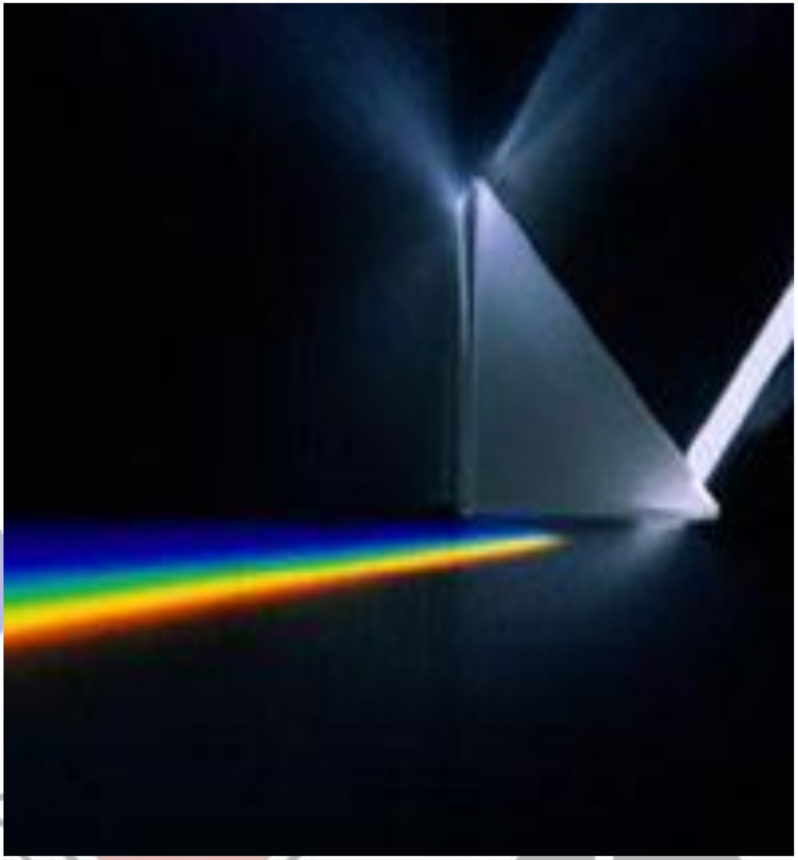
Refraction of Prisms



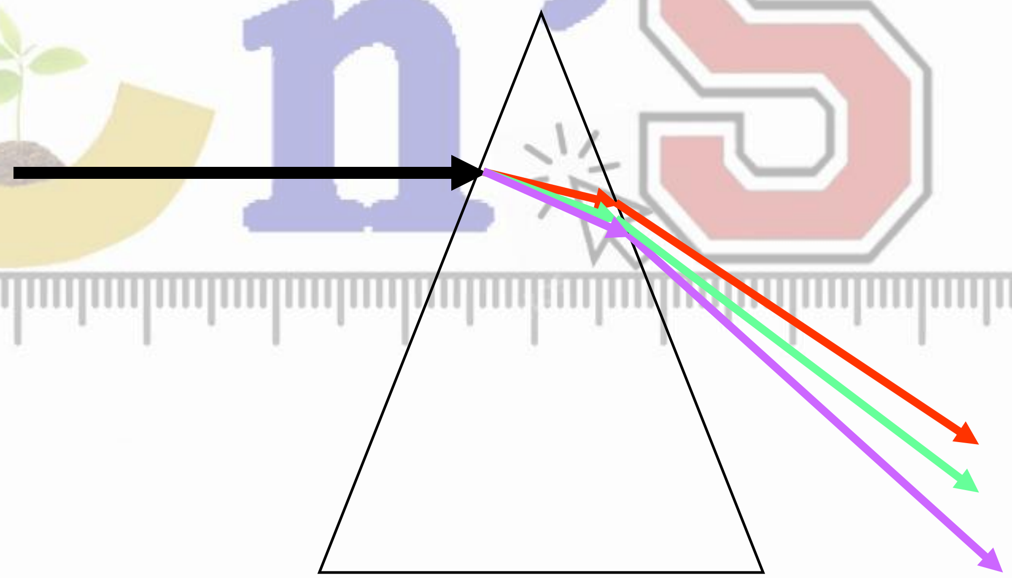
Dispersion by Refraction

Dispersion

The separation of white light into colors due to different refractive indices for different wavelengths.

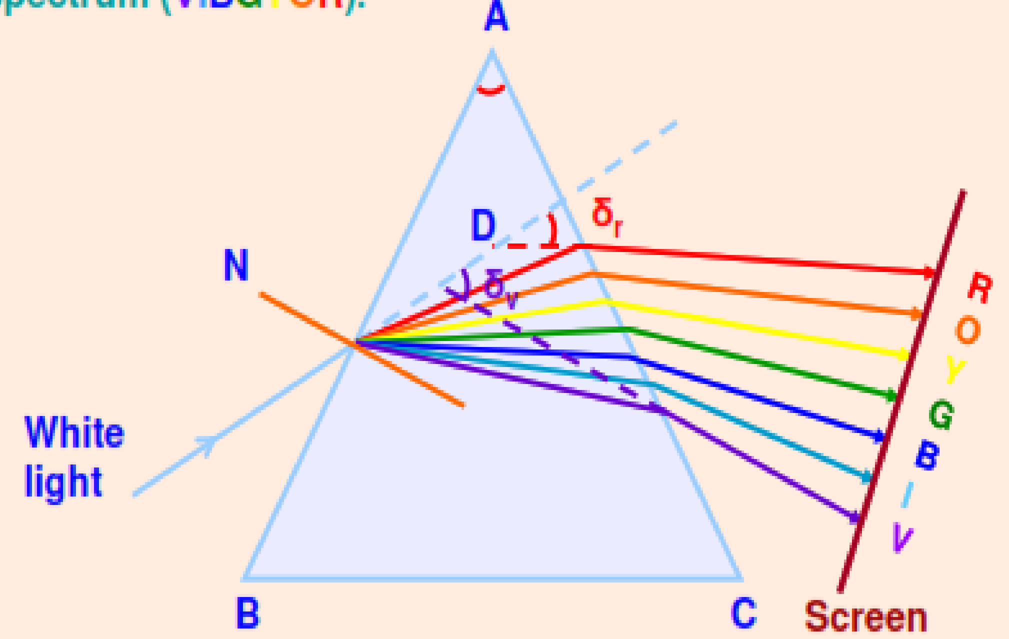
Dispersion

Red refracts least while violet refracts the most.



Dispersion of White Light through Prism:

The phenomenon of splitting a ray of white light into its constituent colours (wavelengths) is called dispersion and the band of colours from violet to red is called spectrum (VIBGYOR).



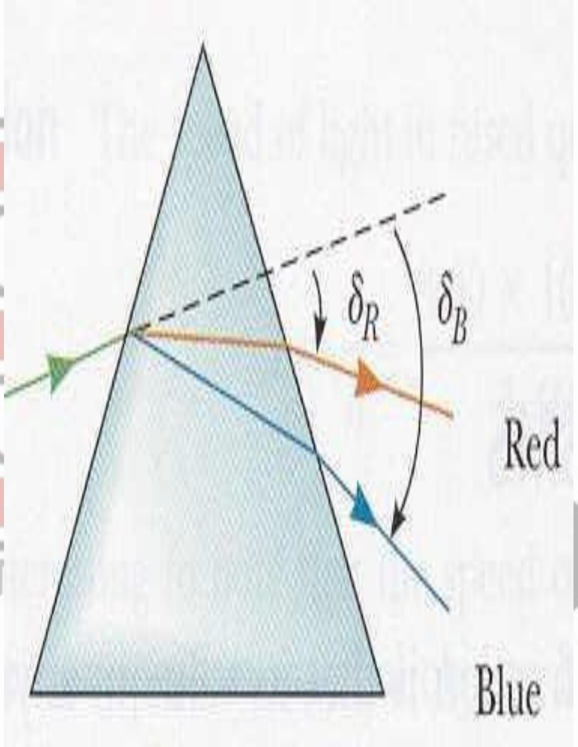
Cause of Dispersion:

$$\mu_v = \frac{\sin i}{\sin r_v} \quad \text{and} \quad \mu_r = \frac{\sin i}{\sin r_r}$$

Since $\mu_v > \mu_r$, $r_r > r_v$

So, the colours are refracted at different angles and hence get separated.

Suppose a beam of white light (a combination of all visible wavelengths) is incident on a prism. a refracting material.
Suppose a beam of white light (a combination of all visible wavelengths) is incident on a prism. Because of dispersion, the rays that emerge from the second face of the prism fan out in a series of colors known as a visible spectrum.



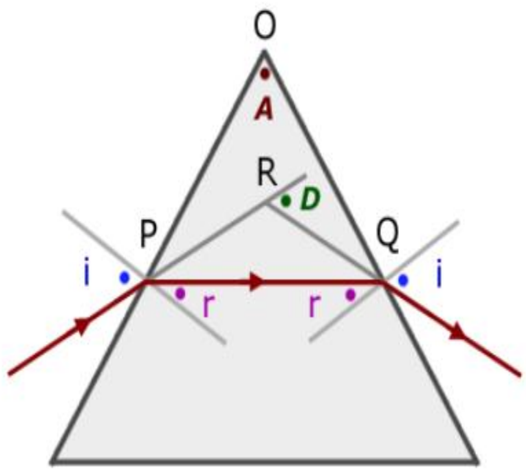
Scattering of Light – Blue colour of the sky and Reddish appearance of the Sun at Sun-rise and Sun-set:

The molecules of the atmosphere and other particles that are smaller than the longest wavelength of visible light are more effective in scattering light of shorter wavelengths than light of longer wavelengths. The amount of scattering is inversely proportional to the fourth power of the wavelength. (Rayleigh Effect)

Light from the Sun near the horizon passes through a greater distance in the Earth's atmosphere than does the light received when the Sun is overhead. The correspondingly greater scattering of short wavelengths accounts for the reddish appearance of the Sun at rising and at setting.

When looking at the sky in a direction away from the Sun, we receive scattered sunlight in which short wavelengths predominate giving the sky its characteristic bluish colour.

MINIMUM DEVIATION



In ΔPQR

$$D = \angle RPQ + \angle PQR \qquad (i$$

around points **P** and **Q** respectively,

$$i = \angle RPQ + r \qquad i = \angle PQR + r$$

$$\angle RPQ = i - r \qquad \angle PQR = i - r$$

substituting into equation (i

$$\therefore\ D = 2(i - r)$$

In ΔOPQ

$$A + \angle OPQ + \angle PQO = 180^\circ$$

$$A + (90^\circ - r) + (90^\circ - r) = 180^\circ$$

$$\underline{A = 2r} \qquad (iii$$

adding together equations (ii & (iii,

$$A + D = 2r + 2(i - r)$$

$$= 2r + 2i - 2r$$

$$= 2i$$

$$\underline{i = \frac{A + D}{2}} \qquad (iv$$

rearranging equation (iii,

$$\underline{r = \frac{A}{2}} \qquad (v$$

Using Snell's Law equation,

$$n_1 \sin(i) = n_2 \sin(r)$$

and substituting for **i** and **r** from equations (iv & (v above,

$$n_1 \sin\left(\frac{A + D}{2}\right) = n_2 \sin\left(\frac{A}{2}\right)$$

n_1 is the refractive index of air which approximates to 1. Hence the equation becomes:

$$\sin\left(\frac{A + D}{2}\right) = n_2 \sin\left(\frac{A}{2}\right)$$

Example

So for a 60° equilateral prism made of glass(n = 1.5 approx.), the minimum deviation angle D is given by:

$$\sin\left(30^\circ + \frac{D}{2}\right) = 1.5 \times \sin(30^\circ)$$
$$= 1.5 \times 0.5$$
$$= 0.75$$

It follows that:

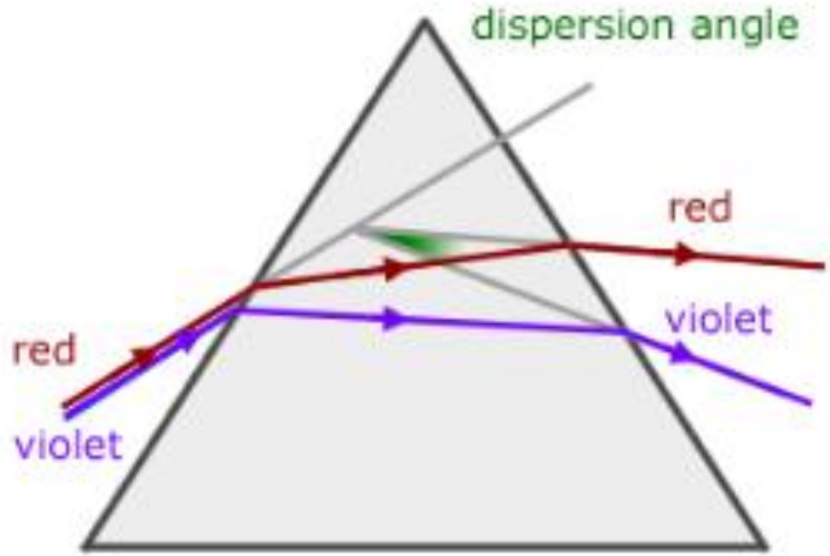
$$30^\circ + \frac{D}{2} = \sin^{-1}(0.75)$$

remembering that $\sin^{-1}(x)$ means the angle whose sine is 'x',

$$30^\circ + \frac{D}{2} = 48.59^\circ$$
$$\frac{D}{2} = 48.59^\circ - 30^\circ$$
$$= 18.59^\circ$$
$$D = 2 \times 18.59^\circ$$
$$\underline{D = 37.18^\circ}$$

Minimum angle of dispersion

This is measure of the angle of 'spread' of a spectrum when it leaves a prism. For minimum angular dispersion, the angle is derived from the difference in deviation between red and violet rays of light.

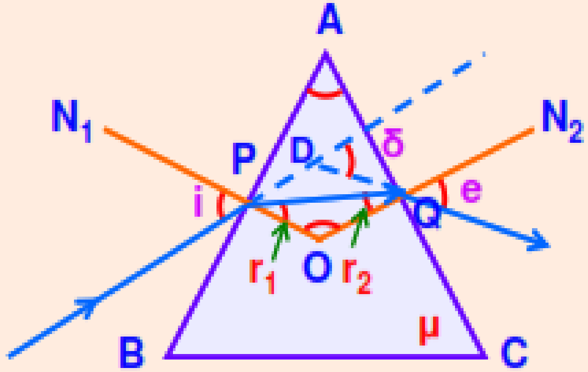


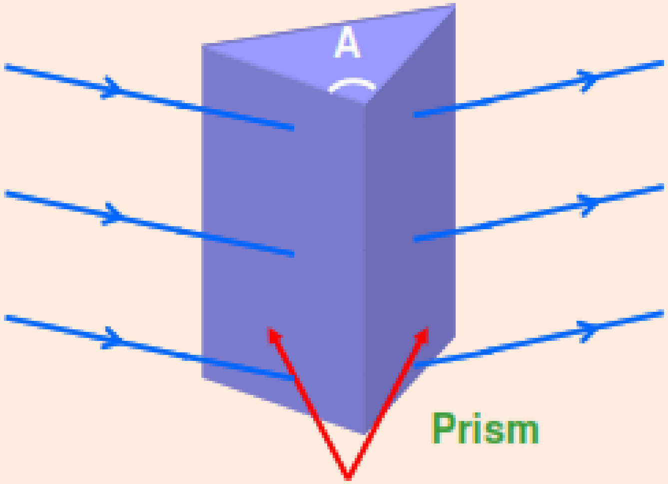
Replacing n_2 with n_r for red, and n_v for violet light in the minimum deviation equation below,

$$\sin\left(\frac{A+D}{2}\right) = n_2 \sin\left(\frac{A}{2}\right)$$

we can calculate values of deviation for each colour. Subtracting the angles gives the minimum dispersion angle for white light.

Refraction of Light through Prism:





In quadrilateral APOQ,

$A + O = 180°$ .... (1)

(since N_1 and N_2 are normal)

In triangle OPQ,

$r_1 + r_2 + O = 180°$ .... (2)

In triangle DPQ,

$\delta = (i - r_1) + (e - r_2)$

$\delta = (i + e) - (r_1 + r_2)$ ....(3)

From (1) and (2),

$A = r_1 + r_2$

From (3),

$\delta = (i + e) - (A)$

or $\boxed{i + e = A + \delta}$

Sum of angle of incidence and angle of emergence is equal to the sum of angle of prism and angle of deviation.

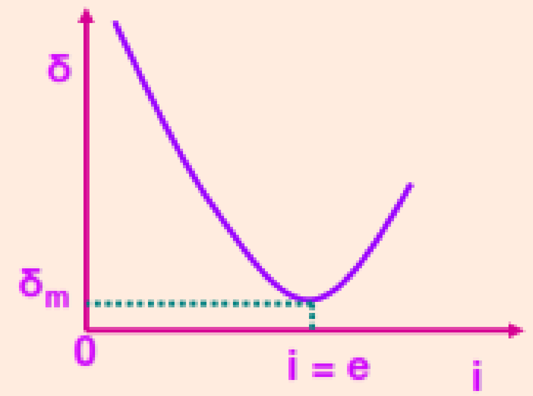
Variation of angle of deviation with angle of incidence:

When angle of incidence increases, the angle of deviation decreases.

At a particular value of angle of incidence the angle of deviation becomes minimum and is called 'angle of minimum deviation'.

At δ_m, $i = e$ and $r_1 = r_2 = r$ (say)

After minimum deviation, angle of deviation increases with angle of incidence.



Refractive Index of Material of Prism:

$A = r_1 + r_2$

$A = 2r$

$\boxed{r = A / 2}$

$i + e = A + \delta$

$2i = A + \delta_m$

$\boxed{i = (A + \delta_m) / 2}$

According to Snell's law,

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin i}{\sin r}$$

$$\therefore \boxed{\mu = \frac{\sin \frac{(A + \delta_m)}{2}}{\sin \frac{A}{2}}}$$

Refraction by a Small-angled Prism for Small angle of Incidence:

$$\mu = \frac{\sin i}{\sin r_1} \quad \text{and} \quad \mu = \frac{\sin e}{\sin r_2}$$

If i is assumed to be small, then r_1, r_2 and e will also be very small.
So, replacing sines of the angles by angles themselves, we get

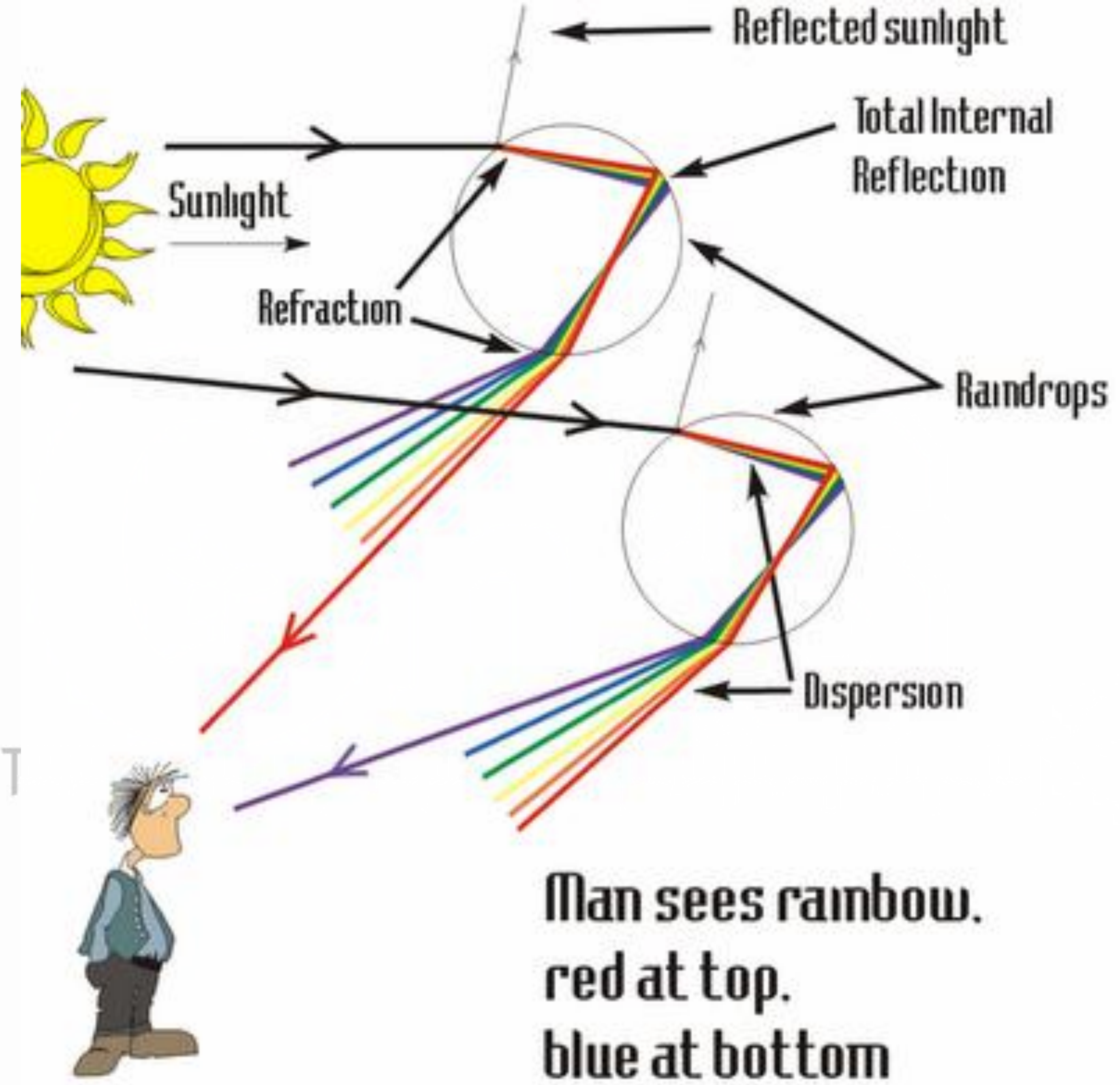
$$\mu = \frac{i}{r_1} \quad \text{and} \quad \mu = \frac{e}{r_2}$$

$$i + e = \mu\,(r_1 + r_2) = \mu\,A$$

But $i + e = A + \delta$

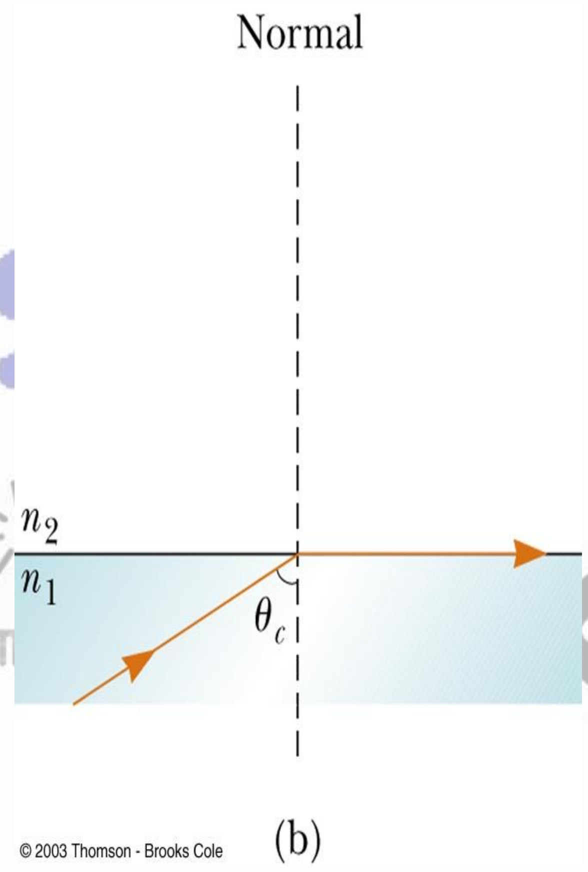
So, $A + \delta = \mu\,A$

or $\boxed{\delta = A\,(\mu - 1)}$


Reflected sunlight
Total Internal
Reflection
Sunlight
Refraction
Raindrops
Dispersion
Man sees rainbow.
red at top.
blue at bottom

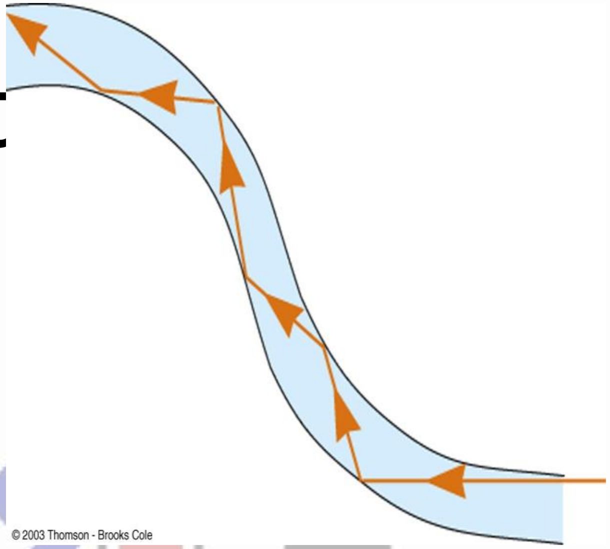
Critical Angle

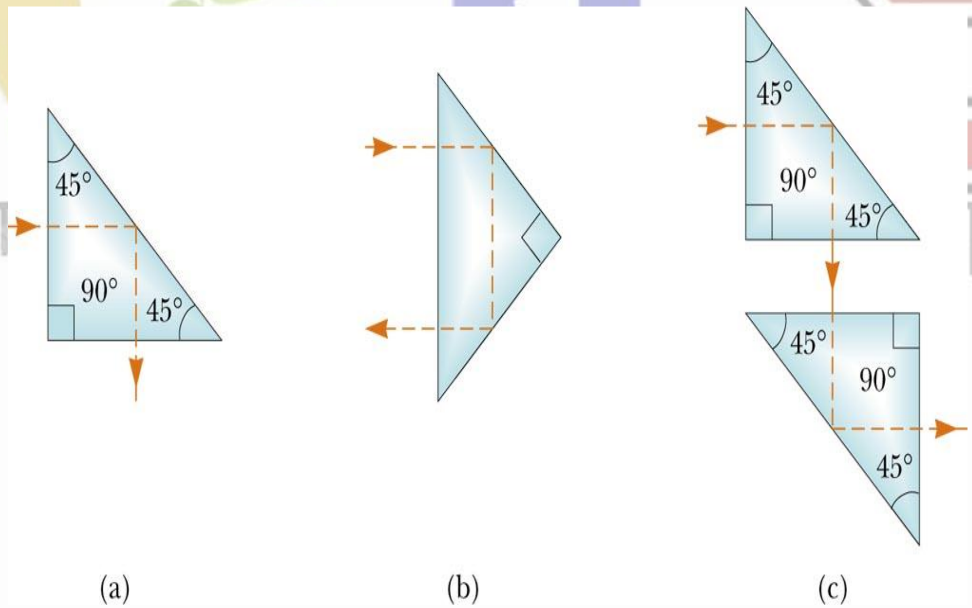
- The critical angle is an angle of incidence that will result in an angle of refraction of 90°



$$\sin\theta_c = \frac{n_2}{n_1} \quad for \quad n_1 > n_2$$

Fiber Opt





Conceptual questions

- 18. If a beam of light with a given cross-section enter a new medium, the cross section of the refracted beam is

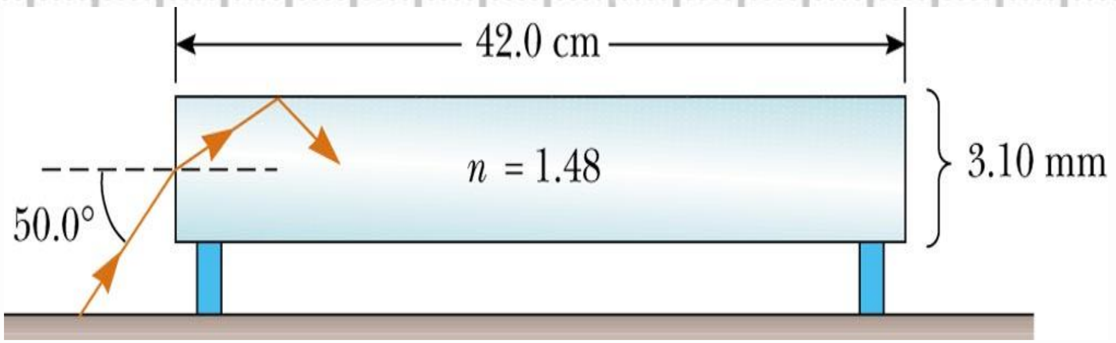
  a) larger

  b) smaller

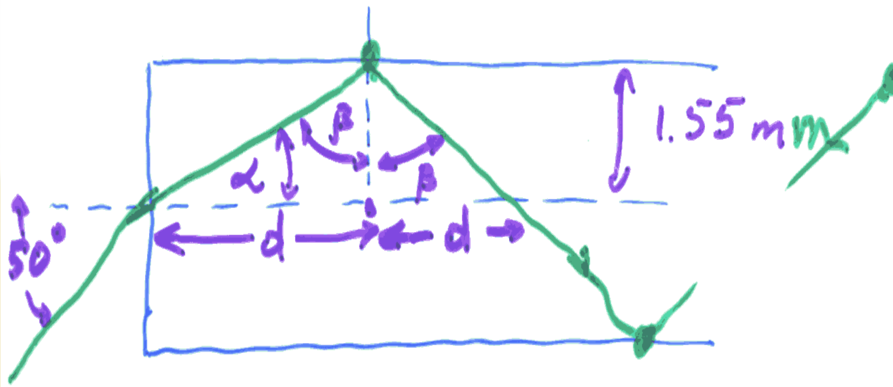
  c) not changed
- 9. In dispersive materials, the angle of refraction for a light ray depends on the wavelength of light. Does the angle of reflection depend on the wavelength?
- 11. Explain why a diamond loses most of its sparkle when submerged in carbon disulfide.
- 16. Explain why an oar partially in water appears to be bent.
- 17. Why do astronomers looking at distant galaxies talk about looking backward in time?

Problem 22.56

A laser beam strikes one end of a slab of material, as in Figure P22.56. The index of refraction of the slab is 1.48. Determine the number of internal reflections of the beam before it emerges from the opposite end of the slab.



Solution



$$1.48 \cdot \sin\alpha = 1.0 \cdot \sin 50.0^\circ$$

$$\alpha = 31.17^\circ$$

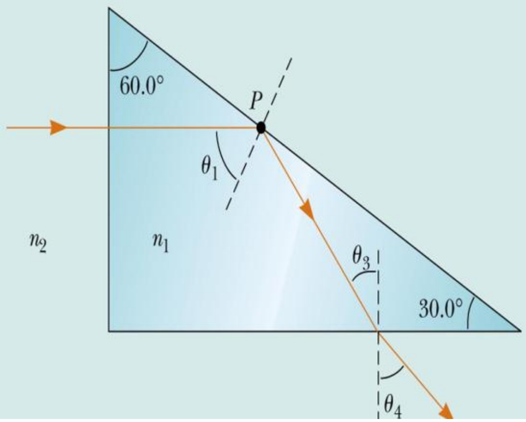
$$\tan\alpha = \frac{1.55\ m}{d}$$

$$d = \frac{1.55}{\tan 31.37^\circ}$$

$$2d = \frac{2 \times 1.55}{\tan 31.37^\circ}$$

$$2d = 5.125\ mm$$

$$N = \frac{L}{2d} = \frac{420\ mm}{5.125\ mm} = 82$$



Problem 21-49

As shown in Figure P22.49, a light ray is incident normally on one face of a 30°-60°-90° block of dense flint glass (a prism) that is immersed in water. (a) Determine the exit angle θ_4 of the ray. (b) A substance is dissolved in the water to increase the index of refraction. At what value of n_2 does total internal reflection cease at point P?

Review questions

1. Visible light of which color bends the most when changing mediums?

- A. yellow
- B. green
- C. violet
- D. red

2. Water has an index of refraction of 1.3. Approximately how fast does light move through the water?

a. 1.0×10^8 m/s

b. 2.3×10^8 m/s

c. 3.0×10^8 m/s

d. 4.9×10^8 m/s

3. The sky is blue because it

a. Absorbs light at 390 nm

b. Reflects light at 390 nm

c. Absorbs light at 700 nm