CSci 423 Homework 5

Due: 1:00 pm, Wednesday, 10/17

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1. (10 points) Exercise 2.6 (b) and (d) on page 129.

   b.

   $S \to aSb|Ra|bR$
   $R \to Ra|Rb|\varepsilon$

   d.

   $S \to T|U\#T|T\#U|U\#T\#U$
   $T \to a|b|aTa|bTb|\#|\#U\#|\varepsilon$
   $U \to U\#W|W$
   $W \to aW|bW|\varepsilon$

2. (5 points) Exercise 2.9 on page 129.

   CFG:

   $S \to RU|RT$
   $R \to aRb|\varepsilon$
   $T \to bTc|\varepsilon$
   $U \to cU|\varepsilon$
   $W \to aW|\varepsilon$

   Yes the grammar is abiguous because there is a string a string that can be derived in more than 1 fashion. abc can be derived:
   1) $S \implies R \implies aRbU \implies abU \implies abc$
   2) $S \implies T \implies WbTc \implies Wbc \implies abc$

3. (10 points) Exercise 2.13 on page 129.
   *Referenced: http://courses.engr.illinois.edu/cs373/fa2010/Problem Sets/hw9sol.pdf *
   a) **Description:** The resulting string of L(G) can result in two situations. One will have one # in between zero or more 0s, where the 0s on the right of the # is twice the number of 0s on the left. The situation results in a string that will have two #'s with any number of 0s within the string.
   b) **Proof:** We will use the closure property

   Consider a homomorphism h_1: $0,1 \to 0,\#$, such that $h_1(0) = 0, h_1(\#) = \#, and h_1(1) = 00$. Let $K_1 = h_1^{-1}(L(G))$.

   Let $K_2 = K_1 \cap L(0^i\#1^i)$. Observe that the only strings in L(G) that have a single # symbol are those belonging to L_2, and given the definition of homomorphism h_1, we can conclude that $K_2 = 0^i\#1^i i \geq 0$.

   Consider the homomorphism $h_2 : 0,1,\# \to 0,1$, where $h_2(0)$ = 0, $h_2(1)$ = 1, and $h_2(\#)$ = ε. Observe that $K_3 = h_2(K_2) = 0^n1^n|n \geq 0$.

   Since K_1, K_2, K_3 are obtained from L(G) through regularity preserving operations and K_3 is non-regular, we can conclude that L(G) is not regular.

4. (5 points) Exercise 2.14 on page 129.

   1. $S_0 \to A$
      $A \to BAB|B|\varepsilon$
      $B \to 00|\varepsilon$
   2. $S_0 \to A$
      $A \to BAB|B|AB|BA|\varepsilon$
      $B \to 00$
   3. $S_0 \to A|\varepsilon$
      $A \to BAB|B|AB|BA|BB$
      $B \to 00$
   4. $S_0 \to BAB|B|AB|BA|BB|\varepsilon$
      $A \to BAB|B|AB|BA|BB$
      $B \to 00$
   5. $S_0 \to UB|0C|AB|BA|BB|\varepsilon$
      $A \to UB|0C|AB|BA|BB$
      $B \to 0C$
      $C \to 0$
      $U \to BA$

5. (10 points) Exercise 2.10 on page 129. (Give the state diagram of your PDA.)
   Description: The PDA checks the case that $i = j$ and in another part that $j = k$. The first part has an a pushed for every a read, and an a popped for every b read. $ will be the only one in the stack if there is an equal number of a's and b's. c's are then read and dealt with at the finish state. In the second part, a's are read and produce nothing, while a b will be pushed for every b read, and popped off for every c read.
   State Diagram: