

1. We assign Frodo, Sam, Gandalf, Legolas, and eight miscellaneous hobbits to four teams of three each — call them Team 1, Team 2, Team 3, and Team 4 — by handing out slips of paper drawn uniformly at random without replacement from a box, just as we did in lecture. What is the probability that one of the teams turns out to be a “power team” consisting of three out of the four main characters? (Thanks to a student from the Spring 2019 offering of ECE 3100 for raising this question.)

2. (Problem 1.55 in the book) Eight rooks are placed randomly on distinct squares of an (8×8) chessboard, with all possible placements equally probable. Find the probability that all the rooks are safe from each other in the sense that no row or column contains more than one rook.

3. A vendor receives lots of crossbows from a manufacturer. Each lot contains $N = 100$ units. The vendor samples K units drawn uniformly without replacement from each shipment and accepts the shipment if none of the tested units is defective.

- Suppose a given shipment contains M defective crossbows. The vendor tests K crossbows from the shipment as described above. Assuming $K \leq 100 - M$, what is the probability that none of the units in the K -sample is defective?
- Show that the number you calculated in (a) is decreasing in M given K .
- Suppose the vendor deems a lot unacceptable if it contains more than five defective units and acceptable otherwise. Explain how you would calculate the minimum sample size K needed to ensure that the vendor is 90% sure not to accept an unacceptable lot after finding no defectives in a K -sample. Note that, given (b), you need only consider the probability that the vendor accepts a lot containing $M = 6$ defective units.

4. (Problem 2.6 from the book) The Red Wings and Maple Leafs are set to face each other in the playoffs. The Leafs win any given game between the two teams with probability p , independent of other games. The general managers need to agree on whether to play a single winner-take-all game or a best-of-3 series. What would the Leafs prefer if $p > 1/2$? Answer the same question if the choice is between a best-of-3 series and a best-of-5 series.

5. (Problem 2.14 in the book) Let the discrete random variable X take each integer value k satisfying $0 \leq k \leq 9$ with probability $1/10$.

- Write a formula for p_X , the pmf of X .
- Find a formula for p_Y , the pmf of the random variable $Y = X \pmod{3}$.
- Find a formula for p_Z , the pmf of the random variable $Z = 5 \pmod{X + 1}$.