CSci 423 Homework 9

Due: 1:00 pm, Wednesday, 11/14
Eric Shih

1. (7 points) Exercise 4.3 on page 183.
A decidable language is closed under the complement. TM $M_{\bar{L}}$ which decides $\bar{L}$ will accept when TM $M_{\bar{L}}$ that decides L rejects, and reject when $M_{\bar{L}}$ accepts. It has been shown that E_{DFA} is decidable where E_{DFA} = $\{< A > |$ is a DFA and $L(A) = \emptyset\}$.
On input $< A >$ where A is a DFA:
   (a) Mark the start state of A.
   (b) Repeat until no new states are marked.
   (c) Mark states that has a transition coming in from any state that is already marked.
   (d) If no accept state is marked, accept. Otherwise, reject.

2. (7 points) Exercise 4.6 on page 183.
Proof by contradiction. Assume that B is countable. Infinate sequences over $0,1$ can be represented by $N = 1,2,3,\ldots.$ A $f(n)$ can be mapped where $n \in N$. $f(n) = (w_{n1}, w_{n2}, w_{n3}, \ldots)$ where w_{ni} is the ith bit in the nth sequence. Using diagonalization, it is possible to find a sequence over $0,1$ that is not in $f(n)$, where the ith bit is the opposite of the ith sequence. This means that the sequences are $\notin B$, giving us a contradiction, proving B to be uncountable.

n	$f(n)$
1	$(w_1 1, w_1 2, w_1 3, w_1 4, \ldots)$
2	$(w_2 1, w_2 2, w_2 3, w_2 4, \ldots)$
3	$(w_3 1, w_3 2, w_3 3, w_3 4, \ldots)$
$\vdots$	$\vdots$

3. (7 points) Exercise 4.7 on page 183.
reference www.cs.ucdavis.edu/ gusfield/cs120f11/HW6key.pdf
To prove that T is countable we provide a function f : T N. Given a triple (i, j, k) from T f is dened as follows: convert each number i, j, and k to their respective binary representation. We will compute the natural number n in its binary representation. For the ath bit of n, we divide a by 3. This will produce a quotient q and a remainder r. If r is 0 then we extract the qth bit from i, if r is 1 then we extra the qth bit from j, if r is 2 then we extract the qth bit from k. Note that if the qth bit of i, j, or k is not dened then use a 0. Note that f maps every i, j, k tuple to a natural number and every natural number can be converted to an i, j, k tuple (by the reverse process). Hence f is a one-to-one function and onto (aka a bijection). Therefore T is countable.

4. (7 points) Problem 4.19 on page 184.
test.scripts.psu.edu/users/a/y/.../Decidable%20Languages(2).docx
Let P = ADFA. We know this to be decidable.
We also know that regular languages are closed under the reverse operation. So, if there is a DFA D for w, there is a DFA D for w^R. We can then construct an NFA N by branching to D and D. Since we have an algorithm to convert NFAs to DFAs, we run it on $< N >$ to get $< M >$. Now we have a

DFA M, and since ADFA is decidable, there is some S such that S = $\{< M >\}$. Since M is a DFA that accepts wR whenever it accepts w, $S = \{< M > |M$ is a DFA that accepts w^R whenever it accepts $w\}$, and S is decidable.

5. (12 points) In class, we learned that A_D is non-TR, A_{TM} and $HALT_{TM}$ are TR but non-TD. What can you say about their complements? Are they non-TR, TR but non-TD, or TD? Justify your answers.

Both $A_{\bar{T}M}$ and $HA\bar{L}T_{TM}$ are not TR since A_{TM} and $HALT_{TM}$ are TR. If both sets were TR, then they would also be TD.
$\bar{A_D}$ is not TD, according to the closure property, because A_D is not TD. $\bar{A_D}$ is, on the other hand, TR. In the case $A_D = \{w_i | w_i \notin L(M_i)\}$, its complement $\bar{A_D} = \{w_i | w_i \in L(M_i)\}$. A TM R can be constructed to take w_i as input, then use the string to code TM M_i. Both are then fed into $A_T M$, which will accept if it accepts. This shows that $\bar{A_D}$ is TR.

Reading Summary 4