ELASTICITY

Q # 1. What do you know about crystalline solids? Describe its properties.

Ans. The solids in which the atoms, ions and molecules are arranged periodically are called crystalline solids. Examples:

- Metals such as copper, zinc, iron etc.
- Ionic compounds such as sodium chloride
- Ceramics such as zirconia are the examples of crystalline solids.

Properties of Crystalline Solids

1. The crystalline solids show the phenomenon of X-ray diffraction.

2. Every crystalline solid has sharp melting point i.e., for every crystal there is a temperature at which the thermal vibrations becomes so great that the structure suddenly breaks up, and the solid melts.

- Q # 2. Write a note on amorphous or glassy solids?
- Ans. The word amorphous means shapeless. Thus in amorphous solids, there is no regular arrangement of molecules like that in crystalline solids. Examples: The ordinary glass is an example of amorphous solids.
- Properties of Amorphous Solids

1. As the atom, ions and molecules are not arranged periodically in amorphous solids, so these solids don't show the phenomenon of X-ray diffraction.

2. The amorphous solids don't have sharp melting point. For example, a glass passes through a paste like state on heating and becomes a very viscous liquid at almost 8000C.

- Q # 3. What are Polymeric Solids? Describe its properties. Polymeric solids are formed by polymerization reaction in which relatively simple molecules are chemically combined into massive long chain molecules. Polymers may be said to be more or less solid materials with a structure that is intermediate between order and disorder. Example: Plastics, synthetic rubber, polythene and nylon etc. are the examples of polymers. Properties of Polymeric Solids Polymeric solids have low specific gravity, but yet they exhibit good strength to weight ratio.
- Q # 4. Define following: i) Unit Cell ii) Crystal Lattice
- Ans. Unit Cell A crystalline solid consists of three dimensional pattern that repeat itself over and over again. This smallest three dimensional basic structure is called unit cell.
- Crystal Lattice The whole structure obtained by the repetition of unit cell is known as crystal lattice.

- Q # 5. What do you know about deformation? Also describe the phenomenon of deformation in crystalline solids.
- Ans. Deformation Any change in shape, volume and length of an object when it is subjected to some external force is called deformation.
- Deformation in Crystalline Solids
- In crystalline solids atoms are arranged in a certain order. When external force is applied on such a body, a distortion results because of displacement of the atoms from their equilibrium position and the body is said to be in state of deformation.
- In deformed crystalline solid, the atoms return to their equilibrium position after the removal of external force. This ability of the body to return to its original shape is called elasticity.

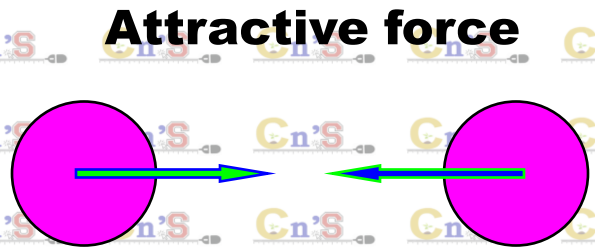
Elastic properties of materials

- Elasticity is the ability of an object to regain its original shape after being deformed by an external force. Steel is more elastic than rubber because steel can withstand large amount of external force before being permanently deformed.
- An **elastic body** is one that returns to its original shape after a deformation.
- An **inelastic body** is one that does **not** return to its original shape after a deformation.

Elasticity

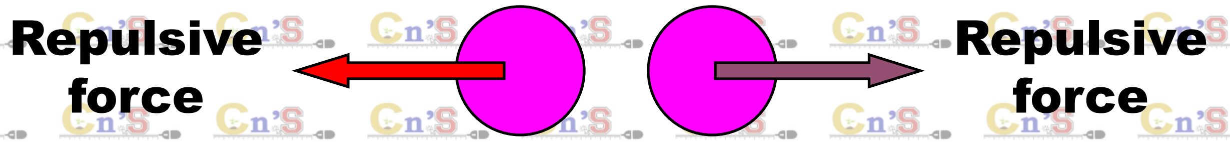
- A force can change the size and shape of an object in various ways: stretching, compressing, bending, and twisting.
- Elasticity is property of matter that enable an object to return to its original size and shape when the force that was acting on it is removed.
- The elasticity of solids is due to the strong inter-molecular forces between the molecules of the solid.

Elasticity

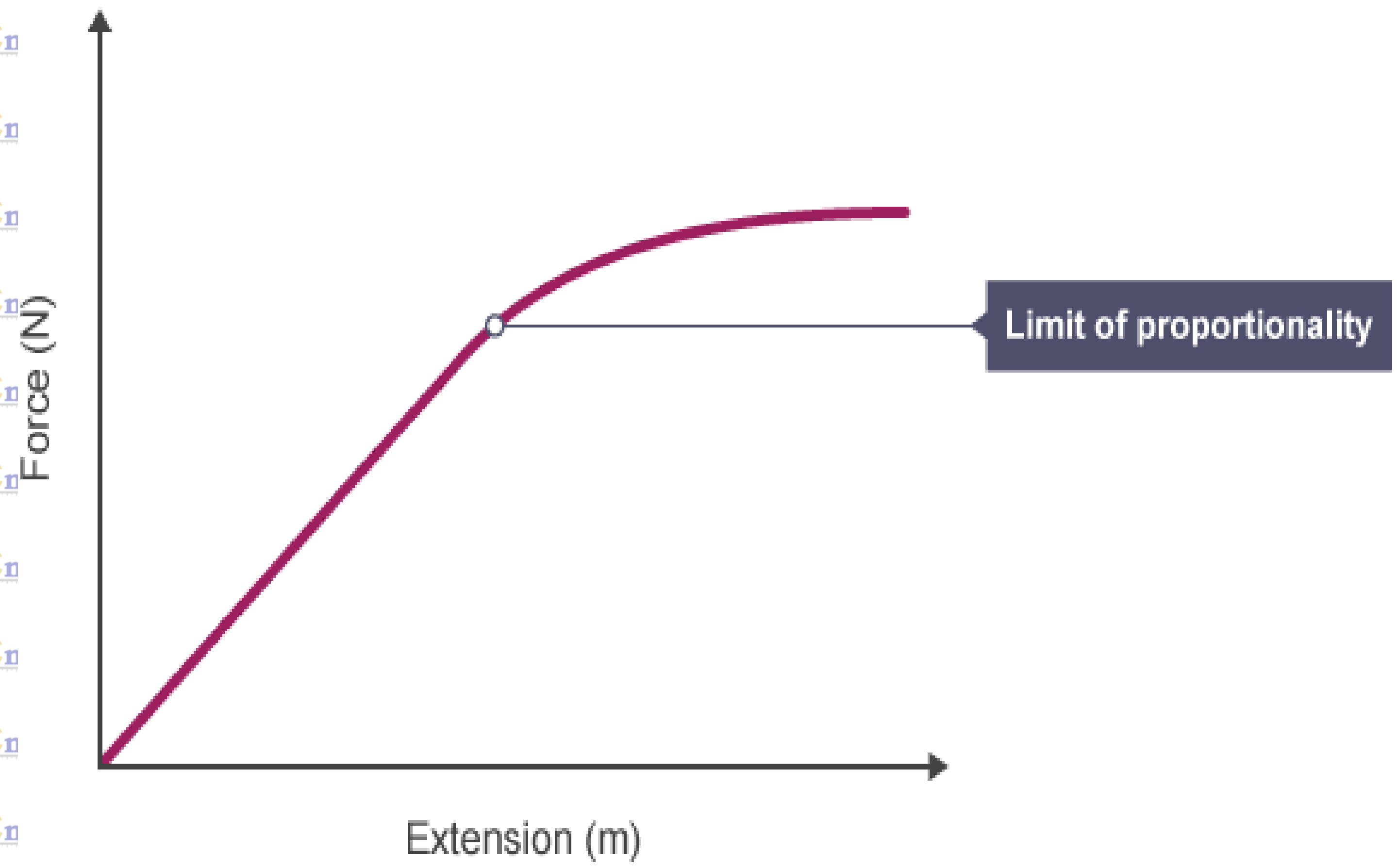


- Stretching a solid causes its molecules to be displaced away from each other.
- A strong attractive intermolecular force acts between the molecules to oppose the stretching.

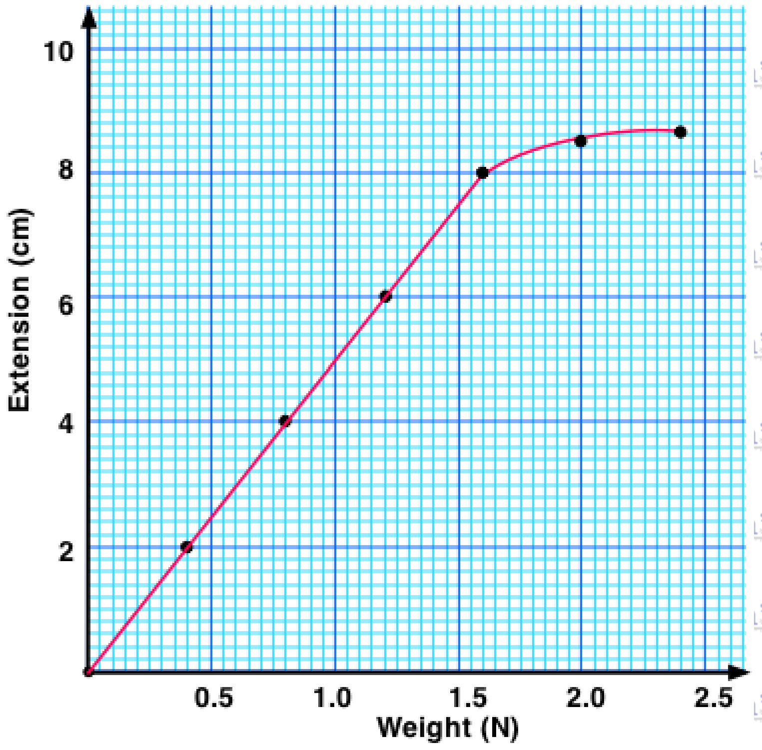
Elasticity



- Compressing a solid causes its molecules to be displaced closer to each other.
- A strong repulsive intermolecular force acts between the molecules to appose the compression.



Force (N)
Limit of proportionality
Extension (m)



Extension (cm)
10
8
6
4
2
0.5
1.0
1.5
2.0
2.5
Weight (N)

Yield point: Beyond the yeild point plastic deformation occurs.

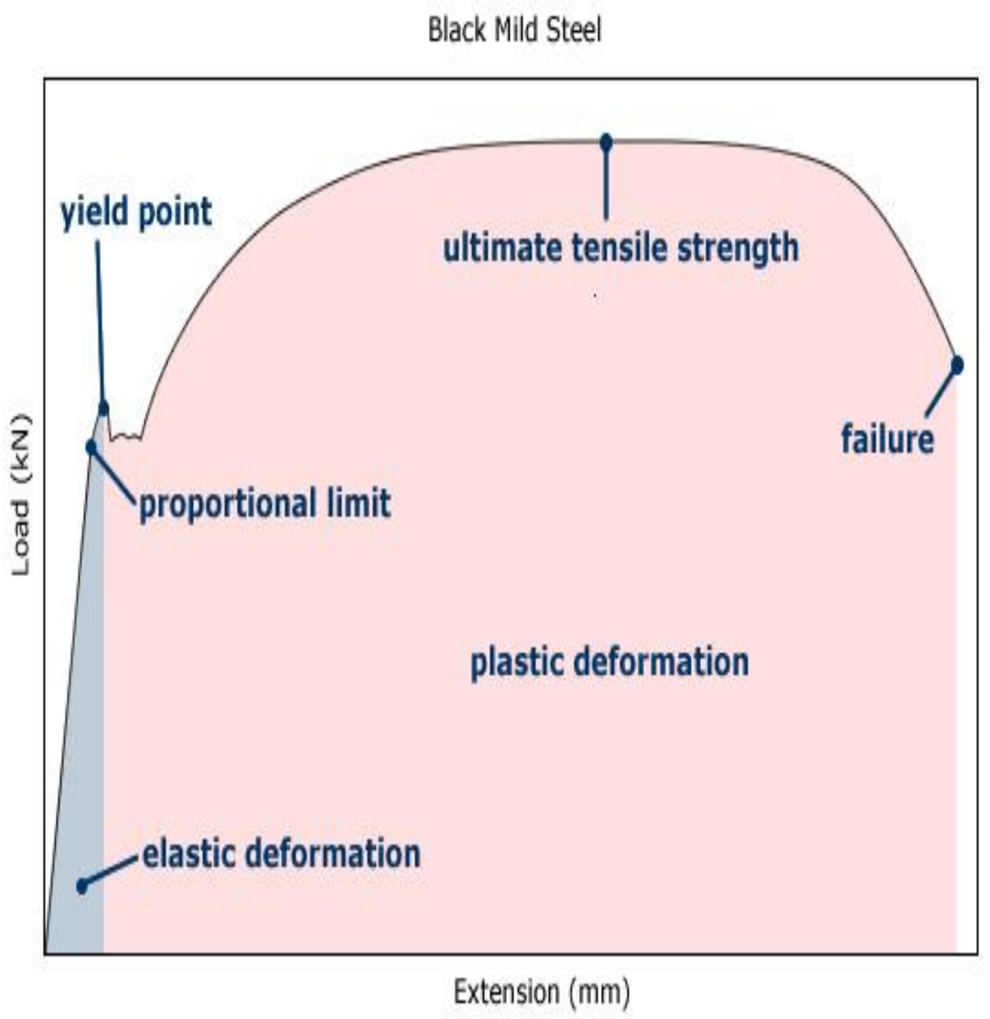
Ultimate tensile strength: This is the maximum load/force the specimen withstood.

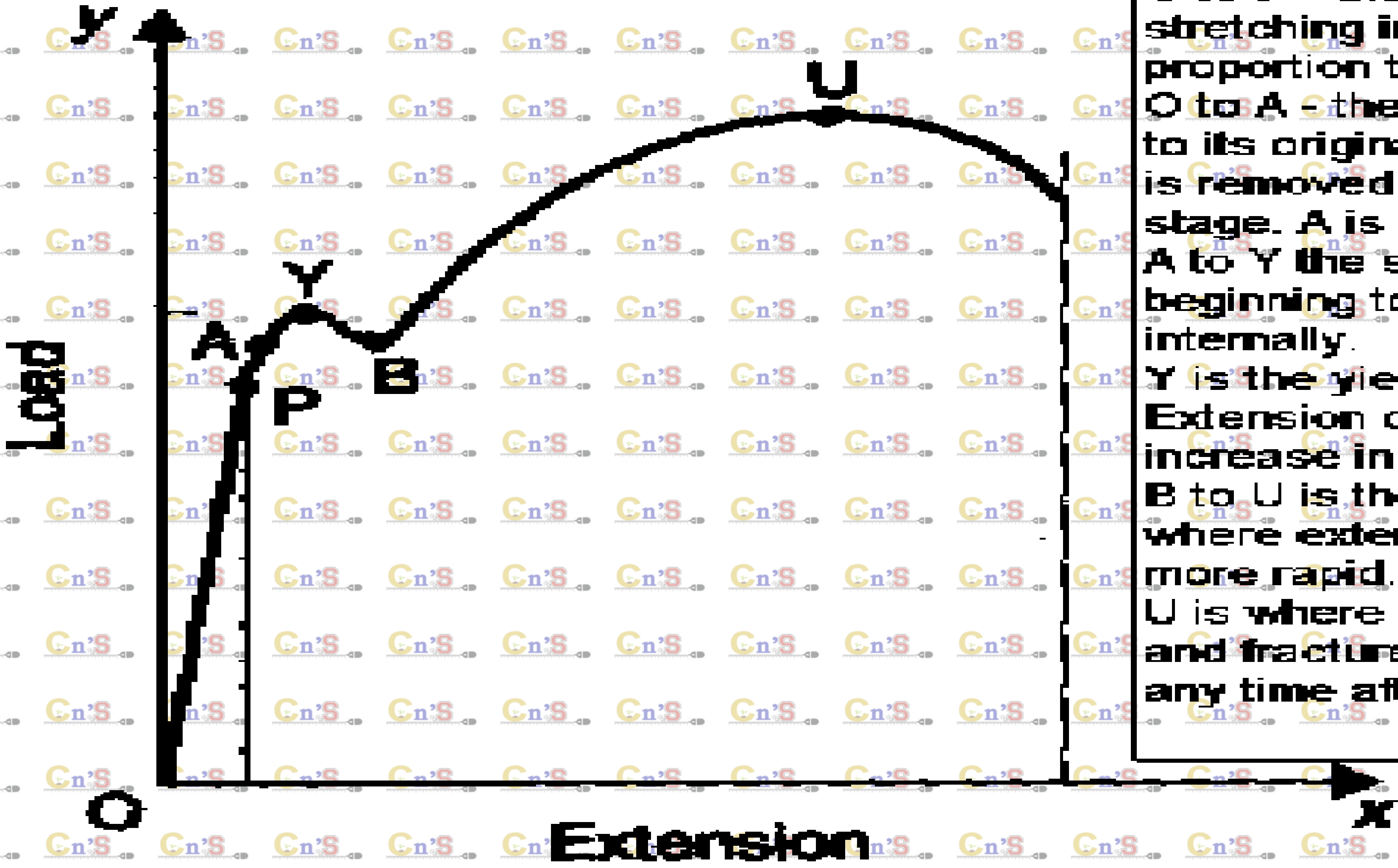
Failure: This is the point where the specimen failed/fractured.

Propertional limit: This is the end of the straight line portion of the graph where Hooks law is obeyed.

Elastic deformation: Within this blue region the material will return to its original shape if the load is removed.

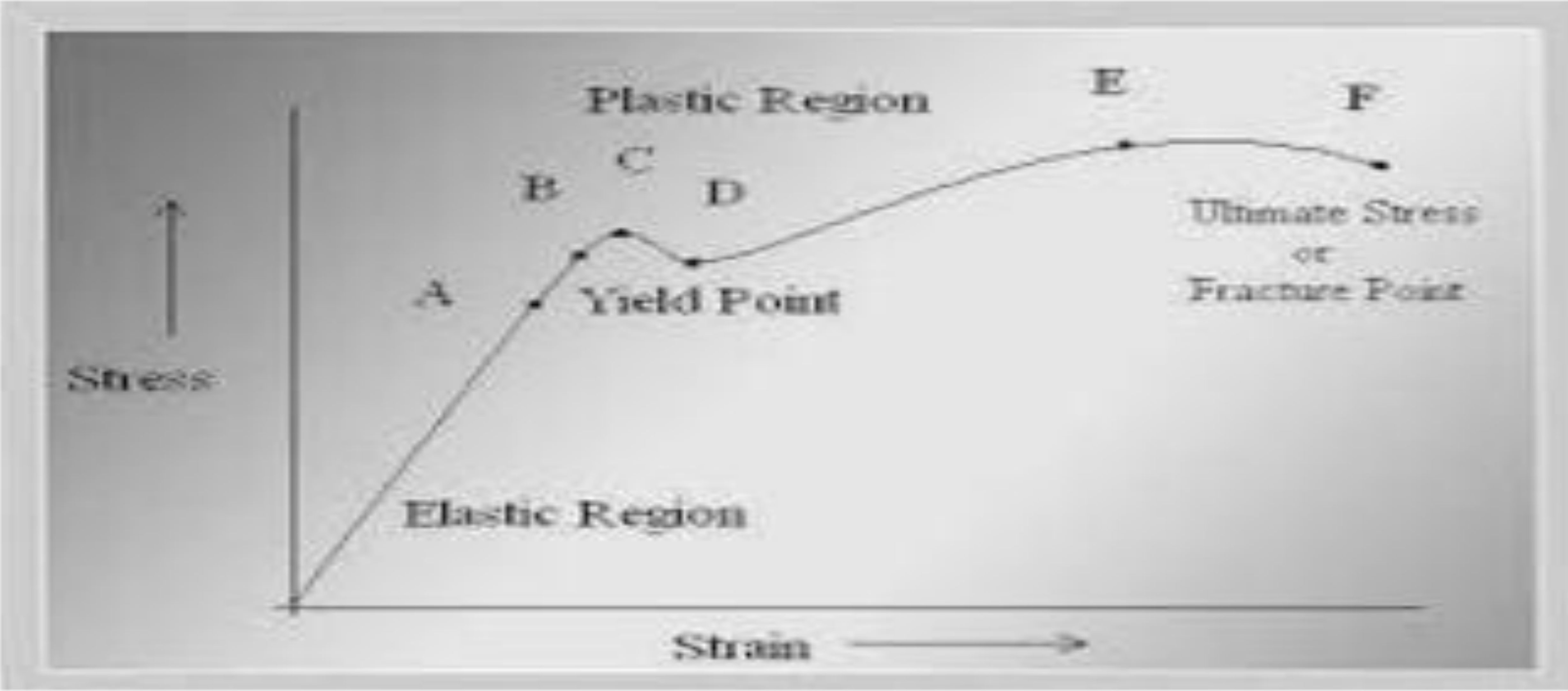
Plastic deformation: Within the red region the material will show a permanent change if the load is removed

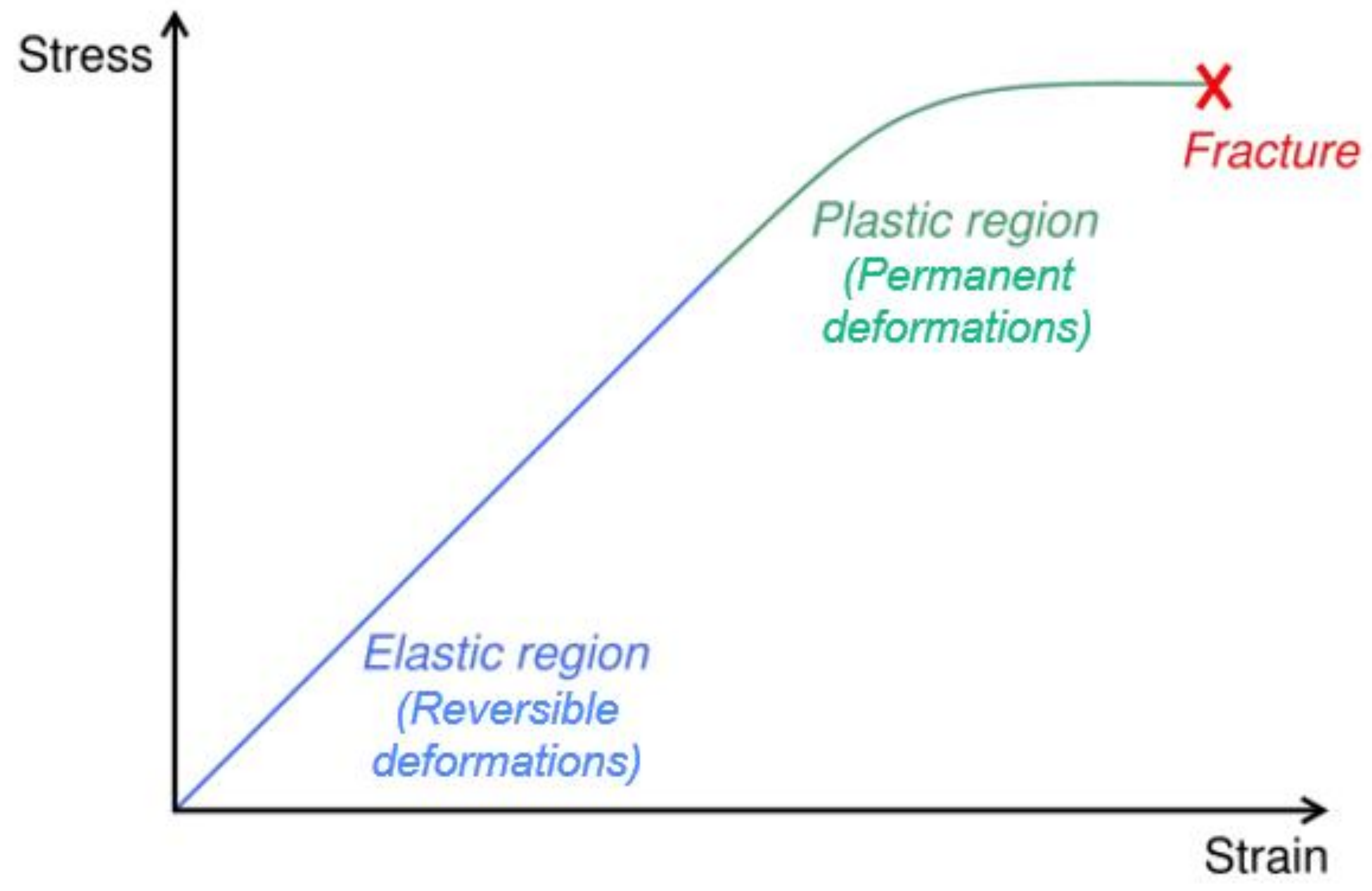


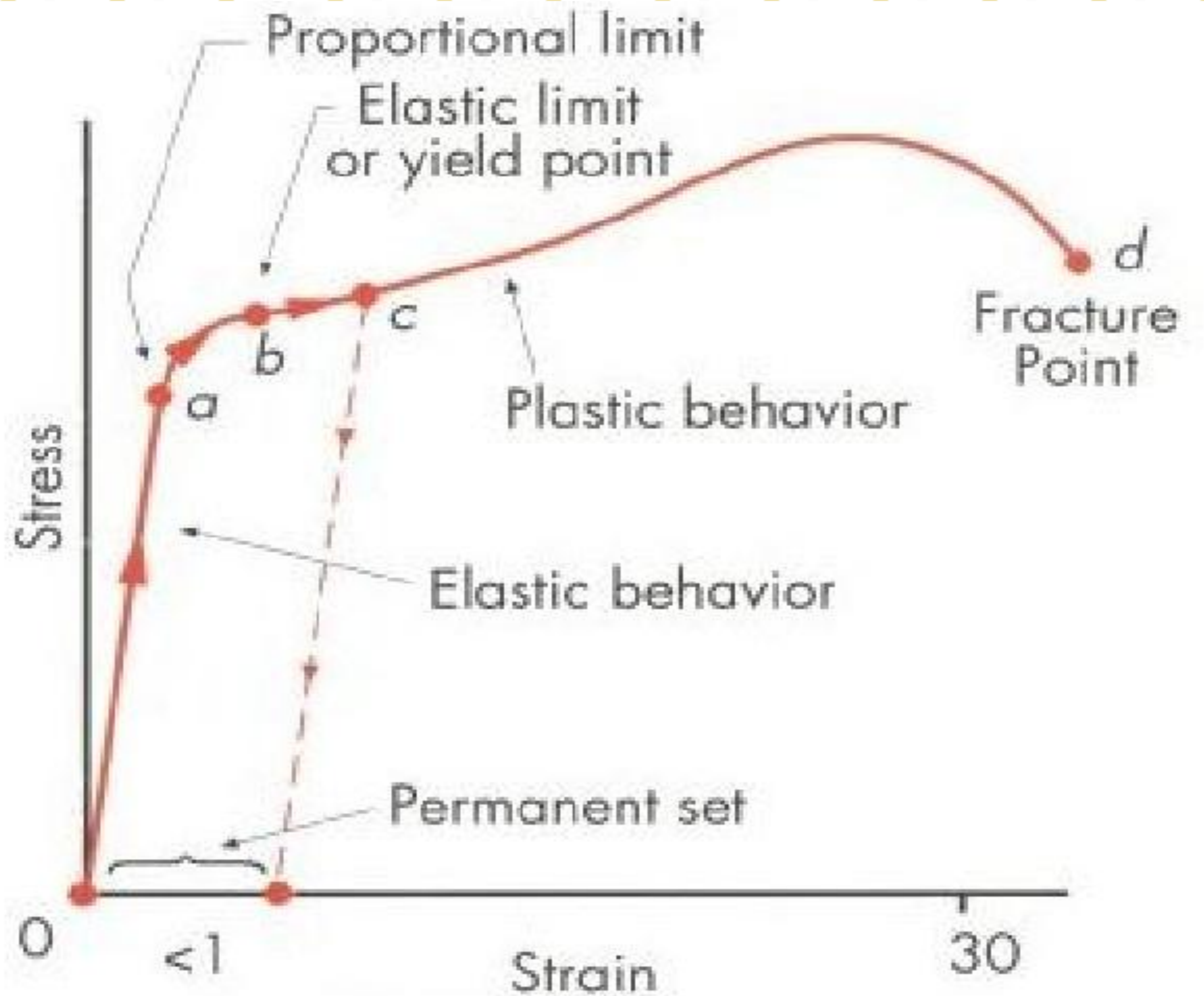


O to P – the steel is stretching in direct proportion to the load.
O to A - the steel will return to its original size if the load is removed – the elastic stage. A is the elastic limit.
A to Y the steel is beginning to change internally.
Y is the yield point. Extension occurs with no increase in load.
B to U is the plastic stage where extension is much more rapid.
U is where waisting occurs and fracture can occur at any time after this.

Typical load - extension graph for mild steel.


Plastic Region
E
F
C
B
D
Ultimate Stress
or
Fracture Point
A
Yield Point
Stress
Elastic Region
Strain


Stress
Strain
Elastic region
(Reversible
deformations)
Plastic region
(Permanent
deformations)
Fracture


Proportional limit
Elastic limit
or yield point
d
Fracture
Point
c
b
a
Plastic behavior
Stress
Elastic behavior
Permanent set
0
<1
Strain
30

Stress strain curve has different regions and points. These regions and points are:

(i). Proportional limit

(ii). Elastic limit

(iii). Yield Stress/ point

(iv). Ultimate stress point

(v). Failure/Fracture or breaking point.

(i) Proportional Limit

The point A in the graph is called the proportional limit. It is the linear region in the strain curve which obeys hookes law

i.e. within this limit the stress is directly proportion to the strain produced in the material.

In this limit the ratio of stress with strain gives us proportionality constant known as young's modulus.

(ii) The Elastic Limit

The point B is the Elastic limit in the graph.

The elastic limit is the maximum stress a body can experience without becoming permanently deformed.

It is the point in the graph upto which the material returns to its original position when the load acting on it is completely removed. Beyond this limit the material cannot return to its original position and a plastic deformation starts to appear in it.

If the stress exceeds the elastic limit, the final length will be longer than the original .

(iii) Yield Point or Yield Stress Point: C

At this point the strain begins with a small or no rise of the stress. Yield point in a stress strain diagram is defined as the point at which the material starts to deform plastically. The int. structure of the material has changed-the crystal planes have slid across each other. After the yield point is passed there is permanent deformation develops in the material and which is not reversible. The stress corresponding to the yield point is called yield point stress. i.e. Mild Steel.

(iv) **The Ultimate Strength**

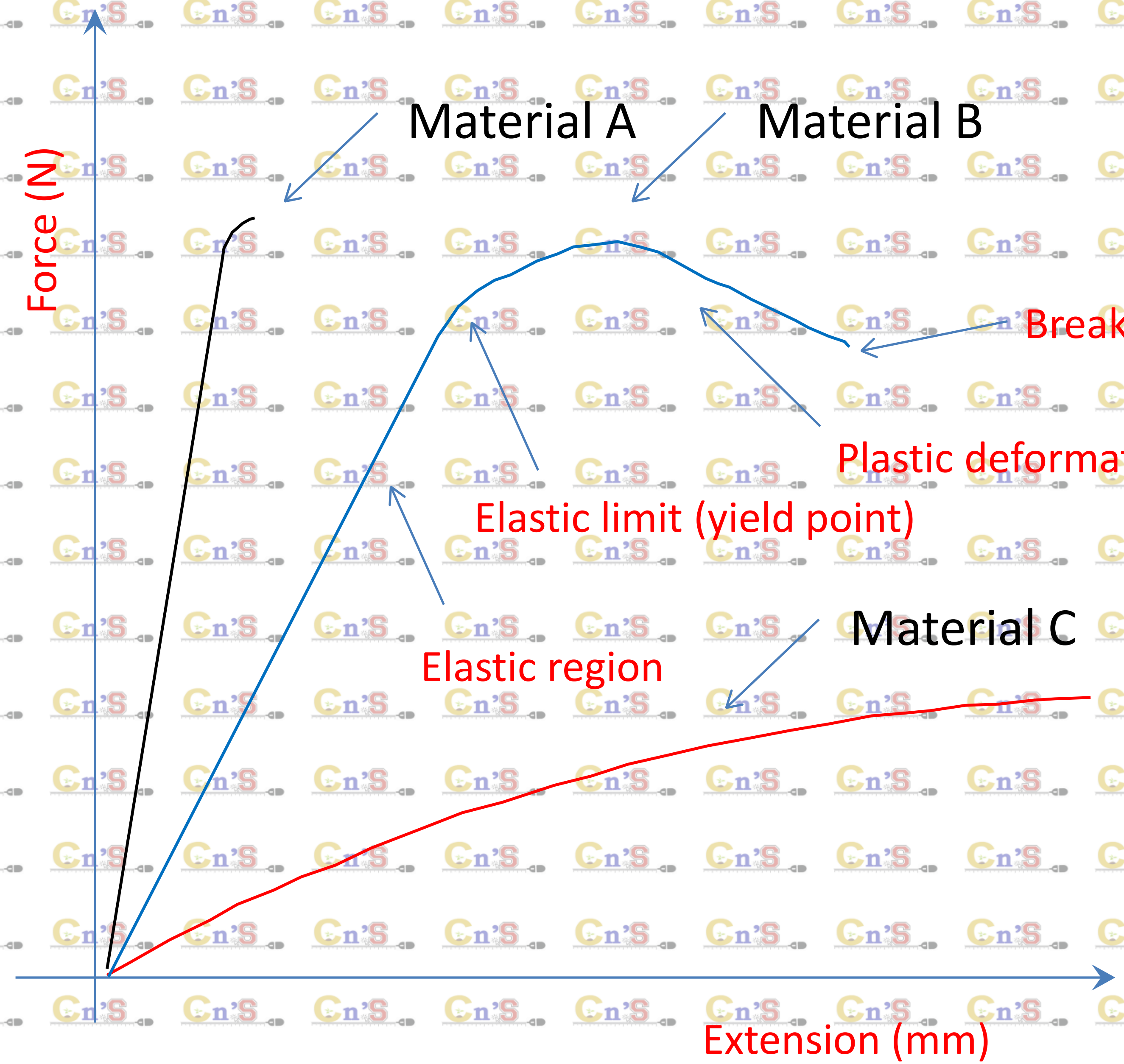
Point U in the graph is the ultimate stress point. The ultimate strength is the greatest stress a body can experience without breaking or rupturing. It is the point corresponding to the maximum stress that a material can handle before failure. It is the maximum strength point of the material that can handle the maximum load.

If the stress exceeds the ultimate strength, the string breaks!

(v). Fracture or Breaking Point:

The point E is the breaking point in the graph.

It is the point in the stress strain curve at which the failure of the material takes place. The fracture or breaking of material takes place at this point.

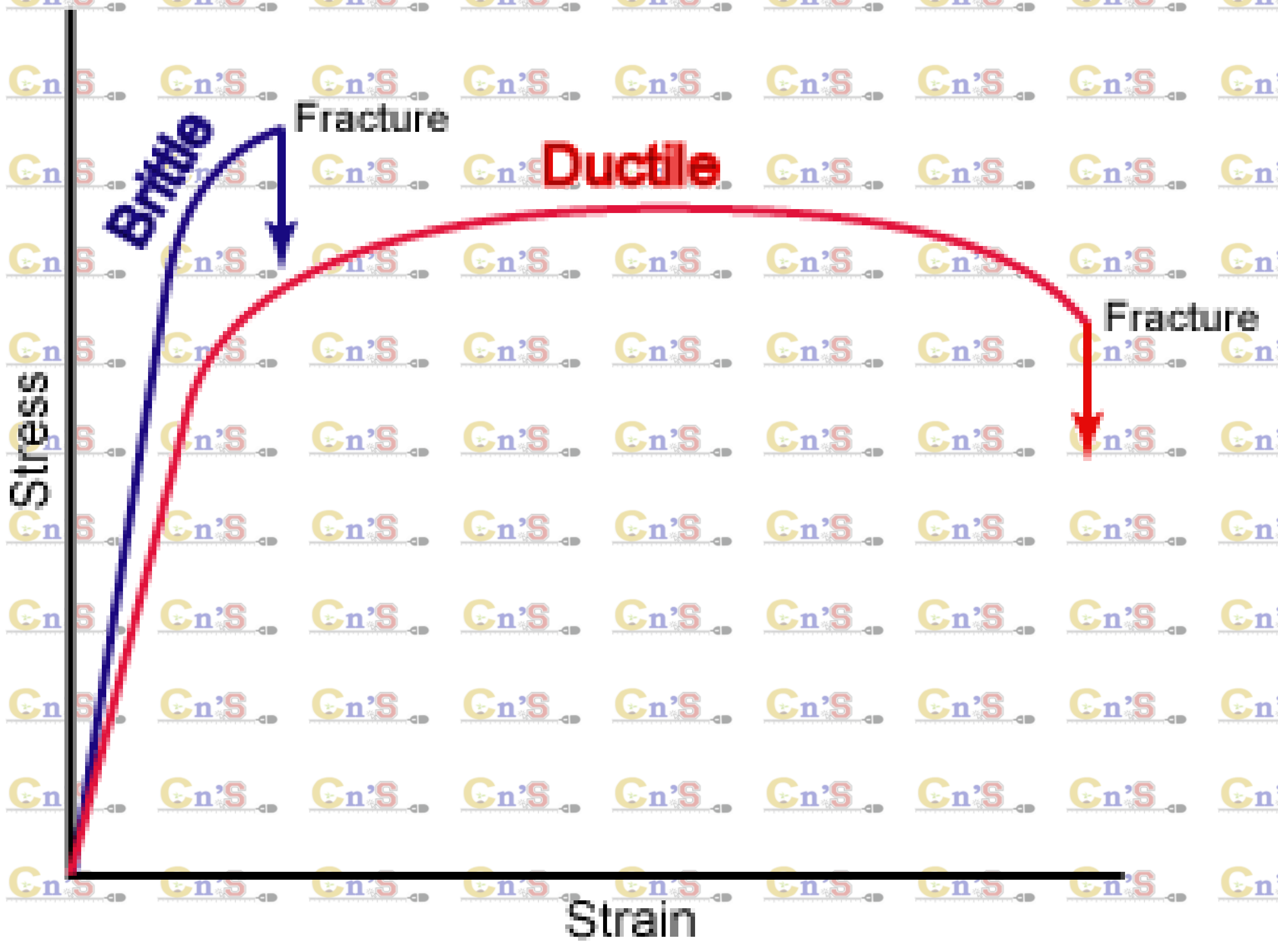


Material A is brittle, Material B ductile and Material C polyme

Material A	**Brittle**	**Cast Iron** **Glass** **Concrete** **Ceramic**	• **A brittle material deforms under load and breaks without significant deformation (often suddenly).** • **Brittle materials absorb relatively little energy prior to fracture.** • **Brittle materials can often withstand a large force.** • **Brittle materials often have no plastic deformation.**

Material B	**Ductile**	• **Copper** • **Steel** • Lead • Aluminium • Gold • Silver	• **A ductile material is easily stretched without breaking or lowering in strength.** • **Ductile materials can also often withstand large forces.** • **Ductile materials have defined areas of elastic and plastic deformation.** • **Materials that are highly ductile can be drawn into long thin wires without breaking.**

Material C	**Polymeric**	**• Plastics • Rubber • Elastomers**	**• Polymeric materials have no elastic (linear) region on force-extension graph. • Polymeric materials tend not to obey Hooke's Law i.e. there is no linear region on the force-extension diagram. • Due to their internal composition of long polymer chains the force-extension graph for polymeric materials often exhibits hysteresis (when loading and unloading).**


Brittle
Fracture
Ductile
Fracture
Stress
Strain

. DUCTILITY

Material which can be permenebtly streached, beyond the Elastic limit, before it breaks.

Ductile materials are those which can be drawn into wires when tension is applied

And can be drawn into sheet when compressive load is applied

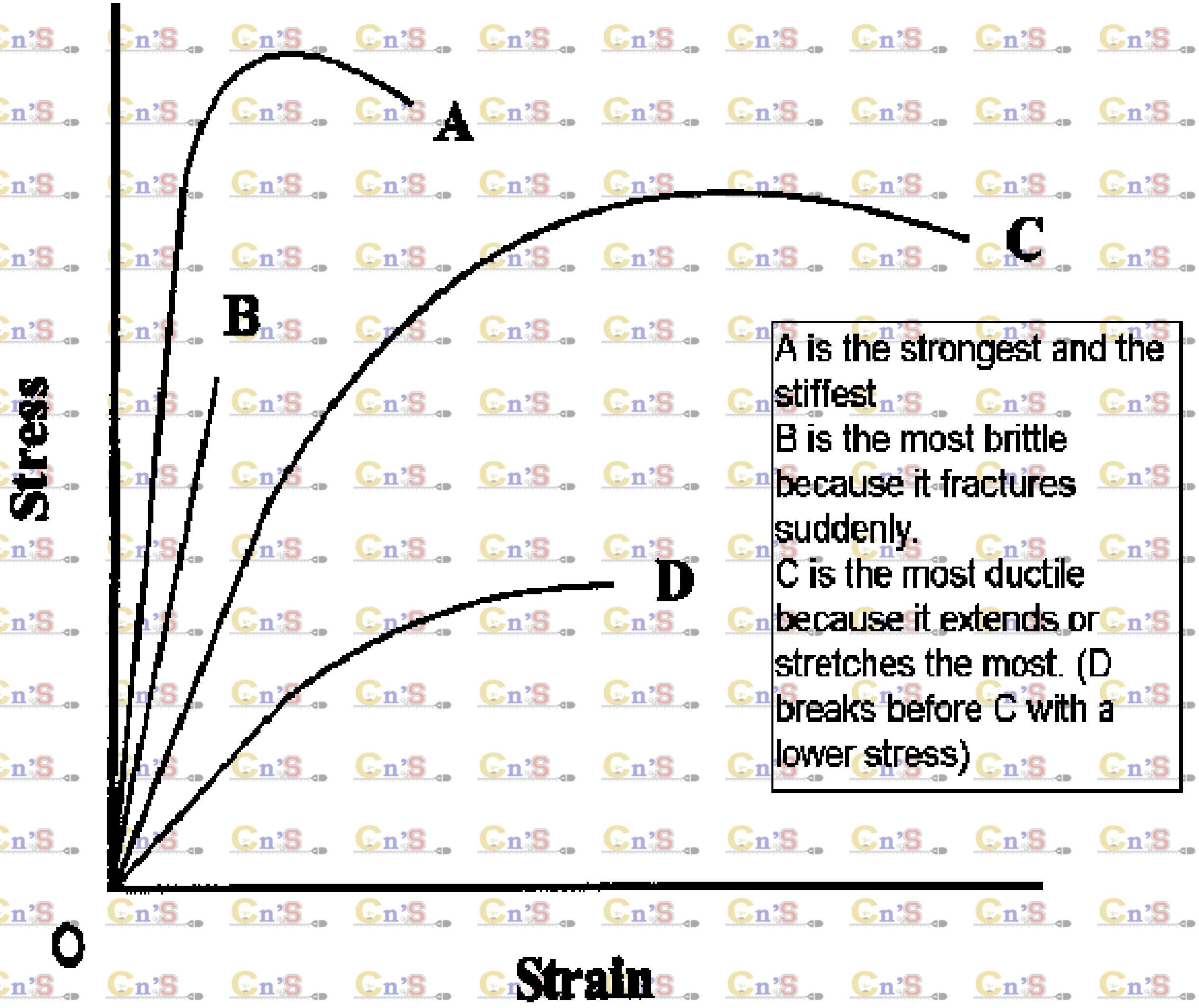
For the above mentioned properties the material should have a tendency to elongate i.e for lower value of stress ,the strain produce is much higher

So the region is called elastic region . As material have to be elastic to possess the property of ductility and so called ductile materials

BRITLENESS

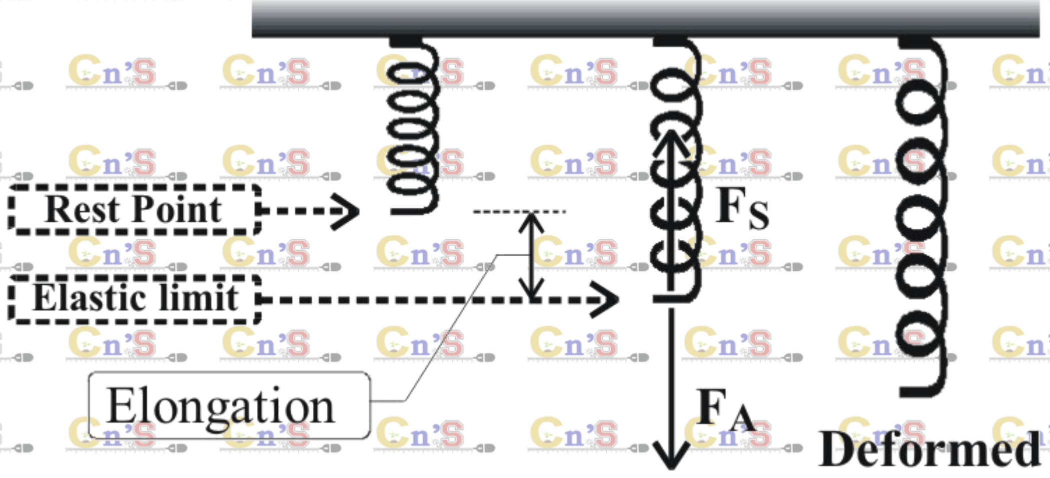
- Substances which breaks just after the elastic limit is reached.
- Brittle materials are those which can't elongate as much the ductile materials because they don't have such a long elastic region and they just have fracture point after point B

A graph showing stress-strain of four materials



Hooke's Law

As illustrated below, the distance a spring is stretched is called the "elongation". Robert Hooke was the first to discover that the spring force is directly proportional to the elongation. Today, we call this law Hooke's Law.



The formula for the spring is: $F_S = kx$ ☜ (Hooke's Law)

Where: F_S = the spring force (N)
k = the spring constant (N/m)
x = the elongation (m)

Relationship Between Force and Extension of a spring.

- Hooke's law states that the extension of a spring is directly proportional to the applied force provided the elastic limit is not exceed.

$$F \alpha x$$

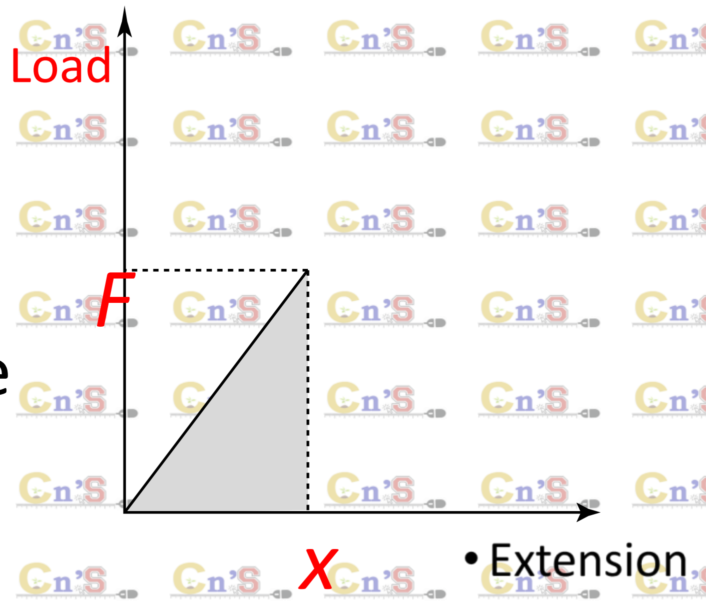
$$F = kx$$

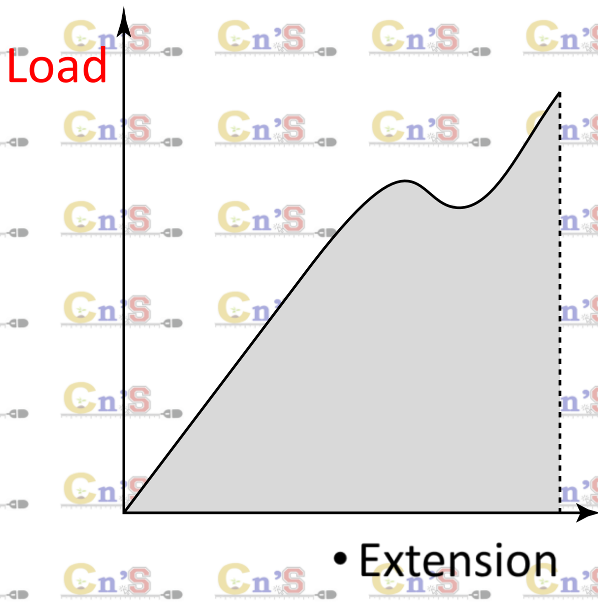
The spring constant k is a measure of the elasticity of the spring.

- Elastic limit- max force that can be applied to a spring such that the spring will be able to restore to its original length when the force is removed.

Energy stored in a deformed material

- The first graph shows the extension of a body that obeys Hooke's law. The work done in stretching the body is equal to force multiplied by distance moved. This is equal to the strain potential energy in the body. The force is not, however, F the maximum force — it is the average force, which is ½F.



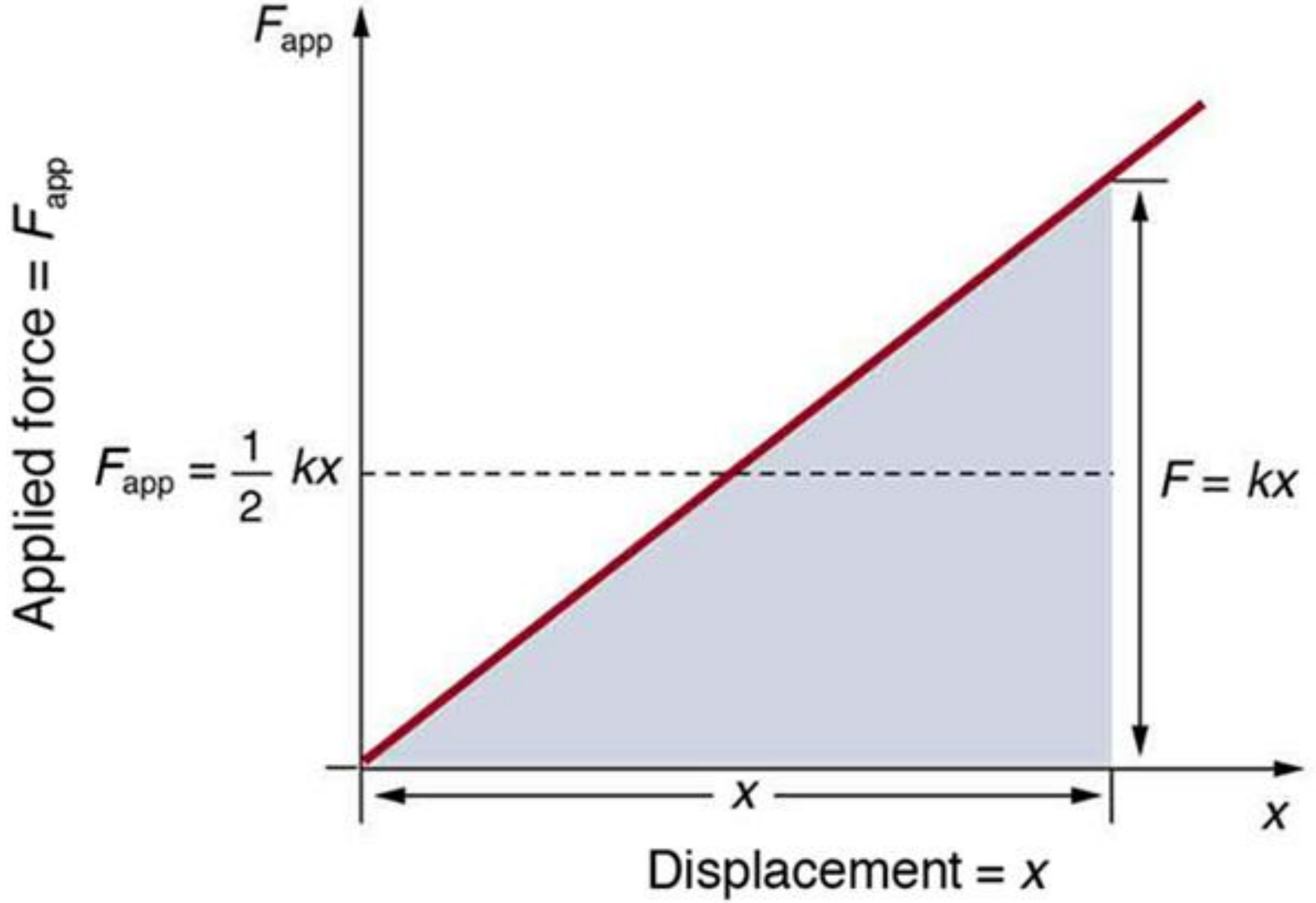


- strain energy = ½Fx

- This is the area of the triangle under the graph.

Substituting for $F = kx$, it can be rewritten as: strain energy = $\frac{1}{2} kx^2$

If the extension is doubled the energy stored is quadrupled, if the extension is tripled the energy stored is multiplied by nine etc.



Method A

$$W = \frac{1}{2}bh = \frac{1}{2}kxx$$

$$W = \frac{1}{2}kx^2$$

Method B

$$W = f \cdot x = \left(\frac{1}{2}kx\right)(x)$$

$$W = \frac{1}{2}kx^2$$

Hooke's Law

The spring constant is different for different springs and depends upon the type of material the spring is made of as well as the thickness of the spring coil. The greater the value of the spring constant, the "stiffer" the spring..

The formula for the potential energy stored in a spring is:

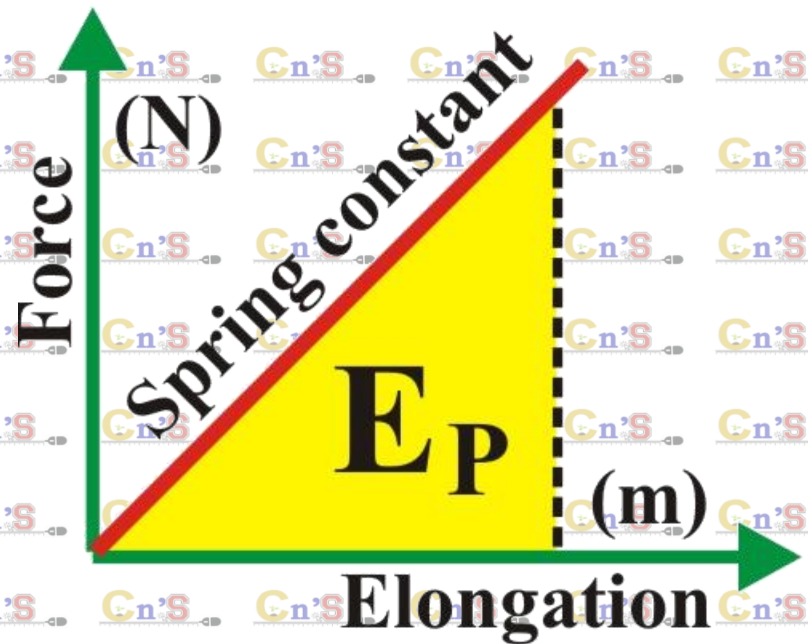
$$E_p = \frac{1}{2} kx^2$$

Where: E_p = Potential energy (in joules)
k = the spring constant (in N/m)
x = the elongation (in metres)

Remember that in an ideal spring, there is no loss of energy (E_p) due to friction.

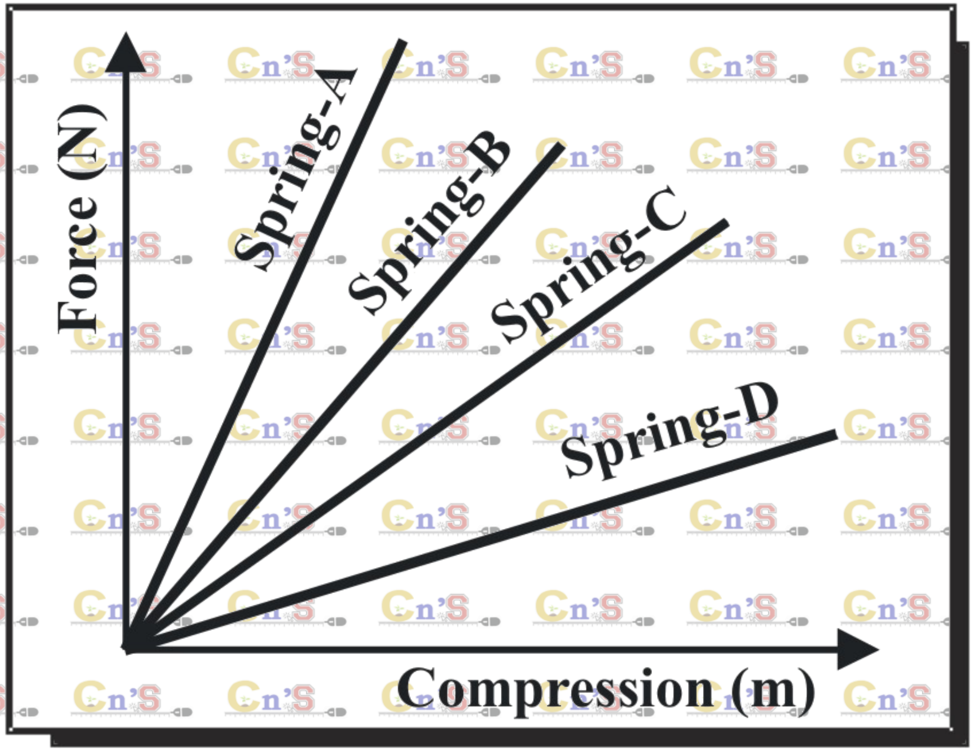
Hooke's Law

When the spring force (F_s) is plotted versus the elongation (x) of the spring, the resulting graph is a linear relation. The slope of the curve represents the *spring constant* while the area under the curve represents the *potential energy* stored in the spring.



The graph displays the force-compression curve of four springs labeled Spring-A, Spring-B, Spring-C and Spring D.

Which spring is the “springiest”?



Spring-D (Least slope)

Question 1

- A spring has a force constant of 25 N cm^{-1}. What is the force that will cause a 3 cm extension of the spring?

$$F = kx$$
$$= 25 \times 3$$
$$= 75 \text{ N}$$

Question 2

- The length of a spring is increased from 23.0 cm to 28.0 cm when a mass of 4 kg was hung from the end of a spring.

  a) What is the load on the spring in newton?

  b) What is the extension of the spring?

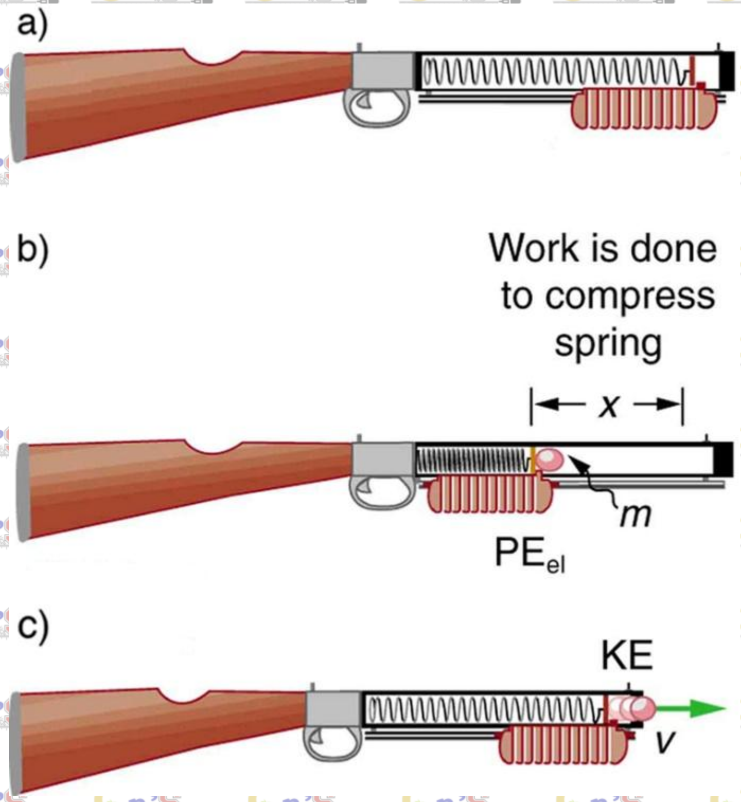
  c) Calculate the force constant of the spring. [Assume g= 10 ms^{-2}]

Ans a) 40 N b) 5.0 cm c) 8 N cm^{-1}

Questions

1. A 4N force on a spring produces an extension of 3cm. What is the extension when the force is increase to 10N?
2. A force of 8N on a spring compresses the spring from 18 cm to 15cm. What is the force constant of the spring?

How much energy is stored in the spring of a tranquilizer gun that has a force constant of 50.0 N/m and is compressed 0.150 m?
If you neglect friction and the mass of the spring, at what speed will a 2.00-g projectile be ejected from the gun?



The spring is uncompressed before being cocked. When the spring has been compressed a distance x , The energy stored in the spring= the elastic potential energy, $(1/2)kx^2$.

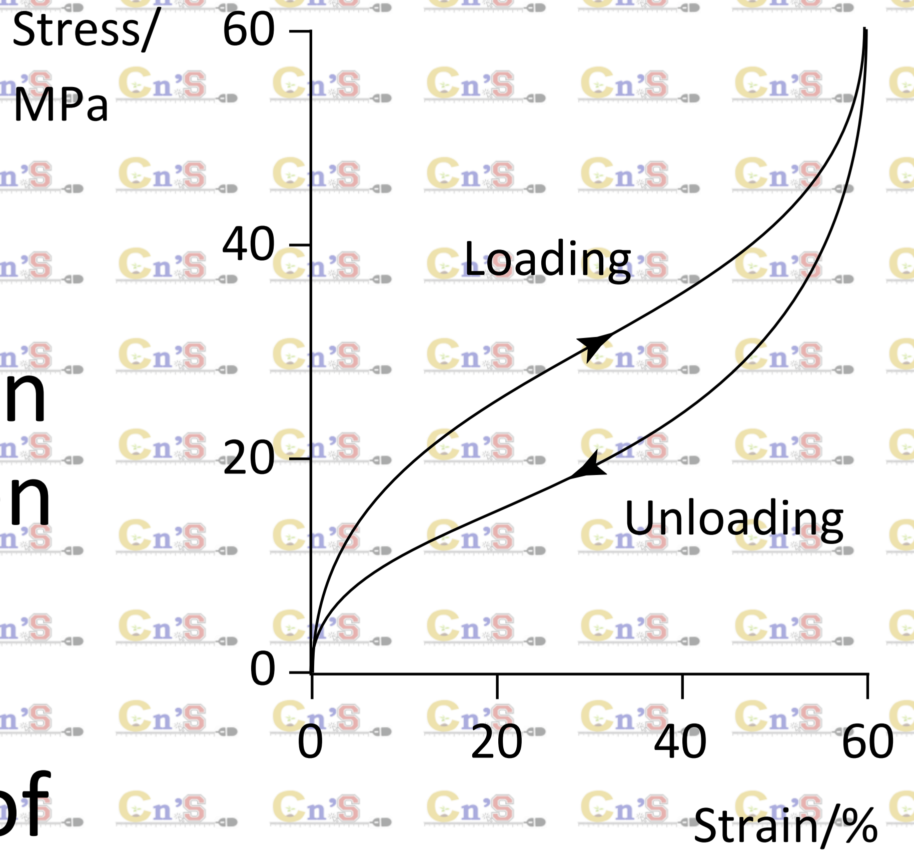
When the spring is released, this elastic potential energy is converted into kinetic energy, $1/2mv^2$

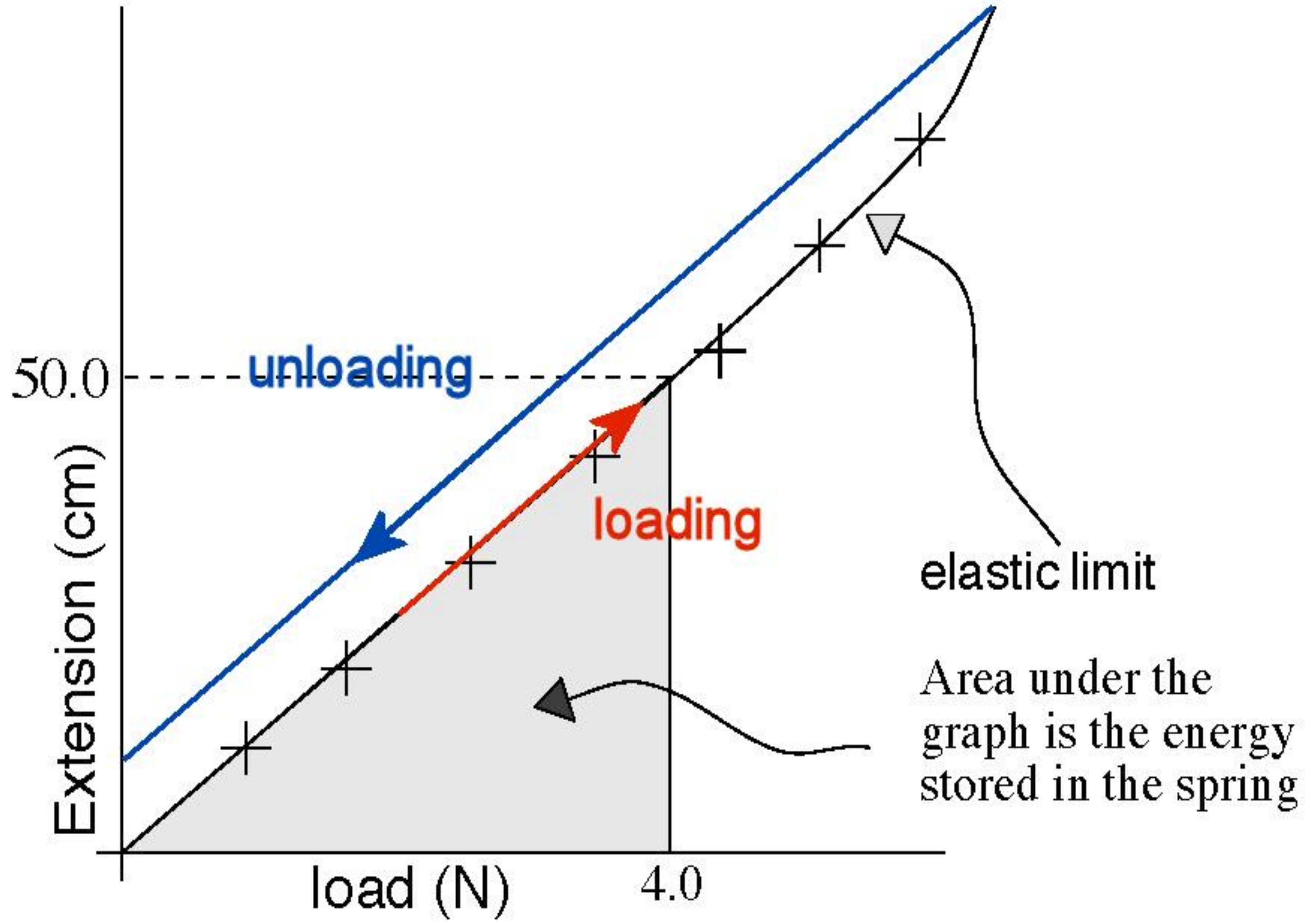
Because there is no friction, the potential energy is converted entirely into kinetic energy.

$1/2mv^2=(1/2)kx^2$

V= 23.7 m/s (more than 80 km/h).

In Polymeric materials i.e. rubber, The area under the graph when loading is larger than when unloading the wire. This means that more energy is stored in the stretched wire than is released when the load is removed. What happens to this energy? It is converted to internal energy in the wire — the temperature of the wire increases. The energy released is equal to the area in the enclosed loop made by the two curves. This is known as elastic hysteresis.




50.0
unloading
loading
Extension (cm)
load (N)
4.0
elastic limit
Area under the graph is the energy stored in the spring

Stress

Deformation is caused by a stress. Deforming force per unit area is called stress.

A stress that produces a change in length is called a tensile stress.

A stress that produces a change in volume is called compressional stress or volume stress.

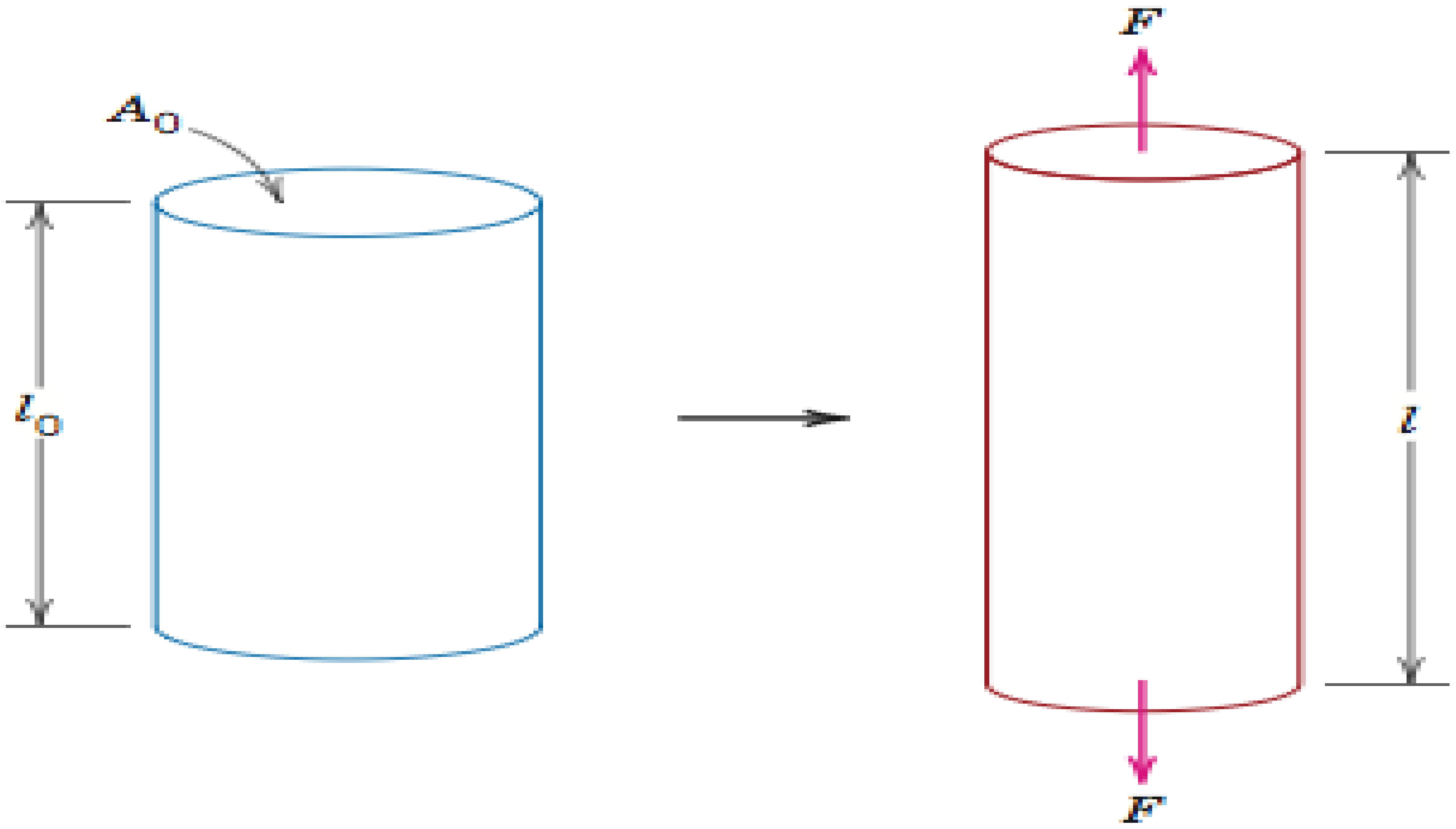
A stress that simply deforms the object at a certain angle is called the shear stress.

Tensile / Longitudinal Stress

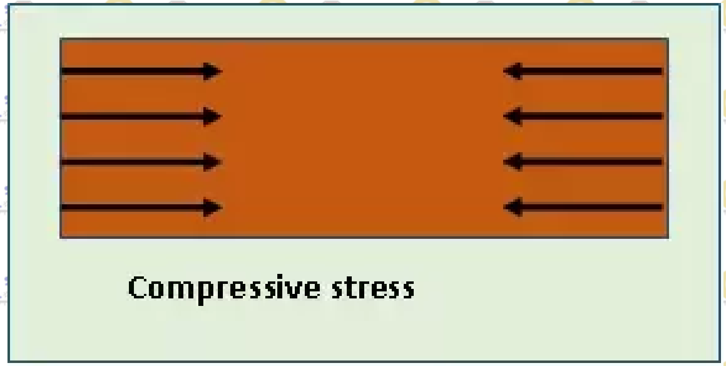
- Stress is the ratio of an applied force F to the area A over which it acts
- Stress = F/A
- The SI unit of stress is N/m2.
- For wires, rods, and bars, there is a longitudinal stress F/A that produces a change in length per unit length.

Tensile Stress

- Tensile stress is a quantity associated with ***stretching*** or tensile forces. Usually, tensile stress is defined as the force per unit area and denoted by the symbol σ. The tensile stress (σ) that develops when an external stretching force (F) is applied on an object is given by σ = F/A where A is the cross sectional area of the object. Therefore, the SI unit of measuring tensile stress is Nm-2 or Pa. Higher the load or tensile force, higher the tensile stress.

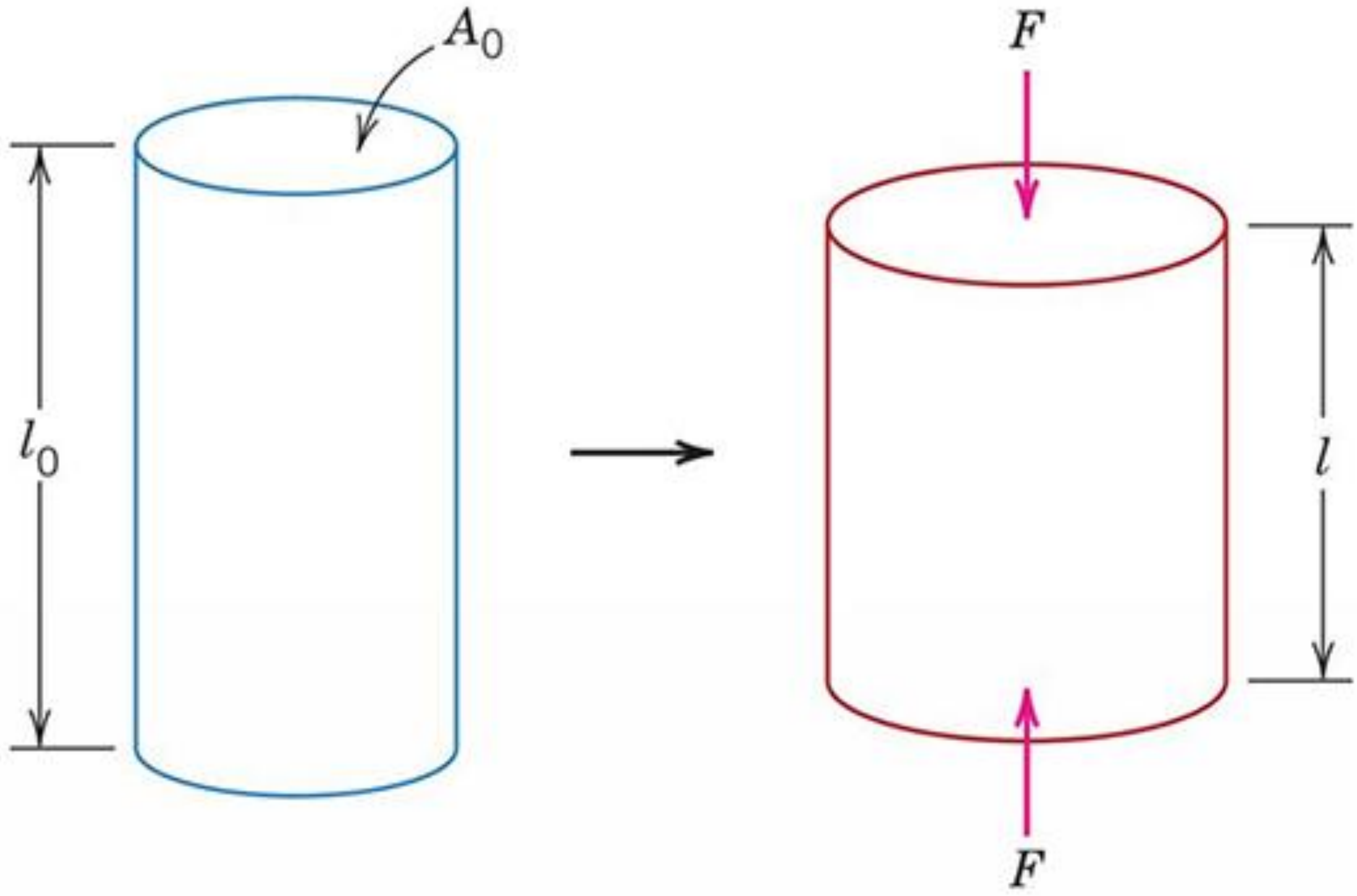


A_0
l_0
F
l
F



Compressive Stress

- Compressive stress is the opposite of tensile stress. An object experiences a compressive stress when a squeezing force is applied on the object. So, an object subjected to a compressive stress is shortened. Compressive stress is also defined as the force per unit area and denoted by the symbol σ. The compressive stress (σ) that develops when an external compressive or squeezing force (F) is applied on an object is given by $\sigma = F/A$. Higher the compressive force, higher the compressive stress. The **main difference** between tensile and compressive stress is that **tensile stress results in elongation whereas compressive stress results in shortening.**



A0
l0
F
l
F

Tensile strain

- strain refers to the effect of the deformation.
- When an object is subjected to a stress, it suffers a strain.

The ratio of change in length to original length is called tensile strain.

If ΔL is the increase in length for an original length L, then:

Tensile strain $= \Delta L / L$

Strain is a simple ratio, it has no units.

The Modulus of Elasticity

- Provided that the elastic limit is not exceeded, an elastic deformation (strain) is directly proportional to the magnitude of the applied force per unit area (stress).

$$\text{Modulus of Elasticity, } E = \frac{\text{stress}}{\text{strain}}$$

This longitudinal modulus of elasticity is called Young's Modulus and is denoted by the symbol E.

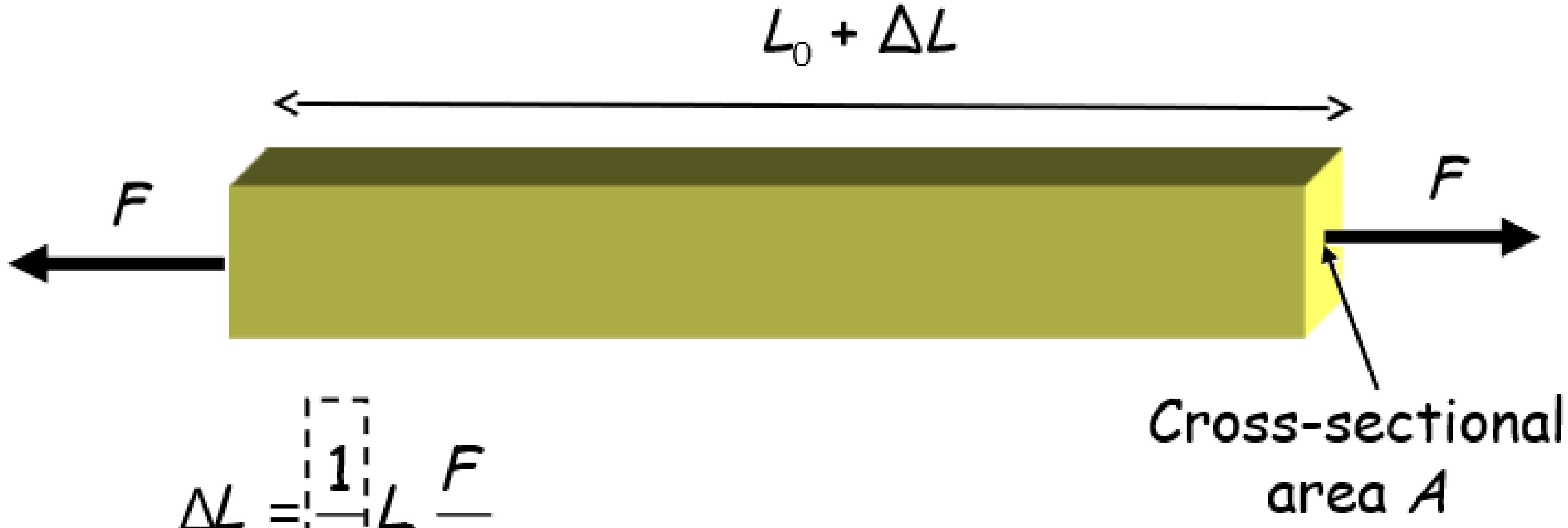
Young's Modulus

For materials whose length is much greater than the width or thickness, we are concerned with the **longitudinal modulus** of elasticity, or **Young's Modulus (Y)**.

$$Young's\ modulus = \frac{longitudinal\ stress}{longitudinal\ strain}$$

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\,\Delta L}$$

$$Units:\ \mathrm{Pa}\ \ or\ \ \frac{\mathrm{lb}}{\mathrm{in.}^2}$$



$$\Delta L = \underbrace{\boxed{\frac{1}{Y}}}_{\text{Proportionality factor}} L_0 \frac{F}{A}$$

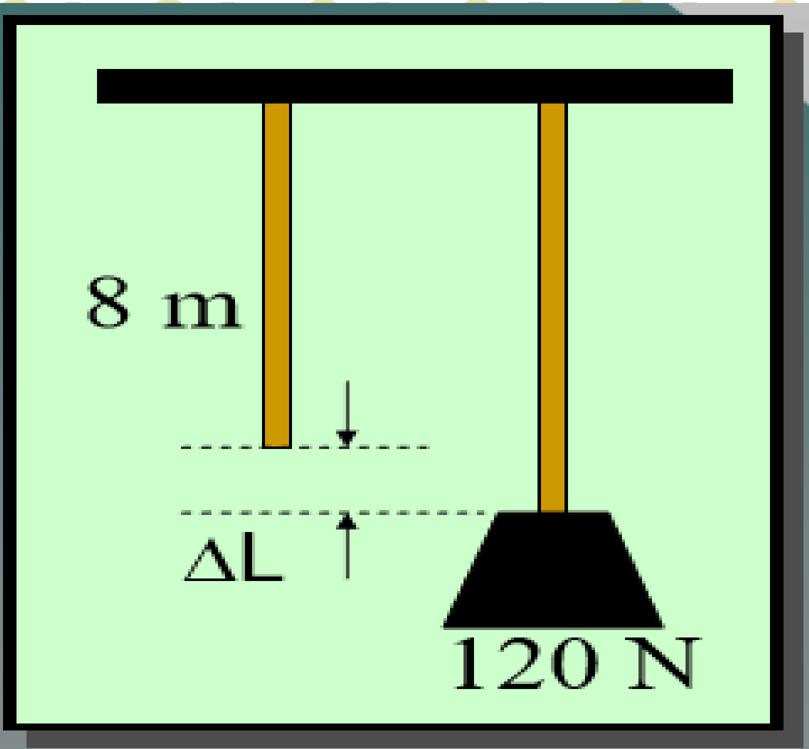
Young's modulus

$$Y = \frac{F/A}{\Delta L / L_0}$$

stress

strain (deformation)

Example 4: Young's modulus for brass is 8.96×10^{11}Pa. A 120-N weight is attached to an 8-m length of brass wire; find the increase in length. The diameter is 1.5 mm.



First find area of wire:

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.0015\text{ m})^2}{4}$$

$$A = 1.77 \times 10^{-6}\text{ m}^2$$

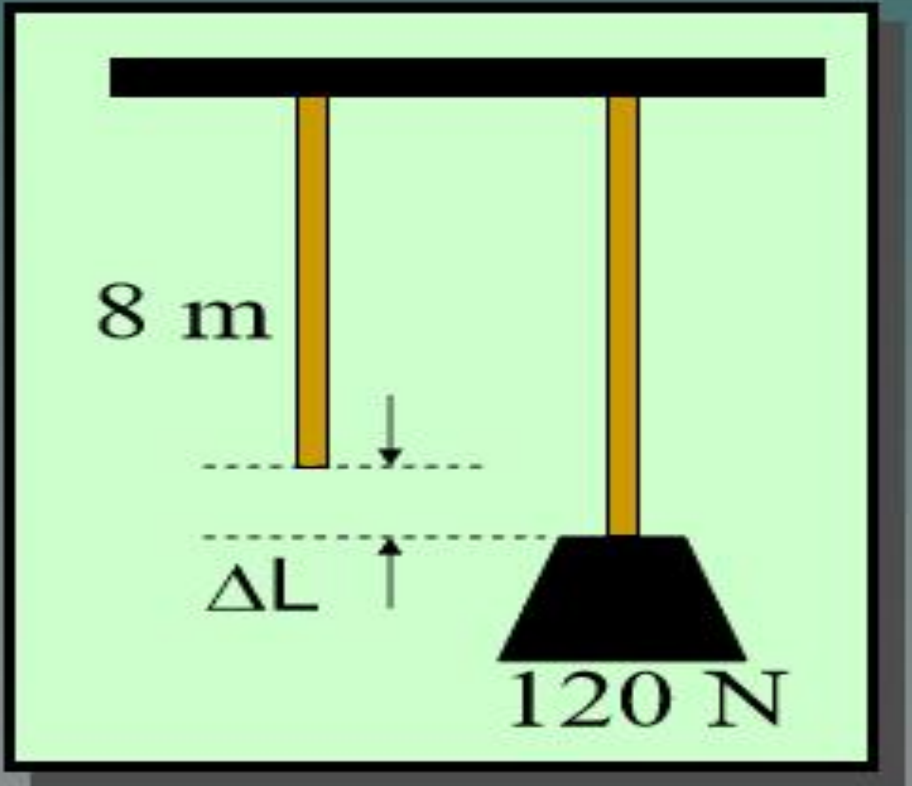
$$\boxed{Y = \frac{FL}{A\Delta L} \quad \text{or} \quad \Delta L = \frac{FL}{AY}}$$

Example 4: (Continued)

$Y = 8.96 \times 10^{11}$ Pa; $F = 120$ N;

$L = 8$ m; $A = 1.77 \times 10^{-6}$ m^2

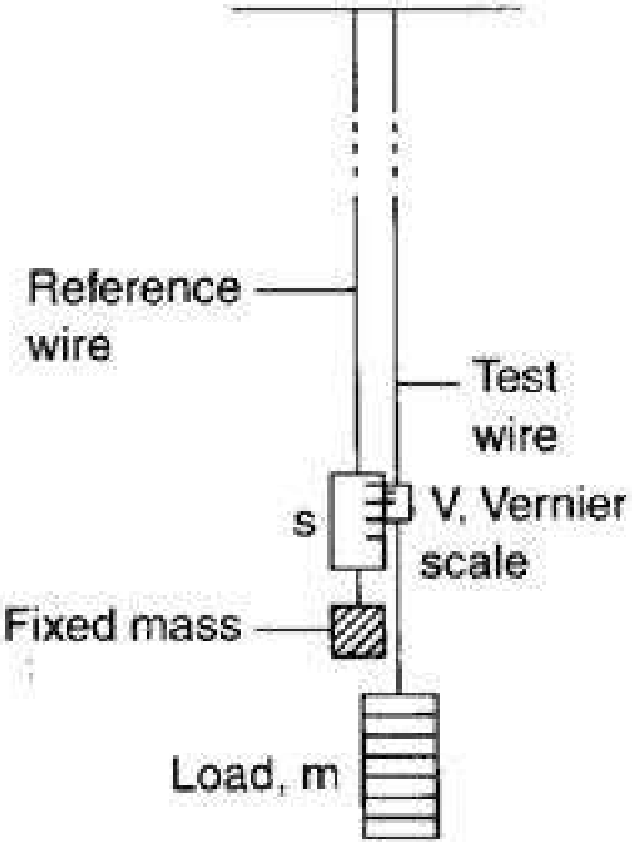
$F = 120$ N; $\Delta L = ?$

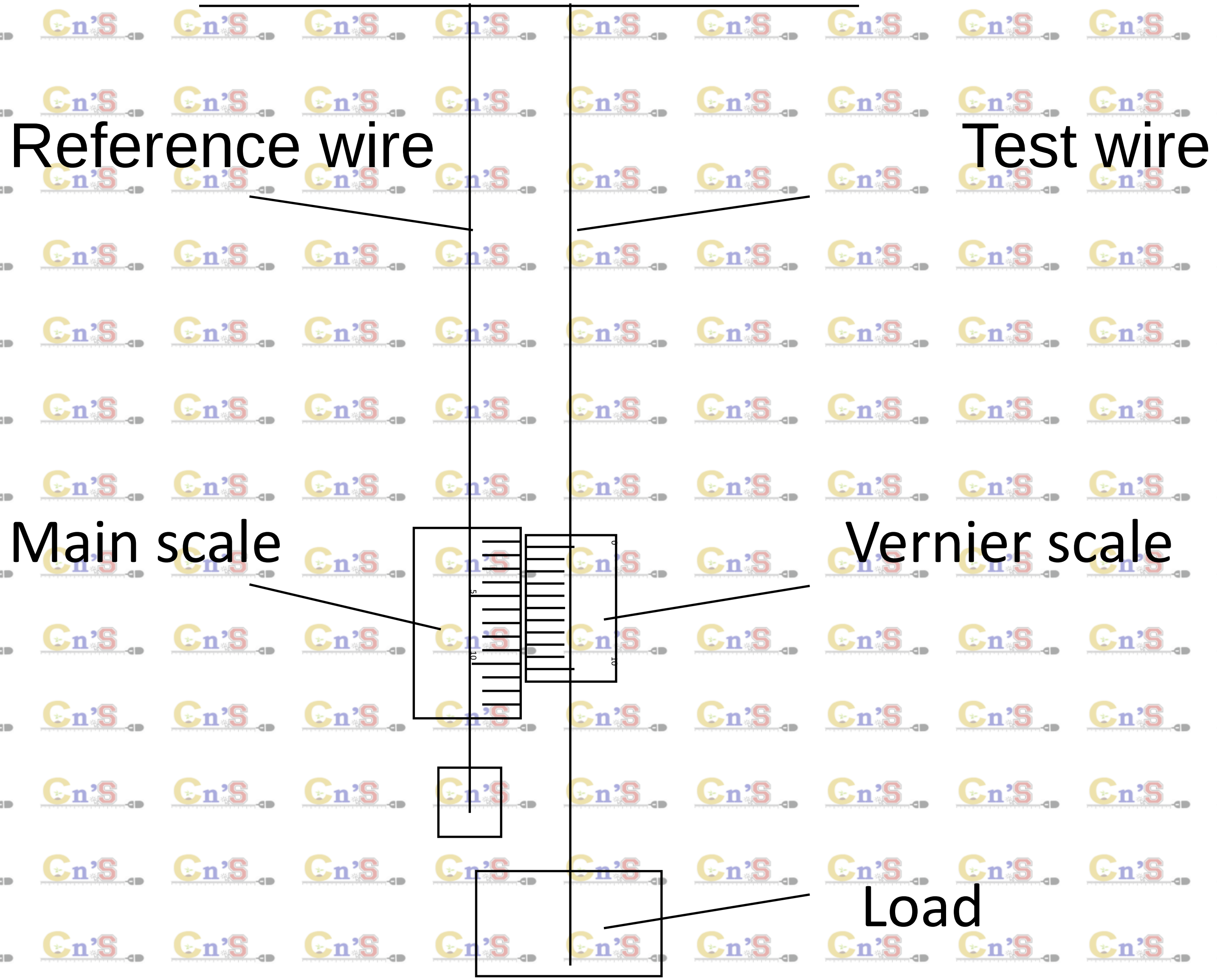


$$Y = \frac{FL}{A\Delta L} \quad \text{or} \quad \Delta L = \frac{FL}{AY}$$

$$\Delta L = \frac{FL}{AY} = \frac{(120\text{ N})(8.00\text{ m})}{(1.77 \times 10^{-6}\text{m}^2)(8.96 \times 10^{11}\text{Pa})}$$

Increase in length: $\boxed{\Delta L = 0.605 \text{ mm}}$

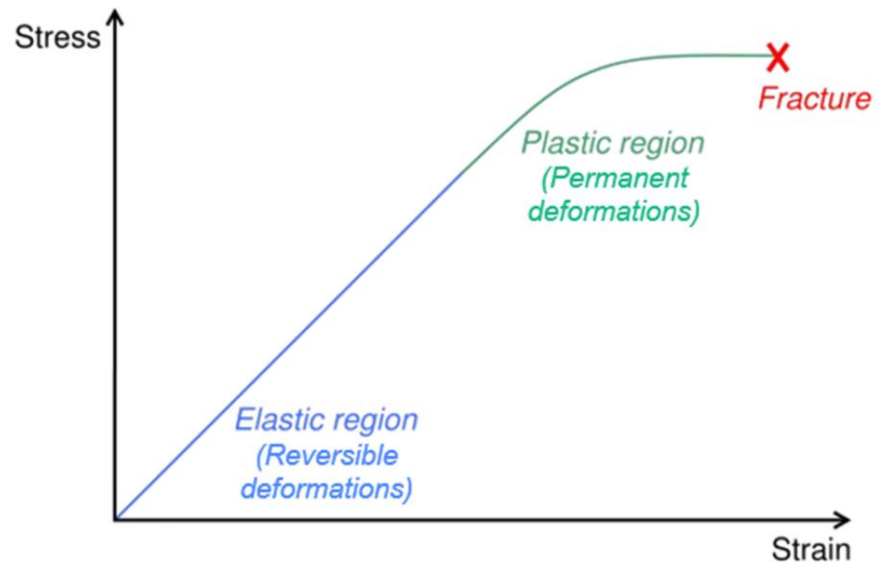




Determination of Young’s Modulus of the Material of a Wire

The readings that need to be taken are shown in the Table

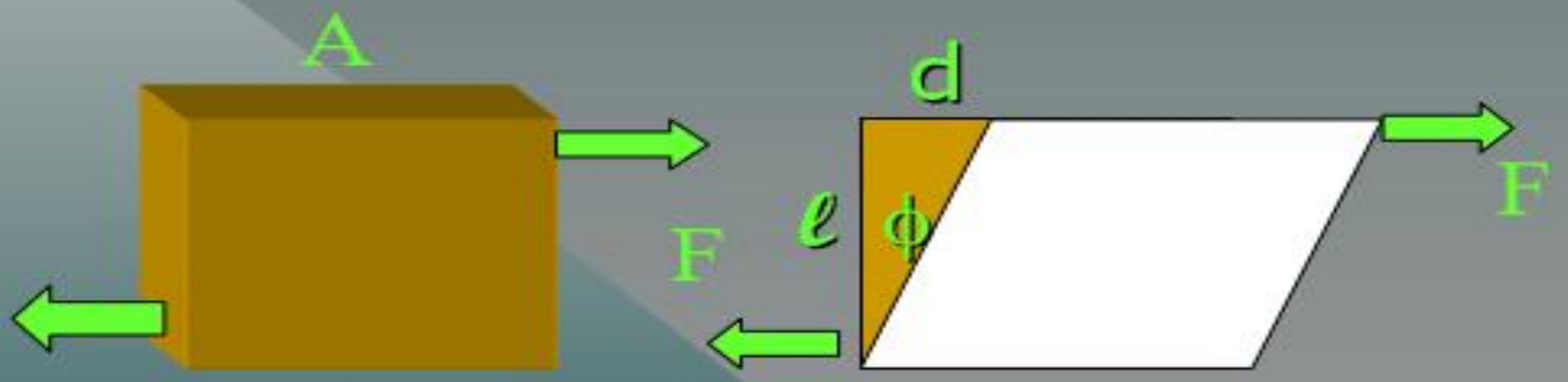
Reading	Reason	Instrument
Length of wire	Direct use	Metre ruler
Diameter of wire	Enables the cross-sectional area to be found	Micrometer screw gauge
Initial and final readings from the vernier slide	The difference between the two readings gives the extension	Vernier scale



Draw the stress–strain graph which gives STRAIGHT LINE, proving Hooke's law.

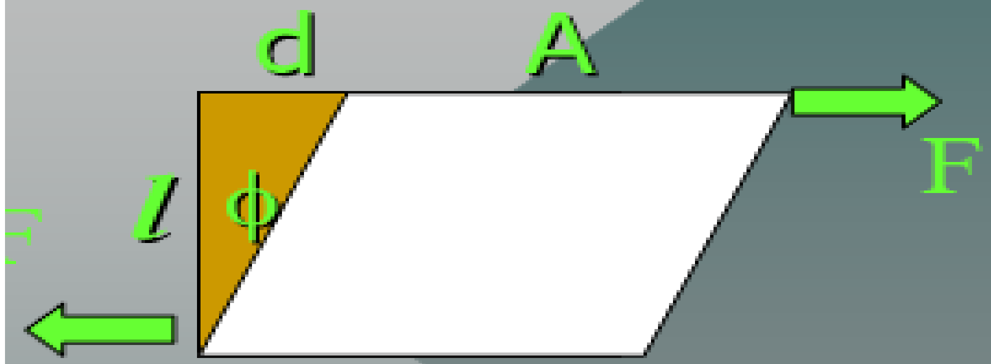
Shear Modulus

A shearing stress alters only the shape of the body, leaving the volume unchanged. For example, consider equal and opposite shearing forces F acting on the cube below:



The shearing force F produces a shearing angle ϕ. The angle ϕ is the strain and the stress is given by F/A as before.

Calculating Shear Modulus



Stress is force per unit area:

$$Stress = \frac{F}{A}$$

The strain is the angle expressed in <u>radians</u>:

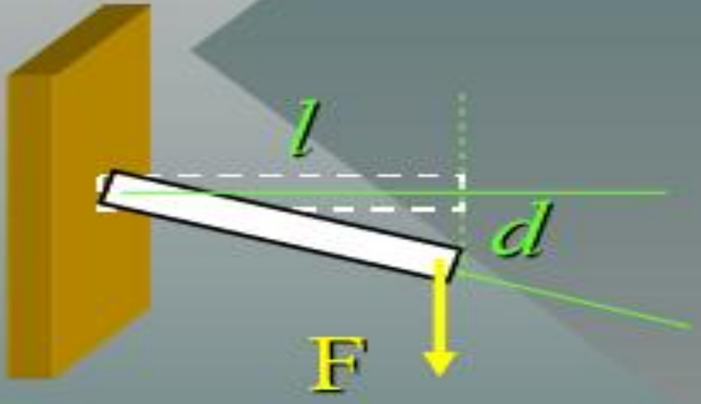
$$Strain = \phi = \frac{d}{l}$$

The shear modulus S is defined as the ratio of the shearing stress *F/A* to the shearing strain ϕ:

The shear modulus:
Units are in Pascals.

$$S = \frac{F/A}{\phi}$$

Example 5. A steel stud ($S = 8.27 \times 10^{10}$Pa) 1 cm in diameter projects 4 cm from the wall. A 36,000 N shearing force is applied to the end. What is the defection *d* of the stud?



$$A = \frac{\pi D^2}{4} = \frac{\pi (0.01\text{ m})^2}{4}$$

Area: $A = 7.85 \times 10^{-5}\text{ m}^2$

$$S = \frac{F/A}{\phi} = \frac{F/A}{d/l} = \frac{Fl}{Ad}; \qquad d = \frac{Fl}{AS}$$

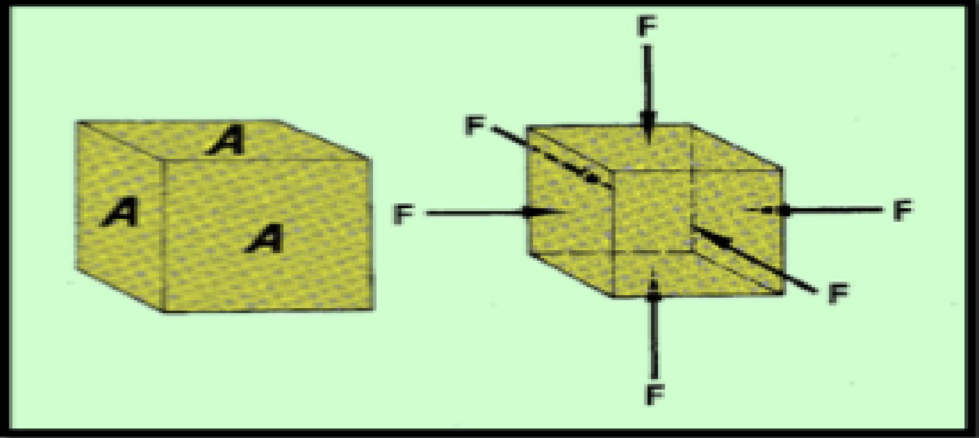
$$d = \frac{(36{,}000\text{ N})(0.04\text{ m})}{(7.85 \times 10^{-5}\text{m}^2)(8.27 \times 10^{10}\text{Pa})}$$

$\boxed{d = 0.222\text{ mm}}$

Volume Elasticity

Not all deformations are linear. Sometimes an applied stress F/A results in a decrease of volume. In such cases, there is a bulk modulus B of elasticity.

$$B = \frac{Volume\ stress}{Volume\ strain} = \frac{-F/A}{\Delta V/V}$$



The bulk modulus is negative because of decrease in V.

The Bulk Modulus

$$B = \frac{Volume\ stress}{Volume\ strain} = \frac{-F/A}{\Delta V/V}$$

Since *F/A* is generally pressure *P*, we may write:

$$B = \frac{-P}{\Delta V/V} = \frac{-PV}{\Delta V}$$

Units remain in Pascals (Pa) since the strain is unitless.

Example 7. A hydrostatic press contains 5 liters of oil. Find the decrease in volume of the oil if it is subjected to a pressure of 3000 kPa. (Assume that B = 1700 MPa.)

$$B = \frac{-P}{\Delta V / V} = \frac{-PV}{\Delta V}$$

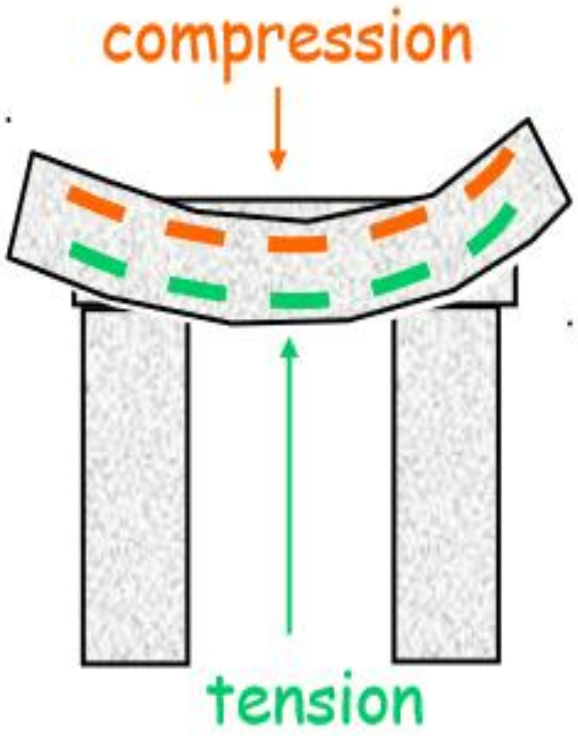
$$\Delta V = \frac{-PV}{B} = \frac{-(3 \text{ x } 10^6\text{Pa})(5 \text{ L})}{(1.70 \text{ x } 10^9\text{Pa})}$$

Decrease in V; milliliters (mL):

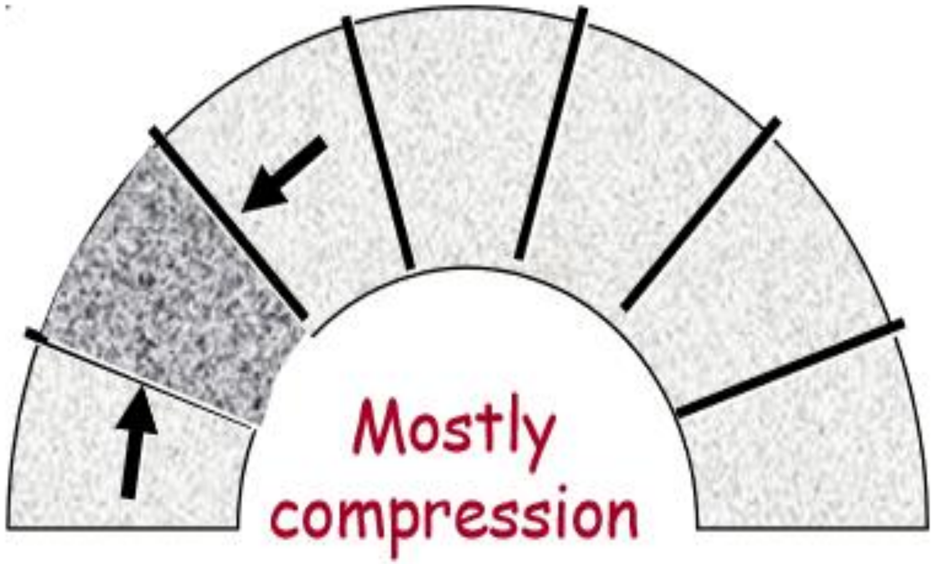
$$\boxed{\Delta V = -8.82 \text{ mL}}$$

Arches

Material	Tensile strength (MPa)	Compressive strength (MPa)
concrete	2	20
marble	-	80
wood (parallel to grain)	40	35



Ok with wood, not stone



Good design for stone