YQuantum: Optimization of Correlated Insured Property Portfolios

Use case by The Hartford and Capgemini’s Quantum Lab

Description of Broad Problem

In the realm of insurance, the risks associated with properties can often be correlated due to various factors such as geographical location, environmental conditions, and socio-economic situations. For an insurance company, it is crucial to minimize the total risk associated with their portfolio of insured properties. This requires identifying a set of properties whose risks are as uncorrelated as possible.

Insurance companies often face the challenge of balancing risk and return when building their portfolios. This involves selecting properties that not only provide a good return on investment but also minimize the risk of large-scale losses due to correlated events. For example, properties in the same geographical area may be susceptible to natural disasters like floods or earthquakes, leading to simultaneous claims. Therefore, diversifying the portfolio by including properties from different regions can significantly reduce the risk of correlated losses.

The primary challenge lies in finding a portfolio of properties that are not correlated with each other. This means developing a method to identify and select properties that have minimal interdependencies in terms of risk factors, thereby reducing the overall risk of the portfolio.

By leveraging advanced techniques like quantum computing and Markowitz portfolio optimization, insurance companies can efficiently identify and select optimal property portfolios.

Where Quantum Can Be Used

Mathematical formulation of the problem

A standard approach involves the Markowitz portfolio optimization technique. This method focuses on minimizing the total risk of the portfolio through a mathematical approach. The objective function for this method can be outlined as follows:

$$\min q \sum_i \sum_j x_i x_j \sigma_{ij} - \sum_i x_i \mu_i$$
$$s.t. \sum_i x_i = B$$

- $q > 0$ is the risk appetite of the decision maker. 5% is a reasonable value for this case.
- x_i is the decision variable indicating whether property i is included in the portfolio.
- σ_{ij} represents the correlation between properties i and j.
- μ_i is the expected return on the property i, i.e., the premium, which we assume to be fixed (and given) in this case.
- B is the budget, i.e., how many properties a decision maker wants in its portfolio. 10% of the possible properties is a reasonable value for this case.

The goal is to find the values of x_i that minimize the total portfolio risk, considering the correlations σ_{ij} between all pairs of properties. This optimization ensures that the selected portfolio has the lowest possible risk due to uncorrelated properties.

Quantum Approaches

The formulation above is not ready for a quantum computer yet, because it has constraints. We can add the term $\lambda(\sum_i x_i - B)^2$ to make sure the objective function is minimal when the constraint $\sum_i x_i = B$ is met. λ is a penalty term that you must tweak to make the solutions feasible, but also non-trivial.
The final formulation is now:

$$\min q \sum_i \sum_j x_i x_j \sigma_{ij} - \sum_i x_i \mu_i + \lambda \left(\sum_i x_i - B \right)^2.$$

Now the formulation is called a quadratic unconstrained binary optimization or *QUBO* problem. Quadratic because of the term $x_i x_j$, unconstrained because there are no constraints, and binary because the x_i's are binary variables. This formulation is a good fit for quantum computers, as it is reminiscent of the Ising model.

Several quantum approaches exist, of which the Quantum Approximate Quantum Algorithm (QAOA) is the most well-known and well-studied [1], [2]. This could be a start for your project, though just implementing the QAOA for the formulation above will not score many points for novelty/innovativeness. Other approaches are VQE[3], α-VQE [4] analog quantum [5], [6], or annealing [7], [8].

By integrating these methods, one can develop a comprehensive solution to minimize the overall risk of an insured property portfolio. This use case demonstrates the power of quantum computing and advanced optimization techniques in addressing complex real-world challenges in the insurance industry.

Expectations of the Challenge

The challenge is to create a solution that minimizes correlation between properties in an insurance portfolio using a quantum computer. Specifically:

- Create a solution (i.e., in Jupyter Notebook, (Python) code or any other technical implementation)
- The solution should be able to take in an input list (provided) of potential properties for the portfolio with associated correlations, run the optimization (using some quantum simulator/hardware at some point in the algorithm), and give a good portfolio that minimizes total risk, while maximizing the return.
- Make the solution (demo) applicable to the stakeholders, so try to get their advice on the matter.

Judging criteria

You will be judged based on the following criteria:

Description	Max points
Technical quality of the solution (i.e., code quality, mathematical foundation, efficiency of code)	30
Applicability to The Hartford (i.e., connection with insurance business, real-world applicability)	30
Innovativeness	20
Comparison to existing classical computing solutions	10
Presentation	10
Total	**100**

Resources

Starting points

Here are some webpages, video's, articles that might get you started

- IBM's Qiskit
  - Introduction to Qiskit https://docs.quantum.ibm.com/guides
  - Portfolio Optimization - Qiskit Finance 0.4.1: https://qiskit-community.github.io/qiskit-finance/tutorials/01_portfolio_optimization.html
  - Solving combinatorial optimization problems using QAOA https://github.com/Qiskit/textbook/blob/main/notebooks/ch-applications/qaoa.ipynb
- Pasqal's Pulser
  - Pulser https://pulser.readthedocs.io/en/stable/
  - QAOA and QAA to solve a QUBO problem https://pulser.readthedocs.io/en/stable/tutorials/qubo.html

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