1. (Problem 1.15 in the book) A coin is tossed twice. Frodo claims that the probability that two heads come up given that the first toss is a head is at least as large as the probability that two heads come up given that at least one head comes up. Is he right? Does the answer to that question depend on p, the probability that a head comes up on a single flip? Can you generalize Frodo's reasoning to situations other than coin flips?

2. (Problem 1.17 in the book) A quality-control inspector tests four items selected uniformly at random from a batch of 100 items. She rejects the batch if at least one tested item comes up defective. What is the probability that she rejects the batch given that it contains five defectives?

3. (Problem 1.27 in the book) Frodo tosses $n + 1$ fair coins and Sam tosses n fair coins. Use the Total Probability Theorem to find the probability that Frodo's tosses yield more heads than Sam's. It helps to consider the possible states of affairs after both have tossed exactly n coins. In particular, it makes sense (and is true) that the two events "Frodo has more heads than Sam after both have completed n tosses" and "Sam has more heads than Frodo after both have completed n tosses" have equal probability. You can prove this formally (think about $n = 1$, $n = 2$, $n = 3$, etc.) or argue it intuitively.

4. Let Ω be a sample space and $\mathbb{P}$ a probability law on Ω. Remember that a conditional probability law on Ω is an actual probability law on Ω. In other words, if $B \subset \Omega$ and $\mathbb{P}(B) \neq 0$, then $\mathbb{P}(A \mid B) \geq 0$ for all $A \subset \Omega$; $\mathbb{P}(\Omega \mid B) = 1$; and $\mathbb{P}(A_1 \cup A_2 \mid B) = \mathbb{P}(A_1 \mid B) + \mathbb{P}(A_2 \mid B)$ when $A_1 \cap A_2 = \phi$. Consider the following situation. Your radar will pick up both standard aircraft and alien spaceships. With probability 0.1 you see a blip on the radar in a given minute when no alien spaceship is present, but if an alien spaceship is present, you see a blip with probability 0.99. History tells you that the probability that an alien spaceship is present during any given minute is 0.05. Find the probability that you fail to detect an alien spaceship when one is present. Also find the probability that you get a "false alarm" in the sense that your radar blips despite the fact no alien spaceship is present.