ECE 3100 Probability Cheat Sheet (Bertsekas & Tsitsiklis, Ch. 1–2.3 + All HW/Discussions)

1. Sets & Sample Spaces (Sec. 1.1)

Experiment: procedure producing exactly one **outcome**. **Sample space** Ω: set of *all* possible outcomes. **Event**: subset $A\subseteq\Omega$; occurs if outcome $\omega\in A$. Outcomes must be **mutually exclusive** and **collectively exhaustive**.
Set operations: $A\cup B$ (union/"or"), $A\cap B$ (intersection/"and"), $A^c=\Omega\setminus A$ (complement/"not A").
Basic identities: $(A^c)^c=A$; $A\cup\Omega=\Omega$; $A\cap\Omega=A$; $A\cup A^c=\Omega$; $A\cap A^c=\varnothing$; $A\cup\varnothing=A$; $A\cap\varnothing=\varnothing$
De Morgan's Laws: $(A\cup B)^c=A^c\cap B^c$; $(A\cap B)^c=A^c\cup B^c$. Generalized: $\left(\bigcup_i A_i\right)^c=\bigcap_i A_i^c$; $\left(\bigcap_i A_i\right)^c=\bigcup_i A_i^c$.
Distributive: $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$; $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.
Decomposition: $A^c=(A^c\cap B)\cup(A^c\cap B^c)$; $(A\cap B)^c=(A^c\cap B)\cup(A^c\cap B^c)\cup(A\cap B^c)$.
Partition: $A_1,\dots,A_n$ partition Ω if $A_i\cap A_j=\varnothing$ for $i\neq j$ and $\bigcup_i A_i=\Omega$.
Disjoint (Mutually Exclusive): $A\cap B=\varnothing$; A and B cannot both occur.
Subset: $A\subseteq B$ means every outcome in A is also in B; $P(A)\le P(B)$.
Expressing events: "At least two of A,B,C": $(A\cap B)\cup(A\cap C)\cup(B\cap C)$. "Exactly one of A,B,C": $(A\cap B^c\cap C^c)\cup(A^c\cap B\cap C^c)\cup(A^c\cap B^c\cap C)$. "At most one of A,B,C": $(A\cap B)^c\cap(A\cap C)^c\cap(B\cap C)^c$. "$A$ or, if not, not B": $A\cup B^c$. "A occurs but B doesn't": $A\cap B^c$. "Neither A nor B": $A^c\cap B^c=(A\cup B)^c$. "A or B but not both" (XOR): $(A\cap B^c)\cup(A^c\cap B)$. "$A$ and C occur, but neither B nor D": $A\cap C\cap B^c\cap D^c$.
Infinite intersections: $\bigcap_{n=0}^{\infty}A_n$ where $A_n=\{m\in\mathbb{N}: m\ge n\}$: every natural number is eventually excluded, so $\bigcap A_n=\varnothing$.

2. Probability Axioms (Sec. 1.2)

(i) **Nonnegativity:** $P(A)\ge 0$ for all A.
(ii) **Normalization:** $P(\Omega)=1$.
(iii) **(Countable) Additivity:** If $A_1,A_2,\dots$ pairwise disjoint, $P(\bigcup_i A_i)=\sum_i P(A_i)$.

Key Consequences

$P(\varnothing)=0$. $P(A^c)=1-P(A)$. $0\le P(A)\le 1$.
Inclusion–Exclusion (2): $P(A\cup B)=P(A)+P(B)-P(A\cap B)$.
Inclusion–Exclusion (3): $P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$.
Alt. form: $P(A\cup B\cup C)=P(A)+P(A^c\cap B)+P(A^c\cap B^c\cap C)$.
Complement rule: $P(\text{at least one of } A,B)=1-P(A^c\cap B^c)$.
Union bound (Boole): $P(A\cup B)\le P(A)+P(B)$; equality iff disjoint. $P(\bigcup_i A_i)\le\sum_i P(A_i)$.
Difference: $P(A\setminus B)=P(A\cap B^c)=P(A)-P(A\cap B)$.
Bonferroni: $P(A\cap B)\ge P(A)+P(B)-1$; general: $P(\bigcap_{i=1}^n A_i)\ge\sum_{i=1}^n P(A_i)-(n-1)$.
Bounds on $P(A\cap B)$: $\max(0,\,P(A)+P(B)-1)\le P(A\cap B)\le\min(P(A),P(B))$. $\max(P(A),P(B))\le P(A\cup B)\le\min(1,\,P(A)+P(B))$.
Exactly one of A,B: $P(A\cap B^c)+P(A^c\cap B)=P(A)+P(B)-2P(A\cap B)$.
Exactly one of A,B,C: $P(A)+P(B)+P(C)-2P(A\cap B)-2P(A\cap C)-2P(B\cap C)+3P(A\cap B\cap C)$.
Exactly 2 of A,B,C: $P(A\cap B)+P(A\cap C)+P(B\cap C)-3P(A\cap B\cap C)$.

Discrete Uniform Law

If Ω finite with $|\Omega|=n$ equally likely outcomes: $P(A)=|A|/|\Omega|=(\#\text{ favorable})/(\#\text{ total})$.

Continuous Uniform Models

Uniform on interval $[a,b]$: $P([c,d])=(d-c)/(b-a)$ for $a\le c\le d\le b$.
Uniform on region $S\subset\mathbb{R}^2$: $P(A)=\text{Area}(A\cap S)/\text{Area}(S)$.
Manhattan distance: $|x|+|y|$. Point uniform on $[0,1]^2$: $P(x+y\le a)$: if $0\le a\le 1$, $=a^2/2$; if $1<a\le 2$, $=1-(2-a)^2/2$.
Meeting problem: Two arrivals uniform on $[0,T]$. $P(|X-Y|\le w)=1-(1-w/T)^2$ for $0\le w\le T$. One arrives first but other late by $>w$: $P=\frac{(T-w)^2}{2T^2}$ (each person).

3. Conditional Probability (Sec. 1.3)

$$P(A\,|\,B)=\frac{P(A\cap B)}{P(B)},\quad P(B)>0.$$

$P(\cdot\,|\,B)$ is itself a valid probability law (satisfies all three axioms). So all rules hold conditionally, e.g. $P(A_1\cup A_2\,|\,B)=P(A_1\,|\,B)+P(A_2\,|\,B)$ when $A_1\cap A_2=\varnothing$. $P(A^c\,|\,B)=1-P(A\,|\,B)$. If equally likely: $P(A\,|\,B)=|A\cap B|/|B|$.
Multiplication rule: $P(A\cap B)=P(B)\,P(A\,|\,B)=P(A)\,P(B\,|\,A)$.
Chain rule: $P(\bigcap_{i=1}^n A_i)=P(A_1)\prod_{k=2}^n P\left(A_k\,\middle|\,\bigcap_{j=1}^{k-1}A_j\right)$.
Two coins: $P(\text{HH}\,|\,\text{1st H})=p$. But $P(\text{HH}\,|\,\text{at least one H})=\frac{p^2}{2p-p^2}\le p$.

4. Total Probability & Bayes' Rule (Sec. 1.3–1.4)

If $A_1,\dots,A_n$ partition Ω with $P(A_i)>0$:
Total Probability: $P(B)=\sum_{i=1}^n P(A_i)\,P(B\,|\,A_i)$.
Bayes' Rule: $P(A_i\,|\,B)=\frac{P(B|A_i)\,P(A_i)}{\sum_{j=1}^n P(B|A_j)\,P(A_j)}$.
Two-event: $P(A\,|\,B)=\frac{P(B|A)\,P(A)}{P(B|A)\,P(A)+P(B|A^c)\,P(A^c)}$.
Terminology: Prior $P(A_i)$—initial belief. **Likelihood** $P(B\,|\,A_i)$—how likely evidence under each hypothesis. **Posterior** $P(A_i\,|\,B)$—updated belief after evidence.
Sequential Bayes: After observing B_1, posterior $P(A_i\,|\,B_1)$ becomes new prior; observe B_2 and apply Bayes again.
False positive/negative: $P(\text{false alarm})=P(\text{detect}\,|\,\text{absent})\,P(\text{absent})$. $P(\text{miss})=P(\text{no detect}\,|\,\text{present})\,P(\text{present})$. **False-Positive Puzzle:** Accurate test can have low positive predictive value if prior probability (prevalence) is very small.
Monty Hall/Prisoner: Posterior depends on guard's/host's randomization strategy. If Gollum to be released (prob 2/3), guard's answer is determined. If not (prob 1/3), guard picks uniformly. By Bayes: $P(\text{released}\,|\,\text{guard says } X)=2/3$ regardless.

5. Independence (Sec. 1.5)

Two Events

A and B are **independent** iff $P(A\cap B)=P(A)\,P(B)$. Equiv. (when defined): $P(A\,|\,B)=P(A)$; $P(B\,|\,A)=P(B)$. If $A\perp\!\!\!\perp B$: $A\perp\!\!\!\perp B^c$, $A^c\perp\!\!\!\perp B$, $A^c\perp\!\!\!\perp B^c$.

Independence vs. Disjointness — Key Comparison

Disjoint ($A\cap B=\varnothing$)	**Independent** ($P(A\cap B)=P(A)P(B)$)
$P(A\cap B)=0$	$P(A\cap B)=P(A)P(B)$
$P(A\cup B)=P(A)+P(B)$	$P(A\cup B)=P(A)+P(B)-P(A)P(B)$
$P(A\,|\,B)=0$ (if $P(B)>0$)	$P(A\,|\,B)=P(A)$
Knowing $B\Rightarrow A$ did not occur	Knowing B gives *no info* about A
$P(A),P(B)>0\Rightarrow$ **NOT** indep.	Can be disjoint only if $P(A)=0$ or $P(B)=0$

Critical: If $P(A)>0$ and $P(B)>0$, disjoint events are **never** independent ($0\neq P(A)P(B)$). Disjoint = *maximally dependent.*
Positive/Negative Association: $P(A\,|\,B)>P(A)\Leftrightarrow P(B\,|\,A)>P(B)$ (symmetric). If $P(A\,|\,B)>P(A)$: A,B are **not** independent and **not** disjoint.
Independent of itself: $P(A)=P(A)^2\Rightarrow P(A)\in\{0,1\}$. If $P(A)=0$ or $P(A)=1$, then A is independent of *every* event.

Multiple Events & Mutual Independence

$A_1,\dots,A_n$ **mutually independent** iff for *every* subset $S\subseteq\{1,\dots,n\}$ with $|S|\ge 2$: $P(\bigcap_{i\in S}A_i)=\prod_{i\in S}P(A_i)$. For 3 events: need all 4 conditions (3 pairwise + triple). 2^n-n-1 conditions total. Pairwise indep. $\not\Rightarrow$ mutual indep.; triple alone $\not\Rightarrow$ pairwise.
Key identity (complements): If $A_1,\dots,A_n$ mutually independent: $P(A_1^c\cap\cdots\cap A_n^c)=\prod_{i=1}^n(1-P(A_i))$; $P(A_1^c\cup\cdots\cup A_n^c)=1-\prod_{i=1}^n P(A_i)$.
Proof: By De Morgan: $A_1^c\cup\cdots\cup A_n^c=(A_1\cap\cdots\cap A_n)^c$. $P=1-P(A_1\cap\cdots\cap A_n)=1-\prod P(A_i)$ by independence.
Conditional Independence given C ($P(C)>0$): $P(A\cap B\,|\,C)=P(A\,|\,C)\,P(B\,|\,C)$; equiv. $P(A\,|\,B\cap C)=P(A\,|\,C)$. Indep. $\not\Rightarrow$ cond. indep., and vice versa.
Independent trials: Coin flips, die rolls, transmissions—each trial's outcome doesn't affect others. Product rule: $P(\text{seq})=\prod P(\text{each})$.

Bernoulli Trials & Reliability

n indep. tosses, $P(\text{head})=p$: $P(k\text{ heads})=\binom{n}{k}p^k(1-p)^{n-k}$.
Series (all must work): $P=p_1p_2\cdots p_m$. **Parallel** (any suffices): $P=1-(1-p_1)\cdots(1-p_m)$.

6. Counting (Sec. 1.6)

Multiplication principle: r stages with $n_1,\dots,n_r$ choices $\Rightarrow\prod n_i$ total.
Permutations: all n: $n!$; $0!=1$. **k-permutations:** $n!/(n-k)!$ (ordered subsets of size k).
Combinations: $\binom{n}{k}=\frac{n!}{k!(n-k)!}$ (unordered subsets). $\binom{n}{0}=\binom{n}{n}=1$, $\binom{n}{k}=\binom{n}{n-k}$, $\sum_{k=0}^n\binom{n}{k}=2^n$. $\binom{n}{k}=0$ if $k<0$ or $k>n$.
Pascal's Rule: $\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}$. $k\binom{n}{k}=n\binom{n-1}{k-1}$; $\sum_{k=0}^n k\binom{n}{k}=n\cdot 2^{n-1}$.
Multinomial: $\frac{n!}{n_1!n_2!\cdots n_r!}$ ways to partition n into groups of sizes $n_1+\cdots+n_r=n$. E.g., anagrams of TATTOO: $6!/(3!\,1!\,2!)=60$.

Sampling summary:

	Ordered	Unordered
With replacement	n^k	$\binom{n+k-1}{k}$
Without replacement	$\frac{n!}{(n-k)!}$	$\binom{n}{k}$

Hypergeometric: N items, M special, draw K w/o replacement. $P(j\text{ special})=\frac{\binom{M}{j}\binom{N-M}{K-j}}{\binom{N}{K}}$, $0\le j\le\min(K,M)$.
Derangements (permutations with no fixed points): $D_n=n!\sum_{k=0}^n\frac{(-1)^k}{k!}\approx\frac{n!}{e}$. $P(\text{all } n\text{ match})=1/n!$; $P(\text{first } m\text{ match})=(n-m)!/n!$.

7. Discrete Random Variables (Sec. 2.1–2.3)

Random variable (r.v.): function $X:\Omega\to\mathbb{R}$. **Discrete:** range is finite or countably infinite.

7.1 PMF (Probability Mass Function) (Sec. 2.2)

$p_X(x)=P(X=x)$. $p_X(x)\ge 0$; $\sum_x p_X(x)=1$. $P(X\in S)=\sum_{x\in S}p_X(x)$. To compute: for each x, collect all outcomes giving $X=x$, sum their probs.

7.2 Common Discrete Distributions

Bernoulli(p): $X\in\{0,1\}$; $p_X(1)=p$, $p_X(0)=1-p$. $E[X]=p$, $\text{var}(X)=p(1-p)$.
Binomial(n,p): $X=\#$ successes in n indep. Bernoulli trials. $p_X(k)=\binom{n}{k}p^k(1-p)^{n-k}$, $k=0,\dots,n$. $E[X]=np$, $\text{var}(X)=np(1-p)$.
Geometric(p): $X=\#$ trials until first success. $p_X(k)=(1-p)^{k-1}p$, $k=1,2,\dots$ $E[X]=1/p$, $\text{var}(X)=(1-p)/p^2$. $P(X>k)=(1-p)^k$. **Memoryless:** $P(X>m+n\,|\,X>m)=P(X>n)$.
Negative Binomial (Pascal): $X=\#$ trials until r-th success. $p_X(k)=\binom{k-1}{r-1}p^r(1-p)^{k-r}$, $k=r,r{+}1,\dots$

$E[X]=r/p$, $\text{var}(X)=r(1-p)/p^2$. Sum of r indep. Geometric(p) r.v.'s.
Discrete Uniform on $\{a,\dots,b\}$: $p_X(k)=1/(b-a+1)$. $E[X]=(a+b)/2$, $\text{var}(X)=(b-a)(b-a+2)/12$. Die: $E[X]=3.5$, $\text{var}(X)=35/12$.
Poisson(λ): $p_X(k)=e^{-\lambda}\lambda^k/k!$, $k=0,1,\dots$ $E[X]=\lambda$, $\text{var}(X)=\lambda$. Approx. Binomial when n large, p small, $\lambda=np$.

7.3 Functions of Random Variables (Sec. 2.3)

If $Y=g(X)$: $p_Y(y)=\sum_{\{x:g(x)=y\}} p_X(x)$. E.g., $Y=|X|$: $p_Y(y)=p_X(y)+p_X(-y)$ for $y>0$.
Example: X uniform on $\{0,\dots,9\}$, $Y=X \bmod 3$: $p_Y(0)=4/10$; $p_Y(1)=3/10$; $p_Y(2)=3/10$.
Product r.v.: $X=H\cdot T$ where $H+T=n$ flips. $X=k(n-k)$ for k heads. $P(X=0)=p^n+(1-p)^n$.

7.4 Expected Value (Mean) (Sec. 2.4)

$E[X]=\sum_x x\,p_X(x)$ (weighted average).
LOTUS (Expected Value Rule): $E[g(X)]=\sum_x g(x)\,p_X(x)$.
Linearity (always): $E[aX+b]=aE[X]+b$. $E[X+Y]=E[X]+E[Y]$ (even if dependent).
$E[XY]=E[X]\,E[Y]$ **only when** X,Y independent. **Warning:** $E[g(X)]\neq g(E[X])$ in general (only if g linear).
n-th moment: $E[X^n]=\sum_x x^n p_X(x)$.

7.5 Variance & Standard Deviation

$\text{var}(X)=E[(X-E[X])^2]=E[X^2]-(E[X])^2$.
$\text{var}(aX+b)=a^2\text{var}(X)$ (shift doesn't change var). $\sigma_X=\sqrt{\text{var}(X)}$. $\text{var}(X)\geq 0$; $=0$ iff X constant.
Independent: $\text{var}(X+Y)=\text{var}(X)+\text{var}(Y)$ (extends to n mutually indep. r.v.'s).
Var of Binomial: $X=X_1+\cdots+X_n$, each $X_i\sim\text{Bern}(p)$, indep. $\text{var}(X)=\sum\text{var}(X_i)=np(1-p)$.

8. Problem-Solving Strategies

Complement: $P(\text{at least one})=1-P(\text{none})$.
Conditioning (Total Prob): Partition into scenarios, compute each conditional, sum.
Sequential Bayes: Posterior from step k becomes prior for step $k+1$.
Tree diagrams: Multiply along paths (chain rule); add across paths for total prob.
Counting method: When Ω finite, equally likely: $P(A)=|A|/|\Omega|$.

9. Useful Formulas & Identities

$\sum_{n=0}^{\infty}\gamma^n=\frac{1}{1-\gamma}$, $|\gamma|<1$; $\sum_{n=0}^{N-1}\gamma^n=\frac{1-\gamma^N}{1-\gamma}$
$\sum_{n=1}^{\infty}n\gamma^{n-1}=\frac{1}{(1-\gamma)^2}$, $|\gamma|<1$
$\lim_{n\to\infty}\left(1+\frac{\alpha}{n}\right)^n=e^{\alpha}$; $(a+b)^n=\sum_{k=0}^n\binom{n}{k}a^kb^{n-k}$
$\sum_{k=1}^n k=\frac{n(n+1)}{2}$; $\sum_{k=1}^n k^2=\frac{n(n+1)(2n+1)}{6}$; $n!\approx\sqrt{2\pi n}(n/e)^n$
Binomial PMF sums to 1: $\sum_{k=0}^n\binom{n}{k}p^k(1-p)^{n-k}=1$.
Geometric PMF sums to 1: $\sum_{k=1}^{\infty}(1-p)^{k-1}p=p\cdot\frac{1}{1-(1-p)}=1$.

10. Worked Examples from HW/Discussion

Probability Bounds (HWII)
Given $P(D)=0.13$, $P(M)=0.37$: $P(D\cup M)\in[0.37,\ 0.50]$; $P(D\cap M)\in[0,\ 0.13]$. Expect closer to max union / min intersection (most people don't overlap).

Repetition Coding (HWII)
Send bit 3 times, majority decode. $P(\text{correct})=p^3+3p^2(1-p)$. $P(\text{error})=3p(1-p)^2+(1-p)^3$ where $p=$ prob bit transmitted correctly.

Quality Control (HWII, DiscIV)
N items, M defective, test K w/o replacement. $P(\text{no defective})=\frac{\binom{N-M}{K}}{\binom{N}{K}}$; decreases as M increases. $P(\text{reject}\,|\,M\text{ defective})=1-\frac{\binom{N-M}{K}}{\binom{N}{K}}$.

Hat/Name Drawing (HWII)
n slips, n students. Total arrangements: $n!$. $P(\text{all match})=1/n!$; $P(\text{first } m \text{ match})=(n-m)!/n!$. $P(\text{first } m \text{ get names of last } m)=m!(n-m)!/n!$.

Conditional Prob. & Independence (HWIII)
If $P(A\,|\,B)>P(A)$: (a) Yes, $P(B\,|\,A)>P(B)$ (symmetric). (b) Not independent. (c) Not disjoint (would need $P(A\,|\,B)=0$).
Sum of dice: $A=\{\text{sum}=12\}$, $B=\{\text{at least one 6}\}$, $C=\{\text{at least one 2}\}$. $P(A)=1/36$, $P(B)=11/36$, $P(A\cap B)=1/36$. Check: $P(A)P(B)=11/1296\neq 1/36$. **Not indep.** $P(A\cap C)=0\neq P(A)P(C)$: not indep. (disjoint with $P>0$!).

Sequential Bayes (HWIII)
Biased die: standard vs loaded. Roll 3, then 6, then 5. After each roll, update prior. After seeing 5: $P(\text{standard}\,|\,5)=1$ (loaded can't produce 5).

Smaug in Caves (HWIII)
$P(\text{cave 1})=p_1$, $P(\text{cave 2})=p_2$. Drone visits cave 1, misses Smaug. $P(\text{cave 1}\,|\,\text{miss})=\frac{p_1(1-d_1)}{p_1(1-d_1)+p_2}$; $P(\text{cave 2}\,|\,\text{miss})=\frac{p_2}{p_1(1-d_1)+p_2}$. Sum to 1.

Win-by-2 Match (HWIII)
Games paired into rounds. Round decisive w.p. $p^2+(1-p)^2$. $P(\text{win match})=\frac{p^2}{p^2+(1-p)^2}$ (geometric series on tied rounds).

Gambler's Ruin
Start \$$k$, win \$1 w.p. p, lose \$1 w.p. $q=1-p$, stop at \$0 or \$$n$. $P(\text{reach } n)=\begin{cases}\frac{1-(q/p)^k}{1-(q/p)^n} & p\neq q\\ k/n & p=q=1/2\end{cases}$

Phone Scratching (HWIV)
Each phone scratched w.p. p indep. $P(\text{first } m \text{ get scratched})=p^m$. $P(\text{exactly } m \text{ scratched})=\binom{n}{m}p^m(1-p)^{n-m}$.

Volleyball Rosters (HWIV)
15 students (6W, 9M), choose 6. Total: $\binom{15}{6}$. Exactly 2W, 4M: $\binom{6}{2}\binom{9}{4}$. At least 2W: $\binom{15}{6}-\binom{9}{6}-\binom{6}{1}\binom{9}{5}$.

Best-of-$(2m-1)$ Series (HWIV, DiscIV)
First to m wins. $P(\text{win series})=\sum_{k=m}^{2m-1}\binom{k-1}{m-1}p^m(1-p)^{k-m}$. If $p>1/2$: prefer *longer* series (more games $\Rightarrow$ skill dominates).
Best-of-7: $P(\text{Rangers win})=(1-p)^4\sum_{j=0}^{3}\binom{j+3}{3}p^j$.

ALOHA Transmission (HWIV)
n transmitters, each sends w.p. p indep. $P(\text{success})=np(1-p)^{n-1}$. Max at $p=1/n$: $P_{\max}=(1-1/n)^{n-1}\to 1/e\approx 0.368$. Over T slots: $P(\geq 1\text{ success})=1-(1-P_s)^T$. Min T for $P\geq\alpha$: $T\geq\lceil\ln(1-\alpha)/\ln(1-P_s)\rceil$. $P(\text{exactly } k \text{ successful slots})=\binom{T}{k}P_s^k(1-P_s)^{T-k}$. $\lim_{\alpha\to 1}T=\infty$ (need infinite slots to guarantee success).

Teams/Power Team (DiscIV)
12 people, 4 teams of 3. Prob 3 of 4 special on same team: $\frac{\binom{4}{3}\cdot 4\cdot\binom{8}{2}\cdot\frac{9!}{(3!)^3}}{\frac{12!}{(3!)^4}}$.

Rooks on Chessboard (DiscIV)
8 rooks on 8×8, all safe (no shared row/col): $P=\frac{8!}{\binom{64}{8}}$.

Radar/Detection (DiscII)
$P(\text{alien})=0.05$, $P(\text{blip}\,|\,\text{alien})=0.99$, $P(\text{blip}\,|\,\text{no alien})=0.1$. $P(\text{alien}\,|\,\text{blip})=\frac{0.99\times 0.05}{0.99\times 0.05+0.1\times 0.95}\approx 0.343$. $P(\text{miss})=0.01\times 0.05=0.0005$; $P(\text{false alarm})=0.1\times 0.95=0.095$.

More Heads (DiscII)
Frodo tosses $n+1$ coins, Sam n coins. $P(\text{Frodo more heads})=1/2$. After n coins each, equal prob of Frodo or Sam ahead; last coin breaks ties.

Multiple Choice Bayes (DiscIII)
Knows answer w.p. 1/2 (picks correct). W.p. 1/4 eliminates 1 wrong (picks from 3). Otherwise guesses from 4. $P(\text{knew}\,|\,\text{correct})=\frac{1/2}{1/2+\frac{1}{4}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{1}{4}}=\frac{24}{37}$.

Judge vs. Jury (DiscIII)
Judge correct w.p. p. Jury: 2 members w.p. p, 1 flips coin. Majority rule. $P(\text{jury})=p^2+p(1-p)+\frac{1}{2}[p(1-p)+(1-p)^2]$. Compare to p.

Power Plants (DiscIII)
Plant k fails w.p. p_k, independently. Any one suffices: $P(\text{blackout})=\prod_k p_k$. Need ≥ 2 running: $P(\text{blackout})=\prod_k p_k+\sum_j(1-p_j)\prod_{k\neq j}p_k$.

Non-Uniform Die (HWI)
Face k has prob $\alpha(k+1)$: $\sum_{k=0}^{n-1}\alpha(k+1)=1\Rightarrow\alpha=\frac{2}{n(n+1)}$. 12-sided: $\alpha=1/78$, $P(k)=(k+1)/78$.

Crayon Drawing (HWI)
Box with 3 crayons. Draw one, return only if cyan, draw again. $P(2\text{nd}=l\,|\,1\text{st}=k)$ depends on whether k returned. Use chain rule for joint prob.

PMF Computation (DiscV)
$p_X(x)=x^2/a$ for $x\in\{-3,\dots,3\}$: $a=28$. $E[X]=0$ (symmetry). $E[X^2]=7$. $\text{var}(X)=7$.

Random Test Score (DiscV)
3 tests (easy $p=0.9$, med $p=0.7$, hard $p=0.5$), 3 questions, chosen uniformly. $p_X(k)=\frac{1}{3}\left[\binom{3}{k}(0.9)^k(0.1)^{3-k}+\binom{3}{k}(0.7)^k(0.3)^{3-k}+\binom{3}{k}(0.5)^3\right]$.

Spinner (HWI)
Uniform on $[0,2\pi)$. Colors in quadrants: $P(\text{any color})=1/4$. Finite: $\Omega=\{R,G,Y,B\}$, $P=1/4$. Infinite: $\Omega=[0,2\pi)$, $P=\theta/(2\pi)$.

Key Checks
- Do probs sum to 1? Is sample space complete?
- Ordered/unordered? With/without replacement?
- For independence: check $P(A\cap B)=P(A)P(B)$ (not just $P(A\,|\,B)$).
- For Bayes: use total probability in denominator.
- Complement often easier than direct calculation.