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CSCI 243 Spring 2011 HW2

1. w = Randy works hard, d = is a dull boy, j = get a job
   (a) w Hypothesis
   (b) $w \to d$ Hypothesis
   (c) d Modus Ponens
   (d) $d \to \neg j$ Hypothesis
   (e) $\neg j$ Modus Ponens

2. (a) $c(x)$ = is in this class, $j(x)$ = can program in java, $p(x)$ = can get a high-paying job
      i. $\forall x(j(x) \to h(x)$ Hypothesis
      ii. $j(doug) \to h(doug)$ Universal Instantiation
      iii. $j(doug)$ Hypothesis
      iv. $p(doug)$ Modus Ponens
      v. $c(doug)$ Hypothesis
      vi. $c(doug) \wedge p(x)$ Conjunction
      vii. $\exists x(c(x) \wedge p(x))$ Existential Generalization

   (b) $c(x)$ = is in this class, $w(x)$ = enjoys whale watching, $o(x)$ = cares about ocean pollution
      i. $\exists x(c(x) \wedge w(x))$ Hypothesis
      ii. $c(y) \wedge w(y)$ Existential Instantiation
      iii. $w(y)$ Simplification
      iv. $c(y)$ Simplification
      v. $\forall x(w(x) \to o(x))$ Hypothesis
      vi. $w(x) \to o(x)$ Universal Instantiation
      vii. $o(y)$ Modus Ponens
      viii. $c(y) \wedge o(y)$ Conjunction
      ix. $\exists x(c(x) \wedge o(x))$ Existential Generalization

   (c) $c(x)$ = is in this class, $p(x)$ = owns a PC, $w(x)$ = can use a word program
      i. $\forall x(c(x) \to p(x))$ Hypothesis
      ii. $c(Zeke) \to p(Zeke)$ Universal Instantiation
      iii. $c(Zeke)$ Hypothesis
      iv. $p(Zeke)$ Modus Ponens
      v. $\forall x(p(x) \to w(x))$ Hypothesis
      vi. $p(Zeke) \to w(Zeke)$ Universal Instantiation
      vii. $w(Zeke)$ Modus Ponens

   (d) $j(x)$ = is in NJ, $o(x)$ = within 50 miles of the ocean, $s(x)$ = has seen the ocean
      i. $\exists x(j(x) \wedge \neg s(x))$ Hypothesis
      ii. $j(y) \wedge \neg s(y)$ Existential Instantiation
      iii. $j(y)$ Simplification
      iv. $\neg s(y)$ Simplification
      v. $\forall x(j(x) \to f(x))$ Hypothesis
      vi. $j(y) \to f(y))$ Universal Instantiation
      vii. $f(y)$ Modus Ponens

viii. $f(y) \wedge \neg s(y))$ Conjunction

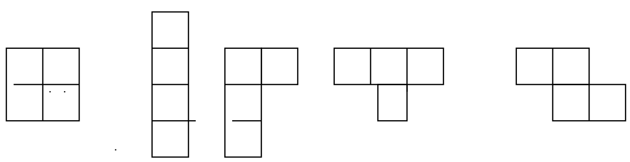
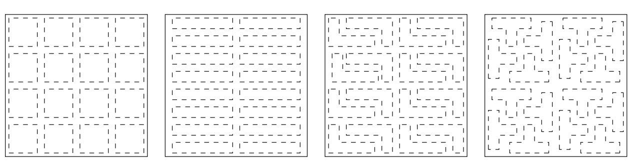
ix. $\exists x(f(x) \rightarrow \neg s(x))$ Existential Generalization

3. (a) False, because they imply that the conclusionmakes the begnning true.
   (b) True, modus tollens is used.
   (c) False, because they have $\neg p$ implying $\neg q$ which cannot be done.

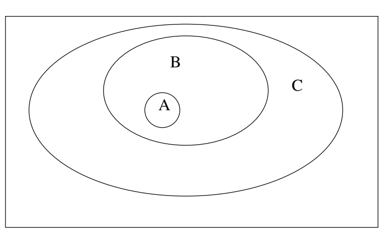
4. (a) if n is odd
   $n = 2k+1 \implies n^3+5$
   $= (2k+1)^3+5$
   $= 8k^3+12k^2+6k+6$
   $= 2(4k^3+6k^2+3k+3)$
   thus n^3+5 is even
   (b) if n is odd
   n^3 will be odd, but adding 5 will make the equation so that odd + odd = even.

5. Prove Using Contrapossitve
   if n is even $\implies n = 2k$
   $5n+6 = 10k+6$
   $5n+6 = 2(5k+3)$
   Thus, $5n+6$ is even

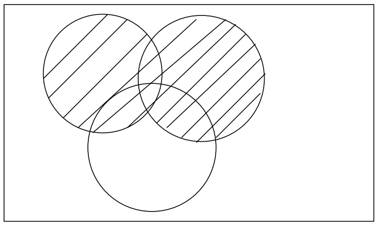
6. Prove by Exhaustion
   $n > 4 \implies n^3 > 100$
   when n = 1,2,3,4 becuase only these are $n^3 > 100$, $n^2+n^3 = 100$ is never true.

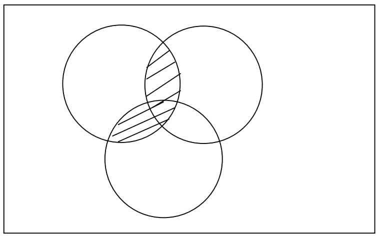
7. (a) 
   (b) 

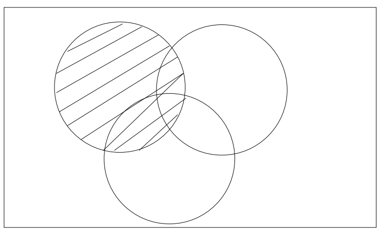
8. (a) Yes
   (b) No, first has 1 element, while the second set has 2 elements.
   (c) No, first set has no elements, second set has 1 element.

9. 

10. One set is a 3-tuple, the other set is a 2-tuple and a 1-tuple. Therefore they are not the equivalent.

11. (a) 

(b) 

(c) 

12. (a) $\cup_{i=1}^{\infty} A_i = Z$
$\cap_{i=1}^{\infty} A_i = \{-1, 0, 1\}$

(b) $\cup_{i=1}^{\infty} A_i = Z - \{0\}$
$\cap_{i=1}^{\infty} A_i = \emptyset$

(c) $\cup_{i=1}^{\infty} A_i = R$
$\cap_{i=1}^{\infty} A_i = [-1, 1]$

(d) $\cup_{i=1}^{\infty} A_i = [1, \infty)$
$\cap_{i=1}^{\infty} A_i = \emptyset$

13. (a) $f(x) = x + 3$
*I got this wrong because I didn't know that I could split up to create a system of equations. I should have set anything less than 0 to one equation and anything greater than 0 to another.

(b) $f(x) = |x| + 1$

(c)

$$f(x) = \begin{cases} \text{x+1} & ifx \geq 0 \\ \text{—x—} & ifx < 0 \end{cases}$$

(d) $f(x) = 1$

14. (a) Since f is one-to-one and onto, whatever g sends into f, the resulting output will have to be one-to-one or onto. It does not matter what g sends f, f will output something that follows its properties.

(b) Since as before, whatever g sends f, the output will still be onto; no matter what the initial input is.

15. (a) $a_0 = 2, a_1 = 3, a_2 = 5, a_3 = 9$

(b) $a_0 = 1, a_1 = 4, a_2 = 27, a_3 = 256$

(c) $a_0 = 0, a_1 = 0, a_2 = 1, a_3 = 1$

(d) $a_0 = 0, a_1 = 1, a_2 = 2, a_3 = 3$

16. (a) each set adds one more 1 and one more 0 with each reiteration of the pattern: 1,1,1...
    (c) every even index has 2^n where n increases $2^{(\frac{n-1}{2})}$ only every other index: 32, 0, 64...
    (e) subtract 7 each time, 22-7n: -34, -41, -48...
    (g) the equation is $2n^3$: 1024, 1458, 2000...

17. (a) $2 + 3 + 4 + 5 + 6 = 20$
    (b) $1 - 2 + 4 - 8 + 16 = 11$

18. (a) $\frac{3(2^9-1)}{2-1} = 1533$
    (b) $\frac{(2^9-1}{2-1} - 1 = 510$

19. (c) $(1 + 1 + 1) + (2 + 2 + 2) + (3 + 3 + 3) = 18$
    (d) $(0 + 0 + 0) + (1 + 2 + 3) + (2 + 4 + 6) = 18$

20. (a) Countable: $f(x) = |x|$
    (b) Countable: $f(x) = xwheneven$
    $f(x) = 1 - xwhenodd$
    (c) Not Countable
    (d) Countable: ???
    *I really did not know the correspondance, but now I understand the equation is a lot like (b) but with different constants. I definitely over-thought this one.

21. If A is not countable within B, then a part of B is not countable, thus all are uncountable.