

Perceptual Similarities Among Wallpaper Group Exemplars

Peter J. Kohler^{1,2}, Shivam Vedak³, and Rick O. Gilmore³

¹ York University, Department of Psychology, Toronto, ON M3J 1P3, Canada

² Centre for Vision Research, York University, Toronto, ON, M3J 1P3, Canada

³Department of Psychology, The Pennsylvania State University, Pennsylvania, USA

Abstract

Symmetries are abundant within the visual environment, and many animal species are sensitive to visual symmetries. Wallpaper groups constitute a class of 17 regular textures that each contain a distinct combination of the four fundamental symmetries, translation, reflection, rotation and glide reflection, and together represent the complete set of possible symmetries in two-dimensional images. Wallpapers are visually compelling and elicit responses in visual brain areas that precisely capture the symmetry content of each group in humans and other primates. Here we ask to what extent *different* exemplars from the *same* wallpaper group are perceptually similar. We used an algorithm to produce a set of well-matched exemplars from 5 of the 17 wallpaper groups and instructed participants to freely sort the exemplars from each group into as many subsets as they wished based on any criteria they saw appropriate. P_1 , the simplest of the 17 groups, was consistently rated more self-similar than any other group, while the other four groups, although varying in symmetry content, were comparable in self-similarity. Our results suggest that except for the most extreme case (P_1), perceived self-similarity of wallpaper groups is not directly tied to categories of symmetry based on group theory.

key words: wallpaper groups, visual perception, behavioral sorting, self-similarity

Introduction

Symmetry ~~has exists in an object or pattern if a transformation can be applied that maps the object/pattern onto itself. In the two-dimensional plane, the set of isometries - distance-preserving transformations, see (Liu et al., 2010) - that can give rise to symmetries are translation, reflection, rotation and glide reflection and their combinations. The wallpaper groups are a set of 17 regular textures, where each has a unique combination of isometries that leave the texture unchanged (Fedorov, 1891; Polya, 1924; Liu et al., 2010). Each wallpaper group therefore contains a distinct combination of four symmetry types (see Figure 1). Symmetries have~~ been recognized as important for human visual perception since the late 19th century (Mach, 1959). ~~In the two spatial dimensions relevant for images, symmetries can be combined in 17 distinct ways, the wallpaper groups (Fedorov, 1891; Polya, 1924; Liu et al., 2010).~~ Wallpaper groups are different from stimuli typically used to probe the role of symmetry in visual perception in two ways: First, they contain combinations of ~~the four fundamental symmetry types~~ translation, reflection, rotation and glide reflection, ~~four symmetry types,~~ rather than just reflection ~~or mirror symmetry~~ (also called mirror symmetry), which

37 have been the focus of most vision research. Second, ~~the symmetries~~ in wallpaper groups symmetries
 38 are repeated to tile the plane, ~~rather than and form textures, instead of being~~ positioned at a single
 39 image location as is usually the case with standard stimuli. These differences, and the important fact that
 40 wallpaper groups together form the complete set of symmetries possible in the two-dimensional image
 41 plane, make wallpapers an interesting stimulus set for studying perception of visual symmetries.

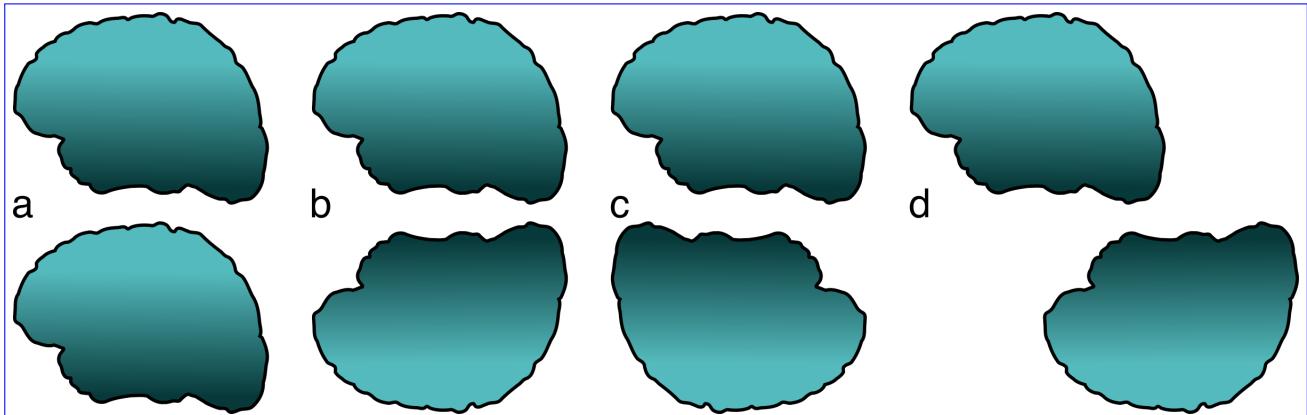


Figure 1: The four fundamental ~~region and lattice structure of the five wallpaper groups used in the study~~. The complete ~~wallpaper is shown in the left-hand column with symmetry types: (a shaded region that is repeated and enlarged in the right-hand column. The colored outline in the enlarged region indicates the repeating lattice for each group) translation, while the shaded area indicates the fundamental region (see text b)~~. For P_1 the fundamental region covers the entire lattice. Note that even though P_6 and $P_{31}M$ have the same fundamental region and lattice shapes~~reflection~~, they differ in terms of the symmetries present within the lattice – most notably (c) rotation (order 2, $P_{31}M$ contains 180°), (d) glide reflection symmetry while P_6 does not. The symmetry content of each group is detailed on – translation followed by reflection over a line parallel to the ~~wallpaper group wikipedia page~~ direction of translation.

42 Brain imaging studies using functional MRI (Kohler et al., 2016) and EEG (Kohler et al., 2018; Kohler
 43 and Clarke, 2021) have shown that the human visual system carries detailed and precise representations of
 44 the symmetries within the individual wallpaper groups. Specifically, response amplitudes scale approximately
 45 linearly with the symmetry content within the wallpaper groups, across all of the possible combinations
 46 of reflection, rotation and glide reflection symmetries. Functional MRI evidence from macaque mon-
 47 keys reveal similar representations in analogous areas of the macaque visual system, and the brain regions
 48 responding to symmetry are largely analogous between humans and monkeys, namely functionally defined
 49 regions V3, V4, VO1 and LOC (Audurier et al., 2021).

50 These representations, complex as they are, The wallpaper group representations that have been
 51 identified using brain imaging are highly complex, but do not appear to be readily available for driving
 52 conscious behaviour: Humans have limited intuitive sense of group membership for wallpaper group
 53 exemplars, as evidenced by behavioral experiments showing that although naïve observers can distinguish
 54 many of the wallpaper groups (Landwehr, 2009), they tend to sort exemplars into fewer (4-12) sets than the
 55 number of wallpaper groups, often placing exemplars from different ~~wallpaper groups in groups into~~ the
 56 same set (Clarke et al., 2011). Wallpaper groups are nonetheless visually compelling, and anecdotally we have
 57 observed that exemplars from a given group can be quite perceptually diverse. This observation inspired
 58 the current study. Here, we use behavioral sorting, a common technique to study perceptual categorization
 59 (Milton et al., 2008; Pothos et al., 2011), to probe the perceptual self-similarity of different exemplars from
 60 the same wallpaper group, ~~and assess the~~. In previous sorting experiments with wallpaper groups (e.g.
 61 (Clarke et al., 2011)) observers were shown exemplars from different wallpaper groups and their ability
 62 to correctly sort exemplars from the same group into the same subset was assessed. Our approach was
 63 different: We wanted to know the extent to which ~~exemplars from the same group would be spontaneously~~

64 organized into subsets, i.e. the self-similarity varies across five groups.

65 We algorithmically generated 20 exemplars from each group that were well-matched in terms of low-level
66 visual properties (see Figures 2 of exemplars from a given group. We selected five distinct wallpaper groups:
67 P_1 , P_3M_1 , P_31M , P_6 and P_6M (see Figure 2). All wallpaper groups consists of a lattice that is repeated to tile
68 the plane. P_1 is the simpleste group, and contains no symmetries other than the translation generated
69 by the repeating lattice. P_6 has rotation symmetries of order 6, 3 and 2, but no other symmetries besides
70 translation. P_3M_1 and P_31M both have rotations of order 3, reflections in 3 distinct directions, and glide
71 reflections in 3 distinct directions, but differ in terms of how these symmetries are organized in the lattice.
72 P_6M is the most complex of the groups, it has rotation symmetries of order 6, 3 and 2, reflections in 6
73 distinct directions, and glide reflections in 6 distinct directions. The lattice structure of the five groups is
74 described in detail on the wallpaper group wikipedia page. The five groups selected have all been found
75 to have high self-similarity (Clarke et al., 2011), and four of them (P_3M_1 , P_31M , P_6 and P_6M) share the same
76 lattice shape (see Figure 2).

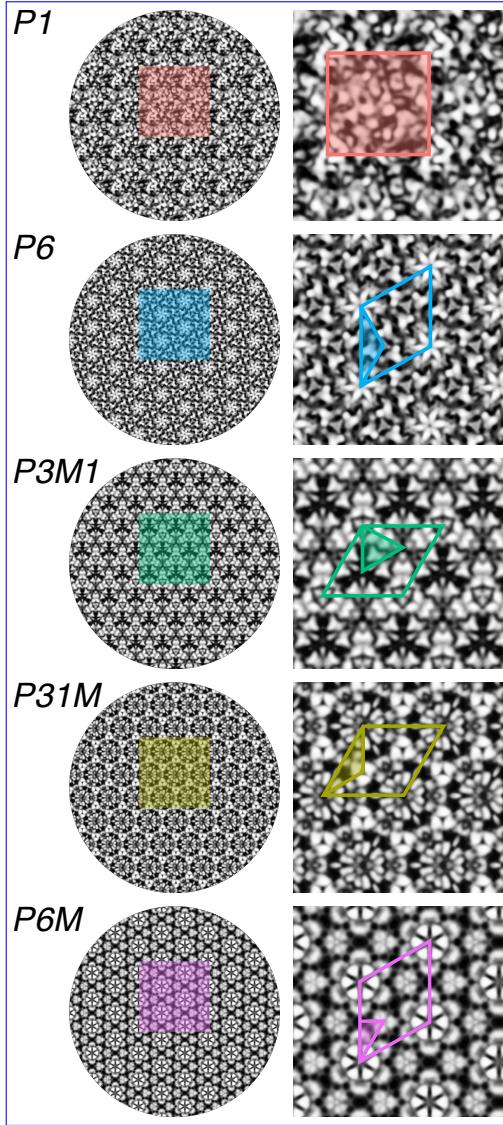


Figure 2: The fundamental region and lattice structure of the five wallpaper groups used in the study. The complete wallpaper is shown in the left-hand column with a shaded region that is repeated and enlarged in the right-hand column. The colored outline in the enlarged region indicates the repeating lattice for each group, while the shaded area indicates the fundamental region (see text). For P_1 the fundamental region covers the entire lattice. Note that even though P_6 and P_{31M} have the same fundamental region and lattice shapes, they differ in terms of the symmetries present within the lattice - most notably, P_{31M} contains reflection symmetry while P_6 does not. The symmetry content of each group is detailed on the [wallpaper group wikipedia page](#).

Participants were given 20 exemplars, all belonging to same group (see Figure 3 for a selection of the exemplars, and the Materials and Methods section for details on how they were generated). We then printed exemplars on white cardstock and gave participants the 20 cards with exemplars from each wallpaper group, and asked them created) and asked to freely sort them into as many subsets as they wished based on any criteria they saw appropriate. Participants sorted exemplars belonging to five different wallpaper groups, one group at a time. This approach allowed us to compare the five wallpaper groups, both in terms of how many subsets participants generated, and also in terms of the Jaccard index, a summary statistic capturing the similarity across exemplar pairs for each group. Within each group, we were also able to identify exemplar pairs that were rated as highly similar and highly dissimilar. Our main conclusion is that P_1 was systematically more self-similar than the any other groups, while the other four other groups could not be distinguished on these measures. We also show that for all five groups,

participants consistently group certain pairs of exemplars together, although the number of consistent pairs varies among groups. Our results open the door to further investigations into the psychological and neural mechanisms that drive perceptual similarity among wallpaper group exemplars, and indeed among exemplars from different classes of structured patterns.

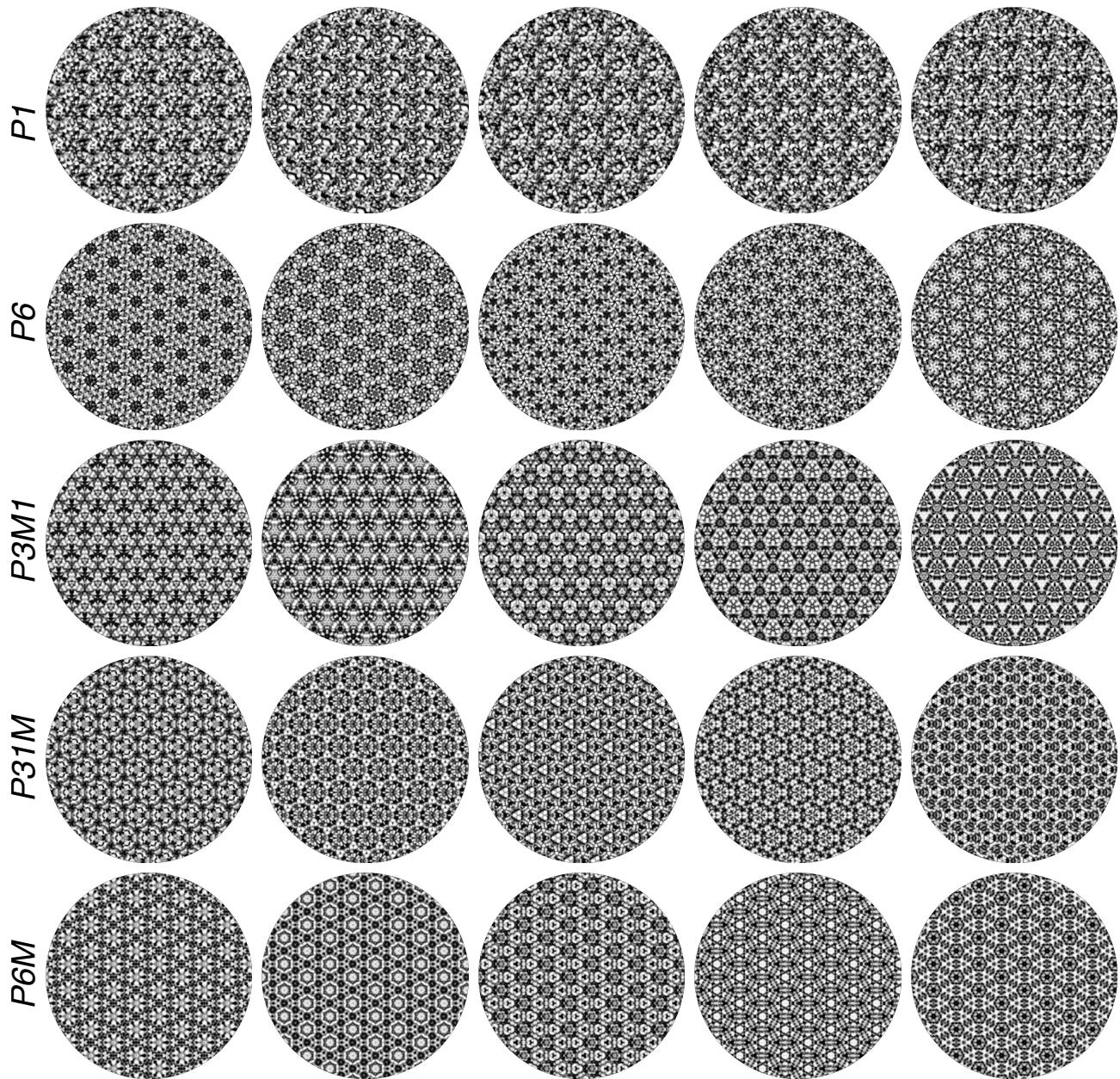


Figure 3: 5 of the 20 exemplars used for each group are shown to highlight the diversity among exemplars.

Results

Wallpaper group P_1 was more self-similar than the other four groups. This was evident in the number of sets generated for this group across participants, which was lower for P_1 (median = 3) than for the four other groups (median = 4-5, see Figure ??₄). We confirmed this observation statistically by running a repeated measures analysis of variance (ANOVA) with group as a fixed factor and participant as a random factor, which revealed a significant effect of group ($F(4,124) = 7.330, p < 0.0001$). Post-hoc pairwise t -tests showed that the mean number of sets was lower for P_1 than all other groups ($p < 0.005$), but no other means differed (see Table 1). Next, we computed the Jaccard index (see Materials and Methods) across participants for every pairwise combination of exemplars in each group. This provides a measure of the similarity between

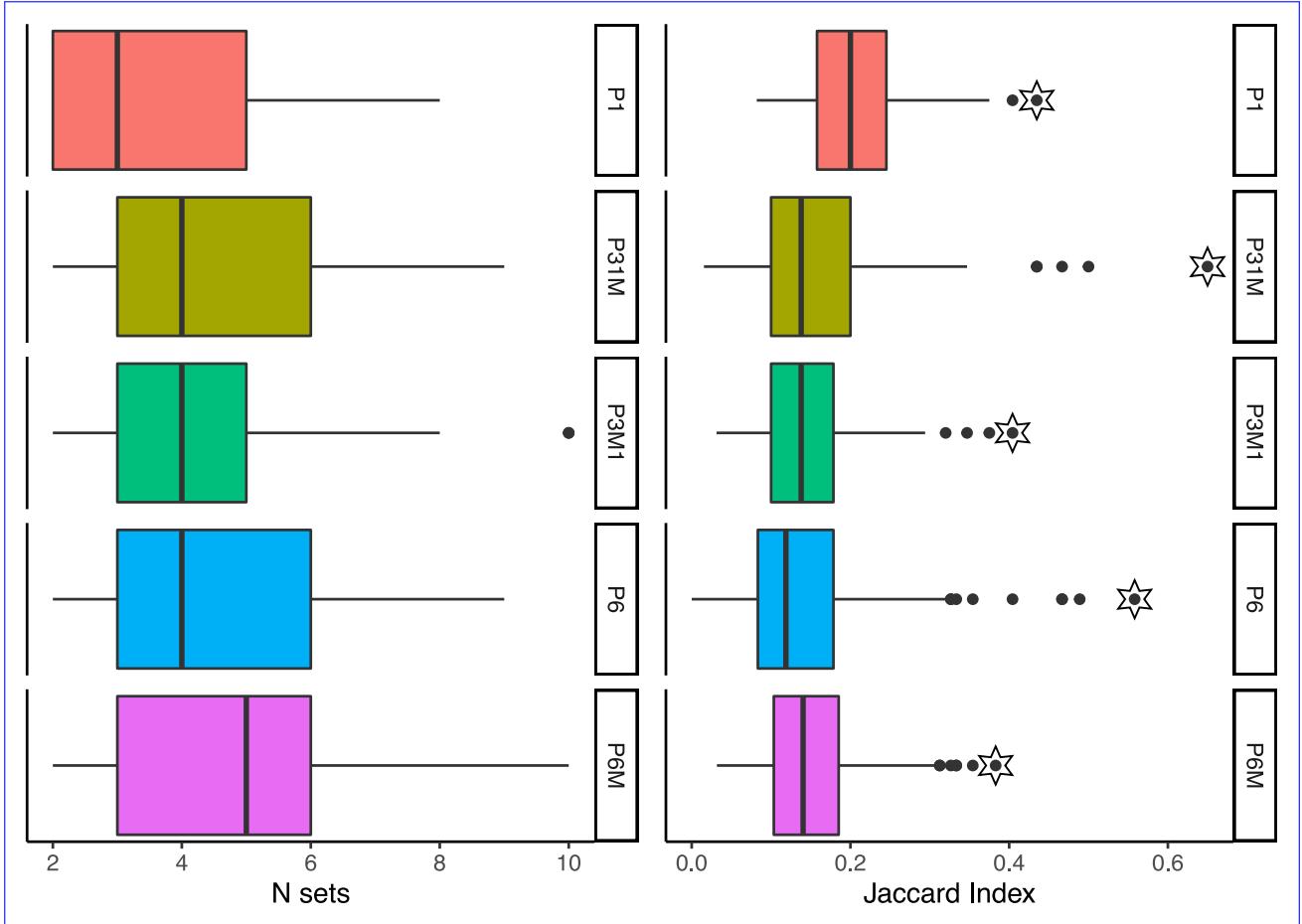


Figure 4: Left panel: Boxplots showing the number of subsets generated by participants for each of the wallpaper groups. Right panel: Boxplots showing Jaccard indices for every pairwise combination of exemplars in each of the wallpaper groups. Note that each data point here is the Jaccard index for a particular exemplar pair calculated across participants unlike the left panel where each data point is a participant. The exemplar pairs with the highest Jaccard indices have been highlighted with stars. Those outlier pairs are explored further in Figure 6. For both panels, the lower box boundary is the 25th percentile. The dark line in the box is the median. The upper box boundary is the 75th percentile. The “whiskers” show -/+ the interquartile range * 1.5.

101 exemplars within each group. P_1 had systematically higher Jaccard indices than the four other groups (see
 102 Figure ??4), as confirmed by ~~an ANOVA with wallpaper group as a factor. The analysis is a repeated measures~~
 103 ~~ANOVA which~~ revealed a statistically significant effect of group ($F(4, 495) = 20.178, p < 0.0001$). Post-hoc
 104 ~~pairwise~~-t-tests showed that P_1 had higher Jaccard indices than all other groups ($p < 0.0001$; see Table 1).
 105 The fact that the group (P_1) for which fewer subsets were generated also had higher Jaccard indices than
 106 the other groups illustrates the inherent link between the two measures. For wallpaper groups where the
 107 20 exemplars are sorted into fewer subsets, each individual exemplar pair is more likely to be a member
 108 of the same subset, and less likely to be a member of distinct subsets. This in turn leads to higher Jaccard
 109 indices. Our pairwise t-tests also showed that P_{31M} had lower Jaccard indices than P_6 ($p = 0.037$). This
 110 effect is relatively weak and does not pass our Bonferroni-corrected threshold for significance ($\alpha < 0.005$), but
 111 may nonetheless possibly reflect real differences in how consistently exemplars were grouped together
 112 across participants. We will explore this idea more in depth shortly, ~~but for now we can conclude that out~~
 113 ~~Out~~ of the five groups tested, P_1 is the only one that can be reliably differentiated based on our measures,
 114 being higher on self-similarity among the exemplars, and thus lower on diversity among exemplars.

pairs	number of sets			Jaccard Index		
	<i>t</i>	<i>p</i>	<i>D</i>	<i>t</i>	<i>p</i>	<i>D</i>
<i>P</i> ₁ vs <i>P</i> _{31M}	-2.981	0.0034	-0.734	5.641	<0.0001	0.579
<i>P</i> ₁ vs <i>P</i> _{3M1}	-3.423	0.0008	-0.843	7.233	<0.0001	0.742
<i>P</i> ₁ vs <i>P</i> ₆	-4.748	<0.0001	-1.169	7.734	<0.0001	0.794
<i>P</i> ₁ vs <i>P</i> _{6M}	-4.553	<0.0001	-1.132	6.946	<0.0001	0.713
<i>P</i> _{31M} vs <i>P</i> _{3M1}	-0.442	0.6595	-0.109	1.592	0.1117	0.163
<i>P</i> _{31M} vs <i>P</i> ₆	-1.767	0.0797	-0.435	2.094	0.0366	0.215
<i>P</i> _{31M} vs <i>P</i> _{6M}	-1.600	0.1120	-0.398	1.305	0.1921	0.134
<i>P</i> _{3M1} vs <i>P</i> ₆	-1.325	0.1875	-0.326	0.502	0.6160	0.051
<i>P</i> _{3M1} vs <i>P</i> _{6M}	-1.163	0.2470	0.289	-0.287	0.7745	-0.029
<i>P</i> ₆ vs <i>P</i> _{6M}	0.150	0.8814	0.037	-0.788	0.4307	-0.081

Table 1: ~~Boxplots showing Jaccard indices for every Results of post-hoc pairwise combination *t*-tests on number of exemplars in each of the wallpaper groups sets and Jaccard Indices.~~ Note that each data point here is Degrees-of-freedom was 945 for the Jaccard index for a particular exemplar pair calculated across participants. ~~Index test. For the number of sets test, unlike Figure ?? were each data point is a degrees-of-freedom had to be adjusted to account for the fact that one participant . The box boundary did not report number of sets for P6M (see Materials and Methods), and whiskers follow the same logic as in Figure ?? ranged between 127.0 and 127.1. The exemplar pairs with the highest Jaccard indices have been highlighted with stars. Those outlier pairs are explored further in Figure 6.~~

consistent pairings		
group	<i>p</i> < 0.01	<i>p</i> < 0.0001
<i>P</i> ₁	6	1
<i>P</i> _{31M}	17	10
<i>P</i> _{3M1}	12	3
<i>P</i> ₆	17	11
<i>P</i> _{6M}	15	4

Table 2: Number of consistent pairings at two different α -levels for the five groups.

In order to quantify the extent to which exemplars were consistently grouped together, we ran a permutation analysis in which exemplar labels were shuffled among the sets generated for each participant (see Materials and Methods). This provides, for each group, the expected distribution of Jaccard indices for every pairwise combination of exemplars, if exemplars were assigned randomly to subsets. And the analysis allows us to compute an empirical *z*-score that expressed the extent to which a given pair of exemplars deviates from random assignment.

Because the random distribution is generated by shuffling exemplars across the specific sets generated

by each participant for each group, this z -score is independent of the number of sets. If for a given group, none of the pairs deviate significantly from the random distribution, it would indicate that no exemplar pairs were consistently grouped together across participants. To estimate the extent to which this is the case, we look at the distribution of z -scores across the pairs for each group, as plotted in Figure 5, and count the number of pairs for each group for which the p -value associated with the threshold exceeds a given α value. At a threshold of $\alpha = 0.01$, several pairs survive for all groups, and even at a much more conservative criterion of $\alpha = 0.0001$ most groups have more than one pairing that survives (see Table 2). It is worth noting that the latter threshold ($\alpha = 0.0001$) is lower than the α associated with a Bonferroni correction within group, given that there are 190 pairs per group:

$$\alpha = \frac{0.05}{190} = 0.00003$$

So we conclude that for several exemplar pairs, participants are consistent in how they tend to pair the exemplars. It is interesting to consider that this measure of consistency might provide another way of differentiating wallpaper groups in terms of perceptual self-similarity. While groups $P_{31}M$, P_3M_1 , P_6 and P_{6M} have comparable Jaccard scores (see Figure 22), they differ in the number of consistent pairings, with $P_{31}M$ and P_6 producing more consistent pairs than the other two (see Figure 5).

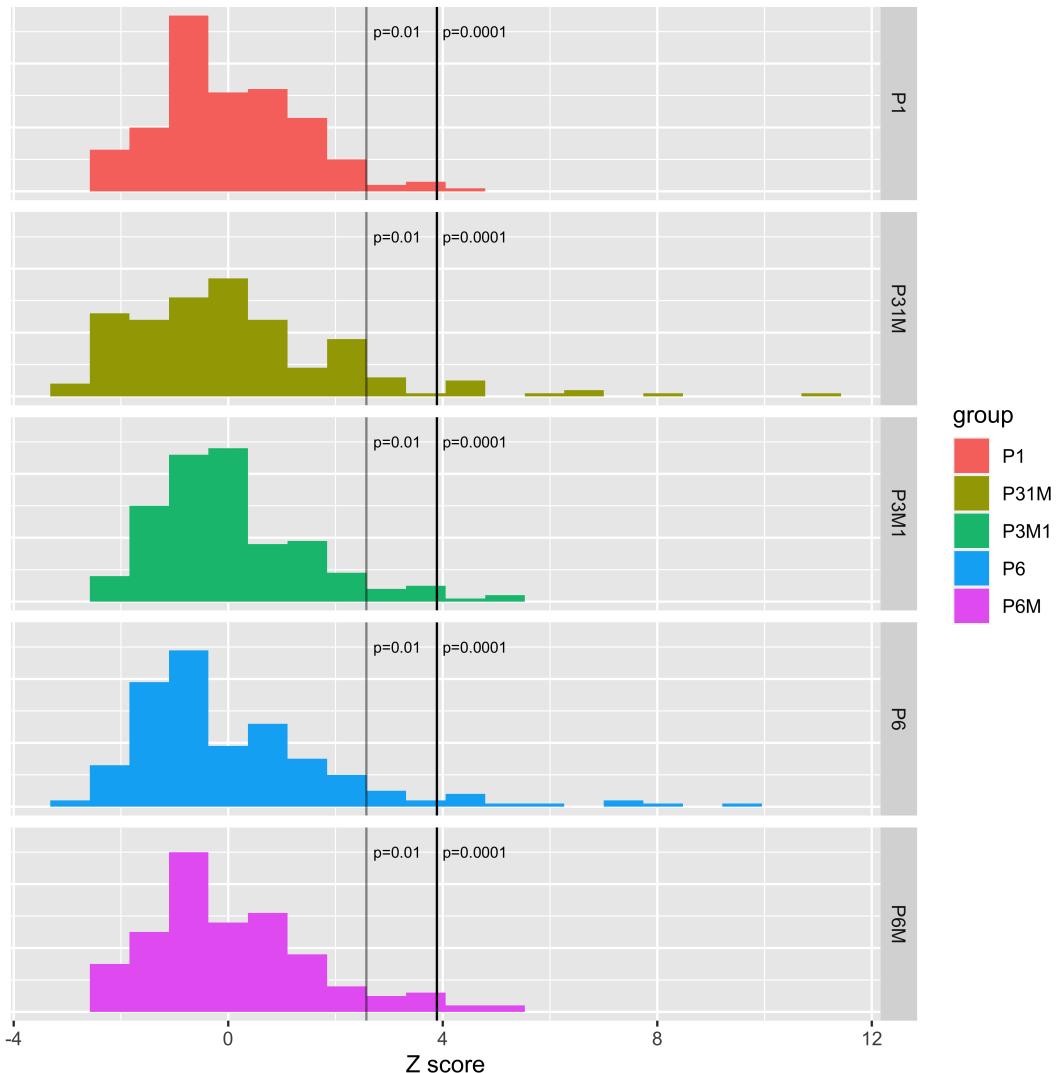


Figure 5: Distribution of z -scores across the 190 pairs in each of the five wallpaper groups. The two lines indicate the z -scores associated with α of 0.01 and 0.0001, respectively.

The Jaccard indices also allow us to focus on exemplar pairs that have a high level of similarity relative to the rest of the pairs in the set. We do this by identifying outliers pairs from each group in term of

138 Jaccard indices, as identified with stars in Figure 224. Because the Jaccard indices are computed across
139 participants, these outliers are also among the pairs most consistently sorted together, as identified in
140 Figure 5. For each exemplar in each outlier pair, we can visualize the pairwise similarity (as measured by
141 the Jaccard index) to every other exemplar in the set (see Figure 6). That is, we can visualize portions
142 of the network of perceived similarity within a set of exemplars. Future work could probe the extent to
143 which networks of perceived similarity have similar structure across wallpaper groups and examine what
144 perceptual features best account for participants' perceptions of exemplar similarity.

145 Discussion

146 Previous work has demonstrated that visual cortex of both humans and macaque monkeys carries highly
147 detailed representations of the symmetries within wallpaper groups, as evidenced by systematic differences
148 in the magnitude of the response elicited by different groups (Kohler et al., 2016; Kohler and Clarke,
149 2021; Audurier et al., 2021). This distinction between groups can also be observed in psychophysical
150 threshold measurements (Kohler and Clarke, 2021), although observers may not have a strong awareness
151 of the wallpaper group membership of individual exemplars (Clarke et al., 2011). In the current study, we
152 explored a new piece of the story about how wallpaper groups are processed by the visual system, namely the
153 issue of how self-similar different exemplars from the *same* wallpaper group appear to untrained observers.
154 We tested this by asking participants to spontaneously sort 20 exemplars from each of five wallpaper groups
155 into different subsets.

156 Our first finding concerns the number of subsets generated for each group. We find that P_1 is divided
157 into fewer subsets than the other four groups. This indicates that the limited complexity of this group,
158 which contains only translation symmetry, has a direct effect of the number of distinct subsets. The rela-
159 tionship between complexity / symmetry content and number of subsets produced is not straightforward,
160 however, as indicated by the fact that $P6M$ is not consistently grouped into more subsets than $P6$, $P3M_1$
161 and $P3_1M$, despite the fact that these other groups all contain fewer symmetries than $P6M$ —and generate
162 weaker brain activity (Kohler and Clarke, 2021). We speculate that this lack of further differentiation is a
163 result of an upper limit on how additional complexity can influence perceptual self-similarity. However,
164 future work with additional wallpaper groups, including groups that are relatively low on complexity by
165 high on self-similarity (e.g., P_2 and PMM , see (Clarke et al., 2011)), is needed to draw firm conclusions about
166 this hypothesis.

167 It is important to note that $P6$, $P3M_1$ and $P3_1M$ all consistently generate weaker brain activity than $P6M$,
168 and produce higher thresholds in a symmetry detection task (Kohler and Clarke, 2021). Our results would
169 therefore suggest that there is no clear relationship between the strength of the visual system's response to
170 symmetries in wallpaper group, and the perceptual self-similarity of each individual group. Future work
171 should explore this more closely, and look for neural correlates of similarity among exemplars from the
172 same group.

173 We also computed Jaccard indices that, for every possible exemplar pair, expresses the frequency of
174 those two exemplars being grouped together. As described above, the average Jaccard index for a group
175 is inherently linked to the number of subsets produced for that group, because fewer subsets mean that
176 exemplars are more likely to be made members of the same pair, and less likely to be made members of
177 the same pair. It is therefore not surprising that we find the same general pattern for Jaccard indices and
178 number of subsets, namely that P_1 has higher indices than the other groups. The advantage of the Jaccard
179 indices, however, is that they allow us to conduct a permutation analysis that quantifies the extent to which
180 pairs of exemplars are consistently grouped together across participants, independent of the number of
181 sets produced for a given group. It is important to note that consistency in the choice of which exemplars
182 to group into subsets is not an unavoidable consequence of our experimental design, and it does not follow

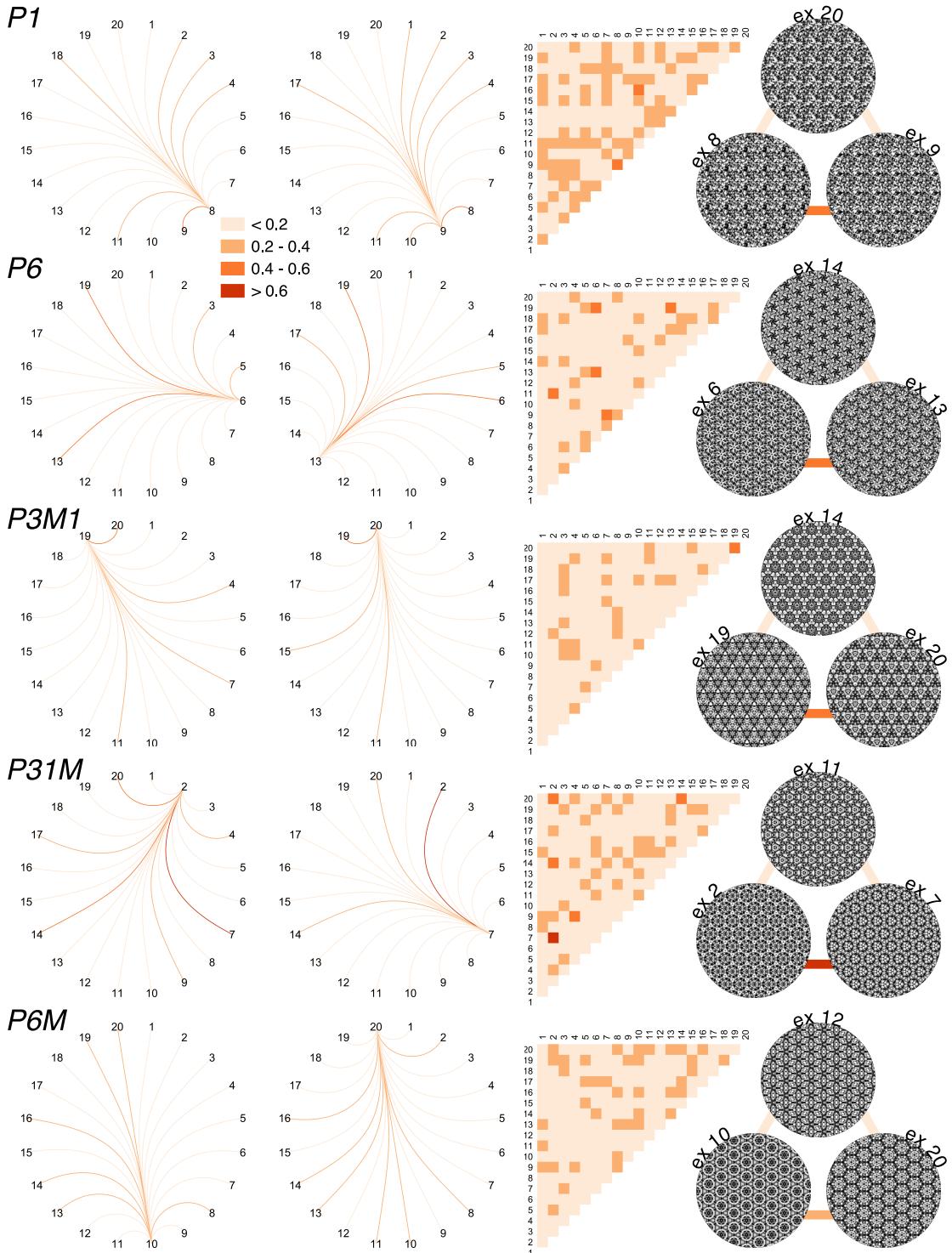


Figure 6: For each wallpaper group, we identified the two most self-similar exemplars, the same pair that is indicated by the right-most datapoint for each group in Figure 5. The two circular network plots are showing the pairwise similarities between those two exemplars and every other exemplar in the set. The pairwise similarities across all exemplars are plotted as a similarity matrix and on the rightmost side of the plot, the two most self-similar exemplars (bottom) are plotted with the exemplar that was least similar to both (top). The connecting lines between the exemplars indicate the similarity.

naturally from the results described so far. It would be perfectly possible for participants to group the sets together, producing fewer subsets for P_1 as observed, but exhibit no consistency across participants at all. That is not what we see, however. Even when setting a conservative threshold, all five groups produce one or more pairs that are consistently grouped together, demonstrating that the sorting of exemplars into

187 subsets is not done randomly or arbitrarily across the participants. Rather, different individuals agree to
188 some extent on which exemplars belong together. Because our measure of consistency is independent of
189 the number of subsets produced for a given group, it allows us to show that although P_1 has the highest
190 overall Jaccard indices (as a result of the fewer sets produced for this group), it in fact produces fewer
191 consistent pairs than other groups (see Table 2). Indeed, participants made few comments about their own
192 sorting strategies, but most observed that P_1 exemplars were the most difficult to sort because of the lack
193 of readily apparent features that were consistent across exemplars.

194 In sum, we find consistencies in the way that untrained human observers sort wallpaper images. Ob-
195 servers sort exemplars with translational symmetry alone (P_1) into smaller numbers of sets than exemplars
196 with rotational rotation or reflection symmetry. On average, pairs of P_1 exemplars are sorted together more
197 often than exemplars from other wallpaper groups. At the same time, some specific exemplar pairs from
198 wallpaper groups with 3- or 6-fold rotational or reflection symmetry are sorted together substantially more
199 often than predicted by chance.

200 We note that that the spontaneous sorting task our observers engaged in has less intrinsic structure
201 than some other tasks used to study similar questions like oddball detection (Landwehr, 2009; Hebart et al.,
202 2020; Landwehr, 2011), and thus may involve somewhat different perceptual and cognitive processes. In
203 particular, wallpaper group exemplars have a reduced dimensionality relative to natural objects. Even so,
204 large scale evaluations of how human observers perceive similarity in natural objects yield dimensions that
205 appear to relate to the strict regularities observed in wallpapers: round shape, patterning, and repetition
206 (Hebart et al., 2020). In future work, it would be interesting to explore whether different behavioral tasks
207 yield comparable similarity spaces, or more generally, how task demands shape similarity judgments.

208 In conclusion, our results suggest that human observers show sensitivity to the dimensions of 2D
209 symmetry (translation, rotation, and reflection) embedded in wallpaper exemplars. However, their sorting
210 behavior shows only weak evidence that group-theoretic measures of symmetry influence the perception
211 of self-similarity. These results contribute to a small, but growing literature on the perception of visual
212 aesthetics (Carneiro et al., 2012; Graham et al., 2010; Friedenberg, 2012; Laine-Hernandez and Westman,
213 2008; Richards, 1972) where symmetry is one of many contributing factors.

214 Materials and Methods

215 Participants

216 33 participants (9 Male, 24 Female), ranging in age between 18 and 35 completed this study. Participants
217 were recruited from the undergraduate participant pool in the Department of Psychology at the The
218 Pennsylvania State University. All participants had self-reported 20/20 or corrected to 20/20 vision.
219 We obtained written consent to participate from all participants under procedures approved by the In-
220 stitutional Review Board of The Pennsylvania State University (#38536). The research was conducted
221 according to the principles expressed in the Declaration of Helsinki. Participants include $n=11$ collected
222 and described in (Vedak, 2014), plus an additional group collected at a later date using the same protocol.

223 Stimulus Generation

224 Five wallpaper groups (P_1 , P_6 , $P_{31}M_1$, $P_{31}M$, P_6 and P_6M) that were selected for use in the study. The
225 selection was motivated partially by the fact that all five groups has previously been shown to be high in
226 self-similarity (Clarke et al., 2011), were selected—and partially by the fact that $P_{31}M_1$, $P_{31}M$, P_6 and P_6M
227 all share the same lattice shape. We also found it interesting that while P_6 , $P_{31}M_1$ and $P_{31}M$ differ in their
228 symmetry content, all are subgroups of P_6M with index 2, which means that P_6M can be generated by
229 adding one additional transformation to P_6 , $P_{31}M_1$ or $P_{31}M$ (Kohler and Clarke, 2021). 20 exemplars from

each of these five wallpaper groups were generated using a modified version of the methodology developed by Clarke and colleagues (Clarke et al., 2011) that we have described in detail elsewhere (Kohler et al., 2016). Briefly, exemplar patterns for exemplars belonging to each group were generated from by starting with a random-noise textures, which were patch, which was then repeated and transformed to cover the plane, according to tile the image plane, in accordance with the symmetry axes and geometric lattice specific to each group. The use of noise textures patches as the starting point for stimulus generation allowed the creation of an almost infinite makes it possible to create an almost unlimited number of distinct exemplars of from each wallpaper group. To make individual exemplars as similar as possible we replaced the power spectrum of each exemplar with the median across exemplars within a group. These images were printed onto white cardstock and cut into squares, allowing participants to manipulate the orientation of the images during the sorting tasks. Five exemplars from each group are shown (in reduced size) in Figure 3.

Procedure

Participants were presented with the 20 exemplars of a single wallpaper group (i.e. P₁, P₃M₁, P₃M₁, P₆, P₆M) and instructed to sort them into subsets by placing them into piles. Participants were advised to sort the exemplars into as many piles as they deemed necessary based on whatever criteria they desired. There were no time constraints placed on this sorting task, and the participants were allowed to move exemplars between piles until they were satisfied with their classification. This method was then repeated for the remaining four wallpaper groups for each participant, with group presentation order randomized between participants. These tasks were carried out on a large table with sufficient space to randomly lay out all twenty exemplars of each set, illuminated by normal overhead room lighting. Upon completion of each sorting task, participants were asked to verbalize which features they used to sort the exemplars. After completion of all five sorting tasks, participants were asked which if they had a distinct method for sorting the images, and if any wallpaper group was particular easy or difficult to sort.

Generating the Jaccard Index

The data was prepared for analysis by creating one binary variable for each subset created by each participant within a sorting task. Then, each exemplar was assigned a value of one (1) if it was included in a subset, or a value zero (0) if it was not. Next, the similarity of each pair of exemplars within a sorting task was calculated using the Jaccard index, a measure of similarity and diversity for binary data. This index is calculated by the equation

$$J = \frac{x}{x + y + z}$$

with x representing the number of subsets that contained both exemplars, and y and z the number of subsets that contain only one exemplar of the pair (Capra, 2005), across participants. Thus, the Jaccard index is the ratio of the number of subsets containing both exemplars of a pair to the number of subsets containing at least one of the exemplars of a pair, thereby excluding subsets with joint absences.

Permutation Statistical Analysis

The permutation analysis We tested for differences between the five wallpaper groups tested in terms of number of sets produced and Jaccard indices, by running repeated measures analyses of variance (rmANOVA) with group as a fixed factor and participant as a random factor. We then tested the extent to which differences between specific pairs of wallpaper groups contributed to any rmANOVA effects found, by running post-hoc paired t-tests comparing every possible pairing of the wallpaper groups, for both number of sets and Jaccard Indices. Because there were 10 possible pairings of the groups, we applied Bonferroni-correction and adjusted our α -level so that each t-test was only considered significant

271 if $p < 0.005$. Number of sets data was missing for one participant for wallpaper group $P6M$, so the degrees
272 of freedom used in the paired t -tests was adjusted using the Kenward-Roger technique—the default in the
273 *emmeans* package in R.

274 We ran a permutation analysis in order to quantify the extent to which pairs of exemplars were
275 consistently grouped together, across participants. This involved generating a randomized dataset, as
276 follows. For each participant and wallpaper group, we randomized which specific exemplars were sorted
277 together. This retained the basic structure of each participants' sorting data—the number of subsets
278 created—but randomized the relationship between specific wallpaper exemplars that were sorted together
279 across the participants. We then created 1,000 such permuted datasets, and calculated the Jaccard index for
280 each exemplar pair within each group for each of the permuted datasets. This permitted the calculation
281 of an empirical Jaccard index based on the permuted data from which distributional statistics like z could
282 be calculated. The observed Jaccard indices for each exemplar pair were then compared to the empirically-
283 derived reference distribution to determine which exemplar pairs were sorted together more frequently
284 than chance would predict.

285 References

- Audurier, P., Héjja-Brichard, Y., De Castro, V., Kohler, P. J., Norcia, A. M., Durand, J.-B., and Cottreau, B. R. (2021). Symmetry Processing in the Macaque Visual Cortex. *Cerebral Cortex*, (bhab358):314–323.
- Capra, M. G. (2005). Factor Analysis of Card Sort Data: An Alternative to Hierarchical Cluster Analysis. *Proceedings of the Human Factors and Ergonomics Society Annual Meeting*, 49(5):691–695. Publisher: SAGE Publications Inc.
- Carneiro, G., da Silva, N. P., Del Bue, A., and Costeira, J. P. (2012). Artistic image classification: An analysis on the PRINTART database. In *Computer Vision – ECCV 2012*, pages 143–157. Springer Berlin Heidelberg.
- Clarke, A. D. F., Green, P. R., Halley, F., and Chantler, M. J. (2011). Similar symmetries: The role of wallpaper groups in perceptual texture similarity. *Symmetry*, 3(4):246–264.
- Fedorov, E. (1891). Symmetry in the plane. In *Zapiski Imperatorskogo S. Peterburgskogo Mineralogicheskogo Obshchestva [Proc. S. Peterb. Mineral. Soc.]*, volume 334, pages 345–390.
- Friedenberg, J. (2012). Aesthetic judgment of triangular shape: compactness and not the golden ratio determines perceived attractiveness. *i-Perception*, 3(3):163–175.
- Graham, D. J., Friedenberg, J. D., Rockmore, D. N., and Field, D. J. (2010). Mapping the similarity space of paintings: Image statistics and visual perception. *Visual cognition*, 18(4):559–573.
- Hebart, M. N., Zheng, C. Y., Pereira, F., and Baker, C. I. (2020). Revealing the multidimensional mental representations of natural objects underlying human similarity judgements. *Nature human behaviour*, 4(11):1173–1185.
- Kohler, P. J., Clarke, A., Yakovleva, A., Liu, Y., and Norcia, A. M. (2016). Representation of maximally regular textures in human visual cortex. *The Journal of Neuroscience*, 36(3):714–729.
- Kohler, P. J. and Clarke, A. D. F. (2021). The human visual system preserves the hierarchy of two-dimensional pattern regularity. *Proceedings of the Royal Society B: Biological Sciences*, 288(1955):20211142. Publisher: Royal Society.
- Kohler, P. J., Cottreau, B. R., and Norcia, A. M. (2018). Dynamics of perceptual decisions about symmetry in visual cortex. *NeuroImage*, 167(Supplement C):316–330.
- Laine-Hernandez, M. and Westman, S. (2008). Multifaceted image similarity criteria as revealed by sorting tasks. *Proceedings of the American Society for Information Science and Technology*, 45(1):1–14.
- Landwehr, K. (2009). Camouflaged symmetry. *Perception*, 38:1712–1720.
- Landwehr, K. (2011). Visual discrimination of the plane symmetry groups. *Symmetry*, 3(2):207–219.

- 342 Liu, Y., Hel-Or, H., Kaplan, C. S., and Van Gool,
343 L. (2010). Computational symmetry in computer
344 vision and computer graphics. *Foundations and*
345 *Trends® in Computer Graphics and Vision*, 5(1–2):³⁵⁹
346 195.
- 347 Mach, E. (1959). *The Analysis of Sensations* (1897).
348 English transl., Dover, New York.
- 349 Milton, F., Longmore, C. A., and Wills, A. J. (200³⁵²).
350 Processes of overall similarity sorting in free cla³⁵³
351 sification. *Journal of experimental psychology. Human*
352 *perception and performance*, 34(3):676–692.
- 353 Polya, G. (1924). Xii. Über die analogie des
354 kristallsymmetrie in der ebene. *Zeitschrift für*
355 *Kristallographie-Crystalline Materials*, 60(1):278–28²⁶⁷
- Pothos, E. M., Perlman, A., Bailey, T. M., Kurtz, K.,
Edwards, D. J., Hines, P., and McDonnell, J. V.
(2011). Measuring category intuitiveness in uncon-
strained categorization tasks. *Cognition*, 121(1):83–
100.
- 361 Richards, L. G. (1972). A multidimensional scal-
ing analysis of judged similarity of complex forms
from two task situations. *Perception & psychophysics*,
12(2):154–160.
- Vedak, S. (2014). The salience of lower-order features
in highly self-similar wallpaper groups. *Honors the-
sis, The Pennsylvania State University*.