

1 Perceptual Similarities Among Wallpaper Group 2 Exemplars

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7 Abstract

8 Symmetries are abundant within the visual environment, and many animals species are
9 sensitive to visual symmetries. Wallpaper groups constitute a class of 17 regular textures
10 that each contain a distinct combination of the four fundamental symmetries, translation,
11 reflection, rotation and glide reflection, and together represent the complete set of possible
12 symmetries in two-dimensional images. Wallpapers are visually compelling and elicit responses
13 in visual brain areas that precisely capture the symmetry content of each group in humans
14 and other primates. Here we ask to what extent exemplars from the same wallpaper group are
15 perceptually similar. We algorithmically produce a set of well-matched exemplars from 5 of the
16 17 wallpaper groups and instructed participants to freely sort the exemplars from each group
17 into as many subsets as they wished based on any criteria they saw appropriate. P_1 , the simplest
18 of the 17 groups, was consistently rated more self-similar than any other group, while the other
19 four groups, although varying in symmetry content, were comparable in self-similarity. Our
20 results suggest that except for the most extreme case (P_1), perceived self-similarity of wallpaper
21 groups is not directly tied to categories of symmetry based on group theory.

22 Introduction

23 Symmetry has been recognized as important for human visual perception since the late 19th cen-
24 tury (Mach, 1959). In the two spatial dimensions relevant for images, symmetries can be combined
25 in 17 distinct ways, *the wallpaper groups* (Fedorov, 1891; Polya, 1924; Liu et al., 2010). Wallpaper
26 groups are different from stimuli typically used to probe the role of symmetry in visual perception
27 in two ways: First, they contain combinations of the four fundamental symmetry types trans-
28 lation, reflection, rotation and glide reflection, rather than just reflection or mirror symmetry,
29 which have been the focus of most vision research. Second, the symmetries in wallpaper groups
30 are repeated to tile the plane, rather than positioned at a single image location as is usually the case.
31 These differences, and the fact that wallpaper groups together form the complete set of symme-
32 tries possible in the two-dimensional image plane, make wallpapers an interesting stimulus set for
33 studying perception of visual symmetries.

34 Brain imaging studies using functional MRI (Kohler
 35 et al., 2016) and EEG (Kohler et al., 2018; Kohler and
 36 Clarke, 2021) have shown that the human visual system
 37 carries detailed and precise representations of the sym-
 38 metries within the individual wallpaper groups. Func-
 39 tional MRI evidence from macaque monkeys reveal sim-
 40 ilar representations in analogous areas of the macaque
 41 visual system (Audurier et al., 2021).

42 These representations, complex as they are, do not
 43 appear to be readily available for driving conscious
 44 behaviour: Humans have limited intuitive sense of
 45 group membership for wallpaper group exemplars, as
 46 evidenced by behavioral experiments showing that al-
 47 though naïve observers can distinguish many of the wall-
 48 paper groups (Landwehr, 2009), they tend to sort exem-
 49 plars into fewer (4-12) sets than the number of wallpaper
 50 groups, often placing exemplars from different wallpaper
 51 groups in the same set (Clarke et al., 2011). Wallpaper
 52 groups are nonetheless visually compelling, and anec-
 53 dotally we have observed that exemplars from a given
 54 group can be quite perceptually diverse. This observa-
 55 tion inspired the current study. Here, we use behavioral
 56 sorting, a common technique to study perceptual catego-
 57 rization (Milton et al., 2008; Pothos et al., 2011), to probe
 58 the perceptual self-similarity of different exemplars from
 59 the same wallpaper group, and assess the extent to which
 60 self-similarity varies across five groups.

61 We algorithmically generated 20 well-matched exem-
 62 plars from each group (see Figures 1 and 2 for a selection
 63 of the exemplars, and the **Materials and Methods** sec-
 64 tion for details on how they were generated) and printed
 65 them out on white cardstock. We then gave participants
 66 the 20 cards with exemplars from each wallpaper group,
 67 and asked them to freely sort them into as many sub-
 68 sets as they wished based on any criteria they saw ap-
 69 propriate. This approach allowed us to compare the five
 70 wallpaper groups, both in terms of how many subsets
 71 participants generated, and also in terms of the *Jaccard*
 72 index, a summary statistic capturing the similarity across
 73 exemplar pairs for each group. Within each group, we
 74 were also able to identify exemplar pairs that were rated

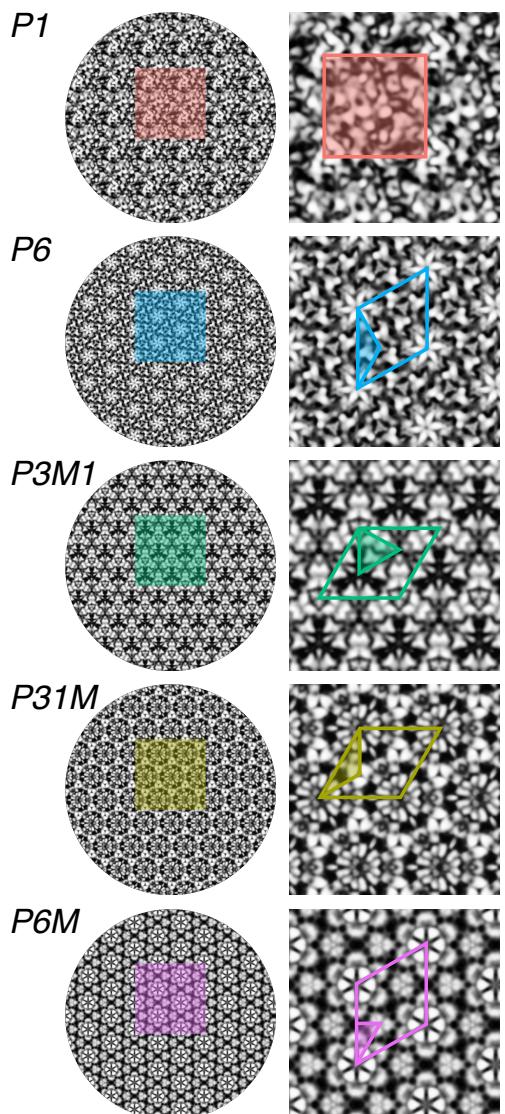


Figure 1: The fundamental region and lat-
 tice structure of the five wallpaper groups
 used in the study. The complete wallpa-
 per is shown in the left-hand column with
 a shaded region that is repeated and en-
 larged in the right-hand column. The col-
 ored outline in the enlarged region indi-
 cates the repeating lattice for each group,
 while the shaded area indicates the funda-
 mental region (see text). For P_1 the funda-
 mental region covers the entire lattice.
 Note that even though P_6 and $P_{31}M$ have
 the same fundamental region and lattice
 shapes, they differ in terms of the symme-
 tries present within the lattice - most no-
 tably, $P_{31}M$ contains reflection symmetry
 while P_6 does not. The symmetry content
 of each group is detailed on the wallpaper
 group wikipedia page.

as highly similar and highly dissimilar. Our main conclusion is that P_1 was systematically more self-similar than the any other groups, while the other four other groups could not be distinguished on these measures. We also show that for all five groups, participants consistently group certain pairs of exemplars together, although the number of consistent pairs varies among groups. Our results open the door to further investigations into the factors that drive perceptual similarity among wallpaper group exemplars, and indeed among exemplars from different classes of structured patterns.

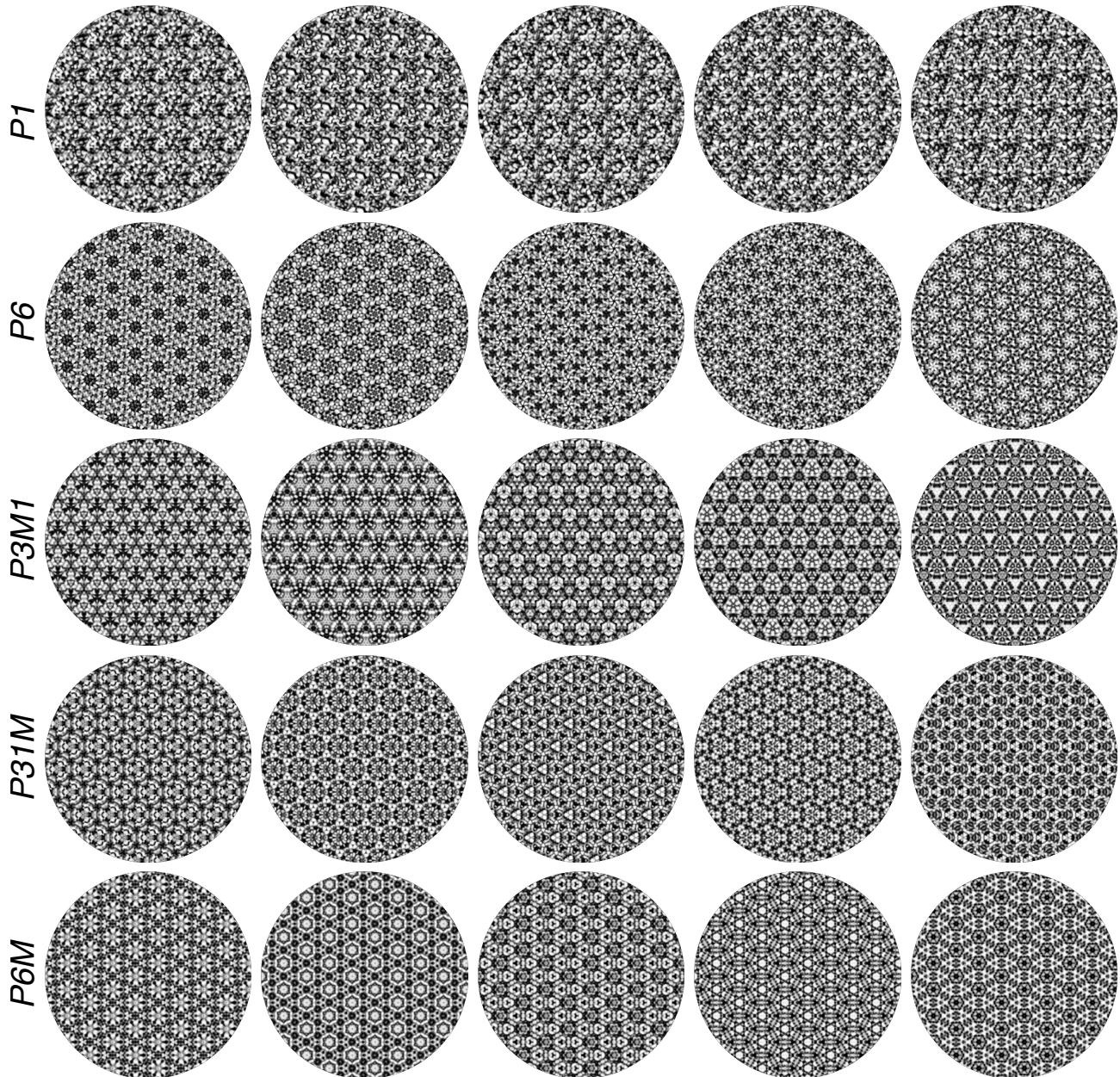


Figure 2: 5 of the 20 exemplars used for each group are shown to highlight the diversity among exemplars.

Results

Wallpaper group P_1 was more self-similar than the other four groups. This was evident in the number of sets generated for this group across participants, which was lower for P_1 (median = 3) than for the other groups (median = 4-5, see Figure 3). We confirmed this observation statistically by running a repeated measures analysis of variance (ANOVA) with group as a fixed

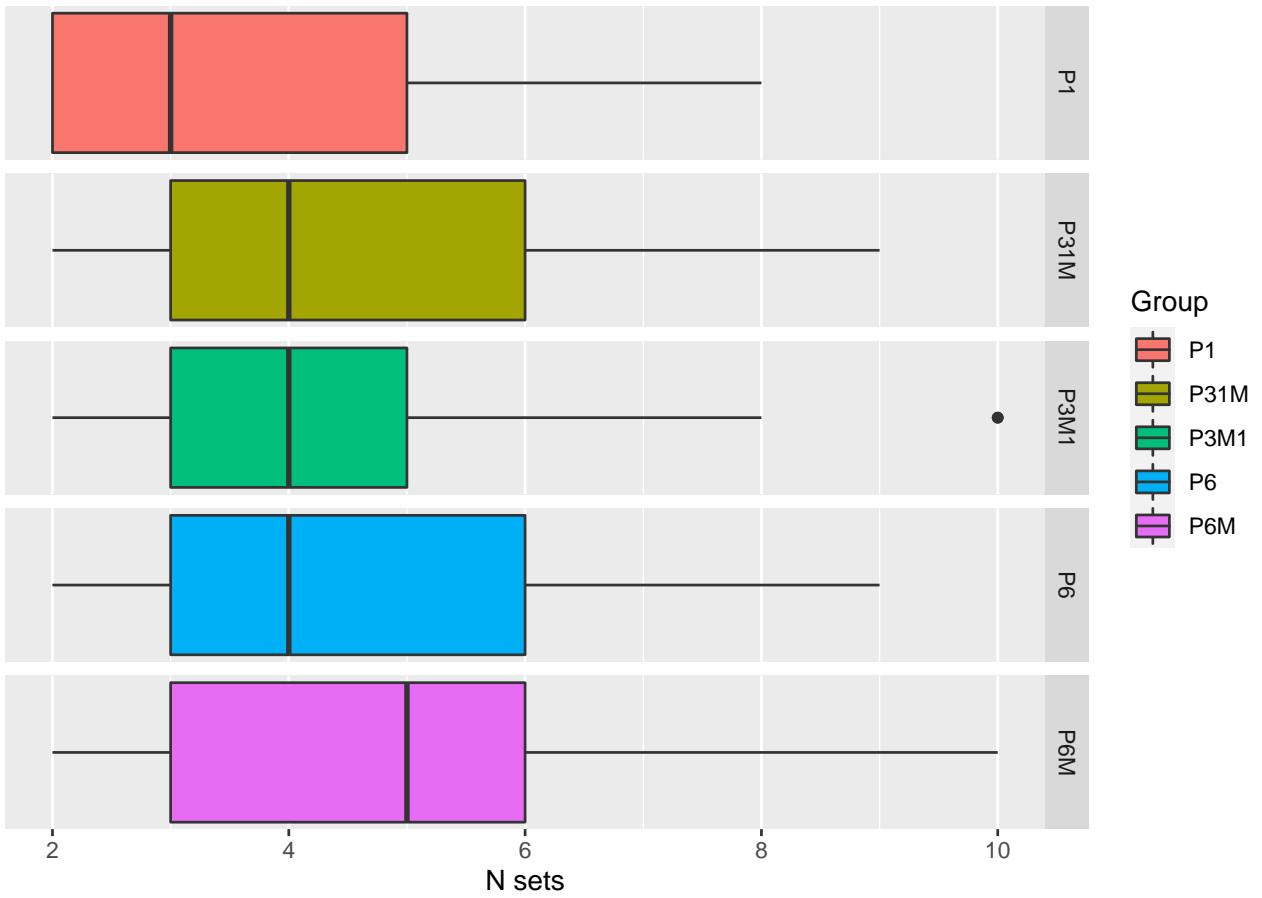


Figure 3: Boxplots showing the number of subsets generated by participants for each of the wallpaper groups. The lower box boundary is the 25th percentile. The dark line in the box is the median. The upper box boundary is the 75th percentile. The “whiskers” show -/+ the interquartile range * 1.5.

factor and participant as a random factor, which revealed a significant effect of group ($F(4,124) = 7.830, p < 0.0001$). Post-hoc pairwise *t*-tests showed that the mean number of sets was lower for *P₁* than all other groups, but no other means differed. Next, we computed the Jaccard index (see Materials and Methods) across participants for every pairwise combination of exemplars in each group. This provides a measure of the similarity between exemplars within each group. *P₁* had systematically higher Jaccard indices than the four other groups (see Figure 4), as confirmed by an ANOVA with wallpaper group as a factor. The analysis revealed a statistically significant effect of group ($F(4, 495) = 20.178, p < 0.0001$). Post-hoc pairwise *t*-tests showed that *P₁* had higher Jaccard indices than all other groups ($p < 0.0001$). The fact that the group (*P₁*) for which fewer subsets were generated also had higher Jaccard indices than the other groups illustrates the inherent link between the two measures. For wallpaper groups where the 20 exemplars are sorted into fewer subsets, each individual exemplar pair is more likely to be a member of the same subset, and less likely to be a member of distinct subsets. This in turn leads to higher Jaccard indices. Our pairwise *t*-tests also showed that *P_{31M}* had lower Jaccard indices than *P₆* ($p = 0.037$). This effect is relatively weak, but may reflect real differences in how consistently exemplars were grouped together across participants. We will explore this idea more in depth shortly, but for now we can conclude that out of the five groups tested, *P₁* is the only one that can be reliably differentiated based on our measures, being higher on self-similarity among the exemplars, and thus lower on

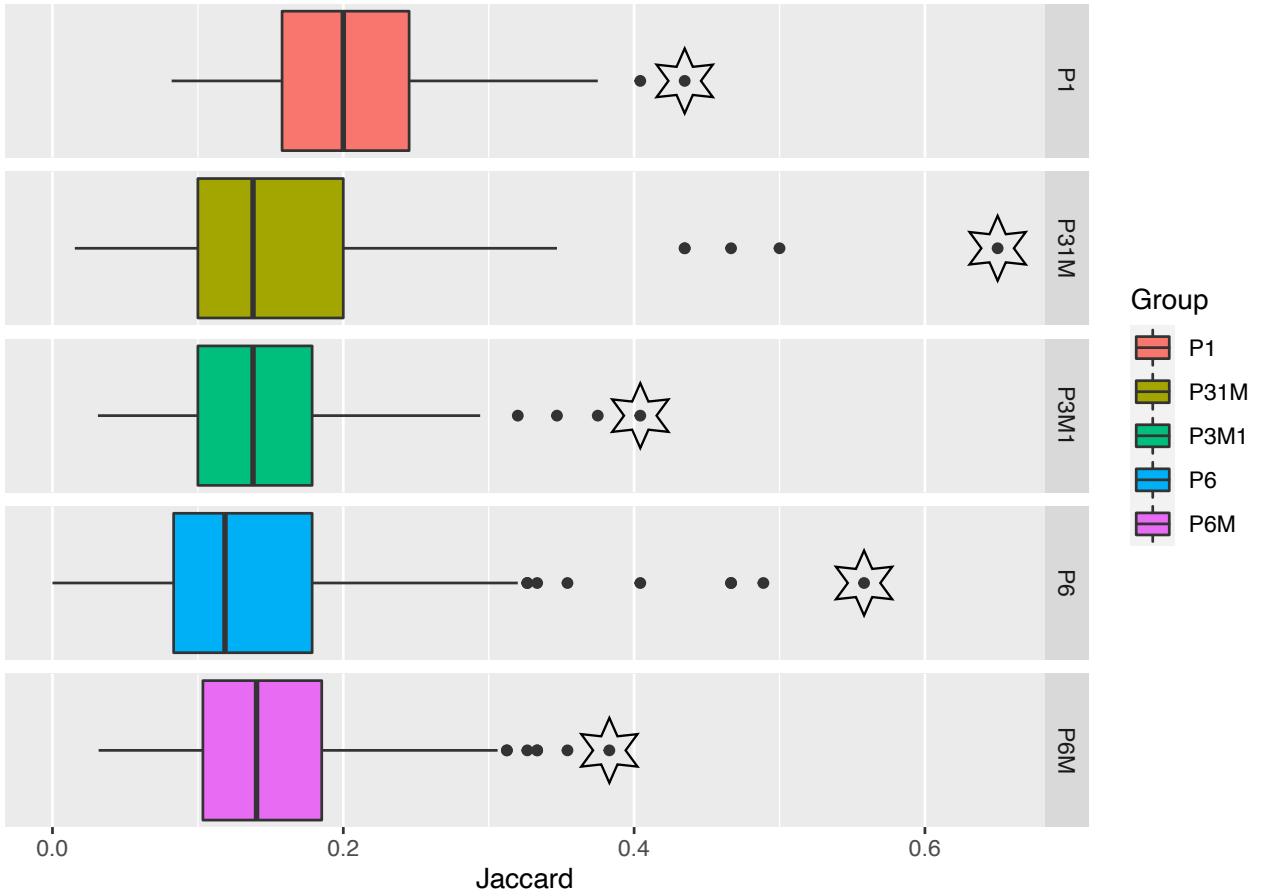


Figure 4: Boxplots showing Jaccard indices for every pairwise combination of exemplars in each of the wallpaper groups. Note that each data point here is the Jaccard index for a particular exemplar pair calculated across participants, unlike Figure 3 where each data point is a participant. The box boundary and whiskers follow the same logic as in Figure 3. The exemplar pairs with the highest Jaccard indices have been highlighted with stars. Those outlier pairs are explored further in Figure 6.

105 diversity among exemplars.

106 In order to quantify the extent to which exemplars were consistently grouped together, we
 107 ran a permutation analysis in which exemplar labels were shuffled among the sets generated
 108 for each participant (see Materials and Methods). This provides, for each group, the expected
 109 distribution of Jaccard indices for every pairwise combination of exemplars, if exemplars were as-
 110 signed randomly to subsets. And the analysis allows us to compute an empirical z -score that
 111 expressed the extent to which a given pair of exemplars deviates from random assignment.
 112 Because the random distribution is generated
 113 by shuffling exemplars across the specific sets
 114 generated by each participant for each group,
 115 this z -score is independent of the number of
 116 sets. If for a given group, none of the pairs de-
 117 viate significantly from the random distribu-
 118 tion, it would indicate that no exemplar pairs
 119 were consistently grouped together across par-
 120 ticipants. To estimate the extent to which this
 121 is the case, we look at the distribution of z -

consistent pairings		
group	$p < 0.01$	$p < 0.0001$
P_1	6	1
P_{31M}	17	10
P_{3M1}	12	3
P_6	17	11
P_{6M}	15	4

Table 1: Number of consistent pairings at two different α -levels for the five groups.

122 scores across the pairs for each group, as plotted
 123 in Figure 5, and count the number of pairs for each group for which the p -value associated
 124 with the threshold exceeds a given α value. At a threshold of $\alpha = 0.01$, several pairs survive for all
 125 groups, and even at a much more conservative criterion of $\alpha = 0.0001$ most groups have more than
 126 one pairing that survives (see Table 1). It is worth noting that the latter threshold ($\alpha = 0.0001$) is
 127 lower than the α associated with a Bonferroni correction within group, given that there are 190
 128 pairs per group:

$$\alpha = \frac{0.05}{190} = 0.0003$$

129 . So we conclude that for several exemplar pairs, participants are consistent in how they tend to
 130 pair the exemplars. It is interesting to consider that this measure of consistency might provide
 131 another way of differentiating wallpaper groups in terms of perceptual self-similarity. While
 132 groups P_{31M} , P_3M_1 , P_6 and P_6M have comparable Jaccard scores (see Figure 4), they differ in the
 133 number of consistent pairings, with P_{31M} and P_6 producing more consistent pairs than the other
 134 two (see Figure 5).

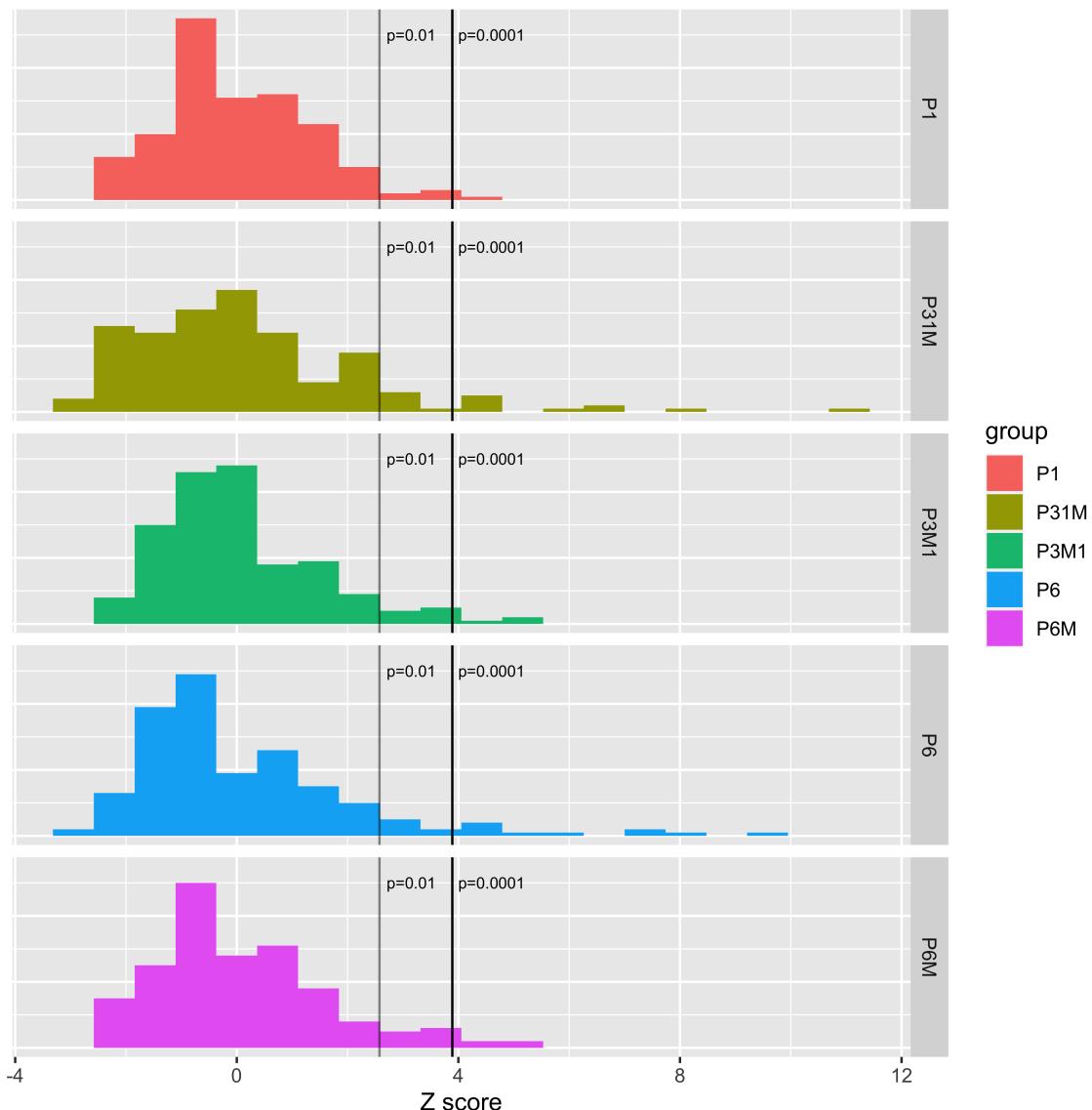


Figure 5: Distribution of z -scores across the 190 pairs in each of the five wallpaper groups. The two lines indicate the z -scores associated with α of 0.01 and 0.0001, respectively.

135 The Jaccard indices also allow us to focus on exemplar pairs that have a high level of similarity
136 relative to the rest of the pairs in the set. We do this by identifying outliers pairs from each
137 group in term of Jaccard indices, as identified with stars in Figure 4. Because the Jaccard indices
138 are computed across participants, these outliers are also among the pairs most consistently sorted
139 together, as identified in Figure 5. For each exemplar in each outlier pair, we can visualize the
140 pairwise similarity (as measured by the Jaccard index) to every other exemplar in the set (see
141 Figure 6). That is, we can visualize portions of the network of perceived similarity within a set of
142 exemplars. Future work could probe the extent to which networks of perceived similarity have
143 similar structure across wallpaper groups and examine what perceptual features best account for
144 participants' perceptions of exemplar similarity.

145 Discussion

146 Previous work has demonstrated that visual cortex of both humans and macaque monkeys car-
147 rries highly detailed representations of the symmetries within wallpaper groups, as evidenced
148 by systematic differences in the magnitude of the response elicited by different groups (Kohler
149 et al., 2016; Kohler and Clarke, 2021; Audurier et al., 2021). This distinction between groups can
150 also be observed in psychophysical threshold measurements (Kohler and Clarke, 2021), although
151 observers may not have a strong awareness of the wallpaper group membership of individual
152 exemplars (Clarke et al., 2011). In the current study, we explored a new piece of the story about
153 how wallpaper groups are processed by the visual system, namely the issue of how self-similar
154 different exemplars from the *same* wallpaper group appear to untrained observers. We tested this
155 by asking participants to spontaneously sort 20 exemplars from each of five wallpaper groups into
156 different subsets.

157 Our first finding concerns the number of subsets generated for each group. We find that P_1 is
158 divided into fewer subsets than the other four groups. This indicates that the limited complexity
159 of this group, which contains only translation symmetry, has a direct effect of the number of dis-
160 tinct subsets. The relationship between complexity / symmetry content and number of subsets
161 produced is not straightforward, however, as indicated by the fact that P_{6M} is not consistently
162 grouped into more subsets than P_6 , P_{2M_1} and P_{3_1M} , despite the fact that these other groups all
163 contain fewer symmetries than P_{6M} - and generate weaker brain activity (Kohler and Clarke,
164 2021). We speculate that this lack of further differentiation is a result of an upper limit on how
165 additional complexity can influence perceptual self-similarity.

166 We also computed Jaccard indices that, for every possible exemplar pair, expresses the fre-
167 quency of those two exemplars being grouped together. As described above, the average Jaccard
168 index for a group is inherently linked to the number of subsets produced for that group, be-
169 cause fewer subsets mean that exemplars are more likely to be made members of the same pair,
170 and less likely to be made members of the same pair. It is therefore not surprising that we find
171 the same general pattern for Jaccard indices and number of subsets, namely that P_1 has higher
172 indices than the other groups. The advantage of the Jaccard indices, however, is that they allow
173 us to conduct a permutation analysis that quantifies the extent to which pairs of exemplars are

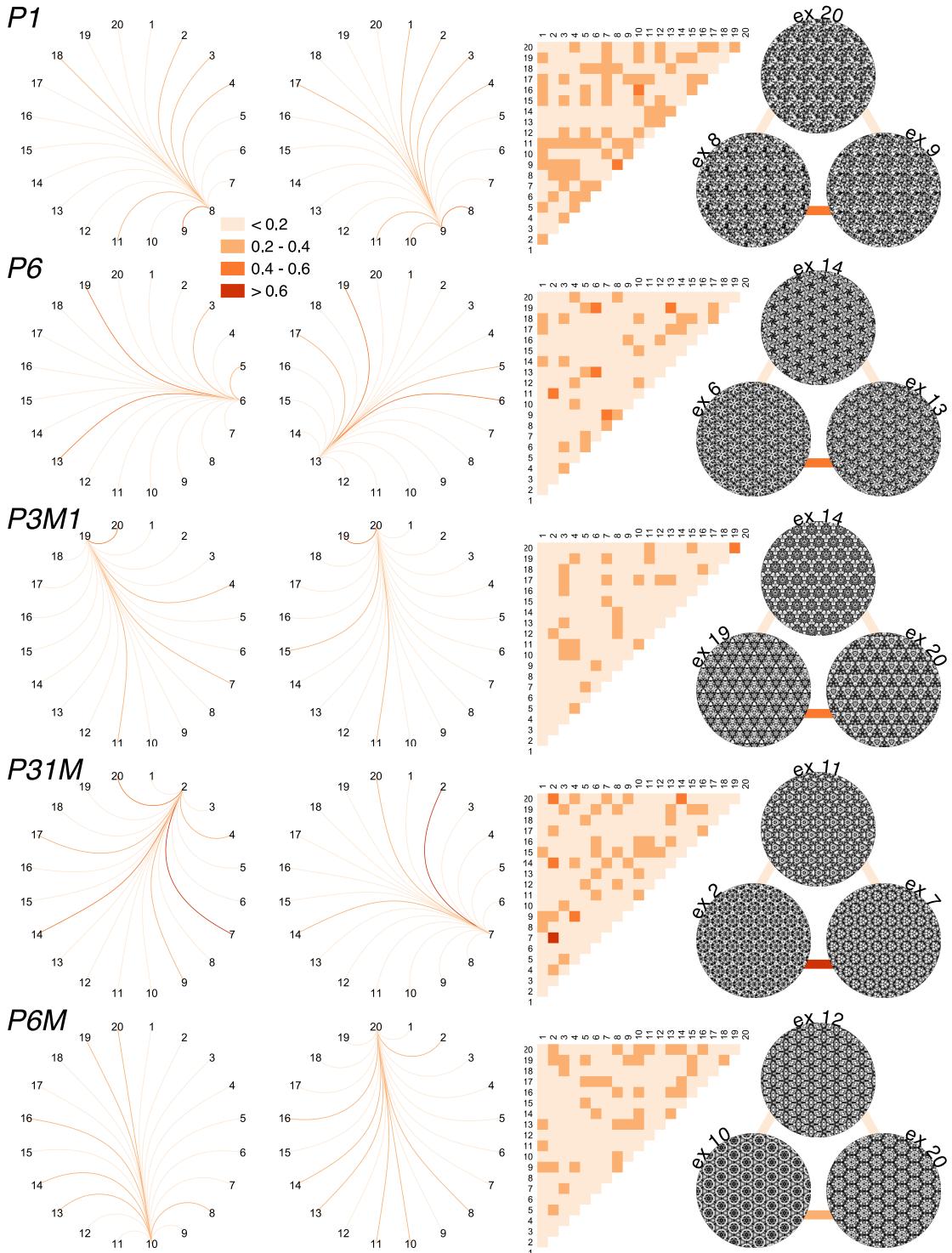


Figure 6: For each wallpaper group, we identified the two most self-similar exemplars, the same pair that is indicated by the right-most datapoint for each group in Figure 5. The two circular network plots are showing the pairwise similarities between those two exemplars and every other exemplar in the set. The pairwise similarities across all exemplars are plotted as a similarity matrix and on the rightmost side of the plot, the two most self-similar exemplars (bottom) are plotted with the exemplar that was least similar to both (top). The connecting lines between the exemplars indicate the similarity.

174 consistently grouped together across participants, independent of the number of sets produced
 175 for a given group. It is important to note that consistency in the choice of which exemplars to
 176 group into subsets is not an unavoidable consequence of our experimental design, and it does
 177 not follow naturally from the results described so far. It would be perfectly possible for partic-

178 ipants to group the sets together, producing fewer subsets for P_1 as observed, but exhibit no
179 consistency across participants at all. That is not what we see, however. Even when setting a
180 conservative threshold, all five groups produce one or more pairs that are consistently grouped
181 together, demonstrating that the sorting of exemplars into subsets is not done randomly or in
182 an arbitrary across the participants. Rather, different individuals agree to some extent on which
183 exemplars belong together. Because our measure of consistency is independent of the number of
184 subsets produced for a given group, it allows us to show that although P_1 has the highest overall
185 Jaccard indices (as a result of the fewer sets produced for this group), it in fact produces fewer
186 consistent pairs than other groups (see Table 1).

187 In sum, we find consistencies in the way that untrained human observers sort wallpaper
188 images. Observers sort exemplars with translational symmetry alone (P_1) into smaller numbers
189 of sets than exemplars with rotational or reflection symmetry. On average, pairs of P_1 exemplars
190 are sorted together more often than exemplars from other wallpaper groups. At the same time,
191 some specific exemplar pairs from wallpaper groups with 3- or 6- fold rotational or reflection
192 symmetry are sorted together substantially more often than predicted by chance.

193 We note that that the spontaneous sorting task our observers engaged in has less intrinsic
194 structure than some other tasks used to study similar questions like oddball detection (Landwehr,
195 2009; Hebart et al., 2020; Landwehr, 2011), and thus may involve somewhat different perceptual
196 and cognitive processes. In particular, wallpaper group exemplars have a reduced dimensionality
197 relative to natural objects. Even so, large scale evaluations of how human observers perceive sim-
198 ilarity in natural objects yield dimensions that appear to relate to the strict regularities observed
199 in wallpapers: round shape, patterning, and repetition (Hebart et al., 2020). In future work, it
200 would be interesting to explore whether different behavioral tasks yield comparable similarity
201 spaces, or more generally, how task demands shape similarity judgments.

202 In conclusion, our results suggest that human observers show sensitivity to the dimensions of
203 2D symmetry (translation, rotation, and reflection) embedded in wallpaper exemplars. However,
204 their sorting behavior shows only weak evidence that group-theoretic measures of symmetry
205 influence the perception of self-similarity. These results contribute to a small, but growing liter-
206 ature on the perception of visual aesthetics (Carneiro et al., 2012; Graham et al., 2010; Friedenberg,
207 2012; Laine-Hernandez and Westman, 2008; Richards, 1972) where symmetry is one of many con-
208 tributing factors.

209 Materials and Methods

210 Participants

211 33 participants (9 Male, 24 Female), ranging in age between 18 and 35 completed this study. All
212 participants had self- reported 20/20 or corrected to 20/20 vision. We obtained written consent to
213 participate from all participants under procedures approved by the Institutional Review Board
214 of The Pennsylvania State University (#38536). The research was conducted according to the
215 principles expressed in the Declaration of Helsinki. Participants include $n=11$ collected and
216 described in (Vedak, 2014), plus an additional group collected at a later date using the same

217 protocol.

218 Stimulus Generation

219 Five wallpaper groups (P_1 , P_3M_1 , P_31M , P_6 and P_6M) that has previously been shown to be high in
220 self-similarity (Clarke et al., 2011), were selected. 20 exemplars from each of these five wallpaper
221 groups were generated using a modified version of the methodology developed by Clarke and
222 colleagues (Clarke et al., 2011) that we have described in detail elsewhere (Kohler et al., 2016).
223 Briefly, exemplar patterns for each group were generated from random-noise textures, which
224 were then repeated and transformed to cover the plane, according to the symmetry axes and
225 geometric lattice specific to each group. The use of noise textures as the starting point for
226 stimulus generation allowed the creation of an almost infinite number of distinct exemplars of
227 each wallpaper group. To make individual exemplars as similar as possible we replaced the power
228 spectrum of each exemplar with the median across exemplars within a group. These images
229 were printed onto white cardstock and cut into squares, allowing participants to manipulate the
230 orientation of the images during the sorting tasks. Five exemplars from each group are shown
231 (in reduced size) in Figure 2.

232 Procedure

233 Participants were presented with the 20 exemplars of a single wallpaper group (i.e. P_1 , P_3M_1 ,
234 P_31M , P_6 , P_6M) and instructed to sort them into subsets by placing them into piles. Participants
235 were advised to sort the exemplars into as many piles as they deemed necessary based on
236 whatever criteria they desired. There were no time constraints placed on this sorting task, and
237 the participants were allowed to move exemplars between piles until they were satisfied with
238 their classification. This method was then repeated for the remaining four wallpaper groups for
239 each participant, with group presentation order randomized between participants. These tasks
240 were carried out on a large table with sufficient space to randomly lay out all twenty exemplars
241 of each set, illuminated by normal overhead room lighting. Upon completion of each sorting
242 task, participants were asked to verbalize which features they used to sort the exemplars. After
243 completion of all five sorting tasks, participants were asked which if they had a distinct method
244 for sorting the images, and if any wallpaper group was particular easy or difficult to sort.

245 Generating the Jaccard Index

246 The data was prepared for analysis by creating one binary variable for each subset created by
247 each participant within a sorting task. Then, each exemplar was assigned a value of one (1) if
248 it was included in a subset, or a value zero (0) if it was not. Next, the similarity of each pair of
249 exemplars within a sorting task was calculated using the Jaccard index, a measure of similarity
250 and diversity for binary data. This index is calculated by the equation

$$J = \frac{x}{x + y + z}$$

251 with x representing the number of subsets that contained both exemplars, and y and z the number
252 of subsets that contain only one exemplar of the pair (Capra, 2005), across participants. Thus,

253 the Jaccard index is the ratio of the number of subsets containing both exemplars of a pair to the
254 number of subsets containing at least one of the exemplars of a pair, thereby excluding subsets
255 with joint absences.

256 Permutation Analysis

257 The permutation analysis involved generating a randomized dataset, as follows. For each partic-
258 ipant and wallpaper group, we randomized which specific exemplars were sorted together. This
259 retained the basic structure of each participants' sorting data—the number of subsets created—
260 but randomized the relationship between specific wallpaper exemplars that were sorted together
261 across the participants. We then created 1,000 such permuted datasets, and calculated the Jac-
262 card index for each exemplar pair within each group for each of the permuted datasets. This
263 permitted the calculation of an *empirical* Jaccard index based on the permuted data from which
264 distributional statistics like z could be calculated. The *observed* Jaccard indices for each exemplar
265 pair were then compared to the empirically-derived reference distribution to determine which
266 exemplar pairs were sorted together more frequently than chance would predict.

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