

# CSci 423 Homework 9

Due: 1:00 pm, Wednesday, 11/14

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1. (7 points) Exercise 4.3 on page 183.

A decidable language is closed under the complement. TM  $M_{\bar{L}}$  which decides  $\bar{L}$  will accept when TM  $M_L$  that decides  $L$  rejects, and reject when  $M_L$  accepts. It has been shown that  $E_{DFA}$  is decidable where  $E_{DFA} = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) = \emptyset \}$ .

On input  $\langle A \rangle$  where  $A$  is a DFA:

- (a) Mark the start state of  $A$ .
  - (b) Repeat until no new states are marked.
  - (c) Mark states that has a transition coming in from any state that is already marked.
  - (d) If no accept state is marked, accept. Otherwise, reject.
2. (7 points) Exercise 4.6 on page 183.
- Proof by contradiction. Assume that  $B$  is countable. Infinite sequences over 0, 1 can be represented by  $N = 1, 2, 3, \dots$ . A  $f(n)$  can be mapped where  $n \in N$ .  $f(n) = (w_{n1}, w_{n2}, w_{n3}, \dots)$  where  $w_{ni}$  is the  $i$ th bit in the  $n$ th sequence. Using diagonalization, it is possible to find a sequence over 0, 1 that is not in  $f(n)$ , where the  $i$ th bit is the opposite of the  $i$ th sequence. This means that the sequences are  $\notin B$ , giving us a contradiction, proving  $B$  to be uncountable.

n	$f(n)$
1	$(w_{11}, w_{12}, w_{13}, w_{14}, \dots)$
2	$(w_{21}, w_{22}, w_{23}, w_{24}, \dots)$
3	$(w_{31}, w_{32}, w_{33}, w_{34}, \dots)$
$\vdots$	$\vdots$

3. (7 points) Exercise 4.7 on page 183.

\*reference [www.cs.ucdavis.edu/~gusfield/cs120f11/HW6key.pdf](http://www.cs.ucdavis.edu/~gusfield/cs120f11/HW6key.pdf)\*

To prove that  $T$  is countable we provide a function  $f: T \rightarrow \mathbb{N}$ . Given a triple  $(i, j, k)$  from  $T$   $f$  is defined as follows: convert each number  $i, j$ , and  $k$  to their respective binary representation. We will compute the natural number  $n$  in its binary representation. For the  $a$ th bit of  $n$ , we divide  $a$  by 3. This will produce a quotient  $q$  and a remainder  $r$ . If  $r$  is 0 then we extract the  $q$ th bit from  $i$ , if  $r$  is 1 then we extract the  $q$ th bit from  $j$ , if  $r$  is 2 then we extract the  $q$ th bit from  $k$ . Note that if the  $q$ th bit of  $i, j$ , or  $k$  is not defined then use a 0. Note that  $f$  maps every  $i, j, k$  tuple to a natural number and every natural number can be converted to an  $i, j, k$  tuple (by the reverse process). Hence  $f$  is a one-to-one function and onto (aka a bijection). Therefore  $T$  is countable.

4. (7 points) Problem 4.19 on page 184.

\*test.scripts.psu.edu/users/a/y/.../Decidable%20Languages(2).docx\*

Let  $P = \text{ADFA}$ . We know this to be decidable.

We also know that regular languages are closed under the reverse operation. So, if there is a DFA  $D$  for  $w$ , there is a DFA  $D^R$  for  $w^R$ . We can then construct an NFA  $N$  by branching to  $D$  and  $D^R$ . Since we have an algorithm to convert NFAs to DFAs, we run it on  $\langle N \rangle$  to get  $\langle M \rangle$ . Now we have a

DFA  $M$ , and since ADFA is decidable, there is some  $S$  such that  $S = \{ \langle M \rangle \}$ . Since  $M$  is a DFA that accepts  $wR$  whenever it accepts  $w$ ,  $S = \{ \langle M \rangle \mid M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w \}$ , and  $S$  is decidable.

5. (12 points) In class, we learned that  $A_D$  is non-TR,  $A_{TM}$  and  $HALT_{TM}$  are TR but non-TD. What can you say about their complements? Are they non-TR, TR but non-TD, or TD? Justify your answers.

Both  $\bar{A}_{TM}$  and  $\bar{HALT}_{TM}$  are not TR since  $A_{TM}$  and  $HALT_{TM}$  are TR. If both sets were TR, then they would also be TD.

$\bar{A}_D$  is not TD, according to the closure property, because  $A_D$  is not TD.  $\bar{A}_D$  is, on the other hand, TR. In the case  $A_D = \{w_i \mid w_i \notin L(M_i)\}$ , its complement  $\bar{A}_D = \{w_i \mid w_i \in L(M_i)\}$ . A TM  $R$  can be constructed to take  $w_i$  as input, then use the string to code TM  $M_i$ . Both are then fed into  $A_{TM}$ , which will accept if it accepts. This shows that  $\bar{A}_D$  is TR.

## **Reading Summary 4**

The Turing machine is far from being bettered by other models. Over 75 years, the Turing Machine is still considered the best machine model available. It can be used to model both modern computing and the natural world. What we have learned over the years in computing is a result of the model.

The Turing machine was so special because it disembodied the machine and broadened the view of computation. It brought the view away from what goes specifically into the machine, but how the machine reacts and works as a whole. There are also challenges to the standard model, which all have an impact on universality. The most generally respected challenge to the model is that “one can apply it ever more widely to what we observe in nature, and let it mold our expectations of what computing machines can achieve.