Investments

Factor models and APT

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Overview

- Introduce Arbitrage Pricing Theory (APT)
  - What is a risk factor?
  - Why might there be multiple priced risk factors?
  - How can we identify factors and factor betas?
  - Arbitrage pricing Theory (APT)
  - Applications of the APT

APT vs. CAPM

- CAPM
  - Expected returns are a function of covariance with the market portfolio

- APT
  - Expected returns are a function of covariance with one or more “risk factors”.
  - Relies on less restrictive assumptions

What do we mean by a factor?

- A factor represents common variation among securities.

- What is a source of common variation?
  - Oil prices?
  - Interest rates?
  - Productivity shocks?

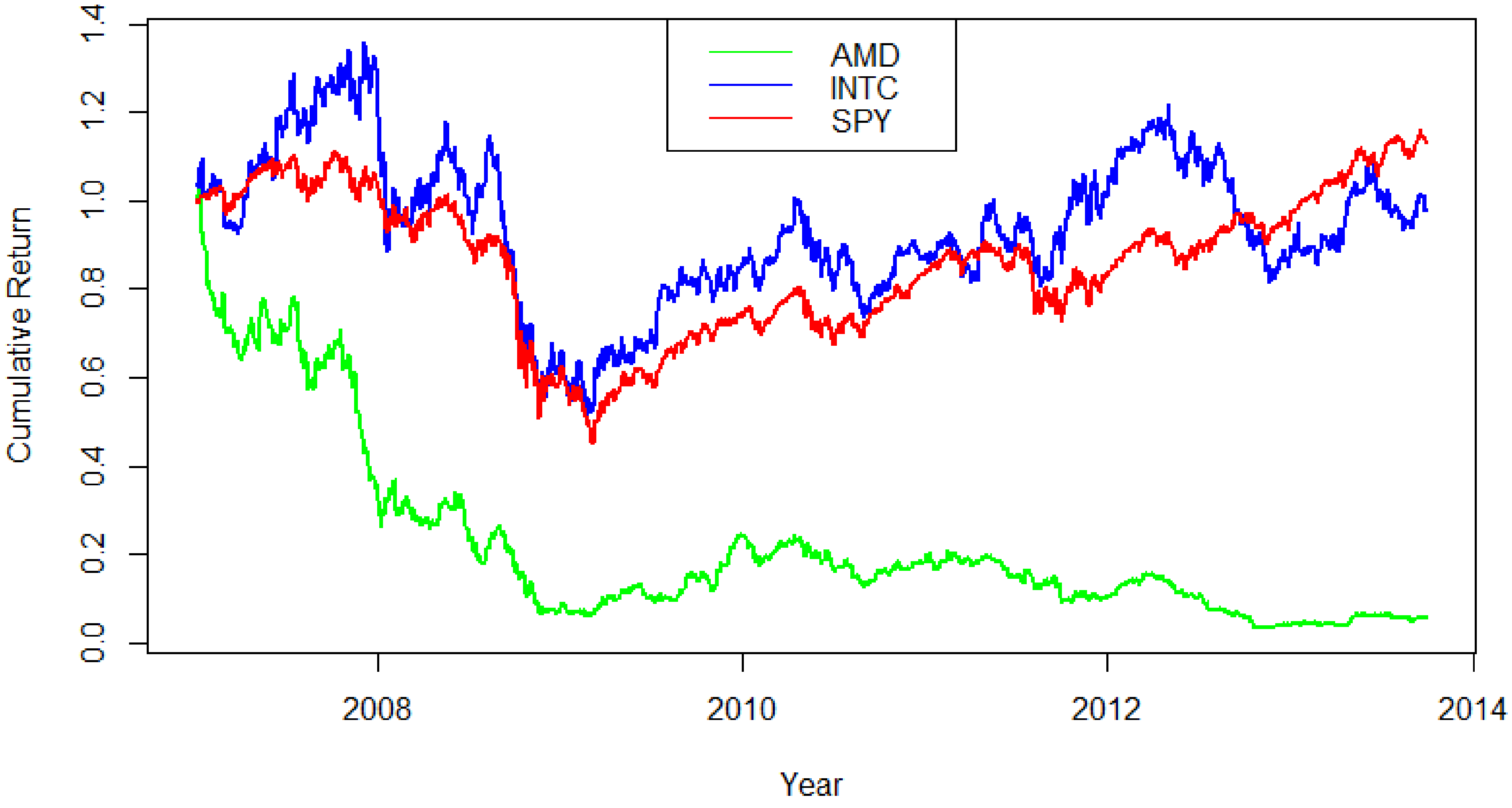
- Affects returns of even broadly diversified portfolios

One factor: the market model

$$\underbrace{\tilde{r}_i}_{\text{Stock return}} - r_f = \underbrace{\alpha_i}_{\bar{r}\text{ that isn't risk}} + \underbrace{\beta_i}_{\text{beta or loading}} \left[\underbrace{\tilde{r}_{mkt}}_{\text{Factor return}} - r_f\right] + \underbrace{\tilde{\epsilon}_i}_{\text{Uncorrelated across securities}}$$

- Is it realistic to assume $\tilde{\epsilon}_i$s are independent?

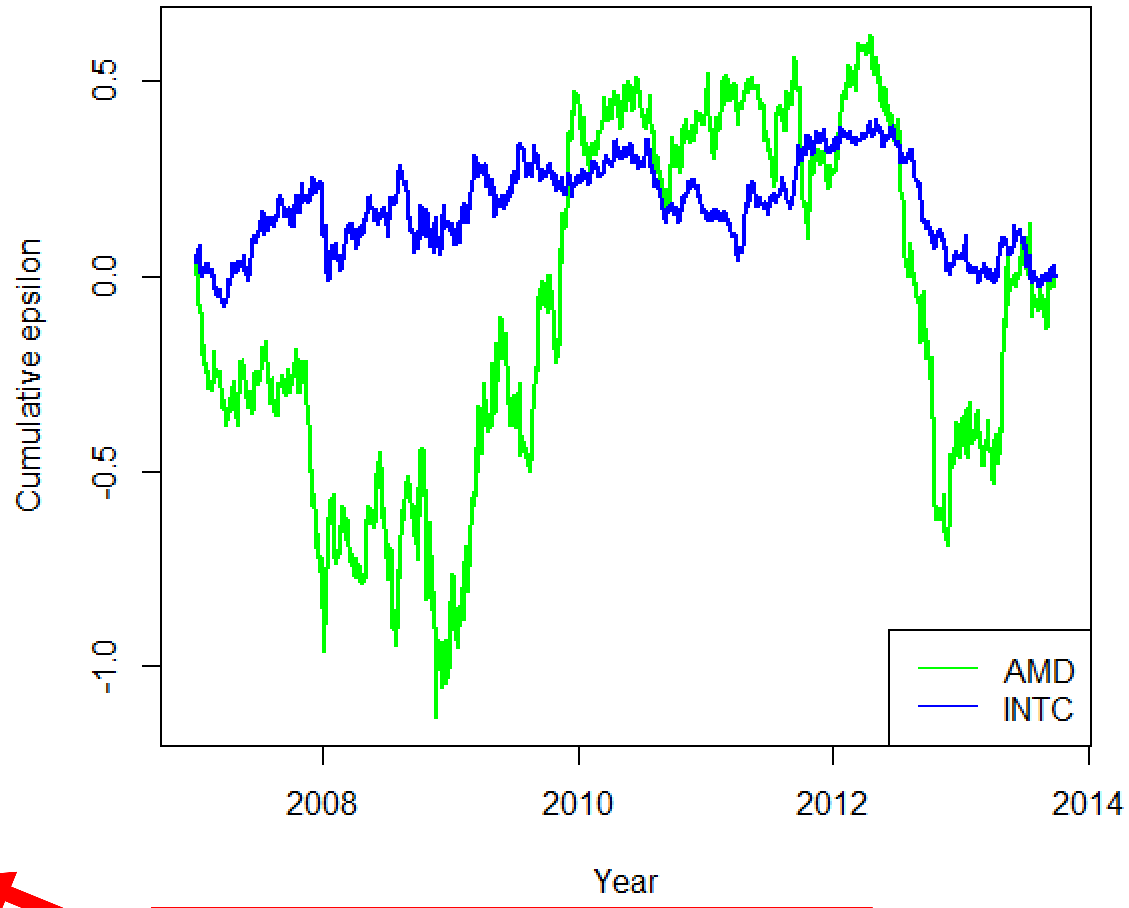
AMD vs. INTC


AMD
INTC
SPY
Cumulative Return
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
2008
2010
2012
2014
Year

AMD vs. INTC

- Do market-model regression for AMD/INTC
- AMD
  - $\sigma = 3.8\%$
  - $\beta_{spy} = 1.42$
- INTC
  - $\sigma = 2.0\%$
  - $\beta_{spy} = 1.02$

Correlation between ϵ_{AMD} and ϵ_{INTC} is 0.2



The epsilons are not independent.
Motivates search for additional factors.

Multi-factor model

$$\tilde{r}_i - r_f = \alpha_i + \beta_{i1}\left[\tilde{r}_{F_1} - r_f\right] + \cdots + \beta_{ik}\left[\tilde{r}_{F_2} - r_f\right] + \tilde{\epsilon}_i$$

- Variation determined by
  - Covariance with K common factors
  - Firm-specific variation

- What is $\tilde{r}_{F_k}$?
  - New information about common variation
    - Macro announcements, interest rate shocks, country defaults, …
  - Standard to use portfolios as factors
    - Market Factor, Value Factor, Size Factor, Momentum Factor, etc.

Factors are useful for hedging

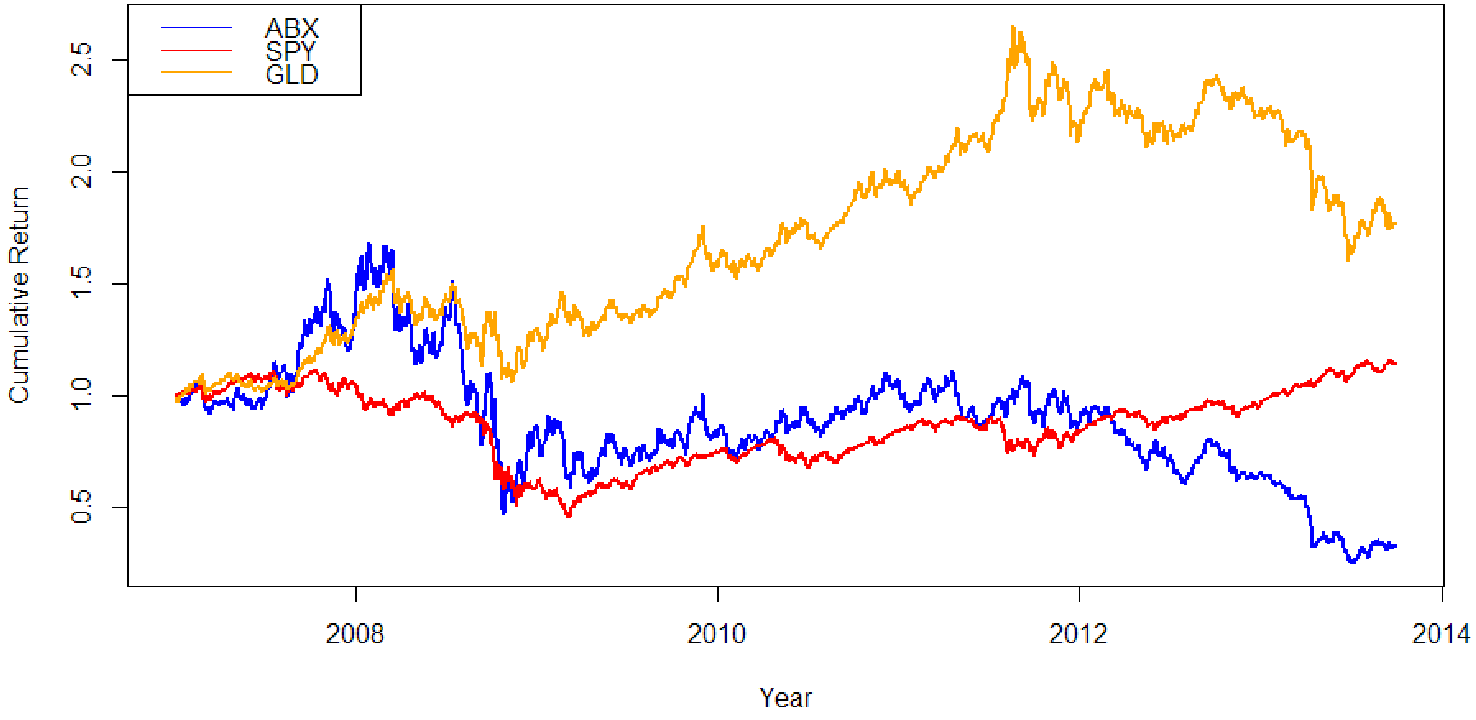
- Consider Barrick Gold (ABX), a gold mining firm.
  - Assume a factor model such as the following, where the factors are the market (SPY) and the price of gold (GLD).

$$\tilde{r}_{abx} - r_f = \alpha_{abx} + \beta_{spy}\left[\tilde{r}_{spy} - r_f\right] + \beta_{GLD}\left[\tilde{r}_{GLD} - r_f\right] + \tilde{\epsilon}_{abx}$$

  - If you form a portfolio with the same β_{spy} and β_{GLD}, then you track Barrick's exposure to these common factors. It's like replication with factors.
  - You can track exposure to both factors or exposure to just one (e.g., GLD).

Barrick Gold Example

- You want to buy Barrick Gold (ABX), but you want to hedge out the exposure to the price of gold.



Barrick Gold Example

- Regress returns of ABX on two factors: SPY and GLD (a gold ETF). In this example SPY and GLD are factors:

$$\tilde{r}_{abx} - r_f = 0.00 + 0.45 \times [\tilde{r}_{spy} - r_f] + 1.62 \times [\tilde{r}_{gld} - r_f] + \tilde{\epsilon}_{abx}$$

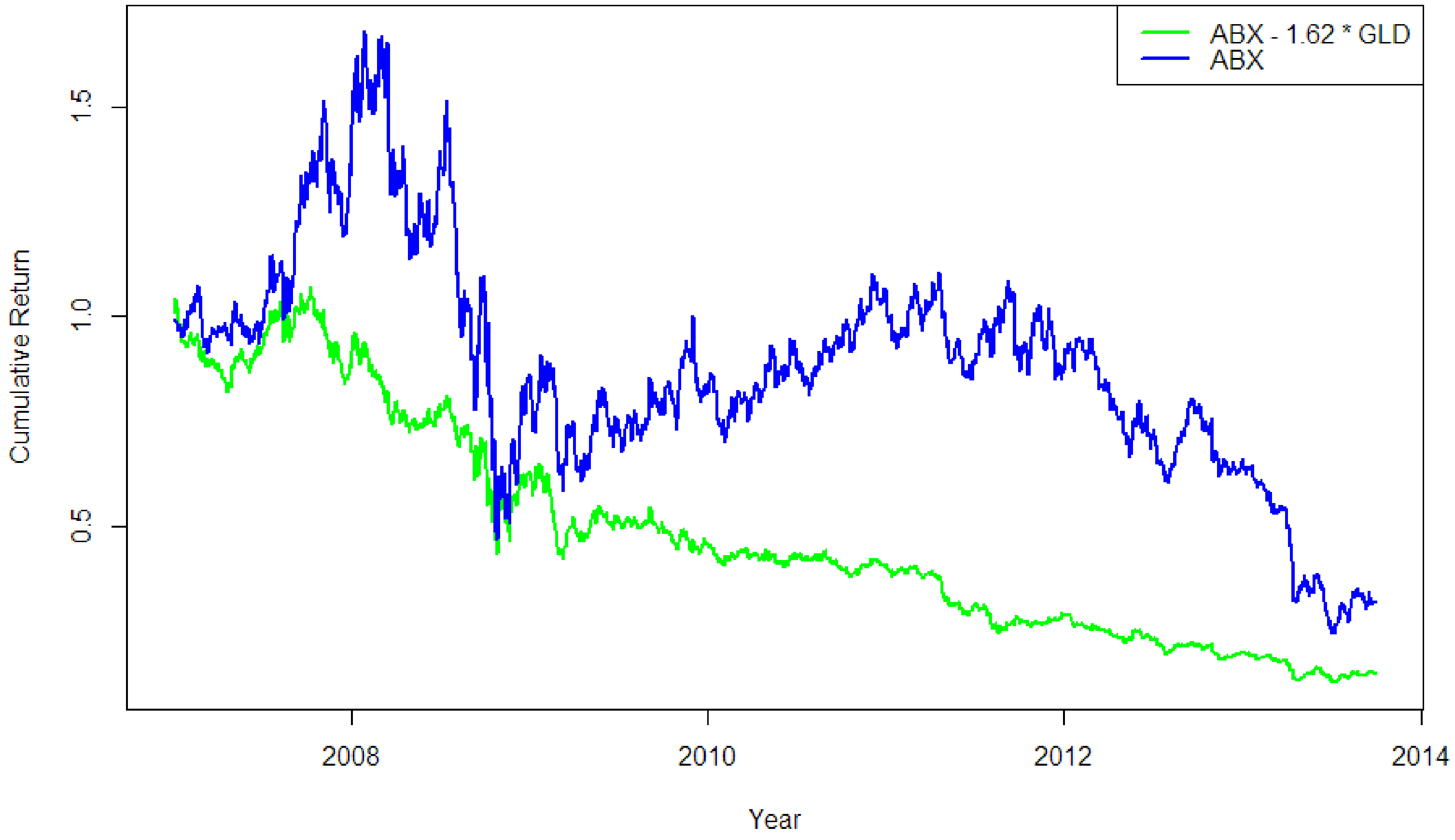
- The portfolio that tracks the GLD exposure has $\beta_{gld} = 1.62$:
  - Consider a tracking portfolio with these weights:

    $w_{gld} = 1.62$ and $w_{rf} = -0.62$.

$$\begin{aligned} \tilde{r}_{track} = w_{gld}\tilde{r}_{gld} + w_{rf}r_{rf} &= 1.62 \times \tilde{r}_{gld} + (-0.62) \times r_{rf} \\ &= r_f + 1.62\left[\tilde{r}_{gld} - r_f\right] \end{aligned}$$

- The tracking portfolio has $\beta_{gld} = 1.62$
- So to hedge out the GLD exposure, short \$1.62 of GLD for every \$1 in ABX and put \$0.62 in the risk-free bond.

Barrick Gold Example



Barrick Gold Example

- Now imagine a portfolio that tracks BOTH SPY and GLD exposure.
  - Weight in GLD = 1.62, weight in SPY = 0.45, weight in risk-free $= 1 - (1.62 + 0.45) = -1.07$

$$\begin{aligned}\tilde{r}_{track} &= 0.45\tilde{r}_{spy} + 1.62\tilde{r}_{gld} + (-1.07)r_{rf} \\ &= 0.45(\tilde{r}_{spy} - r_f) + 1.62(\tilde{r}_{gld} - r_f) + (0.45 + 1.62 - 1.07)r_{rf}\end{aligned}$$

- What is the return of a portfolio long Barrick Gold and short the tracking portfolio?

$$\begin{aligned}\tilde{r}_{hedge} &= 1 \times \tilde{r}_{abx} - 1 \times \tilde{r}_{track} \\ &= \left(r_f + \alpha_{abx} + 0.45(\tilde{r}_{spy} - r_f) + 1.62(\tilde{r}_{gld} - r_f) + \epsilon_{abx}\right) - \\ &\quad \left(0.45(\tilde{r}_{spy} - r_f) + 1.62(\tilde{r}_{gld} - r_f) + r_{rf}\right) \\ &= \alpha_{abx} + \tilde{\epsilon}_{abx}\end{aligned}$$

Barrick Gold Example

- Let's assume the epsilons = 0. In reality, they aren't, but let's assume this for a moment:

$$\rightarrow \tilde{r}_{hedge} = \alpha_{abx} + \tilde{\epsilon}_{abx} = \alpha_{abx}$$

- If epsilons are *always* zero, then we have a zero cost portfolio with certain returns. What should the return be?

$$\rightarrow \alpha_{abx} = 0$$

- Plug back into return equation:

$$\begin{aligned}\tilde{r}_{abx} &= r_f + \alpha_{abx} + \beta_{spy}\left[\tilde{r}_{spy} - r_f\right] + \beta_{GLD}\left[\tilde{r}_{GLD} - r_f\right] + \tilde{\epsilon}_{abx} \\ &= r_{rf} + 1.62\left(\tilde{r}_{gld} - r_{rf}\right) + 0.45\left(\tilde{r}_{spy} - r_{rf}\right)\end{aligned}$$

What did we learn?

- If we have a factor model, we can create a tracking portfolio.
- *If there is no idiosyncratic risk*, then we've replicated the security and we can write down the return equation:

$$\tilde{r}_{abx} = 1 \times r_{rf} + 1.62\left(\tilde{r}_{gld} - r_{rf}\right) + 0.45\left(\tilde{r}_{spy} - r_{rf}\right)$$
$$\rightarrow$$
$$\bar{r}_{abx} = r_{rf} + \underbrace{1.62 \times}_{\text{Factor beta or loading}} \underbrace{\left(\bar{r}_{gld} - r_{rf}\right)}_{\text{Factor premium}} + 0.45 \times \left(\bar{r}_{spy} - r_{rf}\right)$$

- This is the idea behind the APT

How do we kill the epsilons?

- Diversification!!!
- Form a portfolio that diversifies away all non-factor risk. I.E., get rid of all idiosyncratic variation.
- Then the portfolio can be priced relative to the common risk factors.

Variance in factor model

- To see how diversification works, consider variance in a factor model:

$$
\begin{aligned}
var(r_i) &= var\left(\alpha_i + \beta_F\left(\tilde{r}_F - r_f\right) + \tilde{\epsilon}_i\right) \\
&= var(\beta_F \tilde{r}_F + \tilde{\epsilon}_i) \\
&= \beta_F^2 \sigma_F^2 + \sigma_{\epsilon_i}^2 + 2 \times cov(\beta_F \tilde{r}_F, \tilde{\epsilon}_i) \\
&= \underbrace{\beta_F^2 \sigma_F^2}_{\substack{\text{Factor risk, or} \\ \text{Systematic risk}}} + \underbrace{\sigma_{\epsilon_i}^2}_{\substack{\text{Asset-specific “risk”, or} \\ \text{Idiosyncratic “risk”}}}
\end{aligned}
$$

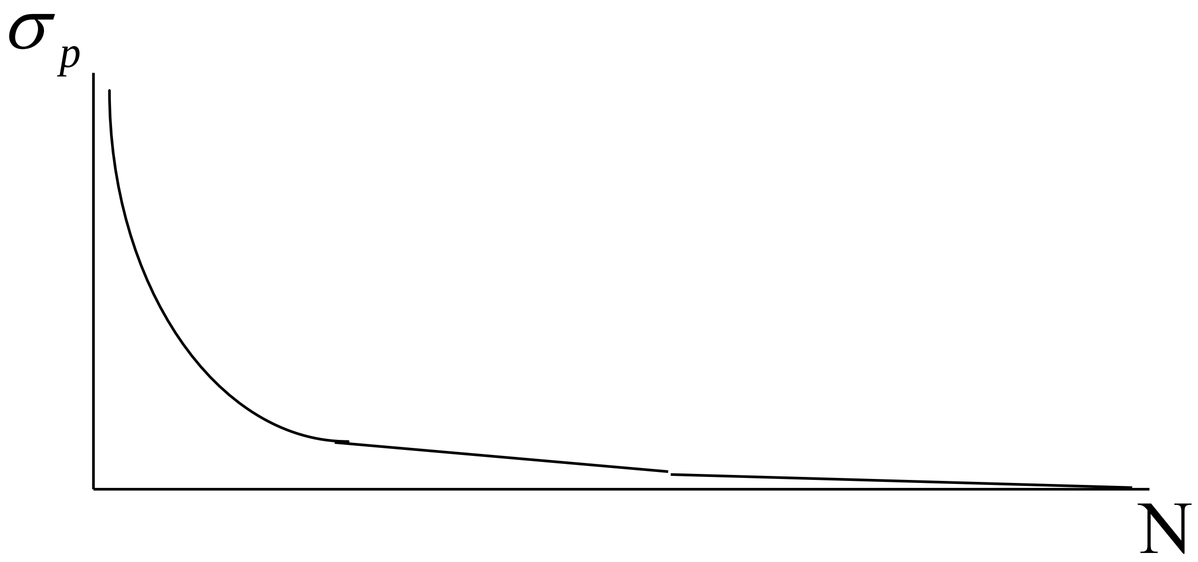
Diversification Example

- Assume all stocks have
  - 15% expected return
  - 50% standard deviation
  - **all stocks' returns are independent** **(i.e. $\rho_{i,j} = 0$)**
- Consider investing ***equally*** in N stocks
- One can show that for the stock portfolio

$$E(r_p) = 0.15, \ \sigma_p = \frac{0.5}{\sqrt{N}}$$

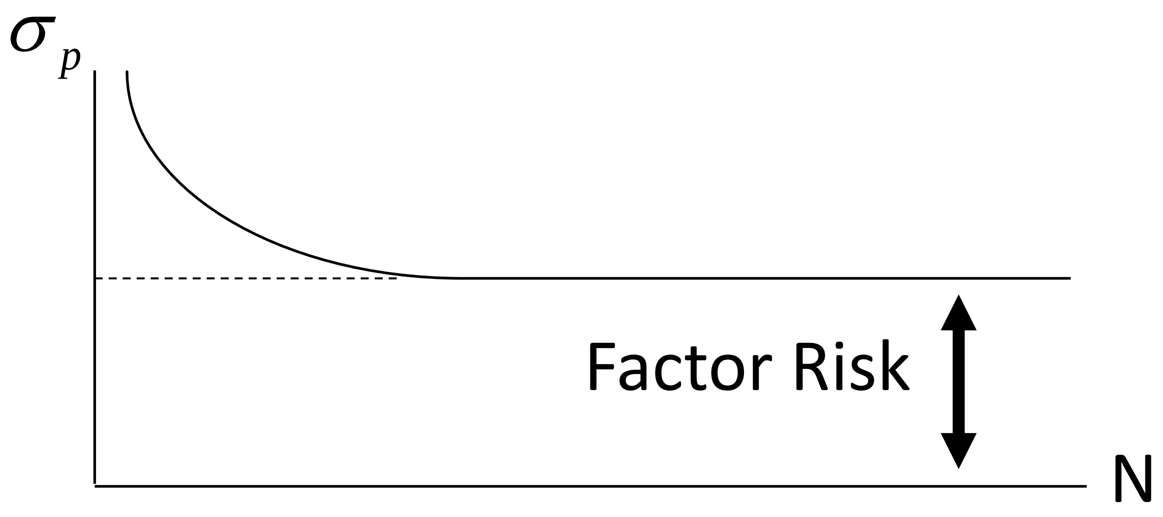
Power of Diversification

- In this naïve diversification example, one can eliminate all risks by holding lots of stocks (N large)



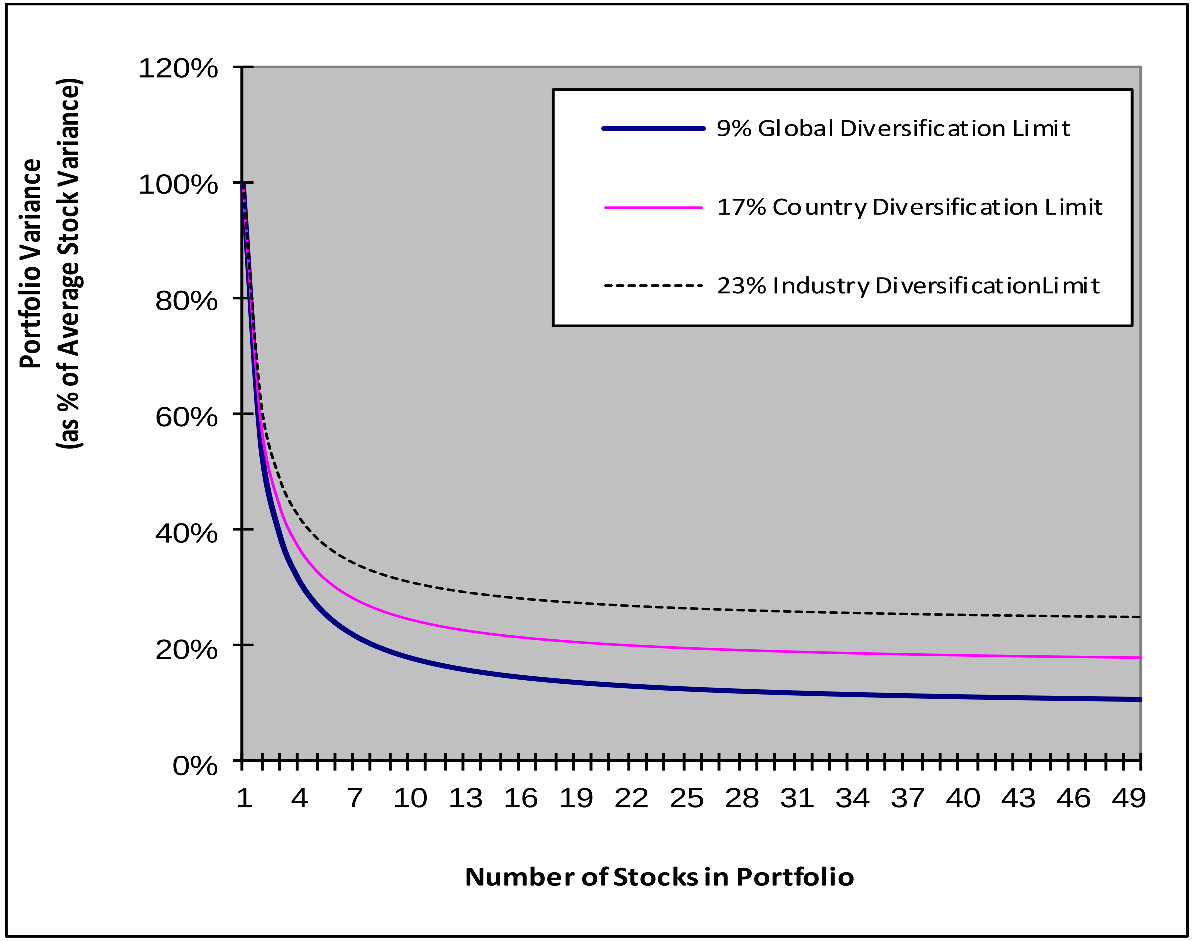
Correlation Effect

- However, in reality, stocks are correlated. It will be impossible to diversify all risk away.
- One can diversify away idiosyncratic risk, but not factor risk.



Global Example

- Griffin and Karolyi (1998) study
- Industry
  - Across industries within a country
- Country
  - Across countries within an industry
- Global
  - Across countries and industries



Using tracking portfolios and arbitrage arguments to get a less restrictive theory of expected returns

APT

What is the APT?

- Developed by Stephen Ross (1976)
  - Motivation

    "… on theoretical grounds it is difficult to justify either the assumption [in mean-variance analysis and CAPM] of normality in returns … or of quadratic preferences … and on empirical grounds the conclusions as well as the assumptions of the theory have also come under attack."

    \- Ross (1977)

- Assumptions
  1. Returns can be described using a factor model
  2. No arbitrage
  3. We can diversify away firm-specific risk
  4. Financial markets are frictionless

Implications

- Consider a broadly diversified portfolio
  - Since broadly diversified, no firm-specific risk

$$\tilde{r} - r_f = \alpha + \beta_1(\tilde{r}_{F_1} - r_f) + \cdots + \beta_K(\tilde{r}_{F_k} - r_f)$$

  - Go long that portfolio and short its tracking portfolio
    - You've hedged out factor exposure—all variation is gone
    - So expected returns of the two portfolio must be the same
      - What if the portfolio and tracking portfolio had different E[r]?

- Therefore

$$\bar{r} - r_f = \beta_1\lambda_1 + \cdots + \beta_K\lambda_K$$

where λ_k is the risk premium of factor k (e.g., $\bar{r}_{F_k} - r_f$).

Violations of APT $\bar{r}$ relationship

- Imagine a violation occurred for a small number of stocks
  - The portfolio will have variation that cannot be hedged with a tracking portfolio
  - I.E., firm-specific risk still exists
  - There is no arbitrage
- Imagine a violation for a very large number of stocks
  - Large number of stocks, so all firm-specific risk is diversified away
  - There is an arbitrage
- Implication
  - APT could be violated for a small number of securities
  - BUT, it should hold for most securities, or else there is an arbitrage.

If APT holds exactly

1. No risk-premium for firm-specific risk

2. Tangency portfolio is a combination of factor portfolios

3. Everyone holds a combination of risk-free asset and factor portfolios
   - But note everyone isn't holding the same portfolio

In Application: We still need to find the factors and factor premiums. This is the central question in asset pricing. It's a tough problem!

Application: FF 3-Factor Model

- Define size and book-to-market as risk factors
  - Build ***portfolios*** based on size and book/market
  - Create factors using ***returns*** on these portfolios
  - Measure ***betas*** of assets against these factors
  - Use these ***betas*** to explain risk premia

- The FF model looks like the APT

$$\bar{r} = r_f + \beta_{mkt}\lambda_{mkt} + \beta_{smb}\lambda_{smb} + \beta_{hml}\lambda_{hml}$$

Calculating discount rates

- Using AMZN from 10/2008 to 6/2013 and factor premiums 6/1926-6/2013

1. AMZN average annual return = 29.7%
2. CAPM = 5.5%

$$\tilde{r}_{amzn} - r_f = 0.02 + 0.72 \times MKT + \tilde{\epsilon}_{amzn}$$

$$\overline{MKT} = 7.7\% \text{ per year}$$

$$\Rightarrow risk\ premium = 7.7\% \times 0.72 = 5.5\%$$

3. FF 3-factor model

$$\tilde{r}_{amzn} - r_f = 0.01 + 1.04 \times MKT + 0.66 \times SMB - 1.61 \times HML + \tilde{\epsilon}_{amzn}$$

- Average returns: MKT = 7.7%, SMB = 2.8%, HML = 4.8%
- Risk premium $= 1.04 \times 7.7\% + .66 \times 2.8\% - 1.61 \times 4.8\% = 2.1\%$

Use of the Fama-French Model: Performance Measurement

- Results show convincingly that returns on portfolios of:
  - ***Small stocks*** behave ***differently*** from ***large stocks***
  - ***Value stocks*** behave differently from growth stocks
- In ***measuring portfolio performance*** therefore it certainly makes sense to adjust for differences in return caused by amount of exposure (beta) to these factors
- This makes sense ***whether or not*** there are consistent ***risk premia*** associated with these factors
- Here FF model is being used as ***factor model of returns***

The Information Ratio

- The contribution of the active portfolio depends on the ratio of its alpha (α_p) to its residual standard deviation (ε_p).
- The information ratio measures the extra return we can obtain from security analysis.
- Recall,

$$S_P = \frac{E(r_P) - r_f}{\sigma_P}$$

- Information Ratio (IR)= $\frac{\alpha_p}{\sigma(\varepsilon_p)}$
- Also, known as the Appraisal ratio

Buffett again

- CAPM vs. FF 1976-2012
  - note all factors are excess returns
- CAPM, annualized $\alpha = 13.3\%$, IR

$$\tilde{r}_{brk} - r_f = 0.0111 + 0.63 \times MKT + \epsilon_{brk}$$
$$\sigma(\epsilon) = 0.063$$

Annualized IR $= \frac{12}{\sqrt{12}} \frac{0.0111}{0.063} = \sqrt{12} \times 0.176 = 0.61$

- FF 3-Factor, annualized $\alpha = 11.6\%$

$$\tilde{r}_{brk} - r_f = 0.0097 + 0.80 \times MKT - 0.29 \times SMB + 0.51 \times HML + \tilde{\epsilon}_{brk}$$
$$\sigma(\epsilon) = 0.060$$

Annualized IR $= \sqrt{12} \frac{0.0097}{0.060} = 0.56$

Interpreting α

- We saw that Berkshire Hatheway had a positive CAPM (or Jensen's) alpha of 13.3% 1976-2012:

$$\tilde{r}_{brk} - r_f = 0.0111 + 0.63 \times MKT + \epsilon_{brk}$$

$$\sigma(\epsilon) = 0.063$$

$$IR = \sqrt{12}\frac{0.0111}{0.063} = 0.61$$

- If CAPM holds, this alpha is a measure of out-performance.
- But if CAPM does not hold, the alpha could be risk!

Solution: Multi-factor model

- Consider Buffett performance eval from (FKP '13):

Coef Est.

T-stat.

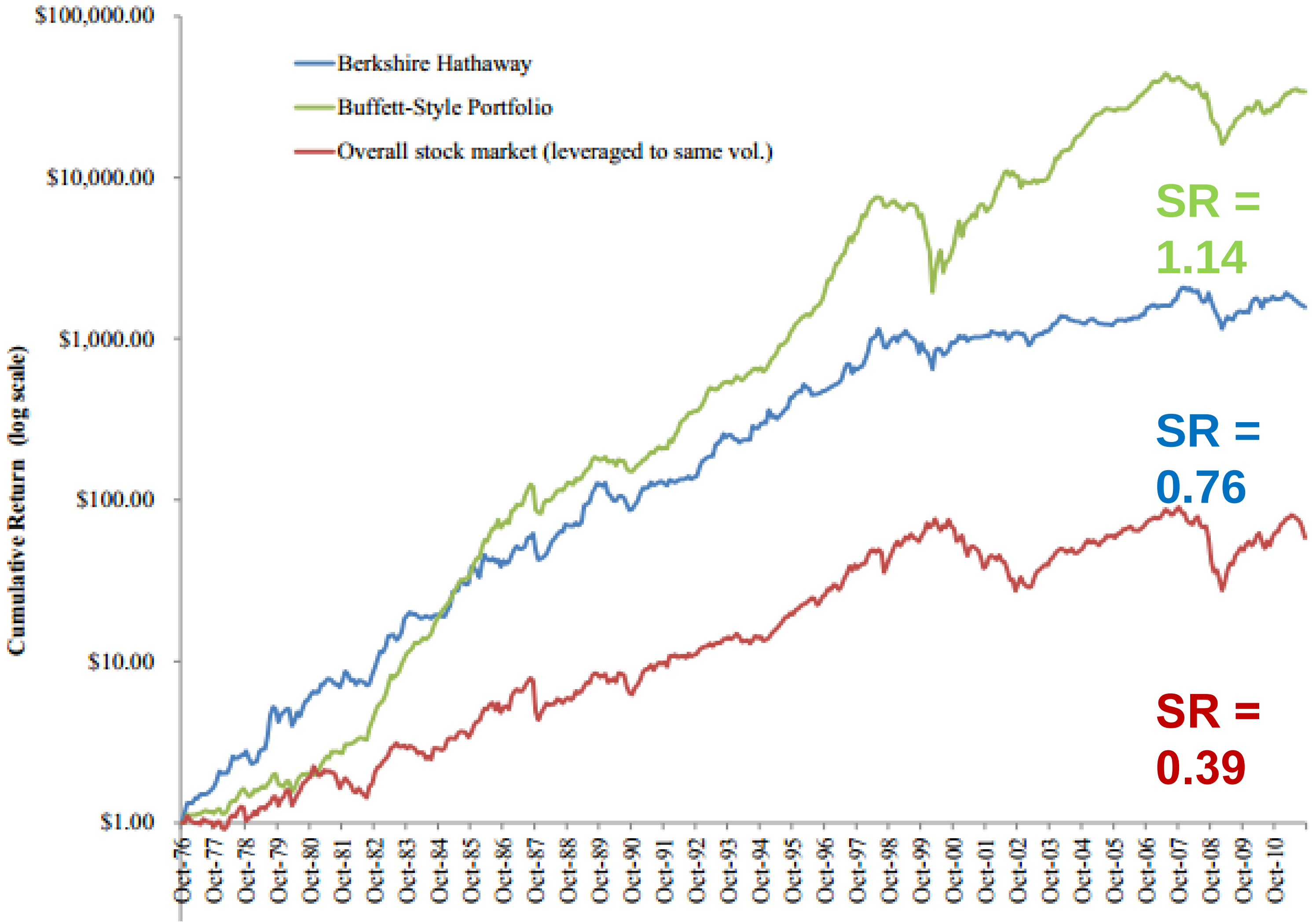
	Berkshire stock 1976 - 2011			13F portfolio 1980 - 2011			Private Holdings 1984 - 20011		
Alpha	**12.1%**	**9.2%**	6.3%	**5.3%**	3.5%	0.3%	5.6%	4.6%	4.9%
	(3.19)	(2.42)	(1.58)	(2.53)	(1.65)	(0.12)	(1.35)	(1.08)	(1.09)
MKT	**0.84**	**0.83**	**0.95**	**0.86**	**0.86**	**0.98**	**0.40**	**0.40**	**0.39**
	(11.65)	(11.70)	(10.98)	(21.55)	(21.91)	(20.99)	(5.01)	(5.01)	(3.94)
SMB	**-0.32**	**-0.32**	-0.15	**-0.18**	**-0.18**	0.00	**-0.29**	**-0.29**	**-0.31**
	-(3.05)	-(3.13)	-(1.15)	-(3.14)	-(3.22)	(0.02)	-(2.59)	-(2.53)	-(2.17)
HML	**0.63**	**0.38**	**0.46**	**0.39**	**0.24**	**0.31**	**0.39**	0.28	0.27
	(5.35)	(2.79)	(3.28)	(6.12)	(3.26)	(4.24)	(3.07)	(1.89)	(1.81)
UMD	0.06	-0.03	-0.05	-0.02	**-0.08**	**-0.10**	0.09	0.04	0.05
	(0.90)	-(0.40)	-(0.71)	-(0.55)	-(1.98)	-(2.66)	(1.13)	(0.52)	(0.55)
BAB		**0.37**	**0.29**		**0.22**	**0.15**		0.16	0.17
		(3.61)	(2.67)		(4.05)	(2.58)		(1.40)	(1.41)
QMJ			**0.43**			**0.44**			-0.05
			(2.34)			(4.55)			-(0.24)
R2 bar	0.25	0.27	0.28	0.57	0.58	0.60	0.08	0.08	0.08

What happened to α?

- Add betting-on-beta and quality-minus-junk factors to standard 4-Factor model
  - α goes from 12.1% to 6.3% and is no longer significantly different from zero.
- Why?
  - Positive loading on BAB factor.
    - Some outperformance due to buying low-beta stocks, which have positive 4-Factor alphas.
  - Positive loading on QMJ factor.
    - Some due to buying high-quality stocks, which also have positive 4-Factor alphas.

    "… It's far better to buy a wonderful company at a fair price than a fair company at a wonderful price…" – Buffett, 1989 Ann. Report

Berkshire vs. tracking portfolios



Buffett's α Conclusions

- Loadings from factor regressions can tell you about a portfolio's "style".
  - e.g., Berskshire buys low-beta, high-quality, value stocks.

- Was Buffett's α due to risk or mispricing?
  - Depends on whether you think low-beta and high-quality stocks are riskier.

- Do we give him credit if his multi-factor α is not significantly different from 0?
  - If the factors are known *ex-ante*, no!
  - But often times, people do pay "2 and 20" or similar for simple easily replicable factor exposures. DFA and AQR make a living from charging low fees for such factor exposures (beta not alpha).

APT Conclusions

- Motivation for APT
  - Asset pricing model relying on less restrictive assumptions
- E[r] of a portfolio with no firm-specific risk
  - Determined by factor betas and factor premiums
- Wide-spread use as performance measurement tool
  - e.g., FF-3 factor, benchmarking hedge-fund returns

Key Concepts

- Factors
- Factor Model
- Factor Betas
- Factor Premium
- Tracking Portfolios
- APT (Arbitrage Pricing Theory)
- Fama-French 3-Factor Model

APPENDIX

Factor Model of a Portfolio

- Portfolio return has form similar to security return

$$\tilde{r}_p - r_f = \alpha_p + \beta_{p1}\left(\tilde{r}_{F_1} - r_f\right) + \cdots + \beta_{pK}\left(\tilde{r}_{F_K} - r_f\right) + \tilde{\epsilon}_P$$

- α and β are weighted averages of portfolio stocks
  - weights on securities $1, 2, \ldots, N = w_1, w_2, \ldots, w_N$
  - Then

$$\alpha_p = \ w_1\alpha_1 + \cdots + w_N\alpha_N$$
$$\beta_{pk} = w_1\beta_{1k} + \cdots + w_N\beta_{Nk}$$

Covariance in a factor model

- Consider two stocks and a 1-factor model

$$\tilde{r}_1 = r_f + \alpha_1 + \beta_1\left(\tilde{r_F} - r_f\right) + \tilde{\epsilon}_1$$
$$\tilde{r}_2 = r_f + \alpha_2 + \beta_2\left(\tilde{r_F} - r_f\right) + \tilde{\epsilon}_2$$

$$\begin{aligned} cov(\tilde{r}_1, \tilde{r}_2) &= cov(\beta_1\tilde{r_F} + \tilde{\epsilon}_1, \beta_2\tilde{r_F} + \tilde{\epsilon}_2) \\ &= cov(\beta_1\tilde{r_F}, \beta_2\tilde{r_F}) + cov(\beta_1\tilde{r_F}, \tilde{\epsilon}_2) + cov(\tilde{\epsilon}_1, \beta_2\tilde{r_F}) \\ &\quad + cov(\tilde{\epsilon}_1, \tilde{\epsilon}_2) \\ &= cov(\beta_1\tilde{r_F}, \beta_2\tilde{r_F}) = \beta_1\beta_2 var(\tilde{r_F}) \end{aligned}$$

- In General (note assumes uncorrelated factors):

$$\sigma_{i,j} = \beta_{i1}\beta_{j1}var\left(\tilde{r}_{F_1}\right) + \cdots + \beta_{iK}\beta_{jK}var(\tilde{r}_{F_K})$$