROW, c in COL)(visit[r,c] >= 1) <= steps;
% reach Jing province
constraint exists(r in ROW, c in COL)(Jing[r,c] /\ visit[r,c] >= 1);

% visit at most one city
constraint not exists(r1,r2 in ROW, c1,c2 in COL)
```

```
                        ((r1 != r2 \/ c1 != c2)
                        /\ terrain[r1,c1] = C /\ terrain[r2,c2] = C
                        /\ visit[r1,c1] >= 1 /\ visit[r2,c2] >= 1);

% can't enter Mountain
constraint not exists(r in ROW, c in COL)(
  terrain[r,c] = M /\ visit[r,c] >= 1
);

% visit only one place in every step
constraint forall(r1,r2 in ROW, c1,c2 in COL)
                    (r1 != r2 \/ c1 != c2
                    -> (visit[r1,c1] = 0
                        \/ visit[r2,c2] != visit[r1,c1]));
% steps form a path
constraint forall(s in 1..steps-1)
                    (exists(r1, r2 in ROW, c1, c2 in COL)
                        (abs(r1-r2) + abs(c1-c2) = 1
                            /\ visit[r1,c1] = s /\ visit[r2,c2] = s+1));
% no shortcuts on path
constraint forall(r1,r2 in ROW, c1,c2 in COL)
                    (abs(r1-r2) + abs(c1-c2) = 1 ->
                    visit[r1,c1] = 0 \/ visit[r2,c2] = 0 \/
                    abs(visit[r1,c1] - visit[r2,c2]) = 1);

% not too much delay
constraint time <= timelimit;
var int: time = sum(r in ROW, c in COL)(
  delay[terrain[r,c]]*(visit[r,c] >= 1)
);

% minimize the number of soldiers traversed
solve minimize sum(r in ROW, c in COL)((visit[r,c] > 0)*soldier[r,c]);

array[TERRAIN] of string: ter = [".", "#", "^", "C", "~"];

output
        [ " " ++ ter[fix(terrain[r,c])]
        ++ if c = ncol then "\n" else "" endif
        | r in ROW, c in COL ]
        ++ ["\n"] ++
        [ if soldier[r,c] > 0 then show_int(2,soldier[r,c]) else " ." endif
            ++ if c = ncol then "\n" else "" endif
        | r in ROW, c in COL ]
        ++ ["\n"] ++
        [ if fix(visit[r,c]) > 0 then show_int(2,visit[r,c]) else " ." endif
```

```
            ++ if c = ncol then "\n" else "" endif
        | r in ROW, c in COL ];
```

An example data file is

```
nrow = 5;
ncol = 5;

start_row = 5;
start_col = 5;

delay = [ 1, 9, 3, 1, 2 ];
timelimit = 8;

terrain = [| P, P, P, P, M
           | P, C, M, P, P
           | P, P, C, P, P
           | P, R, P, C, F
           | M, R, F, P, P |];

Jing = [|  true,  true,  true, false, false
        |  true, false, false, false, false
        |  true, false, false, false, false
        | false, false, false, false, false
        | false, false, false, false, false |];

soldier = [| 3,1,4,8,1
           | 2,1,9,5,4
           | 6,1,4,8,1
           | 3,1,7,1,2
           | 6,1,2,4,1 |];

maxstep = 8;
```

which considers a 5×5 map (in file `to_jing_0.dzn`). Liu Bei's model correctly finds an optimal solution for this small data file. His output also prints in comments the terrain map ("`.`" for Plains, "`#`" for Mountains, "`^`" for Forest, "`C`" for City and "`~`" for River) and soldier maps, as well as a representation of the path. For this data it outputs

```
 . . . . #
 . C # . .
 . . C . .
 . ~ . C ^
 # ~ ^ . .

 3 1 4 8 1
 2 1 9 5 4
```

```
 6 1 4 8 1
 3 1 7 1 2
 6 1 2 4 1

 . . . . .
 . . . . .
 7 6 . . .
 . 5 4 3 .
 . . . 2 1
----------
==========
```

Showing a path of length 7 steps which requires 8 days to traverse and passes 21 squadrons of soldiers.

The aim of this assignment is to improve the given model so that it is much more efficient. This may involve rewriting the constraints, or even changing the decision variables, but the data file format cannot be changed, and the output must have the same format as Liu Bei's model (excluding the parts that are comments). To build a model that solves the largest examples will be very difficult, and is not expected of any but the highest achieving students.

As the **interface to the grader**, your updated model must still contain variable declarations for `steps`, `visit`, and `time`, and it must calculate the correct objective value. Note that you are allowed to use `::output_only` variable declarations, if you want to use a different point of view.

Instructions

Edit the provided `mzn` model files to solve the problems described above. Your implementations can be tested locally by using the **Run + Check** icon in the MiniZinc IDE. Once you are confident that you have solved the problem, submit your solution for grading.

Technical Requirements To complete this assignment you will a new version of the MiniZinc Bundle (`http://www.minizinc.org`). Your submissions to the Coursera grader are currently checked using MiniZinc version 2.5.5.

Handin This assignment contains 5 solution submissions and 1 model submissions. For solution submissions, we will retrieve the best/last solution the solver has found using your model and check its correctness and quality. For model submissions, we will retrieve your model file (.mzn) and run it on some hidden data to perform further tests.

From the MiniZinc IDE, the **Submit to Coursera** icon can be used to submit assignment for grading. Follow the instructions to apply your MiniZinc model(s) on the various assignment parts. You can submit multiple times and your grade will be the best of all submissions.[1]. You can track the status of your submission on the **programming assignments** section of the course website.

[1] Please limit your number of submissions and test on your local machine first to avoid congestion on the Coursera servers