ID3 algorithm for learning classification

Simon Dixon
Queen Mary University of London
based on material by Boris Mailhé and Alan Neymark

Outline

- The classification problem
  - geometrical interpretation
- Attribute selection
  - entropy, mutual information
- Some classifiers
  - Bayesian, SVM, decision trees
- The ID3 algorithm
- Avoiding overfitting

Classification

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- Goal
    - given a set of examples described by several attributes (or *features*) and a set of classes, label each example with a class

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- Applications:
  - biometry: identify people from their voice, face, gait, ...
  - category recognition: e.g. objects in images; speech; musical instrument, style, mood
  - data mining: predict user behaviour depending on their past history
  - medical diagnosis: knowing the symptoms and eventual test results, determine whether a patient has a particular disease

Classification: why is it hard?

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  - the classes represent high-level information (identity, behaviour,...) but only low-level attributes (pixel values, numbers of clicks on a web page,...) are observed
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    - so we need a lot of attributes
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- There are many examples
  - if not, it could be done by hand
  - learning algorithms may have high computational cost

Machine learning for classification

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- The learning step is costly but performed only once
- How can we ensure that the classifier will work on the problem data?
  - we cannot be certain
  - Occam's razor: the simplest solution is often the right one

Classification: geometrical view

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  - each attribute is a dimension
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- Applying a classifier is therefore just checking in which subspace a point lies

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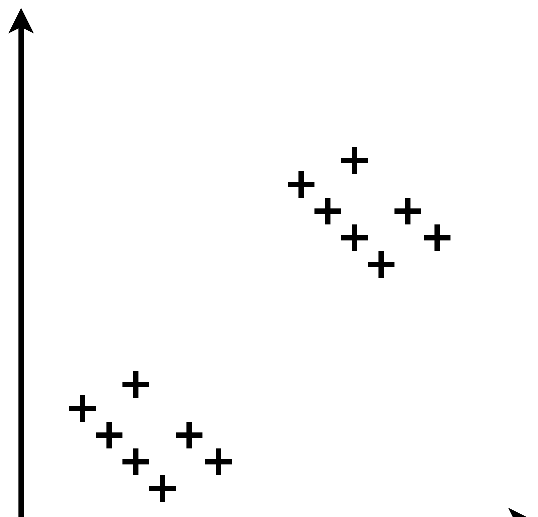

Problem

- Attributes are dimensions (axes)
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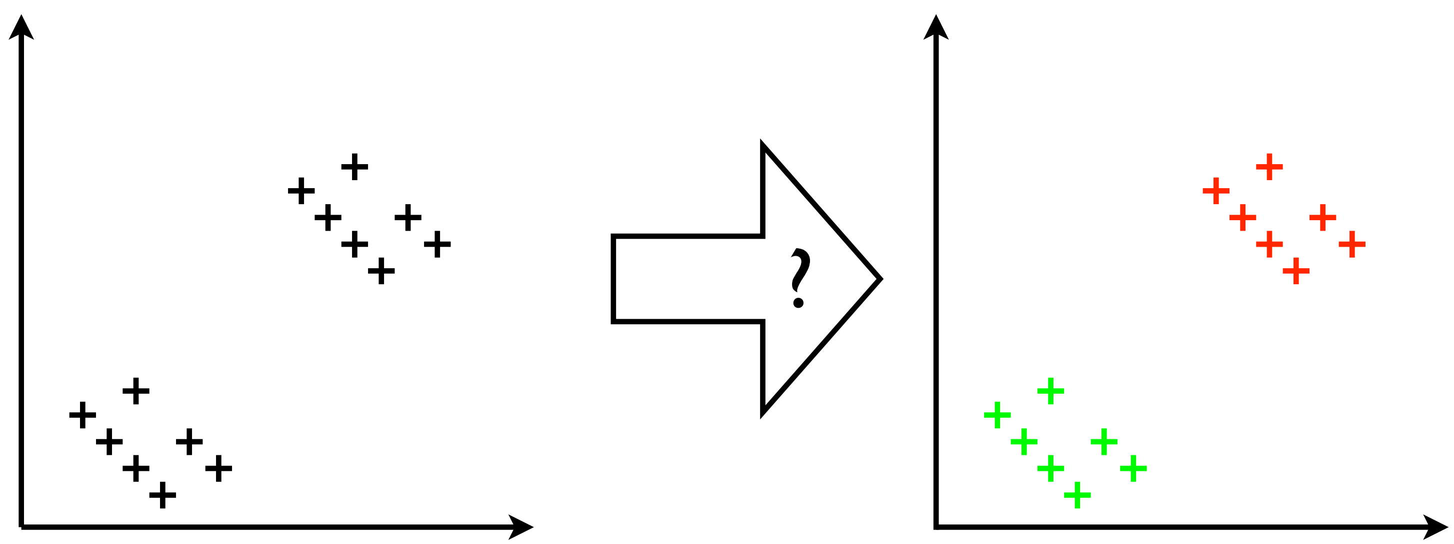

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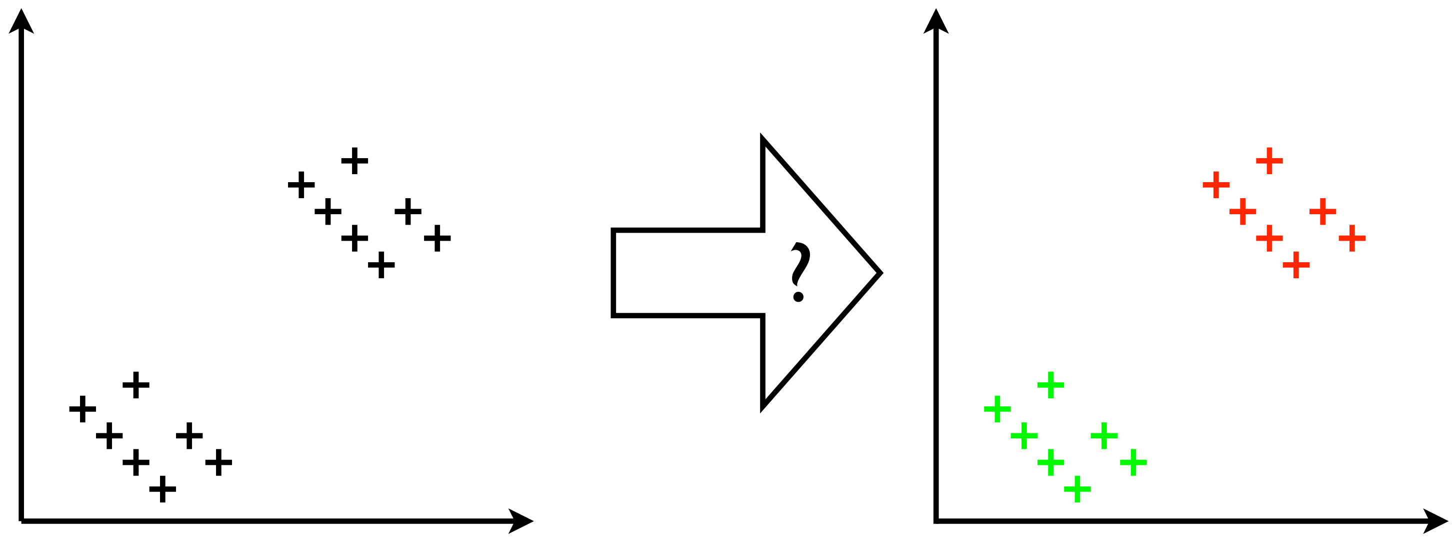

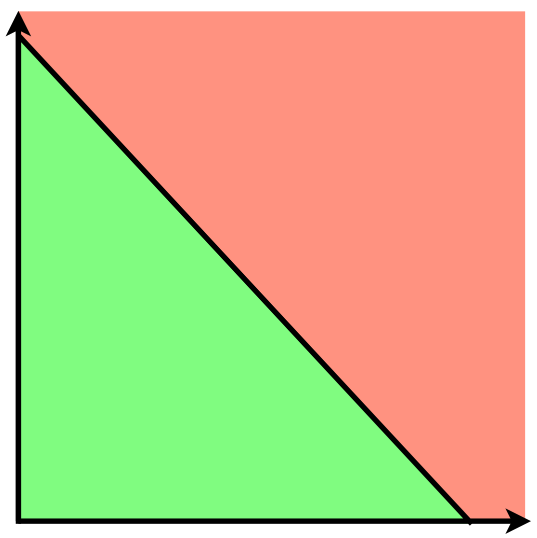

Problem

Classifier

- Attributes are dimensions (axes)
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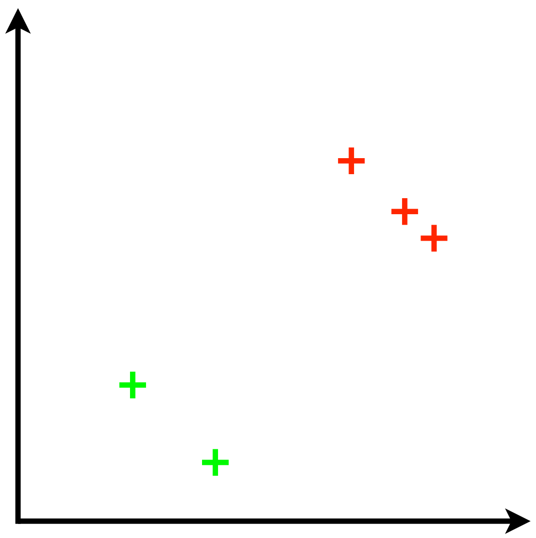
- Colour the whole space

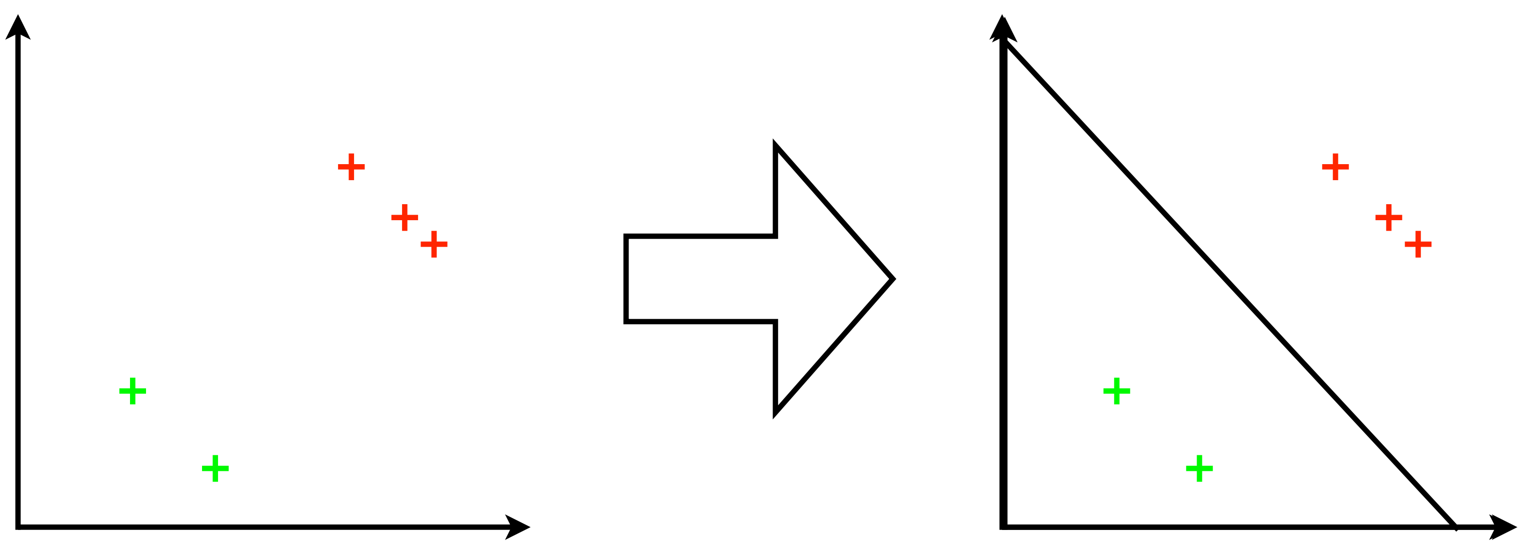
Learning a classifier

Learning a classifier



Select training examples

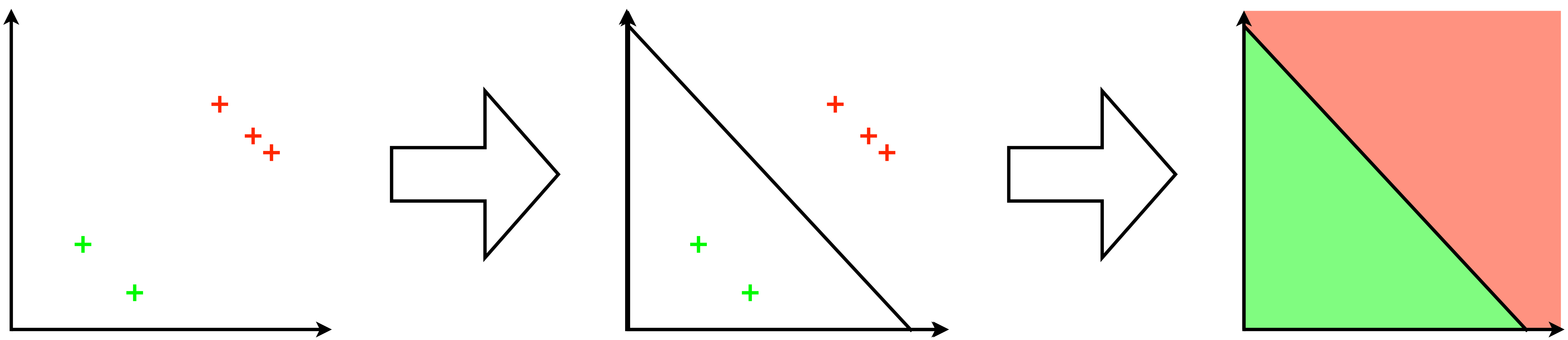
Learning a classifier



Select training examples

Draw the borders

Learning a classifier



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Applying the classifier

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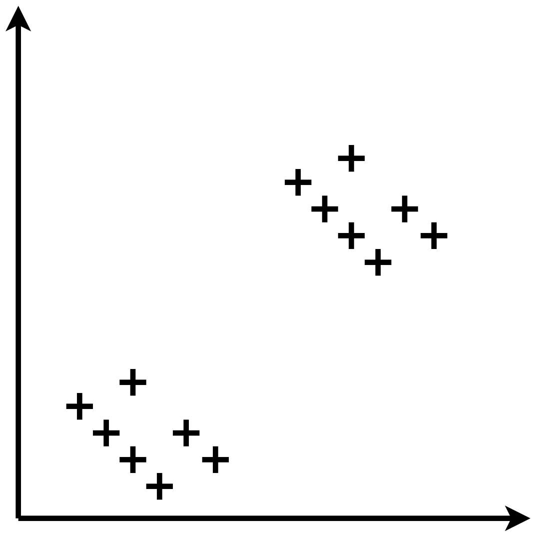

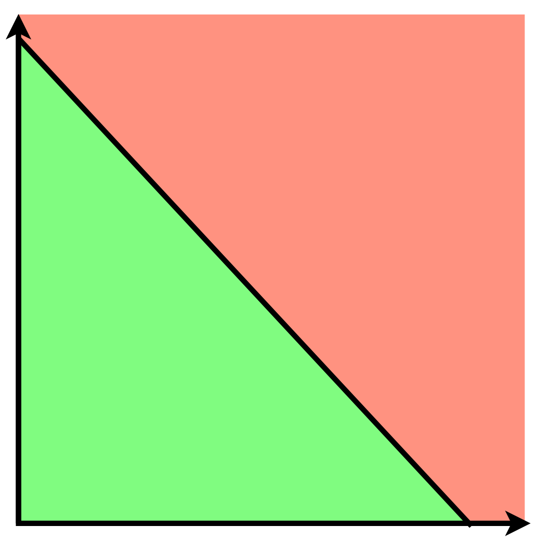

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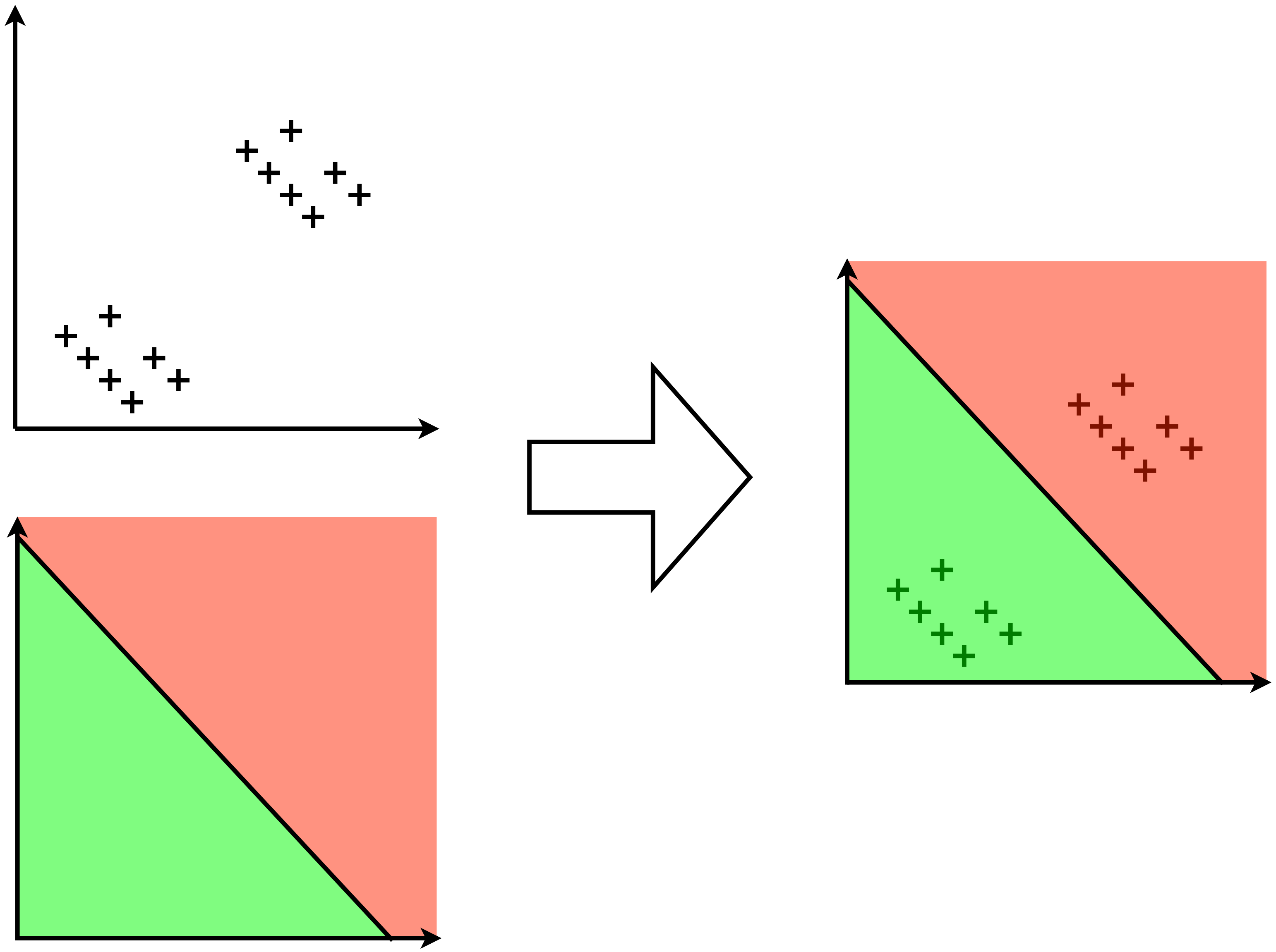

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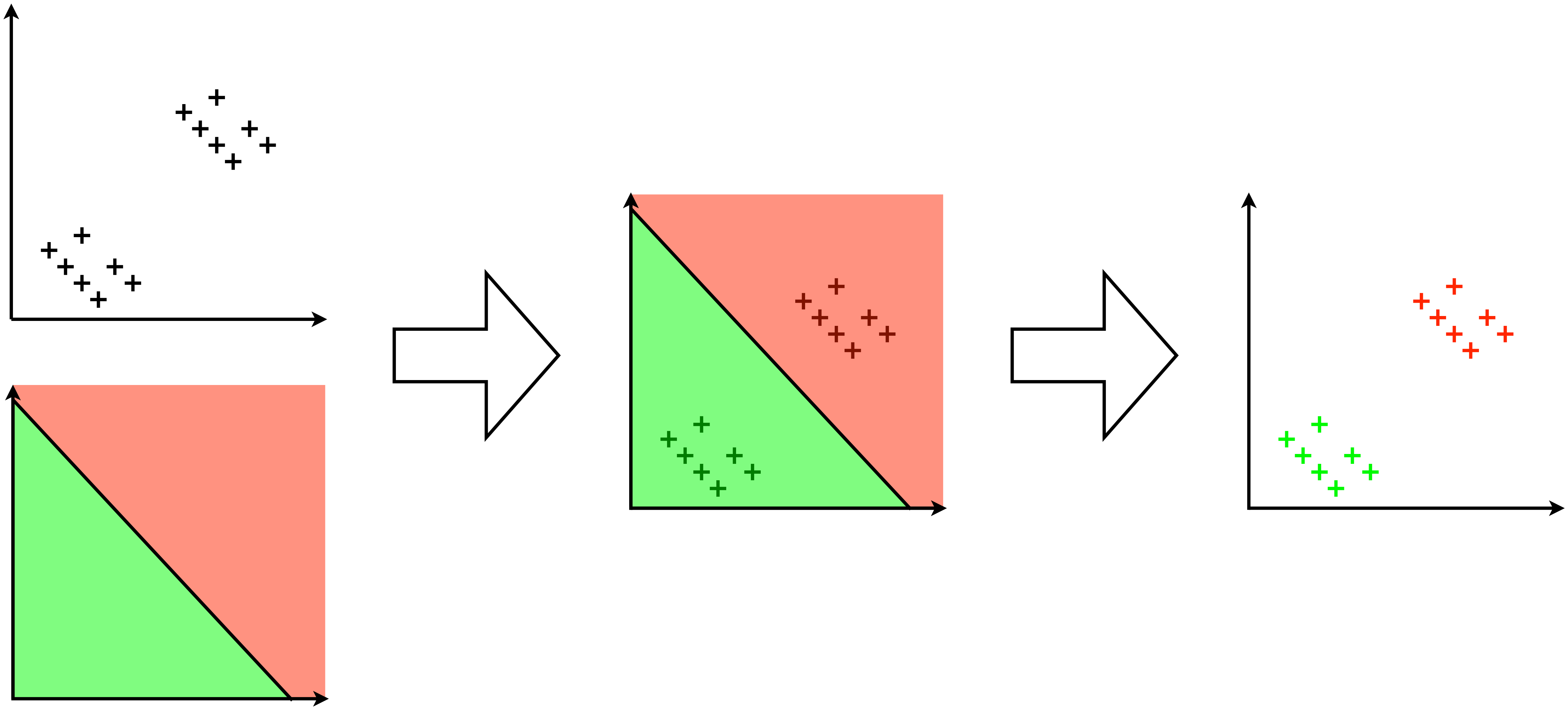

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Structure of the classifier

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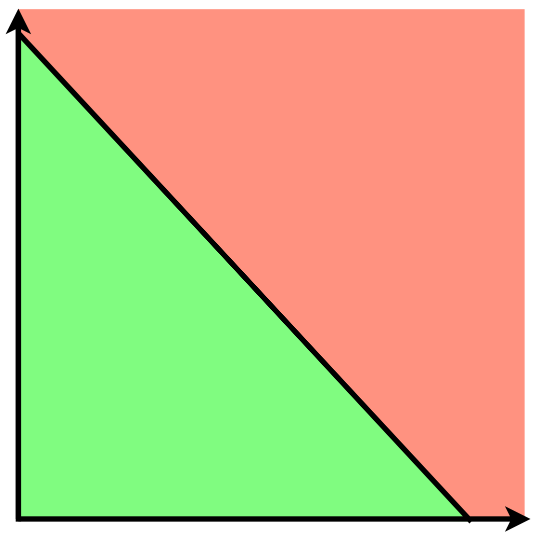

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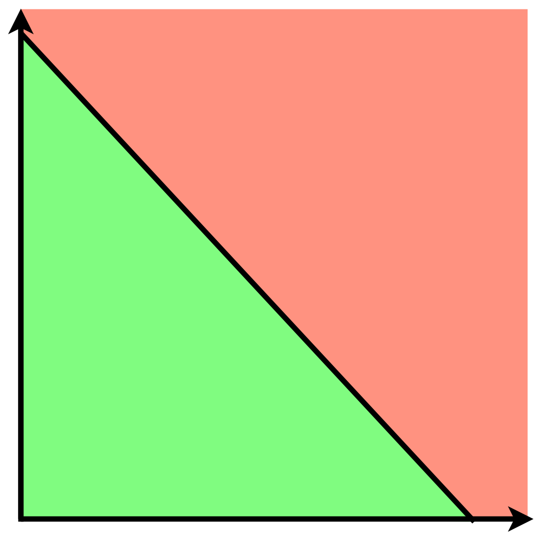

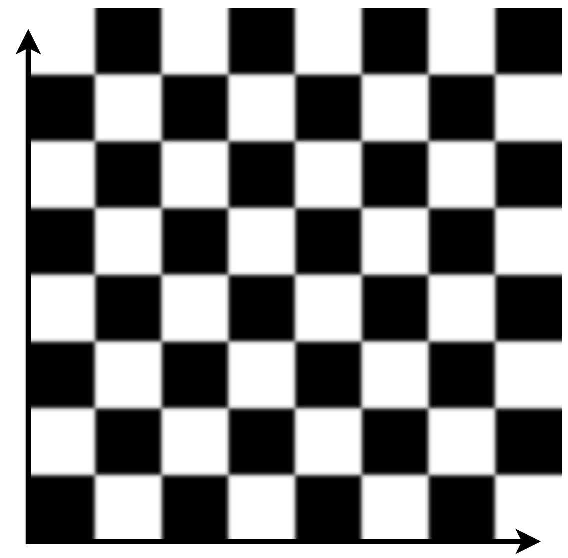

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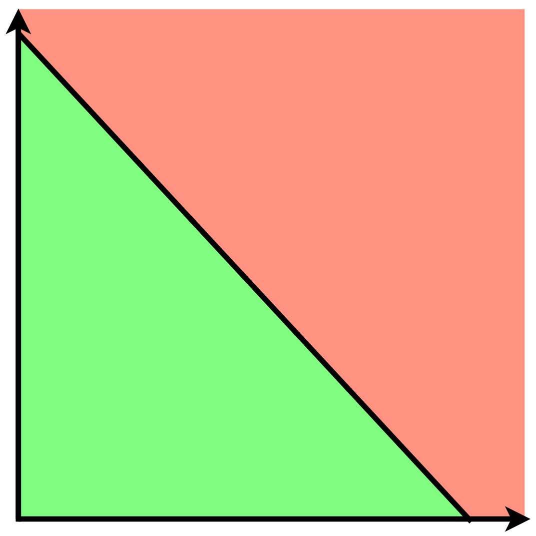

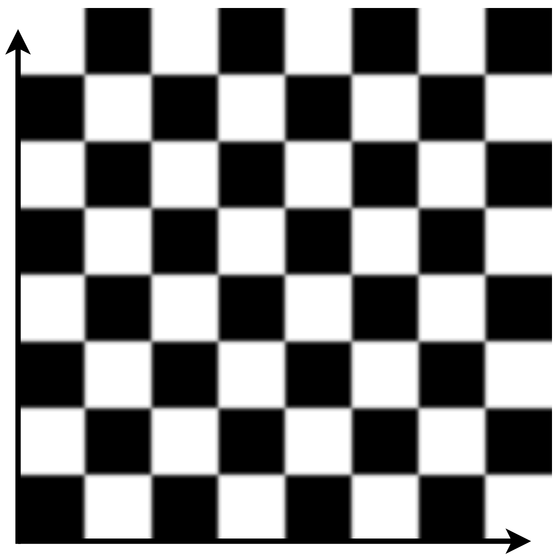

- The subspaces of the classifier do not have to be connected
- The border can be much more complex than a single line
- Generally, the types of borders allowed are set by the algorithm
    - need expert knowledge of the problem
    - there is no single algorithm that solves every problem

Attribute Types

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Attribute Types

- Symbolic attributes
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  - can be enumerated
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- Numeric attributes
  - real or integer valued: e.g. grades, temperature, pixels, audio samples, ...
  - can be measured
  - can use *algebra* to combine different attributes *before* testing them
    - can create new attributes

Attribute selection/engineering

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- Learning is easier if one starts with "good" attributes
  - fewer attributes decrease the cost of learning
  - a good initial guess helps finding the best classifier
- What is a "good" attribute?
  - easiest case (trivial): the classes themselves as an attribute
  - hardest case (insoluble): no attributes at all
  - an attribute that carries a lot of "information"
  - an attribute whose "information" is related to the class

Information theory (Shannon 1948)

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- The more uncertain the distribution, the more information in the content
  - if the coin is double-headed, then there is nothing to transmit

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$$H(X) = -\sum_{x \in X} p_x \log_2 p_x$$

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- Probabilities are multiplicative, information is additive
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- Entropy:

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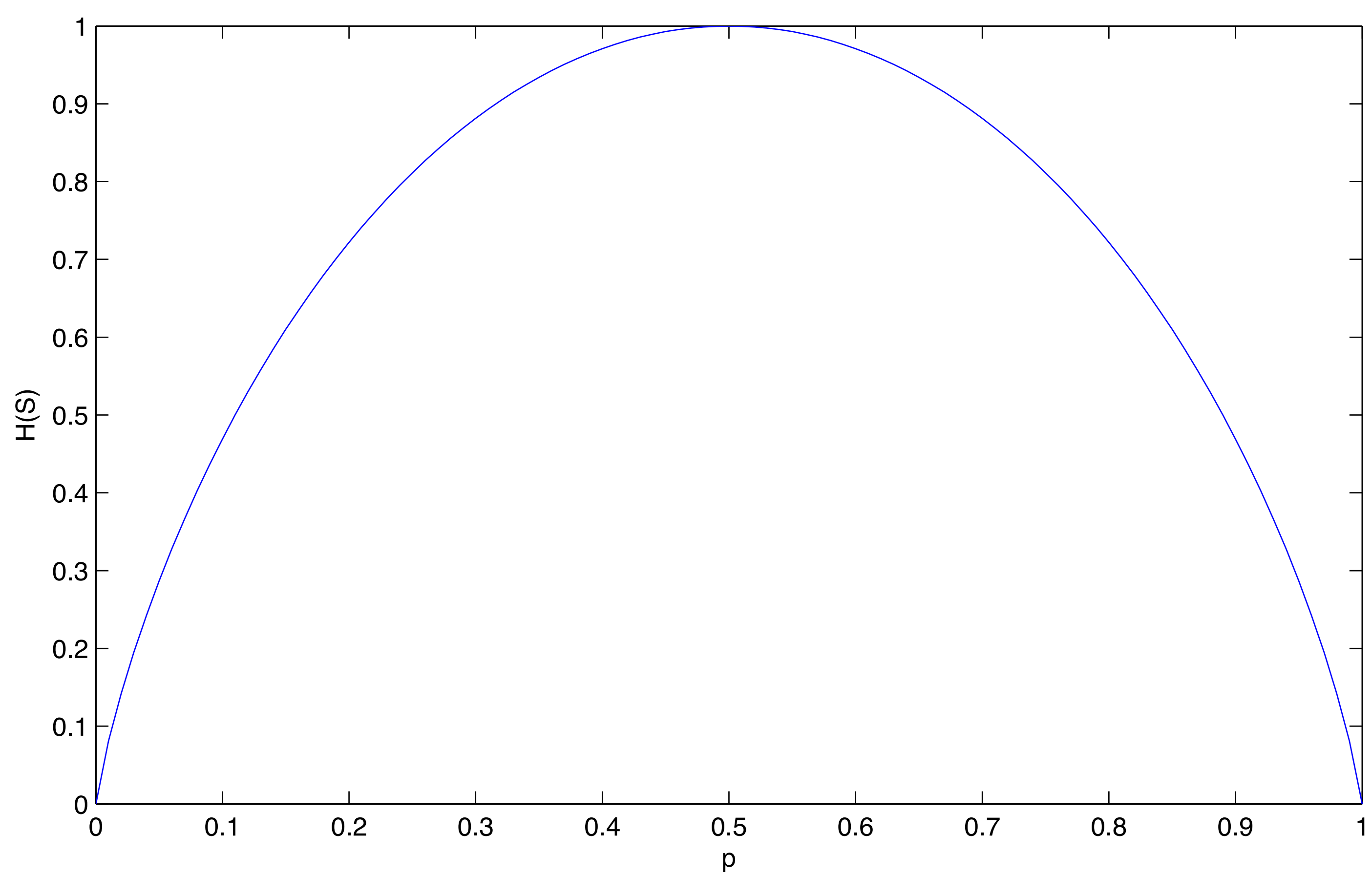

What does it mean?

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- How can one coin-toss hold less than 1 bit of information when there are still 2 possible outcomes?
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- How does it work?
  - If $H(X)$ is less than maximal, then there are rare events
  - Conjunctions of rare events become rarer and rarer as word length increases
  - At some point they become negligible and do not need to be encoded anymore

Mutual information

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  - 0 if the variables are independent
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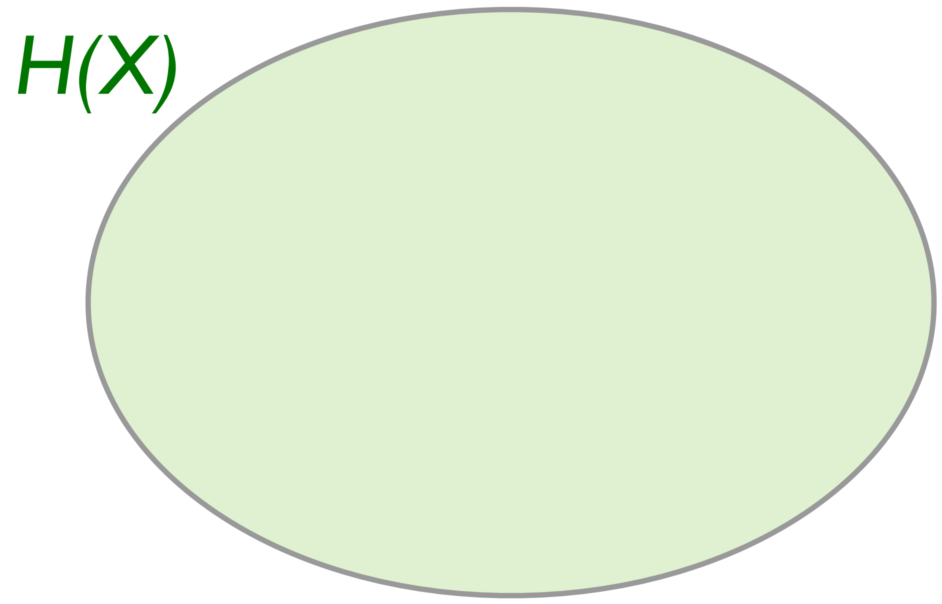

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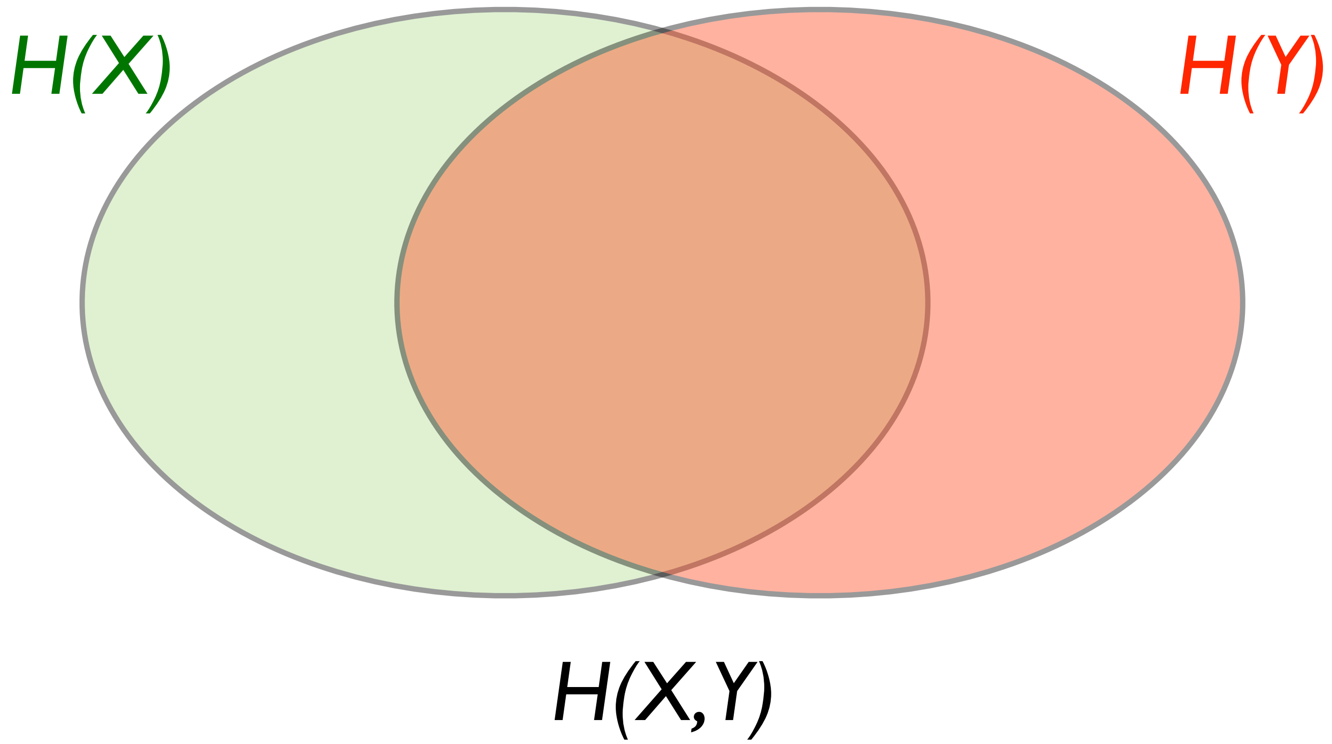

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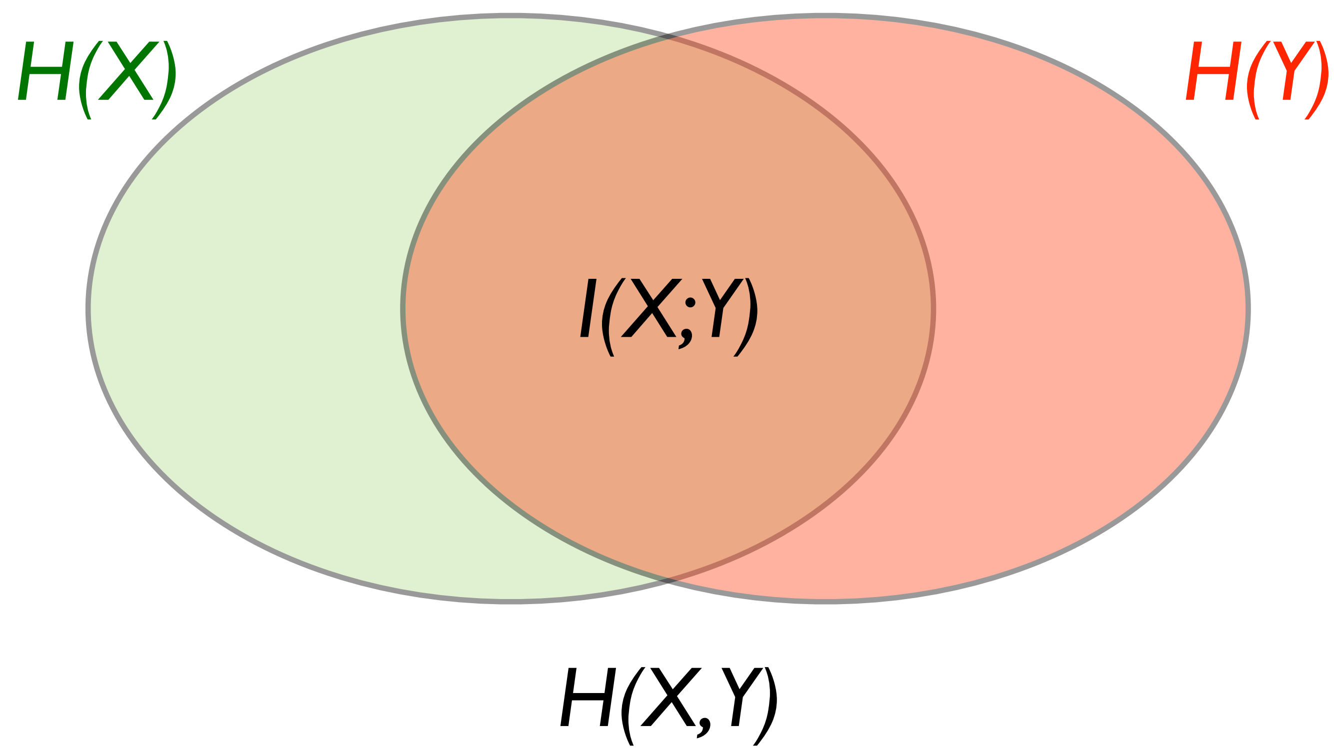

Computing the entropy numerically

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- Note that log(0) is undefined and will give an error if you compute it
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- How to compute $\log_2$:
  - many languages do have a $\log_2$ function (MATLAB, C, C++,...)
  - ... but some do not (Java,...)
  - for any bases *a* and *b*, $\log_b x = \dfrac{\log_a x}{\log_a b}$
  - in our case *b*=2
  - *a* is usually e (2.71828…) or 10 (does it matter?)

Back to attribute selection

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- Finding attributes with high entropy:
  - start with a compressing transform (e.g. time/frequency for sounds, wavelet or DCT for images, movement vectors for video,...)
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- Finding features with high mutual information with the classes:
  - expert knowledge may be available
- Common sense works!
  - e.g., don't classify text documents based on frequency of "a", "the", "and", ...

Some common classifiers

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- Bayesian classifiers
  - Maximum Likelihood learning: for each class C, learn a probability distribution $p(.|C)$ that maximizes $\prod p(m|C)$ for all the m in the training data belonging to C
  - classification: for a point m, find the class C that maximises $p(C|m)$
    - using Bayes' rule: $p(C|m) = \frac{p(C)p(m|C)}{p(m)}$
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    - using Bayes' rule: $p(C|m) = \frac{p(C)p(m|C)}{p(m)}$
    - scales linearly with the number of classes
- Support Vector Machines (SVM)
  - linear classifiers: boundaries are hyperplanes
  - Maximal Margin learning: learn the hyperplane that is the furthest away from making a mistake
    - maximize the distance to the closest (=worst) points of each class (the support vectors)
  - mainly for 2 classes (*binary* classification)

Decision trees

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- Decompose a complex problem into a sequence of simple questions
- Which question to ask next depends on the previous answers
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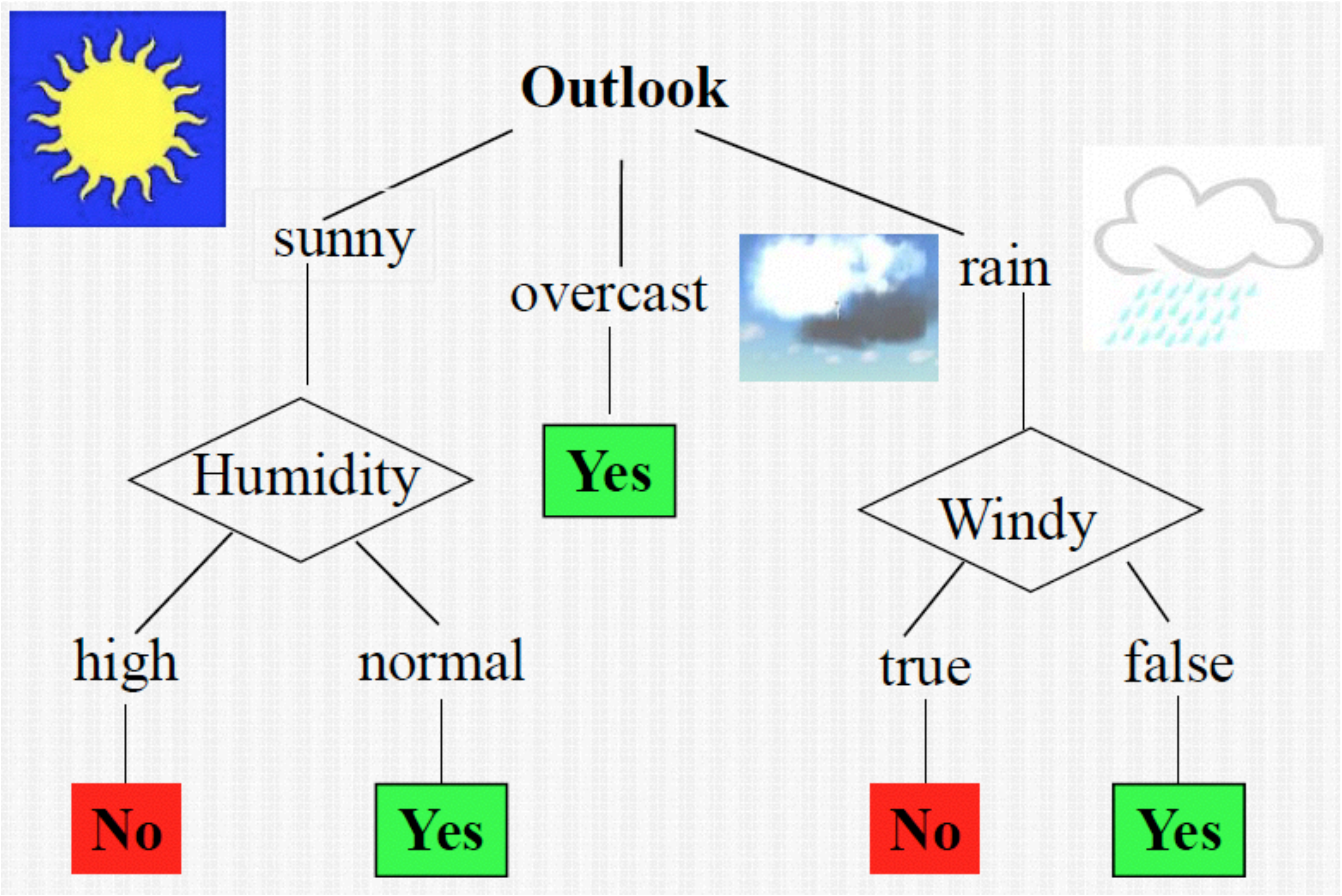
- Tree structure:
  - leaves: decisions
  - nodes: questions
  - branches: possible answers to the parent question

The morning problem

Should I attend the class or not?



Decision trees and classification

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  - the decision to take: which label apply to a data point?

Decision trees and classification

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- Classification can be seen as a decision problem
  - the decision to take: which label apply to a data point?
- Classifiers can be represented as a decision tree
  - a decision tree also partitions the attribute space
  - question: on which side of a given border is the point we're interested in?
  - several trees can represent the same classifier

Learning decision trees

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- Learning problem
  - which questions to ask?

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- If one asks enough questions, one will eventually classify the training set
  - 1 question = 1 new split of the space
  - with enough splits, one can eventually separate each training point from all others
  - but would that classifier generalise well to data it hasn't seen before?

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  - with enough splits, one can eventually separate each training point from all others
  - but would that classifier generalise well to data it hasn't seen before?
- Goals of the learning:
  - the classifier should be general
  - it should be fast to apply
    - build a classifier that needs to ask few questions before taking a decision (a shallow, bushy tree)

ID3 algorithm (Quinlan 1986)

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- ID3 takes a training set and builds a decision tree such that each leaf is homogeneous
  - builds the tree from the root to the leaves
  - greedy algorithm: no backtracking

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```
ID3(Training_Set) =
  if all the points in Training_Set have the same class C
    return Leaf(C);
  elseif no questions remain for Training_Set
    return MajorityClass(Training_Set);
  else
    Question = Find_Best_Question(Training_Set);
    [Set_1,...,Set_n] = Split(Training_set, Question);
    return Tree(Question, ID3(Set_1),...,ID3(Set_n));
```

ID3 algorithm (Quinlan 1986)

- ID3 takes a training set and builds a decision tree such that each leaf is homogeneous
  - builds the tree from the root to the leaves
  - greedy algorithm: no backtracking

```
ID3(Training_Set) =
  if all the points in Training_Set have the same class C
    return Leaf(C);
  elseif no questions remain for Training_Set
    return MajorityClass(Training_Set);
  else
    Question = Find_Best_Question(Training_Set);
    [Set_1,...,Set_n] = Split(Training_set, Question);
    return Tree(Question, ID3(Set_1),...,ID3(Set_n));
```

- How does `Find_Best_Question` work?
  - which questions are allowed?
  - how to select the best one?

Finding the next question to ask

$$H(S) = -\sum_{c \in C} p_c \log_2 p_c, \qquad \text{where } p_c = P(x \in c \mid x \in S)$$

Finding the next question to ask

- ID3 restricts itself to questions that test only one attribute
  - less questions to choose from
  - geometrically, all the borders are orthogonal to one axis
  - works well for symbolic attributes
    - symbolic attributes cannot be combined
    - symbolic attributes can be enumerated

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  - less questions to choose from
  - geometrically, all the borders are orthogonal to one axis
  - works well for symbolic attributes
    - symbolic attributes cannot be combined
    - symbolic attributes can be enumerated
- Each question is chosen so that the resulting subsets are as classified as possible
  - the entropy of the class variable c within a set S measures how classified it is

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$$G(S,Q) = H(S) - \sum_{S_i \in \mathrm{Split}(S,Q)} \frac{|S_i|}{|S|} H(S_i)$$

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$$G(S,Q) = H(S) - \sum_{S_i \in \mathrm{Split}(S,Q)} \frac{|S_i|}{|S|} H(S_i)$$

- Compute *G* for all possible questions *Q* and select the *Q* with largest *G(S,Q)*

Which questions to ask?

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- Symbolic attributes (gender, hair colour,...):
  - only a finite number of possible values for all data
  - one branch for each different value

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- Symbolic attributes (gender, hair colour,...):
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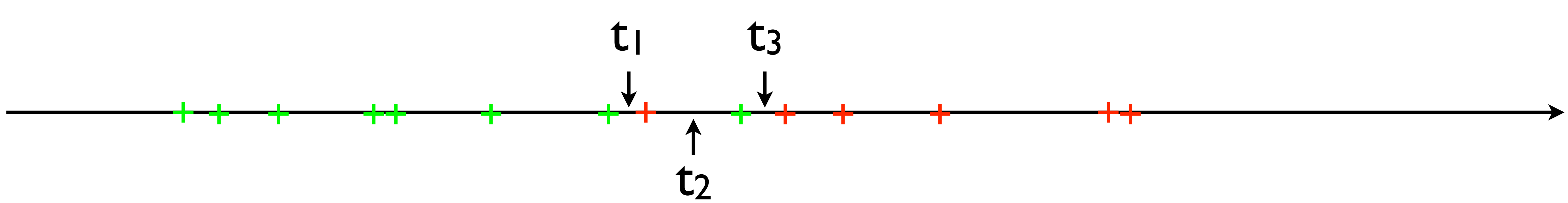
- Numerical attributes (temperature, pixel value,...):
  - infinite range even if only a finite number of values are present in the training set
  - choose a threshold t and ask the question: value < t?
- Threshold choice:
  - the information gain criterion still holds
  - the best threshold must be between two training examples of different classes
  - try all possible thresholds



Example: attend the lecture or not?

Outlook	Temperature	Windy	Attend?
sunny	29	No	Yes
sunny	27	Yes	No
overcast	28	No	Yes
rain	21	Yes	No
rain	20	No	Yes
rain	18	Yes	No
overcast	17	Yes	Yes
sunny	22	No	No

$$
\begin{aligned}
H(S) &= -\frac{4}{8}\log_2\frac{4}{8} - \frac{4}{8}\log_2\frac{4}{8} \\
&= 1
\end{aligned}
$$

Example: split along Outlook

Outlook	Temperature	Windy	Attend?
sunny	29	No	Yes
sunny	27	Yes	No
overcast	28	No	Yes
rain	21	Yes	No
rain	20	No	Yes
rain	18	Yes	No
overcast	17	Yes	Yes
sunny	22	No	No

$$
\begin{aligned}
H(sunny) &= -\frac{1}{3}\log_2\frac{1}{3} - \frac{2}{3}\log_2\frac{2}{3} \approx 0.9183 \\
H(overcast) &= -\frac{2}{2}\log_2\frac{2}{2} - \frac{0}{2}\log_2\frac{0}{2} \approx 0 \\
H(rain) &= -\frac{1}{3}\log_2\frac{1}{3} - \frac{2}{3}\log_2\frac{2}{3} \approx 0.9183 \\
G(Outlook) &= H(S) - \frac{3}{8}H(sunny) - \frac{2}{8}H(overcast) - \frac{3}{8}H(rain) \\
&\approx 0.3113
\end{aligned}
$$

Example: split along Temperature

Outlook	Temperature	Windy	Attend?
sunny	29	No	Yes
sunny	27	Yes	No
overcast	28	No	Yes
rain	21	Yes	No
rain	20	No	Yes
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overcast	17	Yes	Yes
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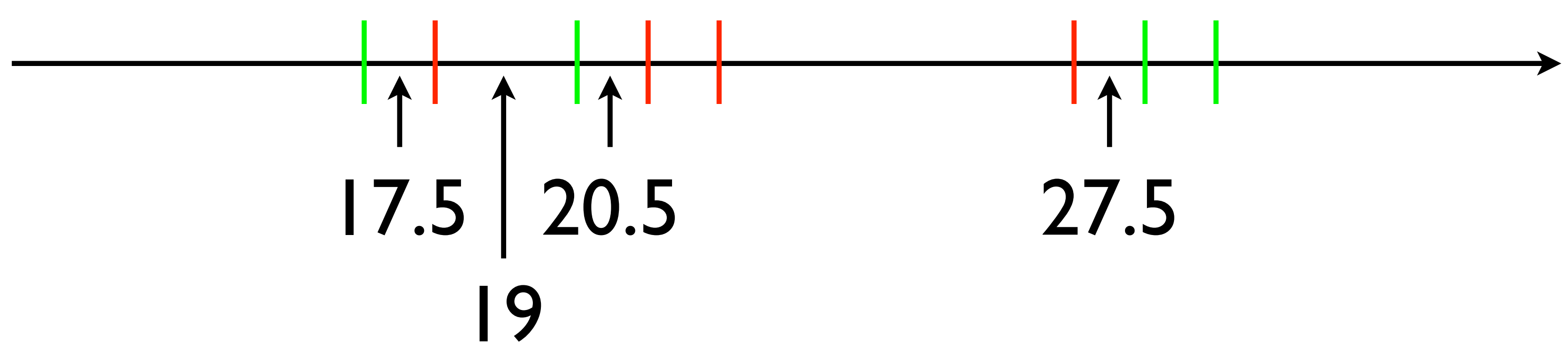

Example: split at t=17.5°C

Outlook	Temperature	Windy	Attend?
sunny	29	No	Yes
sunny	27	Yes	No
overcast	28	No	Yes
rain	21	Yes	No
rain	20	No	Yes
rain	18	Yes	No
overcast	17	Yes	Yes
sunny	22	No	No

$$H(t < 17.5) = -\frac{1}{1}\log_2\frac{1}{1} - \frac{0}{1}\log_2\frac{0}{1} \approx 0$$

$$H(t > 17.5) = -\frac{3}{7}\log_2\frac{3}{7} - \frac{4}{7}\log_2\frac{4}{7} \approx 0.9852$$

$$\begin{aligned} G(t = 17.5) &= H(S) - \frac{1}{8}H(t < 17.5) - \frac{7}{8}H(t > 17.5) \\ &\approx 0.1379 \end{aligned}$$

Example: split at t=19°C

Outlook	Temperature	Windy	Attend?
sunny	29	No	Yes
sunny	27	Yes	No
overcast	28	No	Yes
rain	21	Yes	No
rain	20	No	Yes
rain	18	Yes	No
overcast	17	Yes	Yes
sunny	22	No	No

$$
\begin{aligned}
H(t<19) &= -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2} \approx 1 \\
H(t>19) &= -\frac{3}{6}\log_2\frac{3}{6} - \frac{3}{6}\log_2\frac{3}{6} \approx 1 \\
G(t=19) &= H(S) - \frac{2}{8}H(t<19) - \frac{6}{8}H(t>19) \\
&= 0
\end{aligned}
$$

Example: split at t=20.5°C

Outlook	Temperature	Windy	Attend?
sunny	29	No	Yes
sunny	27	Yes	No
overcast	28	No	Yes
rain	21	Yes	No
rain	20	No	Yes
rain	18	Yes	No
overcast	17	Yes	Yes
sunny	22	No	No

$$H(t < 20.5) = -\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3} \approx 0.9183$$

$$H(t > 20.5) = -\frac{3}{5}\log_2\frac{3}{5} - \frac{2}{5}\log_2\frac{2}{5} \approx 0.9710$$

$$\begin{aligned} G(t = 20.5) &= H(S) - \frac{3}{8}H(t < 20.5) - \frac{5}{8}H(t > 20.5) \\ &\approx 0.0488 \end{aligned}$$

Example: split at t=27.5°C

Outlook	Temperature	Windy	Attend?
sunny	29	No	Yes
sunny	27	Yes	No
overcast	28	No	Yes
rain	21	Yes	No
rain	20	No	Yes
rain	18	Yes	No
overcast	17	Yes	Yes
sunny	22	No	No

$$H(t < 27.5) = -\frac{2}{6}\log_2\frac{2}{6} - \frac{4}{6}\log_2\frac{4}{6} \approx 0.9183$$

$$H(t > 27.5) = -\frac{2}{2}\log_2\frac{2}{2} - \frac{0}{2}\log_2\frac{0}{2} = 0$$

$$G(t = 27.5) = H(S) - \frac{6}{8}H(t < 27.5) - \frac{2}{8}H(t > 27.5)$$

$$\approx 0.3113$$

Example: split along windy

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Outlook	Temperature	Windy	Attend?
sunny	29	No	Yes
sunny	27	Yes	No
overcast	28	No	Yes
rain	21	Yes	No
rain	20	No	Yes
rain	18	Yes	No
overcast	17	Yes	Yes
sunny	22	No	No

$$H(windy) = -\frac{1}{4}\log_2\frac{1}{4} - \frac{3}{4}\log_2\frac{3}{4} \approx 0.8113$$

$$H(\neg windy) = -\frac{3}{4}\log_2\frac{3}{4} - \frac{1}{4}\log_2\frac{1}{4} \approx 0.8113$$

$$\begin{aligned} G(Windy?) &= H(S) - \frac{4}{8}H(windy) - \frac{4}{8}H(\neg windy) \\ &\approx 0.1887 \end{aligned}$$

Example: first iteration

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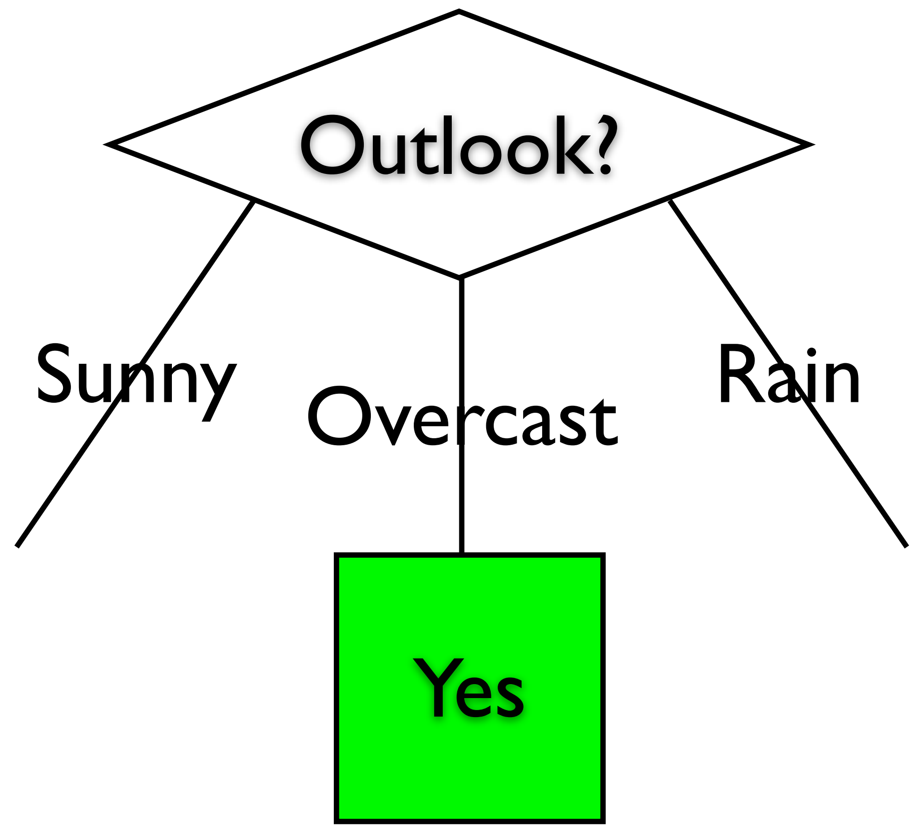
- The best splits were either Outlook or t=27.5°C
- Let us assume we split on Outlook:



Example: first iteration

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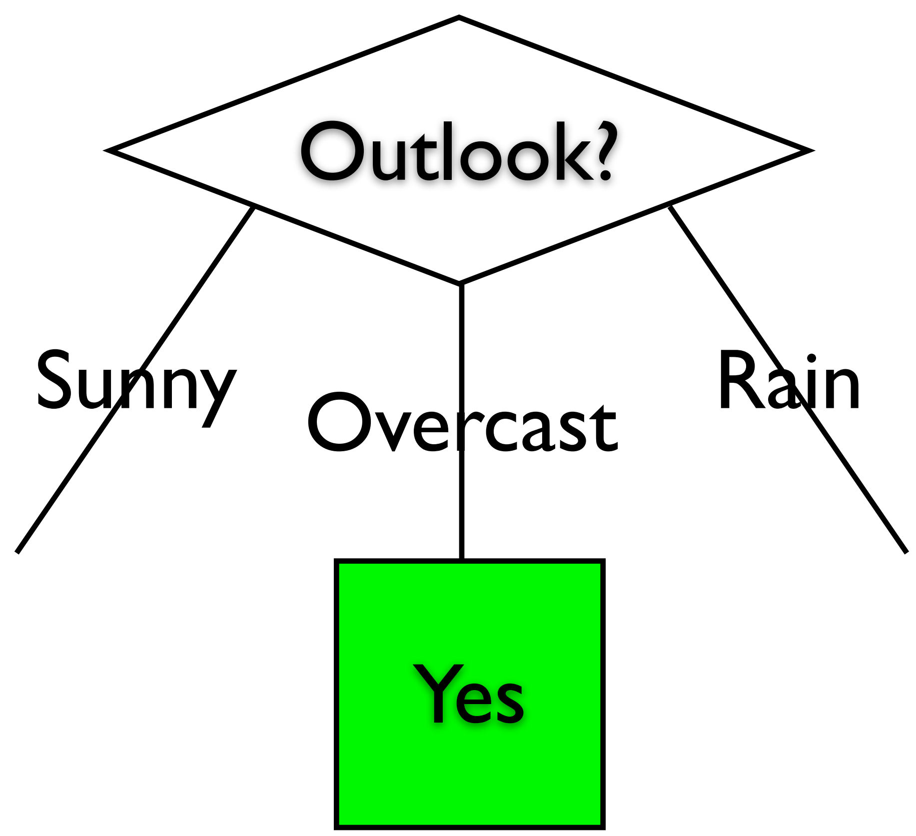
- The best splits were either Outlook or t=27.5°C
- Let us assume we split on Outlook:



- We must then apply the algorithm to the Sunny and Rain branches

Example: the Sunny branch

Outlook	Temperature	Windy	Attend?
sunny	29	No	Yes
sunny	27	Yes	No
sunny	22	No	No

A split at t=28°C classifies the branch

Example: the Rainy branch

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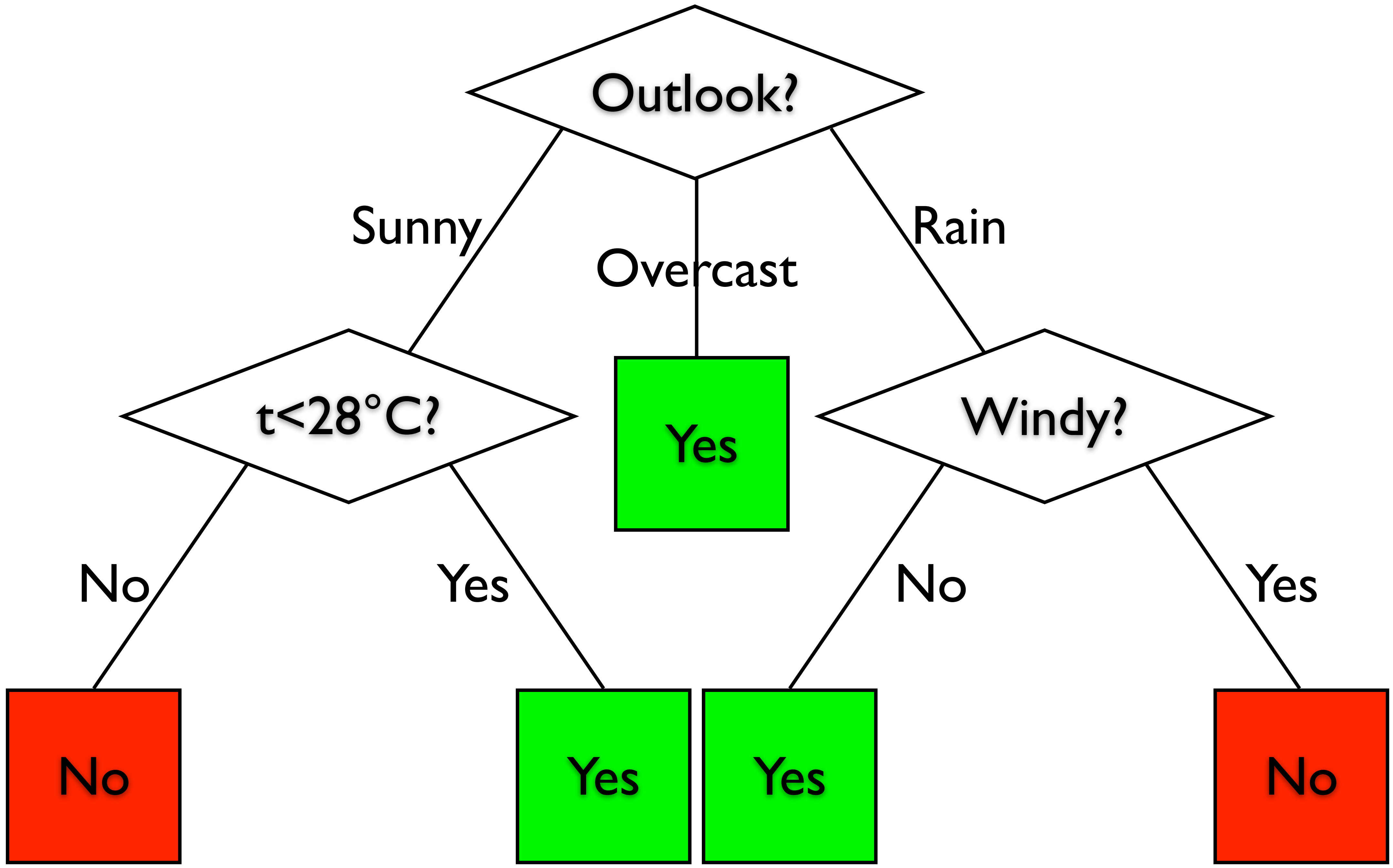
Outlook	Temperature	Windy	Attend?
rain	21	Yes	No
rain	20	No	Yes
rain	18	Yes	No

A split along Windy classifies the branch

Example: the final tree



Strong and weak points of ID3

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- Each question only checks one feature:
  - works well with attributes of different types (e.g. size and gender)
  - designs rectangular regions only, even if it does not fit the data

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- Each question only checks one feature:
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- Generates the shortest possible tree:
  - fast classification after learning
  - helps to generalise the model
  - helps to understand the generated rules

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- Each question only checks one feature:
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  - designs rectangular regions only, even if it does not fit the data
- Generates the shortest possible tree:
  - fast classification after learning
  - helps to generalise the model
  - helps to understand the generated rules
- Training is expensive:
  - try all possible questions at each node
  - for each question, compute the answer with each data point of the subset
  - infinite range attributes generate lots of different questions

A word on overfitting

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- Noise blurs the boundaries between classes

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- Sampling can alter the perception of the problem
  - selection bias
  - rare events

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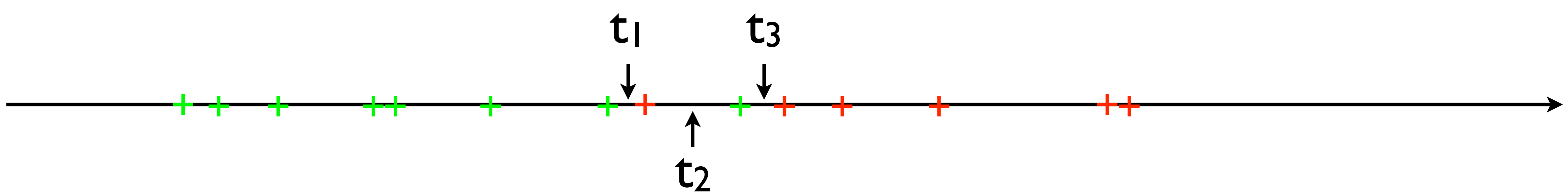
A word on overfitting

- Noise blurs the boundaries between classes
- Sampling can alter the perception of the problem
  - selection bias
  - rare events
- General solution: more data! (not always possible)
- **Overfitting**
  - learning a tree that models properties specific to the training set
    - the tree works on the training set but not on the problem data
    - perfect classification of the training set is usually too much



Reduced-error pruning

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  - one for learning
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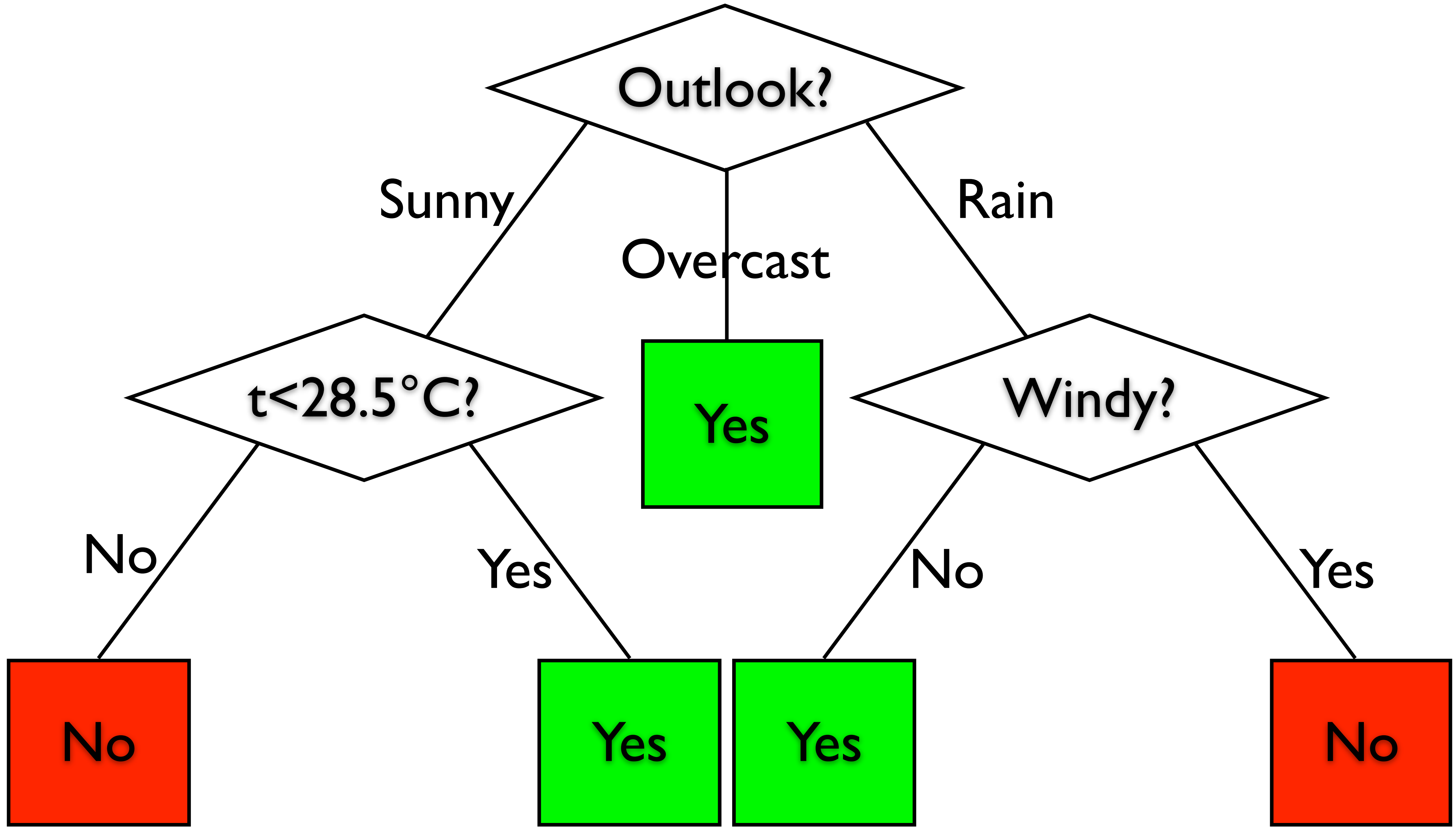
- First build a tree with ID3 on the training set (learning subset)
- Then greedily prune nodes to improve the classification of the pruning set
- What if we don't have enough data to make a pruning set?

Rule pruning

Rule pruning

- The tree can be written as a set of logical rules
  - each path is a single conjunction (“and”) of tests
  - there is a disjunction (“or”) between different paths leading to the same class



Rule post-pruning

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  - Precision = fraction of points of the correct class among the points picked by the rule

Rule post-pruning

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- First convert the tree into rules
- Then greedily prune the rules by removing conditions
- Only prune if the precision of the rule does not decrease
  - Precision = fraction of points of the correct class among the points picked by the rule
- The pruning is done on the training set directly
  - no need to split the set in two
  - more used in practice

Summary

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- Information as a measurable quantity
  - information as structure in data

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Summary

- Information as a measurable quantity
  - information as structure in data
- ID3: a method of using information to decide on structure
- Avoid overfitting
  - pruning to re-generalise