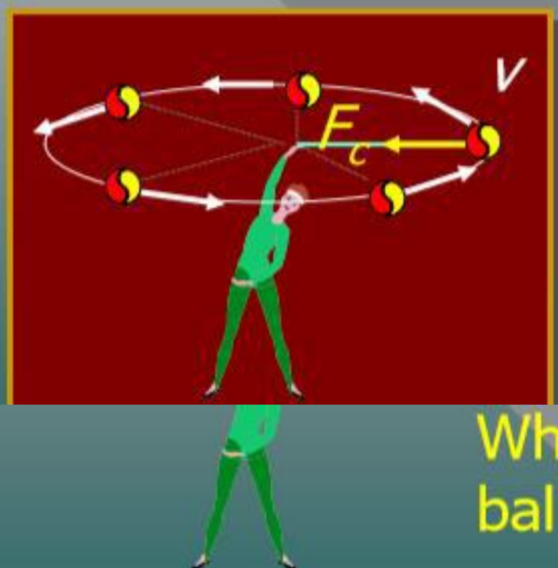
CIRCULAR MOTION

Uniform Circular Motion

<u>Uniform circular motion</u> *is motion along a circular path in which there is no change in speed, only a change in direction.*



- *Constant velocity tangent to path.*
- *Constant force toward center.*

When central force is removed, ball continues in straight line.

Centripetal force is needed to change direction.

Tangential Speed

- •The **tangential speed**(*vt*) of an object in circular motion is the object’s speed along an imaginary line drawn tangent to the circular path.
- •Tangential speed depends on the **distance**from the object to the center of the circular path.
- •When the tangential speed is constant, the motion is described as **uniform circular motion.**

(i) Angular Displacement Angular displacement is the angle subtended by the position vector at the centre of the circular path.

Angular displacement $(\Delta\theta) = (\Delta S/r)$

where Δs is the linear displacement and r is the radius. Its unit is radian.

(ii) Angular Velocity The time rate of change of angular displacement $(\Delta\theta)$ is called angular velocity.

Angular velocity $(\omega) = (\Delta\theta/\Delta t)$

Angular velocity is a vector quantity and its unit is rad/s.

Relation between linear velocity (v) and angular velocity (ω) is given by

$v = r\omega$

$$\Theta = S/r$$

$$\omega = \Theta/t$$

$$V = S/t$$

$$V = \omega r$$

Centripetal Acceleration

In circular motion, an acceleration acts on the body, whose direction is always towards the centre of the path. This acceleration is called centripetal acceleration.

Centripetal acceleration $a = \frac{v^2}{r} = r\omega^2$

Centripetal acceleration is also called radial acceleration as it acts along radius of circle.

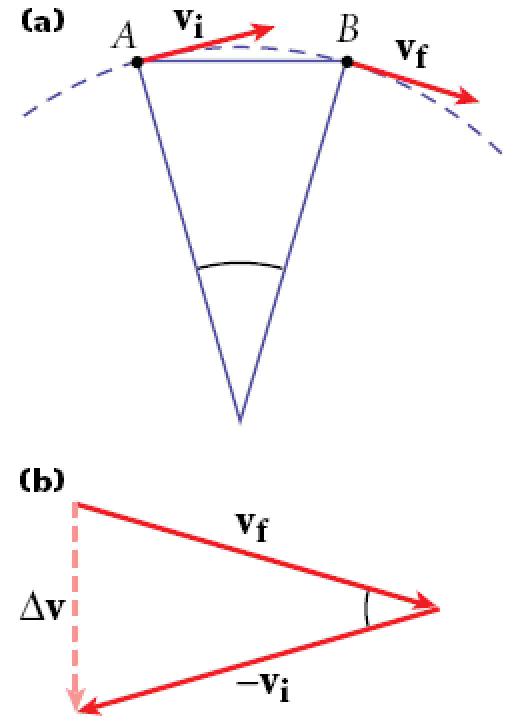
Its unit is in m/s^2 and it is a vector quantity.

Centripetal Acceleration

- • The acceleration of an object moving in a circular path and at constant speed is due to a **change in direction.**
- • An acceleration of this nature is called a **centripetal acceleration.**
- CENTRIPETAL ACCELERATION ac vt 2 r centripetal acceleration = (tangential speed)2 radius of circular path
- **Circular Motion**

Centripetal Acceleration

- •**(a)**As the particle moves from *A*to *B*, the direction of the particle's velocity vector changes.
- •**(b)**For short time intervals, **Δv**is directed toward the center of the circle.
- •**Centripetal acceleration is always directed toward the center of a circle.**



- •The **centripetal acceleration**results from a **change in direction.**

- •In circular motion, an acceleration due to a **change in speed**is called **tangential acceleration.**
- •To understand the difference between centripetal and tangential acceleration, consider a car traveling in a circular track.
- •Because the car is moving in a circle, the car has a **centripetal**component of acceleration.
- •If the car’s speed changes, the car also has a **tangential** component of acceleration.

Centripetal Force

It is that force which complex a body to move in a circular path.

It is directed along radius of the circle towards its centre.

For circular motion a centripetal force is required, which is not a new force but any force present there can act as centripetal force.

Centripetal force $F = \frac{mv^2}{r} = mr\omega^2$

where, m = mass of the body, c = linear velocity,

ω = angular velocity and r = radius.

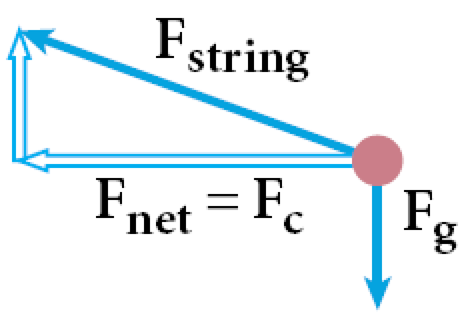
Work done by the centripetal force is zero because the centripetal force and displacement are at right angles to each other.

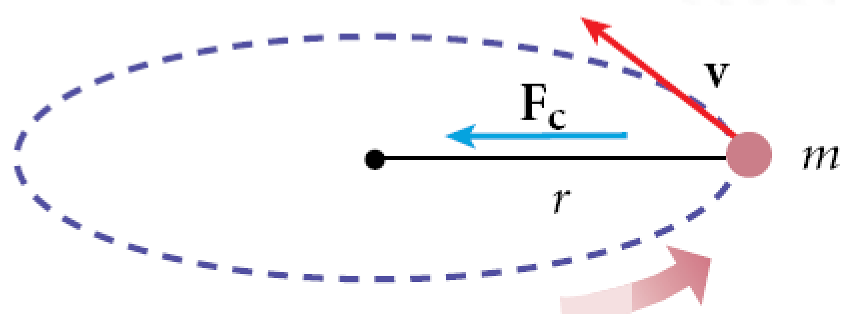
Centripetal Force

- •Consider a ball of **mass *m***that is being whirled in a horizontal circular path of **radius *r***with constant speed.

- •The force exerted by the string has horizontal and vertical components. The **vertical**component is equal and opposite to the **gravitational force**. Thus, the **horizontal component**is the **net force.**

- •This net force, which is isdirected toward the center of the circle, is a **centripetal force.**





- Newton's second law can be combined with the equation for centripetal acceleration to derive an equation for centripetal force:

$$a_c = \frac{v_t^2}{r}$$

$$F_c = ma_c = \frac{mv_t^2}{r}$$

$$\text{centripetal force} = \frac{\text{mass} \times (\text{tangential speed})^2}{\text{radius of circular path}}$$

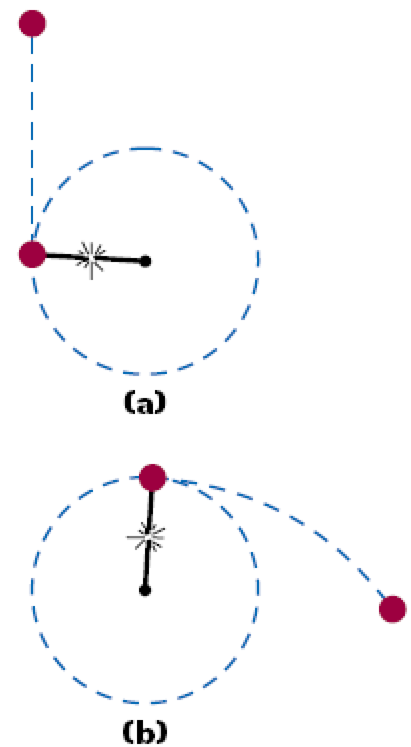
- **Centripetal force** is simply the name given to the **net force** on an object in uniform circular motion.
- **•Any type of force or combination of forces can provide this net force.**
- –For example, **friction** between a race car’s tires and a circular track is a centripetal force that keeps the car in a circular path.
- –As another example, **gravitational force** is a centripetal force that keeps the moon in its orbit.
- **Circular Motion**

Examples of some incidents and the cause of centripetal force involved.

S.No.	Incidents	Force providing Centripetal Force
1	Orbital motion of planets.	Gravitational force between planet and sun.
2	Orbital motion of electron.	Electrostatic force between electron and necleus.
3	Turning of vehicles at turn.	Frictional force acting between tyres of vehicle and road.
4	Motion of a stone in a circular path, tied with a string.	Tension in the string.

- If the centripetal force vanishes, the object stops moving in a circular path.

- •A ball that is on the end of a string is whirled in a vertical circular path.
- *–If the string breaks at the position shown in **(a)**,the ball will move vertically upward in free fall.*
- *–If the string breaks at the top of the ball's path, as in **(b)**,the ball will move along a parabolic path.*



Describing a Rotating System

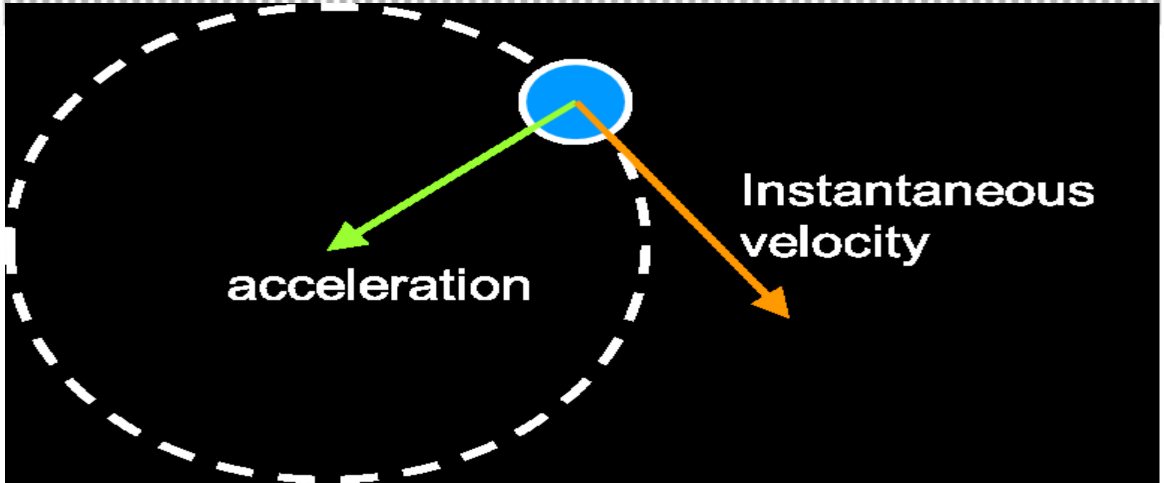
- •To better understand the motion of a rotating system, consider a car traveling at high speed and approaching an exit ramp that curves to the left.
- •As the driver makes the sharp left turn, the passenger slides to the right and hits the door.
- •What causes the passenger to move toward the door?

- As the car enters the ramp and travels along a curved path, the passenger, because of inertia, tends to move along the original straight path.
- •If a sufficiently large centripetal force acts on the passenger, the person will move along the same curved path that the car does. The origin of the centripetal force is the force of friction between the passenger and the car seat.
- •If this frictional force is not sufficient, the passenger slides across the seat as the car turns underneath.

Circular Motion

- Uniform circular motion occurs when an object travels in a circular path at a constant speed.
- Uniform circular motion
- Q. Explain why a ball moving in a circle must be accelerating towards the centre.
- A.
- -Magnitude of velocity is constant
- -Direction of velocity changes

- -Thus v is changing so it must be accelerating



Centripetal acceleration is calculated using the formula…

$$\boxed{a = \frac{v^2}{r}}$$

v = tangential velocity at any instant (ms^{-1})
r = radius of circle (m)
a = centripetal acceleration (ms^{-2})

Thus, using F = ma

$$\boxed{\text{Centripetal force} = \frac{mv^2}{r}}$$

Note that using the speed formula

$T = 2\pi/\omega$ $V = \omega r$

$$\boxed{v = \frac{2\pi r}{T}}$$

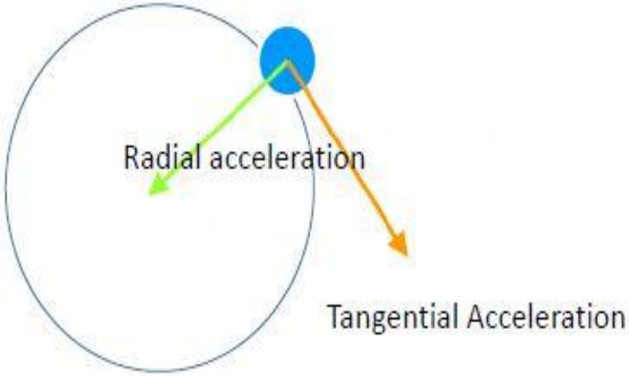
$$\boxed{T = \frac{1}{f}}$$

Tangential Acceleration

Non uniform circular motion

When velocity is not constant the particle has a Tangential Acceleration in addition to Radial Acceleration.

$$\alpha = \omega / t$$



$$a_t = V/t$$
$$= \omega r / t$$
$$= r.\ \omega / t$$
$$= r.\ \alpha$$

In Non uniform circular motion the Resultant Acceleration is the vector sum of both accelerations. Resultant Accn = $\sqrt{(a_r^2 + a_t^2)}$

E.g.1

A stone of mass 0.5kg on the end of a string 250cm long is swung in a horizontal plane at a constant speed of $10ms^{-1}$. What is the tension in the string?

$$F = \frac{mv^2}{r}$$

$$= \frac{0.5 \times 10^2}{2.5}$$

$$= \underline{20N}$$

1. A 500 kg car is driving in a 25 m circle at 18 m/s. • Calculate the period of the car. •Calculate the centripetal acceleration. • Calculate the centripetal force.
2. A 5.2 kg rock is attached to a 1.8 m string and is swung in a horizontal circle. •Calculate the maximum speed of the rock if the string breaks under a tension of 215 N. • Calculate the period and frequency of the rock.
3. Two identical objects are placed on a rotating disc. One object is twice as far from the centre as the other. How d o there velocities compare? $V=2\pi R/T$

A Closer object moves twice as fast.

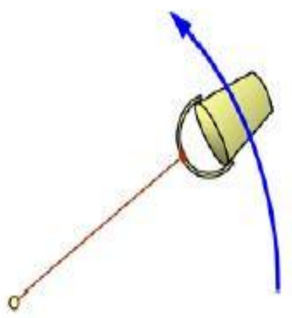
B Closer object moves twice as slow.

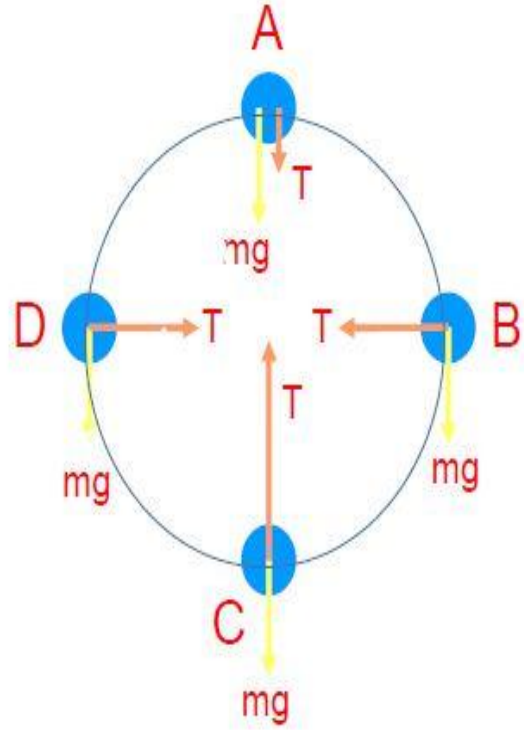
C The have the same speed.

Vertical Circular Motion

When moving at a constant speed in a vertical circle, the centripetal acceleration and centripetal force must both be constant.

E.g. A bucket swinging vertically on a string:





At A:

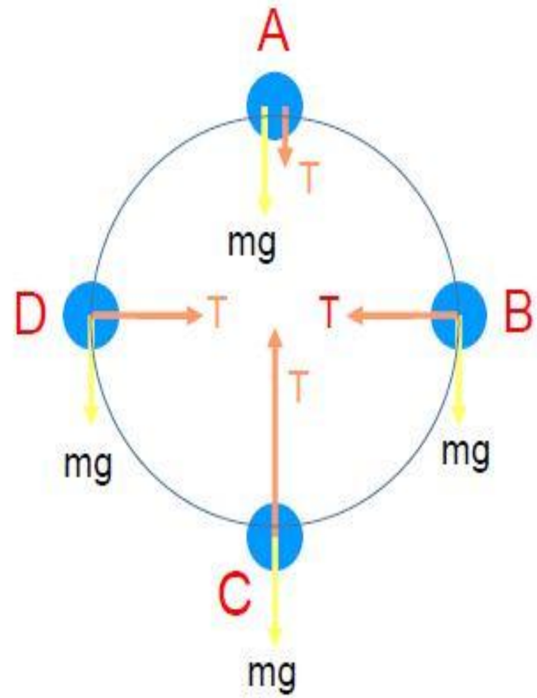
The tension is smallest because part of the centripetal force is provided by the weight.

At C:

The tension is greatest because the tension must hold up the weight as well as supply the centripetal force.

At B and D:

The tension provides all of the centripetal force as the weight is perpendicular to the tension.

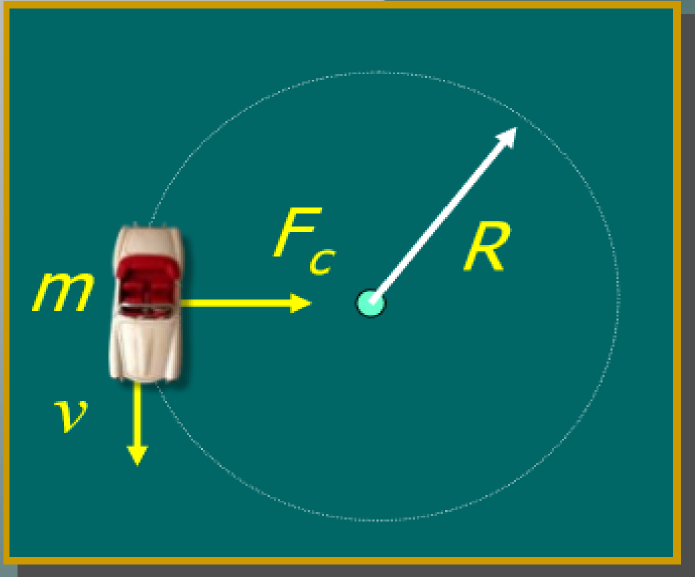


At A: $mv^2/r = mg + T$

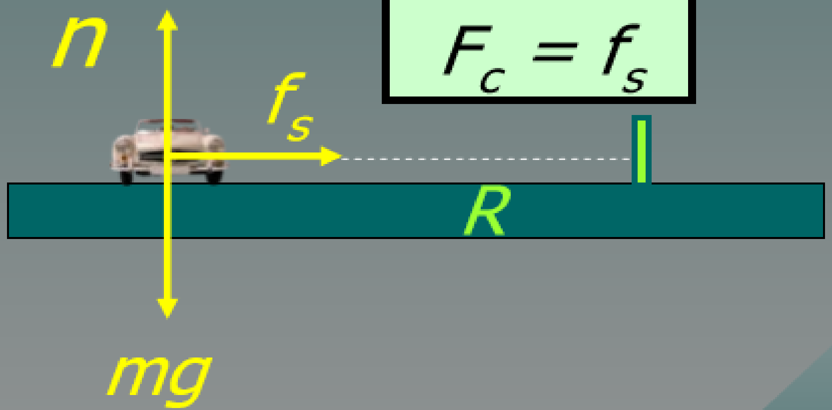
At C: $mv^2/r = T - mg$

At B and D: $mv^2/r = T$

Car Negotiating a Flat Turn



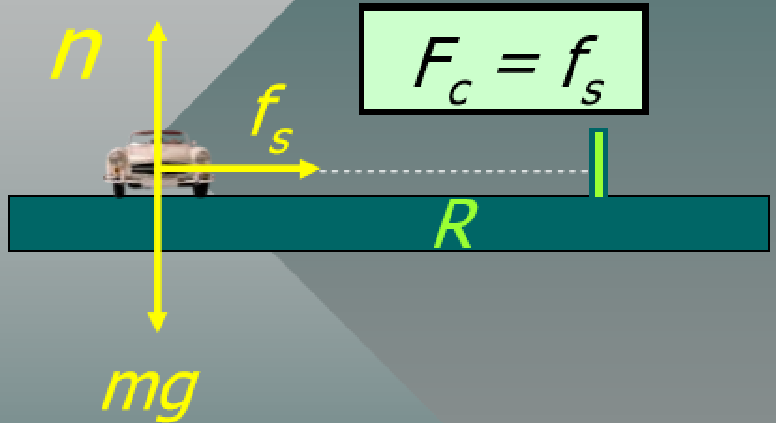
The centripetal force F_c is that of static friction f_s:

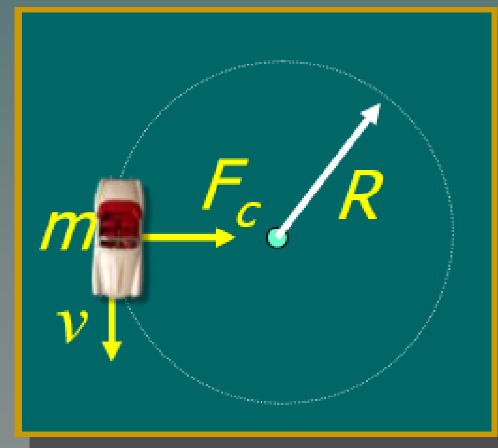


$$\boxed{F_c = f_s}$$

The central force F_C and the friction force f_s are not two different forces that are equal. There is just one force on the car. The nature of this central force is static friction.

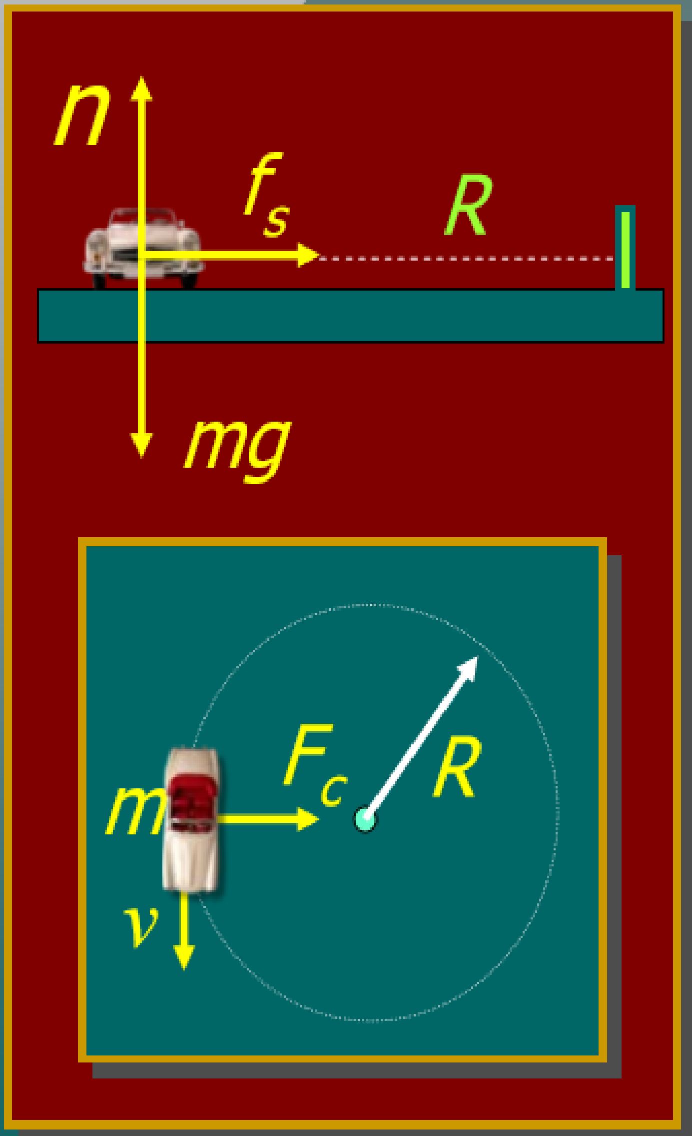
Finding the maximum speed for negotiating a turn without slipping.





The car is on the verge of slipping when F_C is equal to the maximum force of static friction f_s.

$$F_c = f_s \qquad F_c = \frac{mv^2}{R} \qquad f_s = \mu_s mg$$



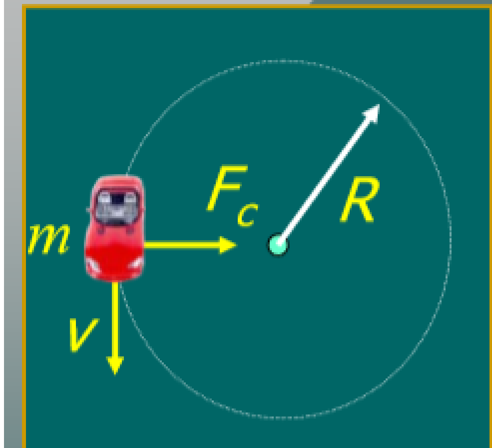
$$F_c = f_s$$

$$\frac{\cancel{m}v^2}{R} = \mu_s \cancel{m}g$$

$$\boxed{v = \sqrt{\mu_s g R}}$$

Velocity v is maximum speed for no slipping.

Optimum Banking Angle



By banking a curve at the optimum angle, the normal force n can provide the necessary centripetal force without the need for a friction force.

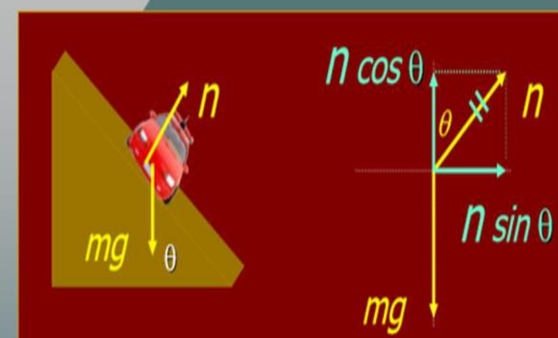
Optimum Banking Angle (Cont.)

$$n \sin\theta = \frac{mv^2}{R}$$

$$n \cos\theta = mg$$

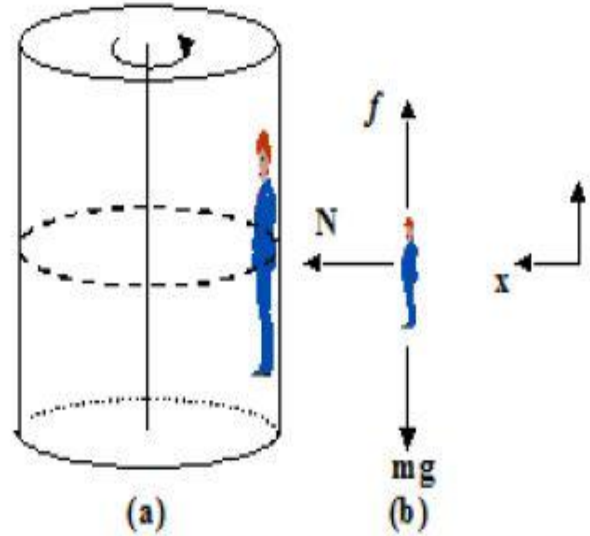
$$\tan\theta = \frac{\dfrac{\cancel{m}v^2}{R}}{\dfrac{\cancel{m}g}{1}} = \frac{v^2}{gR}$$

Optimum Banking Angle (Cont.)



Optimum Banking Angle θ $\quad \tan\theta = \dfrac{v^2}{gR}$

In a carnival ride called the rotor, people stand on a ledge inside a large cylinder that rotates about a vertical axis. When it reaches a high enough rotational speed, the ledge drops away. Find the minimum coefficient of friction for the people not to slide down. Take the radius to be 2 m and the period of revolution to be 2s



A person on a "rotor" ride. The force of friction balances the weight. The centripetal force is provided by the normal force **N**.

Solution

The forces acting on a person is shown in the figure (b).

Applying Newton's Second Law

$f=mg$ (1)

$N=m\omega2R$ (2)

Since $f=\mu N$

$\mu=g\omega2R=gT24\pi2R$

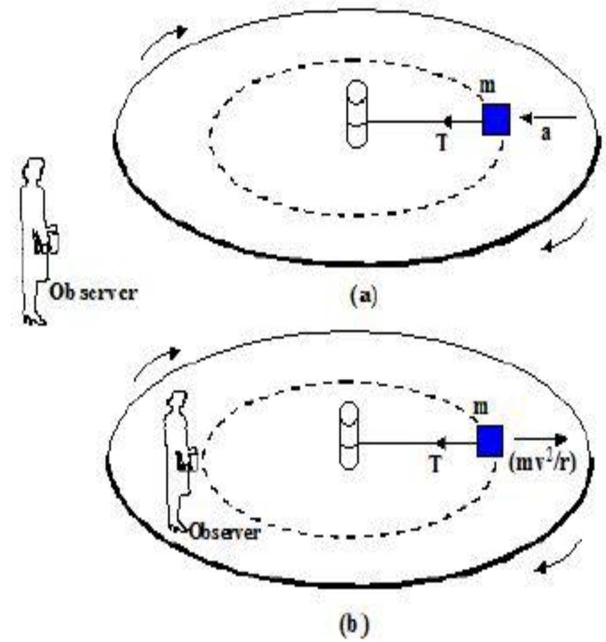
Here $T = 2s$; $R = 2m$; $g = 10\ m/s2$

$\mu = 0.5$

A block is tied to the centre post of a rotating platform by a string.

(a) An inertial observer sees the block moving in a circle with centripetal force provided by the tension in the string.

(b) According to a non-inertial observer on the platform, the block is not accelerating/moving.

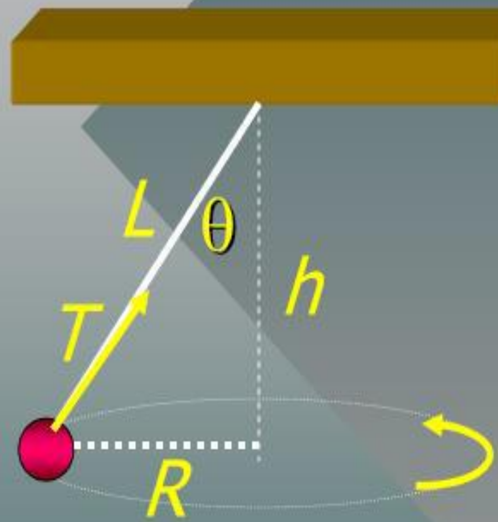


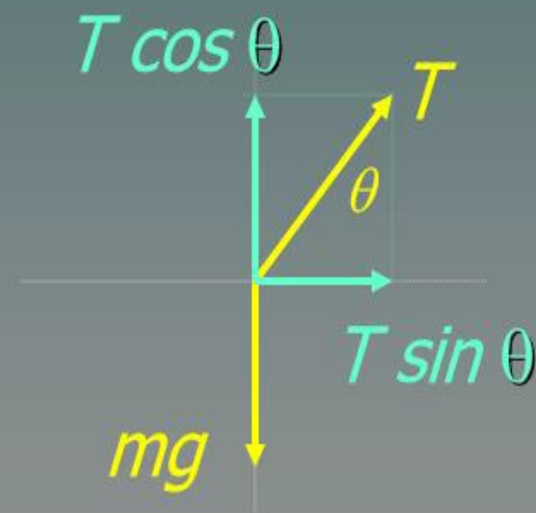
The Conical Pendulum

A conical pendulum consists of a mass m revolving in a horizontal circle of radius R at the end of a cord of length L.

Note: The inward component of tension $T \sin \theta$ gives the needed central force.

Angle θ and velocity v:



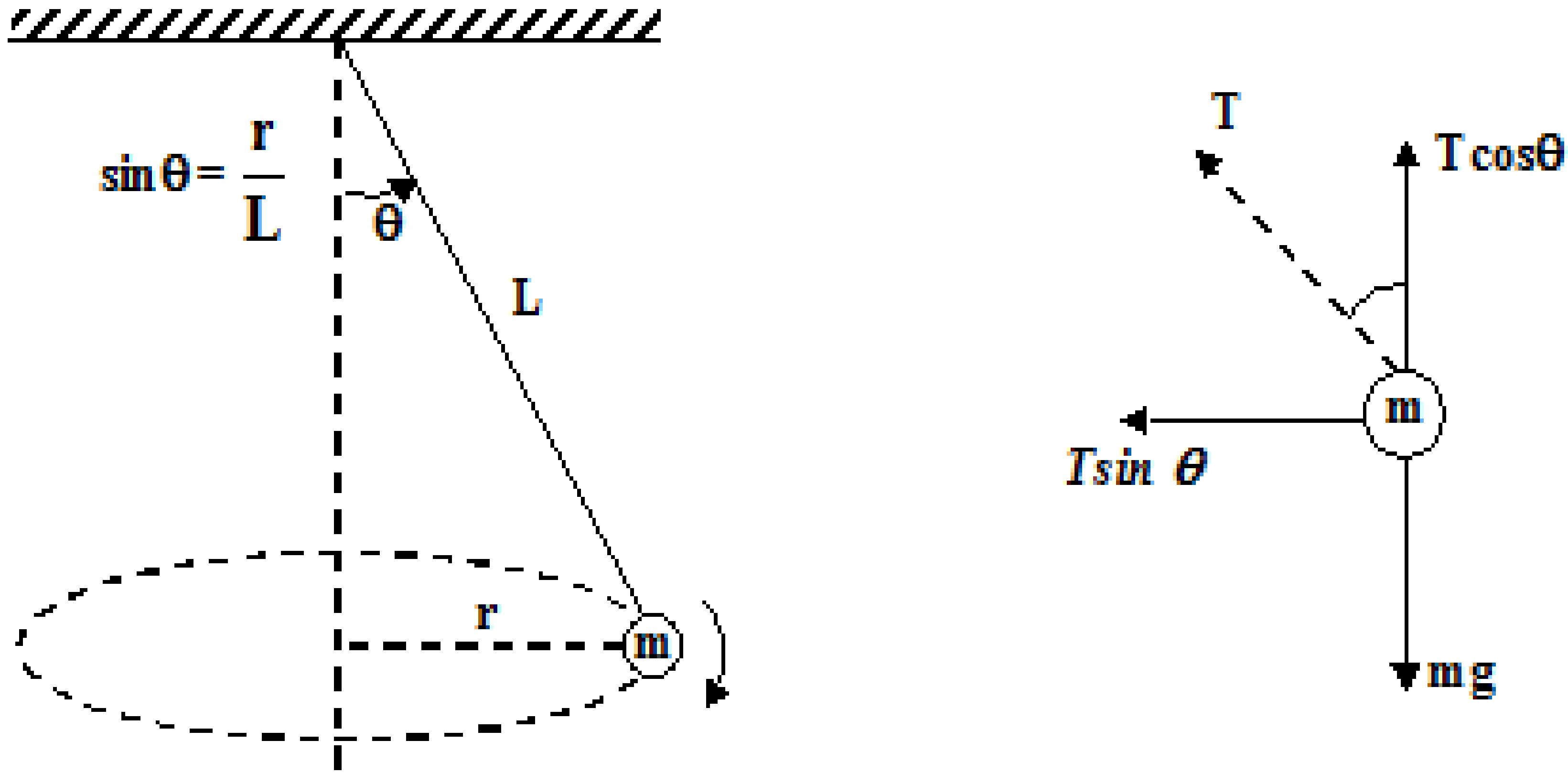


Solve two equations to find angle θ

$$T\sin\theta = \frac{mv^2}{R}$$

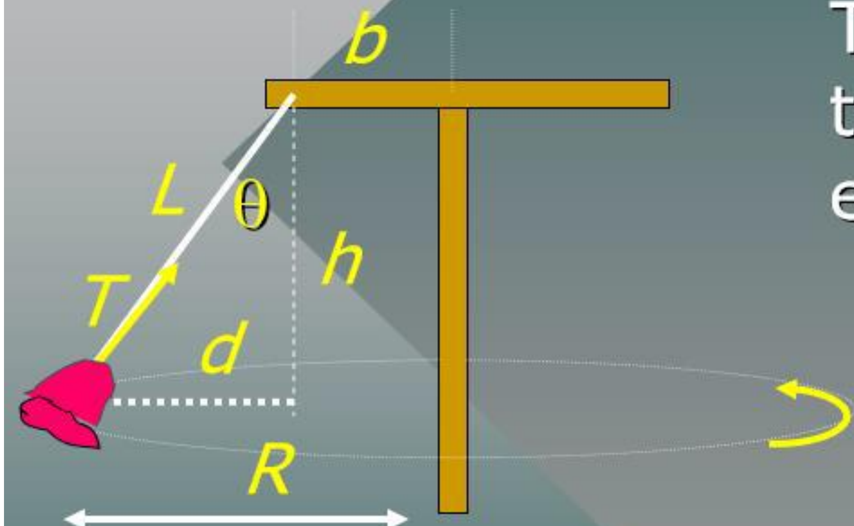
$$T\cos\theta = mg$$

$$\boxed{\tan\theta = \frac{v^2}{gR}}$$



(a) (b)

Swinging Seats at the Fair



This problem is identical to the other examples except for finding R.

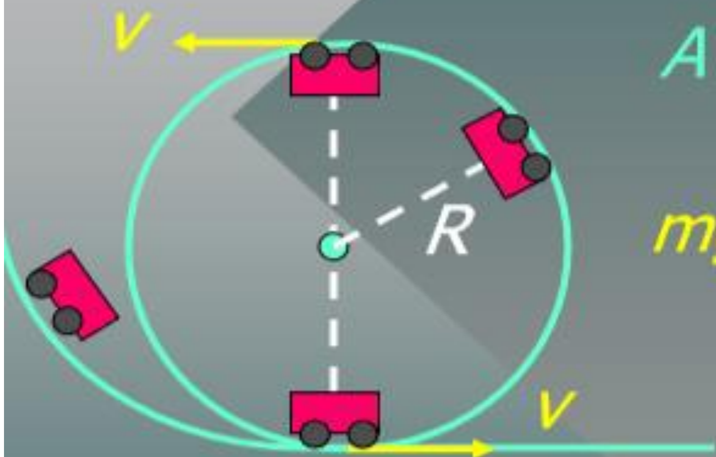
$$R = d + b$$

$$R = L \sin\theta + b$$

$$\tan\theta = \frac{v^2}{gR} \quad \text{and} \quad v = \sqrt{gR \tan\theta}$$

The Loop-the-Loop

Same as cord, n replaces T



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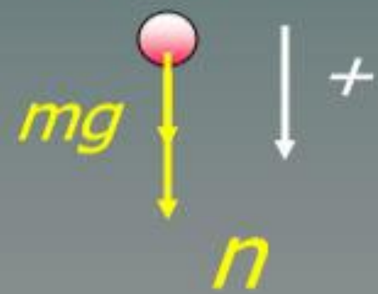
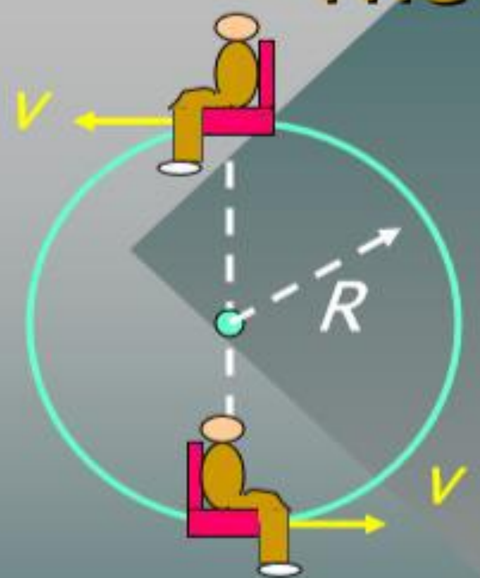


$$\boxed{n = \frac{mv^2}{R} - mg}$$

AT BOTTOM:

$$\boxed{n = \frac{mv^2}{R} + mg}$$

The Ferris Wheel



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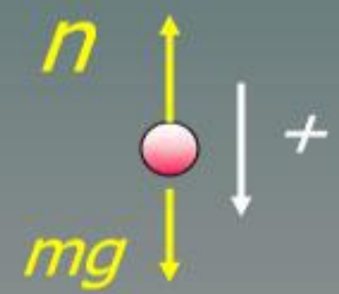

$$mg - n = \frac{mv^2}{R}$$

$$\boxed{n = mg - \frac{mv^2}{R}}$$

AT BOTTOM:

$$\boxed{n = \frac{mv^2}{R} + mg}$$