HYDRODYNAMICS

P2

Objectives

- Explain following terms related to fluid flow.
- Steady flow
- If the flow of a fluid is steady, then all the fluid particles that pass any given point follow the same path at the same speed.
- Turbulent flow
- In this type of flow the speed and direction of the fluid particles passing any point vary with time.
- Line of flow
- The path followed by a particle of the fluid
- Streamline
- A streamline is a curve whose tangent at any point is along the direction of the velocity of the fluid particles at that point. Streamline never cross. In steady flow the streamlines coincide with lines of flow.

- Tube of flow
- This is a tabular region of a flowing fluid whose boundaries are defined by a set of streamlines.
- Incompressible fluid
- This is a fluid in which changes in pressure produce no change in the density of the fluid.
- Liquids can be considered to be incompressible and gases subject only to small pressure differences can also be taken to be incompressible.
- Present the equation of continuity.
- Present Bernoulli’s equation and the conditions for validity.
- Discuss applications of Bernoulli’s equation.
- Explain phenomena that can be explained using Bernoulli’s equation.

Ideal Fluid

- The Ideal Fluid
  - no “viscosity” - no flow resistance (no internal friction)
  - incompressible - density constant
  - Steady flow

Consider an ideal fluid moving with steady flow - velocity at each point in the flow is constant in time

- In this case, fluid moves on streamlines

- streamlines do not meet or cross

- velocity vector is tangent to streamline

- volume of fluid follows a tube of flow bounded by streamlines

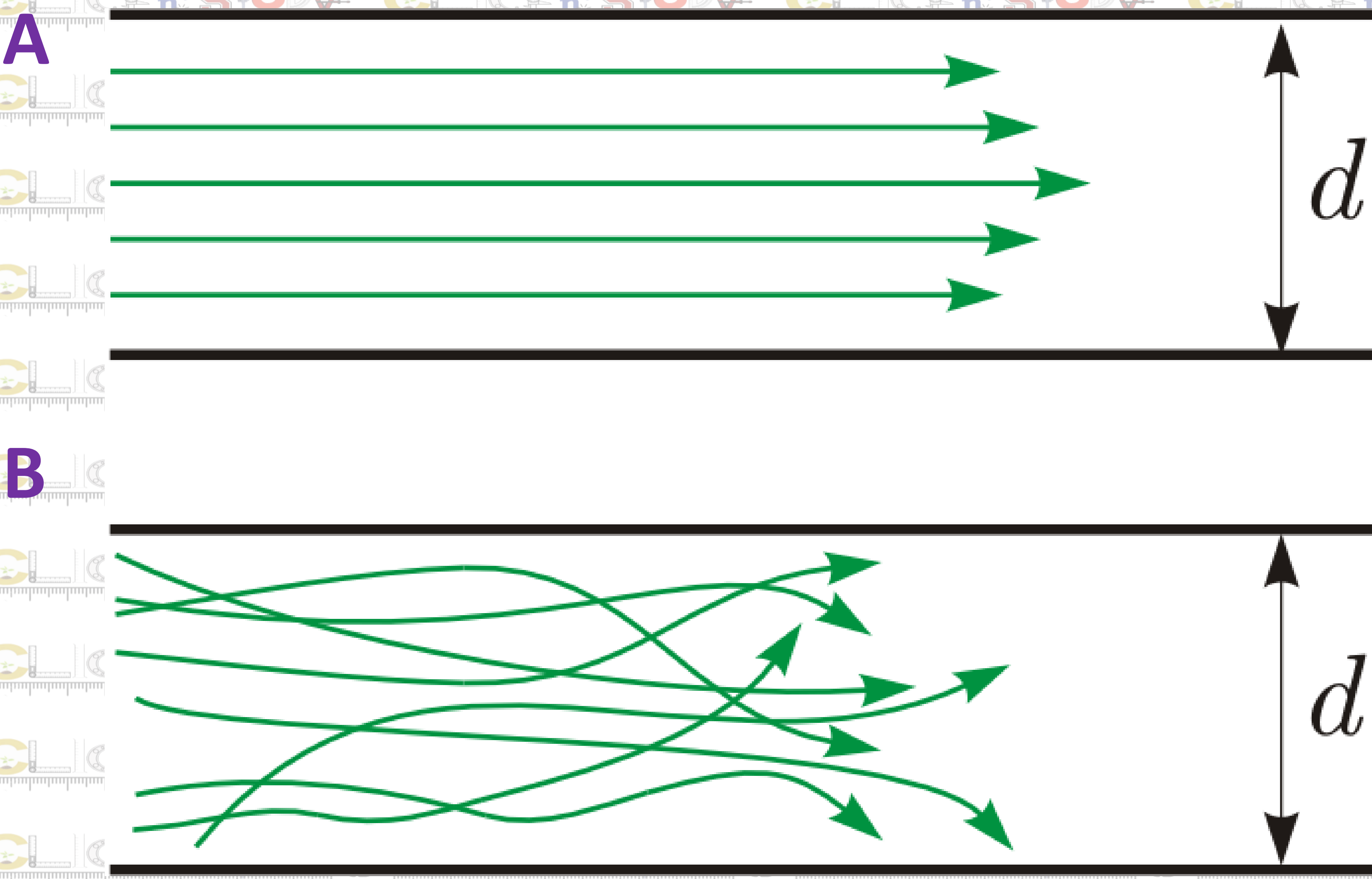
- Flow obeys continuity equation

← volume flow rate $Q = A \cdot v$ is constant along flow tube.

$$\boxed{A_1 v_1 = A_2 v_2}$$

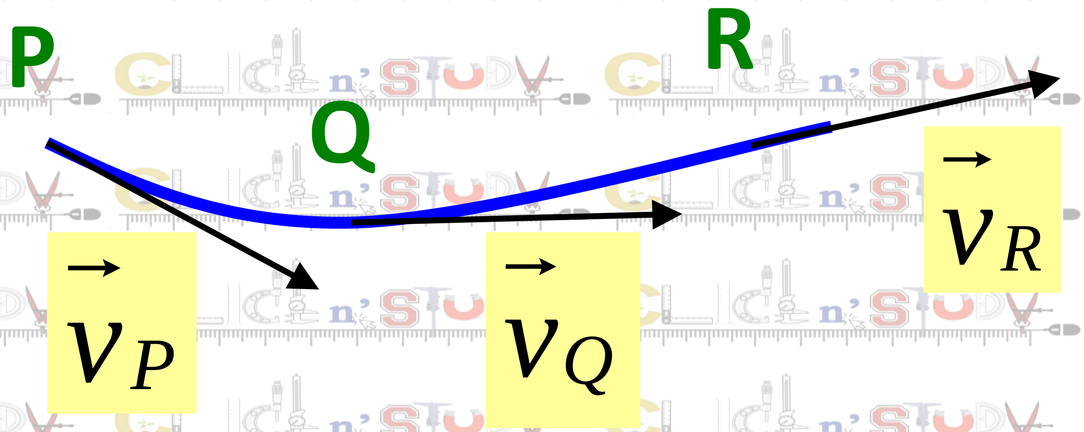
- follows from mass conservation if flow is incompressible.

The flow of a fluid can be either **laminar** or **turbulent**.

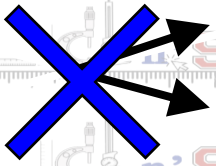


Streamlines

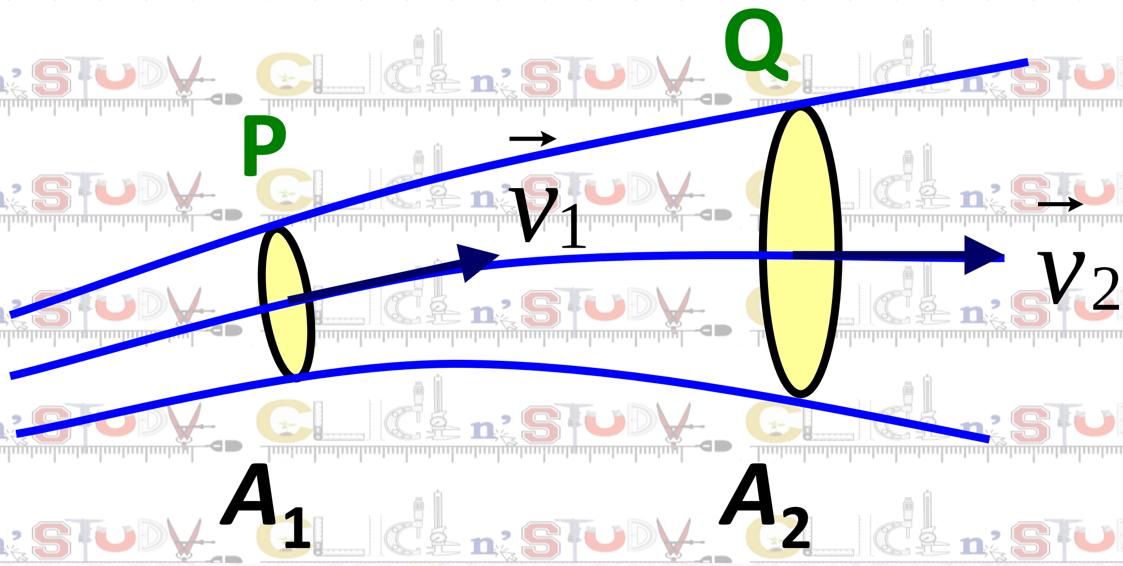
Paths of particles

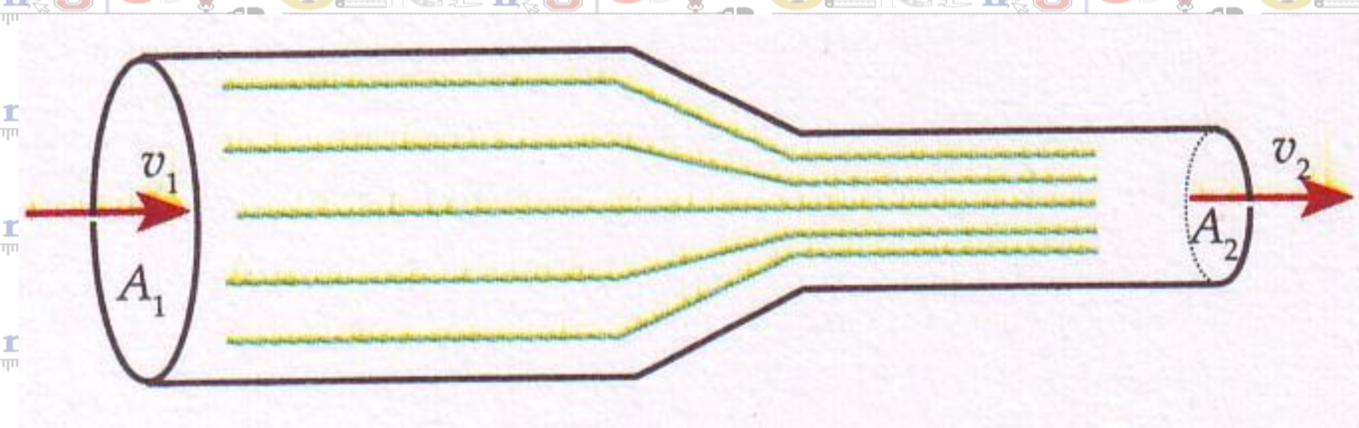


- P → Q → R
- *v* tangent to the streamline
- No crossing of streamlines



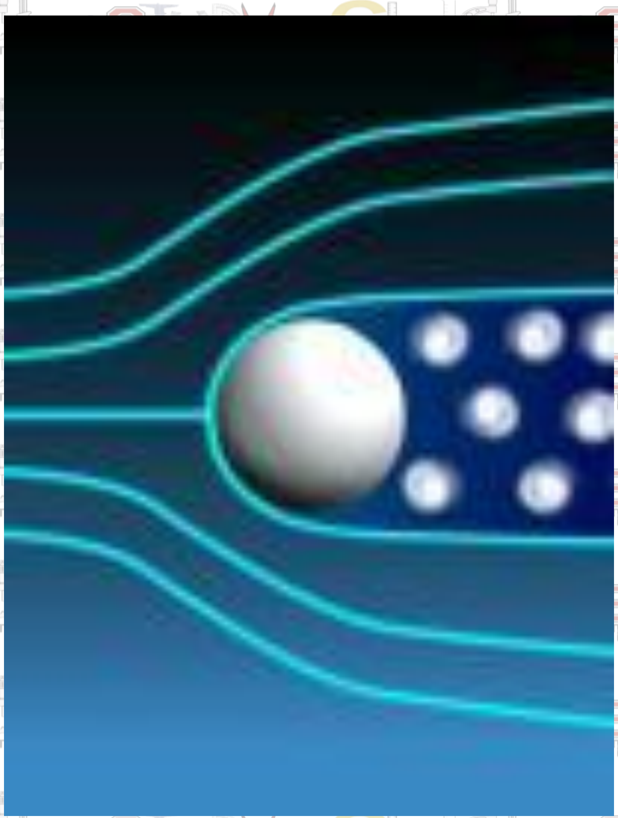
Tube of flow: bundle of streamlines





Laminar or Streamlined Flow

- A slow, smooth flow over a surface, in which the paths of individual particles do not cross.
  - Each path is called a *streamline*



Laminar or Streamlined Flow

- The fluid speed at the surface is **0** and increases speed farther from the surface.
- Fluid moves in theoretical layers, or laminates, with increasing speed away from the surface.
- Drag is produced by the friction between successive layers of fluid. This is called **frictional drag.**

Turbulent Flow

- Irregular flow with eddies and whorls causing fluid to move in different directions.
  - Turbulence is produced by high speeds, irregular shapes that aren't streamlined and sharp bends in the path of a fluid.

Turbulence produces the visible wake behind moving **boats** and the invisible wake behind a moving **airplane or car**.

- In laminar or streamline flow, each particle of fluid follows a smooth path, the streamline.
- Air flow over this Corvette is laminar until the end: the air cannot curve in completely at the back, it breaks away forming a turbulent wake.

Conservation of Mass

- When a fluid flows, mass is conserved.

- If there are no outlets or inlets, the same mass per unit time will flow everywhere in the stream

- Liquids are not compressible, so mass conservation is identical to volume conservation for a liquid.

- **The volume of a liquid flowing in a pipe, per unit time, is constant throughout the pipe.**

Volume flow rate (Q)

- The rate at which volume crosses a section of the tube:

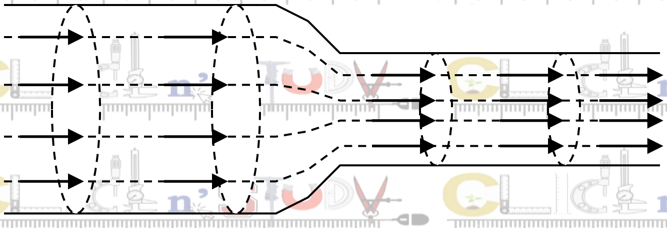
$$Q = \frac{V}{t}$$

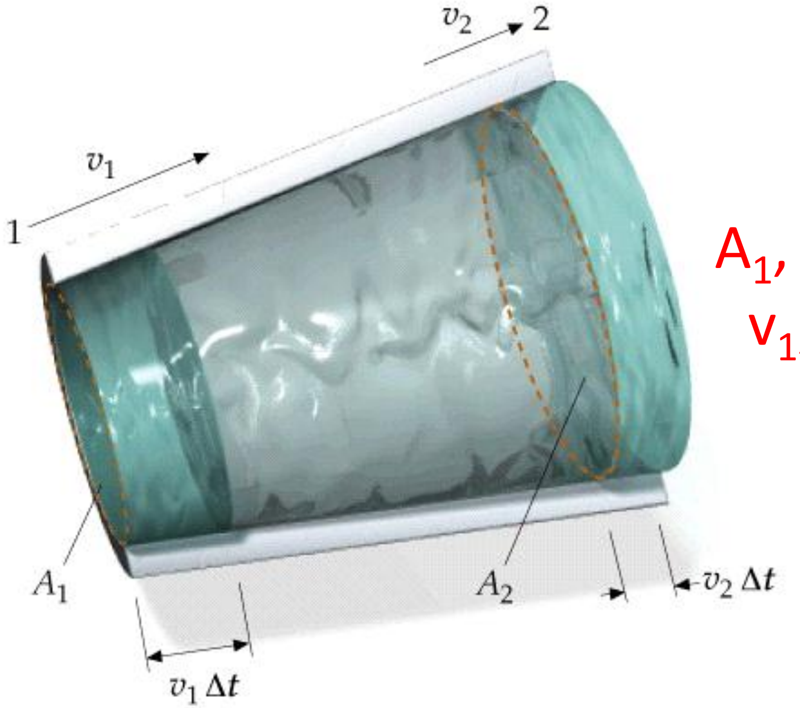
$$Q = A\,v$$

Q = Volume flow rate (m^3/s)
V = volume of fluid in motion (m^3)
t = time (s)
v = the velocity of fluid flow (m/s)

Continuity equation

- The mass of a moving fluid doesn't change as it flows.





$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

A_1, A_2: cross sectional areas at points 1 and 2
v_1, v_2: speed of fluid flow at points 1 and 2

$$A_1 v_1 = A_2 v_2$$

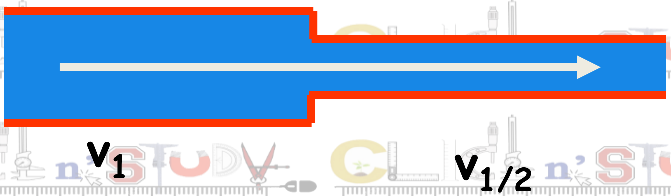
$$Q_1 = Q_2$$
$$A_1\, v_1 = A_2\, v_2$$

Conservation of Mass

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 = \text{constant}$$

$$A_1 v_1 = A_2 v_2 = \text{constant, for incompressible fluid}$$

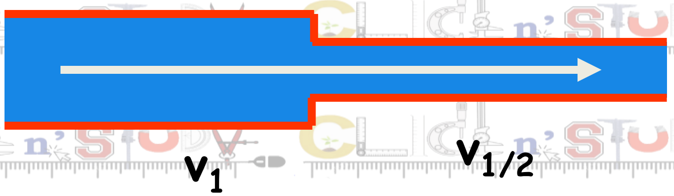
1). A housing contractor saves some money by reducing the size of a pipe from 1” diameter to 1/2” diameter at some point in your house.



If the water moving in the pipe is an ideal fluid, relative to its speed in the 1” diameter pipe, how fast is the water going in the 1/2” pipe?

a) 2 v_1 b) 4 v_1 c) 1/2 v_1 c) 1/4 v_1

2) What is the pressure in the 1/2” pipe relative to the 1” pipe?



a) smaller b) same c) larger

THE EQUATION OF CONTINUITY

The continuity equation expresses that an ideal fluid flow, the product of fluid flow speed & its cross sectional area is constant.

Example

1. The average velocity of water flow in a pipe with diameter 4 cm is 4 m/s. Calculate the amount of fluid (water) flowing per second (Q)!

Solution

d = 4 cm → r = 2 cm = 2×10^{-2} m

v = 4 m/s

Q = …?

Q = A v = $\pi r^2 v$

= 3.14 (2×10^{-2} m) x 4 m/s

= 5.024 m^3/s

THE EQUATION OF CONTINUITY

2. A pipe with diameter 12 cm and the narrowing end point with diameter 8 cm. If the velocity of flow in the large diameter is 10 cm/s, calculate the velocity of flow at the narrowing end.

Solution

$d_1 = 12$ cm → r = 6 cm = 6×10^{-2} m

$d_2 = 8$ cm → r = 4 cm = 2×10^{-2} m

$A_1 = \pi r_1^2 = 3{,}14 \times (6\text{ cm})^2 = 113{,}04\text{ cm}^2$

$A_1 = \pi r_1^2 = 3{,}14 \times (4\text{ cm})^2 = 50{,}24\text{ cm}^2$

$V_1 = 10$ cm/s and $v_2 = \ldots\ldots?$

$$A_1 v_1 = A_2 v_2$$

$113.04\text{ cm}^2 \times 10\text{ cm/s} = 50.24\text{ cm}^2$

$$v_2 = \frac{1130.4}{50.24}$$

$$v_2 = 22.5\ \mathrm{cm}/\mathrm{s}$$

BERNOULLI'S PRINCIPLE

Continuity equation:

$$\boxed{A_1v_1 = A_2v_2}$$

Bernoulli's principle:

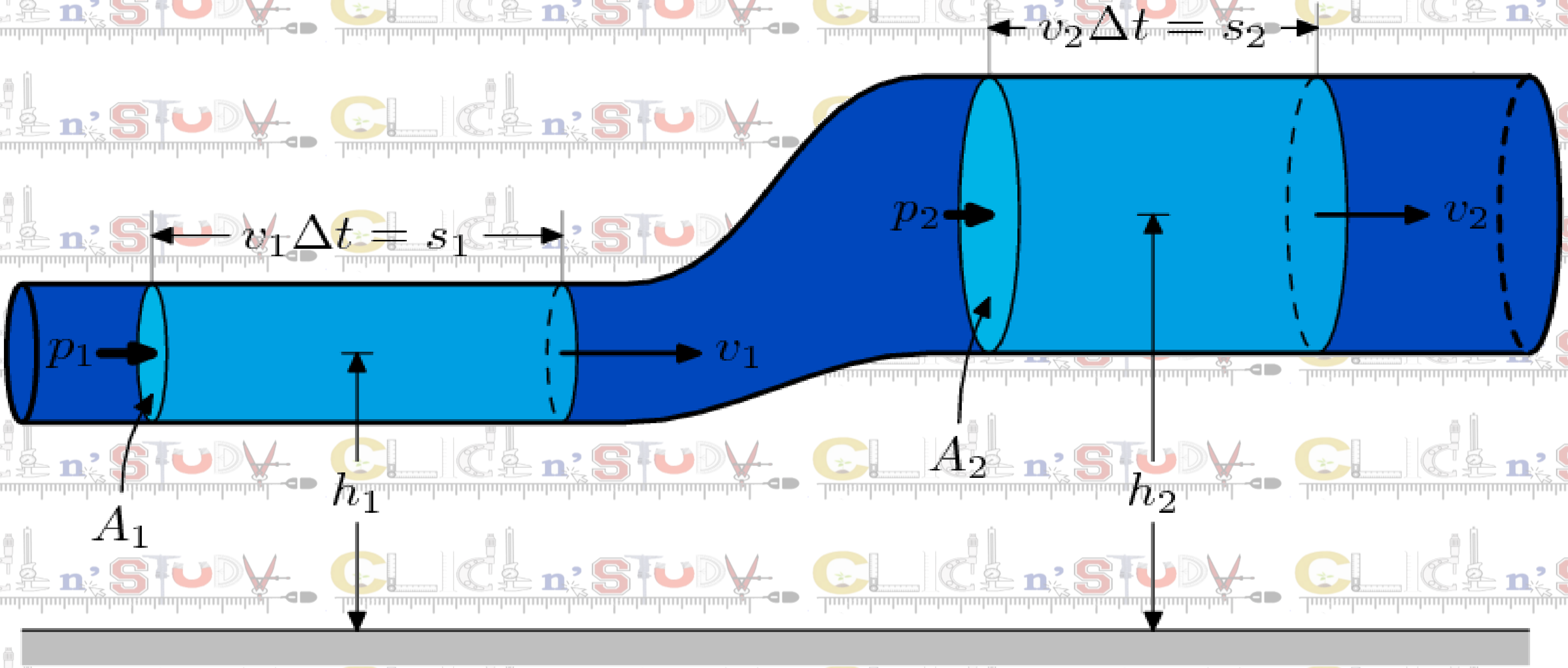
"The pressure in a fluid decreases as the fluid's velocity increases."

Bernoulli's equation:

$$\boxed{P + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant}}$$

Bernoulli's equation at two different points of varying height

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$$



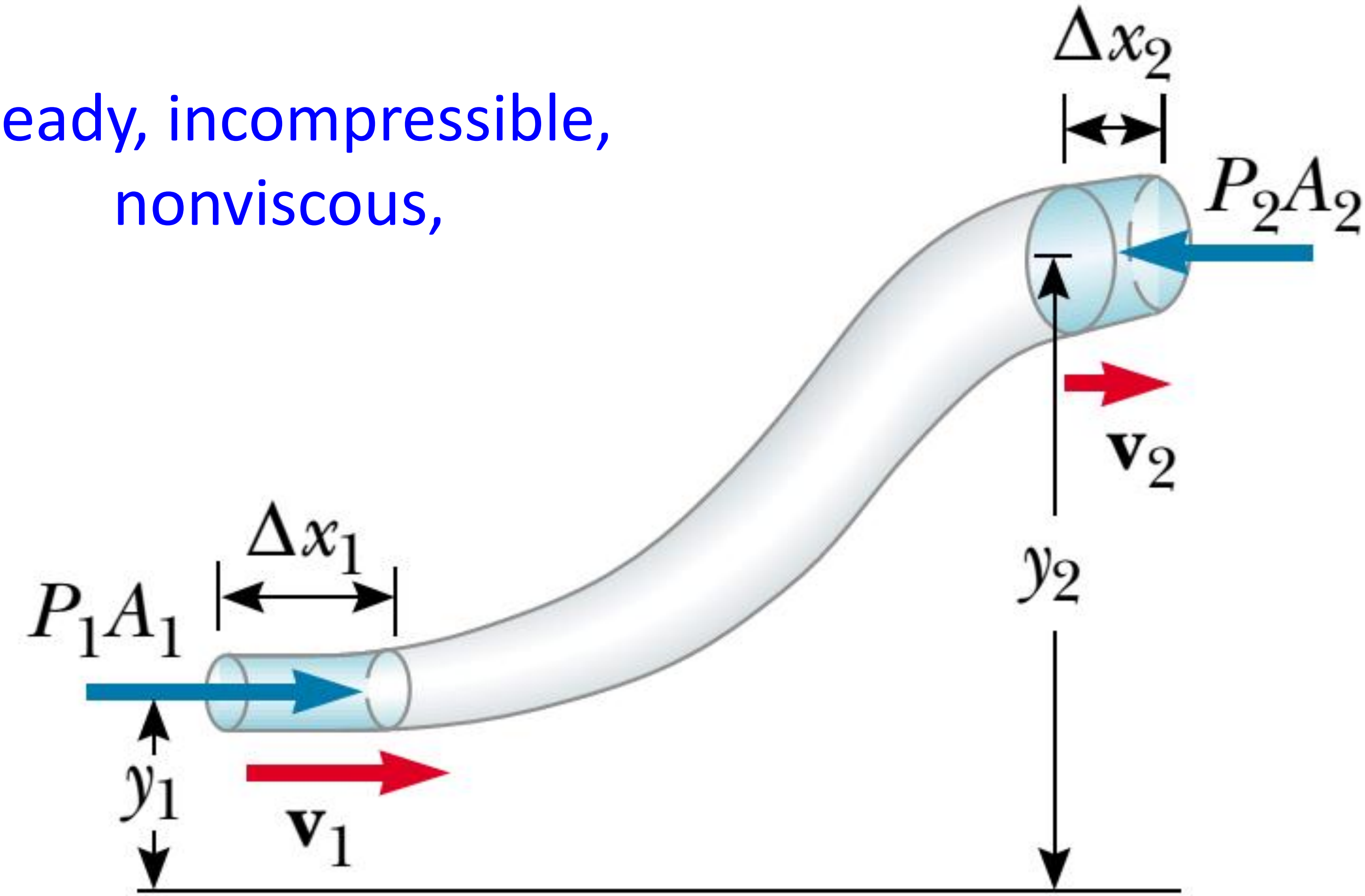
Source: http://commons.wikimedia.org/wiki/File:BernoullisLawDerivationDiagram.svg

Bernoulli's Theorem

- $P + \rho gh + ½ \rho v^2 = \text{Constant}$
- P : pressure (must be in Pa)
- ρ : density of fluid (kg/m^3)
- g: acceleration due to gravity ($9.8\ m/s^2$)
- h: height above lowest point (m)
- v: speed of fluid flow at a point in the pipe (m/s)

Conservation of Energy

Steady, incompressible,
nonviscous,



Bernoulli's Equation

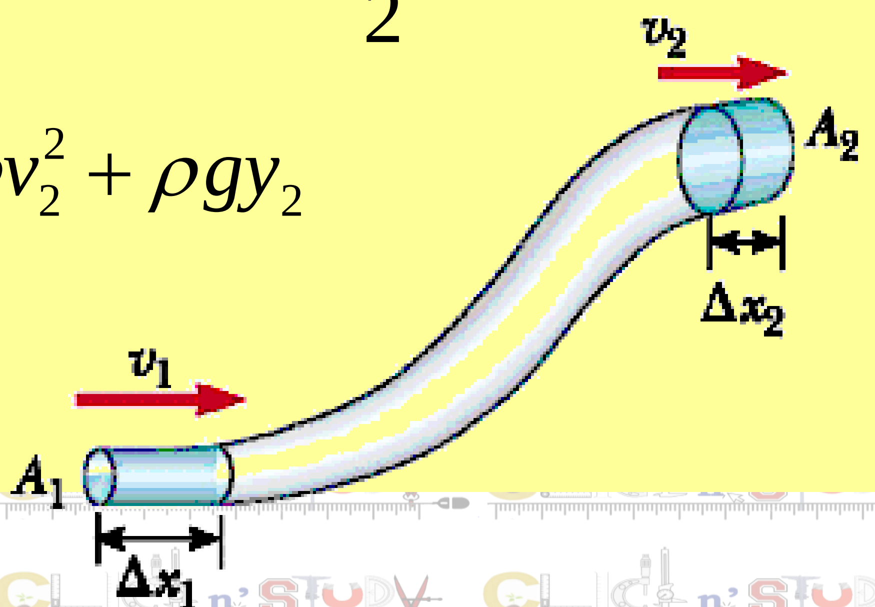
kinetic E, potential E, external work

$$\delta m = \rho A_1 \delta x_1 = \rho A_2 \delta x_2$$

$$p_1 A_1 \delta x_1 - p_2 A_2 \delta x_2 = \frac{1}{2}\delta m v_2^2 + \delta m g y_2 - \frac{1}{2}\delta m v_1^2 - \delta m g y_1$$

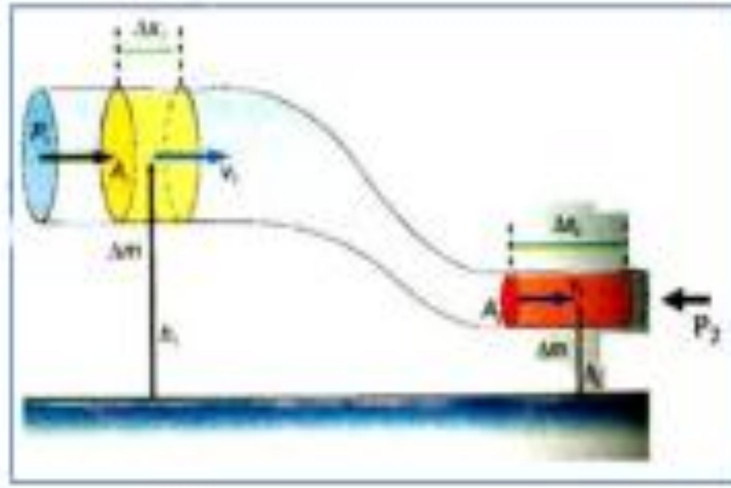
$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$



Q # 17. Derive the expression of Bernoulli's equation for the case of an ideal fluid.

Ans. Consider a fluid that is incompressible, non viscous and flows in a steady state manner through a pipe of non-uniform size as shown in the figure:



The force on the upper end of the fluid is P_1A_1, where P_1 is the pressure and A_1 is the cross-sectional area at the upper end. The work done on the fluid in moving through a distance Δx_1, will be:

$$W_1 = F_1\Delta x_1 = P_1A_1\Delta x_1$$

Similarly the work done on the fluid at the lower end is:

$$W_2 = -F_2\Delta x_2 = -P_2A_2\Delta x_2$$

Where P_2 is the pressure, A_2 is the area of cross-section of lower end and Δx_2 is the distance moved by the fluid in the same time interval t. The work done W_2 is taken to be negative as this work is done against the fluid force.

- Work done $= P_1V - P_2V$

Change in PE $= mgh_2 - mgh_1$

- $K = 1/2mv_2^2 - 1/2\ mv_1^2$

- Net work done= Change in KE + Change in PE

Part of this work is utilized by the fluid in changing its K.E. and a part is used in changing its gravitational P.E.

$$\text{Change in K. E.} = \Delta \text{K. E.} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\text{Change in P. E.} = \Delta \text{P. E.} = mgh_2 - mgh_1$$

Where h_1 and h_2 are the heights of the upper and lower ends resnectively.

Applying the law of conservation of energy to this

volume of fluid, we get

$$(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$$

$$(P_1 - P_2)\frac{1}{\rho} = m\left[\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 + gh_2 - gh_1\right]$$

$$(P_1 - P_2) = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho gh_2 - \rho gh_1$$

Rearranging the above equation:

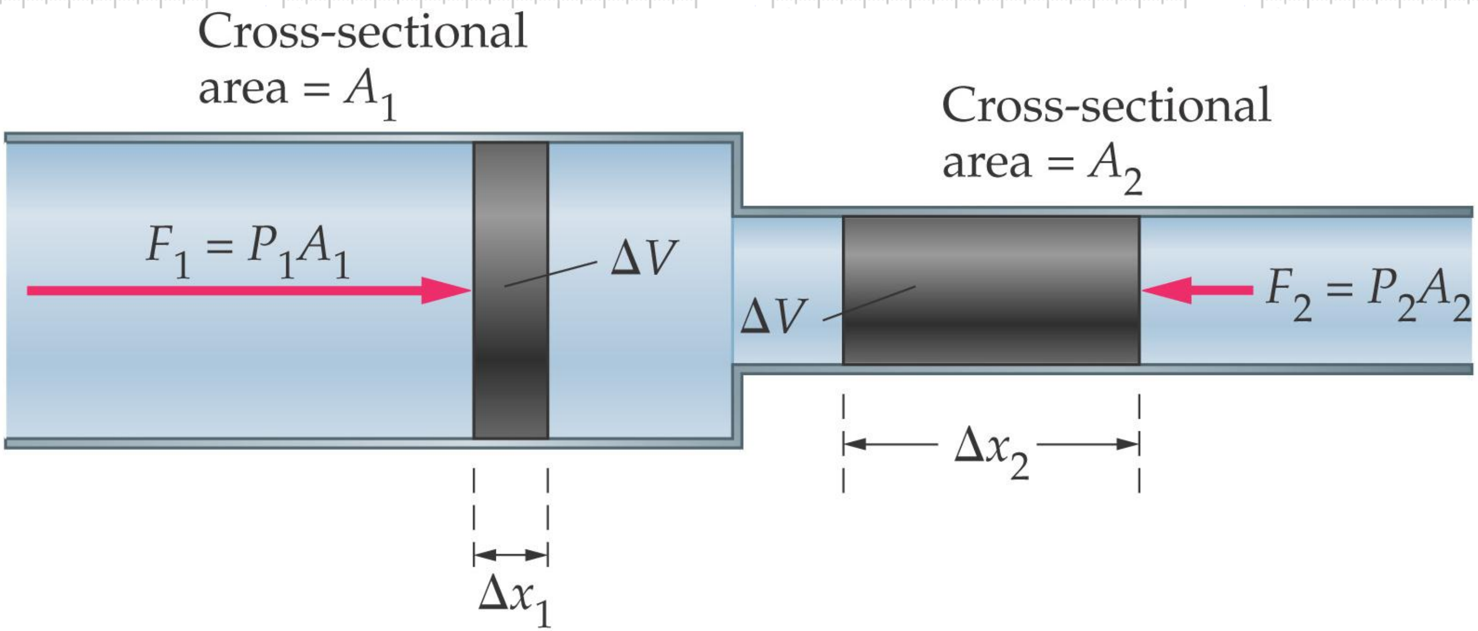
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

This is Bernoulli's equation and is often expressed as:

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

B) Bernoulli's Equation

When a fluid moves from a wider area of a pipe to a narrower one, its speed increases; therefore, work has been done on it.



$$\Delta W_{\text{total}} = (P_1 - P_2)\Delta V$$

Bernoulli's Equation

The kinetic energy of a fluid element is:

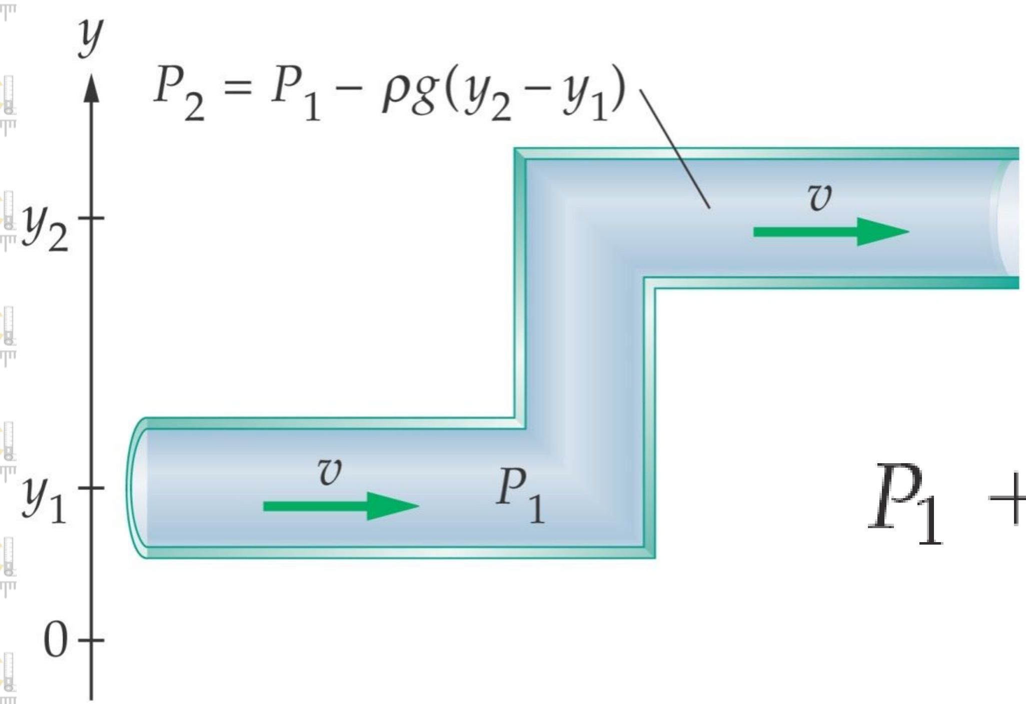
$$K = \frac{1}{2}(\Delta m)v^2 = \frac{1}{2}(\rho \Delta V)v^2$$

Equating the work done to the increase in kinetic energy gives:

$$P_1 + \frac{1}{2}\rho {v_1}^2 = P_2 + \frac{1}{2}\rho {v_2}^2$$

Bernoulli's Equation

If a fluid flows in a pipe of constant diameter, but changes its height, there is also work done on it against the force of gravity.



Equating the work done with the change in potential energy gives:

$$P_1 + \rho g y_1 = P_2 + \rho g y_2$$

Bernoulli's Equation

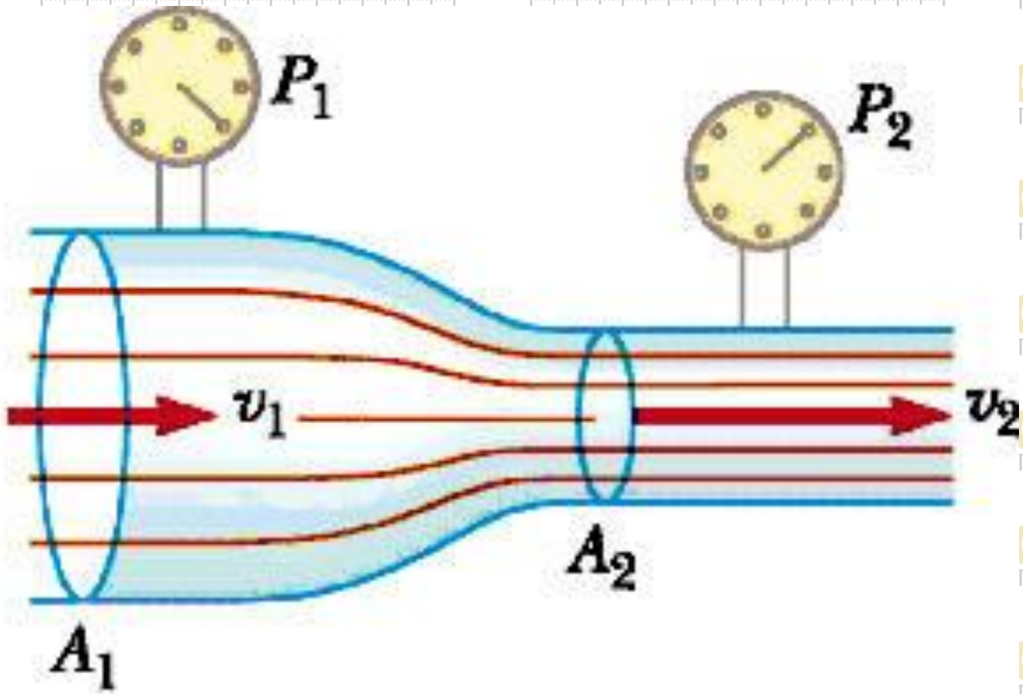
The general case, where both height and speed may change, is described by Bernoulli's equation:

Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho v_1{}^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2{}^2 + \rho g y_2$$

This equation is essentially a statement of conservation of energy in a fluid.

What Accelerates the Fluid?



Acceleration due to pressure difference.

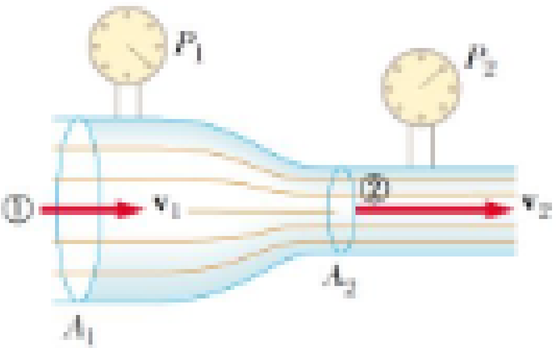
Bernoulli's Principle = Conservation of energy

Q # 22. Describe the relationship between speed and pressure of the fluid.

Ans. A result of Bernoulli's equation is that

" *Where the speed is high, the pressure will be low*"

Explanation



Suppose that water flows through a pipe system as shown in the figure. Clearly, the water flows faster at point 2 than it does at point 1.

Consider the flow speed at point 1 is 0.20 ms^{-1} and at point 2 it is 2 ms^{-1}. Applying the Bernoulli's equation and noting that the average P.E. is the same at both places, we have:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Substituting $v_1 = 0.20\ ms^{-1}$, $v_2 = 2\ ms^{-1}$ and $\rho = 1000\ kgm^{-3}$, we get:

$$P_1 - P_2 = 1980\ Nm^{-2}$$

This shows that the pressure in the narrow pipe where the streamline are closer together i much smaller than in the wider pipe.

There are 2 special cases related to Bernoulli's equation

1. The fluid at a rest or not flowing ($v_1 = v_2 = 0$)

$$p_1 - p_2 = \rho g (h_2 - h_1)$$

This equation expresses the hydrostatic pressure in the liquid at certain height.

2. The fluid in motion in a horizontal pipe ($h_1 = h_2 = h$)

$$p_1 - p_2 = \frac{1}{2}\rho (v_2{}^2 - v_1{}^2)$$

This equation expresses if $v_2 > v_1$, then $p_1 > p_2$

p_1 and p_2 = pressure at point 1 and 2 (N/m^2)
h_1 and h_2 = the height of place 1 and 2 (m)
v_1 and v_2 = the velocity at point 1 and 2 (m/s)
ρ = density of fluid (kg/m^3)
g = gravitational acceleration (m/s^2)

BERNOULLI'S PRINCSIPLE

$$p_1 - p_2 = \frac{1}{2}\rho(v_2{}^2 - v_1{}^2)$$

, it means if the the velocity of fluid flow in a place is big then its pressure is small and holds the contrary.

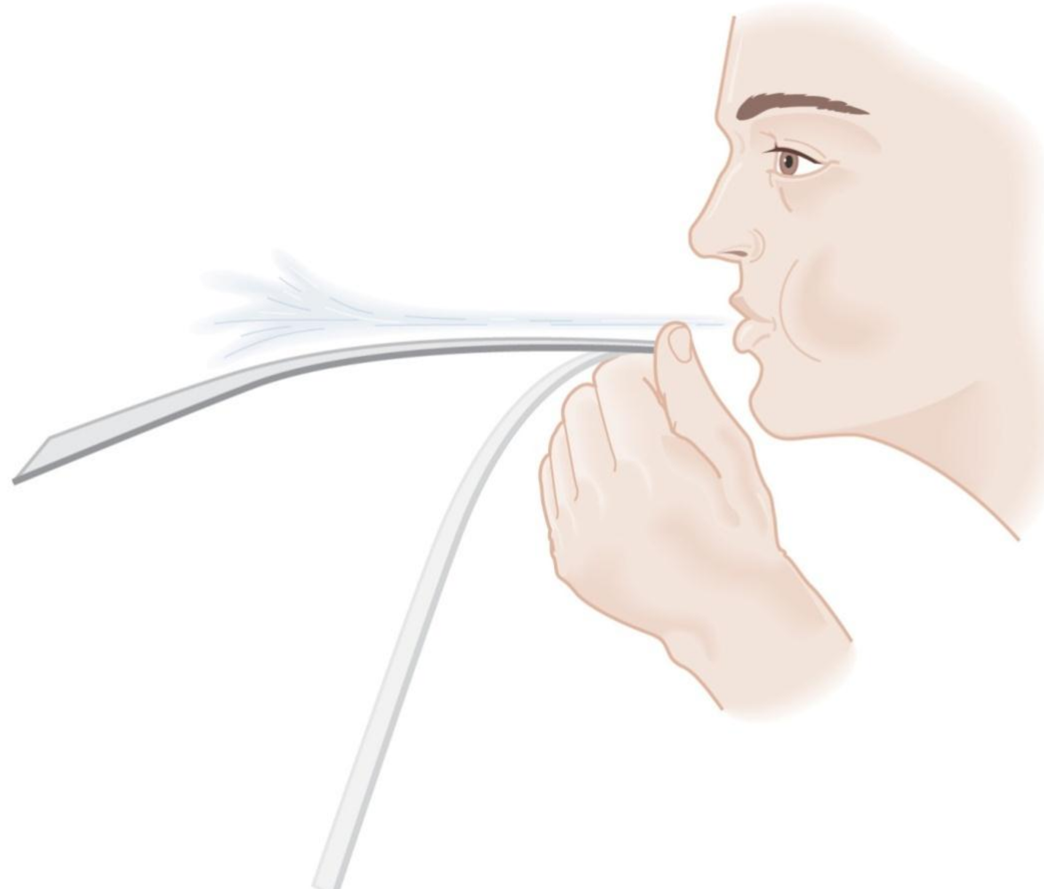
Note:

p_1 and p_2 = pressure at point 1 and 2 (N/m^2)

ρ = density of fluid (kg/m^3)

Applications of Bernoulli's Equation

Hold a sheet of paper by its end, so that it would be horizontal if it were stiff, and blow across the top. The paper will rise, due to the higher speed, and therefore lower pressure, above the sheet.



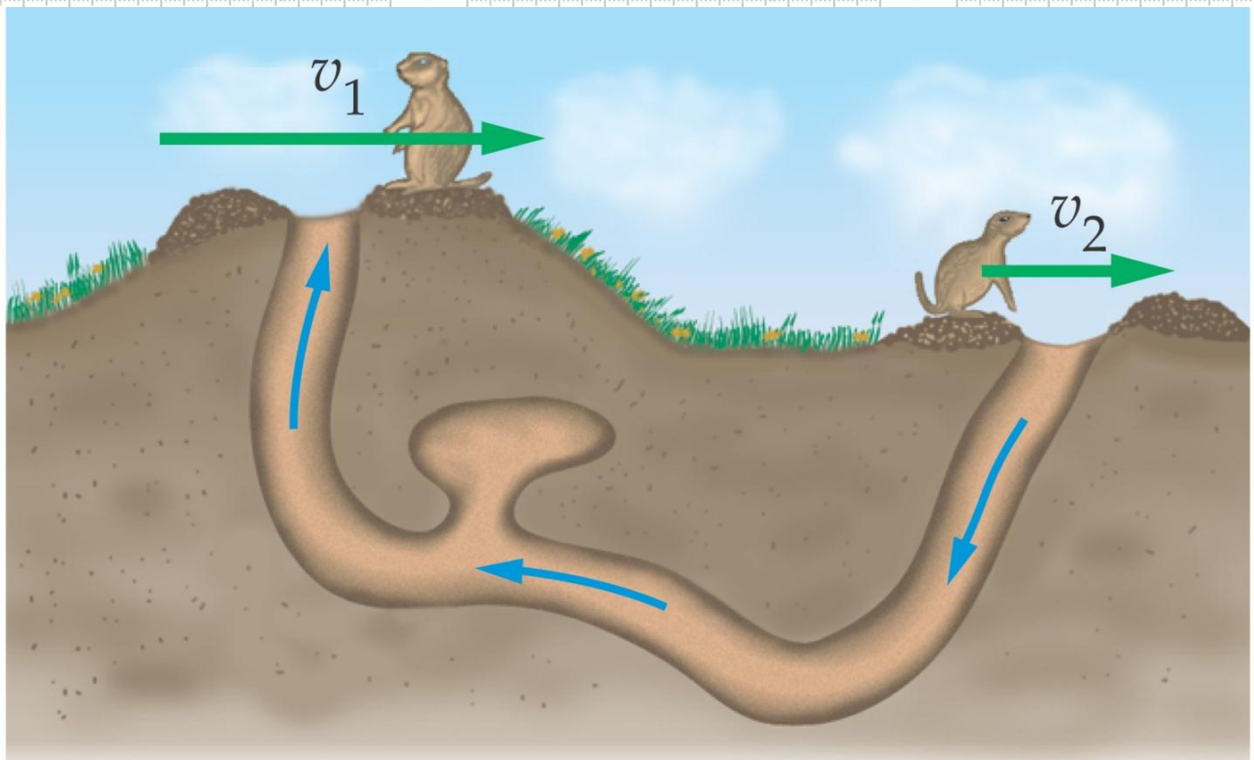
MCQ 1997/ 35

Applications of Bernoulli's Equation

This lower pressure at high speeds is what rips roofs off houses in hurricanes and tornadoes, and causes the roof of a convertible to expand upward. It even helps prairie dogs with air circulation!

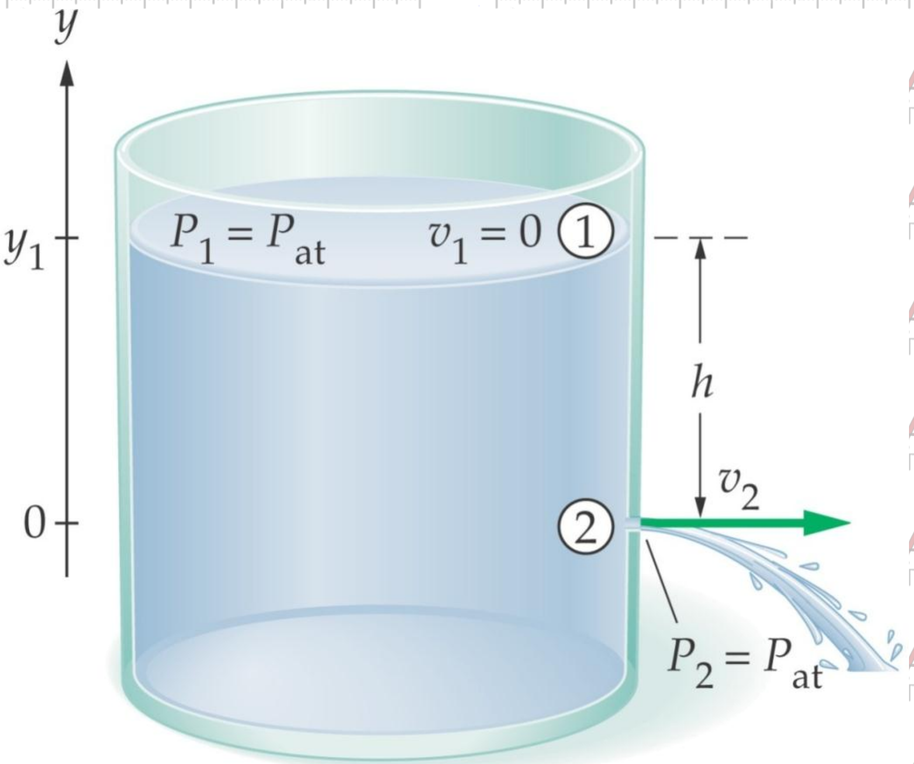
MCQ 2003/23

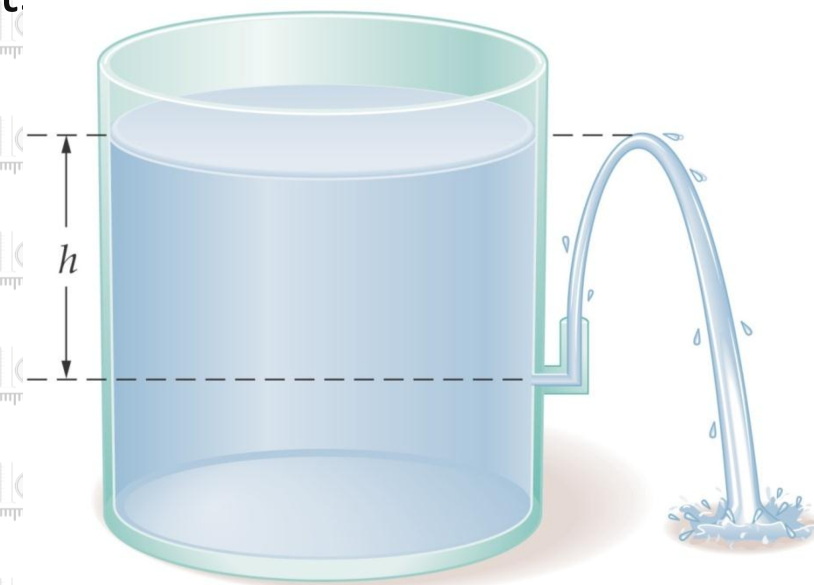
MCQ 2008/17



Applications of Bernoulli's Equation

If a hole is punched in the side of an open container, the outside of the hole and the top of the fluid are both at atmospheric pressure. Since the fluid inside the container at the level of the hole is at higher pressure, the fluid has a horizontal velocity as it exits





If the fluid is directed upwards instead, it will reach the height of the surface level of the fluid in the container, provided NO VISCOSE FRICTIONAL FORCES IN THE FLUID

Flow from a tank hole

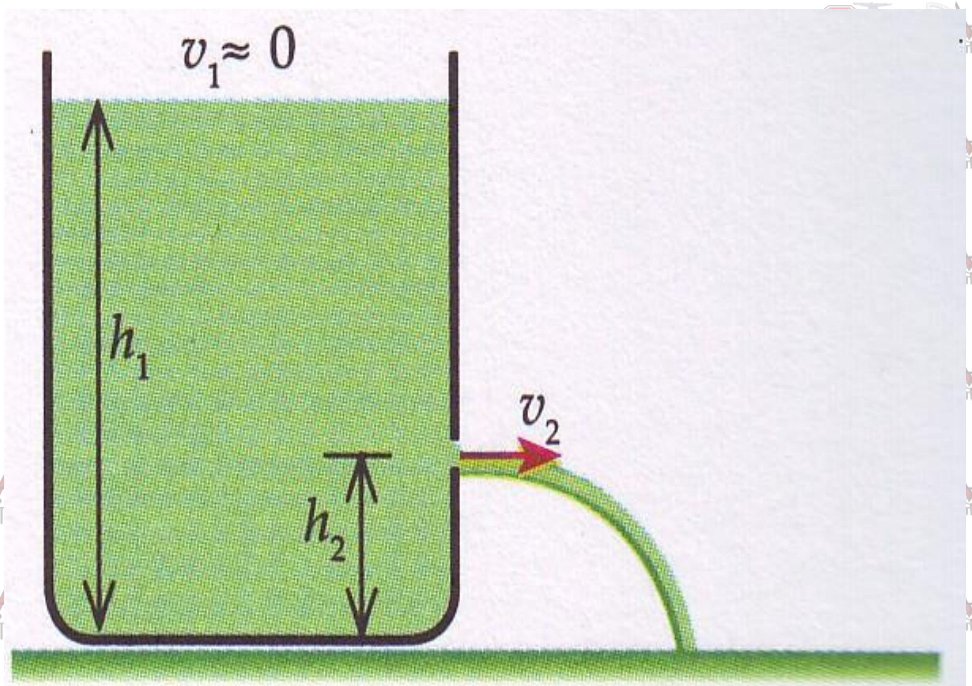
The speed $\mathbf{v_2}$ is much larger than $\mathbf{v_1}$ of the top surface of water. We can therefore, take $\mathbf{v_1}$ approximately equal to zero. Hence the Bernoulli's equation can be written as:

$$P_1 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\therefore P_1 = P_2 = \text{Atmospheric Pressure}$$

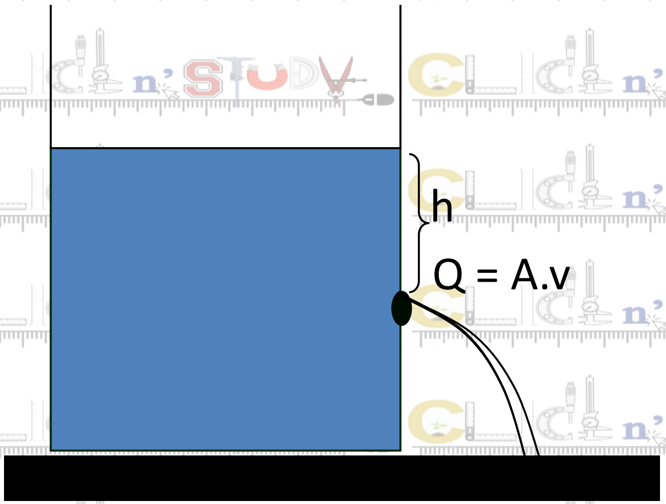
$$\rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$$

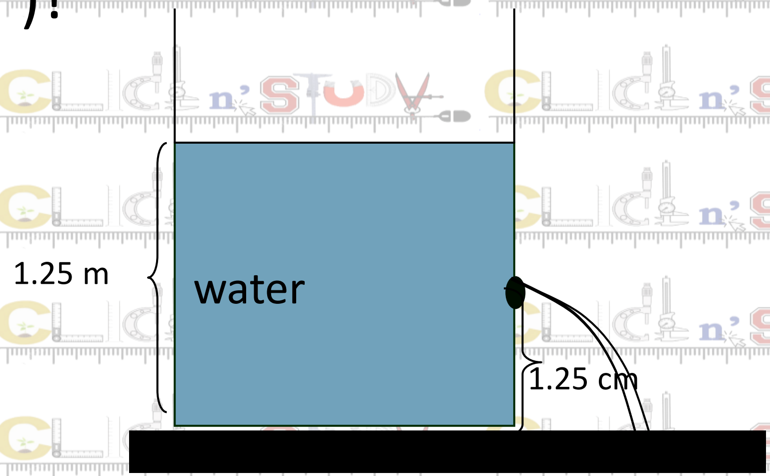
$$v_2 = \sqrt{2g(h_1 - h_2)}$$



MCQ 2007/51

A tank contains water of 1.25 m high. There is a leak hole in the tank 45 cm from the bottom of tank. How far the place of water falls measured from the tank ($g = 10\ m/s^2$)?





$$\boxed{v = \sqrt{2gh}}$$

$$\boxed{Q = A\sqrt{2gh}}$$

The velocity of water flow from leak hole

$$v = \sqrt{2\,g\,(h_1 - h_2)}$$

$$= \sqrt{210\,m/s^2\,(125\,m - 0.45\,m)}$$

$$= \sqrt{20\,m/s^2(0.80\,m)}$$

$$= \sqrt{16\,m^2/s^2} = 4\,m/s$$

The water path is partially a parabolic motion with elevation angle α = 0° ($v_{0 \text{ is in}}$ horizontal direction)

$$y = v_0 \sin \alpha t + \tfrac{1}{2} g t^2$$

$$0.45\,m = 0 + \tfrac{1}{2}(10\,m/s^2)t^2$$

$$0.45\,m = 5\,m/s^2 t^2$$

$$t = \sqrt{\frac{0.45\,m}{5\,m/s^2}}$$

$$t = \sqrt{0.9\,s^2}$$

$$t = 0.3\,s$$

$$x = v_0(\cos\alpha)t$$

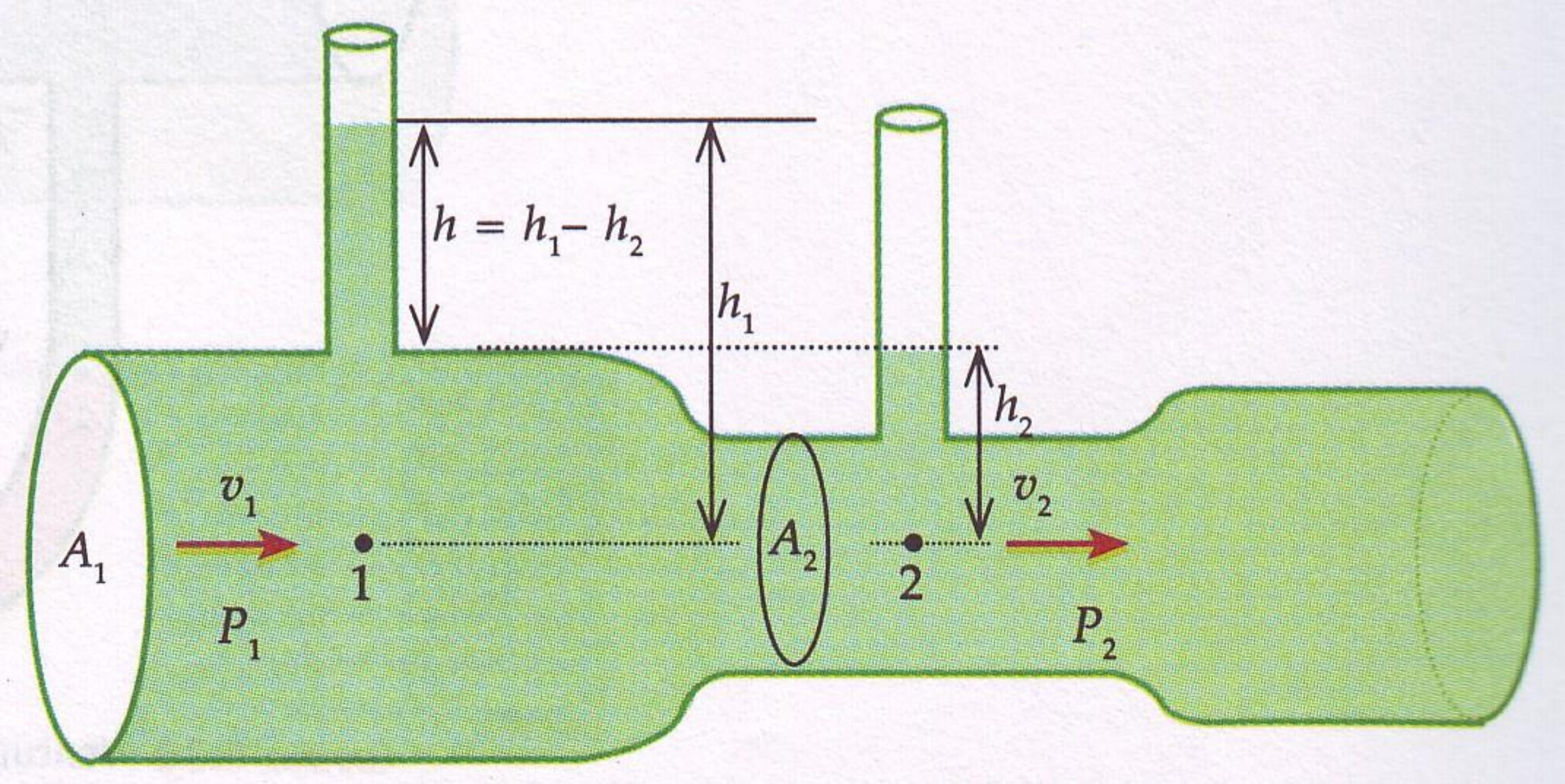
$$= (4\,m/s)(1)(0.3\,s)$$

$$= 1.2\,m$$

Thus, the water falls at 1.2 m from the tank.

Venturi Meter

The venturimeter



The bernoulli's Equation in this case will be in the form of :

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2)$$

$$P_1 - P_2 = \rho g h$$

and

$$A_1 v_1 = A_2 v_2$$

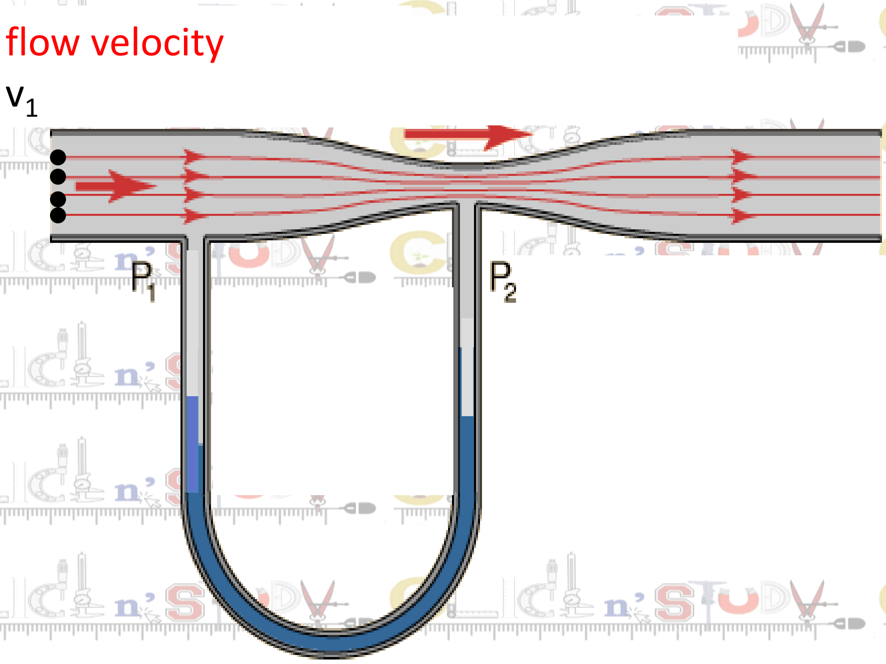
We will get :

$$v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

Or :

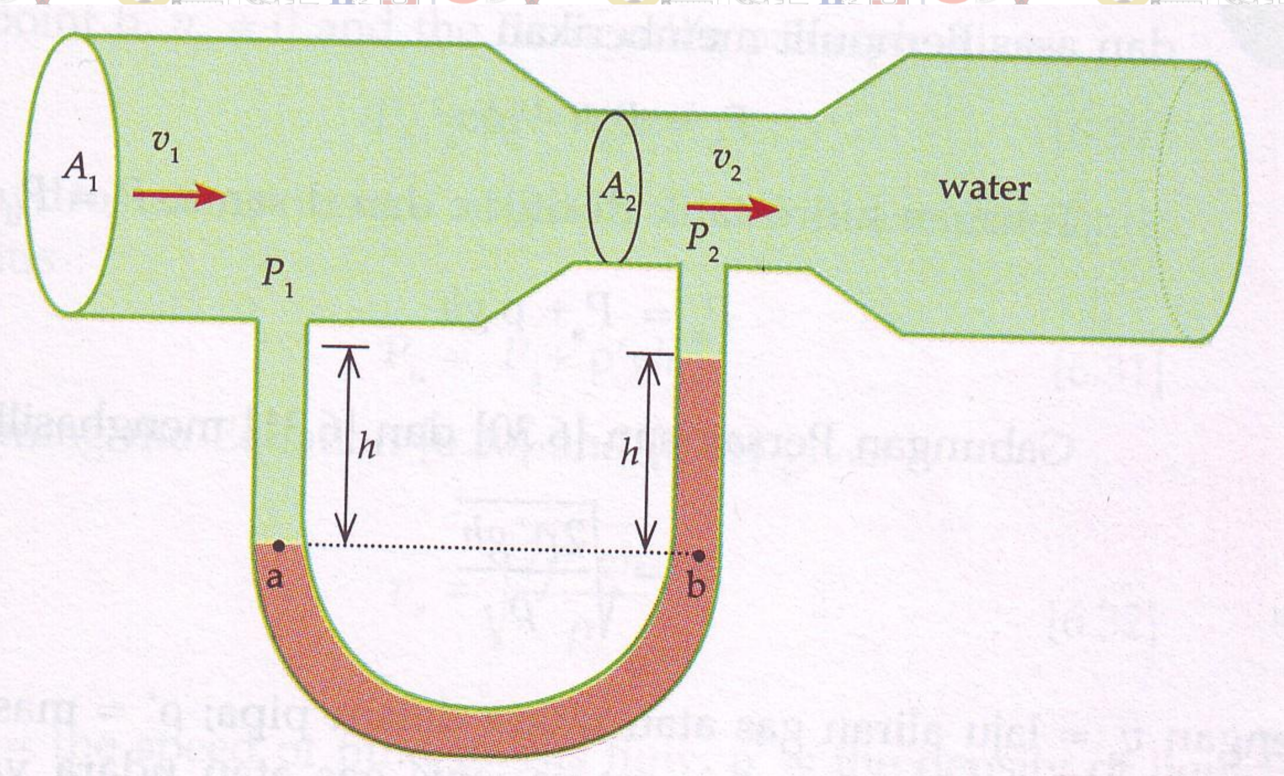
$$v_2 = \sqrt{\frac{2gh}{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

Venturimeter



$$v_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho[(A_1 / A_2)^2 - 1]}}$$

Venturimeter with manometer



$$v_1 = A_2\sqrt{\frac{2(\rho' - \rho)gh}{\rho(A_1^2 - A_2^2)}}$$

$$v_2 = A_1\sqrt{\frac{2(\rho' - \rho)gh}{\rho(A_1^2 - A_2^2)}}$$

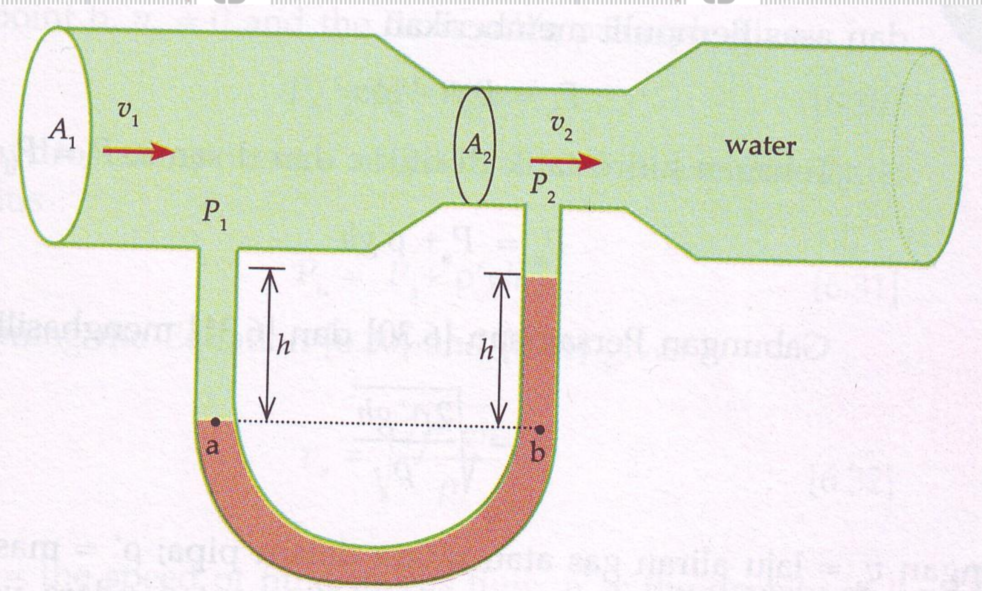
Example

Diagram shows a venturimeter with a manometer. The rate of flow of water which flow through the venturi is 3,200 cm^3/s. the cross section area 1 and area 2 each is 40 cm^2 and 16 cm^2. The density of mercury 13.6 g/cm^3

a. what is the speed of the water at the area 1 and area 2 ?

b. what is the difference of pressure between pipe1 & pipe 2

c. what is the difference of mercury high at the manometer ?



Q # 30. Derive the Venturi relation for an ideal fluid.

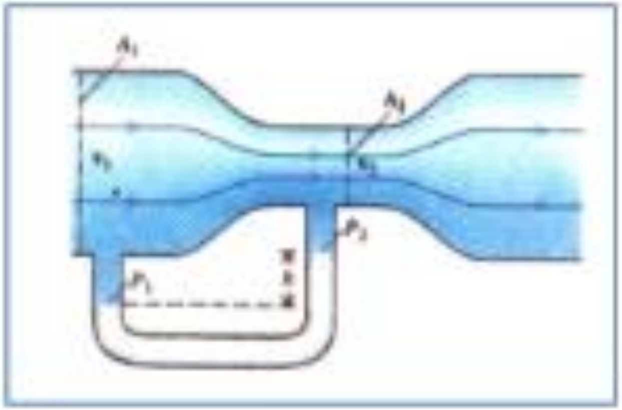
Ans. If one part of a horizontal pipe has a much smaller diameter than the other, we write Bernoulli equation in more convenient form. For horizontal position of pipe, ρgh terms become equal. Thus the Bernoulli equation is expressed as:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

As the cross-sectional area A_2 is small as compared to the area A_1, then from equation of continuity $v_1 = \left(\frac{A_2}{A_1}\right) v_2$, will be small as compared to v_2. Thus we can neglect v_1 on the right hand side of the equation. Hence



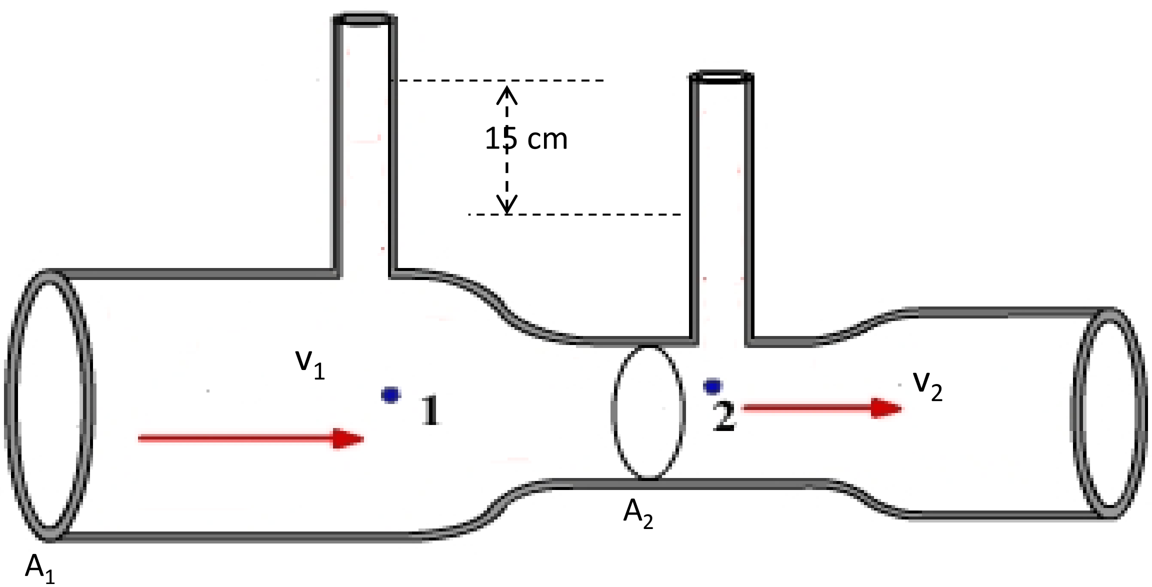
$$P_1 - P_2 = \frac{1}{2}\rho\, v_2^2$$

This is known as Venturi relation, which is used in Venturi meter, a device used to measure speed of liquid flow.

Example

A venturimeter with the big section area 10 cm^2 and small section area 5 cm^3 is used to measure the velocity of water flow. If the height difference of water surface is 15 cm.

Calculate the velocity of water flow in the big and small section ($g = 10\ m/s^2$)?



Solution

A_1 = 10 cm^2 = 10 x 10^{-4} m^2

A_2 = 5 cm2 = 5 x 10^{-4} m^2

h = 15 cm = 15 x 10^2 m

g = 10 m/s^2 , v_2 = …?

$$v = \sqrt{\frac{2\,g\,h}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

$$= \sqrt{\frac{2\times 10\; m/s^2 \times 15\times 10^{-2}\; m}{\left(\frac{10\times 10^{-4}\; m^2}{5\times 10^{-4}\; m^2}\right)^2 - 1}}$$

To determine the velocity v_2 use the equation of continuity.

$$A_1 v_1 = A_2 v_2$$

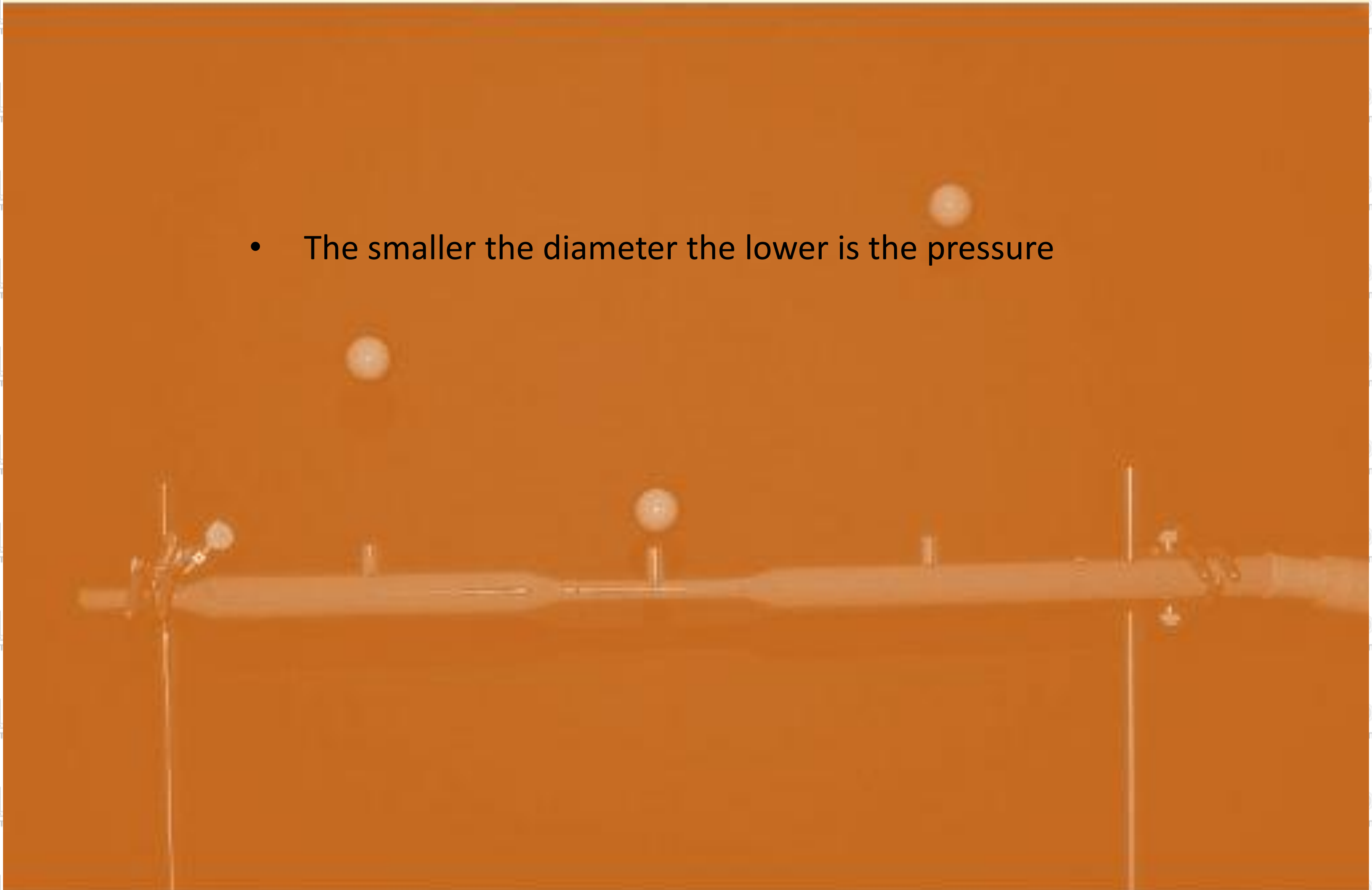
$$v_2 = \frac{A_1}{A_2} v_1$$

$$= \frac{10\times 10^{-4}\; m^2}{5\times 10^{-4}\; m^2} \times 1\; m/s$$

$$= 2\; m/s$$

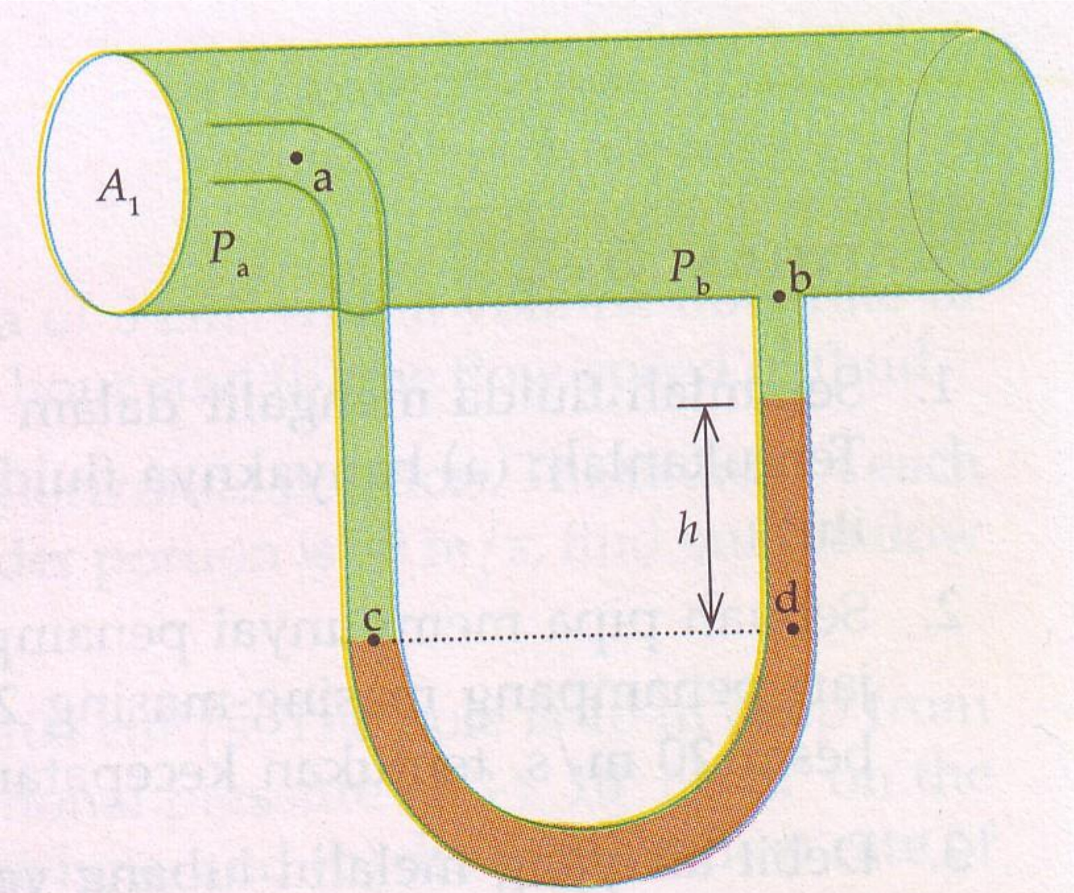
Thus, the velocityof flow in the big and small section are 1 m/s and 2 m/s

- The smaller the diameter the lower is the pressure



MCQ 2001/32 MCQ 2004/19 MCQ 2005 /37 MCQ 2009/16

Pitot Tube: A pilot tube is an instrument used to measure the speed of a gas or air flow.



Bernoulli's principle gives :

$$P_b = P_a + \tfrac{1}{2}\rho v_a^2$$

$V_b = 0$

The hydrolic pressure of point c and d are the same, $P_c = P_d$

$$P_b = P_a + \rho' g h$$

The combination of two equation above :

$$v_a = \sqrt{\frac{2\rho' g h}{\rho}}$$

Pitot tube

$$v = \sqrt{\frac{2\rho' gh}{\rho}}$$

Note:

h = height difference of liquid column surface in the manometer (m)

g = gravitation acceleration (m/s^2)

ρ = density of gas (kg/m^3)

ρ' = density of liquid in manometer (kg/m^3)

v = speed of air or gas flow (m/s)

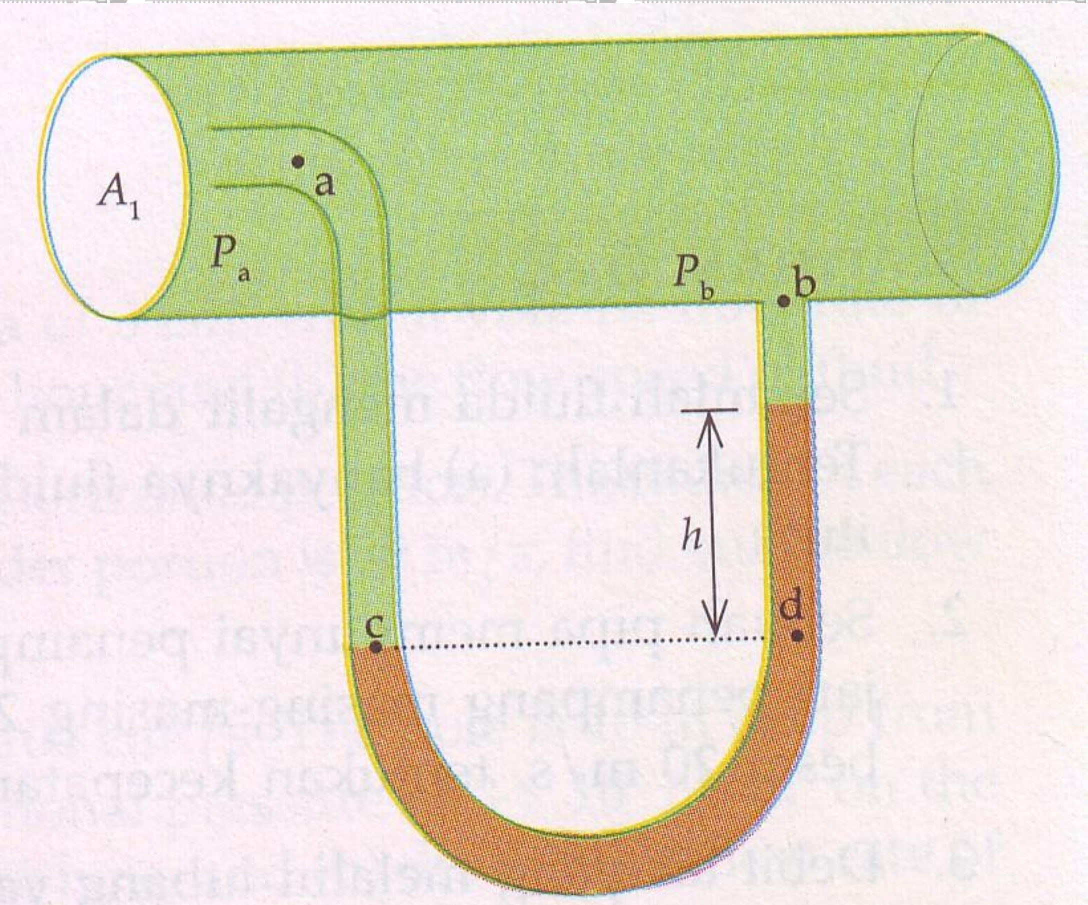
Note:

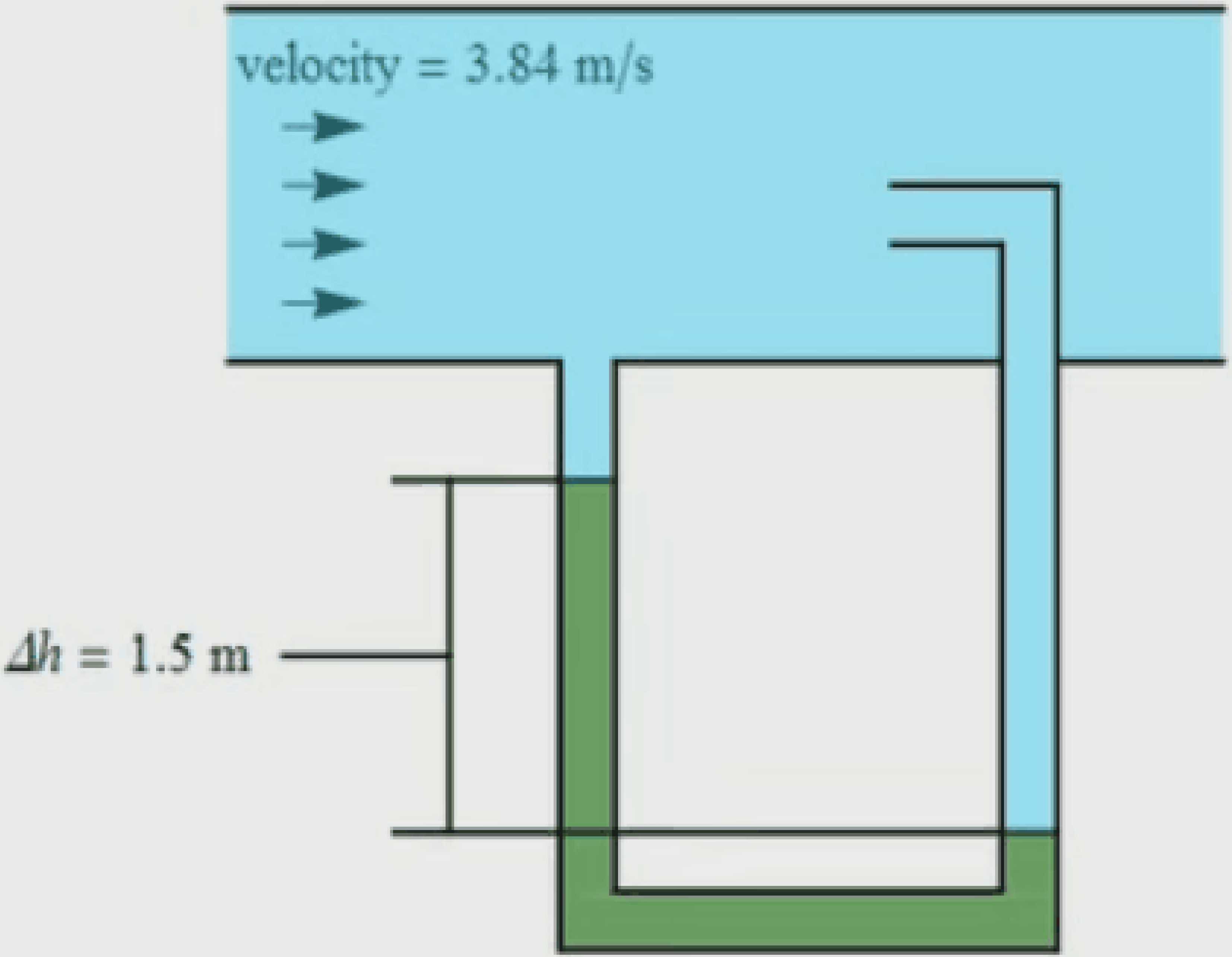
The velocity at the height of the bend in the tube (stagnation point) is zero. Some kinetic energy density of the fluid flowing through the pipe is converted into pressure, resulting in a change in manometer height.

Q. When the air flows through a Pitot tube, The difference in hight between mercury columns in mamometer is 2 cm. Determine the flow speed of air.

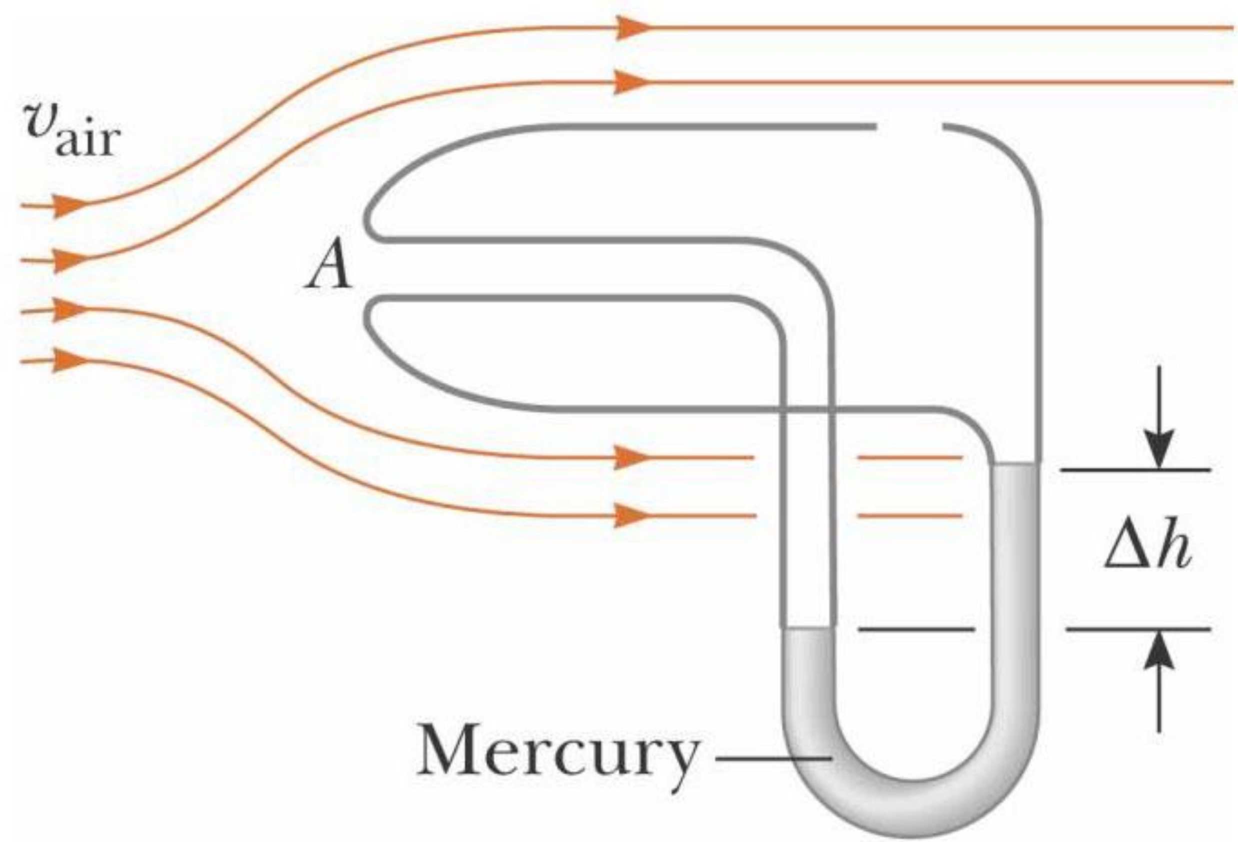
ρ_{air} - 1.29 kg/m^3

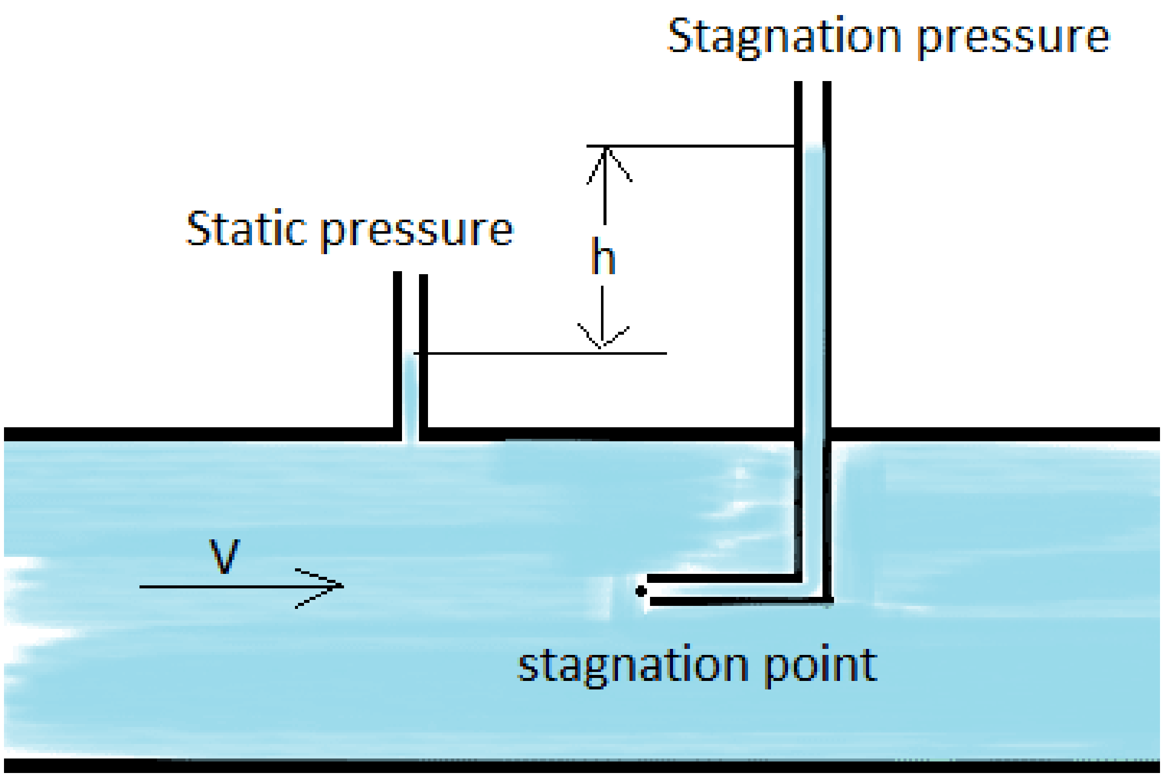
$\rho_{mercury}$ - 13.6 g/cm^3

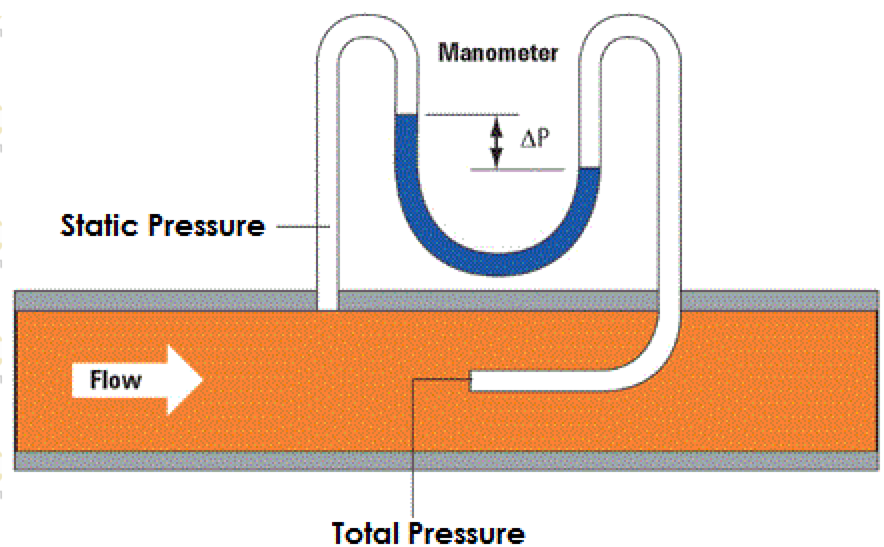


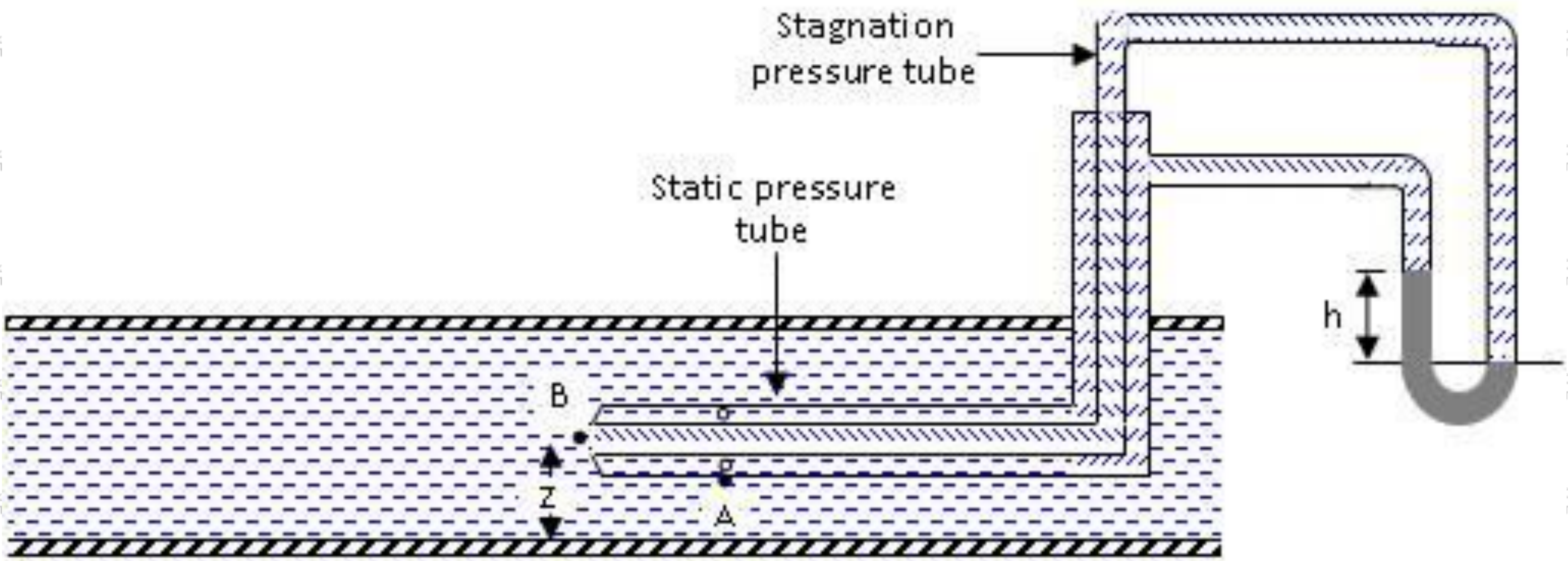

velocity = 3.84 m/s
Δh = 1.5 m

- A **Pitot tube** can be used to determine the velocity of air flow by measuring the difference between the total pressure and the static pressure.
- If the fluid in the tube is mercury, density $\rho_{Hg} = 13\,600$ kg/m^3, and $h = 5.00$ cm, find the speed of air flow. (Assume that the air is stagnant at point *A* and take $\rho_{air} = 1.25$ kg/m^3.)




Stagnation pressure
Static pressure
h
V
stagnation point


Manometer
ΔP
Static Pressure
Flow
Total Pressure



- Pitot tube is used to measure flow velocity. Outer body of pilot tube consist of ports at point A, for sensing the static pressure of fluid. At point B fluid vel. become zero and inner tube is for sensing the stagnation pressure. The outlet C & D is connected to U-tube manometer for measuring the pressure difference between the points A and B.The flow velocity is given by : $V = \sqrt{2gh}$

THE APPLICATION OF BERNOULLI'S PRINCIPLE

Example

A pitot tube is used to measure the speed of oxigen gas flow with density 1.43 kg/m^3 in a pipe. If the height difference of liquid at both feet of manometer is 5 cm and liquid density is 13600 kg/m^3.

Calculate the speed of gas flow at that pipe! (g = 10 m/s^2)

Solution

ρ = 1,43 kg/m^3

ρ'= 13600 kg/m^3

h = 5 cm = 0,05 m

g = 10 m/s^2

v =...?

$$v = \sqrt{\frac{2\rho' gh}{\rho}}$$

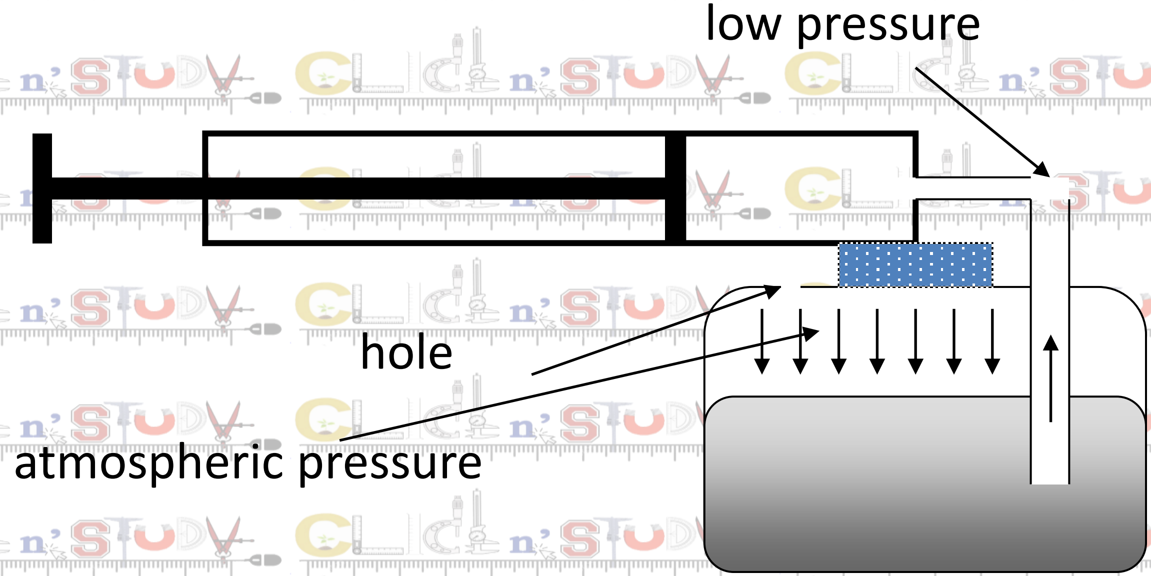
$$= \sqrt{\frac{2 \times 13600\ kg/m^3 \times 10\ m/s^2 \times 0.05\ m}{1.43\ kg/m^3}}$$

$$= 97.52\ m/s$$

Thus, the speed of oxygen flow in the pipe is 97,52 m/s

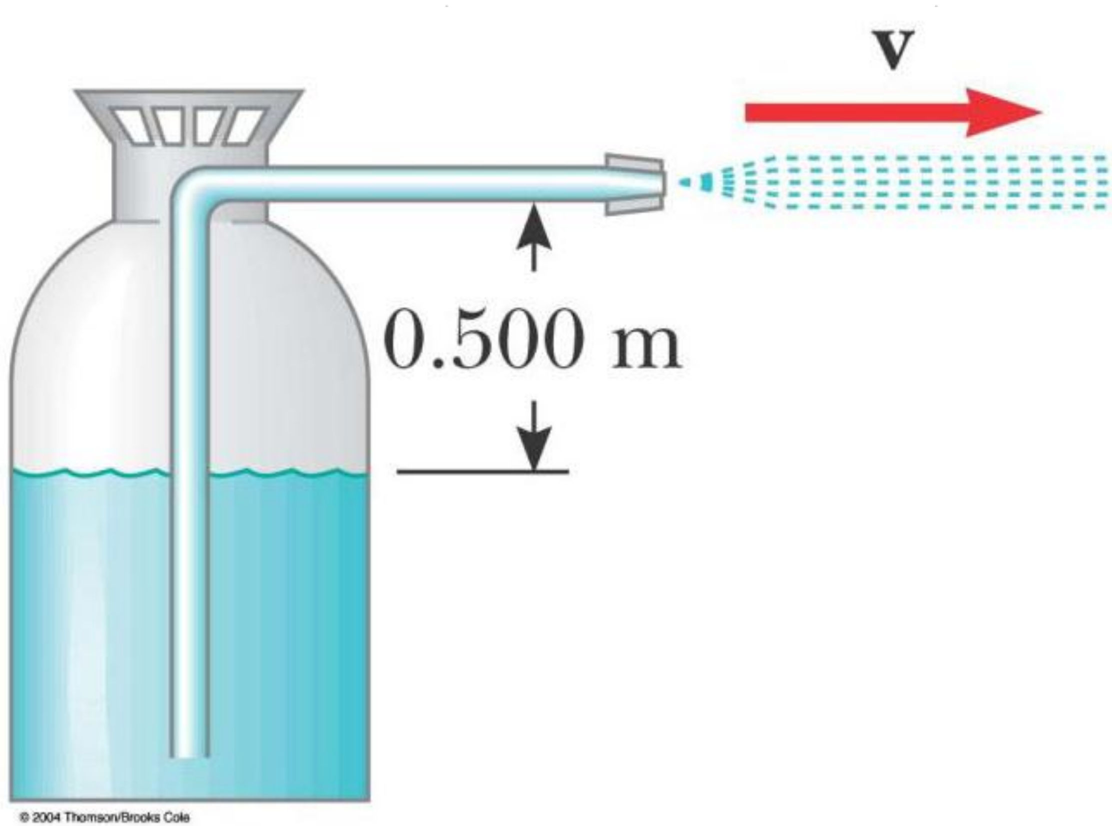
Mosquito sprayer

If the piston is pumped, the air from the cylinder tube is forced out through the narrow hole. The air coming out from this narrow hole has height velocity so that decreasing the air pressure above the nozzle.



Because the air pressure above the nozzle is smaller than that of on liquid surface in the tube, then the liquid will spray out through the nozzle.

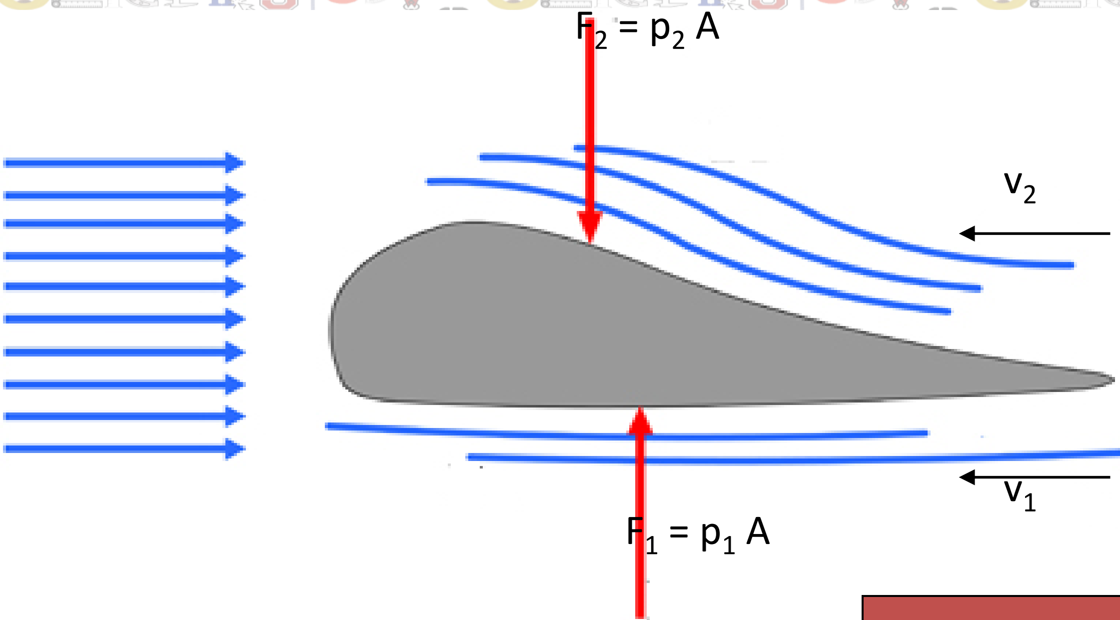
Q. Water is forced out of a fire extinguisher by air pressure, as shown in Figure below. How much gauge air pressure in the tank (above atmospheric) is required for the water jet to have a speed of 30.0 m/s when the water level in the tank is 0.500 m below the nozzle?



Lift on an Aero plane

- The lift on an aero plane is explained on the basis of relationship between pressure and velocity. The wing of the aero plane is designed to deflect the air so that the streamlines are closer together above the wing than below it. Thus, air is travelling faster on the upper side of the wing than on the lower. As the result, the pressure will be lower at the top of the wing, and the wing will be forced upward.

Lift force of airplane's wings



According to Bernoulli's principle, if the speed of air flow at the up side of wing is larger than that of at the downside, then the air pressure at upside of wing smaller than that of at the downside.

$$F_1 - F_2 = (p_1 - p_2)A$$

F_1 = pushing force of plane to the upward (N)
F_2 = pushing force of plane to the downward (N)
$F_1 – F_2$ = lift force of plane (N)
p_1 = pressure at the downside (N/m^2)
p_2 = pressure at the upside (N/m^2)
A = section area of wing (m^2)

The equation of lift force above can also be expressed as follow:

$$F_1 - F_2 = \frac{1}{2}\rho({v_2}^2 - {v_1}^2)A$$

Note:

F_1 = pushing force of plane to the upward (N)

F_2 = pushing force of plane to the downward (N)

$F_1 – F_2$ = lift force of plane (N)

v_1 = the velocity of air below the wing (m/s)

v_2 = the velocity of air above the wing (m/s)

ρ = the density of air (kg/m^3)

Example

If the velocity of air flow at the downside of a plane's wings is 60 m/s, what is the velocity at the upside of the plane's wings if the upward pressure obtained is 10 N/m^2?

(ρ = 1.29 kg/m^3)

Solution

$p_2 - p_1 = 10$ N/m $v_2 = 60$ m/s $h_1 = h_2$ $v_1 = \ldots?$

$$p_1 + \tfrac{1}{2}\rho v_1^{\,2} + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^{\,2} + \rho g h_2$$

$$\tfrac{1}{2}\rho(v_1^{\,2} - v_2^{\,2}) = p_2 - p_1$$

$$v_1^{\,2} - v_2^{\,2} = \frac{2(p_2 - p_1)}{\rho}$$

$$v_1^{\,2} = v_2^{\,2} + \frac{2(p_2 - p_1)}{\rho}$$

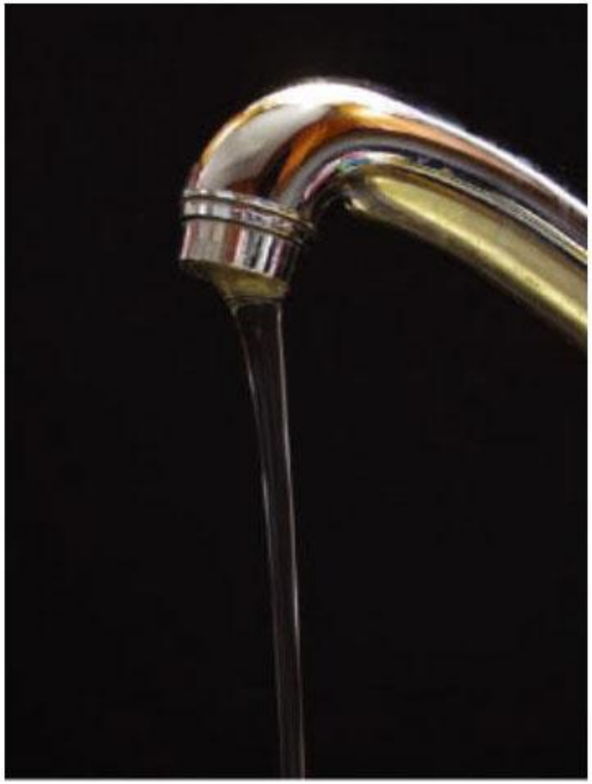
$$= (60\ m/s)^2 + \frac{2\,(10)\ N/m^2}{1.29}$$

$$v_1 = \sqrt{3615.5\ m^2/s^2}$$

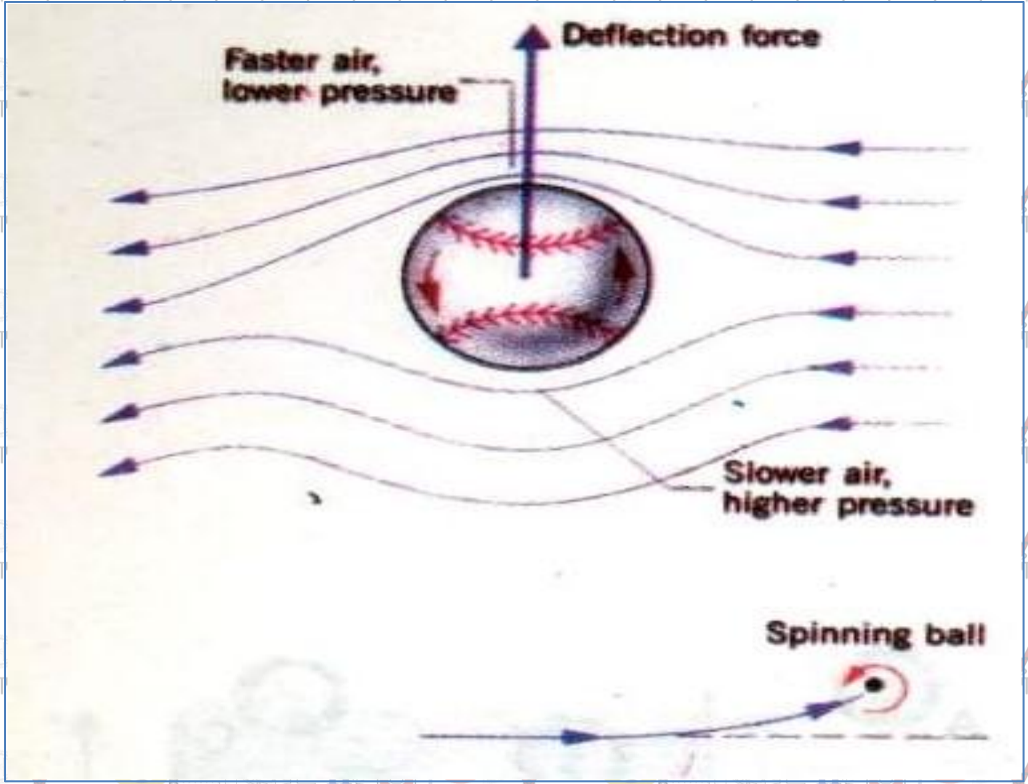
$$= 60.13\ m/s$$

The velocity of air flow at the upside of the plane's wings is 60.13 m/s

Q. Figure on the right shows a stream of water in steady flow from a kitchen faucet. At the faucet the diameter of the stream is 0.960 cm. The stream fills a 125-cm^3 container in 16.3 s. Find the diameter of the stream 13.0 cm below the opening of the faucet.



Swing of the tennis ball in air



- When a tennis ball is hit by the racket in such a way that it spins as well as moves forward, the velocity of air on one side of the ball increases due to spin and hence the pressure decreases. This gives an extra curvature to the ball known as swing which deceives an opponent player.

- **Q # 5. State Bernoulli's relation for a liquid in motion and describe some of its applications?**

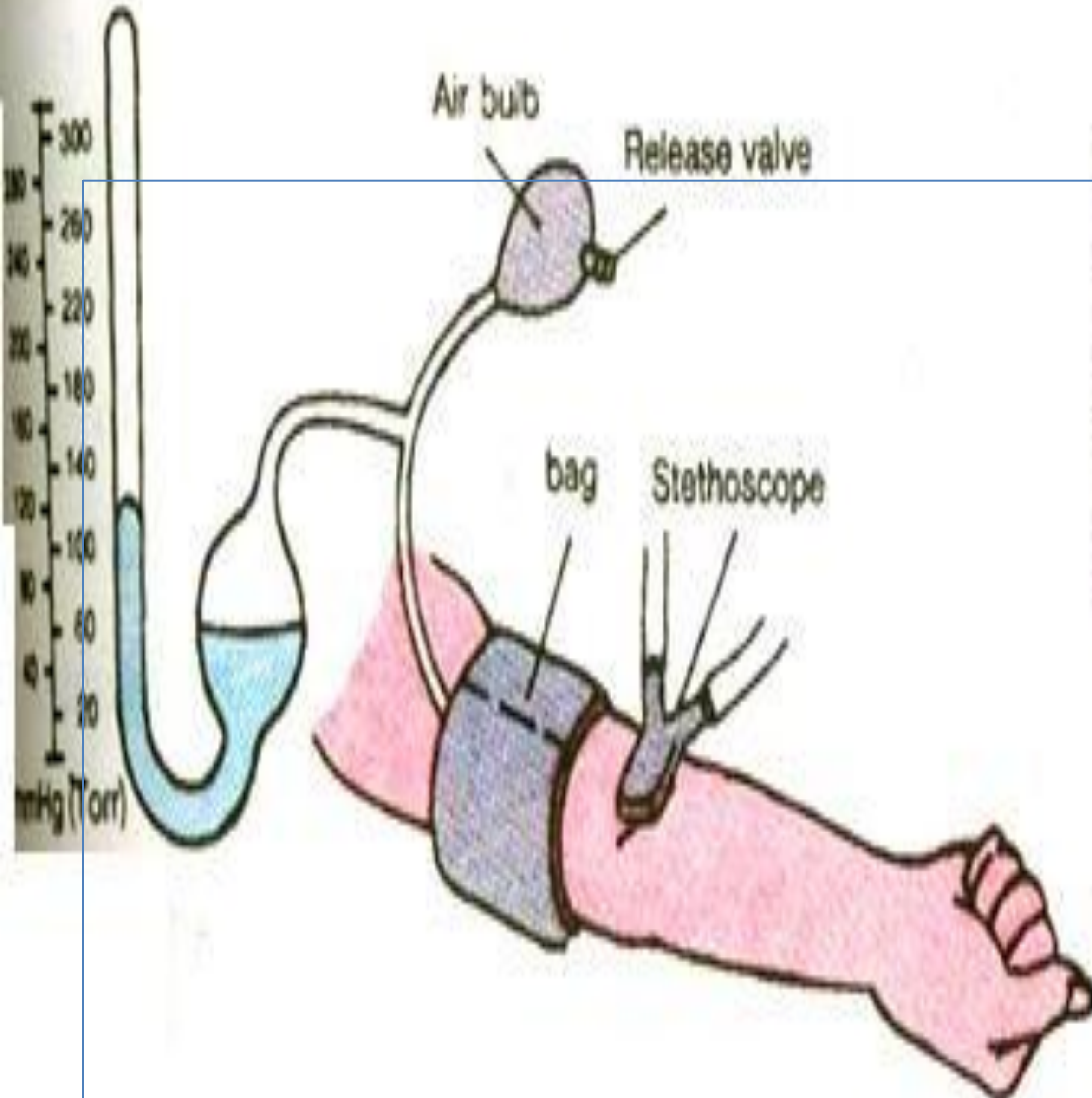
Ans. The principle states that the sum of pressure, the kinetic and potential energy per unit volume for an ideal fluid remains constant at every point of its path. Mathematically, it is described as

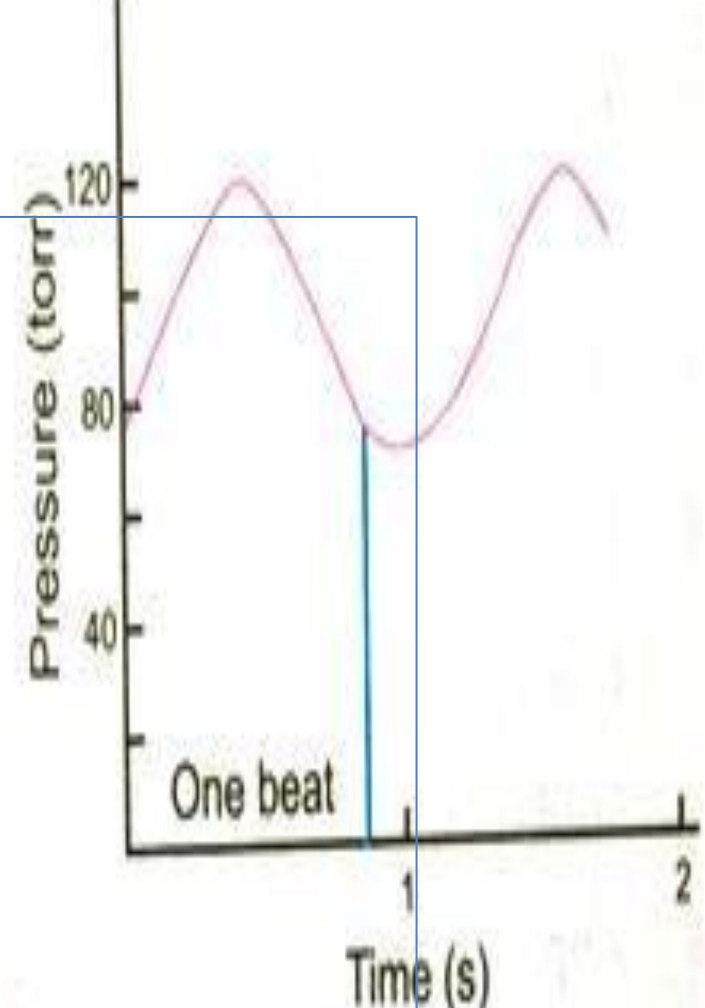
$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$

- Where P is the pressure, v is the velocity and **ρ** is density of the fluid.
- The Bernoulli relation is important in nozzle design and in flow measurements.
- **Q # 6. A person is standing near a fast moving train. Is there any danger that he will fall towards it?**
- **Ans.** When a person is standing near a fast moving train, then the air between them is also fast. According to Bernoulli, where the speed is high, pressure will be low. So the pressure between the person and train will be low as compared to the pressure of side way. So there will be a chance of force acting on the person from high pressure region to the low pressure region and that he may fall towards train.

- **Q # 8. Two row boats moving parallel in the same direction are pulled towards each other. Explain?**
- Ans. When two boats are moving parallel in the same direction, then the water between them is also flowing fast. According to Bernoulli, where the speed is high, pressure will be low. So the pressure between the tow boats decreases as compared to the pressure of side way. So the side way high pressure pushes the two boats towards each other.
- **Q # 9. Explain, how the swing is produced in a fast moving cricket ball?**
- **Ans.** When the cricket ball is thrown in such a way that is spins as well as moves forward, the velocity of the air on one side of the ball increases due to the spins and hence the pressure decreases. This gives an extra curvature to the ball known as swing which deceives opponent player.

- **Q # 10. Explain the working of a carburetor of a motor car using Bernoulli's principal.**
- **Ans.** The petrol tank is attached with Carburetor's pipe through a very small inlet. Air moves very fast through this pipe. As a result, pressure in the pipe decreases as compared to the pressure in the petrol tank which is atmospheric pressure. So the petrol moves from the tank to the air pipe (i.e., from high pressure to low pressure), and a correct mixture of petrol and air reaches the engine.
- **Q # 11. For what position will the maximum blood pressure in the body have the smallest value**
- **(a) Standing up right (b) Sitting (c) Lying Horizontally (d)Standing on one's head?**
- **Ans.** The option (c) is correct. The blood pressure will have the smallest value when a person lying horizontally.
- **Q # 12. In orbiting space station, would the blood pressure in the major arteries in the leg ever be greater than the blood pressure in major arteries in the neck.**
- **Ans.** In an orbiting space station, due to weightlessness, the blood pressure in the major arteries in the leg will be equal to the blood pressure in major arteries in the neck.


Air bulb
Release valve
bag
Stethoscope
mmHg (Torr)


Pressure (torr)
120
80
40
One beat
1
2
Time (s)

- **Q # 32. How the blood pressure of a person is measured?**
- **Ans.** The blood pressure of a person is measured using a device called sphygmomanometer. It consists of an inflatable bag, that is wound around the arm of a patient and external pressure on the arm is increased by inflating the bag. Blood pressure is measured in torr or in mm of Hg.
- When the external pressure applied becomes larger than the systolic pressure, the vessels collapse, cutting off the flow of the blood. Opening the release valve on the ball gradually decrease the external pressure.
- A stethoscope detects the instant at which the external pressure becomes equal to the systolic pressure. At this point, the blood flow through the vessel with very high speed. As a result the flow is initially turbulent.
- As the pressure drops, the external pressure eventually equals the diastolic pressure. The flow of the blood switches from turbulent to laminar, and gurgle in the stethoscope disappears. This is the signal to record the diastolic pressure.