CSci 423 Homework 9

Due: 1:00 pm, Wednesday, 11/14
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1. (7 points) Exercise 4.3 on page 183.
   A decidable language is closed under the complement. TM $M_{\bar{L}}$ which decides $\bar{L}$ will accept when TM $M_{\bar{L}}$ that decides L rejects, and reject when $M_{\bar{L}}$ accepts. It has been shown that E_{DFA} is decidable where E_{DFA} = $\{< A > |$ is a DFA and $L(A) = \emptyset\}$.
   On input $< A >$ where A is a DFA:

   (a) Construct a DFA B that recognizes $L(\bar{A})$, by swapping accept and non-accepting states.

   (b) Run the TM E on input $< B >$, where E is the decider of E_DFA.

   (c) If E accepts, accept. If E rejects, reject.

2. (7 points) Exercise 4.6 on page 183.
   Proof by contradiction. Assume that B is countable. Infinate sequences over $0,1$ can be represented by $N = 1,2,3,....$ A $f(n)$ can be mapped where $n \in N$. $f(n) = (w_{n1}, w_{n2}, w_{n3}, ...)$ where w_{ni} is the ith bit in the nth sequence. Using diagonalization, it is possible to find a sequence over $0,1$ that is not in $f(n)$, where the ith bit is the opposite of the ith sequence. This means that the sequences are $\notin B$, giving us a contradiction, proving B to be uncountable.

n	$f(n)$
1	$(w_{11}, w_{12}, w_{13}, w_{14}, ...)$
2	$(w_{21}, w_{22}, w_{23}, w_{24}, ...)$
3	$(w_{31}, w_{32}, w_{33}, w_{34}, ...)$
$\vdots$	$\vdots$

3. (7 points) Exercise 4.7 on page 183.
   reference staffwww.dcs.shef.ac.uk/people/J.Marshall/alc/studyguides/Selected.pdf
   Construct a function f :: T $\rightarrow$ N which is uniquely invertible. Remember the unique factorisation theorem, which states that every number in N is expressible as a unique product of primes. Consequently we may choose three arbitrary primes $p \neq q \neq r$ and define our function $f(i, j, k) = p^i q^j r^k$. From the unique prime factorisation theorem we see that any choice of i, j, k uniquely determines a value of f, and so f is invertible.

4. (7 points) Problem 4.19 on page 184.
   reference http://www.scribd.com/doc/47299154/SolutionManualIntroductionSipser.pdf For any language A, let $A^R = \{w^R | w \in A\}$. If $< M >$ in S, then $L(M) = L(M)^R$. The following TM T decides S.
   T = On the input $< M >$, where M is a DFA:

   (a) Construct DFA N that recognizes $L(M)^R$.

   (b) Run TM F from Theorem 4.5 on $< M, N >$, were F is the Turing machine deciding EQ_{DFA}.

   (c) If F accepts, accept. If F rejects, reject.

5. (12 points) In class, we learned that A_D is non-TR, A_{TM} and $HALT_{TM}$ are TR but non-TD. What can you say about their complements? Are they non-TR, TR but non-TD, or TD? Justify your answers.

Both $\bar{A_{TM}}$ and $HA\bar{L}T_{TM}$ are not TR since A_{TM} and $HALT_{TM}$ are TR. If both sets were TR, then they would also be TD.
$\bar{A_D}$ is not TD, according to the closure property, because A_D is not TD. $\bar{A_D}$ is, on the other hand, TR. In the case $A_D = \{w_i | w_i \notin L(M_i)\}$, its complement $\bar{A_D} = \{w_i | w_i \in L(M_i)\}$. A TM R can be constructed to take w_i as input, then use the string to code TM M_i. Both are then fed into $A_T M$, which will accept if it accepts. This shows that $\bar{A_D}$ is TR.

Reading Summary 4

The turing machine is far from being bettered by other models. Over 75 years, the Turing Machine is still considered the best machine model available. It can be used to model both modern computing and the natural world. What we have learned over the years in computing is a result of the model.

The Turing machine was so special because it disembodied the machine and broadened the view of computation. It brought the view away from what goes specifically into the machine, but how the machine reacts and works as a whole. There are also challenges to the standard model, which all have an impact on universality; they are labelled into sections called reductionalists, impressionists, remodelers, and incomputability theorists(recursion theorists). The most generally respected challenge to the model, reductionalists, is explained by saying “one can apply it ever more widely to what we observe in nature, and let it mold our expectations of what computing machines can achieve.

The impressionists are a large force in the community of real-world ”computation“. Real-world computation is how nature and people in the real world compute. The old definition of computation being steps carried out by a computer is now being challenged by the idea that computation is something can be a non-stop natural process. Some believe that this old idea is a result of a ”Mathematical Bias“ because the founders of computing were mathematicians.

To capture much of the computational aspects of nature, people have turned to creating models to be analyzed. Hypercomputational packages is one model that is subject to much disagreement. Some think that this idea of computational models that go beyond the Turing Machine are impossible and wrong, while others believe that hypercomputational packages are the way forward.

Finally, the halting set is used by many scholars to explain incomputability. They believe that incomputability is simply the halting set with an existential quantifier added. As it is described, ”All the fuss and descriptive variety associated with incomputability in nature and its models merely cloak avatars of halting sets for different machines.“

Even with the amount of discourse between scholars regarding the future of the Turing Machine, there is no doubt that it’s impact is huge among the subject of computability and will continue to play a factor in the future.