Bank balance sheets and asset performance

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Jaromir Benes jaromir.benes@gimm.institute
Tomas Motl tomas.motl@gimm.institute

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Workshop repository:
https://github.com/gimm-institute/july-2023-rwanda-workshop.git

Simplified aggregate bank balance sheets

The model explicitly describes balance sheet of the aggregate banking sector in the following form:

	Assets	Liabilities	
le_t	Net loans	Non-equity liabilities	d_t
$+\sum l_t^k$	+ Gross loans	+ Local currency	$+\,d_t^{\mathrm{lcy}}$
$-\sum a_t^k$	− Allowances for CL	+ Foreign currency	$+\,d_t^{\mathrm{fcy}}$
ona_t	Other net assets	Balance sheet capital	bk_t

- Bank loans can be denominated in local or foreign currency
- Non-equity liabilities d_t include deposits as well as other sources of financing.
- All allowances here assumed to be contra-assets, and netted against gross exposure values before risk weighting.

MULTIPERIOD LOANS WITH GEOMETRIC PAYDOWN

Motivation

- Real-world loan portfolios include a whole range of loan durations (maturities)
- Stock-flow relationships between the stock of credit and new credit critical for the cost of deleveraging process
- Need to simplify to keep the model tractable, but still maintain key features of the real-world
- Assume a composite portfolio of loans with geometrically decreasing paydowns[1]
  - Represents a mix of durations
  - Needs one single parameter
  - Has a recursive representation
  - Calibrated using Macaulay duration

Lifetime of a single riskless pool

Each hypothetical loan follows the same life-cycle:

	Origination	Time 1	Time 2	...
Closing book value	l	$(1-\theta)l$	$(1-\theta)^2 l$	...
Paydown		$\theta\, l$	$\theta\,(1-\theta)l$	...
Interest		$rl_0\, l$	$rl_1\,(1-\theta)l$	...
Total cash flow generated		$(\theta + rl_0)l$	$(\theta + rl_1)\,(1-\theta)l_0$	...

Parameter θ determines the average maturity of the loan portfolio, and can be calibrated using e.g. Macaulay duration.

The effective interest rate on the stock of outstanding loans (stock rate), rl_t may vary over time and be disconnected from θ in general.

Calibrating loan duration

Steady-state Macaulay duration

$$MD \quad = \quad 1 \cdot \frac{\theta + rl}{1 + rl} \quad + \quad 2 \cdot \frac{(\theta + rl)\,(1-\theta)}{(1+rl)^2} \quad + \quad \cdots \quad = \frac{1 + rl}{\theta + rl}$$

Time evolution of a dynamic riskless loan portfolio

Dynamic here means inclusive of the inflow of new loans.

Assuming no credit events and no exchange rate valuation, the loan portfolio evolves as:

$$l_t = (1 - \theta) l_{t-1} + l_t^{\Delta}$$

where l_t^{Δ} is the amount of new loans extended in time t.

Time t cash flows (paydown + interest) generated by the loan portfolio collected at t:

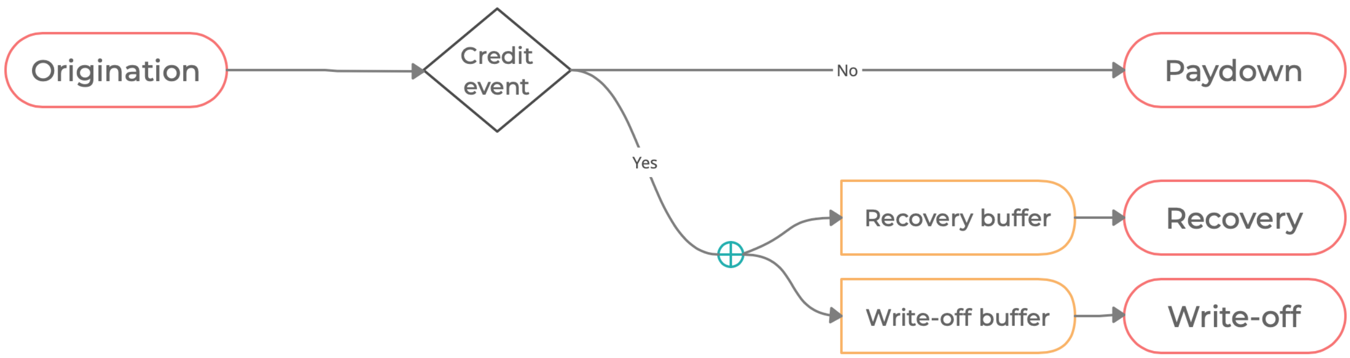
$$l_{t-1} \left(\theta + r l_{t-1} \right)$$

CREDIT RISK AND LOAN PERFORMANCE

Credit events

- Introduce a theoretical structure that is simple enough to remain analytically tractable
- Make losses proportional not to the book value but the present value $\longrightarrow$ simplify present value calculations (e.g. IFRS9 expected credit loss)[2]

Static snapshot of possible paths for a given loan



Classification of gross loans

Gross loans are classified either as **performing** or **nonperforming**.

$$l_t = lp_t + ln_t$$

Non-performing loans break down into a recovery buffer (to be gradually recovered, i.e. generating cashflows), lnc_t, and a write-off buffer (to be gradually written-off), lnw_t

$$ln_t = lnc_t + lnw_t$$

This is a simplification for modeling purposes. Do not think of individual loans as being classified as lnc_t or lnw_t; rather the book value of each loan is split between these two categories on default. The recoverable amount is not known on default in real-world setting, but we simplify.

Total pool of loans generating cash flows

$$lc_t = lp_t + lnc_t$$

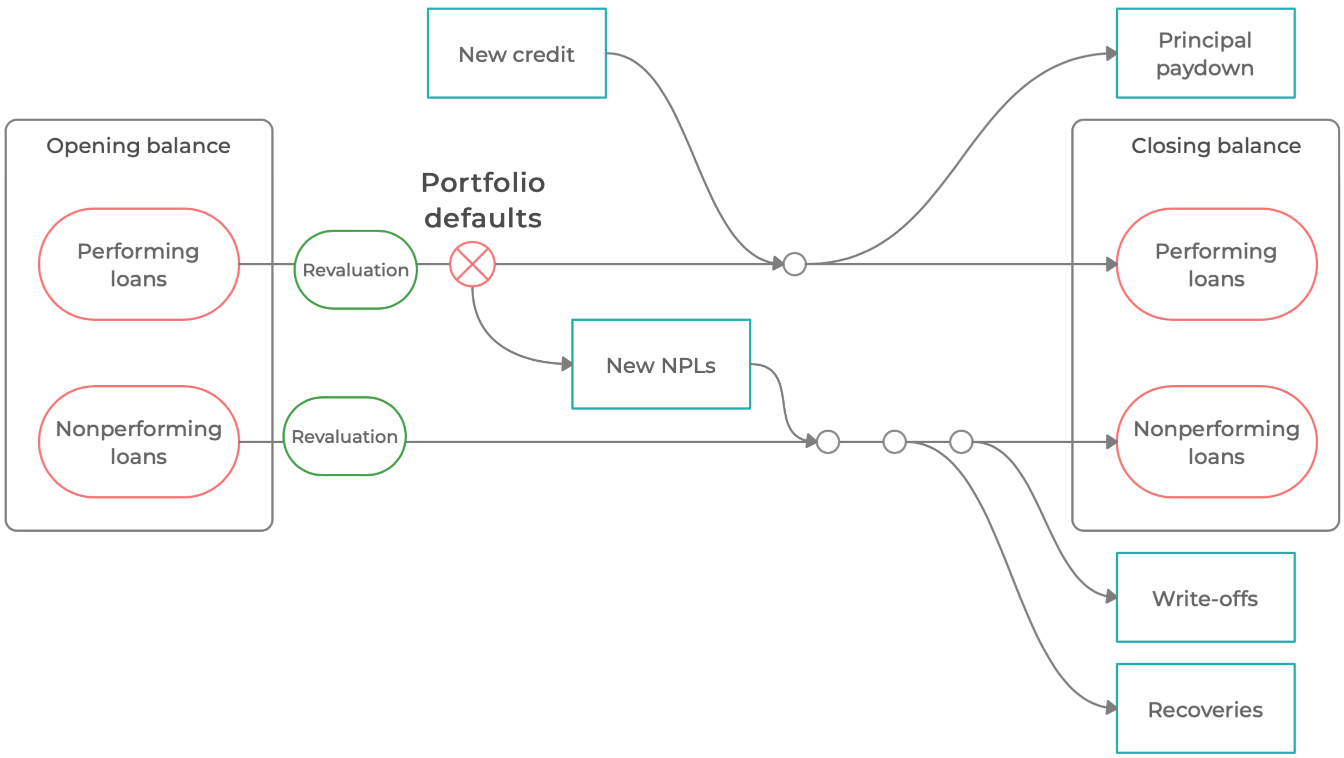
Flows in gross loan portfolio

Portfolio division	Cash Flows	Credit event
Performing lp_t	Paydown, interest	(outflow) $ln^\Delta_t = q_t \; lp^0_t \rightarrow$
Nonperforming recovery buffer lnc_t	Collections, repossession, resale	(inflow) $\rightarrow (1-\lambda) \; ln^\Delta_t$
Nonperforming writeoff buffer lnw_t	None	(inflow) $\rightarrow \lambda \; ln^\Delta_t$

where

- ln^Δ_t is the amount of loans that experience a credit event and become non-performing in time t
- q is the share of performing loans experiencing a credit event (default)
- λ is parameter governing the share of the newly non-performing loans which falls into the write-off buffer lnw_t

Stock-flow dynamics in risky loan portfolio



Time evolution of dynamic risky loan portfolio

Assuming no exchange rate valuation in all that follows below

Performing loans

Closing balance rolled over from previous time $t-1$

$$lp_{t-1}$$

Balance after **new information** at new time t: new information (defaults) arrives, adjust the balance for new information

$$lp_t^0 = lp_{t-1} - ln_t^\Delta$$

$$ln_t^\Delta = q_t \; lp_{t-1}$$

Throughout-period cash flows generated by the performing loans: paydown plus interest income

$$(\theta_{lp} + rl_{t-1}) \; lp_t^0$$

Closing balance after paydown and inclusive of new credit l_t^Δ

$$lp_t = (1-\theta_{lp}) \; lp_t^0 + l_t^\Delta$$

Nonperforming loans: Recovery buffer

Closing balance rolled over from previous time $t-1$

$$lnc_{t-1}$$

Balance after **new information** at new time t: new information (defaults) arrives, adjust the balance for new information

$$lnc_t^0 = lnc_{t-1} + (1-\lambda) \; ln_t^\Delta$$

Throughout-period cashflow generated by the recovery buffer

$$rec_t \; lnc_t^0$$

The recovery rates are mimicked by hypothetical paydowns θ_{lnc} (may be different from θ_{lp}) and interest payments; this assumption is to

- distribute the collections, repossession of collateral and/or resale over time
- preserve the net present value equal the current book value (that is why we include the interest payments) as a simplifying assumption

$$rec_t = \theta_{lnc} + rl_{t-1}$$

Closing balance after recovery cash flows

$$lnc_t = (1 - \theta_{lnc})\, lnc_t^0$$

Nonperforming loans: Write-off buffer

Closing balance rolled over from previous time $t-1$

$$lnw_{t-1}$$

Balance **after new information** at new time t: new information arrives, adjust the balance for new information

$$lnw_t^0 = lnw_{t-1} + \lambda\, ln_t^\Delta$$

Throughout-period write-offs

$$w_t$$

Closing balance

$$lnw_t = lnw_t^0 - w_t$$

PROVISIONING AND WRITE-OFFS

Provisioning

The stock of allowances, a_t, is determined by expected credit losses; the balancing item here is the impact of changes in allowances on period profit/loss after correction for write-offs

$$a_t = a_{t-1} - w_t + a_t^{\Delta}$$

- a_t is the stock of allowances for expected credit losses
- w_t is period write-offs (flow)
- a_t^{Δ} is the impact of new provisions (flow) on period profit/loss

Write-off process

Write off a certain proportion of the exiting buffer each period

$$w_t = \omega \, lnw_{t-1}$$

- $\omega \in (0, 1)$ is a parameter determining the speed of the write-off process
- lnw_{t-1} is the write-off buffer balance at the close of last period

SEGMENTATION AND VALUATION

Portfolio segmentation

Consider total gross loans consisting of a number K of segments (subportfolios, subclasses), each differing in its

- risk parameters
- responsiveness to macroeconomic conditions
- share of foreign exchange denomination

$$l_t = l_t^1 + \cdots + l_t^K = \sum_{k=1}^{K} l_t^k$$

Each loan segment is tracked separately. Equations presented above exist in K variants for each loan segment separately.

Exchange rate valuation

For practical modeling:

- We introduce an effective (nominal) exchange rate that would be sufficiently representative of movements on foreign currency denominated asset classes
- All balance sheet quantities will be expressed (reported, tracked) in *local currency units* independently of their currency of *denomination*

Parameterizing foreign currency denomination

For each loan segment, we define the steady-state degree of foreign exchange denomination (exposure) by $\sigma_k \in [0,\, 1]$:

$$\sigma_k = \frac{l_t^{k,fcy}}{l_t^k}$$

- $\sigma_k = 0$ means fully home currency denomination
- $\sigma_k = 1$ means fully foreign currency denomination
- $0 < \sigma_k < 1$ means mixed currency denomination

The loan portfolio segment l^k is adjusted for exchange rate valuation whenever crossing a time period using the following exchange rate valuation impact indicator (depending on the parameter σ_k)

$$j_t^k = (1 - \sigma_k) + \sigma_k \, \frac{e_t}{e_{t-1}} = 1 + \sigma_k \left(\frac{e_t}{e_{t-1}} - 1 \right)$$

Time evolution of foreign currency denominated loan segments

Closing balance from previous time $t-1$

$$lp_{t-1}^{k}$$

Balance after new information at a new time t: new information arrives **including the new level of the exchange rate** adjust the balance for new information

$$\begin{aligned} ln_t^{\Delta k} &= j_t^k \; q_t \; lp_{t-1}^k \\ lp_t^{0k} &= j_t^k \; lp_{t-1}^k - ln_t^{\Delta k} \end{aligned}$$

Period cash flows generated by the portfolio: paydown plus interest income

$$\left(\theta_{lp}^k + rl_{t-1}^k\right) lp_t^{0k}$$

Closing balance after paydown and inclusive of new lending

$$lp_t^k = (1-\theta_{lp}^k) lp_t^{0k} + l_t^{\Delta k}$$

Open net foreign positions

Part of the non-equity liabilities is denominated in foreign currency:

$$d_t = d_t^{\mathrm{lcy}} + d_t^{\mathrm{fcy}}$$

The banks can have open net foreign positions, expressed as a share of capital:

$$d_t^{\mathrm{fcy}} = \sum_{k=1}^{K} l_t^k \cdot \sigma_k + \mathit{onfx}_t \cdot bk_t$$

and onfx_t is an exogenou process around its steady-state value

$$\mathit{onfx}_t = \rho_{\mathit{onfx}_t}\, \mathit{onfx}_{t-1} + (1 - \rho_{\mathit{onfx}_t})\, \mathit{onfx}_{ss} + \varepsilon_{\mathit{onfx}_t, t}$$

1. For instance, Rudebusch and Swanson (2008) Examining the bond premium puzzle with a DSGE model. JME 55 (2008), S111--S126↩
2. For instance, Duffie and Singleton (1999) Modeling Term Structure of Defaultable Bonds. *The Review of Financial Studies*, 12(4).↩