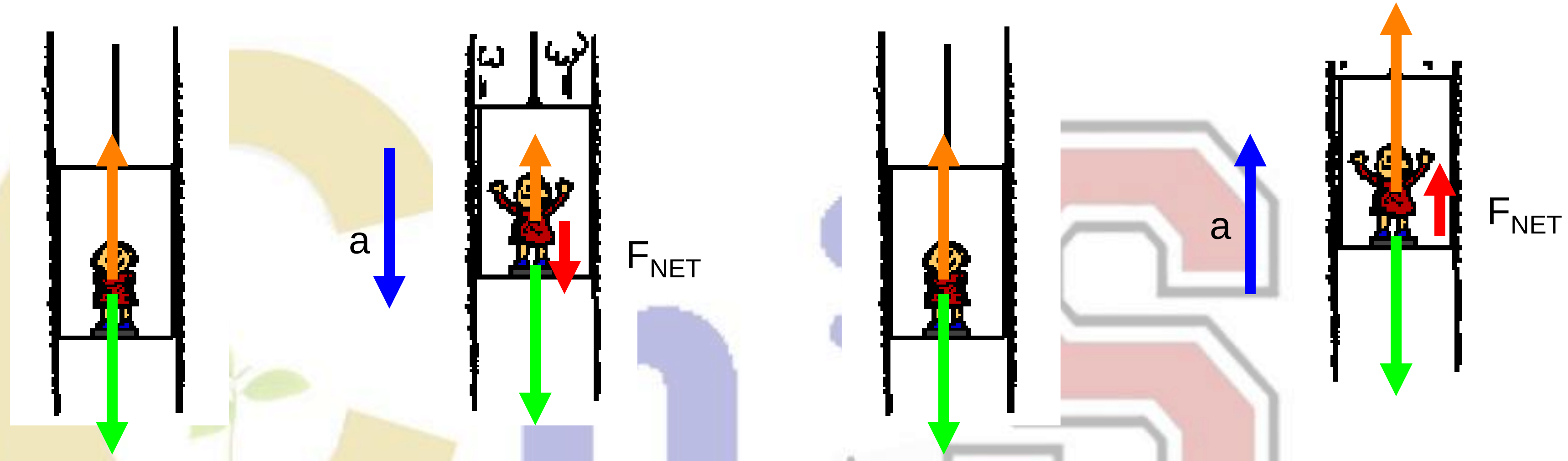
Equilibrium of Forces

Apparent Weight in an Elevator

Coming Down - Concept FBD’s



Elevator at rest

Zero Acceleration

$F_{NET} = 0$

$F_N = F_g$

Elevator starts accelerating downwards

$F_{NET} = m\,a$ (down)

$F_N < F_g$

Elevator at constant speed downwards

Zero Acceleration

$F_{NET} = 0$

$F_N = F_g$

Elevator is slowing down to a stop while moving down

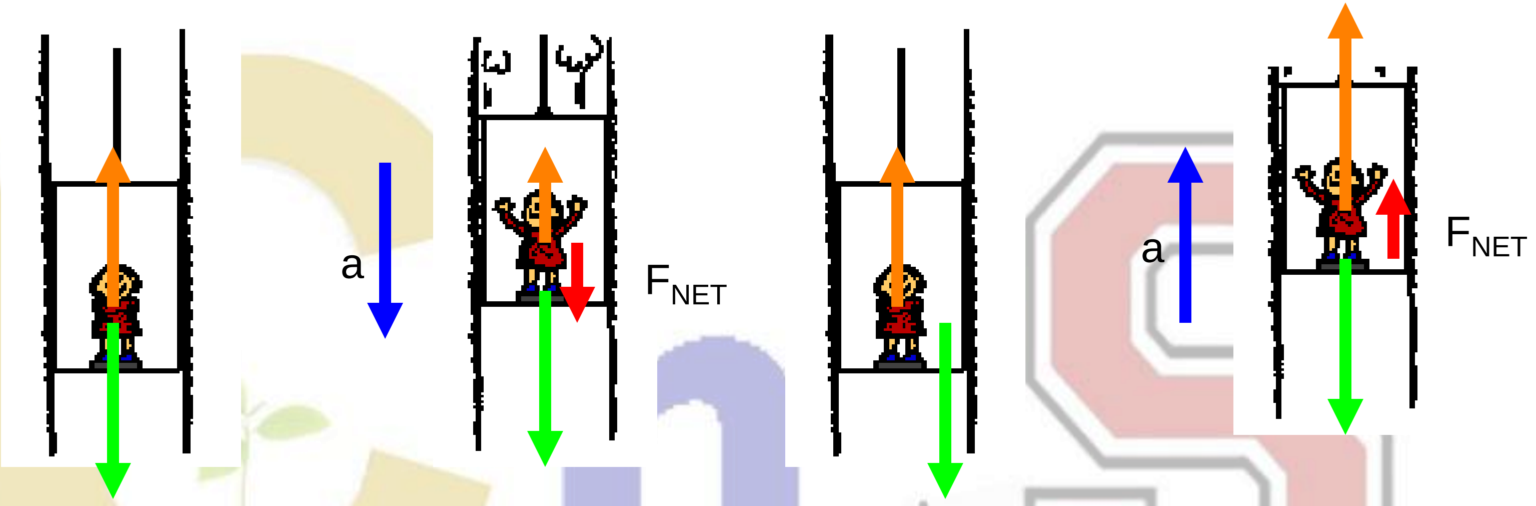
$F_{NET} = m\,a$ (up)

$F_N > F_g$

The normal Force (orange) is what she feels

Apparent Weight in an Elevator

Coming Down - Theory



$F_{NET} = 0$

$F_N = F_g = \mathbf{mg}$

Weigh normal

$F_{NET} = m\,a$ (down)

$F_{NET} = F_g - F_N$

$mg - F_N = ma$

$F_N = mg - ma$

$\mathbf{F_N = m(g - a)}$

Weigh less

$F_{NET} = 0$

$F_N = F_g = \mathbf{mg}$

Weigh normal

$F_{NET} = m\,a$ (up)

$F_{NET} = F_N - F_g$

$F_N - mg = ma$

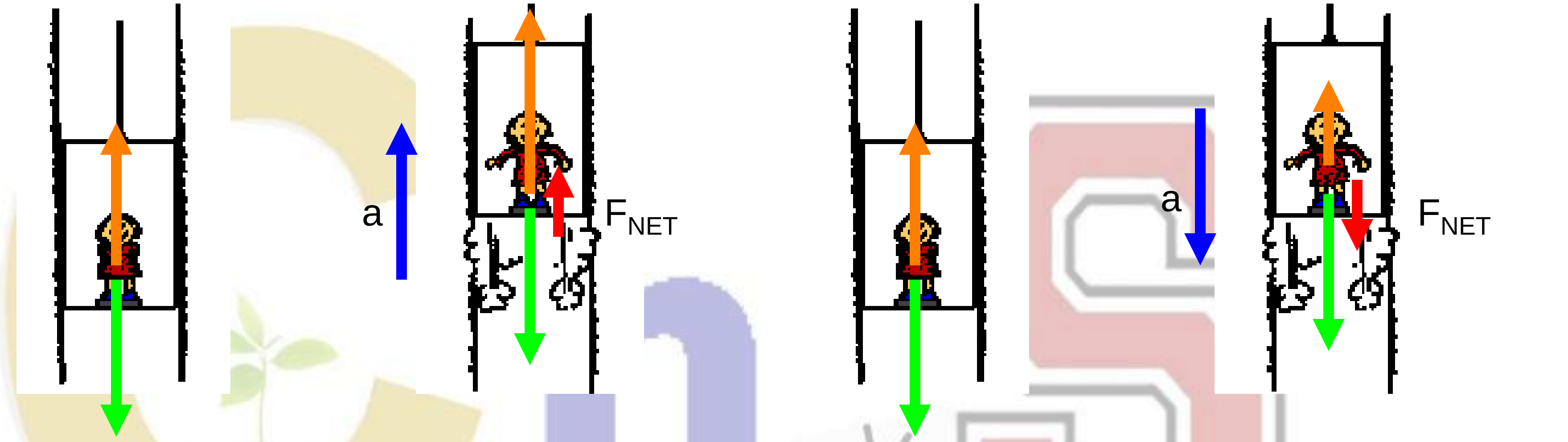
$F_N = ma + mg$

$\mathbf{F_N = m(g + a)}$

Weigh more

Apparent Weight in an Elevator

Going Up - Concept FBD's



Elevator at rest

Zero Acceleration

$F_{NET} = 0$

$F_N = F_g$

Elevator starts accelerating upwards

$F_{NET} = m\ a$ (up)

$F_N > F_g$

Elevator at constant speed upwards

Zero Acceleration

$F_{NET} = 0$

$F_N = F_g$

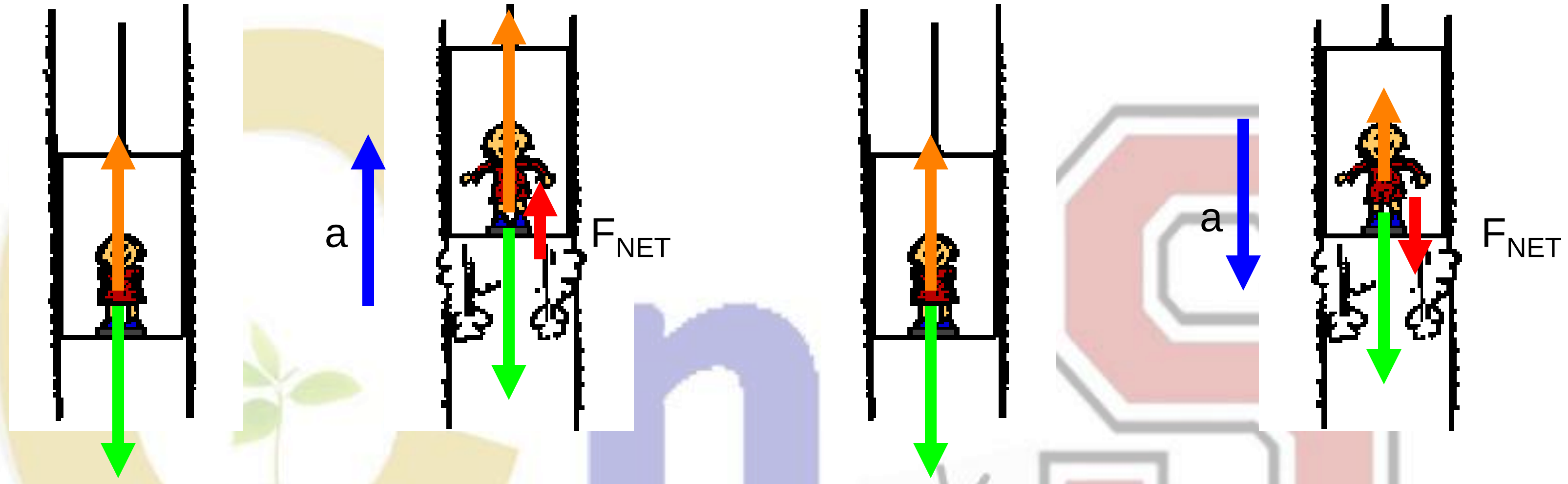
Elevator is slowing down to a stop while moving up

$F_{NET} = m\ a$ (down)

$F_N < F_g$

Apparent Weight in an Elevator

Going Up - Theory



$F_{NET} = 0$

$F_N = F_g = \mathbf{mg}$

Weigh normal

$F_{NET} = m\,a$ (up)

$F_{NET} = F_N - F_g$

$F_N - mg = ma$

$F_N = ma + mg$

$\mathbf{F_N = m(g + a)}$

Weigh more

$F_{NET} = 0$

$F_N = F_g = \mathbf{mg}$

Weigh normal

$F_{NET} = m\,a$ (down)

$F_{NET} = F_g - F_N$

$mg - F_N = ma$

$F_N = mg - ma$

$\mathbf{F_N = m(g - a)}$

Weigh less

Accelerating Lifts

- A man weighs himself with a scale in an elevator. While the elevator is at rest, he measures a weight of 800 N.
  - What weight does the scale read if the elevator accelerates upward at 2.0 m/s^2?
  - What weight does the scale read if the elevator accelerates downward at 2.0 m/s^2?

- Upward:

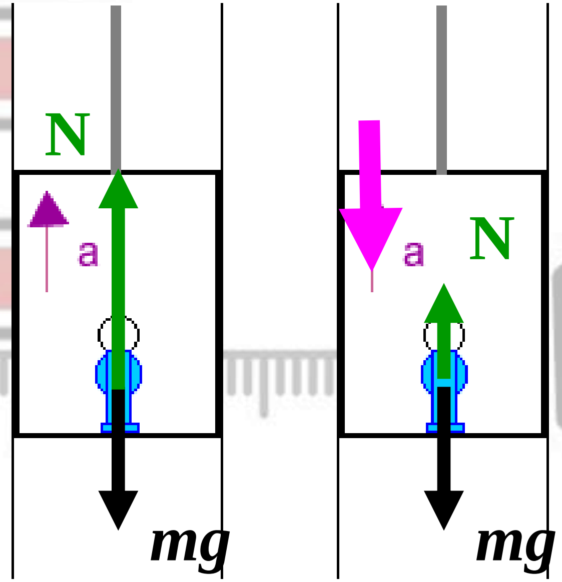
$$\sum F_y = N - mg = ma$$

$$N = mg + ma = m(g + a)$$

$$m = \frac{w}{g} = \frac{800\,\mathrm{N}}{9.8\,\mathrm{m/s}^2} = 80\,\mathrm{N}$$

$$N = 80(2.0 + 9.8) = 944N$$

$$\boxed{N > mg}$$

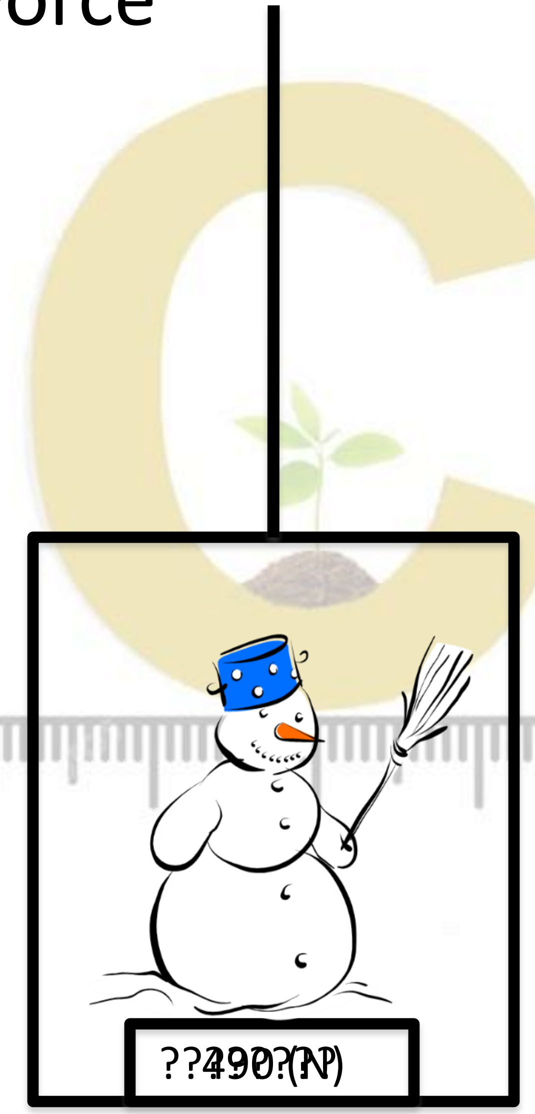


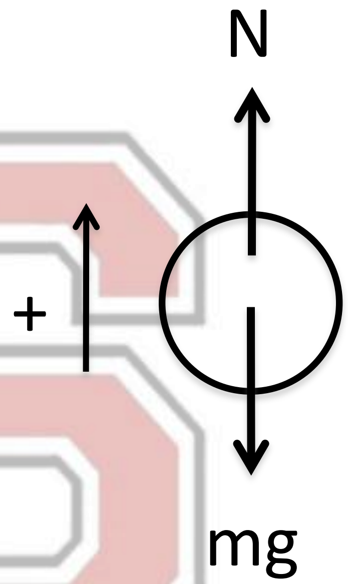
- Downward: $N = 80(-2.0 + 9.8) = 624\,\mathrm{N}$

$$\boxed{N < mg}$$

What will the scale read if the elevator is at rest?

Snowman mass = 50 kg, Scale Reading is Normal Force





$$\Sigma F = ma$$

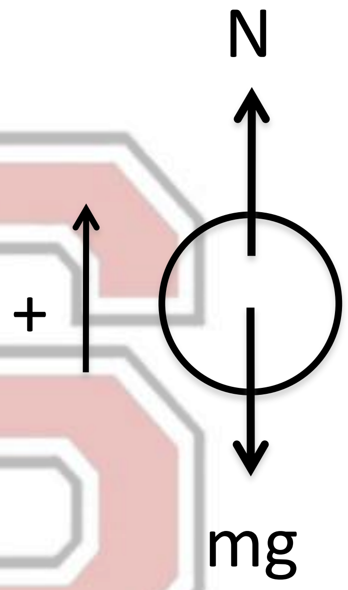
$$a = 0$$

$$N - mg = 0$$

$$N = mg = 50kg \times 9.8 \, {}^{N}\!/_{kg} = 490(N)$$

What will the scale read if the elevator has an upward acceleration of 2 m/s² ?

Snowman mass = 50 kg
(he doesn't melt)



$$\Sigma F = ma$$

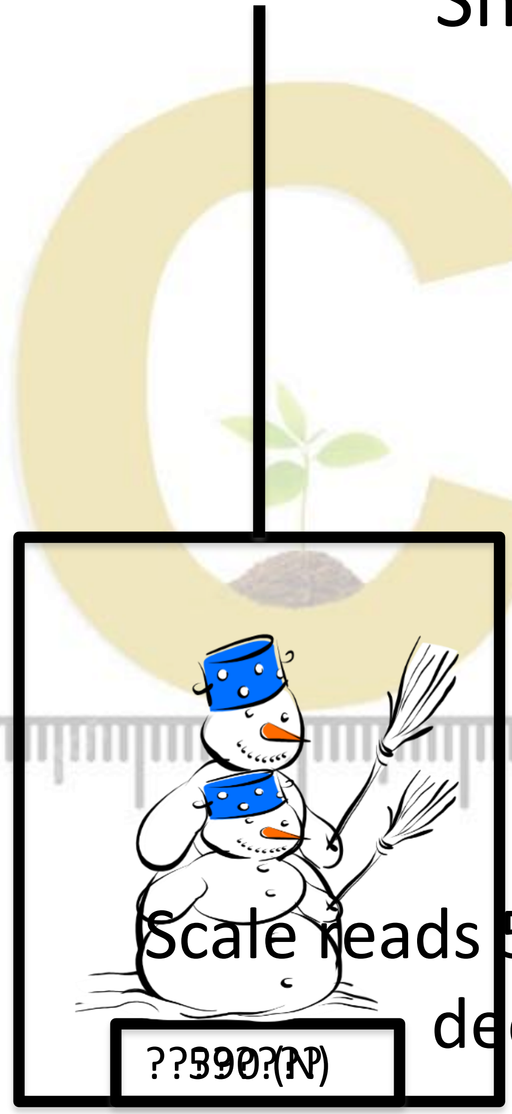
$$N - mg = ma$$

$$N = ma + mg$$

$$a = +2m/s^2$$

$$N = 50kg \times 2m/s^2 + 50kg \times 9.8N/kg$$

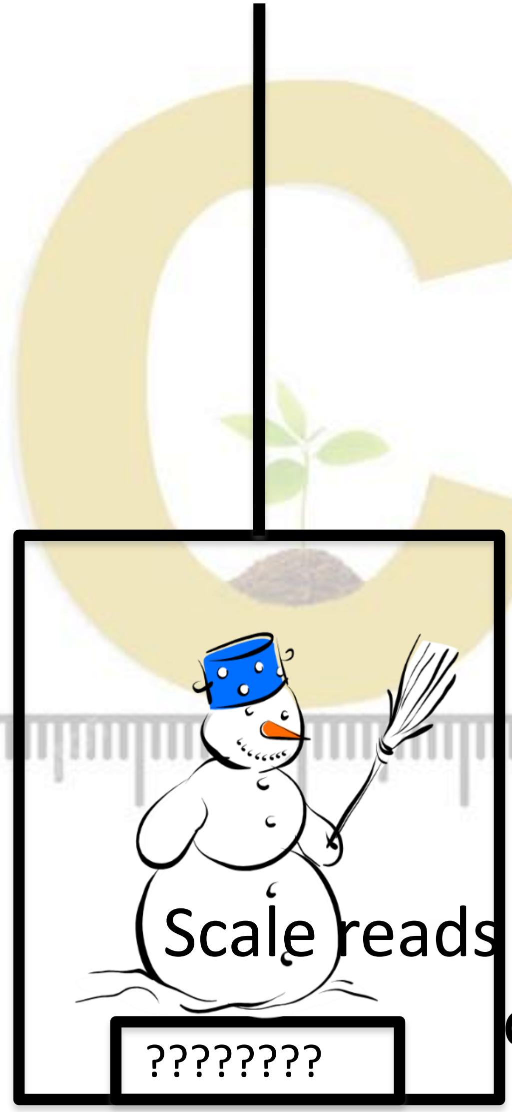
$$N = 590(N)$$

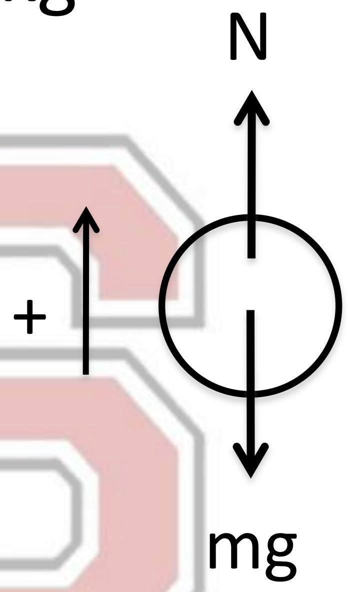


Scale reads 590(N) increasing speed on the way up OR decreasing speed on the way down

What will the scale read if the elevator is moving at a constant velocity?

Snowman mass = 50 kg





$$\Sigma F = ma$$

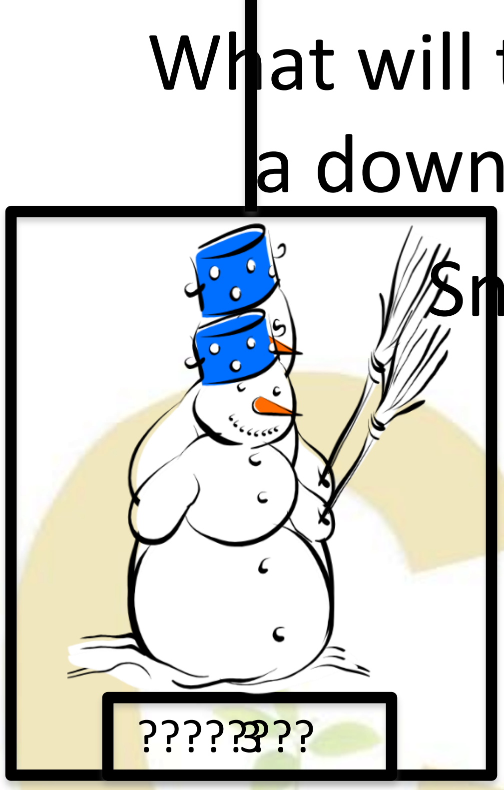
$$a = 0$$

$$N - mg = 0$$

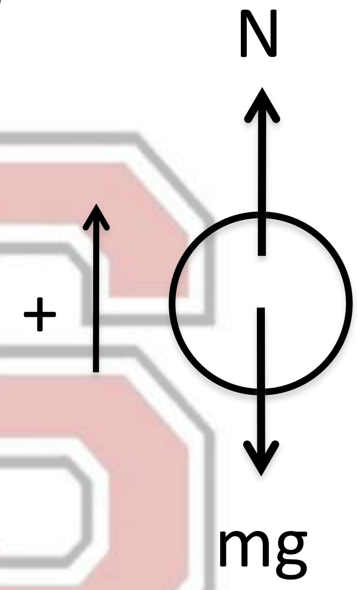
$$N = mg = 50kg \times 9.8 \,{}^{N}\!/_{kg} = 490(N)$$

Scale reads 490(N) for constant velocity on the way up or way down (and when at rest)

What will the scale read if the elevator has a downward acceleration of 3 m/s^2 ?



Snowman mass = 50 kg

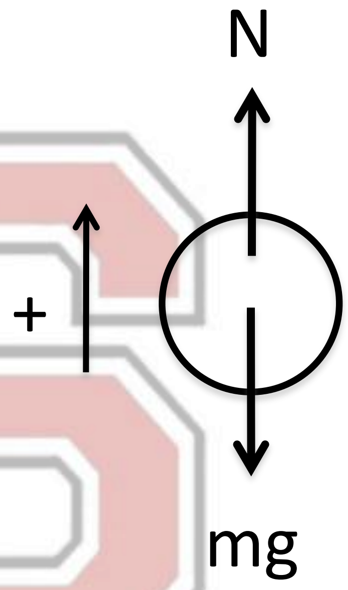


$$\Sigma F = ma$$
$$N - mg = ma$$
$$N = ma + mg$$
$$a = -3m/s^2$$
$$N = 50kg \times (-3)m/s^2 + 50kg \times 9.8N/kg$$
$$N = 340(N)$$

Scale reads 340(N) increasing speed on the way down OR decreasing speed on the way up

What will the scale read if the elevator has a downward acceleration of 9.8 m/s^2 ?

Snowman mass = 50 kg



$$\Sigma F = ma$$

$$N - mg = ma$$

$$N = ma + mg$$

$$a = -9.8 m/s^2$$

$$N = 50kg \times (-9.8) m/s^2 + 50kg \times 9.8 N/kg$$

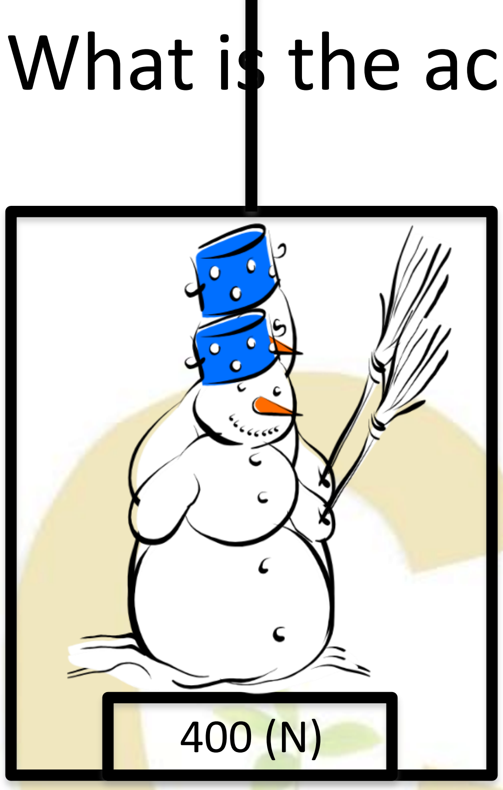
$$N = 0(N)!!!$$

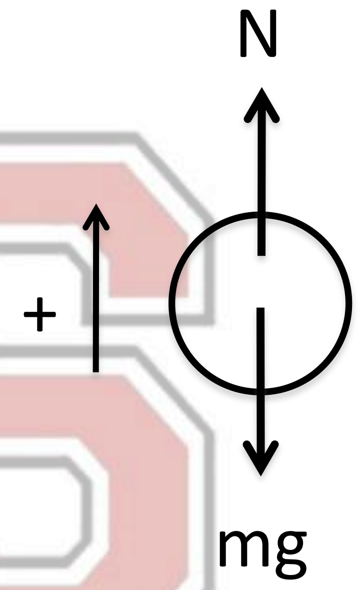
Scale reads 0(N) increasing speed on the way down OR decreasing speed on the way up

What Has to Happen for an Elevator to have a Downward Acceleration of 9.8m/s^2?

What is the acceleration if the scale reads 400 (N)?

Snowman mass = 50 kg





$$\Sigma F = ma$$

$$N - mg = ma$$

$$a = (N - mg)/m$$

$$a = (400(N) - 50kg \times 9.8N/kg)/50$$

$$a = -1.8m/s^2$$

Elevator is either increasing speed on the way down or decreasing speed on the way up

According to Rene Descartes, the human body is a mechanical system designed by the hands of God.

Main points of last lectures

Newton's Laws:

1. If $\sum \vec{F} = 0$, velocity doesn't change.
2. $\sum \vec{F} = m\vec{a}$
3. $\vec{F}_{12} = -\vec{F}_{21}$

Newton's Third Law

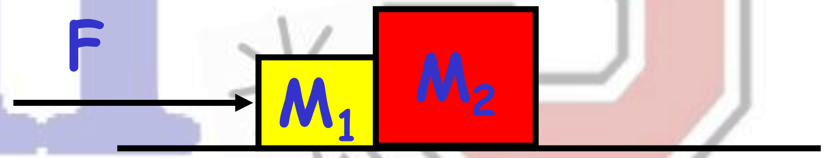
- Single isolated force cannot exist
- For every action there is an equal and opposite reaction
- Action and Reaction Forces act on different objects

Example .1

Two blocks sit on a frictionless table. The masses are M_1=2 kg and M_2=3 Kg. A horizontal force F=5 N is applied to Block 1.

1. What is the acceleration of the blocks?

2. What is the force of block 1 on block 2?



1. $a = 1\ m/s^2$
2. $F_{21} = 3$ N

Mechanical Forces

Gravity: w=mg
Normal forces
Strings, ropes and Pulleys
Friction
Springs

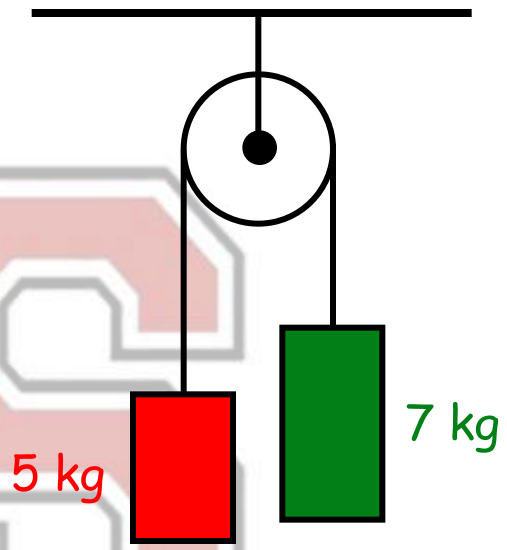
Rules for Ropes and Pulleys

- Force from rope points AWAY from object
  - (Rope can only pull)
- Magnitude of the force is Tension
  - Tension is same everywhere in the rope
- Tension does not change when going over pulley

Approximations: Neglect mass of rope and pulley, neglect friction in pulley

Example 2

a) Find acceleration
b) Find T, the tension in the string
c) Find force ceiling must exert on pulley



a) $a = g/6 = 1.635\ m/s^2$
b) $T = 57.2\ N$
c) $F_{pulley} = 2T = 114.5\ N$

Example 3

Which statements are correct? Assume the objects are in static equilibrium.

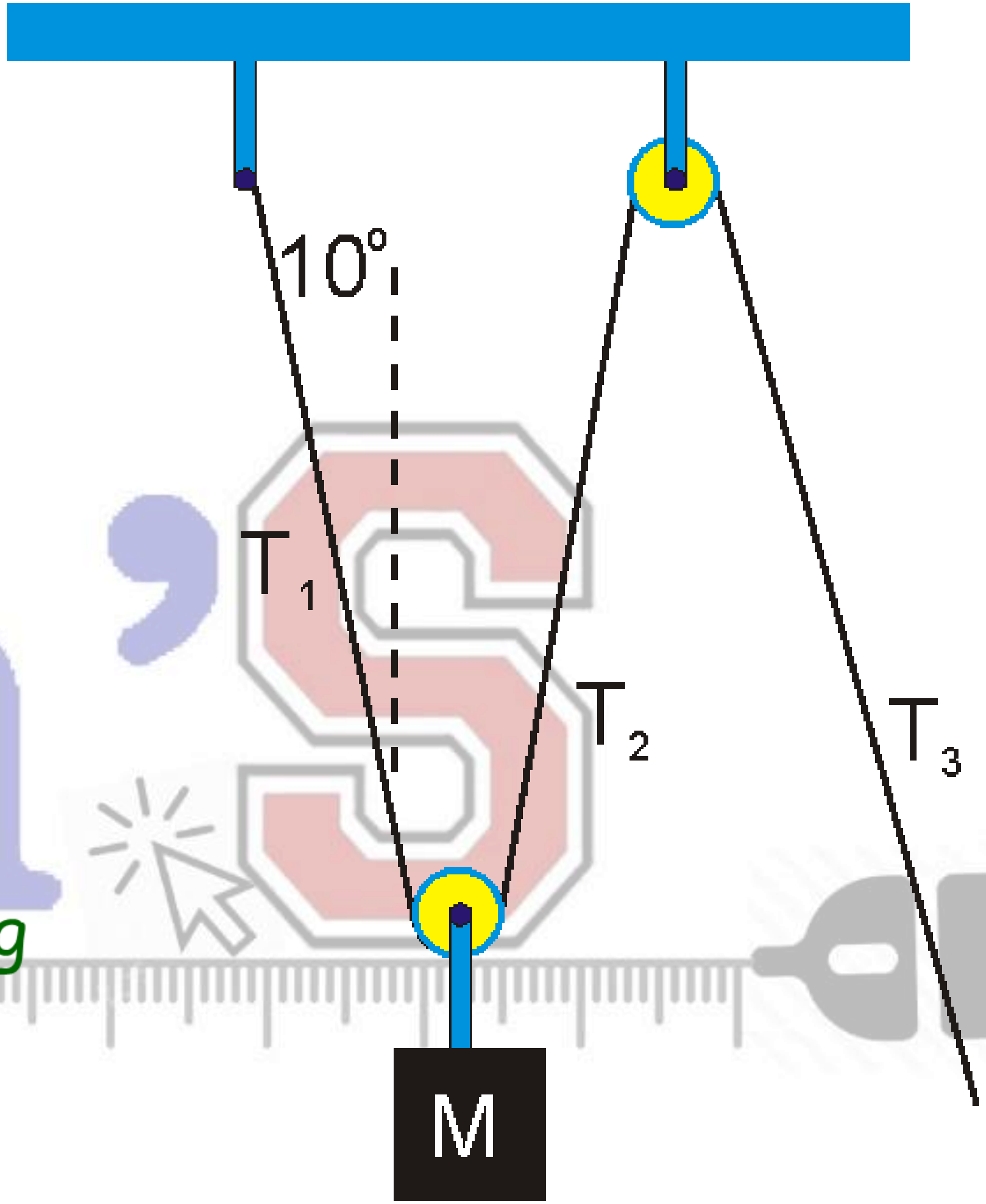
T_1 is ______ T_2

T_2 is ______ T_3

T_3 is ______ Mg

T_1+T_2 is ______ Mg

A) Less than
B) Equal to
C) Greater than



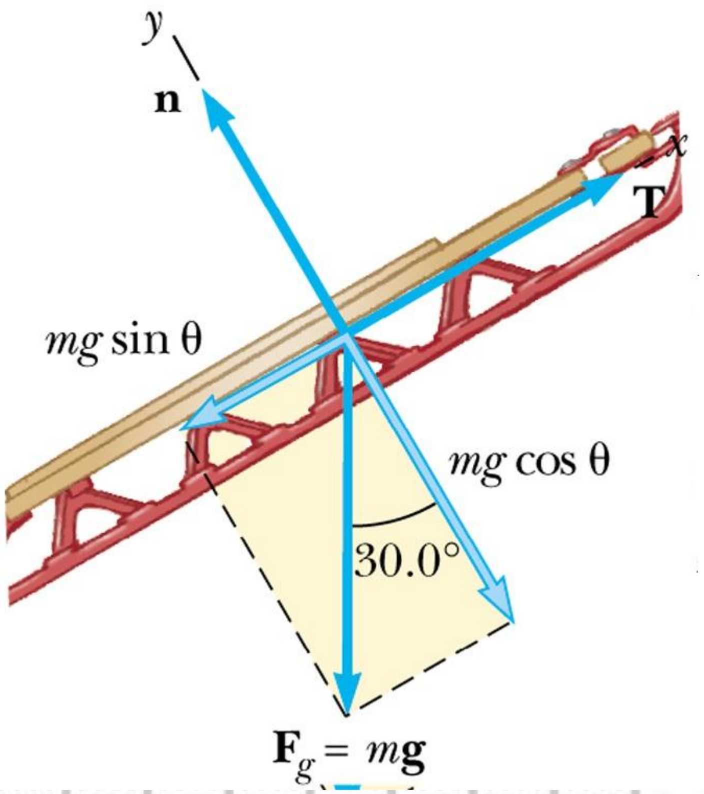
$\cos(10^o)=0.985$
$\sin(10^o)=0.173$

Inclined Planes

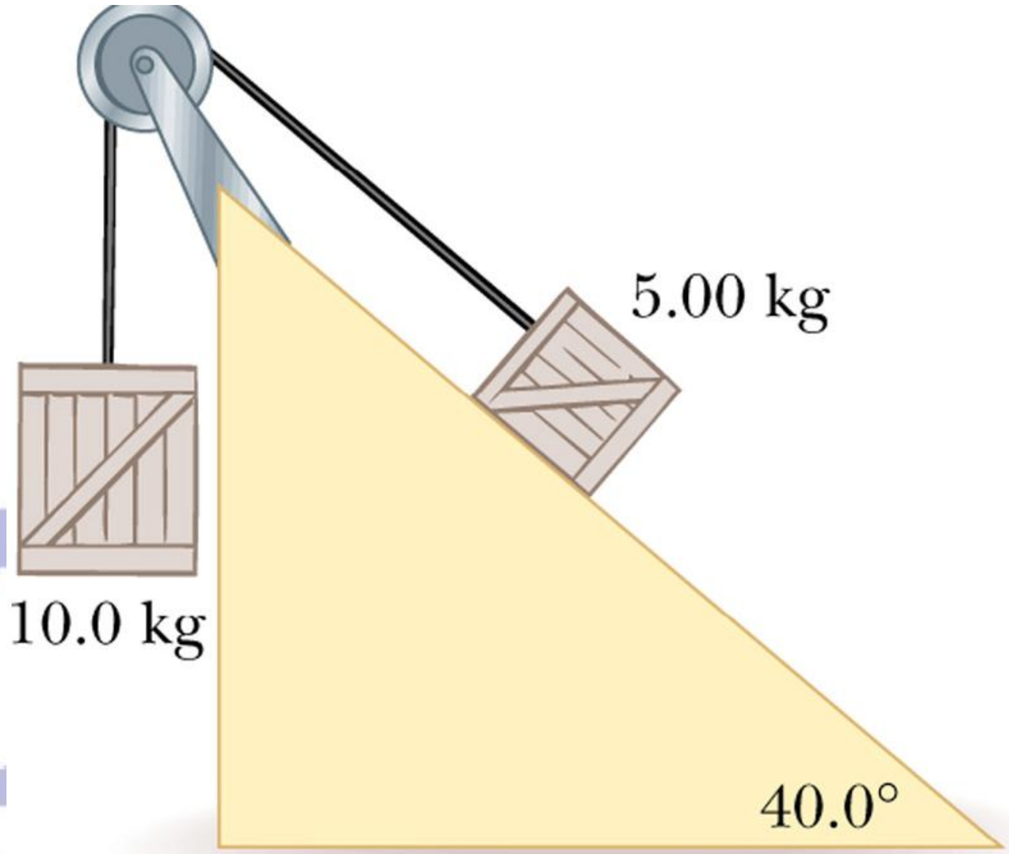
- Choose x along the incline and y perpendicular to incline
- Replace force of gravity with its components

$$F_{g,x} = mg\sin\theta$$

$$F_{g,y} = mg\cos\theta$$



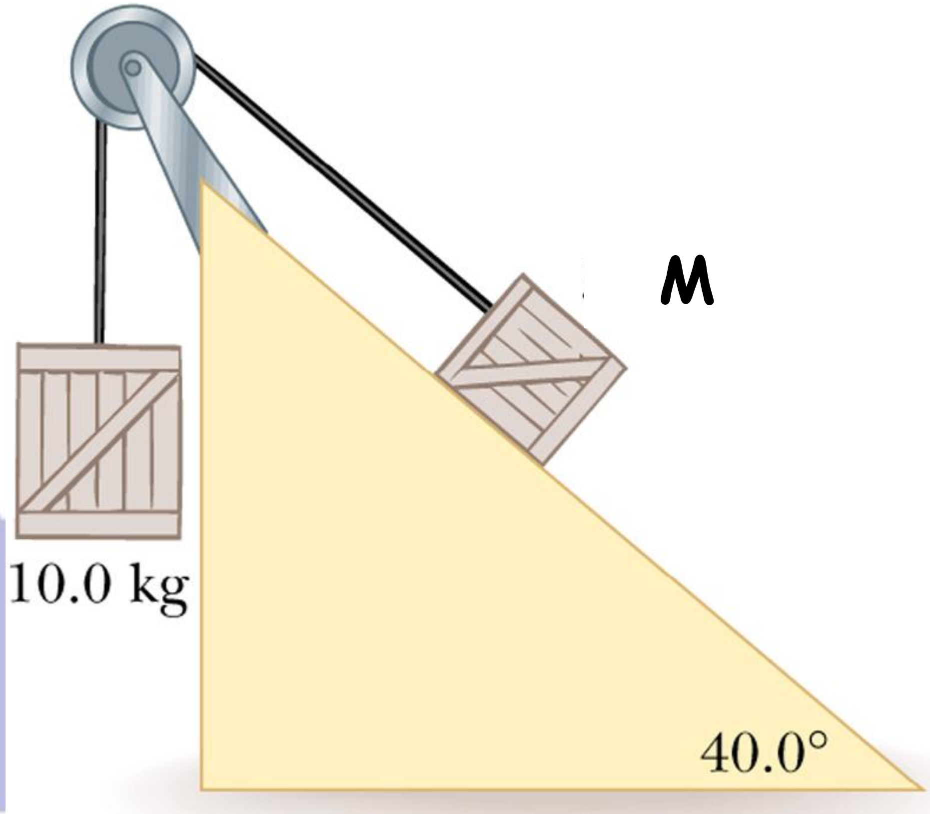
Example 4.



Find the acceleration and the tension

$a = 4.43\ m/s^2$, $T = 53.7\ N$

Example 5



Find M such that the box slides at constant v

M=15.6 kg

EQUILIBRIUM OF A RIGID OBJECT

If a rigid object is in Equilibrium

I) **Then the resultant force is zero in all directions.**
II) **The Total Torque is zero about any axis.**

The second condition is called the The Principle of Moments

Equilibrium means that…

- …there is no rotation.
- …there is no acceleration.
- …there is no net force acting on the object.

The First Condition of Equilibrium:
If the sum of all forces acting concurrently on a body is equal to zero, then the body must be in equilibrium. Mathematically:

$$\sum F = 0$$

$$\sum F_x = \sum F_y = 0$$

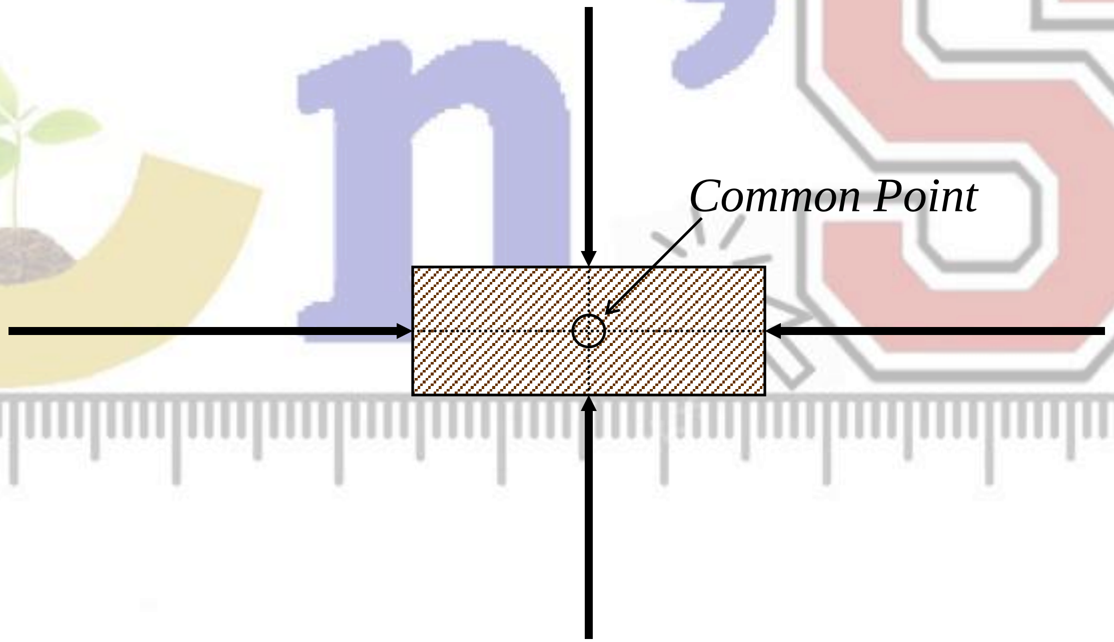
FORCE SYSTEMS

- Concurrent
- Nonconcurrent

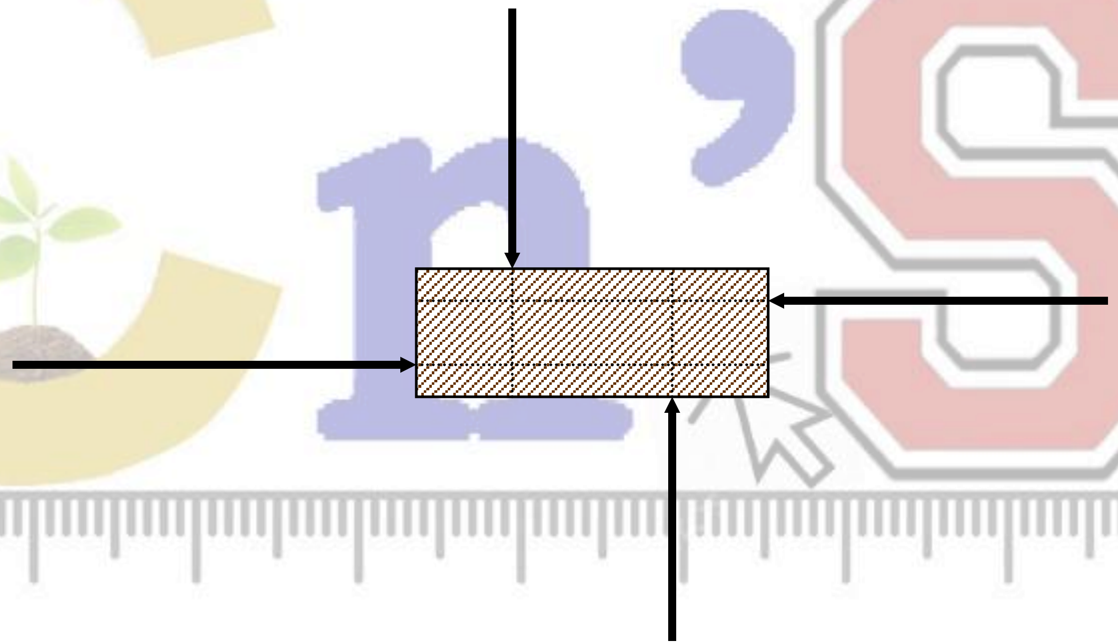
- Concurrent system occur when the lines of actions of the forces acting on a body intersect at a common point.

- Nonconcurrent system occurs when the forces are acting at different points.

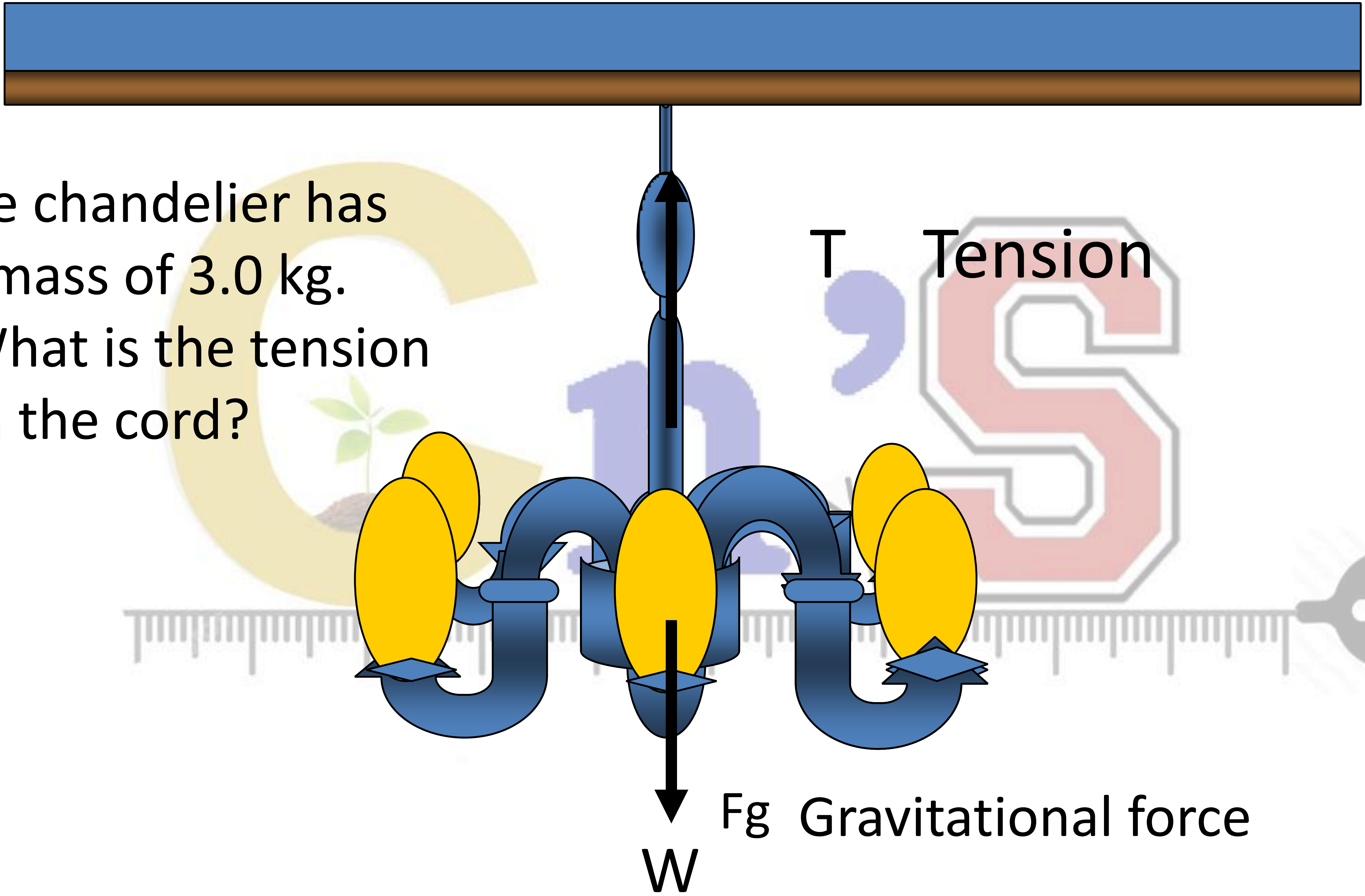
- <u>Concurrent forces</u> are forces whose line of action all pass through a common point.



- For <u>Non-Concurrent Forces</u> you get a rotation.

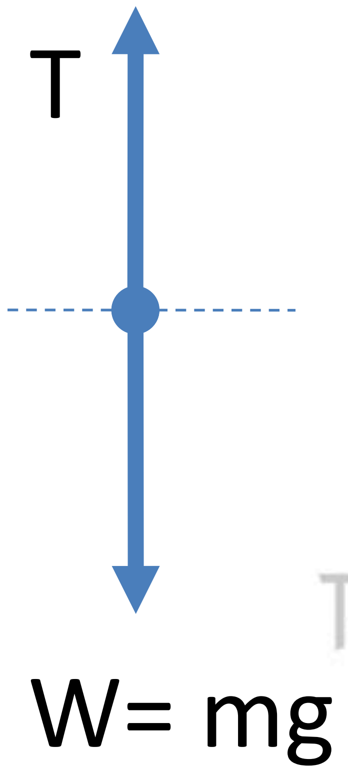


Example no.1



The chandelier has a mass of 3.0 kg. What is the tension in the cord?

Free-body Diagram



$\Sigma F = 0$

$\Sigma F_x = 0$

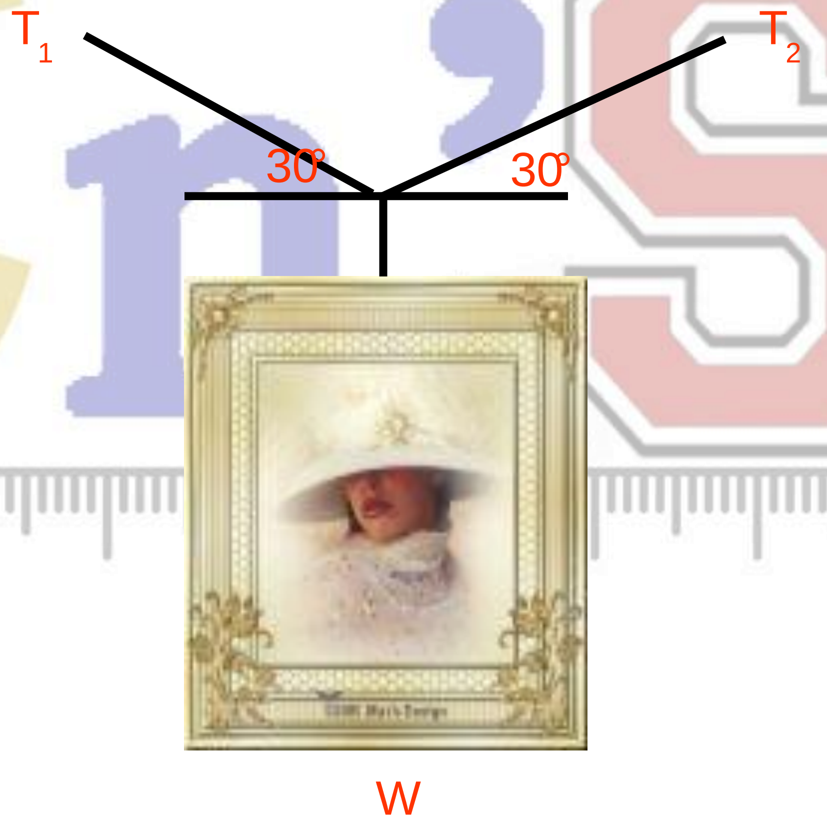
$\Sigma F_y = T – W = 0$

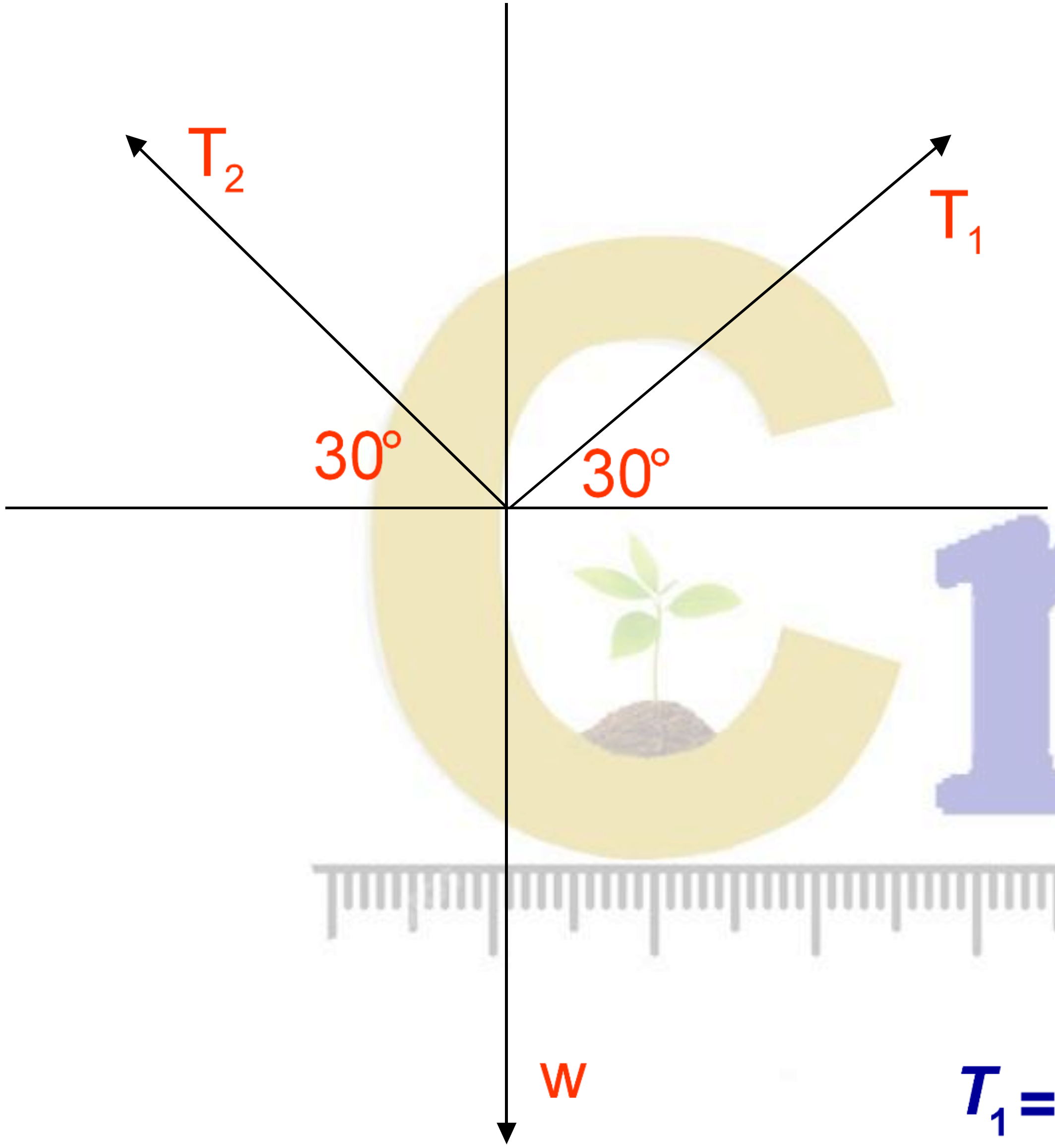
$T – mg = 0T = mg = (3.0\ kg)(9.8 m/s^2)$

$T = 29.4\ N$

EXAMPLE 2

You hang your picture frame by means of vertical string. Two strings in turn support this string. Each string makes 30° with an overhead horizontal beam. Find the tension in the strings. w = 55N.





Applying the first condition for equilibrium, we have:

$\sum F_x = 0$: $T_1 \cos 30° - T_2 \cos 30° = 0$

$T_1 = T_2$

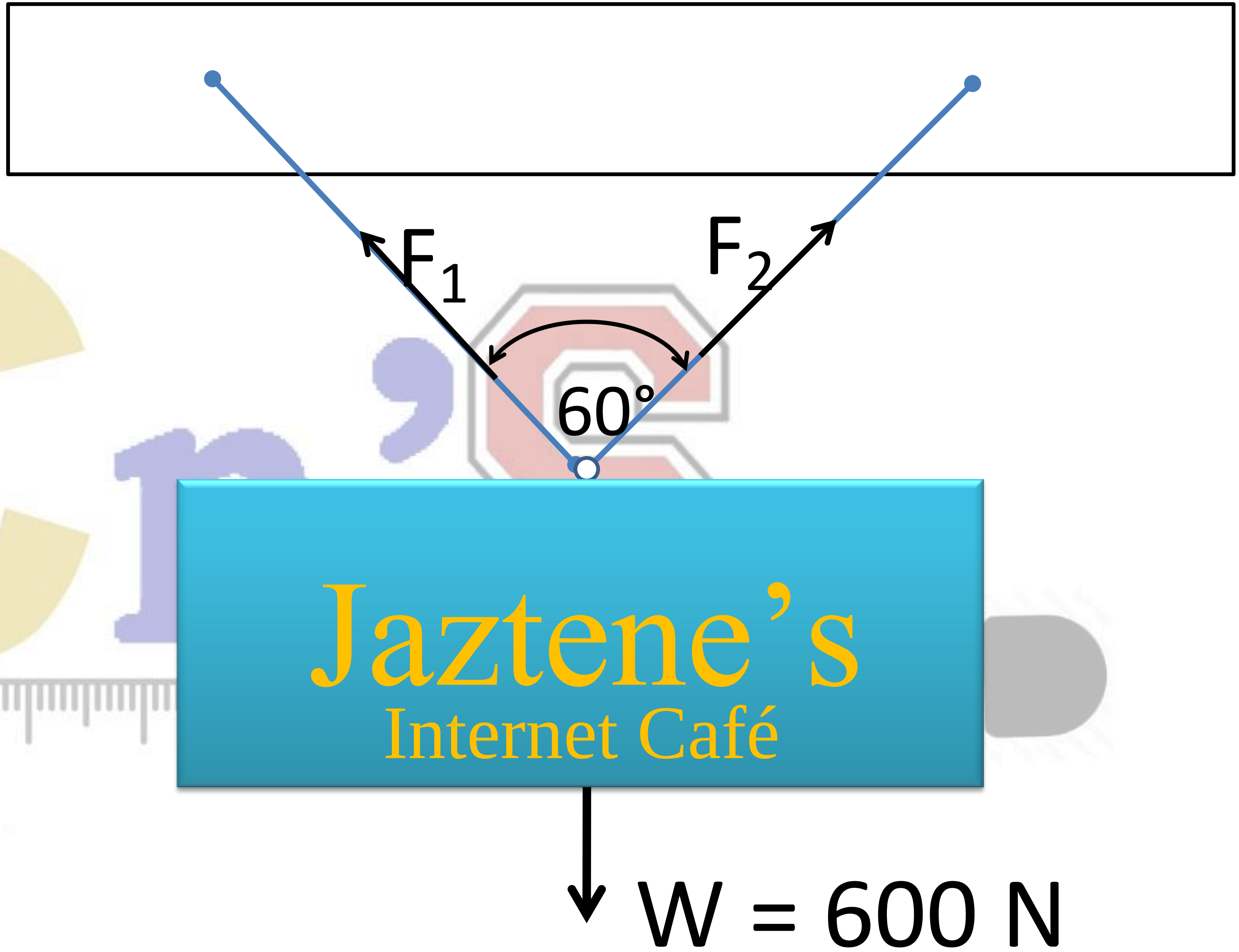
$\sum F_y = 0$: $T_1 \sin 30° + T_2 \sin 30° - 55N = 0$

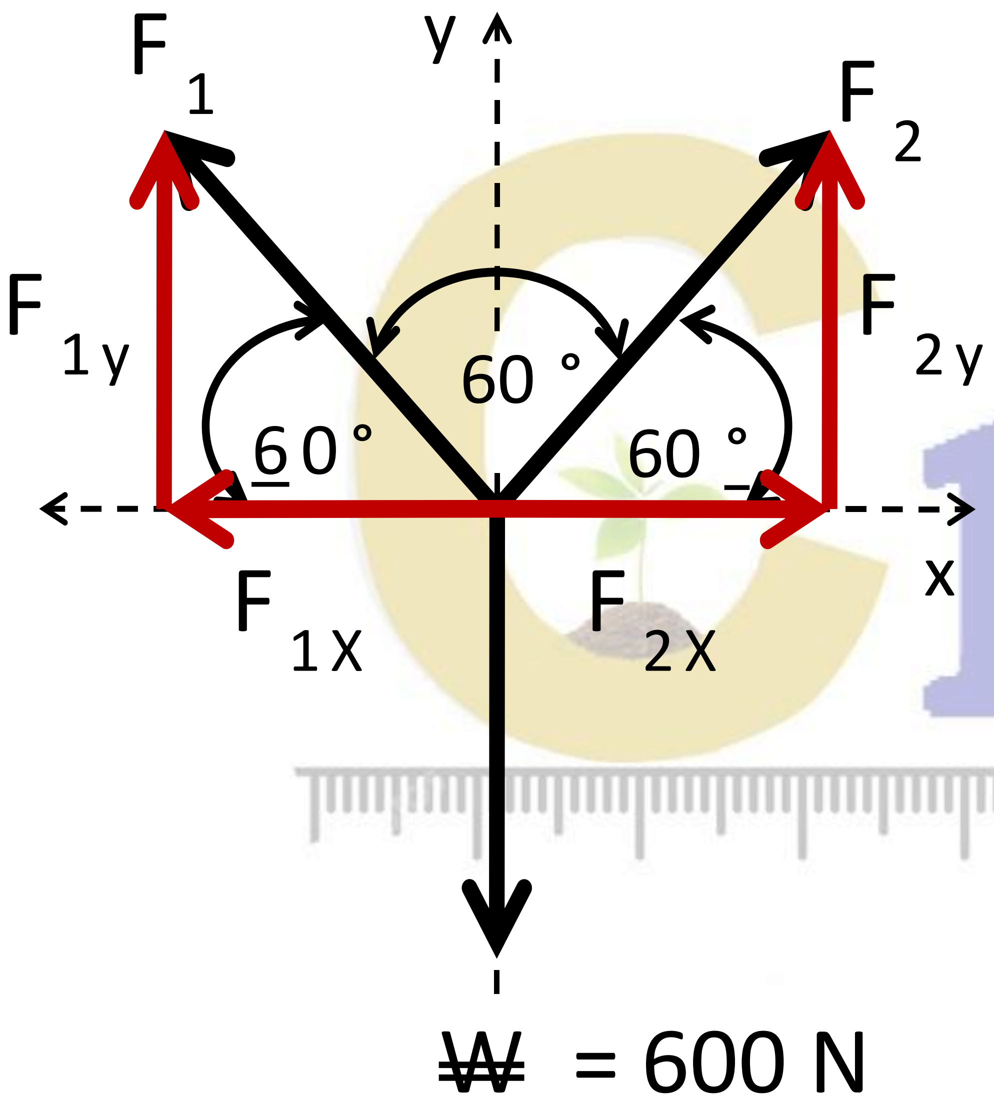
$T_1 (.5) + T_1 (.5) – 55\ N = 0$

$T_1 = 55\ N$

$T_2 = 55\ N$

Find F_1 and F_2





$$\Sigma F_X = -F_{1x} + F_{2x} = 0$$

$$-F_1 \cos 60° + F_2 \cos 60° = 0$$

$$F_1 = F_2 \text{ ----- eq. 1}$$

$$\Sigma F_y = F_{1y} + F_{2x} = 0$$

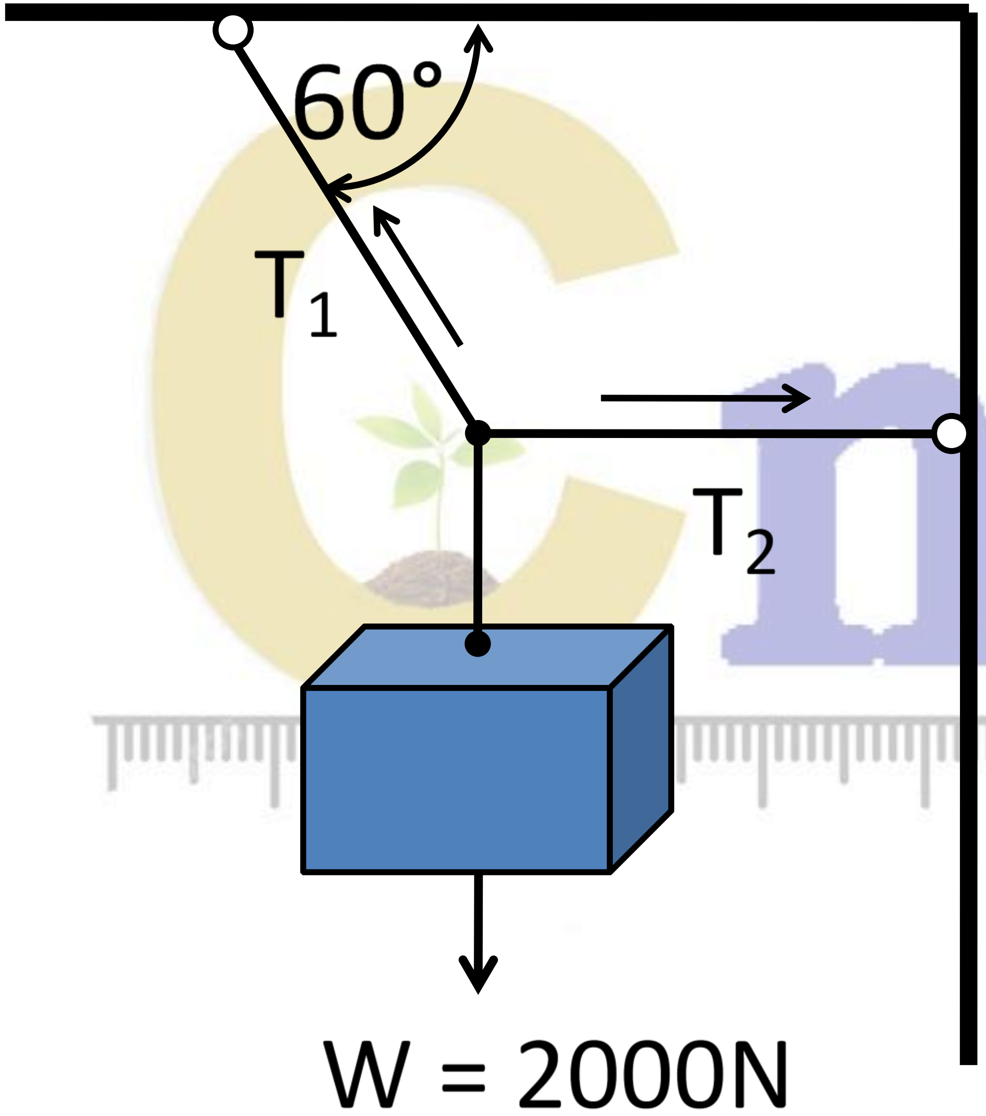
$$F_1 \sin 60° + F_2 \sin 60° - W = 0$$

$$2F1 \sin 60° = 600N$$

$$F1 = 600N / 2 \sin 60° \quad = 600N\ 1.73$$

$$F1 = F2 = 347\ N$$

3. Determine the tension in the cords supporting the 2000-N load?



$$\Sigma F = 0$$

$$\Sigma F_X = -T_{1x} + T_2 = 0$$

$$-T_1 \cos 30° + T_2 = 0$$

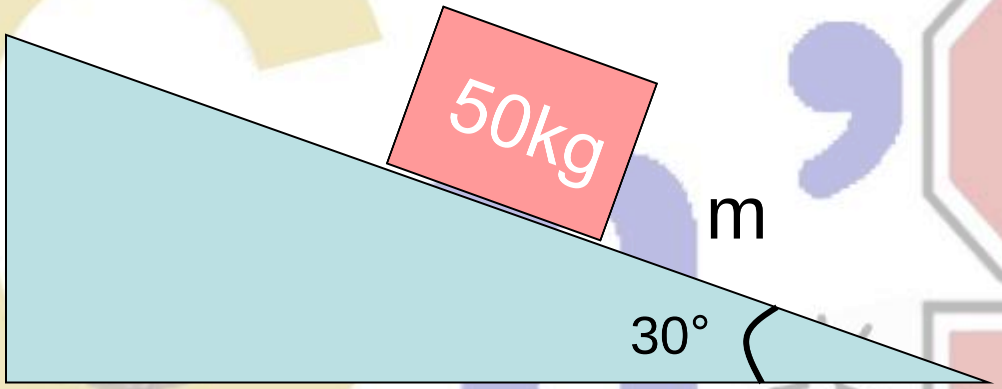
$$T_2 = T_1 \cos 30° \quad \text{---- eq. 1}$$

$$\Sigma F_y = T_{1y} - W = 0$$

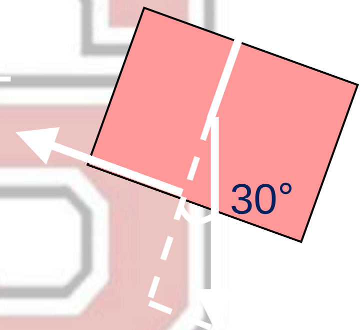
$$T_1 \sin 30° - W = 0$$

$$\boxed{\begin{array}{l} T_1 = 4000\ N \\ T_2 = 2000\ N \end{array}}$$

$T_1 \sin 30° = 2000N$



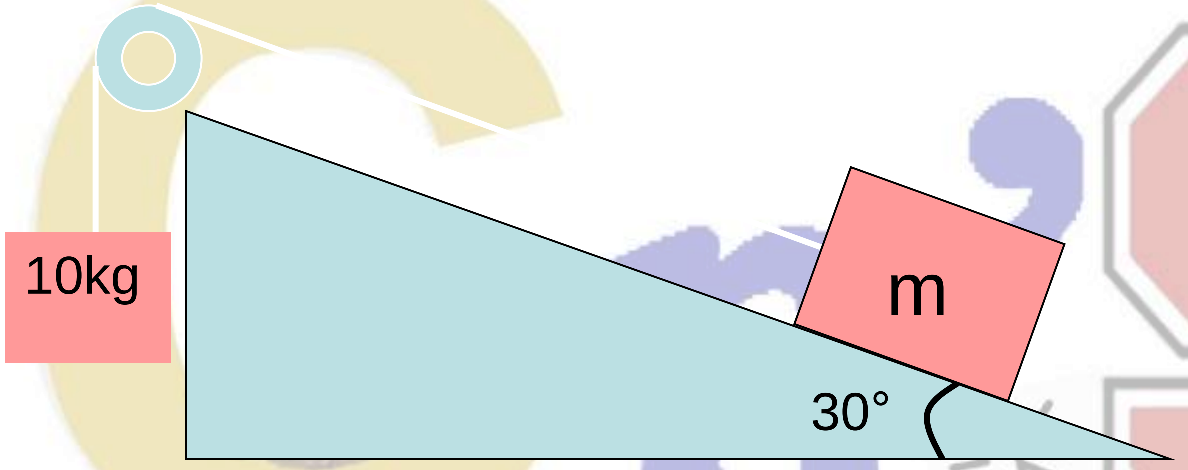
FBFD for m:



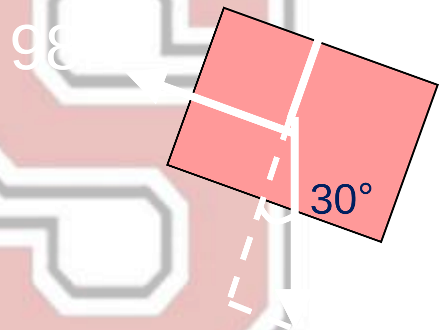
Equilibrium parallel to slope:

m

30°



FBFD for m:



Equilibrium parallel to slope:

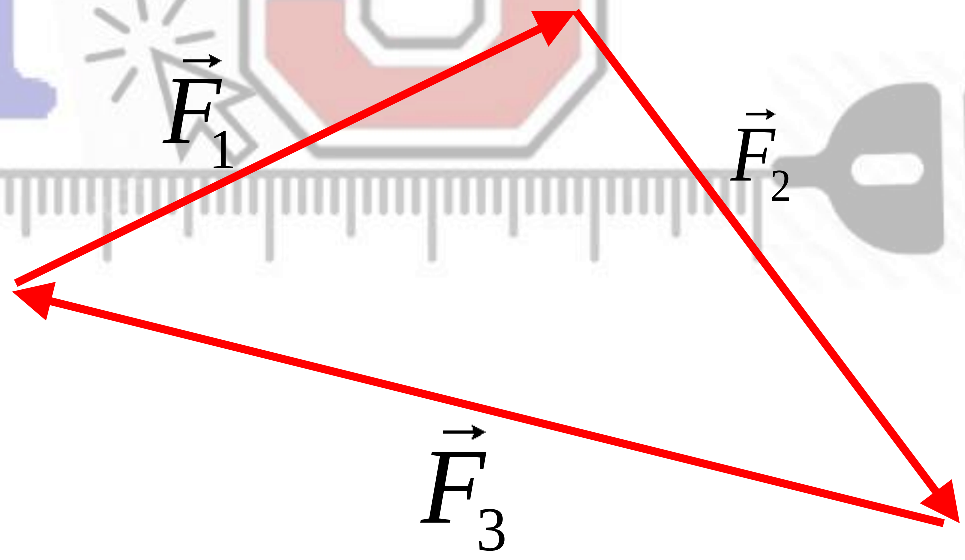
Extension: Now determine the normal reaction force by considering equilibrium perpendicular to the slope.

Equilibrium of Forces

The graphical method of vector addition for the forces acting on the body, always produces a closed loop:

Graphically:

$$\sum \vec{F} = 0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$



This is an example of the theorem known as the triangle of forces.

●● When a body is in equilibrium under the action of three non-parallel forces, then

(i) the forces can be represented in magnitude and direction by the sides of a triangle

(ii) the lines of action of the forces pass through the same point.

When more than three forces are in equilibrium the first statement still holds but the triangle is then a polygon. The second is not necessarily true.

? Lami’s theorem states that when three forces acting at a point as shown in the diagram are in equilibrium then

F F F 1 2 3 sin sin α β γ = = sin

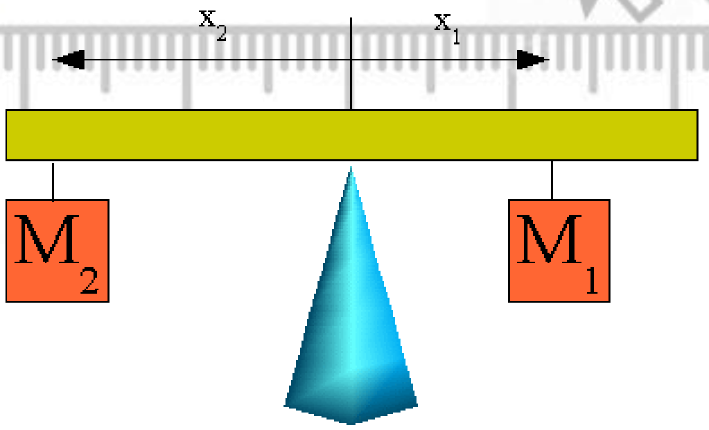
.

Sketch a triangle of forces and say how the angles in the triangle are related to α, β and γ. Hence explain why Lami’s theorem is true.

ROTATIONAL EQUILIBRIUM

A necessary condition for a body to be in rotational equilibrium is that the sum of the torques with their proper signs about point must be zero. $\sum t = 0$

The condition is known as the *second condition for equilibrium*.



TORQUE

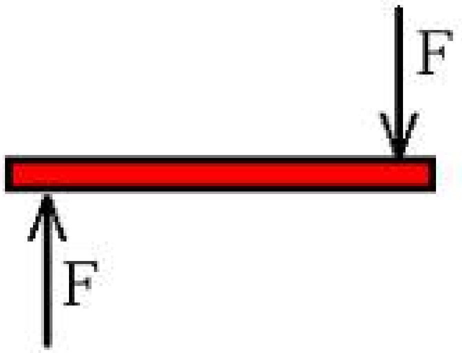
In physics, a TORQUE (τ) is a vector that measures the tendency of a force to rotate an object about some axis The magnitude of a torque is defined as force times its lever arm . Just as a force is a push or a pull, a torque can be thought of as a twist.

The SI unit for torque is newton meters (N m). In

The symbol for torque is τ, the Greek letter *tau* .

Static Equilibrium

- Consider a light rod subject to the two forces of equal magnitude as shown in figure. Choose the correct statement with regard to this situation:



(A) The object is in force equilibrium but not torque equilibrium.

(B) The object is in torque equilibrium but not force equilibrium

(C) The object is in both force equilibrium and torque equilibrium

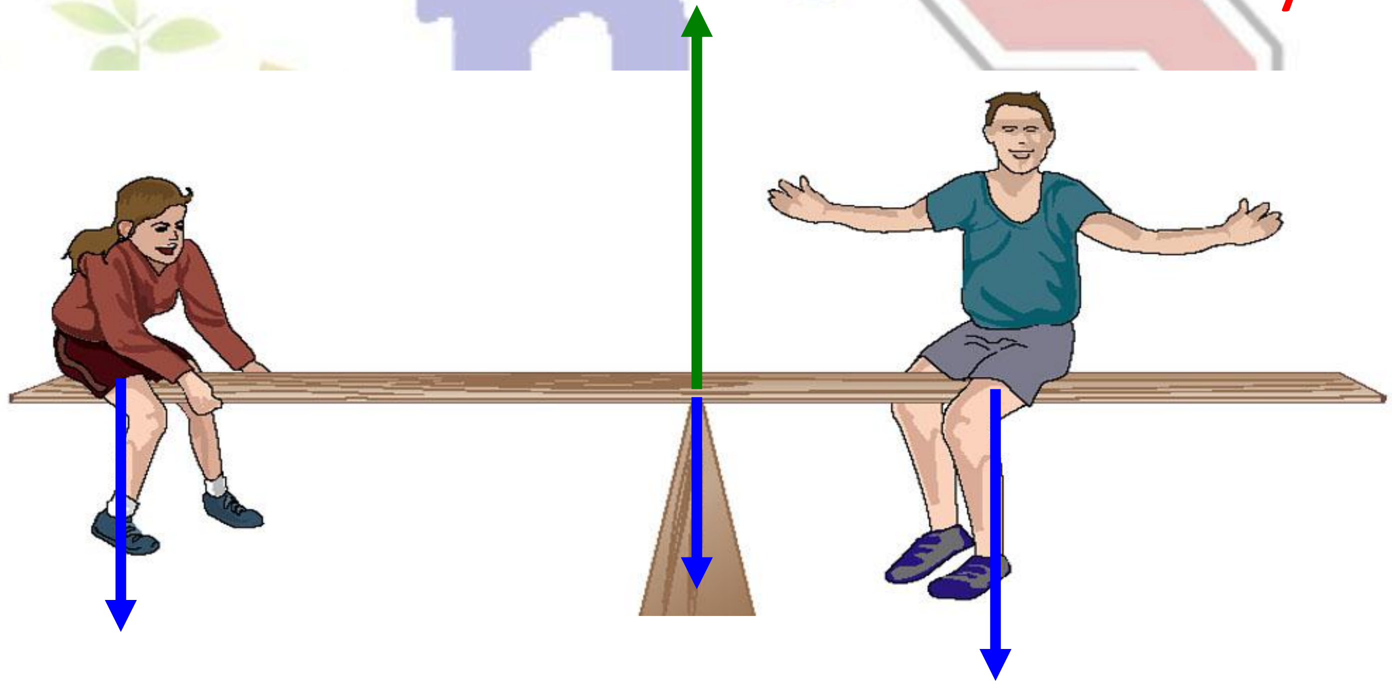
(D) The object is in neither force equilibrium nor torque equilibrium

- If an object is in equilibrium and the net torque is zero about one axis, then the net torque must be zero about any other axis

- A seesaw consisting of a uniform board of mass m_{pl} and length L supports at rest a father and daughter with masses M and m, respectively. The support is under the center of gravity of the board, the father is a distance d from the center, and the daughter is a distance 2.00 m from the center.

A) Find the magnitude of the upward force **n** exerted by the support on the board.

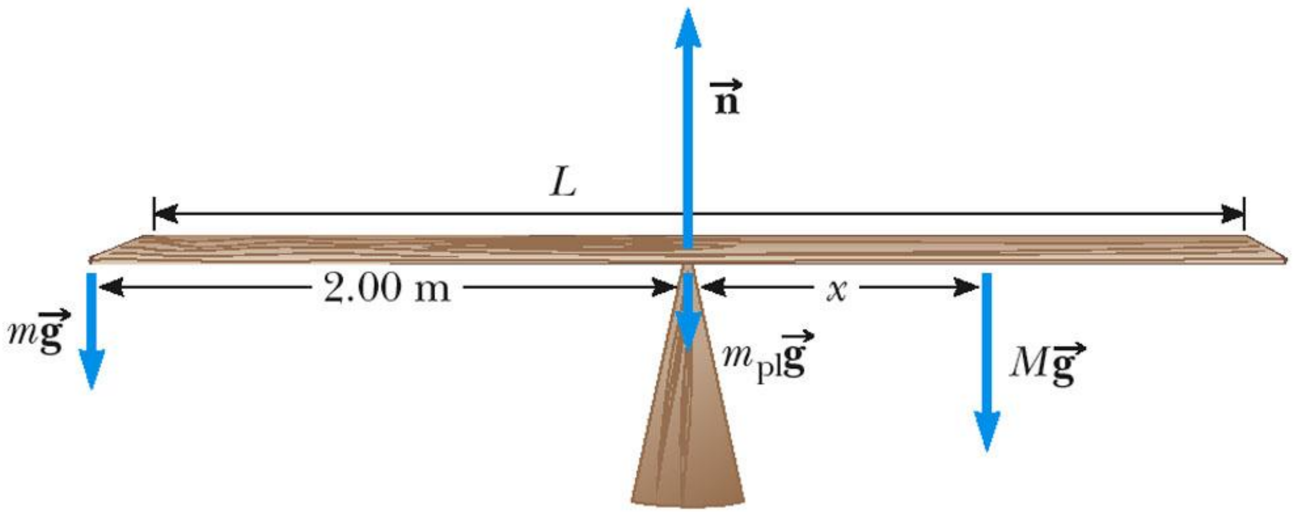
B) Find where the father should sit to balance the system at rest.



$$F_{net,x} = \sum F_{ext,x} = 0$$
$$F_{net,y} = \sum F_{ext,y} = 0$$
$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$

$$F_{net,y} = n - mg - Mg - m_{pl}g = 0$$
$$n = mg + Mg + m_{pl}g$$

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$
$$= mgd - Mgx + 0 + 0 = 0$$
$$mgd = Mgx$$
$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M} < 2.00\ \mathrm{m}$$



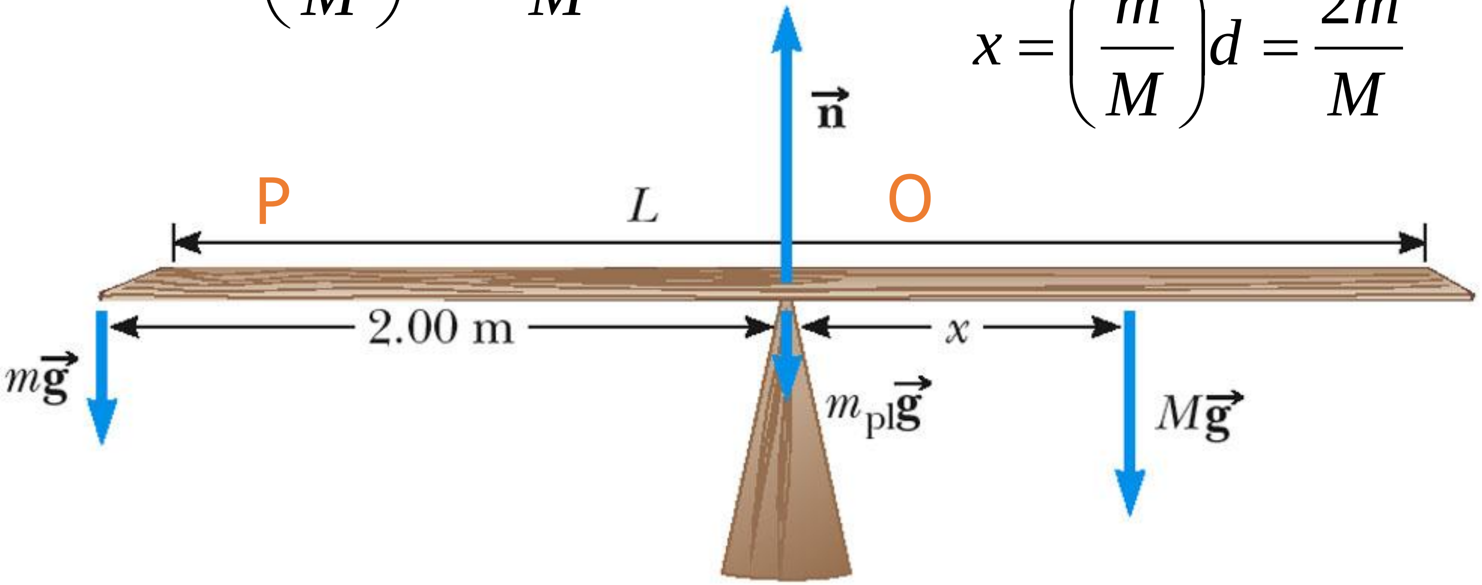
B) Find where the father should sit to balance the system at rest.

Rotation axis O

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$
$$= mgd - Mgx + 0 + 0 = 0$$
$$mgd = Mgx$$
$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M}$$

Rotation axis P

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$
$$= 0 - Mg(d + x) - m_{pl}gd + nd = 0$$
$$-Mgd - Mgx - m_{pl}gd + (Mg + mg + m_{pl}g)d = 0$$
$$mgd = Mgx$$
$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M}$$



$$F_{net,x} = \sum F_{ext,x} = 0$$
$$F_{net,y} = \sum F_{ext,y} = 0$$
$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$