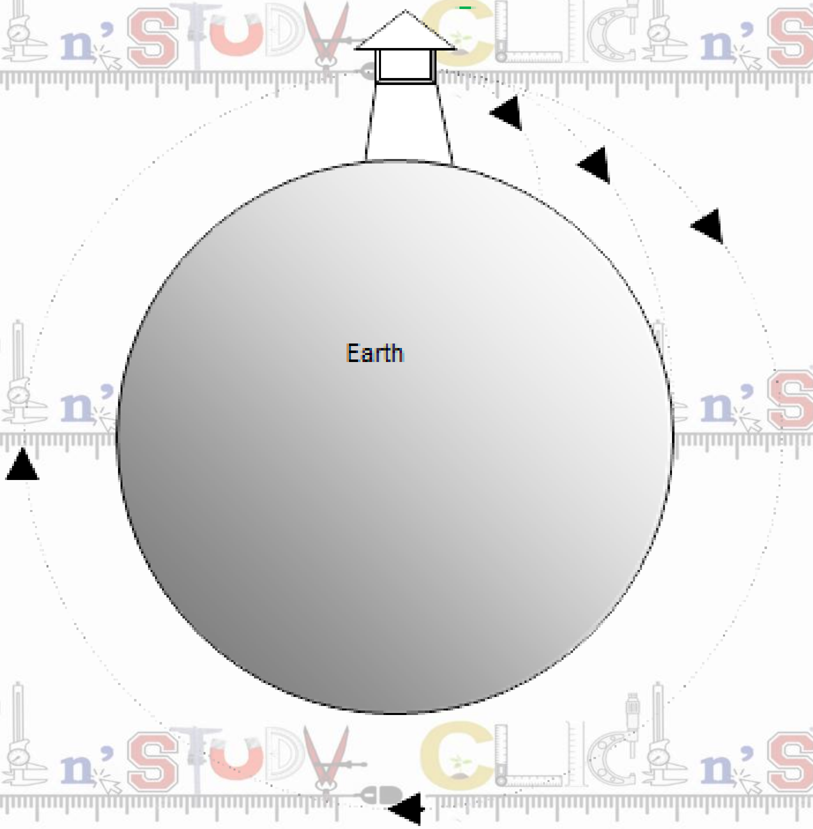
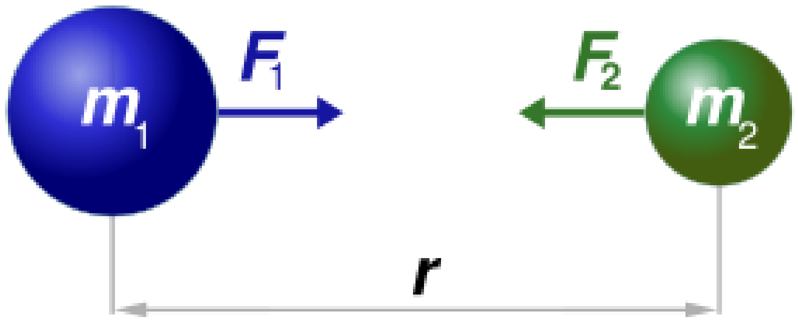
GRAVITATION FIELDS



Newton's law of gravitation

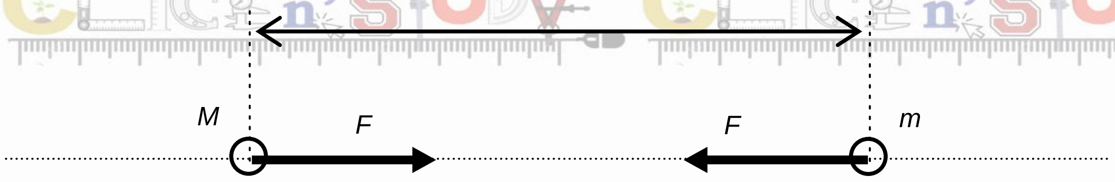
- Newton 's law of gravitation states that the force of attraction between particles is directly proportional to their mass and inversely proportional to the square of distance apart.



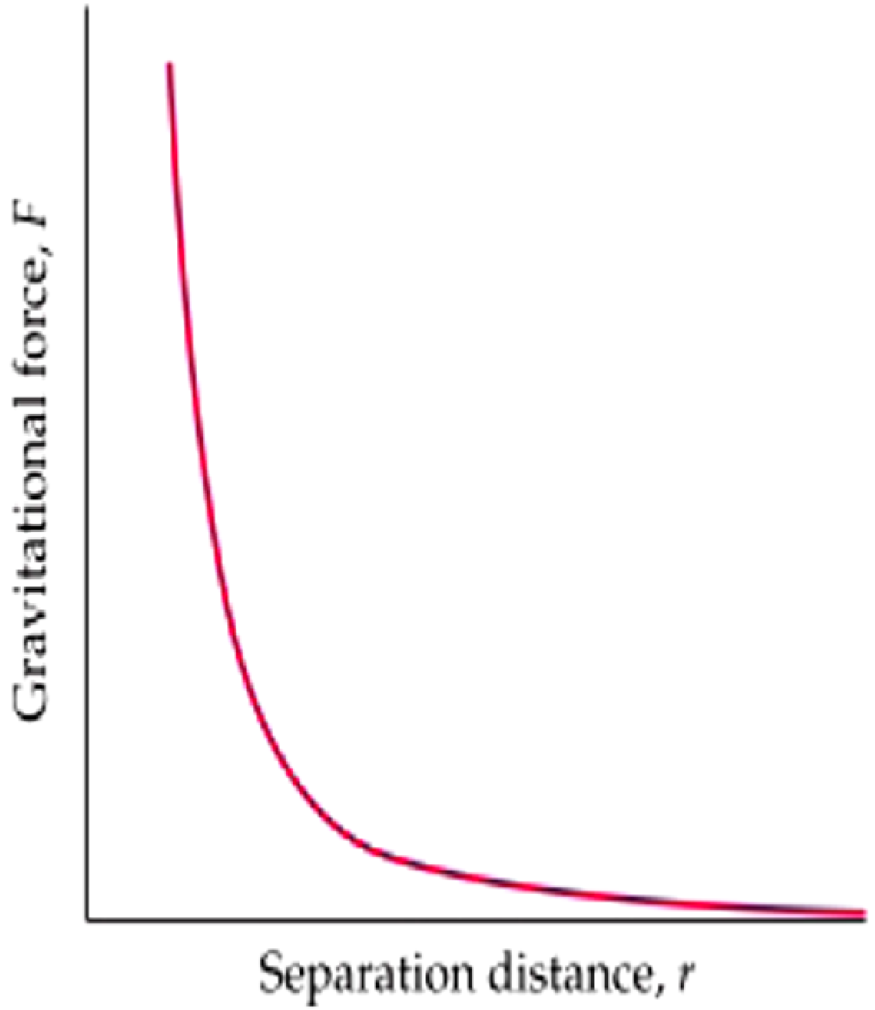
$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

where r is the distance between the masses and G is a constant referred as the **universal gravitation constant.**

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$
$$= 6.67 \times 10^{-11}\ \mathrm{m^3/kg \cdot s^2}.$$



- The two forces in the diagram are action-reaction pair because each force is acting on the object by the other object and they are of the same type of force.
- In that case, when Earth pulls you down, why did you not pull Earth up?
- You did! But the mass of Earth is relatively much bigger than your mass and hence its acceleration towards you is relatively much smaller.



The law of gravity applies to all objects small or large.

$$F = -\frac{Gm_1 m_2}{r^2}$$

- where The negative sign shows that the force is an attractive force.

- **Eg 1**: Determine the force of gravitational attraction between the earth 5.98 x 10^{24}kg and a 70 kg boy who is standing at sea level, a distance of 6.38 x 10^{6} m from earth's center.
- m_1 = 5.98 x 10^{24}kg , m_2 = 70 kg, r = 6.38 x 10^{6} m,
- G = 6.6726 x 10^{-11}N-m^2/kg^2

$$F = \frac{Gm_1m_2}{r^2} = \frac{(6.673 \times 10^{-11})(5.98 \times 10^{24})(70)}{(6.38 \times 10^6)^2}$$

$$= \frac{2.79 \times 10^{16}}{4.07 \times 10^{13}} = 685.43\ N$$

The Gravitational Field Strength

- During the 19^{th}century, the motion of the "***field***" entered physics (via Michael Faraday).
- Objects with mass create an ***invisible disturbance in the space around them*** that is felt by other massive objects - this is a ***gravitational field***. Gravitational Field
- Every object sets up a gravitational field around itself due to its mass. When two objects enter each other's gravitational fields, they will be attracted towards each other. Hence, a gravitational field is a region of space in which any object lies in it experiences a gravitational force towards the object that creates the field, due to its mass. (magnetic fields and electric fields are examples .)

- So, since the Sun is very massive, it creates an intense gravitational field around it, and the ***Earth responds to the field***.

- Definition: The gravitational field strength, g at a particular point in the gravitational field is defined as gravitational force per unit mass acting on a small test mass placed at that point.
- ***Gravitational Field Strength, $g = F/m$***
- Why must the test mass be small?

The test mass must be physically small so that it does not distort or change the gravitational field generated by the source mass.

LO (d)

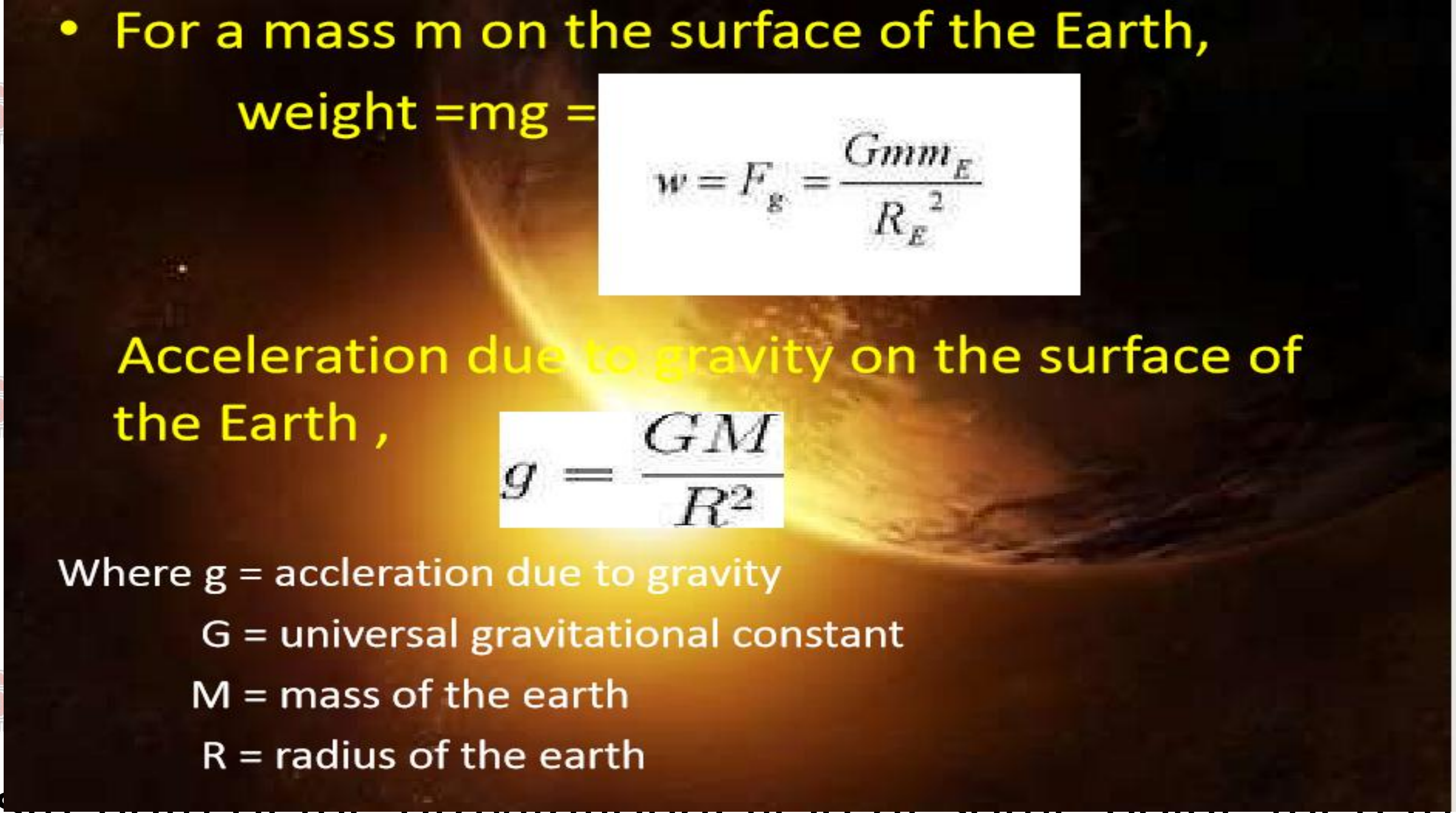
Based on (i) Newton's law of gravitation, where $|F| = \frac{G\,M\,m}{r^2}$ (gravitational force acting on a point mass, m by the source mass, M); and (ii) gravitational field strength, g is the gravitational force, F per unit mass acting on the small test mass, m, we may derive that the gravitational field strength,

$$g = \frac{F}{m}$$

$$= \frac{G\,M\,m}{r^2} \Big/ m$$

$$= \frac{GM}{r^2}$$

- Near the surface of the Earth, $g = F/m = 9.8$ N/kg $= 9.8$ m/s^2.
- In general, ***$g = GM/r^2$***, where M is the mass of the object creating the field, r is the distance from the object's center, and $G = 6.67 \times 10^{-11}$ Nm^2/kg^2.



If g is the strength of the gravitational field at some point, then the gravitational force on an object of mass m at that point is ***F_{grav} = mg.*** The gravitational field strength of Earth is approximately constant at 9.81 (in N/kg), near its surface. It is also known as the acceleration of free fall or the acceleration due to gravity, g (in m/s2).

Questions

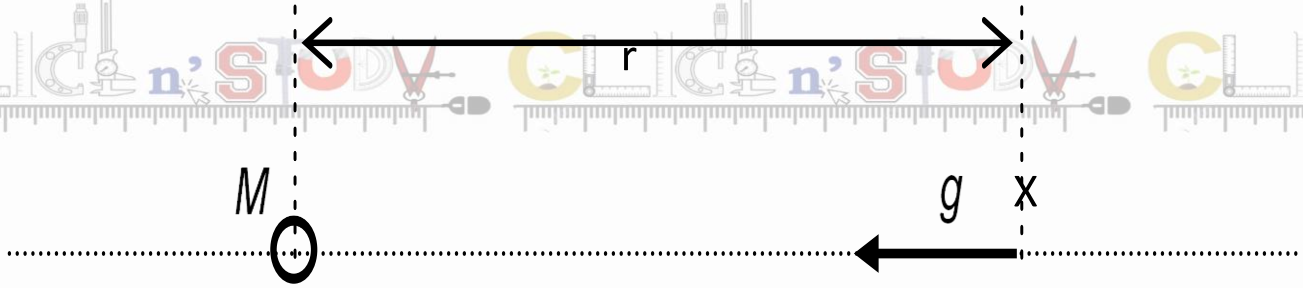
1. A point mass produces a gravitational field strength of 3.4×10^{-2}N/kg at a distance of 4.9×10^{2}m away. What is the mass of the point?

- **Answer**: g = Gm/r^2Thus , m = gr2/G = 1.2×10^{14} kg

2. Estimate the gravitational field strength at the surface of an interstellar body whose density is 5.5×10^{3}kg/m^3 and radius is 6.4×10^{6} m.

Answer : g = Gm/r2 and m = density x volume = D x V = D x 4/3 x p x r3
Substitute for m in the field equation above.

Thus, g = 4pGDr/3 = 4 x p x 6.67 x 10-11 x 5.5 x 103 x 6.4 x 106/3 = 9.8 N/kg



•Consider a point X in the field set up by mass *M* located at a distance *r* from the center of the mass *M*, the <u>magnitude</u> of the gravitational field strength at point X due to mass *M* is **, $g = GM/r^2$**

- Note:

1) Gravitational field strength, g is a vector quantity, and it is in the same direction as the gravitational force.

2) The gravitational field strength, g of the source mass, M is independent of the mass of the test mass.

3) As distance r of the test mass from source mass increases, g decreases in an inverse square law manner. Hence gravitational field is also known as an inverse square law field.

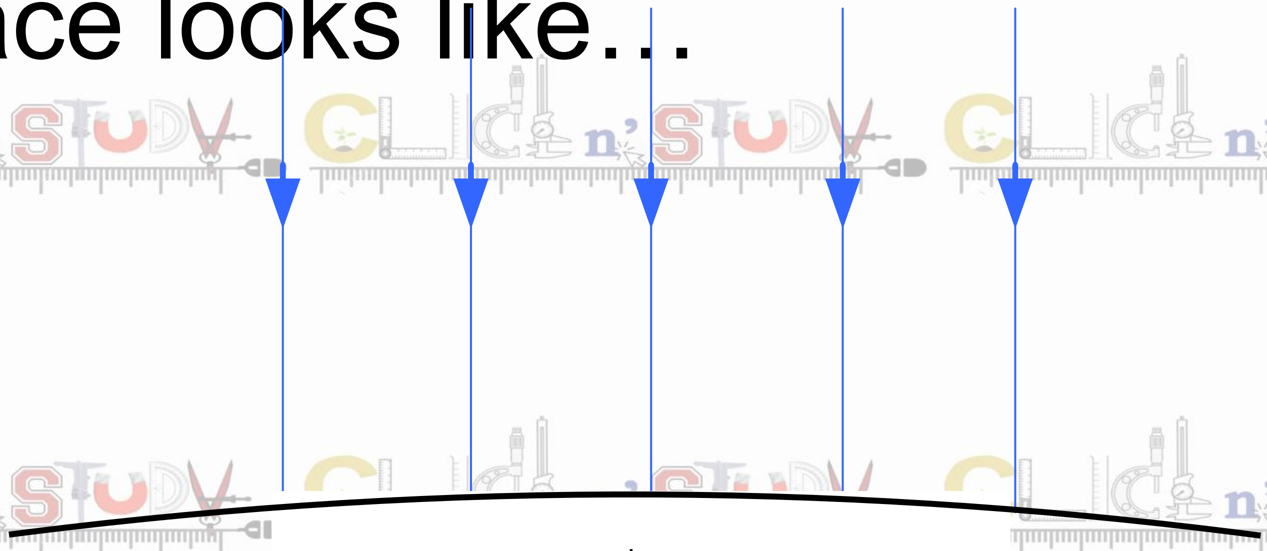
G is the Universal Gravitational Constant.

It is a scalar quantity with dimension $L^3M^{-1}T^{-2}$

g is the acceleration due to gravity .

It is a vector quantity with dimension LT^{-1}

Gravitational field is invisible and is represented by imaginary field lines. The Earth's gravitational field near to Earth's Surface looks like…



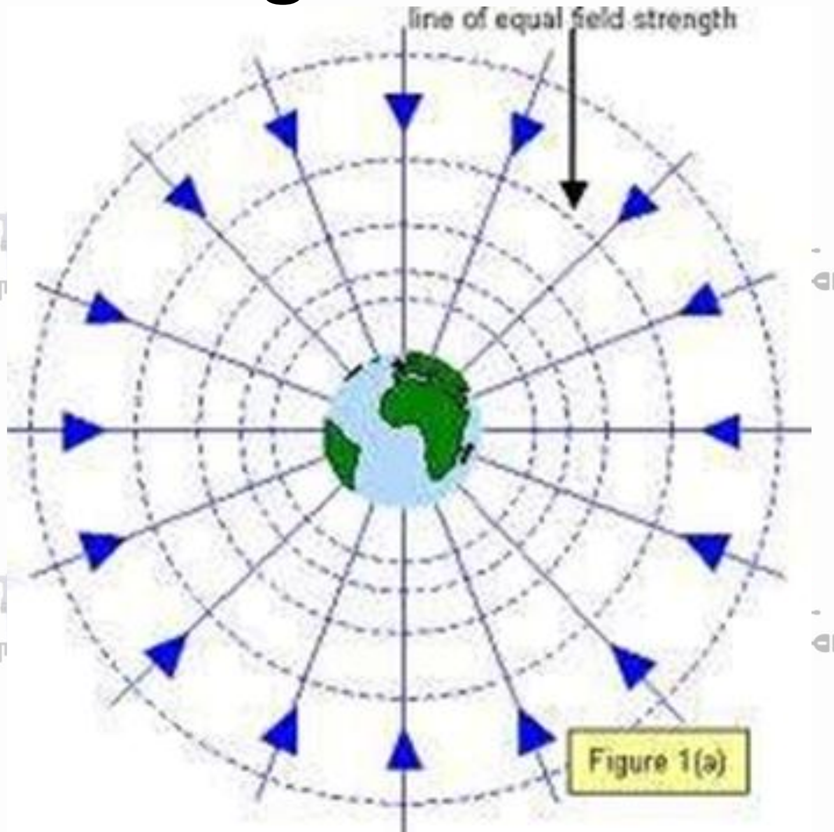
- The gravitational field near Earth's surface is uniform.
- The closer the field lines, the stronger the field strength.
- The field lines should be drawn parallel to each other and of equal spacing.

The Earth’s gravitational field over large distances from the Earth

- The gravitational field around Earth is non uniform.
- The field lines should be drawn radially pointing towards the center of Earth.
- The field lines get further apart

(field strength decreases) as it gets

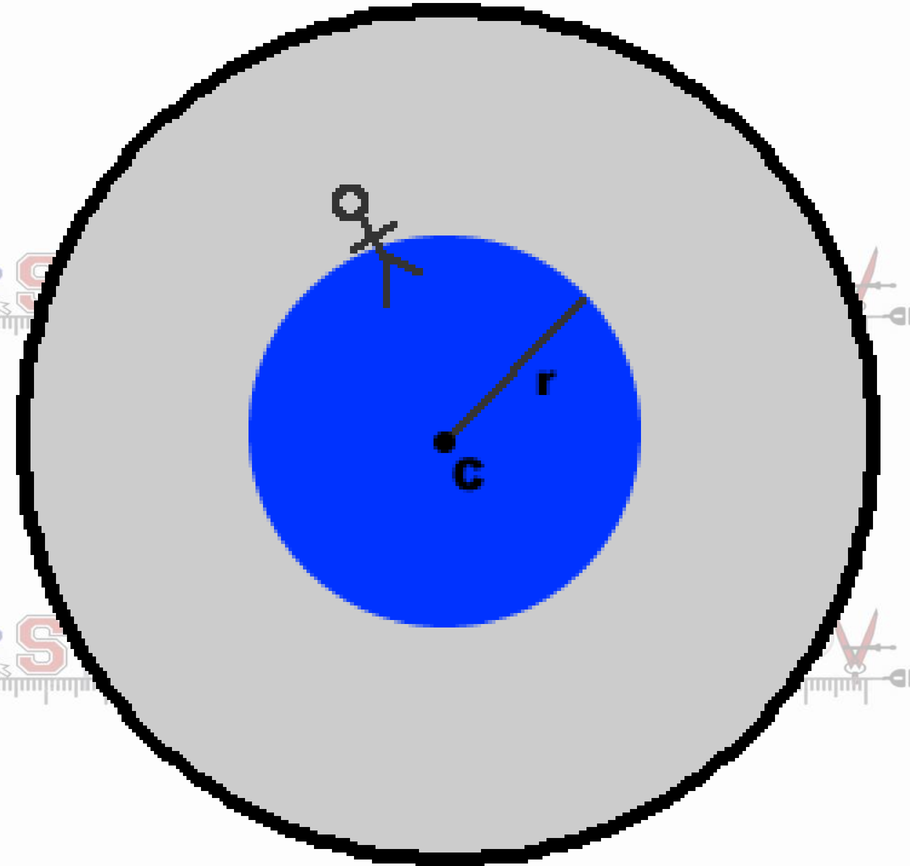
further from Earth.



Gravitational Field Inside a Planet

- If you are located a distance r from the center of a planet:
  - all of the planet's mass inside a sphere of radius r pulls you *toward the center* of the planet.
  - All of the planet's mass outside a sphere of radius r exerts ***no*** net gravitational force on you.

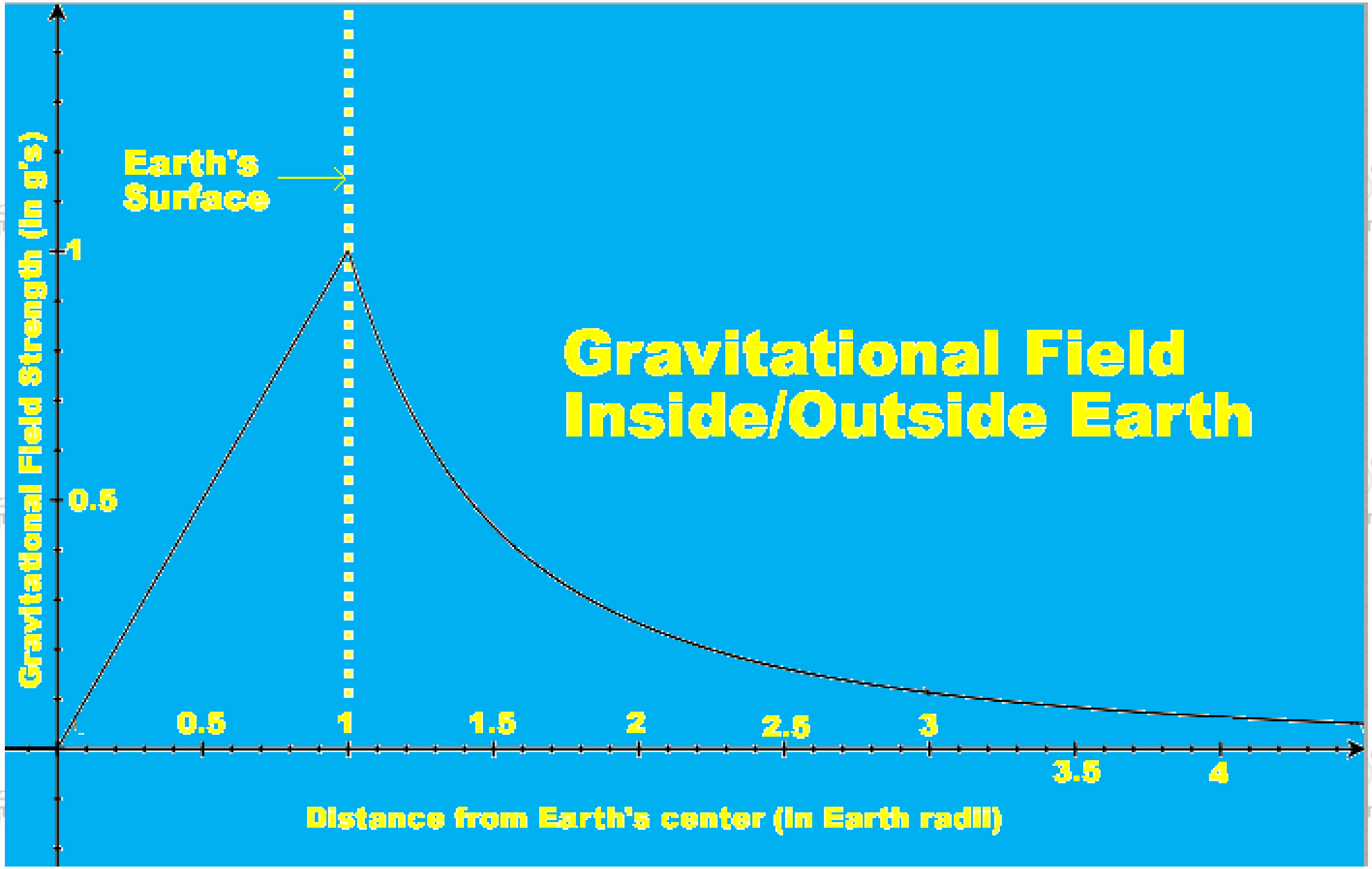
- The blue-shaded part of the planet pulls you toward point C.
- The grey-shaded part of the planet does not pull you at all.

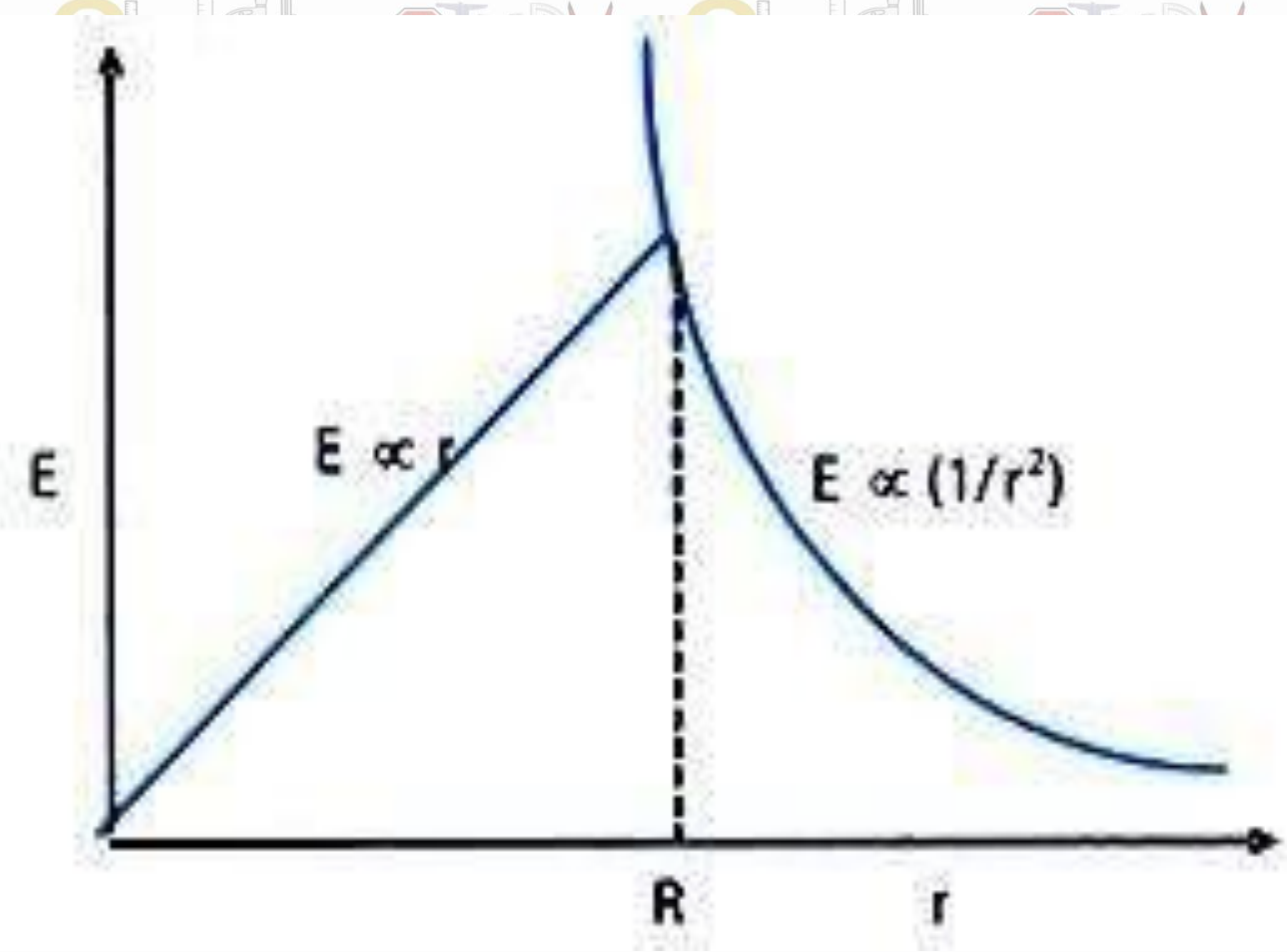


Gravitational Field Inside a Planet

- Half way to the center of the planet, g has one-half of its surface value.

- At the center of the planet, g = 0 N/kg.





The variation of the acceleration due to gravity g' with distance r from the center of the Earth is illustrated by the graph above. *To prove, refer notes.*

Gravitational Potential

- The Strength of the gravitational force at a point in a gravitational field is described by the gravitational field strength **E** or **g** is a **Vector Quantity.**

- Another quantity associated with the point in the gravitational field is the gravitational Potential. It is a **Scalar Quantity**.

- Gravitational Potential V
  - Amount of work done in bringing a body of unit mass from infinity to the given point
  - Gravitational Potential = $-\frac{GM}{r}$

1)The work done or Energy released is negative. The work to be done or E absorbed is positive.

2) Gravitational potential is a scalar quantity. (i.e. it has no direction and a negative V means it is less than zero).

3) This expression implies that V is also always negative (less than zero) and by convention, the gravitational potential at infinity is also taken to be zero (maximum V value).

4) Similar to gravitational field strength, gravitational potential is also independent of the mass of the test mass.

5) As distance r of the point mass from source mass increases, V increases.

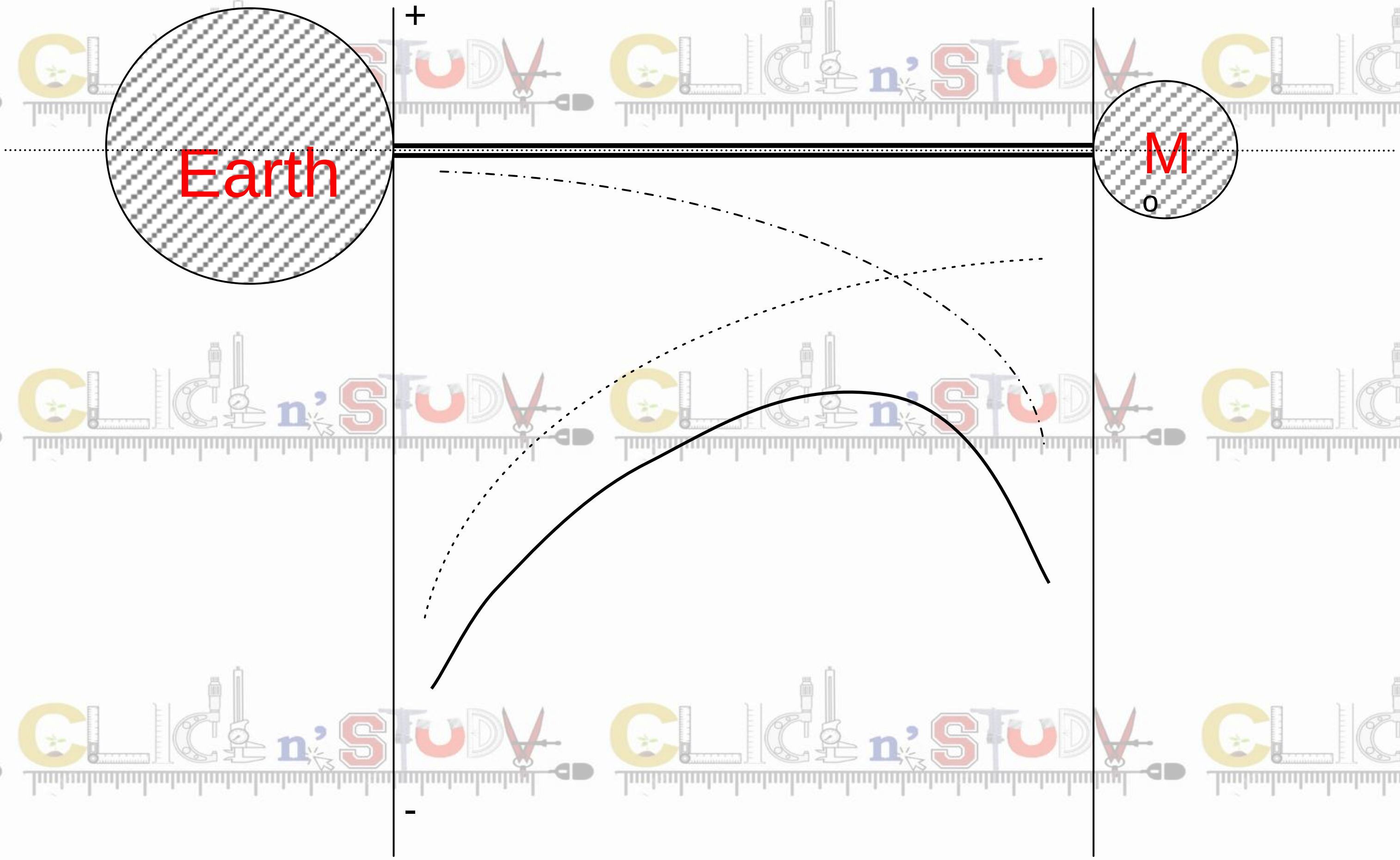
6) units: J kg-1

- The gravitational potential V at a point P in a gravitational field is defined as the work done per unit mass to bring a body *from infinity to P*. The unit for gravitational potential is Jkg^{-1}.
- The gravitational potential energy U of a body at a point P in a gravitational field is defined as the work done to bring the body *from infinity to P*. The unit for gravitational potential energy is J.
- Hence the gravitational potential energy U of a body of mass m at a point where the gravitational potential, V is given by

$$U = mV$$

Conversely,

$$V = \frac{U}{m}$$



Gravitational potential due to **Earth**

Gravitational potential due to Moon

Net gravitational potential along the line of centers is equal to the **scalar addition** of the gravitational potentials due to the E and M.

$$U = -\frac{GMm}{r}$$

where

U is the gravitational potential energy of the system [J]

G, the constant of universal gravitation, is 6.67×10^{-11} N m^2 kg^{-2}

M is one of the point mass [kg]

m is the other point mass [kg]

r is the ***centre–to–centre distance*** between the two point masses [m]

$$U = -\frac{GMm}{r}$$

To understand the formula for calculating GPE, *U*:

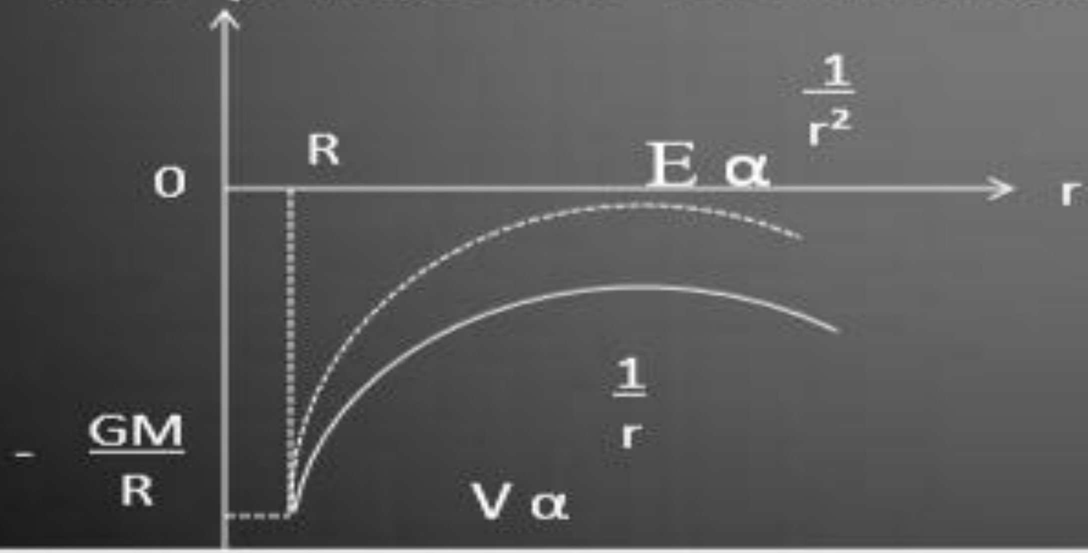
- Gravitational potential energy is a **scalar** quantity (i.e. it has no direction and a negative U value simply means it is *less than zero*).

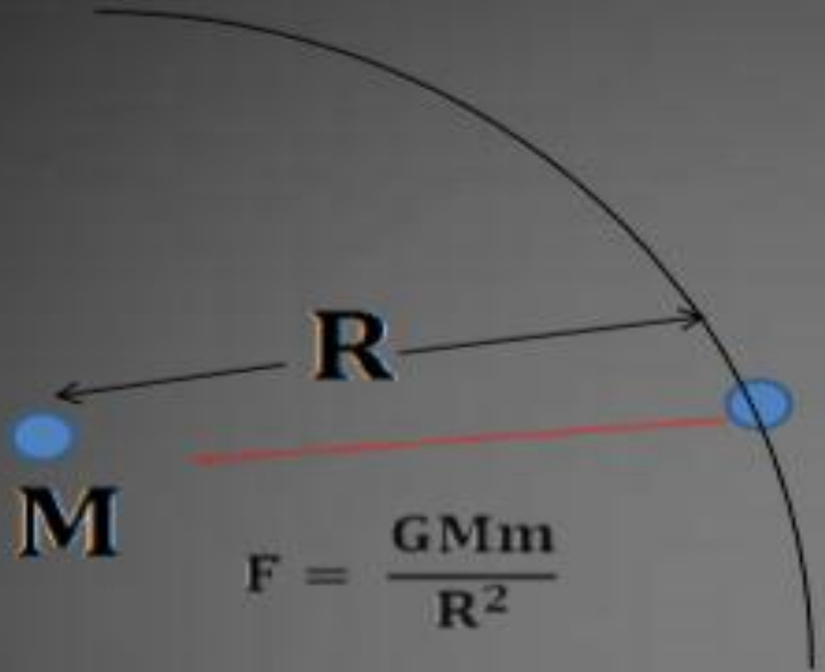
- This expression implies that U is **always negative** (less than zero) and the larger the r, the smaller the value of $\frac{GMm}{r}$ and hence the larger the value of $U = -\frac{GMm}{r}$. (For eg. – 2 is larger than – 4)

- When the object is moved to an infinitely far place where $r = \infty$, U becomes zero (which implies maximum gravitational potential energy, since zero is larger than any negative values).

- By standard convention, infinity is taken as the reference level, which has zero gravitational potential energy. But please note that this zero GPE is the maximum U, not the minimum U!

- On the surface on the Earth, r = R
  - Gravitational potential, $V = -\frac{GM}{R}$
  - Gravitational potential energy, $U = -\frac{GMm}{R}$
- The graph illustrates the variation of the gravitational potential V with distance r from the centre of the Earth.





$$F = \frac{GMm}{R^2}$$

$$mg = \frac{GMm}{R^2}$$

$$g = \frac{GM}{R^2}$$

$$gR^2 = GM$$

Where g = accerelation due to gravity

R = constant radius of earth

G = universal gravitational constant

M = mass of Earth

EXAMPLE :

A planet of mass M and radius r rotates about its axis with an angular velocity large enough to substances on its equator just able to stay on its surface . Find in terms of M , r and G the period of rotation of the planet.

$$g' = g - r\omega^2$$

$$g' = 0$$

$$g = r\omega^2$$

$$g = r\left(\frac{2\pi}{t}\right)^2$$

$$g = \frac{4\pi^2 r}{t^2}$$

$$GM = gR^2$$

$$GM = \left(\frac{4\pi^2 r}{T^2}\right) r^2$$

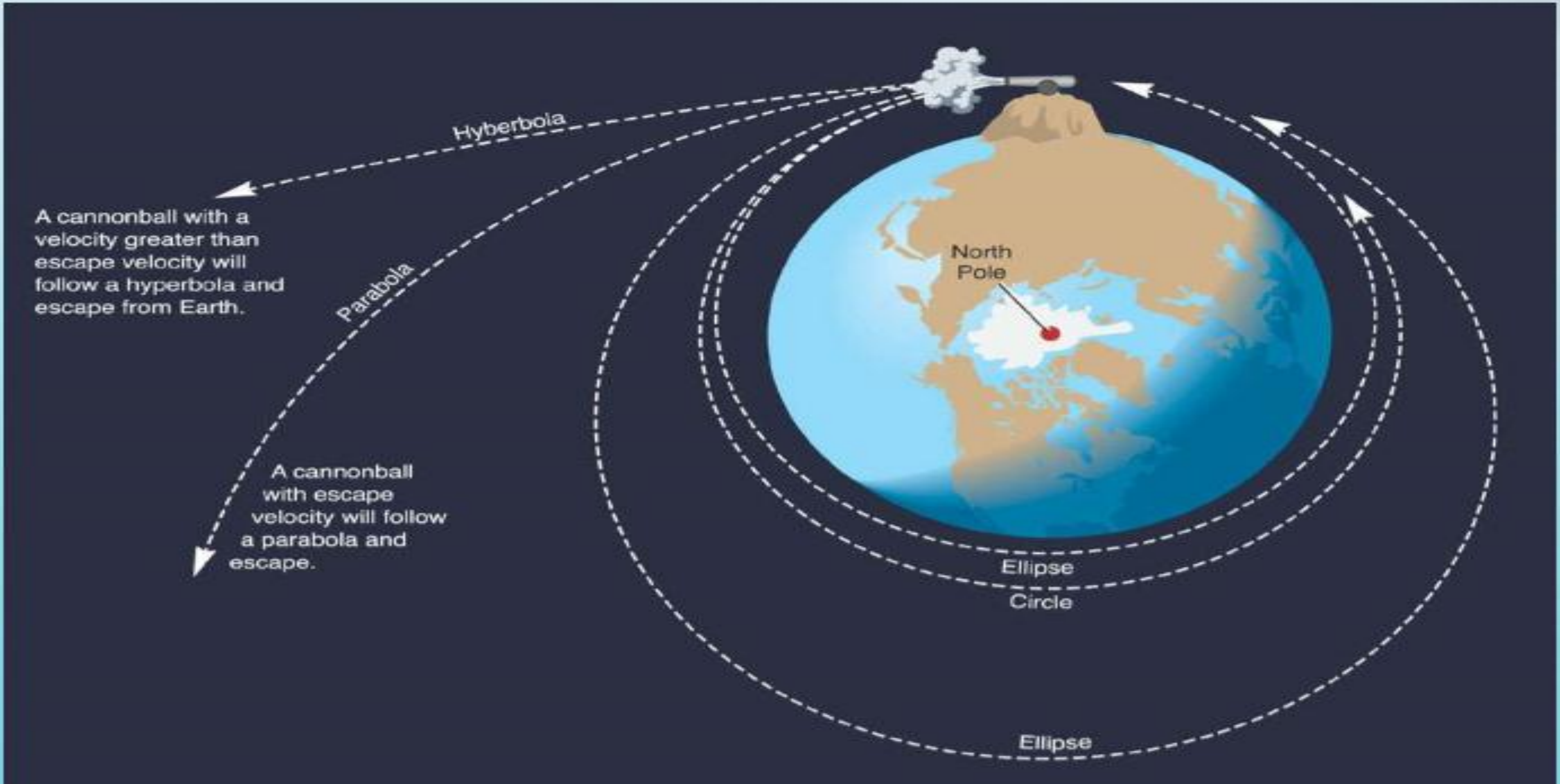
$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

Satellite Motion in Circular Orbits

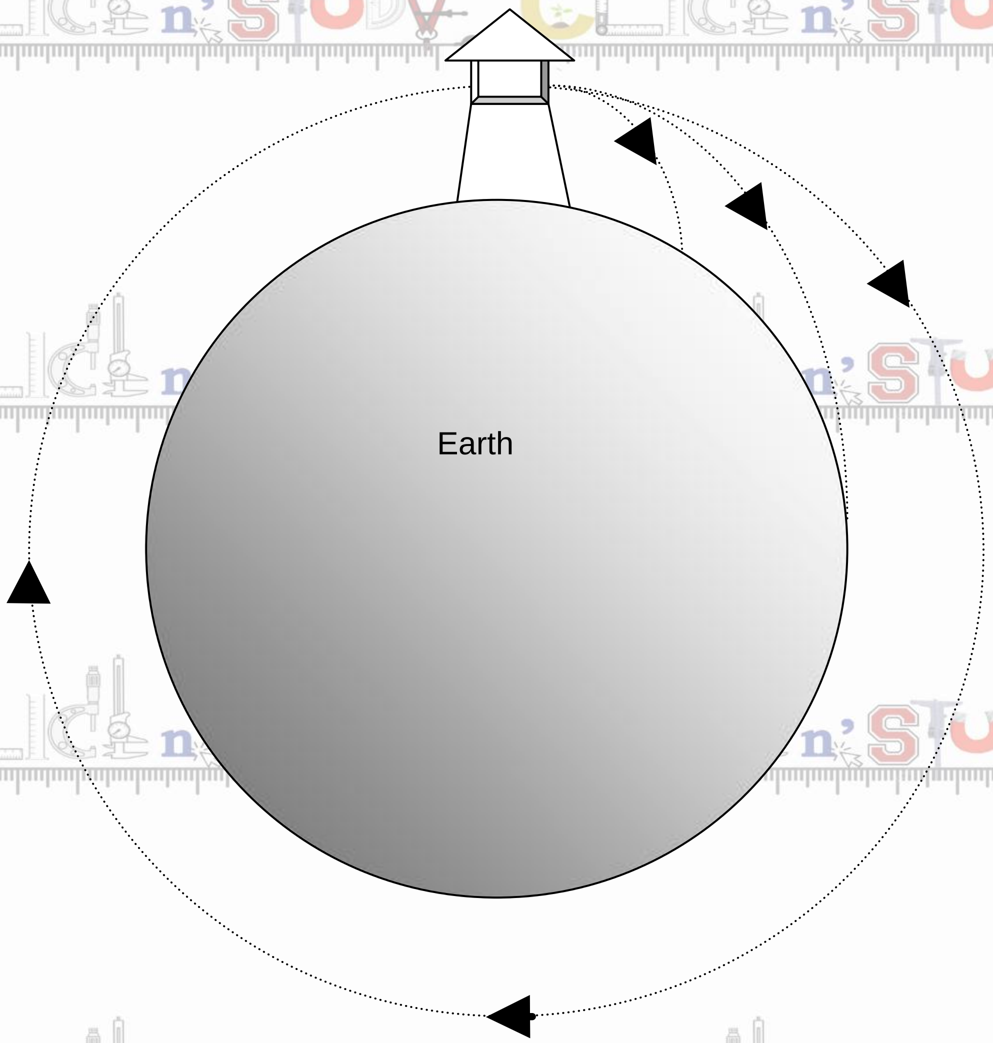
Satellite is a body that revolves round a planet. Satellites can be categorized as natural satellites or man-made satellites. The moon, the planets and comets are examples of natural satellites. Examples of man-made satellites are Sputnik I , Measat I ,II and III which are communication satellites.

In order to launch satellite into orbit , rockets are used. When rocket that carries the satellite reaches the required height , the satellite is launched into circular orbit with a certain velocity v that is tangential to intended orbit.

- Orbital velocity is the velocity required to put the satellite into its orbit around earth



An object projected horizontally near the Earth's surface follows a parabolic trajectory as shown on the right. As the speed of projection increases, the object will reach a speed where the trajectory follows the curvature of the Earth's surface. If air resistance is negligible, the object will orbit round the Earth continuously and will never hit the Earth's surface.



- Many man-made satellites move in circular orbits around the Earth. The first man-made satellite, the “Sputnik 1”, was launched by Soviet Union in 1957. Since then, hundreds of satellites have been launched into orbit around the Earth. The only force acting on the satellite in a circular orbit is the Earth’s gravitational force, which is directed towards the center of Earth (also the center of its circular orbit). Since the satellite moves perpendicular to the gravitational force, its magnitude of velocity remains constant while its direction changes. This means that the satellite is travelling in a uniform circular motion with constant distance from the satellite to the center of Earth.

Since the centripetal force required for the circular motion of the satellite is provide by the gravitational attraction of the Earth on the satellite,

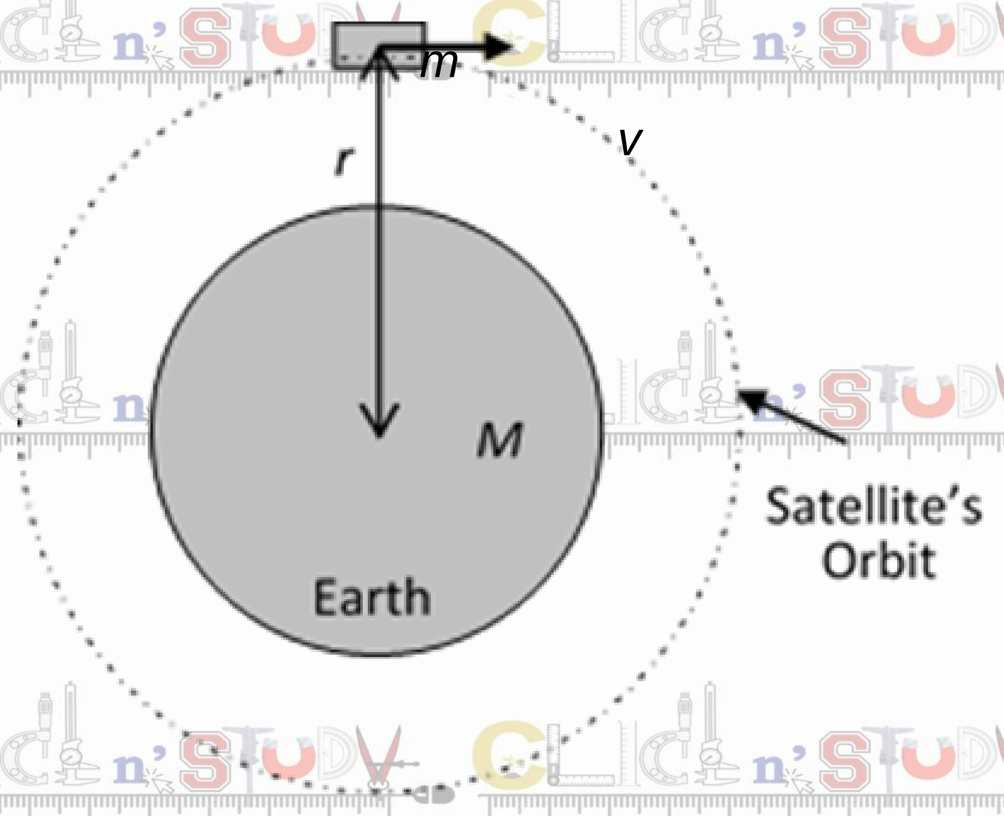
$$(mv^2) / r = G(Mm) / r^2$$

Velocity of satellite,

$$v = [(GM) / r]^{1/2}$$

since $GM = gR^2$,

$$v = [(gR^2) / r]^{1/2}$$



For a satellite orbiting close to the Earth's surface , the radius of the satellite's orbit is approximate to the radius of the surface.

$$(r = R)$$

$$V = 7.92 \times 10^3 \ ms^{-1}$$

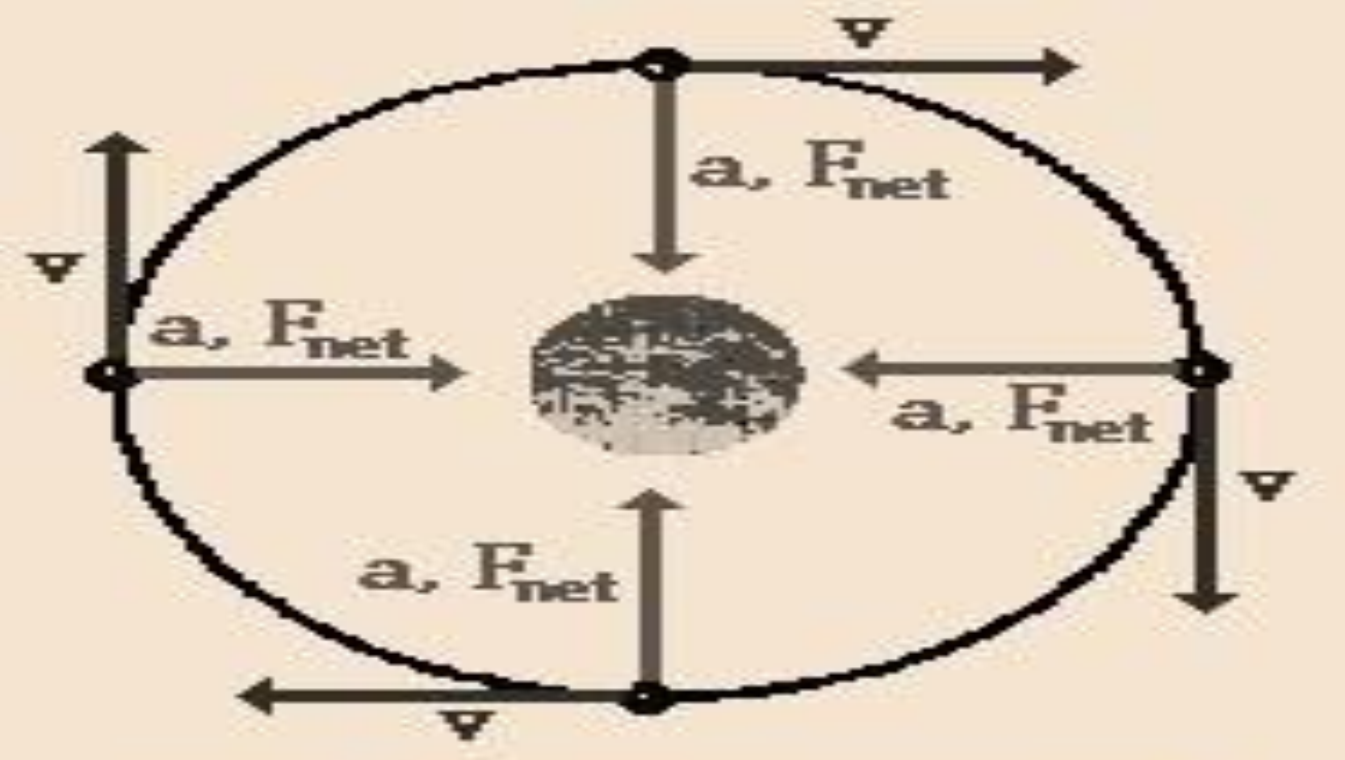
Period of satellite's orbital , T = 85 minutes

where v denotes the orbiting speed of satellite.

The above formula can be used to calculate the speed required for any object to orbit around a planet of mass M at a constant distance r.

What will happen to the orbiting satellite if it starts to slow down?

The gravitational force will be higher than the required centripetal force to keep it in the uniform circular motion. Hence the satellite will be pulled closer towards Earth and move in a smaller circular orbit.



Satellites encounter inward forces and accelerations and tangential velocities.

Energy of a Satellite

Total energy of the satellite ,

$$E = U + K$$

$$E = -(GMm)/r + (GMm)/2r$$

As an object approaches the Earth ,

*Its kinectic energy K increases as the gravitational pull of the Earth on it increase.

*Its gravitational potential energy U decrease , becoming more and more negative.

*Its total energy E also increase.

A satellite <u>in orbit</u> possesses both kinetic energy, E_K, (by virtue of its motion) and gravitational potential energy, E_P, (by virtue of its position within the Earth's gravitational field).

Hence, total energy of an orbiting satellite, $E_T = E_P + E_K$

$$= -\frac{GMm}{r} + \frac{1}{2}mv^2 \qquad \text{--- Equation (1)}$$

Recall that for a satellite <u>in orbit</u>, its gravitational force acts as the centripetal force:

$$\Sigma F = \frac{mv^2}{r} \qquad \rightarrow \qquad \frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\text{i.e.} \quad \boxed{E_K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r}} \qquad \text{--- Equation (2)}$$

Substituting equation (2) into (1),

Hence total energy of an orbiting satellite, $E_T = E_P + E_K$

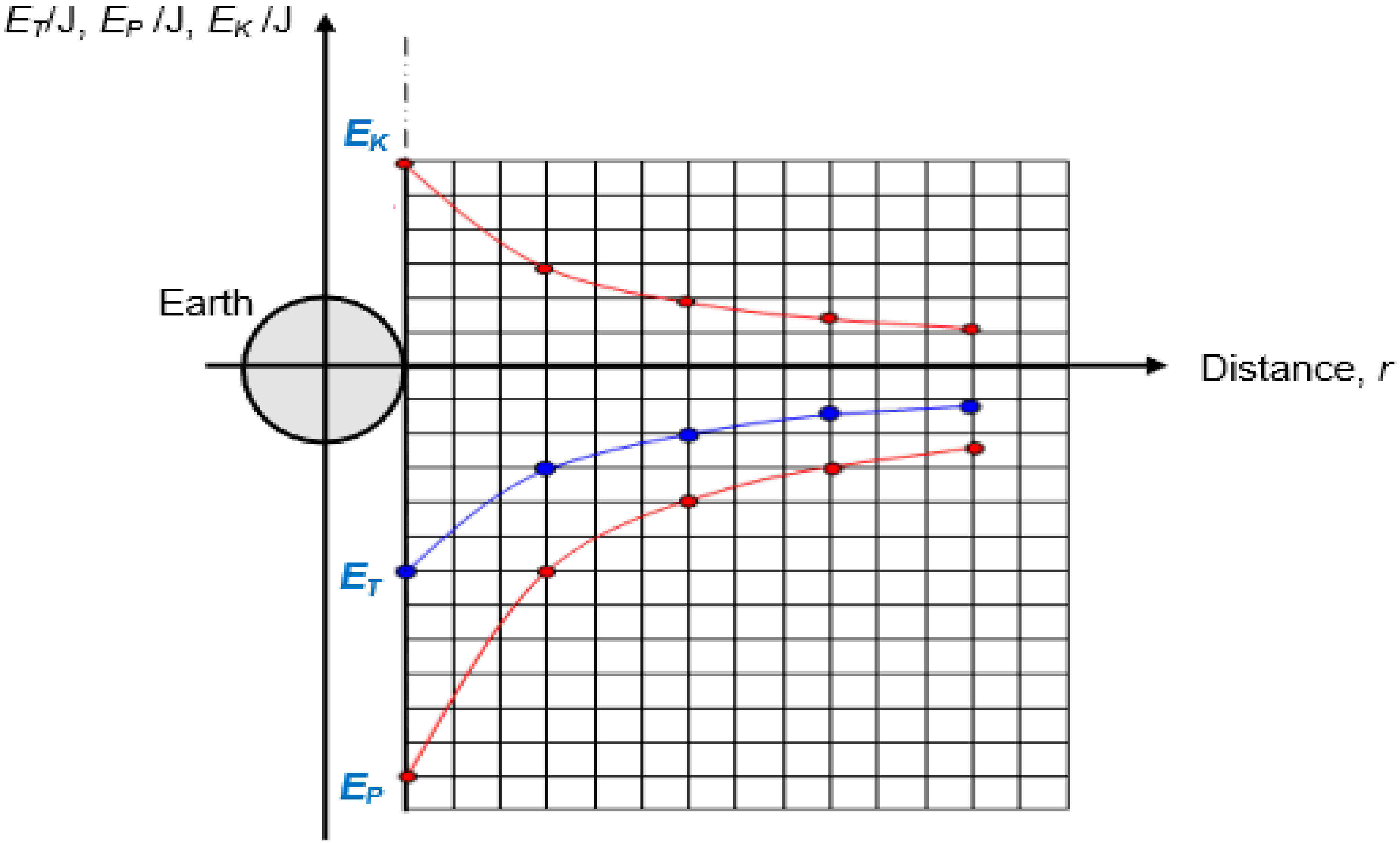
$$E_T = -\frac{GMm}{r} + \frac{1}{2}mv^2$$

$$E_T = -\frac{GMm}{r} + \frac{GMm}{2r}$$

$$\boxed{E_T = -\frac{GMm}{2r}}$$

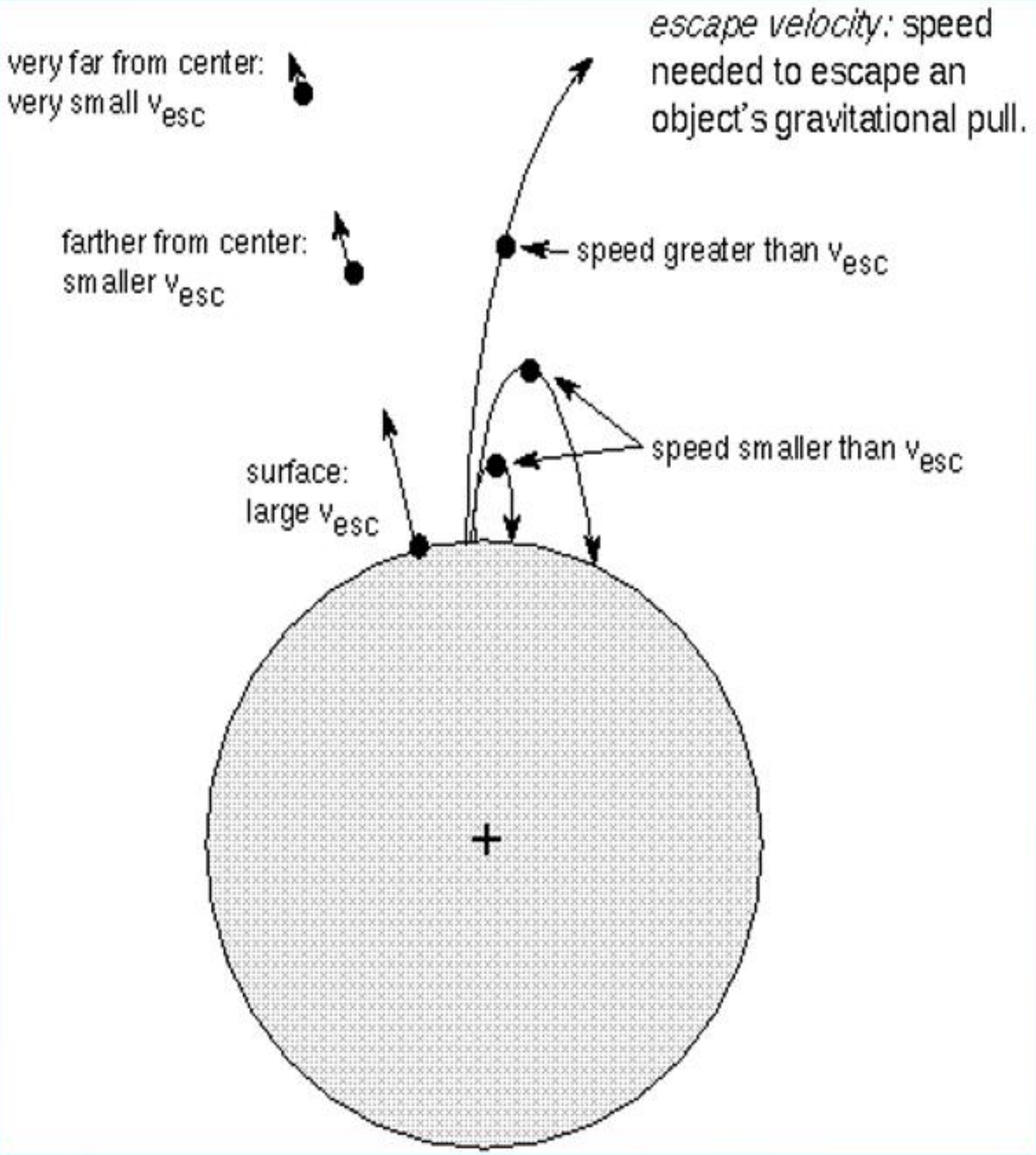
[Note: $E_T = -E_K$ or $\frac{1}{2} E_P$]

A typical graph showing the relationship between E_T, E_P and E_K with respect to the distance, r, from centre of Earth, is shown below. Label the graphs accordingly.



Escape Velocity

- If we throw a ball into air, it rises to a certain height and falls back. If we throw it with a greater velocity , it will rise higher before falling down. If we throw with sufficient velocity , it will never come back.i.e. It will escape from the gravitational pull of the earth.
- The minimum velocity required to do so is called escape velocity.
- When a rocket is launched from the earth's surface , it can escape from gravitational pull of the earth if it has sufficient energy to travel to infinity.
- The energy required comes from the kinetic energy when the rocket is launched.
- Consider the earth to be a sphere of mass M and radius R with center O.



- Using total energy concept we can derive the equation for escape velocity.
- At any point the Total energy must be zero.
- We know that Kinetic energy = $\frac{1}{2} mv_e^2$
- Potential energy = $\frac{-GMm}{R}$
- K.E+P.E = 0
- $\frac{1}{2} mv_e^2 + \frac{-GMm}{R} = 0$
- $v_e^2 = \frac{2GM}{R}$
- $V_e = \sqrt{\frac{2GM}{R}}$
- Multiply and divide by R
- $Ve = \sqrt{(2gR)}$

Expression for escape velocity Using the principle of conservation of energy

Kinetic energy required by the body on the Earth's surface = Gain in gravitational potential energy of the body when it moves from the Earth 's surface to infinity.

$$\frac{1}{2}mv_e^2 = U_\infty - U_R$$

$$\left(U_\infty = -\frac{GMm}{\infty} = 0\right)$$

$$= 0 - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$$

$$V_e = \sqrt{\frac{2GM}{R}}$$

Why the moon has no atmosphere?

It is because the speeds of the air particles are higher than their escape speeds and hence they can escape from the moon surface

Due to the small value of g. The escape velocity in the moon is 2.38 km/s . The air molecules have thermal velocity is greater than the escape velocity and therefore air molecules escape.

1. A tennis ball of mass 10g is attached to the end of a 0.75m string and is swung in a circle around someone's head at a frequency of 1.5 Hz. What is the tension in the string?

$$\omega = 2\pi f = 2\pi \times 1.5 = 3\pi \text{ rad s}^{-1}$$

$$F = T = m\omega^2 r = 0.01 \times (3\pi)^2 \times 0.75 = 0.0675\pi^2 = 0.666 \text{ N}$$

2. A planet orbits a star in a circle. Its year is 100 Earth years, and the distance from the star to the planet is 70 Gm from the star. What is the mass of the star?

100 days = 100 x 365.24 x 24 x 60 x 60 = 3155673600s

$$f = \frac{1}{T} = \frac{1}{3155673600} = 3.17 \times 10^{-10} \text{ Hz}$$

$$\omega = 2\pi f = 2\pi \times 3.17 \times 10^{-10} = 1.99 \text{ nrad s}^{-1}$$

$$\frac{GM}{r^2} = \omega^2 r$$

$$M = \frac{\omega^2 r^3}{G} = \frac{(1.99 \times 10^{-9})^2 \times (70 \times 10^9)^3}{6.67 \times 10^{-11}} = 2.04 \times 10^{25} \text{ kg}$$

3. A 2000 kg car turns a corner, which is the arc of a circle, at 20kmh^{-1}. The centripetal force due to friction is 1.5 times the weight of the car. What is the radius of the corner?

20kmh^{-1} = 20000 / 3600 = 5.56ms^{-1}

$$W = 2000 \times 9.81 = 19620 \text{ N}$$

$$F_r = 1.5 \times 19620 = 29430 \text{ N}$$

$$29430 = \frac{mv^2}{r} = \frac{2000 \times 5.56^2}{r} = \frac{61728}{r}$$

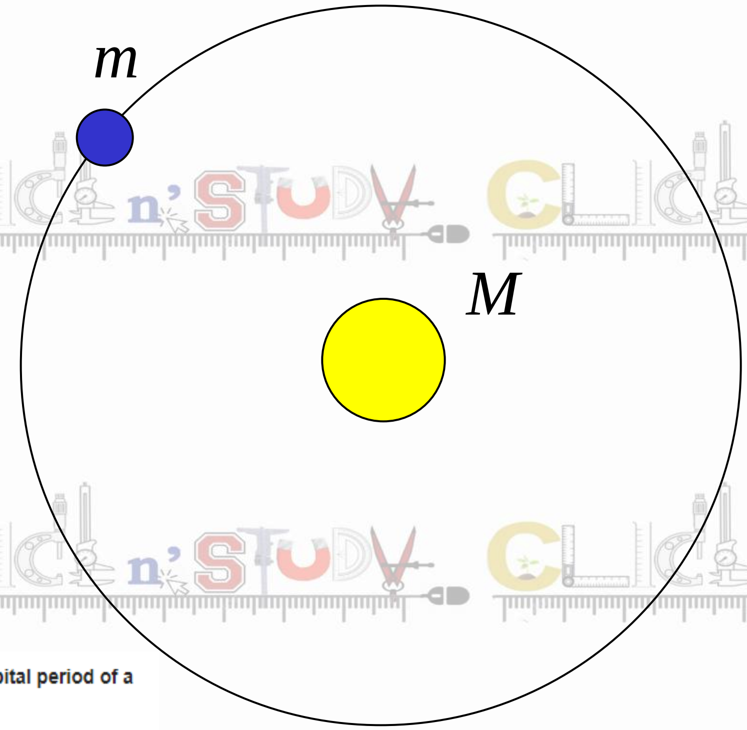
$$r = \frac{61728}{29430} = 2.1 \text{ m}$$

The square of the orbital period of any planet is proportional to cube of the average distance from the Sun/Moon to the planet.

$$F_{\text{grav}} = \frac{GMm}{R^2} = ma_{\text{cent}} = m\omega^2 R$$

$$\omega = \frac{2\pi}{T}$$

$$\Rightarrow \boxed{\frac{R^3}{T^2} = \frac{GM}{4\pi^2}}$$



4. Using the formulae for centripetal acceleration and gravitational field strength, and the definition of angular velocity, derive an equation linking the orbital period of a planet to the radius of its orbit.

$$\omega^2 r = \frac{GM_{star}}{r^2}$$

$$\omega^2 r^3 = GM_{star}$$

$$\omega = \frac{2\pi}{T}$$

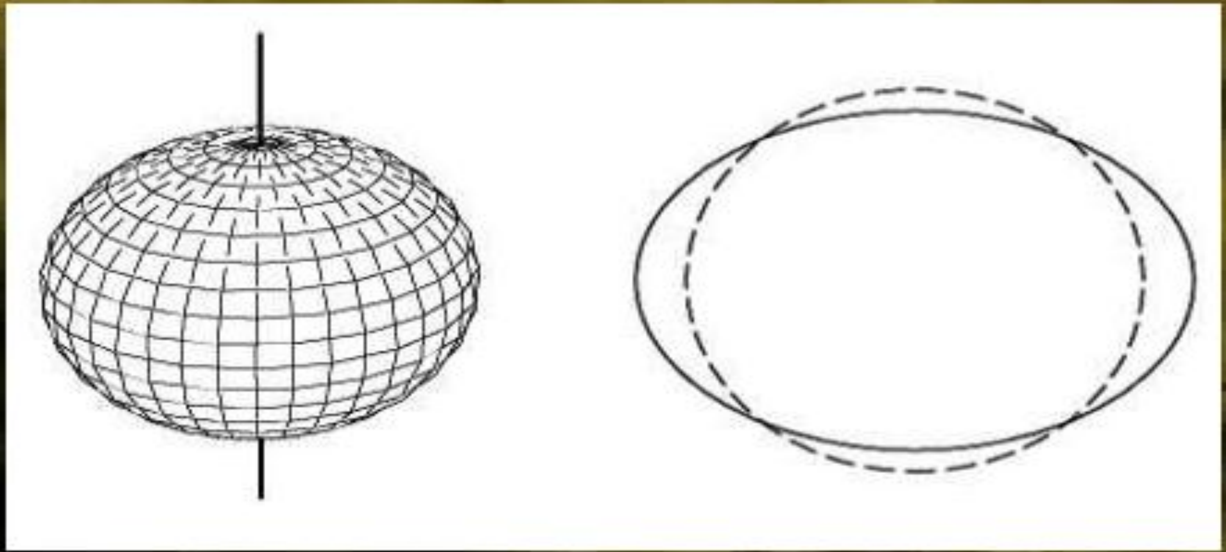
$$\frac{4\pi^2 r^3}{T^2} = GM_{star}$$

$$T^2 = \frac{4\pi^2 r^3}{GM_{star}}$$

So, orbital period squared is proportional to radius of orbit cubed. Incidentally, this is Kepler's Third Law in the special case of a circular orbit (a circle is a type of ellipse).

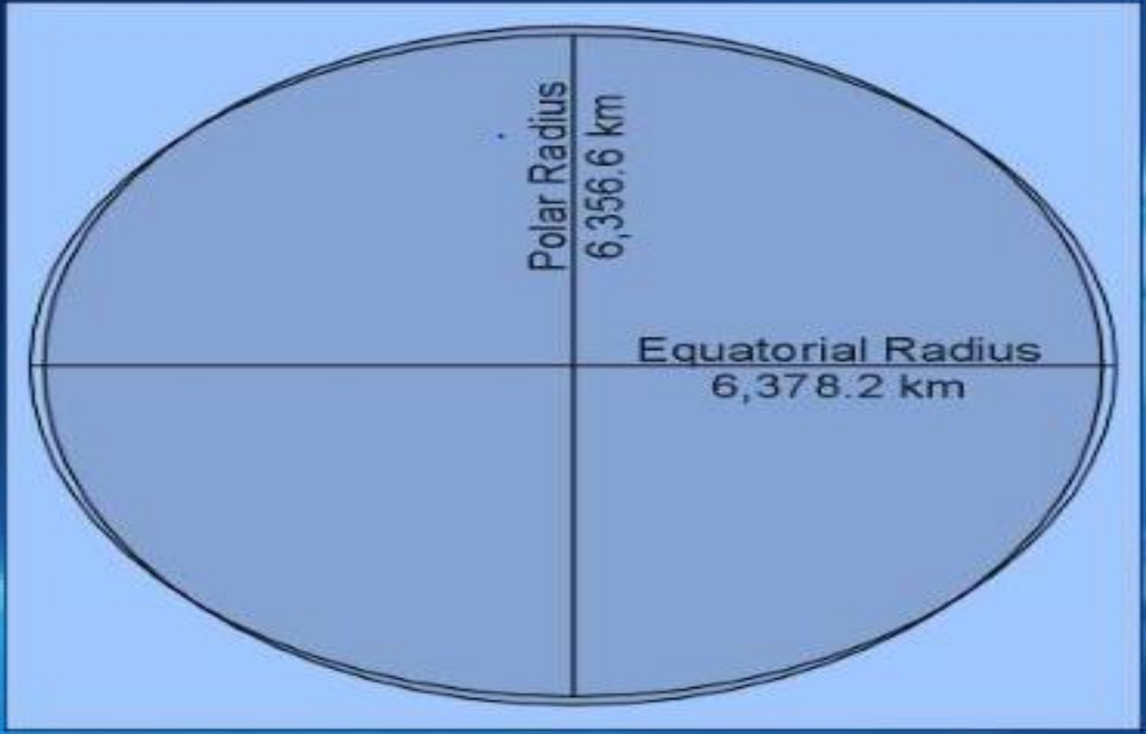
Variation of g with latitude

- The acceleration due to gravity, and the gravitational field strength on the surface of the Earth is not the same at different points on the Earth's surface because :-
- The Earth is not a perfect sphere, but it is an ellipsoid as shown in below.
- The rotation of the Earth about its axis.



- **The Earth's rotation on its axis causes it to bulge at the equator and flatten at the poles, forming an ellipsoidal shape.**

The figure below shows that the radius of the Earth at the equator R_1 is greater than the distance of the poles from the centre of the Earth R_2.



$g \propto 1/r^2$

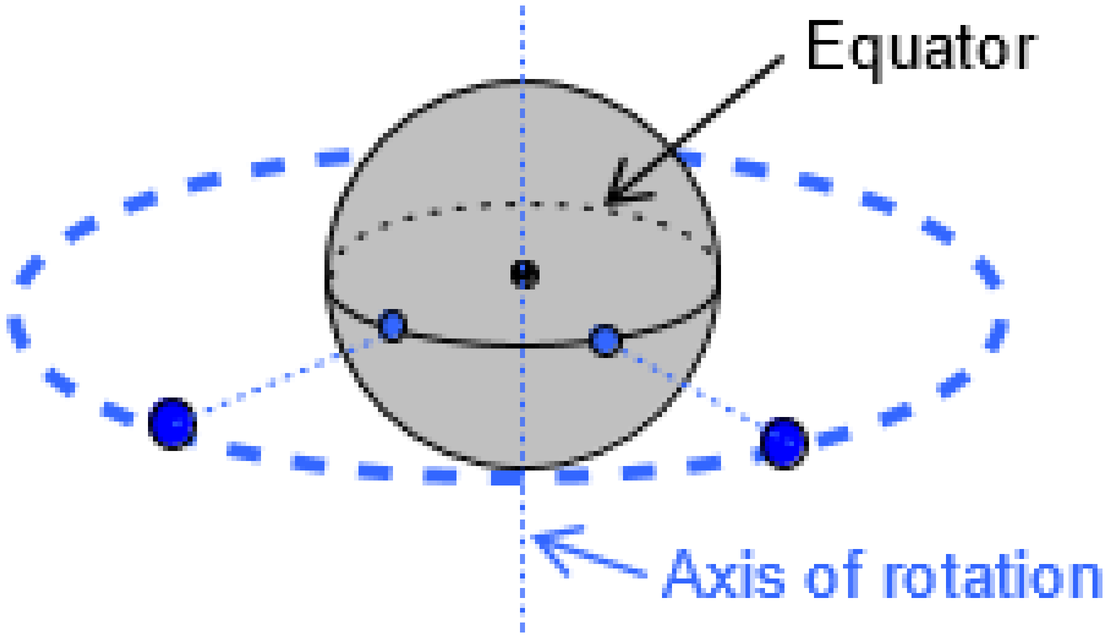
The value of g at the equator is less than the value of g at the poles.

Geostationary Satellites

A geostationary (Earth) satellite is a satellite that rotates around the Earth such that it is always positioned above the same point on the Earth's surface. Hence from the point of view of an observer standing at that point on Earth's surface, the geostationary satellite appears to be always 'stationary' above him/her (when actually, both observer and satellite are rotating at the same angular speed).

In order for a satellite to be moving in a geostationary orbit, it needs to meet the following conditions:

- Geostationary satellites must be **placed vertically above the equator** (so that its axis of rotation is the same as the Earth's);
- They must move **from west to east** (so that it moves in the same direction as the rotation of the Earth about its own axis);
- The **satellite's orbital period** must be equal to **24 hrs** (so that it is the same as the Earth's rotational period about its own axis).

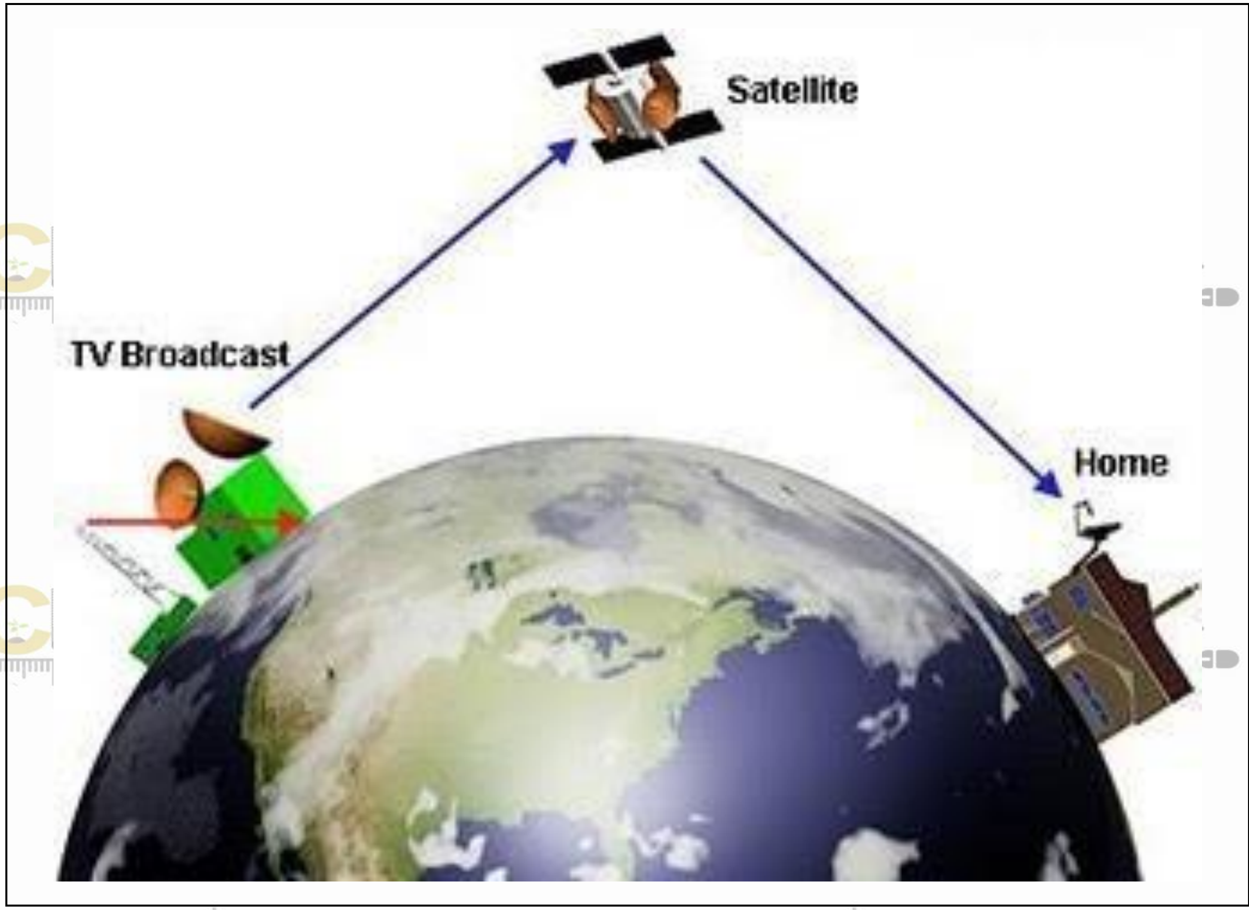


Uses of geostationary (Earth) satellite

- In communicating radio , T.V and telephone signals across the world.
- In studying the upper regions of the atmosphere.
- In Forecasting weather.
- In studying meteorites.
- In studying solar radiation and cosmic rays.
- And used in GPS (Global positioning System).

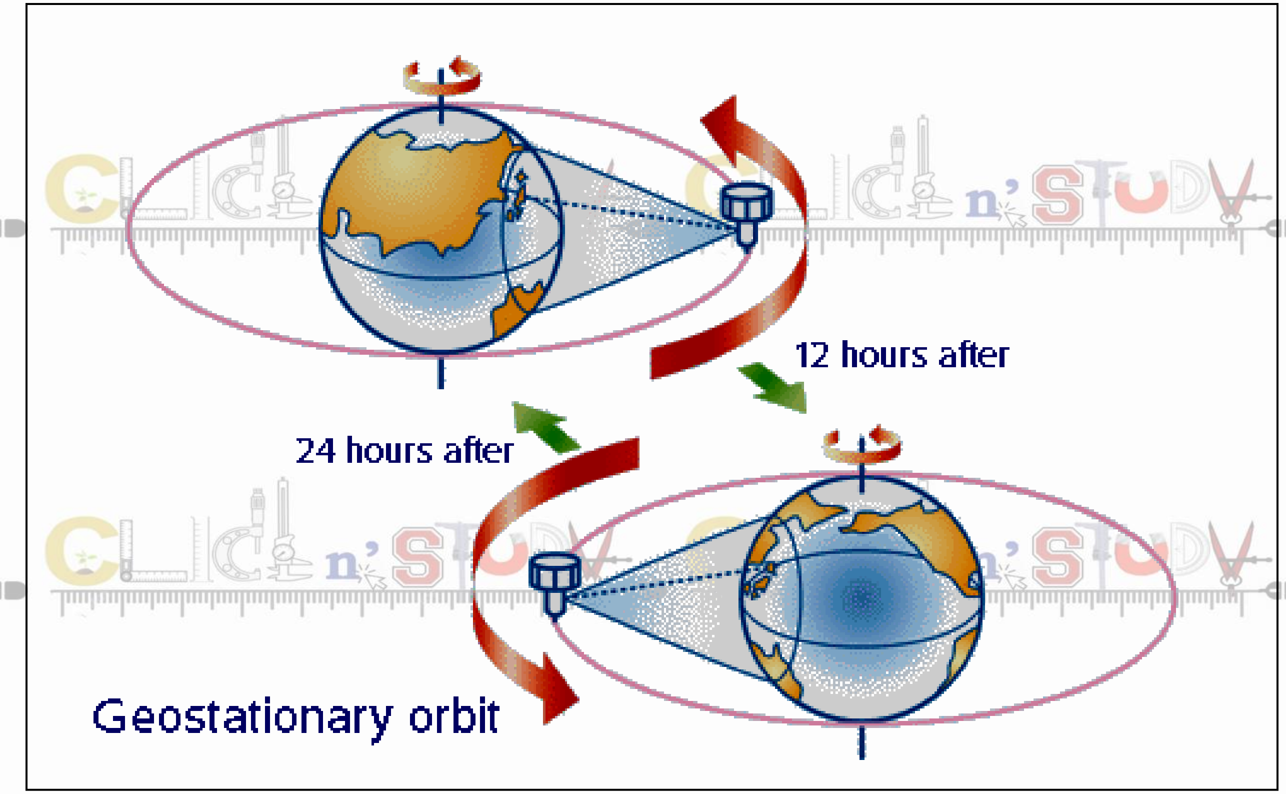
Advantages of geostationary satellites:

- A geostationary satellite is ideal for telecommunication & weather forcasting purposes since it remains 'stationary' above the same spot on the Earth's surface at all times (continuously observe the same area). The distance between the satellite and the transmitting station on Earth is kept relatively constant and a clear line of 'vision' between the transmitter and the receiver allows continuous and uninterrupted signal



Advantages of geostationary satellites:

- GSS use directional antennas for comn with gd stations.Since it is always at the same position relative to the Earth's surface, there is no need to keep adjusting the direction of the satellite dish to transmit or receive signals to or from the geostationary satellite.
- As geostationary satellites are positioned at a high altitude (a distance of 35 700 km away from the surface of the Earth), it can view and scan a large section of the Earth surface continuously. Hence, they are ideal for meteorological applications and remote imaging.



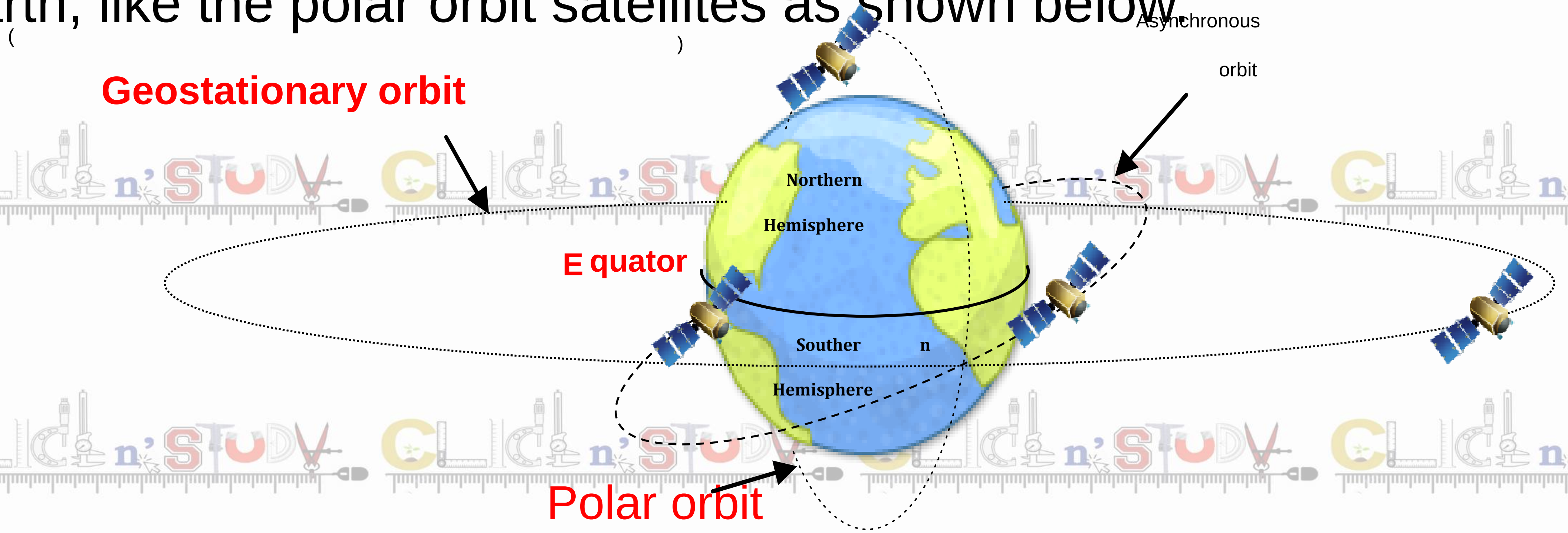
Disadvantages of geostationary satellites:

- The number of satellites that can be maintain in a Geostationary orbits without interfering one another is limited.
- As geostationary satellites are positioned at such a high altitude, the resolution of the images may not be as good as those captured by the lower orbiting satellites, specially at locations away from the equator.
- An EM signal emitted from a gd stn, travels at a speed of light.Because of its high altitude, there may be a delay in the reception of the signals by another stn, resulting in a lag time for live international broadcast or video conferencing.

- The transmitting stations in countries positioned at latitudes higher than 60 degrees may not be able to receive strong signals from geostationary satellites, as the signals would have to pass through a larger amount of atmosphere. This is true for countries beyond the 60 degrees latitude ‘belt’, both on the northern and southern sides.

- The damage caused by the EM radiation from the Sun when GSS comes closer to the sun especially when the Ssun passes through the equatorial plane during late March and September.

Besides geostationary satellites which are placed at a large distance from Earth, there are other types of satellite which orbit at lower altitudes from Earth, like the polar orbit satellites as shown below.



Satellites in polar orbits rotate around the Earth over the poles, in a constant plane perpendicular to the equator. Polar satellites have much lower altitudes (about 850 km) and they serve to provide detailed information about the weather and cloud formation. However satellites in this type of orbit can view only a narrow strip of Earth's surface on each orbit. Strips of images must be "stitched together," to produce a larger view.

- **Advantages of low altitude orbit satellites (eg. polar orbit):**

- Due to their lower altitudes, these satellites can capture images of the Earth’s surface with higher resolution. Polar satellites have the advantage of photographing close-up images of Earth.

- There is reduced lag time or delay between the transmission and reception of the signal.

- It needs less energy & resources to place a satellite into a Low Earth Orbit & less powerful amplifiers for successful comn.

Disadvantages of low altitude orbit satellites (eg. polar orbit):

- It is not possible to view the same spot on the Earth's surface continuously by a single satellite in a polar orbit because of its high speed. A typical low orbit satellite takes only 2 hours to make one revolution round the Earth. In order to have a continuous relay of data, there must be a series or chain of satellites in the same orbit so that one 'takes over' the predecessor's function.

- Because the satellite changes its location constantly with respect to the Earth's surface, the direction of the satellite dish would need to be adjusted constantly as well. LAOS use non directional antennas.