Imperial College London

NOTES

IMPERIAL COLLEGE LONDON

DEPARTMENT OF COMPUTING

422 Computational Finance

Author:
Thomas Teh (CID: 0124 3008)

Date: February 7, 2017

1 Theory of Interest

1. Relationship between effective interest rate r_{eff} and nominal rate (m period compounding), $r^{(m)}$:

$$1 + r_{eff} = \left(1 + \frac{r^{(m)}}{m}\right)^m$$

2. **Definitions.** An ideal bank

   - applies the same interest rates for borrowing and lending
   - no transaction costs
   - has the same rate for any size of principal

3. **Definitions.** An ideal bank has an interest value that is independent of the length of time of which it applies, it is called a constant ideal bank.

4. **Theorem.** The cash flow streams $\{x_i\}_{i=1}^n$ and $\{y_i\}_{i=1}^n$ are equivalent for a constant ideal bank with interest rate r if and only if their PVs are equal.

5. **Definitions.** The spot rate s_t is the annualized interest rate charged for money held from the present until time t. Properties of spot rate

   - Long commitments tend to offer higher interest rates than short commitments
   - The spot rate curve undulates around in time.
   - Spot rate curve is normally curved if it is increasing; and inverted if it is decreasing.
   - Spot rate curve is smooth.

6. **Definitions.** Forward rate between times t_1 and t_2 is denoted by f_{t_1,t_2}. It is the interest rate charged for borrowing money at time t_1 which is to be repaid at t_2. f_{t_1,t_2} is agreed on today.

$$(1 + s_{t_2})^{t_2} = (1 + s_{t_1})^{t_1}(1 + f_{t_1,t_2})^{t_2-t_1}$$

7. The forward rate $f_{1,2}$ is

   - the implied rate for money loaned for 1 year, a year from now
   - the market expectation today of what the 1-year spot rate will be next year

2 Fixed Income Securities

1. Important sequences:

   - Geometric Progression

$$S_n = \sum_{k=1}^{n} ar^{k-1} = \frac{a(1-r^n)}{1-r}$$
$$S_\infty = \sum_{k=0}^{\infty} ar^{k-1} = \frac{1}{1-r}$$

   - Arithmetic - Geometric Progression

$$S_n = \sum_{k=1}^{n} [a+(k-1)d]\, r^{k-1} = \frac{a-[a+(n-1)d]\, r^n}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2}$$

2. Price of various basic securities

   - Annuities

$$a_{\overline{n}|} = \frac{1-v^n}{r} \qquad a_{\overline{n}|}^{(m)} = \frac{1-v^n}{r^m}$$

   - Perpetuity

$$a_{\overline{\infty}|} = \frac{1}{r} \qquad a_{\overline{\infty}|} = \frac{1}{r^m}$$

   - Varying annuity

$$(Ia)_{\overline{n}|} = Pa_{\overline{n}|} + D\left[\frac{a_{\overline{n}|} - nv^n}{i}\right]$$

   - Bond

$$P = NCa_{\overline{n}|} + Nv^n$$

3. **Definition.** A bond's **yield to maturity** if the flat interest rate at which the PV of the CFs is equal to the current price. The bond's **current yield** is $\frac{NC}{P}$.

   - There's an inverse relationship between price and yield
   - The longer the time to maturity, the more sensitive is the price of the bond to the yield (think of duration!)
   - If yield to maturity is the same as the coupon rate, the bond price would be the face value

4. **Duration.** Duration measures the sensitivity of the bond with respect to the interest rate

$$D_{mac} = \frac{\sum_{t=1}^{n} t \times PV_t}{\sum_{t=1}^{n} PV_t}$$
$$D_{mod} = -\frac{1}{P(r_0)} \left.\frac{dP(r)}{dr}\right|_{r=r_0} = \frac{D_{mac}}{(1+r_0)}$$

5. **Convexity.** Convexity is defined as

$$C = \frac{1}{P(r_0)} \left.\frac{d^2P(r)}{dr^2}\right|_{r=r_0} = \frac{1}{P(r_0)} \sum_{t=1}^{n} t^2 PV_t$$

6. Estimation of bond price change

$$\Delta P \approx -D_{mod} P(r_0) \Delta r + \frac{1}{2} P(r_0)(\Delta r)^2$$

7. **Immunization.** Let A be assets and L be liabilities.
   - $PV(A) = PV(L)$
   - $D(A) = D(L)$
   - $C(A) > C(L)$

Macaulay Duration
Modified Duration