Linear Motion
And Projectile

Objectives

Describing Motion with

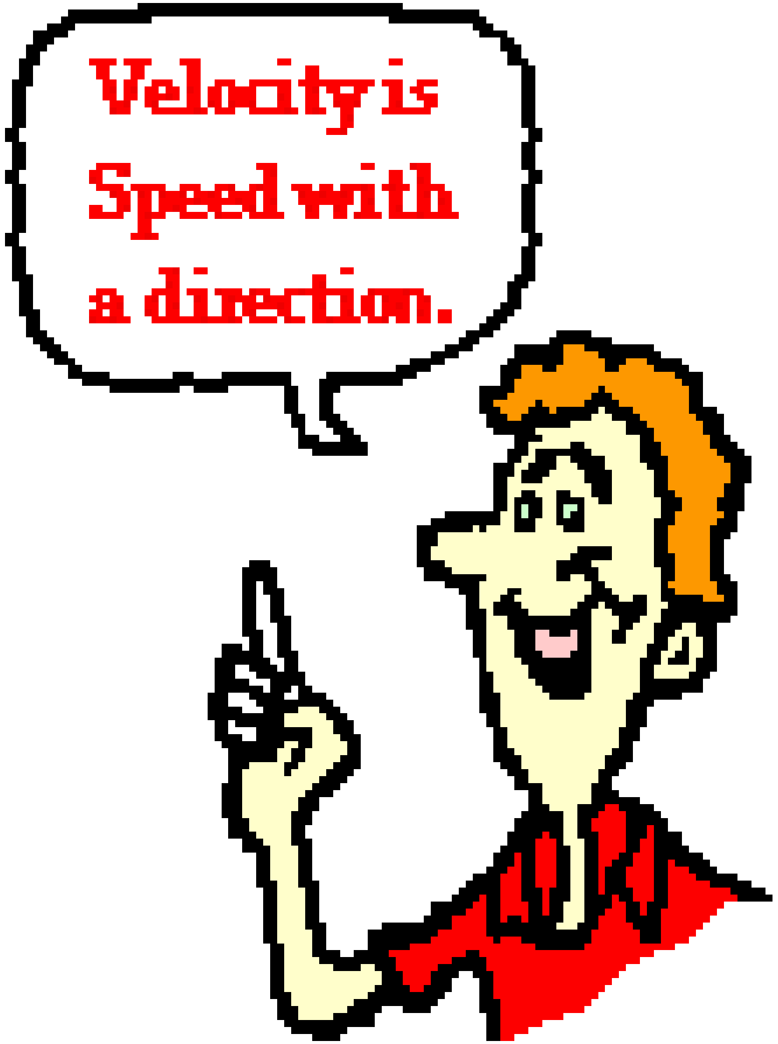
- Displacement vs. Time Graphs
- Velocity vs. Time Graphs
- Meaning of Slope for a s-t Graph
- Determining the Slope on a s-t Graph
- Meaning of Slope for a v-t Graph
- Determining the Slope on a v-t Graph
- Determining the Area on a v-t Graph

Speed and Velocity

Speed is a scalar quantity

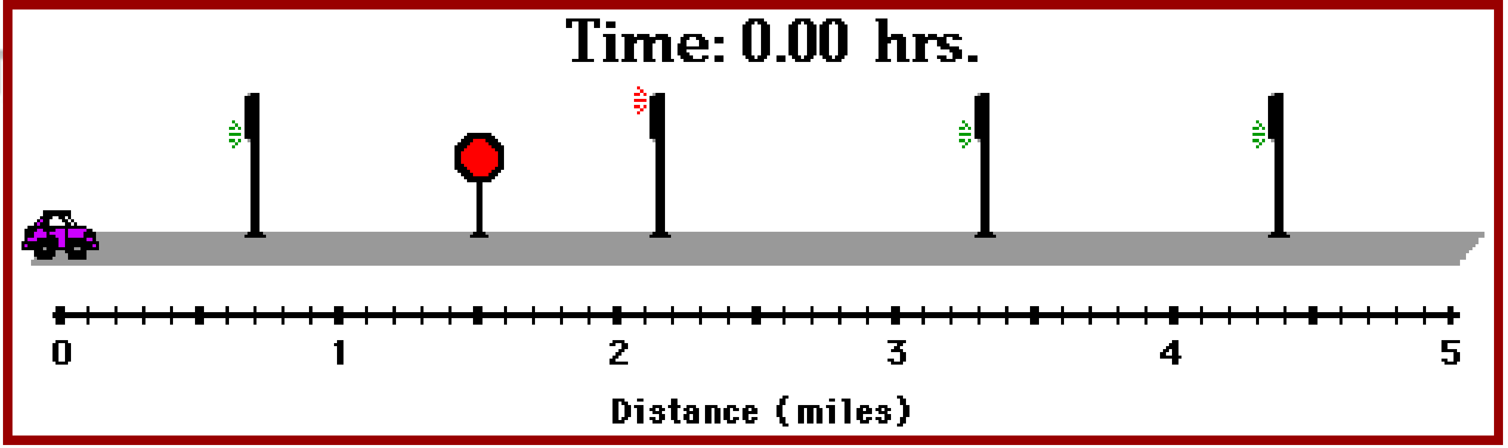
Velocity is a vector quantity

So an airplane moving towards the west with a speed of 300 mi/hr has a velocity of 300 mi/hr, west. Note that speed has no direction (it is a scalar) and the velocity at any instant is simply the speed value with a direction.



Average vs. Instantaneous Speed

During a typical trip to school, your car will undergo a series of changes in its speed. If you were to inspect the speedometer readings at regular intervals, you would notice that it changes often. The speedometer of a car reveals information about the instantaneous speed of your car. It shows your speed at a particular instant in time.



The instantaneous speed of an object is not to be confused with the average speed. Average speed is a measure of the distance traveled in a given period of time; it is sometimes referred to as the distance per time ratio. Suppose that during your trip to school, you traveled a distance of 5 miles and the trip lasted 0.2 hours (12 minutes). The average speed of your car could be determined as

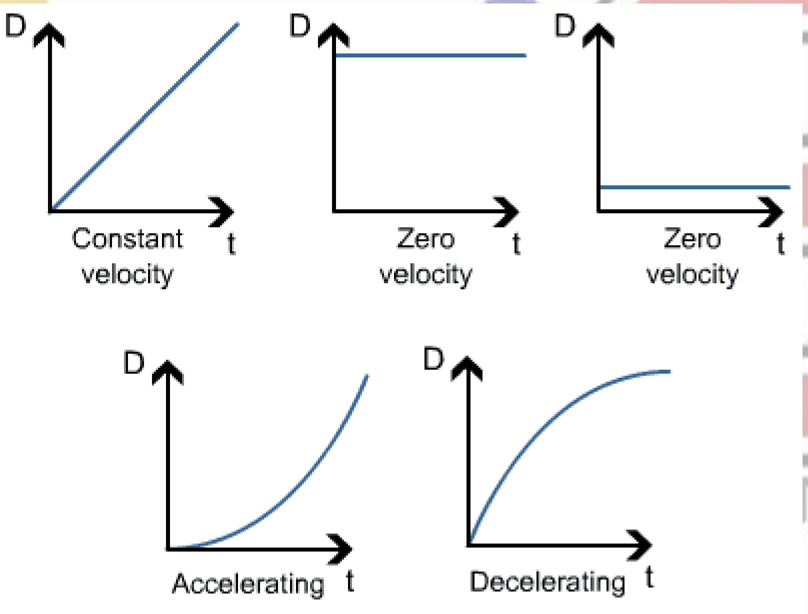
$$\text{Ave. Speed} = \frac{5 \text{ miles}}{0.2 \text{ hours}} = 25 \text{ miles/hour}$$

- Q: While on vacation, Lisa Carr traveled a total distance of 440 miles. Her trip took 8 hours. What was her average speed?

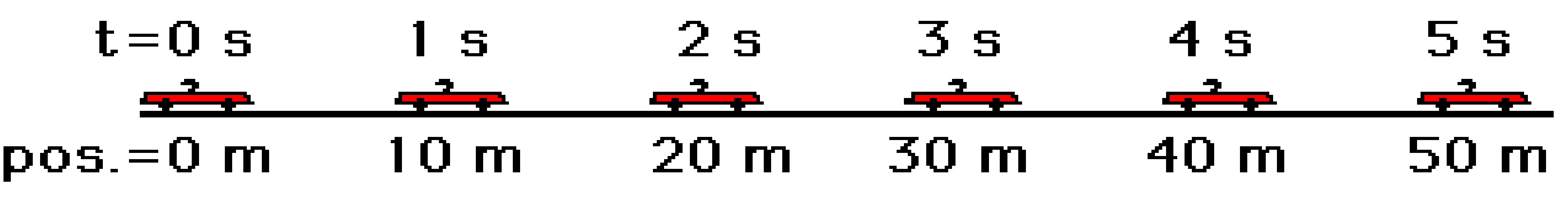
- To compute her average speed, we simply divide the distance of travel by the time of travel.

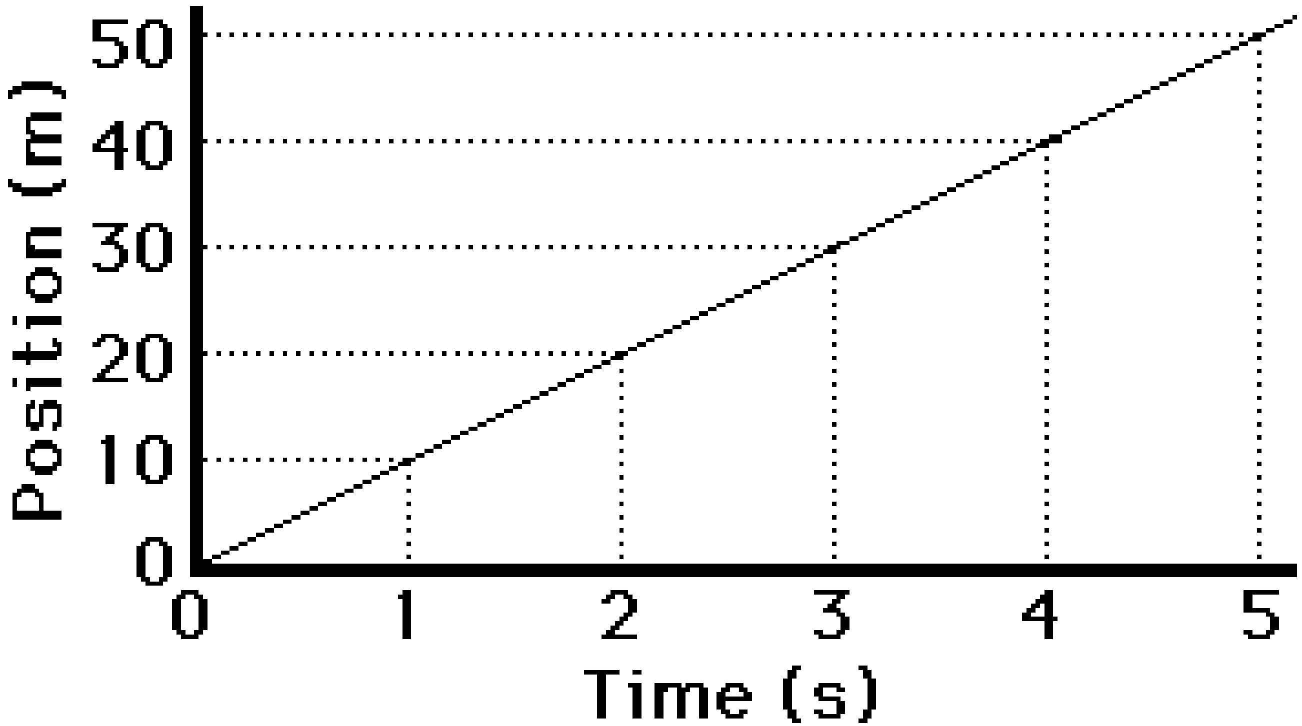
$$v = \frac{d}{t} = \frac{440\ \text{mi}}{8\ \text{hr}} = 55\ \text{mi/hr}$$

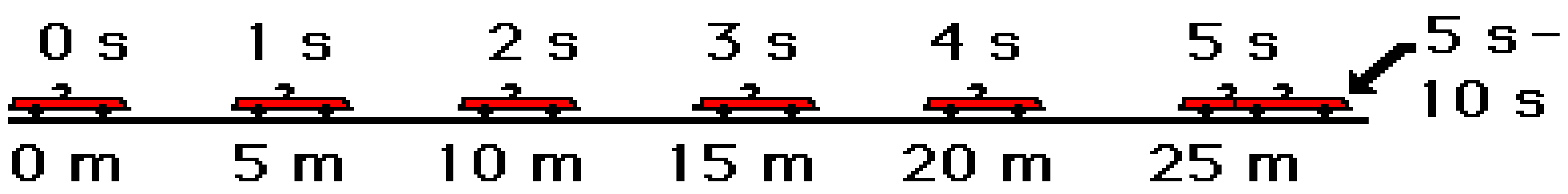
- Lisa averaged a speed of 55 miles per hour. She may not have been traveling at a constant speed of 55 mi/hr. She undoubtedly, was stopped at some instant in time (perhaps for a bathroom break or for lunch) and she probably was going 65 mi/hr at other instants in time. Yet, she averaged a speed of 55 miles per hour. The above formula represents a shortcut method of determining the average speed of an object.

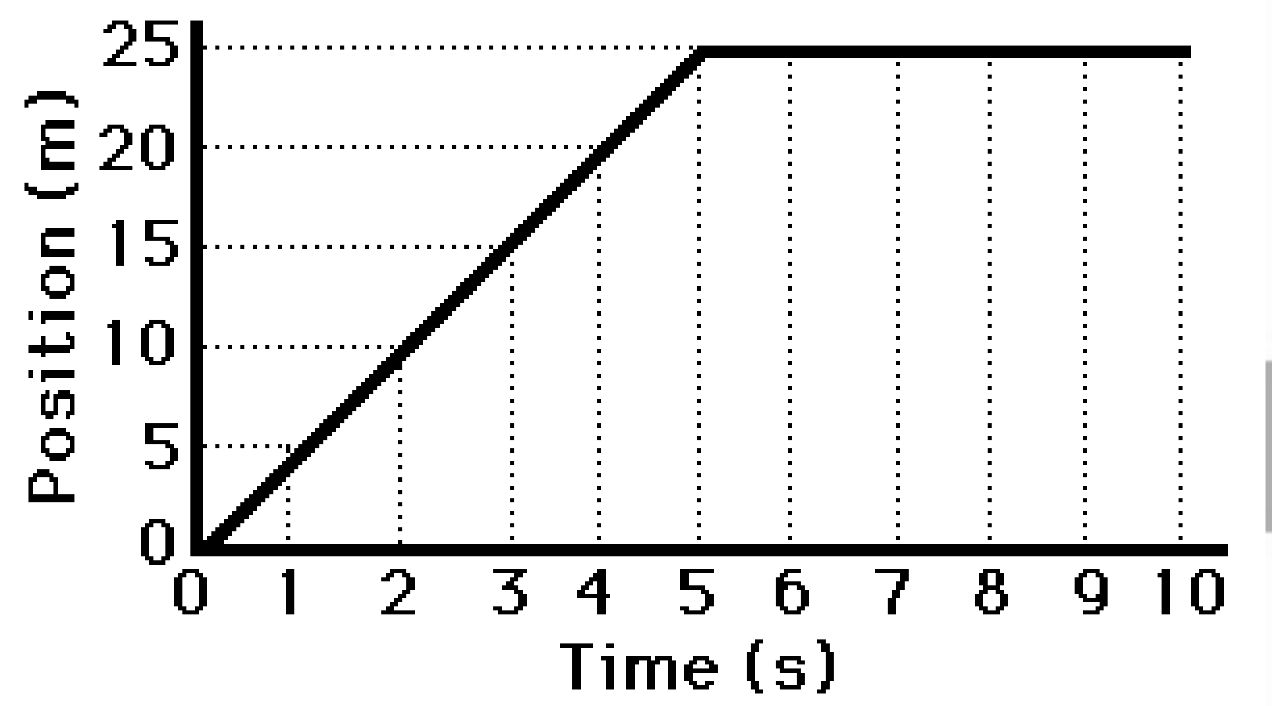

D
t
Constant velocity
D
t
Zero velocity
D
t
Zero velocity
D
t
Accelerating
D
t
Decelerating

The Meaning of Shape for a p-t Graph

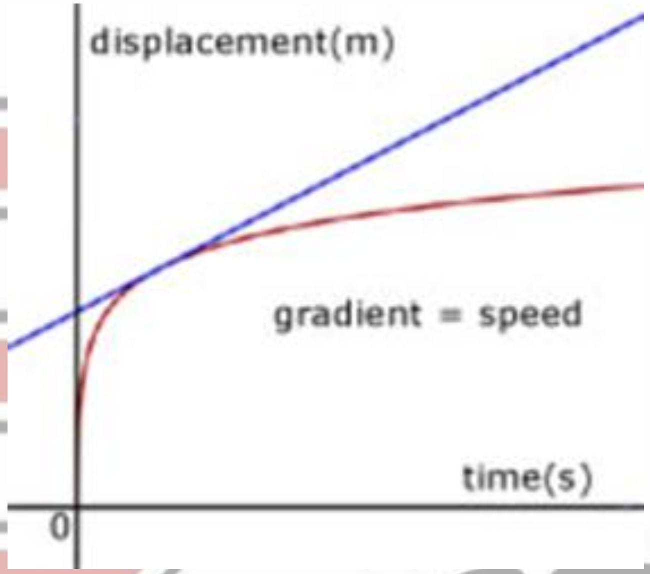




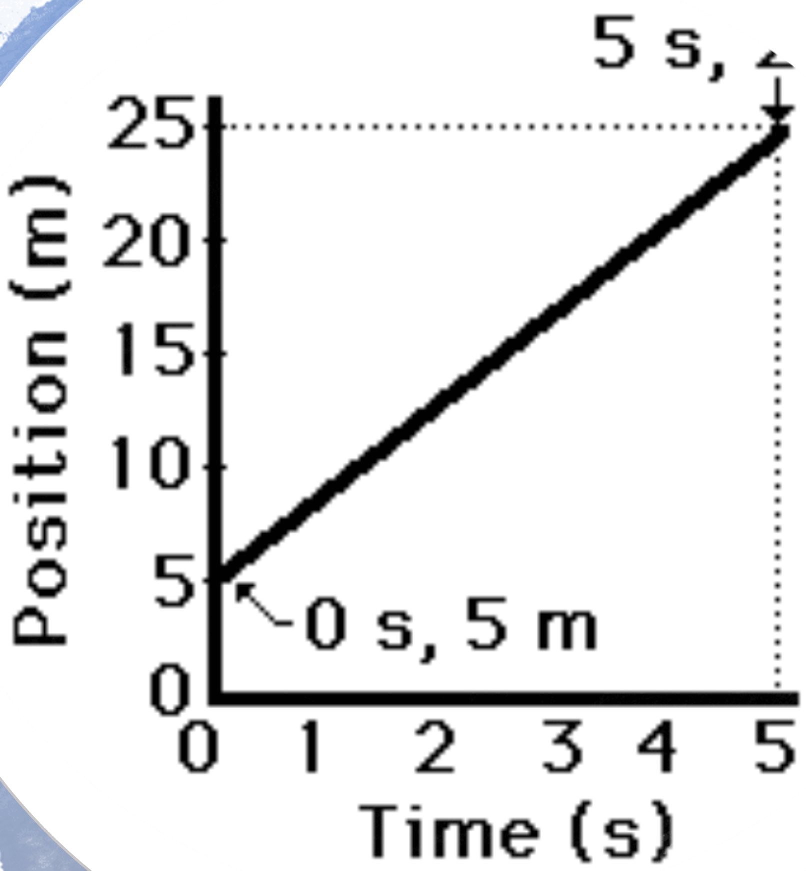

0 s
1 s
2 s
3 s
4 s
5 s
5 s–10 s
0 m
5 m
10 m
15 m
20 m
25 m


Position (m)
0
5
10
15
20
25
0
1
2
3
4
5
6
7
8
9
10
Time (s)

Displacement-Time graphs



For a displacement-time graph, the gradient at a point is equal to the velocity (speed) .



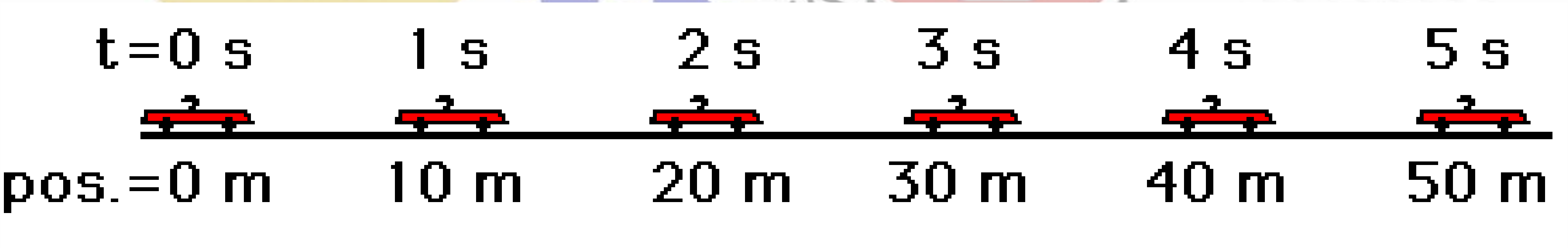
1. Determine the velocity (i.e., slope) of the object as portrayed by the graph below.

- The velocity (i.e., slope) is **4 m/s**. If you think the slope is 5 m/s, then you're making a common mistake. You are picking one point (probably 5 s, 25 m) and dividing y/x. Instead you must pick two points (as discussed in this part of the lesson) and divide the change in y by the change in x.

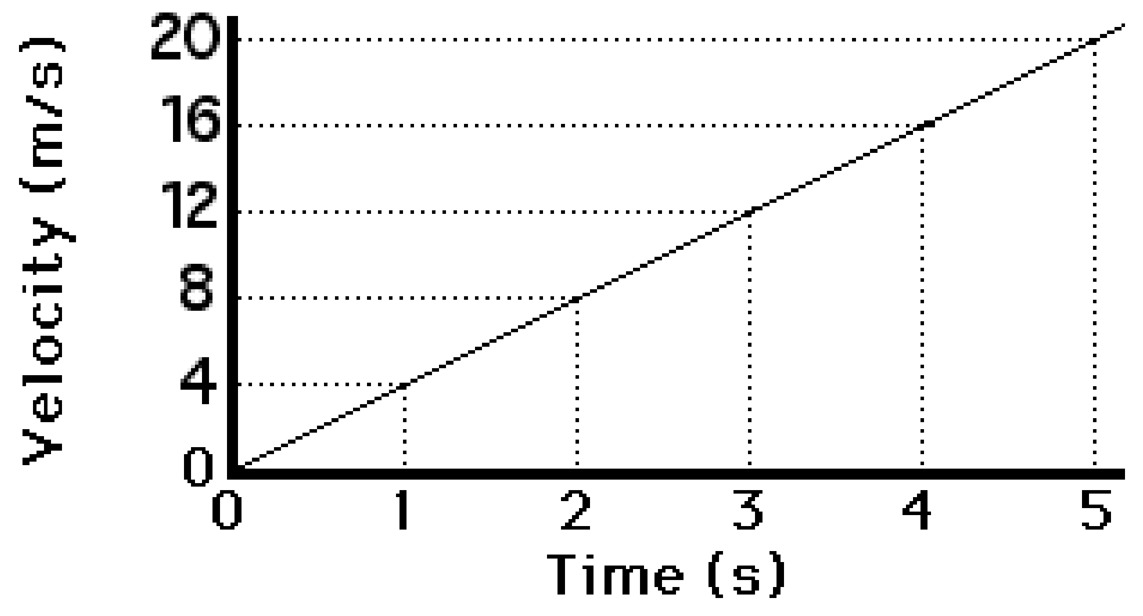
velocity versus time graphs

- **Constant Velocity versus Changing Velocity**

Consider a car moving with a constant, rightward (+) velocity - say of +10 m/s. As learned in an earlier lesson, a car moving with a constant velocity is a car with zero acceleration.

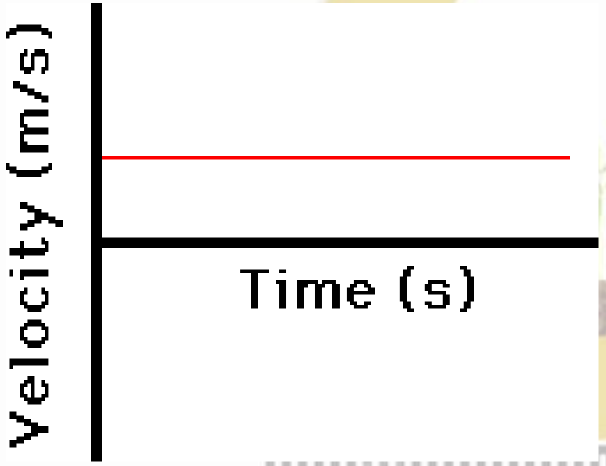


The slope of the line is positive, corresponding to the positive acceleration. Furthermore, only positive velocity values are plotted, corresponding to a motion with positive velocity.

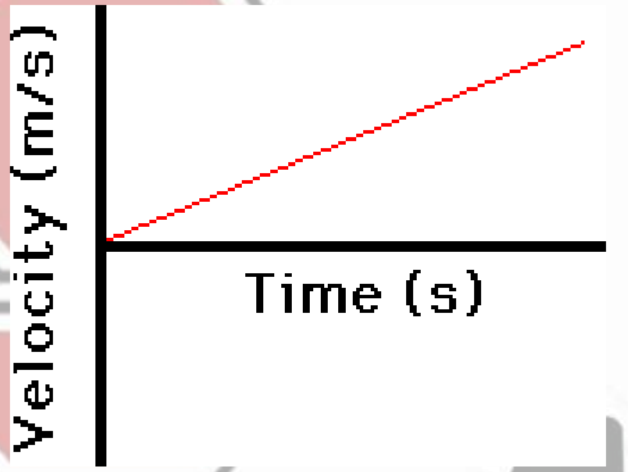


The velocity vs. time graphs for the two types of motion - constant velocity and changing velocity (acceleration)

Positive Velocity
Zero Acceleration



Positive Velocity
Positive Acceleration



The slope of the line on a velocity-time graph reveals useful information about the acceleration of the object. If the acceleration is zero, then the slope is zero (i.e., a horizontal line). If the acceleration is positive, then the slope is positive (i.e., an upward sloping line). If the acceleration is negative, then the slope is negative (i.e., a downward sloping line).

Acceleration

Acceleration is a vector quantity that is defined as the rate at which an object changes its velocity. An object is accelerating if it is changing its velocity.

Sports announcers will occasionally say that a person is accelerating if he/she is moving fast. Yet acceleration has nothing to do with going fast. A person can be moving very fast and still not be accelerating. Acceleration has to do with changing how fast an object is moving. If an object is not changing its velocity, then the object is not accelerating.

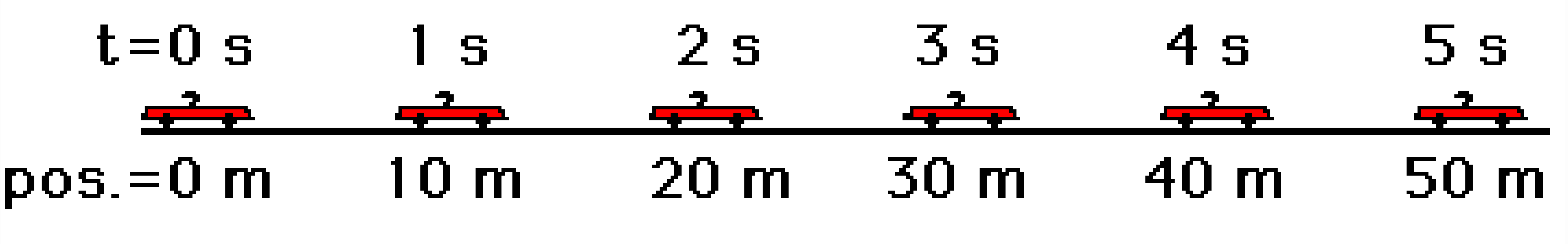
Sometimes an accelerating object will change its velocity by the same amount each second. As mentioned in the previous paragraph, the data table above show an object changing its velocity by 10 m/s in each consecutive second. This is referred to as a constant acceleration since the velocity is changing by a constant amount each second.

An object with a constant acceleration should not be confused with an object with a constant velocity. Don't be fooled! If an object is changing its velocity -whether by a constant amount or a varying amount - then it is an accelerating object. And an object with a constant velocity is not accelerating.

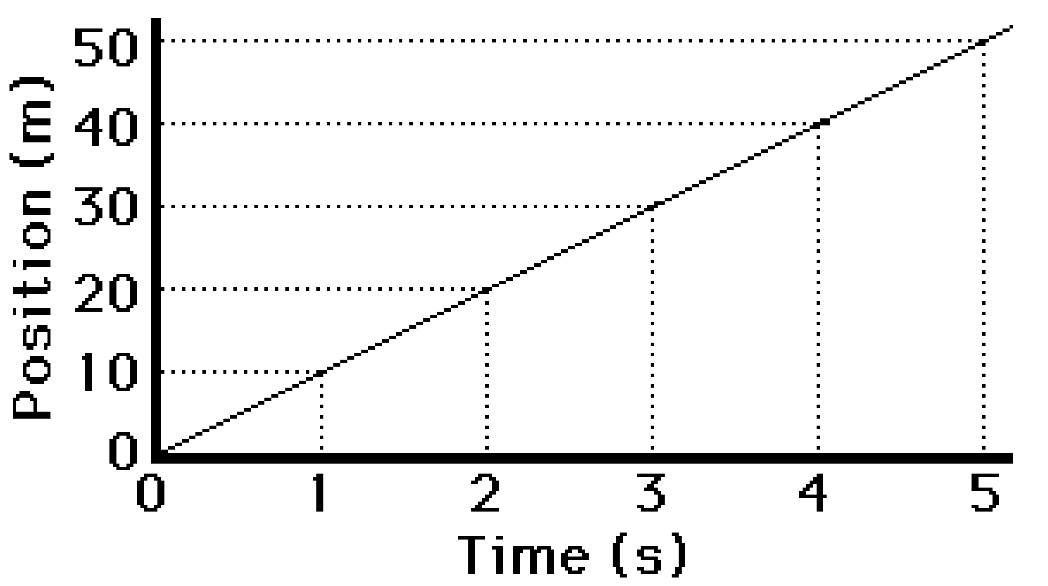
Time	Velocity
0 s	0 m/s, N
1 s	10 m/s, N
2 s	20 m/s, N
3 s	30 m/s, N
4 s	40 m/s, N
5 s	50 m/s, N

Contrasting a Constant and a Changing Velocity

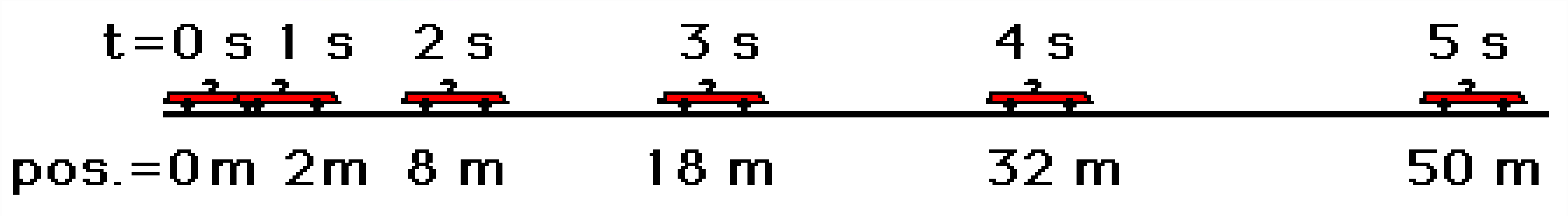
To begin, consider a car moving with a constant, rightward (+) velocity - say of +10 m/s.



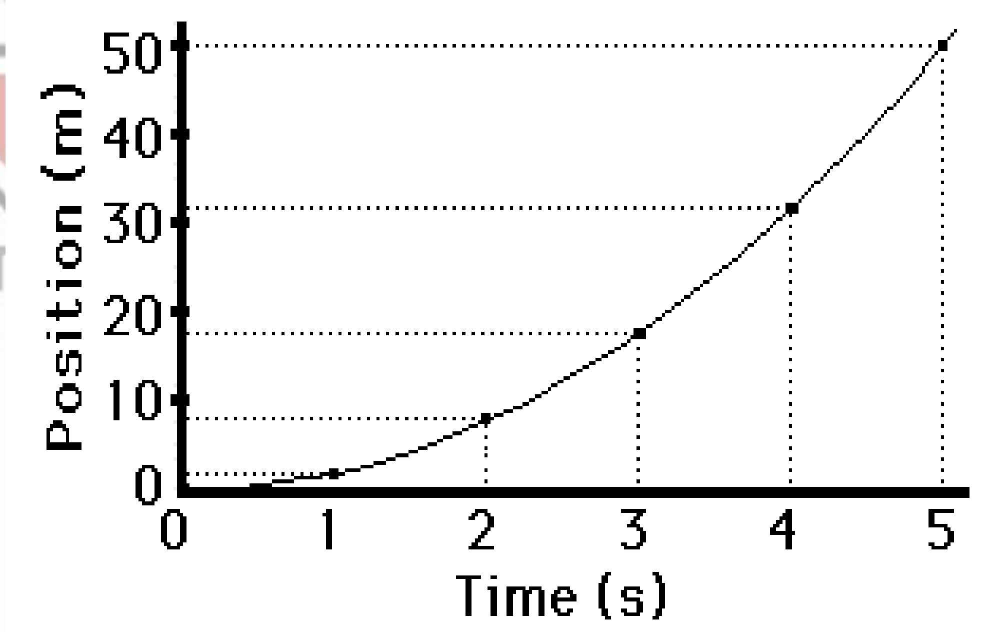
A constant, positive velocity results in a line of constant and positive slope when plotted as a position-time graph.



Now consider a car moving with a rightward (+), changing velocity - that is, a car that is moving rightward but speeding up or *accelerating*.

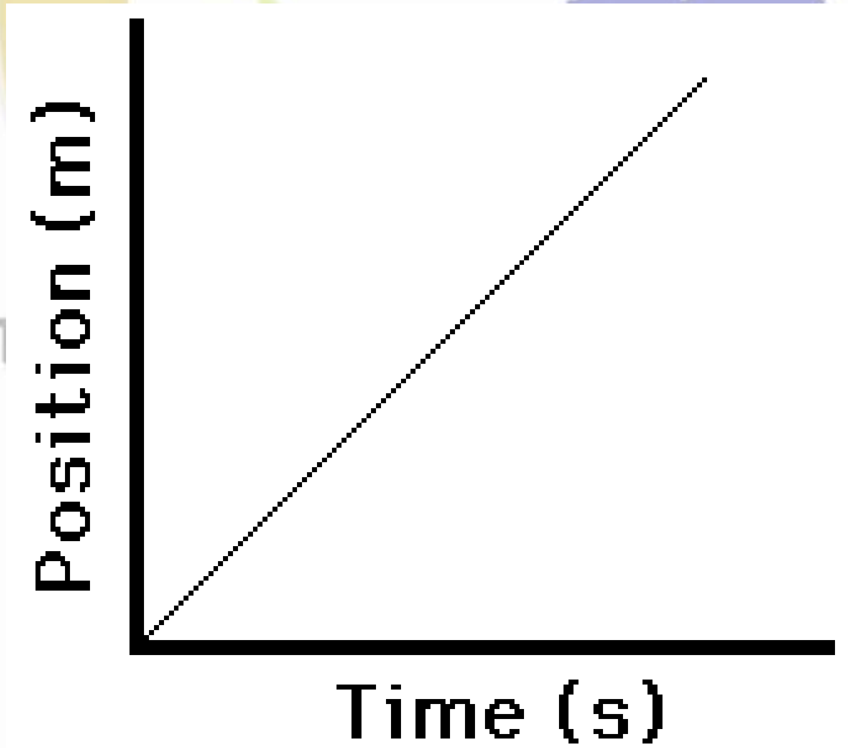


If the position-time data for such a car were graphed, then the resulting graph would look like the graph at the right. Note that a motion described as a changing, positive velocity results in a line of changing and positive slope when plotted as a position-time graph.

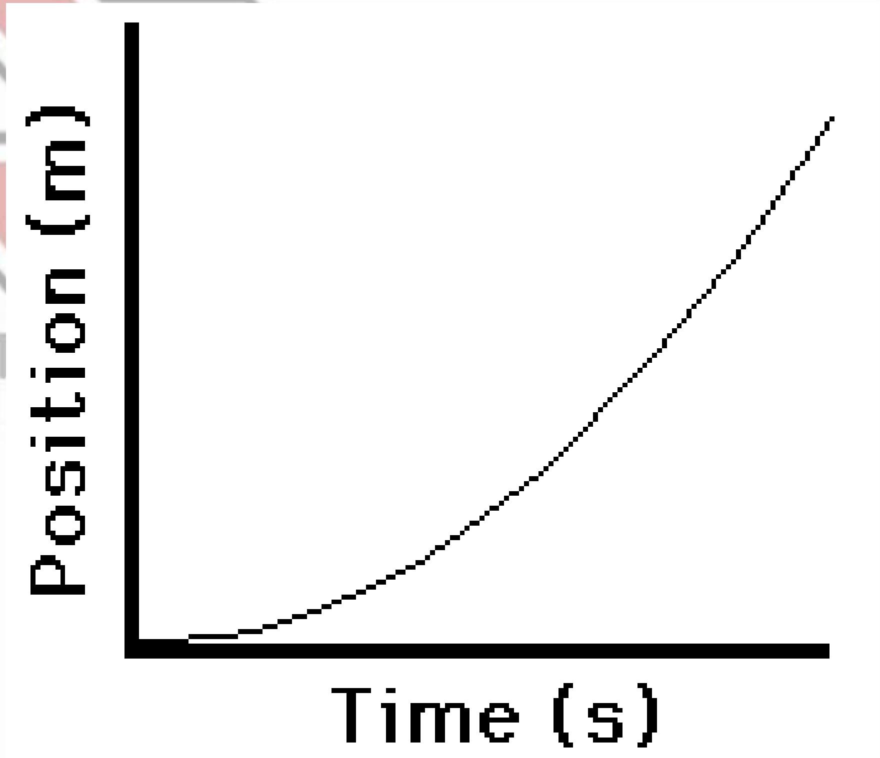


The position vs. time graphs for the two types of motion - constant velocity and changing velocity (acceleration) - are depicted as follows.

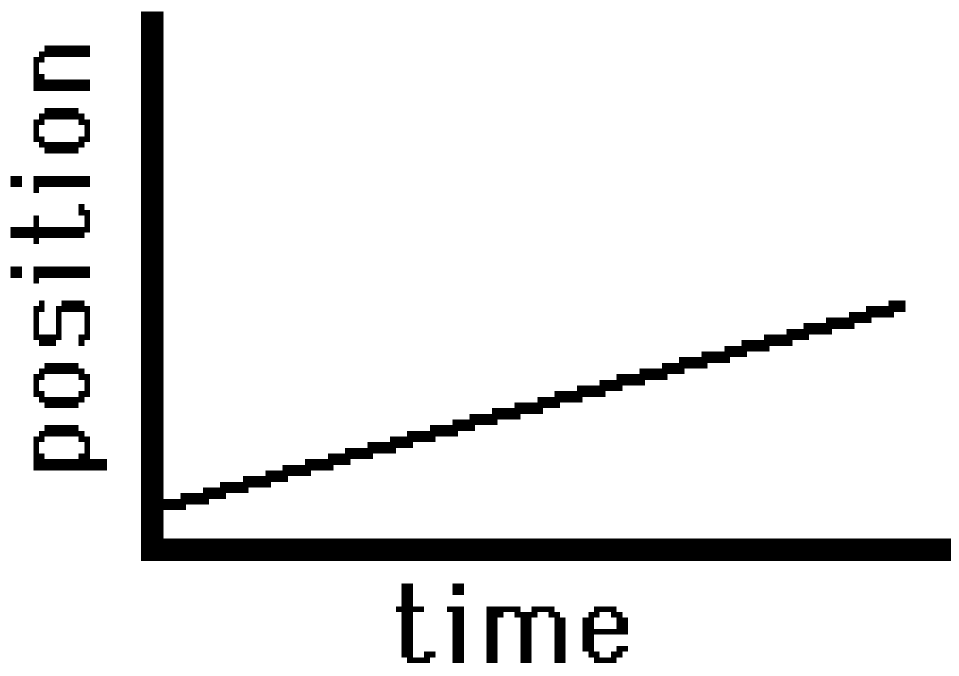
Constant Velocity
Positive Velocity



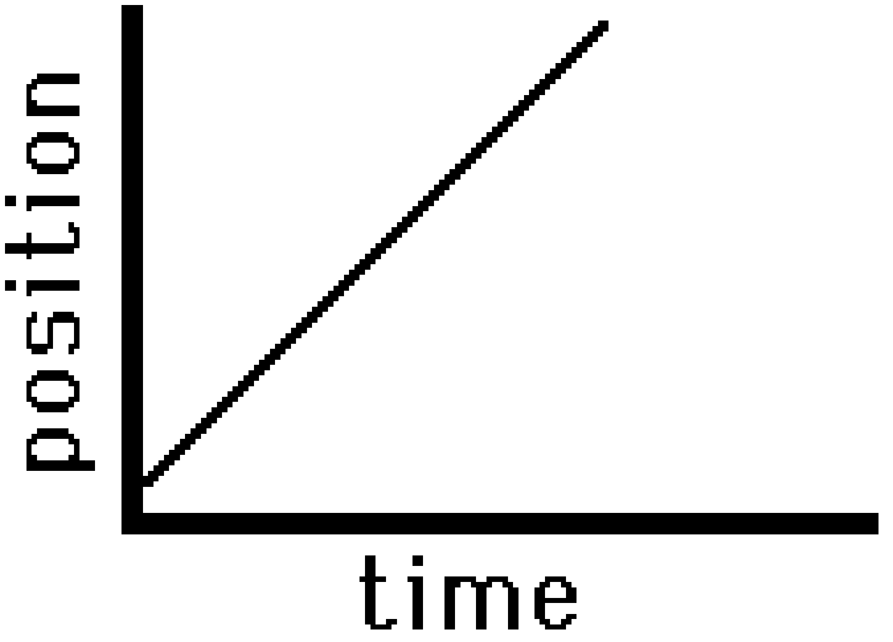
Positive Velocity
Changing Velocity (acceleration)



Slow, Rightward(+)
Constant Velocity

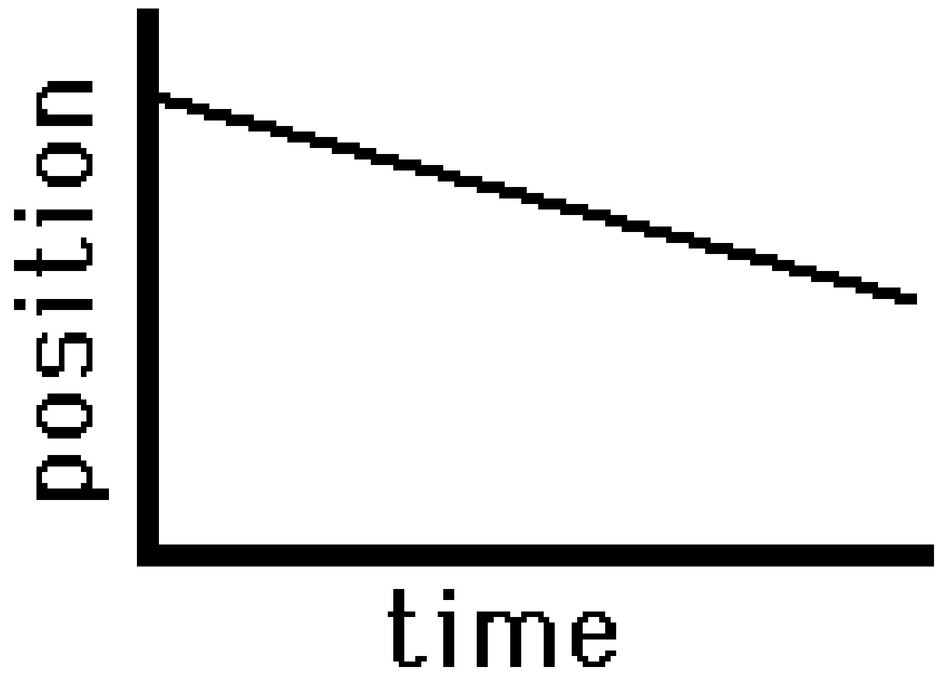


Fast, Rightward(+)
Constant Velocity

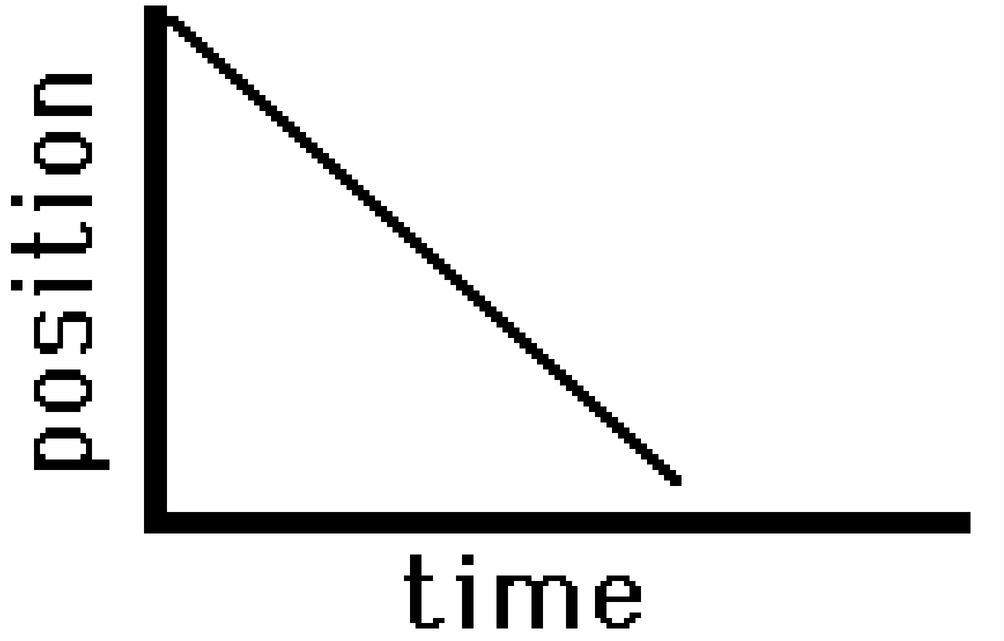


This larger slope is indicative of a larger velocity.

Slow, Leftward(-)
Constant Velocity



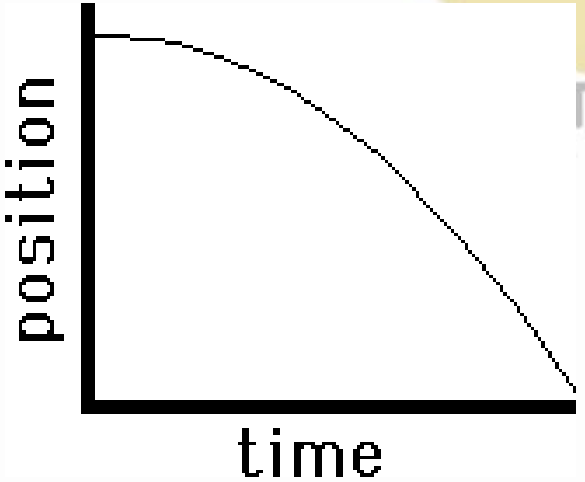
Fast, Leftward(-)
Constant Velocity



The graph on the left is representative of an object that is moving with a negative velocity (as denoted by the negative slope), a constant velocity (as denoted by the constant slope) and a small velocity (as denoted by the small slope).

Representing an Accelerated Motion

- Curved lines have changing slope; they may start with a very small slope and begin curving sharply (either upwards or downwards) towards a large slope. In either case, the curved line of changing slope is a sign of accelerated motion (i.e., changing velocity). Applying the principle of slope to the graph on the left, one would conclude that the object depicted by the graph is moving with a negative velocity (since the slope is negative).

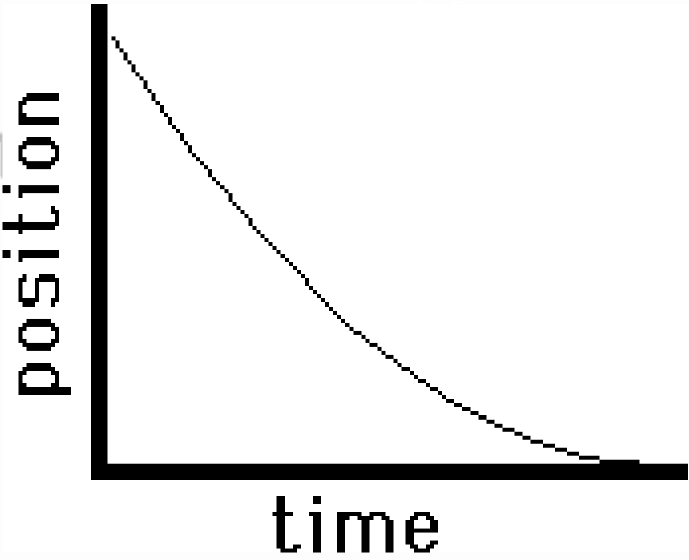


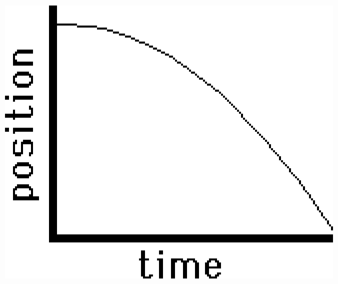
Negative (-) Velocity

Slow to Fast

Leftward (-) Velocity

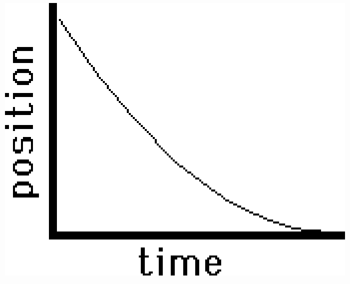
Fast to Slow





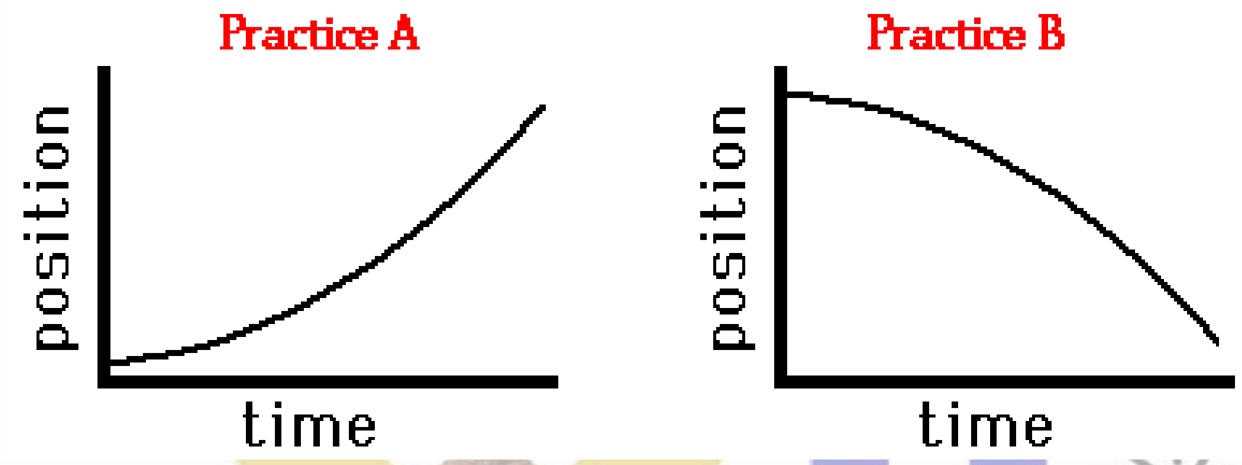
Negative (-) Velocity
Slow to Fast

Leftward (-) Velocity
Fast to Slow



- Furthermore, the object is starting with a small velocity (the slope starts out with a small slope) and finishes with a large velocity (the slope becomes large). That would mean that this object is moving in the negative direction and speeding up (the small velocity turns into a larger velocity). This is an example of negative acceleration - moving in the negative direction and speeding up. The graph on the right also depicts an object with negative velocity (since there is a negative slope). The object begins with a high velocity (the slope is initially large) and finishes with a small velocity (since the slope becomes smaller). So this object is moving in the negative direction and slowing down. This is an example of positive acceleration.

- Use the principle of slope to describe the motion of the objects depicted by the two plots below. In your description, be sure to include such information as the direction of the velocity vector (i.e., positive or negative), whether there is a constant velocity or an acceleration, and whether the object is moving slow, fast, from slow to fast or from fast to slow. Be complete in your description.



Answer

- The object has a positive or rightward velocity (note the + slope). The object has a changing velocity (note the changing slope); it is accelerating. The object is moving from slow to fast since the slope changes from small big.

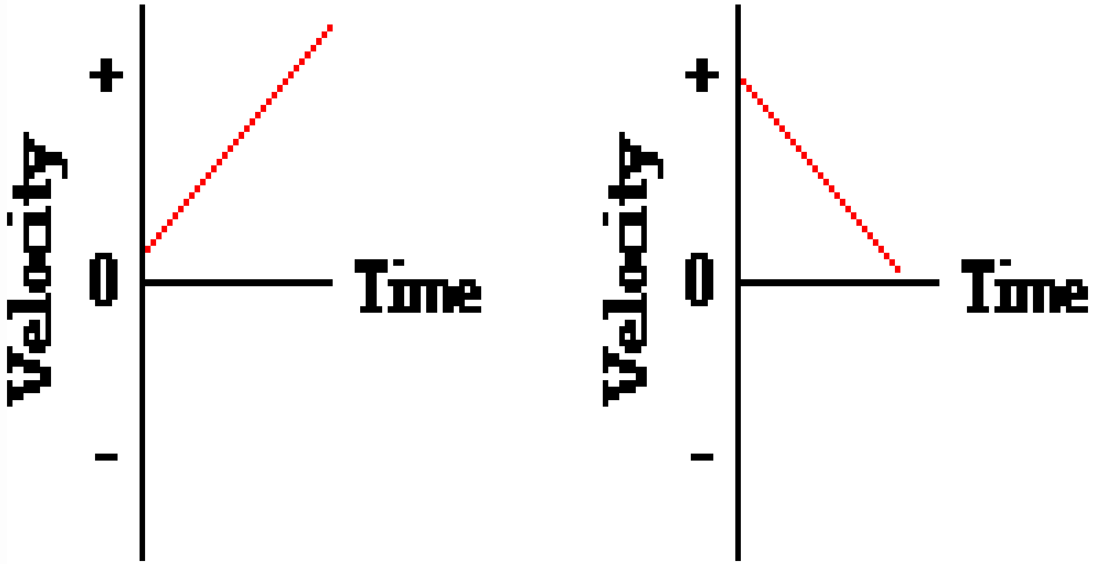
- The object has a negative or leftward velocity (note the - slope). The object has a changing velocity (note the changing slope); it has an acceleration. The object is moving from slow to fast since the slope changes from small to big.

Positive Velocity versus Negative Velocity

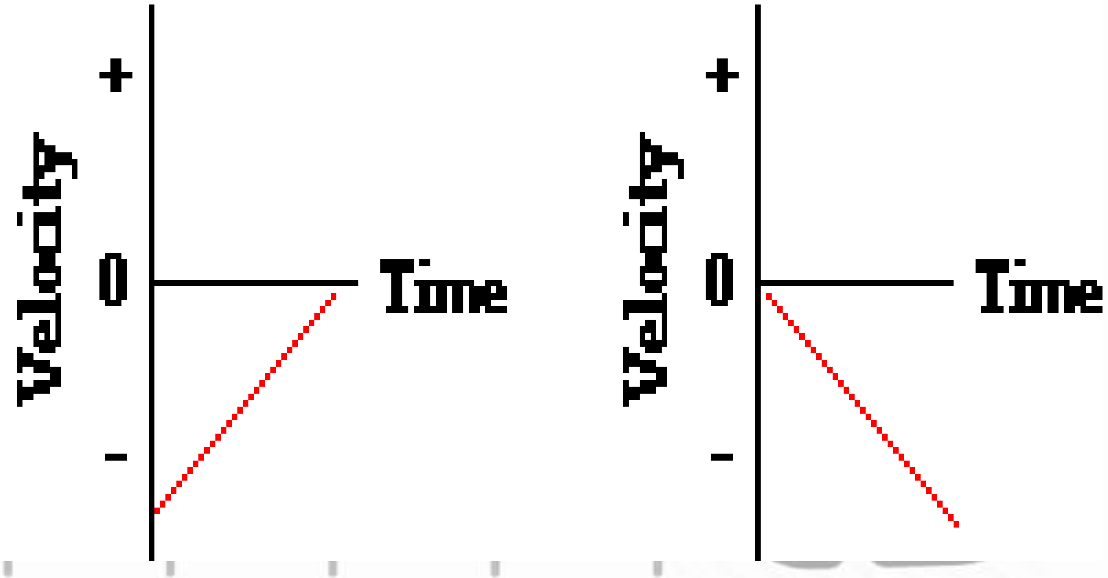
- The answers to these questions hinge on one's ability to read a graph. Since the graph is a velocity-time graph, the velocity would be positive whenever the line lies in the positive region (above the x-axis) of the graph. Similarly, the velocity would be negative whenever the line lies in the negative region (below the x-axis) of the graph.

A positive velocity means the object is moving in the positive direction; and a negative velocity means the object is moving in the negative direction. So one knows an object is moving in the positive direction if the line is located in the positive region of the graph (whether it is sloping up or sloping down). And one knows that an object is moving in the negative direction if the line is located in the negative region of the graph (whether it is sloping up or sloping down). And finally, if a line crosses over the x-axis from the positive region to the negative region of the graph (or vice versa), then the object has changed directions.

These objects are moving with a positive velocity.

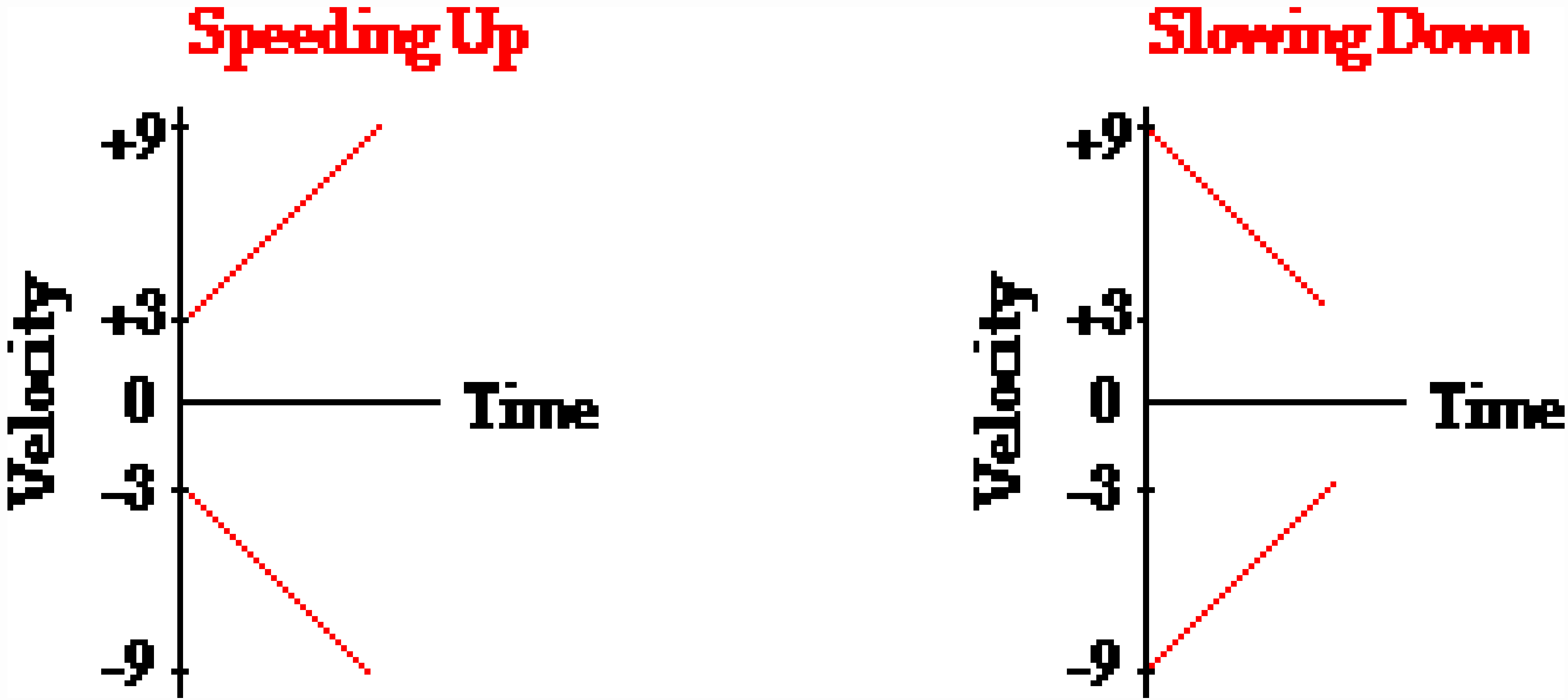


These objects are moving with a negative velocity.



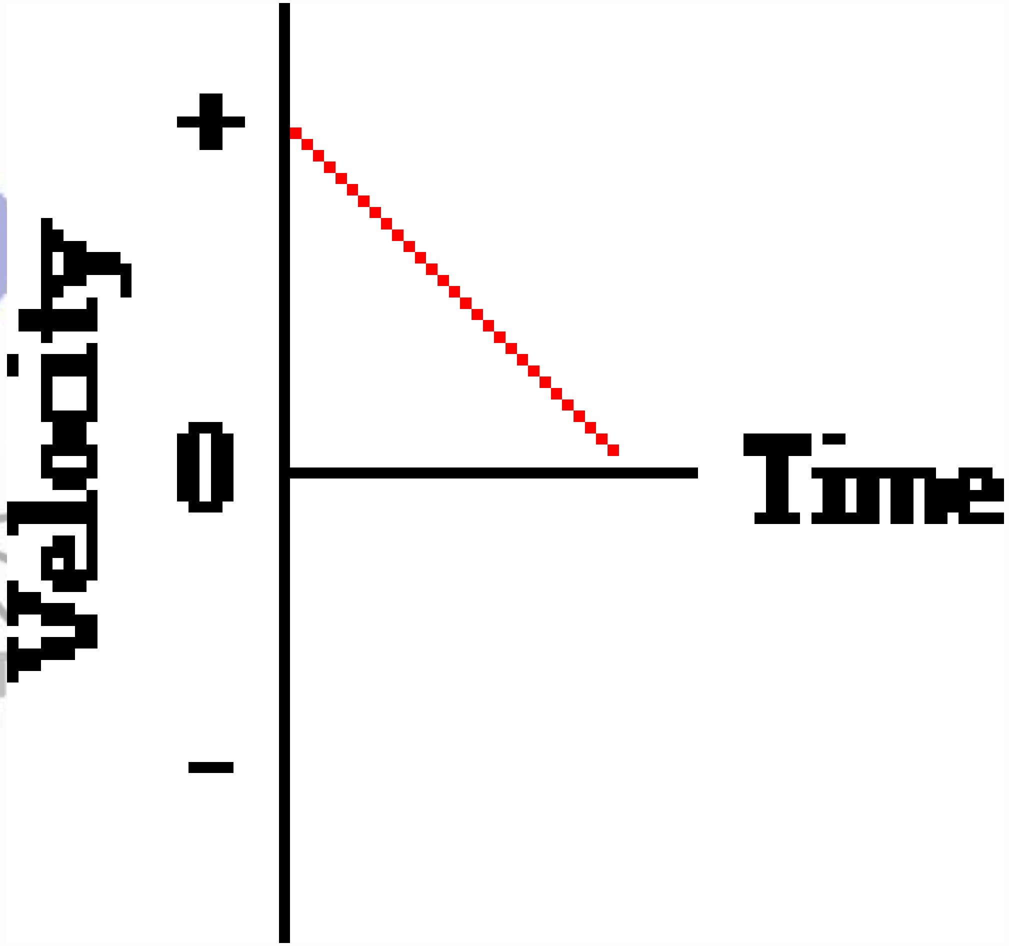
Speeding Up versus Slowing Down

Now how can one tell if the object is speeding up or slowing down? Speeding up means that the magnitude (or numerical value) of the velocity is getting large. For instance, an object with a velocity changing from +3 m/s to + 9 m/s is speeding up. Similarly, an object with a velocity changing from -3 m/s to -9 m/s is also speeding up. In each case, the magnitude of the velocity (the number itself, not the sign or direction) is increasing; the speed is getting bigger. Given this fact, one would believe that an object is speeding up if the line on a velocity-time graph is changing from near the 0-velocity point to a location further away from the 0-velocity point. That is, if the line is getting further away from the x-axis (the 0-velocity point), then the object is speeding up. And conversely, if the line is approaching the x-axis, then the object is slowing down.


Speeding Up
Slowing Down
Velocity
+9
+3
0
-3
-9
Time
Velocity
+9
+3
0
-3
-9
Time

Consider the graph at the right. The object whose motion is represented by this graph is ... (include all that are true):

1. moving in the positive direction.
2. moving with a constant velocity.
3. moving with a negative velocity.
4. slowing down.
5. changing directions.
6. speeding up.
7. moving with a positive acceleration.
8. moving with a constant acceleration.



Answers:

a: TRUE since the line is in the positive region of the graph.

b. FALSE since there is an acceleration (i.e., a changing velocity).

c. FALSE since a negative velocity would be a line in the negative region (i.e., below the horizontal axis).

d. TRUE since the line is approaching the 0-velocity level (the x-axis).

e. FALSE since the line never crosses the axis.

f. FALSE since the line is not moving away from x-axis.

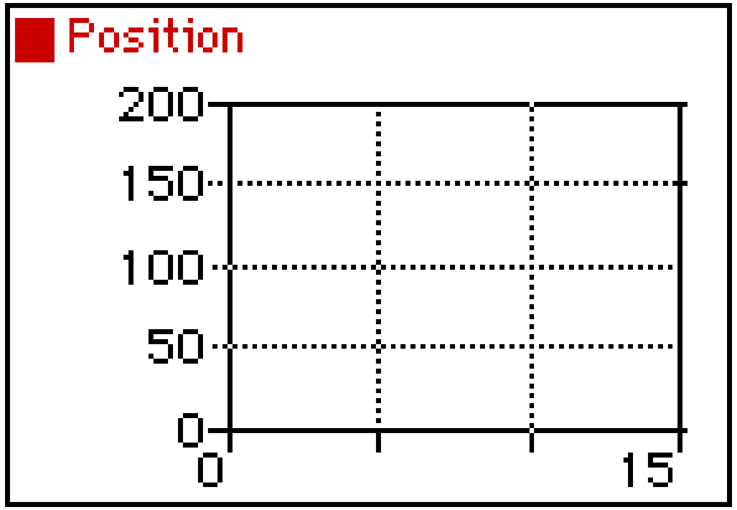
g. FALSE since the line has a negative or downward slope.

h. TRUE since the line is straight (i.e, has a constant slope).

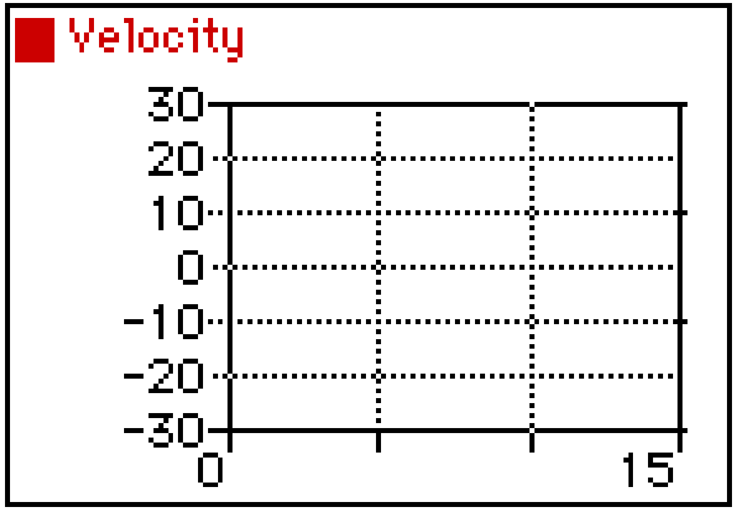
Constant Positive Velocity

Observe that the object below moves with a constant velocity in the positive direction. The *dot diagram* shows that each consecutive dot is the same distance apart (i.e., a constant velocity). The position-time graph shows that the slope is both constant (meaning a constant velocity) and positive (meaning a positive velocity). The velocity-time graph shows a horizontal line with zero slope (meaning that there is zero acceleration); the line is located in the positive region of the graph (corresponding to a positive velocity). The acceleration-time graph shows a horizontal line at the zero mark (meaning zero acceleration).

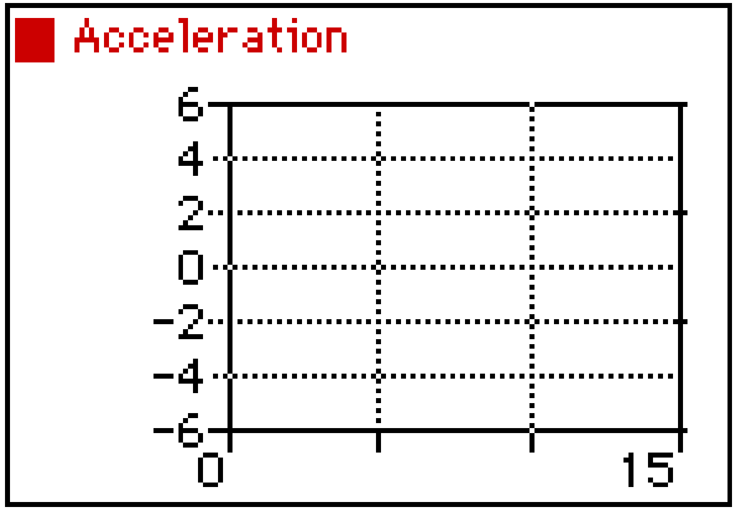
Position-Time Graph



Velocity-Time Graph



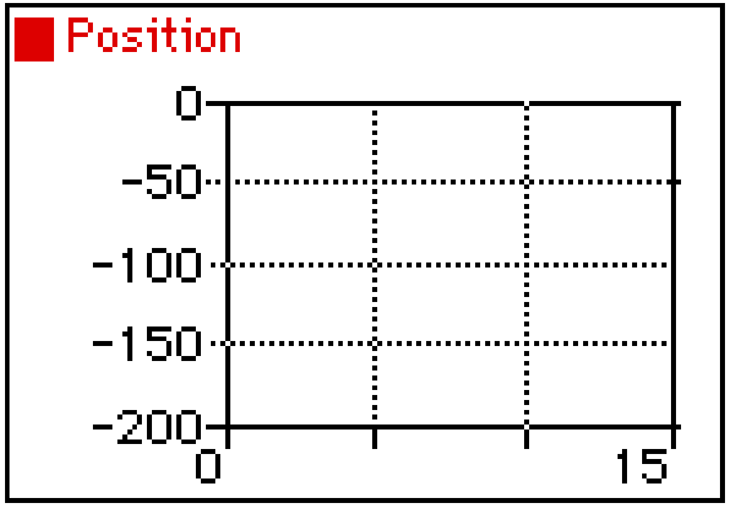
Acceleration-Time Graph



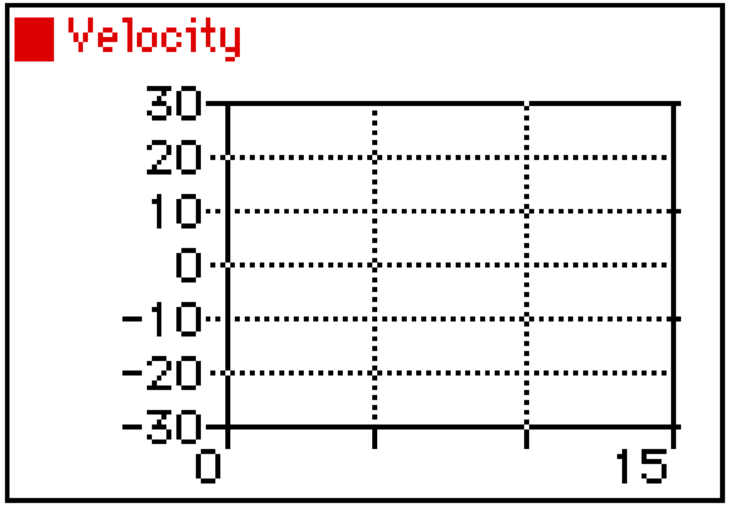
Constant Negative Velocity

Observe that the object below moves with a constant velocity in the negative direction. The *dot diagram* shows that each consecutive dot is the same distance apart (i.e., a constant velocity). The position-time graph shows that the slope is both constant (meaning a constant velocity) and negative (meaning a negative velocity). The velocity-time graph shows a horizontal line with zero slope (meaning that there is zero acceleration); the line is located in the negative region of the graph (corresponding to a negative velocity). The acceleration-time graph shows a horizontal line at the zero mark (meaning zero acceleration).

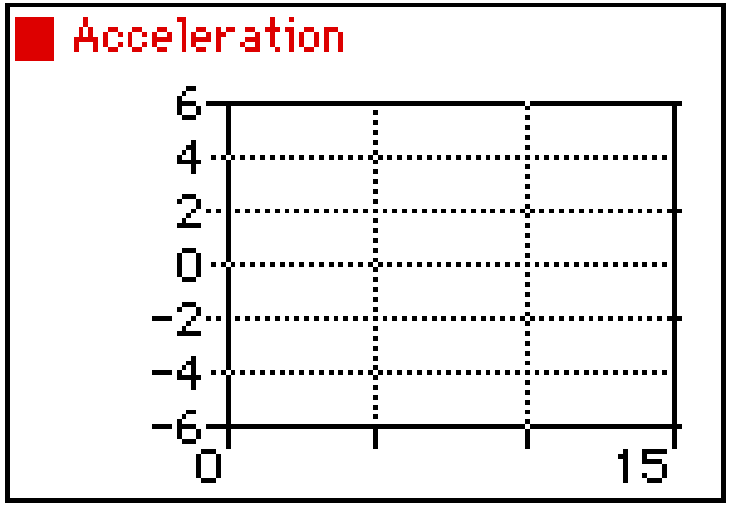
Position-Time Graph



Velocity-Time Graph

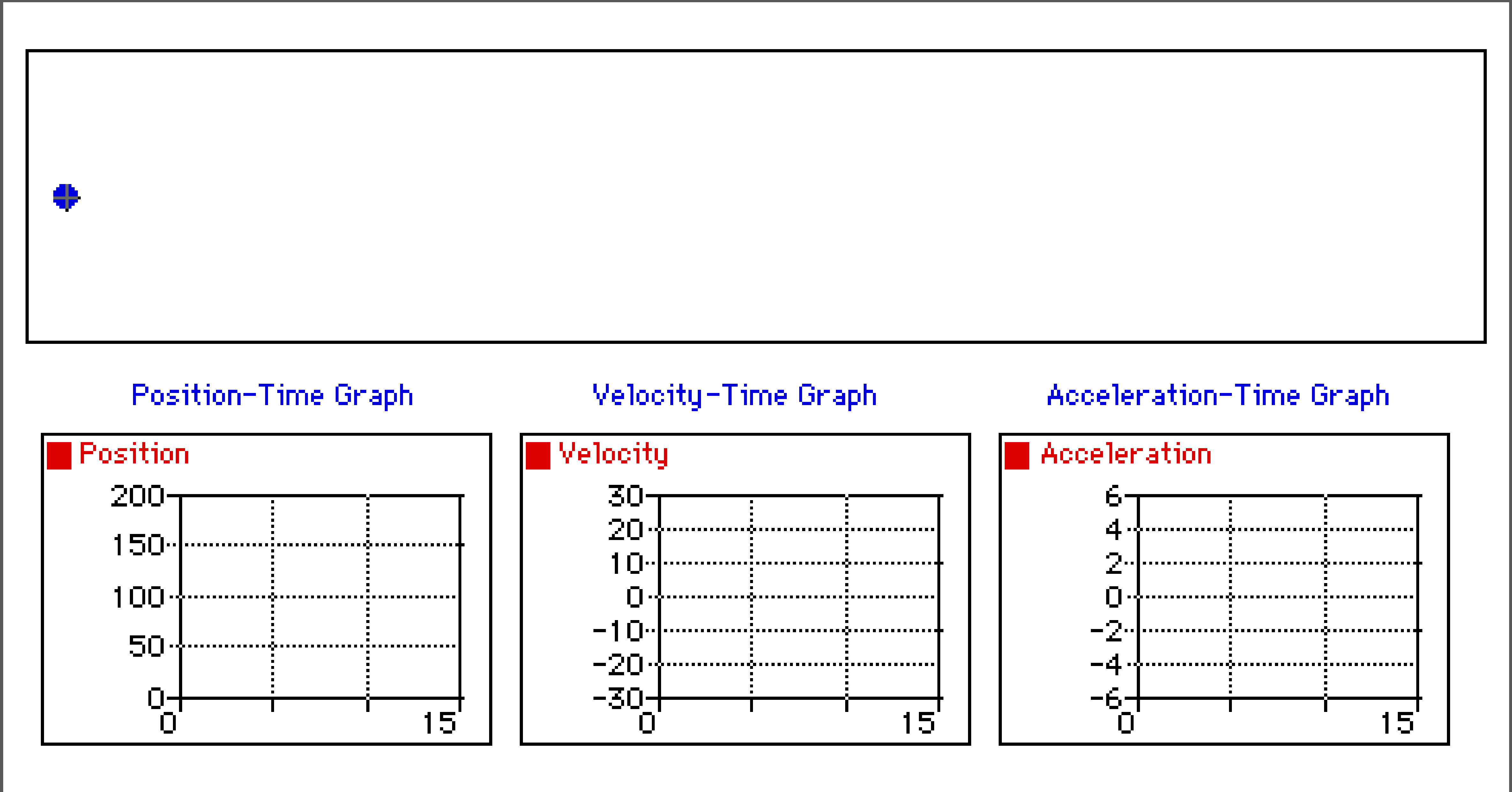


Acceleration-Time Graph



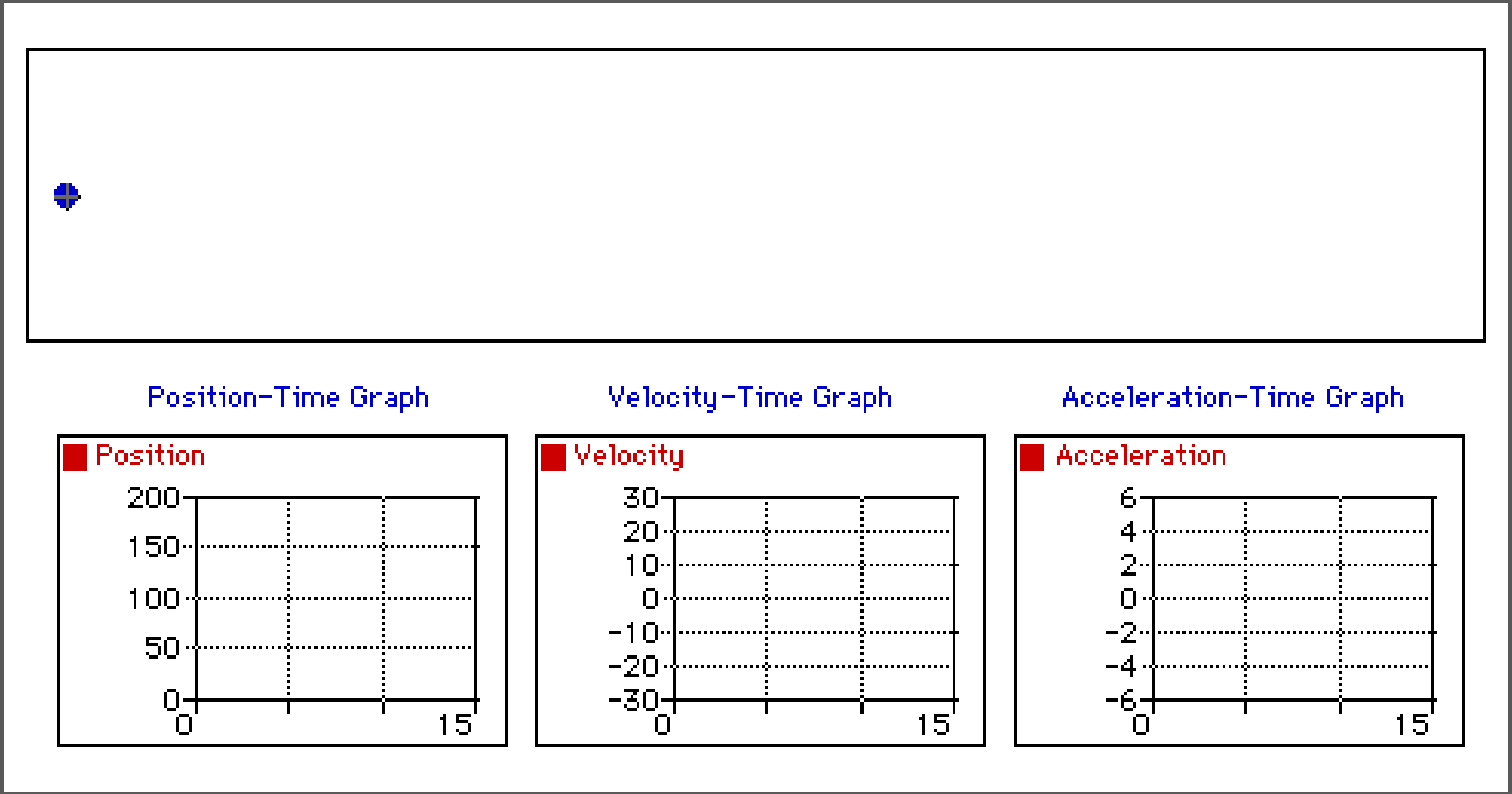
Positive Velocity and Positive Acceleration

- Observe that the object below moves in the positive direction with a changing velocity. An object which moves in the positive direction has a positive velocity. If the object is speeding up, then its acceleration vector is directed in the same direction as its motion (in this case, a positive acceleration). The *dot diagram* shows that each consecutive dot is not the same distance apart (i.e., a changing velocity). The position-time graph shows that the slope is changing (meaning a changing velocity) and positive (meaning a positive velocity). The velocity-time graph shows a line with a positive (upward) slope (meaning that there is a positive acceleration); the line is located in the positive region of the graph (corresponding to a positive velocity). The acceleration-time graph shows a horizontal line in the positive region of the graph (meaning a positive acceleration).


Position-Time Graph
Velocity-Time Graph
Acceleration-Time Graph
Position
200
150
100
50
0
0
15
Velocity
30
20
10
0
-10
-20
-30
0
15
Acceleration
6
4
2
0
-2
-4
-6
0
15

Positive Velocity and Negative Acceleration

- Observe that the object below moves in the positive direction with a changing velocity. An object which moves in the positive direction has a positive velocity. If the object is slowing down then its acceleration vector is directed in the opposite direction as its motion (in this case, a negative acceleration). The *dot diagram* shows that each consecutive dot is not the same distance apart (i.e., a changing velocity). The position-time graph shows that the slope is changing (meaning a changing velocity) and positive (meaning a positive velocity). The velocity-time graph shows a line with a negative (downward) slope (meaning that there is a negative acceleration); the line is located in the positive region of the graph (corresponding to a positive velocity). The acceleration-time graph shows a horizontal line in the negative region of the graph (meaning a negative acceleration).

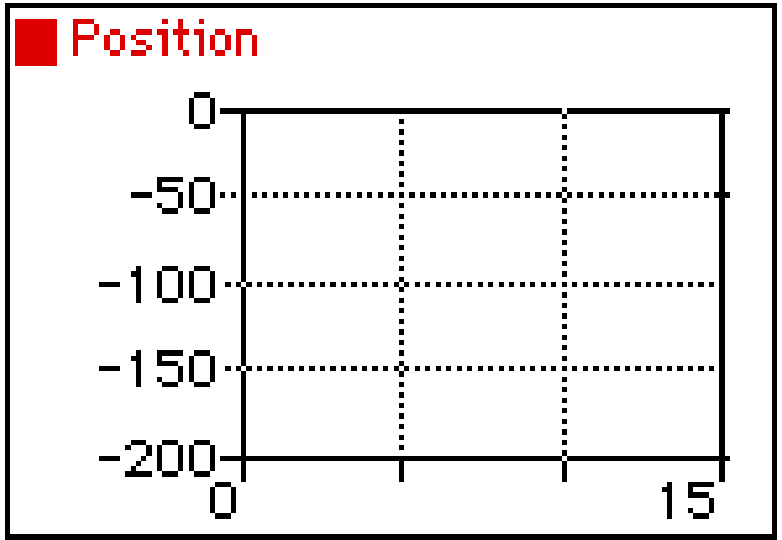


Position-Time Graph
Velocity-Time Graph
Acceleration-Time Graph
Position
200
150
100
50
0
0
15
Velocity
30
20
10
0
-10
-20
-30
0
15
Acceleration
6
4
2
0
-2
-4
-6
0
15

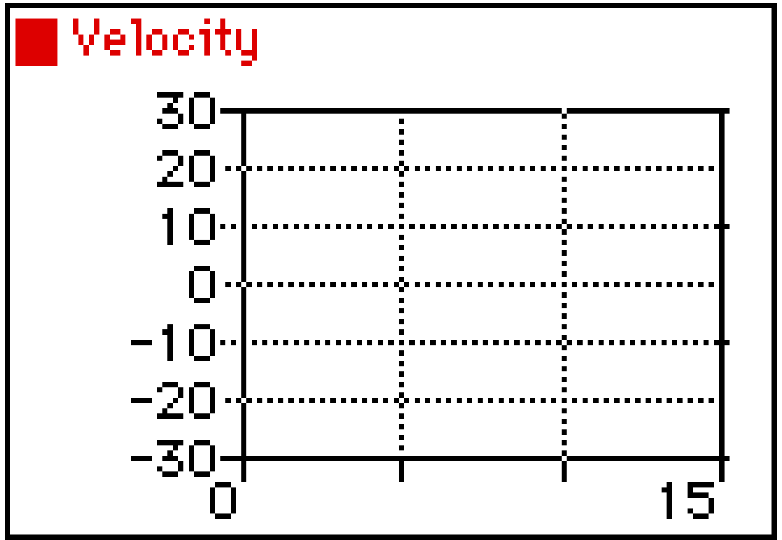
Negative Velocity and Negative Acceleration

- Observe that the object below moves in the negative direction with a changing velocity. An object which moves in the negative direction has a negative velocity. If the object is speeding up then its acceleration vector is directed in the same direction as its motion (in this case, a negative acceleration). The *dot diagram* shows that each consecutive dot is not the same distance apart (i.e., a changing velocity). The position-time graph shows that the slope is changing (meaning a changing velocity) and negative (meaning a negative velocity). The velocity-time graph shows a line with a negative (downward) slope (meaning that there is a negative acceleration); the line is located in the negative region of the graph (corresponding to a negative velocity). The acceleration-time graph shows a horizontal line in the negative region of the graph (meaning a negative acceleration).

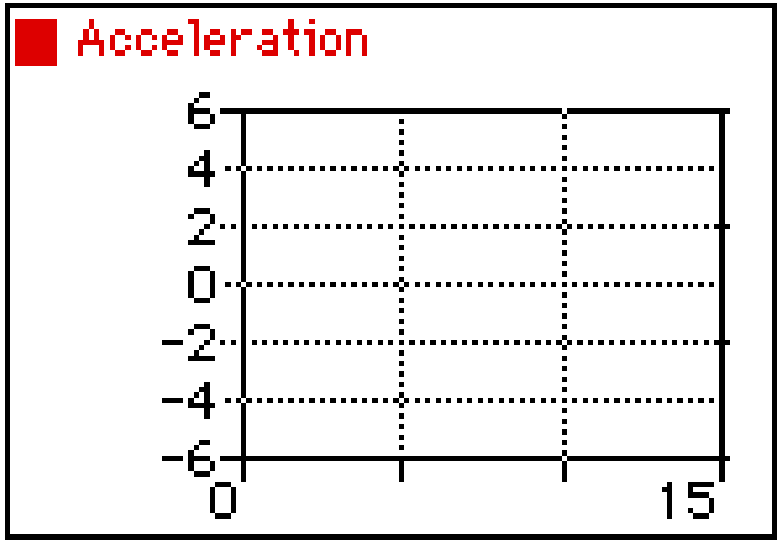
Position-Time Graph



Velocity-Time Graph



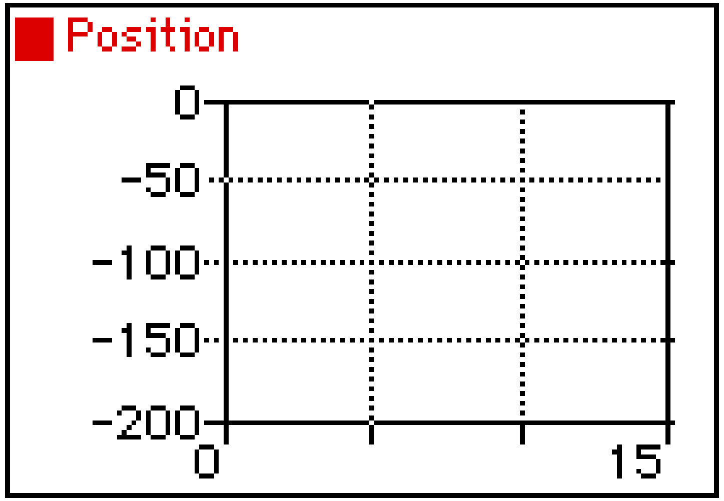
Acceleration-Time Graph



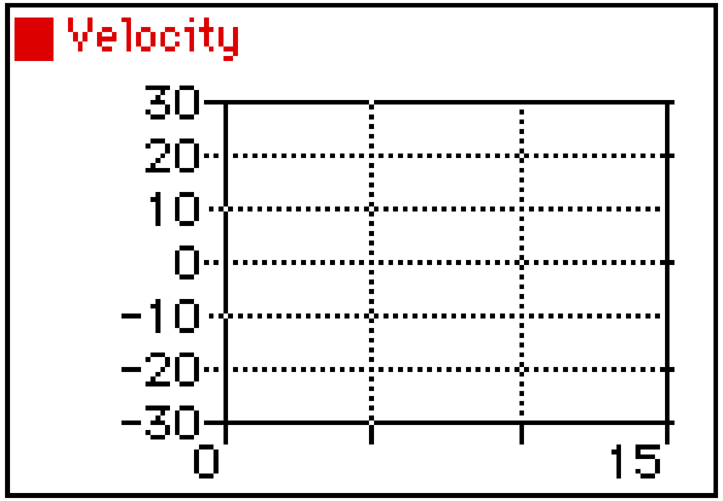
Negative Velocity and Positive Acceleration

- Observe that the object below moves in the negative direction with a changing velocity. An object which moves in the negative direction has a negative velocity. If the object is slowing down then its acceleration vector is directed in the opposite direction as its motion (in this case, a positive acceleration). The *dot diagram* shows that each consecutive dot is not the same distance apart (i.e., a changing velocity). The position-time graph shows that the slope is changing (meaning a changing velocity) and negative (meaning a negative velocity). The velocity-time graph shows a line with a positive (upward) slope (meaning that there is a positive acceleration); the line is located in the negative region of the graph (corresponding to a negative velocity). The acceleration-time graph shows a horizontal line in the positive region of the graph (meaning a positive acceleration).

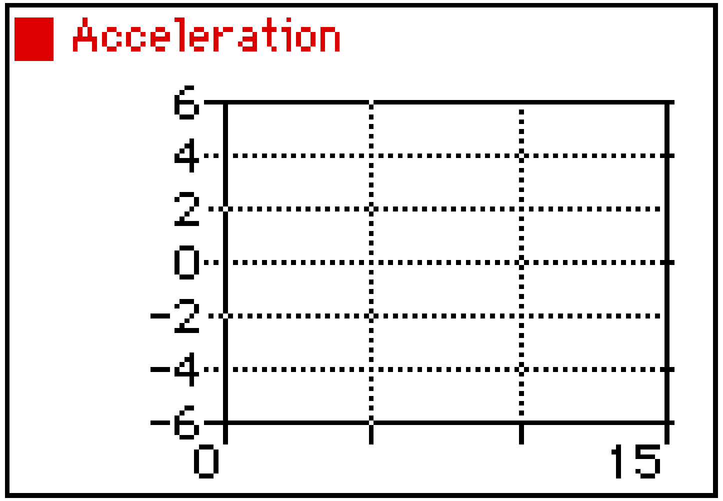
Position-Time Graph



Velocity-Time Graph

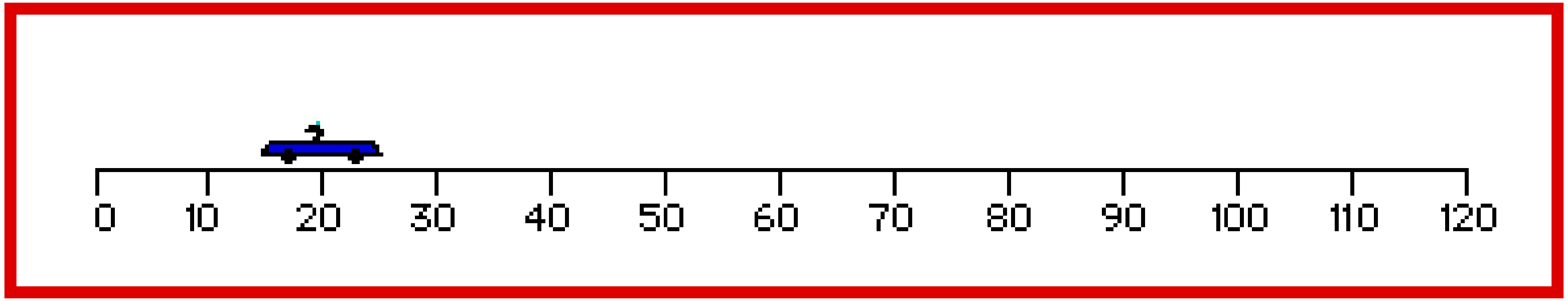


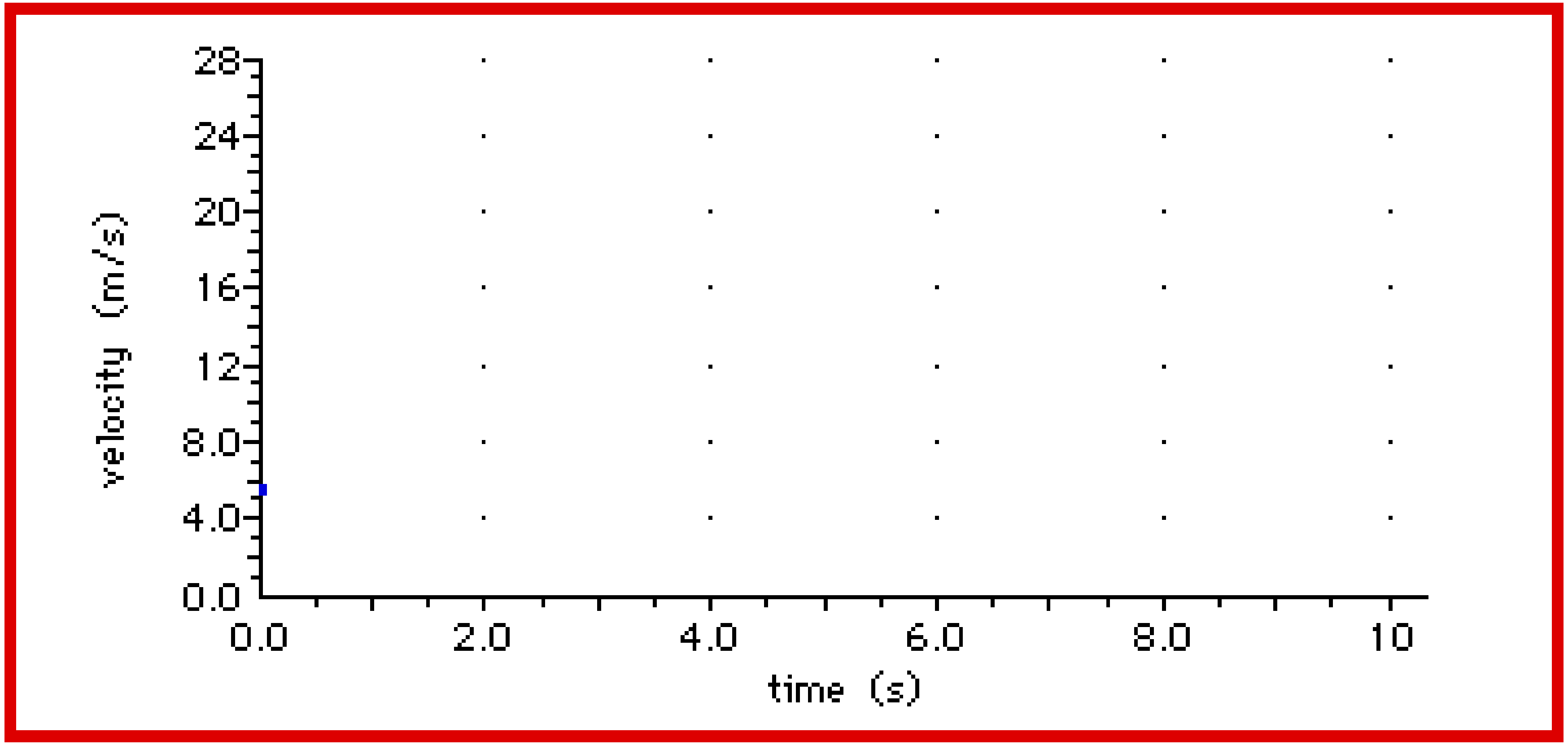
Acceleration-Time Graph



The Passing Lane

- Observe the two cars below. The blue car starts ahead of" the red car. (The red car actually starts *off the screen*.) Since the red car is moving faster, it eventually catches up with and passes the blue car. Observe the velocity-time graphs for these two cars. Each car's motion is represented by a horizontal line, indicating a constant velocity. Observe that even though the cars pass each other, the lines on the velocity-time graphs do not intersect. Since the cars never have the same velocity, the lines on the velocity-time graph never cross. The lines would intersect for a [position vs. time graph](); the fact that the red car passes the blue car means that there is an instant in which they occupy the same *position*. The two cars have the same position at seven seconds; yet they never have the same velocity at any instant in time.


0
10
20
30
40
50
60
70
80
90
100
110
120


velocity (m/s)
28
24
20
16
12
8.0
4.0
0.0
0.0
2.0
4.0
6.0
8.0
10
time (s)

Which car or cars (red, green, blue) are undergoing an acceleration?

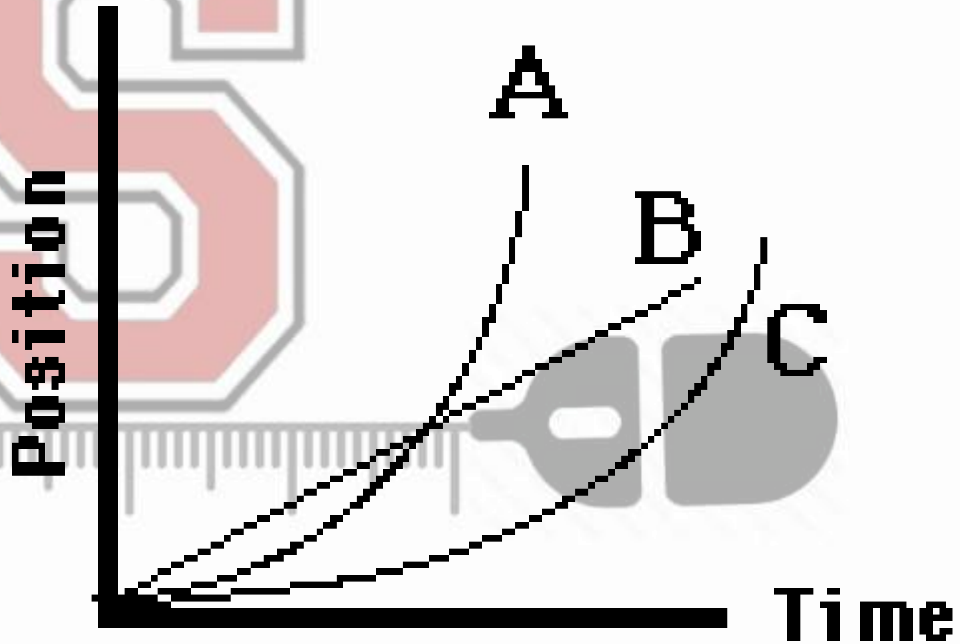
If you inspect each car individually, you will more likely notice that only the green and the blue cars accelerate. The red car moves with a constant speed, covering the same distance in each second of the animation.

The green and the blue cars are speeding up, thus covering an increasing distance in each second of the animation.

The blue car has a greater acceleration. It is changing its velocity at a more drastic rate.

The red car is moving with a constant velocity and must correspond to object B which has a constant slope.

The green and blue cars have a changing velocity and must correspond to lines with a changing slopes - objects A and C. The green car is object C which has a more gradually changing slope than object A (blue car).



A free-falling object that is accelerating at a constant rate will cover different distances in each consecutive second.

Accelerating Objects are Changing Their Velocity ...

... by a constant amount each second ...

Time (s)	Velocity (m/s)
0	0
1	4
2	8
3	12
4	16

...in which case, it is referred to as a constant acceleration.

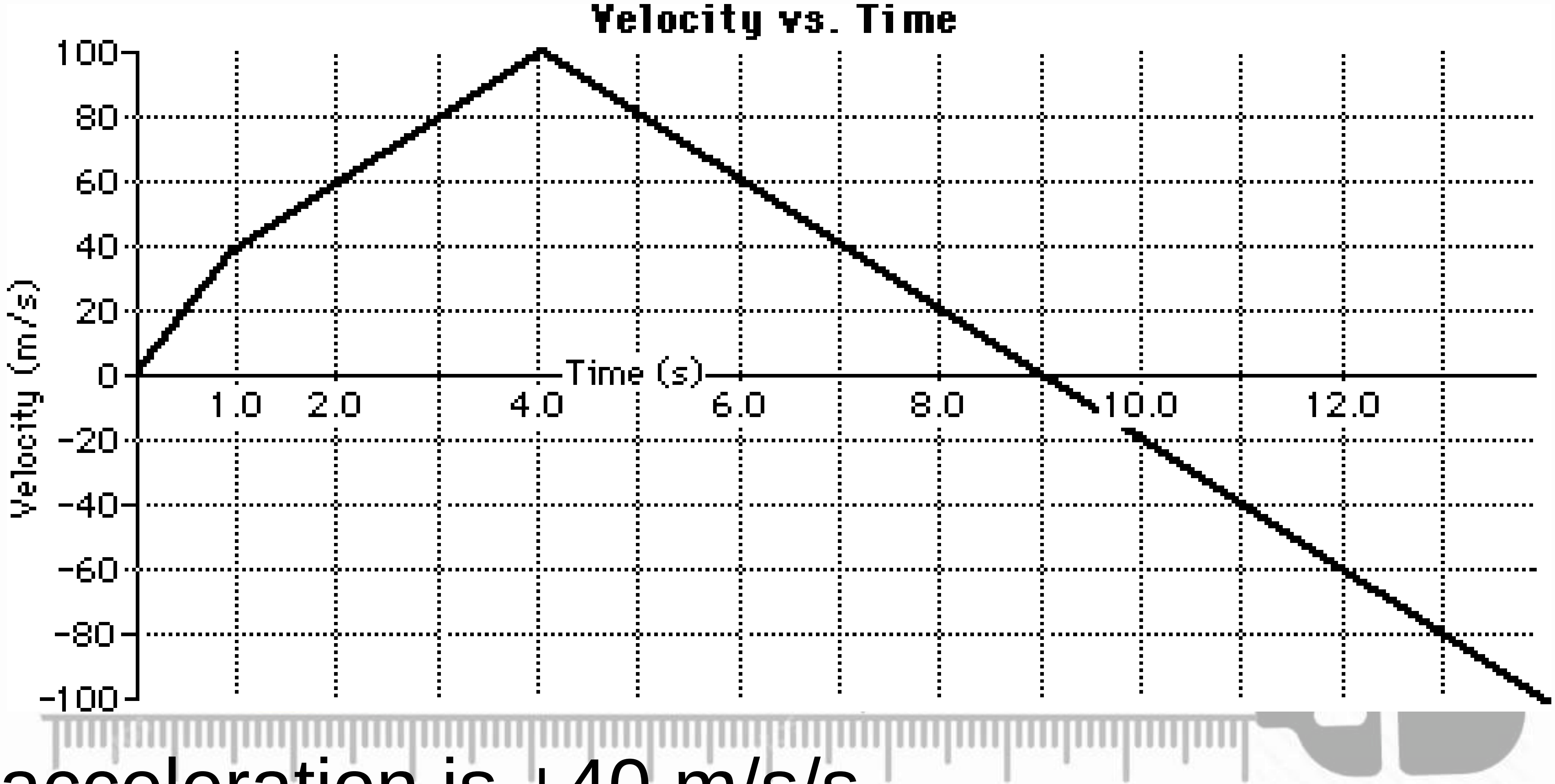
... or by a changing amount each second ...

Time (s)	Velocity (m/s)
0	0
1	1
2	4
3	5
4	7

...in which case, it is referred to as a non-constant acceleration.

$$\text{Ave. acceleration} = \frac{\Delta\,\text{velocity}}{\text{time}} = \frac{v_f - v_i}{t}$$

- The velocity-time graph for a two-stage rocket is shown below. Use the graph and your understanding of slope calculations to determine the acceleration of the rocket during the listed time intervals. When finished, use the buttons to see the answers. (Help with Slope Calculations)
- t = 0 - 1 second
- t = 1 - 4 second
- t = 4 - 12 second



The acceleration is +40 m/s/s.
The acceleration is **+20 m/s/s**.
The acceleration is **-20 m/s/s**.

- This general principle can be applied to determine whether the sign of the acceleration of an object is positive or negative, right or left, up or down, etc. Consider the two data tables below. In each case, the acceleration of the object is in the positive direction. In Example A, the object is moving in the positive direction (i.e., has a positive velocity) and is speeding up. When an object is speeding up, the acceleration is in the same direction as the velocity. Thus, this object has a positive acceleration. In Example B, the object is moving in the negative direction (i.e., has a negative velocity) and is slowing down. According to our general principle, when an object is slowing down, the acceleration is in the opposite direction as the velocity. Thus, this object also has a positive acceleration.

Example A

Time (s)	Velocity (m/s)
0	0
1	2
2	4
3	6
4	8

Example B

Time (s)	Velocity (m/s)
0	-8
1	-6
2	-4
3	-2
4	0

These are both examples of positive acceleration.

- This same general principle can be applied to the motion of the objects represented in the two data tables below. In each case, the acceleration of the object is in the negative direction. In Example C, the object is moving in the positive direction (i.e., has a positive velocity) and is slowing down. According to our principle, when an object is slowing down, the acceleration is in the opposite direction as the velocity. Thus, this object has a negative acceleration. In Example D, the object is moving in the negative direction (i.e., has a negative velocity) and is speeding up. When an object is speeding up, the acceleration is in the same direction as the velocity. Thus, this object also has a negative acceleration.

So to say that an object has a negative acceleration as in Examples C and D is to simply say that its acceleration is to the left or down (or in whatever direction has been defined as negative). Negative accelerations do not refer acceleration values that are less than 0. An acceleration of -2 m/s/s is an acceleration with a magnitude of 2 m/s/s that is directed in the negative direction.

Example C

Time (s)	Velocity (m/s)
0	8
1	6
2	4
3	2
4	0

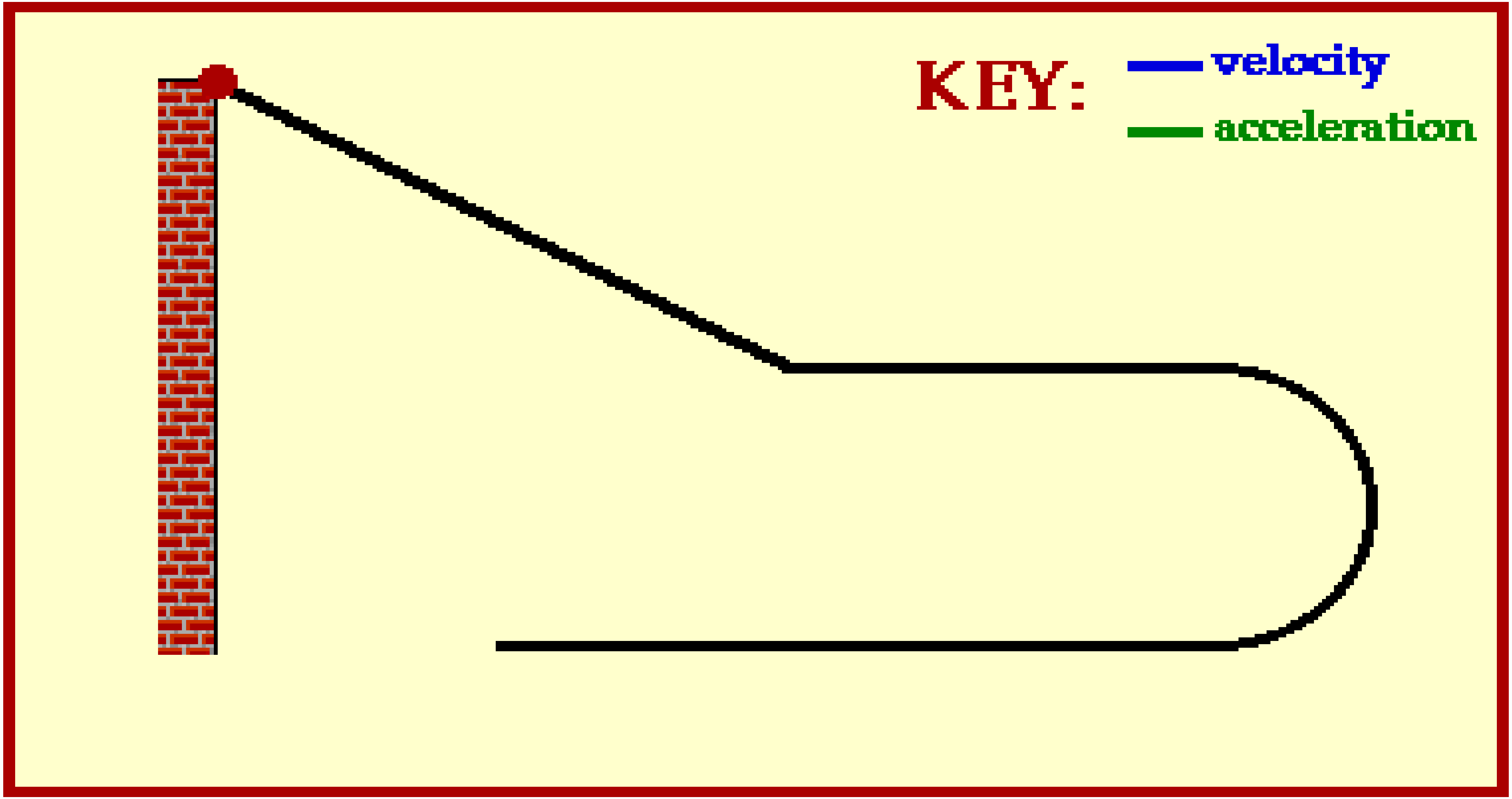
Example D

Time (s)	Velocity (m/s)
0	0
1	-2
2	-4
3	-6
4	-8

These are both examples of negative acceleration.

Direction of Acceleration and Velocity

- Consider the motion of a Hot Wheels car down an incline, across a level and straight section of track, around a 180-degree curve, and finally along a final straight section of track. Such a motion is depicted in the animation below. The car gains speed while moving down the incline - that is, it accelerates. Along the straight sections of track, the car slows down slightly (due to air resistance forces). Again the car could be described as having an acceleration. Finally, along the 180-degree curve, the car is changing its direction; once more the car is said to have an acceleration due to the change in the direction. Accelerating objects have a changing velocity - either due to a speed change (speeding up or slowing down) or a direction change.


KEY:
velocity
acceleration

- This simple animation above depicts some additional information about the car's motion. The velocity and acceleration of the car are depicted by vector arrows. The direction of these arrows are representative of the direction of the velocity and acceleration vectors. Note that the velocity vector is always directed in the same direction which the car is moving. A car moving eastward would be described as having an eastward velocity. And a car moving westward would be described as having a westward velocity.

- The direction of the acceleration vector is not so easily determined. As shown in the animation, an eastward heading car can have a westward directed acceleration vector. And a westward heading car can have an eastward directed acceleration vector. So how can the direction of the acceleration vector be determined? A simple rule of thumb for determining the direction of the acceleration is that an object which is slowing down will have an acceleration directed in the direction opposite of its motion. Applying this rule of thumb would lead us to conclude that an eastward heading car can have a westward directed acceleration vector if the car is slowing down.

Use the equation for acceleration to determine the acceleration for the following two motions.

Practice A

Time (s)	Velocity (m/s)
0	0
1	2
2	4
3	6
4	8

Practice B

Time (s)	Velocity (m/s)
0	8
1	6
2	4
3	2
4	0

Answer: **a = 2 m/s/s**

Use $a = (v_f - v_i) / t$ and pick any two points.

a = (8 m/s - 0 m/s) / (4 s)

a = (8 m/s) / (4 s)

a = 2 m/s/s

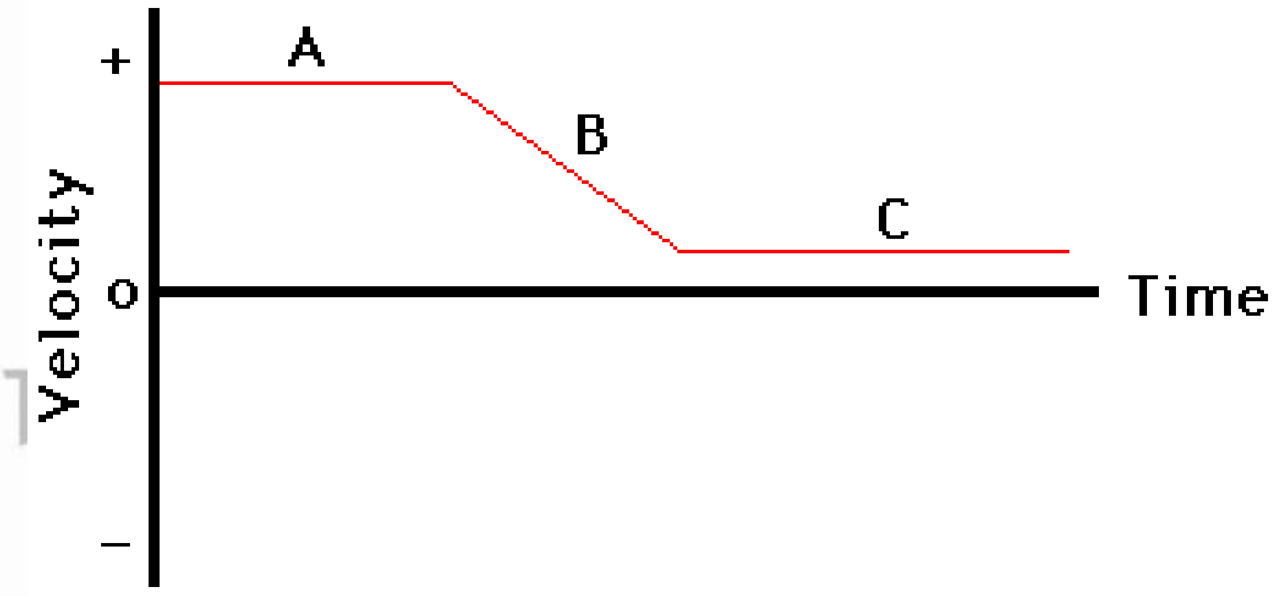
Answer: **a = -2 m/s/s**

Use $a = (v_f - v_i) / t$ and pick any two points.

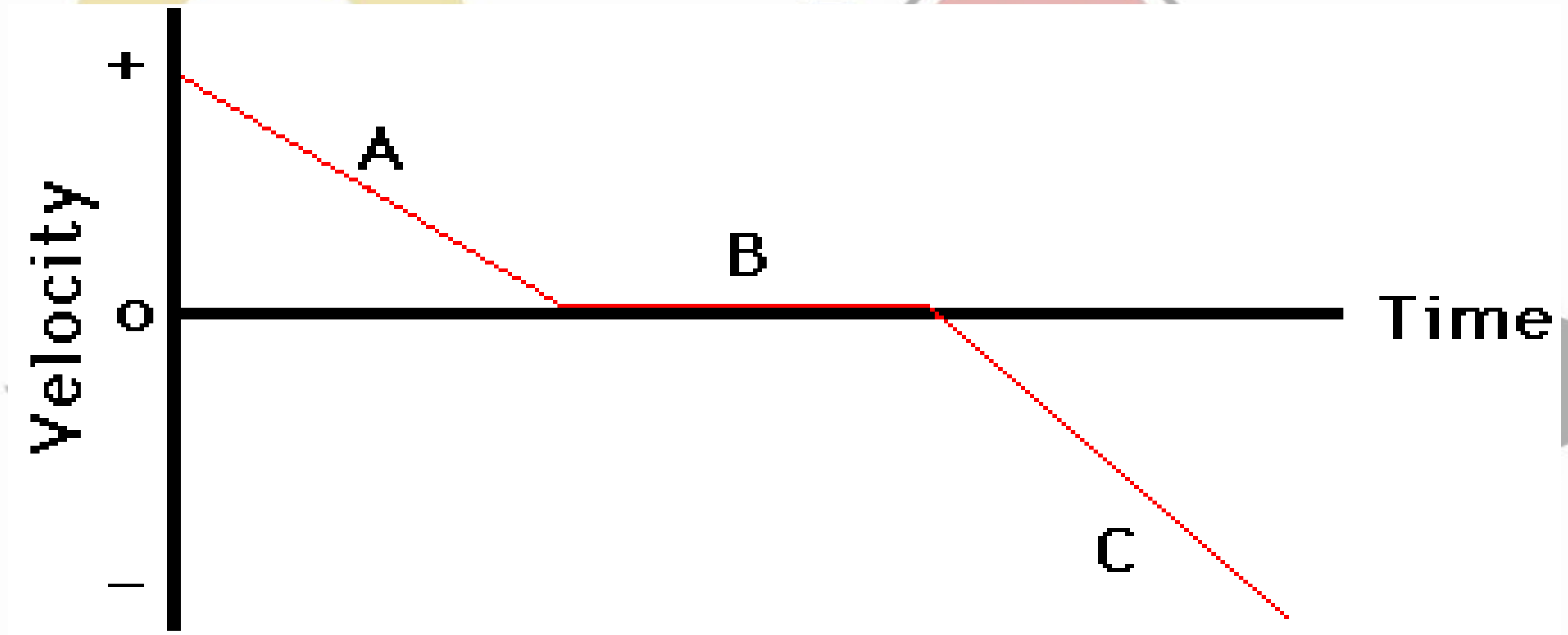
a = (0 m/s - 8 m/s) / (4 s) a = (-8 m/s) / (4 s)

a = -2 m/s/s

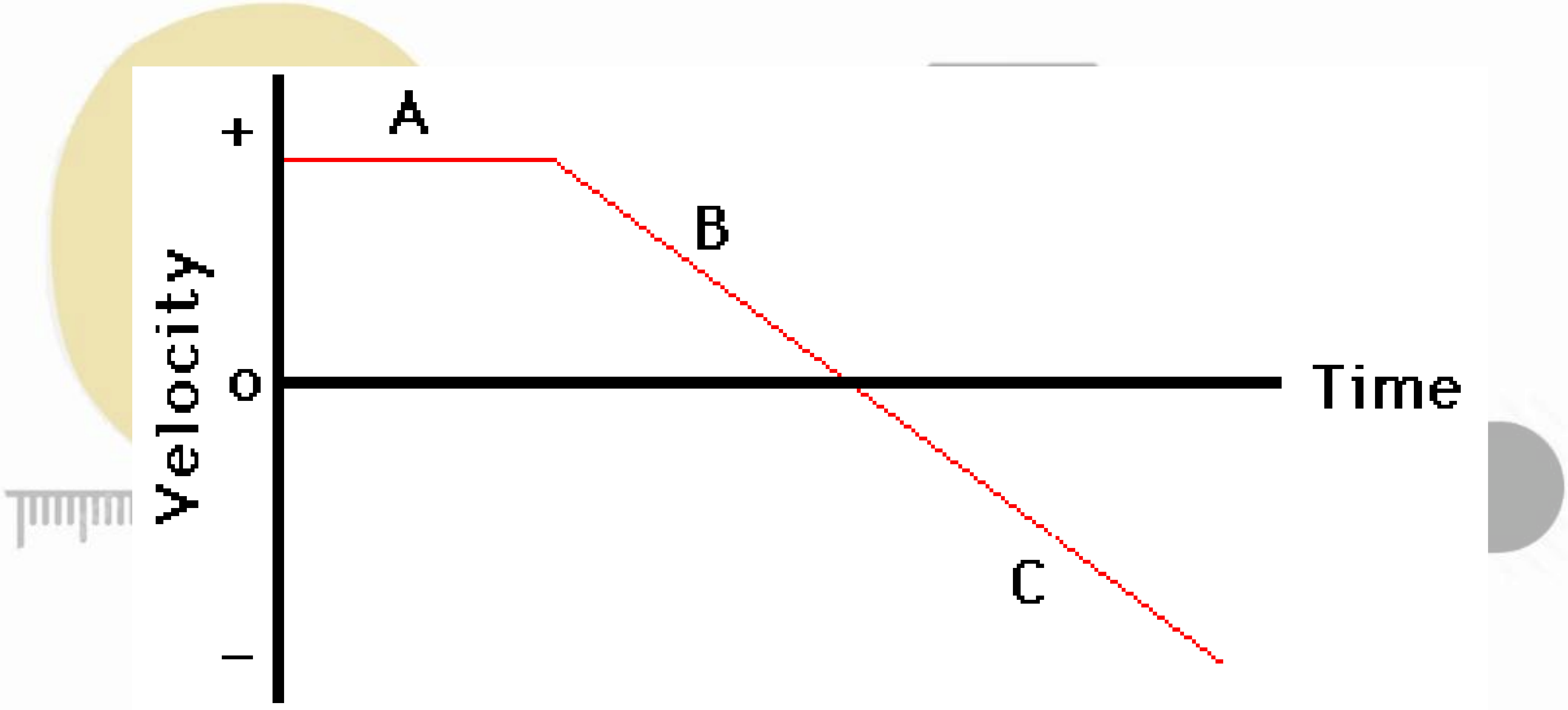
- Describe the motion depicted by the following velocity-time graphs. In your descriptions, make reference to the direction of motion (+ or - direction), the velocity and acceleration and any changes in speed (speeding up or slowing down) during the various time intervals (e.g., intervals A, B, and C). When finished, view the answers.



The object moves in the + direction at a constant speed - zero acceleration (interval A). The object then continues in the + direction while slowing down with a negative acceleration (interval B). Finally, the object moves at a constant speed in the + direction, slower than before (interval C).



The object moves in the + direction while slowing down; this involves a negative acceleration (interval A). It then remains at rest (interval B). The object then moves in the - direction while speeding up; this also involves a negative acceleration (interval C).



- The object moves in the + direction with a constant velocity and zero acceleration (interval A). The object then slows down while moving in the + direction (i.e., it has a negative acceleration) until it finally reaches a 0 velocity (stops) (interval B). Then the object moves in the - direction while speeding up; this corresponds to a - acceleration (interval C).

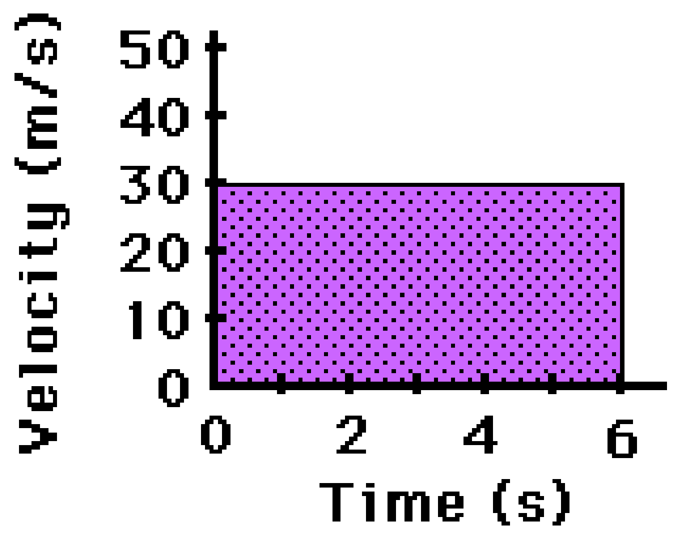
Determining the Area on a v-t Graph

- For velocity versus time graphs, the area bound by the line and the axes represents the displacement. The diagram below shows three different velocity-time graphs; the shaded regions between the line and the time-axis represent the displacement during the stated time interval.

Rectangle

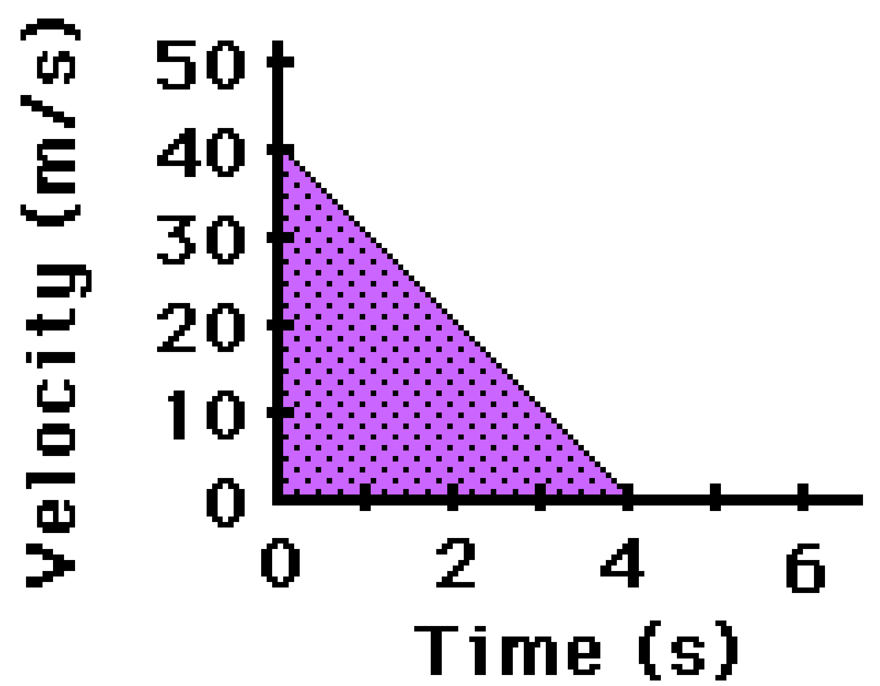
Area = b • h

The shaded area is representative of the displacement during from 0 seconds to 6 seconds. This area takes on the shape of a rectangle can be calculated using the appropriate equation.



Triangle

Area = ½ • b • h



The shaded area is representative of the displacement during from 0 seconds to 4 seconds. This area takes on the shape of a triangle can be calculated using the appropriate equation.

Trapezoid

Area = ½ • b • (h1 + h2)

The shaded area is representative of the displacement during from 2 seconds to 5 seconds. This area takes on the shape of a trapezoid can be calculated using the appropriate equation.