CSci 423 Homework 7

Due: 1:00 pm, Wednesday, 10/31
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1. (20 points) Problem 2.30 (a) and (d) on page 131.

   a) Use the pumping lemma to show that the following language is not context free:
   $A = \{0^n1^n0^n1^n | n \geq 0\}$
   Assume A is context free. The pumping lemma applies. Let p be the pumping length. Let string $s = 0^p1^p0^p1^p \in A$ and $|s| \geq p$ with $s = uvxyz$, $|vy| > 0$ and $|vxy| \leq p$ and $\forall i \geq 0$, $uv^ixy^iz \in A$.
   Pumping s, all blocks must be equal in length or a p‘ that is different from p will result, rendering $s \notin A$. Therefore, $\forall i \geq 0$, $uv^ixy^iz \in A$ is true if and only if none of the ps change or ps change in $0^p1^p0^p1^p$. Since $|vy| > 0$ and $|vxy| \leq p$ the substring vxy can be:
   1. one type of symbol, 0 or 1, and a maximum length of p.
   2. at most 2 types of symbols, 0 or 1, no matter where vxy spans $0^p1^p0^p1^p$.
   Therefore, when $i = 0$, and $s = uxz$, there will be one or two ps and $s \notin A$. This is a contradiction, proving that A is not context free.

   d) Use the pumping lemma to show that the following language is not context free:
   $B = \{t_1\#t_2\#\ldots\#t_k | k \geq 2, each t_i \in \{a,b\}*, and t_i = t_j for some i \neq j\}$
   Assume B is context free. The pumping lemma applies. Let p be the pumping length. Let string $s = a^pb^p\#a^pb^p \in B$ and $|s| \geq p$ with $s = uvxyz$, $|vy| > 0$ and $|vxy| \leq p$ and $\forall i \geq 0$, $uv^ixy^iz \in B$.
   Let $i = 0$:

   Case 1: vxy is contained in the first a^pb^p. With $s = uv^0xy^0z$, there are fewer symbols on the left of the # than on the right. Thus $s \notin B$.

   Case 2.1: If either v or y contains the # symbol, than we can pump s down to uv^0xy^0z, which will not contain the # symbol and hence will not be in L

   Case 2.2: If x contains the # symbol v is a substring of 1^p , and y is a substring of 0^p (since $|vxy| \leq p$). Pumping s down to uv^0xy^0z reduces either the number of ones in t_1 or the number of zeros in t_2 or both. As a result, $t_1 \neq t_2$ for uv^0xy^0z, so the string is not in L.

   Case 3: Similar to case 1, vxy is contained in the second a^pb^p. With $s = uv^0xy^0z$, there will be fewer symbols on the right of the # than on the left. Thus $s \notin B$.

   Since within each case $s \notin B$, there is a contradiction, proving that B is not context free.

2. (10 points) Problem 2.32 on page 131. (Hint: The following selection of s won't work: $s = 1^p2^p3^p4^p$. Try other block orders.)
   Assume C is context free. The pumping lemma applies. Let p be the pumping length. Let string $s = 1^p3^p2^p4^p \in C$ and $|s| \geq p$ with $s = uvxyz$, $|vy| > 0$ and $|vxy| \leq p$ and $\forall i \geq 0$, $uv^ixy^iz \in C$.
   Let $i = 0$:

Case 1: vxy is within 1^p, 2^p, 3^p, or 4^p. With $s = uv^0xy^0z$, there could be a p' that does not have equal number of values. Thus making $s \notin C$.

Case 2: vxy spans $1 \rightarrow 3$, $3 \rightarrow 2$, or $2 \rightarrow 4$. $1 \rightarrow 3$ would cause a different number of $1sthan2s$ as well as $3sand4s$. $3 \rightarrow 2$ would cause a different number of $3sthan4s$ as well as $2sthan1s$. $2 \rightarrow 4$ would cause a different number of $2sthan1s$ and $4sthan3s$. Thus however the string is seperated, $s \notin C$.

Since all cases result in $s \notin C$, there is a contradiction, proving that C is not context free.

3. (10 points) Problem 2.42 on page 131. (Hint: $s = 1^{p+1}\#1^{p+2}\#...\#1^{3p}$).u
   Assume Y is context free. The pumping lemma applies. Let p be the pumping length. Let string $s = 1^{p+1}\#1^{p+2}\#...\#1^{3p} \in Y$ and $|s| \geq p$ with $s = uvxyz$, $|vy| > 0$ and $|vxy| \leq p$ and $\forall i \geq 0$, $uv^ixy^iz \in C$. Let $i = 2$ for Case 1, $i = 0$ for Case 2, and $i = 3$ for Case 3:

Case 1: If v or y contains only 1s, is within a block of 1s, or is along two sets of 1s from 1^{p+1} to 1^{2p-1}. Since $|vxy|$ is bound by $|p|$ and $|vy| > 0$, uv^2xy^2z will be reproduced again in a t_j. Thus making $s \notin Y$.

Case 2: If v or y contains only 1s, is within a block of 1s, or is along two sets of 1s from 1^{2p} to 1^{3p}. Since $|vxy|$ is bound by $|p|$ and $|vxy| > 0$, uv^0xy^0z will be reproduced again in a t_j. Thus making $s \notin Y$.

Case 3: If v or y contains a #. If there is only a # in v or y it would repeat correctly after 3 iterations, but after 3 iterations, that number would be repeated in the next block of 1s. Thus making $s \notin Y$.

Since all cases result in $s \notin Y$, there is a contradiction, proving that Y is not context free.