ECE 3100 **HOMEWORK ASSIGNMENT II** **Spring 2026**

1. Suppose that 13% of Cornell engineering students like deadmau5 and 37% like Miley Cyrus.

(a) Find the lowest and highest possible probabilities that a Cornell engineering student selected at random likes either deadmau5 or Miley Cyrus or both.
(b) Find the lowest and highest possible probabilities that a Cornell engineering student selected at random likes both deadmau5 and Miley Cyrus.
(c) In (a) and (b), do you expect the actual probability of the indicated event to be closer to its minimum possible value or to its maximum possible value? Why?

2. Frodo wants to send Gandalf a one-bit message over a noisy communication channel. The channel can either pass a bit through without change or flip the bit with some probability $p \in (0, 1)$. You can visualize Saruman administering the channel as follows: each time a bit comes in, he flips a coin with heads probability p and flips the bit if and only if heads comes up. Because of the threat of bit-flipping, Frodo uses so-called threefold repetition coding: when the message he wants to transmit is 0, he sends the sequence 0, 0, 0; when the message he wants to transmit is 1, he sends 1, 1, 1. If, for example, Frodo sends 1, 1, 1 and Saruman flips HTT, the channel output will be 0, 1, 1. At the receiving end, Gandalf decodes the received 3-bit string using so-called majority-rule decoding: if he receives 2 or more 0's, he decodes it as a 0; if he receives 2 or more 1's, he decodes it as a 1. Consider the following random experiment: Frodo flips a fair coin to decide what message he wants to send. He uses threefold repetition coding, the channel behaves as described above, and Gandalf uses majority-rule decoding. The outcome of the random experiment is the pair $(\mathrm{bit}_1, \mathrm{bit}_2)$, where bit_1 is Frodo's message and bit_2 is how Gandalf decodes what he receives. For this experiment, $\Omega = \{(0,0), (0,1), (1,0), (1,1)\}$.

(a) Find the probability that the outcome is $(1, 1)$.
(b) Find the probability of error, that is the probability that $\mathrm{bit}_1 \neq \mathrm{bit}_2$.

To prepare for the last two problems, you might want to read Section 1.6 of the book, titled Counting, which begins on page 44.

3. A common quality-control practice is to test a few products chosen uniformly at random without replacement from a given batch. If the number of defective products in the sample exceeds a certain threshold, either the batch is thrown out or all products in the batch are tested individually, which is costly. If the number of defective samples falls below the threshold, the batch is deemed to satisfy the quality-control standard. Key user choices are the sample size and the threshold triggering action. Make decisions on these parameters hinges on answering the following question: given a batch of size N containing M defective products, what is the probability that a sample of size K will contain exactly j defective products?

(a) Given $N \geq K$, how many different samples of size K from a batch of size N are there? Given such a sample, what is the probability that we draw it when drawing uniformly without replacement from the batch?
(b) When a batch of size N contains $M \leq N$ defective products, how many samples of size K from the batch contain exactly j defective products, where $0 \leq j \leq K$?
(c) In the notation of (a) and (b), suppose $N = 50$, $M = 10$, and $K = 10$. What is the probability that exactly 5 items in a given sample are defective?

4. In a hat are n slips of paper, each bearing the name of one student in a class of size n. Every student's name is on exactly one slip in the hat. One by one, each student draws a slip from the hat, uniformly from whatever slips remain in the hat.

(a) How many ways can the drawing end up? In other words, how many ways can the slips be doled out, one to each student?

(b) What is the probability that every student draws their own name?

(c) What is the probability that the first m students draw their own names, where $m < n$?

(d) What is the probability that every student among the first m to draw gets the name of one of the last m students to draw, where $m < n$?