

Perceptual Similarities Among Wallpaper Group Exemplars

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Abstract

Symmetries are abundant within the visual environment, and many animals species are sensitive to visual symmetries. Wallpaper groups a class of 17 regular textures that each contain a distinct combination of the four fundamental symmetries, translation, reflection, rotation and glide reflection, and together represent the complete set of possible symmetries in two-dimensional images. Wallpapers are visually compelling and elicit responses in visual brain areas that precisely capture the symmetry content of each group, in humans and other primates. Here we ask to what extent exemplars from the same wallpaper group are perceptually similar. We algorithmically produce a set of well-matched exemplars from 5 of the 17 wallpaper groups and instructed participants to freely sort the exemplars from each group into as many subsets as they wished based on any criteria they saw appropriate. P_1 , the simplest of the 17 groups, was consistently rated more self-similar than any other group, while the other four groups, although varying in symmetry content, were comparable in self-similarity. Our results suggest that except for the most extreme case (P_1), self-similarity of wallpaper groups is not directly tied to symmetry content.

Introduction

Symmetry has been recognized as important for human visual perception since the late 19th century (Mach, 1959). In the two spatial dimensions relevant for images, symmetries can be combined in 17 distinct ways, the *wallpaper groups* (Fedorov, 1891; Polya, 1924; Liu et al., 2010). Wallpaper groups are different from the stimuli typically used to probe the role of symmetry in visual perception in two ways: First, they contain combinations of the four fundamental symmetry types translation, reflection, rotation and glide reflection, rather than just reflection or mirror symmetry, which has been the focus of most vision research. Second, the symmetries in wallpaper groups are repeated to tile the plane, rather than positioned at a single image location as is usually the case. These differences, and the fact that wallpaper groups together form the complete set of symmetries possible in the two-dimensional image plane, make wallpapers an interesting stimulus set for studying perception of visual symmetries.

34 Brain imaging studies using functional MRI (Kohler
 35 et al., 2016) and EEG (Kohler et al., 2018; Kohler and
 36 Clarke, 2021) has shown that the human visual system
 37 carries detailed and precise representations of the sym-
 38 metries within the individual wallpaper groups, and
 39 functional MRI evidence from macaque monkeys re-
 40 veal similar representations in analogous areas of the
 41 macaque visual system (Audurier et al., 2021).

42 These representations, complex as they are, do not
 43 appear to be readily available for driving conscious
 44 behaviour: Humans have limited intuitive sense of
 45 group membership for wallpaper group exemplars, as
 46 evidenced by behavioral experiments showing that al-
 47 though naïve observers can distinguish many of the wall-
 48 paper groups (Landwehr, 2009), they tend to sort exem-
 49 plars into fewer (4-12) sets than the number of wallpaper
 50 groups, often placing exemplars from different wallpaper
 51 groups in the same set (Clarke et al., 2011). Wallpaper
 52 groups are nonetheless visually compelling and anec-
 53 dotally we have observed that exemplars from a given
 54 group can be quite perceptually diverse. This obser-
 55 vation inspired the current study, in which we use the
 56 behavioral sorting approach to probe the perceptual self-
 57 similarity of different exemplars from the same wallpa-
 58 per group, and assess the extent to which self-similarity
 59 varies across five groups.

60 We algorithmically generated 20 well-matched exem-
 61 plars from each group (see Figures 1 and 2 for a selection
 62 of the exemplars, and the **Materials and Methods** sec-
 63 tion for details on how they were generated) and printed
 64 them out on white cardstock. We then gave participants
 65 the 20 cards with exemplars from each wallpaper group,
 66 and asked them to freely sort them into as many sub-
 67 sets as they wished based on any criteria they saw ap-
 68 propiate. This approach allowed us to compare the five
 69 wallpaper groups, both in terms of how many subsets
 70 participants generated, and also in terms of *jaccard in-*
 71 *dex*, a summary statistic capturing the similarity across
 72 exemplar pairs for each group. Within each group, we
 73 were also able to identify exemplar pairs that were rated
 74 as highly similar and highly dissimilar. Our main con-

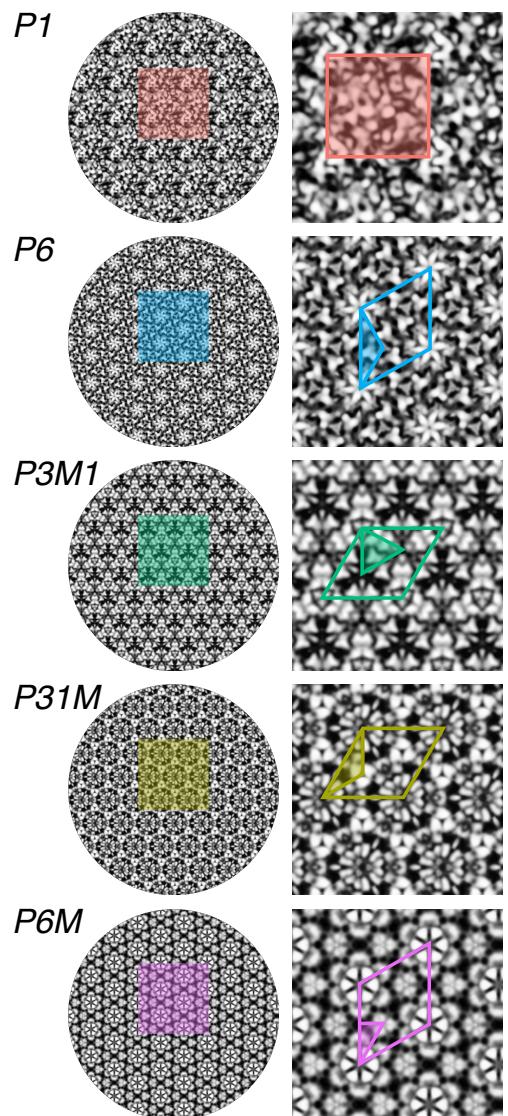


Figure 1: The fundamental region and lat-
 tice structure of the five wallpaper groups
 used in the study. The complete wallpa-
 per is shown in the left-hand column with
 a shaded region that is repeated and en-
 larged in the right-hand column. The col-
 ored outline in the enlarged region indi-
 cates the repeating lattice for each group,
 while the shaded area indicates the funda-
 mental region (see text). For P_1 the funda-
 mental region covers the entire lattice.
 Note that even though P_6 and $P_{31}M$ have
 the same fundamental region and lattice
 shapes, they differ in terms of the symme-
 tries present within the lattice - most no-
 tably, $P_{31}M$ contains reflection symmetry
 while P_6 does not. The symmetry content
 of each group is detailed on the wallpaper
 group wikipedia page.

75 clusion is that P_1 was systematically less self-similar than the any other groups, while the other
76 four other groups could not be distinguished on these measures.

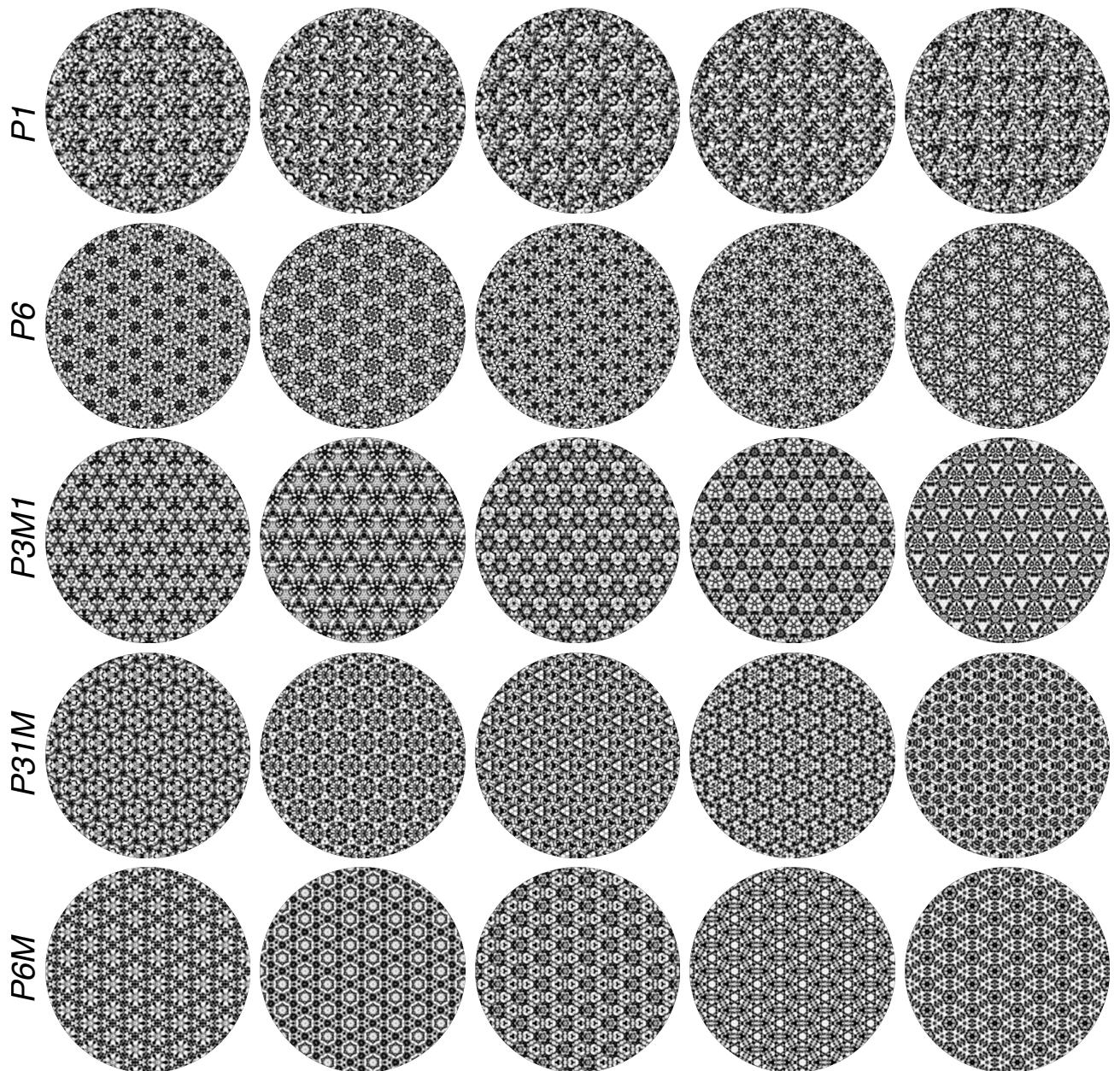


Figure 2: 5 of the 20 exemplars used for each group are shown to highlight the diversity among exemplars.

77 Results

78 Wallpaper group P_1 was less self-similar than the other four groups. This was evident in the
79 number of sets generated for this group across participants, which was lower for P_1 (median
80 = 3) than for the other groups (median = 4.5, see Figure 3). We confirmed this observation
81 statistically by running a repeated measures analysis of variance (ANOVA) with group as a fixed
82 factor and participant as a random factor, which revealed a significant effet of group ($F(4,124) =$
83 7.330, $p < 0.0001$). Post-hoc pairwise t -tests showed that the mean number of sets was lower for
84 P_1 than all other groups, but no other means differed. Next, we computed the Jaccard index (see
85 Materials and Methods) across participants for every pairwise combination of exemplars in each
86 group. This provides a measure of the similarity between exemplars within each group. P_1 had

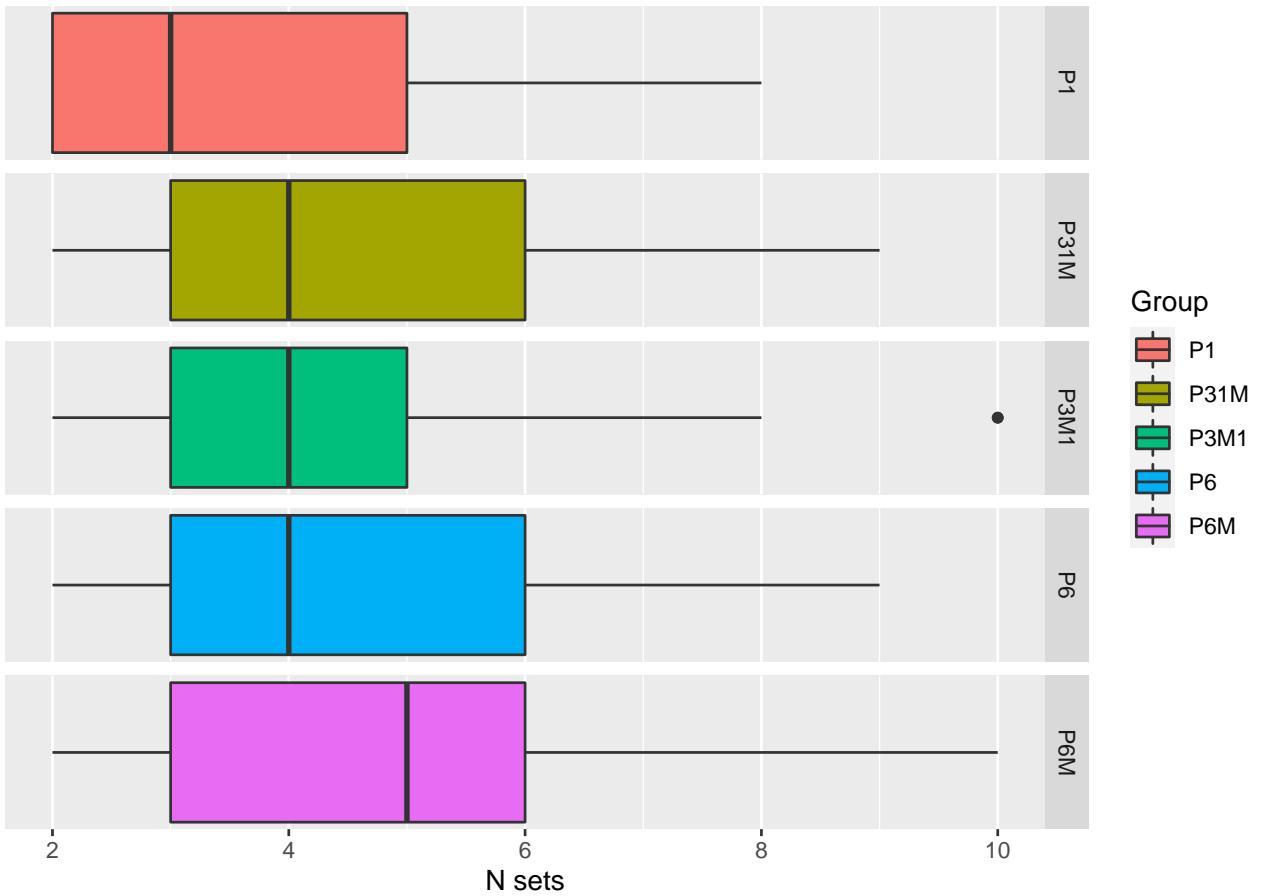


Figure 3: Boxplots showing the number of subsets generated by participants for each of the wallpaper groups. The lower box boundary is the 25th percentile. The dark line in the box is the median. The upper box boundary is the 75th percentile. The “whiskers” show -/+ the interquartile range * 1.5.

87 systematically higher Jaccard indices than the four other groups (see Figure 4), as confirmed by an
 88 ANOVA with wallpaper group as a factor. The analysis revealed a statistically significant effect of
 89 group ($F(4, 495) = 20.178, p < 0.0001$). Post-hoc pairwise t -tests showed that P_1 had higher Jaccard
 90 indices than all other groups ($p < 0.0001$). The fact that the group (P_1) for which fewer subsets
 91 were generated also had higher Jaccard indices than the other groups illustrates the inherent link
 92 between the two measures: For wallpaper groups where the 20 exemplars are sorted into fewer
 93 subsets, each individual exemplar pair are more likely to be members of the same subset, and
 94 less likely to be members of distinct subsets, which in turn leads to higher Jaccard indices. Our
 95 pairwise t -tests also showed that P_{31M} had lower Jaccard indices than P_6 ($p = 0.037$). This effect
 96 is relatively weak, but may reflect real differences in how consistently exemplars were grouped
 97 together across participants. We will explore this idea more in depth shortly, but for now we can
 98 conclude that out of the five groups tested, P_1 is the only one that can be reliably differentiated
 99 based on our measures, being higher on self-similarity among the exemplars, and thus lower on
 100 diversity among exemplars.

101 The Jaccard indices also allow us to focus on exemplar pairs that have a high level of similarity
 102 relative to the rest of the pairs in the set. We do this by identifying outliers from each group, as
 103 identified with stars in 4. For each exemplar in each outlier pair, we can visualize the pairwise
 104 similarity (as measured by the Jaccard index) to every other exemplar in the set (see Figure

105 5). In order to quantify the extent to which exemplars were consistently grouped together, we
 106 ran a permutation analysis in which exemplar labels were shuffled among the sets generated
 107 for each participant (see Materials and Methods). This provides, for each group, the expected
 108 distribution of Jaccard indices for every pairwise combination of exemplars, if exemplars were
 109 assigned randomly to subsets, and allows us to compute an empirical z -score that expressed
 110 the extent to which a given pair of exemplars deviates from random assignment. Because the
 111 random distribution is generated by shuffling exemplars across the specific sets generated by
 112 each participant for each group, this z -score is independent of number of sets. If for a given
 113 group, none of the pairs deviate significantly from the random distribution, it would indicate that
 114 participants were not consistently grouping any pair of exemplars together.

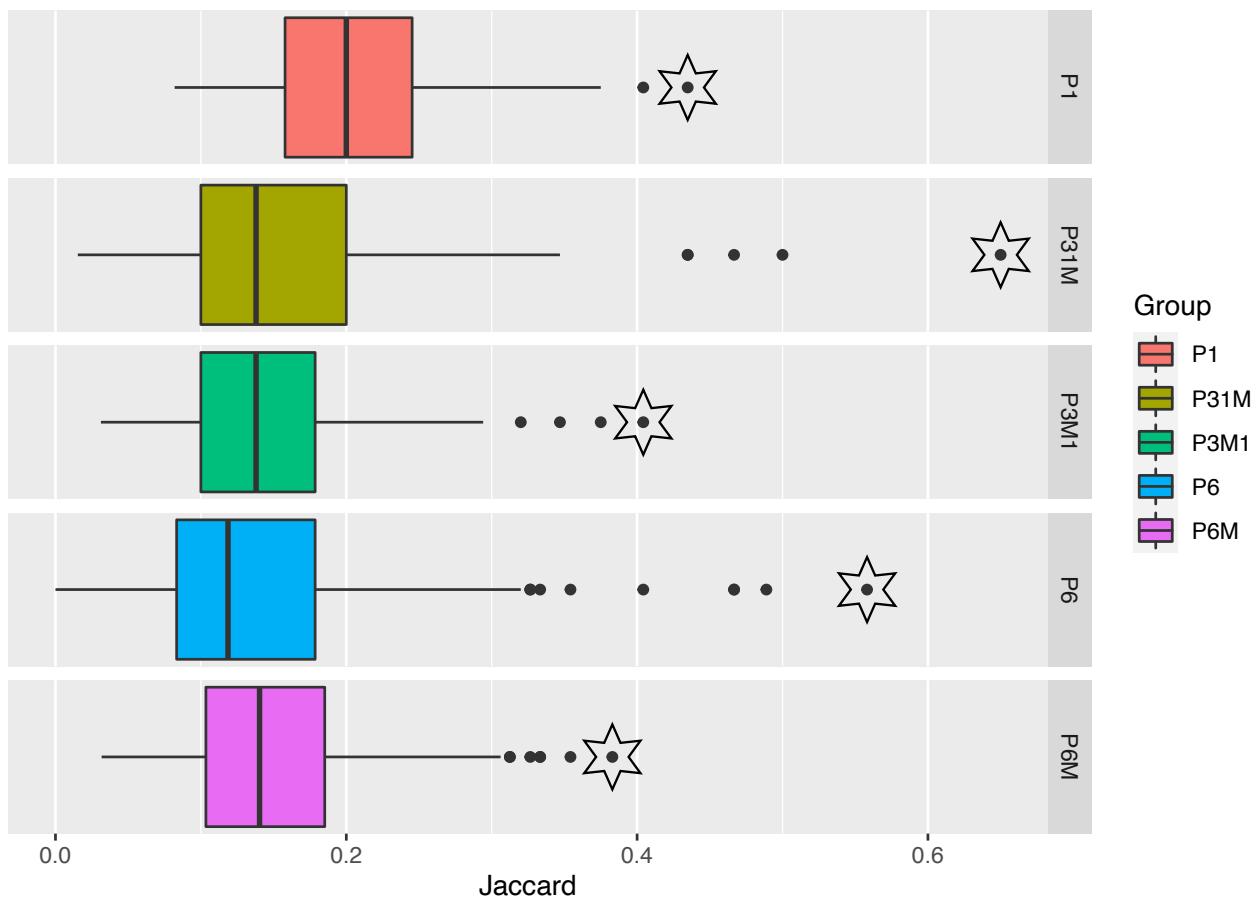


Figure 4: Boxplots showing Jaccard indices for every pairwise combination of exemplars in each of the wallpaper groups. Note that each data point here is the Jaccard index for a particular exemplar pair calculated across participants, unlike Figure 3 where each data point is a participant. The box boundary and whiskers follow the same logic as in Figure 3. The exemplar pairs with the highest Jaccard indices have been highlighted with stars. Those outlier pairs are explored further in Figure 5.

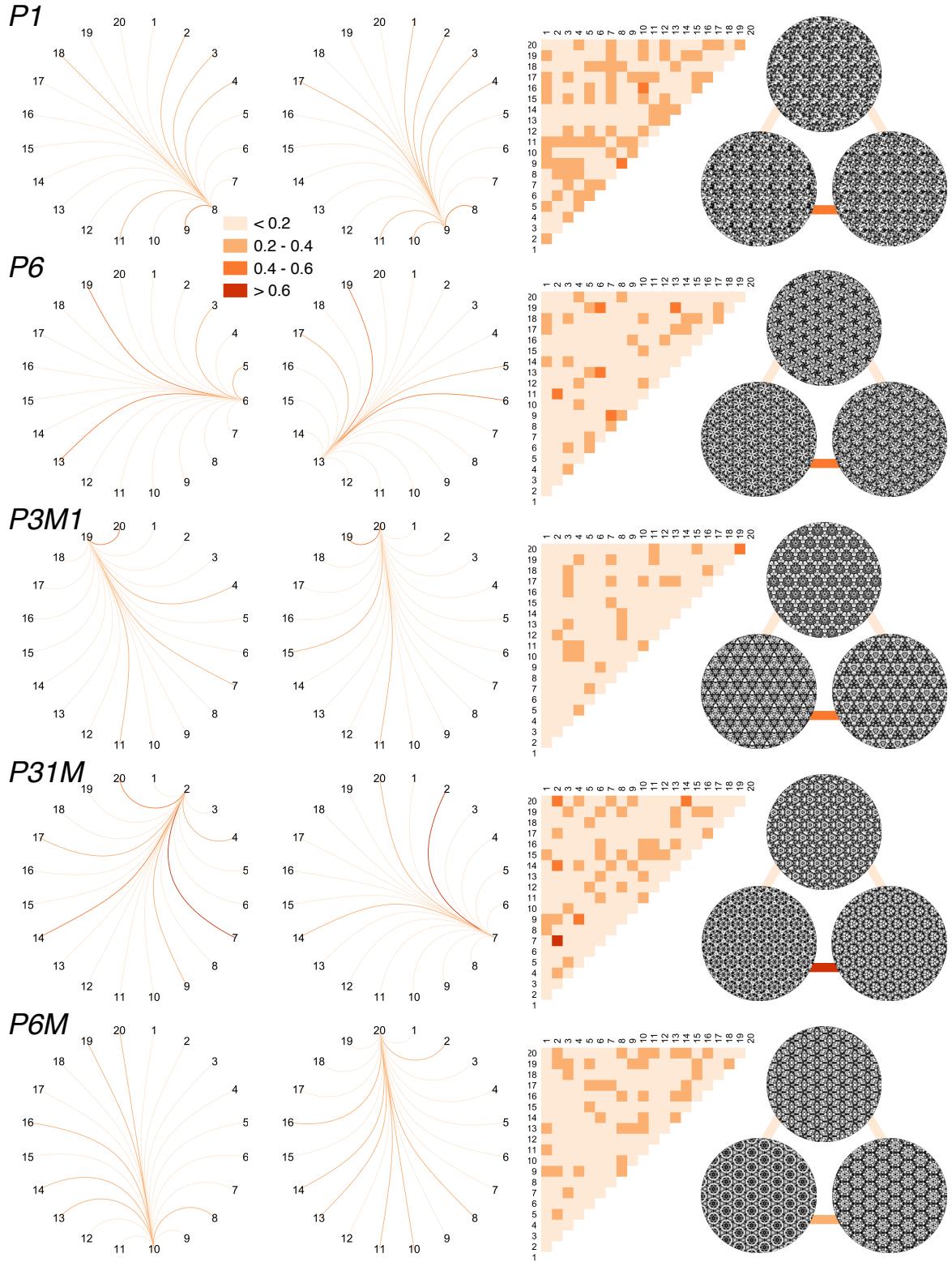


Figure 5: For each wallpaper group, we identified the two most self-similar exemplars. The two circular network plots are showing the pairwise similarities between those two exemplars and every other exemplar in the set. The pairwise similarities across all exemplars are plotted as a similarity matrix and on the rightmost side of the plot, the two most self-similar exemplars (bottom) are plotted with the exemplar that was least similar to both (top). Note that the connecting lines between the exemplars indicate the similarity.

115 Discussion

116 Materials and Methods

117 Participants

118 33 participants (9 Male, 24 Female), ranging in age between 18 and 35 completed this study. All
119 participants had self reported 20/20 or corrected to 20/20 vision. We obtained written consent to
120 participate from all participants under procedures approved by the Institutional Review Board
121 of The Pennsylvania State University (#38536). The research was conducted according to the
122 principles expressed in the Declaration of Helsinki.

123 Stimulus Generation

124 Five wallpaper groups (P_1 , P_3M_1 , $P_{31}M$, P_6 and P_{6M}) that has previously been shown to be high in
125 self-similarity (Clarke et al., 2011), were selected. 20 exemplars from each of these five wallpaper
126 groups were generated using a modified version of the methodology developed by Clarke and
127 colleagues(Clarke et al., 2011) that we have described in detail elsewhere(Kohler et al., 2016).
128 Briefly, exemplar patterns for each group were generated from random-noise textures, which
129 were then repeated and transformed to cover the plane, according to the symmetry axes and
130 geometric lattice specific to each group. The use of noise textures as the starting point for
131 stimulus generation allowed the creation of an almost infinite number of distinct exemplars of
132 each wallpaper group. To make individual exemplars as similar as possible we replaced the power
133 spectrum of each exemplar with the median across exemplars within a group. These images
134 were printed onto white cardstock and cut into squares, allowing participants to manipulate the
135 orientation of the images during the sorting tasks. Five exemplars from each group are shown
136 (in reduced size) in 2.

137 Procedure

138 Participants were presented with the 20 exemplars of a single wallpaper group (i.e. P_1 , P_3M_1 ,
139 $P_{31}M$, P_6 , P_{6M}) and instructed to sort them into subsets by placing them into piles. Participants
140 were advised to sort the exemplars into as many piles as they deemed necessary based on
141 whatever criteria they desired. There were no time constraints placed on this sorting task, and
142 the participants were allowed to move exemplars between piles until they were satisfied with
143 their classification. This method was then repeated for the remaining four wallpaper groups for
144 each participant, with group presentation order randomized between participants. These tasks
145 were carried out on a large table with sufficient space to randomly lay out all twenty exemplars
146 of each set, illuminated by normal overhead room lighting. Upon completion of each sorting
147 task, participants were asked to verbalize which features they used to sort the exemplars. After
148 completion of all five sorting tasks, participants were asked which if they had a distinct method
149 for sorting the images, and if any wallpaper group was particular easy or difficult to sort.

150 Generating the Jaccard Index

151 The data was prepared for analysis by creating one binary variable for each subset created by
152 each participant within a sorting task. Then, each exemplar was assigned a value of one (1) if
153 it was included in a subset, or a value zero (0) if it was not. Next, the similarity of each pair of
154 exemplars within a sorting task was calculated using the Jaccard index, a measure of similarity
155 and diversity for binary data. This index is calculated by the equation

$$J = \frac{x}{x + y + z}$$

156 with x representing the number of subsets that contained both exemplars, and y and z the number
157 of subsets that contain only one exemplar of the pair Capra (2005), across participants. Thus, the
158 Jaccard index is the ratio of the number of subsets containing both exemplars of a pair to the
159 number of subsets containing at least one of the exemplars of a pair, thereby excluding subsets
160 with joint absences.

161 Permutation Analysis

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