

1. (Extension of Problem 2.14 in the book) Let the random variable X take each integer value k satisfying $0 \leq k \leq 9$ with probability $1/10$.

- (a) Find the expected value of the random variable $Y = X \bmod 3$.
- (b) Find the expected value of the random variable $Y = 5 \bmod (X + 1)$.

2. (Problem 2.16 in the book) Let X be a random variable with pmf

$$p_X(x) = \begin{cases} x^2/a & \text{when } x = -3, -2, -1, 0, 1, 2, 3 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find a and $\mathbb{E}(X)$.
- (b) What is the pmf of the random variable $Z = (X - \mathbb{E}(X))^2$?
- (c) Find the variance of X using the result of part (c).
- (d) Find the variance of X using the Expected Value Rule

$$\mathbb{E}(g(X)) = \sum_x g(x)p_X(x) .$$

3. Three tests consist of three questions each, with each question worth 1 point. Frodo can answer each question on the easy test independently with probability 0.9, on the medium test with probability 0.7, and on the hard test with probability 0.5. Frodo chooses one test uniformly at random and answers the questions on that test. Find the pmf of his score X .

4. Legolas, equipped with a huge supply of arrows, engages in target practice. On any given shot, his arrow hits the bullseye with probability $p \in (0, 1)$, and all shots are independent of each other. Before starting practice, he resolves to continue shooting until he has hit the bullseye r times. Let X be the number of arrows he expends during his practice session.

- (a) What is the set of possible values X can take on?
- (b) Find the pmf of X .