Fluid Mechanics

P1

Objectives

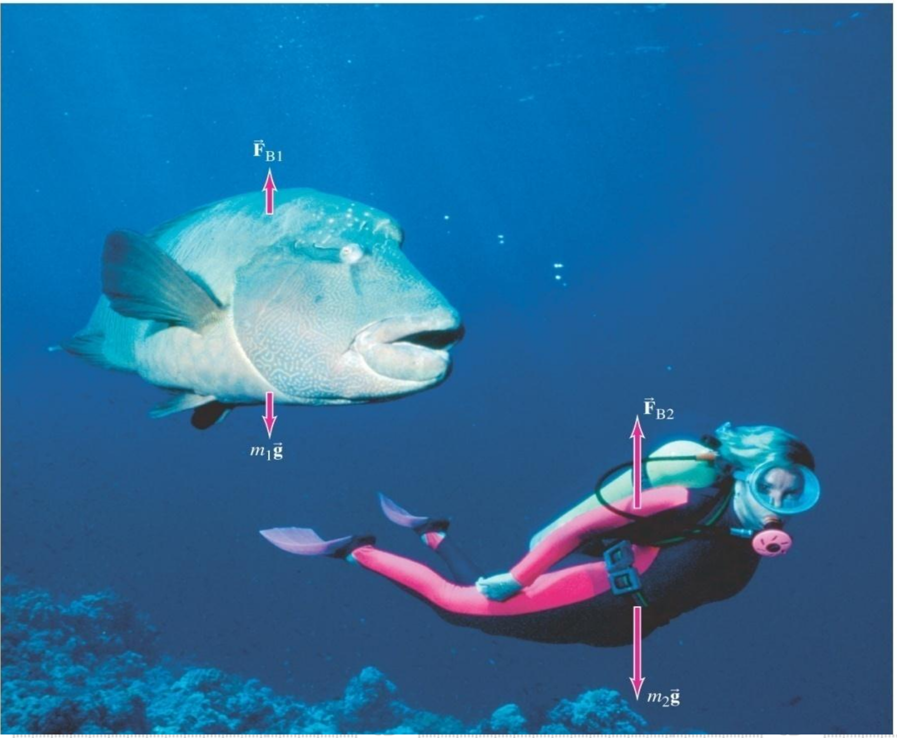
- Recall the definitions of density and relative density.
- Introduce homogeneous and incompressible fluids.
- Derive the expression $p = h\ g$ for hydrostatic pressure at a point in a homogeneous liquid at rest.
- Explain that the pressure in a fluid increases with depth. All points at the same depth in the fluid are at the same pressure.
- The force perpendicular to the surface is independent of the orientation of the surface.
- State that the pressure in a liquid acts equally in all directions.
- Explain the comparison of densities of two liquids using a U tube and hare's apparatus.
- State Pascal's principle.
- Explain how a force can be increased using hydraulic pressure, apparatus.
- Conduct a discussion to identify the uses of Pascal's principle.
- Recall upthrust exerts on a body immerse in a liquid.
- State Archimedes's principle

- State the principle of flotation.
- Discuss the conditions for flotation.
- Introduce centre of Buoyancy.
- Describe the structure of hydrometer.
- Explain the use of hydrometer.
- Guide students to compare densities of various liquids using hydrometer.

- Comparison of densities using U tube.
- Comparison of densities using hare’s apparatus.
- Determination of density of a liquid using weighted test tube.

Defining a Fluid

- A **fluid** is a nonsolid state of matter in which the atoms or molecules are free to move past each other.
- Both liquids and gases are considered fluids because they can flow and change shape.
- Liquids have a definite volume; gases do not.



Density

Density – mass per unit volume

$$\rho = \frac{m}{V}$$

Mass in kilograms

Volume in m^3

Density of pure water is $1000 kg/m^3$

Water at 4°C has a density of $1 g/cm^3 = 1000 kg/m^3$.

Salt water is $1025 kg/m^3$

Densities of Common Substances

Densities of Some Common Substances*

Substance	ρ (kg/m^3)
Hydrogen	0.0899
Helium	0.179
Steam (100°C)	0.598
Air	1.29
Oxygen	1.43
Carbon dioxide	1.98
Ethanol	0.806×10^3
Ice	0.917×10^3
Fresh water (4°C)	1.00×10^3
Sea water (15°C)	1.025×10^3
Iron	7.86×10^3
Mercury	13.6×10^3
Gold	19.3×10^3

*All densities are measured at 0°C and 1 atm unless otherwise noted.

Fluid Pressure

- Deep sea divers wear atmospheric diving suits to resist the forces exerted by the water in the depths of the ocean.
- You experience this pressure when you dive to the bottom of a pool, drive up a mountain, or fly in a plane.

$$P = \frac{F}{A}$$

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

- The SI unit for pressure is the pascal, Pa.
- It is equal to 1 N/m^2.
- The pressure at sea level is about 1.01 x 10^5 Pa.

This pressure does not crush us, as our cells maintain an internal pressure that balances it.

- 1 atm = 1.013 x 10^5 Pa = $1.013 \times 10^5 \, N/m^2$

$$1 \text{ bar} = 1.00 \times 10^5 \, N/m^2$$

$$1 \text{ bar} = 10^5 \text{ Pa} = 0.1 \text{ MPa} = 100 \text{ kPa}$$

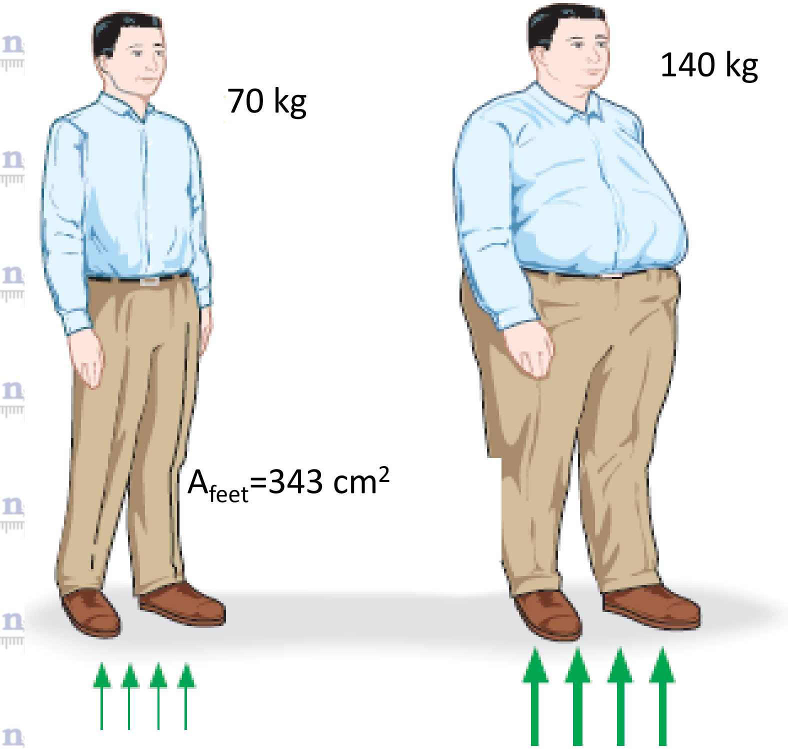
$$1 \text{ atm} = 101{,}325 \text{ Pa} = 101.325 \text{ kPa} = 1.01325 \text{ bars}$$

$$\begin{aligned} 1 \text{ kgf/cm}^2 &= 9.807 \text{ N/cm}^2 = 9.807 \times 10^4 \text{ N/m}^2 = 9.807 \times 10^4 \text{ Pa} \\ &= 0.9807 \text{ bar} \\ &= 0.9679 \text{ atm} \end{aligned}$$

Pressure: A normal force exerted by a fluid per unit area

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

Some basic pressure gages.



$P = (70 \times 9.81/1000)\ \text{kN} / 0.0343\ \text{m}^2 = 20\ \text{kPa}$

The normal stress (or "pressure") on the feet of a chubby person is much greater than on the feet of a slim person.

Pascal’s Principle

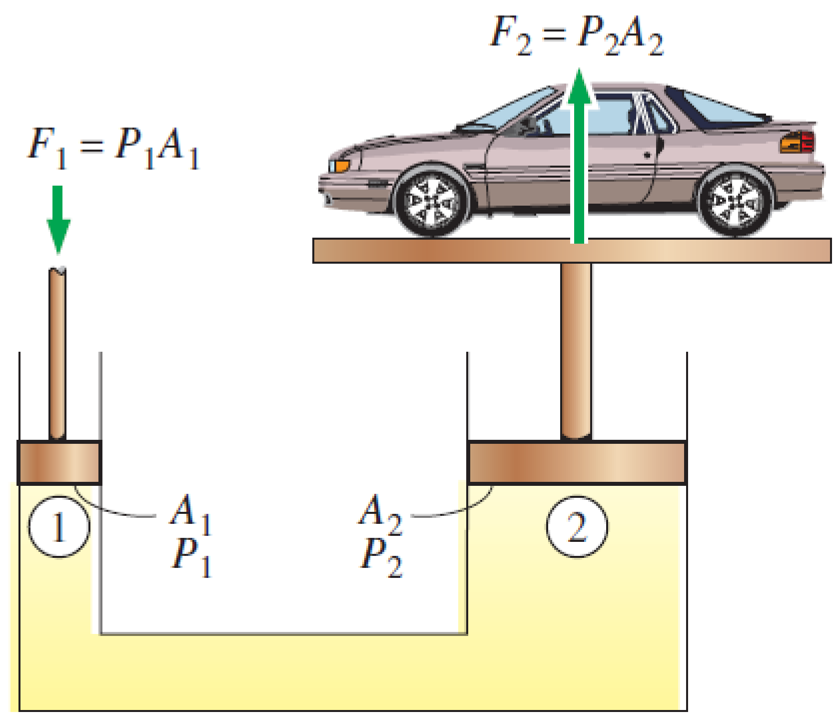
- Pascal’s principle states that pressure applied to a fluid in a closed container is transmitted equally to every point of the fluid and to the walls of the container.

- When you pump a bike tire, you apply force on the pump that in turn exerts a force on the air inside the tire.
- The air responds by pushing not only on the pump but also against the walls of the tire.
- As a result, the pressure increases by an equal amount throughout the tire.

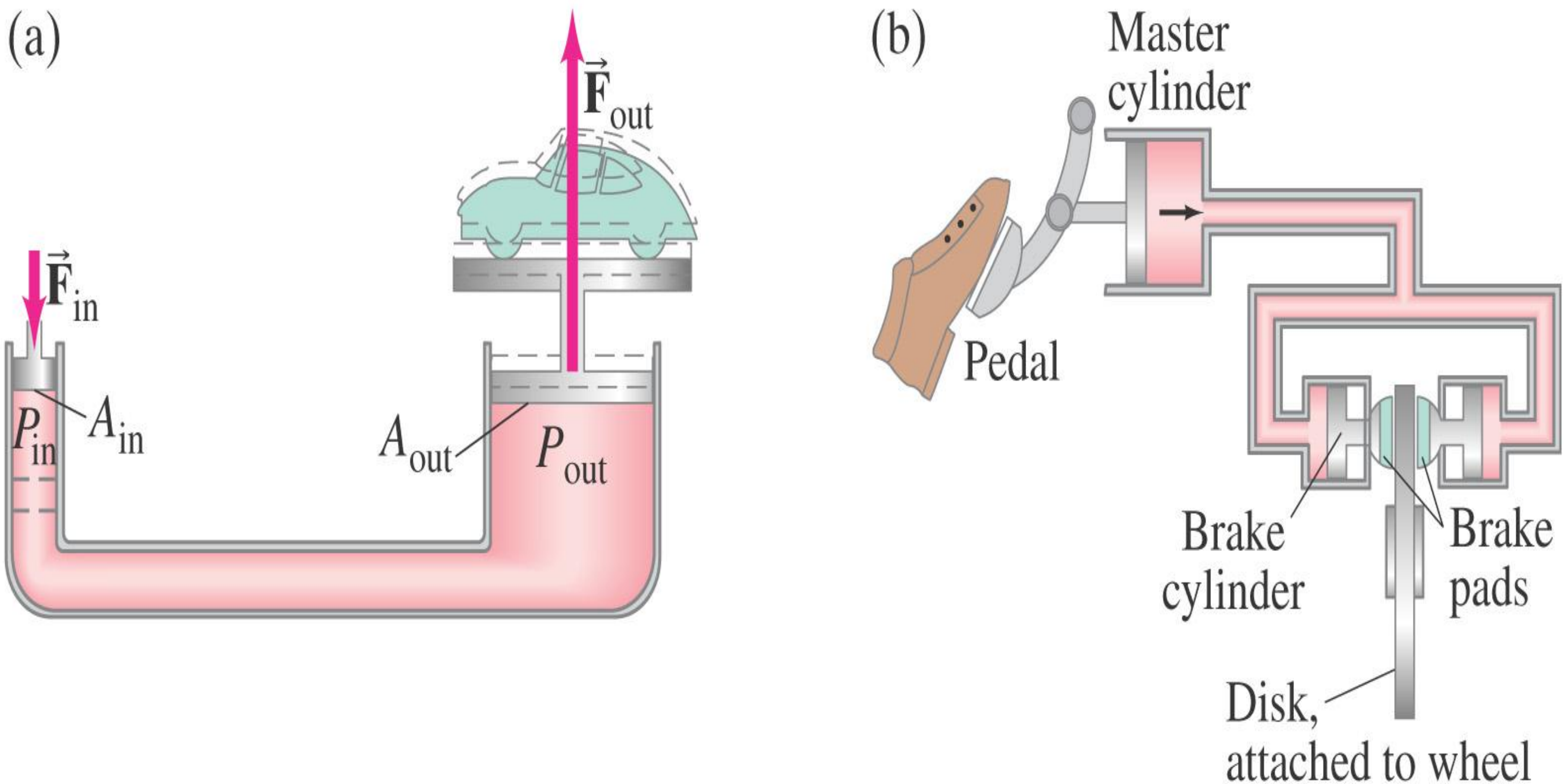
Pascal's law: The pressure applied to a confined fluid increases the P throughout by the same amount. i.e. hydraulic lifts and hydraulic brakes.

$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

The area ratio A_2/A_1 is called the *idea* *mechanical advantage* of the hydraul



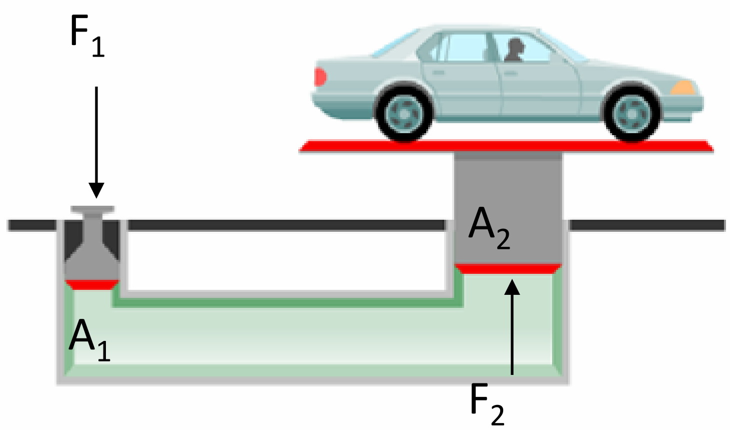
Lifting of a large weight by a small force by the application of Pascal's law.



(a)
$\vec{F}_{out}$
$\vec{F}_{in}$
A_{in}
P_{in}
A_{out}
P_{out}
(b)
Master cylinder
Pedal
Brake cylinder
Brake pads
Disk, attached to wheel

Pascal’s Principle

- A hydraulic lift uses Pascal's principle.
- A small force is applied (F_1) to a small piston of area (A_1) and cause a pressure increase on the fluid.
- This increase in pressure (P_{inc}) is transmitted to the larger piston of area (A_2) and the fluid exerts a force (F_2) on this piston.



$$P_{inc} = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = F_1 \frac{A_2}{A_1}$$

Example

- The small piston of a hydraulic lift has an area of $0.20\ m^2$. A car weighing 1.20×10^4 N sits on a rack mounted on the large piston. The large piston has an area of $0.90\ m^2$. How much force must be applied to the small piston to support the car?

Solution

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \longrightarrow F_1 = F_2 \frac{A_1}{A_2}$$

- $F_1 = (1.20 \times 10^4 \text{ N}) (0.20 \text{ m}^2 / 0.90 \text{ m}^2)$
- $F_1 = 2.7 \times 10^3 \text{ N}$

Example

- In a car lift, compressed air exerts a force on a piston with a radius of 5.00 cm. This pressure is transmitted to a second piston with a radius of 15.0 cm.
  - How large of a force must the air exert to lift a 1.33×10^4 N car?
- A person rides up a lift to a mountain top, but the person's ears fail to "pop". The radius of each ear drum is 0.40 cm. The pressure of the atmosphere drops from 10.10×10^5 Pa at the bottom to 0.998×10^5 Pa at the top.
  - What is the pressure difference between the inner and outer ear at the top of the mountain?
  - What is the magnitude of the net force on each eardrum?

Pressure

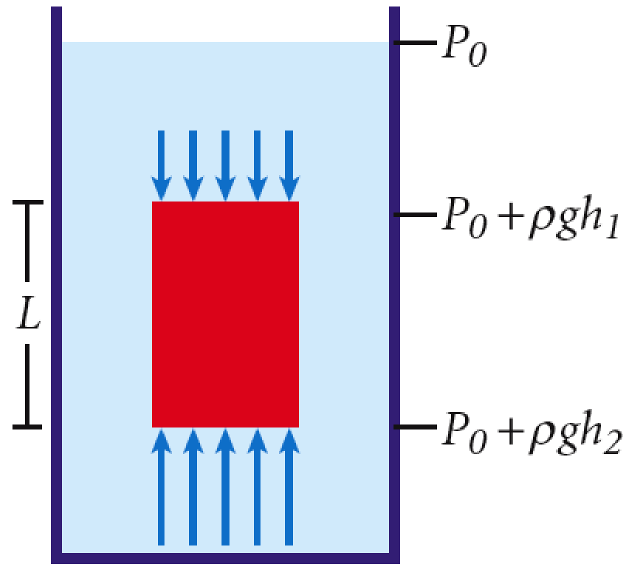
- Pressure varies with depth in a fluid.
- The pressure in a fluid increases with depth.

$$P = P_0 + \rho g h$$

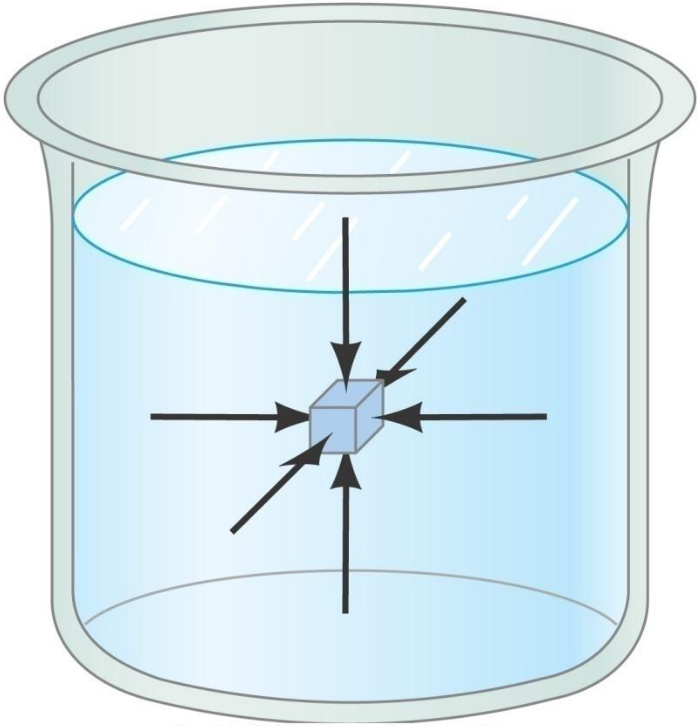
absolute pressure =

atmospheric pressure +

$(\text{density} \times \text{free-fall acceleration} \times \text{depth})$



Pressure in Fluids



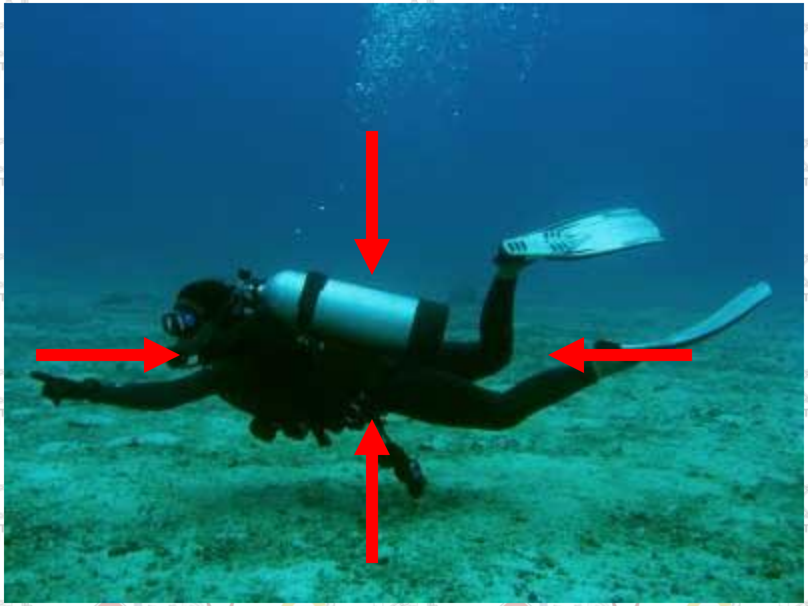
Pressure is the same in every direction in a fluid at a given depth; if it were not, the fluid would flow.

Pressure in Fluids

Pressure in a fluid

Pressure is equal on all sides

If not – object would accelerate



10.4 Atmospheric Pressure and Gauge Pressure

How does a person suck drink up a straw?

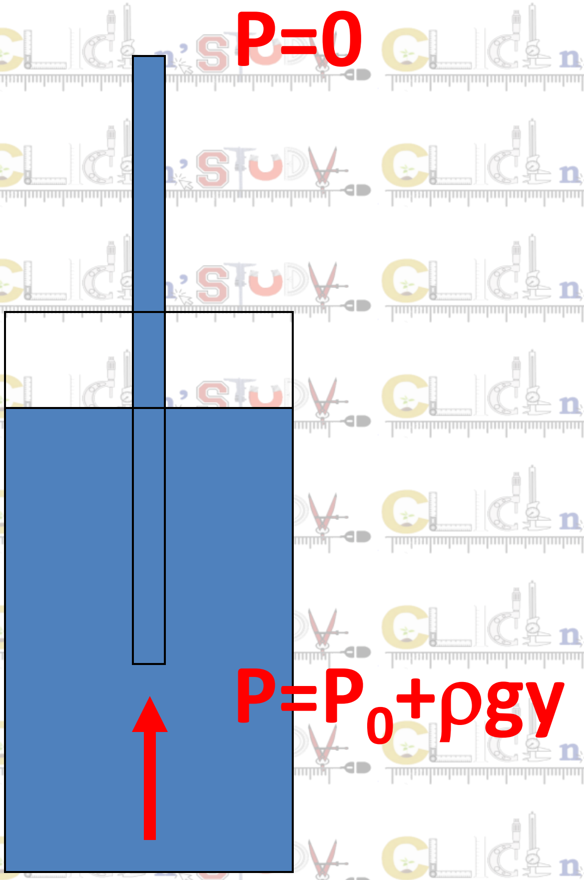
Pressure at the top is reduced

What is the pressure at the the bottom?

Since $$P = \frac{F}{A}$$

Then $$F = PA$$

Area at the top and bottom are the same so net force up



Atmospheric Pressure and Gauge Pressure

Most pressure gauges measure the pressure above the atmospheric pressure – this is called the gauge pressure.

The absolute pressure is the sum of the atmospheric pressure and the gauge pressure.

$$P = P_A + P_G$$

10.4 Atmospheric Pressure and Gauge Pressure

Nothing in physics ever sucks

Pressure is reduced at one end

Gauge Pressure – beyond atmospheric pressure

Absolute pressure

$$P = P_A + P_G$$

Absolute pressure: The actual pressure at a given position. It is measured relative to absolute vacuum (i.e., absolute zero pressure).

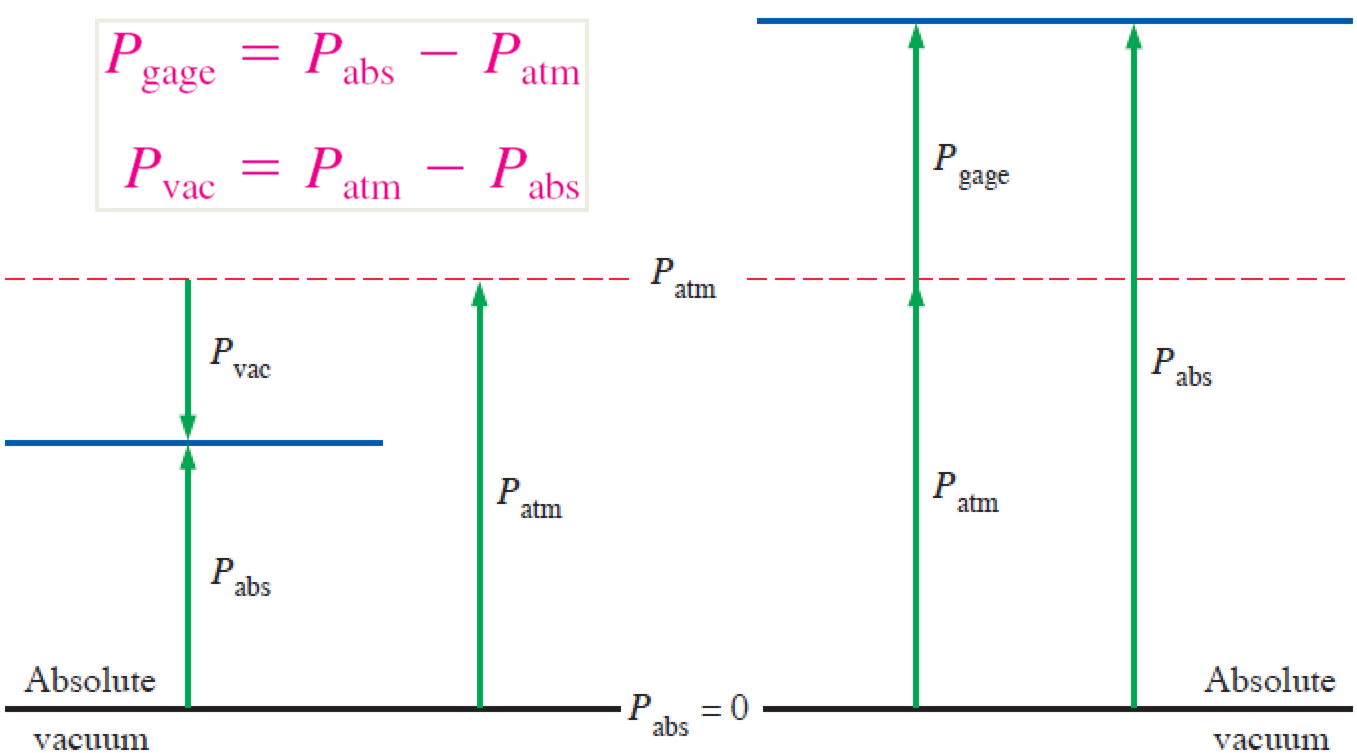
Gage pressure: The difference between the absolute pressure and the local atmospheric pressure. Most pressure-measuring devices are calibrated to read zero in the atmosphere, and so they indicate gage pressure.

Vacuum pressures: Pressures below atmospheric pressure.

Throughout this text, the pressure P will denote *absolute pressure* unless specified otherwise.

$$P_{\text{gage}} = P_{\text{abs}} - P_{\text{atm}}$$

$$P_{\text{vac}} = P_{\text{atm}} - P_{\text{abs}}$$



EXAMPLE 3–1 Absolute Pressure of a Vacuum Chamber

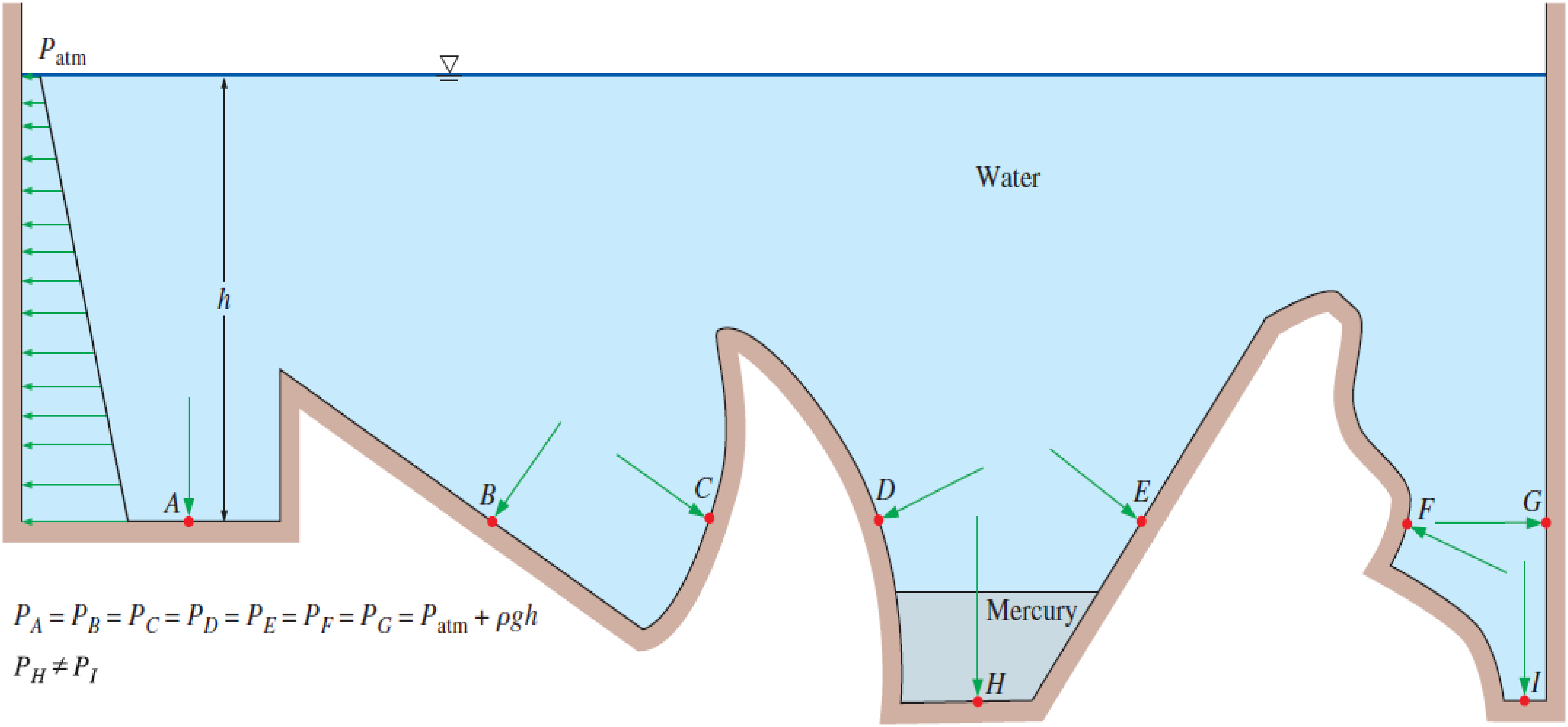
A vacuum gage connected to a chamber reads 40 kPa at a location where the atmospheric pressure is 100 kPa. Determine the absolute pressure in the chamber.

SOLUTION The gage pressure of a vacuum chamber is given. The absolute pressure in the chamber is to be determined.

Analysis The absolute pressure is easily determined from Eq. 3–2 to be

$$P_{\text{abs}} = P_{\text{atm}} - P_{\text{vac}} = 100 - 40 = 60 \text{ kPa}$$

Discussion Note that the *local* value of the atmospheric pressure is used when determining the absolute pressure.

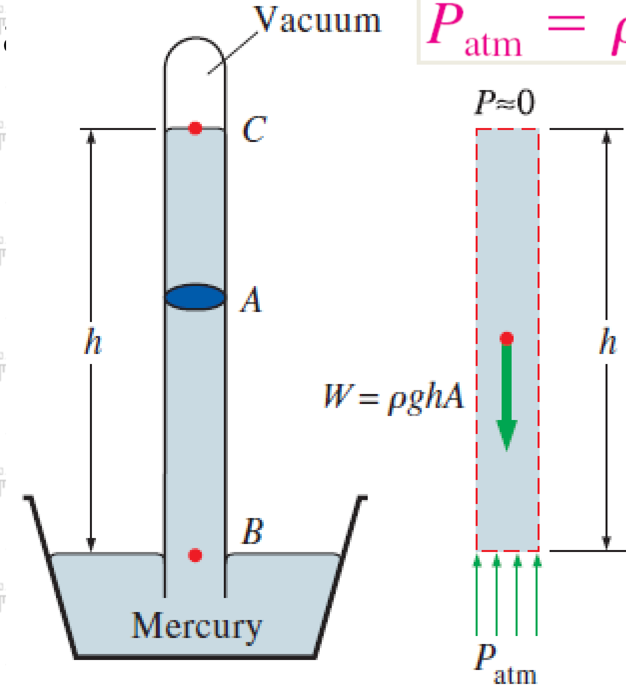


The pressure is the same at all points on a horizontal plane in a given fluid regardless of geometry, provided that the points are interconnected by the same fluid.

PRESSURE MEASURING DEVICES- The Barometer

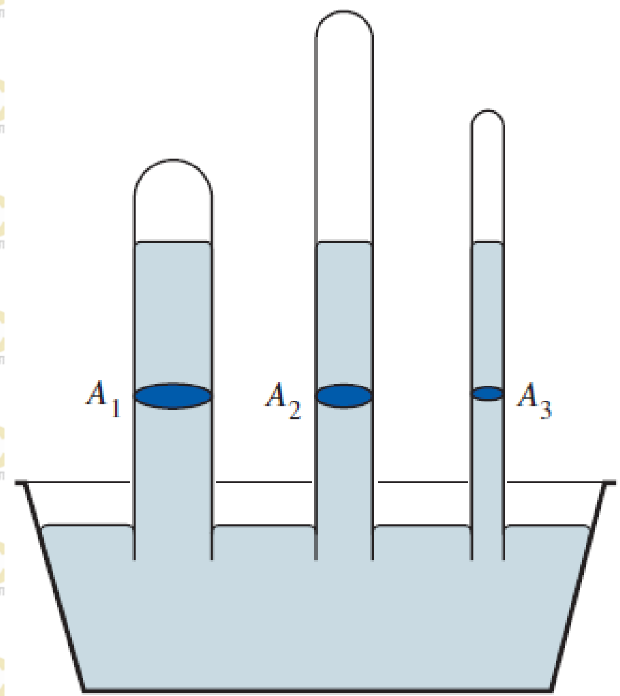
- Atmospheric pressure is measured by a device called a barometer; thus, the atmospheric pressure is often referred to as the *barometric pressure*.
- A frequently used pressure unit is the *standard atmosphere*, which is defined as the pressure produced by a column of mercury 760 mm in height at 0°C (ρ_{Hg} = 13,595 kg/m^3) under standard gravitational m/s^2).

$$P_{atm} = \rho g h$$

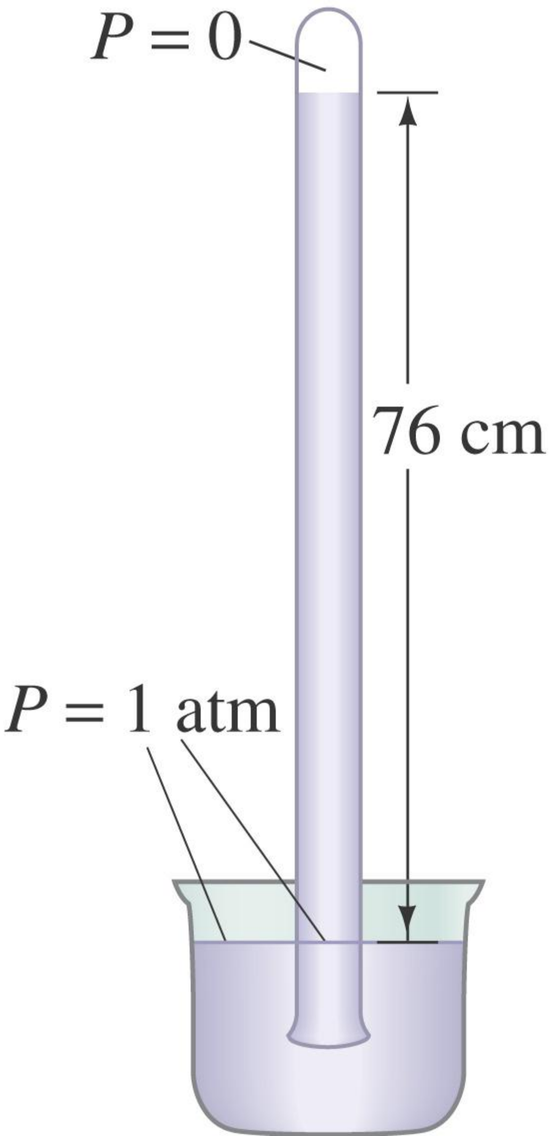


The basic barometer.

The length or the cross-sectional area of the tube has no effect on the height of the fluid column of a barometer, provided that the tube diameter is large enough to avoid surface tension (capillary) effects.



Measurement of Pressure; Gauges and the Barometer



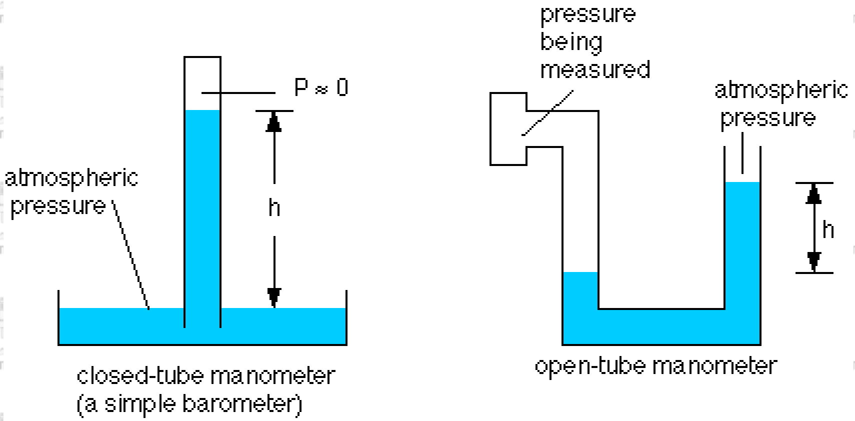
This is a mercury barometer, developed by Torricelli to measure atmospheric pressure. The height of the column of mercury is such that the pressure in the tube at the surface level is 1 atm.

Therefore, pressure is often quoted in millimeters (or inches) of mercury.

Any liquid can serve in a Torricelli-style barometer, but the most dense ones are the most convenient. This barometer uses water.

10.6 Barometer

How does a manometer work?



Closed tube

$$P = \rho g \Delta h$$

Open tube

$$P = P_0 + \rho g \Delta h$$

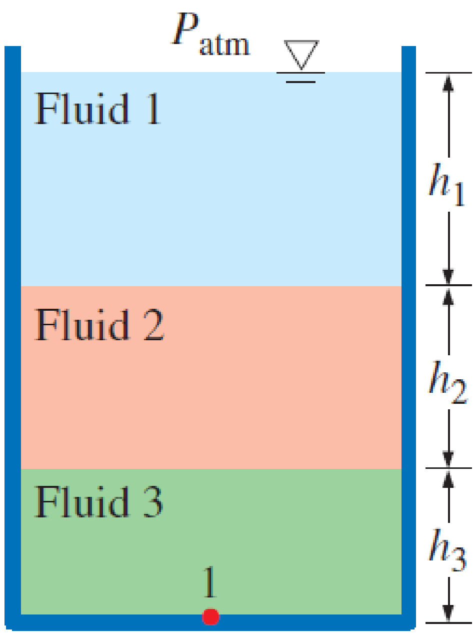
- A manometer is a tube-like device which measures atmospheric measure. There are two types: closed tube and open tube, but both measure pressure by comparing the pressure exerted by the atmosphere at one end of the tube with a known pressure at the other. Manometer tubes are typically filled with mercury.
- **Barometer** Barometers also measure atmospheric pressure. Mercury barometers are a type of closed-tube manometer, while aneroid barometers use a small, spring balance to take the measurement. In the past, mercury barometers were

common in family homes where people used them to predict the weather based on the air pressure reading. Rising air pressure meant good weather was on the way, while falling pressure might bring rain.

- A barometer measures the pressure of the air around you. A manometer is anything that measures pressure. Therefore, all barometers are manometers. If you are measuring the difference in air pressure between two parts of a system, that would be referred to as a manometer instead of as a barometer.

The Manometer

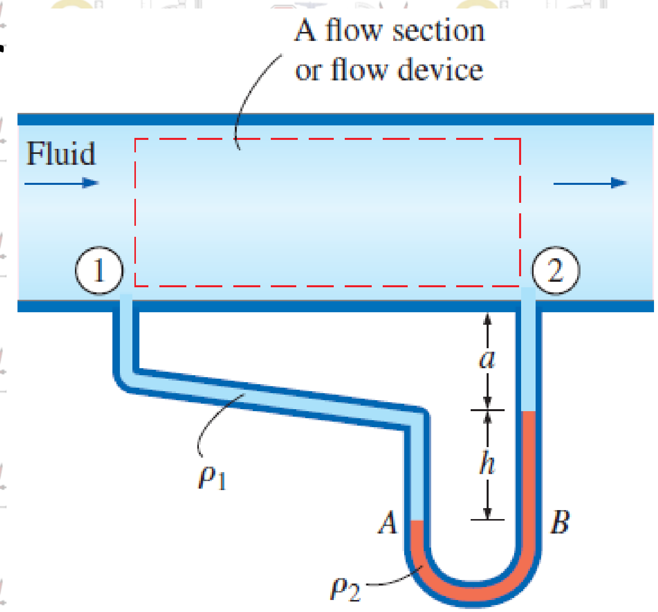
It is commonly used to measure small and moderate pressure differences. A manometer contains one or more fluids such as mercury, water, alcohol, or oil.



$$P_{atm} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 = P_1$$

In stacked-up fluid layers, the pressure change across a fluid layer of density ρ and height h is $\rho g h$.

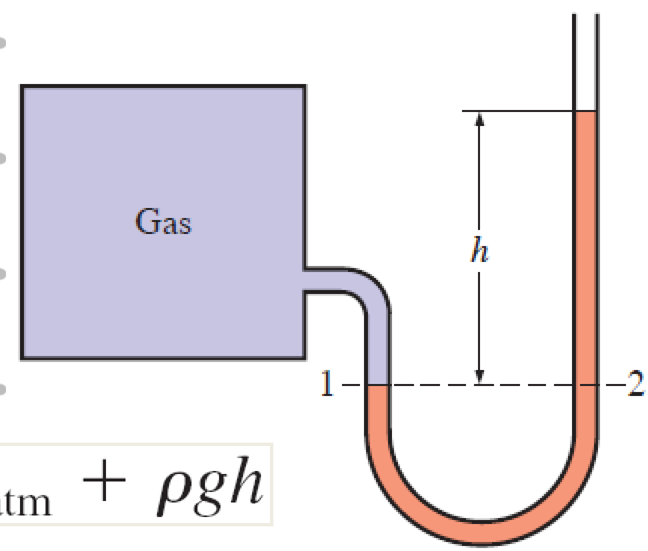
Measuring the pressure drop across a flow section or a flow device by a differential manometer.



$$P_1 + \rho_1 g(a + h) - \rho_2 g h - \rho_1 g a = P_2$$

$$P_1 - P_2 = (\rho_2 - \rho_1) g h$$

The basic manometer.



$$P_2 = P_{atm} + \rho g h$$

A simple U-tube manometer, with high pressure applied to the right side.

Comparison of densities using U tube.

- Balancing two liquid columns
- If two liquids of different density that do not mix are poured into a beaker the two limbs of a U tube the one with the greater density will "fall" to the bottom with the one of lower density floating on top. (Vinegar and oil are a good example. Vinegar is more dense than oil and so will float on top of it).

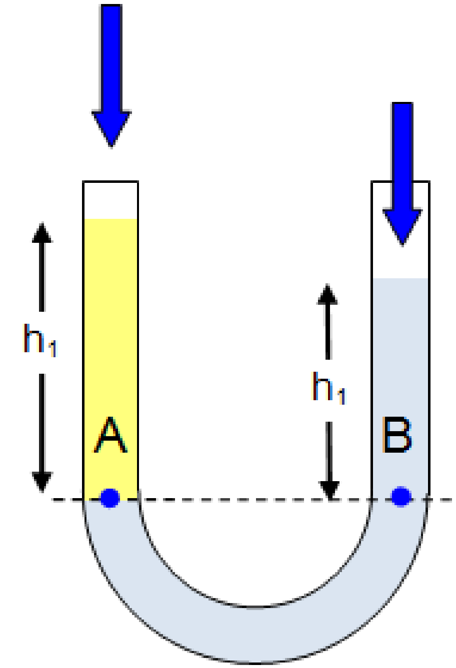
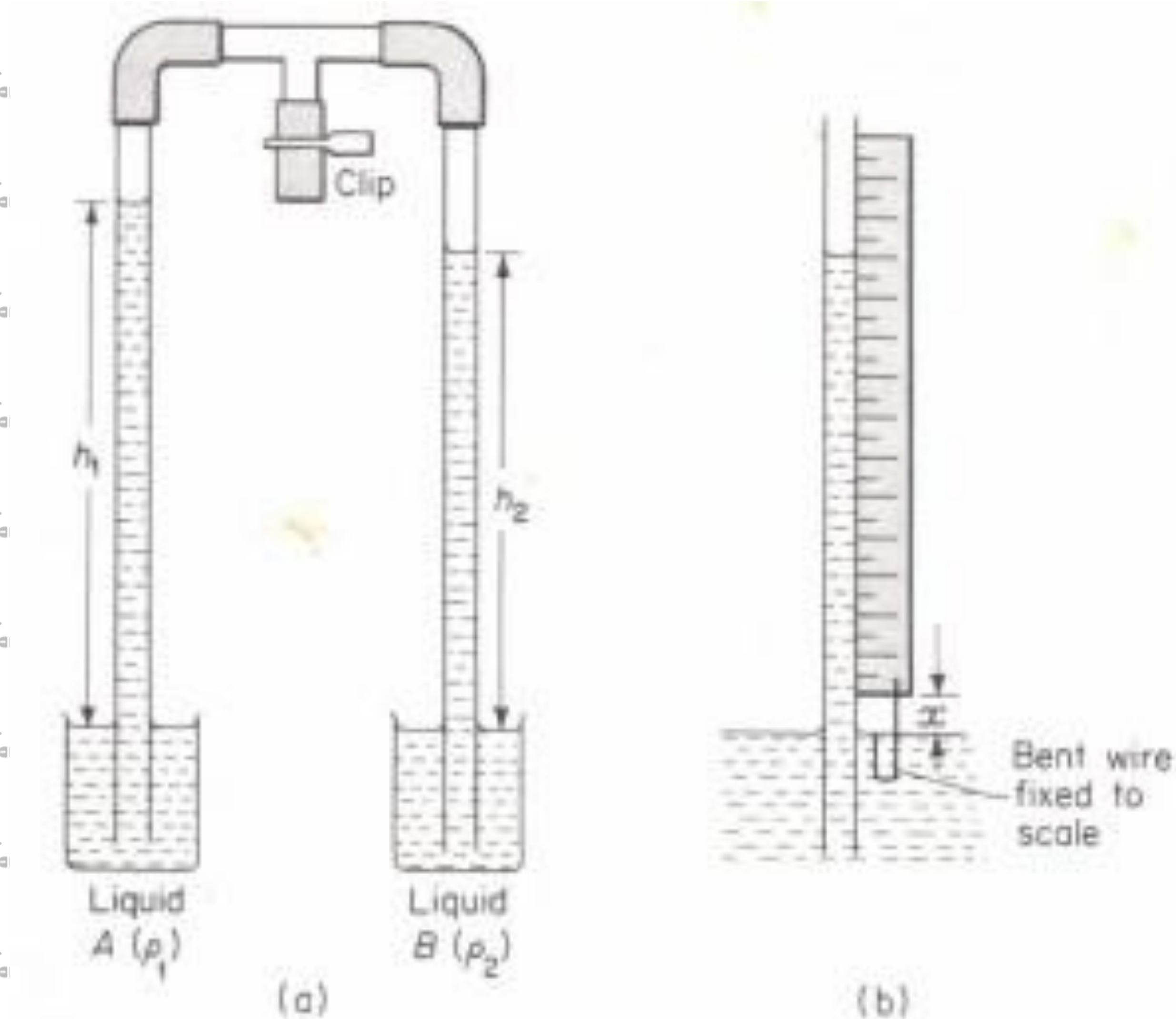


Figure 2

If these two liquids are poured into the two limbs of a U tube they will take up the positions shown in the diagram with the denser liquid at the bottom. Since the pressure at a given depth in a liquid is the same at all points the heights of the two liquid columns above X must exert equal pressures.

Therefore: $h_1\rho_1 g = h_2\rho_2 g$

To compare the densities of two liquids by means of Hare's apparatus



- The apparatus consists of two vertical wide-bore glass tubes connected at the top by a glass T-piece. These tubes dip into beakers containing the two liquids of densities PI and P2
- Some air is sucked out of the tubes through the centre limb of the T-piece and the clip closed. Removal of air causes a reduction of pressure inside, with the result that atmospheric pressure pushes the liquids up the tubes. The liquids rise until the pressures exerted at the base of each column are each equal to atmospheric pressure

- A certain amount of difficulty may arise when measuring the height of the columns, owing to the meniscus which forms when a boxwood scale touches surface of the liquid. This may be overcome by the use of a bent wire attached to lower end of the scale, as shown in Fig. 11.8 (b). The scale is adjusted until the wire is just level with the liquid surface. The scale reading of the liquid level is then taken, and added to the distance x between the tip of the wire & the zero of the scale. Several pairs of values of hl and h2 are taken, entered in a suitable table an mean value of the ratio of the densities calculated.
- Alternatively, we may plot a graph of h2 against h, and obtain the ratio fro
gradient.

- The pressure at the base of liquid A is then

$$P_A = P + \rho_A g h_1$$

- The pressure at the base of liquid B is then

$$P_A = P + \rho_B g h_2$$

- Equating these gives $P + \rho_A g h_1 = P + \rho_B g h_2$

- Hence $\rho_A g h_1 = \rho_B g h_2 \rightarrow \rho_A h_1 = \rho_B h_2 \rightarrow \dfrac{\rho_1}{\rho_2} = \dfrac{h_2}{h_1}$.

The pressure at the base of a column is made up of two parts:

(i) the pressure, P, of the air in the tube above the liquid,

and

(ii) the pressure, $h\rho g$, of the liquid column itself (see page 108).

$$\text{hence} \qquad P + h_1\rho_1 g = P + h_2\rho_2 g$$

$$\text{thus} \qquad h_1\rho_1 = h_2\rho_2$$

$$\text{or} \qquad \frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}$$

If liquid B is water, then $\frac{\rho_1}{\rho_2}\left(\text{or } \frac{h_2}{h_1}\right)$ will be equal to the relative density of liquid A.

EXAMPLE 3–5 Measuring Pressure with a Manometer

A manometer is used to measure the pressure of a gas in a tank. The fluid used has a specific gravity of 0.85, and the manometer column height is 55 cm, as shown in Fig. 3–20. If the local atmospheric pressure is 96 kPa, determine the absolute pressure within the tank.

SOLUTION The reading of a manometer attached to a tank and the atmospheric pressure are given. The absolute pressure in the tank is to be determined.

Assumptions The density of the gas in the tank is much lower than the density of the manometer fluid.

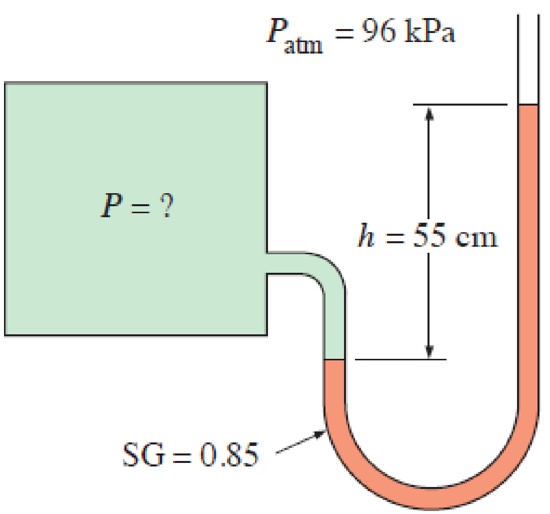
Properties The specific gravity of the manometer fluid is given to be 0.85. We take the standard density of water to be 1000 kg/m³.

Analysis The density of the fluid is obtained by multiplying its specific gravity by the density of water,

$$\rho = \text{SG}\,(\rho_{H_2O}) = (0.85)(1000\ \text{kg/m}^3) = 850\ \text{kg/m}^3$$

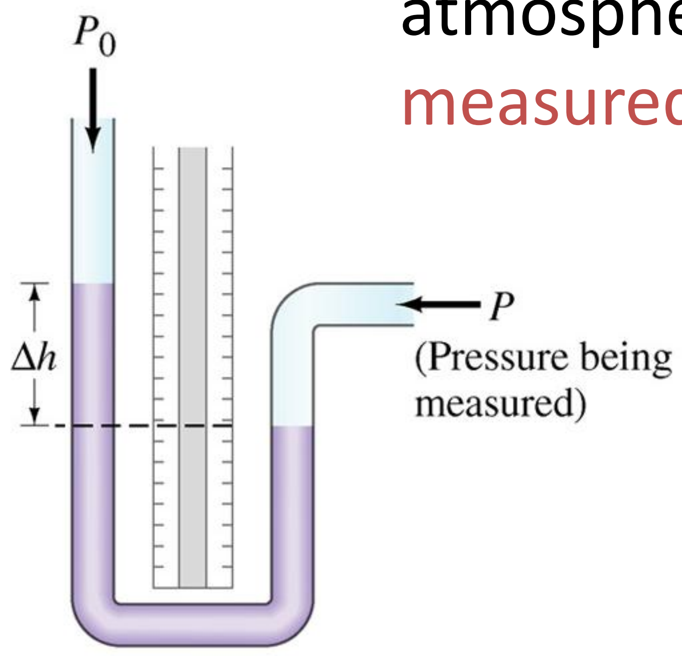
Then from Eq. 3–13,

$$\begin{aligned} P &= P_{\text{atm}} + \rho g h \\ &= 96\ \text{kPa} + (850\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.55\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= \mathbf{100.6\ kPa} \end{aligned}$$



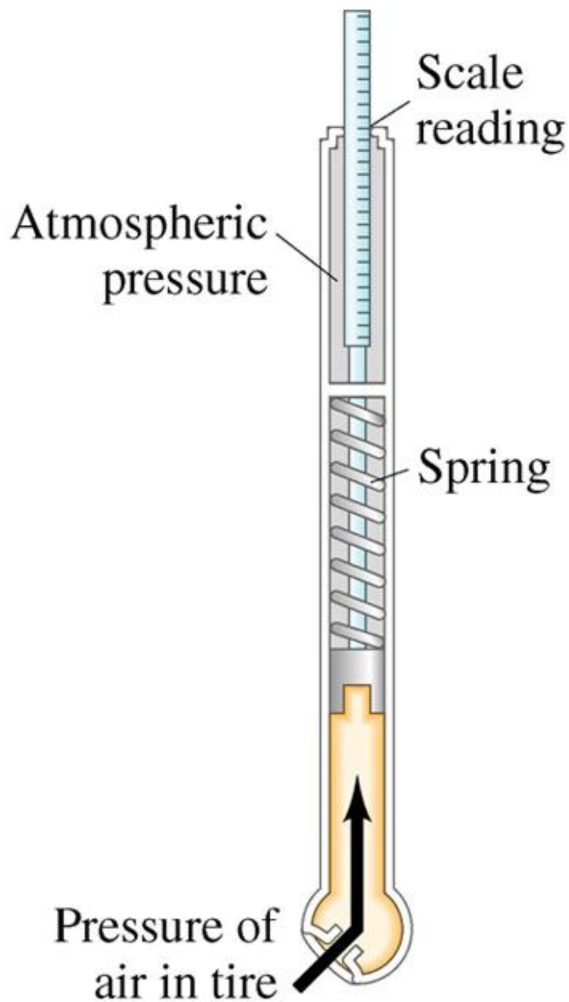
Discussion Note that the gage pressure in the tank is 4.6 kPa.

There are a number of different types of pressure gauges. This one is an open-tube manometer. The pressure in the open end is atmospheric pressure; the pressure being measured will cause the fluid to rise until the pressures on both sides at the same height are equal.

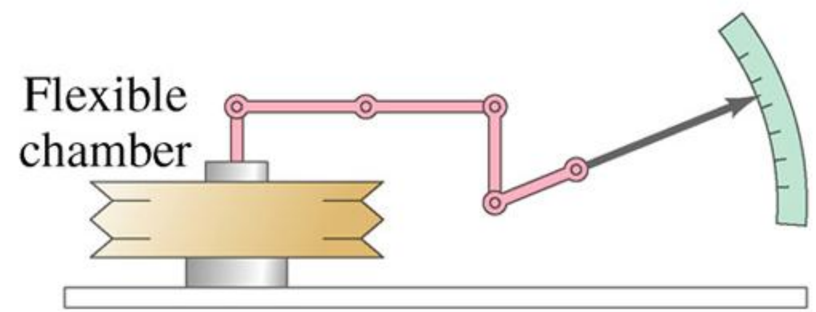


(a) Open-tube manometer

Here are two more devices for measuring pressure: the aneroid gauge and the tire pressure gauge.



(c) Tire gauge



(b) Aneroid gauge (used mainly for air pressure, and then called an aneroid barometer)

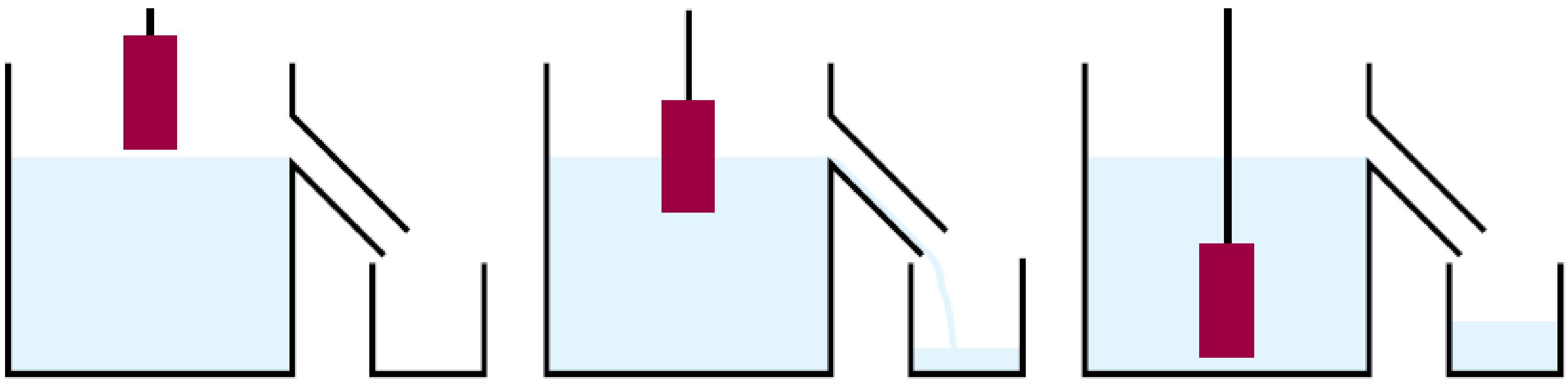
Buoyant force/ Upthurst

- The **Buoyant force/ Upthurst** is the upward force exerted by a liquid on an object immersed in or floating on the liquid.

- Buoyant forces can keep objects afloat.

Buoyant Force and Archimedes' Principle

- The Brick, when added will cause the water to be displaced and fill the smaller container.
- What will the volume be inside the smaller container?
- The same volume as the brick!



Buoyant Force and Archimedes' Principle

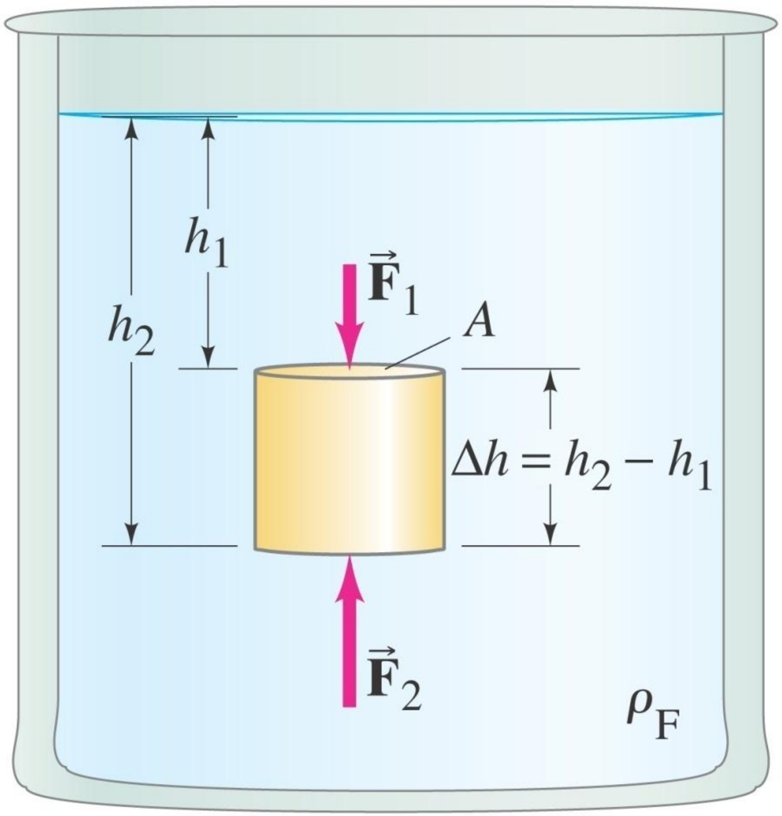
- Archimedes' principle: *Any object completely or partially submerged in a fluid experiences an upward buoyant force equal in magnitude to the weight of the fluid displaced by the object.*

$$U\ (F_B) = m_f g = = V\rho_F g$$

magnitude of buoyant force = weight of fluid displaced

Buoyancy

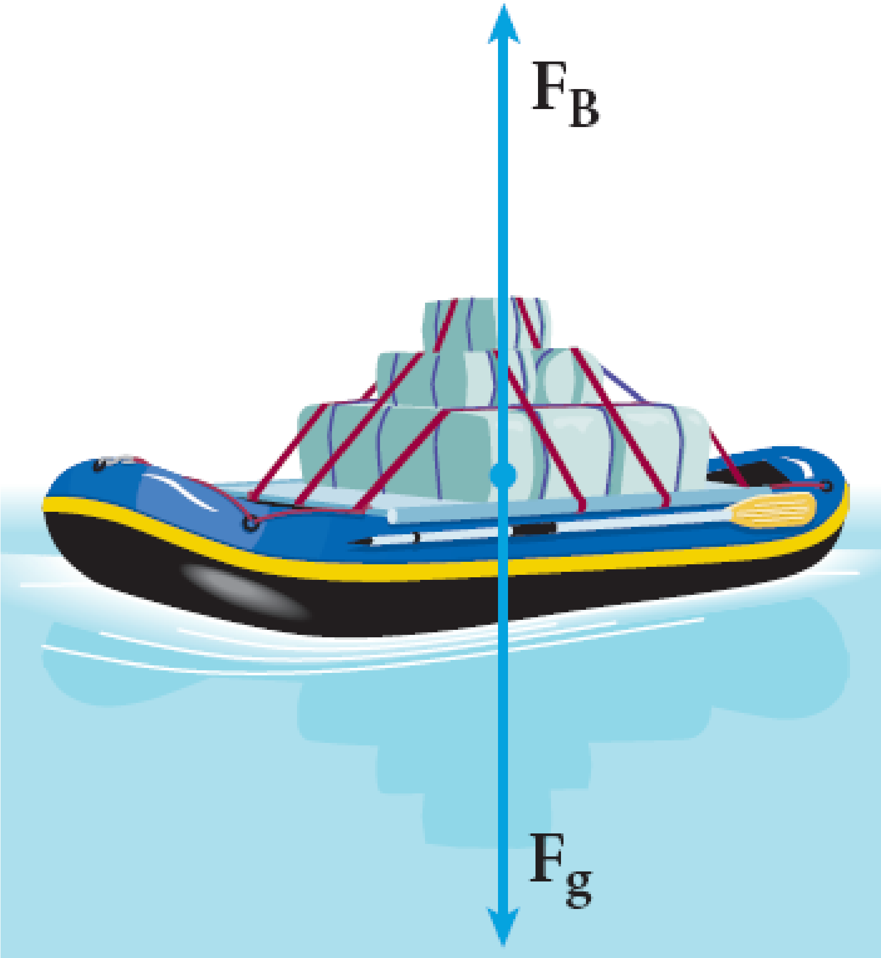
This is an object submerged in a fluid. There is a net force on the object because the pressures at the top and bottom of it are different.



The buoyant force is found to be the upward force on the same volume of water:

$$
\begin{aligned}
F_B = F_2 - F_1 &= \rho_F g A (h_2 - h_1) \\
&= \rho_F g A \, \Delta h \\
&= \rho_F V g \\
&= m_F g,
\end{aligned}
$$

Buoyant Force



- The raft and cargo are floating because their weight and buoyant force are balanced.
- $F_B = F_g$
- $F_g = mg$
- $F_B = m_F g = \rho_F g V$

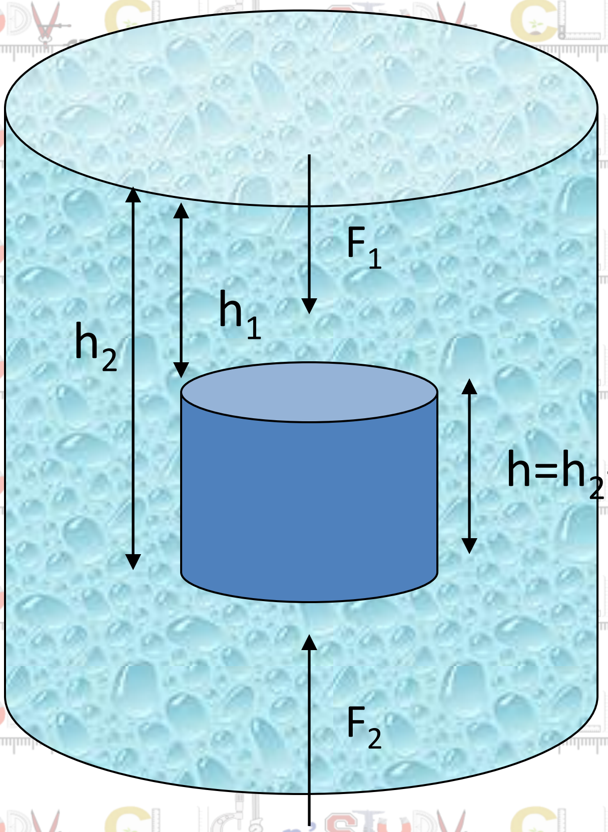
For an object with density ρ_O submerged in a fluid of density ρ_f, the buoyant force F_B obeys the following ratio:

$$\frac{F_g(object)}{F_B} = \frac{\rho_O}{\rho_f}$$

Buoyancy and Archimedes' Principle

Buoyant force (F_B)

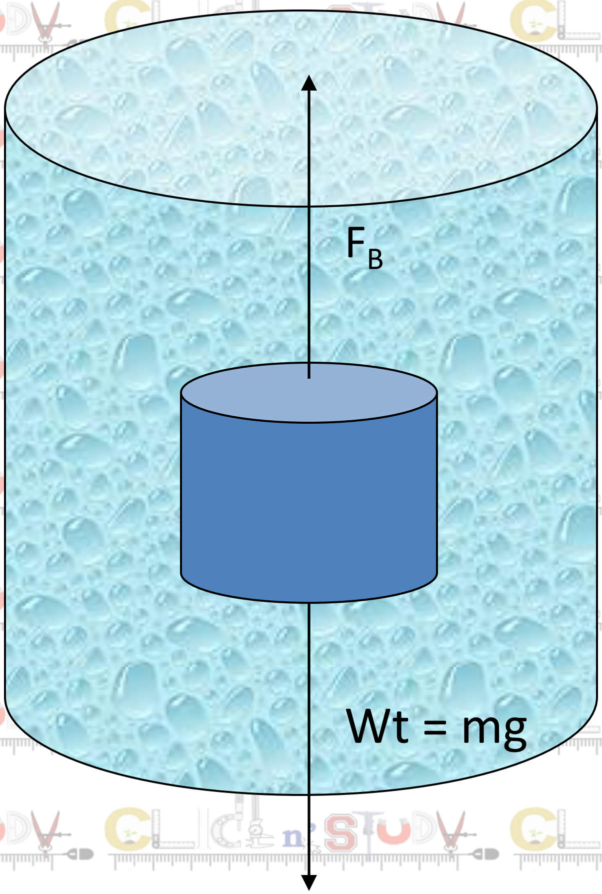
- The net force due to the force of the fluid down (F_1) and up (F_2)
- $F_B = F_2 - F_1$
- Since $F = PA = \rho_F ghA$
- $F_B = \rho_F gA(h_2 - h_1)$
- $F_B = \rho_F gAh = \rho_F gV$



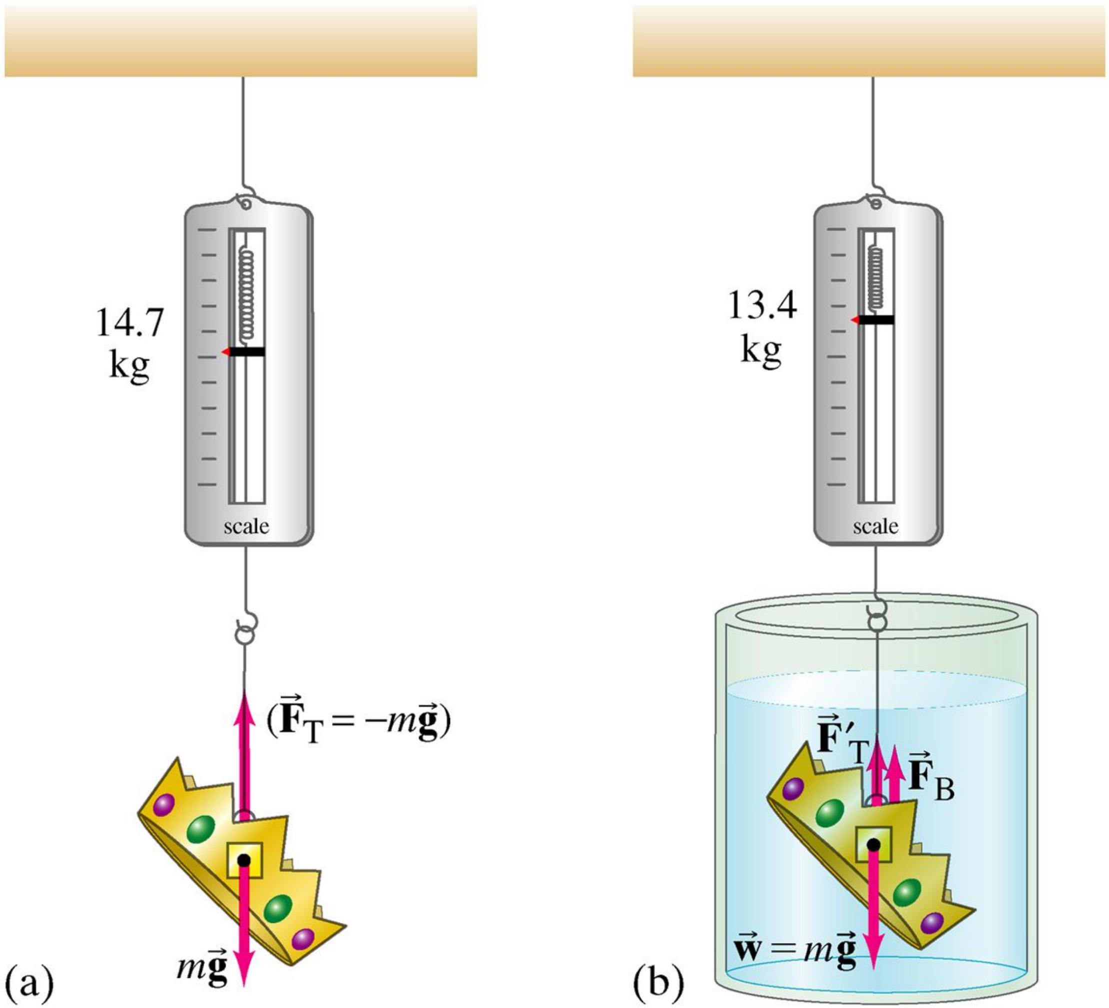
Apparent weight

Archimedes' Principle

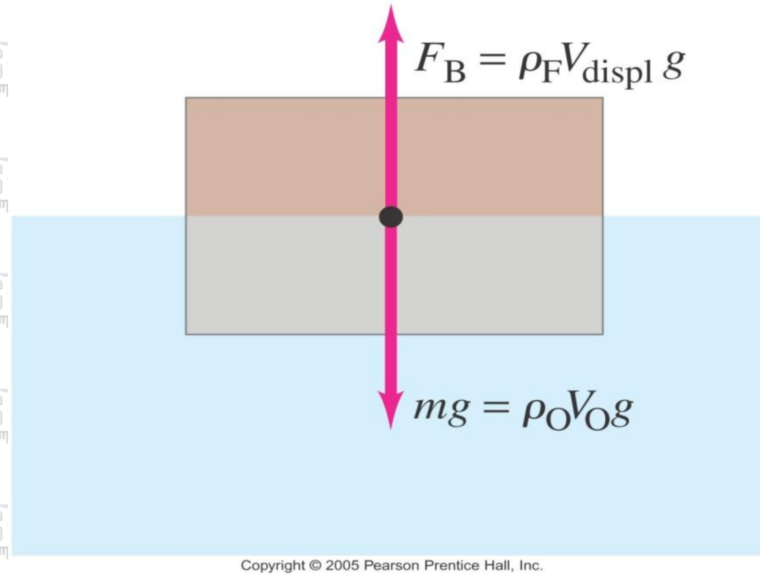
- So when an object is weighed in water its apparent weight (w') is equal to its actual weight (w) minus its buoyant force (F_B)
- $w' = w - F_B$
- $w/(w—w') = \rho_o / \rho_F$



The net force on the object is then the difference between the buoyant force and the gravitational force.

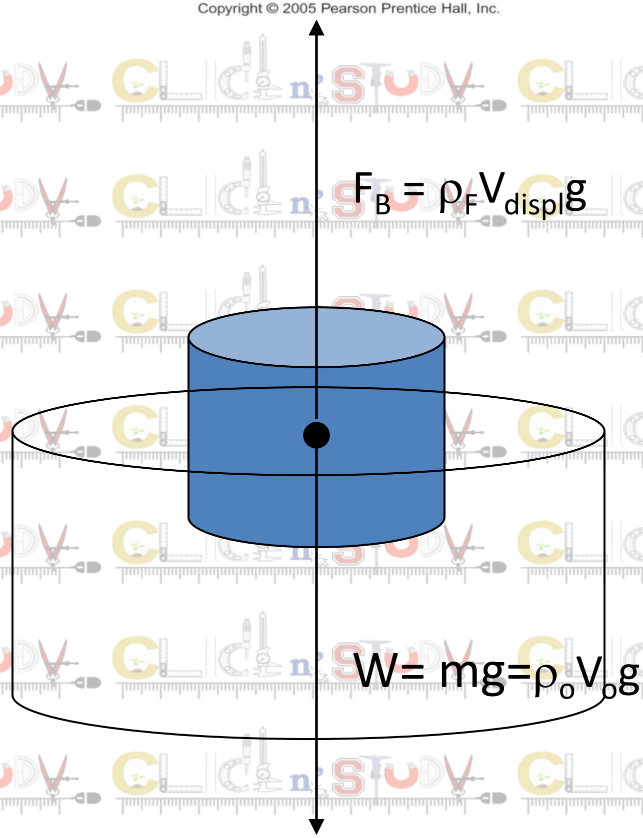


For a body to float in a liquid, the weight of the liquid displaced by the immersed portion of the body must be equal to it's own weight.



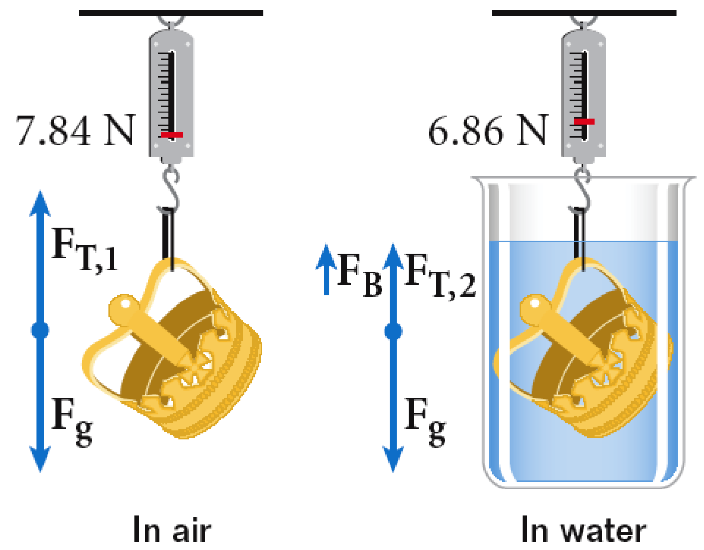
Archimedes' Principle

- Also relates to objects floating in fluid
- Object floats in a fluid if its density is less than the density of the fluid
- The amount submerged can be calculated by
- $V_{displ}/V_o = \rho_o/\ \rho_F$



Example

- *A bargain hunter purchases a “gold” crown at a flea market. After she gets home, she hangs the crown from a scale and finds its weight to be 7.84 N. She then weighs the crown while it is immersed in water, and the scale reads 6.86 N. Is the crown made of pure gold? Explain.*



Ans; From the table, the density of gold is 19.3×10^3 kg/m^3. Because 8.0×10^3 kg/m^3 $< 19.3 \times 10^3$ kg/m^3, the crown cannot be pure gold.

Buoyancy and Archimedes' Principle

Example: When a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown made of gold? (ρ=19300 kg/m^3)

If W_A is the apparent weight

$$W_A = W - B$$

Weight can be written as

$$W = mg = \rho_g V g$$

Using these equations

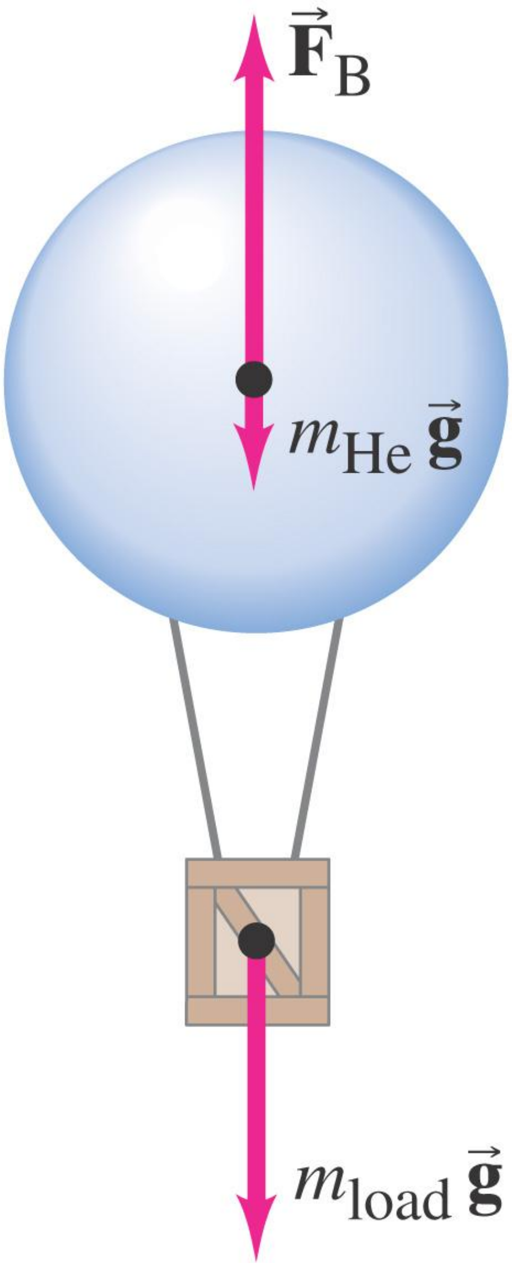
$$W - W_A = \rho_{H_2O} V g$$

$$\frac{W \rho_{H_2O}}{W - W_A} = \rho_g$$

$$\frac{(14.7)(1000)}{(14.7 - 13.4)} = 11300 \frac{kg}{m^3}$$

John Ninomiya flying a cluster of 72 helium-filled balloons over Temecula, California in April of 2003. The helium balloons displace approximately 230 m^3 of air, providing the necessary buoyant force. Don't try this at home!

Buoyancy and Archimedes' Principle



This principle also works in the air; this is why hot-air and helium balloons rise.

RELATIVE DENSITY -Specific Gravity

Specific Gravity – the ratio of the density of the substance to the density of water at 4°C.

Prove that:

$$RD = \frac{\text{Wt of the object in air}}{\text{Loss of wt in water}}$$

RD has no unit

RD of Water is 1

Salt water would be 1.025

Example

- A piece of metal weighs 50.0 N in air and 36.0 N in water and 41.0 N in an unknown liquid. Find the densities of the following:
  - The metal
  - The unknown liquid
- A 2.8 kg rectangular air mattress is 2.00 m long and 0.500 m wide and 0.100 m thick. What mass can it support in water before sinking?
- A ferry boat is 4.0 m wide and 6.0 m long. When a truck pulls onto it, the boat sinks 4.00 cm in the water. What is the weight of the truck?

Multiple Choice

1. Which of the following is the correct equation for the net force acting on a submerged object?

   A. $F_{net} = 0$

   B. $F_{net} = (\rho_{object} - \rho_{fluid})gV_{object}$

   C. $F_{net} = (\rho_{fluid} - \rho_{object})gV_{object}$

   D. $F_{net} = (\rho_{fluid} + \rho_{object})gV_{object}$

2. How many times greater than the lifting force must the force applied to a hydraulic lift be if the ratio of the area where pressure is applied to the lifted area is 1/7 ?

F. 1/49

G. 1/7

H. 7

J. 49

3. A typical silo on a farm has many bands wrapped around its perimeter, as shown in the figure below. Why is the spacing between successive bands smaller toward the bottom?

A. to provide support for the silo's sides above them

B. to resist the increasing pressure that the grains exert with increasing depth

C. to resist the increasing pressure that the atmosphere exerts with increasing depth

D. to make access to smaller quantities of grain near the ground possible

4. A fish rests on the bottom of a bucket of water while the bucket is being weighed. When the fish begins to swim around in the bucket, how does the reading on the scale change?

F. The motion of the fish causes the scale reading to increase.

G. The motion of the fish causes the scale reading to decrease.

H. The buoyant force on the fish is exerted downward on the bucket, causing the scale reading to increase.

J. The mass of the system, and so the scale reading, will remain unchanged.

Use the passage below to answer questions 5–6.

A metal block (ρ = 7900 kg/m^3) is connected to a spring scale by a string 5 cm in length. The block's weight in air is recorded. A second reading is recorded when the block is placed in a tank of fluid and the surface of the fluid is 3 cm below the scale.

5. If the fluid is oil (ρ < 1000 kg/m^3), which of the following must be true?

A. The first scale reading is larger than the second reading.

B. The second scale reading is larger than the first reading.

C. The two scale readings are identical.

D. The second scale reading is zero.

Use the passage below to answer questions 5–6.

A metal block (ρ = 7900 kg/m^3) is connected to a spring scale by a string 5 cm in length. The block's weight in air is recorded. A second reading is recorded when the block is placed in a tank of fluid and the surface of the fluid is 3 cm below the scale.

6. If the fluid is mercury (ρ = 13 600 kg/m^3), which of the following must be true?

F. The first scale reading is larger than the second reading.

G. The second scale reading is larger than the first reading.

H. The two scale readings are identical.

J. The second scale reading is zero.

Use the passage below to answer questions 7–8.

Water near the top of a dam flows down a spillway to the base of the dam. Atmospheric pressure is identical at the top and bottom of the dam.

7. If the speed of the water at the top of the spillway is nearly 0 m/s, which of the following equations correctly describes the speed of the water at the bottom of the spillway?

A. $v_{bottom} = \sqrt{2g\rho_{water}\left(h_{top} - h_{bottom}\right)}$

B. $v_{bottom} = \sqrt{2g\left(h_{top} - h_{bottom}\right)}$

C. $v_{bottom} = 2g\left(h_{top} - h_{bottom}\right)$

D. $v_{bottom} = 2g\rho_{water}\left(h_{top} - h_{bottom}\right)$

Use the passage below to answer questions 7–8.

Water near the top of a dam flows down a spillway to the base of the dam. Atmospheric pressure is identical at the top and bottom of the dam.

8. If the cross-sectional area of the spillway were half as large, how many times faster would the water flow out of the spillway?

F. 1/4

G. 1/2

H. 2

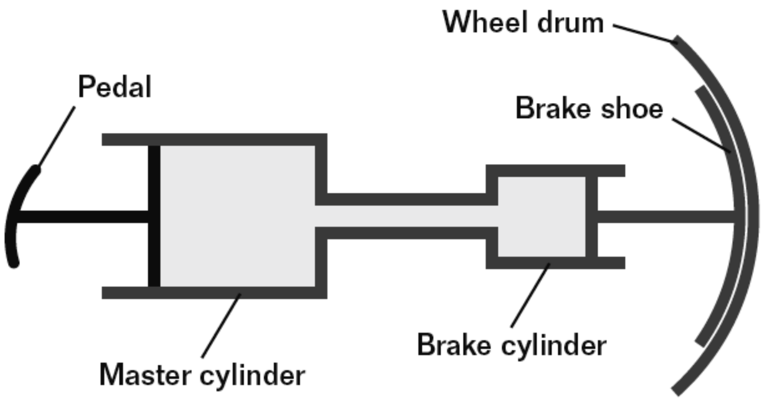
J. 4

Short Response

9. Will an ice cube float higher in water or in mercury? Explain your answer.

10. The approximate inside diameter of the aorta is 1.6 cm, and that of a capillary is 1.0×10^{-6} m. The average flow speed is about 1.0 m/s in the aorta and 1.0 cm/s in the capillaries. If all the blood in the aorta eventually flows through the capillaries, estimate the number of capillaries.

11. A hydraulic brake system is shown below. The area of the piston in the master cylinder is 6.40 cm^2, and the area of the piston in the brake cylinder is 1.75 cm^2. The coefficient of friction between the brake shoe and wheel drum is 0.50. What is the frictional force between the brake shoe and wheel drum when a force of 44 N is exerted on the pedal?



Base your answers to questions 12–14 on the information below.

Oil, which has a density of 930.0 kg/m^3, floats on water. A rectangular block of wood with a height, *h*, of 4.00 cm and a density of 960.0 kg/m^3 floats partly in the water, and the rest floats under the oil layer.

12. What is the balanced force equation for this situation?

Base your answers to questions 12–14 on the information below.

Oil, which has a density of 930.0 kg/m^3, floats on water. A rectangular block of wood with a height, *h*, of 4.00 cm and a density of 960.0 kg/m^3 floats partly in the water, and the rest floats under the oil layer.

13. What is the equation that describes *y*, the thickness of the part of the block that is submerged in water?

Base your answers to questions 12–14 on the information below.

Oil, which has a density of 930.0 kg/m^3, floats on water. A rectangular block of wood with a height, *h*, of 4.00 cm and a density of 960.0 kg/m^3 floats partly in the water, and the rest floats under the oil layer.

14. What is the value for *y*?

Pressure at a Point

$$\sum F_x = ma_x = 0: \qquad P_1\,\Delta y \Delta z - P_3 \Delta y l \sin\theta = 0$$

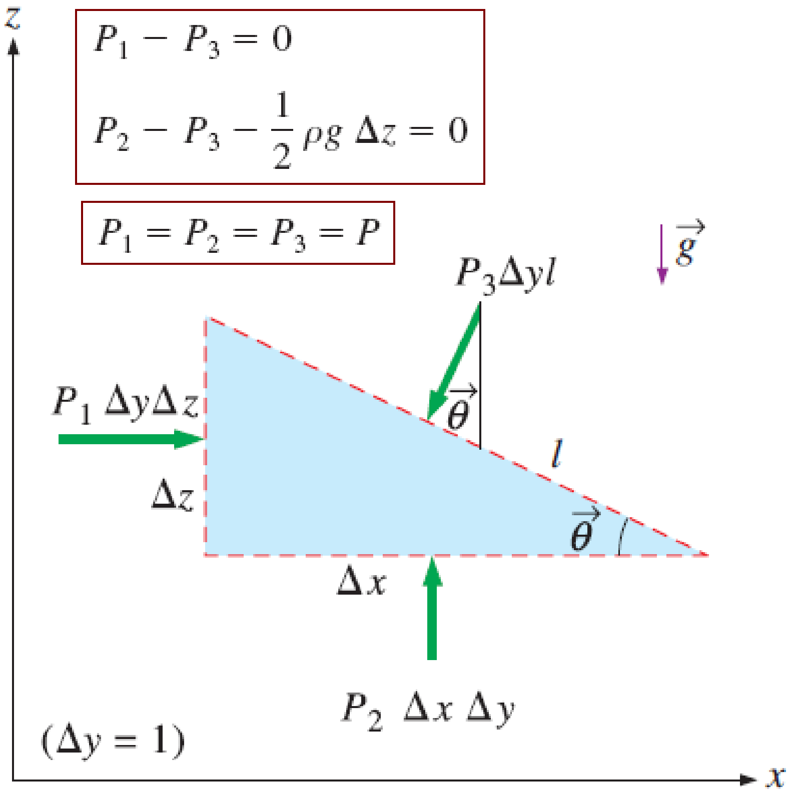
$$\sum F_z = ma_z = 0: \qquad P_2\,\Delta y \Delta x - P_3 \Delta y l \cos\theta - \frac{1}{2}\rho g\,\Delta x\,\Delta y\,\Delta z = 0$$

$$W = mg = \rho g\,\Delta x\,\Delta y\,\Delta z/2 \qquad \Delta z = l\sin\theta, \quad \Delta x = l\cos\theta$$

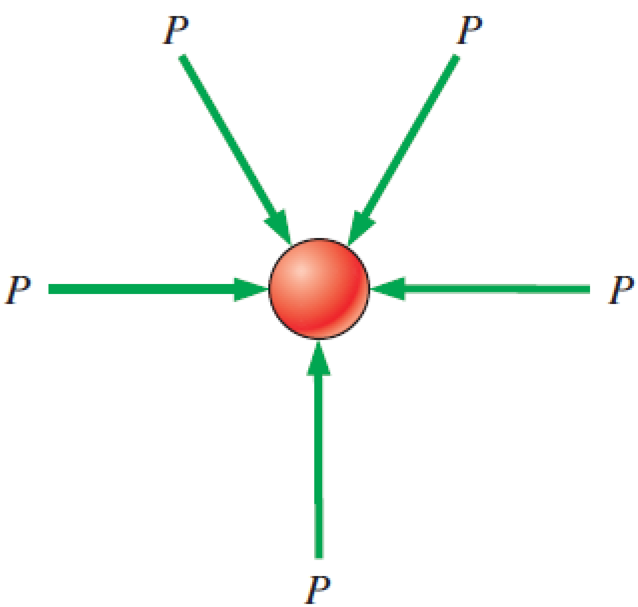
Pressure is the *compressive force* per unit area but it is not a vector. *Pressure at any point in a fluid is the same in all directions.* Pressure has magnitude but not a specific direction, and thus it is a scalar quantity.

$$\boxed{\begin{array}{l} P_1 - P_3 = 0 \\ P_2 - P_3 - \frac{1}{2}\rho g\,\Delta z = 0 \end{array}}$$

$$\boxed{P_1 = P_2 = P_3 = P}$$



Forces acting on a wedge-shaped fluid element in equilibrium.



Pressure is a *scalar* quantity, not a vector; the pressure at a point in a fluid is the same in all directions.

Variation of Pressure with Depth

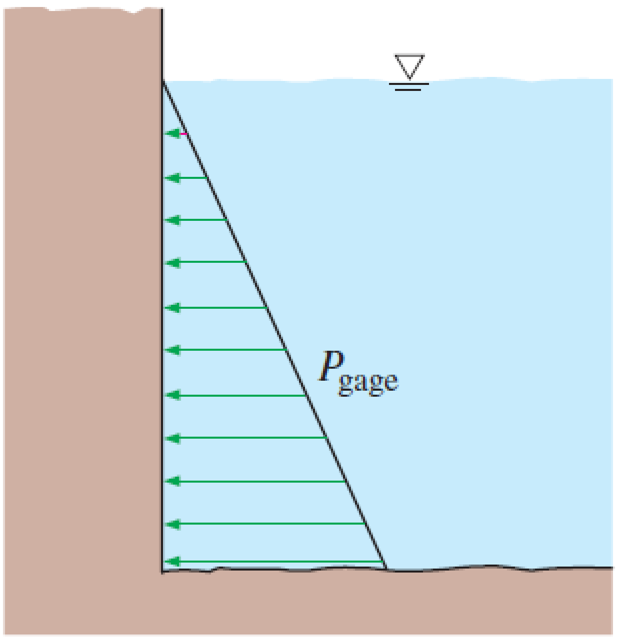
$$\Delta P = P_2 - P_1 = \rho g\, \Delta z = \gamma_s\, \Delta z$$

$$P_{\text{below}} = P_{\text{above}} + \rho g|\Delta z| = P_{\text{above}} + \gamma_s|\Delta z|$$

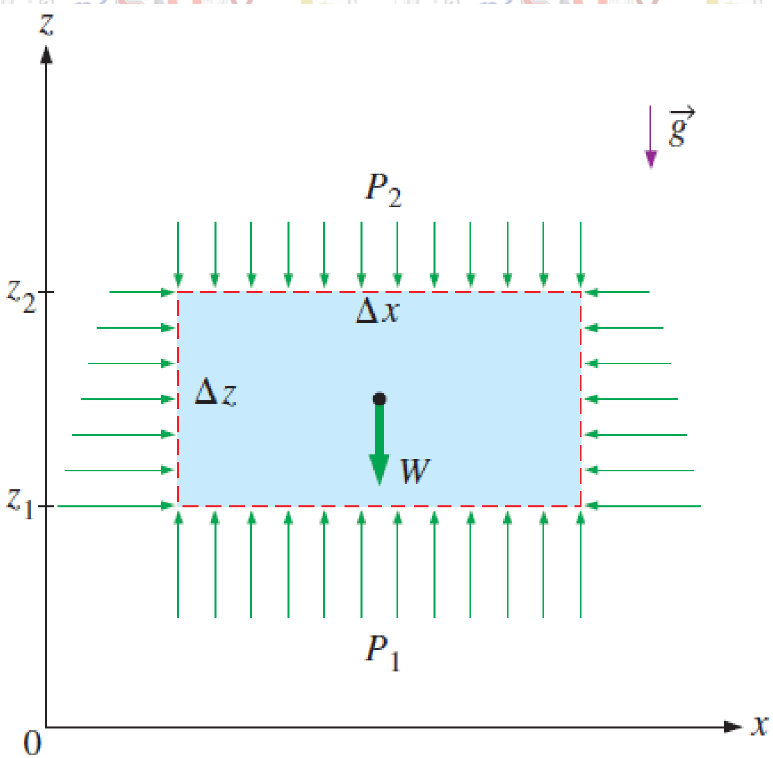
$$P = P_{\text{atm}} + \rho g h \quad \text{or} \quad P_{\text{gage}} = \rho g h$$

When the variation of density with elevation is known

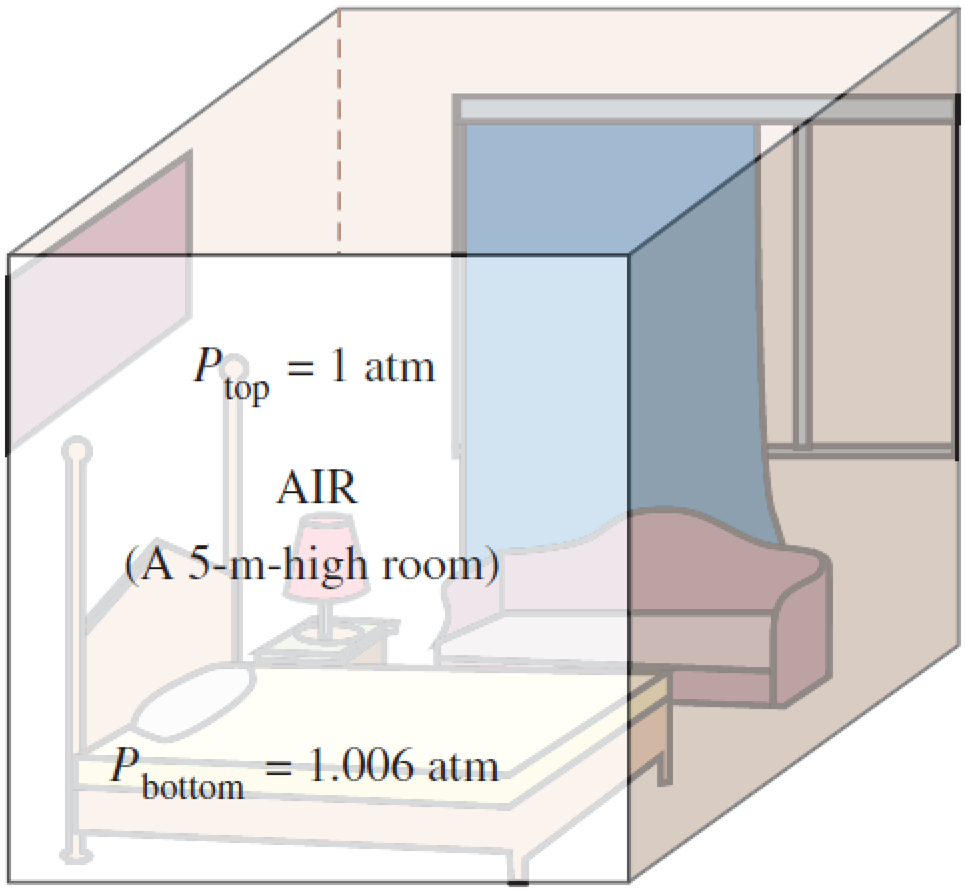
$$\Delta P = P_2 - P_1 = -\int_1^2 \rho g\, dz$$



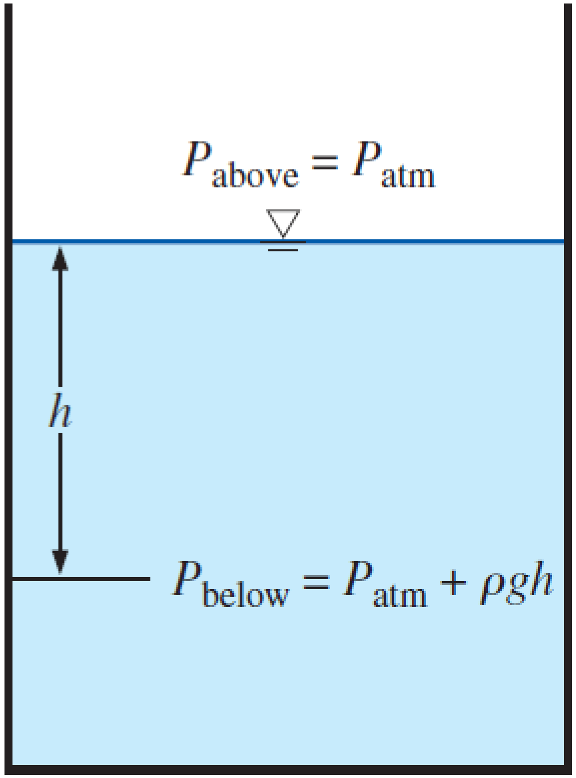
The pressure of a fluid at rest increases with depth (as a result of added weight).



Free-body diagram of a rectangular fluid element in equilibrium.



In a room filled with a gas, the variation of pressure with height is negligible.



Pressure in a liquid at rest increases linearly with distance from the free surface.

EXAMPLE 3–2 **Measuring Atmospheric Pressure with a Barometer**

Determine the atmospheric pressure at a location where the barometric reading is 740 mm Hg and the gravitational acceleration is $g = 9.805\ \text{m/s}^2$. Assume the temperature of mercury to be 10°C, at which its density is 13,570 kg/m³.

SOLUTION The barometric reading at a location in height of mercury column is given. The atmospheric pressure is to be determined.
Assumptions The temperature of mercury is assumed to be 10°C.
Properties The density of mercury is given to be 13,570 kg/m³.
Analysis From Eq. 3–12, the atmospheric pressure is determined to be

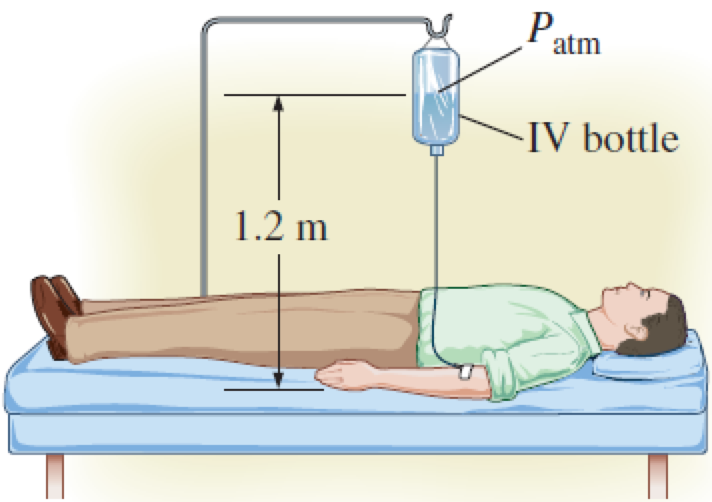
$$
\begin{aligned}
P_{\text{atm}} &= \rho g h \\
&= (13{,}570\ \text{kg/m}^3)(9.805\ \text{m/s}^2)(0.740\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\
&= \mathbf{98.5\ kPa}
\end{aligned}
$$

Discussion Note that density changes with temperature, and thus this effect should be considered in calculations.

EXAMPLE 3–3 **Gravity Driven Flow from an IV Bottle**

Intravenous infusions usually are driven by gravity by hanging the fluid bottle at sufficient height to counteract the blood pressure in the vein and to force the fluid into the body (Fig. 3–15). The higher the bottle is raised, the higher the flow rate of the fluid will be. (*a*) If it is observed that the fluid and the blood pressures balance each other when the bottle is 1.2 m above the arm level, determine the gage pressure of the blood. (*b*) If the gage pressure of the fluid at the arm level needs to be 20 kPa for sufficient flow rate, determine how high the bottle must be placed. Take the density of the fluid to be 1020 kg/m^3.

SOLUTION It is given that an IV fluid and the blood pressures balance each other when the bottle is at a certain height. The gage pressure of the blood and elevation of the bottle required to maintain flow at the desired rate are to be determined.



Assumptions **1** The IV fluid is incompressible. **2** The IV bottle is open to the atmosphere.

Properties The density of the IV fluid is given to be $\rho = 1020 \text{ kg/m}^3$.

Analysis (*a*) Noting that the IV fluid and the blood pressures balance each other when the bottle is 1.2 m above the arm level, the gage pressure of the blood in the arm is simply equal to the gage pressure of the IV fluid at a depth of 1.2 m,

$$
\begin{aligned}
P_{\text{gage, arm}} &= P_{\text{abs}} - P_{\text{atm}} = \rho g h_{\text{arm}-\text{bottle}} \\
&= (1020 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.20 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2}\right) \\
&= \mathbf{12.0 \text{ kPa}}
\end{aligned}
$$

(*b*) To provide a gage pressure of 20 kPa at the arm level, the height of the surface of the IV fluid in the bottle from the arm level is again determined from $P_{\text{gage, arm}} = \rho g h_{\text{arm}-\text{bottle}}$ to be

$$
\begin{aligned}
h_{\text{arm}-\text{botttle}} &= \frac{P_{\text{gage, arm}}}{\rho g} \\
&= \frac{20 \text{ kPa}}{(1020 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1000 \text{ kg}\cdot\text{m/s}^2}{1 \text{ kN}}\right)\left(\frac{1 \text{ kN/m}^2}{1 \text{ kPa}}\right) \\
&= \mathbf{2.00 \text{ m}}
\end{aligned}
$$

Discussion Note that the height of the reservoir can be used to control flow rates in gravity-driven flows. When there is flow, the pressure drop in the tube due to frictional effects also should be considered. For a specified flow rate, this requires raising the bottle a little higher to overcome the pressure drop.

EXAMPLE 3–4 Hydrostatic Pressure in a Solar Pond with Variable Density

Solar ponds are small artificial lakes of a few meters deep that are used to store solar energy. The rise of heated (and thus less dense) water to the surface is prevented by adding salt at the pond bottom. In a typical salt gradient solar pond, the density of water increases in the gradient zone, as shown in Fig. 3–16, and the density can be expressed as

$$\rho = \rho_0\sqrt{1 + \tan^2\left(\frac{\pi}{4}\frac{s}{H}\right)}$$

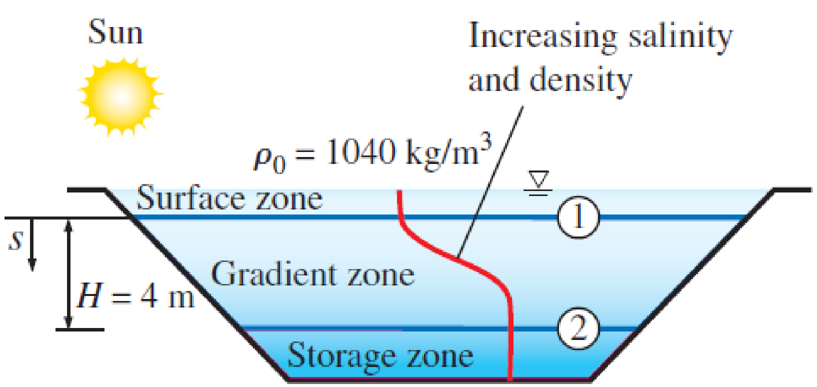
where ρ_0 is the density on the water surface, s is the vertical distance measured downward from the top of the gradient zone ($s = -z$), and H is the thickness of the gradient zone. For $H = 4$ m, $\rho_0 = 1040$ kg/m^3, and a thickness of 0.8 m for the surface zone, calculate the gage pressure at the bottom of the gradient zone.

SOLUTION The variation of density of saline water in the gradient zone of a solar pond with depth is given. The gage pressure at the bottom of the gradient zone is to be determined.

Assumptions The density in the surface zone of the pond is constant.

Properties The density of brine on the surface is given to be 1040 kg/m^3.

Analysis We label the top and the bottom of the gradient zone as 1 and 2, respectively. Noting that the density of the surface zone is constant, the



gage pressure at the bottom of the surface zone (which is the top of the gradient zone) is

$$P_1 = \rho g h_1 = (1040\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.8\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg·m/s}^2}\right) = 8.16\ \text{kPa}$$

since 1 kN/m² = 1 kPa. Since $s = -z$, the differential change in hydrostatic pressure across a vertical distance of ds is given by

$$dP = \rho g\, ds$$

Integrating from the top of the gradient zone (point 1 where $s = 0$) to any location s in the gradient zone (no subscript) gives

$$P - P_1 = \int_0^s \rho g\, ds \quad \rightarrow \quad P = P_1 + \int_0^s \rho_0 \sqrt{1 + \tan^2\left(\frac{\pi}{4}\frac{s}{H}\right)}\, g\, ds$$

Performing the integration gives the variation of gage pressure in the gradient zone to be

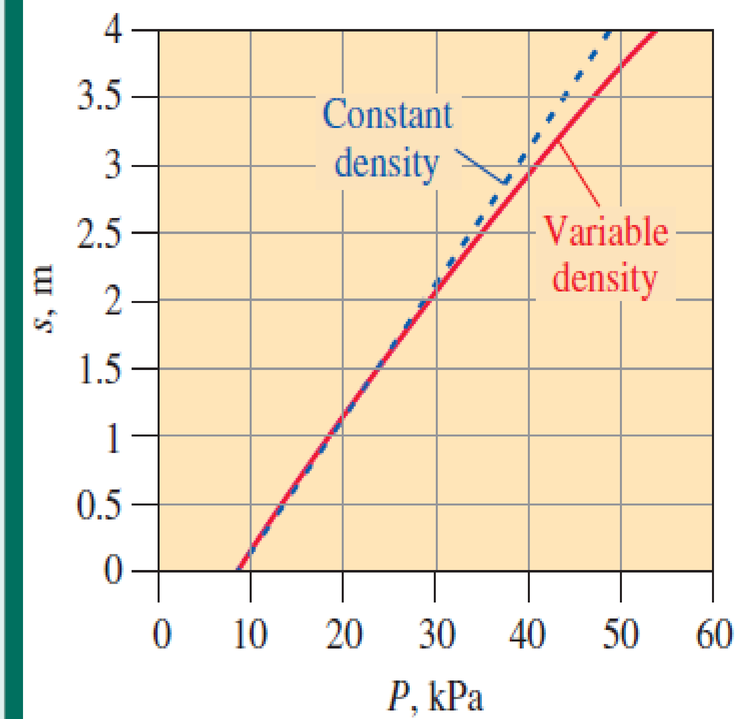
$$P = P_1 + \rho_0 g \frac{4H}{\pi} \sinh^{-1}\left(\tan \frac{\pi}{4}\frac{s}{H}\right)$$

Then the pressure at the bottom of the gradient zone ($s = H = 4$ m) becomes

$$P_2 = 8.16\ \text{kPa} + (1040\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\frac{4(4\ \text{m})}{\pi} \sinh^{-1}\left(\tan \frac{\pi}{4}\frac{4}{4}\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg·m/s}^2}\right)$$

$$= \mathbf{54.0\ kPa\ (gage)}$$

Discussion The variation of gage pressure in the gradient zone with depth is plotted in Fig. 3–17. The dashed line indicates the hydrostatic pressure for the case of constant density at 1040 kg/m³ and is given for reference. Note that the variation of pressure with depth is not linear when density varies with depth. That is why integration was required.



The variation of gage pressure with depth in the gradient zone of the solar pond.