CENTER OF GRAVITY

The *center of gravity* of a body is the point where its entire weight maybe assumed concentrated.

CENTER of GRAVITY of a GROUP of BODIES

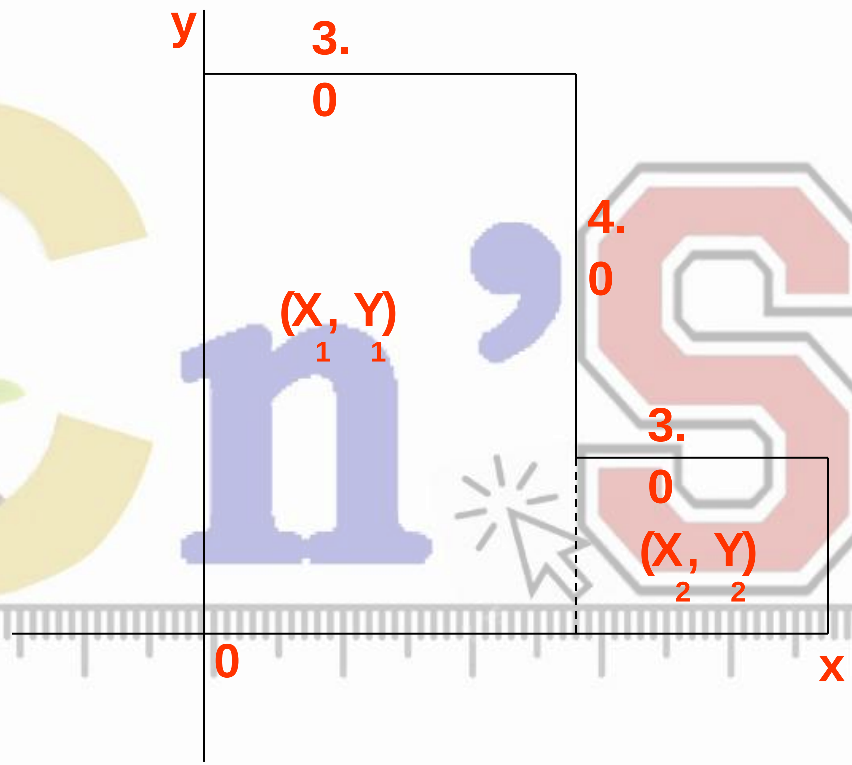
The center of gravity of a group of bodies whose centers of gravity are known from a fixed point is defined as the sum of the products of weight of individual body and it center of gravity divided by the total weight. In symbols:

$$x_{CG} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum m_i x_i}{\sum m_i}$$

Center of Gravity

- The torque due to the gravitational force on an object of mass M is the force Mg acting at the center of gravity of the object
- If g is uniform over the object, then the center of gravity of the object coincides with its center of mass
- If the object is homogeneous and symmetrical, the center of gravity coincides with its geometric center

Find The *center of gravity* of the wood



SOLUTION

The center of gravity of the wood will specified by two coordinates: $\bar{x}$ and $\bar{y}$. We divide the plate into regularly shaped parts. Let A_1 be the area of the bigger rectangle and A_2 the area of the smaller rectangle. The center of gravity of A_1 labeled as (x_1,y_1) is the (1.5 m, 3.0 m). The center of gravity of A_2 labeled as (x2, y2) is (4.5 m, 1.0 m).

A_1 = (3 m) (6 m) = 18 m2

A_2 = (3 m)(6 m) = 18 m2

W_1 = (2.4 kg/m2) (18 m2) = 43.2 N

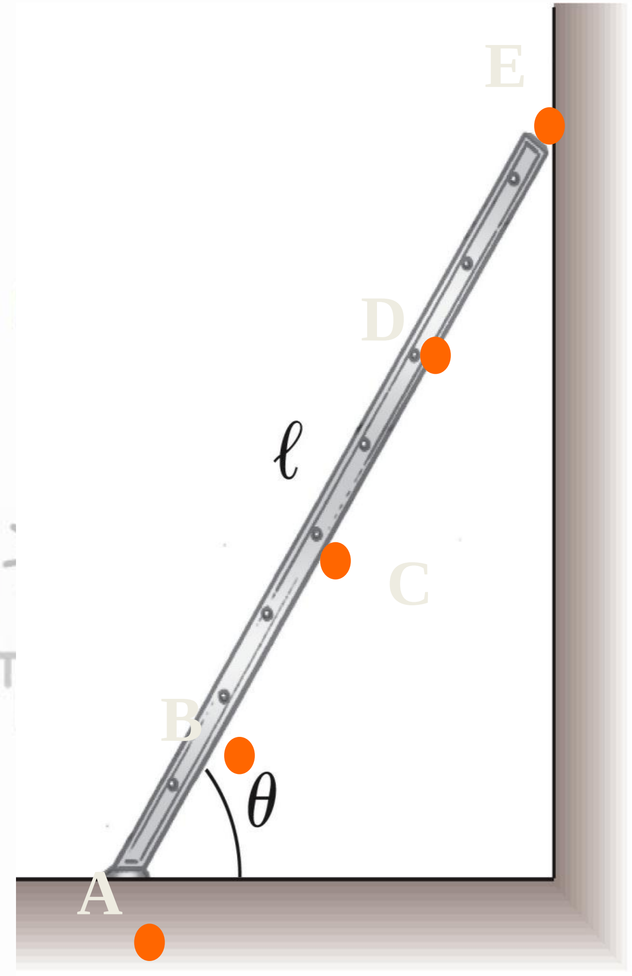
W_2 = (2.4 kg/m2) (6.m2) = 14.4 N

$$\bar{x} = \frac{(43.2\ \text{N})(1.5\ \text{m}) + (14.4\ \text{N})(4.5\ \text{m})}{43.2\ \text{N} + 14.4\ \text{N}} = 2.3\ \text{m}$$

$$\bar{y} = \frac{(43.2\ \text{N})(3\ \text{m}) + (14.4\ \text{N})(1\ \text{m})}{43.2\ \text{N} + 14.4\ \text{N}} = 2.5\ \text{m}$$

CG of a Ladder

- A uniform ladder of length l rests against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. Find the minimum angle θ at which the ladder does not slip.



(a)

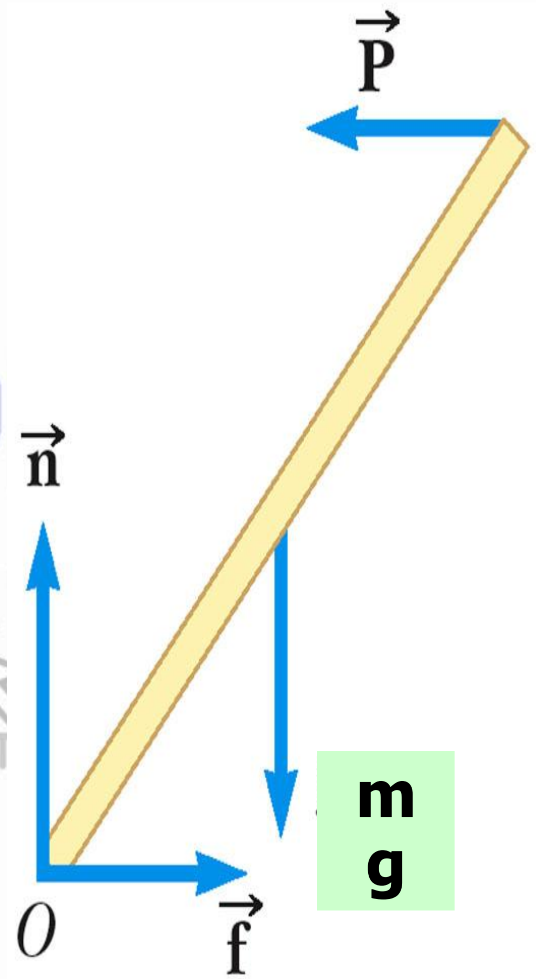
$$\sum F_x = f_x - P = 0$$

$$\sum F_y = n - mg = 0$$

$$P = f_x$$

$$n = mg$$

$$P = f_{x,\max} = \mu_s n = \mu_s mg$$

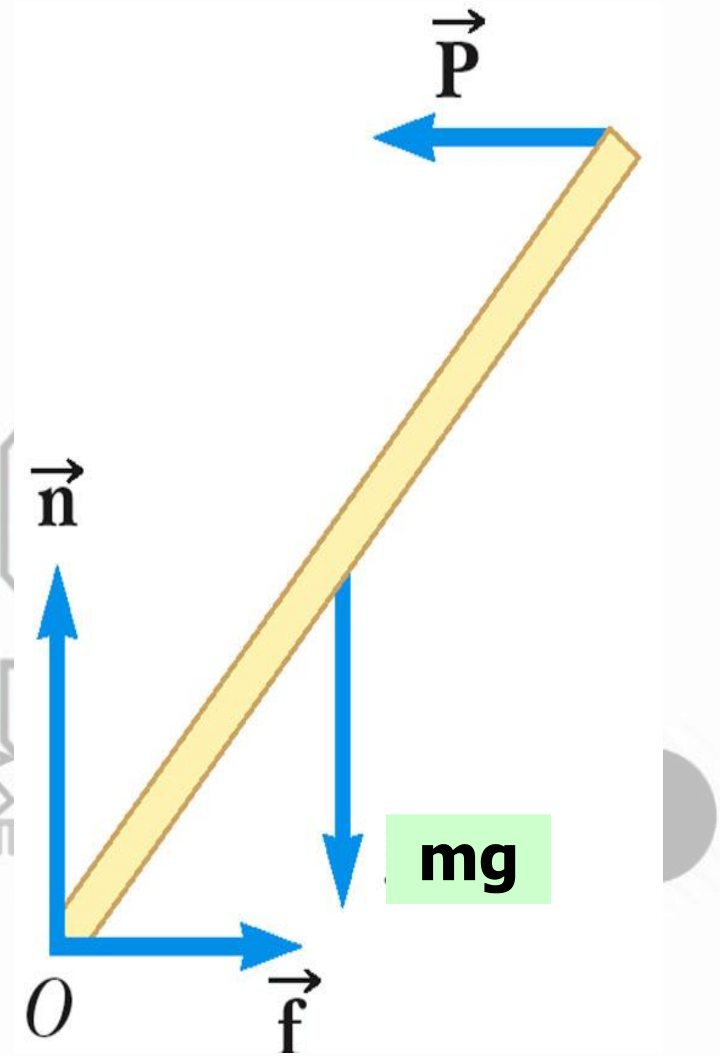


$$\sum \tau_O = \tau_n + \tau_f + \tau_g + \tau_P$$

$$= 0 + 0 + Pl \sin\theta_{\min} - mg\frac{l}{2}\cos\theta_{\min} = 0$$

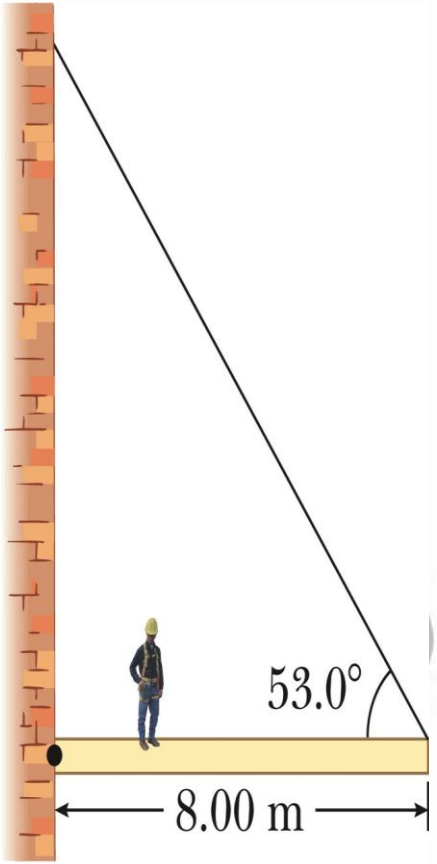
$$\frac{\sin\theta_{\min}}{\cos\theta_{\min}} = \tan\theta_{\min} = \frac{mg}{2P} = \frac{mg}{2\mu_s mg} = \frac{1}{2\mu_s}$$

$$\theta_{\min} = \tan^{-1}(\frac{1}{2\mu_s}) = \tan^{-1}[\frac{1}{2(0.4)}] = 51^{\circ}$$

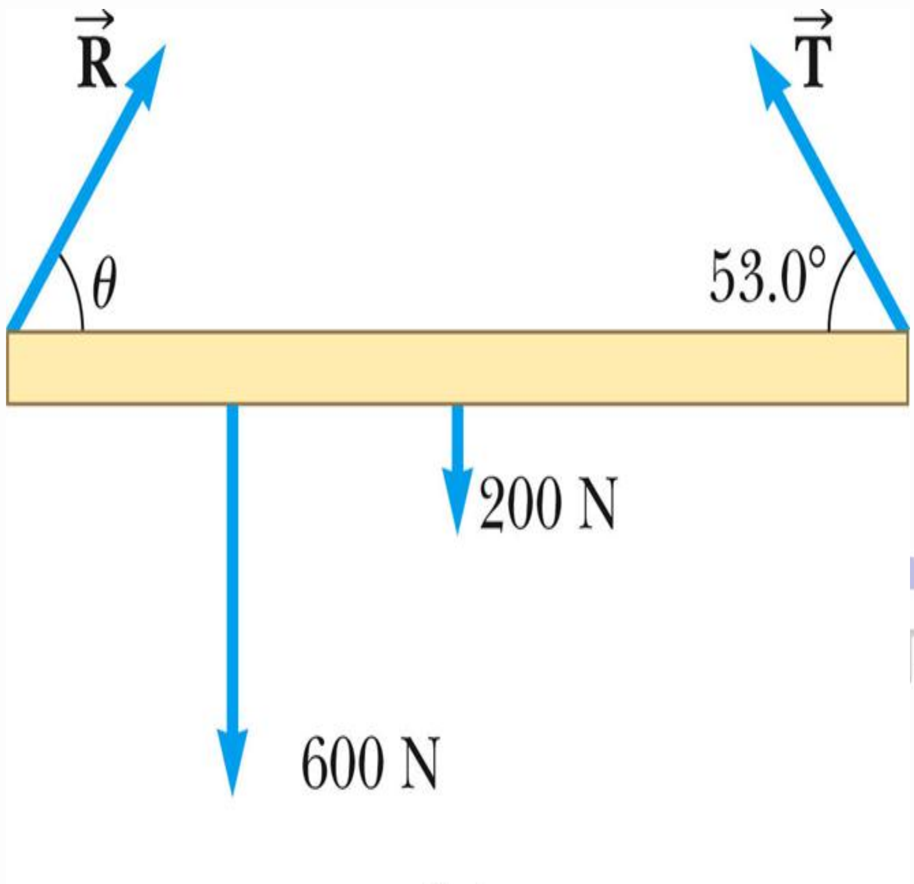


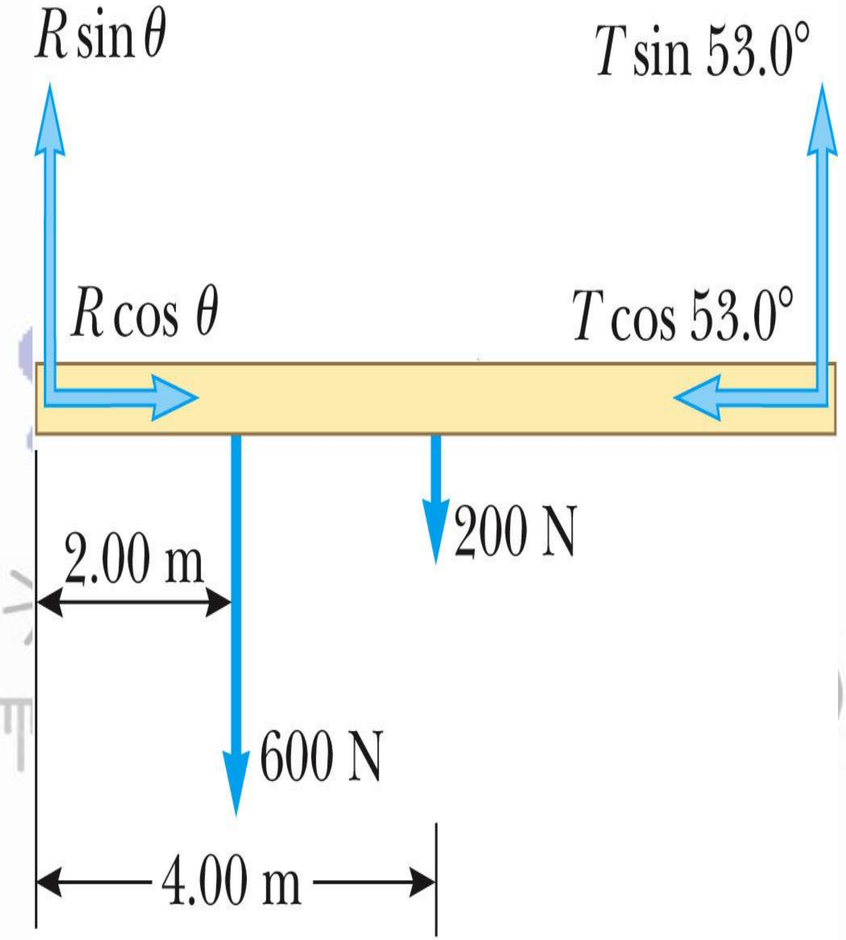
Horizontal Beam Example

- A uniform horizontal beam with a length of l = 8.00 m and a weight of W_b = 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of $\phi = 53°$ with the beam. A person of weight W_p = 600 N stands a distance d = 2.00 m from the wall. Find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.



(a)


$\vec{R}$
$\vec{T}$
θ
53.0°
200 N
600 N
(b)


$R \sin \theta$
$T \sin 53.0°$
$R \cos \theta$
$T \cos 53.0°$
200 N
2.00 m
600 N
4.00 m
(c)

$$\sum \tau_z = (T\sin\phi)(l) - W_p d - W_b\left(\frac{l}{2}\right) = 0$$

$$T = \frac{W_p d + W_b\left(\frac{l}{2}\right)}{l\sin\phi} = \frac{(600N)(2m) + (200N)(4m)}{(8m)\sin 53^\circ} = 313N$$

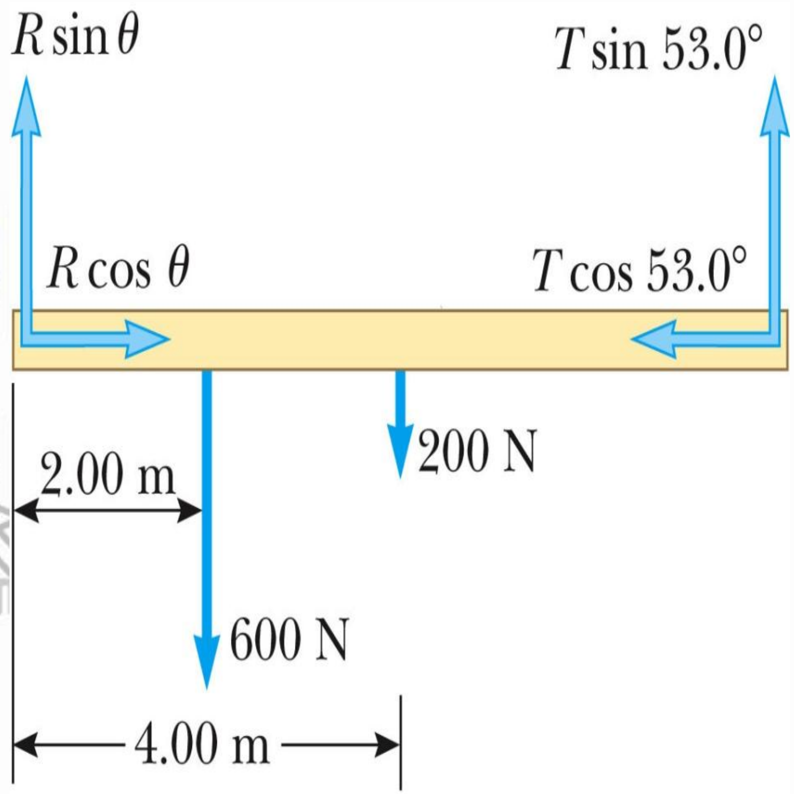
$$\sum F_x = R\cos\theta - T\cos\phi = 0$$

$$\sum F_y = R\sin\theta + T\sin\phi - W_p - W_b = 0$$

$$\frac{R\sin\theta}{R\cos\theta} = \tan\theta = \frac{W_p + W_b - T\sin\phi}{T\sin\phi}$$

$$\theta = \tan^{-1}\left(\frac{W_p + W_b - T\sin\phi}{T\sin\phi}\right) = 71.7^\circ$$

$$R = \frac{T\cos\phi}{\cos\theta} = \frac{(313N)\cos 53^\circ}{\cos 71.7^\circ} = 581N$$



(c)

STABILITY

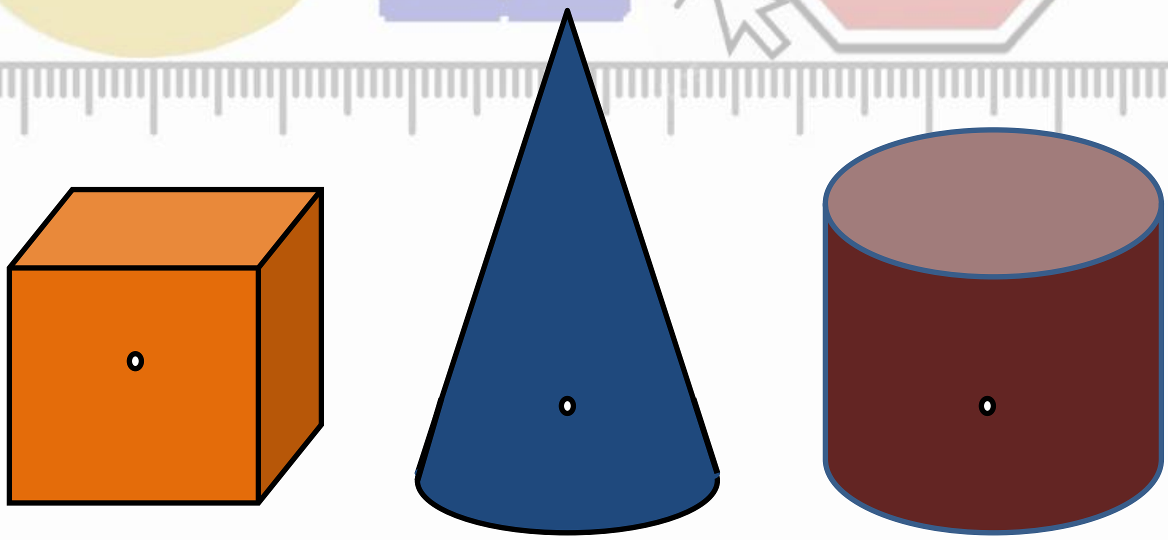
Three types of equilibrium:

UNSTABLE – the great example of this is a cone
that is balance on its apex but when disturbed slightly, it will fall over
STABLE – the condition of an object to return it is original position
when slightly disturbed.
NEUTRAL – the condition where an object is lying on its side
and displace but manages to remains its equilibrium about
its new position

STABLE EQUILIBRIUM

For stable objects:

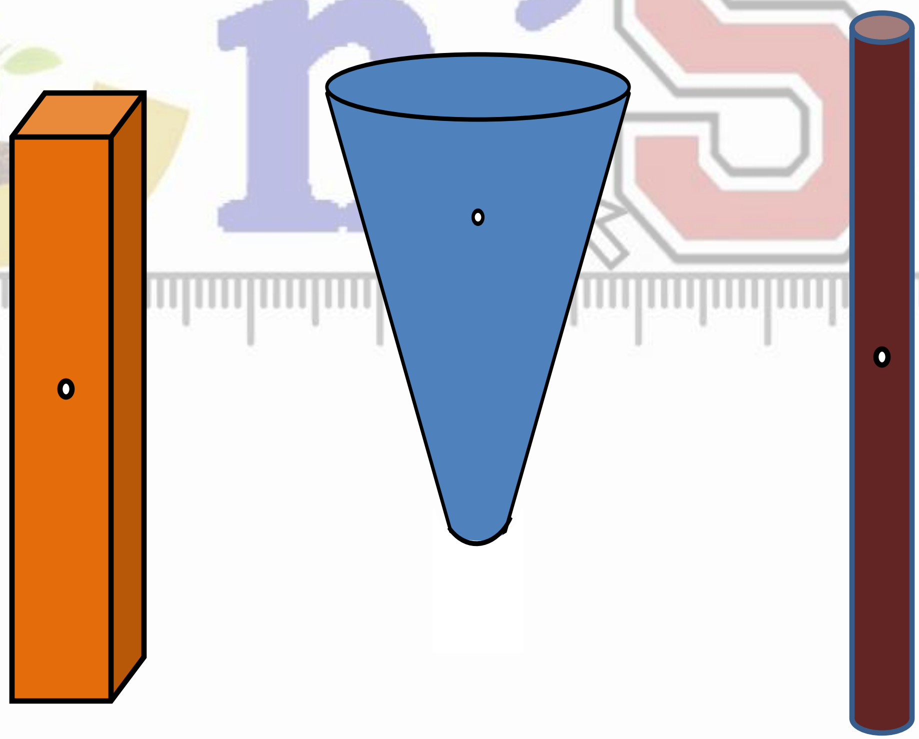
- the C.G. is at lowest possible position.
- the C.G. needs to be raised in order to topple the object.
- they are difficult to topple over.



UNSTABLE EQUILIBRIUM

For unstable objects:

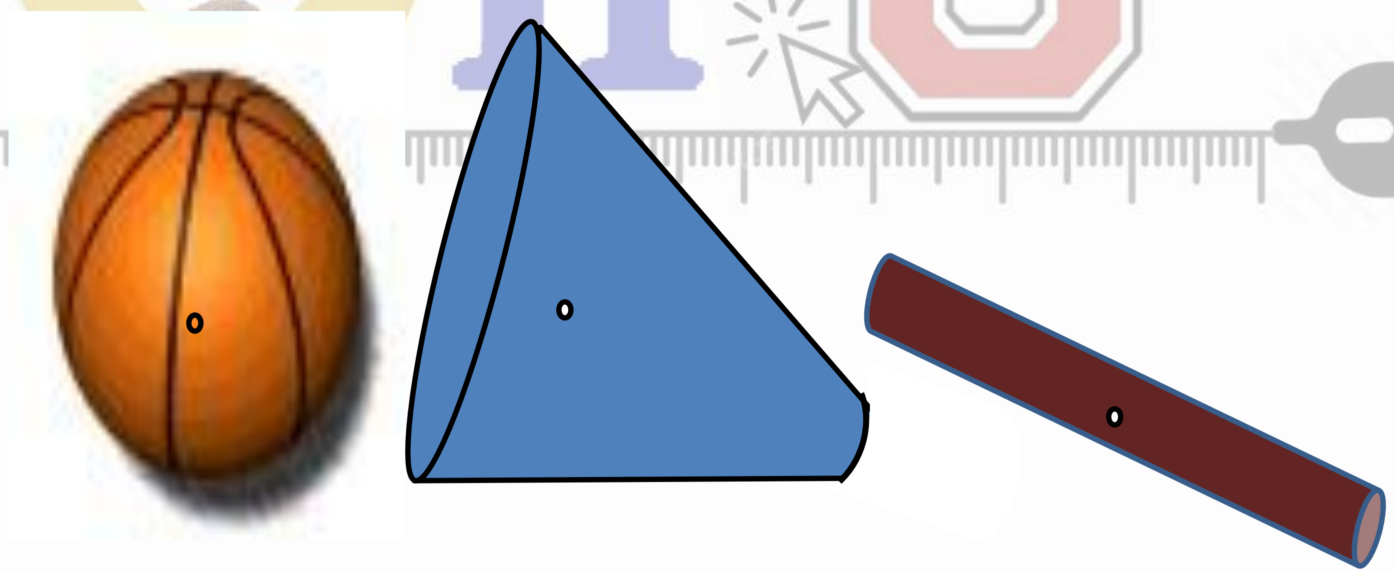
- the C.G. is at the highest possible position.
- the C.G. is lowered in order to topple the object.



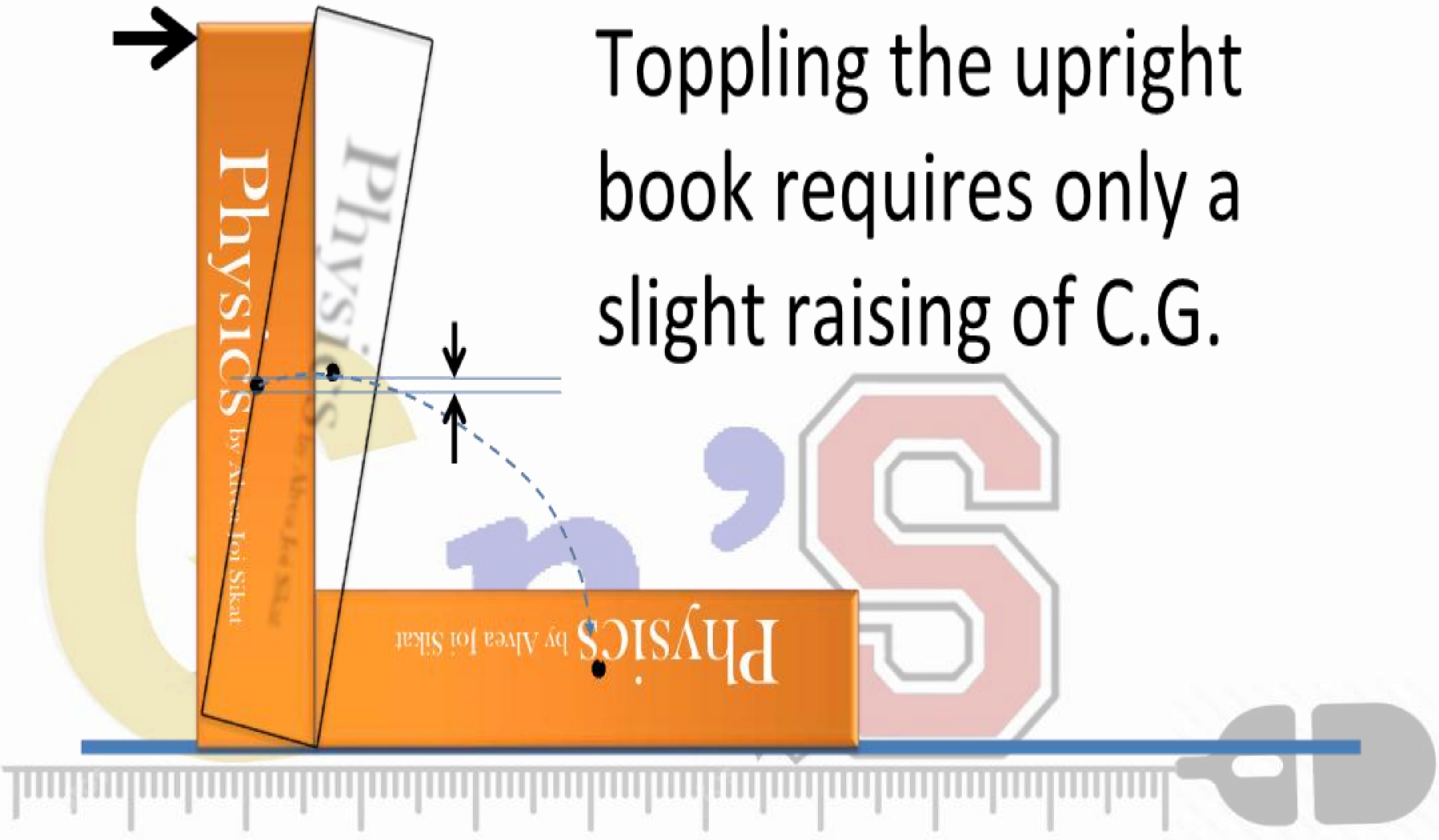
NEUTRAL EQUILIBRIUM

For objects with neutral equilibrium:

- the C.G. is neither lowered nor raised when the object is toppled.
- they roll from one side to another.

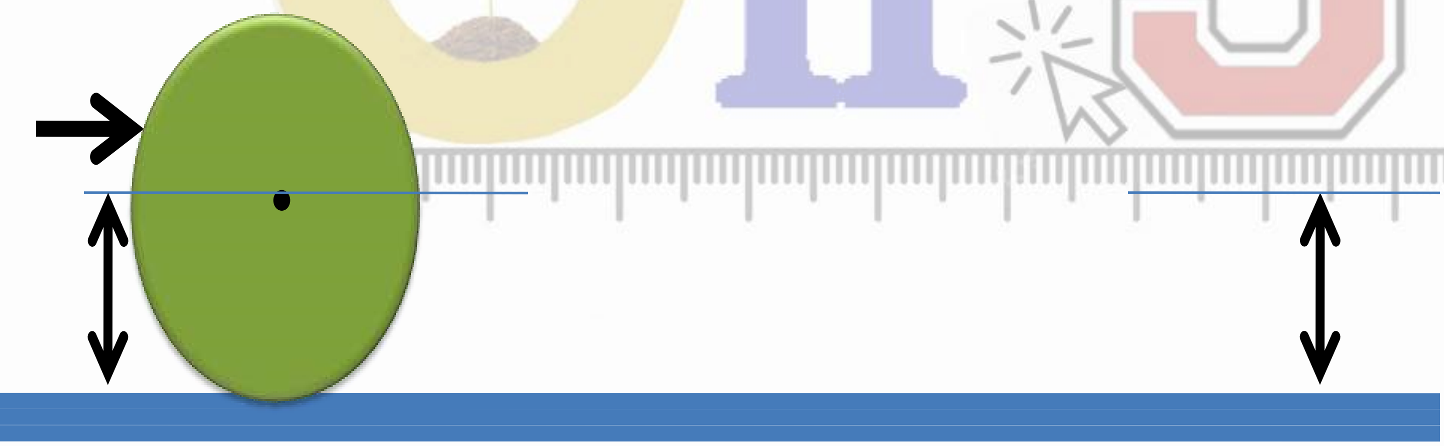


Toppling the upright book requires only a slight raising of C.G.



Toppling the flat book requires a relatively
large raising of its C.G.

Toppling the cylinder does not change the height of its C.G.



3 FACTORS FOR STABILITY

1. Mass of the object
2. Location of the center of gravity
3. Area of the base of support

Another Example

Given: W=50 N, L=0.35 m,

x=0.03 m

Find the tension in the muscle

F = 583 N