ECE 3100 **HOMEWORK ASSIGNMENT I** **Spring 2026**

1. All events named or described in this problem are nonempty subsets of a given sample space Ω. Express each described event in terms of unions, intersections, and complements with respect to Ω of the named events. Draw a Venn diagram illustrating each case. Note that many possible Venn diagrams work for each case, but maybe you can come up with "maximally general" ones.

(a) At least two of events A, B, and C occur.
(b) Events A and C occur, but neither B nor D does.
(c) Exactly one of the events A, B, or C occurs.
(d) Either event A occurs or, if not, event B also does not occur.
(e) At most one of the events A, B, or C occurs.

2. A spinner such as the kind used in games like Twister is colored as in the accompanying diagram. In terms of the labels around the outside, red covers the wedge $0 \leq \theta < \pi/2$, green the wedge $\pi/2 \leq \theta < \pi$, yellow the wedge $\pi \leq \theta < 3\pi/2$, and blue the wedge $3\pi/2 \leq \theta < 2\pi$. In a certain game of chance, what's relevant about the spinner is what color it lands on after being spun. Assume that the spinner is equally likely to land pointing in any direction $0 \leq \theta < 2\pi$.

(a) Construct a finite sample space Ω along with probability law $\mathbb{P}$ on Ω that suffice to model this situation. In terms of your Ω and $\mathbb{P}$, what is the event "spinner lands on blue"? What is the event B that "spinner lands on neither yellow nor red" and what is $\mathbb{P}(B)$?
(b) Repeat part (a) with the word "finite" replaced by "infinite."

3. Given that 23% of Cornell engineering students like Radiohead, 29% like Ed Sheeran, and 61% like neither, what is the probability that a Cornell engineering student drawn at random likes

(a) both Radiohead and Ed Sheeran?
(b) either Radiohead or Ed Sheeran?

4. Let A and B be disjoint events in a sample space Ω, and let $\mathbb{P}$ be a probability law on Ω satisfying $\mathbb{P}(A) = 0.35$ and $\mathbb{P}(B) = 0.55$. Find the probability that

(a) both A and B occur.
(b) A doesn't occur but B does.
(c) neither A nor B occurs.

5. This problem alludes to conditional probabilities. Although we haven't covered those in section yet, all you need to know here is that for any events A and B with $\mathbb{P}(B) > 0$,

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} \, .$$

When rolling a twelve-sided die with faces $k = 0, 1, 2, \ldots, 11$, the probability that the die lands with face k on the bottom turns out to equal $\alpha(k+1)$ for some number α.

(a) Find α. You might want to use the identity $\sum_{m=1}^{n}(m) = n(n+1)/2$.

(b) Let B be the event that an odd number k lands on the bottom, and let A be the event that 3, 4, or 7 lands on the bottom. Find $\mathbb{P}(A \mid B)$.
(c) With A and B as in part (c), find $\mathbb{P}(B \mid A)$.

6. This problem is about a box containing crayons. Whatever the state of the box, you are equally likely to draw any of the crayons in it. The three available crayons have colors cyan, magenta, and fuchsia.

(a) The box contains all three crayons. An experiment consists of drawing one crayon from the box, returning it to the box, and drawing a second crayon. The outcome of the experiment is the result of the two draws — the order matters. Construct a sample space Ω and reasonable probability law $\mathbb{P}$ that suffice to model this experiment and specify $\mathbb{P}(\{s\})$ for each $s \in \Omega$.
(b) Change the experiment as follows: again, start with all three crayons in the box. Draw a crayon, and return that crayon to the box if and only if it's cyan. Construct a good sample space for this experiment. Also construct a reasonable probability law. This is more subtle, and will rely a bit on conditional-probability reasoning. You may assume the following fact: the probability that you draw kl, where k and l represent crayon colors, possibly the same, is the product of
   - the probability that you draw k on the first draw
   - the probability that you draw l on the second draw given that you drew k on the first draw

   The second quantity will sometimes be $1/3$ and sometimes $1/2$.