Investments

Capital Asset Pricing Model

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This Lecture

- Introduce the Capital Asset Pricing Model (CAPM)
  - What is the model?
  - What does the model imply about the optimal portfolio?
  - How does one estimate beta?
  - Uses for performance analysis

Recap: Portfolio Theory

- Mean-variance analysis
  - Portfolio returns
    - Function of portfolio securities' returns
  - Portfolio variance
    - Function of portfolio securities' covariances
- Validity of focusing on only the mean and variance
  - Assumptions
    1. Investors only care about mean and variance, **OR**
    2. Returns are normally distributed
  - Does it have some problems? Sure, but it still teaches us:
    - Portfolios dominate individual securities.
    - Covariances are more important than variances.
    - How to construct mean-variance efficient portfolios.

Applying mean-variance analysis

- To a limited number of assets (e.g., asset classes)
  - Fairly straightforward—works well.
    - When N is small, easy to calculate means and covariances
    - Means, variances, covariances of broad asset classes are relatively stable
- With a large number of assets (e.g., individual stocks)
  - More complicated—harder to do well.
    - Calculating means/covariances may take a lot of computation
    - Mean/variances/covariances may be unstable—"Past performance no guarantee of future results"
  - But, the process can be simplified with more assumptions
    - CAPM (today)
    - Factor models, APT (later)

Can the tangency portfolio help get at a better estimate of risk premia?

- When we have r_f and tangency portfolio *T*,

$$\frac{\bar{r}_i - r_f}{cov(\widetilde{r}_i, r_T)} = K \text{ for all } i$$

- Implication

$$\frac{\bar{r}_i - r_f}{cov(\widetilde{r}_i, \tilde{r}_T)} = \frac{\bar{r}_T - r_f}{cov(\tilde{r}_T, \tilde{r}_T)}$$

$$\Rightarrow \bar{r}_i - r_f = \frac{cov(\tilde{r}_i, \tilde{r}_T)}{var(\tilde{r}_T)} (\bar{r}_T - r_f)$$

Risk vs. return in terms of β on Tangency Portfolio

- Beta

$$\beta = \frac{cov(\tilde{r}_i, \tilde{r}_T)}{var(\tilde{r}_T)}$$

- Expected return

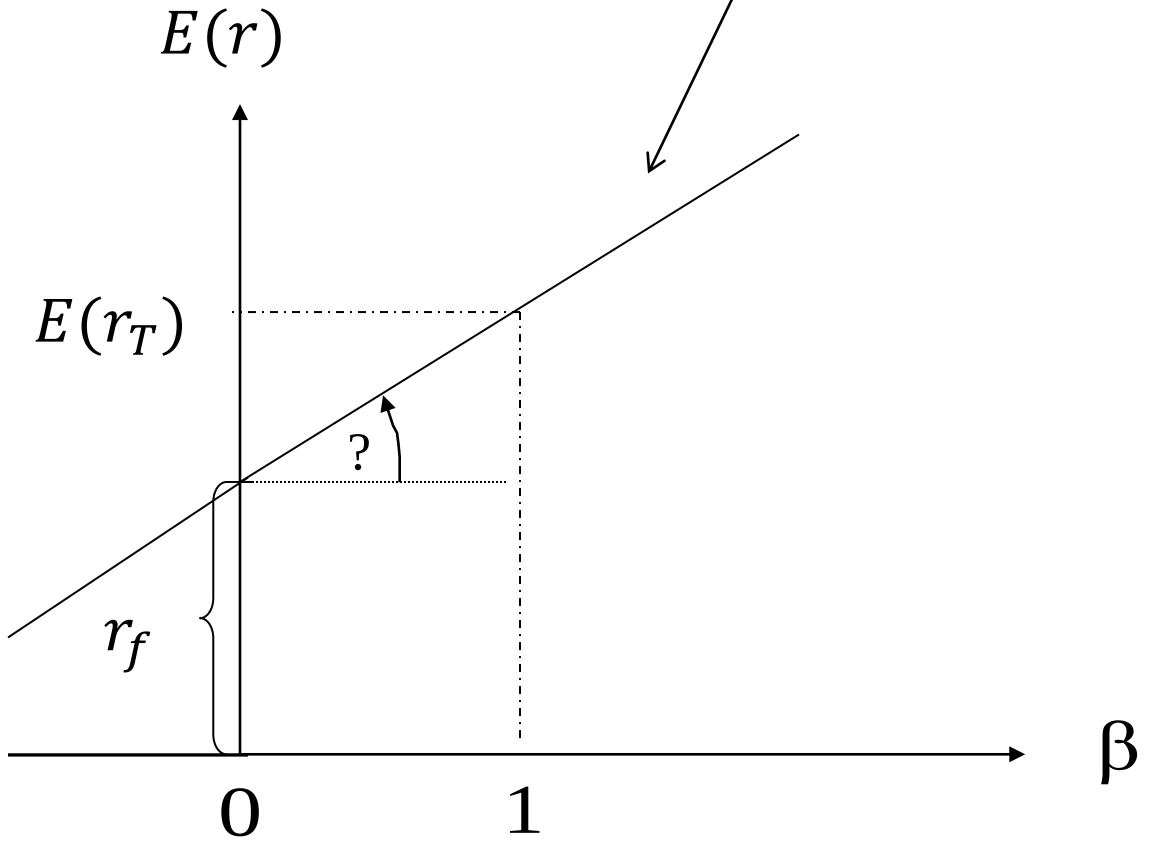
$$\bar{r}_i - r_f = \beta(\bar{r}_T - r_f)$$

This equation works so long as we know the tangency portfolio.

It is called the **Security Market Line** (SML)

Securities Market Line (SML)

$$\overline{r_i} - r_f = \beta(\bar{r}_T - r_f)$$



Security Market Line (SML)

- The Security Market Line (SML)
  - represents the ***linear*** expected returns - β relationship
- SML is plotted in beta/return space
  - as opposed to being plotted in standard deviation/return space (as in the CML)
- Only β affects expected returns
  - Because only covariance affects portfolio variance

Great! We have a security's risk premium in terms of beta on the tangency portfolio.

But we still have this nagging issue of …

how exactly do we figure out what the tangency portfolio is?

CAPM—AN EQUILIBRIUM THEORY OF THE TANGENCY PORTFOLIO

Capital Asset Pricing Model (CAPM)

- The goal of the theory is to make it easier to identify the tangency portfolio.
- Assumptions
  - Start with mean-variance analysis assumptions
    - Only means and variances matter
    - We have perfect financial markets
  - Add one more assumption
    - Investors have the same beliefs about means/sd/cov
- Result
  - The market portfolio is the tangency portfolio.

Let’s explain why, but first, what is the market portfolio?

What is the market portfolio?

- Market portfolio
  - Portfolio in which securities' weights are market values:

$$w_i = \frac{\text{Market Value}_i}{\sum_{i=1}^{N} \text{Market Value}_i}$$

  - Includes all risky assets in the economy
    - Stocks, bonds, real-estate, labor income, etc.

- Real-life proxies
  - S&P 500
  - All US stocks
  - All global stocks

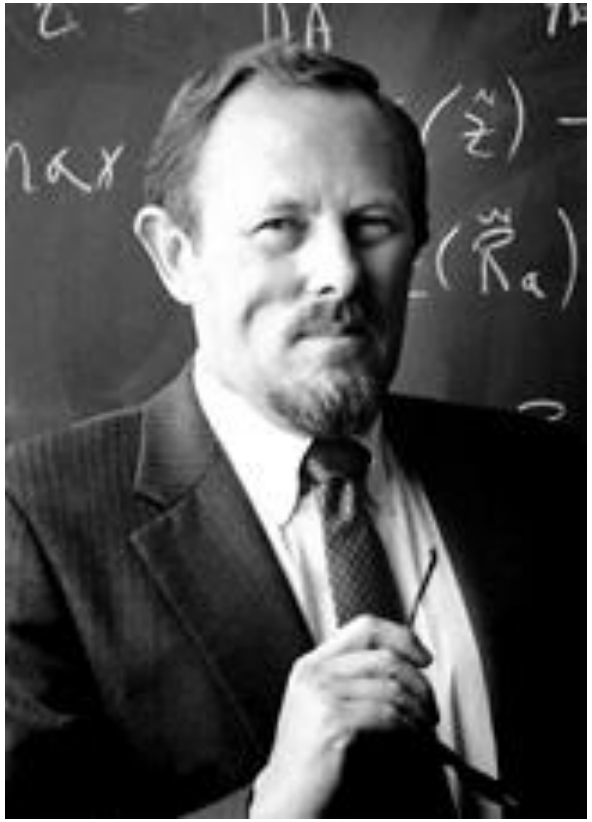
Why is the market portfolio tangent?

- Same beliefs → agree on tangency portfolio (P_T)
- Everybody holds risky assets in same proportions
  - Because risky holdings are proportional to the tangency portfolio (from two-fund separation).
    - I hold 20% risk-free bond, 80% tangency port
    - You hold -30% risk-free bond, 130% tangency port
- Thus, sum of everybody's holdings has these properties:
  1. Proportional to tangent portfolio (by two-fund sep.).
  2. Equals the market portfolio (by definition).

Therefore, the market portfolio is proportional to tangency portfolio

William F. Sharpe

- One of the originators of the CAPM
- CAPM paper submitted to Journal of Finance in 1962
  - Paper was initially considered irrelevant and rejected
  - Published in 1964 after change in editorial staff (At that time, model was independently developed by John Lintner and Jack Treynor)
- Recipient of the Nobel Prize in Economic Sciences (1990)
- Honorary degree from LBS (2008)



CAPM Theory Summary

- The CAPM prescribes a method of determining the relationship between risk and equilibrium expected returns
- An ***equilibrium model***
- Principal results:
  - *The market portfolio is the tangency portfolio—it has the highest Sharpe Ratio*
  - *The market portfolio is an efficient portfolio, and all investors will hold a combination of the market and the risk-free asset.*
  - *The risk premium on an asset will be proportional to the market risk premium and to the asset beta computed with respect to the market portfolios*

Now that we know the tangency portfolio, we can

1. Solve for our optimal portfolio holdings.
2. Calculate asset risk premia

1. Portfolio holdings in the CAPM

- All investors hold some combination of the market portfolio and risk-free asset

- Why important?
  - Hard to find tangency portfolio directly
    - With 100 securities, need to calculate 4,950 correlations
    - Especially important at time of CAPM development

2. Expected returns in the CAPM

$$E[r_i] = r_f + \beta_i\left[E(r_m) - r_f\right], \text{where}$$

$$\beta_i = \frac{cov(r_i, r_m)}{\sigma_m^2}$$

- First model to specify exactly which risks are priced
  - Market, or systematic, risks
- What is a priced risk?
  - Something is priced if exposure to it gives you extra expected return.
  - So here, market exposure (measured by CAPM β) is priced.

More intuitively

- Expected return on a share of stock =
  Risk-free rate +
  Compensation for taking on risk

  Compensation for taking on risk =
  Measure or risk (Beta) x
  Price of risk

Risk-free rate? Easy to look up.
Compensation for taking on risk? Need to estimate:

1. Beta
2. Price of risk

We measure CAPM risk with beta

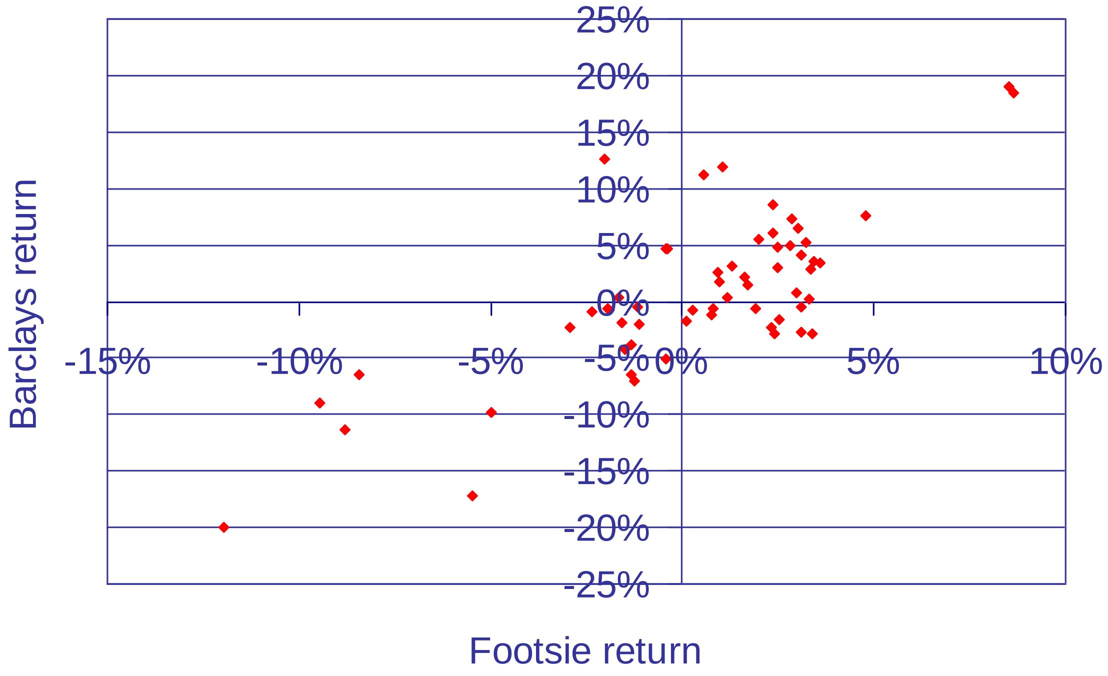
- Beta captures the extent to which a share contributes to portfolio risk
  - High beta = high risk
  - Low beta = low risk
- Intuitively, beta is a measure of a share's cyclicality
  - Beta tells you how much stock price moves for x% change in the market.
  - If beta = 1.5, then 1% change in market leads to 1.5% change in stock

Betas are estimated

- Different services come up with different betas, because they use different approaches to measure co-movement with the market:
  - How many years of data?
  - What periodicity?
  - Which market?
- Recommendations:
  - 2-5 years (2 years if firm is changing, 5 if stable).
  - Weekly is best, monthly second best.
  - Use the market relevant to your investors.
  - Consider "Bloomberg Adjustment"
    - 2/3 x data beta + 1/3 x 1 = adjusted beta

How to estimate beta?

- Obtain data on monthly returns on the share and the market:
  - E.g. Barclays shares and Footsie 100
- Make a scatter plot of monthly returns on the individual security you want to compute the beta and the monthly returns on the market.



How to estimate beta?

- Beta is the slope of a regression line which best fits the scatter plot of monthly returns on the shares of the firm and on the market index. The regression line has the formula:

$$R_i = \alpha + \beta_i \times R_M$$

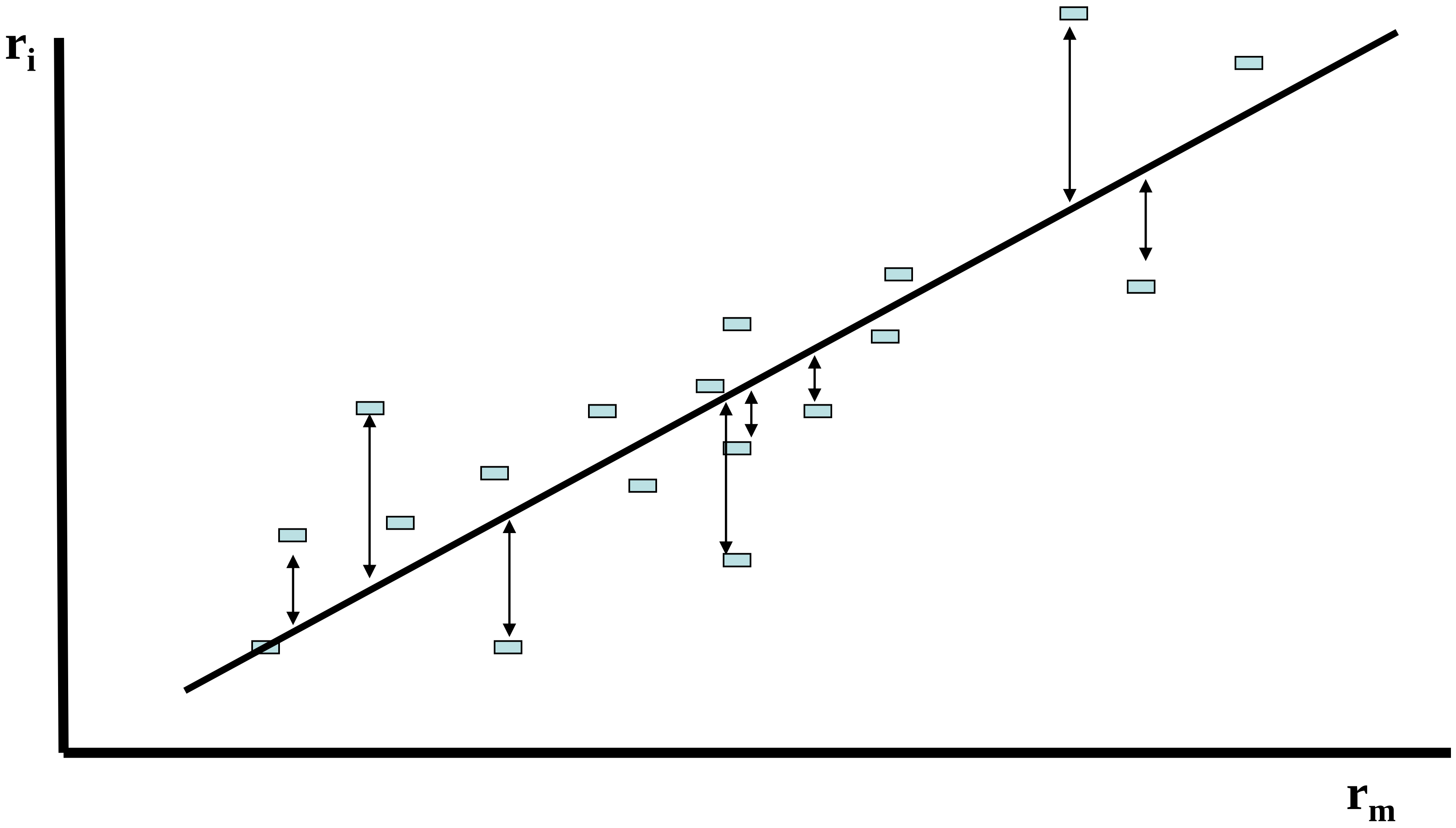
where the slope β_i is defined as: $\beta_i = \dfrac{Cov(R_i, R_M)}{Var(R_M)}.$

- Since $Cov(A,B) = \rho_{A,B}\sigma_A\sigma_B$ then:

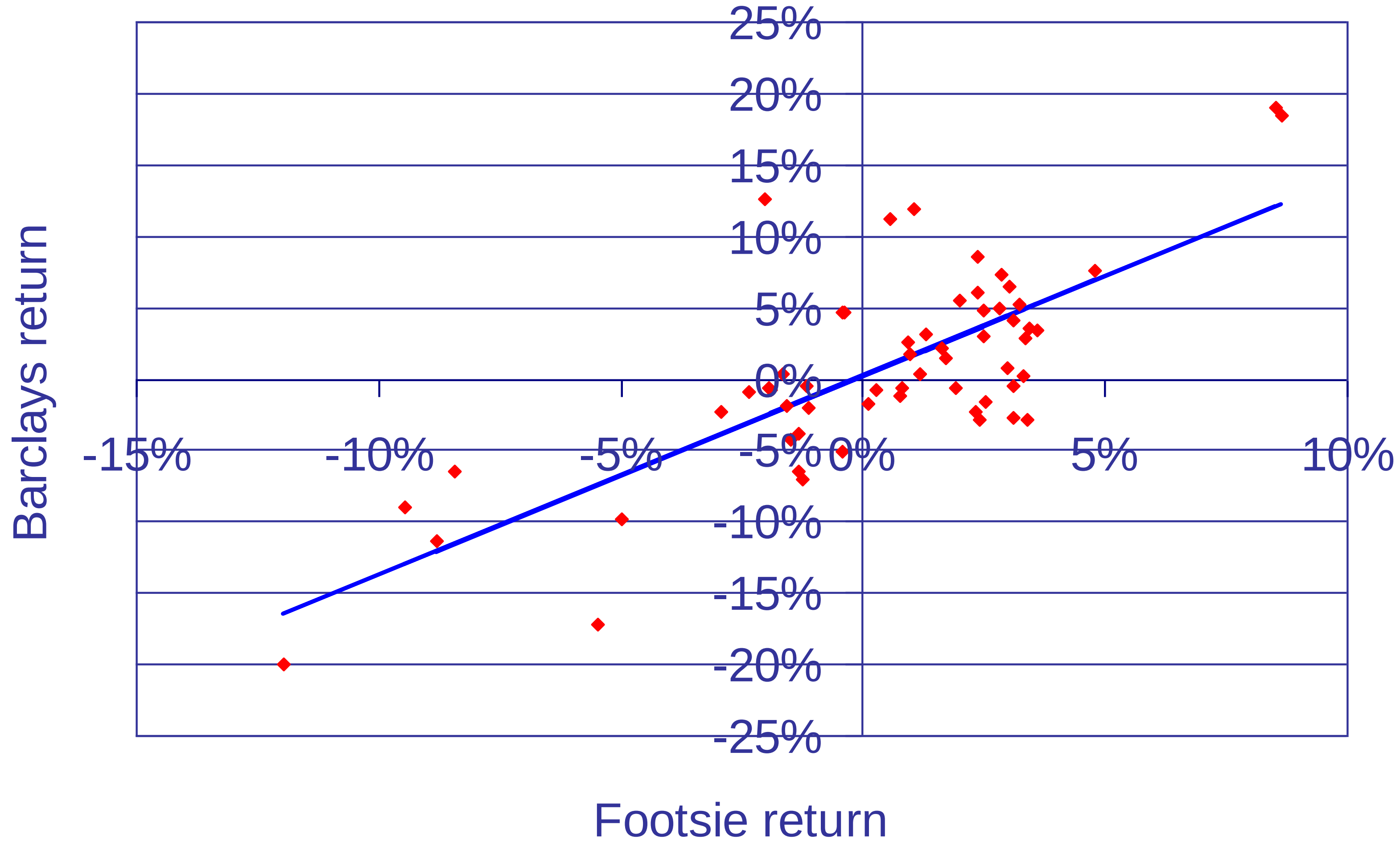
$$\beta_i = \frac{\rho_{R_i,R_{Market}}\sigma_{R_i}\sigma_{R_{Market}}}{\sigma^2_{R_{Market}}} = \frac{\rho_{R_i,R_{Market}}\sigma_{R_i}}{\sigma_{R_{Market}}}.$$

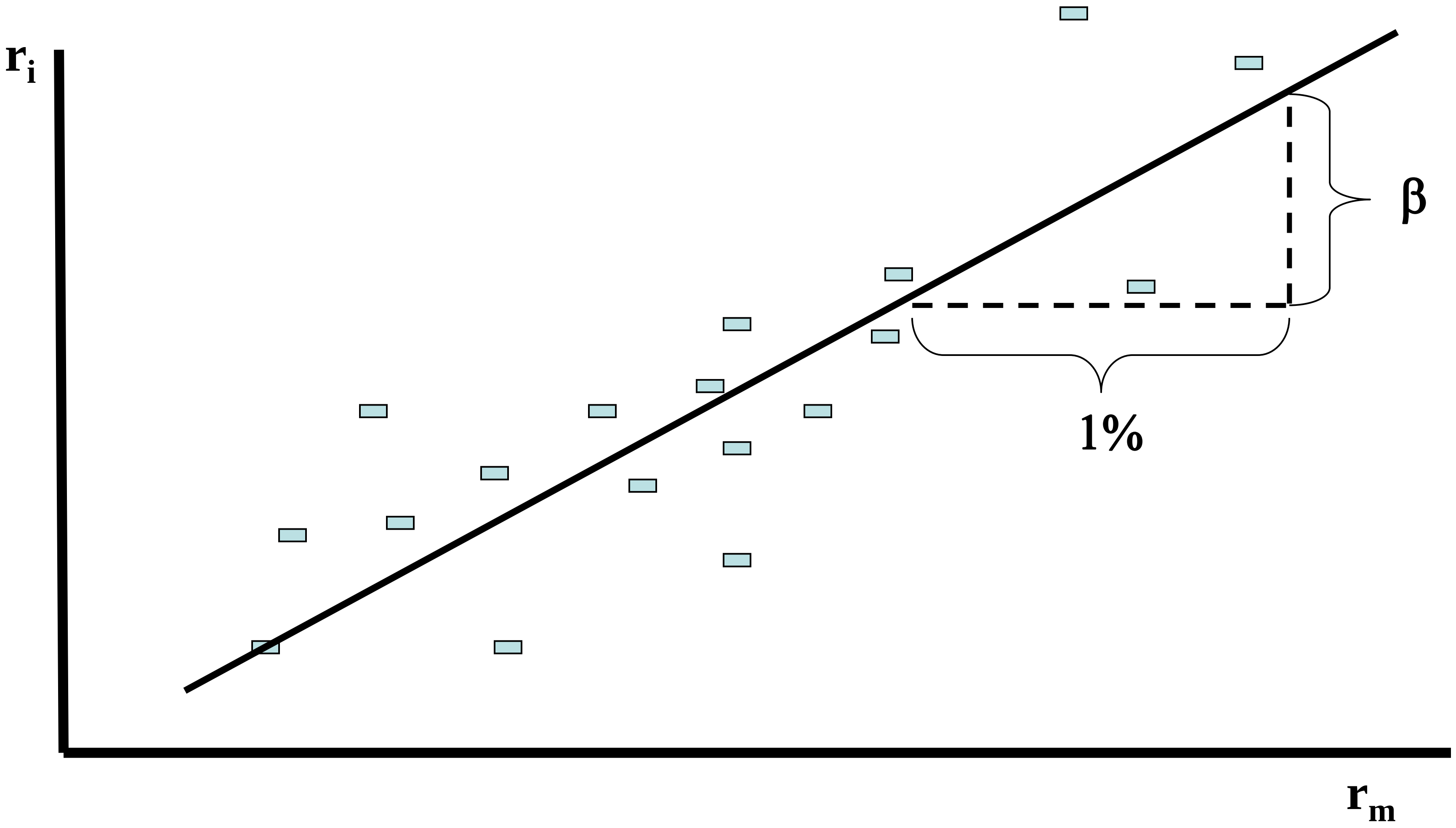
- A very volatile stock can have a low beta if its correlation with the market is low. Can you think of examples?

Draw line that minimizes squared error



Beta is the slope of the regression line




r_i
β
1%
r_m

Regression output (from excel)

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.75595855
R Square	0.57147333
Adjusted R Square	0.563955318
Standard Error	0.046209779
Observations	59

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.162315783	0.162315783	76.01389146	4.46281E-12
Residual	57	0.12171459	0.002135344		
Total	58	0.284030374			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	0.001563598	0.006048032	0.258530022	0.796929159	-0.010547377	0.013674573	-0.010547377	0.013674573
X Variable 1	1.396329821	0.160155379	8.718594581	4.46281E-12	1.075624209	1.717035434	1.075624209	1.717035434

Price of Risk

- The price of risk is the extra return expected from investing in shares compared to safe assets:
  - In the CAPM it is the difference between the average rate of return on stocks (the market return) and the risk-free rate
  - Around 5%
  - Values somewhere between 4 and 6% are reasonable

- What are safe assets: Government debt
  - Use the yield on short-term UK or US government debt. Look-up online.

Expected return on Barclays shares

We estimated Barclays equity has beta of 1.4

Assume:

- Market risk-premium is 5%
- Risk-free rate is 4.5%

4.5% + 1.4 (5%) = 11.5%

This is the cost of equity for Barclays.

When you value the shares of Barclays, this is the rate you should use to discount cash flows to equity holders.

Regressions in Excel

- Excel
  - Make sure that the regression module is installed
    (File → Options → Add-Ins → Analysis Toolpack)
  - Implementation
    (Data → Data Analysis → Regression)

- I often use R http://cran.r-project.org/
  - Steeper learning curve, but more powerful

Application of the CAPM

PERFORMANCE EVALUATION

Performance Evaluation

- In asset management, a major problem is how to appropriately judge a fund’s performance
  - Usually with historical data
- What are some potential methods?
  1. Highest average return
  2. Sharpe Ratio
  3. CAPM: ***Jensen’s alpha***

Jensen's alpha

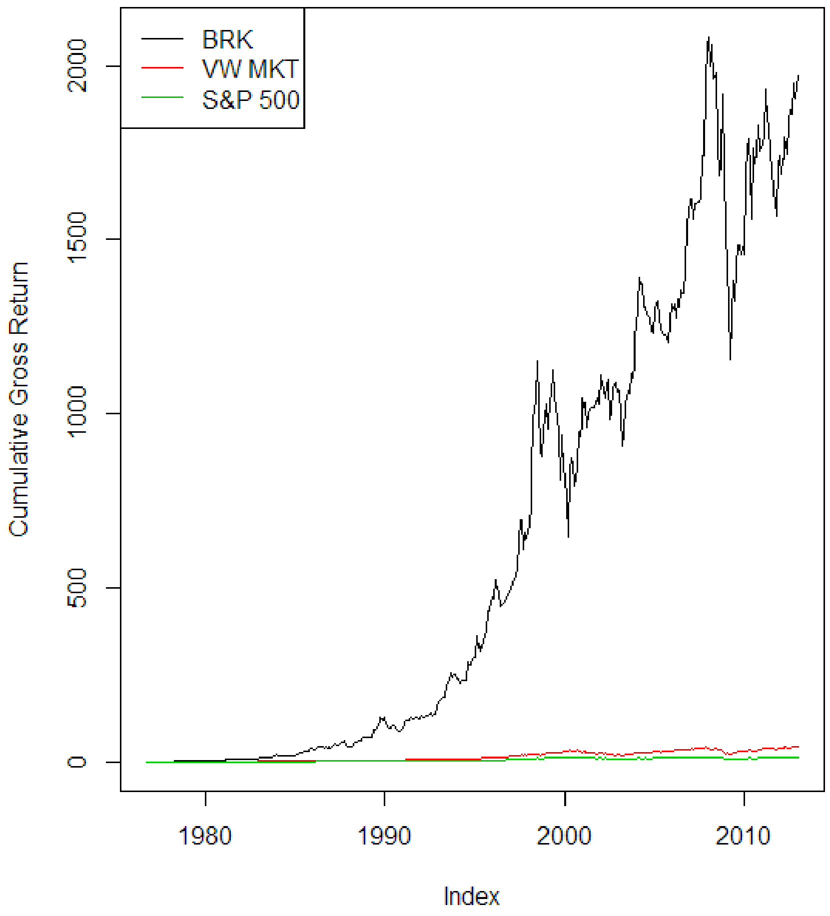
- We typically benchmark a fund's returns against an index (S&P 500). What do we want from a good manager?
  - High average returns that are unrelated to the market
  - Why? We can get market exposure through a S&P 500 fund.
- How to quantify this? Compute alpha from:

$$r_p - r_f = \alpha_p + \beta_p (r_m - r_f) + \varepsilon_p$$

What is a good alpha?

- For hedge funds, the goal is to produce alpha, returns unrelated to the market.
- What makes a good manager? Generally speaking a manager that has an alpha over a couple of percentage points is doing a good job.
- What is Warren Buffett's alpha?

Warren's Results



Berkshire's Corporate Performance vs. the S&P 500

Book Values →

Year	Annual Percentage Change in Per-Share Book Value of Berkshire (1)	in S&P 500 with Dividends Included (2)	Relative Results (1)-(2)
1965	23.8	10.0	13.8
1966	20.3	(11.7)	32.0
1967	11.0	30.9	(19.9)
1968	19.0	11.0	8.0
1969	16.2	(8.4)	24.6
1970	12.0	3.9	8.1
1971	16.4	14.6	1.8
1972	21.7	18.9	2.8
1973	4.7	(14.8)	19.5
1974	5.5	(26.4)	31.9
1975	21.9	37.2	(15.3)
1976	59.3	23.6	35.7
1977	31.9	(7.4)	39.3
1978	24.0	6.4	17.6
1979	35.7	18.2	17.5
1980	19.3	32.3	(13.0)
1981	31.4	(5.0)	36.4
1982	40.0	21.4	18.6
1983	32.3	22.4	9.9
1984	13.6	6.1	7.5
1985	48.2	31.6	16.6
1986	26.1	18.6	7.5
1987	19.5	5.1	14.4
1988	20.1	16.6	3.5
1989	44.4	31.7	12.7
1990	7.4	(3.1)	10.5
1991	39.6	30.5	9.1
1992	20.3	7.6	12.7
1993	14.3	10.1	4.2
1994	13.9	1.3	12.6
1995	43.1	37.6	5.5
1996	31.8	23.0	8.8
1997	34.1	33.4	0.7
1998	48.3	28.6	19.7
1999	0.5	21.0	(20.5)
2000	6.5	(9.1)	15.6
2001	(6.2)	(11.9)	5.7
2002	10.0	(22.1)	32.1
2003	21.0	28.7	(7.7)
2004	10.5	10.9	(0.4)
2005	6.4	4.9	1.5
2006	18.4	15.8	2.6
2007	11.0	5.5	5.5
2008	(9.6)	(37.0)	27.4
2009	19.8	26.5	(6.7)
2010	13.0	15.1	(2.1)
2011	4.6	2.1	2.5
2012	14.4	16.0	(1.6)
Compounded Annual Gain – 1965-2012	19.7%	9.4%	10.3
Overall Gain – 1964-2012	586,817%	7,433%	

Warren’s alpha

- Market Model regression using monthly returns 1976-2012:

$$E\left(r_{Buffet} - r_f\right) = 0.011 + 0.62 \times E\left(r_{VWMKT} - r_f\right)$$

Or, using S&P 500

$$E\left(r_{Buffet} - r_f\right) = 0.013 + 0.68 \times E\left(r_{S\&P} - r_f\right)$$

- Berkshire outperformed the equilibrium return by an annualized 13.2% (15.6% using S&P) per year, and its exposure to risk is 2/3 that of the market!

Summary of CAPM

- Investors don’t like uncertainty
- Diversification reduces risk, so investors have diversified portfolios
- A security’s beta indicates how adding that security to a ***diversified portfolio*** will alter the portfolio’s risk
- **Conclusion:** investors must care about a security’s beta, implying ***higher beta risk*** must be rewarded with ***higher expected returns***

What's next...

- Empirical validity of the CAPM. Does the CAPM hold well in practice?
- If not, what else helps?
- Factor models and the APT