MAGNETIC FIELD

- Demonstrate the force acting on a current carrying conductor placed in a

magnetic field using the current-balance.

- Obtain the expression F=BIl identifying the symbols.
- Define magnetic flux density and give units.
- State Fleming's left hand rule.
- Assign students to find the direction of the force, changing the direction of the current and the direction of magnetic flux density.
- Explain that the force is given by F=BIlsinθ when the conductor is inclined to

the field at angle "θ ".

- State that there is no force when the conductor is parallel to the field.

- Give the expression F=Bqv for the force exerted on a charged particle moving

perpendicular to a magnetic field.

- Discuss the instances where the particle is moving with an angle to the field

and moving parallel to the field.

- Find the direction of the force on a moving positive charged particles and a moving negative charged particles.
- Explain Hall-effect & derive an expression for Hall-voltage.
- Discuss application of Hall-effect and give examples. Demonstrate magnetic effect of an electric current using simple activities.
- Show that the direction of the field near a current carrying conductor can be obtained by using Maxwell’s coke screw rule.

- Express Bio-Savart law introducing the terms, δB =
- Derive an expression for the magnetic flux density at the centre of a flat current carrying circular loop.
- Give expressions for the magnetic flux density for the following instances.

▯ Near a current carrying infinitely long thin straight conductor

▯ Along the axis of a current carrying long solenoid

- To draw the magnetic field patterns of above instances.
- Explain that a mutual force acts on two parallel conductors when current flows in the same direction and in opposite directions.
- Derive the expression.
- Define the unit "Ampere".

$$dB = \frac{\mu_0 \, I \, dl \sin\theta}{4\pi \, r^2}$$

Magnetic Force on a Current-Carrying Conductor

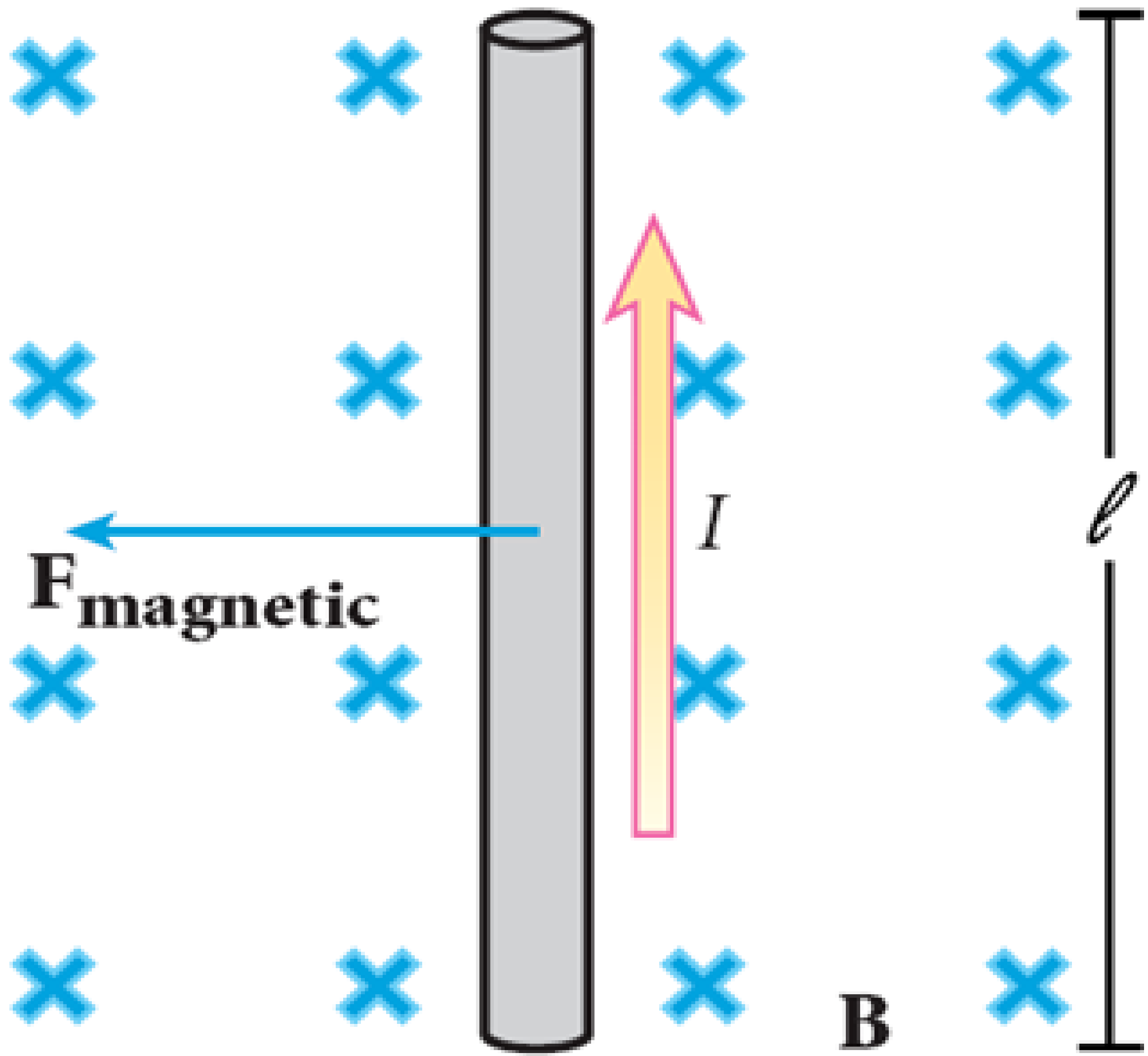
- A current-carrying wire in an external magnetic field undergoes a magnetic force.

- The force on a current-carrying conductor perpendicular to a magnetic field is given by:

$$F_{magnetic} = BI\ell$$

magnitude of magnetic force = (magnitude of magnetic field) × (current) × (length of conductor within B)

Force on a Current-Carrying Wire in a Magnetic Field



Force on a current-carrying conductor in a uniform Magnetic Field:

If a current carrying conductor is placed in an external magnetic field, the magnetic field of conductor will interact with the external magnetic field, as the result of which the conductor may experience a force.The magnitude of magnetic force depends upon the following factors:

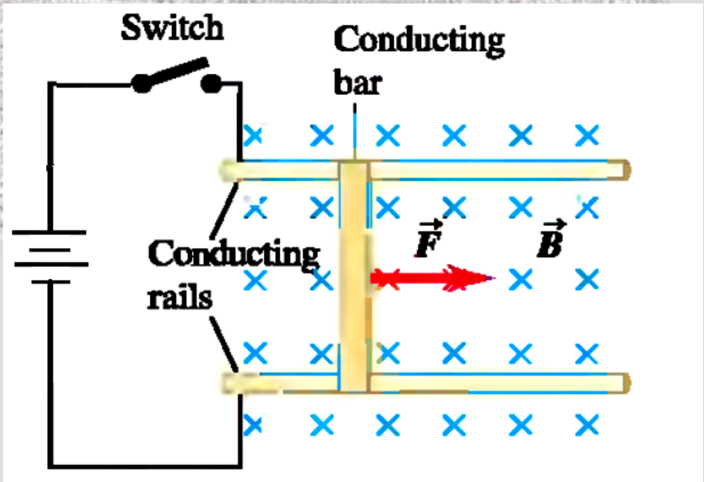
i. The magnetic force is directly proportional to the current flowing through conductor. $F \alpha I$

ii. The force is directly proportional to the length of the conductor inside the magnetic field. $F \alpha l$

iii. The force is directly proportional to the strength of applied magnetic field. $F \alpha B$

iv. The magnetic force on current carrying conductor is directly proportional to sinΘ . where Θ is the angle between conductor and the field.

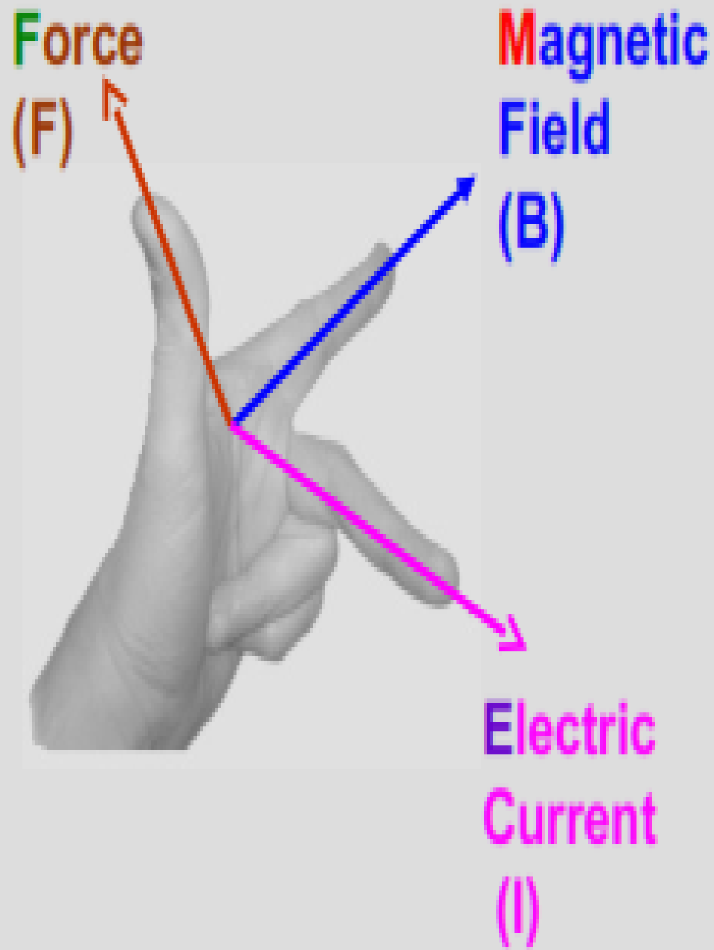
$$F = I l B \sin \theta$$



Combining all these factors,

Fleming's Left Hand Rule:

If the central finger, fore finger and thumb of left hand are stretched mutually perpendicular to each other and the central finger points to current, fore finger points to magnetic field, then thumb points in the direction of motion (force) on the current carrying conductor.



TIP:

Remember the phrase 'e m f' to represent electric current, magnetic field and force in anticlockwise direction of the fingers of left hand.

Charged Particles in a Magnetic Field

- A charge moving through a magnetic field experiences a force proportional to the charge, velocity, and the magnetic field strength (magnetic flux density).
- F = B q v

$$B = \frac{F_{magnetic}}{qv}$$

$$\text{magnetic field} = \frac{\text{magnetic force on a charged particle}}{\text{(magnitude of charge)(speed of charge)}}$$

Chapter 19

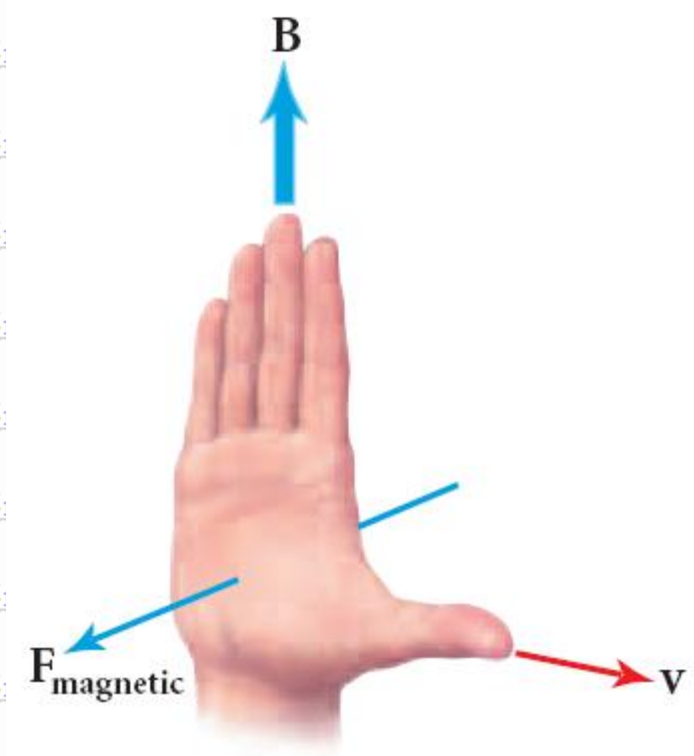
Moving charge in a magnetic field

- The direction of the magnetic force on a moving charge is always perpendicular to both the magnetic field and the velocity of the charge.

- Fleming's left-hand rule or the alternative right-hand rule can be used to find the direction of the magnetic force.

- A charge moving through a magnetic field follows a circular path.

Alternative Right-Hand Rule: Force on a Moving Charge



Particle in a Magnetic Field

Questons

A proton moving east experiences a force of 8.8×10^{-19} N upward due to the Earth's magnetic field. At this location, the field has a magnitude of 5.5×10^{-5} T to the north. Find the speed of the particle.

Use the definition of magnetic field strength.

Given:

$q = 1.60 \times 10^{-19}$ C

$B = 5.5 \times 10^{-5}$ T

$F_{magnetic} = 8.8 \times 10^{-19}$ N

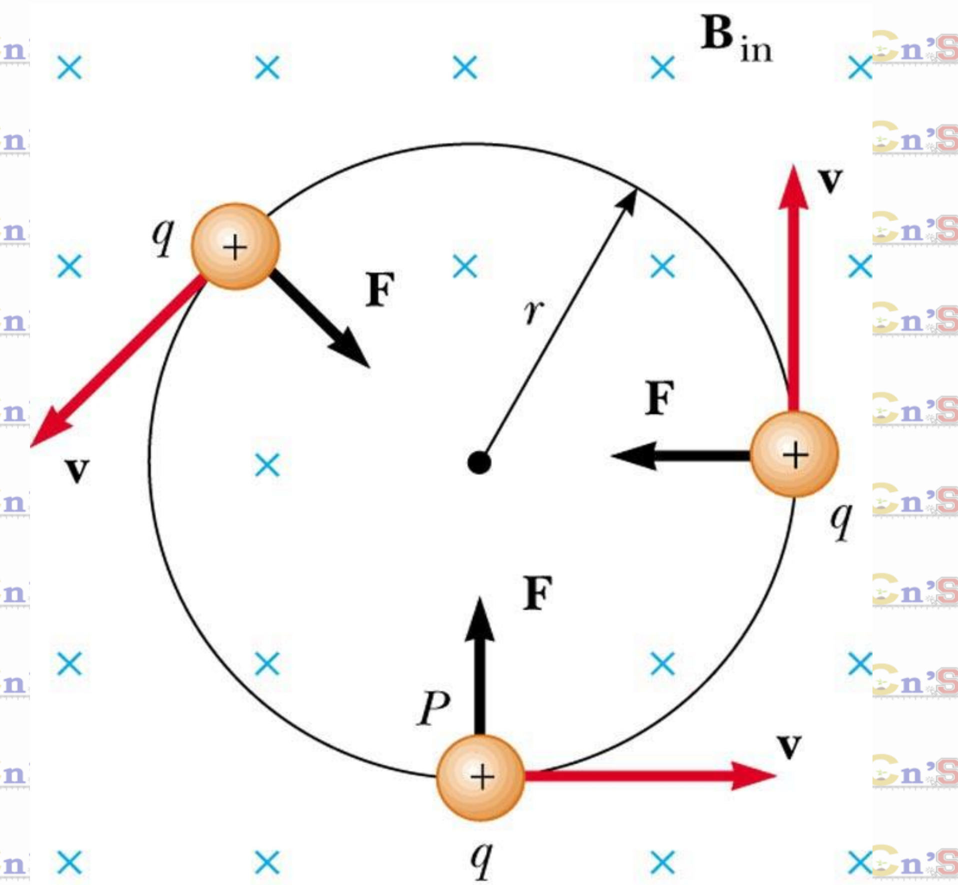
$$B = \frac{F_{magnetic}}{qv}$$

$$v = \frac{F_{magnetic}}{qB} = \frac{8.8 \times 19^{-19}\ \text{N}}{(1.60 \times 19^{-19})(5.5 \times 10^{-5})}$$

$$\boxed{v = 1.0 \times 10^{5}\ \text{m/s}}$$

Force on a Charged Particle in a Magnetic Field

- Consider a particle moving in an external magnetic field so that its velocity is perpendicular to the field
- The force is always directed toward the center of the circular path
- The magnetic force causes a centripetal acceleration, changing the direction of the velocity of the particle



Force on a Charged Particle

- Equating the magnetic and centripetal forces:

$$F = qvB = \frac{mv^2}{r}$$

- Solving for r:

$$r = \frac{mv}{qB}$$

  – r is proportional to the momentum of the particle and inversely proportional to the magnetic field

Examples

- Prove that the T, time to complete one revolution = $2\pi m/qB$
- What is the relationship between Speed and Radius?
- Prove that for an electron, $e/m = 2V/B^2.r^2$

Q # 7. If a charge particle moves in a straight line through some region of space, can you say that the magnetic field in the region is zero.

Ans. The magnitude of magnetic force on a charge particle can be expressed as:

$$F = qvB\sin\theta$$

Where θ is the angle between **B** and **v**. So if the particle moves in a straight line through some region of space then it means that the charge particle is not experiencing magnetic force which might be due to one of the following reasons:

i. Magnetic field strength B in the region is zero
ii. Magnetic field is parallel or anti-parallel to the direction of motion.

Multiple Choice

2. Which of the following statements is most correct?

F. The north pole of a freely rotating magnet points north because the magnetic pole near the geographic North Pole is like the north pole of a magnet.

G. The north pole of a freely rotating magnet points north because the magnetic pole near the geographic North Pole is like the south pole of a magnet.

H. The north pole of a freely rotating magnet points south because the magnetic pole near the geographic South Pole is like the north pole of a magnet.

J. The north pole of a freely rotating magnet points south because the magnetic pole near the geographic South Pole is like the south pole of a magnet.

3. If you are standing at Earth’s magnetic north pole and holding a bar magnet that is free to rotate in three dimensions, which direction will the south pole of the magnet point?

A. straight up

B. straight down

C. parallel to the ground, toward the north

D. parallel to the ground, toward the south

Chapter 19

4. How can you increase the strength of a magnetic field inside a solenoid?

F. increase the number of coils per unit length

G. increase the current

H. place an iron rod inside the solenoid

J. all of the above

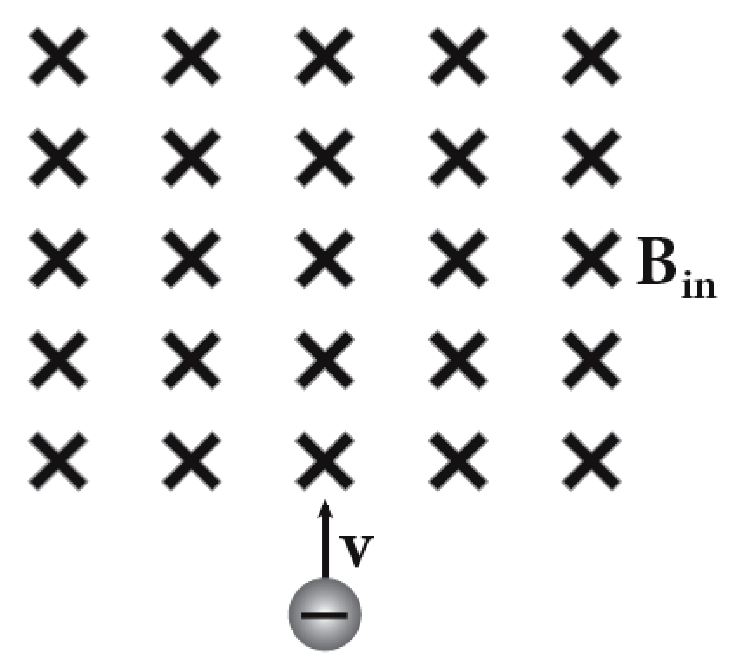
5. How will the electron move once it passes into the magnetic field?

A. It will curve to the right and then continue moving in a straight line to the right.

B. It will curve to the left and then continue moving in a straight line to the left.

C. It will move in a clockwise circle.

D. It will move in a counterclockwise circle.



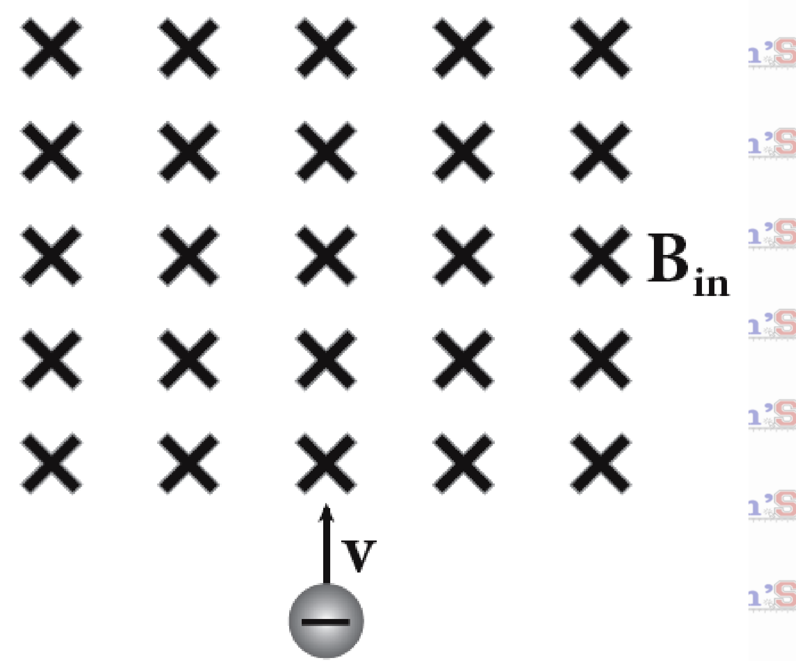
6. What will be the magnitude of the force on the electron once it passes into the magnetic field?

F. qvB

G. $-qvB$

H. qB/v

J. Bil



7. An alpha particle ($q = 3.2 \times 10^{-19}$ C) moves at a speed of 2.5×10^{6} m/s perpendicular to a magnetic field of strength 2.0×10^{-4} T. What is the magnitude of the magnetic force on the particle?

A. 1.6×10^{-16} N

B. -1.6×10^{-16} N

C. 4.0×10^{-9} N

D. zero

Use the passage below to answer questions 8–9.

A wire 25 cm long carries a 12 A current from east to west. Earth's magnetic field at the wire's location has a magnitude of 4.8×10^{-5} T and is directed from south to north.

9. What is the direction of the magnetic force on the wire?

A. north

B. south

C. up, away from Earth

D. down, toward Earth

Multiple Choice, *continued*

Use the passage below to answer questions 8–9.

A wire 25 cm long carries a 12 A current from east to west. Earth's magnetic field at the wire's location has a magnitude of 4.8×10^{-5} T and is directed from south to north.

8. What is the magnitude of the magnetic force on the wire?

F. 2.3×10^{-5} N

G. 1.4×10^{-4} N

H. 2.3×10^{-3} N

J. 1.4×10^{-2} N

Use the passage below to answer questions 8–9.

A wire 25 cm long carries a 12 A current from east to west. Earth's magnetic field at the wire's location has a magnitude of 4.8×10^{-5} T and is directed from south to north.

8. What is the magnitude of the magnetic force on the wire?

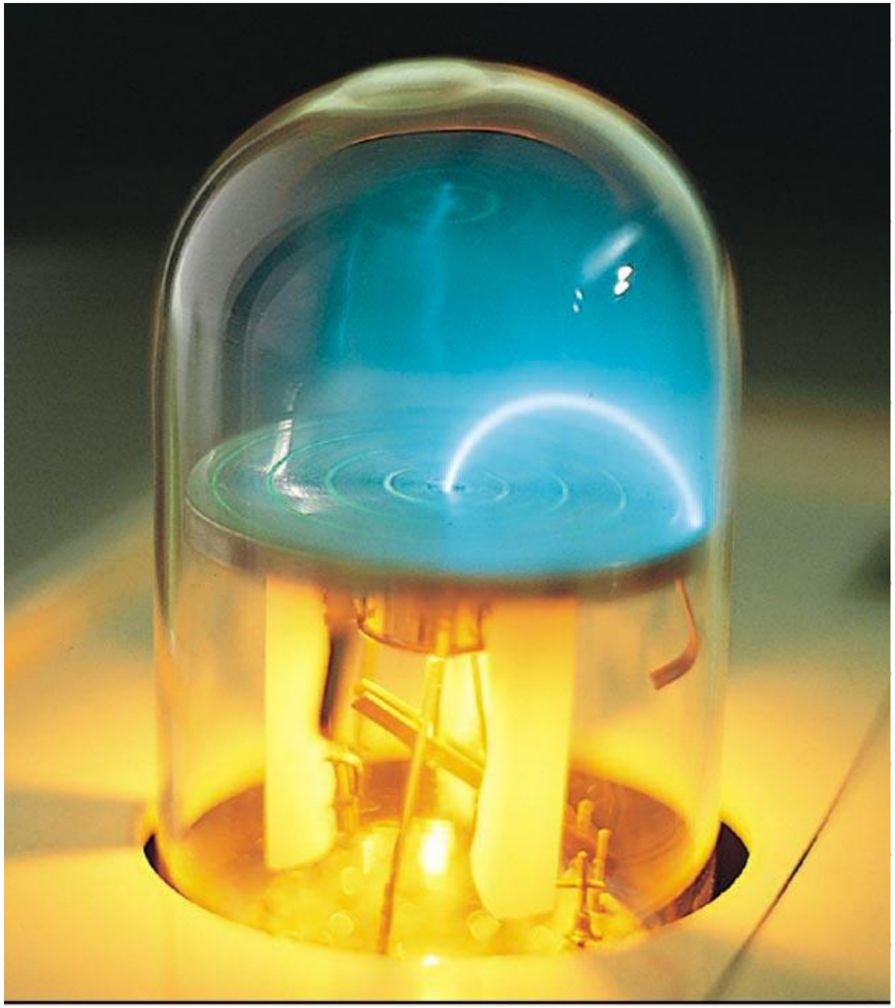
F. 2.3×10^{-5} N

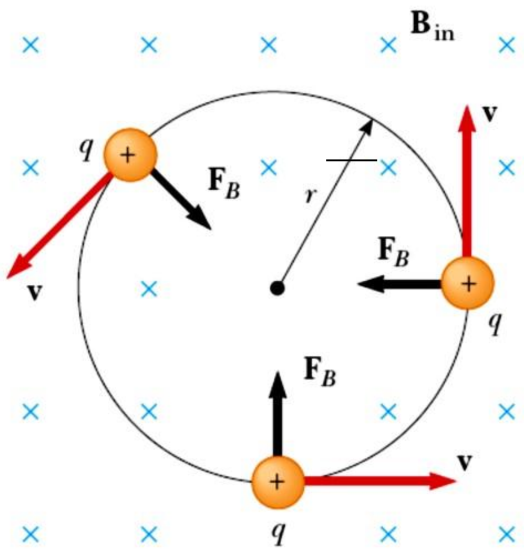
G. 1.4×10^{-4} N

H. 2.3×10^{-3} N

J. 1.4×10^{-2} N

Bending an Electron Beam in an External Magnetic Field





Derive the expression to find out e/m of an electron.

Let a narrow beam of electrons moving with a constant speed v be projected at right angles to a known magnetic field **B**. The magnetic force experienced by the beam of electron will be:

F= -e(v B)

The direction of the force will be perpendicular to both **v** and **B.** As the electron is experiencing a force that acts at right angle to velocity, so it will change the direction of velocity. Thus the electrons are subjected to a constant force F=evB at the right angle to the direction of motion. Under the action of this force, the electrons will move in the circle as shown in the figure.

As the electron moves in the circle, the necessary Cen.force F is provided by the evB magnetic force

$$evB = \frac{mv^2}{r}$$

. Thus we have:

$$\frac{e}{m} = \frac{v}{Br} \quad \text{----------} \quad (I)$$

This equation shows that if the values of v and r is known, e/m of the electron is determined.

To find out the value of r, a glass tube is filled with a gas such as hydrogen at low pressure. The glass tube is placed in a region of uniform magnetic field of known value. As the electrons are shot into this tube, they begin to move along a circle under the action of magnetic force. As the electron move, they collide with the atoms of gas. This excites the atoms due to which they emit light and their path becomes visible as a circular ring of light. The diameter of ring can be easily measured. In order to measure the velocity v of electrons, we should know the potential difference through which the electrons are accelerated before entering into magnetic field. If V is this potential difference, the energy gained by the electrons during their acceleration is Ve. This appears as kinetic energy of electrons:

$$\frac{1}{2}mv^2 = Ve$$

$$v = \sqrt{\frac{2Ve}{m}}$$

Substituting the value of v in equation (1), we get:

$$\frac{e}{m} = \frac{1}{Br}\sqrt{\frac{2Ve}{m}}$$

$$\sqrt{\frac{e}{m}} = \frac{1}{Br}\sqrt{2V}$$

Squaring both sides:

$$\frac{e}{m} = \frac{2V}{B^2r^2}$$

This is the required expression to find the e/m of electron.

Q # 11. How can you use a magnetic field to separate isotopes of chemical element?

Ans. If the ions of isotopes of an element are projected in a magnetic field of known strength B, the ions move in circular path of radius r. The e/m of the ion is given by the expression:

$$\frac{e}{m} = \frac{v}{Br} \Rightarrow r = \frac{v}{B} \times \frac{m}{e}$$

If v, B and e of the ions are constant, then

$$r \propto m$$

So the ions of different mass will have different radii of curvature and hence they can be separated in magnetic field.

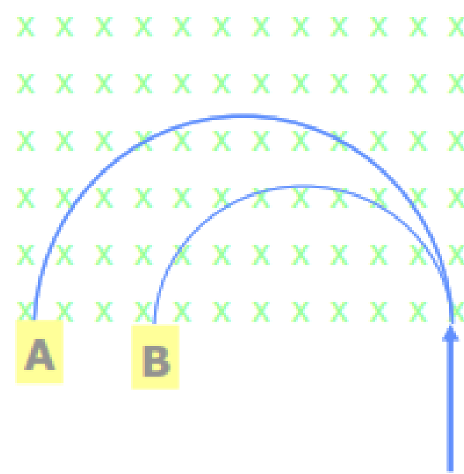
Magnetic Force

→ Two particles of the same charge enter a magnetic field with the same speed. Which one has the bigger mass?

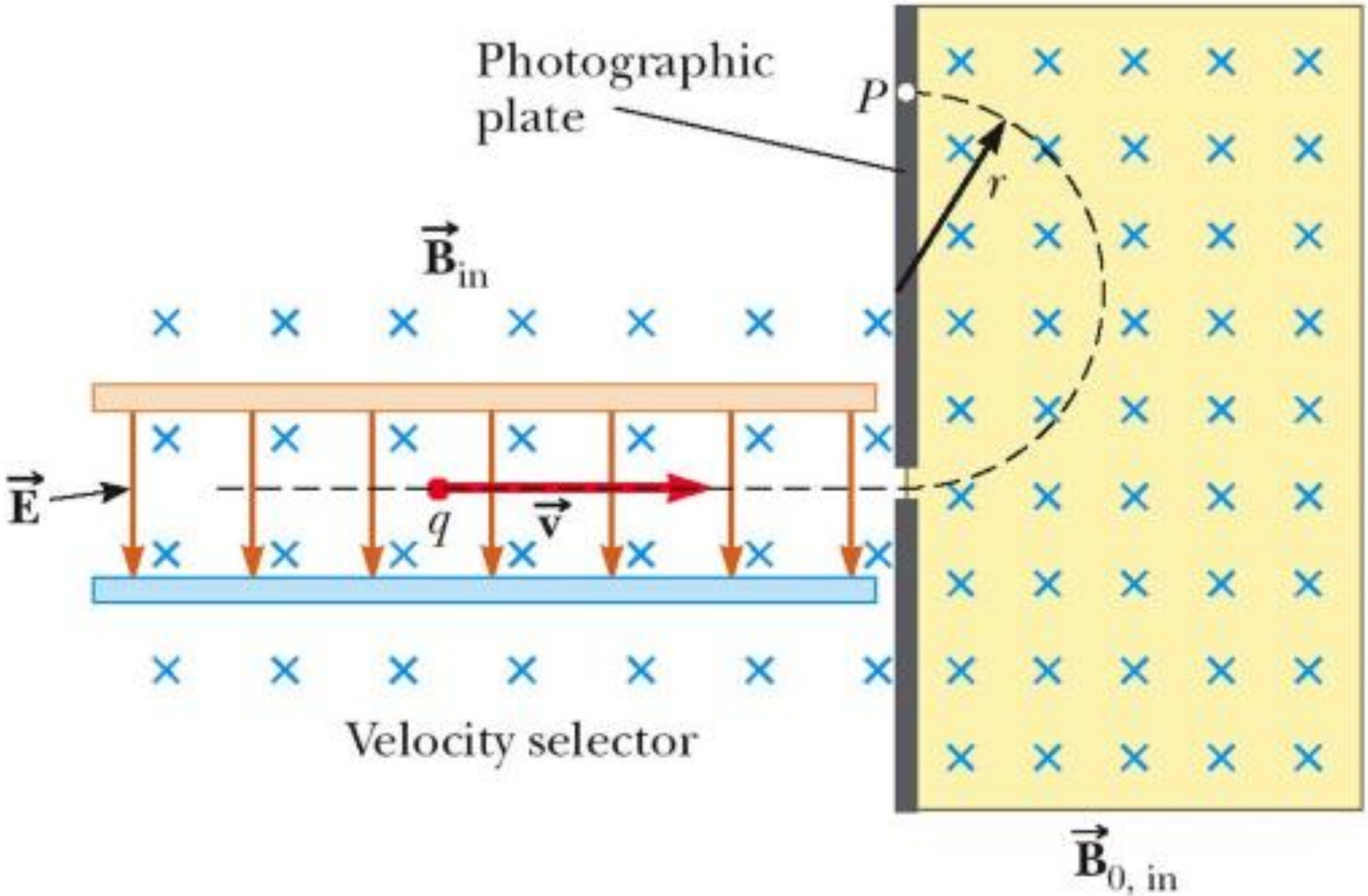
- A
- B
- Both masses are equal
- Cannot tell without more info

$$R = \frac{mv}{qB}$$

Bigger mass means bigger radius



Mass Spectrometer



Mass Spectrometer Operation

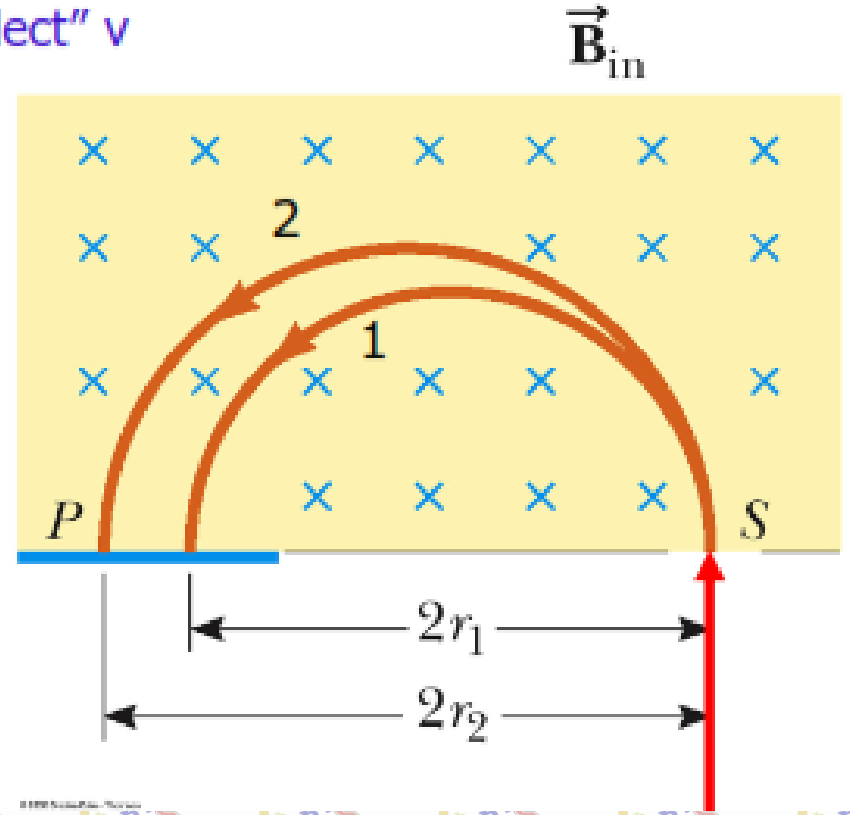
- Positive ions first enter a "velocity selector" where $E \perp B$ and values are adjusted to allow only undeflected particles to enter mass spectrometer.
  - Balance forces in selector $\Rightarrow$ "select" v

$$qE = qvB$$
$$v = E / B$$

  - Spectrometer: Determine mass from v and measured radius r

$$r_1 = \frac{m_1 v}{qB}$$

$$r_2 = \frac{m_2 v}{qB}$$



Mass Spectrometer Example

➔A beam of deuterons travels right at v = 5 x 10^5 m/s

- What value of B would make deuterons go undeflected through a region where E = 100,000 V/m pointing up vertically?

$$eE = evB$$

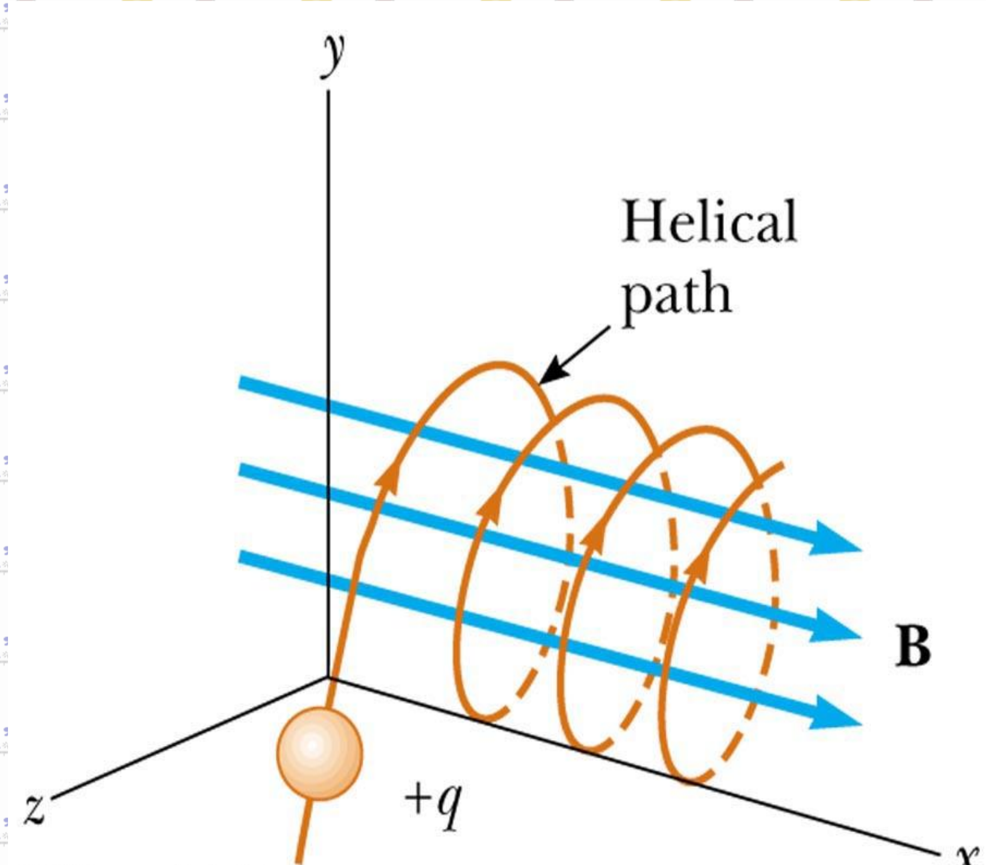
$$B = E/v = 10^5/5\times10^5 = \boxed{0.2\,\mathrm{T}}$$

- If the electric field is suddenly turned off, what is the radius and frequency of the circular orbit of the deuterons?

$$\frac{mv^2}{R} = evB \quad \Rightarrow R = \frac{mv}{eB} = \frac{\left(3.34\times10^{-27}\right)\left(5\times10^5\right)}{\left(1.6\times10^{-19}\right)(0.2)} = \boxed{5.2\times10^{-2}\,\mathrm{m}}$$

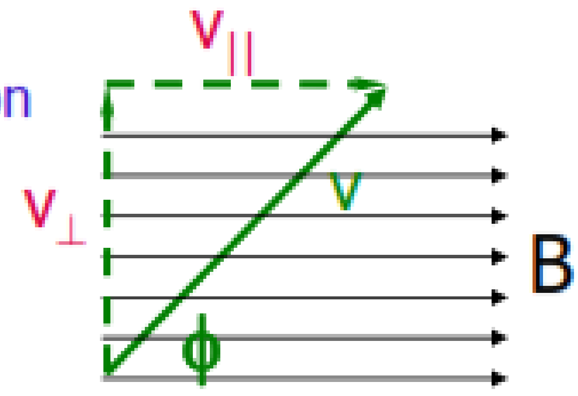
$$f = \frac{1}{T} = \frac{v}{2\pi R} = \frac{5\times10^5}{(6.28)\left(5.2\times10^{-2}\right)} = \boxed{1.5\times10^6\,\mathrm{Hz}}$$

- If the particle's velocity is *not* perpendicular to the field, the path followed by the particle is a spiral
  - The spiral path is called a *helix*

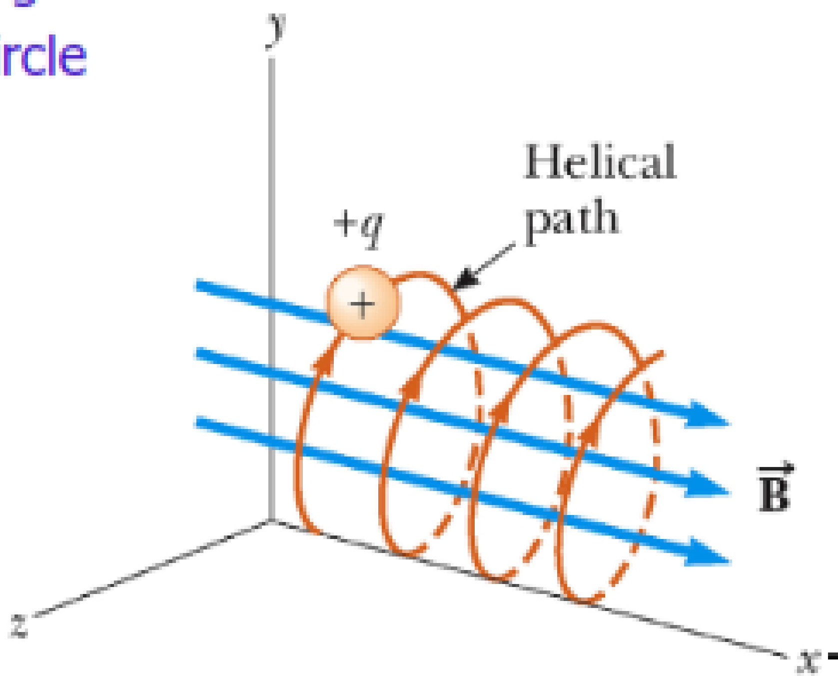


Helical Motion in B Field

- ➔ Velocity of particle has 2 components
  - $\vec{v} = \vec{v}_{||} + \vec{v}_{\perp}$ (parallel to B and perp. to B)
  - Only $v_{\perp} = v \sin\phi$ contributes to circular motion
  - $v_{||} = v \cos\phi$ is unchanged
- ➔ So the particle moves in a helical path
  - $v_{||}$ is the constant velocity along the B field
  - $v_{\perp}$ is the velocity around the circle



$$R = \frac{mv_{\perp}}{qB}$$

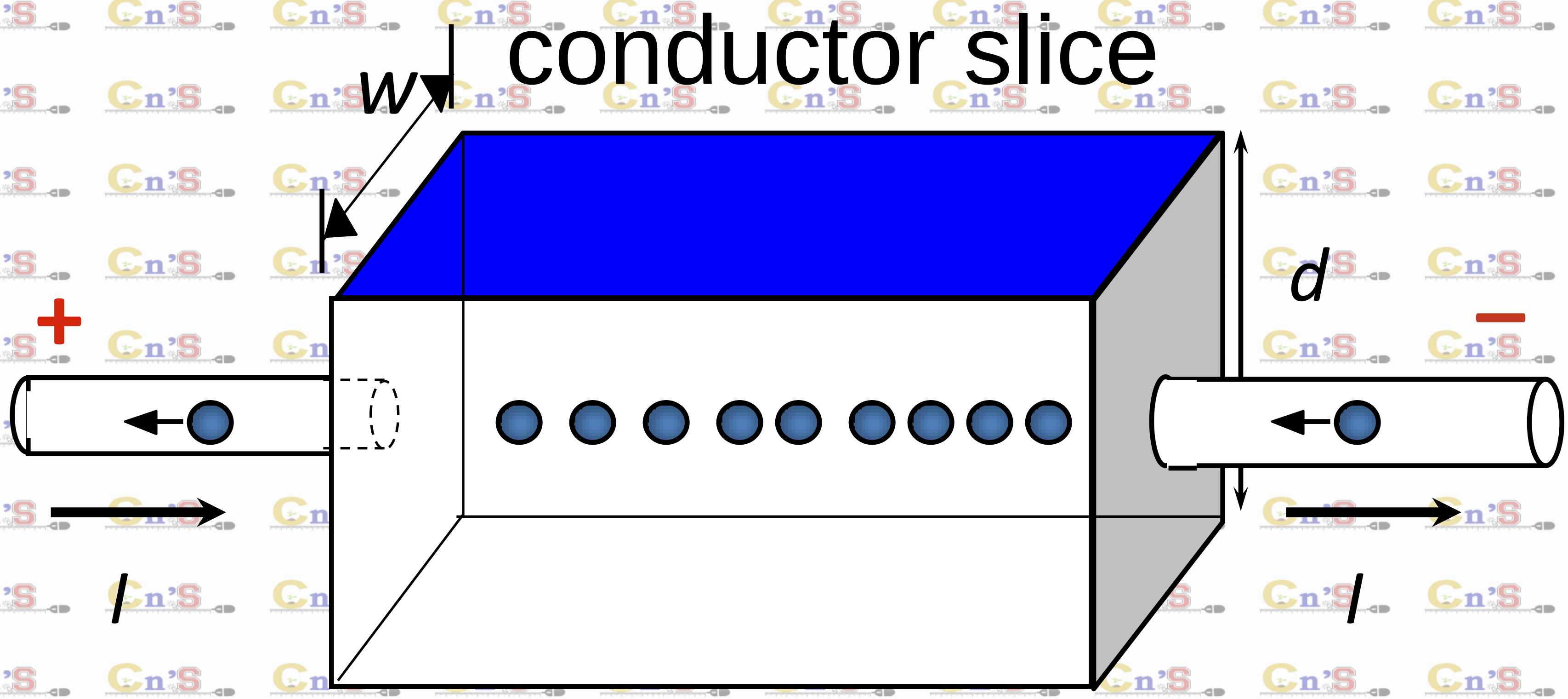


Hall Voltage/ EMF

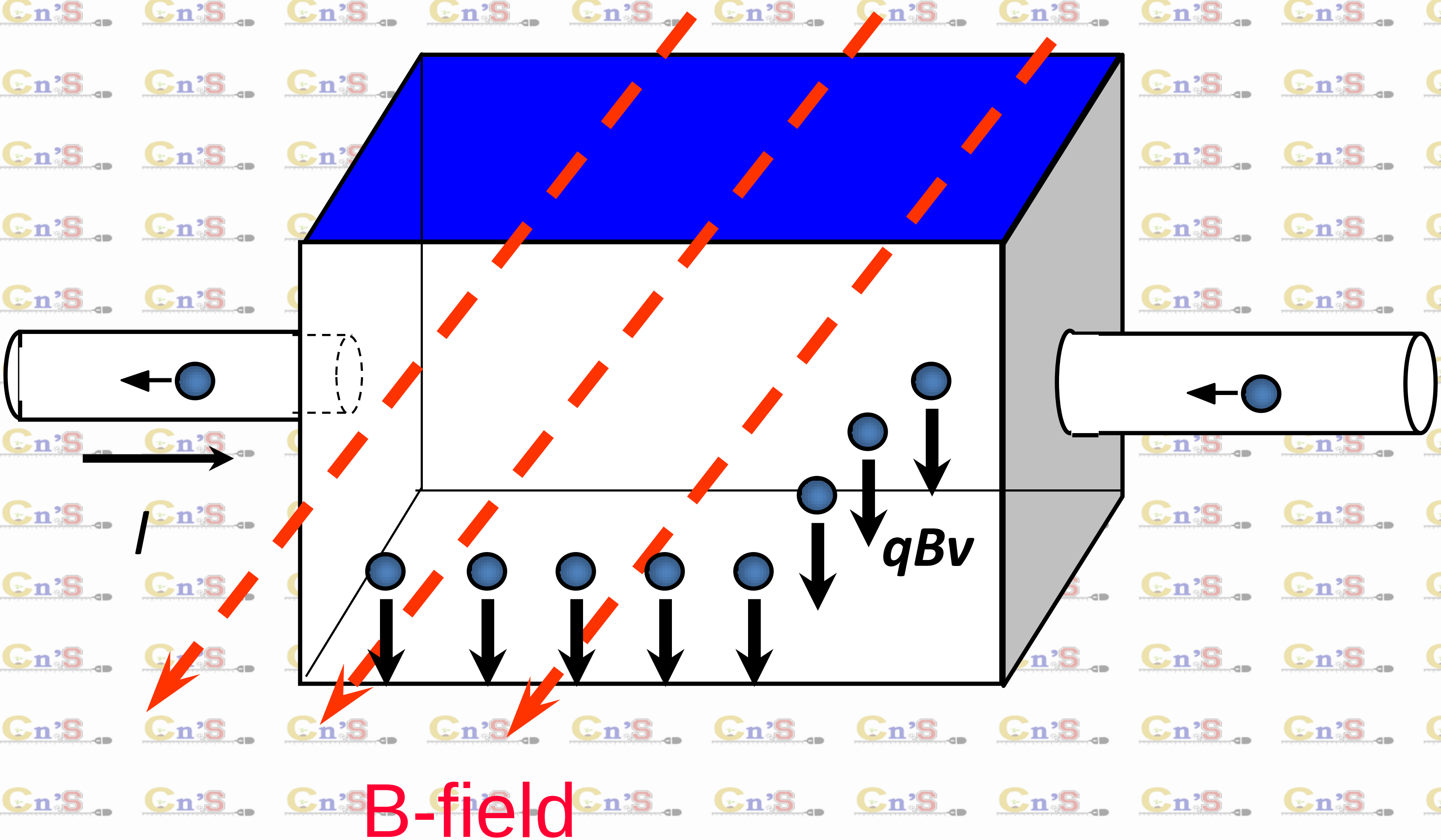
What happens to charges moving through a conductor in a magnetic field?

The field is perpendicular to the electron drift velocity and to the width of the conductor. The conventional current is to the right. Electrons carry the current and move to the left. Moving electrons feel a magnetic force toward one side of the conductor, leaving a net positive charge on the other side. This separation of charge *creates a voltage* , known as the **Hall EMF**, *across* the conductor. The creation of a voltage *across* a current-carrying conductor by a magnetic field is known as the **Hall effect.**

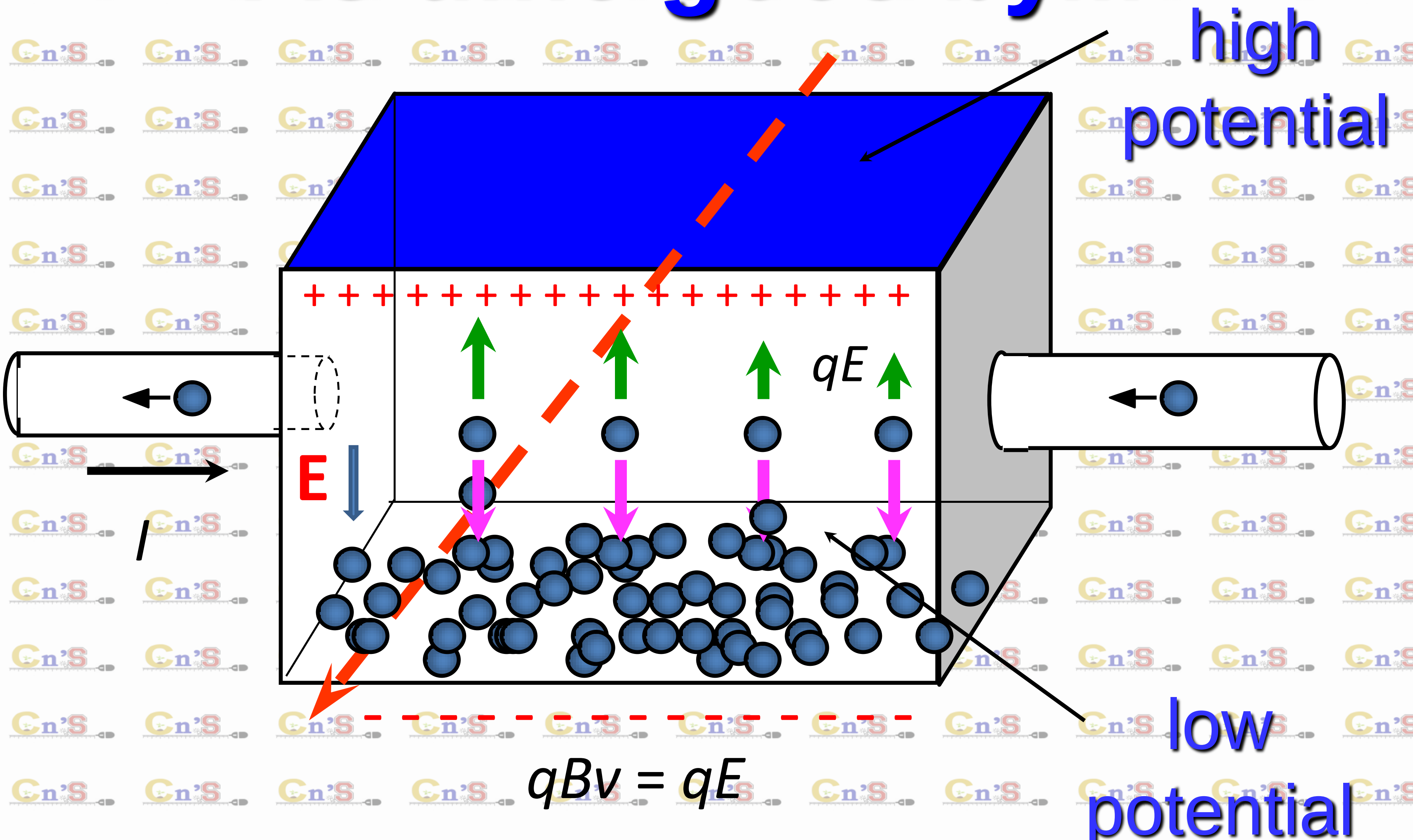
When electrons flow without magnetic field...



When the magnetic field is turned on ...



As time goes by...



The magnitude of the Hall voltage.

Suppose that the mobile charges each possess a charge q and move along the metal with the drift velocity v_d. The magnetic force on a given mobile charge is $q\,v_d\,B$ In a steady-state, this force is balanced by the electric force due to the build up of charges on the upper and lower edges of the metal. If the Hall voltage is V_H, and the width of the metal is w, then the electric field pointing from the upper to the lower edge of the conductor is of magnitude $E = V_H/w$. Now, the electric force on a mobile charge is qE.

This force acts in opposition to the magnetic force.

In a steady state,

$$q\,E = \frac{q\,V_H}{w} = q\,v_d\,B,$$

$$V_H = v_d\,w\,B.$$

Drift velocity

When electric field is established across the ends of a conductor, the free electrons modify their random motion and drift slowly with a constant velocity in the direction opposite to E. This constant velocity is known as drift velocity.

The drift velocity of electron in a conductor of constant cross-sectional area is given by

- V = I/ nAq or $I = qnwdv_d,$
- Where, 'v' is the drift velocity of the electrons
- 'I' is the current flowing through the conductor (amperes)
- 'A' is the cross-section area of the conductor (m^2)
- 'q' is the charge on the charge carrier (coulombs, c)
- n = Number of free electrons in a unit volume.
- w is the thickness of the conductor.

Hall Voltage

$$V_H = \frac{IB}{qnd}.$$

- Hall Voltage is represented by V_H.

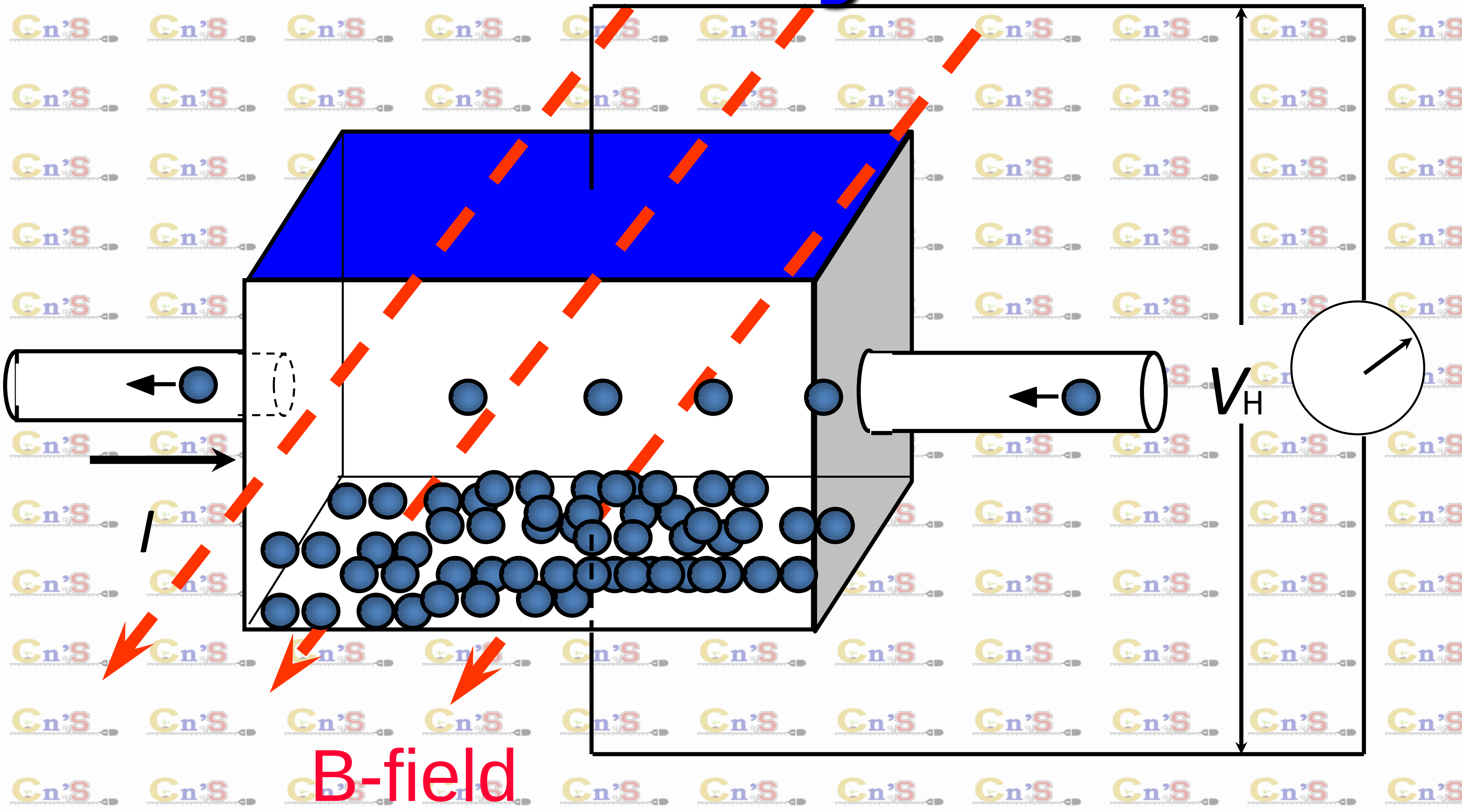
Where:

- **I** – Current flowing through the Sensor
- **B** – Magnetic Field Strength
- **q** – Charge
- **n** – number of charge carriers per unit volume
- **d** – Thickness of the Sensor

$$V_H = \frac{I\,B}{q\,n\,d}.$$

- It is clear that the Hall voltage is proportional to the current, and the magnetic field-strength, and is inversely proportional to the number density of mobile charges, and the thickness of the conductor.
- Thus, in order to construct a sensitive Hall probe (*i.e.*, one which produces a large Hall voltage in the presence of a small magnetic field), we need to take a thin ribbon of some material which possesses relatively few mobile charges per unit volume (*e.g.*, a semiconductor), and then run a large current through it.

Finally…



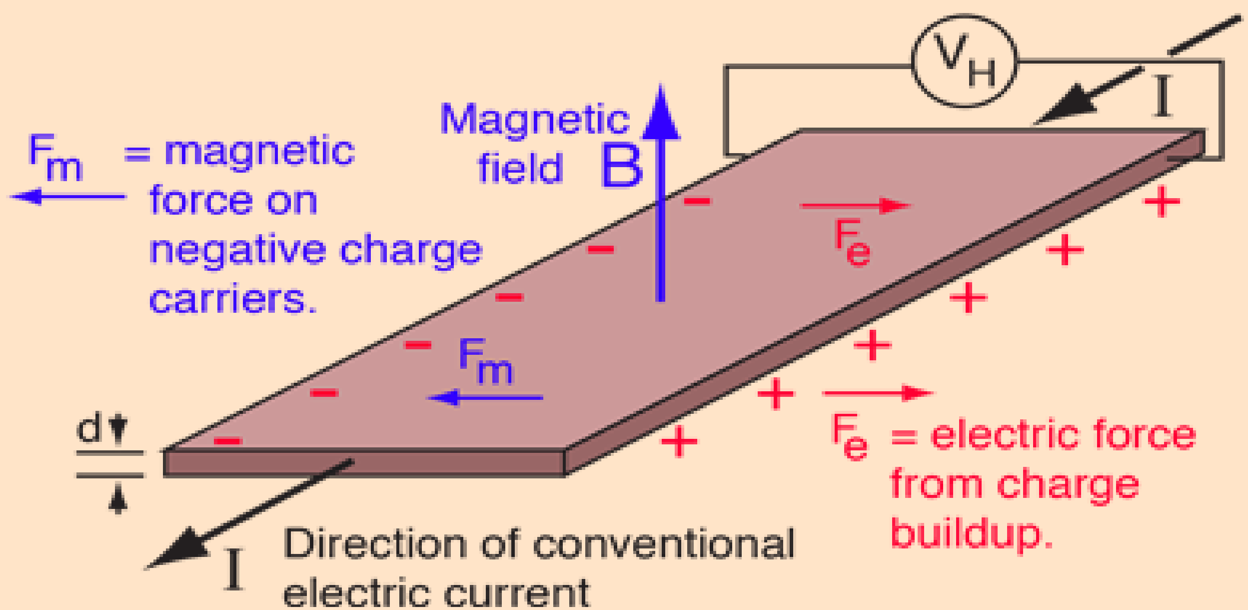
Finally……

.An equilibrium quickly developes in which the E force on each e builds up until it just cancels the M force. When this happens the force due to B, & the force due to E, are in balance. Then the drifting e move along the conductor, at v_d, with no further collection of e at the bottom.& thus no further increase in the Electric field, E

Hall Effect

If an electric current flows through a conductor in a magnetic field, the magnetic field exerts a transverse force on the moving charge carriers which tends to push them to one side of the conductor. This is most evident in a thin flat conductor as illustrated. A buildup of charge at the sides of the conductors will balance this magnetic influence, producing a measurable voltage between the two sides of the conductor. The presence of this measurable transverse voltage is called the Hall effect after E. H. Hall who discovered it in 1879.

Note that the direction of the current I in the diagram is that of conventional current, so that the motion of electrons is in the opposite direction. That further confuses all the "right-hand rule" manipulations you have to go through to get the direction of the forces.



The Hall voltage is given by

$$V_H = IB/ned$$ Show

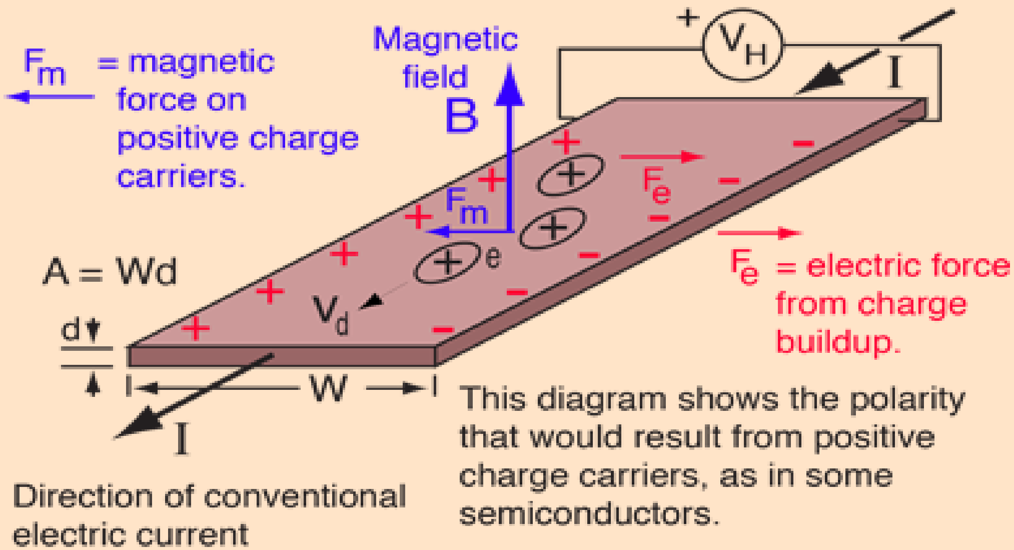
where n = density of mobile charges and e = electron charge.

The Hall effect can be used to measure magnetic fields with a Hall probe.

Hall Voltage for Positive Charge Carriers

The transverse voltage (Hall effect) measured in a Hall probe has its origin in the magnetic force on a moving charge carrier.

The magnetic force is $F_m = ev_dB$ where v_d is the drift velocity of the charge.



The current expressed in terms of the drift velocity is

$$I = neAv_d$$ Show

where **n** is the density of charge carriers. Then

$$F_m = eIB/neA$$

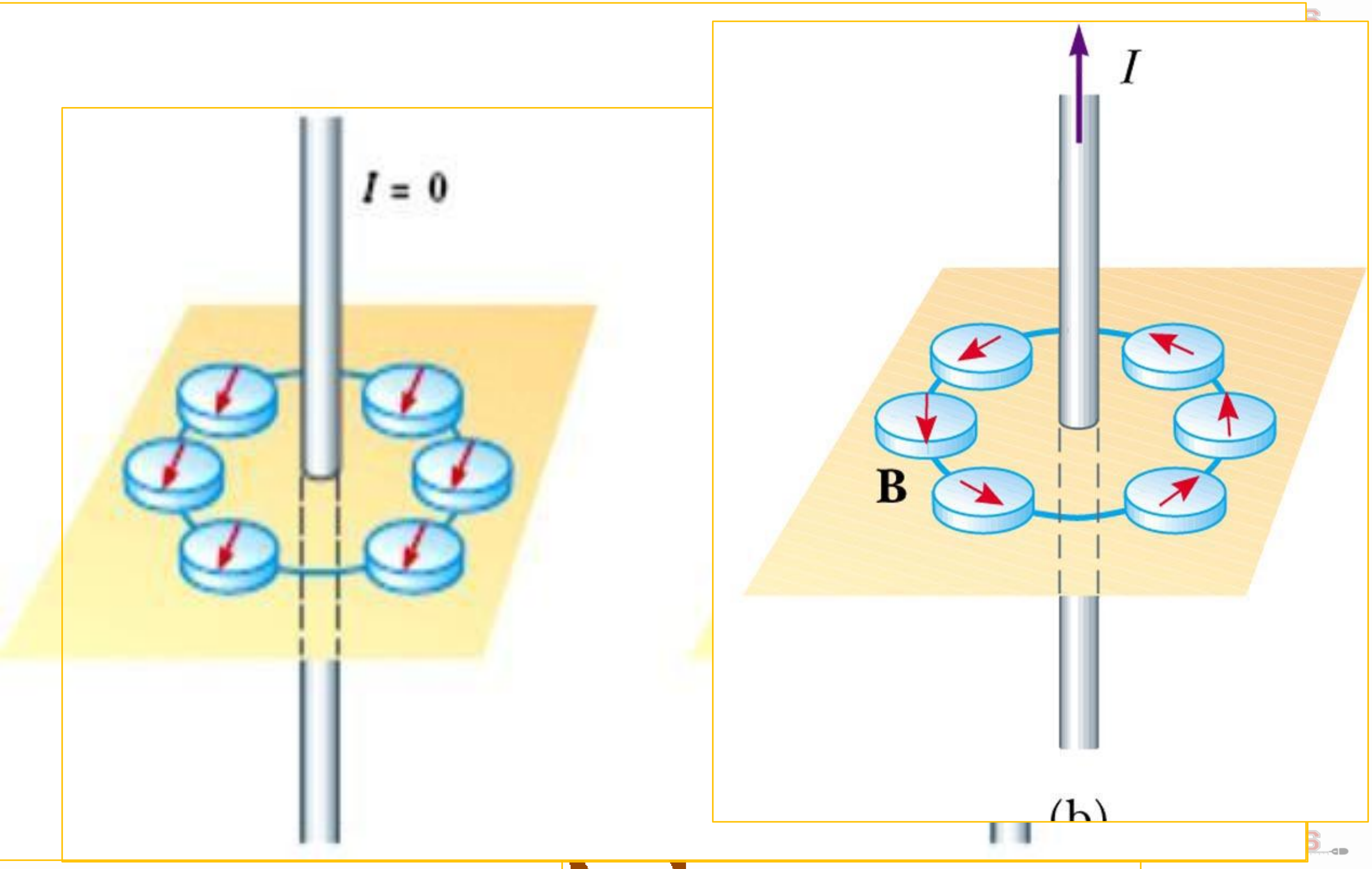
At equilibrium

$$F_m = F_e = V_He/W$$ Show

And substituting gives

$$V_H = IB/ned$$

Magnetic Field around a Long Straight Wire



When the heavy current is passed through a straight conductor: A magnetic field is set up in the region surrounding a current carrying wire.

i. The lines of force are circular and their direction depends upon the direction of current.

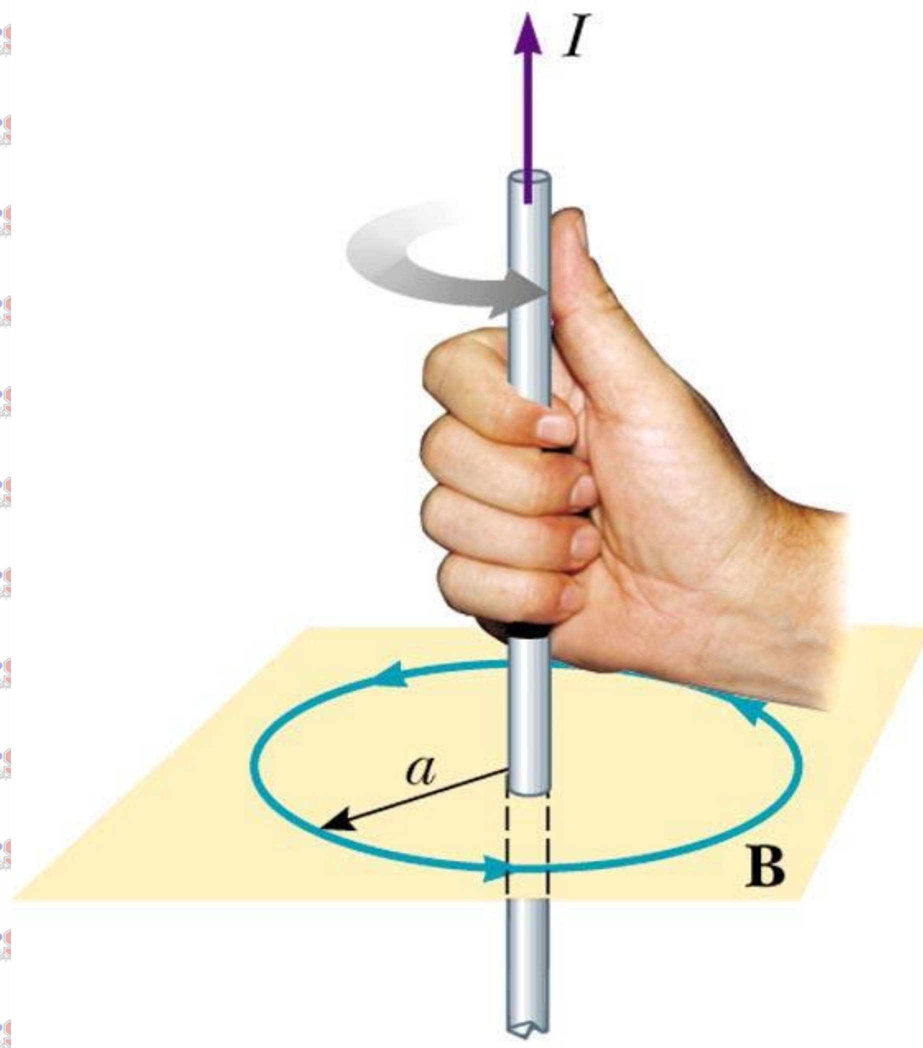
ii. The magnetic field lasts only as long as the current is flowing through the wire.

iii. The direction of magnetic lines of force can be find out by right hand rule.

Direction of the Field of a Long Straight Wire

Right Hand Rule

- Grasp the wire in your right hand
- Point your thumb in the direction of the current
- Your fingers will curl in the direction of the field



Maxwell's Cork Screw Rule or Right Hand Screw Rule:

If the forward motion of an imaginary right handed screw is in the direction of the current through a linear conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.

Right Hand Thumb Rule or Curl Rule:

If a current carrying conductor is imagined to be held in the right hand such that the thumb points in the direction of the current, then the tips of the fingers encircling the conductor will give the direction of the magnetic lines of force.

Biot – Savart's Law:

The strength of magnetic field dB due to a small current element d*l* carrying a current I at a point P distant r from the element is directly proportional to I, d*l*, sin θ and inversely proportional to the square of the distance (r^2) where θ is the angle between d*l* and r.

i) $dB \propto I$

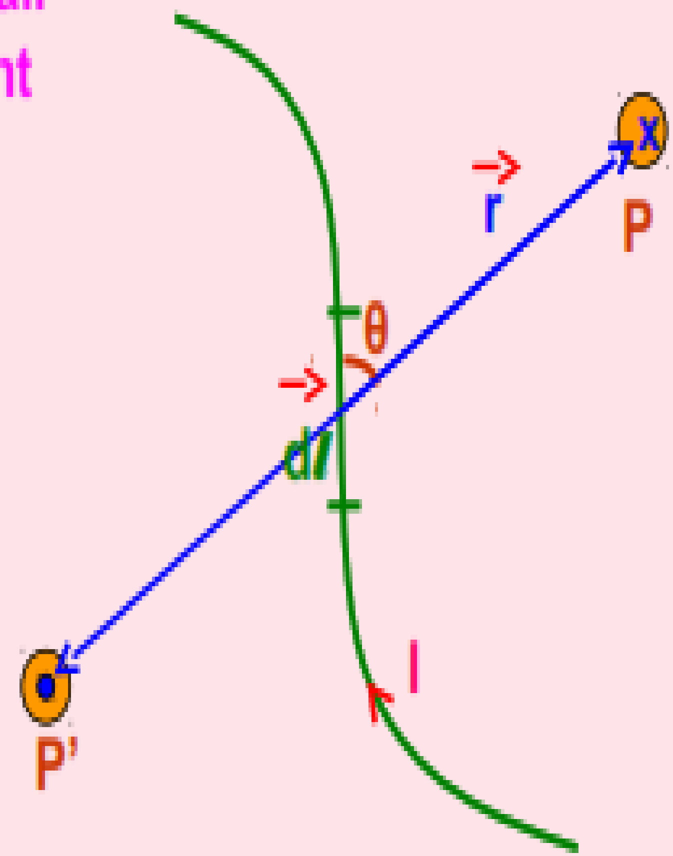
ii) $dB \propto dl$

iii) $dB \propto \sin\theta$

iv) $dB \propto 1/r^2$

$$dB \propto \frac{I\,dl\,\sin\theta}{r^2}$$

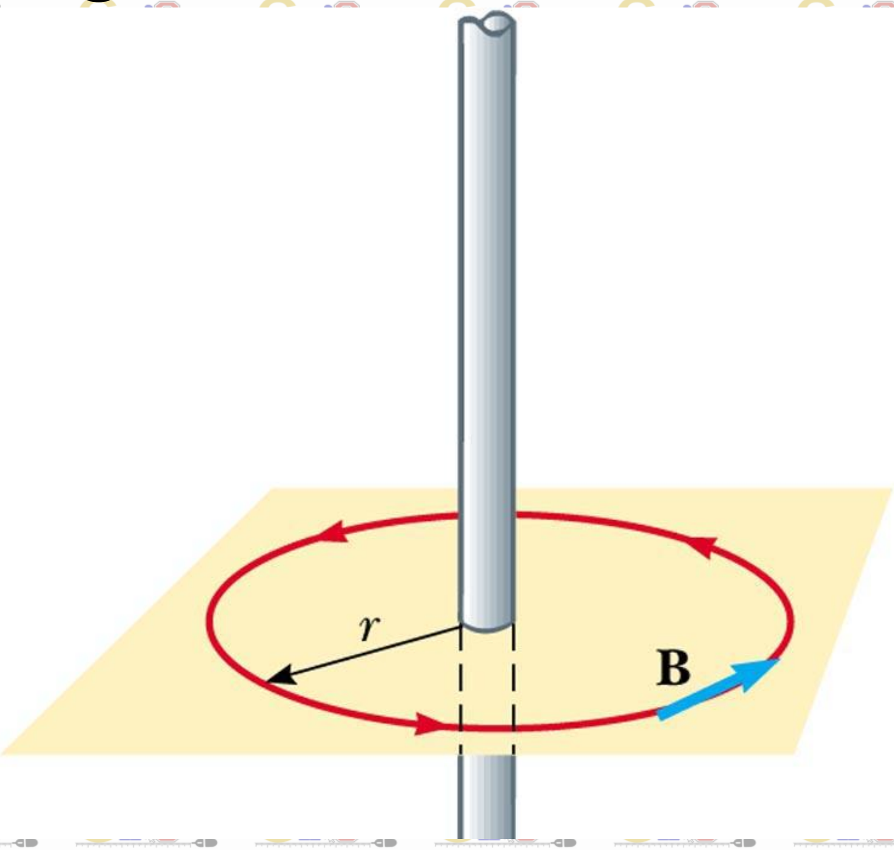
$$\boxed{dB = \frac{\mu_0}{4\pi}\,\frac{I\,dl\,\sin\theta}{r^2}}$$



Magnitude of the Field of a Long Straight Wire

- The magnitude of the field at a distance r from a wire carrying a current of I is

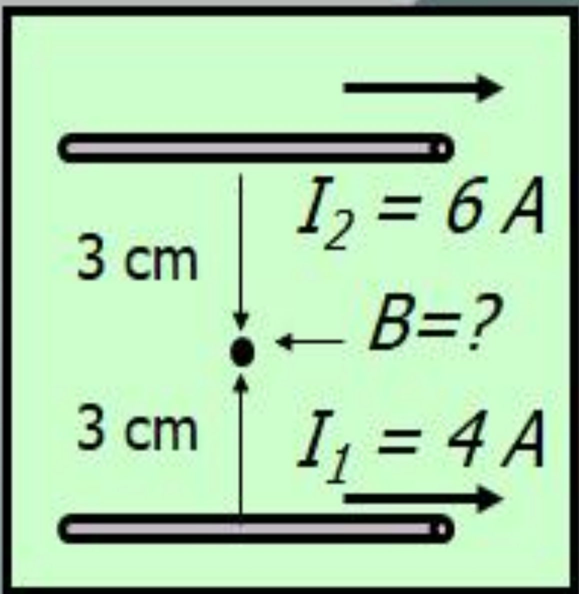
$$B = \frac{\mu_o I}{2\pi r}$$



- $\mu_o = 4\,\pi \times 10^{-7}$ T m / A

μ_o is called the *permeability of free space*

Find resultant B at midpoint.



$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

B_1 is positive

B_2 is negative

$$B_1 = \frac{(4\pi \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}})(4\ \text{A})}{2\pi(0.03\ \text{m})} = +26.7\ \mu\text{T}$$

$$B_2 = \frac{(4\pi \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}})(6\ \text{A})}{2\pi(0.03\ \text{m})} = -40.0\ \mu\text{T}$$

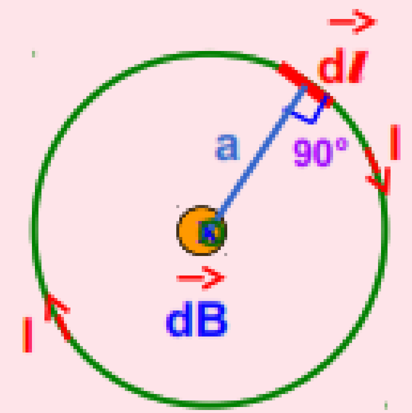
Resultant is vector sum: $B_R = \Sigma B$

$B_R = 26.7\ \mu\text{T} - 40\ \mu\text{T} = -13.3\ \mu\text{T}$

B_R is into paper: $\boxed{B = -13.3\ \mu\text{T}}$

2) B at the centre of the loop:

The plane of the coil is lying on the plane of the diagram and the direction of current is clockwise such that the direction of magnetic field is perpendicular and into the plane.

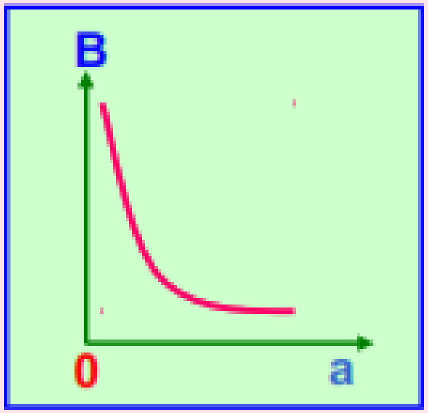


$$dB = \frac{\mu_0 \ I \ dl \sin\theta}{4\pi \quad a^2} \qquad dB = \frac{\mu_0 \ I \ dl}{4\pi \ a^2}$$

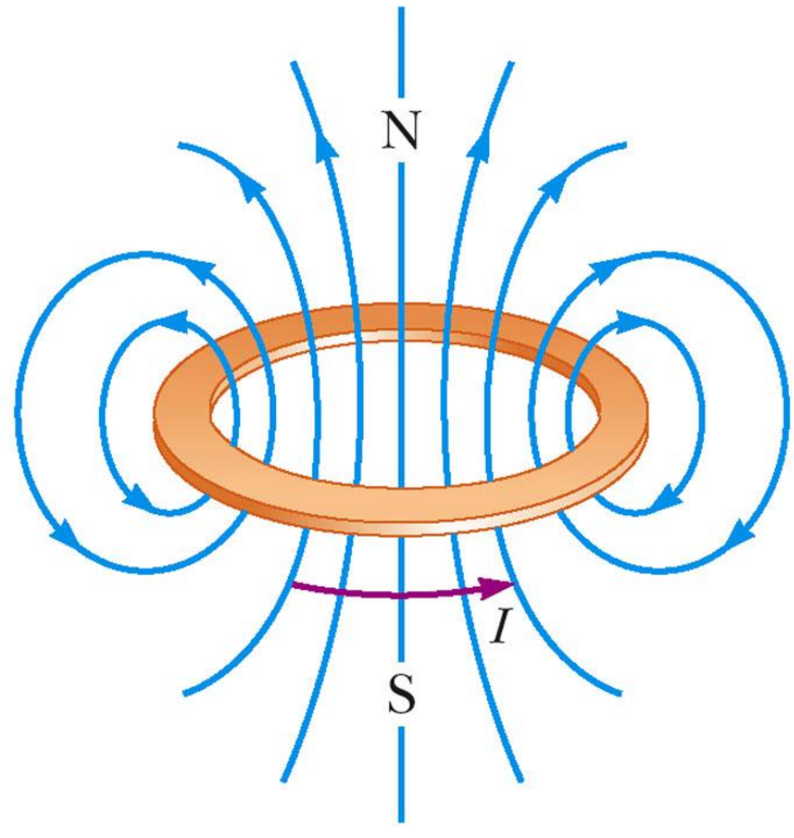
The angle θ between d*l* and a is 90° because the radius of the loop is very small and since sin 90° = 1

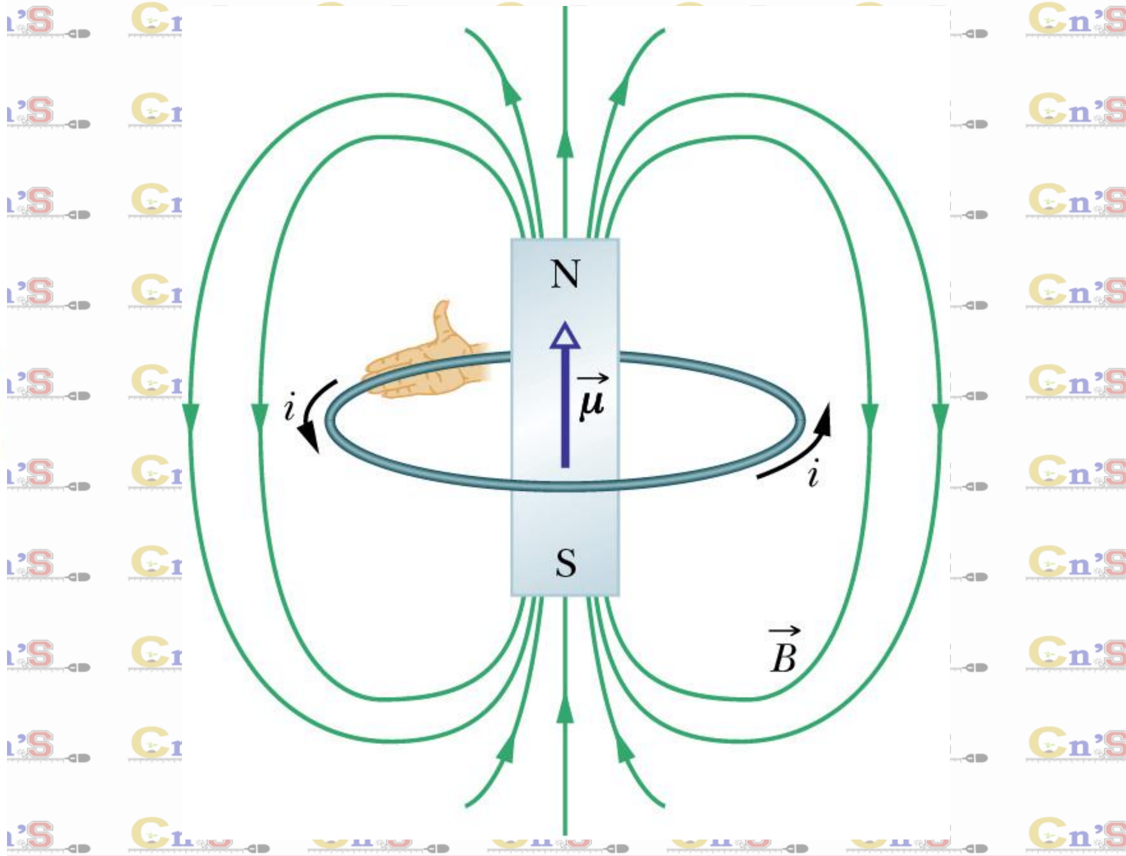
$$B = \int dB = \int \frac{\mu_0 \ I \ dl}{4\pi \ a^2}$$

$$\boxed{B = \frac{\mu_0 \ I}{2a}}$$



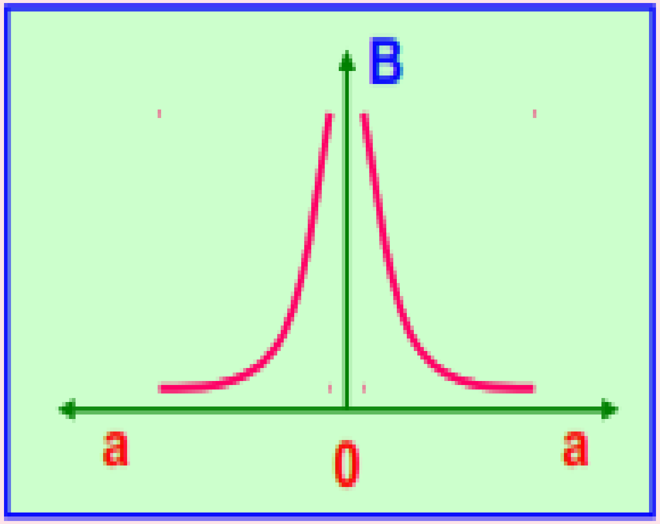
(μ_0 , I, a are constants and $\int dl = 2\pi a$)



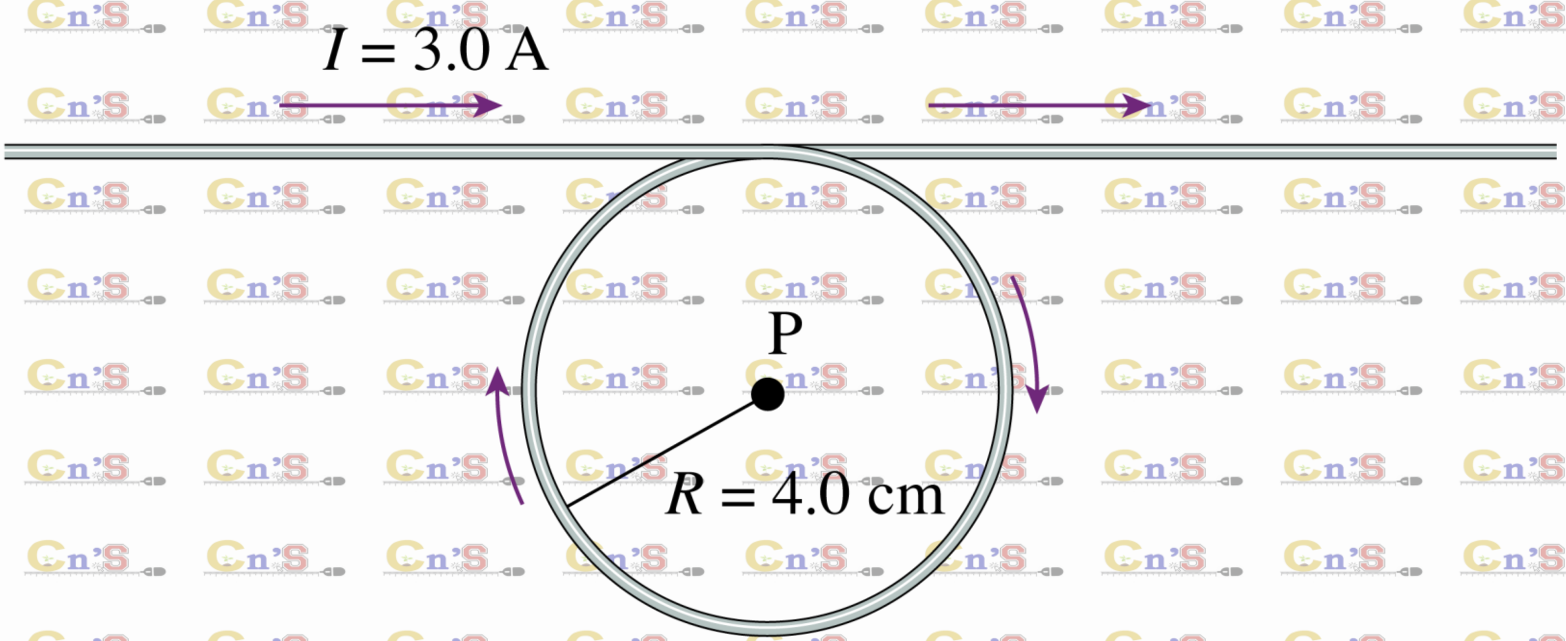


If the straight wire is infinitely long,

$$B = \frac{\mu_0 I}{2\pi a}$$



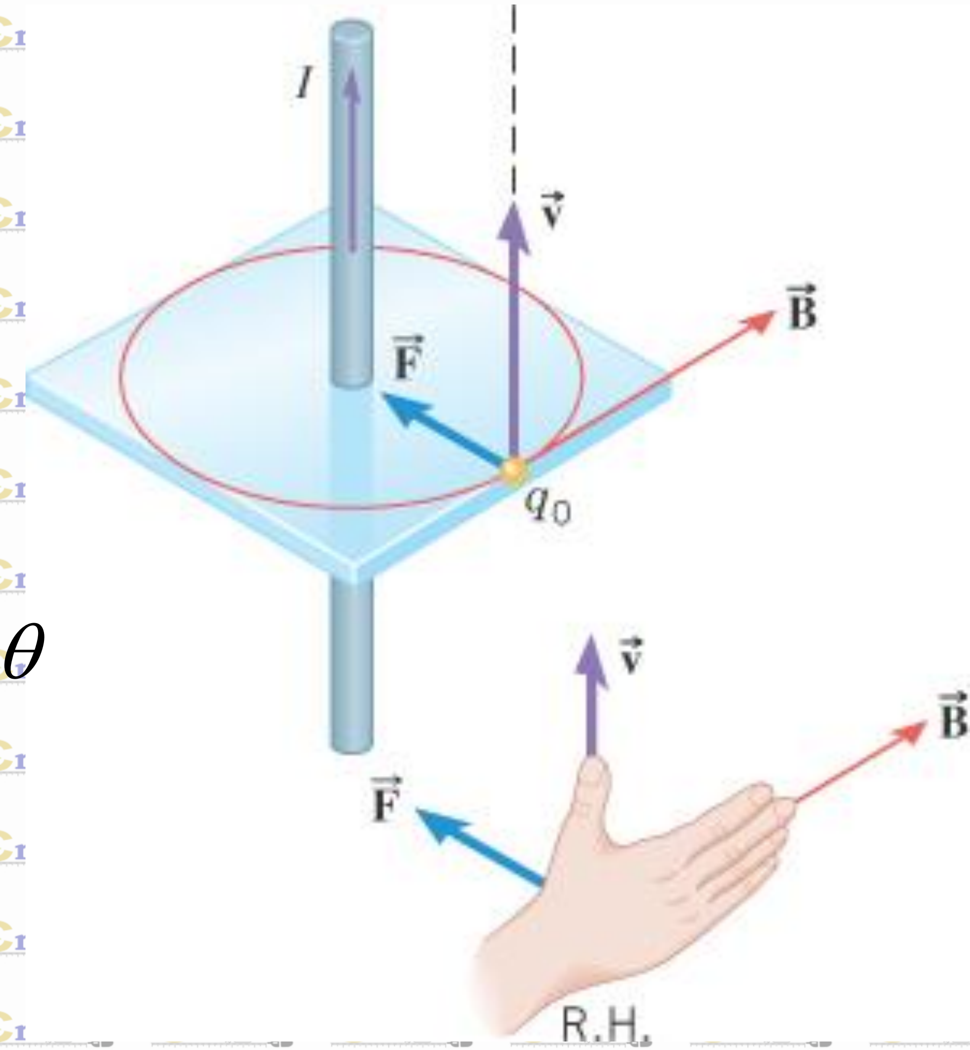
What is the direction and magnitude of the magnetic field at point P, at the center of the loop?



A Current Exerts a Magnetic Force on a Moving Charge

The long straight wire carries a current of 3.0 A. A particle has a charge of $+6.5\times10^{-6}$ C and is moving parallel to the wire at a distance of 0.050 m. The speed of the particle is 280 m/s.

Determine the magnitude and direction of the magnetic force on the particle.



$$F = qvB\sin\theta = qv\left(\frac{\mu_o I}{2\pi r}\right)\sin\theta$$

$$B = \frac{\mu_o I}{2\pi r}$$

$\vec{\mathbf{B}}$ for a Curved Wire Segment

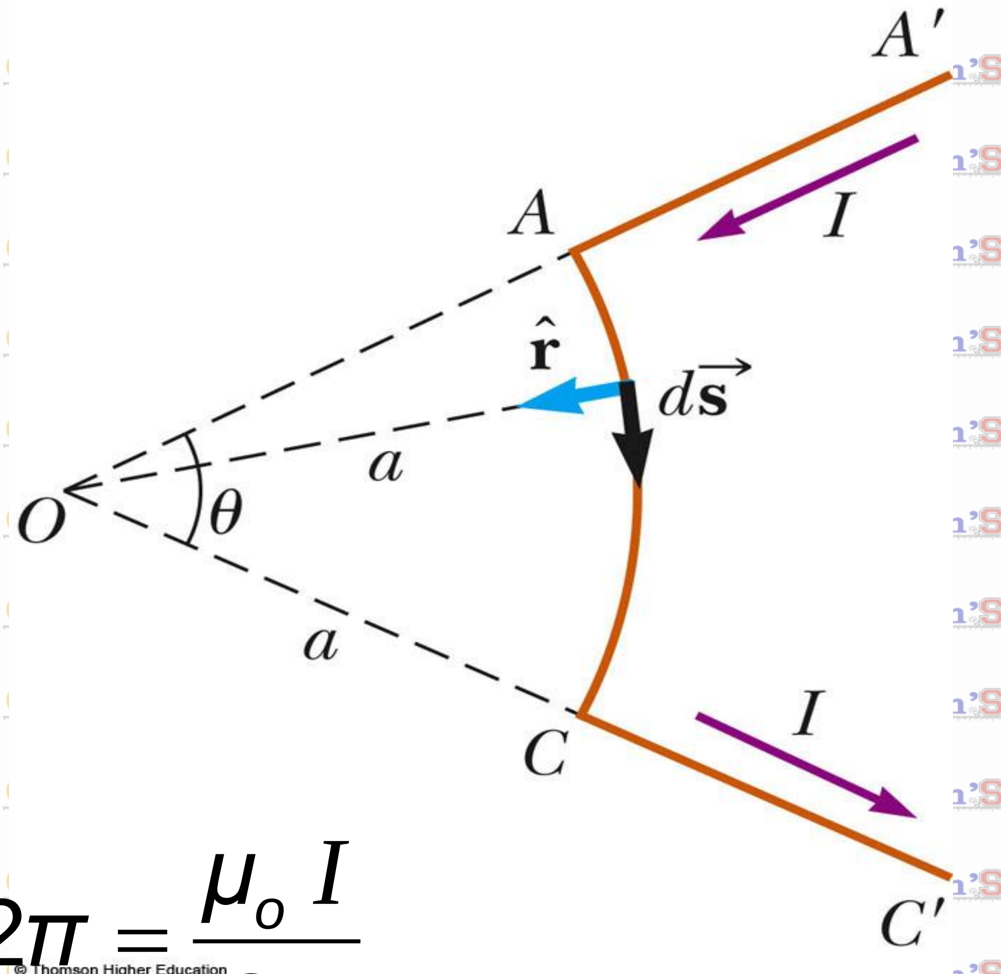
The field at point *O* due to the wire segment

$$B = \frac{\mu_o I}{4\pi R}\theta$$

– θ will be in radians

For a Circular Loop

$\theta = 2\pi$



$$B = \frac{\mu_o I}{4\pi a}\theta = \frac{\mu_o I}{4\pi a}2\pi = \frac{\mu_o I}{2a}$$

Field at Center of Partial Loop

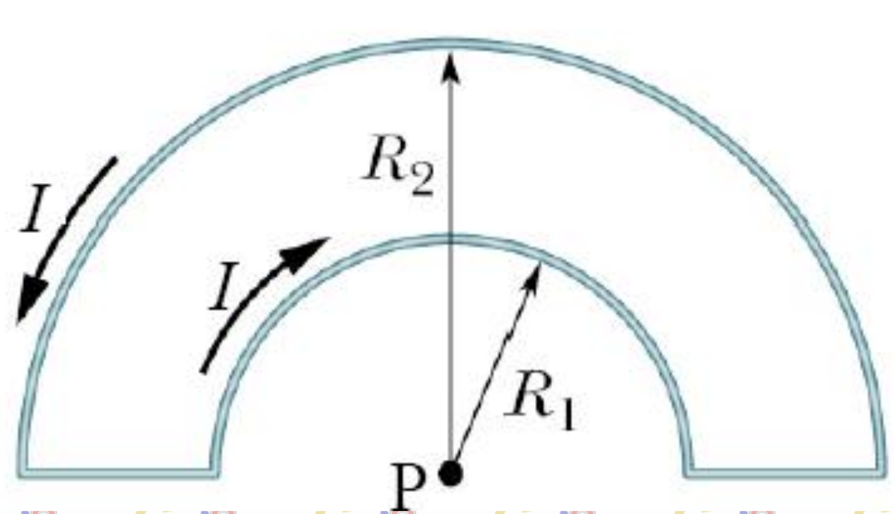
Double Arc

Find *B* at point P.

Only the arcs contribute.

Suppose loop covers angle ϕ

$$B = \frac{\mu_0 i}{2R}\left(\frac{\phi}{2\pi}\right)$$



Where $\phi = \pi$ (half circle)

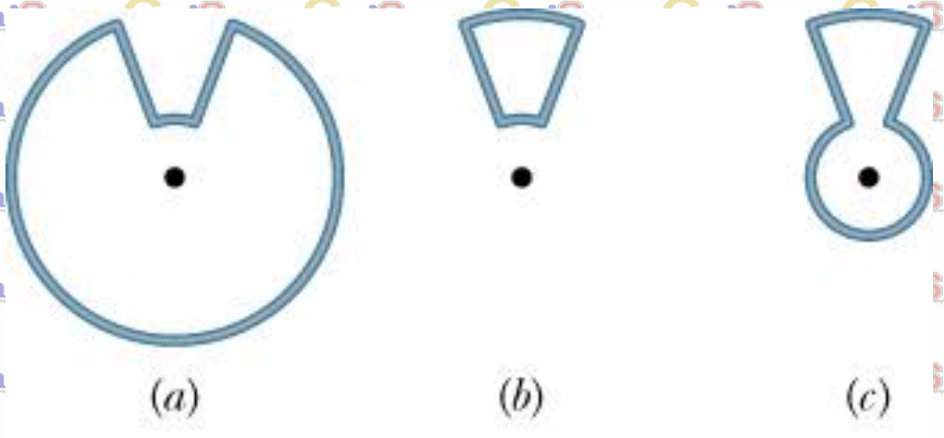
Define direction into page as positive

$$B = \frac{\mu_0 i}{2R_1}\left(\frac{\pi}{2\pi}\right) - \frac{\mu_0 i}{2R_2}\left(\frac{\pi}{2\pi}\right)$$

$$B = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Partial Loops

- Note on problems when you have to evaluate a B field at a point from several partial loops
  - Only loop parts contribute, proportional to angle (previous slide)
  - Straight sections aimed at point contribute exactly nothing
  - Be careful about signs, e.g.in (b) fields partially cancel, whereas in (a) and (c) they add



B for Lines and Arcs

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

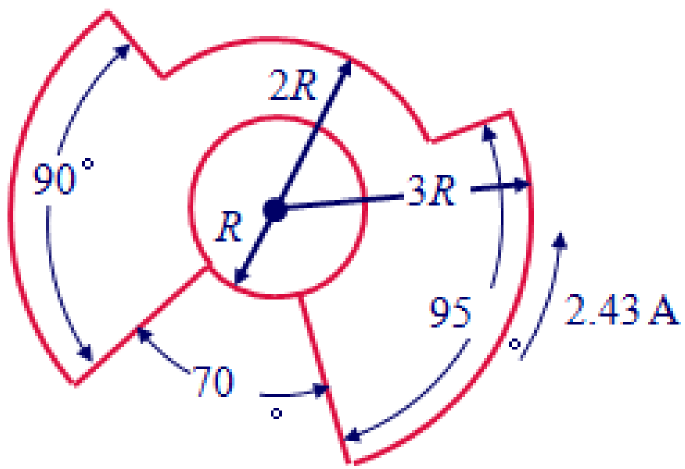
circular arc

How would you determine B in the center of this loop of wire?

Say R = 10 cm, i = 2.43 A. Since 95° = 1.658 radians, 90° = 1.571 radians, 70° = 1.222 radians, 105° = 1.833 radians, we have

$$B = 10^{-7}(2.43)\left[\frac{1.658}{3R} + \frac{1.833}{2R} + \frac{1.571}{3R} + \frac{1.222}{R}\right]$$

$$= \frac{7.812\times10^{-7}}{0.1}\,\mathrm{T} = 7.812\ \mu\mathrm{T}$$ (out of page)



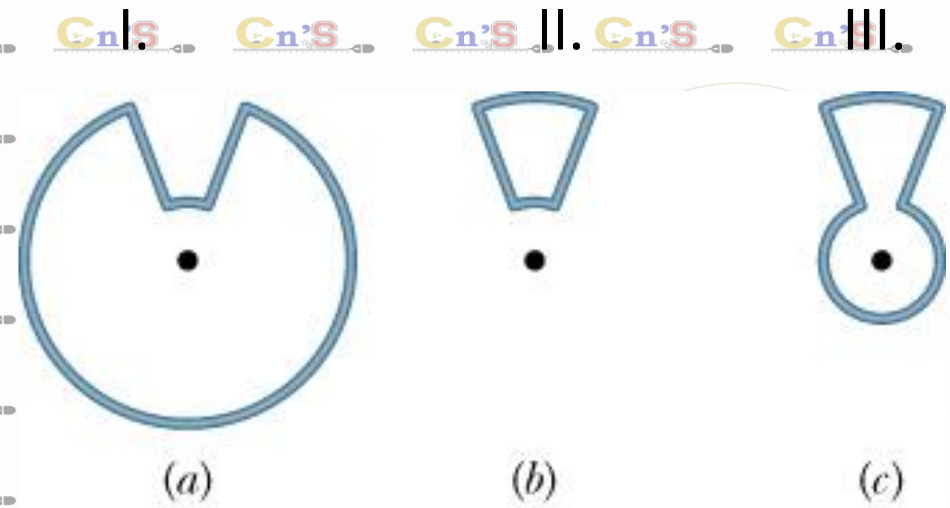
$$B = 10^{-7}(2.43)\left[\frac{1.658}{3R} + \frac{1.833}{2R} + \frac{1.571}{3R} - \frac{5.062}{R}\right]$$

$$= -\frac{7.458\times10^{-7}}{0.1}\,\mathrm{T} = -7.458\ \mu\mathrm{T}$$ (into page)

Magnetic Field from Loops

2. The three loops below have the same current. Rank them in terms of magnitude of magnetic field at the point shown, greatest first.

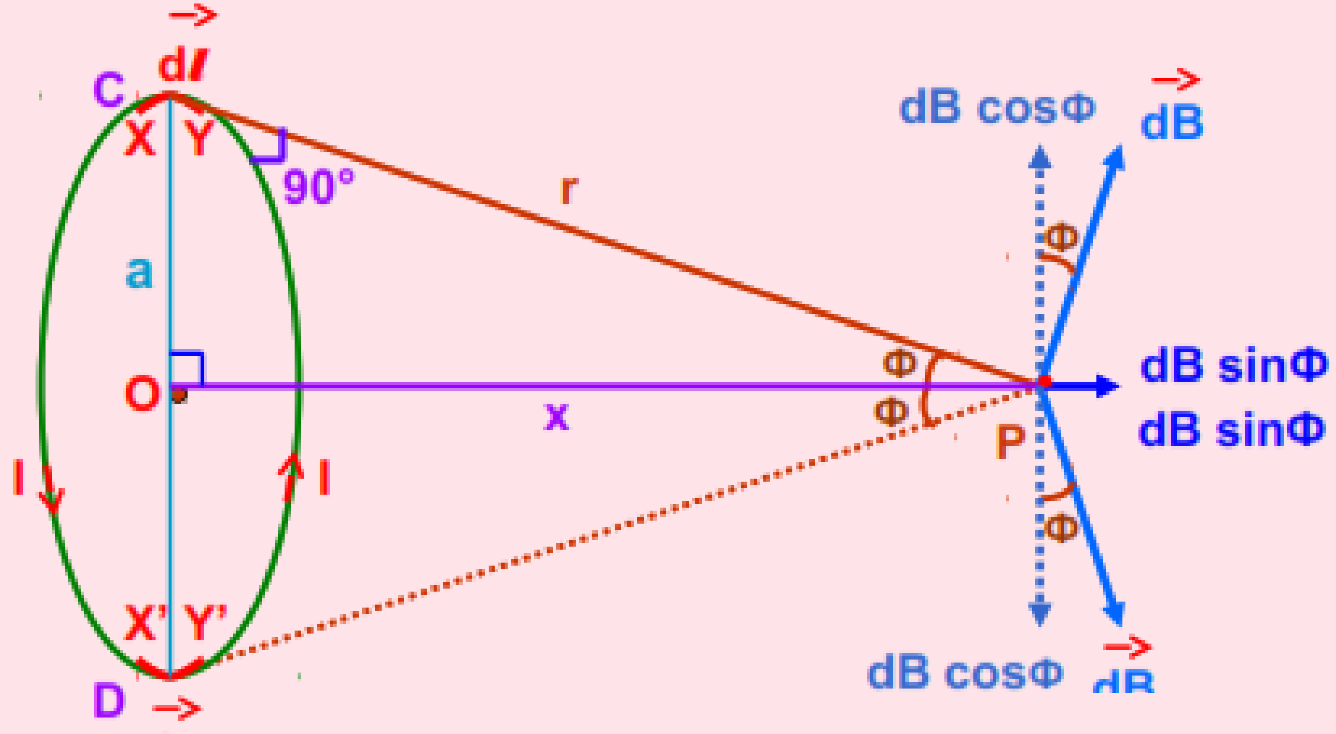
A. I, II, III.
B. II, I, III.
C. III, I, II.
D. III, II, I.
E. II, III, I.



MCQ 93-94-96-97-00-01-03

Magnetic Field due to a Circular Loop carrying current:

1) At a point on the axial line:



$$B_P = B_O \sin^3 \Phi$$

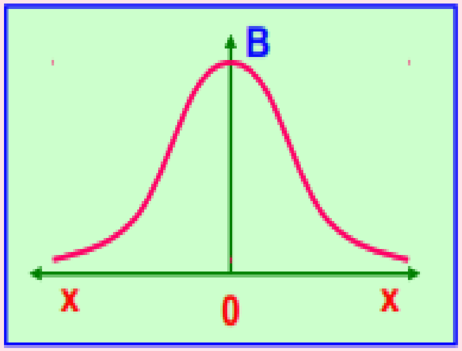
$$B = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$$

Special Cases:

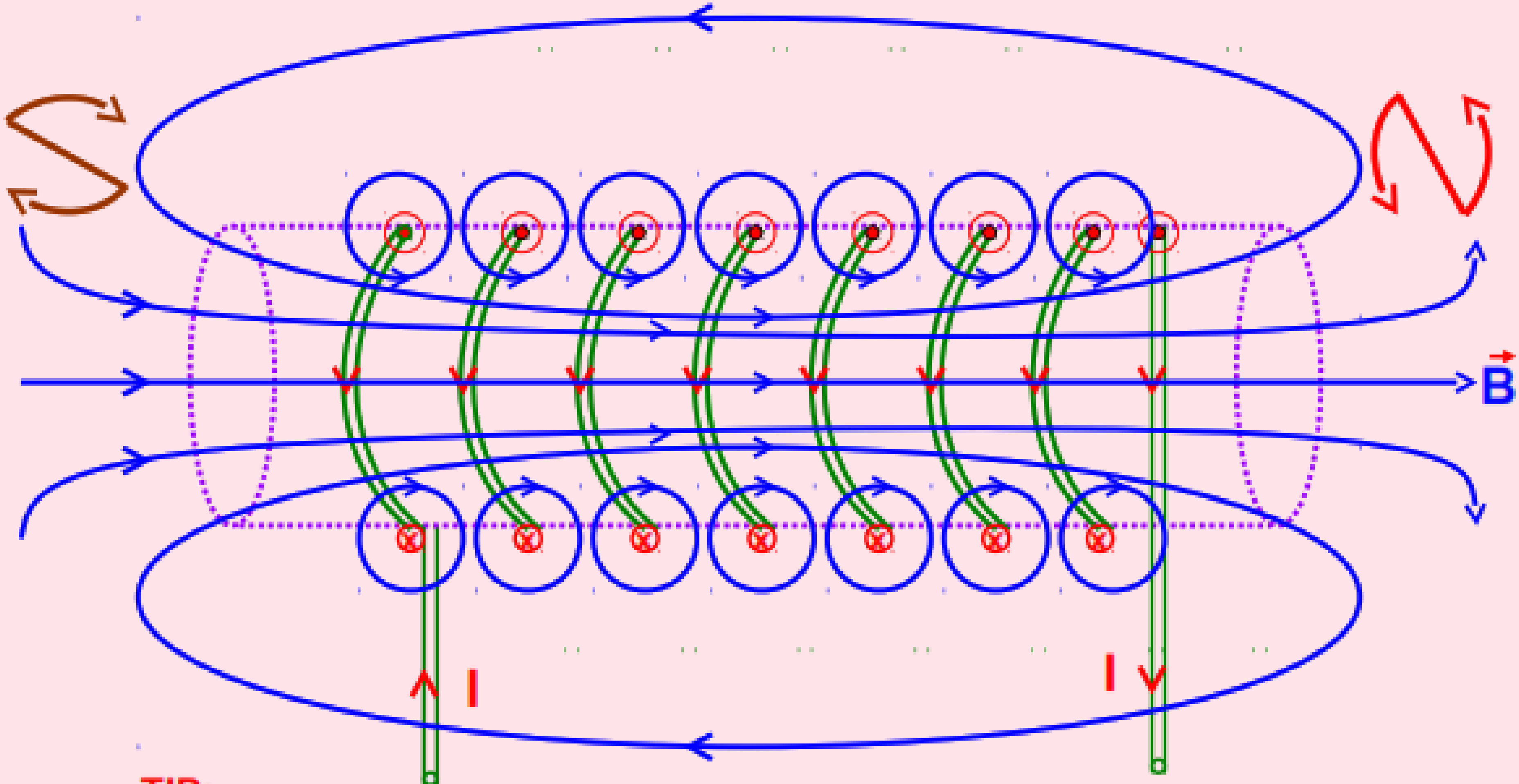
i) At the centre O, x = 0. $\therefore$ $B = \dfrac{\mu_0 I}{2a}$

ii) If the observation point is far away from the coil, then a << x. So, a^2 can be neglected in comparison with x^2.

$\therefore$ $B = \dfrac{\mu_0 I a^2}{2x^3}$



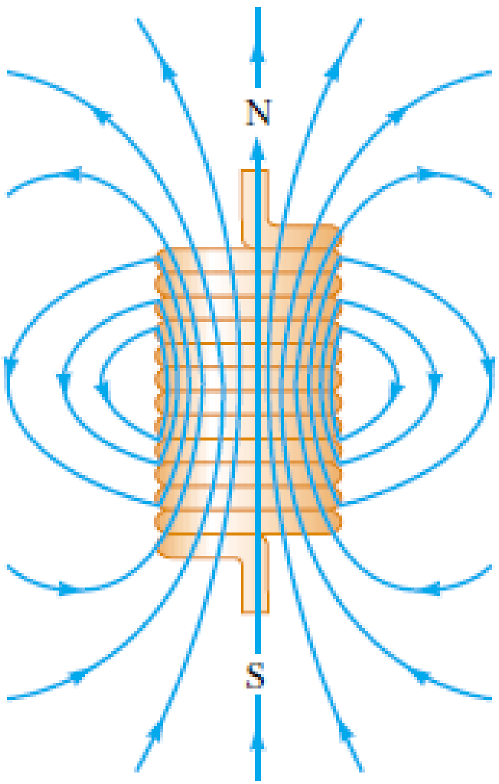
Magnetic Field due to a Solenoid:

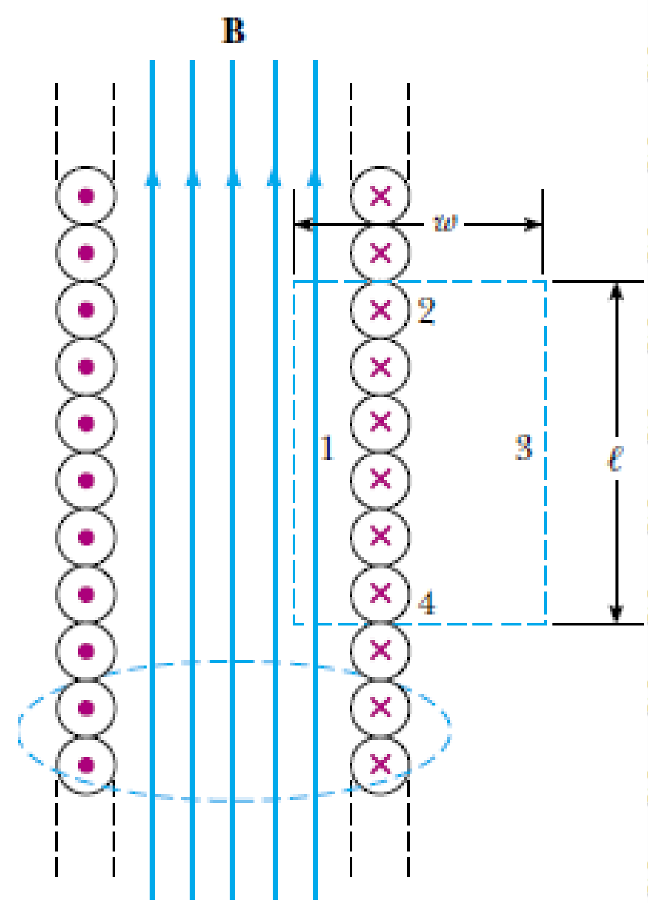


TIP:

When we look at any end of the coil carrying current, if the current is in anti-clockwise direction then that end of coil behaves like North Pole and if the current is in clockwise direction then that end of the coil behaves like South Pole.

A solenoid is a cylindrical frame tightly wounded by an insulated wire. The magnetic field produced by the current carrying solenoid is like the field of a bar magnet. The magnetic field strength outside the solenoid is negligible as compared to the field inside it.

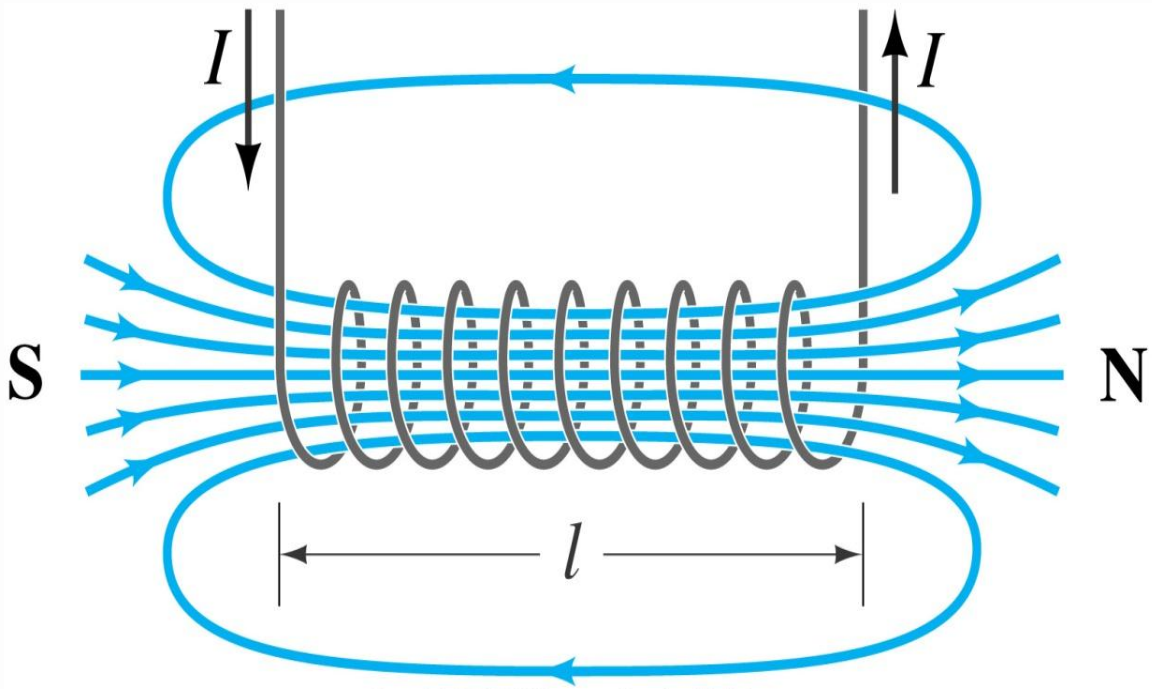




Solenoids

A solenoid is a long coil of wire. If it is tightly wrapped, the magnetic field in its interior is almost uniform:

$$B = \mu_0 I N / l$$



Q # 3. Describe the charge in the magnetic field inside a solenoid carrying steady current I, if (a) the length of the solenoid is doubled but the number of turns remains the same and (b) the number of turns are doubled, but the length remains the same.

Ans. The magnetic field strength **B** inside a current carrying conductor can be find out by the expression:

$$B = \mu_0 n I \quad \text{-----------} \quad (1)$$

Where I is the current flowing through conductor and n is the number of turns per unit length i.e., $n = \frac{N}{L}$. Thus

$$B = \frac{\mu_0 N I}{L}$$

(a) When Length of solenoid is doubled by keeping the number of turns constant, then magnetic field strength:

$$B' = \frac{\mu_0 N I}{2L} \Longrightarrow B' = \frac{B}{2}$$

Thus on doubling the length of solenoid by keeping the turns constant, the magnetic field strength becomes one half of its original value.

(b) When number of turns of solenoid is doubled by keeping the length of solenoid constant, then magnetic field strength:

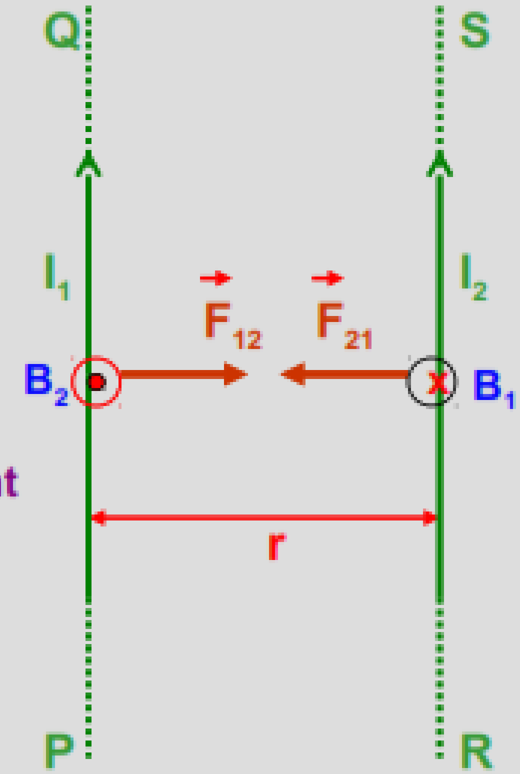
$$B'' = \frac{\mu_0 (2N) I}{L} \Longrightarrow B'' = 2B$$

Thus on doubling the number of turns of solenoid by keeping its length constant, the magnetic field strength becomes doubled of its original value.

Forces between two parallel infinitely long current-carrying conductors:

Magnetic Field on RS due to current in PQ is

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \quad \text{(in magnitude)}$$



Force acting on RS due to current I_2 through it is

$$F_{21} = \frac{\mu_0 I_1}{2\pi r} I_2 \, l \sin 90^\circ \quad \text{or} \quad F_{21} = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

B_1 acts perpendicular and into the plane of the diagram by Right Hand Thumb Rule. So, the angle between l and B_1 is 90˚ . l is length of the conductor.

Magnetic Field on PQ due to current in RS is

$$B_2 = \frac{\mu_0 I_2}{2\pi r} \quad \text{(in magnitude)}$$

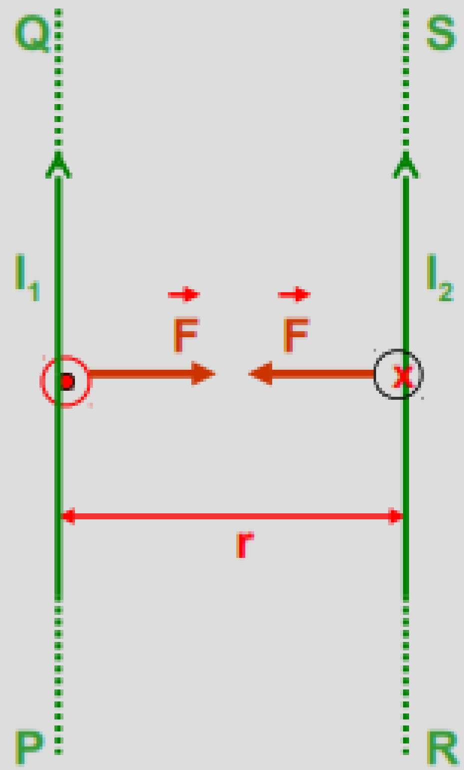
Force acting on PQ due to current I_1 through it is

$$F_{12} = \frac{\mu_0 I_2}{2\pi r} I_1 \, l \sin 90^\circ \quad \text{or} \quad F_{12} = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

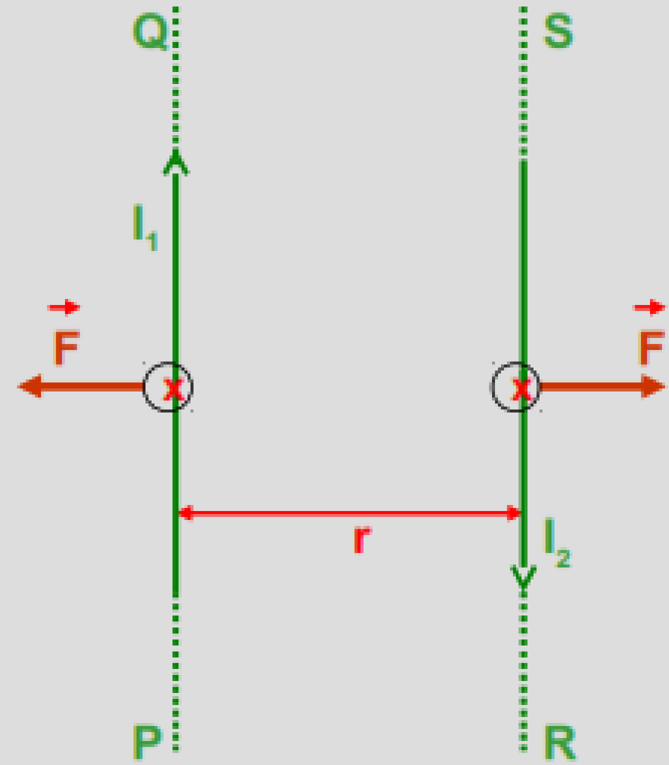
(The angle between l and B_2 is 90˚ and B_2 Is emerging out)

$$\boxed{F_{12} = F_{21} = F = \frac{\mu_0 I_1 I_2 l}{2\pi r}}$$

Force per unit length of the conductor is $\boxed{F / l = \frac{\mu_0 I_1 I_2}{2\pi r}}$ N / m



By Fleming's Left Hand Rule, the conductors experience force towards each other and hence attract each other.



By Fleming's Left Hand Rule, the conductors experience force away from each other and hence repel each other.

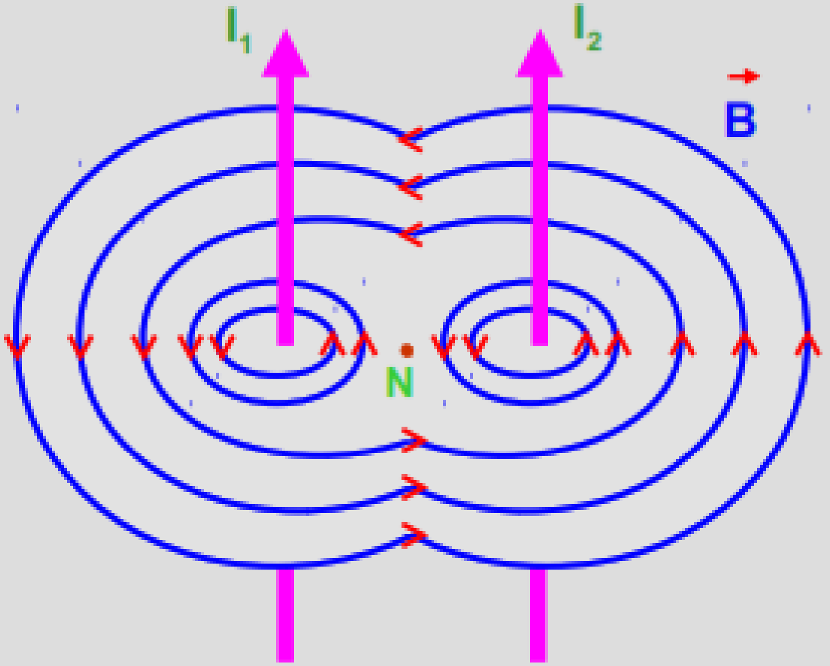
Definition of Ampere:

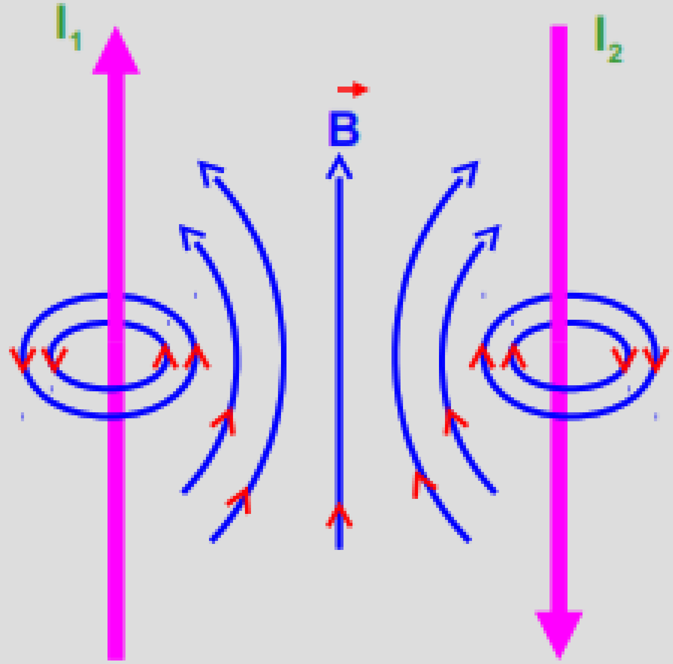
Force per unit length of the conductor is $F / l = \dfrac{\mu_0 I_1 I_2}{2\pi r}$ N / m

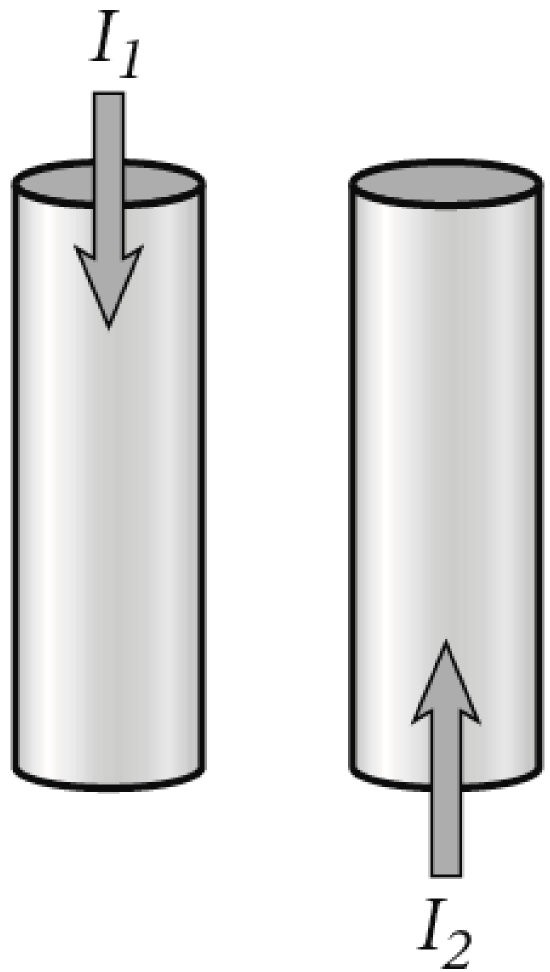
When $I_1 = I_2 = 1$ Ampere and $r = 1$ m, then $F = 2 \times 10^{-7}$ N/m.

One ampere is that current which, if passed in each of two parallel conductors of infinite length and placed 1 m apart in vacuum causes each conductor to experience a force of 2×10^{-7} Newton per metre of length of the conductor.

Representation of Field due to Parallel Currents:







- *Wire 1 carries current I_1 and creates magnetic field B_1.*
- *Wire 2 carries current I_2 and creates magnetic field B_2.*

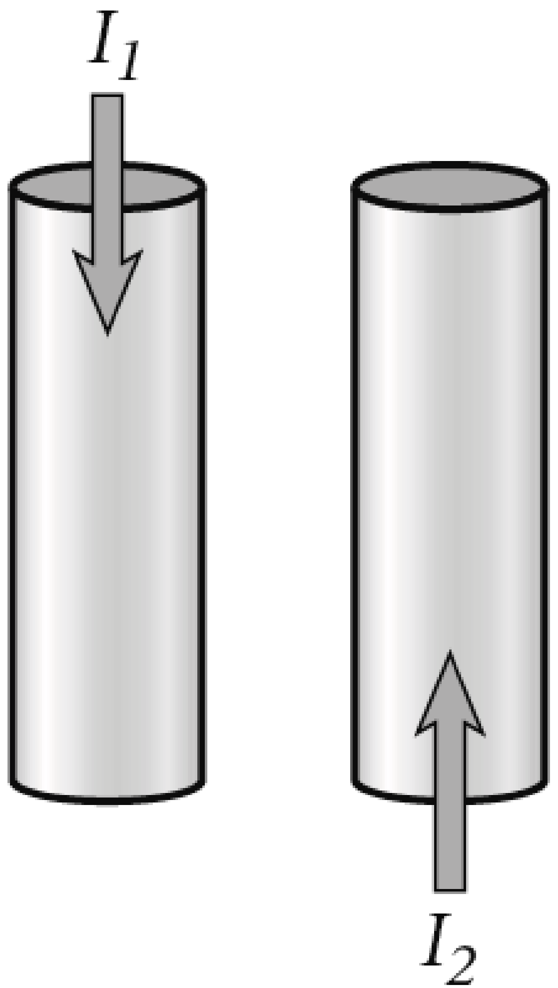
10. What is the direction of the magnetic field B_1 at the location of wire 2?

F. to the left

G. to the right

H. into the page

J. out of the page



- *Wire 1 carries current I_1 and creates magnetic field B_1.*
- *Wire 2 carries current I_2 and creates magnetic field B_2.*

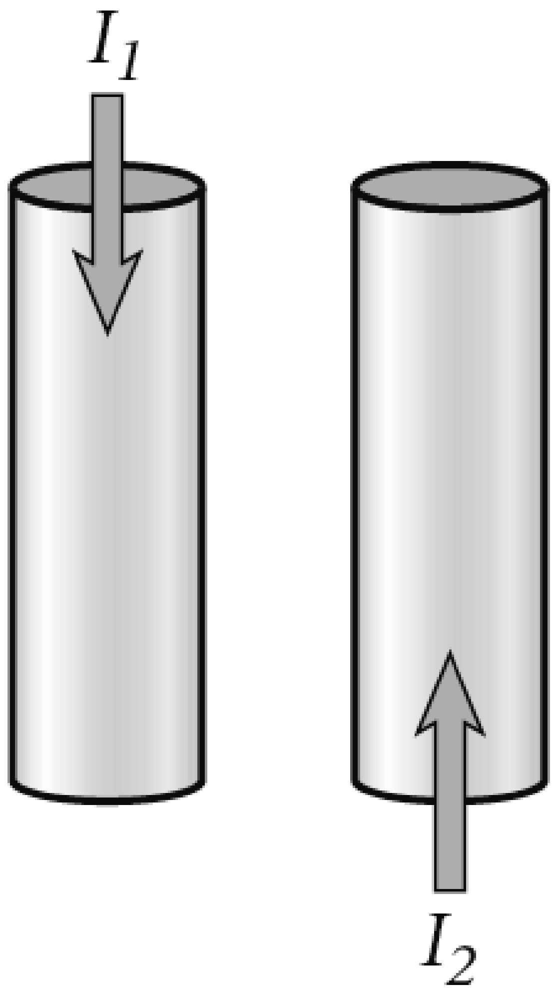
11. What is the direction of the force on wire 2 as a result of B_1?

A. to the left

B. to the right

C. into the page

D. out of the page



- *Wire 1 carries current I_1 and creates magnetic field B_1.*
- *Wire 2 carries current I_2 and creates magnetic field B_2.*

12. What is the magnitude of the magnetic force on wire 2?

F. $B_1 I_1 \ell_1$

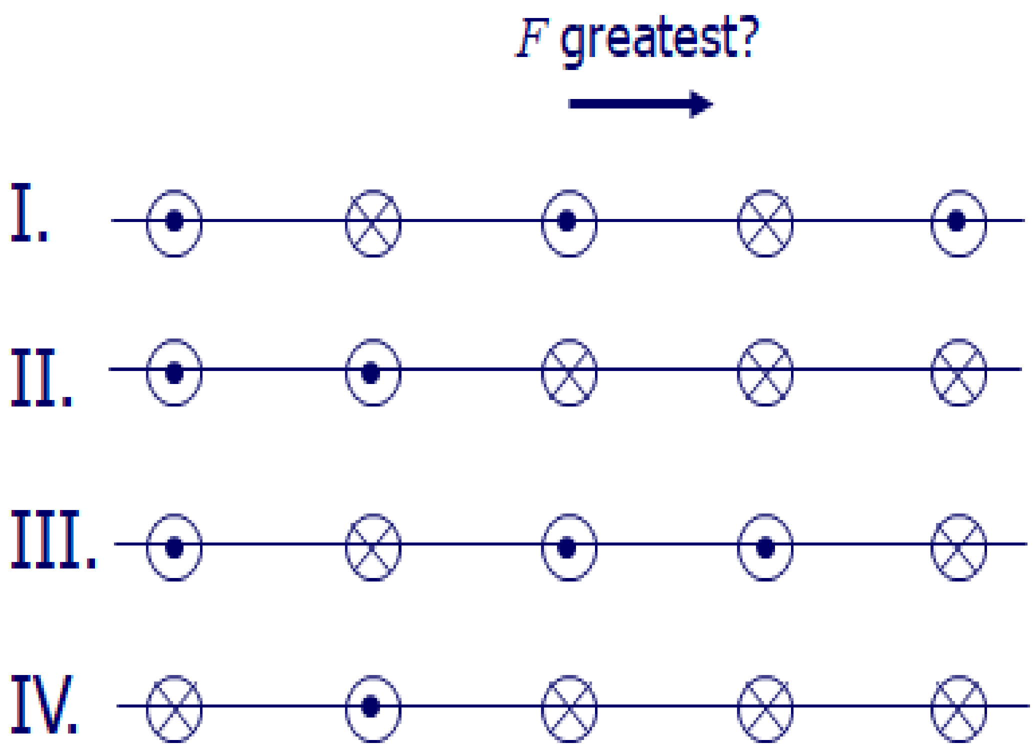
G. $B_1 I_1 \ell_2$

H. $B_1 I_2 \ell_2$

J. $B_2 I_2 \ell_2$

Forces on Parallel Currents

3. Which of the four situations below has the greatest force to the right on the central conductor?

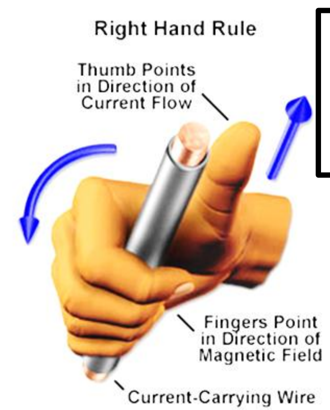


A. I.
B. II.
C. III.
D. IV.
E. Cannot determine.

Formula Summary Magnetic Fields & Forces

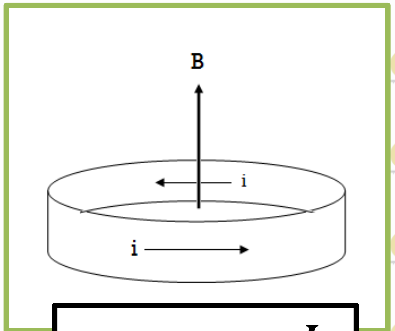
$\mu_0 = 1.26 \times 10^{-6}$ T·m / A

Straight Wire



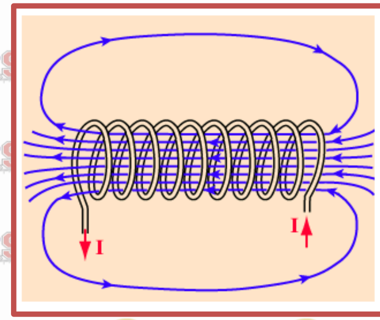
$$B = \frac{\mu_0 I}{2\pi r}$$

Current Loop



$$B = \frac{\mu_0 I}{2R}$$

Coil (solenoid)



N = number of turns

I = current in amps

L = length of coil

$$B = \frac{\mu_0 I N}{L}$$

Force on a charged particle in a B-Field

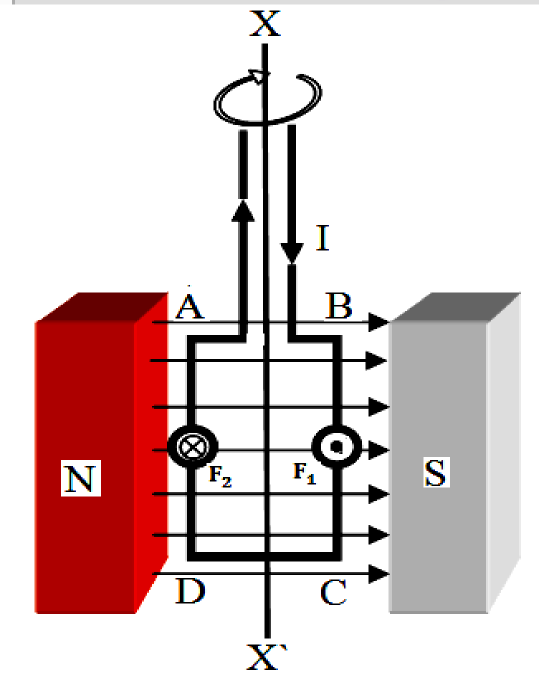
$$F = qvB$$

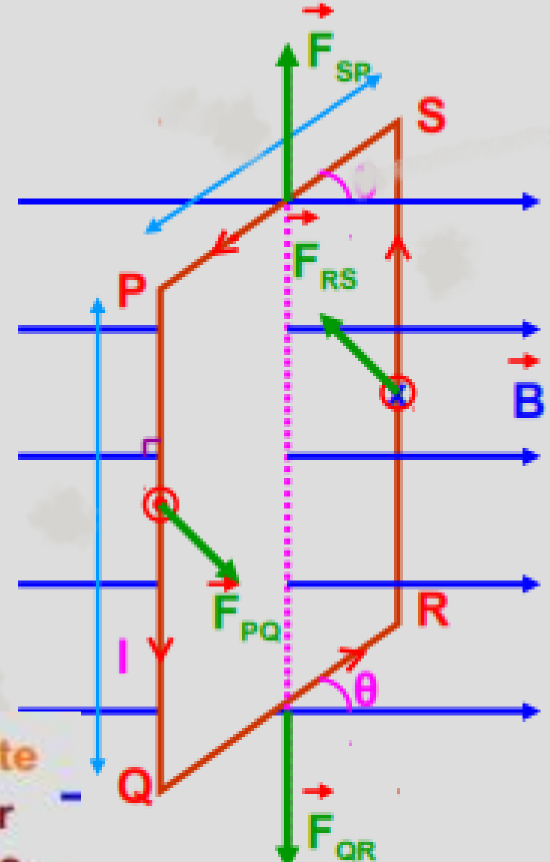
Force on a current in a B-Field

$$F = BiL$$

- Explain how a torque is produced on a current carrying rectangular coil placed in a uniform magnetic field.
- Guide students to derive expressions for the magnitude of the torque when the plane of the coil is parallel, inclined and perpendicular to the magnetic field.
- Explain how a radial magnetic field is obtained.
- Explain that the magnitude of the torque is constant on a rectangular coil placed in a radial magnetic field.
- Guide students to obtain the expression for the magnitude of the torque.
- Explain the structure and the action of a moving coil galvanometer.
- Explain that the coil is in equilibrium when the torque due to the current and the restoring torque in the torsional string (hair spring) are equal in magnitude.
- Obtain $C\theta = BINA$ identifying terms.
- Show that the deflection of the galvanometer is directly proportional to the current through it.
- Explain that the scale of the galvanometer is linear.
- Define the current sensitivity of a moving coil galvanometer
- Discuss the factors affecting current sensitivity.
- Explain the structure and action of a direct current (dc) motor having one armature coil diagrams. • Identify main parts of a dc motor.

Torque experienced by a Current Loop (Rectangular) in a uniform Magnetic Field:

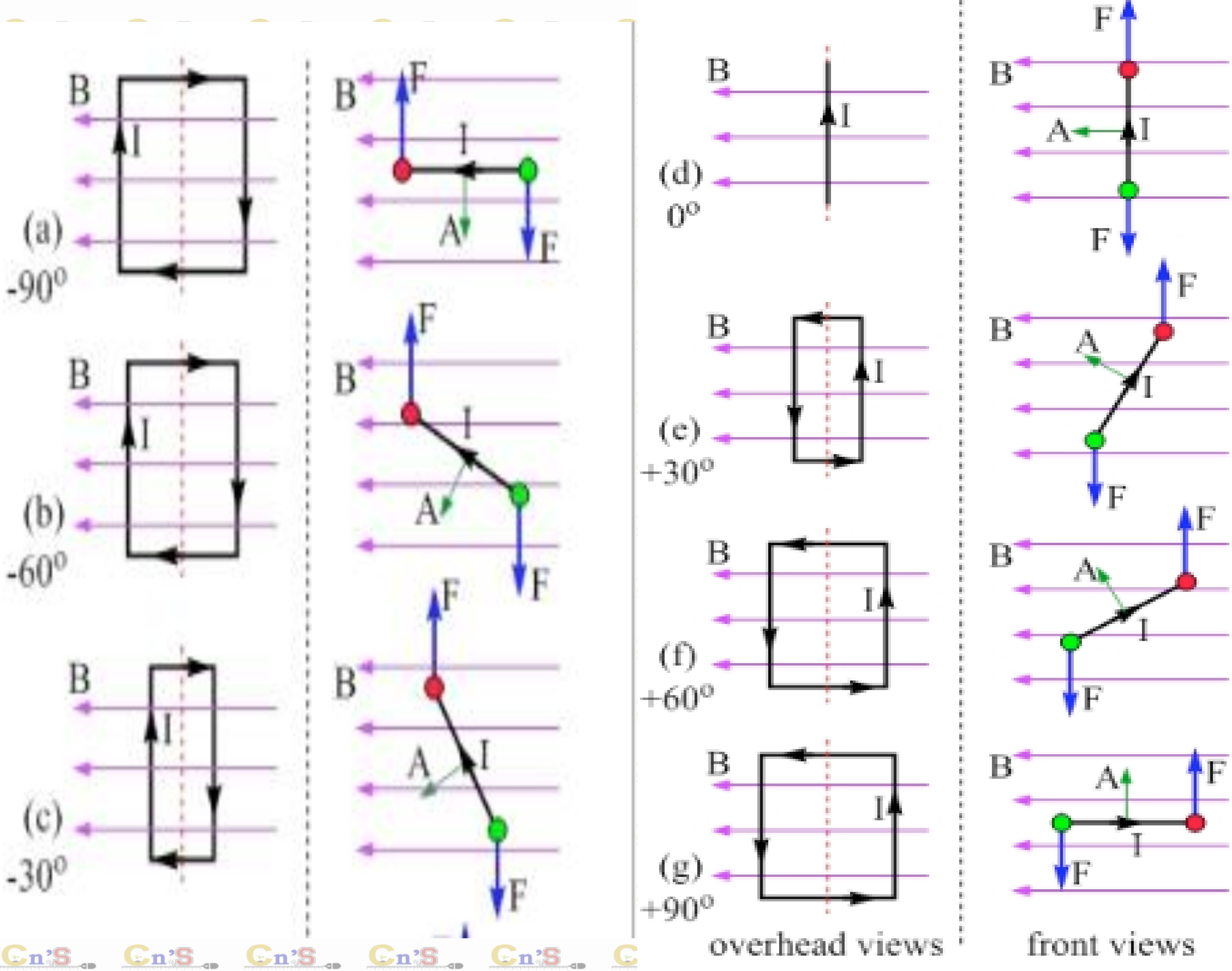




Forces $\vec{F}_{SP}$ and $\vec{F}_{QR}$ are equal in magnitude but opposite in direction and they cancel out each other. Moreover they act along the same line of action (axis) and hence do not produce torque.

Forces $\vec{F}_{PQ}$ and $\vec{F}_{RS}$ being equal in magnitude but opposite in direction cancel out each other and do not produce any translational motion. But they act along different lines of action and hence produce torque about the axis of the coil.

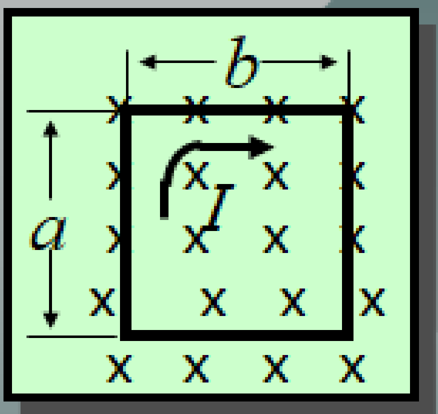
COUPLE PRODUCES NO RESULTANT FORCE, BUT A TORQUE



(a)
-90°
(b)
-60°
(c)
-30°
(d)
0°
(e)
+30°
(f)
+60°
(g)
+90°
B
I
F
A
overhead views
front views

Torque on Current Loop

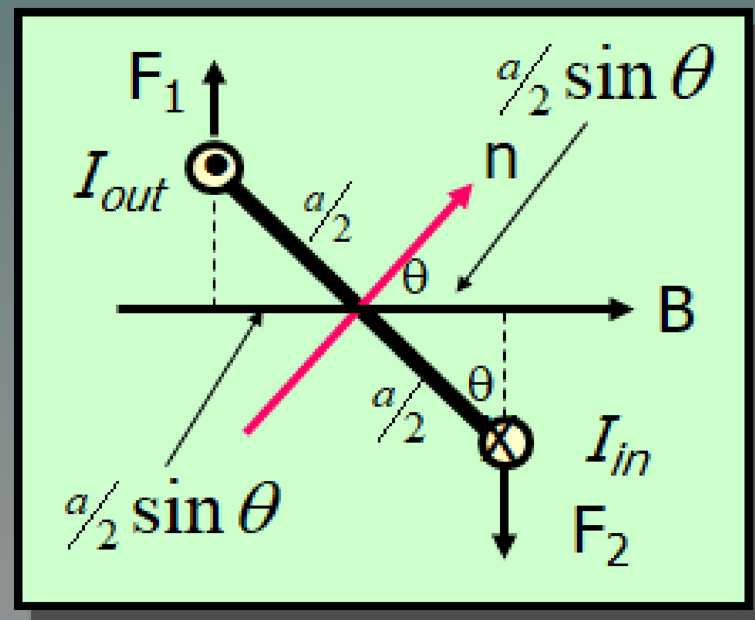
Recall that torque is product of force and moment arm.



The moment arms for F_1 and F_2 are:

$$\tfrac{a}{2}\sin\theta$$

$$F_1 = F_2 = IBb$$



$$\tau_1 = (IBb)(\tfrac{a}{2}\sin\theta)$$

$$\tau_2 = (IBb)(\tfrac{a}{2}\sin\theta)$$

$$\tau = 2(IBb)(\tfrac{a}{2}\sin\theta) = IB(ab)\sin\theta$$

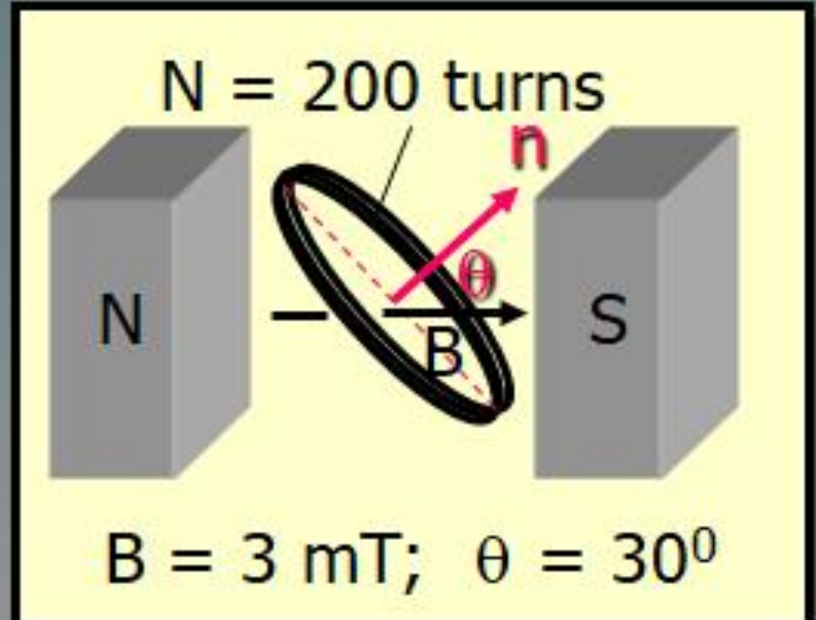
$$\boxed{\tau = IBA\sin\theta}$$

In general, for a loop of *N* turns carrying a current *I*, we have:

$$\boxed{\tau = NIBA\sin\theta}$$

Example 2: A 200-turn coil of wire has a radius of 20 cm and the normal to the area makes an angle of 30^0 with a 3 mT B-field. What is the torque on the loop if the current is 3 A?

$$\boxed{\tau = NIBA\sin\theta}$$



$$A = \pi R^2 = \pi(-.2\,\mathrm{m})^2$$

$A = 0.126\ \mathrm{m}^2$; N = 200 turns

B = 3 mT; $\theta = 30^0$; $I = 3$ A

$$\tau = NIBA\sin\theta = (200)(3\ \mathrm{A})(0.003\,\mathrm{T})(0.126\,\mathrm{m}^2)\sin 30^0$$

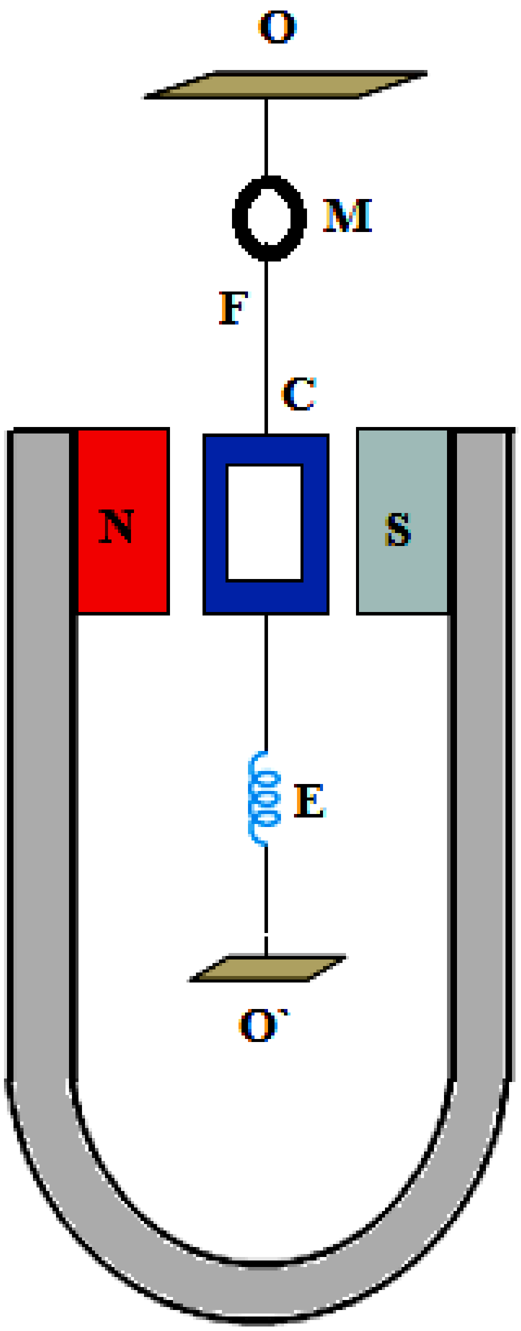
Resultant torque on loop:

$$\boxed{\tau = 0.113\ \mathrm{N{\cdot}m}}$$

A galvanometer is an electrical instrument used to detect the passage of current.

Its working depends upon the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque which can be described by the formula:

$\Gamma = BINA\cos\theta$ Where N is the number of turns in the coil, A is its area, I is the current passing through it, B is the magnetic field in which the coil is placed and θ is the angle which the plane of the coil makes with B. Due to action of the torque, the coil rotates and thus it detects the current.



A rectangular coil C is suspended between concaved shaped N and S poles of horseshoe magnet with the help of a fine metallic suspension wire. The suspension wire F is also used as one current lead to the coil. The other terminal of the coil is connected to a loosely wound spiral E which serve as the second current lead. The pole pieces of the magnet are made concave to make the field radial and stronger.

Working

When the current is passed through the coil, it is acted upon by a couple which tends to rotate the coil. This couple is known as deflecting couple and is given by $BINA\cos\theta$. As the coil is placed in radial magnetic field in which the plane of the coil is always parallel to the field, so θ is always zero.

This makes $\cos\alpha = 1$ and thus,

$$Deflecting\ Couple = NIAB$$

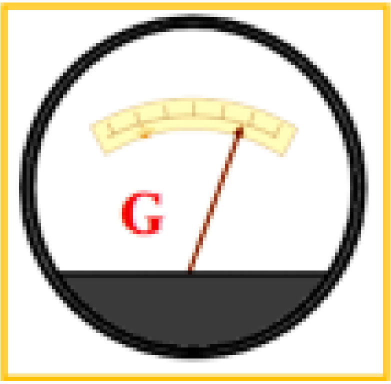
As the coil turns under the action of deflecting couple, the suspension wire is twisted which gives rise to a torsional couple. It tends to untwist the suspension and restore the coil to its original position. This couple is known as restoring couple.

The restoring couple of the suspension wire is proportional to the angle of deflection θ as long as the suspension wire obeys Hook's law. Thus

$$Restoring\ Torque = c\theta$$

Where constant c is called torsional couple and is defined as the couple of untwist.

Under the effect of these two couples, coil comes to rest when



$$Deflecting\ Torque = Restoring\ Torque$$

$$NIAB = c\theta$$

$$I = \frac{c}{NAB}\theta \quad \text{----------} \quad (1)$$

Thus $I\ \alpha\ \theta$ since $\frac{c}{NAB} = constant.$

Thus the current passing through the coil is directly proportional to the angle of deflection.

Moving Coil or Suspended Coil or D' Arsonval Type Galvanometer:

Torque experienced by the coil is

$\tau = N I A B \sin \Phi$

Restoring torque in the coil is

$\tau = k \alpha$ (where k is restoring torque per unit angular twist, α is the angular twist in the wire)

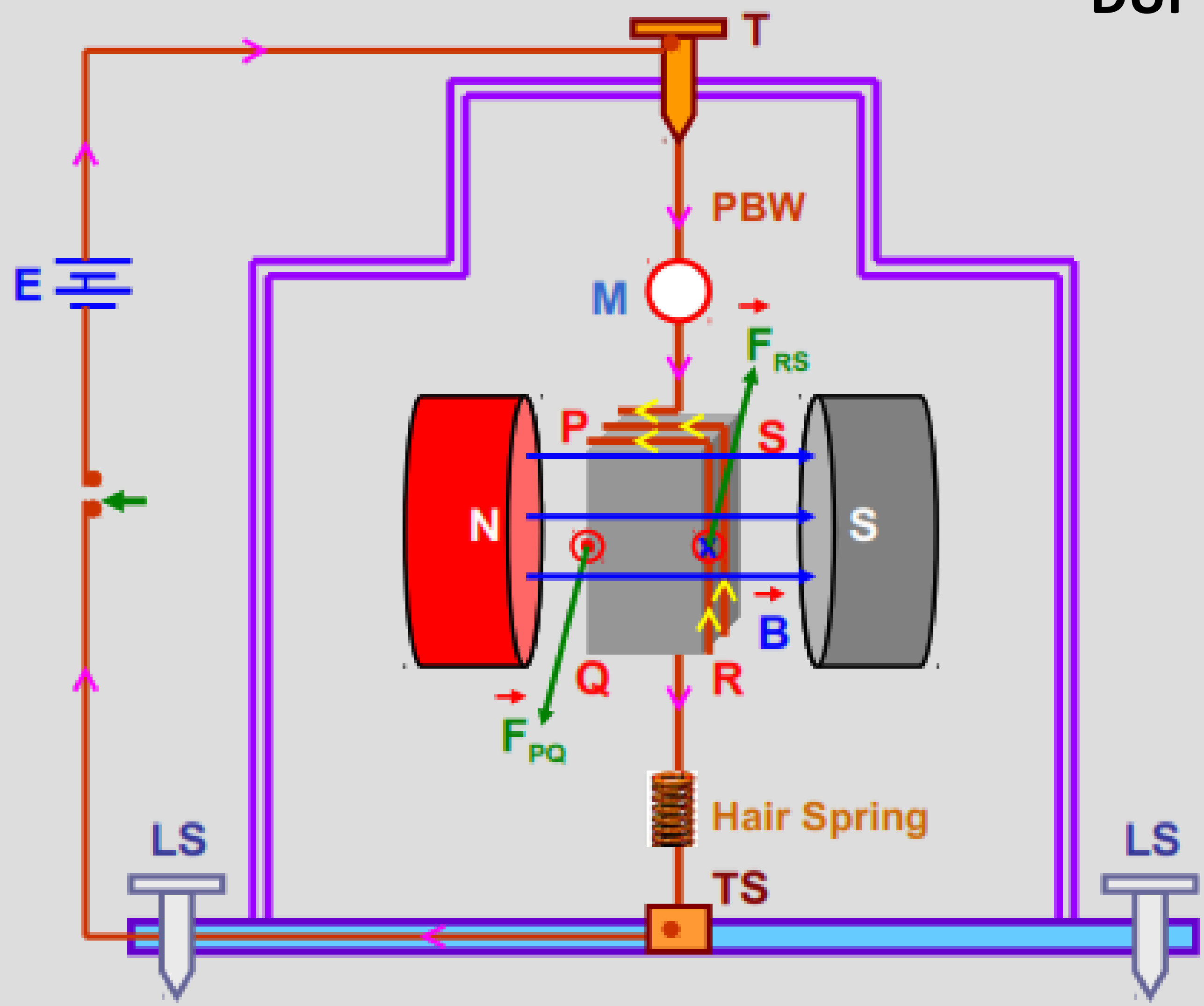
At equilibrium,

$N I A B \sin \Phi = k \alpha$

$$\therefore \quad I = \frac{k}{N A B \sin \Phi} \alpha$$

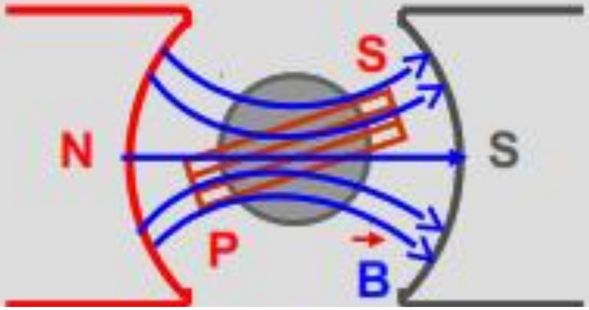
The factor $\sin \Phi$ can be eliminated by choosing Radial Magnetic Field.

DUP



T – Torsion Head, TS – Terminal screw, M – Mirror, N,S – Poles pieces of a magnet, LS – Levelling Screws, PQRS – Rectangular coil, PBW – Phosphor Bronze Wire

Radial Magnetic Field:



The (top view PS of) plane of the coil PQRS lies along the magnetic lines of force in whichever position the coil comes to rest in equilibrium.

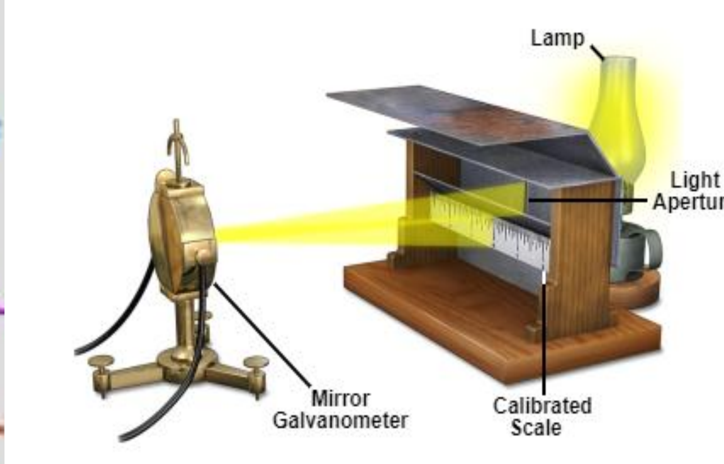
So, the angle between the plane of the coil and the magnetic field is 0°.

or the angle between the normal to the plane of the coil and the magnetic field is 90°.

i.e. $\sin \Phi = \sin 90° = 1$

Take c=k

L



$\therefore$ $\boxed{I = \frac{k}{NAB}\alpha}$ or $\boxed{I = G\alpha}$ where $G = \frac{k}{NAB}$ is called Galvanometer constant

Current Sensitivity of Galvanometer:

It is the defection of galvanometer per unit current.

$$\boxed{\frac{\alpha}{I} = \frac{NAB}{k}}$$

Voltage Sensitivity of Galvanometer:

It is the defection of galvanometer per unit voltage.

$$\boxed{\frac{\alpha}{V} = \frac{NAB}{kR}}$$

EXTRA

Mirror Galvanometer

- The key features of the galvanometer are hidden inside the instrument and not visible in this picture. They consist of a wire coil inside of which a tiny mirror is suspended by a silk thread. Attached to this mirror is one or more permanent magnets. When a current runs through the coil, it becomes an electromagnet, the field of which interacts with the permanent magnet's field in such a way as to cause those magnets to turn – and with them the mirror to which it is attached.
- This is where the second part of the device, a **calibrated scale**, comes in. A **lamp** is positioned behind the scale; its light passes through an **aperture** in the scale that is directed at the galvanometer's mirror. The mirror reflects that light back onto the scale; when the magnet/mirror is deflected by an electric current, the light deflects with it, measured by the scale.

Sensitivity of Galvanometer

Sensitivity of the galvanometer is the measure of the ability of galvanometer to detect small amount of current with bigger Θ. It is obvious from equation

$$I = \frac{c}{NAB}\theta \qquad \Theta/I = \frac{NAB}{c}$$

that a galvanometer can be made more sensitive if NAB/c is made larger. Thus, to increase the sensitivity of galvanometer, c may be decreased or B, A and N may be increased.

The torque is Maximum when the plane of the coil is parallel to the magnetic field.

The torque is Zero when the plane of the coil is perpendicular to the magnetic field.

Γ = BINA sin Φ If Φ is the angle, which the normal to the plane of the coil makes with B

If $\Phi = 0°$, then $0 = \tau$.

If $\Phi = 90°$, then τ is maximum. i.e. $\tau_{max} = N\,I\,A\,B$

Units: B in Tesla, I in Ampere, A in m^2 and τ in Nm.

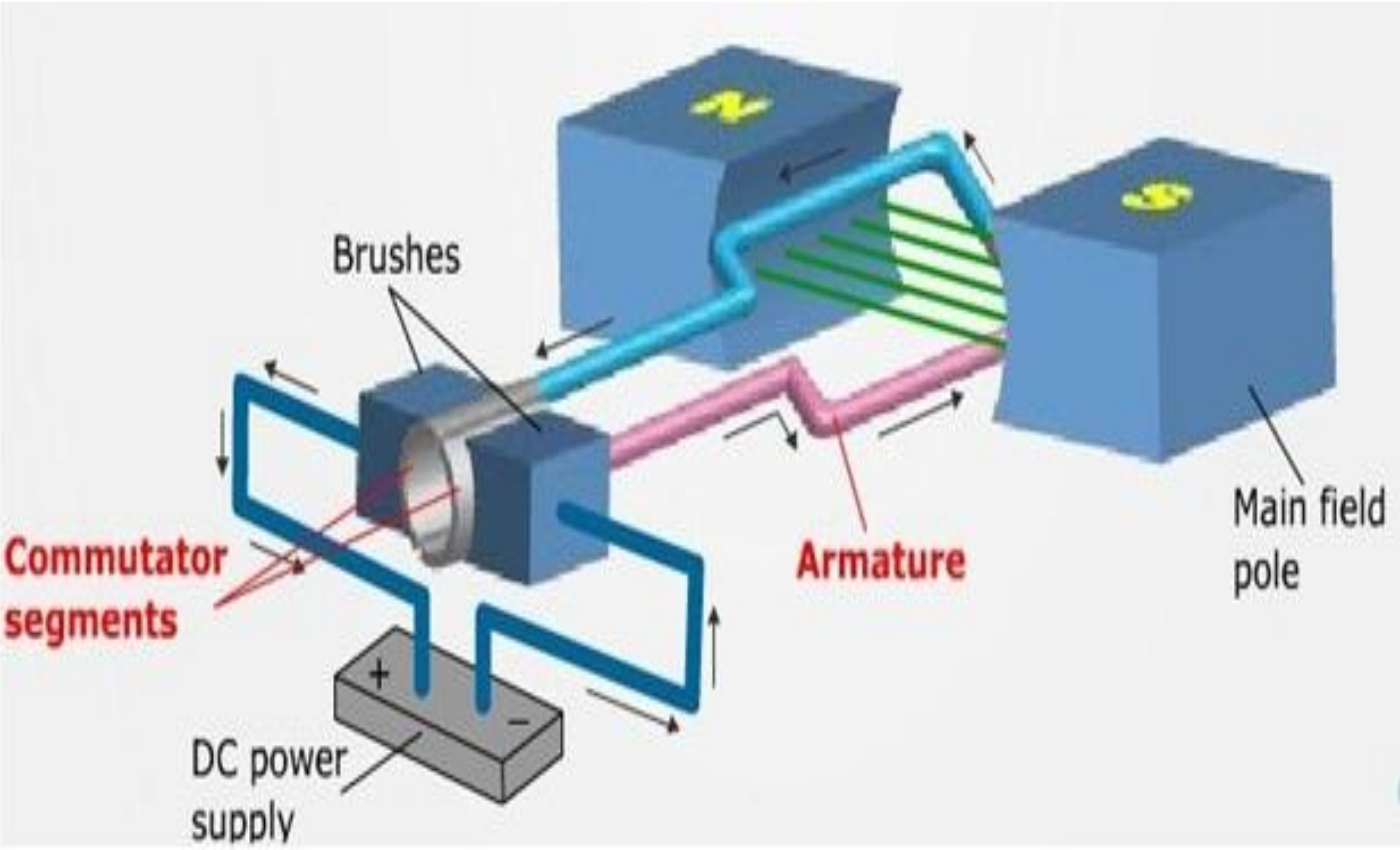
The above formulae for torque can be used for any loop irrespective of its shape.

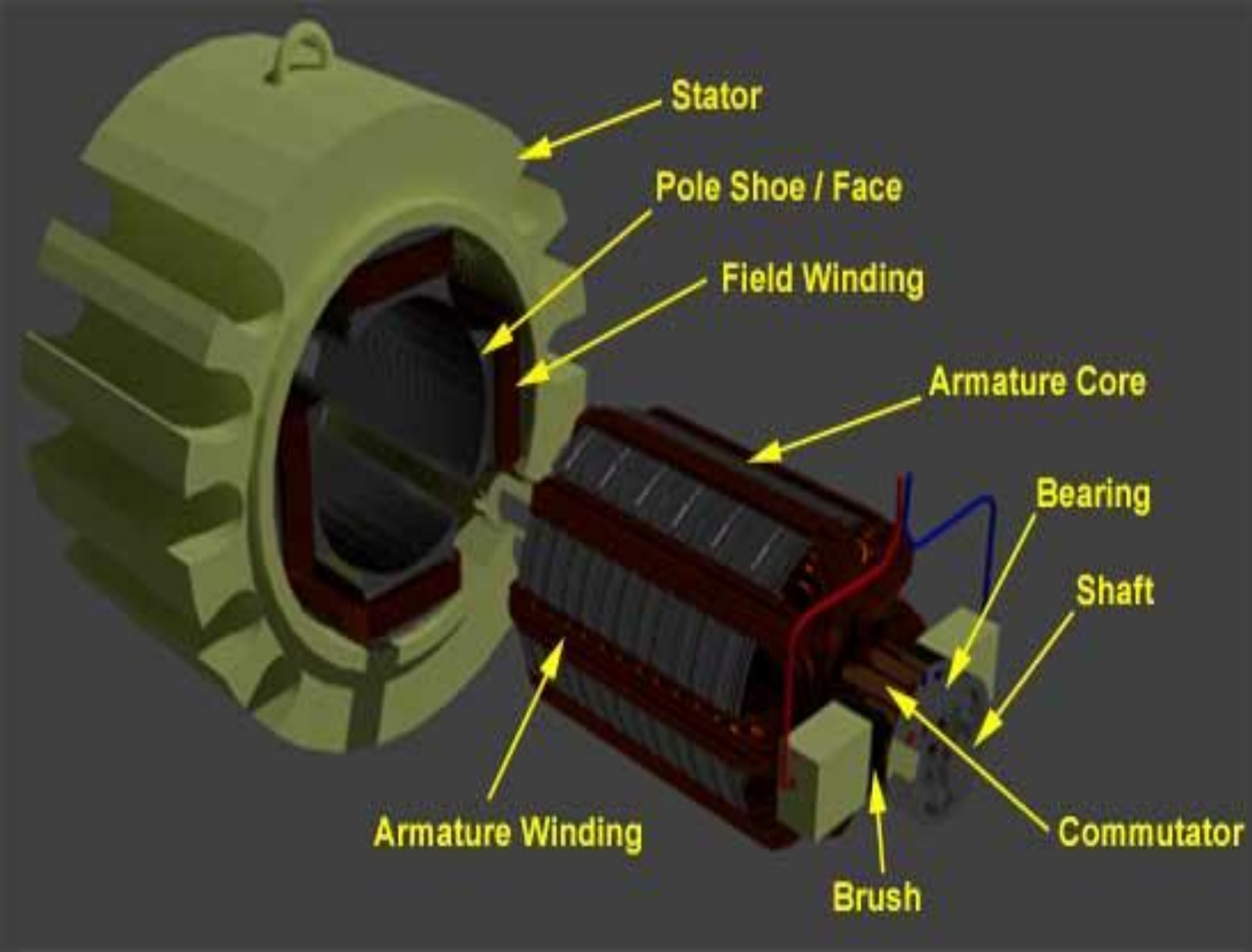
DC motors

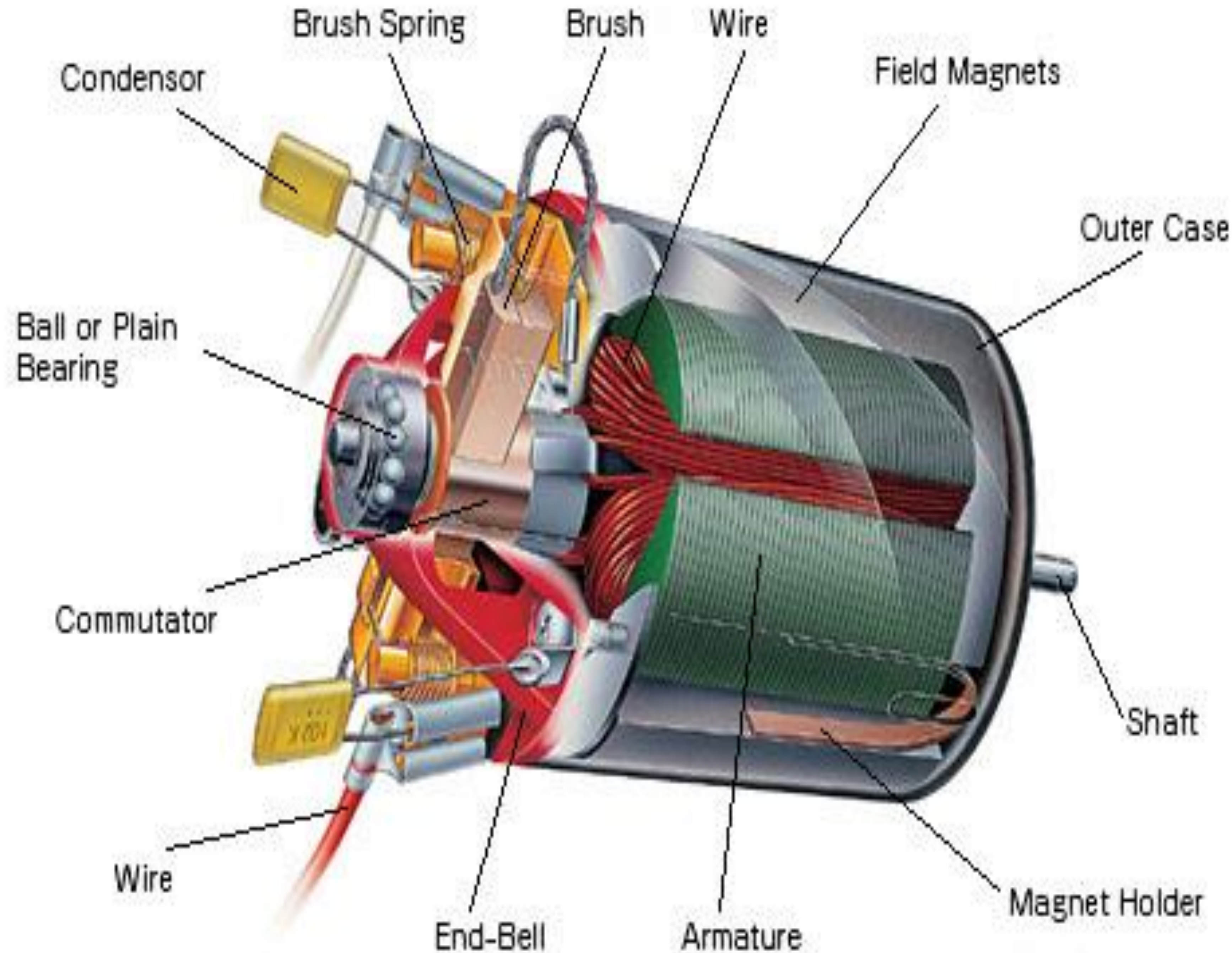
DC motors consist of a coil of wire in between two permanent magnets.

- Current flows through the wire and it experiences a turning effect due to the forces exerted on it in the magnetic field. The turning effect can be increased by:
  - increasing the current
  - using a stronger magnetic field
  - increasing the number of turns on the coil.
- A split ring commutator is used to ensure that the direction that the current flows in the coil reverses every half turn.

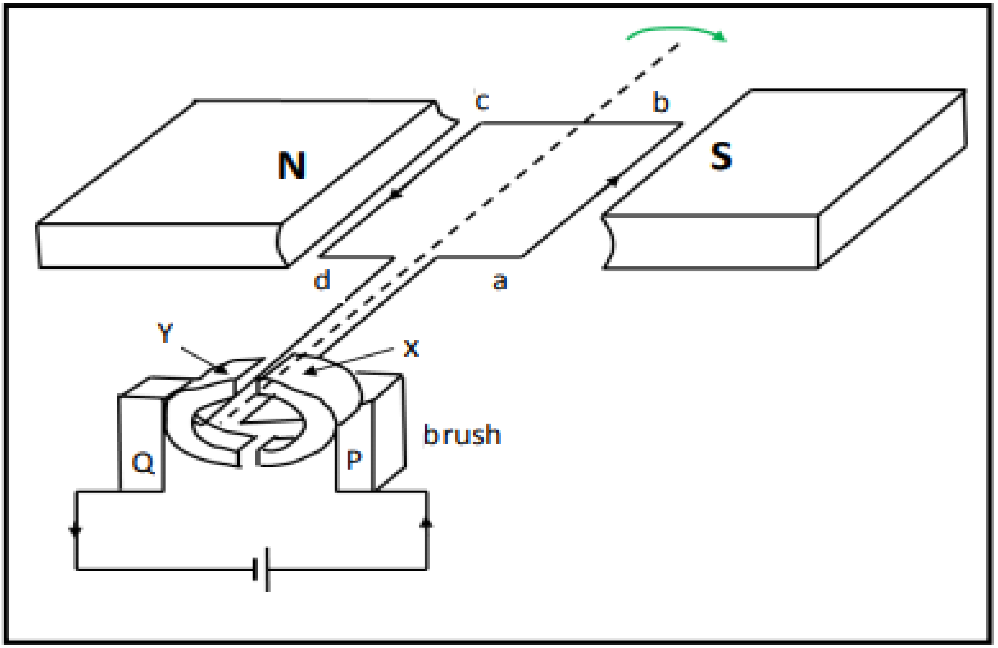
Direct current (dc) motor

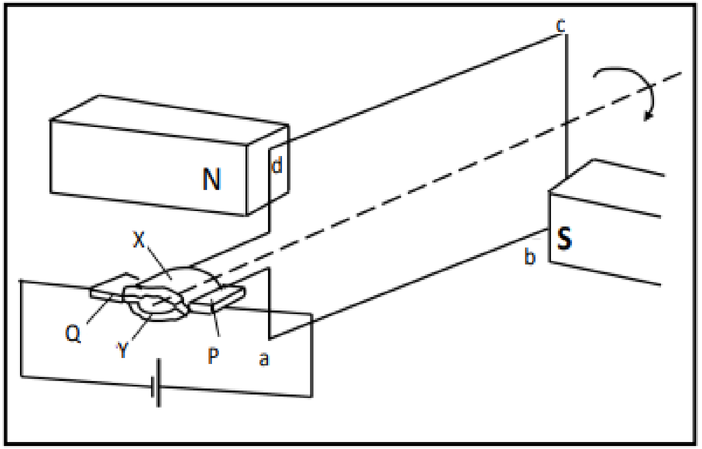


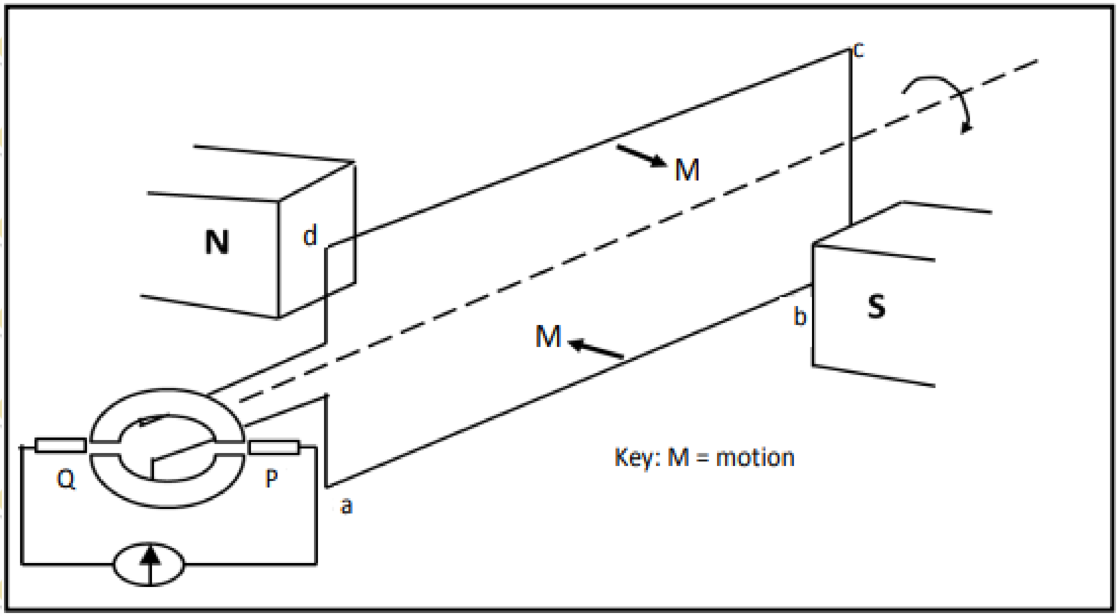

Stator
Pole Shoe / Face
Field Winding
Armature Core
Bearing
Shaft
Armature Winding
Commutator
Brush


Brush Spring
Brush
Wire
Condensor
Field Magnets
Outer Case
Ball or Plain Bearing
Commutator
Shaft
Wire
End-Bell
Armature
Magnet Holder

- The current flows through brush P and then to one half of the split-ring commutator marked X. From the split-ring X, it passes through and out the coil via the other half of the split-ring Y and the brush Q. The force on side AB will be down wards while the force on the side CD will be upwards. The torque then rotates the coil in the clockwise
- direction. The coil
- rotates until it
- reaches the vertical
- position



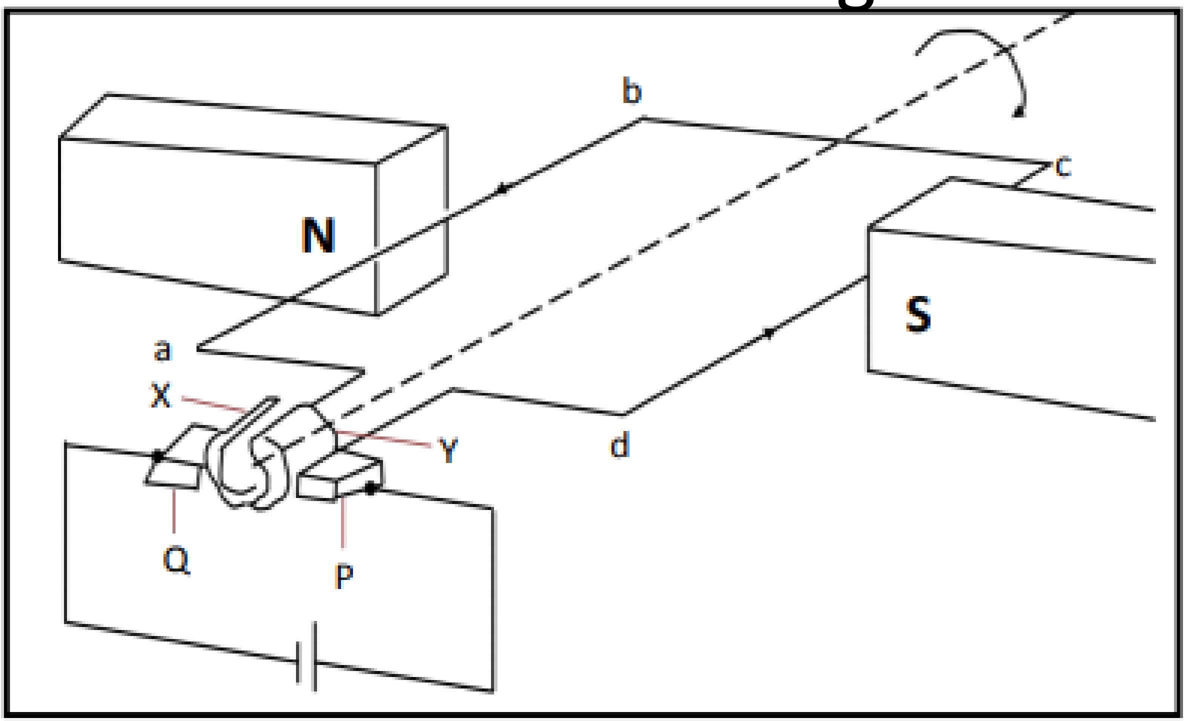




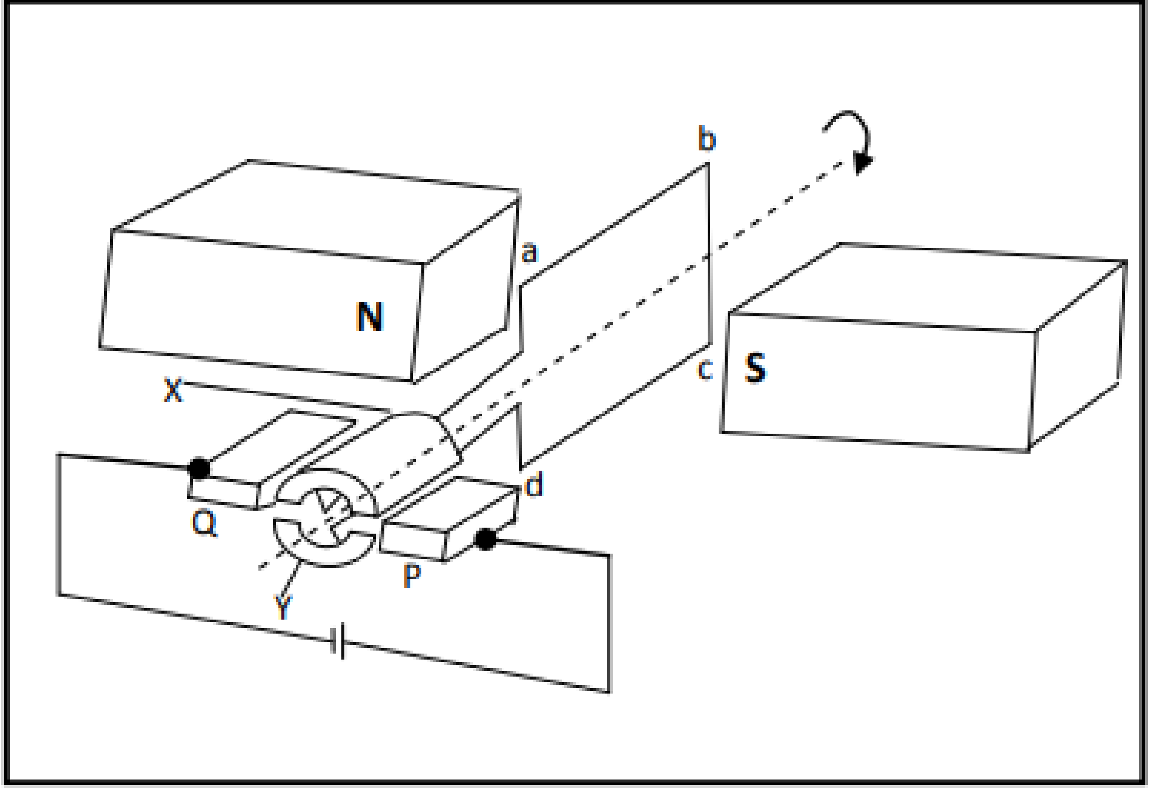
You will now notice that at this vertical position, the two split-rings X and Y are no longer in contact with the brushes P and Q. Hence, no current flows through the coil. But the coil continues to rotate even though there is no turning effect. This is because of its inertia.

At this position, the split-ring X and the side AB of the coil will be in contact with the brush Q. Also, the split-ring Y and the side CD will be in contact with the brush P. The current flows in the direction DCBA. The current now travels from B to A of one side of the coil and from D to C on the other side of the coil. This means that, the forces created will also change directions.

The force on side BA of the coil is upwards & CD downwards.



So the split-ring commutator is a device that reverses current at every half turn along the sides of the rotating coil through the brushes. Hence, the coil is rotating in a clockwise direction.



- In this position, the contact between the carbon brushes and the commutator is broken again. Current flow is cut off but the coil continues to rotate because of its inertia. The processes are repeated and the motor continues to rotate until the current is switched off.

Question. Prove that $\frac{joule}{tesla} = (ampere) \times (meter)^2$

$$L.H.S. = \frac{joule}{tesla} = \frac{(newton) \times (meter)}{tesla}$$

$$\because work\ W = (force\ F) \times (displacement\ d)$$

$$= \frac{(ampere \times meter \times tesla) \times (meter)}{tesla}$$

$$\because force\ F = (current\ I) \times (length\ L) \times (magnetic\ induction\ B)$$

$$= \frac{ampere \times (meter)^2 \times tesla}{tesla} = ampere \times (meter)^2 = R.H.S.$$