

CSci 423 Homework 5

Due: 1:00 pm, Wednesday, 10/17

Eric Shih

1. (10 points) Exercise 2.6 (b) and (d) on page 129.

b.

$$S \rightarrow aSb|Ra|bR$$

$$R \rightarrow Ra|Rb|\epsilon$$

d.

$$S \rightarrow T|U\#T|T\#U|U\#T\#U$$

$$T \rightarrow a|b|aTa|bTb|\#\#U\#|\epsilon$$

$$U \rightarrow U\#W|W$$

$$W \rightarrow aW|bW|\epsilon$$

2. (5 points) Exercise 2.9 on page 129.

CFG:

$$S \rightarrow R|T$$

$$R \rightarrow aRbU|\epsilon$$

$$T \rightarrow WbTc|\epsilon$$

$$U \rightarrow cU|\epsilon$$

$$W \rightarrow aW|\epsilon$$

Yes the grammar is ambiguous because there is a string a string that can be derived in more than 1 fashion. abc can be derived:

$$1) S \Rightarrow R \Rightarrow aRbU \Rightarrow abU \Rightarrow abc$$

$$2) S \Rightarrow T \Rightarrow WbTc \Rightarrow Wbc \Rightarrow abc$$

3. (10 points) Exercise 2.13 on page 129.

*Referenced: [http://courses.engr.illinois.edu/cs373/fa2010/Problem Sets/hw9sol.pdf](http://courses.engr.illinois.edu/cs373/fa2010/Problem%20Sets/hw9sol.pdf) *

a) **Description:** The resulting string of $L(G)$ can result in two situations. One will have one $\#$ in between zero or more 0s, where the 0s on the right of the $\#$ is twice the number of 0s on the left. The situation results in a string that will have two $\#$'s with any number of 0s within the string.

b) **Proof:** We will use the closure property

Consider a homomorphism $h_1: 0, 1 \rightarrow 0, \#$, such that $h_1(0) = 0, h_1(\#) = \#, \text{ and } h_1(1) = 00$. Let $K_1 = h_1^{-1}(L(G))$.

Let $K_2 = K_1 \cap L(0^i\#1^i)$. Observe that the only strings in $L(G)$ that have a single $\#$ symbol are those belonging to L_2 , and given the definition of homomorphism h_1 , we can conclude that $K_2 = 0^i\#1^i | i \geq 0$.

Consider the homomorphism $h_2: 0, 1, \# \rightarrow 0, 1$, where $h_2(0) = 0, h_2(1) = 1$, and $h_2(\#) = \epsilon$. Observe that $K_3 = h_2(K_2) = 0^n 1^n | n \geq 0$.

Since K_1, K_2, K_3 are obtained from $L(G)$ through regularity preserving operations and K_3 is non-regular, we can conclude that $L(G)$ is not regular.

4. (5 points) Exercise 2.14 on page 129.

1. $S_0 \rightarrow A$
 $A \rightarrow BAB|B|\varepsilon$
 $B \rightarrow 00|\varepsilon$
2. $S_0 \rightarrow A$
 $A \rightarrow BAB|B|AB|BA|\varepsilon$
 $B \rightarrow 00$
3. $S_0 \rightarrow A|\varepsilon$
 $A \rightarrow BAB|B|AB|BA|BB$
 $B \rightarrow 00$
4. $S_0 \rightarrow BAB|B|AB|BA|BB|\varepsilon$
 $A \rightarrow BAB|B|AB|BA|BB$
 $B \rightarrow 00$
5. $S_0 \rightarrow UB|0C|AB|BA|BB|\varepsilon$
 $A \rightarrow UB|0C|AB|BA|BB$
 $B \rightarrow 0C$
 $C \rightarrow 0$
 $U \rightarrow BA$

5. (10 points) Exercise 2.10 on page 129. (Give the state diagram of your PDA.)

Description: The PDA checks the case that $i = j$ and in another part that $j = k$. The first part has an a pushed for every a read, and an a popped for every b read. $\$$ will be the only one in the stack if there is an equal number of a 's and b 's. c 's are then read and dealt with at the finish state. In the second part, a 's are read and produce nothing, while a b will be pushed for every b read, and popped off for every c read.

State Diagram:

