VISCOSITY

Objectives

- Explain the difference between streamline flow and turbulent flow.
- Introduce velocity gradient and shear stress for a laminar flow.
- Give the expression $F = -\eta A \frac{dv}{dx}$ for a laminar flow.
- Define coefficient of viscosity and give units and dimensions.
- Give Poiseuille's equation for a steady flow.
- Explain the conditions for the validity of the above equation.
- Verify the equation using dimensions.
- Determination of coefficient of viscosity by using Poiseuille's formula

- Describe the forces acting on a spherical object moving through a viscous medium.
- Explain that the magnitude of the viscous force increases with the speed of the object.
- Show that the object reaches a terminal velocity due to that reason.
- State Stokes' law as the expression, $F = 6\pi r\eta v$ and introduce the terms.
- Guide students to derive expressions for the terminal velocities of the objects moving up and down in a viscous medium.
- Give the velocity-time graph for an object moving in a viscous medium.
- Conduct a discussion to identify the practical uses related to viscosity.
- Guide students to solve problems related to viscosity.

VISCOSITY

- Viscous fluids tend to cling to a solid surface.
- Syrup and honey are more viscous than water.
- Grease is more viscous than engine oils.
- Liquids are more viscous than gases.
- Lava is an example of a very viscous material.

When real fluids flow they have a certain amount of internal friction called **viscosity**. It exists in both liquids and gases and is essentially a friction force between different layers of fluid as they move past one another. In liquids the viscosity is due to the cohesive forces between the molecules whilst in gases the viscosity is due to collisions between the molecules.

(a) With increase in pressure, the viscosity of liquids (except water) increases while that of gases is practically independent of pressure. The viscosity of water decreases with increase in pressure.

(b) Difference between viscosity and solid friction : Viscosity differs from the solid friction in the respect that the viscous force acting between two layers of the liquid depends upon the area of the layers, the relative velocity of two layers and distance between two layers, but the friction between two solid surfaces is independent of the area of surfaces in contact and the relative velocity between them.

(a) The viscosity of gases increases with increase of temperature, because on increasing temperature the rate of collision/diffusion increases.

(b) The viscosity of liquid decreases with increase of temperature, because the cohesive force between the liquid molecules decreases with increase of temperature

VISCOSITY

- **Viscosity**

Viscosity is the property of the fluid (liquid or gas) by virtue of which an internal frictional force comes into play when the fluid is in motion in the form of layers having relative motion. It opposes the relative motion of the different layers. Viscosity is also called as fluid friction.

- The viscous force directly depends on the area of the layer and the velocity gradient.

$$F = -\eta A \frac{dv}{dx}$$

Velocity gradient:

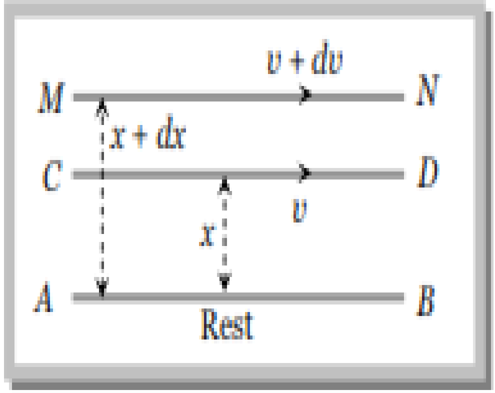
The difference in velocity between adjacent layers of the fluid is known as a velocity gradient. And the SI unit of Velocity Gradient is s^{-1}

- **Coefficient of Viscosity**

Coefficient of viscosity of a liquid is equal to the tangential force required to maintain a unit velocity gradient between two parallel layers of liquid each of area unity.

$$\eta = \frac{F}{A\left(\frac{dv}{dx}\right)}$$

Consider the two layers *CD* and *MN* of the liquid at distances x and $x + dx$ from the fixed surface *AB*, having the velocities v and $v + dv$ respectively. Then $\frac{dv}{dx}$ denotes the rate of change of velocity with distance and is known as velocity gradient.



According to Newton's hypothesis, the tangential force F acting on a plane parallel layer is proportional to the area of the plane A and the velocity gradient $\frac{dv}{dx}$ in a direction normal to the layer, *i.e.*,

$$F \propto A \quad \text{and} \quad F \propto \frac{dv}{dx} \quad \therefore \quad F \propto A\frac{dv}{dx}$$

or $$F = -\eta A\frac{dv}{dx}$$

Where η is a constant called the coefficient of viscosity. Negative sign is employed because viscous force acts in a direction opposite to the flow of liquid.

If $A = 1, \frac{dv}{dx} = 1$ then $\eta = F$.

Hence the coefficient of viscosity is defined as the viscous force acting per unit area between two layers moving with unit velocity gradient.

Unit and dimension of η

We have,

$$F = \eta A \frac{dv}{dx} \text{(in magnitude)}$$

or , $$\eta = \frac{F}{A * \frac{dv}{dx}}$$

$$= \frac{N}{m^2 * s^{-1}}$$

$$= Nsm^{-1}$$ -2 POISEUILLE

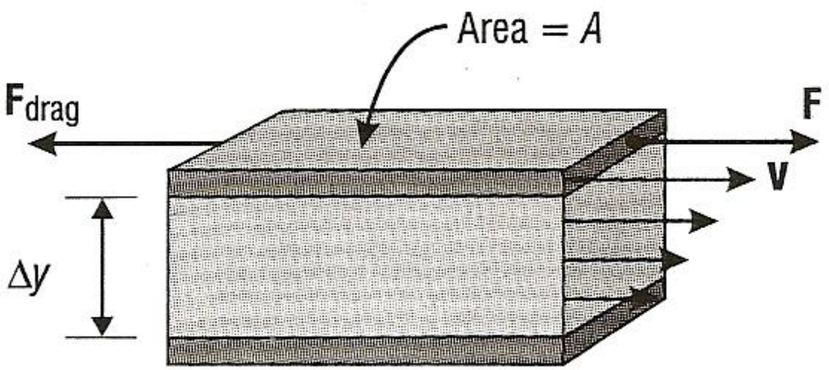
$$= 1\text{deca} - \text{poise (S.I)}$$

And,

$$\eta = [M^1 L^1 T^{-2} * T^1 L^{-2}]$$

$$= [M^1 L^{-1} T^{-1}]$$

Viscosity



- When the plate moves to the right at constant speed, no ***net*** force is acting on the plate.
- Therefore, the fluid exerts a force of friction, or drag force ***F*** ***drag*** on the plate to the left, opposing motion. The magnitude of the drag force equals ***F***.

- As long as the plate speed ***v*** is not so large that turbulence occurs, the fluid flow between the plates is laminar.
- The force ***F*** required to maintain a constant speed for most fluids in laminar flow is found to be:
  - Proportional to ***A*** and ***v***, and
  - Inversely proportional to the thickness of the fluid layer, Δy

$$F_{\text{drag}} = F = \eta \frac{Av}{\Delta y}$$

Viscosities of Common Fluids

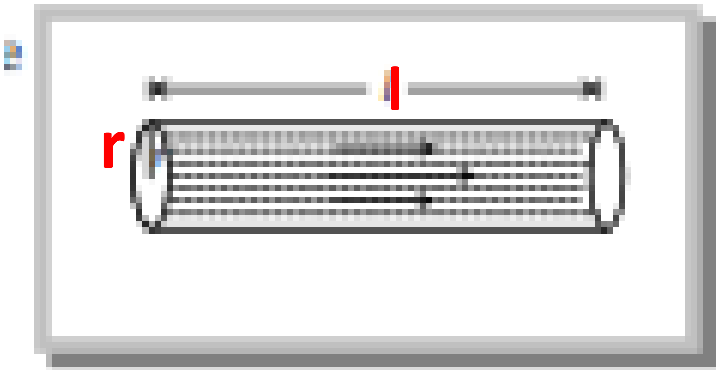
- Viscosity of most liquids decreases as temperature increases.
- Viscosity of most gases increase with temperature

Example:

- Cold honey is thick with a high viscosity
- Hot honey is watery with a low viscosity

Fluid	Temperature °C	Viscosity Pa • s
Gases		
Air	0	1.7×10^{-5}
	20	1.9×10^{-5}
	100	2.2×10^{-5}
Water Vapor	100	1.3×10^{-5}
Liquids		
Water	0	0.0018
	20	0.0010
	100	0.00028
Blood	37	~0.005
Cooking Oil	20	~0.01
Motor Oil	20	1
Corn Syrup	20	8
Molten Lava	950	1000

Poiseuille studied the stream-line flow of liquid in capillary tubes. He found that if a pressure difference (P) is maintained across the two ends of a capillary tube of length $'l'$ and radius r, then the volume of liquid coming out of the tube per second is



(i) Directly proportional to the pressure difference (P).

(ii) Directly proportional to the fourth power of radius (r) of the capillary tube

(iii) Inversely proportional to the coefficient of viscosity (η) of the liquid.

(iv) Inversely proportional to the length (l) of the capillary tube.

i.e. $V \propto \frac{P r^4}{\eta l}$ or $V = \frac{K P r^4}{\eta l}$

$$\therefore V = \frac{\pi P r^4}{8 \eta l}$$

[Where $K = \frac{\pi}{8}$ is the constant of proportionality]

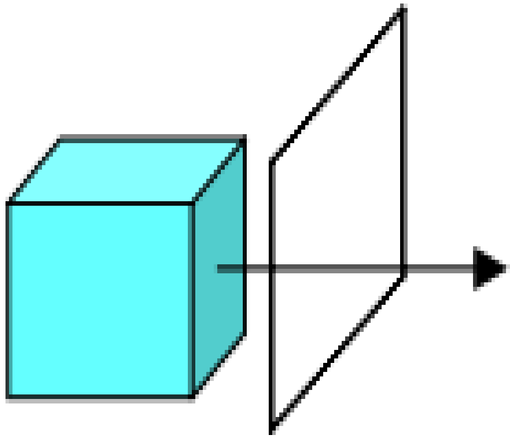
This is known as Poiseuille's equation.

This equation also can be written as,

$V = \frac{P}{R}$ where $R = \frac{8\eta l}{\pi r^4}$

R is called as liquid resistance.

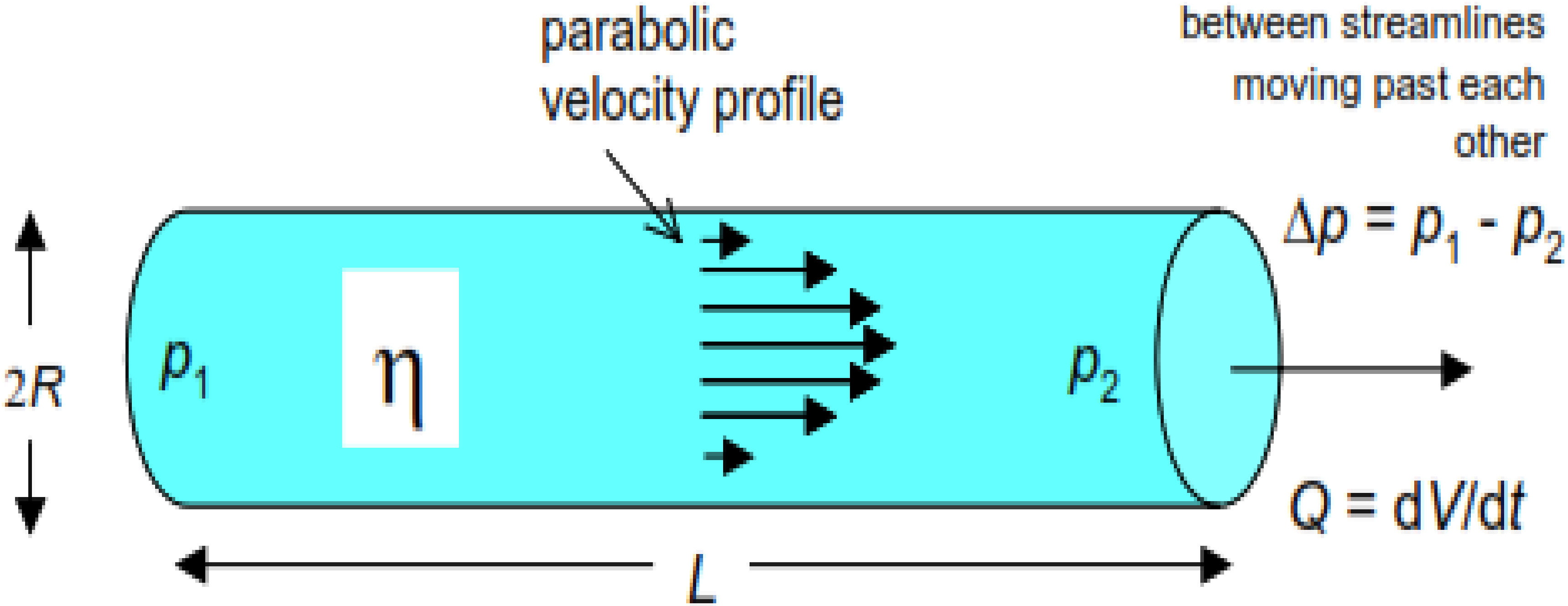
Poiseuille's Law: laminar flow of a newtonian fluid through a pipe



volume flow rate $Q = dV/dt$

$$Q = \frac{dV}{dt} = \frac{\Delta p \, \pi R^4}{8 \eta L}$$

$p_1 > p_2 \Rightarrow$ pressure drop along pipe $\Rightarrow$ energy dissipated (thermal) by friction between streamlines moving past each other



The **volume flow rate**

$$Q = dV/dt$$

of a fluid of viscosity η, through a pipe of radius R and length L, when driven by a pressure difference Δp is given by

$$dV/dt = Q = \Delta p \, \pi \, R^4 / (8 \, \eta \, L)$$

This is known as **Poiseuille's law**. Poiseuille's law only applies to newtonian fluids. Non-newtonian liquids do not obey Poiseuille's law because their viscosities are velocity dependent. The assumption of streamlined (laminar) flow is built in to Poiseuille's law.

Alternative view of Poiseuille's Law

Consider an electrical circuit in which a potential *V* between the ends of a resistance *R* results in a current *I*. Then the flow is determined by the ratio of potential to resistance.

flow (current) = potential / resistance

$$I = V / R$$

Poiseuille's Law can be arranged in this form

flow = potential / resistance

$$Q = \Delta p \,/\, (8\,\eta\, L \,/\, \pi R^4)$$

flow $\Rightarrow$ Q potential $\Rightarrow$ Δp

resistance $\Rightarrow$ $(8\,\eta\, L \,/\, \pi R^4)$

resistance $\propto L$ resistance $\propto \eta$ resistance $\propto (1\,/\,R^4)$

Just as electrical energy is dissipated when an electrical current flows, energy is dissipated when a fluid flows through a pipe. In the electrical circuit this is manifest by the drop in potential around the circuit whereas for the flow in the pipe there is a drop in pressure along the pipe.

(1) Series combination of tubes

(i) When two tubes of length $l_1,\ l_2$ and radii $r_1,\ r_2$ are connected in series across a pressure difference P,

Then $P = P_1 + P_2$ ...(i)

Where P_1 and P_2 are the pressure difference across the first and second tube respectively

(ii) The volume of liquid flowing through both the tubes i.e. rate of flow of liquid is same.

Therefore $V = V_1 = V_2$

i.e., $V = \dfrac{\pi P_1 r_1^4}{8\eta l_1} = \dfrac{\pi P_2 r_2^4}{8\eta l_2}$ ...(ii)

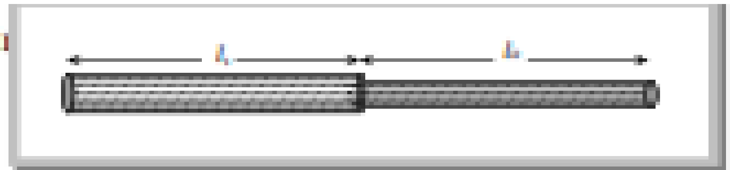
Substituting the value of P1 and P2 from equation (ii) to equation (i) we get

$$P = P_1 + P_2 = V\left[\frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}\right] \therefore V = \frac{P}{\left[\frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}\right]} = \frac{P}{R_1 + R_2} = \frac{P}{R_{eff}}$$

Where R_1 and R_2 are the liquid resistance in tubes

(iii) Effective liquid resistance in series combination

$$R_{eff} = R_1 + R_2$$



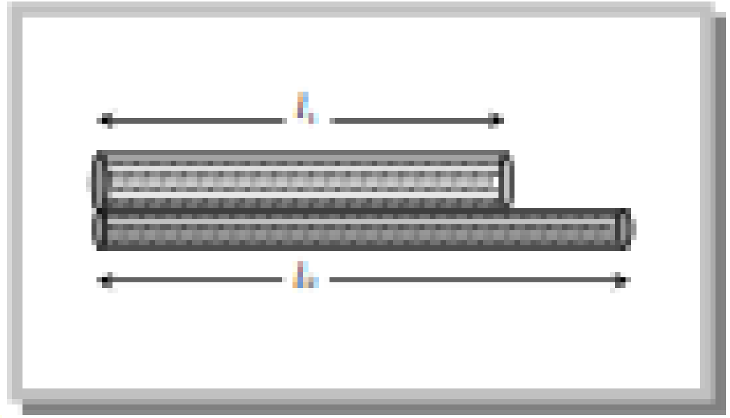
(2) Parallel combination of tubes

(i) $P = P_1 = P_2$

(ii) $V = V_1 + V_2 = \dfrac{P\pi r_1^4}{8\eta l_1} + \dfrac{P\pi r_2^4}{8\eta l_2}$

$$= P\left[\frac{\pi r_1^4}{8\eta l_1} + \frac{\pi r_2^4}{8\eta l_2}\right]$$

$$\therefore V = P\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{P}{R_{eff}}$$



(iii) Effective liquid resistance in parallel combinatio

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ or } R_{eff} = \frac{R_1 R_2}{R_1 + R_2}$$

‹ Stoke's Law And Terminal Velocity

Stoke's Law and Terminal Velocity.

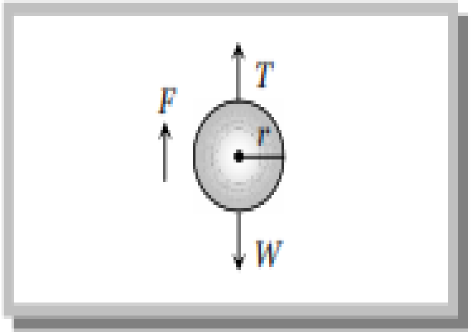
- When a body moves through a fluid, the fluid in contact with the body is dragged with it. This establishes relative motion in fluid layers near the body, due to which viscous force starts operating. The fluid exerts viscous force on the body to oppose its motion. The magnitude of the viscous force depends on the shape and size of the body, its speed and the viscosity of the fluid. Stokes established that if a sphere of radius r moves with velocity v through a fluid of viscosity η, the viscous force opposing the motion of the sphere is $F = 6\pi r\eta v$
- This law is called Stokes law.

$$F_{drag} = 6\pi\, r\, v\, \eta$$

Viscosity

Radius Speed

If a spherical body of radius r is dropped in a viscous fluid, it is first accelerated and then it's acceleration becomes zero and it attains a constant velocity called terminal velocity.



Force on the body

(i) Weight of the body (W) = mg = (*volume* × *density*) × $g = \frac{4}{3}\pi r^3 \rho g$

(ii) Upward thrust (T) = weight of the fluid displaced

= (*volume* × *density*) of the fluid × $g = \frac{4}{3}\pi r^3 \sigma g$

(iii) Viscous force (F) = $6\pi\eta r v$

When the body attains terminal velocity the net force acting on the body is zero. $\therefore W - T - F = 0$ or $F = W - T$

$$\Rightarrow \qquad 6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

$\therefore$ Terminal velocity $v = \frac{2}{9}\frac{r^2(\rho - \sigma) g}{\eta}$

Terminal Velocity

- The constant speed that occurs when the drag force equals the gravitational force acting on an object.

· **Terminal Velocity**

It is maximum constant velocity acquired by the body while falling freely in a viscous medium. This is attained when the apparent weight is compensated by the viscous force.

It is given by

$$v = \frac{2r^2(\rho-\sigma)\,g}{9\eta}$$

Terminal velocity

(i) Terminal velocity depend on the radius of the sphere so if radius is made n - fold, terminal velocity will become n 2 times.

(ii) Greater the density of solid greater will be the terminal velocity

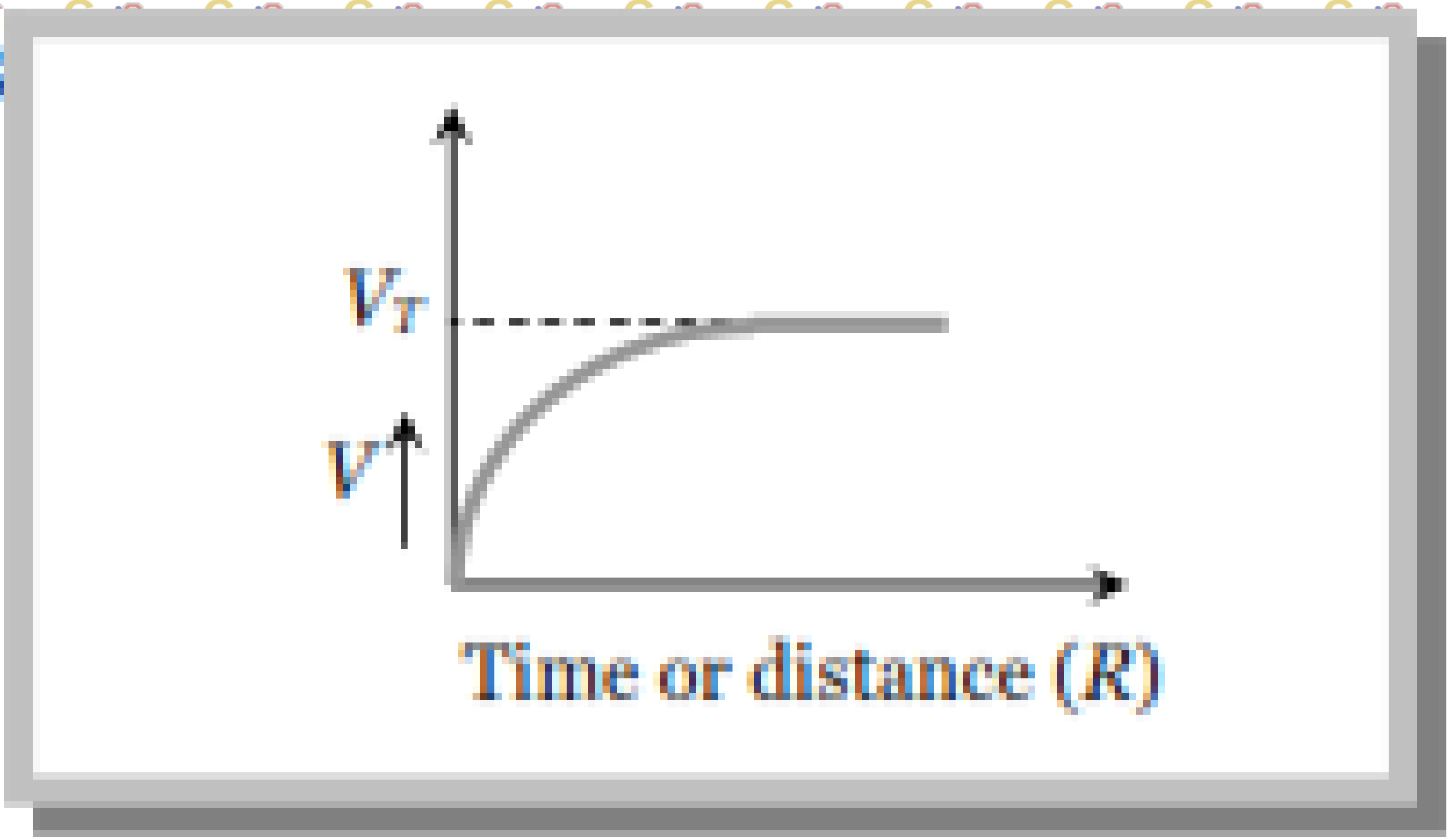
(iii) Greater the density and viscosity of the fluid lesser will be the terminal velocity.

(iv) If ρ > σ then terminal velocity will be positive and hence the spherical body will attain constant

velocity in downward direction.

(v) If ρ < σ then terminal velocity will be negative and hence the spherical body will attain constant velocity in upward direction. Example : Air bubble in a liquid and clouds in sky.

Terminal velocity graph :



Q # 1. Explain what do you know about the term „Viscosity“?

Ans. The amount of force required to slide one layer of liquid over another layer is called as viscosity. It is denoted by the Greek word .

Substances that do not flow easily, such as honey, has large coefficient of viscosity and the substances which flow easily, like water, have small coefficient of viscosity.

Q # 2. What is meant by drag force? What are the factors upon which the drag force acting upon a small sphere of radius r, moving down through a liquid, depends?

Ans. An object moving through a fluid experience a retarding force called the drag force. The drag force depends upon the velocity of object in a fluid, i.e., the drag force increases as the speed of the particle increases. The other factors upon which the drag force drag force depends are the shape and size of material.

Q: Why fog droplets appear to be suspended in air?

Ans. When the magnitude of the drag force on the fog droplet becomes equal to its weight, the net force acting on the droplet is zero. In such a case, the droplet starts falling with a constant speed and appears to be suspended in air.

MULTIPLE CHOICE QUESTIONS

1. The SI unit of coefficient of viscosity is:
   a) $kg\ m^{-1}\ s^{-1}$
   b) $kg\ m^{-2}\ s^{-1}$
   c) $kg\ m^{-2} s^{-2}$
   d) $kg\ m^{2}\ s$

2. An object moving through a fluid experience a retarding force called
   a) Gravitational force
   b) Terminating force
   c) Frictional force
   d) Drag force

3. The drag force increases as the speed of the object
   a) Decreases
   b) Increases
   c) Remain constant
   d) None of these

4. The drag force F on a sphere of radius r moving slowly with speed v through a fluid of viscosity η is
   a) $6\pi\eta\ r^2\ v$
   b) $6\pi\eta\ r\ \ v$
   c) $6\pi^2\eta\ r\ \ v$
   d) $6\pi\eta\ r\ v^2$

5. Stokes law is applicable if body has ________ shape.
   a) Rough
   b) Square
   c) Circular
   d) Spherical

6. When weight of an object falling freely becomes equal to the drag force, then the body will move with
   a) Increasing speed
   b) Decreasing speed
   c) Constant speed
   d) None of these

7. The maximum constant velocity of an object falling vertically downward is called:
   a) Final velocity
   b) Terminal velocity
   c) Initial velocity
   d) None of these

8. If radius of droplet becomes half then its terminal velocity will be
   a) Half
   b) Double
   c) One forth
   d) Four times

9. The flow of ideal fluid is always:
   a) Turbulent
   b) Streamline
   c) Irregular
   d) Straight line

10. Turbulent flow is:
    a) Unsteady and regular
    b) Steady and regular
    c) Unsteady and irregular
    d) Steady and regular

Answers of MCQs of Chapter # 6

Q. No.	Ans	Q. No.	Ans
1	a	11	a
2	d	12	b
3	b	13	c
4	b	14	d
5	d	15	c
6	c	16	a
7	b	17	b
8	c	18	b
9	b	19	d
10	c	20	c

A small sphere of mass m is dropped from a great height. After it has fallen 100 m, it has attained its terminal velocity and continues to fall at that speed. The work done by air friction against the sphere during the first 100 m of fall is

(a) Greater than the work done by air friction in the second 100 m

(b) Less than the work done by air friction in the second 100 m

(c) Equal to 100 mg

(d) Greater than 100 mg

In the first 100 m body starts from rest and its velocity goes on increasing and after 100 m it acquire maximum velocity (terminal velocity). Further, air friction i.e. viscous force which is proportional to velocity is low in the beginning and maximum at $V = V_T$

Hence work done against air friction in the first 100 m is less than the work done in next 100 m.

Two drops of the same radius are falling through air with a steady velocity of 5 *cm* per *sec*. If the two drops coalesce, the terminal velocity would be **[MP PMT 1990]**

(a) 10 *cm* per *sec* (b) 2.5 *cm* per *sec* (c) $5\times(4)^{1/3}\,cm$ per *sec* (d) $5\times\sqrt{2}\,cm$ per sec

If two drops of same radius *r* coalesce then radius of new drop is given by *R*

$$\frac{4}{3}\pi R^3=\frac{4}{3}\pi r^3+\frac{4}{3}\pi r^3 \Rightarrow R^3=2r^3 \Rightarrow R=2^{1/3}r$$

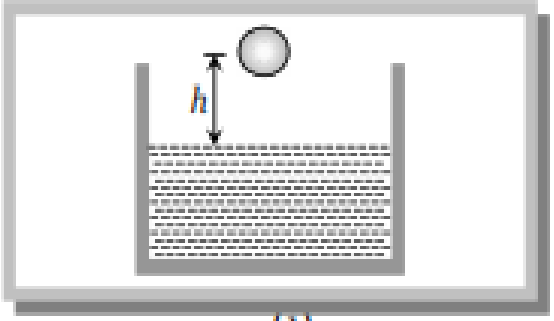
If drop of radius *r* is falling in viscous medium then it acquire a critical velocity v and $v\propto r^2$

$$\frac{v_2}{v_1}=\left(\frac{R}{r}\right)^2=\left(\frac{2^{1/3}r}{r}\right)^2 \Rightarrow v_2=2^{2/3}\times v_1=2^{2/3}\times(5)=5\times(4)^{1/3}\,m/s$$

A ball of radius *r* and density ρ falls freely under gravity through a distance *h* before entering water. Velocity of ball does not change even on entering water. If viscosity of water is η, the value of *h* is given by

(a) $\frac{2}{9}r^2\left(\frac{1-\rho}{\eta}\right)g$ (b) $\frac{2}{81}r^2\left(\frac{\rho-1}{\eta}\right)g$

(c) $\frac{2}{81}r^4\left(\frac{\rho-1}{\eta}\right)^2 g$ (d) $\frac{2}{9}r^4\left(\frac{\rho-1}{\eta}\right)^2 g$



Velocity of ball when it strikes the water surface $v=\sqrt{2gh}$ ....(i)

Terminal velocity of ball inside the water $v=\frac{2}{9}r^2g\frac{(\rho-1)}{\eta}$ ....(ii)

Equating (i) and (ii) we get $\sqrt{2gh}=\frac{2}{9}\frac{r^2g}{\eta}(\rho-1) \Rightarrow h=\frac{2}{81}r^4\left(\frac{\rho-1}{\eta}\right)^2 g$

These hot-air balloons float because they are filled with air at high temperature and are surrounded by denser air at a lower temperature.