ELECTROSTATICS

Learning outcomes:

- uses Coulomb's law to calculate the electrostatic force between two charges.
- states that all charges create electric fields.
- defines electric field intensity.
- uses the equation $F = EQ$ to find the force on a charge placed in an electrostatic (electric) field.
- uses the concept of electric field lines to illustrate the electric field
- draws electric field lines in various electric fields.
- explains the properties of electric field lines.
- calculates the field intensity at a point in an electric field using Coulomb's law.
- finds resultant electric field intensity at a point due to distribution of point charges.
- graphically represents the variation of electric field intensity with the distance from a point charge.

Electric Charge

- Electric Charge: a property of matter that creates a force between objects
- Charge can be positive (+) or negative (-)
- Like charges REPEL
- Opposite charges ATTRACT

Electric Charge

- An object's charge depends on imbalance of protons (+) and electrons (-)
- More protons than electrons → positive
- More electrons than protons → negative

- Units of charge: coulombs (C)
- Protons and electrons have exactly the same amount of charge: 1.6×10^{-19} C
  - Differ in sign (+ or -)

Electrostatics – Elementary Charge

- Since protons and electrons are the smallest whole particles, the charge on any object is a multiple of 1.6×10^{-19} C

- Elementary Charge (e) = 1.6×10^{-19} C

1C of charge is made up of 6.25×10^{18} electrons.

- **Conductor**: **Materials which allow electric charge to flow freely**
  - **Metals are good conductors because their outer electrons are not bound tightly**
  - **Insulator**: **Materials which do not allow electric charge to flow freely (i.e. glass, rubber)**
  - When insulators are charged by rubbing, only the rubbed area becomes charged.
    - There is no tendency for the charge to move into other regions of the material

- **Semiconductor**: Materials that can be made to behave as either a conductor or an insulator of electricity
  - i.e.) germanium, silicon
- **Superconductor**: Material that has infinite conductivity at low temperatures so that charge flows through it without resistance

Conservation of Charge

- The net charge in an isolated system remains constant
- ***Electrons are never created nor destroyed, but are simply transferred from one material to another***

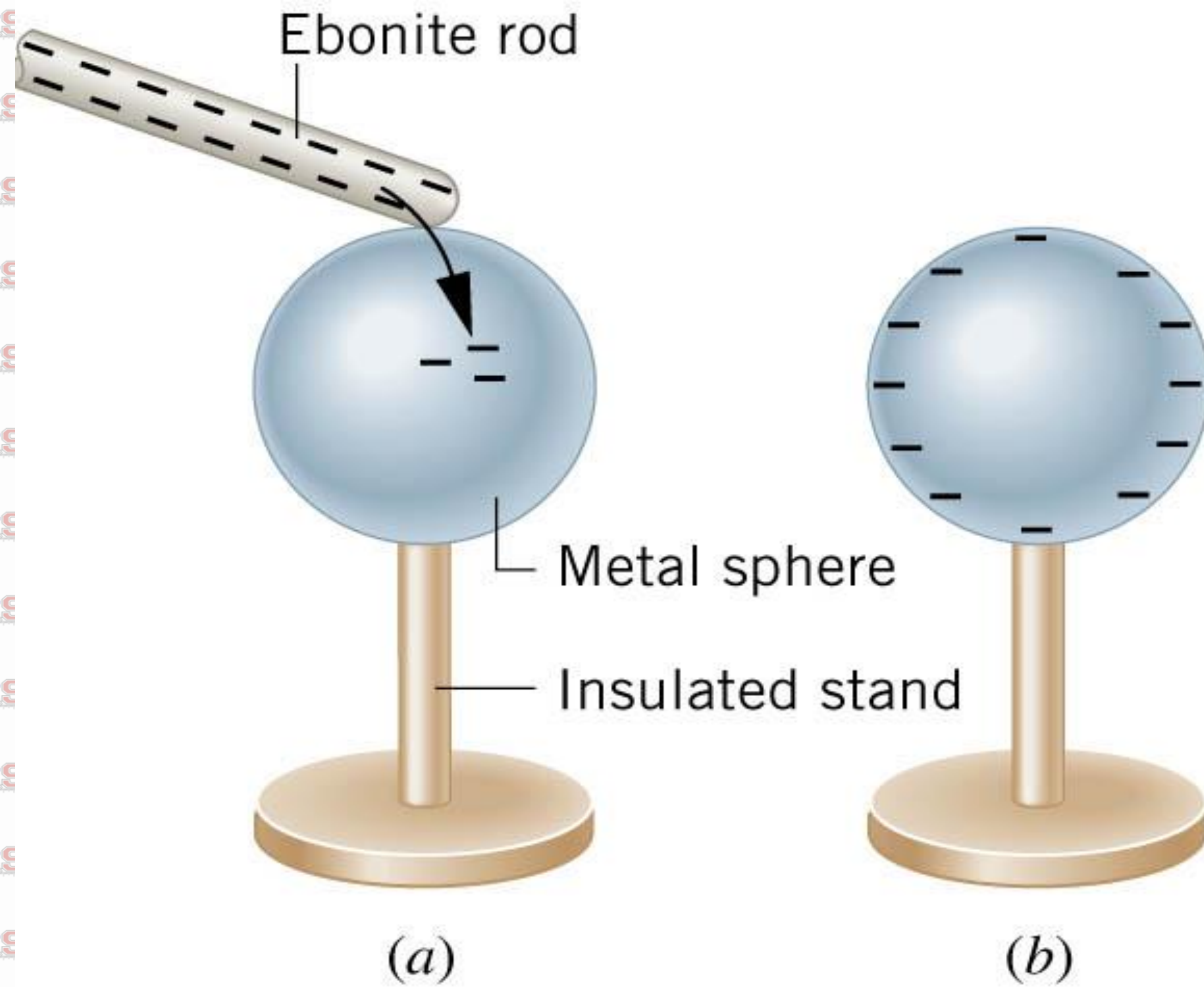
Objects are electrically charged in one of three ways:

- By friction, when electrons are rubbed from one object to another
- By contact, when electrons are transferred through direct contact without rubbing
- Through induction when electrons are gathered or dispersed by the presence of a nearby charge (without physical contact)

Triboelectric Series

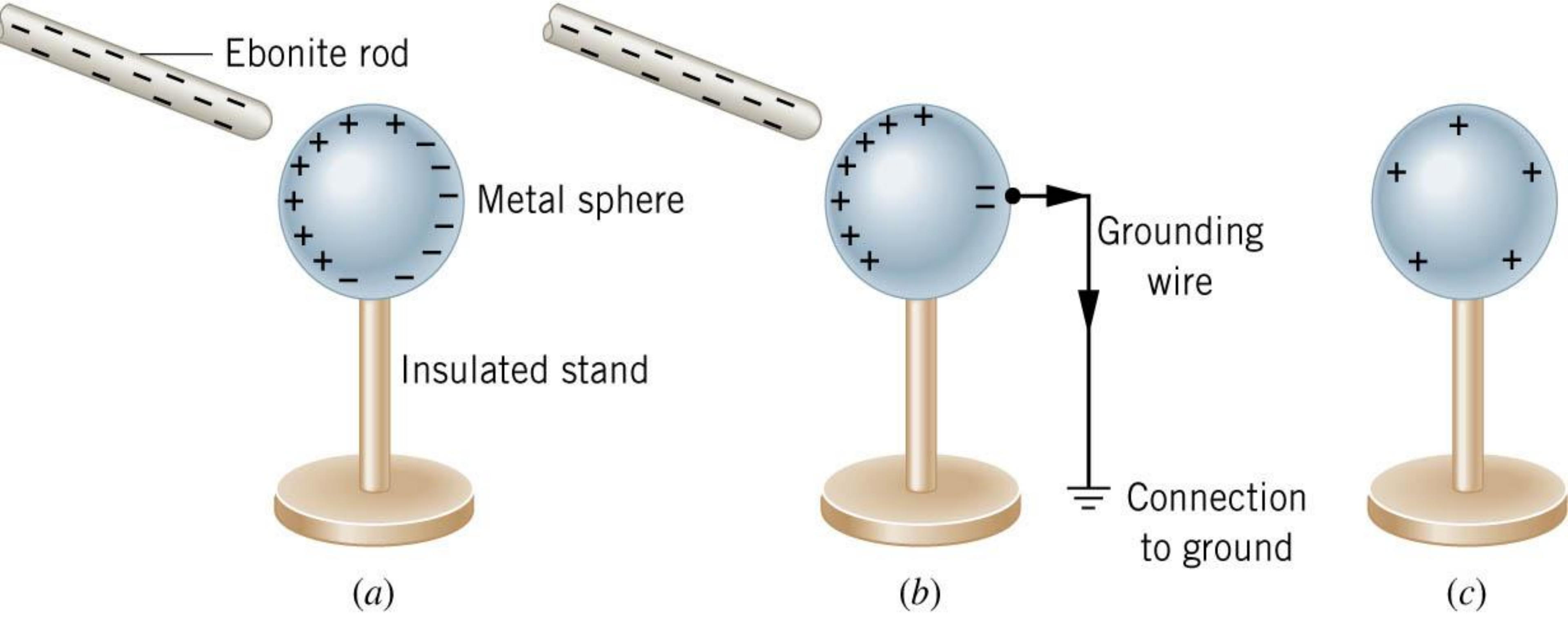
- Asbestos
- Fur (rabbit)
- Glass i.e. +
- Mica
- Wool
- Quartz
- Fur (cat)
- Lead
- Silk i.e. _
- Human skin , Aluminum
- Cotton
- Wood
- Amber
- Copper, Brass
- Rubber
- Sulfur
- Celluloid --India Rubber

If two materials are rubbed together, the one higher on the list should give up electrons and become positively charged.



Charging by contact.

Cn'S



Charging by induction.

Grounding

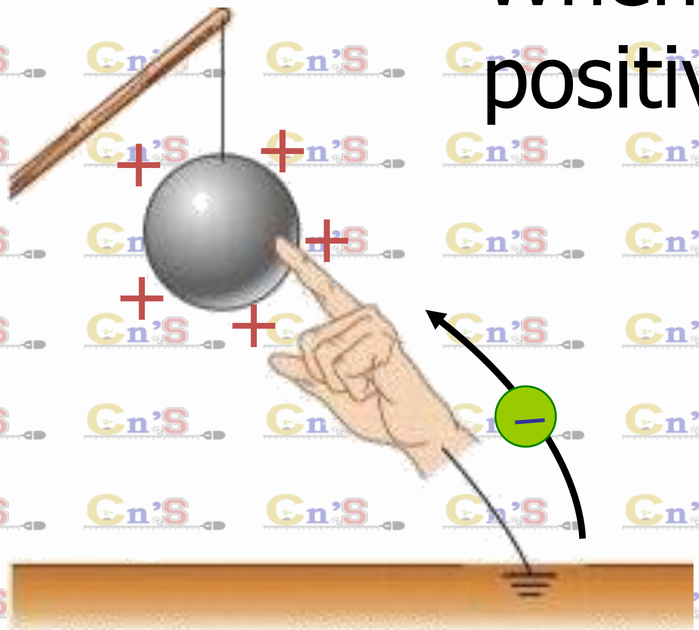
What is grounding?

An object is grounded when it is connected to the earth through a connecting wire.

If a charged conductor is grounded, it will become neutral.

Grounding

b How does grounding occur?



When we touch a metal ball of positive charge...

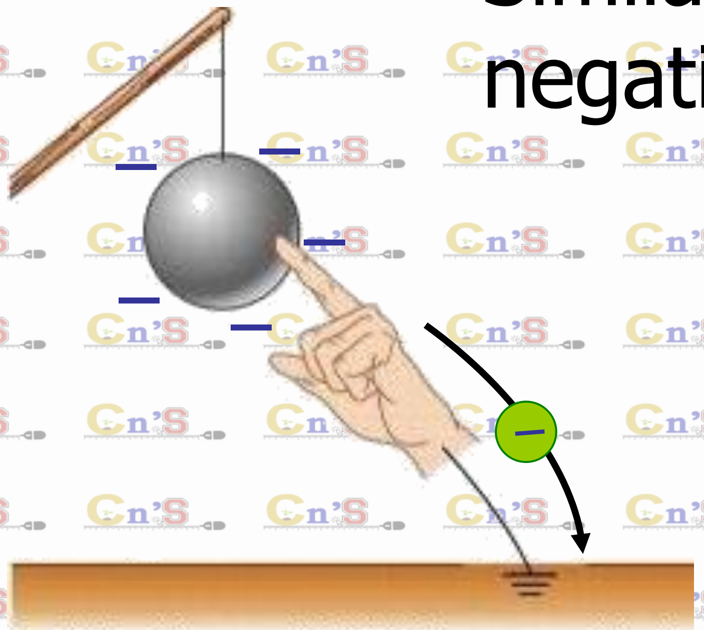
electrons flow from the earth to the metal ball to neutralize the metal ball.

Metal ball becomes neutral.

Grounding

How does grounding occur?

Similarly, if the metal ball is of negative charge...



extra electrons flow from the metal ball to the earth and the ball becomes neutral.

Frictional Electricity:

Frictional electricity is the electricity produced by rubbing two suitable bodies and transfer of electrons from one body to other.

Electrons in glass are loosely bound in it than the electrons in silk. So, when glass and silk are rubbed together, the comparatively loosely bound electrons from glass get transferred to silk.
As a result, glass becomes positively charged and silk becomes negatively charged.

Electrons in fur are loosely bound in it than the electrons in ebonite. So, when ebonite and fur are rubbed together, the comparatively loosely bound electrons from fur get transferred to ebonite.
As a result, ebonite becomes negatively charged and fur becomes positively charged.

It is very important to note that the electrification of the body (whether positive or negative) is due to transfer of electrons from one body to another.

i.e. If the electrons are transferred from a body, then the deficiency of electrons makes the body positive.

If the electrons are gained by a body, then the excess of electrons makes the body negative.

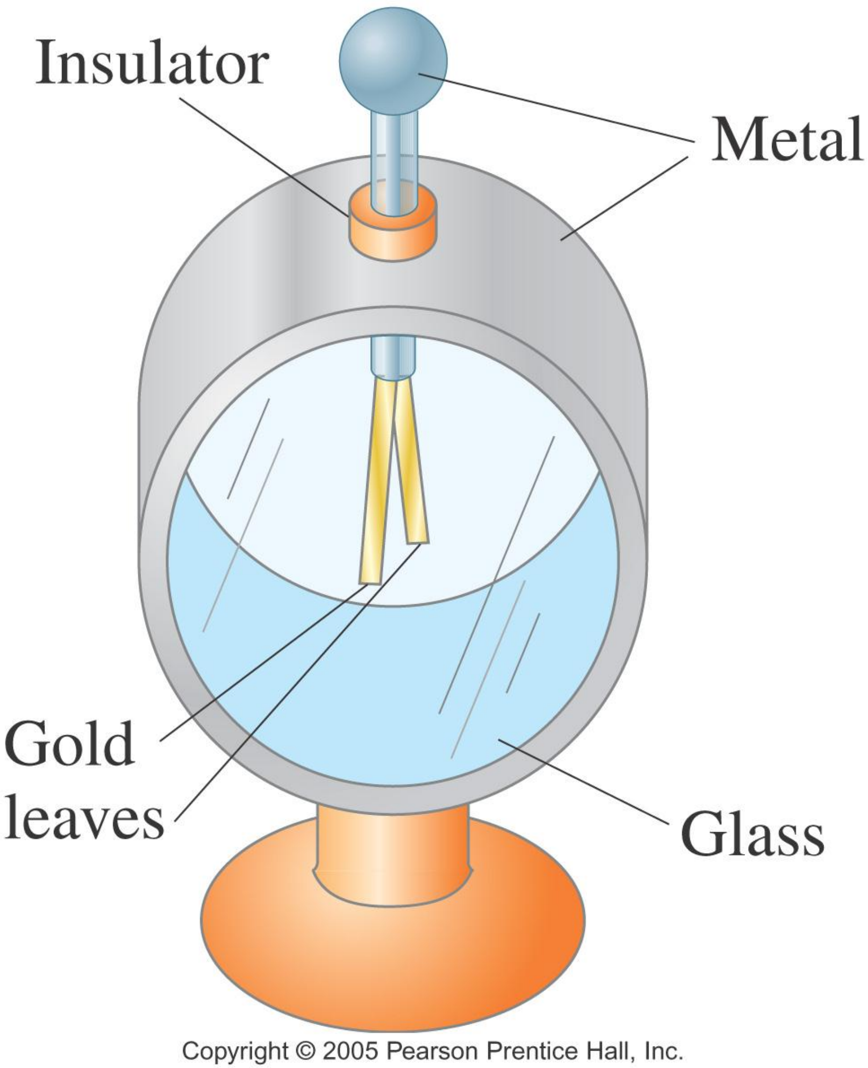
If the two bodies from the following list are rubbed, then the body appearing early in the list is positively charges whereas the latter is negatively charged.

Fur, Glass, Silk, Human body, Cotton, Wood, Sealing wax, Amber, Resin, Sulphur, Rubber, Ebonite.

Column I (+ve Charge)	Column II (-ve Charge)
Glass	Silk
Wool, Flannel	Amber, Ebonite, Rubber, Plastic
Ebonite	Polythene
Dry hair	Comb

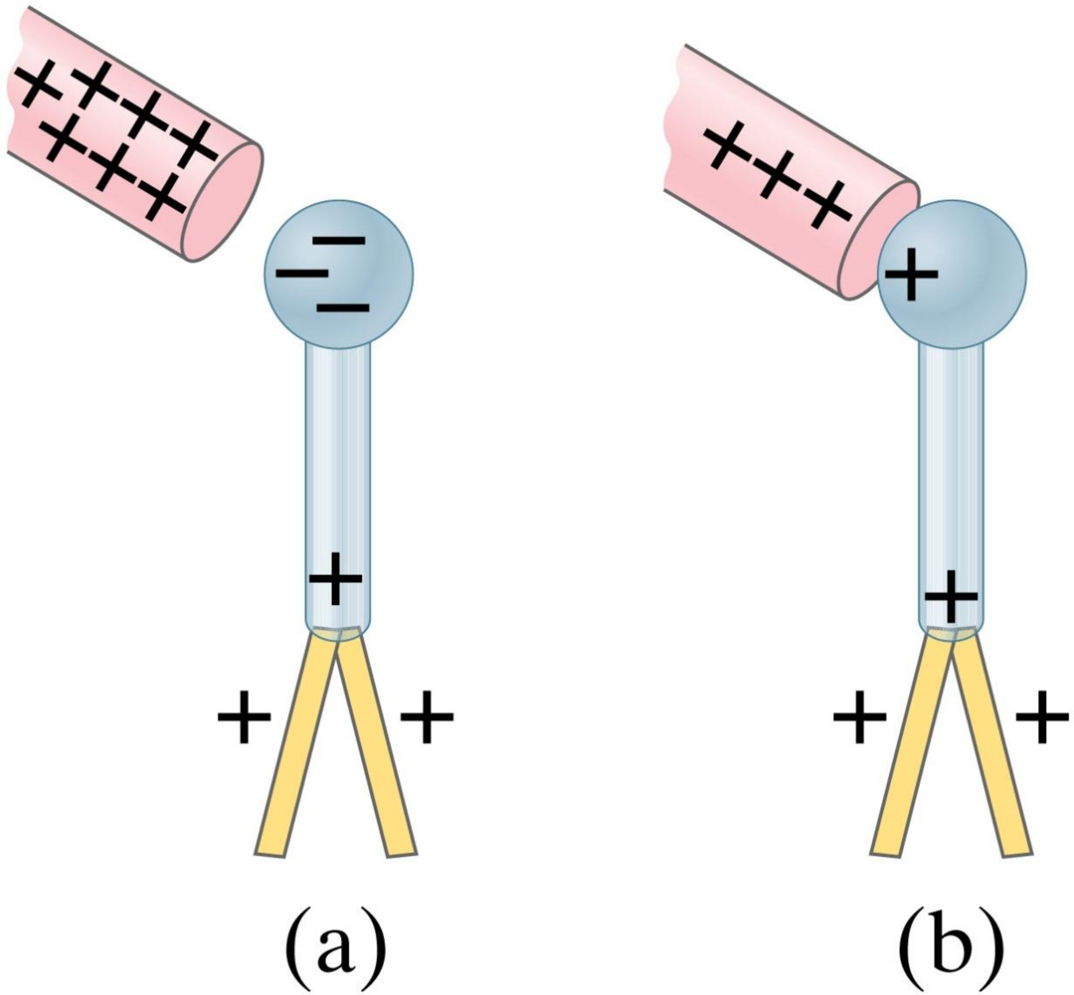
Induced Charge; the Electroscope

The electroscope can be used for detecting charge:



Induced Charge; the Electroscope

The electroscope can be charged either by conduction or by induction.



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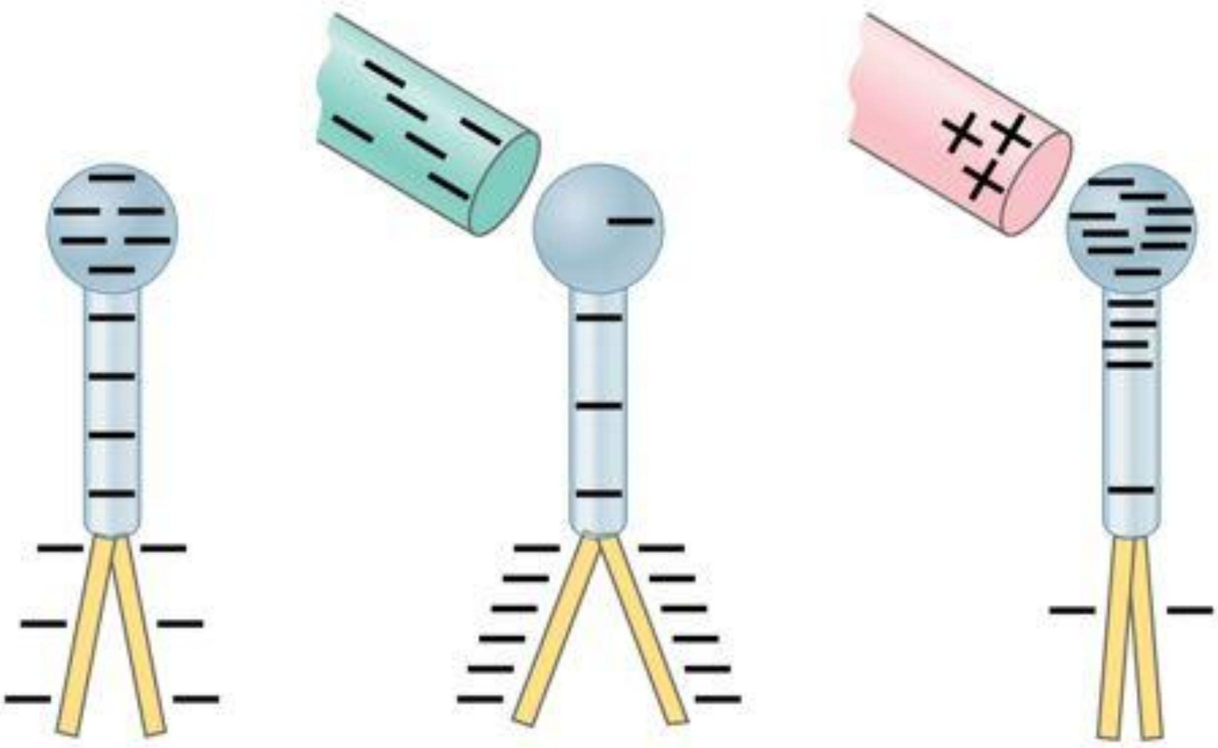
Induced Charge; the Electroscope

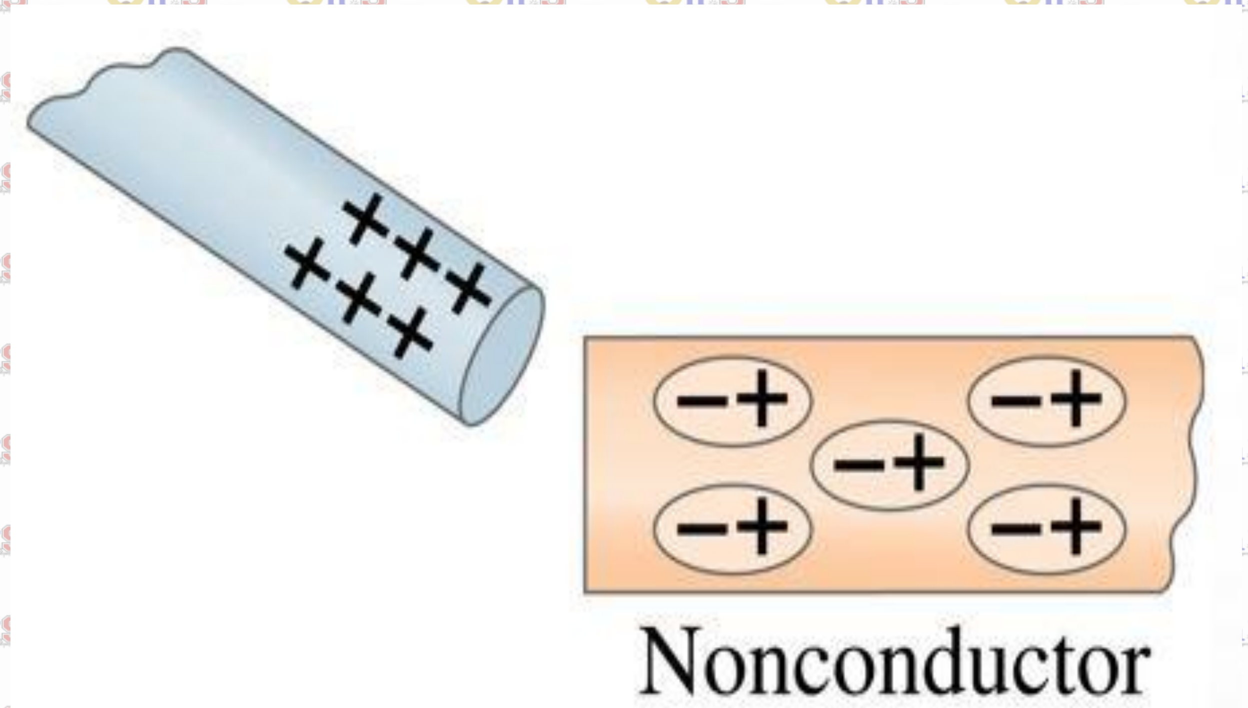
The charged electroscope can then be used to determine the sign of an unknown charge.



(a) (b) (c)

Induced Charge; the Electroscope

Nonconductors won't become charged by conduction or induction, but will experience charge rearrangement. The atoms or molecules become polarized.



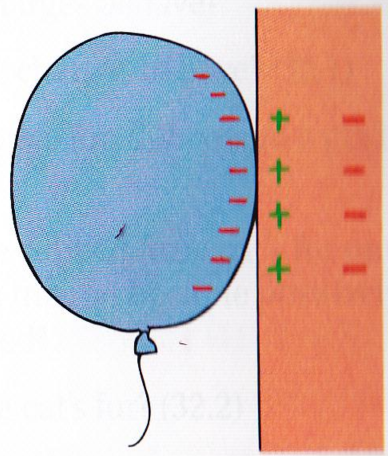
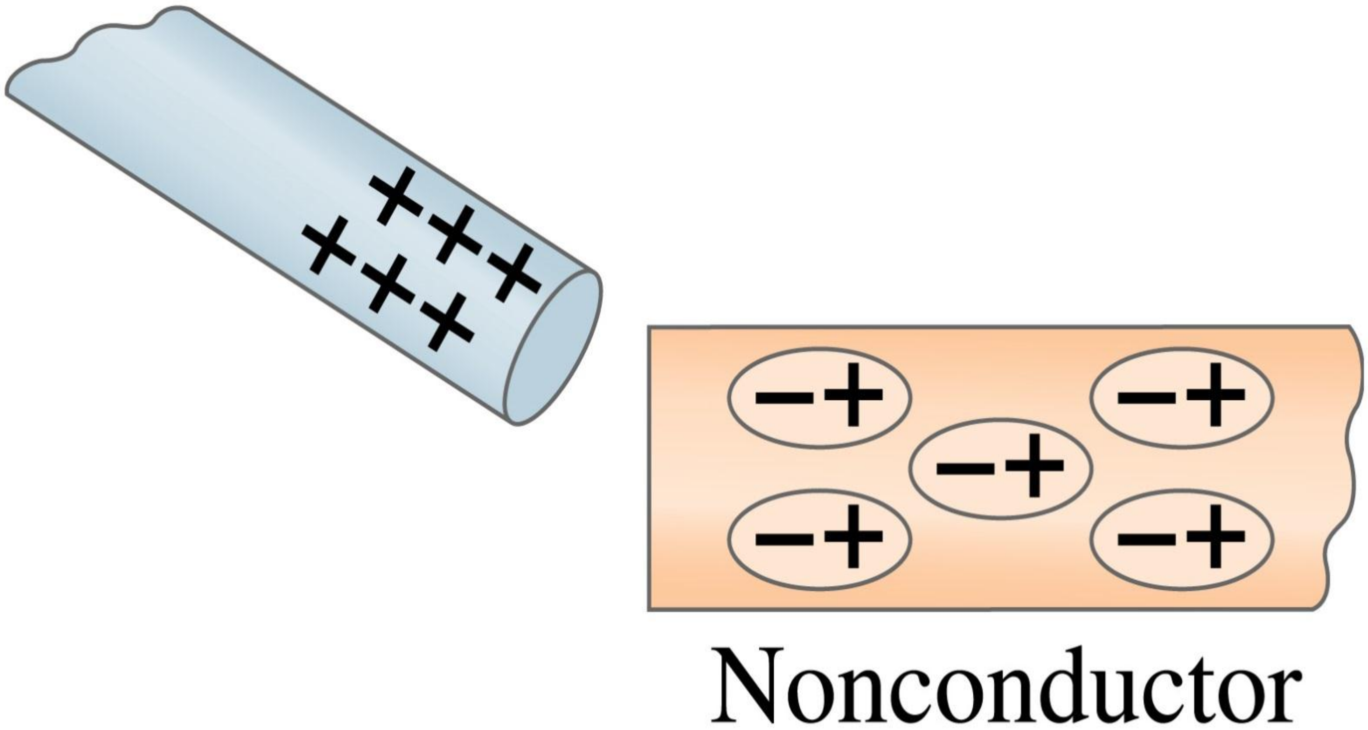


Figure 32.13 ▲
The negatively charged balloon polarizes molecules in the wooden wall and creates a positively charged surface, so the balloon sticks to the wall.

Induced Charge; the Electroscope

Nonconductors won't become charged by conduction or induction, but will experience charge separation:



***True or False? Explain your reasoning*.**

1. An object that is positively charged contains all protons and no electrons.

False

Positively charged objects have electrons; they simply possess more protons than electrons.

2. An object that is electrically neutral contains only neutrons.

False

Electrically neutral atoms simply possess the same number of electrons as protons.

QUICK QUIZ

If a suspended object A is attracted to object B, which is charged, we can conclude that (a) object A is uncharged, (b) object A is charged, (c) object B is positively charged, or (d) object A may be either charged or uncharged.

Electric Force

- Electric Force: force of attraction or repulsion between objects due to charge
- Depends on CHARGE and DISTANCE
  - Increase charge → force increases
  - Increase distance → force decreases
- Forces can act over a distance through a FIELD
- Electric Field: region around a charged object in which other charged objects experience an electric force

Coulomb's Law:

$$F = k\frac{q_1 q_2}{r^2}$$

Where:

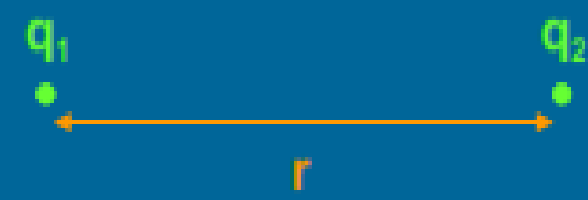
- F = electric force in Newtons
- k = constant (just a #) = $9.0\times10^9\ Nm^2/C^2$
- q_1 = charge of object #1 in Coulombs (C)
- q_2 = charge of object #2 in Coulombs (C)
- r = distance between two charges in meters

Coulomb's Law – Force between two point electric charges:

The electrostatic force of interaction (attraction or repulsion) between two point electric charges is directly proportional to the product of the charges, inversely proportional to the square of the distance between them and acts along the line joining the two charges.

Strictly speaking, Coulomb's law applies to stationary point charges.

$F \propto q_1 q_2$



$F \propto 1 / r^2$

or $F \propto \dfrac{q_1 q_2}{r^2}$ or $F = k \dfrac{q_1 q_2}{r^2}$

where k is a positive constant of proportionality called electrostatic force constant or Coulomb constant.

In vacuum, $k = \dfrac{1}{4\pi\varepsilon_0}$ where ε_0 is the permittivity of free space

In medium, $k = \dfrac{1}{4\pi\varepsilon}$

where ε is the absolute electric permittivity of the dielectric medium

The dielectric constant or relative permittivity or specific inductive capacity or dielectric coefficient is given by

$$\boxed{K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}}$$

∴ In vacuum, $F = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q_1 q_2}{r^2}$

In medium, $F = \dfrac{1}{4\pi\varepsilon_0 \varepsilon_r} \dfrac{q_1 q_2}{r^2}$

$\varepsilon_0 = 8.8542 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

$\dfrac{1}{4\pi\varepsilon_0} = 8.9875 \times 10^9\ N\ m^2\ C^{-2}$ or $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ N\ m^2\ C^{-2}$

Coulomb's law strictly applies only to point charges.

Superposition: for multiple point charges, the forces on each charge from every other charge can be calculated and then added as vectors.

$$\vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots$$

- If the calculated force is:
    - **<u>Negative</u>**
        - –The force is ***attractive*** between particles
    - **<u>Positive</u>**
        - –The force is ***repulsive*** between particles

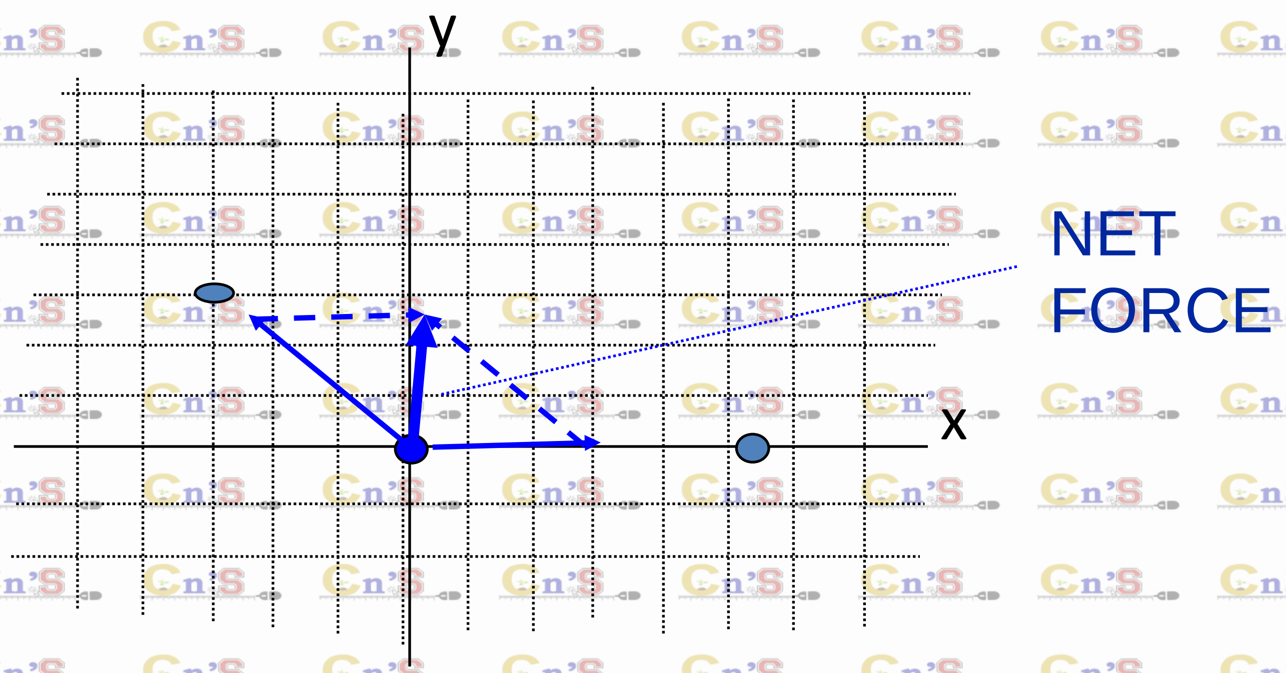
Cn'S

Superposition of forces from two charges

Blue charges fixed , negative, equal charge (-q)

What is force on positive red charge +q ?

Find resultant:



Example

Suppose that two point charges, each with a charge of +1.00 Coulomb are separated by a distance of 1.00 meter. Determine the magnitude of the electrical force of repulsion between them.

$$F = k\frac{|Q_1||Q_2|}{r^2} \qquad F = \frac{(9.0\times10^9)(1)(1)}{1^2} = 9.0\times10^9\,N$$

The force of repulsion of two +1.00 Coulomb charges held 1.00 meter apart is 9 billion Newton. This is an incredibly large force that compares in magnitude to the weight of more than 2000 jetliners.

2. Determine the magnitude and direction of the electric force on the electron of a hydrogen atom exerted by the single proton that is the atom's nucleus. Assume the average distance between the revolving electron and the proton

Given: $|Q_1| = |Q_2| = 1.6\times10^{-19}\,C$ $\quad r = 0.53\times10^{-10}\,m$

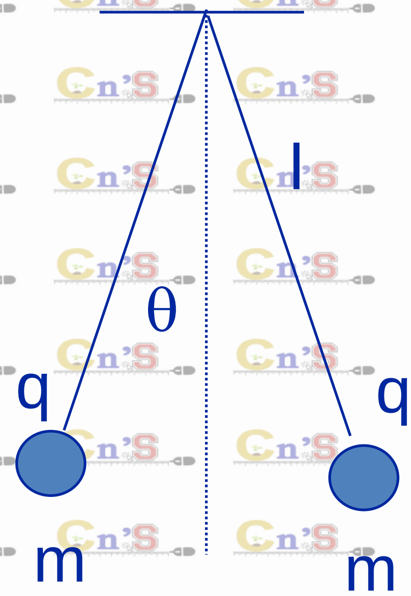
$$F = k\frac{|Q_1||Q_2|}{r^2} = \frac{(9.0\times10^{9}\,N\cdot m^2/C^2)(1.6\times10^{-19}\,C)(1.6\times10^{-19}\,C)}{(0.53\times10^{-10}\,m)^2} = 8.2\times10^{-8}\,N$$

Example

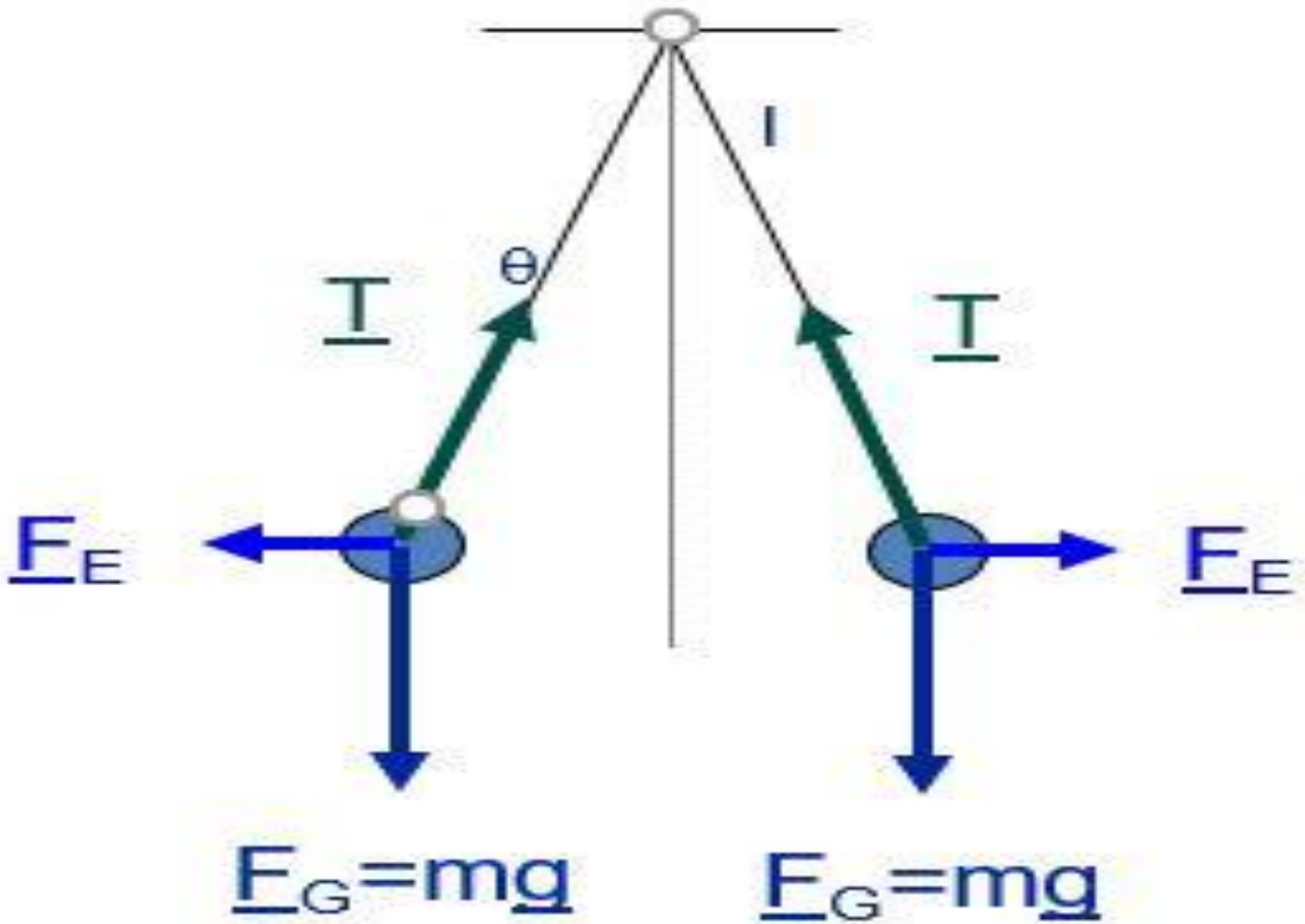
Two identical balls, with mass m and charge q, hang from similar strings of length l.

After equilibrium is reached, find the charge q as a function of θ and l

What forces are acting on the charged balls ?



Draw vector force diagram while identifying the forces.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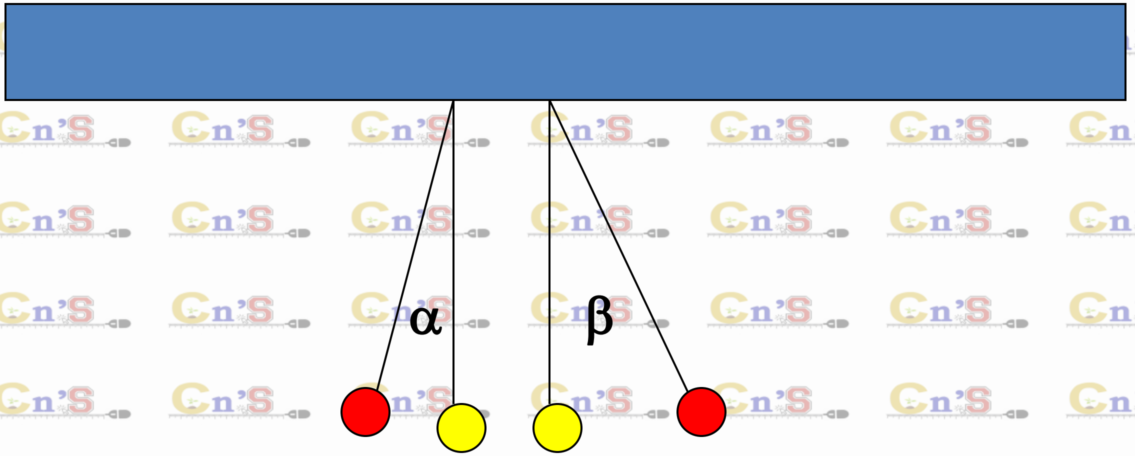
F_G=mg



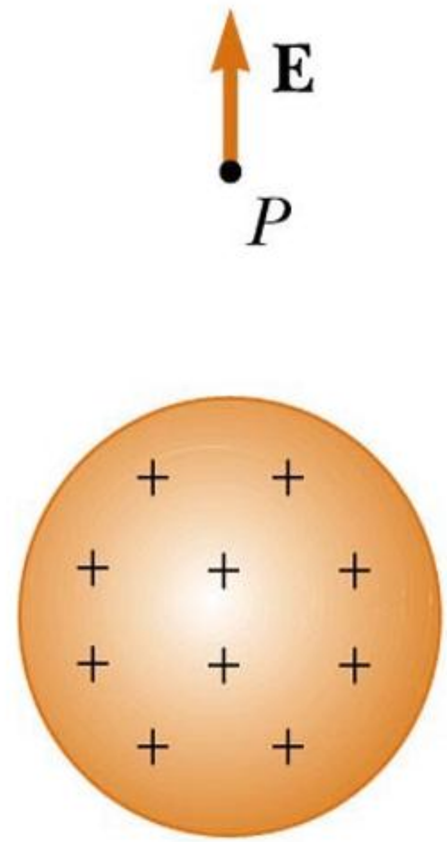
Which angle is bigger, α or β?
Why?

Electric Fields

- An electric field is the region around a charge in which the electrostatic force is felt by other charges.
- Electric fields are drawn using “field lines.”
- By convention, electric field lines always extend from a positively-charged object to a negatively-charged object, from a positively-charged object to infinity, or from infinity to a negatively-charged object.
- Electric field lines are most dense around objects with the greatest amount of charge.
- Electric field lines never cross each other.
- At locations where electric field lines meet the surface of an object, the lines are perpendicular to the surface.

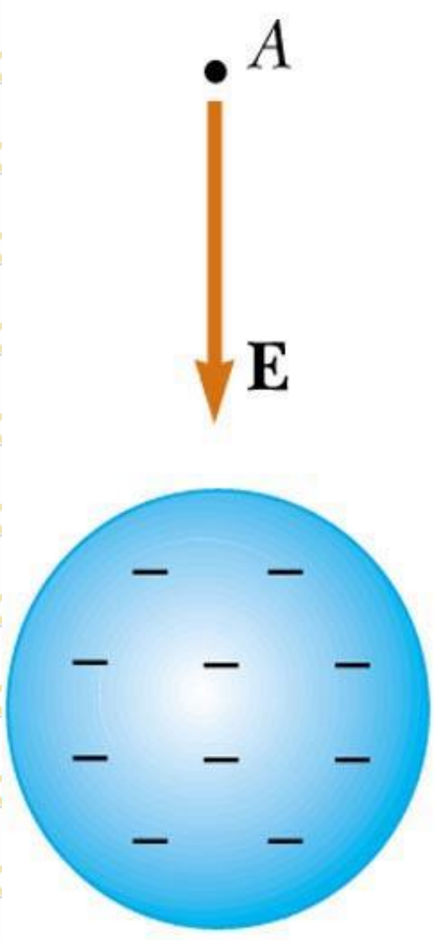
Direction of Electric Field

- The electric field produced by a positive charge is directed away from the charge
  - A positive unit charge would be repelled from the positive source charge

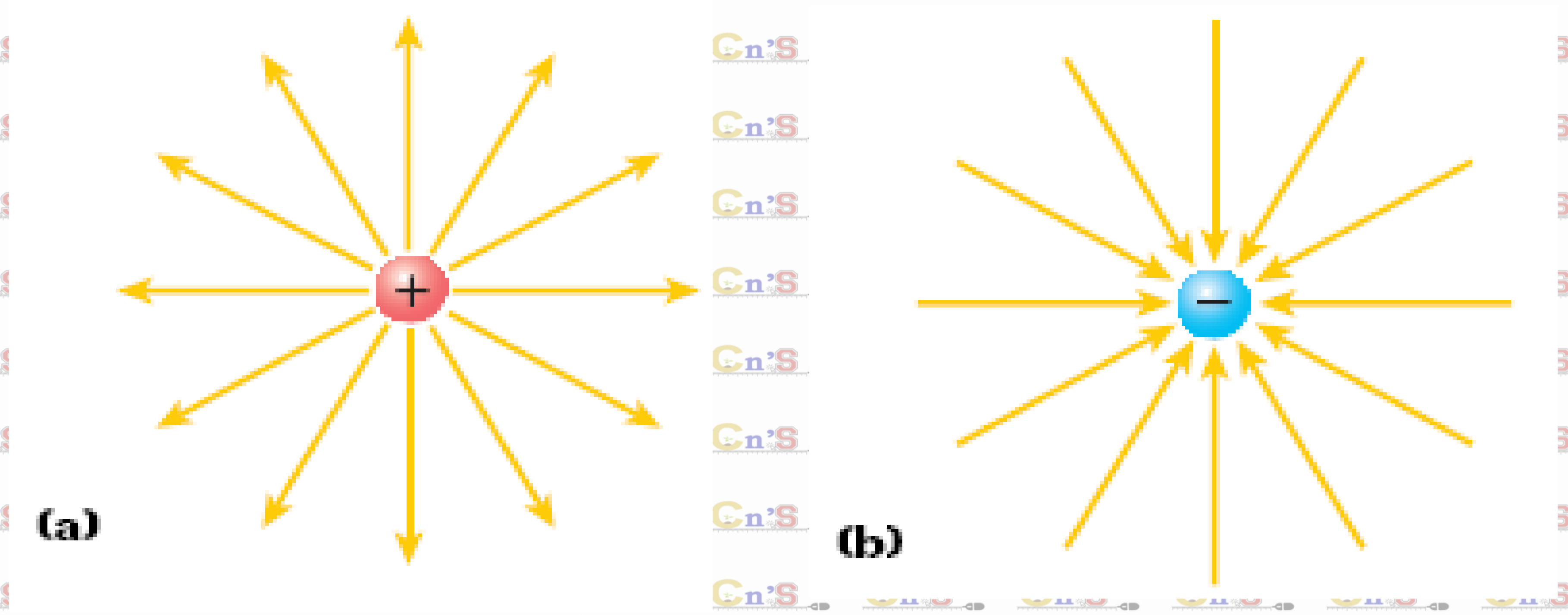


Direction of Electric Field, Cont

- The electric field produced by a negative charge is directed toward the charge
  - A positive unit charge would be attracted to the negative source charge

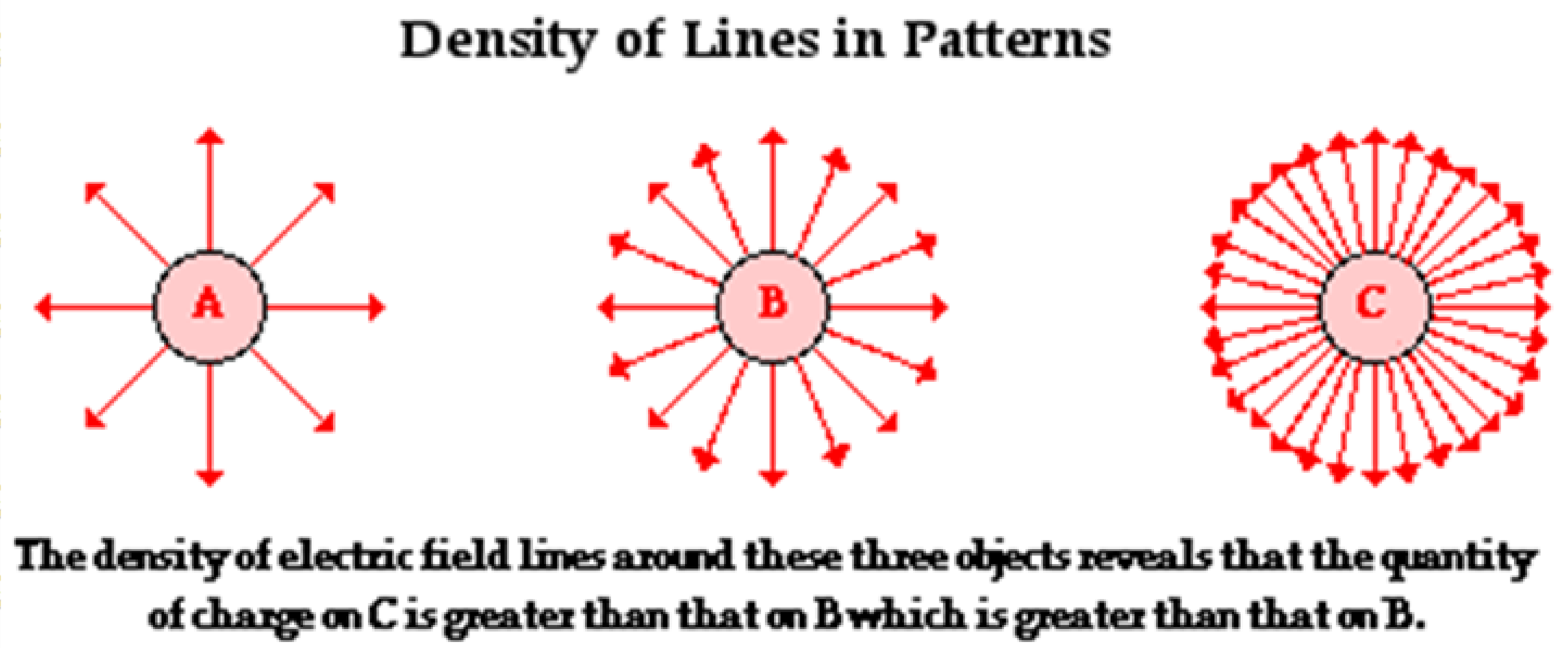


Electric Field Lines



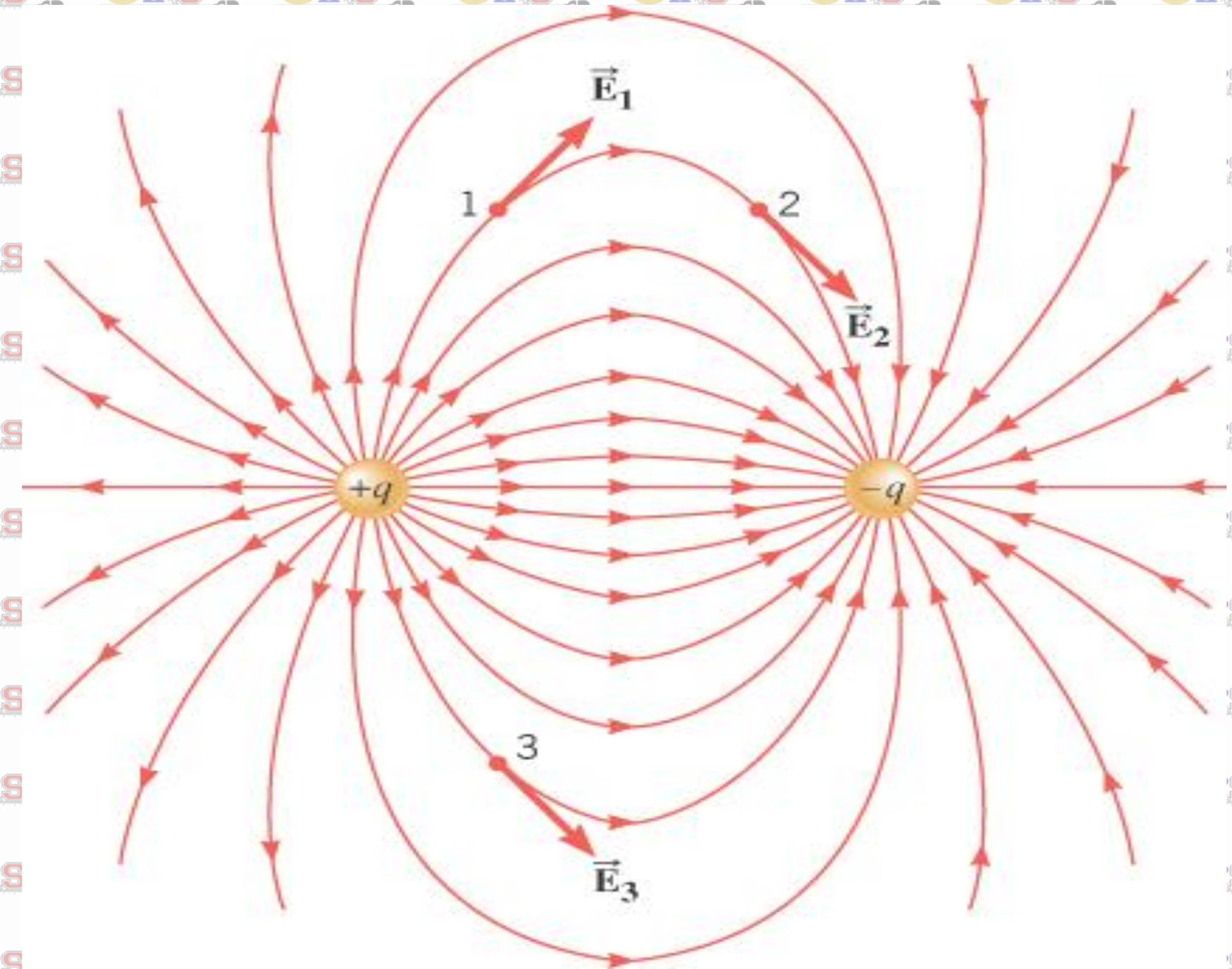
Cn'S

Electric Field Lines



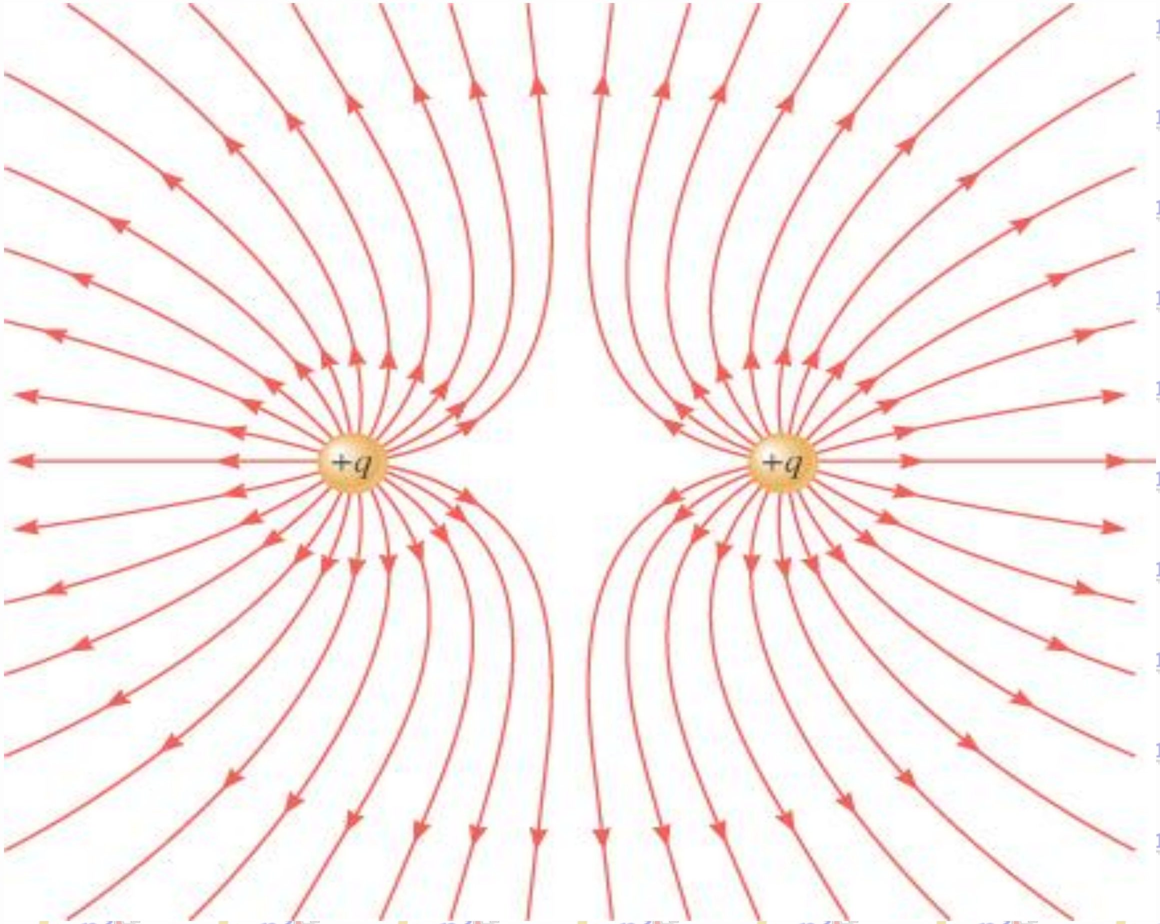
- The number of field lines that meet a charged particle represents the size of the charge, higher charge amounts have more field lines.
- The closer together field lines are to each other, the greater the strength of the field at that point.

Electric Field Lines –



The number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.

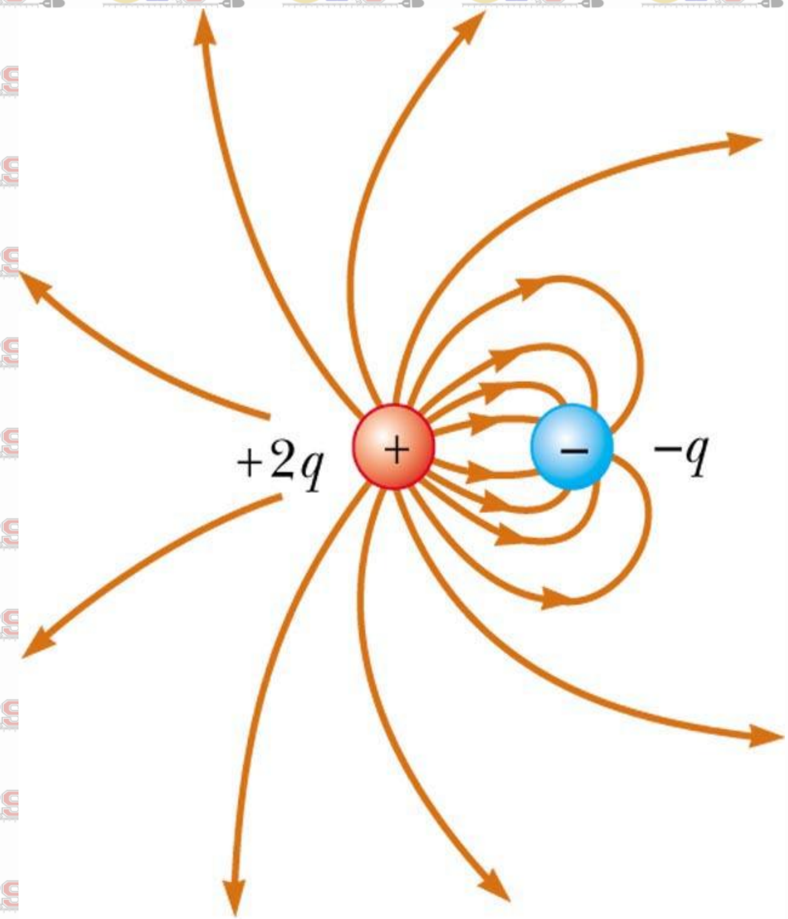
Electric Field Lines –



Now sketch two negative charges!!

Electric Field Patterns

- Unequal and unlike charges
- Note that two lines leave the +2q charge for each line that terminates on -q



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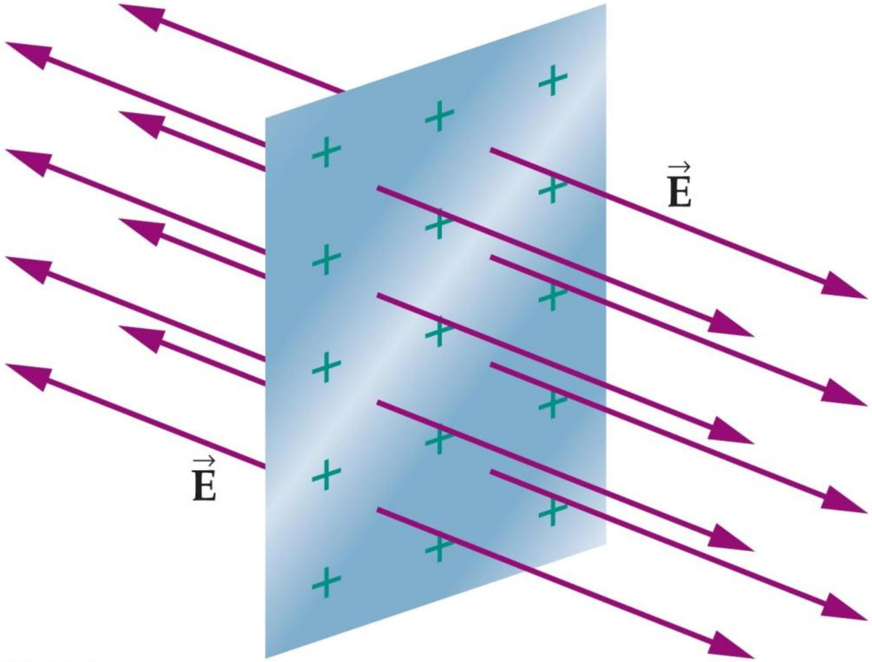
The Electric Field

- A simple but particularly important field picture results when charge is spread uniformly over a very large (essentially infinite) plate, as illustrated in the figure below.



- The electric field is uniform in this case, in both direction and magnitude. The field points in a single direction—perpendicular to the plate. Most remarkably, the magnitude of the electric field doesn't depend on the distance from the plate.

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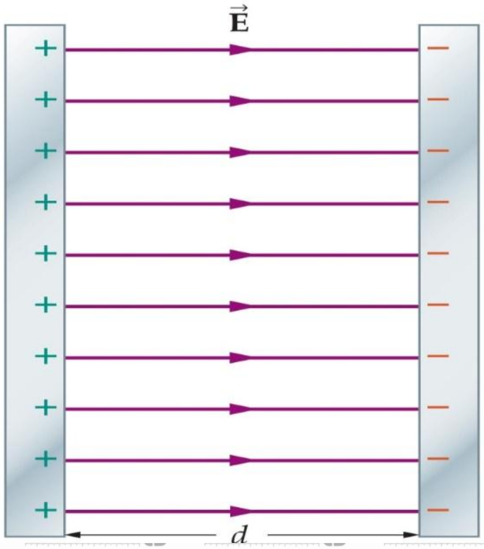
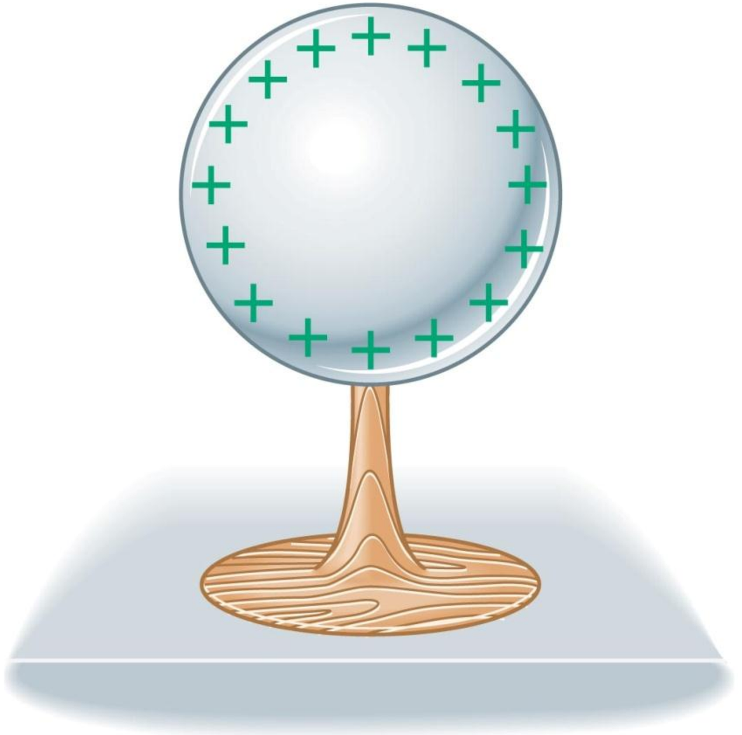
- If two plates with opposite charge are placed parallel to each other and separated by a finite distance, the result is a parallel-plate capacitor. An example is shown in the figure below.



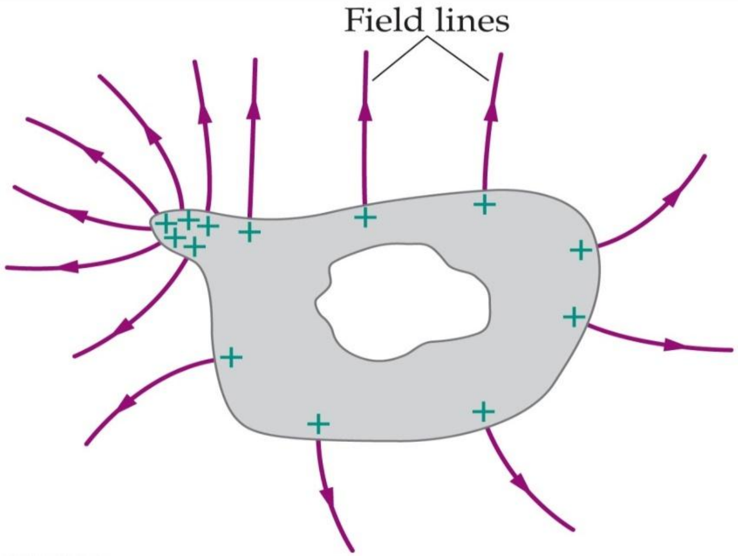
- The field in this case is uniform between the plates and zero outside the plates. This case is ideal, which is exactly true for infinite plates and a good approximation for large plates.

- Conductors contain an enormous number of electrons that are free to move about. This simple fact has some rather interesting consequences. For one, any excess charge placed on a conductor moves to its outer surface, as is indicated in the figure belov



- In this way the individual charges are spread as far apart from one another as possible.
- On a conducting sphere, excess charge placed on the sphere distributes itself uniformly on the surface. None of the excess charge is within the volume of the conductor.
- The distribution of charge on the surface of a conductor guarantees that the electric field within the conductor is zero. This effect is referred to as shielding. Shielding occurs whether the conductor is solid or hollow.

- Shielding is put to use in numerous electrical devices, which often have a metal foil or wire mesh enclosure surrounding the sensitive electrical circuits.
- Related to shielding is the fact that electric field lines always contact a conductor at right angles to its surface. In addition, the field lines crowd together where a conductor has point or a sharp projection, as illustrated in the following figure. The result is an intense electric field at a sharp metal point.



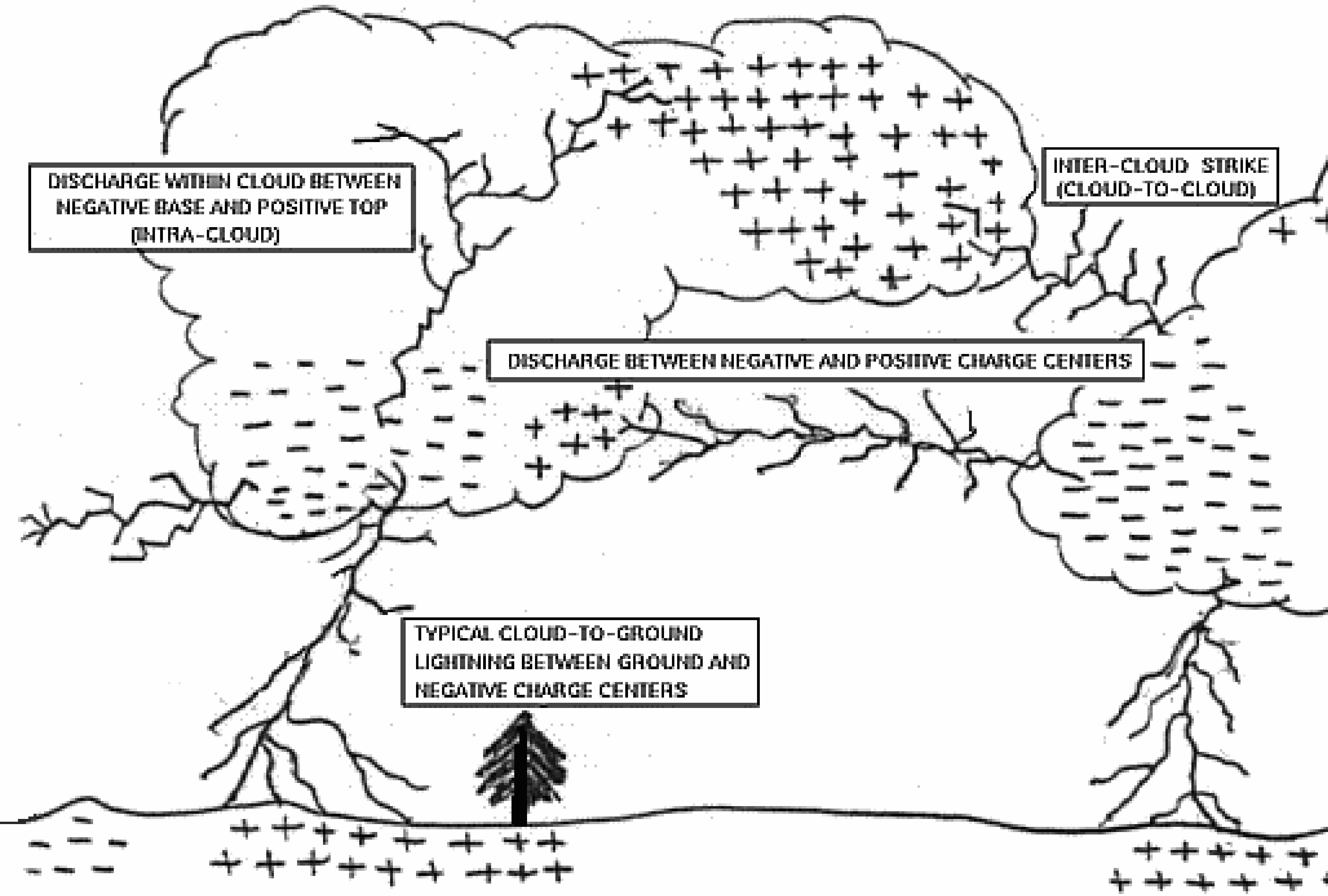
- The crowding of field lines at a point is the basic principle behind the operation of lightning rods. During an electrical storm the electric field at the tip of a lightning rod becomes so intense that electric charge is given off into the atmosphere. In this way a lightning rod discharges the area near the house, thus preventing lightning from striking the house, which would transfer a large amount of charge in one sudden blast.

Lightning

- **kills more than 60 people and**
- **injures more than 400 people a year in the US**
- **one mile every five seconds**
- **about 20,000 C**
- **Voltage of up to 1.2×10^8 volts**
-

Lightning

- Electrical discharge between oppositely charged clouds or the ground
- Results from charging by induction – negatively charged clouds induce positive charge in ground below them


DISCHARGE WITHIN CLOUD BETWEEN NEGATIVE BASE AND POSITIVE TOP (INTRA-CLOUD)
INTER-CLOUD STRIKE (CLOUD-TO-CLOUD)
DISCHARGE BETWEEN NEGATIVE AND POSITIVE CHARGE CENTERS
TYPICAL CLOUD-TO-GROUND LIGHTNING BETWEEN GROUND AND NEGATIVE CHARGE CENTERS

Suggested learning/teaching process:

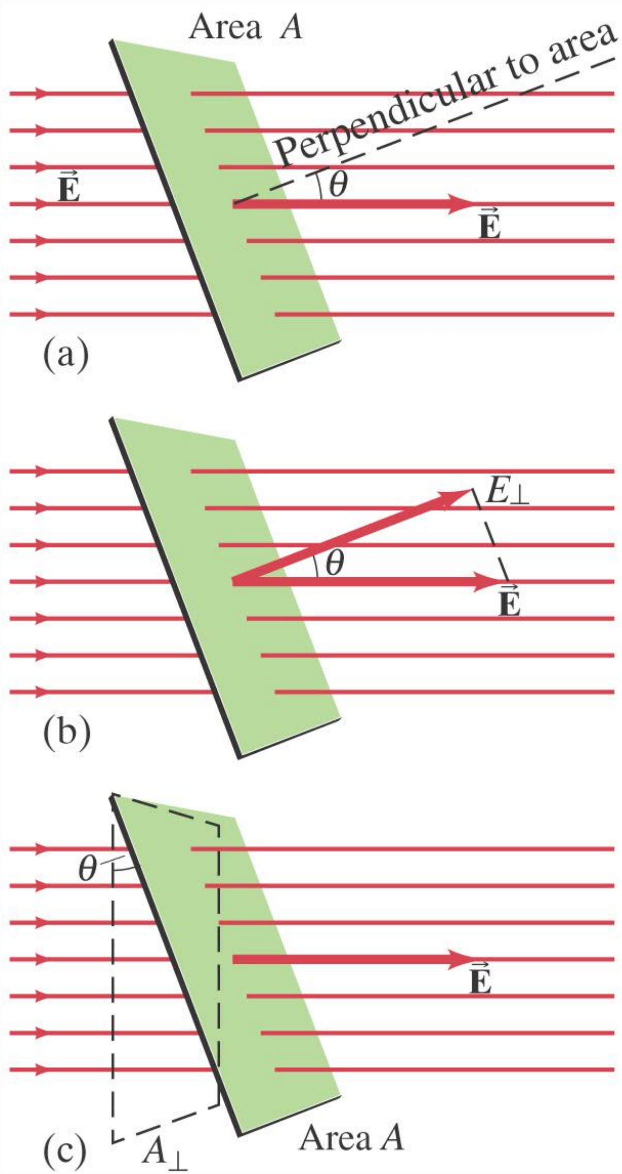
- Introduce the flux model.
- Introduce the flux as $\emptyset_E = EA$.
- Express Gauss's theorem.
- Derive expressions for field intensities of the following fields using Gauss's theorem and the flux model.
  - Near a point charge, $E = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}$
  - Near a charged conducting thin infinite plate, $E = \frac{\sigma}{\varepsilon}$
  - Inside a charged conducting sphere, $E = 0$
  - On the conducting charged sphere and away from the sphere

$$E = \frac{1}{4\pi\varepsilon}\frac{Q}{R^2}, E = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}$$

  - Near charged conducting thin wire of infinite length, $E = \frac{\lambda}{2\pi\varepsilon r}$

- Graphically interpret the variation of field intensity with distance from the centre of the conducting charged sphere.

16.10 Gauss's Law



Electric flux:

$$\Phi_E = EA\cos\theta$$

$$= E_\perp A = EA_\perp \qquad (16\text{-}7)$$

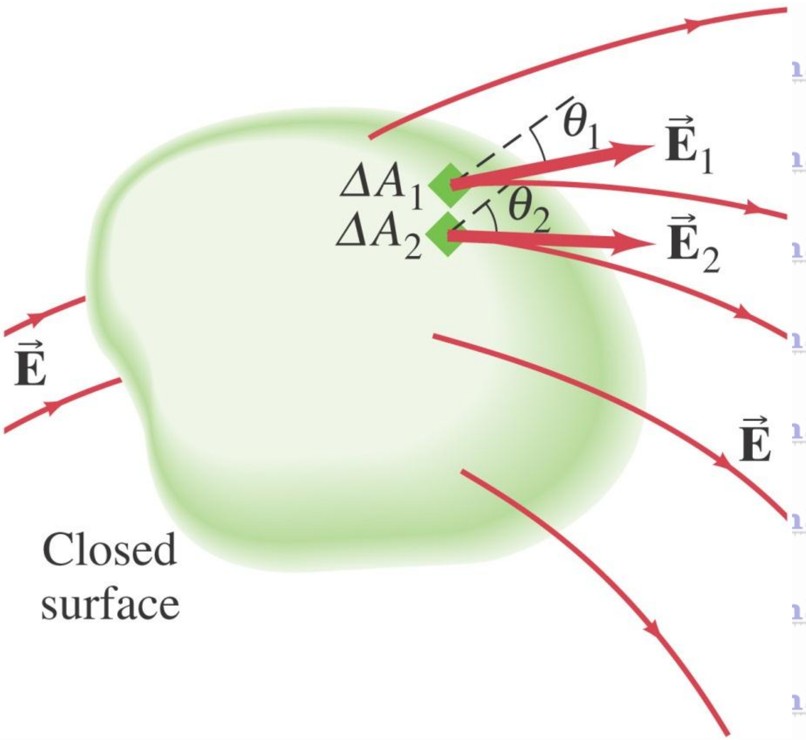
Electric flux through an area is proportional to the total number of field lines crossing the area.

16.10 Gauss's Law

Flux through a closed surface:

$$\Phi_E = E_1\,\Delta A_1 \cos\theta_1 + E_2\,\Delta A_2 \cos\theta_2 + \cdots$$

$$= \sum E\,\Delta A \cos\theta = \sum E_{\perp}\,\Delta A,$$



Gauss's Law

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

This is a useful tool for simply determining the electric field, but only for certain situations where the charge distribution is either rather simple or possesses a high degree of symmetry.

Gauss's Law

Gauss's Law relates the electric flux through a *closed* surface with the charge Q_{in} inside that surface.

The net number of field lines through the surface is proportional to the charge enclosed, and also to the flux, giving Gauss's law:

$$\sum_{\substack{\text{closed} \\ \text{surface}}} E_{\perp} \, \Delta A = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Cn'S

- **Electric flux is the rate of flow of the electric field through a given surface.**
- It is the amount of electric field penetrating a surface. And that surface can be open or closed.

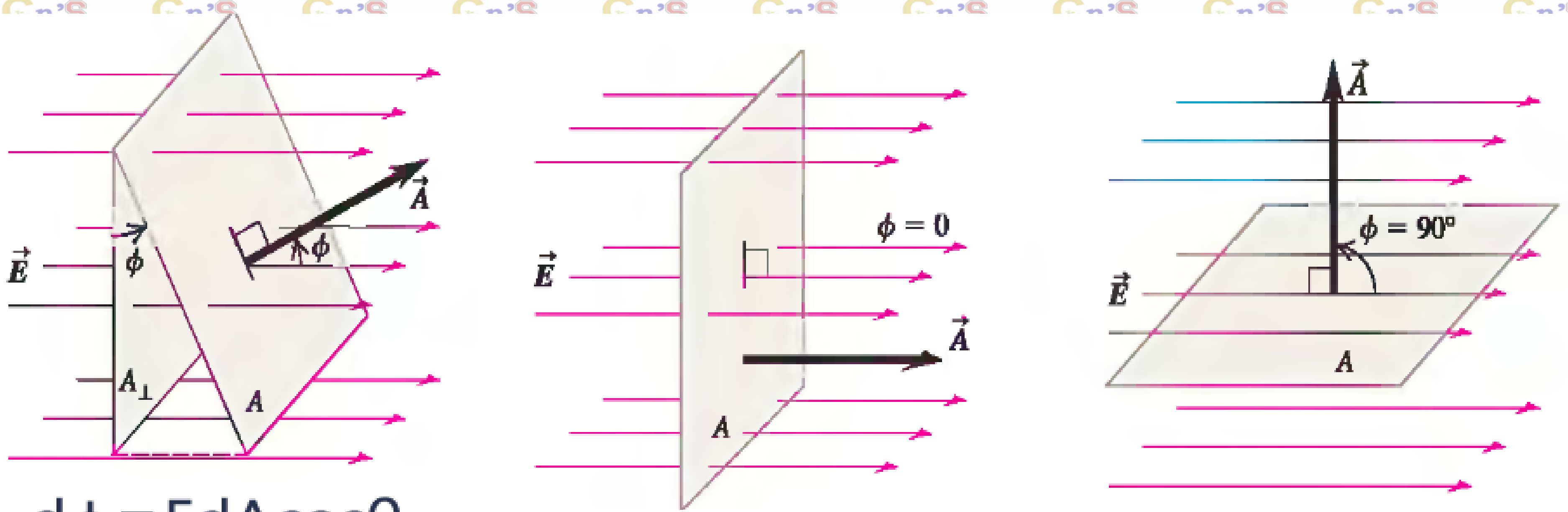
$$d\phi = \vec{E} \cdot d\vec{A}$$

$$d\phi = EdA\cos\theta$$

It is a scalar quantity and the end result can be **positive** or **negative**. If the flux is going from the inside to the outside, we call that a positive flux, if it is going from the outside to the inside, that's a negative flux.

$$[\Phi] = \left[\frac{Nm^2}{C}\right]$$

The unit of electric flux is Newton meters squared per Coulomb (Nm^2/C).



$$d\phi = EdA\cos\theta$$
$$d\phi = EdA\cos 60°$$
$$d\phi = \frac{EdA}{2}$$

$$d\phi = EdA\cos\theta$$
$$d\phi = EdA\cos 0°$$
$$d\phi = EdA$$

$$d\phi = EdA\cos\theta$$
$$d\phi = EdA\cos 90°$$
$$d\phi = 0$$

When the area is parallel to the electric field, the angle θ between them is 90°. Cos90° is 0, so the electric flux here will be 0. This means that nothing goes through that rectangle, so **zero flux**.

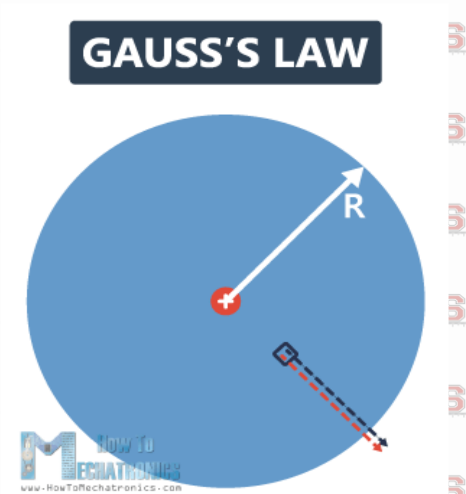
- Now we can calculate the total flux going through this closed surface. The total flux is equal to the integral of dϕ over that entire surface, which we write as the integral over that closed surface of E·dA.

- The total flux can be positive, negative, or equal to zero. If the same amount of flux is entering and leaving the surface, we have **zero total flux**. If more flux is leaving than entering the surface, then we have **positive total flux**. Opposite, if more flux is entering than leaving the surface, we have a **negative total flux**.

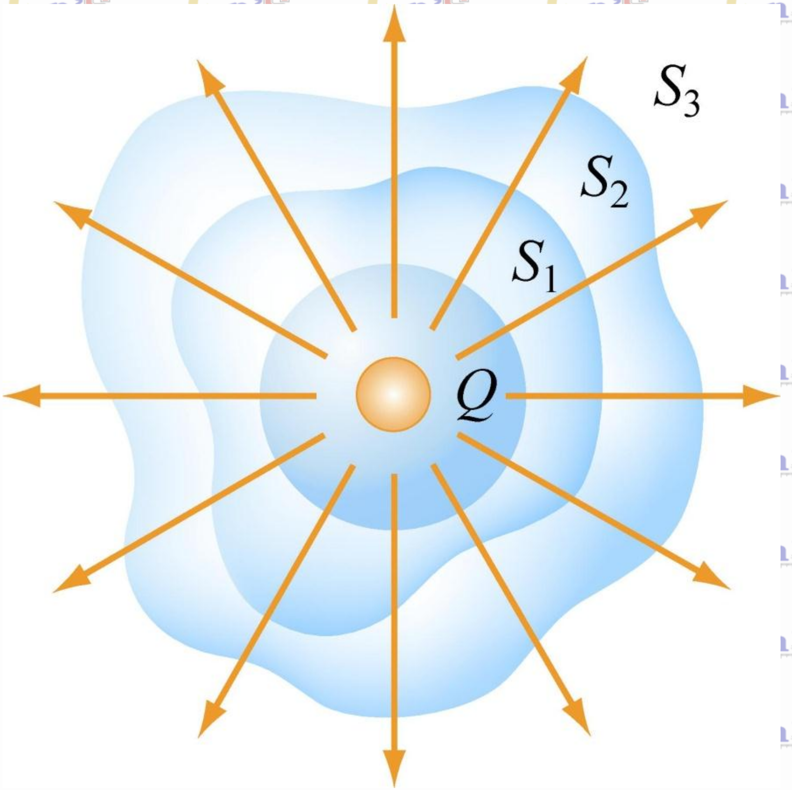
We have a point charge +Q in the center of a sphere with radius R. Now, we'll take a small segment dA, which vector is perpendicular to the surface and is radially outward. The electric field generated by Q at that point is also radially outward. This means that dA and E anywhere on the surface of this sphere are parallel to each other, the angle between them θ is 0°, and cos0° is 1.

$$d\phi = EdA$$

$$\phi = \int d\phi = \oint EdA = E\oint dA = EA$$

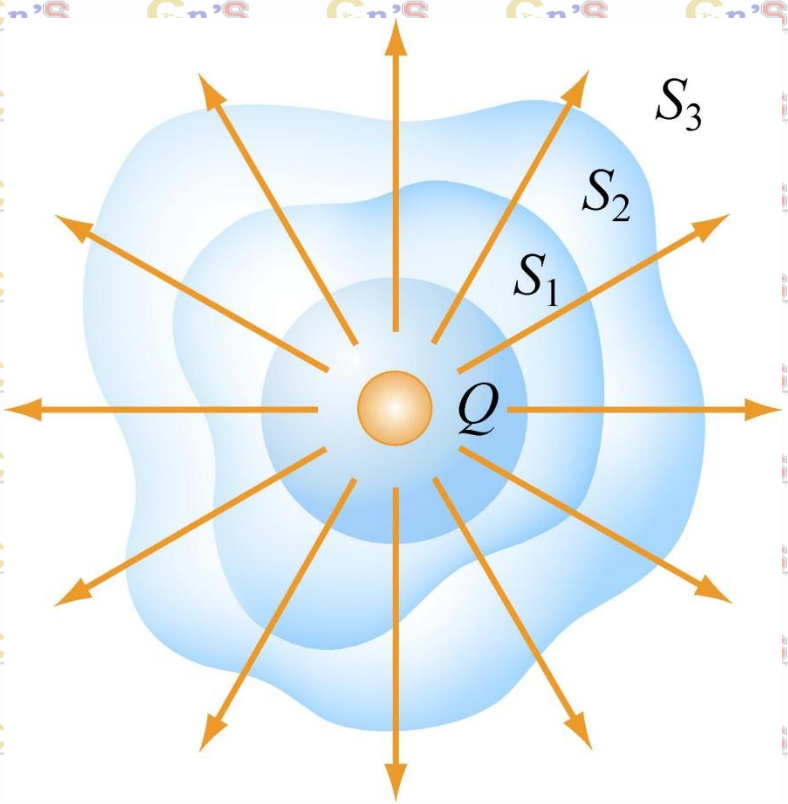


Gauss’ s Law – The Idea



The total “flux” of field lines penetrating any of these closed surfaces is the same and depends only on the amount of charge inside

Arbitrary Gaussian Surfaces



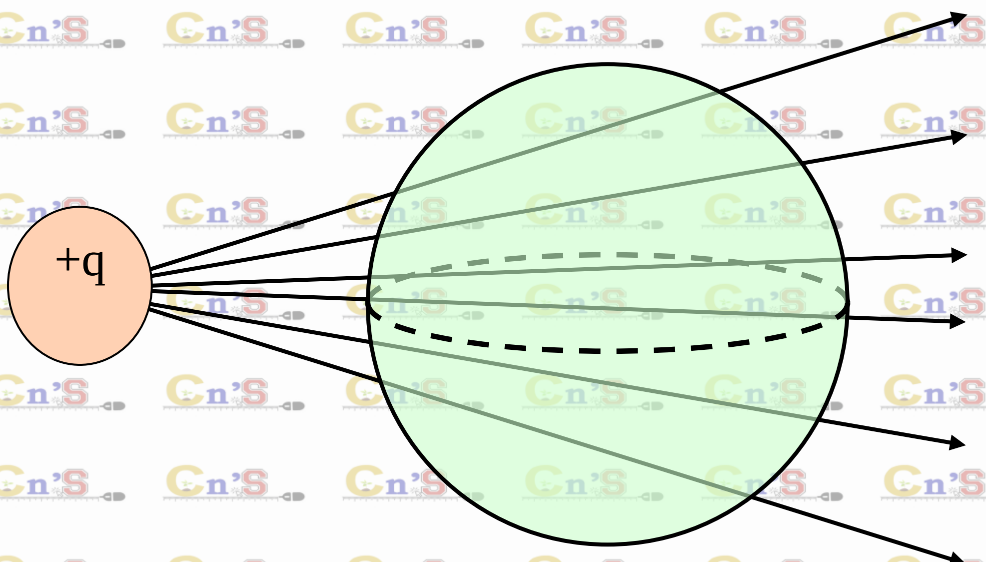
$$\Phi_E = \underset{\substack{\text{closed} \\ \text{surface S}}}{\oiint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q}{\varepsilon_0}$$

True for all surfaces such as S_1, S_2 or S_3

Why? As area gets bigger E gets smaller

Concept Question: Flux thru Sphere

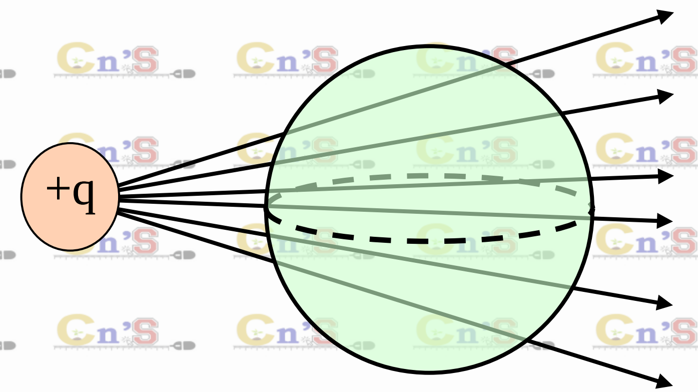
The total flux through the below spherical surface is



1. positive (net outward flux).
2. negative (net inward flux).
3. zero.
4. Not well defined.

Concept Question Answer: Flux thru Sphere

Answer: 3. The total flux is zero



We know this from Gauss' s Law:

$$\Phi_E = \underset{\substack{\text{closed}\\ \text{surface S}}}{\oint\!\!\oint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{enclosed}}{\varepsilon_0}$$

No enclosed charge → no net flux.

Flux in on left cancelled by flux out on right

Electric Field of Point Charge

- Electric field at distance R from a point charge Q is

$$E = k\frac{Q}{R^2} = \frac{1}{4\pi E_0}\frac{Q}{R^2}$$

- Electric field from many charges: superposition (vector sum)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$$

Conductors in Electrostatic Equilibrium

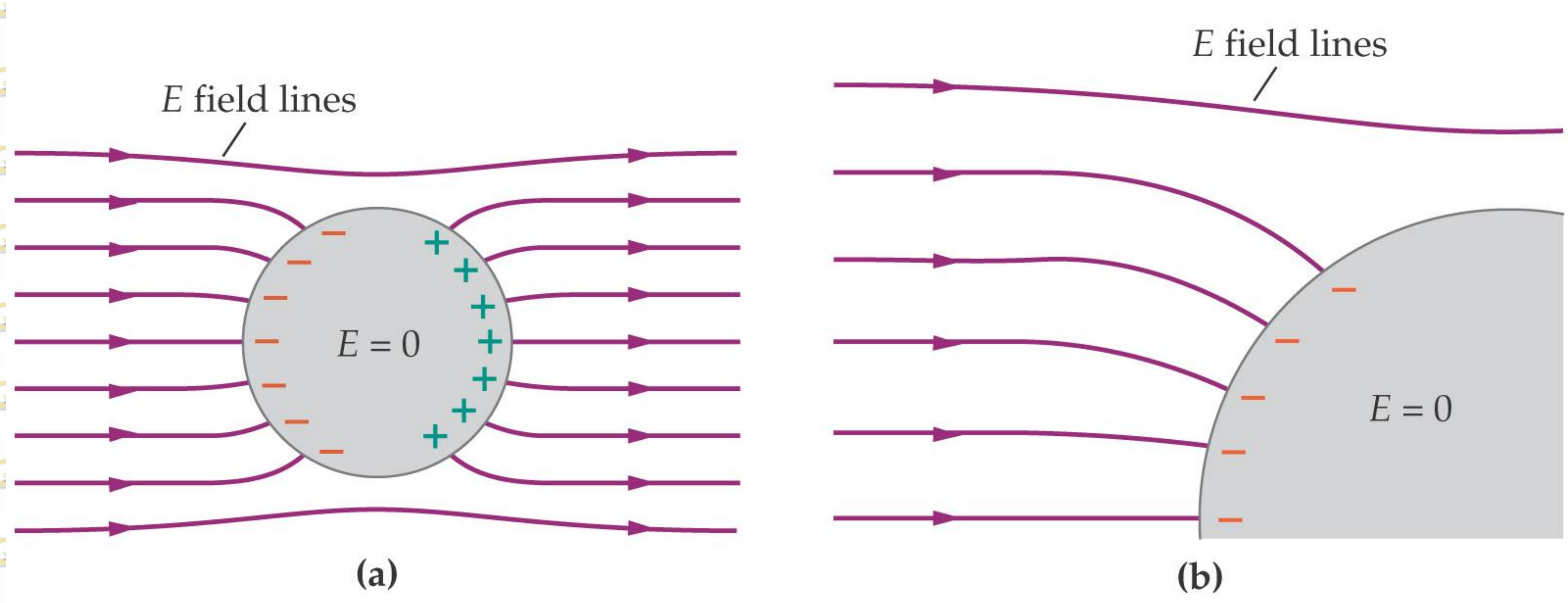
By electrostatic equilibrium we mean a situation where there is no *net* motion of charge within the conductor

- The electric field is zero everywhere inside the conductor

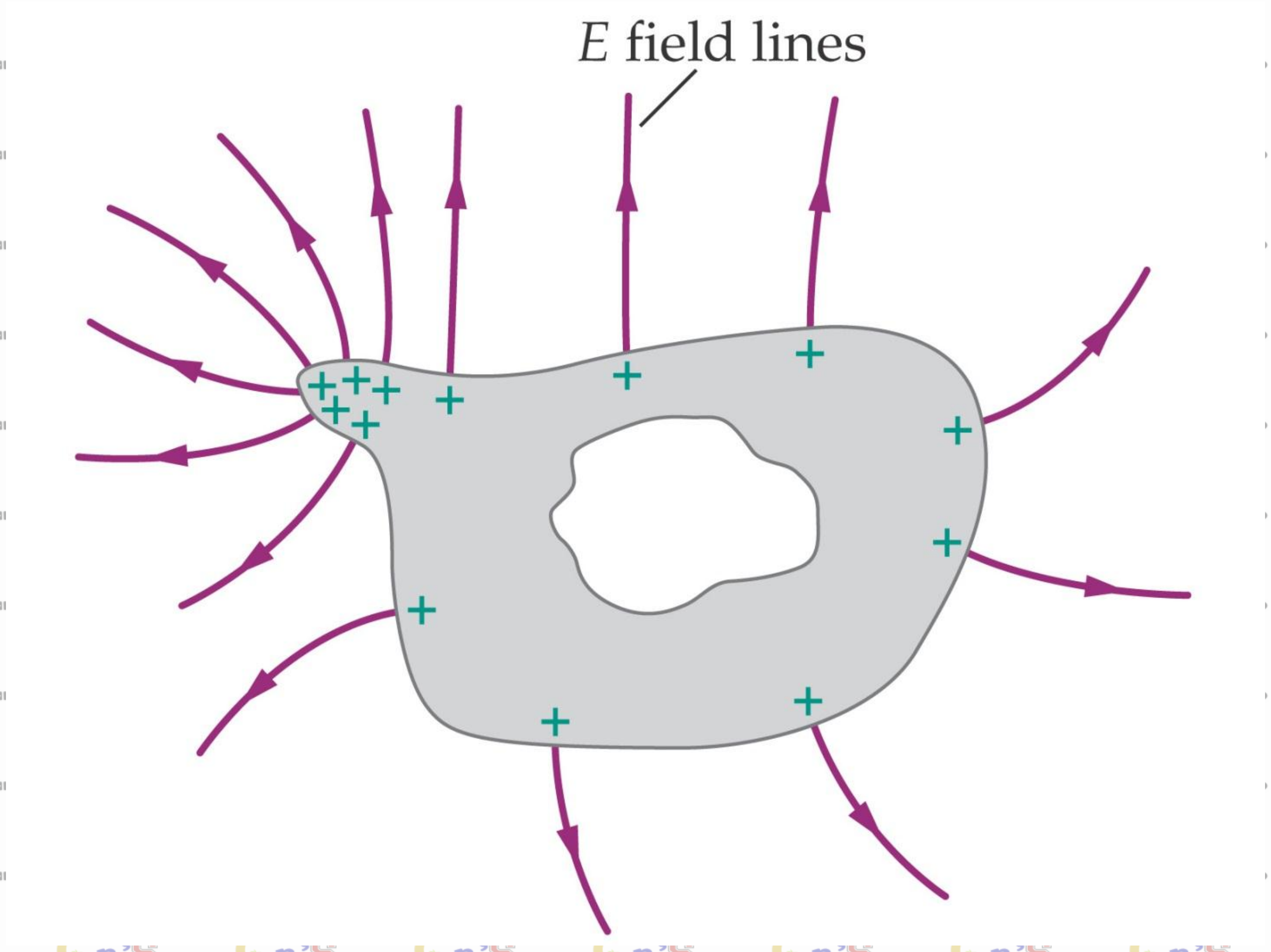
- Any excess charge on an isolated conductor resides entirely on its surface

- The electric field just outside a charged conductor is perpendicular to the conductor’s surface

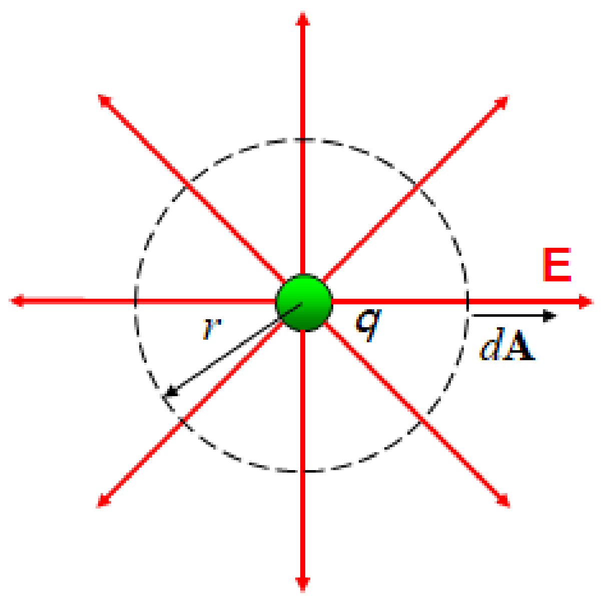
The electric field is always perpendicular to the surface of a conductor – if it weren't, the charges would move along the surface.



The electric field is stronger where the surface is more sharply curved.



Use Gauss' Law to calculate the electric field from an isolated point positive charge q.



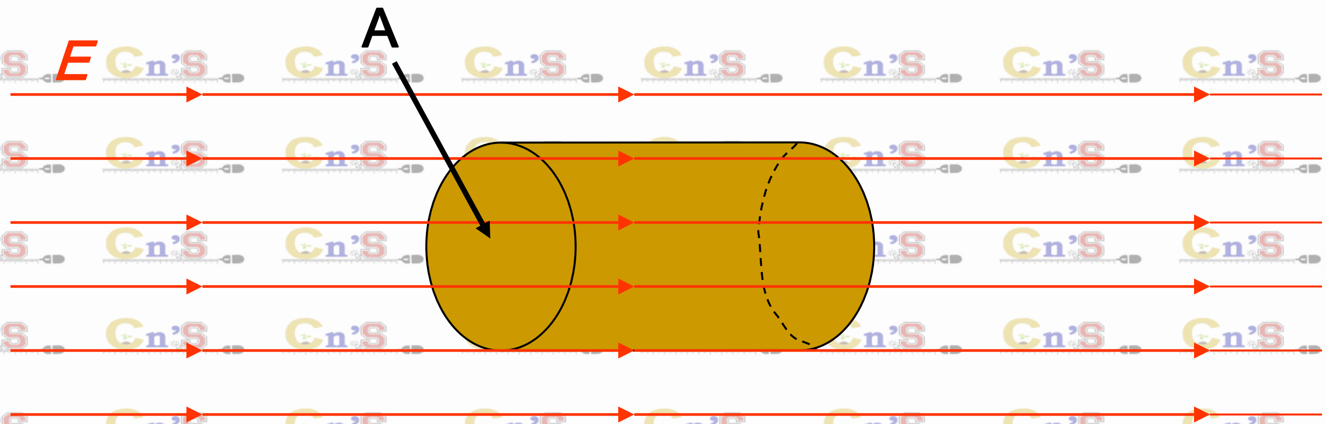
To apply Gauss' Law, we construct a "Gaussian Surface" enclosing the charge.

The Gaussian surface should mimic the symmetry of the charge distribution.

For this example, choose for our Gaussian surface a sphere of radius r, with the point charge at the center.

Worked Example 1

Compute the electric flux through a cylinder with an axis parallel to the electric field direction.



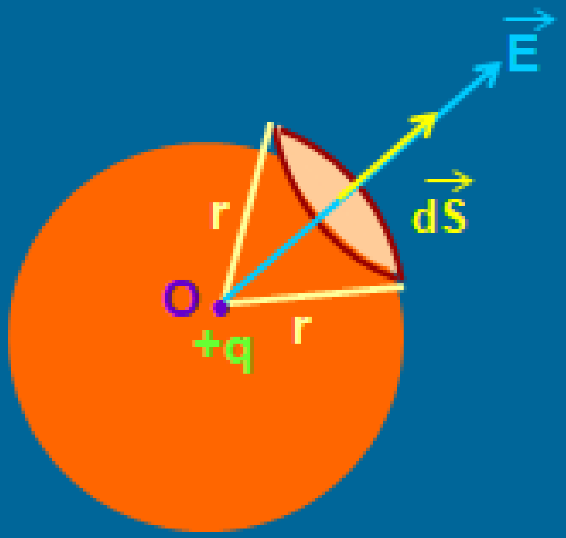
The flux through the curved surface is zero since $\mathbf{E}$ is perpendicular to $d\mathbf{A}$ there. For the ends, the surfaces are perpendicular to $\mathbf{E}$, and $\mathbf{E}$ and $\mathbf{A}$ are parallel. Thus the flux through the left end (*into* the cylinder) is $-EA$, while the flux through right end (*out* of the cylinder) is $+EA$. Hence the net flux through the cylinder is zero.

Deduction of Coulomb's Law from Gauss's Theorem:

From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

Since $\vec{E}$ and $\vec{dS}$ are in the same direction,



$$\therefore \quad \Phi_E = \oint_S E \; dS = \frac{q}{\varepsilon_0}$$

$$\text{or} \quad \Phi_E = E \oint_S dS = \frac{q}{\varepsilon_0}$$

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0} \qquad \text{or} \qquad E = \frac{q}{4\pi\varepsilon_0 \; r^2}$$

If a charge q_0 is placed at a point where E is calculated, then

$$\boxed{F = \frac{q q_0}{4\pi\varepsilon_0 \; r^2}}$$

which is Coulomb's Law.

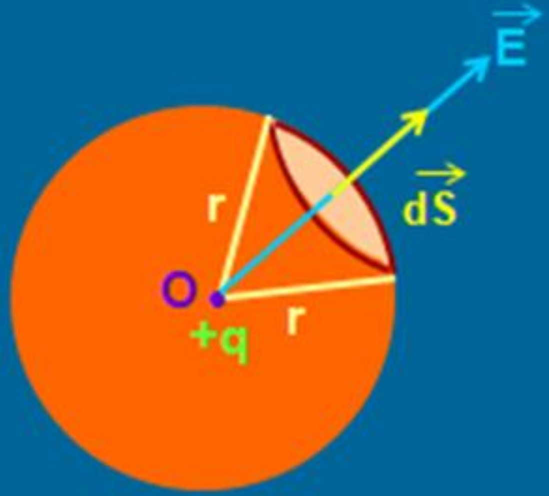
Gauss's Theorem:

The surface integral of the electric field intensity over any closed hypothetical surface (called Gaussian surface) in free space is equal to $1 / \varepsilon_0$ times the net charge enclosed within the surface.

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i$$

Proof of Gauss's Theorem for Spherically Symmetric Surfaces:

$$d\Phi = \vec{E} \cdot \vec{dS}$$



$$\therefore \quad d\Phi = \frac{1}{4\pi\varepsilon_0} \frac{q\, dS}{r^2}$$

$$\Phi_E = \oint_S d\Phi = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \oint_S dS = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} 4\pi r^2 = \frac{q}{\varepsilon_0}$$

Extra Gauss's Law and Coulomb's Law:

Figure 23-8 shows a positive point charge *q*, around which a concentric spherical Gaussian surface of radius r is drawn. Divide this surface into differential areas ***dA***.

The area vector ***dA*** at any point is perpendicular to the surface and directed outward from the interior.

From the symmetry of the situation, at any point the electric field, ***E***, is also perpendicular to the surface and directed outward from the interior.

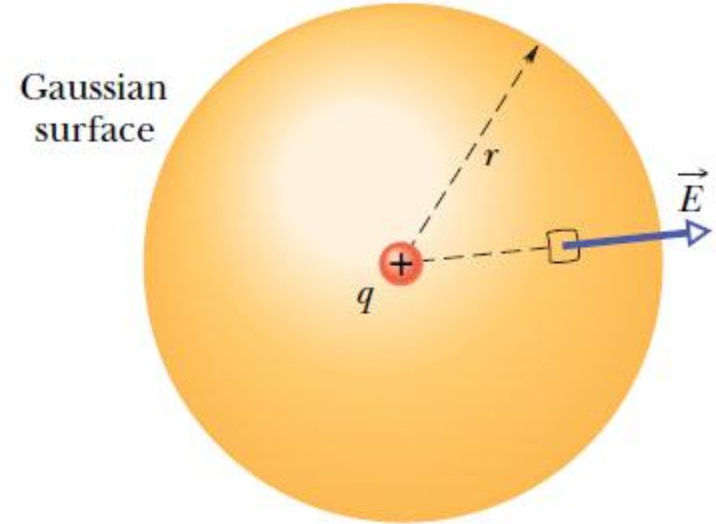


Fig. 23-8 A spherical Gaussian surface centered on a point charge q.

Thus, since the angle θ between ***E*** and ***dA*** is zero, we can rewrite Gauss' law as

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{\text{enc}}.$$

$$\varepsilon_0 E \oint dA = q.$$

$$\varepsilon_0 E(4\pi r^2) = q$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}.$$

This is exactly what Coulomb's law yielded.

Use Gauss' Law to calculate the electric field due to a long line of charge, with linear charge density λ.

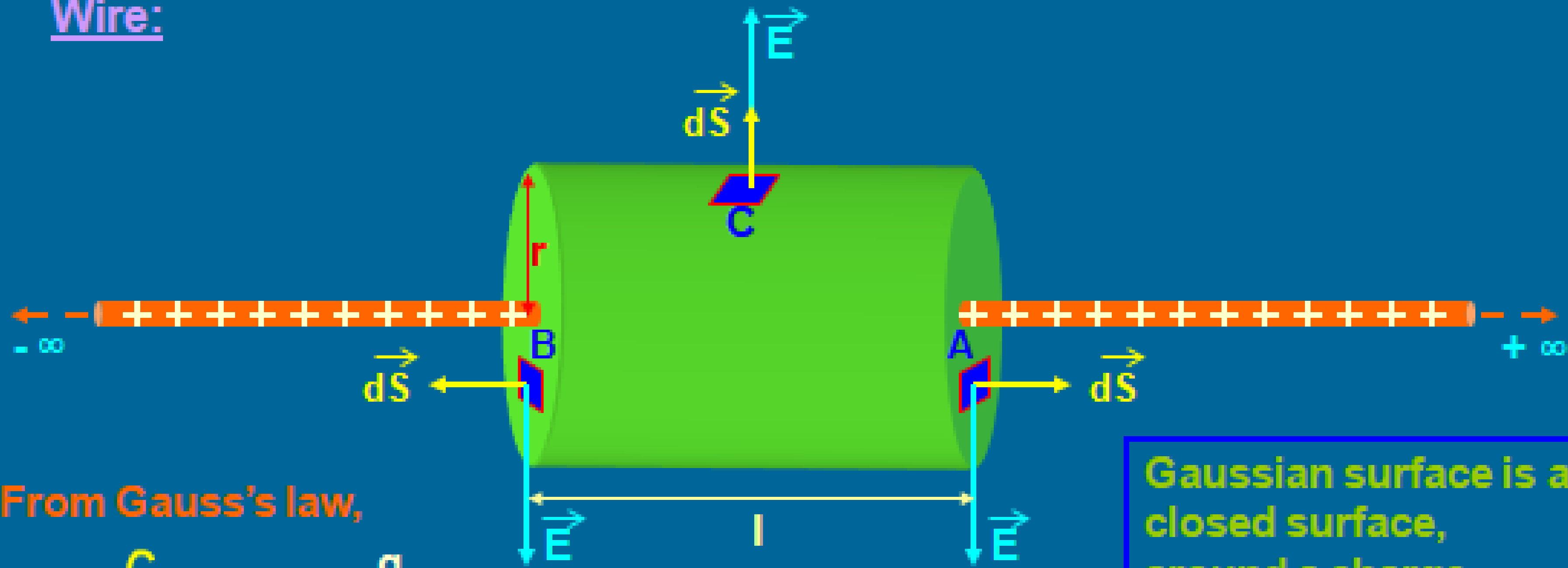
$$E_{\text{line}} = \frac{\lambda}{2\pi\varepsilon_0 r}.$$

Use Gauss' Law to calculate the electric field due to an infinite sheet of charge, with surface charge density σ.

$$E_{\text{sheet}} = \frac{\sigma}{2\varepsilon_0}.$$

Applications of Gauss's Theorem:

1. Electric Field Intensity due to an Infinitely Long Straight Charged Wire:



Gaussian surface is a closed surface, around a charge distribution, such that the electric field intensity has a single fixed value at every point on the surface.

From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

$$\oint_S \vec{E} \cdot \vec{dS} = \int_A \vec{E} \cdot \vec{dS} + \int_B \vec{E} \cdot \vec{dS} + \int_C \vec{E} \cdot \vec{dS}$$

$$\oint_S \vec{E} \cdot \vec{dS} = \int_A E\, dS \cos 90^\circ + \int_B E\, dS \cos 90^\circ + \int_C E\, dS \cos 0^\circ = E \int_C dS \ = E \times 2 \pi r l$$

$$\frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$ (where λ is the liner charge density)

$$\therefore \quad E \times 2 \pi r l = \frac{\lambda l}{\varepsilon_0}$$

or $$E = \frac{1}{2 \pi \varepsilon_0} \frac{\lambda}{r}$$

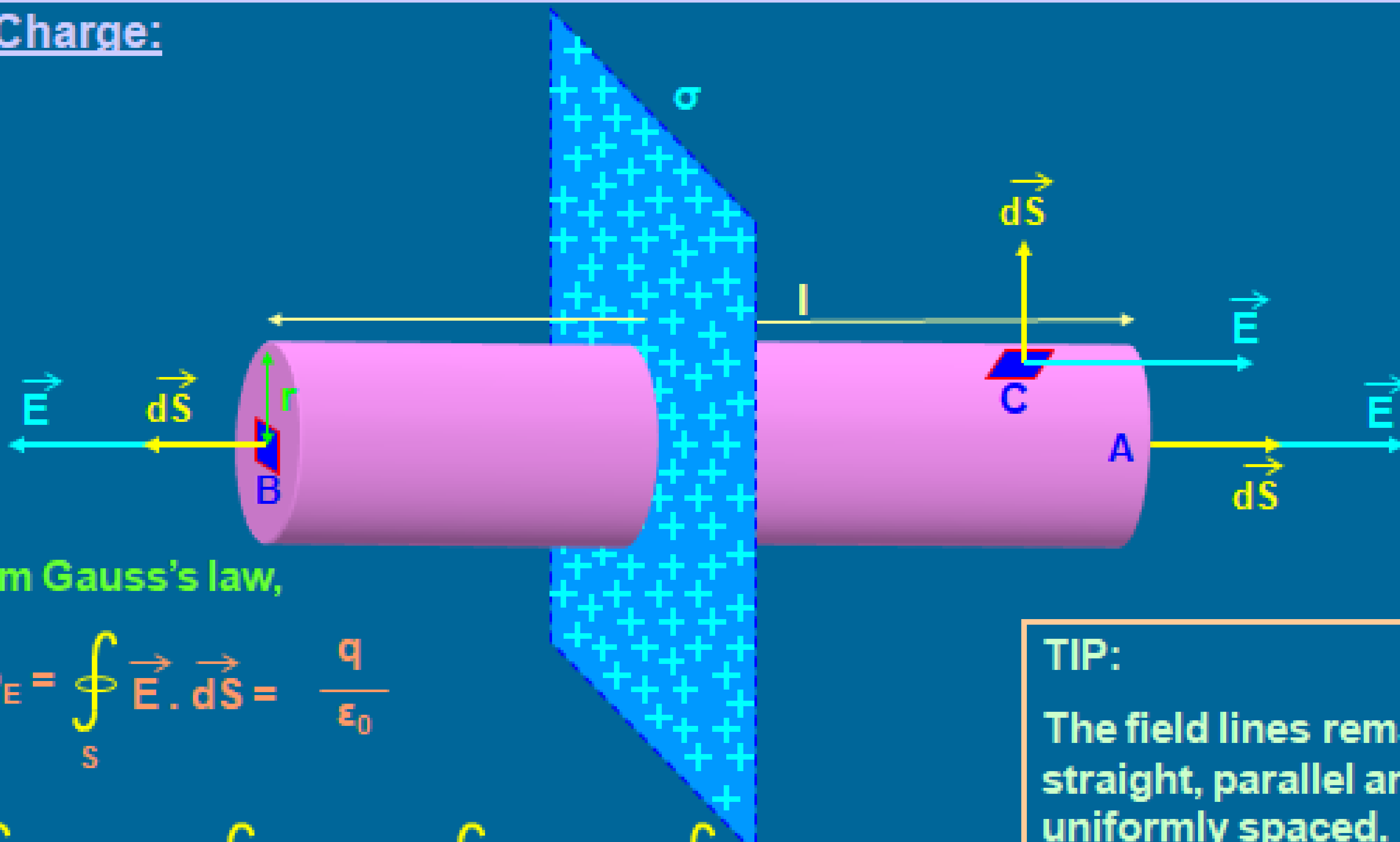
or $$\boxed{E = \frac{1}{4 \pi \varepsilon_0} \frac{2\lambda}{r}}$$

The direction of the electric field intensity is radially outward from the positive line charge. For negative line charge, it will be radially inward.

Note:

The electric field intensity is independent of the size of the Gaussian surface constructed. It depends only on the distance of point of consideration. i.e. the Gaussian surface should contain the point of consideration.

2. Electric Field Intensity due to an Infinitely Long, Thin Plane Sheet of Charge:



From Gauss's law,

$$\Phi_E = \oint_S \vec{E}\cdot\vec{dS} = \frac{q}{\varepsilon_0}$$

TIP:

The field lines remain straight, parallel and uniformly spaced.

$$\oint_S \vec{E}\cdot\vec{dS} = \int_A \vec{E}\cdot\vec{dS} + \int_B \vec{E}\cdot\vec{dS} + \int_C \vec{E}\cdot\vec{dS}$$

$$\oint_S \vec{E}\cdot\vec{dS} = \int_A E\,dS\cos 0^\circ + \int_B E\,dS\cos 0^\circ + \int_C E\,dS\cos 90^\circ = 2E\int dS = 2E \times \pi r^2$$

$$\frac{q}{\varepsilon_0} = \frac{\sigma \pi r^2}{\varepsilon_0}$$ (where σ is the surface charge density)

$$\therefore \quad 2E \times \pi r^2 = \frac{\sigma \pi r^2}{\varepsilon_0}$$

or $$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

The direction of the electric field intensity is normal to the plane and away from the positive charge distribution. For negative charge distribution, it will be towards the plane.

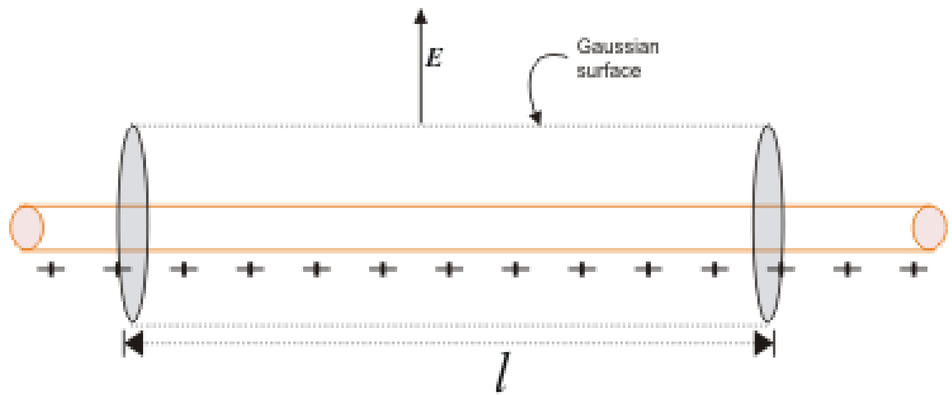
Note:

The electric field intensity is independent of the size of the Gaussian surface constructed. It neither depends on the distance of point of consideration nor the radius of the cylindrical surface.

If the plane sheet is thick, then the charge distribution will be available on both the sides. So, the charge enclosed within the Gaussian surface will be twice as before. Therefore, the field will be twice.

$$\therefore \quad \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Electric Field due to Infinite Wire



- Let us consider an infinitely long wire with linear charge density **λ** and length L. To calculate electric field, we assume a cylindrical Gaussian surface. As the electric field E is radial in direction, the flux through the end of the cylindrical surface will be zero.
- This is because the electric field and area vector are perpendicular to each other. As the electric field is perpendicular to every point of the curved surface, we can say that its magnitude will be constant.

- The surface area of the curved cylindrical surface is 2πrl. The electric flux through the curve is
- E × 2πrl
- According to Gauss's Law

$$\Phi = \frac{q}{\varepsilon_0}$$

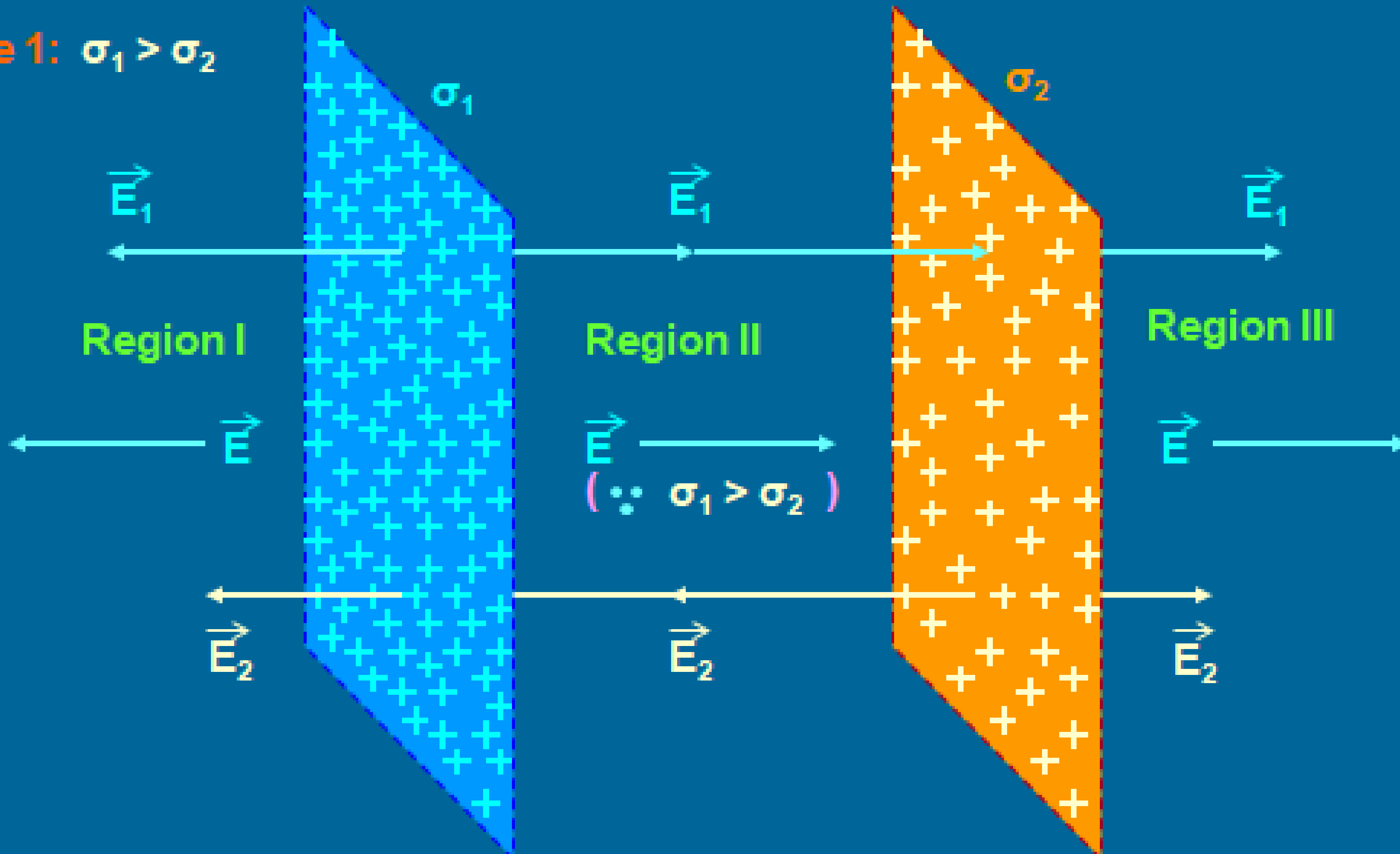
$$E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

- You need to remember that the direction of the electric field is radially outward if linear charge density is positive. On the other hand, it will be radially inward if the linear charge density is negative.

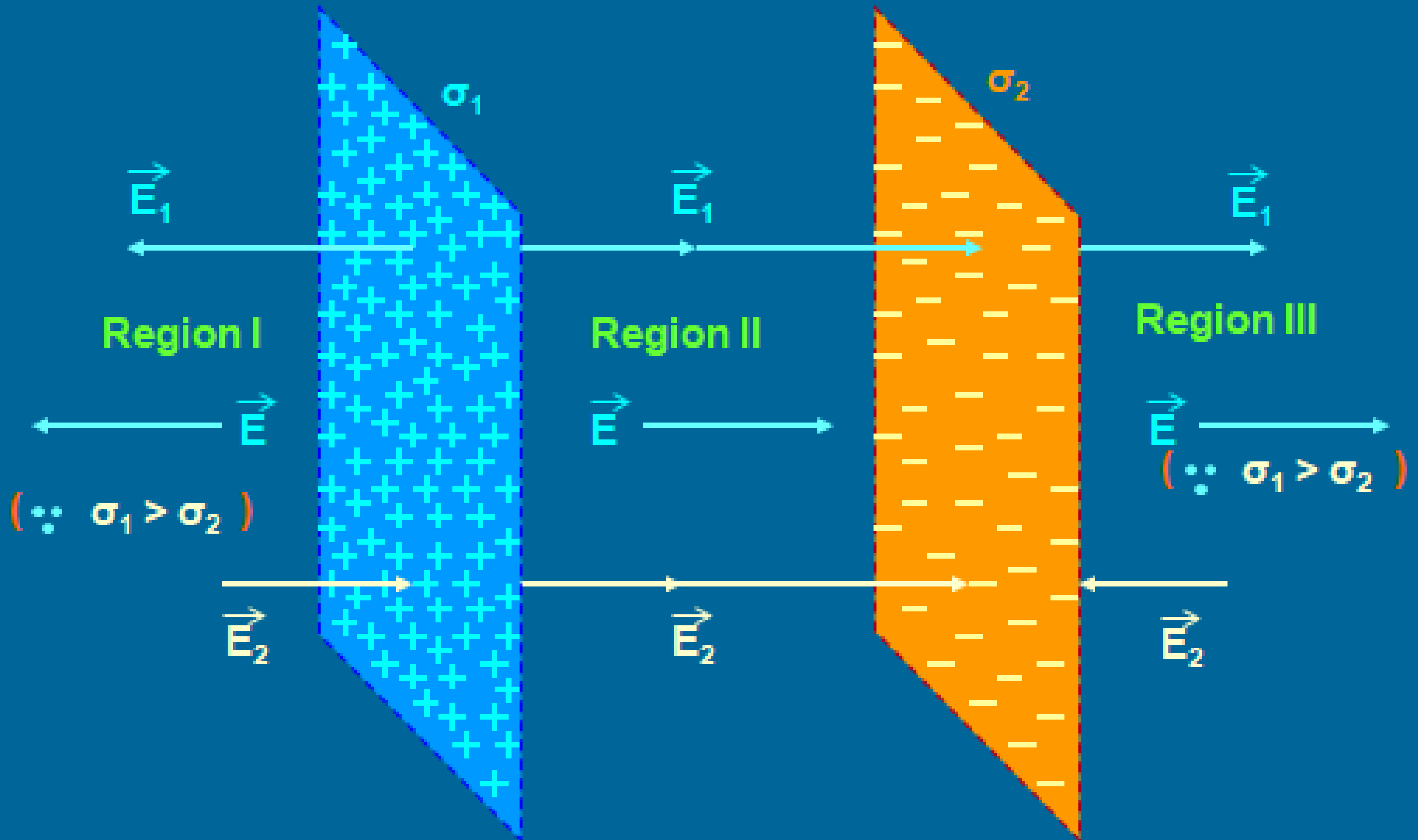
3. Electric Field Intensity due to Two Parallel, Infinitely Long, Thin Plane Sheet of Charge:

Case 1: $\sigma_1 > \sigma_2$



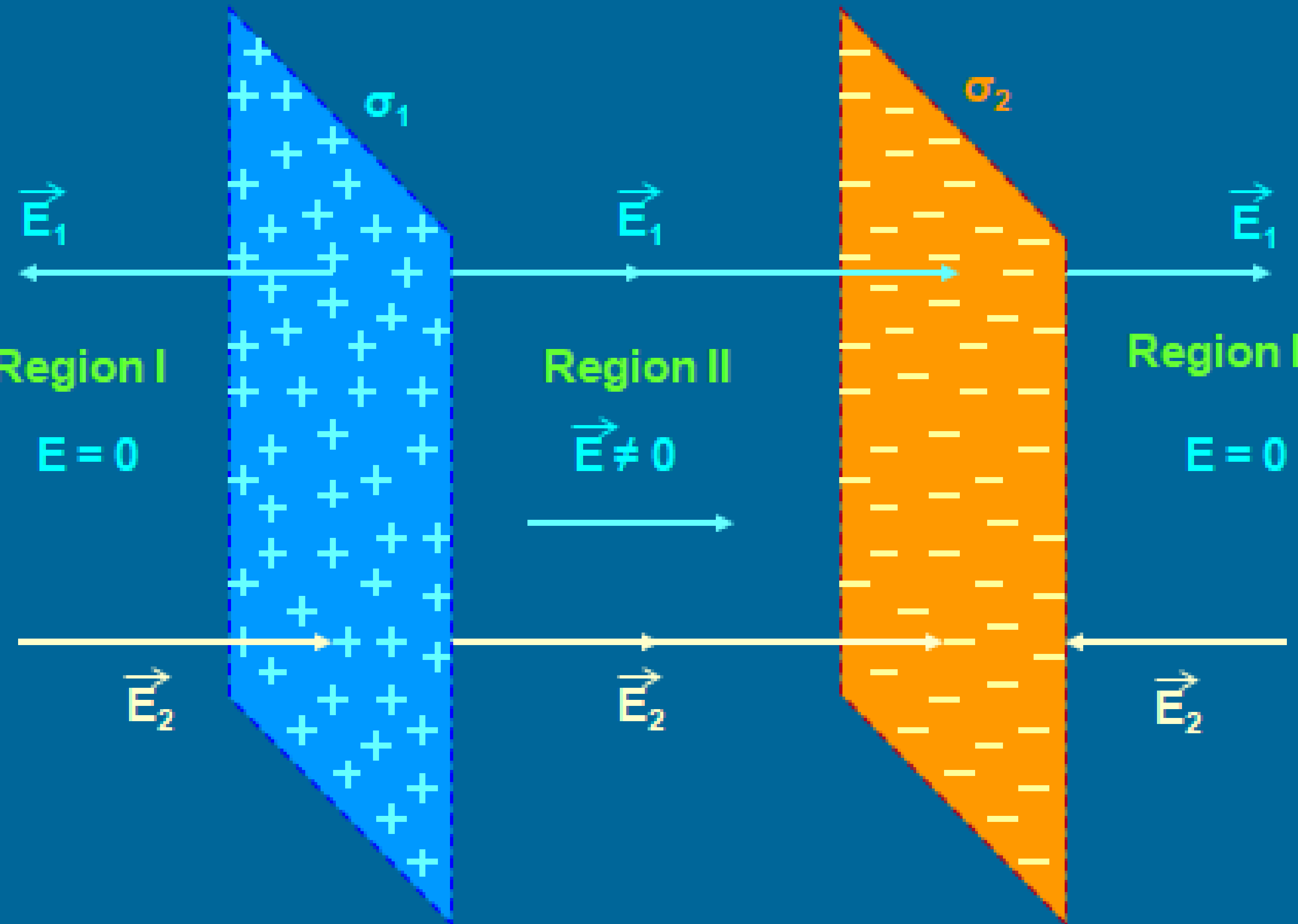
$E = E_1 + E_2$ $E = \dfrac{\sigma_1 + \sigma_2}{2\,\varepsilon_0}$	$E = E_1 - E_2$ $E = \dfrac{\sigma_1 - \sigma_2}{2\,\varepsilon_0}$	$E = E_1 + E_2$ $E = \dfrac{\sigma_1 + \sigma_2}{2\,\varepsilon_0}$

Case 2: $+\sigma_1$ & $-\sigma_2$



$E = E_1 - E_2$ $E = \dfrac{\sigma_1 - \sigma_2}{2\varepsilon_0}$	$E = E_1 + E_2$ $E = \dfrac{\sigma_1 + \sigma_2}{2\varepsilon_0}$	$E = E_1 - E_2$ $E = \dfrac{\sigma_1 - \sigma_2}{2\varepsilon_0}$

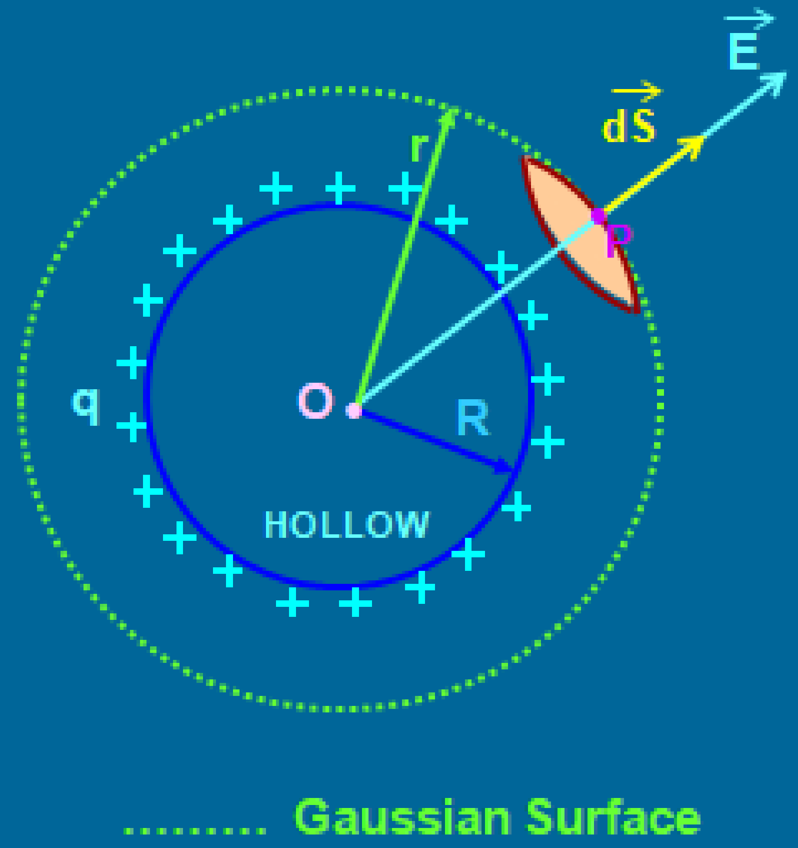
Case 3: $+\sigma$ & $-\sigma$



Region I	Region II	Region III
$E = E_1 - E_2$ $E = \dfrac{\sigma_1 - \sigma_2}{2\varepsilon_0} = 0$	$E = E_1 + E_2$ $E = \dfrac{\sigma_1 + \sigma_2}{2\varepsilon_0} = \dfrac{\sigma}{\varepsilon_0}$	$E = E_1 - E_2$ $E = \dfrac{\sigma_1 - \sigma_2}{2\varepsilon_0} = 0$

4. Electric Field Intensity due to a Uniformed Charged This Spherical Shell:

i) At a point P outside the shell:



From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

Since $\vec{E}$ and $\vec{dS}$ are in the same direction,

$$\therefore \quad \Phi_E = \oint_S E \; dS = \frac{q}{\varepsilon_0}$$

$$\text{or} \quad \Phi_E = E \oint_S dS = \frac{q}{\varepsilon_0}$$

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0} \quad \text{or} \quad \boxed{E = \frac{q}{4\pi\varepsilon_0 \; r^2}}$$

Since $q = \sigma \times 4\pi R^2$, $\quad \therefore \quad \boxed{E = \frac{\sigma \; R^2}{\varepsilon_0 \; r^2}}$

Electric field due to a uniformly charged thin spherical shell at a point outside the shell is such as if the whole charge were concentrated at the centre of the shell.

ii) At a point A on the surface of the shell:

From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

Since $\vec{E}$ and $\vec{dS}$ are in the same direction,

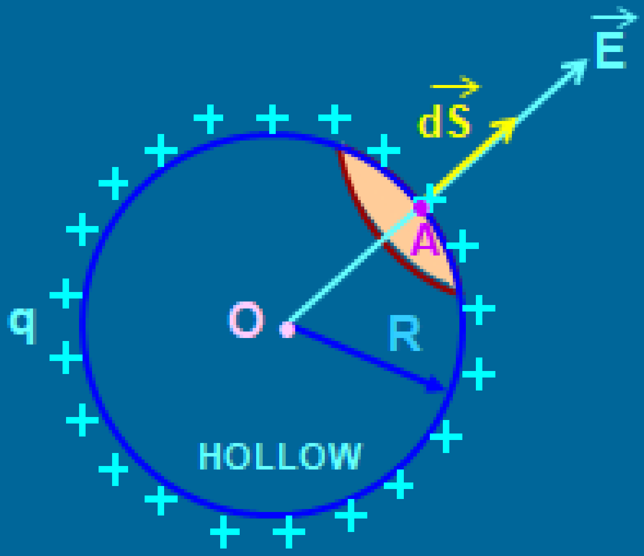
$$\therefore \quad \Phi_E = \oint_S E \, dS = \frac{q}{\varepsilon_0}$$

$$\text{or} \quad \Phi_E = E \oint_S dS = \frac{q}{\varepsilon_0}$$

$$E \times 4\pi R^2 = \frac{q}{\varepsilon_0} \quad \text{or} \quad \boxed{E = \frac{q}{4\pi\varepsilon_0 R^2}}$$

Since $q = \sigma \times 4\pi R^2$,

$$\therefore \quad \boxed{E = \frac{\sigma}{\varepsilon_0}}$$



Electric field due to a uniformly charged thin spherical shell at a point on the surface of the shell is maximum.

iii) At a point B inside the shell:

From Gauss's law,

$$\Phi_E = \oint_S \vec{E}\,.\,\vec{dS} = \frac{q}{\varepsilon_0}$$

Since $\vec{E}$ and $\vec{dS}$ are in the same direction,

$$\therefore \quad \Phi_E = \oint_S E\ dS = \frac{q}{\varepsilon_0}$$

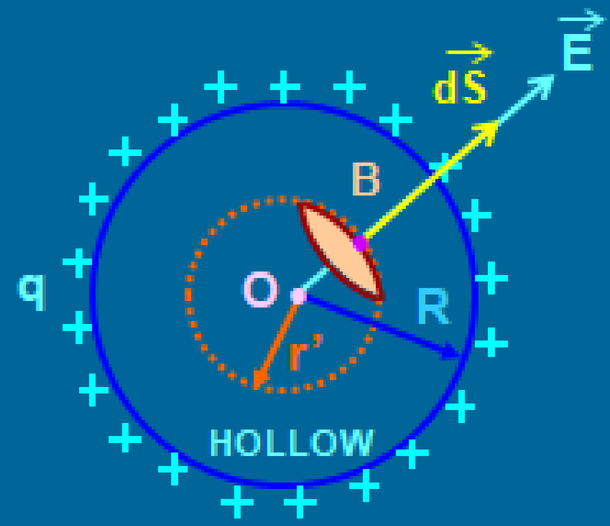
$$\text{or} \quad \Phi_E = E \oint_S dS = \frac{q}{\varepsilon_0}$$

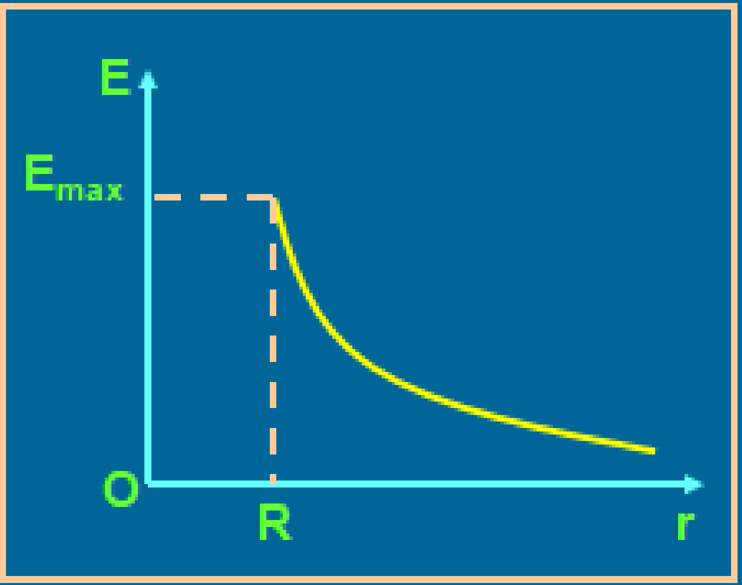
$$E \times 4\pi r'^2 = \frac{q}{\varepsilon_0} \quad \text{or} \quad \boxed{E = \frac{0}{4\pi\varepsilon_0 r'^2}}$$

(since q = 0 inside the Gaussian surface)

$$\therefore \quad \boxed{E = 0}$$

This property E = 0 inside a cavity is used for electrostatic shielding.





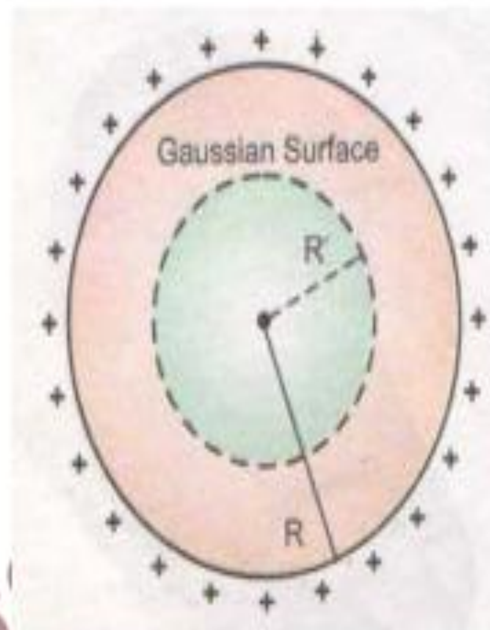
END

Q # 15. Calculate the electric field intensity due to a hollow charged sphere.

Ans. Consider a hollow charged conducting sphere of radius 'R' is given a positive charge 'Q', as shown in the figure below:

We want to find out electric field intensity at point 'P' inside the hollow charged sphere. For this, we consider a spherical Gaussian surface which passes through the point P.

It can be seen that the charge enclosed by the Gaussian surface is zero. Then by applying the Gauss's law, we have



$$\Phi_e = \frac{q}{\varepsilon_0} = 0 \quad \text{----------------- (1)}$$

Also

$$\Phi_e = \mathbf{E}.\mathbf{A} \quad \text{----------------- (2)}$$

Comparing eq. (1) and (2), we get

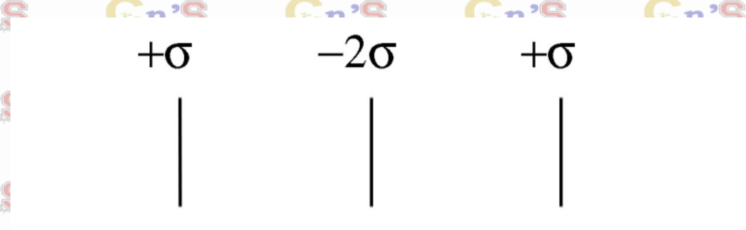
$$\Phi_e = \mathbf{E}.\mathbf{A} = 0$$

As $\mathbf{A} \neq 0$,

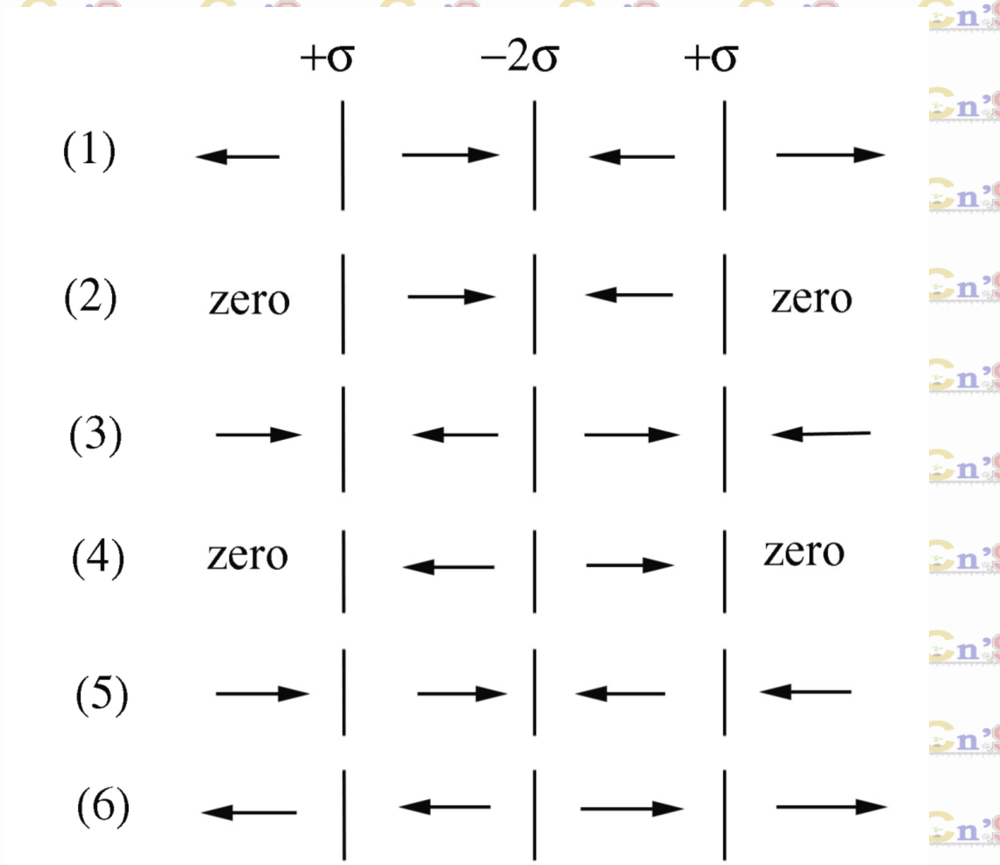
$$\text{Therefore } \mathbf{E} = 0$$

Thus the interior of a hollow charge sphere is a field free region.

Question: Superposition

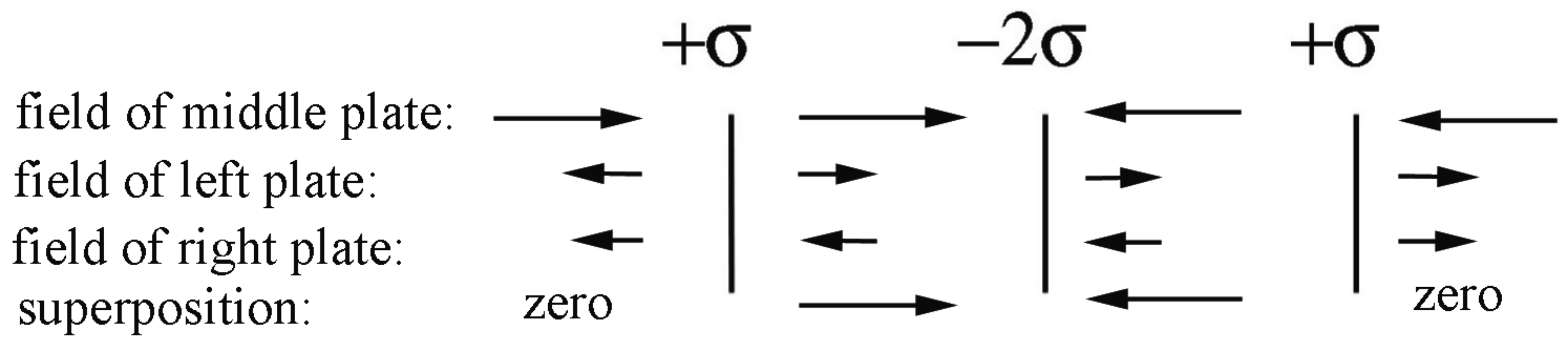


Three infinite sheets of charge are shown above. The sheet in the middle is negatively charged with charge per unit area -2σ, and the other two sheets are positively charged with charge per unit area $+\sigma$. Which set of arrows (and zeros) best describes the electric field?

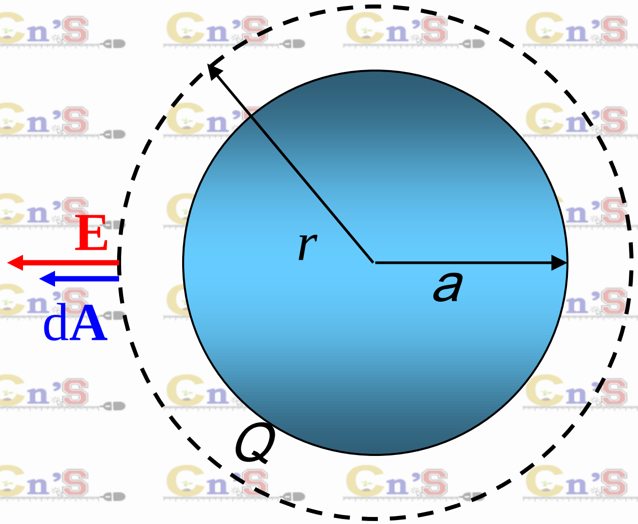


Answer: Superposition

Answer 2 . The fields of each of the plates are shown in the different regions along with their sum.



An insulating sphere of radius *a* has a uniform charge density ρ and a total positive charge *Q*. Calculate the electric field outside the sphere.



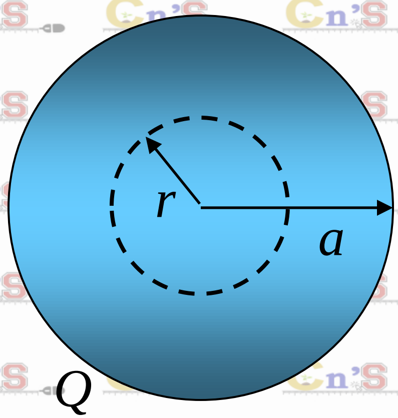
Since the charge distribution is spherically symmetric we select a spherical Gaussian surface of radius *r* > *a* centered on the charged sphere. Since the charged sphere has a positive charge, the field will be directed radially outward. On the Gaussian sphere **E** is always parallel to d**A**, and is constant.

❶ $$\text{Left side: } \oint \vec{E}\cdot d\vec{A} = \oint E\,dA = E\oint dA = E\left(4\pi r^2\right)$$

❷ $$\text{Right side: } \frac{Q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

❸ $$E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0} \quad \text{or} \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = k_e\frac{Q}{r^2}$$

Find the electric field at a point inside the sphere.



Now we select a spherical Gaussian surface with radius $r < a$. Again the symmetry of the charge distribution allows us to simply evaluate the left side of Gauss's law just as before.

❶ $$\boxed{\text{Left side: } \oint \vec{E} \cdot d\vec{A} = \oint E\, dA = E \oint dA = E\left(4\pi r^2\right)}$$

The charge inside the Gaussian sphere is no longer Q. If we call the Gaussian sphere volume V' then

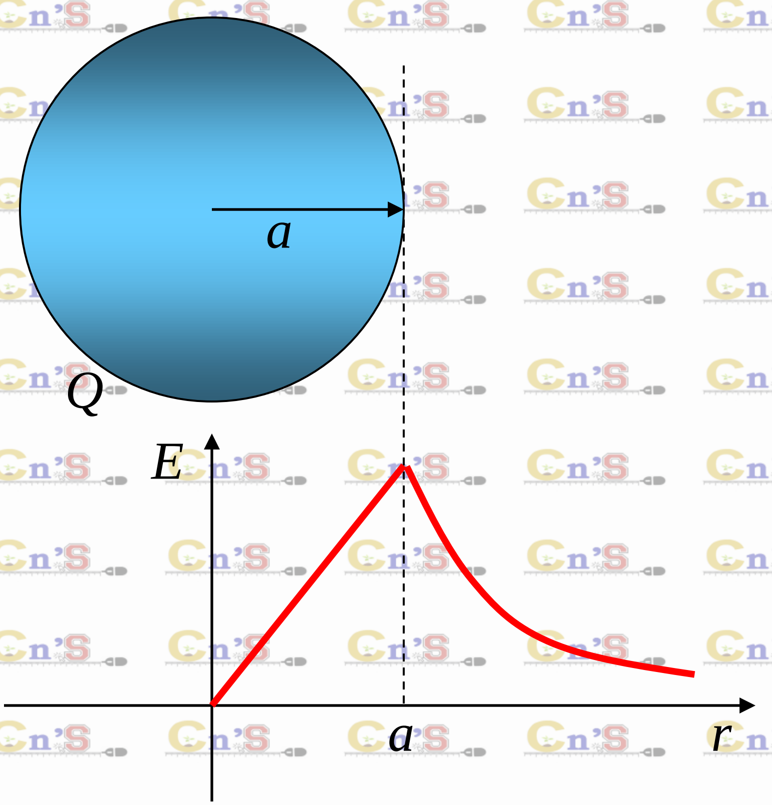
❷ $$\boxed{\text{Right side: } Q_{in} = \rho V' = \rho \frac{4}{3}\pi r^3}$$

❸ $$\boxed{E\left(4\pi r^2\right) = \frac{Q_{in}}{\varepsilon_0} = \frac{4\rho\pi r^3}{3\varepsilon_0}}$$

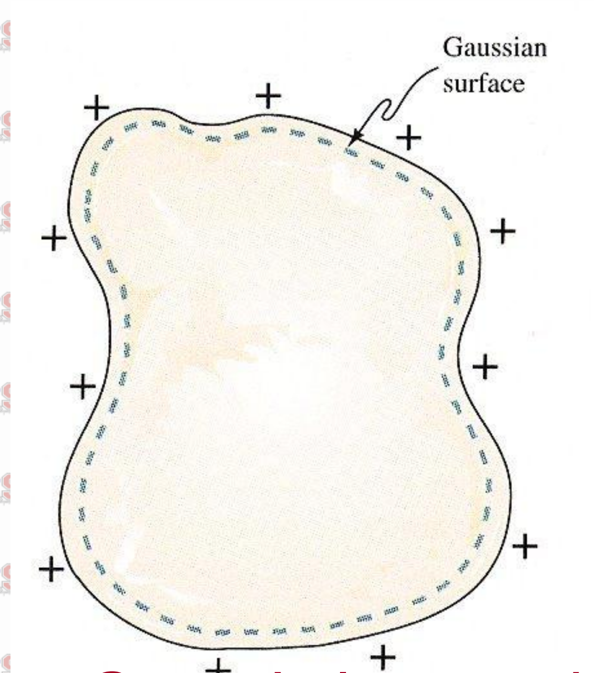
❹ $$\boxed{E = \frac{4\rho\pi r^3}{3\varepsilon_0\left(4\pi r^2\right)} = \frac{\rho}{3\varepsilon_0} r \quad \text{but} \quad \rho = \frac{Q}{\frac{4}{3}\pi a^3} \quad \text{so} \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a^3} r = k_e \frac{Q}{a^3} r}$$

We found for $r > a$, $E = k_e \dfrac{Q}{r^2}$

and for $r < a$, $E = \dfrac{k_e Q}{a^3} r$



Any net charge on an isolated conductor must reside on its surface and the electric field just outside a charged conductor is perpendicular to its surface (and has magnitude σ/ε_0). Use Gauss's law to show this.

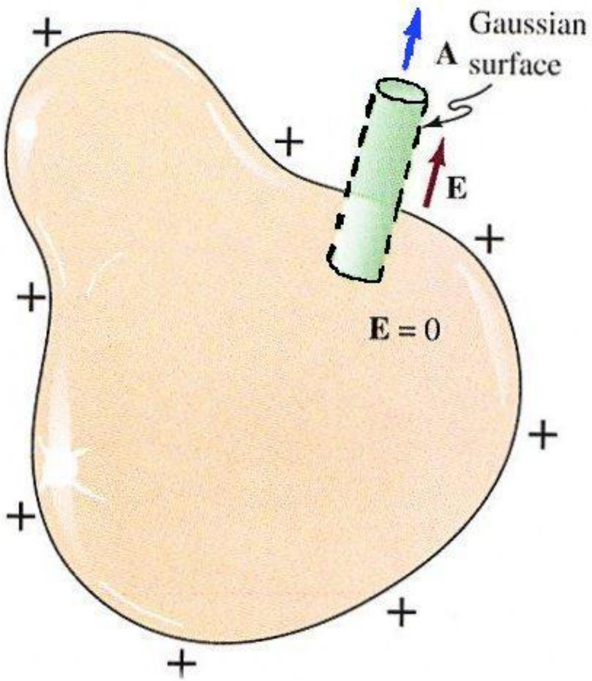


For an arbitrarily shaped conductor we can draw a Gaussian surface inside the conductor. Since we have shown that the electric field inside an isolated conductor is zero, the field at every point on the Gaussian surface must be zero.

$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}}$$

From Gauss's law we then conclude that the net charge inside the Gaussian surface is zero. Since the surface can be made arbitrarily close to the surface of the conductor, *any net charge must reside on the conductor's surface.*

We can also use Gauss's law to determine the electric field just outside the surface of a charged conductor. Assume the surface charge density is σ.

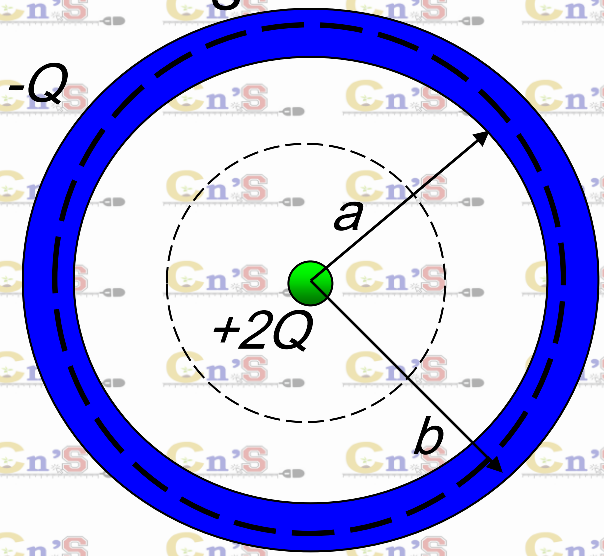


Since the field inside the conductor is zero there is no flux through the face of the cylinder inside the conductor. If **E** had a component along the surface of the conductor then the free charges would move under the action of the field creating surface currents. Thus **E** is perpendicular to the conductor's surface, and the flux through the cylindrical surface must be zero. Consequently the net flux through the cylinder is *EA* and Gauss's law gives:

$$\Phi_E = EA = \frac{Q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \text{ or } E = \frac{\sigma}{\varepsilon_0}$$

Worked Example

A conducting spherical shell of inner radius a and outer radius b with a net charge $-Q$ is centered on point charge $+2Q$. Use Gauss's law to find the electric field everywhere, and to determine the charge distribution on the spherical shell.



First find the field for $0 < r < a$

This is the same as Ex. 2 and is the field due to a point charge with charge $+2Q$.

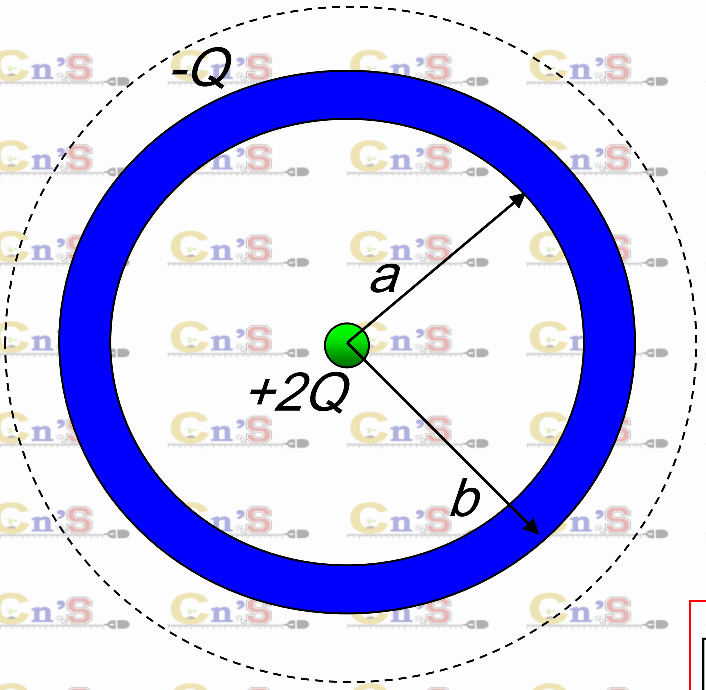
$$\boxed{E = k_e \frac{2Q}{r^2}}$$

Now find the field for $a < r < b$

The field must be zero inside a conductor in equilibrium. Thus from Gauss's law Q_{in} is zero. There is a $+2Q$ from the point charge so we must have $Q_a = -2Q$ on the inner surface of the spherical shell. Since the net charge on the shell is $-Q$ we can get the charge on the outer surface from $Q_{net} = Q_a + Q_b$.

$$Q_b = Q_{net} - Q_a = -Q - (-2Q) = +Q.$$

Find the field for $r > b$



From the symmetry of the problem, the field in this region is radial and everywhere perpendicular to the spherical Gaussian surface. Furthermore, the field has the same value at every point on the Gaussian surface so the solution then proceeds exactly as in Ex. 2, but $Q_{in}=2Q-Q$.

$$\oint \vec{E}\cdot d\vec{A} = \oint E\, dA = E\oint dA = E\left(4\pi r^2\right)$$

Gauss's law now gives:

$$E\left(4\pi r^2\right) = \frac{Q_{in}}{\varepsilon_0} = \frac{2Q-Q}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \quad \text{or} \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = k_e\frac{Q}{r^2}$$