ECE 3100 **HOMEWORK ASSIGNMENT IV** **Spring 2026**

1. Prior to taking an exam, members of a class of n students leaves their cell phones in a box with the TA. After the exam, the TA redistributes the phones at random, so all assignments of phones to students are equally probable. Suppose that every phone deposited in the box has probability p of getting scratched independent of what happens to other phones and independent of who deposits the phone or who receives the phone from the TA after the exam. What is the probability that

(a) the first m students will receive scratched phones from the TA, where $m \leq n$?
(b) exactly m students will receive scratched phones from the TA, where $m \leq n$?

2. ECE graduate students participate in an annual volleyball tournament. Suppose 15 students are willing to participate this year.

(a) How many distinct 6-player rosters exist, with players drawn from among the willing participants?
(b) Six of the willing participants are women and nine are men. How many of the rosters you found in (a) consist of exactly two women and four men?
(c) How many rosters satisfy the constraint that they include at least two women?
(d) Suppose one of the graduate students writes a program to generate a roster randomly, so all 6-player rosters are equally probable program outputs. What is the probability that the program's outputted roster includes exactly two women? What is the probability that it includes at least two women?

3. The NY Rangers and Montréal Canadiens play a best-of-seven series in the NHL Eastern Conference finals. The first team to win four games takes the series. The outcomes of the games are independent, and in a given game the Canadiens win with probability p. What is the probability that the Rangers win the series?

4. Frodo makes n independent flips of a coin that has probability p of coming up heads. Assume n is even, and define the random variable X as the product of number of heads obtained in the n flips with the number of tails obtained.

(a) Find the probability of the event $\{X = 0\}$.
(b) What are the possible values of X?
(c) Find the pmf of X when $n = 4$.

5. Consider a communication system with n transmitters. In each time slot, each of the transmitters sends a message with probability p, where p is the same for all transmitters and the transmitters act independently of each other. Successful transmission occurs in a given time slot when exactly one transmitter sends a message during that time slot, thus avoiding collisions between messages.

(a) Find the probability of successful transmission in a given time slot.
(b) What value of p maximizes the probability you computed in (a)? What is the maximum value of the successful-transmission probability. Discuss your result.
(c) Find the limit as $n \to \infty$ of the probability you computed in (a). Discuss your result.

(d) Suppose the system adds transmitters one by one. Every time a new transmitter joins, every transmitter, including the new one, adjusts its transmission probability so as to maximize the probability of successful transmission for the new expanded system. What is the limit as $n \to \infty$ of the probability of successful transmission in this case? You may want to use the fact that

$$\lim_{n\to\infty} \left(1 + \frac{\alpha}{n}\right)^{n-1} = \lim_{n\to\infty} \left(1 + \frac{\alpha}{n}\right)^{n} = e^{\alpha}$$

for every real number α.

6. Suppose the communication system in the previous problem operates for T time slots. The transmitters' actions are independent of each other in each time slot, and what happens in a given time slot is independent of what happens in all the other time slots.

(a) What is the probability that successful transmission occurs in exactly k time slots, where $1 \leq k \leq T$?

(b) What is the probability that successful transmission occurs in at least one time slot? (Suggestion: think complements.)

(c) Suppose $0 < \alpha < 1$. As a function of α, p, and n, what is the minimum number of time slots T for which the probability you computed in (b) is at least α? Intuition tells us that more time slots would make for a higher probability of at least one successful transmission.

(d) Find the limit as $\alpha \to 1$ of the lower bound on T that you calculated in part (c).