Work, energy and power

Objectives: :

- Define kinetic energy and potential energy, along with the appropriate units in each system.
- Describe the relationship between work and kinetic energy, and apply the WORK-ENERGY THEOREM.
- Define and apply the concept of POWER, along with the appropriate units.

The Ninja … *Georgia, has a height of* 122 ft *and a* …
of 5 … *ght changes into kinetic ene*…

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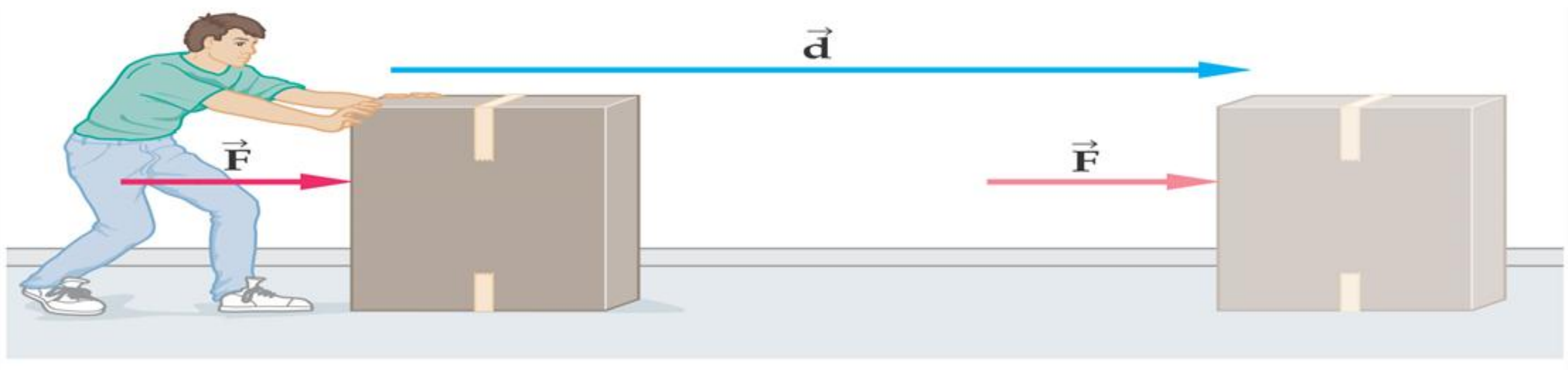
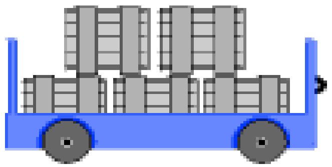


Work (force is parallel to distance)



Force (N)

Work (joules) → **W = F x d**

Distance (m)

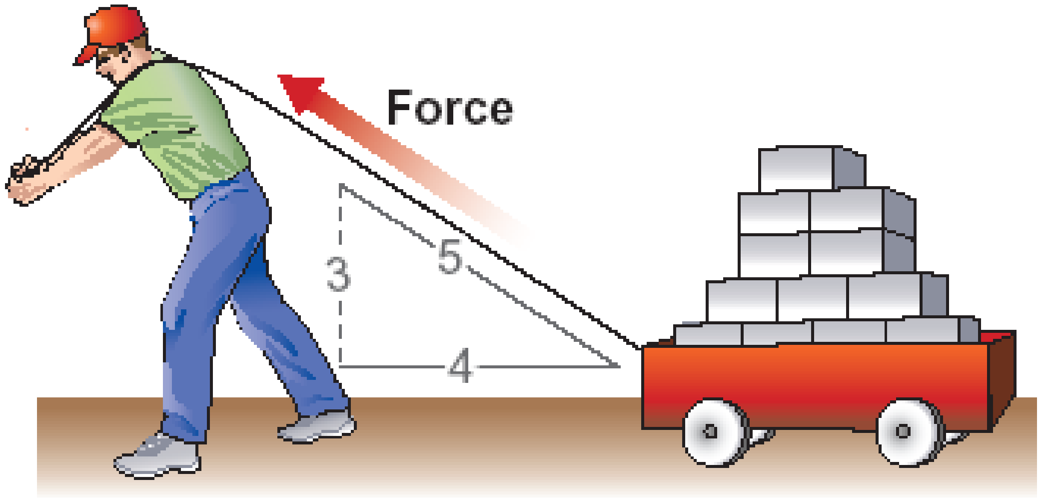
$$W = F \cdot x$$

SI unit = Joule

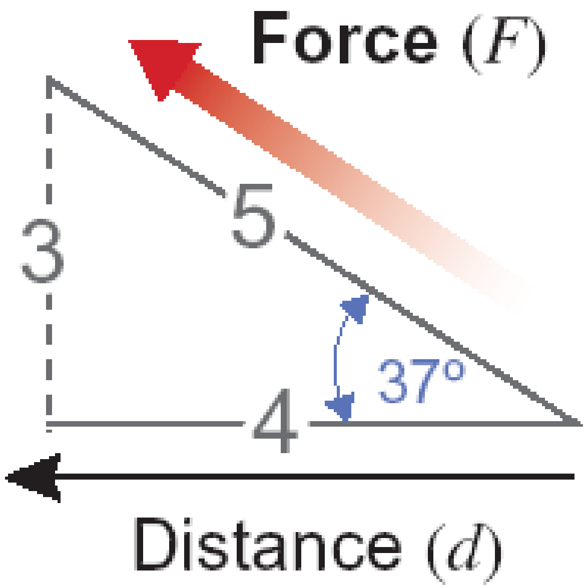
1 J = 1 N·m = 1 kg·m^2/s^2

Force at an Angle to the Distance

PROBLEM



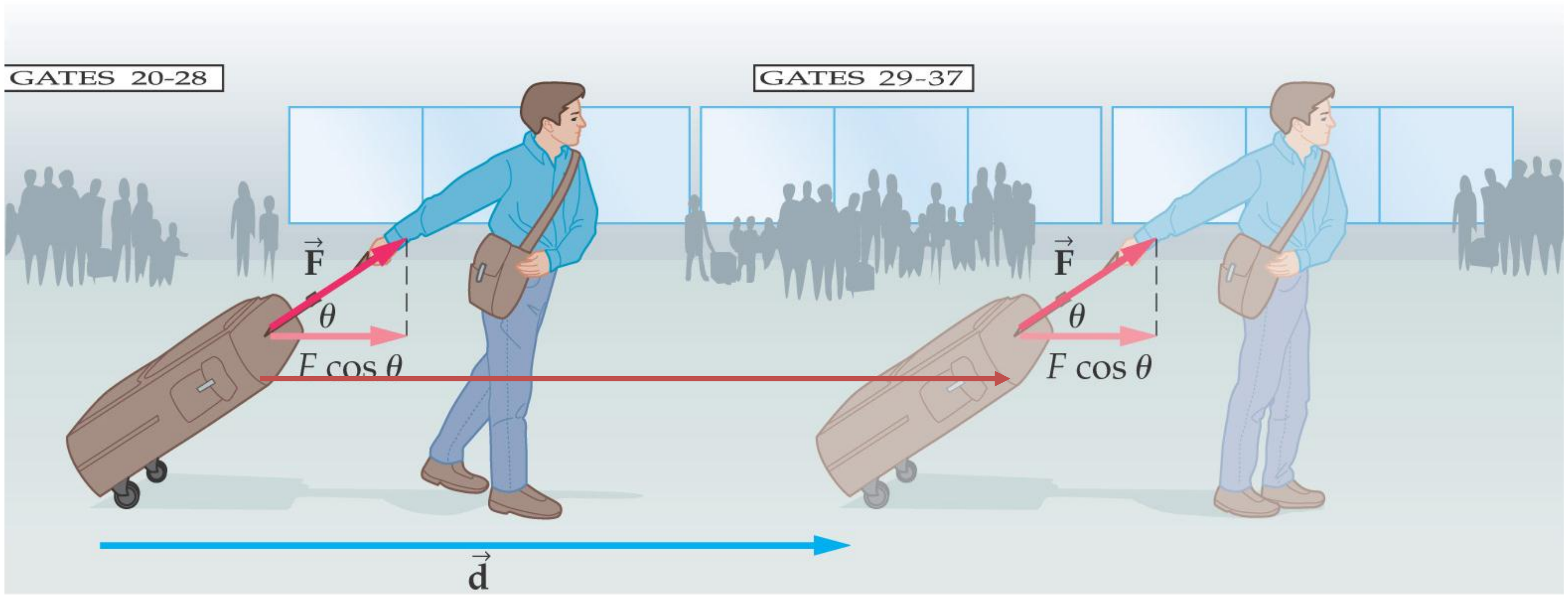
ANALYSIS



SOLUTION

$$W = Fd \times \left(\frac{4}{5}\right) = Fd\cos 37^{\circ}$$

Work…



$\boxed{W = F\,d\cos\theta}$ definition of work

d - the displacement of the point of application of the force

θ - is the angle between the force and the displacement vectors

$\boxed{W = F\,d\cos\theta}$

- A force does no work on the object if the force does not move through a displacement
- The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application $\cos 90 = 0$

When we walk holding a heavy suitcase we therefore do zero work *on the suitcase*. On the other hand, if we walk upstairs or uphill holding the same suitcase we do some work *on the suitcase* (it is important to specify on which object the work is done).

example

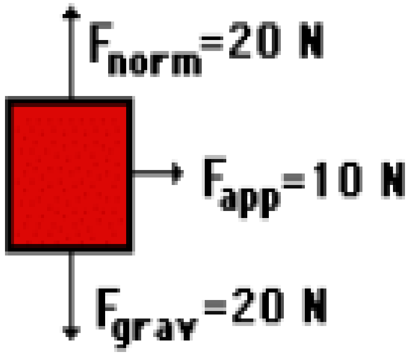
A 10-N forces is applied to push a block across a friction free surface for a displacement of 5.0 m to the right. The work done by the normal (reaction) force is:

(a) 20J

(b) 200J

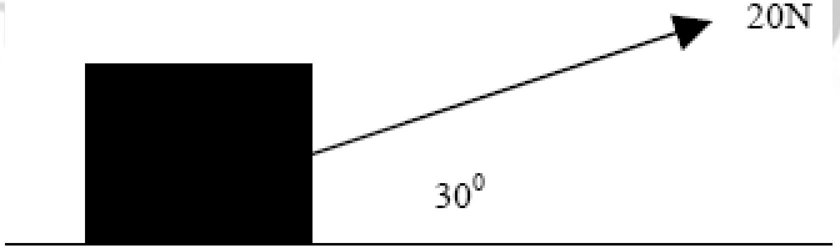
(c) 2000J

(d) 0J



example. Compute the work done by a 20 N force applied to a 10 kg block as shown below if the block moves 20 meters as a result of the application of the force.

$$W = Fs\cos\theta = 20N(20m)(\cos 30^0) = 346J$$

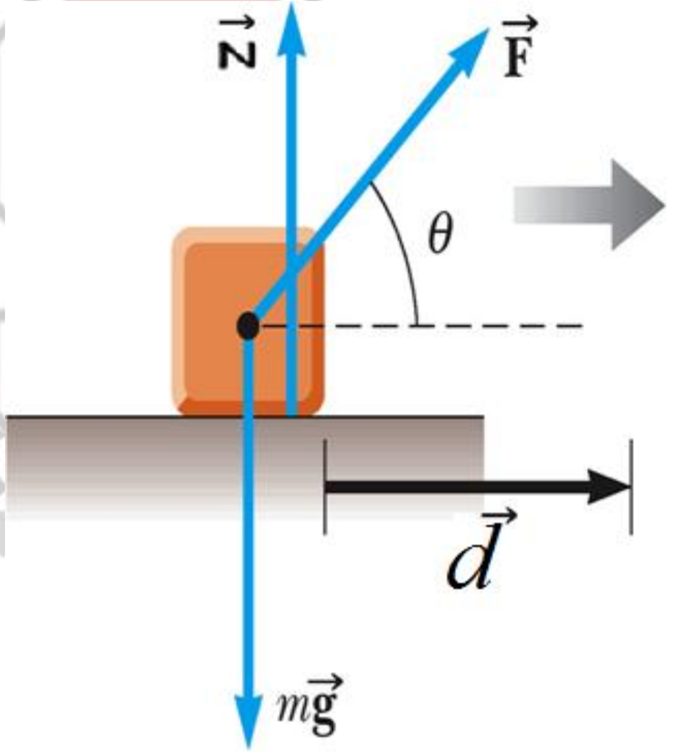


- The normal force $\vec{N}$,, and the gravitational force, $m\mathbf{g}$, do no work on the object

  $\cos\theta = \cos 90° = 0$

- The force $\vec{\mathbf{F}}$ does do work on the object
  - There is a component $\vec{\mathbf{F}}$ along the direction of motion

$$W = F\,d\cos\theta$$



Example :

A horizontal force F pulls a 10 kg carton across the floor at constant speed. If the coefficient of sliding friction between the carton and the floor is 0.30, how much work is done by F in moving the carton by 5m?

The carton moves with constant speed. Thus, the carton is in horizontal equilibrium.

$F = f = \mu_k N = \mu_k mg.$

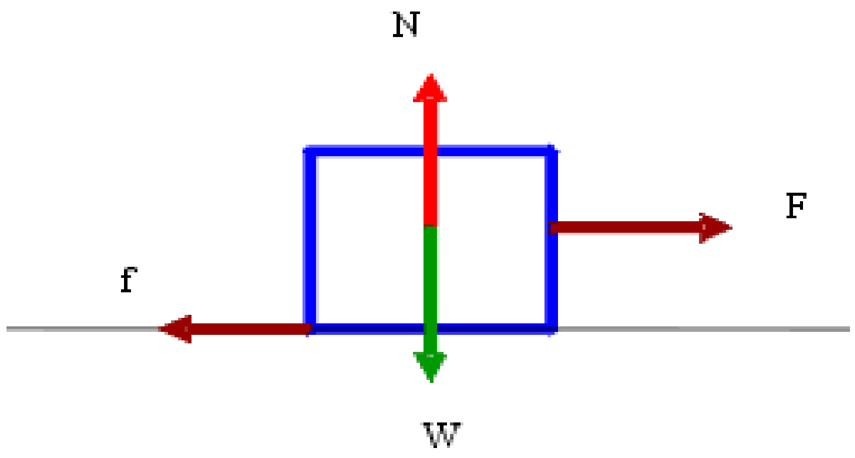
Thus $F = 0.3 \times 10 \times 9.8$

$= 29.4$ N

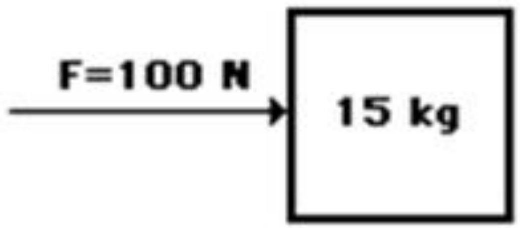
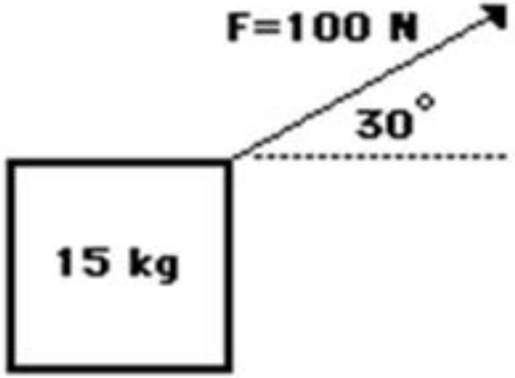
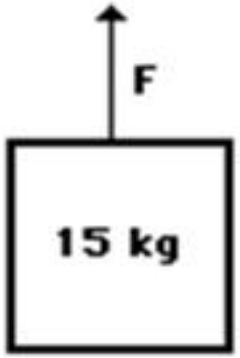
Therefore work done $W = FS$

$=(29.4 \cos 0^o)$

$=147$ J



Work Calculations

Diagram A	Diagram B	Diagram C
		
A 100 N force is applied to move a 15 kg object a horizontal distance of 5 meters at constant speed.	A 100 N force is applied at an angle of 30° to the horizontal to move a 15 kg object at a constant speed for a horizontal distance of 5 m.	An upward force is applied to lift a 15 kg object to a height of 5 meters at constant speed.

W= F x d

=100N X 5m

=500 N m

W=F x d cos 30^0

= 100N X 5m X .87

= 750 N m

W=F x d

=15Kg(10m/s$^{2)}$ X 5m

= 413 N m

Example

Three student push their car which run out of gas. If each student pushes with a force of 500 N and the car pushed a distance of 40 m, find the work done by the student s

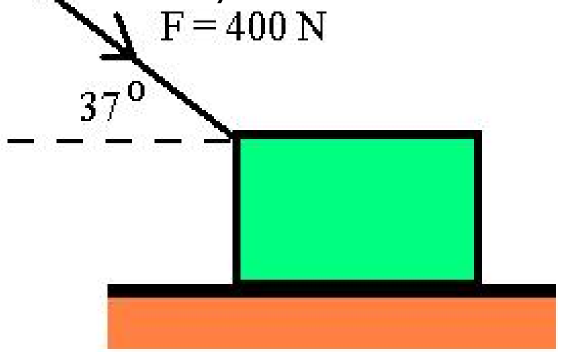
$$Work = F_d d = 1500\ N \times 40\ m = 6000$$

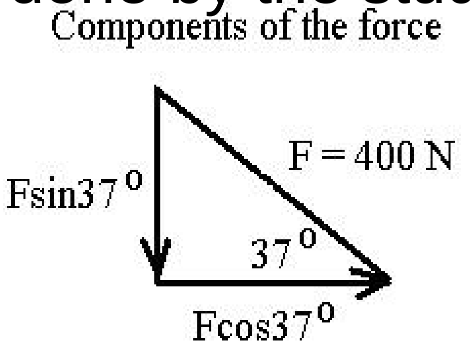
$$Work = 6000\ J$$

A man exert a horizontal force 160 N on a car that moves a distance of 10m calculate the work done by the force

$$Work = F\ d = 160\ N \times 10\ m = 1600\ J$$

A student pushes on a heavy box with a force of 400 N and moves it a distance of 5 m . If the force applied at an angle of 37 shown below , find the work done by the student





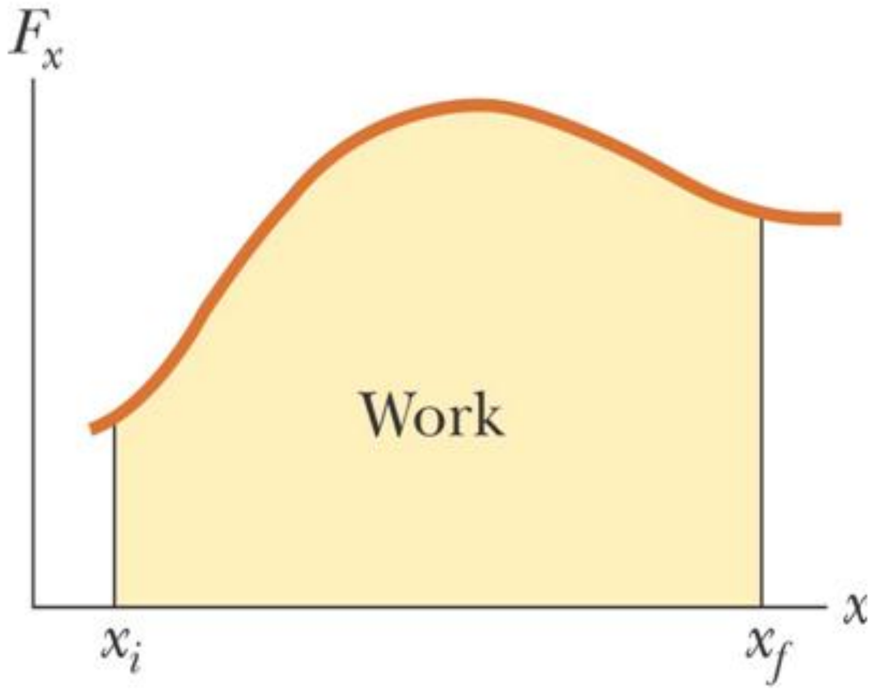
$$Work = F\ d$$

$$F = F\cos 37^{\circ} = (400\ N)(0.8) = 320\ N$$

$$Work = F_d d = 320\ N \times 5\ m = 1600\ Joules$$

Work Done by a Varying Force

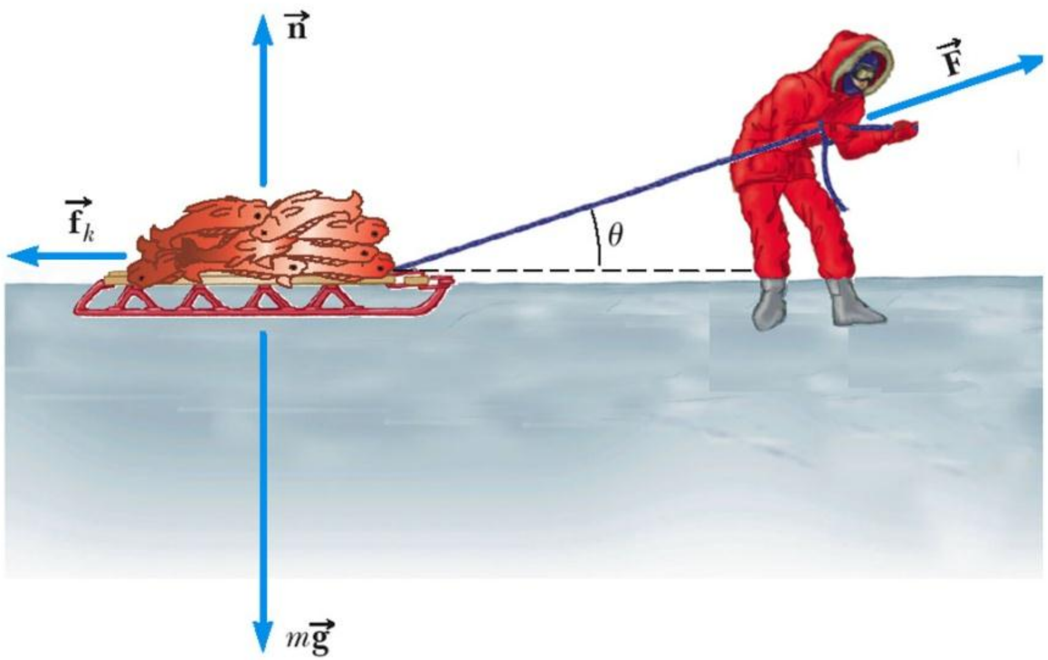
- The work done is equal to the area under the curve



(b)

example

- An Eskimo returning pulls a sled as shown. The total mass of the sled is 50.0 kg, and he exerts a force of 1.20×10^2 N on the sled by pulling on the rope. How much work does he do on the sled if θ = 30° and he pulls the sled 5.0 m ?



$$W = (F\cos\theta)\Delta x$$

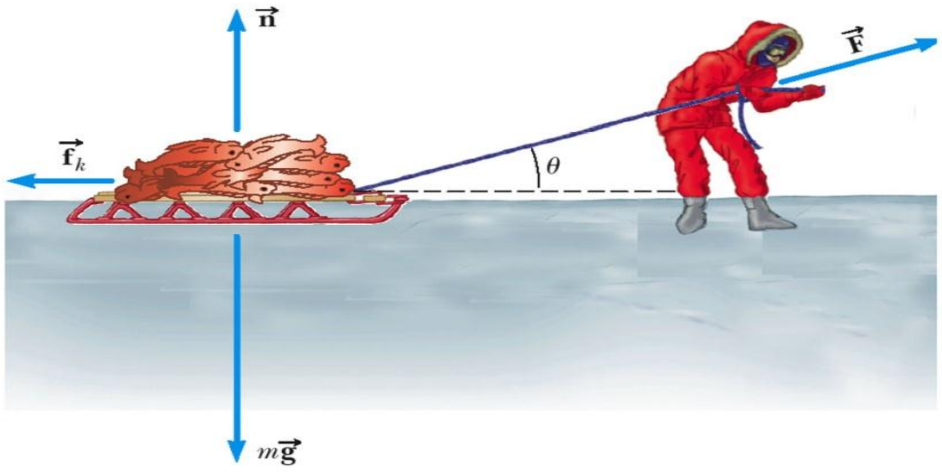
$$= (1.20 \times 10^2 N)(\cos 30^\circ)(5.0m)$$

$$= 5.2 \times 10^2 J$$

- Suppose μ_k = 0.200, How much work done on the sled by friction, and the net work if θ = 30° and he pulls the sled 5.0 m ?

$$F_{net,y} = N - mg + F\sin\theta = 0$$

$$N = mg - F\sin\theta$$



$$W_{fric} = (f_k\cos 180^\circ)\Delta x = -f_k\Delta x$$

$$= -\mu_k N\Delta x = -\mu_k(mg - F\sin\theta)\Delta x$$

$$= -(0.200)(50.0kg\cdot 9.8m/s^2$$

$$-1.2\times 10^2 N\sin 30^\circ)(5.0m)$$

$$= -4.3\times 10^2 J$$

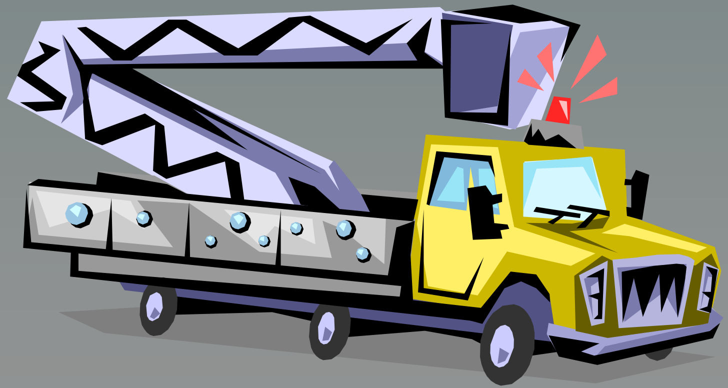
$$W_{net} = W_F + W_{fric} + W_N + W_g$$

$$= 5.2\times 10^2 J - 4.3\times 10^2 J + 0 + 0$$

$$= 90.0J$$

Energy

<u>Energy</u> is anything that can be converted into work; i.e., anything that can exert a force through a distance.



Energy is the capability for doing work.

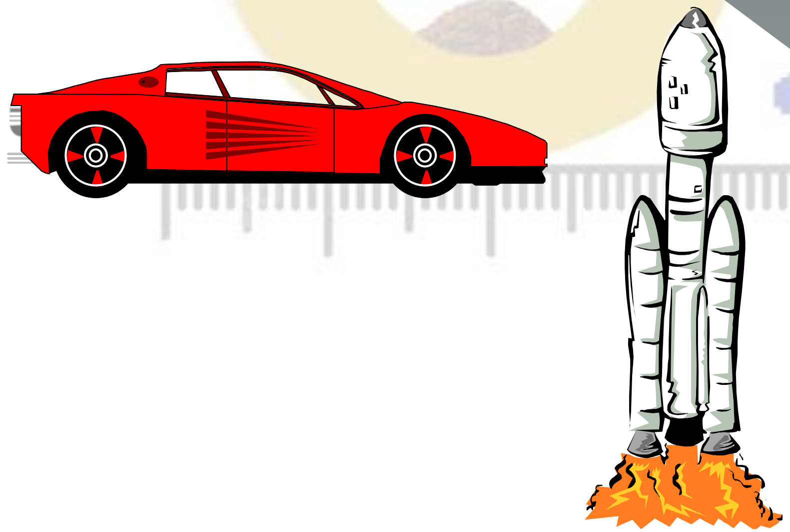
Energy of a mass in motion

Moving energy
(Dynamics)
Dependant on
velocity

Kinetic Energy

Kinetic Energy: Ability to do work by virtue of motion.

A speeding car or a space rocket



The kinetic energy K of an object of mass m moving with a speed v is:

$$K \equiv \frac{1}{2} m v^2$$

Kinetic energy is the energy of particle due to its motion.

Kinetic energy is a scal

the same units

)

An object's kinetic energy depends on its mass and its speed.

What is KE if mass doubled?

(doubled)

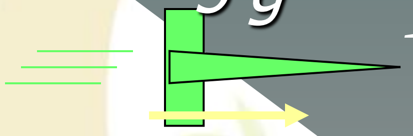
The faster something is moving and the heavier it is, the more work it can do, so …

If 'v' tripled then KE will increase by **9 times.**

If 'v' quadruples, then KE will increase by **16 times**

Examples of Kinetic Energy

What is the kinetic energy of a 5-g bullet traveling at 200 m/s?

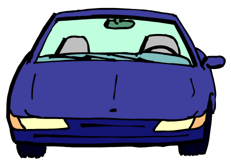


$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.005\text{ kg})(200\text{ m/s})^2$$

$$\boxed{K = 100\text{ J}}$$

What is the kinetic energy of a 1000-kg car traveling at 14.1 m/s?

$$(1000\text{ kg})(14.1\text{ m/s})^2$$



$$\boxed{K = 99.4\text{ J}}$$

example

How fast is a trolley moving if it has 180.5J of kinetic energy?

M= 4 kg

$$k = \frac{1}{2}mv^2$$

$$2\ k = mv^2$$

$$v^2 = \frac{2\ k}{m}$$

$$v = \sqrt{\frac{2\ k}{m}} = \sqrt{\frac{2 \times 180.5}{4}}$$

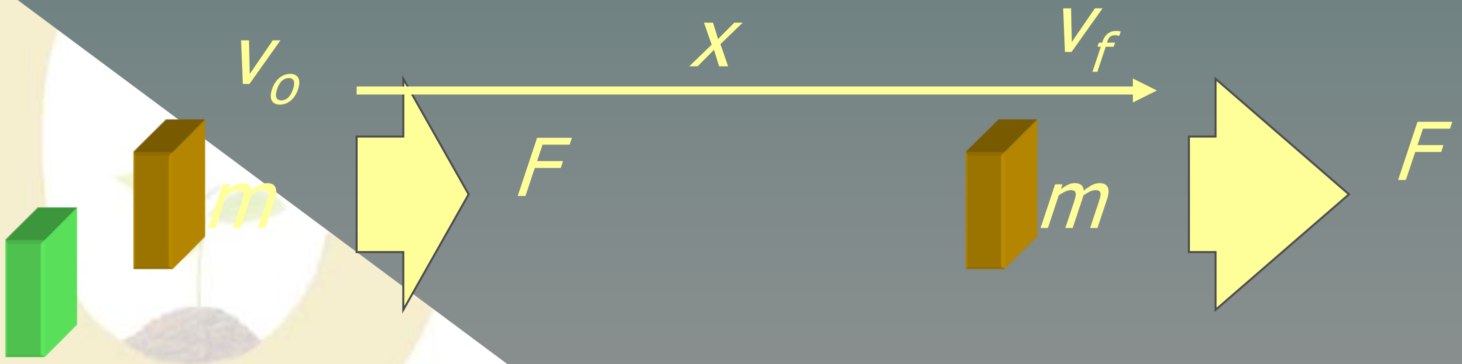
$$= \sqrt{\frac{391}{4}}$$

$$= \sqrt{90.25}$$

so $\underline{v\ =\ 9.5 m/s}$

Work and Kinetic Energy

A resultant force changes the velocity of an object and does work on that object.



$$Work = Fx = (ma)x; \qquad a = \frac{v_f^2 - v_0^2}{2x}$$

$$Work = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2$$

The Work-Energy Theorem

Work is equal to the change in ½mv²

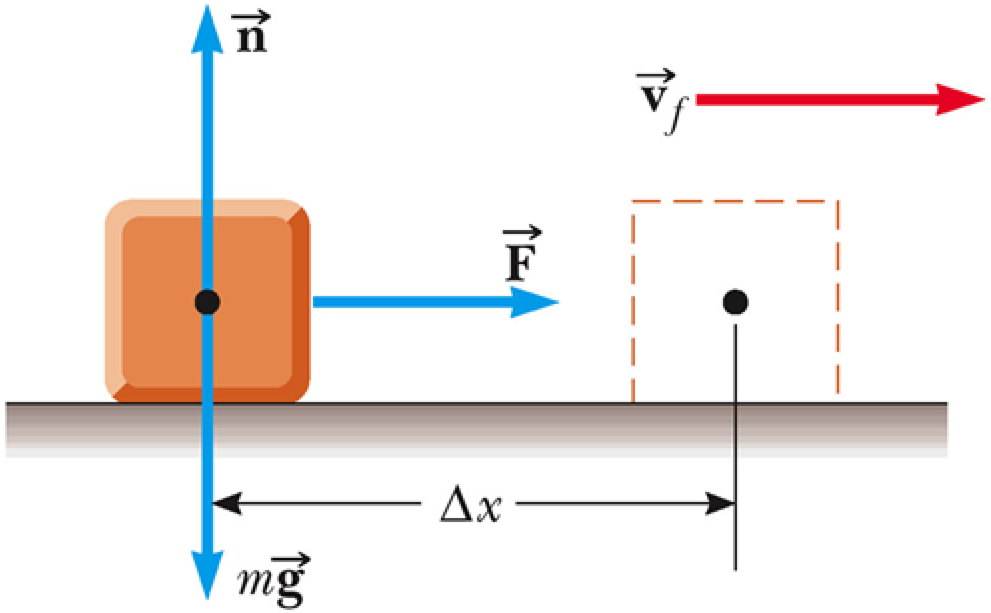
$$Work = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2$$

If we define kinetic energy as ½mv² then we can state a very important physical principle:

The Work-Energy Theorem: The work done by a resultant force is equal to the change in kinetic energy that it produces.

Work-Kinetic Energy Theorem – Example

- Three external forces interact with the block.
- The normal and gravitational forces do no work since they are perpendicular to the direction of the displacement.
- There is no friction and so resultant external force is $\vec{\mathbf{F}}$.



Kinetic Energy and the Work-Kinetic Energy Theorem

- A change in kinetic energy is one possible result of doing work to transfer energy into a system.
- If the net work done on a particle is positive, the speed (and hence the kinetic energy) of the particle increases.
- If the net work done on a particle is negative, the speed (and hence the kinetic energy) of the object decreases.

$$W_{net} = \Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2.$$

A moving hammer strikes a nail and comes to rest. The hammer exerts a force F on the nail; the nail exerts a force -F on the hammer (Newton's third law). The work done on the nail by the hammer is positive (W_n = Fd >0). The work done on the hammer by the nail is negative (W_h = -Fd).

Example

Find the acceleration of the block and determine its final speed using the constant acceleration kinematic equations.

M=6.00 kg

F= 12.0 N

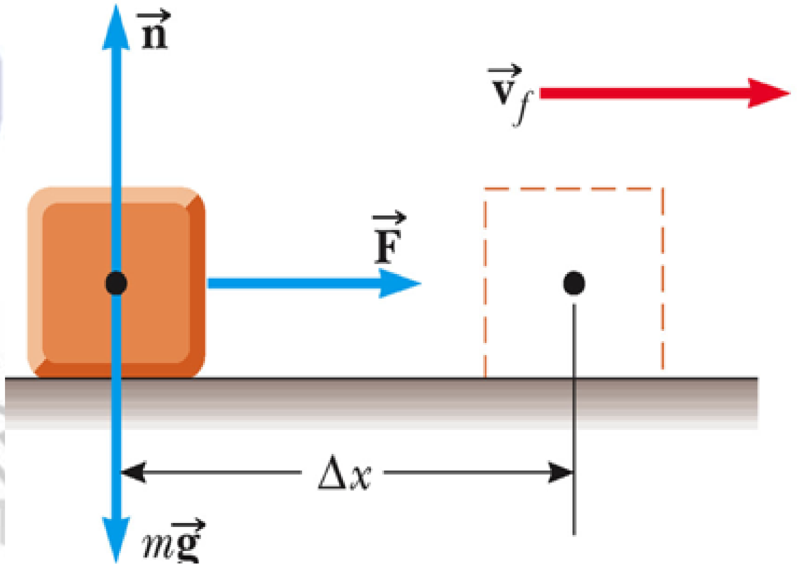
Δ X = 3.00 m

- The work done by the force $\vec{F}$ is:

$$W_F = F\,\Delta x = (12.0\ \mathrm{N})(3.00)\ \mathrm{m}$$
$$= 36.0\ \mathrm{J}$$

- Apply Work-Kinetic Energy Theorem:

$$W_{net} = W_F = K_f - K_i = \frac{1}{2}mv^2 - 0$$



$$v_f = \sqrt{\frac{2W_{net}}{m}} = \sqrt{\frac{2(36.0J)}{6.00kg}} = 3.46\ m/s$$

Example 1: A 20-g projectile strikes a mud bank, penetrating a distance of 6 cm before stopping. Find the stopping force F if the entrance velocity is 80 m/s.

$$\text{Work} = \cancelto{0}{\tfrac{1}{2} m v_f^2} - \tfrac{1}{2} m v_o^2$$

$$F x = -\tfrac{1}{2} m v_o^2$$

$$F(0.06\text{ m}) \cos 180^0 = -\tfrac{1}{2}(0.02\text{ kg})(80\text{ m/s})^2$$

$$F(0.06\text{ m})(-1) = -64\text{ J}$$

$$\boxed{F = 1067\text{ N}}$$

Work to stop bullet = change in K.E. for bullet

Example 2: A bus slams on brakes to avoid an accident. The tread marks of the tires are 80 m long. If $\mu_k = 0.7$, what was the speed before applying brakes?

$Work = \Delta K$

ΔK

$Work = F(\cos\theta)\,x$

$f = \mu_k n = \mu_k mg$

$Work = -\mu_k mg\,x$

$\Delta K = \cancel{\tfrac{1}{2} mv_f^2}^{\,0} - \tfrac{1}{2} mv_o^2$

$-\tfrac{1}{2} mv_o^2 = -\mu_k mg\,x$

$v_o = \sqrt{2\mu_k g x}$

$v_o = \sqrt{2(0.7)(9.8\ \text{m/s}^2)(25\ \text{m})}$

$\boxed{v_o = 59.9\ \text{ft/s}}$

Example 3 :

A 6.0-kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m.

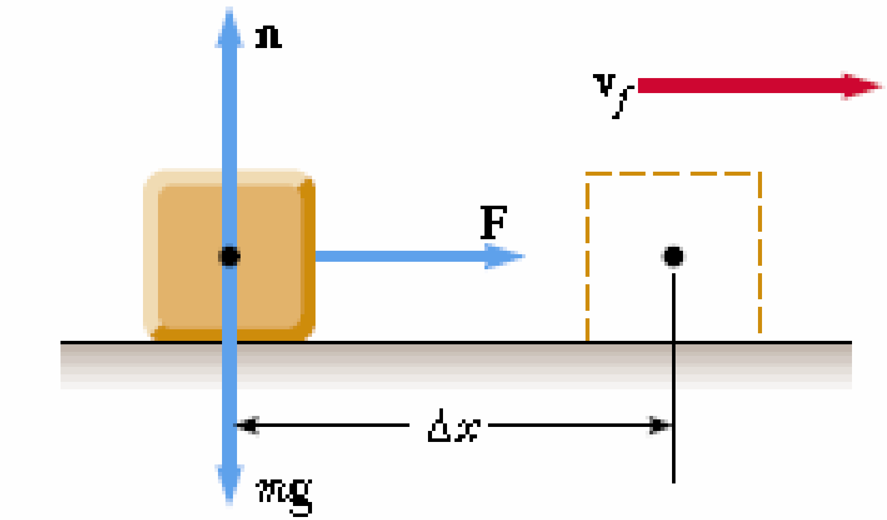
W =F d cos 0 =12x 3x1 = 36J

Δk = w

0.5m V^2 =W =36 J

$$\frac{1}{2}\times 6(v)^2 = 36J$$

$$v = \sqrt{12} = 3.46\, m/s$$



Example

What acceleration is required to stop a 1000kg car traveling 28 m/s in a distance of 100 meters?

$$\frac{1}{2}m\Delta v^2 = ma\Delta x$$

$$\frac{1}{2}\times(28)^2 = a\times 100$$

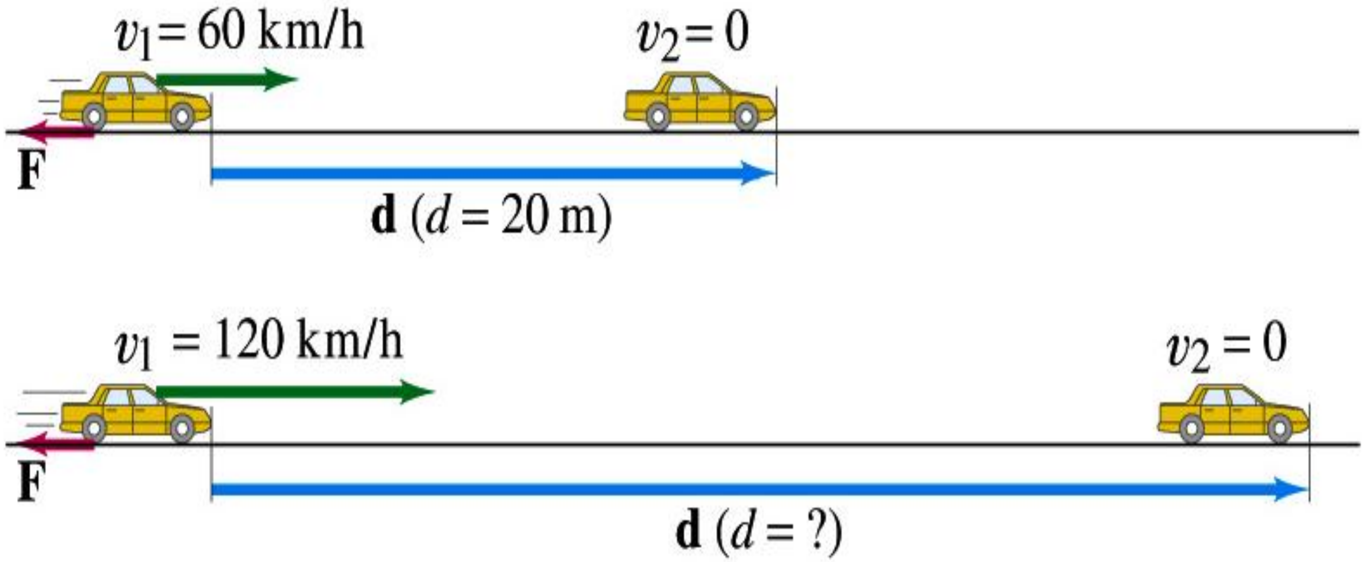
$$a = \frac{784}{200} = 3.92m/s^2$$

A car traveling 60.0 km/h to can brake to a stop within a distance of 20.0 m. If the car is going twice as fast, 120 km/h, what is its stopping distance ?

Example

(a)

(b)



(1) $W_{net} = F\, d_{(a)} \cos 180^o$

$= -F\, d_{(a)} = 0 - m\, v_{(a)}^2 / 2$

→ $-F \times (20.0\text{ m}) = -m\,(16.7\text{ m/s})^2 / 2$

(2) $W_{net} = F\, d_{(b)} \cos 180^o$

$= -F\, d_{(b)} = 0 - m\, v_{(b)}^2 / 2$

→ $-F \times (d) = -m\,(33.3\text{ m/s})^2 / 2$

(3) F & m are common. Thus, d = 80.0 m

example

If a Saturn V rocket with an Apollo spacecraft attached has a combined mass of 2.9×10^5 kg and is to reach a speed of $11.2\,\frac{\text{km}}{\text{s}}$, how much kinetic energy will it then have?

(Convert some units first.) The speed of the rocket will be

$$v = \left(11.2\,\frac{\text{km}}{\text{s}}\right)\left(\frac{10^3\,\text{m}}{1\,\text{km}}\right) = 1.12 \times 10^4\,\frac{\text{m}}{\text{s}}\ .$$

We know its mass: $m = 2.9 \times 10^5$ kg. Using the definition of kinetic energy, we have

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(2.9 \times 10^5\,\text{kg})(1.12 \times 10^4\,\tfrac{\text{m}}{\text{s}})^2 = 1.8 \times 10^{13}\,\text{J}$$

The rocket will have 1.8×10^{13} J of kinetic energy.

example

If an electron (mass $m = 9.11 \times 10^{-31}$ kg) in copper near the lowest possible temperature has a kinetic energy of 6.7×10^{-19} J, what is the speed of the electron?

$$\text{kinetic energy, } K = \frac{1}{2}mv^2$$

$$v^2 = \frac{2K}{m} = \frac{2(6.7 \times 10^{-19}\ \text{J})}{(9.11 \times 10^{-31}\ \text{kg})} = 1.47 \times 10^{12}\ \frac{\text{m}^2}{\text{s}^2}$$

$$v = 1.21 \times 10^{6}\ \frac{\text{m}}{\text{s}}$$

Problem Say a hockey player hits a $.50\,kg$ puck with a force of $100N$ over a distance of $.30m$. What speed does the puck leave the stick with (neglecting friction)?

Δ k= $W = (100\,N)(.3m) = 30\,J$

Notice that a joule is a newton-meter. The work is also the change in kinetic energy. The initial kinetic energy is zero, because the puck starts at zero velocity. The change in kinetic energy is therefore

$$\Delta KE = \frac{1}{2}mv^2$$

where v is the final speed. Comparing the two, we see that

$$v^2 = \frac{2W}{m} = \frac{2 \times 30\,N}{.5\,kg} = 120m^2/s^2$$

Taking the square root gives $v = 11m/s$.

I sweep the ice with a $1\,kg$ broom, using a force of $20N$ at an angle of 45^o with the horizontal. How far do I have to push it to give it a speed of $4m/s$, neglecting friction?

The change in kinetic energy of the broom is

$$\Delta KE = \frac{1}{2}mv^2 = \frac{1}{2}1\,kg(4m/s)^2 = 8J$$

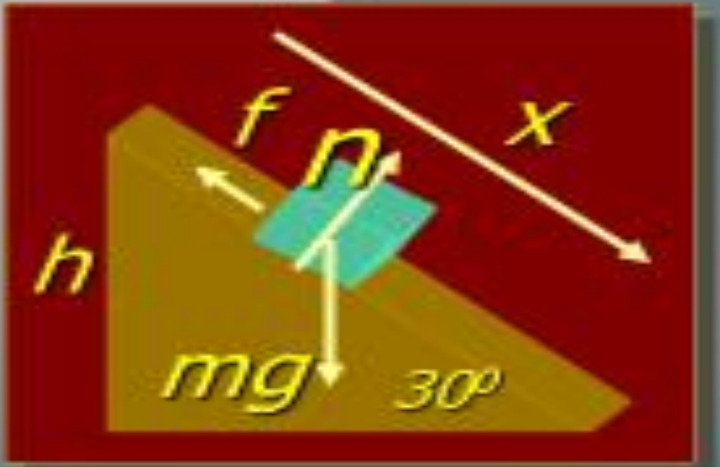
The angle between the force of my pushing is and the displacement is 45^o, so the work in the frictionless case is

$$W = Fd\cos(45^o) = (20\,N)d\frac{1}{\sqrt{2}}$$

Since $W = \Delta KE$, we have

$$d_{frictionless} = \frac{\Delta KE}{F\cos(45^o)} = \frac{8\,J}{20/\sqrt{2}\;N} = .57\,m = 57cm$$

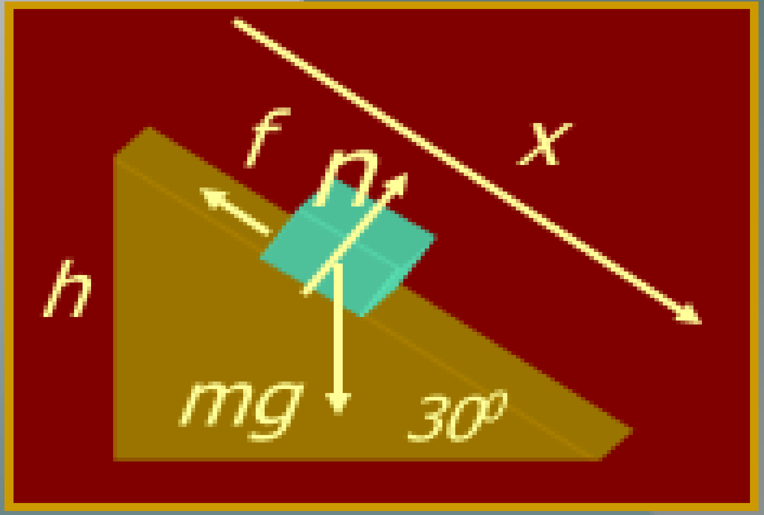
Example 3: A 4-kg block slides from rest at top to bottom of the 30^0 inclined plane. Find velocity at bottom. (h = 20 m and μ_k = 0.2)

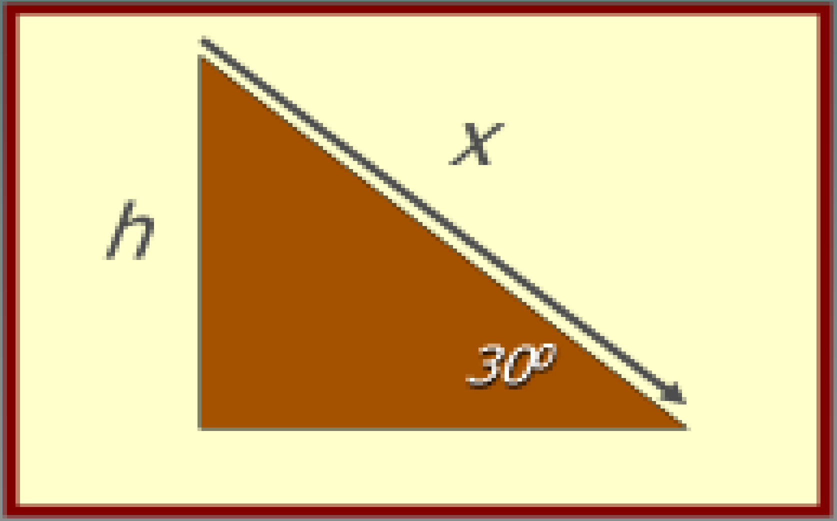


Plan: *We must calculate both the resultant work and the net displacement x. Then the velocity can be found from the fact that Work = ΔK.*

Resultant work = (Resultant force down the plane) x (the displacement down the plane)

Example 3 (Cont.): We first find the net displacement *x* down the plane:



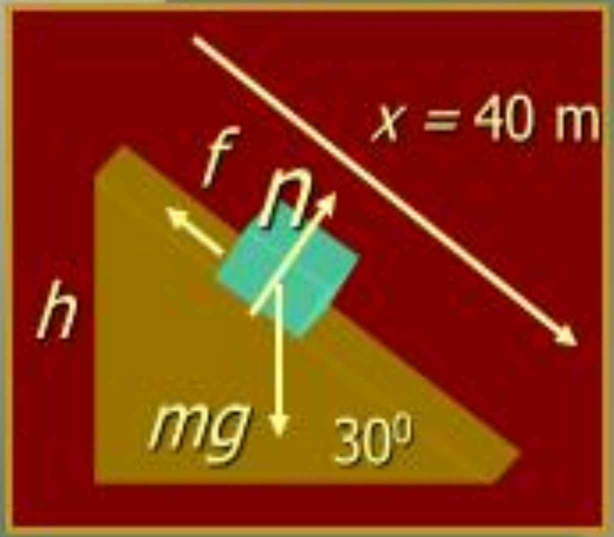


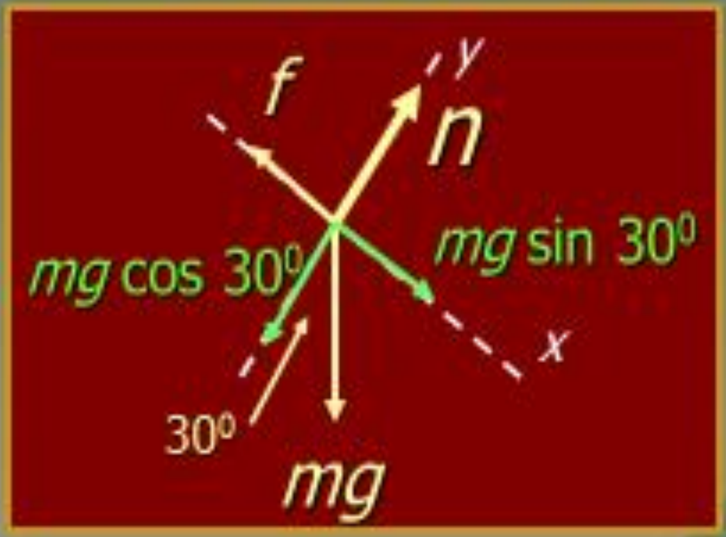
From trig, we know that the Sin 30⁰ = h/x and:

$$\sin 30^0 = \frac{h}{x} \qquad x = \frac{20\text{ m}}{\sin 30^0} = 40\text{ m}$$

Example 3(Cont.): Next we find the resultant work on 4-kg block. (x = 40 m and μ_k = 0.2)

Draw free-body diagram to find the resultant force:

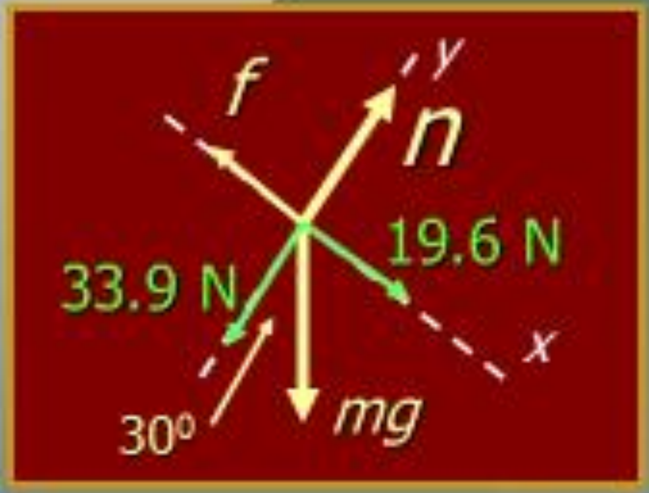




$$W_x = (4\ \text{kg})(9.8\ \text{m/s}^2)(\sin 30^0) = 19.6\ \text{N}$$

$$W_y = (4\ \text{kg})(9.8\ \text{m/s}^2)(\cos 30^0) = 33.9\ \text{N}$$

Example 3(Cont.): Find the resultant force on 4-kg block. (x = 40 m and μ_k = 0.2)



Resultant force down plane: 19.6 N - f

Recall that $f_k = \mu_k n$

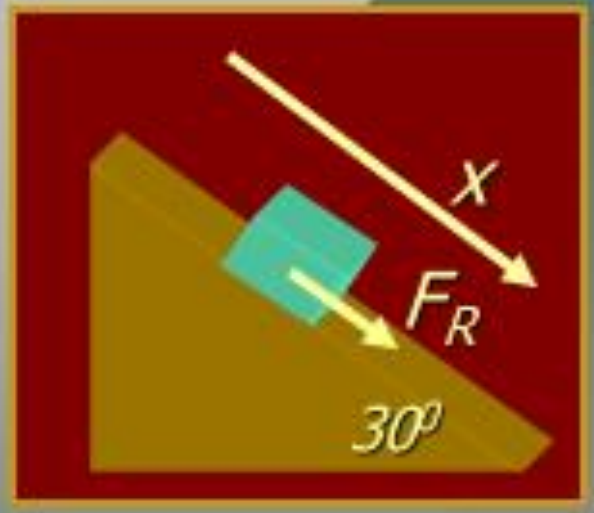
$\Sigma F_y = 0$ or <u>n = 33.9 N</u>

Resultant Force = 19.6 N – $\mu_k n$; and $\mu_k = 0.2$

Resultant Force = 19.6 N – (0.2)(33.9 N) = 12.8 N

Resultant Force Down Plane = 12.8 N

Example 3 (Cont.): The resultant work on 4-kg block. (x = 40 m and F_R = 12.8 N)



$$(Work)_R = F_R x$$

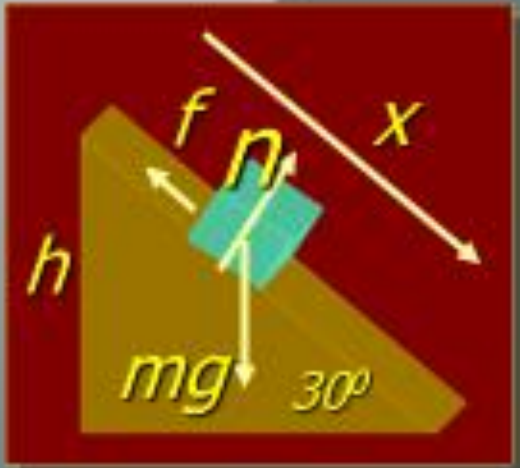
$$Net\ Work = (12.8\text{ N})(40\text{ m})$$

$$Net\ Work = 512\text{ J}$$

Finally, we are able to apply the work-energy theorem to find the final velocity:

$$Work = \tfrac{1}{2}mv_f^2 - \cancelto{0}{\tfrac{1}{2}mv_0^2}$$

Example 3 (Cont.): A 4-kg block slides from rest at top to bottom of the 30^0 plane. Find velocity at bottom. (h = 20 m and μ_k = 0.2)



Resultant Work = 512 J

Work done on block equals the change in K. E. of block.

$$\tfrac{1}{2} m v_f^2 - \cancel{\tfrac{1}{2} m v_o^2}^{\,0} = \text{Work}$$

$$\tfrac{1}{2} m v_f^2 = 512 \text{ J}$$

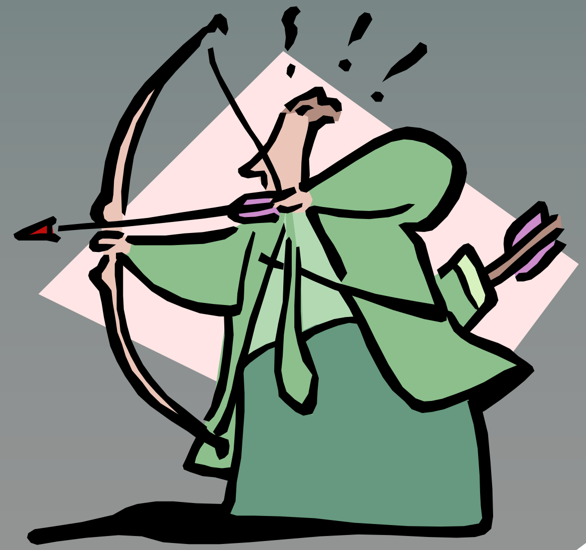
$$\tfrac{1}{2}(4 \text{ kg}) v_f^2 = 512 \text{ J}$$

$$\boxed{v_f = 16 \text{ m/s}}$$

Potential Energy

Potential Energy: *Ability to do work by virtue of position or condition – Potential is stored energy (Statics) Dependant on height*

A suspended weight



A stretched bow

Potential Energy

- Gravitational Potential Energy = weight X height

$$PE = mgh$$

Work done against gravity

Mass (k g)

Height object raised (m)

Work (joules) → $W = mgh$

Gravity (m/sec^2)

Example : What is the potential energy of a 50-kg person in a skyscraper if he is 480 m above the street below?

Gravitational Potential Energy

What is the P.E. of a 50-kg person at a height of 480 m?

$U = mgh = (50\ \text{kg})(9.8\ \text{m/s}^2)(480\ \text{m})$

$\boxed{U = 235\ \text{kJ}}$

example

How much potential energy is lost by a 5Kg object to kinetic energy due a decrease in height of 4.5 m

PE = mgh
PE = (5Kg)(10 m/s^2)(4.5 m)
PE = 225 Kg m^2/s^2
PE = 225 J

example

A ball rolls off a table and hits the floor at 5m/s. What is the height of the table.

Initial energy = Ep

Final energy = Ek

But conservation of energy tells us Ep = Ek.

$$\text{So } mgh = \frac{1}{2}mv^2$$

$$2gh = v^2$$

$$h = \frac{v^2}{2g} = \frac{25}{20}$$

$$\underline{h = 1.25\text{m}}$$

Types of Forces

Conservative Forces

- A force is conservative if the work it does is independent of the path.
  - The work depends only upon the initial and final positions of the object
  - Any conservative force can have a potential energy function associated with it
- Examples of conservative forces include:
  - Gravity
  - Spring force
  - The Electric force between charged objects.

Work Done by Gravity

Slide block down incline

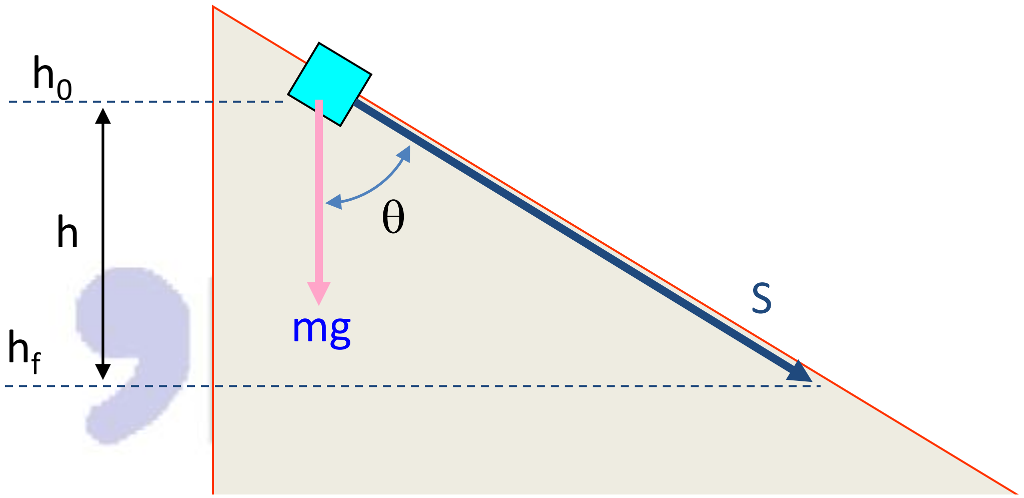
$$W_g = (mg)(S)\cos\theta$$

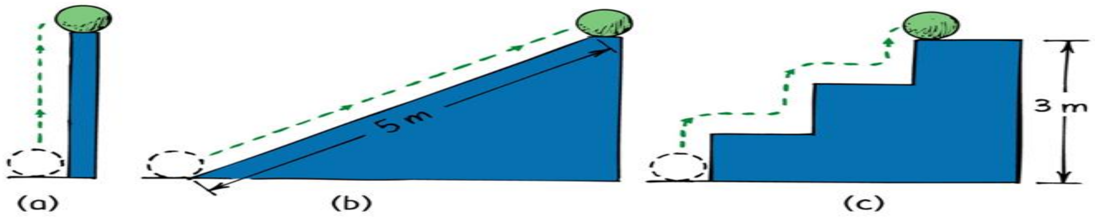
$$S = h/\cos\theta$$

$$W_g = mg(h/\cos\theta)\cos\theta$$

$$W_g = mgh$$

with $h = h_0 - h_f$





$\Rightarrow$Work done by gravity is <u>independent of path</u> taken between h_0 and h_f

=> <u>The gravitational force is a conservative force.</u>

Non conservative Forces

- A force is non conservative if the work it does on an object depends on the path taken by the object between its final and starting points.
- Mechanical energy can be lost or increased.
- Examples of non conservative forces
  - kinetic friction. Work done by the Friction depends on the path.

    When a body slides up & then back down to the starting point,

    The total work done by the friction is not zero.
  - Fluid(Air) Resistance

Law of Conservation of Energy

- If we ignore non conservative forces (friction and the such), the implication is that no non-mechanical energies are present (heat, sound, light, etc) therefore…

$$\Delta KE + \Delta PE = 0$$

Law of conservation of mechanical energy:

$$K_i + U_i = K_f + U_f$$

"In a system in which only conservative forces do work, the total mechanical energy remains constant"

$$E_f = E_i$$

- Energy is never created or destroyed. It only changes form.

Work done by Friction

$\triangle KE + \triangle PE = W(friction)$

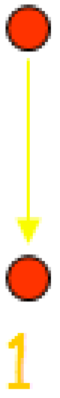
Ex: Mark skateboards down a curved playground ramp. R=3m. Total mass= 25kg. His speed at the bottom=6m/s. What work was done by the friction?

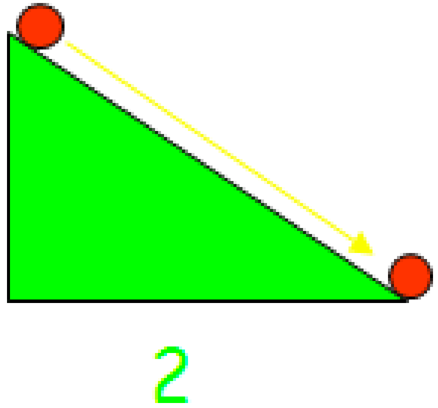
Total Energy of a system, including non mechanical form of energy, is conserved.

$\triangle KE + \triangle PE + \triangle U = 0$

Imagine that you are comparing three different ways of having a ball move down through the same height. In which case does the ball reach the bottom with the highest speed?

1. Dropping
2. Slide on ramp (no friction)
3. All the same



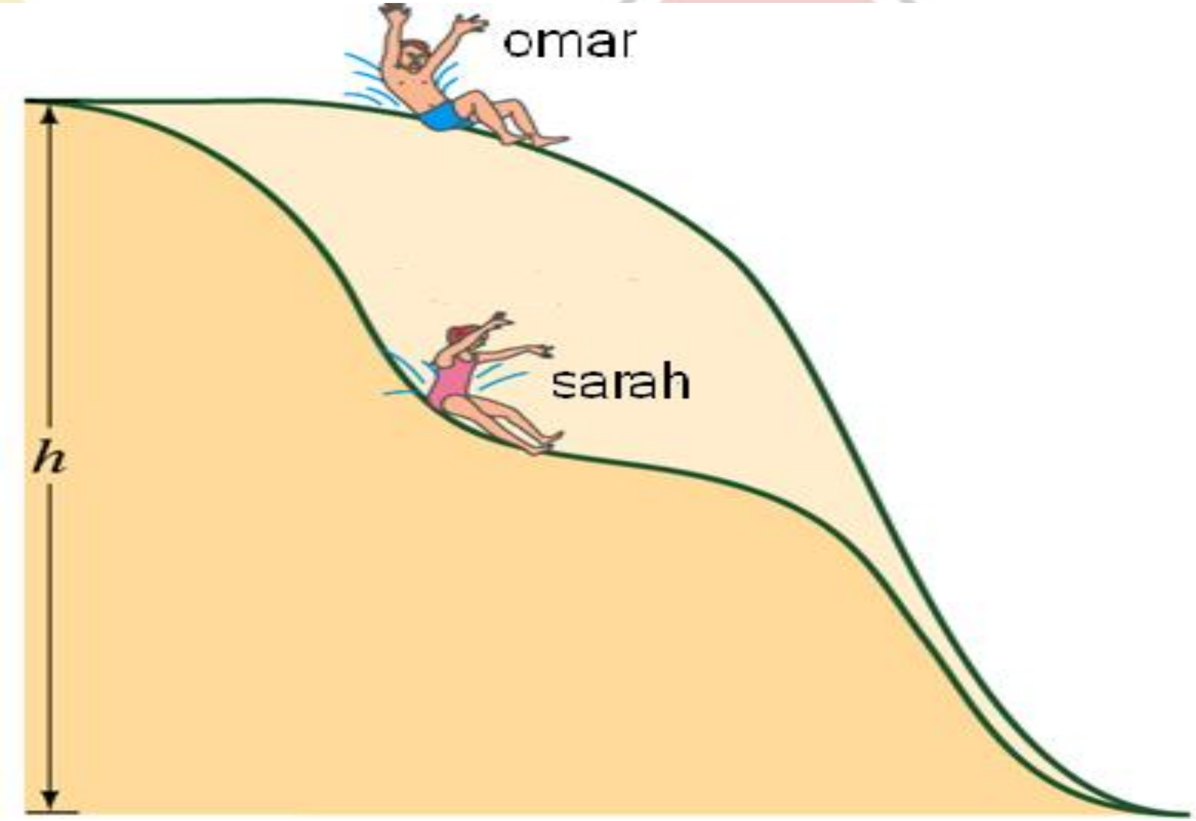


In all three experiments, the balls fall from the same height and therefore the same amount of their gravitational potential energy is converted to kinetic energy. If their kinetic energies are all the same, and their masses are all the same, the balls must all have the same speed at the end.

Who is going faster at the bottom?

- Assume no friction
- Assume both have the same speed pushing off at the top

same



Two marbles, one twice as heavy as the other, are dropped to the ground from the roof of a building. Without air resistance, just before hitting the ground, the heavier marble has

A. as much kinetic energy as the lighter one.
B. twice as much kinetic energy as the lighter one.
C. half as much kinetic energy as the lighter one.
D. four times as much kinetic energy as the lighter one.

Continued from Concept Test , which marble has a greater speed just before hitting the ground?

A. The heavier one.
B. The lighter one.
C. They have the same speed.
D. Need more information.

$$\Delta KE = -\Delta PE$$

$$\frac{1}{2}\cancel{m}v^2 - 0 = -(0 - \cancel{m}gh)$$

$$\Rightarrow v = \sqrt{2gh}$$

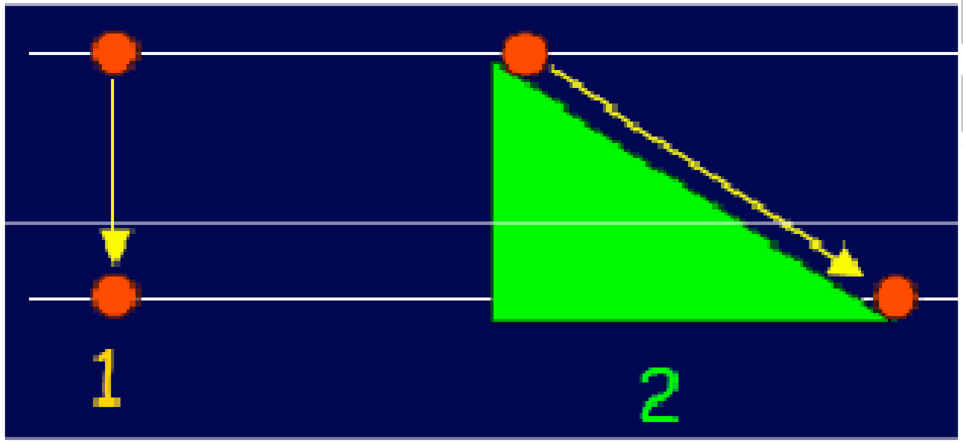
Imagine that you are comparing two different ways of having a ball move down through the same height. In which case does the ball reach the bottom with the highest speed?

A. Dropping
B. Slide on ramp (no friction)
C. Both the same

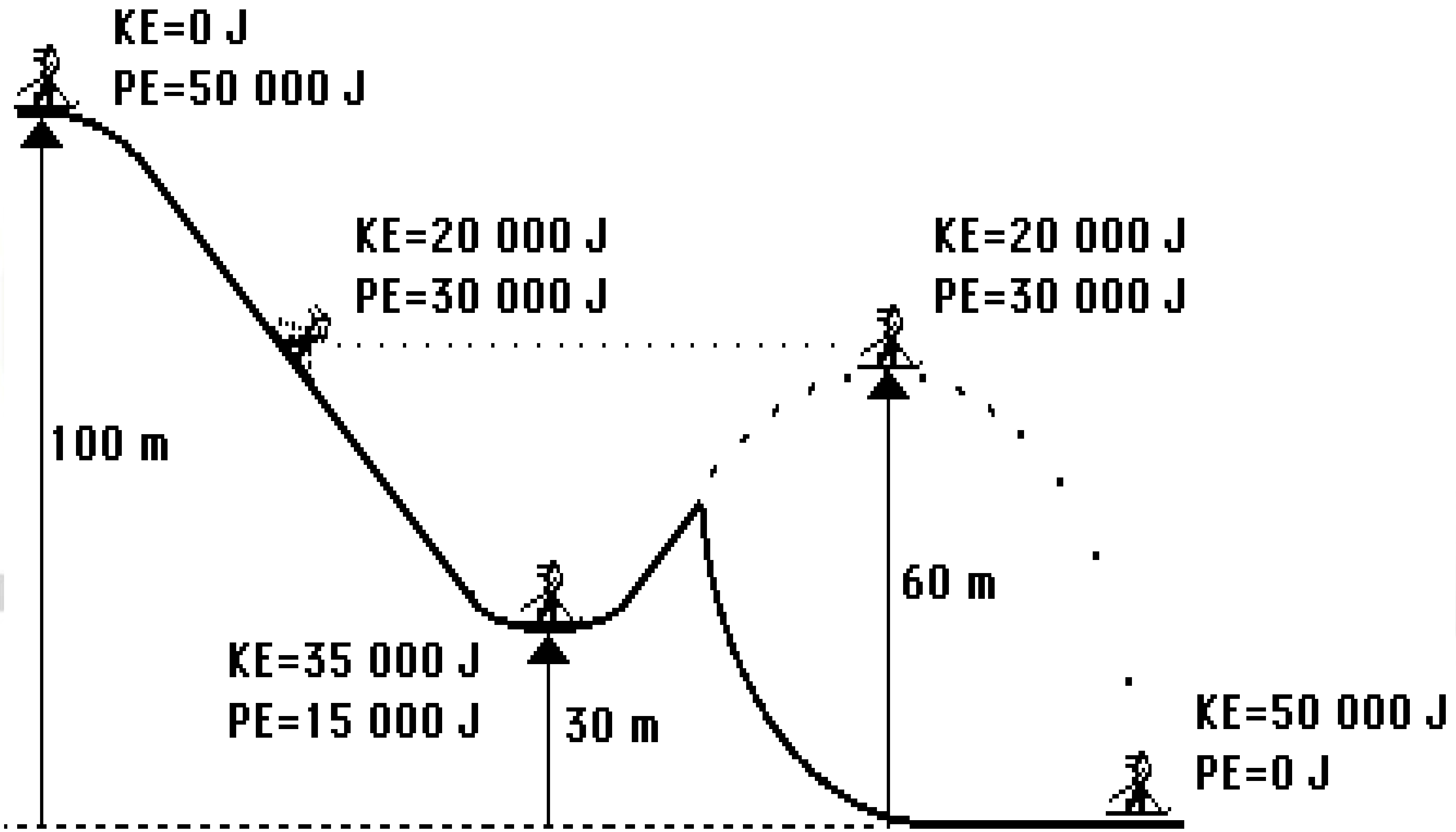
$$KE_i + PE_i = KE_f + PE_f$$

$$\Rightarrow 0 + \cancel{m}gh = \frac{1}{2}\cancel{m}v_f^2 + 0$$

$$\Rightarrow v_f = \sqrt{2gh}$$



Weight =500 N


KE=0 J
PE=50 000 J
KE=20 000 J
PE=30 000 J
KE=20 000 J
PE=30 000 J
100 m
60 m
KE=35 000 J
PE=15 000 J
30 m
KE=50 000 J
PE=0 J

example

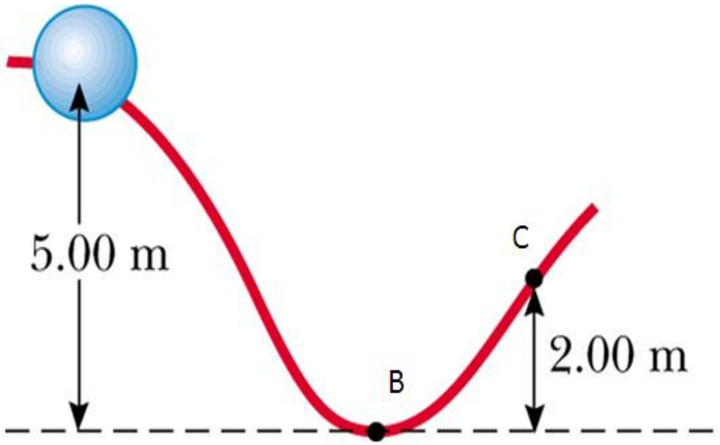
A 0.400-kg bead slides on a curved wire, starting from rest at point **A** in Figure . If the wire is frictionless, find the speed of the bead (a) at **B** and (b) at **C**.

At B

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

$$mgh_i + 0 = 0 + \frac{1}{2}mv_f^2$$

$$(2\ x\ x9.8\ x\ 5)^{1/2} = v$$



At C

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

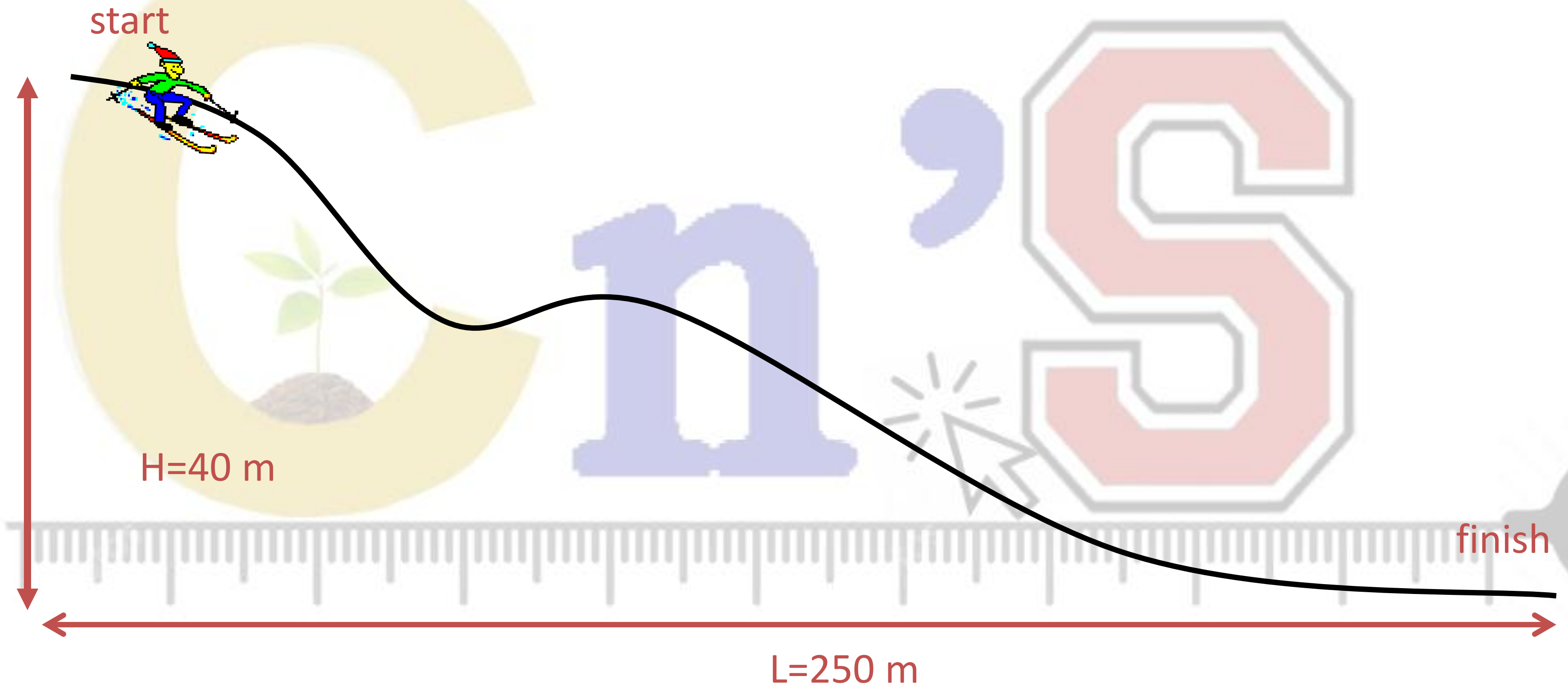
$$0 + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

$$v_i^2 - 2gh_f = v_f^2$$

$$(v_i^2 - 2gh_f)^{1/2} = v_f$$

Example

A skier slides down the frictionless slope as shown. What is the skier's speed at the bottom?



28.0 m/s

Example ?

Suppose the initial kinetic and potential energies of a system are 75J and 250J respectively, and that the final kinetic and potential energies of the same system are 300J and -25J respectively. How much work was done on the system by non-conservative forces?

1. 0J
2. 50J
3. -50J ⟵ correct
4. 225J
5. -225J

Work done by non-conservative forces equals the difference between final and initial kinetic energies plus the difference between the final and initial gravitational potential energies.

W = (300-75) + ((-25) - 250) = 225 - 275 = -50J.

Doing Work to Decrease Energy

- When you catch a ball, its kinetic energy is reduced (or absorbed) by the negative work you do on it
- Your muscles do negative work on your limbs and absorb energy when you land from a jump or fall
- Average force you must exert to absorb energy in catching a ball or landing from a jump or fall depends on how much energy must be absorbed and the displacement over which the force is absorbed
- Safety and protective equipment used in many sports utilizes the work/energy principle to reduce potentially damaging impact forces
- Examples of shock absorbing or energy absorbing materials
  - Landing pads (gymnastics, high jumping, and pole vaulting) increase displacement of the athlete during the impact period
  - Sand (long jumper), water (diver), midsole material in shoes (runner)

[redacted]

Two blocks of mass m_1 and m_2 ($m_1 > m_2$) slide on a frictionless floor and have the same kinetic energy when they hit a long rough stretch ($\mu > 0$), which slows them down to a stop. Which one goes farther?

1) m_1

2) m_2

3) they will go the same distance

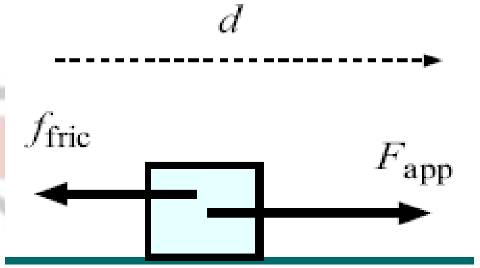
With the same ΔKE, both blocks must have the same work done to them by friction. The friction force is less for m_2 so stopping distance must be greater.

example

A 40 kg box initially at rest is pushed 5.0 m along a rough horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between the box and floor is 0.30, find (a) the work done by the applied force, (b) the energy lost due to friction, (c) the change in kinetic energy of the box, and (d) the final speed of the box.

a-

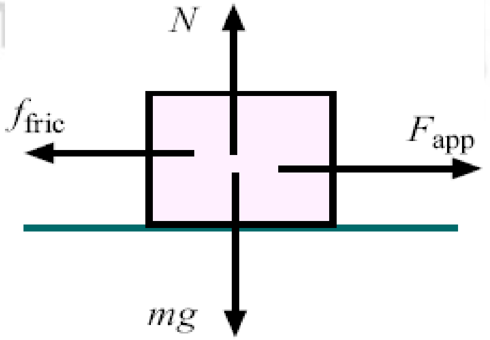
$$W_{\text{app}} = Fd\cos\phi = (130\,\text{N})(5.0\,\text{m})\cos 0^\circ = 6.5\times 10^2\,\text{J}$$



b-

The vertical forces acting on the box are gravity (mg, downward) and the floor's normal force (N, upward). It follows that $N = mg$ and so the magnitude of the friction force is

$$f_{\text{fric}} = \mu N = \mu mg = (0.30)(40\,\text{kg})(9.80\,\tfrac{\text{m}}{\text{s}^2}) = 1.2\times 10^2\,\text{N}$$



The friction force is directed opposite the direction of motion ($\phi = 180°$) and so the work that it does is

$$\begin{aligned} W_{\text{fric}} &= Fd\cos\phi \\ &= f_{\text{fric}}\, d\cos 180° = (1.2 \times 10^2\,\text{N})(5.0\,\text{m})(-1) = -5.9 \times 10^2\,\text{J} \end{aligned}$$

or we might say that 5.9×10^2 J is *lost* to friction.

(c) Since the normal force and gravity do no work on the box as it moves, the net work done is

$$W_{\text{net}} = W_{\text{app}} + W_{\text{fric}} = 6.5 \times 10^2\,\text{J} - 5.9 \times 10^2\,\text{J} = 62\,\text{J} \ .$$

By the work–Kinetic Energy Theorem, this is equal to the change in kinetic energy of the box:

$$\Delta K = K_f - K_i = W_{\text{net}} = 62\,\text{J} \ .$$

(d) Here, the initial kinetic energy K_i was *zero* because the box was initially at rest. So we have $K_f = 62\,\text{J}$. From the definition of kinetic energy, $K = \frac{1}{2}mv^2$, we get the final speed of the box:

$$v_f^2 = \frac{2K_f}{m} = \frac{2(62\,\text{J})}{(40\,\text{kg})} = 3.1\,\frac{\text{m}^2}{\text{s}^2}$$

so that

$$v_f = 1.8\,\frac{\text{m}}{\text{s}}$$

A person pulls a 50-kg crate 40 m along a horizontal floor by a constant force $F_P = 100$ N, which acts at a 37° angle as shown. The floor is smooth and exerts no friction force. Determine (a) the work done by each force acting on the crate, and (b) the net work done on the crate.

SOLUTION (*a*) The work done by the gravitational and normal forces is zero, since they are perpendicular to the displacement $\vec{\mathbf{x}}$ ($\theta = 90°$ in Eq. 7–1):

$$W_G = mgx\cos 90° = 0$$
$$W_N = F_N x\cos 90° = 0.$$

The work done by $\vec{\mathbf{F}}_P$ is

$$W_P = F_P x\cos\theta = (100\,\mathrm{N})(40\,\mathrm{m})\cos 37° = 3200\,\mathrm{J}.$$

(*b*) The net work can be calculated in two equivalent ways:

(1) The net work done on an object is the algebraic sum of the work done by each force, since work is a scalar:

$$\begin{aligned} W_{net} &= W_G + W_N + W_P \\ &= 0 + 0 + 3200\,\mathrm{J} = 3200\,\mathrm{J}. \end{aligned}$$

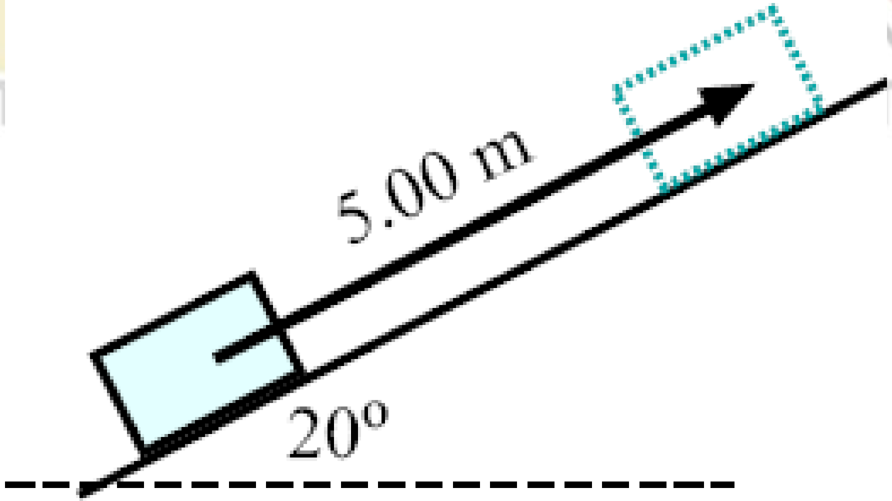
(2) The net work can also be calculated by first determining the net force on the object and then taking its component along the displacement: $(F_{net})_x = F_P\cos\theta$. Then the net work is

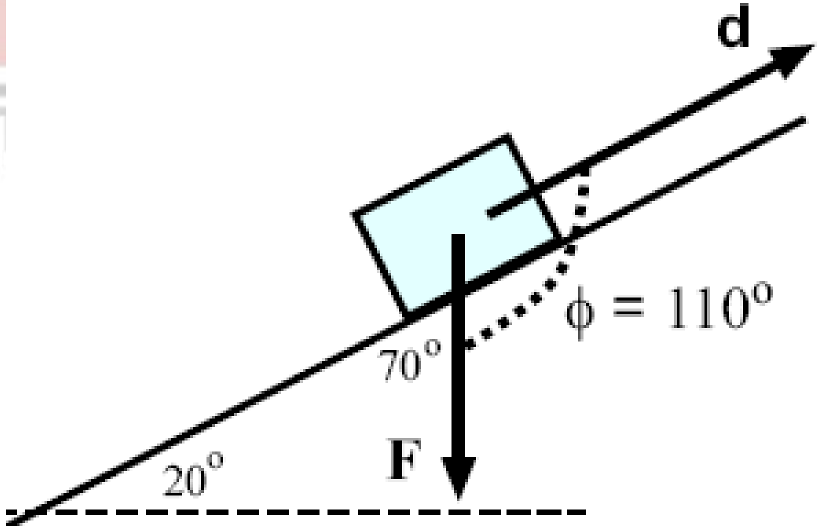
$$\begin{aligned} W_{net} &= (F_{net})_x x = (F_P\cos\theta)x \\ &= (100\,\mathrm{N})(\cos 37°)(40\,\mathrm{m}) = 3200\,\mathrm{J}. \end{aligned}$$

In the vertical (y) direction, there is no displacement and no work done.

example

A crate of mass 10.0 kg is pulled up a rough incline with an initial speed of $1.50\,\frac{\text{m}}{\text{s}}$. The pulling force is 100 N parallel to the incline, which makes an angle of $20.0°$ with the horizontal. The coefficient of kinetic friction is 0.400, and the crate is pulled 5.00 m. (a) How much work is done by gravity? (b) How much energy is lost due to friction? (c) How much work is done by the 100 N force? (d) What is the change in kinetic energy of the crate? (e) What is the speed of the crate after being pulled 5.00 m?





(a) We can calculate the work done by gravity in two ways. First, we can use the definition: $W = \mathbf{F} \cdot \mathbf{d}$. The magnitude of the gravity force is

$$F_{\text{grav}} = mg = (10.0\,\text{kg}(9.80\,\tfrac{\text{m}}{\text{s}^2}) = 98.0\,\text{N}$$

and the displacement has magnitude 5.00 m. We see from geometry (see Fig. 6.4(b)) that the angle between the force and displacement vectors is 110°. Then the work done by gravity is

$$W_{\text{grav}} = Fd\cos\phi = (98.0\,\text{N})(5.00\,\text{m})\cos\,110^\circ = \boxed{-168\,\text{J}}\ .$$

Another way to work the problem is to plug the right values into Eq. 6.10. From simple geometry we see that the change in height of the crate was

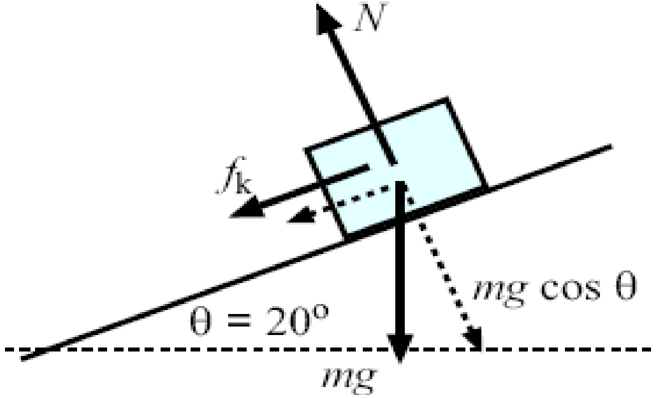
$$\Delta y = (5.00\,\text{m})\sin\,20^\circ = +1.71\,\text{m}$$

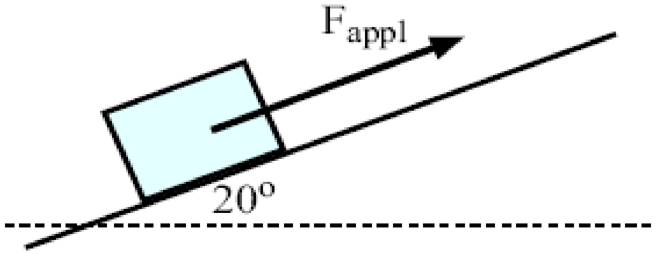
Then the work done by gravity was

$$W_{\text{grav}} = -mg\Delta y = -(10.0\,\text{kg})(9.80\,\tfrac{\text{m}}{\text{s}^2})(1.71\,\text{m}) = -168\,\text{J}$$

(b) To find the work done by friction

$$\begin{aligned} f_{\text{k}} &= \mu_{\text{k}} N = \mu mg\cos\theta \\ &= (0.400)(10.0\,\text{kg})(9.80\,\tfrac{\text{m}}{\text{s}^2})\cos\,20^\circ = 36.8\,\text{N} \end{aligned}$$





This force points exactly opposite the direction of the displacement **d**, so the work done by friction is

$$W_{\text{fric}} = f_k d \cos 180^\circ = (36.8\,\text{N})(5.00\,\text{m})(-1) = -184\,\text{J}$$

(c) The 100 N applied force pulls in the direction up the slope, which is *along* the direction of the displacement **d**. So the work that is does is

$$W_{\text{appl}} = Fd \cos 0^\circ = (100\,\text{N})(5.00\,\text{m})(1) = 500.\,\text{J}$$

$$\begin{aligned} W_{\text{net}} &= W_{\text{grav}} + W_{\text{fric}} + W_{\text{appl}} \\ &= -168\,\text{J} - 184\,\text{J} + 500.\,\text{J} = 148\,\text{J} \end{aligned}$$

the normal force of the surface is perpendicular to the motion, it did no work.

(e) The initial kinetic energy of the crate was

$$K_i \;=\; \tfrac{1}{2}(10.0\,\mathrm{kg})(1.50\,\tfrac{\mathrm{m}}{\mathrm{s}})^2 \;=\; 11.2\,\mathrm{J}$$

If the final speed of the crate is v, then the change in kinetic energy was:

$$\Delta K = K_f - K_i = \tfrac{1}{2}mv^2 - 11.2\,\mathrm{J}\ .$$

Using our answer from part (d), we get:

$$\Delta K \;=\; \tfrac{1}{2}mv^2 - 11.2\,\mathrm{J} = 148\,\mathrm{J} \qquad \Longrightarrow \qquad v^2 = \frac{2(159\,\mathrm{J})}{m}$$

So then:

$$v^2 = \frac{2(159\,\mathrm{J})}{(10.0\,\mathrm{kg})} = 31.8\,\frac{\mathrm{m}^2}{\mathrm{s}^2} \qquad \Longrightarrow \qquad v = 5.64\,\frac{\mathrm{m}}{\mathrm{s}}\ .$$

The final speed of the crate is $5.64\,\frac{\mathrm{m}}{\mathrm{s}}$.

Problem . Two objects are connected by a light string passing over a light frictionless pulley as in the Figure. The object of mass m_1 is released from rest at height h. Using the principle of conservation of energy, (a) determine the speed of m_2 just as m_1 hits the ground.

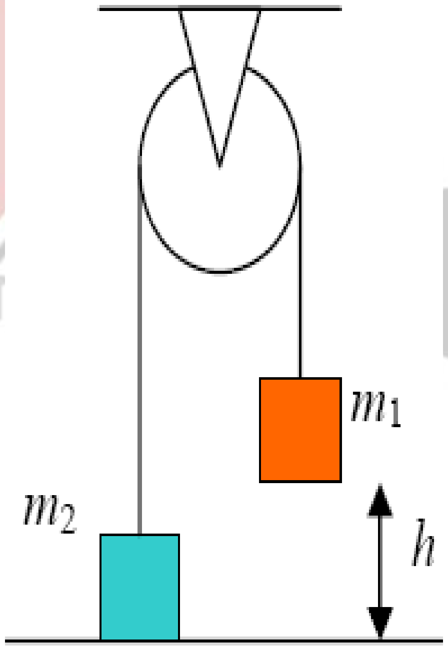
$$K_{1i} + K_{2i} + U_{1i} + U_{2i} = K_{1f} + K_{2f} + U_{1f} + U_{2f}$$

$$\Rightarrow 0 + 0 + m_1 g h_{1i} + m_2 g h_{2i} = \tfrac{1}{2} m_1 v_f^2 + \tfrac{1}{2} m_2 v_f^2 + m_1 g h_{1f} + m_2 g h_{2f}$$

$$\Rightarrow \tfrac{1}{2}(m_1 + m_2) v_f^2 = m_1 g (h_{1i} - h_{1f}) + m_2 g (h_{2i} - h_{2f})$$

$$\Rightarrow \tfrac{1}{2}(m_1 + m_2) v_f^2 = m_1 g (h) + m_2 g (-h) = (m_1 - m_2) g h$$

$$\Rightarrow v_f = \sqrt{\frac{2(m_1 - m_2) g h}{(m_1 + m_2)}}$$

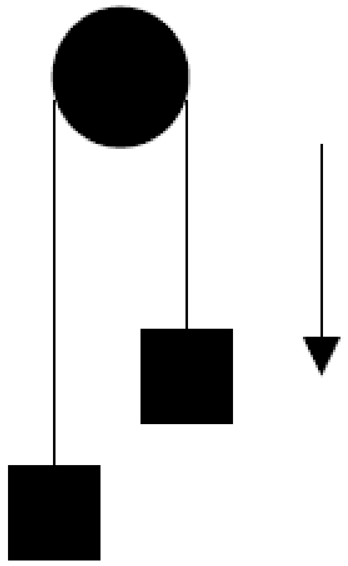


example: Use the *work-energy principle* to determine the final speed of a 5 kg mass (m_1) attached via a light cord over a massless, frictionless pulley to another mass of 3.5 kg (m_2), when the 5 kg mass has fallen (starting from rest) a distance of 2.5 meters.

$$W = \Delta KE_{system} = KE_{f_{sys}} - KE_{i_{sys}} = \sum \vec{F} \cdot \vec{s}$$

$$\sum \vec{F} \cdot \vec{s} = KE_{f_{sys}} - 0 = \frac{1}{2}(m_1 + m_2)v_f^{\,2} - 0$$

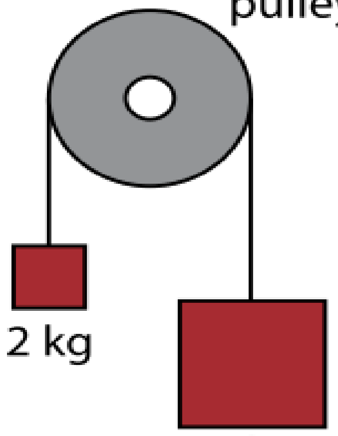
$$v_f = \sqrt{\frac{2\left(\sum \vec{F} \cdot \vec{s}\right)}{m_1 + m_2}} = \sqrt{\frac{2\left[(m_1 g - m_2 g) \cdot s\right]}{m_1 + m_2}} = \sqrt{\frac{2\left[(m_1 - m_2) g \cdot s\right]}{m_1 + m_2}}$$



$$v_f = \sqrt{\frac{2\left[(5kg - 3.5kg)(9.8m \cdot s^{-2}) \cdot 2.5m\right]}{8.5kg}} = 2.94m \cdot s^{-1}$$

Examples

1 Masses of 6.0 kg and 2.0 kg are connected by a light inextensible string passing over a smooth pulley. The string is taut when the masses are released. The smaller mass accelerates upwards and the bigger mass accelerates downwards. Using the Principle of Conservation of Energy, calculate the speed of the masses when the larger one has descended 2.0 m.



Solution

The 2.0 kg mass is accelerating upwards and is gaining both kinetic and potential energy. The 6.0 kg mass is accelerating downwards and is gaining kinetic energy and losing potential energy.

Let the speed of each mass be v (in ms^{-1}) when the larger one has descended 2.0 m.

Net loss in energy of 6.0 kg mass = $\text{p.e.} - \text{k.e.} = mgh - \frac{1}{2}mv^2 = 6 \times g \times 2 - \frac{1}{2} \times 6 \times v^2$
$= 12g - 3v^2$

Net gain in energy of 2.0 kg mass = $\text{p.e.} + \text{k.e.} = mgh + \frac{1}{2}mv^2 = 2 \times g \times 2 + \frac{1}{2} \times 2 \times v^2$
$= 4g + v^2$

By the Principle of Conservation of Energy:
the net loss in energy of the 6.0 kg mass = net gain in energy of the 2.0 kg mass

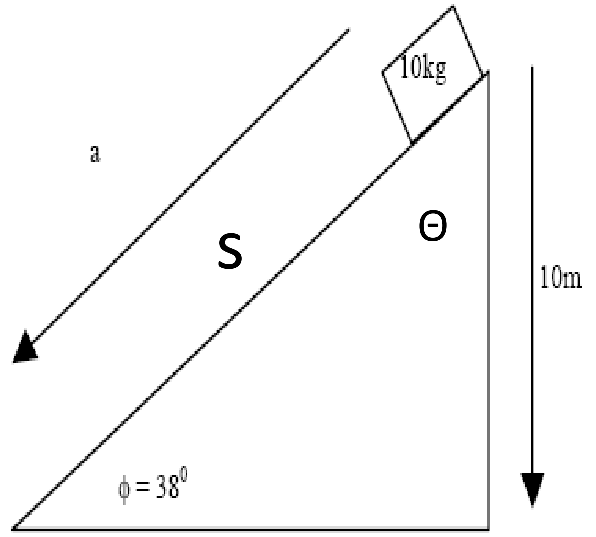
$12g - 3v^2 = 4g + v^2$, which rearranges to give

$8g = 4v^2$, which simplifies to

$v = \sqrt{(2g)} = \sqrt{(2 \times 9.81)} = \mathbf{4.4\ ms^{-1}}$

example

If the surface is smooth what is the speed of the block at the bottom of the incline?



$$\Delta KE = W \Rightarrow \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = Fs\cos\theta$$

$$\frac{1}{2}mv_f^2 - 0 = (mg\sin\phi)(s)(\cos\theta)$$

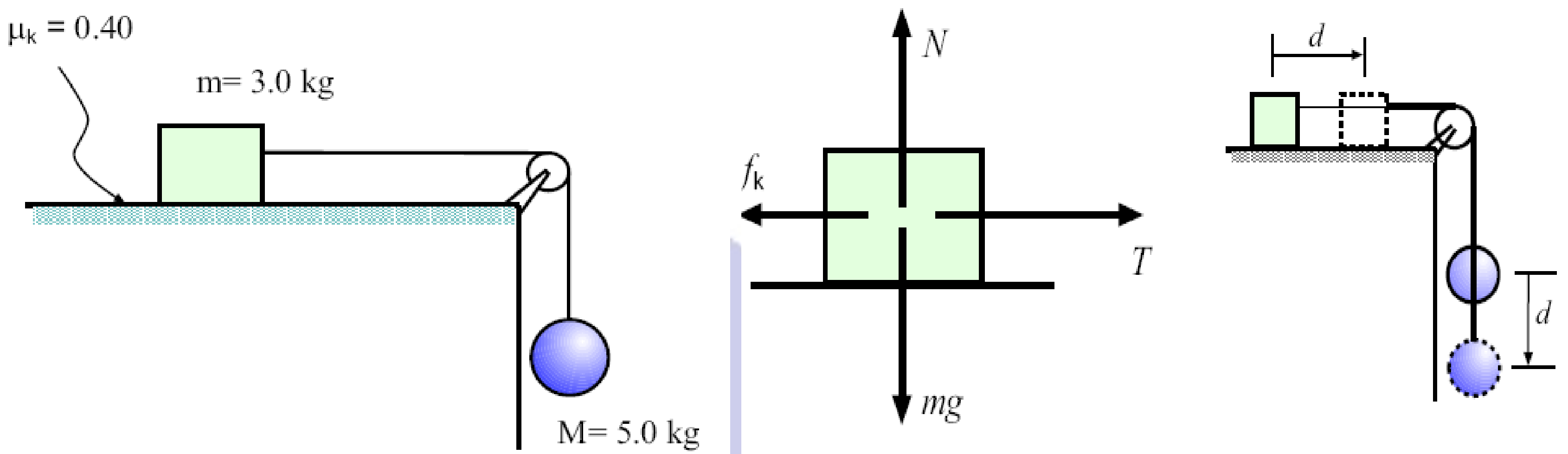
$$\frac{1}{2}v_f^2 = (g\sin\phi)(s)$$

Notice that $(sin\phi)(s)$ = 10 meters, or the initial height of the block above the ground.

$$v_f = \sqrt{2gh} = \sqrt{2(9.8m\cdot s^{-2})(10m)} = 14m\cdot s^{-1}$$

example

The coefficient of friction between the 3.0 kg mass and surface in the Fig. is 0.40. The system starts from rest. What is the speed of the 5.0 kg mass when it has fallen 1.5 m?



When the system starts to move, both masses accelerate; because the masses are connected by a string, *they always have the same speed*. The block (m) slides on the rough surface, and friction does work on it. Since its height does not change, its potential energy does not change, but its kinetic energy increases. The hanging mass (M) drops freely; its potential energy decreases but its kinetic energy increases.

the force of kinetic friction on m has magnitude $\mu_k N = \mu_k mg$.

the work done by friction is

$$W_{\text{fric}} = f_k d \cos\phi = (\mu_k mg)(d)(-1) = -(0.40)(3.0\,\text{kg})(9.80\,\tfrac{\text{m}}{\text{s}^2})(1.5\,\text{m}) = -17.6\,\text{J}\,.$$

Mass m's change in kinetic energy is $\Delta K = \frac{1}{2}(3.0\,\text{kg})v^2 - 0 = (1.5\,\text{kg})v^2$

Mass M's change in kinetic energy is $\Delta K = \frac{1}{2}(5.0\,\text{kg})v^2 - 0 = (2.5\,\text{kg})v^2$

it has a *change in height* given by $-d$, its change in (gravitational) potential energy

$$\Delta U = Mg\Delta y = (5.0\,\text{kg})(9.80\,\tfrac{\text{m}}{\text{s}^2})(-1.5\,\text{m}) = -73.5\,\text{J}$$

Adding up the changes for both masses, the total change in mechanical energy of this system is

$$\begin{aligned}\Delta E &= (1.5\,\text{kg})v^2 + (2.5\,\text{kg})v^2 - 73.5\,\text{J} \\ &= (4.0\,\text{kg})v^2 - 73.5\,\text{J}\end{aligned}$$

$\Delta E = W_{\text{fric}}$ and get:

$$(4.0\,\text{kg})v^2 - 73.5\,\text{J} = -17.6\,\text{J}$$

$$(4.0\,\text{kg})v^2 = 55.9\,\text{J} \qquad \Longrightarrow \qquad v^2 = \frac{55.9\,\text{J}}{4.0\,\text{kg}} = 14.0\,\tfrac{\text{m}^2}{\text{s}^2}$$

$$v = 3.74\,\tfrac{\text{m}}{\text{s}}$$

OR

$$K_{1i} + K_{2i} + U_{1i} + U_{2i} - \mu\, m_1 g\, d = K_{1f} + K_{2f} + U_{1f} + U_{2f}$$

$$\Rightarrow 0 + 0 + m_1 g h_{1i} + m_2 g h_{2i} - \mu\, m_1 g\, d = \tfrac{1}{2} m_1 v_f^{\,2} + \tfrac{1}{2} m_2 v_f^{\,2} + m_1 g\ _{1f} + m_2 g\ _{2f}$$

$$\Rightarrow \tfrac{1}{2}(m_1 + m_2) v_f^{\,2} = m_1 g(h_{1i} - h_{1f}) + m_2 g(h_{2i} - h_{2f}) - \mu\, m_1 g\, d$$

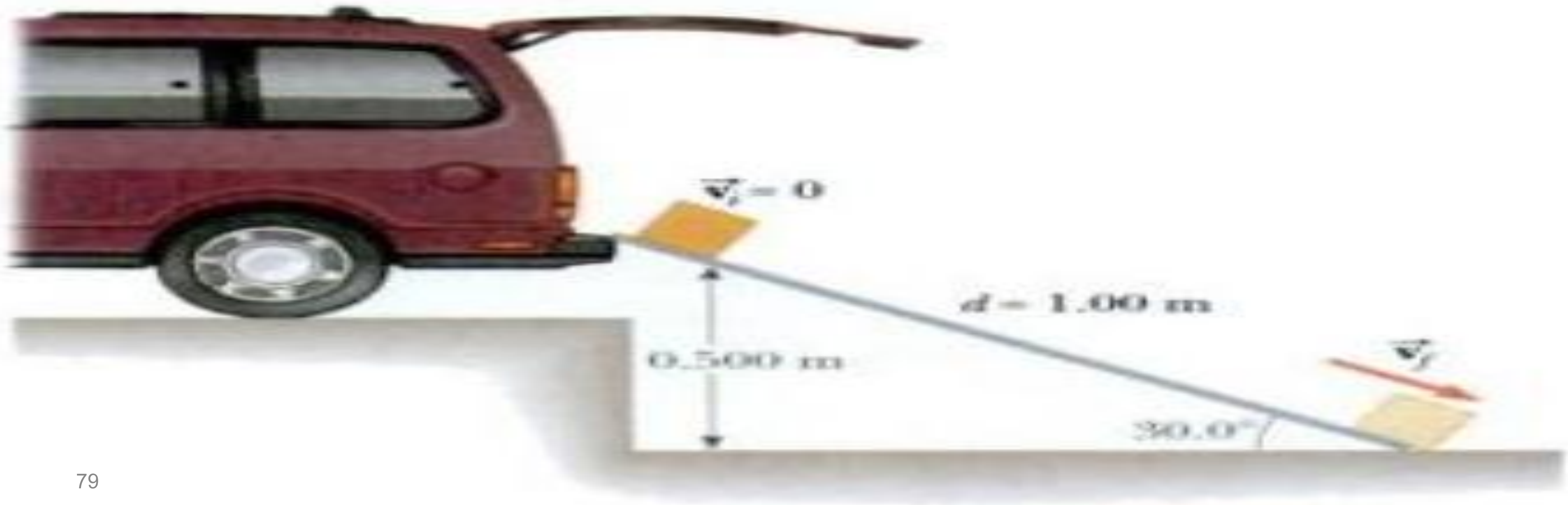
$$\Rightarrow \tfrac{1}{2}(m_1 + m_2) v_f^{\,2} = 0 \qquad + m_2 g(\ h) - \mu\, m_1 g\, d$$

$$v_f^{\,2} = \frac{\mu\, m_1 g\, d + m_2 g(\ h)}{\tfrac{1}{2}(m_1 + m_2)} = \frac{5 \times 9.8 \times 1.5 - 0.4 \times 3 \times 9.8 \times 1.5}{0.5\,(3 + 5)}$$

$$v_f = 3.74\,\tfrac{\text{m}}{\text{s}}$$

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.0° as shown in Figure 8.10. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp.

(A) Use energy methods to determine the speed of the crate at the bottom of the ramp.



Evaluate the total mechanical energy of the system when the crate is at the top:

$$E_i = K_i + U_i = 0 + U_i = mgy_i$$

$$= (3.00\ \text{kg})(9.80\ \text{m/s}^2)(0.500\ \text{m}) = 14.7\ \text{J}$$

Write an expression for the final mechanical energy:

$$E_f = K_f + U_f = \tfrac{1}{2}mv_f^2 + 0$$

$$\Delta E_{\text{mech}} = E_f - E_i = \tfrac{1}{2}mv_f^2 - mgy_i = -f_k d$$

Solve for v_f^2:

$$(1)\quad v_f^2 = \frac{2}{m}(mgy_i - f_k d)$$

Substitute numerical values and solve for v_f:

$$v_f^2 = \frac{2}{3.00\ \text{kg}}[14.7\ \text{J} - (5.00\ \text{N})(1.00\ \text{m})] = 6.47\ \text{m}^2/\text{s}^2$$

$$v_f = 2.54\ \text{m/s}$$

(B) How far does the crate slide on the horizontal floor if it continues to experience a friction force of magnitude 5.00 N?

SOLUTION

Analyze This part of the problem is handled in exactly the same way as part (A), but in this case we can consider the mechanical energy of the system to consist only of kinetic energy because the potential energy of the system remains fixed.

Evaluate the mechanical energy of the system when the crate leaves the bottom of the ramp:

$$E_i = K_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(3.00\text{ kg})(2.54\text{ m/s})^2 = 9.68\text{ J}$$

Apply Equation 8.16 with $E_f = 0$:

$$E_f - E_i = 0 - 9.68\text{ J} = -f_k d$$

Solve for the distance d:

$$d = \frac{9.68\text{ J}}{f_k} = \frac{9.68\text{ J}}{5.00\text{ N}} = 1.94\text{ m}$$

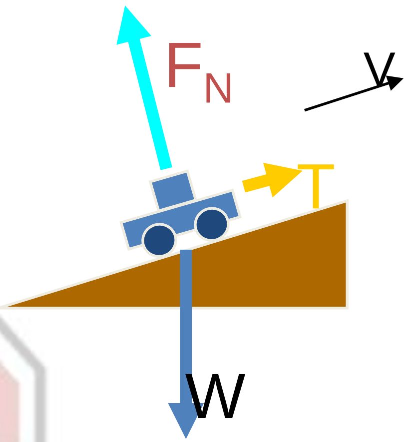
example

You are towing a car up a hill with constant velocity.

A)The work done on the car by the normal force is:

1. positive
2. negative
3. zero ← correct

The normal force is perpendicular to the displacement, hence, does no work.



b)The work done on the car by the gravitational force is:

1. positive
2. negative ← correct
3. zero

With the surface defined as the x-axis, the x component of gravity is in the opposite direction of the displacement, therefore work is negative.

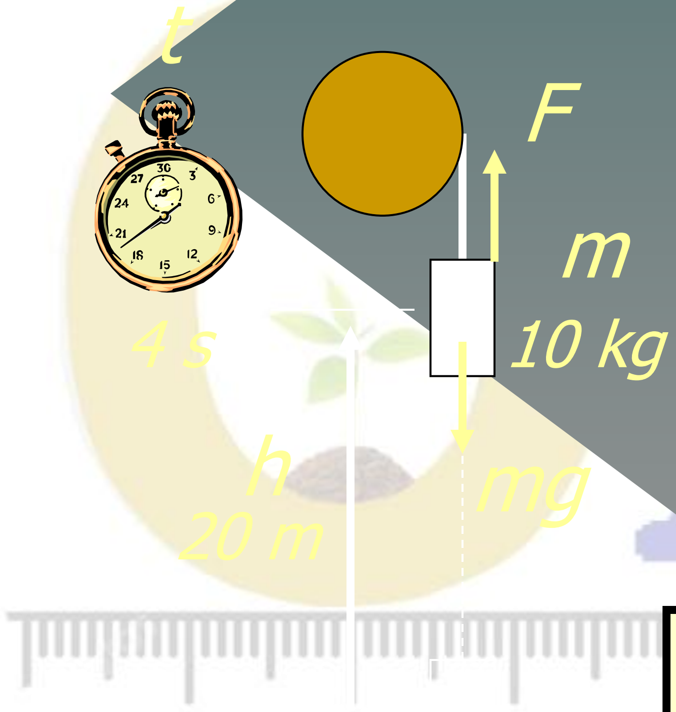
C) The work done on the car by the tension force is:

1. positive ← correct
2. negative
3. zero

Tension is in the same direction as the displacement.

Power

Power is defined as the rate at which work is done: $(P = dW/dt)$



$$\text{Power} = \frac{\text{Work}}{\text{time}} = \frac{F \cdot r}{t}$$

$$P = \frac{mgr}{t} = \frac{(10\text{kg})(9.8\text{m/s}^2)(20\text{m})}{4\text{ s}}$$

$$P = 490\,\text{J/s} \quad \text{or} \quad 490 \text{ watts (W)}$$

Power of 1 W *is work done at rate of* 1 J/s

Units of Power

- *One watt (W) is work done at the rate of one joule per second.*

  1 W = 1 J/s and 1 kW = 1000 W

- Utility sells energy in kilowatt-hours
  1 KWh = 1000 Joules/second times 3600 Seconds = 3.6 X 106 Jo

- *horsepower is work done at the rate of*
  550 ft lb/s)

Example of Power

What power is consumed in lifting a 70-kg *robber* 1.6 m in 0.50 s?

$$P = \frac{Fh}{t} = \frac{mgh}{t}$$

$$P = \frac{(70\text{ kg})(9.8\text{ m/s}^2)(1.6\text{ m})}{0.50\text{ s}}$$

Power Consumed: P = 2220 W

Example 4: A 100-kg cheetah moves from rest to 30 m/s in 4 s. What is the power?

Recognize that work is equal to the change in kinetic energy:

$$Work = \tfrac{1}{2}mv_f^2 - \cancel{\tfrac{1}{2}mv_0^2} \qquad P = \frac{Work}{t}$$

m = 100 kg

$$P = \frac{\tfrac{1}{2}mv_f^2}{t} = \frac{\tfrac{1}{2}(100\text{ kg})(30\text{ m/s})^2}{4\text{ s}}$$

$\boxed{\textit{Power Consumed: } P = 1.22\text{ kW}}$

Power and Velocity

Recall that average or constant velocity is distance covered per unit of time $v = x/t$.

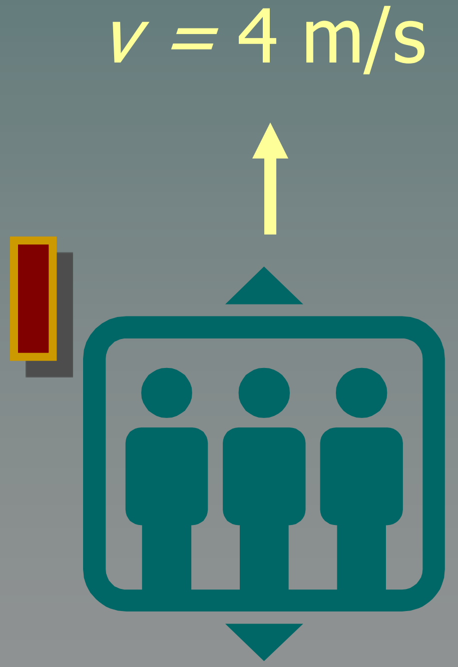
$$P = \frac{Fx}{t} = F\frac{x}{t}$$

$$\boxed{P = F\bar{v}}$$

If power varies with time, then calculus is needed to integrate over time. (Optional)

$$\boxed{Work = \int P(t)\,dt}$$

Example 5: What power is required to lift a 900-kg elevator at a constant speed of 4 m/s?



$$P = F\,\bar{v} = mg\,\bar{v}$$

$$P = (900\text{ kg})(9.8\text{ m/s}^2)(4\text{ m/s})$$

$$\boxed{P = 35.3\text{ kW}}$$

Example 6: What work is done by a 4-hp mower in one hour? The conversion factor is needed: 1 hp = 550 ft lb/s.

$$4\text{hp}\left(\frac{550\text{ft}\cdot\text{lb/s}}{1\text{hp}}\right) = 2200\,\text{ft}\cdot\text{lb/s}$$

$$P = \frac{Work}{t}; \quad Work = Pt$$

$$Work = (2200\,\text{ft}\cdot\text{lb/s})(60\,\text{s})$$

Work = 132,000 ft lb

Summary

Potential Energy: Ability to do work by virtue of position or condition.

$$U = mgh$$

Kinetic Energy: Ability to do work by virtue of motion. (Mass with velocity)

$$K = \frac{1}{2}mv^2$$

The Work-Energy Theorem: The work done by a resultant force is equal to the change in kinetic energy that it produces.

$$\text{Work} = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_o^2$$

Summary (Cont.)

- *Power is defined as the rate at which work is done: (P = dW/dt)*

$$P = \frac{Work}{t}$$

$$Power = \frac{Work}{time} = \frac{F \cdot r}{t}$$

- $P = F\,\bar{v}$

Power of 1 W *is work done at rate of 1* J/s

Example:

Killer whales are able to accelerate up to 30 mi/h in a matter of seconds. Disregarding the considerable drag force of water, calculate the average power a killer whale with mass 8000 kg would need to generate to reach a speed of 12.0 m/s in 6.00 s?

$$\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2}\cdot 8000N(12m/s)^2 - 0$$

$$= 5.76\times 10^5 J$$

$$P = \frac{W}{\Delta t} = \frac{5.76\times 10^5 J}{6.00s} = 9.60\times 10^4 W$$

example

A 100-watt light bulb that is on 10-hours a day for 30 days. How much energy will it use? How much did it cost to operate that lamp if the price is \$.09053 per kWh?

· 10 hours per day x 10 days = 300 hours (energy is all about time of operation)

· 300 Hours x 100 Watts = 30,000 watt hours (energy is also about connected load or Watts)

· A kilowatt (kW) is 1000 watts

· 30,000 watt hours ÷ 1000 watts = 30 kilowatt hours (kWh as on the electric bill

30 kWh x \$.09053 per kWh = \$2.72 cost of operation

Power Delivered by an Elevator Motor

A 1000-kg elevator carries a maximum load of 800 kg. A constant frictional force of 4000 N retards its motion upward. What minimum power must the motor deliver to lift the fully loaded elevator at a constant speed of 3 m/s?

$$F_{net,y} = ma_y$$

Since the speed is constant, so a = 0

$$T - f - Mg = 0$$

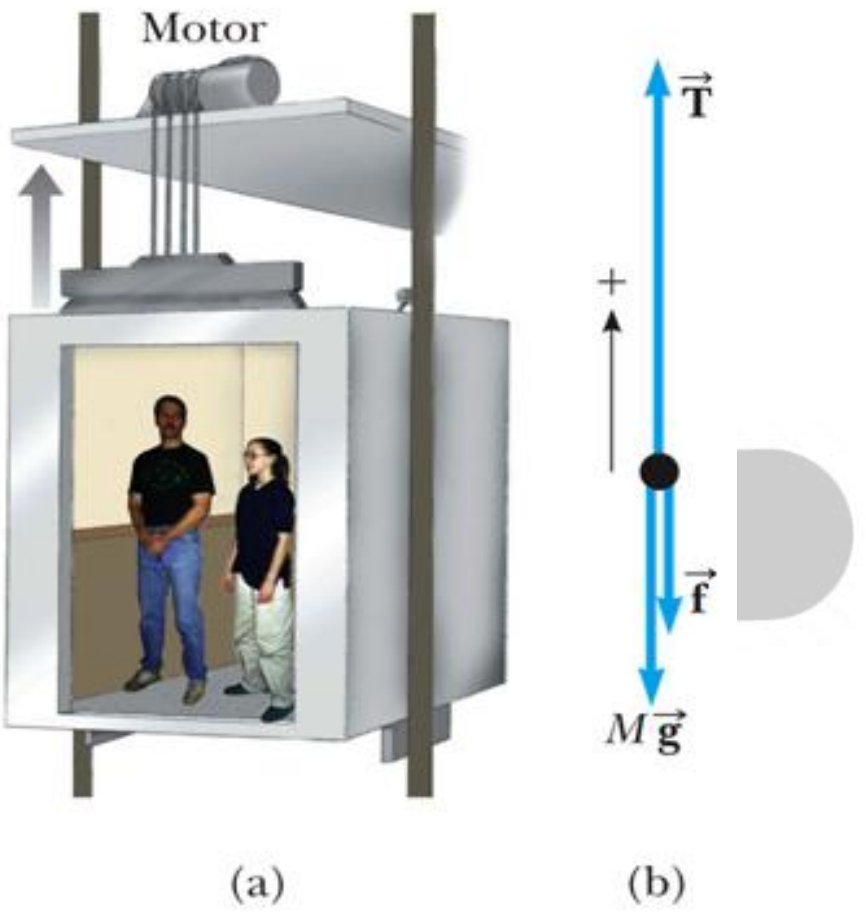
where M = total mass of the system

M= 1000+800=1800 Kg

$$T = f + Mg = 2.16\times10^4\, N$$

$$P = Fv = (2.16\times10^4\, N)(3m/s)$$

$$= 6.48\times10^4 W$$



Summary

- Work: $W = Fd\cos\theta = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}.$
- Kinetic energy is energy of motion: $K = \frac{1}{2}mv^2.$
- Work-energy principle: The net work done on an object equals the change in its kinetic energy. $W_{net} = \Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2.$
- Conservative force: work depends only on end points
- Gravitational potential energy: $U_{grav} = mg\,(h - h_0).$
- Total mechanical energy is the sum of kinetic and potential energies.

$\Delta k = -\Delta u$

$K+u = k_o + u_o$

- Additional types of energy are involved when non conservative forces act.
- Total energy (including all forms) is conserved. $K+u = k_o + u_o + w_a$
- Power: rate at which work is done, or energy is transformed:

$$P = \frac{dW}{dt} = \frac{dE}{dt}, \qquad P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}.$$