Wokflow nets (in one page)[1]

Antonio Brogi, Stefano Forti
Department of Computer Science
University of Pisa, Italy

Petri Nets

Petri nets are directed bipartite graphs with two types of nodes: Places and transitions. Directed arcs connect pairs of nodes having different type.

A ***Petri net*** is a triple (P, T, F) where P is a finite set of *places*, T is a finite set of *transitions*, with $P \cap T = \emptyset$, and $F \subseteq (P \times T) \cup (T \times P)$ is a finite set of *arcs*.

A place $p \in P$ is called an *input place* of a transition $t \in T$ if and only if there exists a directed arc from p to t. A place $p \in P$ is called an *output place* of a transition $t \in T$ if and only if there exists a directed arc from t to p. The sets of input and output places of $t \in T$ are denoted by $\bullet t$ and by $t\bullet$, respectively.

A *state* M of a Petri net is a distribution of *tokens* over places, namely $M \in \{\ f \mid f : P \to \mathbb{N}\}$. Given two states M_1 and M_2, we write $M_1 \geq M_2$ if and only if $\forall p \in P : M_1(p) \geq M_2(p)$.

A transition t is *enabled* if and only if each input place p of t contains at least one token.

An enabled transition may *fire*. When a transition t fires, t consumes one token from each input place of t and it produces one token in each output place of t. The notation $M_1 \xrightarrow{t} M_2$ is used to denote that transition t is enabled in state M_1 and that firing t in M_1 leads to M_2.

A state M_n is *reachable* from a state M_0 (denoted by $M_0 \xrightarrow{*} M_n$) if and only if there is a firing sequence $t_1, t_2, ..., t_n$ such that $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} ... \xrightarrow{t_n} M_n$.

We use (PN, M) to denote a Petri net PN with initial state M. A state M' is a *reachable state* of (PN, M) if and only if M' is reachable from M.

A Petri net (PN, M) is ***live*** if and only if for every reachable state M' and every transition t, there is a state M'' reachable from M' where t is enabled.

A Petri net (PN, M) is ***bounded*** if and only if for each place p there is a $n \in \mathbb{N}$ such that for each reachable state M' the number of tokens in p in M' is less than n.

Workflow Nets

A Petri net $PN = (P, T, F)$ is a ***workflow net*** if and only if:
(1) PN contains a unique *source place* $i \in P$ such that $\bullet i = \emptyset$,
(2) PN contains a unique *sink place* $o \in P$ such that $o\bullet = \emptyset$, and
(3) Adding a transition $\bar{t}$ connecting o to i ($\bullet\bar{t} = \{i\}$ and $\bar{t}\bullet = \{o\}$) makes PN strongly connected.

A workflow net $WN = (P, T, F)$ is ***sound*** if and only if:
(1) $\forall M : (\{i\} \xrightarrow{*} M) \Rightarrow (M \xrightarrow{*} \{o\})$, and
(2) $\forall M : (\{i\} \xrightarrow{*} M \wedge M \geq \{o\}) \Rightarrow (M = \{o\})$, and
(3) $\forall t \in T\ \exists M, M' : \{i\} \xrightarrow{*} M \xrightarrow{t} M'$
where $\{i\}$ and $\{o\}$ denote the states containing only one token in i and o, respectively.

Theorem. A workflow net N is sound if and only if $(\check{N}, \{i\})$ is live and bounded, where $\check{N}$ is N extended with a transition from the sink place o to the source place i.

[1]This note is meant to be used only by the students of the *Advanced Software Engineering* course of the *Master's Degree in Computer Science* and of the *Master's Degree in Computer Science and Networking* at the University of Pisa.