# CSci 423 Homework 11

Due: 1:00 pm, Wednesday, 11/28
Eric Shih

1. (10 points) Problem 5.17 on page 212. (Hint: Give a simple algorithm) If the alphabet is unary, the dominoes only differ in the number of 1s that each has on the top and bottom. This case is solved by:

   M=Given a collection of dominoes

   (a) If some domino has the same number of 1s on top and bottom, there is a trivial match, so accept.
   (b) If all the dominoes have more 1s on top than on bottom, there is no possibility of a match, so reject. Likewise, if all the dominoes have less 1s on top than on bottom, reject.
   (c) Find one domino with more 1s on top than on bottom (say a difference of a 1s), and one domino with more 1s on bottom than on top (say a difference of b 1s). Choosing b of the first domino and a of the second should make an an equal number of 1s on both top and bottom, and hence a match, thus accept.

2. (10 points) This problem considers an attempt at a polynomial reduction from one problem to another that does not work. Your task is to find the flaw. A bipartite graph is an undirected graph in which every cycle has even length. We attempt to show that the Hamiltonian cycle (a cycle that passes through each node exactly once) problem polynomially reduces to the Hamiltonian cycle problem in bipartite graphs. We need a function T: {graphs}→{bipartite graphs} such that T can be computed in polynomial time and for any graph G, G has Hamiltonian cycle iff $T(G)$ has a Hamiltonian cycle. Let $T(G)$ be the bipartite graph obtained by inserting a new vertex on every edge. What is wrong with this transformation?

   This transformation has a condition that for any graph G, G has a Hamiltonian cycle iff T(T) has a Hamiltonian cycle. A counter-example will be used to show the error. If G is the graph shown here:
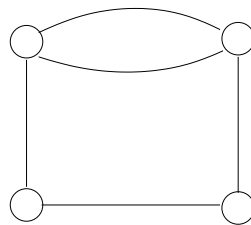
   Graph G

   G does have a Hamiltonian cycle, but there is a T(G) that does not have a Hamiltonian path.
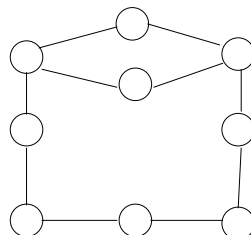
Graph T(G)

From here, it is obvious that there is no path that visits each vertex once while also finishing at the starting vertex. Thus it does not have a Hamiltonian cycle showing what is wrong with the transformation.

3. (10 points) The SET INTERSECTION (SI) problem is defined as follows:
   INSTANCE: Finite sets $A_1, \ldots, A_m$ and $B_1, \ldots, B_n$.
   QUESTION: Is there a set T such that $|T \cap A_i| \geq 1$ for $i = 1, \ldots, m$ and $|T \cap B_j| \leq 1$ for $j = 1, \ldots, n$?
   Show that SAT polynomially reduces to SI.
   reference research.cs.queensu.ca/ cisc365/2010F/365%20SetIntersectionNPC.pdf

   For a CNF expression, T can be transformed as follows:

   (a) For each clause in the CNF expression, it is turned to set A_i. Each literal in A_i becomes an element of the set. The unnegated literal x_i will become element T_i. Negated literal $\bar{x}_i$ will become element F_i. m in the set intersection will equal the number of clauses in the CNF expression

   (b) For each boolean variable, set B_j can be created with elements T_j and F_j. n from the set intersection will be the number of boolean variables.

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