CSci 423 Homework 10

Due: 1:00 pm, Wednesday, 11/28
Eric Shih

1. (10 points) Prove that $A = \{w_{2i} | w_{2i} \notin L(M_i)\}$ is not Turing-recognizable, where w_{2i} is the $2i$th string in the lexicographic order of binary strings and M_i is the TM whose binary cod e isw_i.

   Assume that A is Turing Recognizable, such that $\exists$ TM M such that $A = L(M)$. $M = M_i$ with code w_i for some i. $w_{2i} \in A_D$ if and only if $w_{2i} \notin L(M_i)$ by definition of A. $w_{2i} \in A$ if and only if $w_{2i} \in L(M_1)$ by $A = L(M_{2i})$. This gives us a contradiction proving that A is not Truing Recognizable.

2. (10 points) Exercise 5.1 on page 211. (Hint: Reduction from ALL_{CFG} on page 197.)
   reference www.cs.uml.edu/ giam/91.304/Spring06/tf11

   Assume it is decidable. Then there exists a TM R that decides EQ_{CFG} . A construction of TM S that uses R to decide ALL_{CFG} follows.

   S = On input G , where G is a CFG:

   1. Run R on input G, G1 where G1 is a CFG that generates all strings.

   2. If R accepts, accept; if R rejects, reject.

   If R decides EQ_{CFG} , S decides ALL_{CFG}. But ALL_{CFG} is undecidable, so EQ_{CFG} must also be undecidable.

3. (10 points) Problem 5.12 on page 211. (Hint: Reduction from A_{TM}.)
   reference www.cs.uml.edu/ giam/91.304/Spring06/tf11

   First modify M so that all of its transitions that WRITE a blank over a non blank are replaced by writing a new, different symbol. Then the modified M will accept exactly the same set of strings without ever writing a blank over a non- blank. The encompassing TM - S of the last problem must:

   a) change each M as indicated,

   b) run M and at all acceptances make sure that a blank is now written over a non-blank: you may have to move left one or more positions, but, since the initial input was NOT blank and you never overwrote a blank with a non-blank, there must be at least one non-blank for you to overwrite.

   The submachine that carries out a) and b) is the desired machine R. Now run this machine through the T that is supposed to decide the blank/non-blank. Any machine R that writes a blank must have resulted from a modified M stopping; any machine R that is rejected (did not write a blank) must have come from an M that did NOT stop. So we have a decision for A_{TM}.

4. (10 points) Prove that $ES_{TM} = \{$code of M: TM M accepts the empty string$\}$ is undecidable. (Hint: Reduction from A_{TM}.)

   We will show TM M to be undecidable by reducing A_{TM} to M. Assume that M can decide ES_{TM}. Then M will be used to construct TM M_A to show that A_{TM} is decidable.

M_A = On input $< M, w >$

1. Construct a TM M_v that runs on an input string v such that:
   (a) Replace every symbol with a blank symbol.
   (b) Write w on its input tape.
   (c) Run M on w. If M accepts w, M accepts, otherwise M rejects.
2. Run M_v on M. If M accepts, accept, otherwise, reject.

This will decide A_{TM}, which is a contradiction because A_{TM} is undecidable, which proves ES_{TM} to be undecidable.