1. Let $\mathbb{N}$ be the natural numbers, and for each $n \in \mathbb{N}$ let

$$A_n = \{m \in \mathbb{N} : m \geq n\} = \{n, n+1, n+2, \ldots\} .$$

Show that

$$A_0 \cap A_1 \cap A_2 \cap \cdots = \cap_{n=0}^{\infty} A_n = \phi .$$

2. (Problem 1.2 in the book) Let A and B be subsets of another set Ω. Let A^c and B^c be the complements of A and B respectively in Ω.

(a) Show that

$$A^c = (A^c \cap B) \cup (A^c \cap B^c)$$

and

$$B^c = (A \cap B^c) \cup (A^c \cap B^c) \ .$$

(b) Show that

$$(A \cap B)^c = (A^c \cap B) \cup (A^c \cap B^c) \cup (A \cap B^c) \ .$$

(c) You roll a six-sided die once. Let A be the set of outcomes where the roll yields an odd number and B the set of outcomes where the roll yields a number less than 4. Verify that the equality in (b) holds.

3. Let A and B be events, and suppose that the probability that neither A nor B occurs is 1/3. What is the probability that one or both of A or B occurs?

4. (Problem 1.7 in the book) Consider the following experiment. You roll a four-sided die with faces numbered 1 through 4 repeatedly until an even number comes up, at which point you stop. If an even number never comes up, you keep rolling forever. The outcome of the experiment is the sequence of rolls you make. What is a good sample space Ω for this experiment? What event in Ω corresponds to "first even number appears on roll n"? What event in Ω corresponds to the situation when you continue rolling forever?

5. Given a point (x, y) in the plane $\mathbb{R}^2$, the *Manhattan distance* between (x, y) and $(0, 0)$ is $|x| + |y|$. Consider an experiment where you select a point from the unit square $[0, 1] \times [0, 1]$ "at random" in the sense that every point in the unit square is equally likely to be selected. What is a good sample space Ω for this experiment? What is an appropriate probability law $\mathbb{P}$? Given $0 \leq a \leq 2$, using your probability law find the probability that the Manhattan distance between the chosen point and $(0, 0)$ is at most a.

6. Frodo and Sam agree to meet up at an appointed spot between 2 and 3 PM Shire time. Each of their arrival times between 2 and 3 PM is equally likely.

(a) Construct a good sample space Ω and probability law $\mathbb{P}$ for this experiment.
(b) Using the probability law from (a), find the probability that Frodo arrives before 2:30 and Sam arrives before 2:43.
(b) Suppose they agree that if one arrives and the other doesn't arrive within 13 minutes, the meeting is off. Using the probability law from (a), find the probability that Frodo arrives first but Sam is too late for the meeting.
(c) Under the agreement in (b), find the probability that Frodo and Sam meet up successfully.