1. (Variation on Problem 1.24 in the book) This is an "optimistic" version of the Monty Hall problem in the sense that desirable prizes lurk behind two doors instead of 1. A version of Bayes' Rule that works for this problem: if A_1 and A_2 are events that partition Ω — i.e. $A_1 \cap A_2 = \phi$ and $A_1 \cup A_2 = \Omega$ — then for any event B

$$\mathbb{P}(A_k \mid B) = \frac{\mathbb{P}(B \mid A_k)\mathbb{P}(A_k)}{\mathbb{P}(B \mid A_1)\mathbb{P}(A_1) + \mathbb{P}(B \mid A_2)\mathbb{P}(A_2)} \text{ for } k = 1, 2 .$$

Gollum is one of three prisoners, two of whom are to be released. Gollum asks a guard to tell him the identity of a prisoner other than himself who's slated for release. The guard refuses, saying, "At your present state of knowledge, your probability of being released is 2/3, but after you know my answer, your probability of being released will become 1/2, since two prisoners' fates will be unknown, including yours, and exactly one of those prisoners will be released. Do you really want me to give you information that lowers your probability of release?" The guard reasons incorrectly. Let's sort that out.

(a) The given information doesn't suffice to assign conditional probabilities like the one to which the guard refers. What's missing? Note that if Gollum is slated to be released, the guard's answer to the Gollum's question is determined. What if not?

(b) Suppose that when Gollum is not to be released, the guard chooses uniformly at random which of the other two prisoners to name when answering Gollum's question. What is the conditional probability that Gollum will be released given the guard's answer to Gollum's question?

2. A problem on a multiple-choice exam has four choices. A student knows the correct answer with probability 1/2, and in that case he picks it. With probability 1/4 he can eliminate exactly one incorrect choice, in which case he picks uniformly at random from among the remaining three choices. Otherwise, he picks uniformly at random from among the four choices. What is the probability that he knew the answer given that he makes the correct choice?

3. Suppose a certain courtroom judge makes a correct ruling with probability p. Meanwhile, a three-person jury has the following properties. The three members of the jury make their decisions independently; two members make the correct decision with probability p; the third flips a fair coin to make her decision; and the jury renders its ruling by majority rule. Which of the judge or the jury has a higher probability of ruling correctly?

4. (Problem 1.36 in the book) A power utility can supply electricity to a city from n different power plants. Power plant k fails with probability p_k for $1 \le k \le n$, and plants fail independently of each other.

(a) Suppose that any one plant suffices to power the entire city. What is the probability that the city will experience a blackout?

(b) Suppose instead that at least two power plants need to be up and running for the city to avoid a blackout. What is the probability of a blackout in this case?

5. Under what circumstances, if any, can an event be independent of itself?