Velocity of Sound

Objectives

- Transverse stationary waves in a string.
- Various modes of vibration in a stretched string.
- The simplest mode of vibration (fundamental) in a string.
- Over tones and harmonics in a string.
- Relationship between the length of the string and the wavelength for each mode of vibration.
- The speed of a transverse wave in a stretched string, $v = \frac{T}{m}$

- The fundamental tone in a string.

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

- The speed of longitudinal wave in a medium $V = \frac{E}{\rho}$. (in a rod)

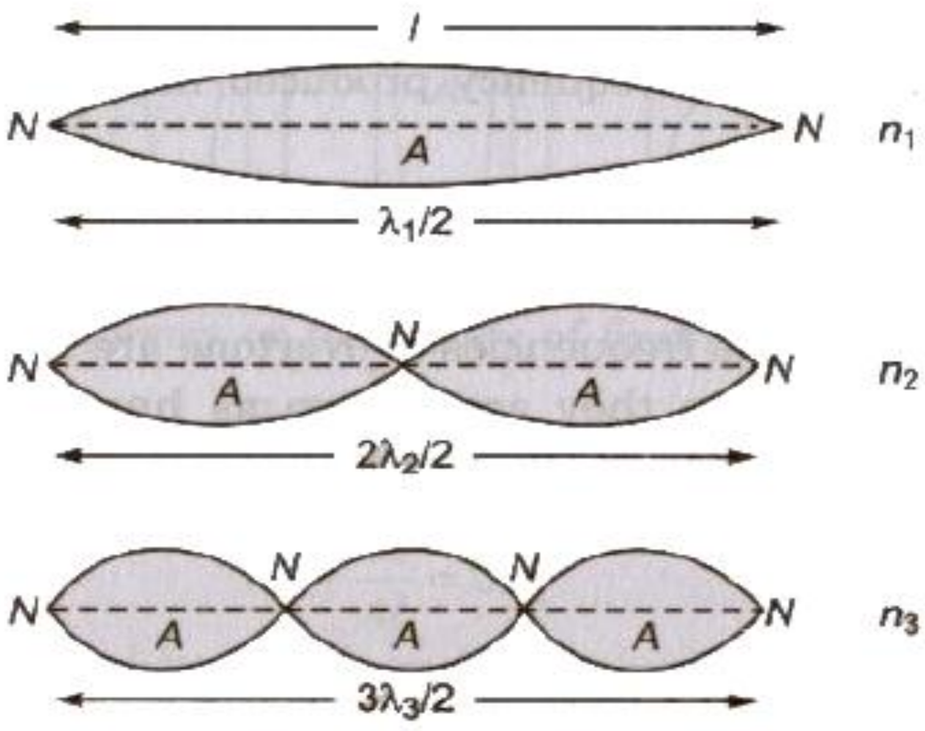
- Stationary waves formed in a rod
  One end clamped
  Clamped in the middle

Q . How the stationary waves are produced in stretched string?

- When the string is plucked at its middle point, two transverse waves will originate from this point. One of waves will moves towards the left end of the string and the other towards the right end. When these waves reach the two clamped ends, they are reflected back thus giving rise to stationary waves and the string vibrate in one loop.
- When the string is plucked from one quarter of its length, the stationary waves will be setup and the string vibrates in two loops.
- When the string is plucked from one-sixth of its length, the stationary waves will be setup and the string vibrates in three loops.
- Similarly, by plucking the string properly, it can be made to vibrate in 4, 5, 6,…..,n loops and the frequency of these modes of vibration will be 4f , 5 f , 6 f ,…., nf , respectively.

Vibrations in a Stretched String

Velocity of a transverse wave in a stretched string.



$$v = \sqrt{\frac{T}{m}}$$

Where,T is tension in the string and m is mass per unit length of the string.

Normal modes of a string

Wavelength: $\lambda_n = \dfrac{2L}{n} \qquad n = 1, 2, 3, \ldots$

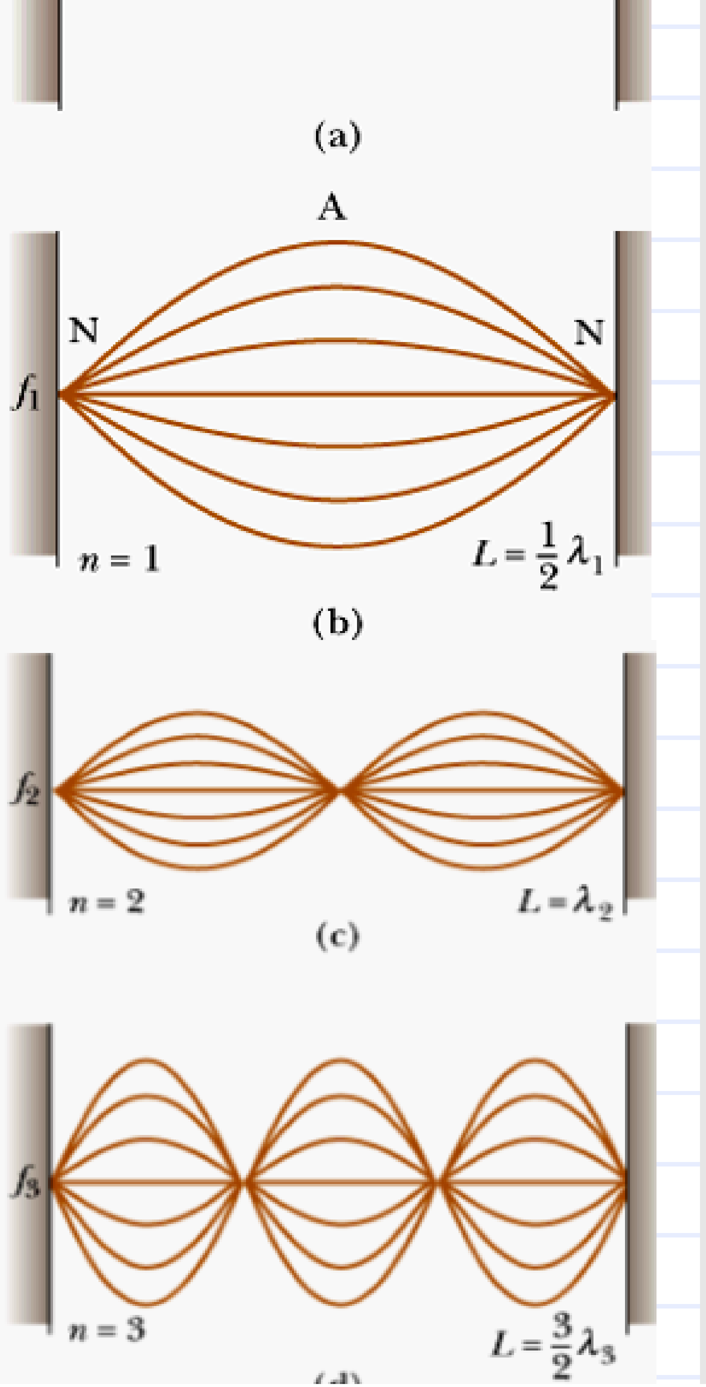
Frequency: $f_n = \dfrac{v}{\lambda_n} = n\dfrac{v}{2L} \qquad n = 1, 2, 3, \ldots$

Using : $v = \sqrt{T/\mu}$

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \qquad n = 1, 2, 3, \ldots$$

$$f_n = n \cdot f_1$$

$f_1 = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$ is called the fundamental frequency



FUNDAMENTAL FREQUENCY ("FIRST HARMONIC") STANDING WAVE

- **Has the lowest frequency and longest wavelength.**

$$\lambda_1 = 2L$$

Fundamental frequency:

$$f_1 = \frac{v}{\lambda_1} = \frac{1}{2L}v = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

HARMONICS OR RESONANT FREQUENCIES

$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$	fundamental or first harmonic
$f_2 = 2f_1$	second harmonic
$f_3 = 3f_1$	third harmonic
$f_4 = 4f_1$	fourth harmonic (14-54)

It is easy to understand why the wave speed depends on the tension in the string. If a string under tension is pulled sideways and released, the tension is responsible for accelerating a particular segment back toward its equilibrium position. The acceleration and wave speed increase with increasing tension in the string. Likewise, the wave speed is inversely dependent on the mass per unit length of the string. Thus, wave speed is directly dependent on the tension and inversely dependent on the mass per unit length. The exact relationship of the wave speed, v, the tension, F_T and the mass per per length, μ, is

$$v = \sqrt{\frac{F_T}{\mu}}$$

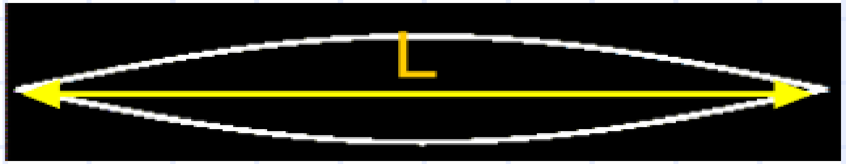
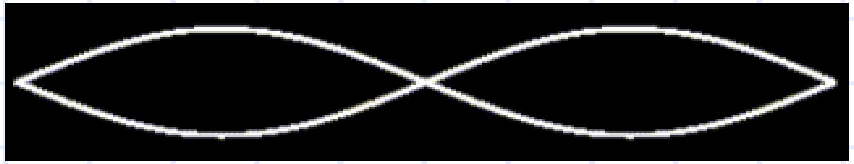
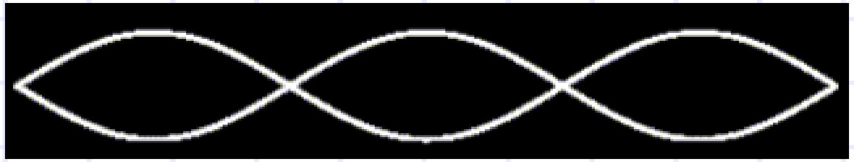
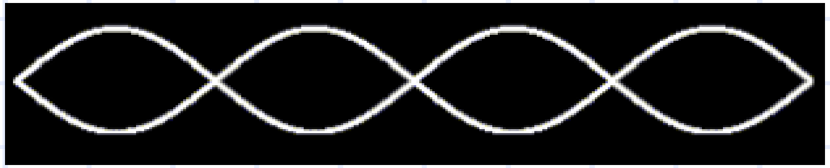
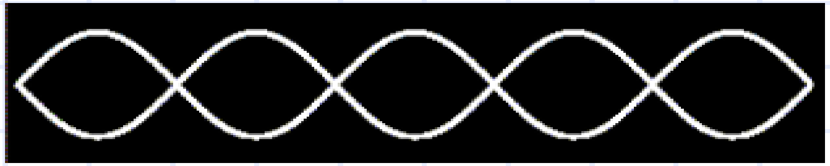
We san increase the speed of a wave on a streched string by increasing the tension in the string. If we wrap a string with a metalic winding, as is done to the bass strings of pianos and guitars, we decrease the speed of a transmitted wave.

Standing waves can be set up in a streched string by connecting one end of the string to a stationary clamp and connecting the other end to a vibrating object. In this situations, traveling waves reflect from the ends, creating waves traveling in both directions on the string. The incident and reflected waves combine according to the superposition principle. If the string is vibrated at exactly the right frequency, the wave appears to stand. A node occurs where the two traveling waves always have the same magnitude of displacement but of opposite sign, so that the net displacement is zero at this point. But midway between two nodes, at an antinude, the string vibrates with the largest amplitude. Note, that the ends of the string must be nodes because these points are fixed. The characteristic frequencies of standing waves in a streched string of length L are

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}} \quad n = 1, 2, 3, \ldots$$

where F_T is the tension in the string, μ is its mass per unit length.

Modes of vibration of strings

Picture of Standing Wave	Name	Structure
 $L = \frac{1}{2}\lambda_1$ $f_1 = v/2L$	1st Harmonic or Fundamental	1 Antinode 2 Nodes
 $L = \lambda_2$ $f_2 = v/L$	2nd Harmonic or 1st Overtone	2 Antinodes 3 Nodes
 $L = 1\frac{1}{2}\lambda_3$ $f_3 = 3v/2L$	3rd Harmonic or 2nd Overtone	3 Antinodes 4 Nodes
 $L = 2\lambda_4$ $f_4 = 2v/L$	4th Harmonic or 3rd Overtone	4 Antinodes 5 Nodes
 $L = 2\frac{1}{2}\lambda_5$ $f_5 = 5v/2L$	5th Harmonic or 4th Overtone	5 Antinodes 6 Nodes

Vibrations of String:

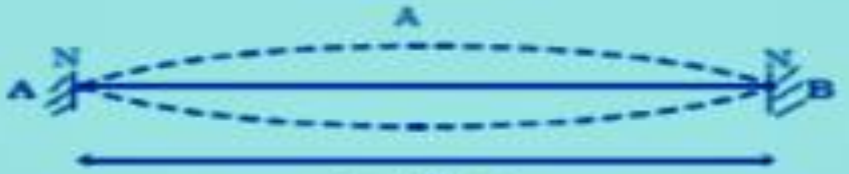
- If 'T' is the tension in the string and 'm' is its linear density, then the velocity of the transverse wave along its length of a stretched wire is given by the expression

$$V = \sqrt{\frac{T}{m}}$$

- Now let us consider a string of finite length and fixed at its two ends. The wave disturbance, originating at the point at which the string is plucked, travels up to the ends and is reflected back. The incident and reflected waves interfere to produce a stationary wave. The particles of the string, at the two ends, which are fixed, always remain at rest. Hence, the two ends of the string always become nodes. However, the other particles in the string can vibrate in different ways giving rise to what is called the different modes of vibration of the string.
-

Vibrations of String (Fundamental Mode):

- In the following figure, the string is shown to be vibrating in the simplest mode called the fundamental mode. In this mode, the frequency of vibration is the least. The two ends are nodes and there is an antinode exactly midway between the two ends. In other words, one complete loop is formed on the string. Since the distance between the two consecutive nodes is λ/2, the length of the string(*l*).



$l = \lambda/2$

$$l = \frac{\lambda}{2}$$

$$\therefore \lambda = 2l$$

$$\text{Now, } v = n.\lambda$$

$$\therefore v = n.2l$$

$$\text{But } v = \sqrt{\frac{T}{m}}$$

$$\therefore n.2l = \sqrt{\frac{T}{m}}$$

$$\therefore n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

This is the frequency of the fundamental mode of a vibration of the stretched string.

Factors that determine the fundamental frequency of a vibrating string

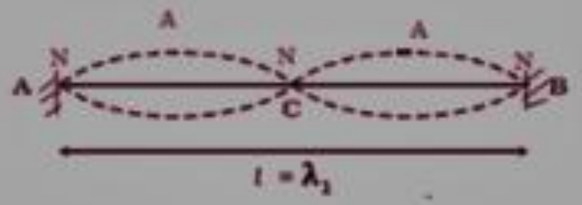
- The frequency of vibration depends on
  - the mass per unit length of the string,
  - the tension in the string and,
  - the length of the string.
- The fundamental frequency is given by

$$f_o = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where T = tension
μ = mass per unit length
L = length of string

Vibrations of String (First Overtone):

- In the following figure, the string is shown to have broken up into two complete loops, there is a node midway between the two nodes and an antinode at a distance equal to a quarter of the length of the string from each end. This mode of vibration is called the first overtone. If the wavelength corresponding to this mode is λ_1 and frequency is n_1.



$$\lambda_1 = l$$

$$\therefore v = n_1\lambda_1 = n_1 l$$

$$\text{Now } v = \sqrt{\frac{T}{m}}$$

$$n_1\lambda_1 = \sqrt{\frac{T}{m}}$$

$$\therefore n_1 = \frac{1}{\lambda_1}\sqrt{\frac{T}{m}}$$

$$\therefore n_1 = \frac{1}{l}\sqrt{\frac{T}{m}}$$

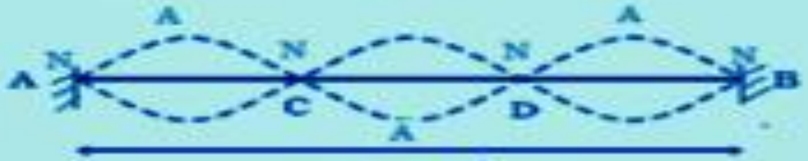
$$\therefore n_1 = 2\left[\frac{1}{2l}\sqrt{\frac{T}{m}}\right]$$

$$\therefore n_1 = 2n$$

This is the frequency of the first overtone of a vibration of the stretched string.

Vibrations of String (Second Overtone):

- In the following figure, the string is shown to have broken up into three complete loops. This is the second overtone. If the wavelength corresponding to this mode is λ_2 and frequency is n_2. Then.



$l = (3/2)\lambda_2$

$$l = \frac{3\lambda_2}{2}$$

$$\therefore \lambda_2 = \frac{2l}{3}$$

$$\text{Now } v = \sqrt{\frac{T}{m}}$$

$$n_2\lambda_2 = \sqrt{\frac{T}{m}}$$

$$\therefore n_2 = \frac{1}{\lambda_2}\sqrt{\frac{T}{m}}$$

$$n_2 = \frac{3}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore \quad n_2 = 3\left[\frac{1}{2l}\sqrt{\frac{T}{m}}\right]$$

$$\therefore \quad n_2 = 3n$$

This is the frequency of the second overtone of a vibration of the stretched string.

Thus the second overtone is the third harmonic.

- Find the first four harmonics of a 1-m-long string if the string has a mass per unit length of $2 \cdot 10^{-3}$ kg/m and is under a tension of 80 N.

$$\begin{aligned}
f_n &= \frac{n}{2L}\sqrt{\frac{F_T}{\mu}} \quad n = 1, 2, 3, 4 \\
f_1 &= \frac{1}{2(1\ \mathrm{m})}\sqrt{\frac{80\ \mathrm{N}}{2 \cdot 10^{-3}\ \mathrm{kg/m}}} = 100\ \mathrm{Hz} \\
f_2 &= 200\ \mathrm{Hz} \\
f_3 &= 300\ \mathrm{Hz} \\
f_4 &= 400\ \mathrm{Hz}
\end{aligned}$$

Q#3: WHAT IS THE MASS OF A STRING WITH LENGTH 2 M, TENSION OF 500 N AND FUNDAMENTAL OF 10 HZ? (C) 0.625 g

Q#4: IF A STRING HAS A FOURTH HARMONIC OF 20 HZ WHAT IS THE FUNDAMENTAL? (20 Hz/4= 5Hz)

QUESTION PART 1

Tom wants to make a violin for his sister as a birthday present. Violins usually make sound frequencies ranging from 200~3000Hz. He has a few 30 cm long strings with linear mass densities:

- 2.8×10^{-4} kg/m
- 4.0×10^{-4} kg/m
- 0.62 g/m

Which string should he use to make the violin in order to get a fundamental frequency of 700Hz if the tension in the string is kept at 70 N?

f_1 = 700 Hz T = 70 N L = 30 cm = 0.30 m

$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

Solve for μ

$$f_1 \times 2L = \sqrt{\frac{T}{\mu}}$$

$$(f_1 \times 2L)^2 = \frac{T}{\mu}$$

$$\mu = \frac{T}{(f_1 \times 2L)^2} = \frac{70\,N}{(700\,Hz \times 2 \times 0.30m)^2} = 3.97 \times 10^{-4}\ \text{kg/ m}$$

$$\approx 4.0 \times 10^{-4}\ \text{kg/m}$$

Q -part two

The violin string broke after a few weeks, but Tom doesn’t have anymore of the same string. If he uses a string with linear mass density of 4.7×10^{-4} kg/m, what should the tension be in the string in order to produce the same sound frequency (700 Hz)?

Solution

f_1 = 700 Hz $\mu = 4.7 \times 10^{-4}$ kg/m L = 30 cm = 0.30 m

$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

Solve for T

$$f_1 \times 2L = \sqrt{\frac{T}{\mu}}$$

$$(f_1 \times 2L)^2 = \frac{T}{\mu}$$

$$T = (f_1 \times 2L)^2 \times \mu = (700Hz \times 2 \times 0.30m)^2 \times 4.7 \times 10^{-4} \text{ kg/m}$$

$= 82.9$ N

≈ 83 N

Qn

A violin string that should be tuned to concert A (440Hz) is slightly mistuned. When the violin string is played in its fundamental mode along with a concert A tuning fork, 3 beats per second are heard.

(a) What are the possible values of the fundamental frequency of the string?

(b) Suppose the string were played in its first overtone simultaneously with a tuning fork with 880Hz. How many beats per second would be heard?

(c) When the tension of the string is increased slightly, the number of beats per second in the fundamental mode increases. What was the original frequency of the fundamental?

Laws of Vibrating String:

Law of Length:

- If the tension in the string and its mass per unit length of a wire remains constant, then the frequency of transverse vibration of a stretched string is inversely proportional to the vibrating length.

Law of Tension:

- If the vibrating length and mass per unit length of a wire remain constant then, the frequency of transverse vibration of a stretched string is directly proportional to the square root of the tension in the string.

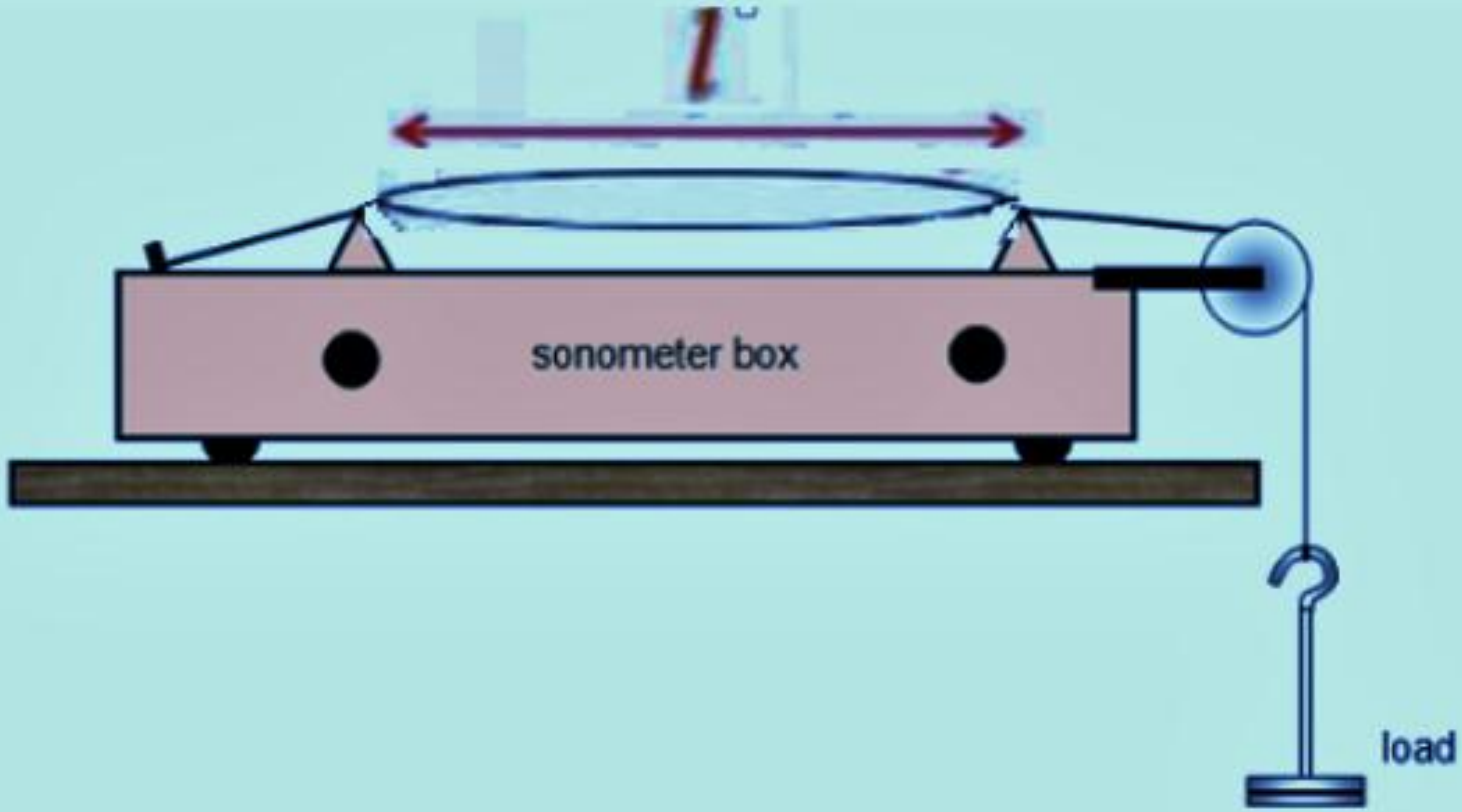
Law of Mass:

- if the vibrating length and tension in the string remain constant then, the frequency of a transverse vibration of a stretched string is inversely proportional to the square root of its mass per unit length

Sonometer:

Construction:

- A sonometer consists of a hollow rectangular wooden box to which a uniform wire is attached at one end. The other end of the wire is passed over two horizontal knife edges or bridges and then over a pulley. A weight hanger is suspended from the free end of the wire. By placing different weights in the weight hanger, the tension in the wire can be suitably adjusted.



- The points at which the wire rests on the knife edges cannot vibrate at all. Hence, when the wire is set up into vibrations, these two points become nodes and the wire vibrates in the fundamental mode. The frequency of vibration of the wire can be varied by either changing the positions of the knife edges by changing the vibrating length or by placing different weights in the pan by changing the tension.

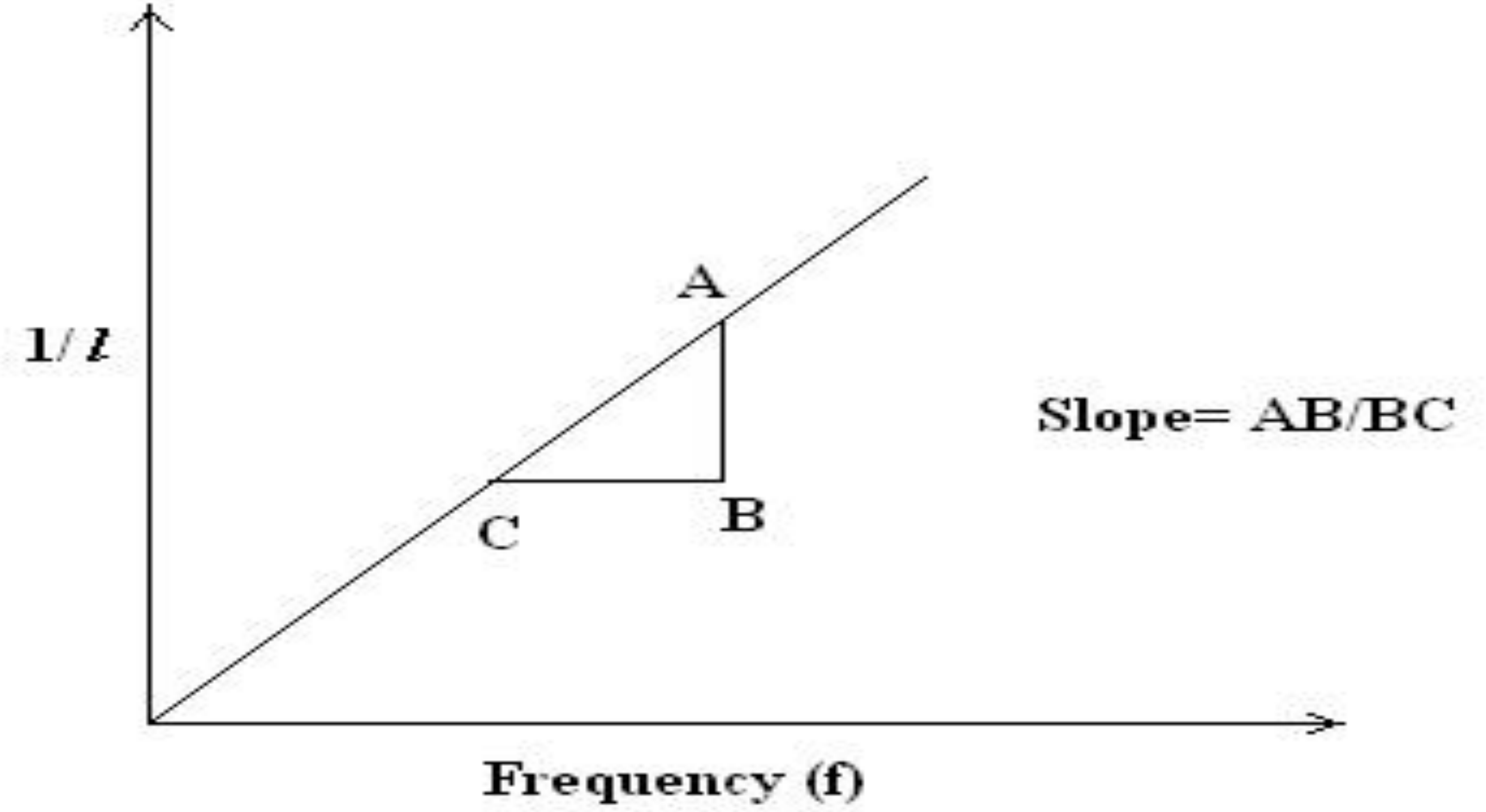
Use of Sonometer to Determine the Unknown Frequency of a Tuning Fork:

- To determine the unknown frequency of a tuning fork, the tension T in the wire is kept constant and the vibrating length between the knife edges is so adjusted, that the fundamental frequency of the wire becomes the same as that of the fork.
- To test this a small paper rider is placed on the wire midway between the knife edges where an antinode is formed.
- The fork is set up into vibration and its stem is placed on the wooden box. The length of the wire is adjusted till it vibrates in unison with the fork. When this happens, the centre of the vibrating wire vibrates with maximum amplitude due to resonance, and the paper rider is thrown off.
- Then the frequency of the tuning fork which is the same as the fundamental frequency of the wire is given by

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Use of Sonometer to Verify the Law of Length:

- If the tension in the string and its mass per unit length of a wire remains constant, then the frequency of transverse vibration of a stretched string is inversely proportional to the vibrating length.
- To verify , the given wire (m = constant) is kept under constant tension (T = constant). A set of tuning forks having different frequencies n_1, n_2, n_3, n_4 etc. is taken.
- The length of the wire, vibrating in unison with each fork, is determined in turn using a paper rider or hearing beats. Let the lengths corresponding to the frequencies be l_1, l_2, l_3, l_4 etc.
- Then, it is found that, within the limits of experimental error, $n_1 l_1 = n_2 l_2 = n_3 l_3 = n_4 l_4$ = constant. Thus in general $n\,l$ = constant or $n \propto 1/l$
- If a graph of n against $1/l$ is plotted, it comes out as a straight line.

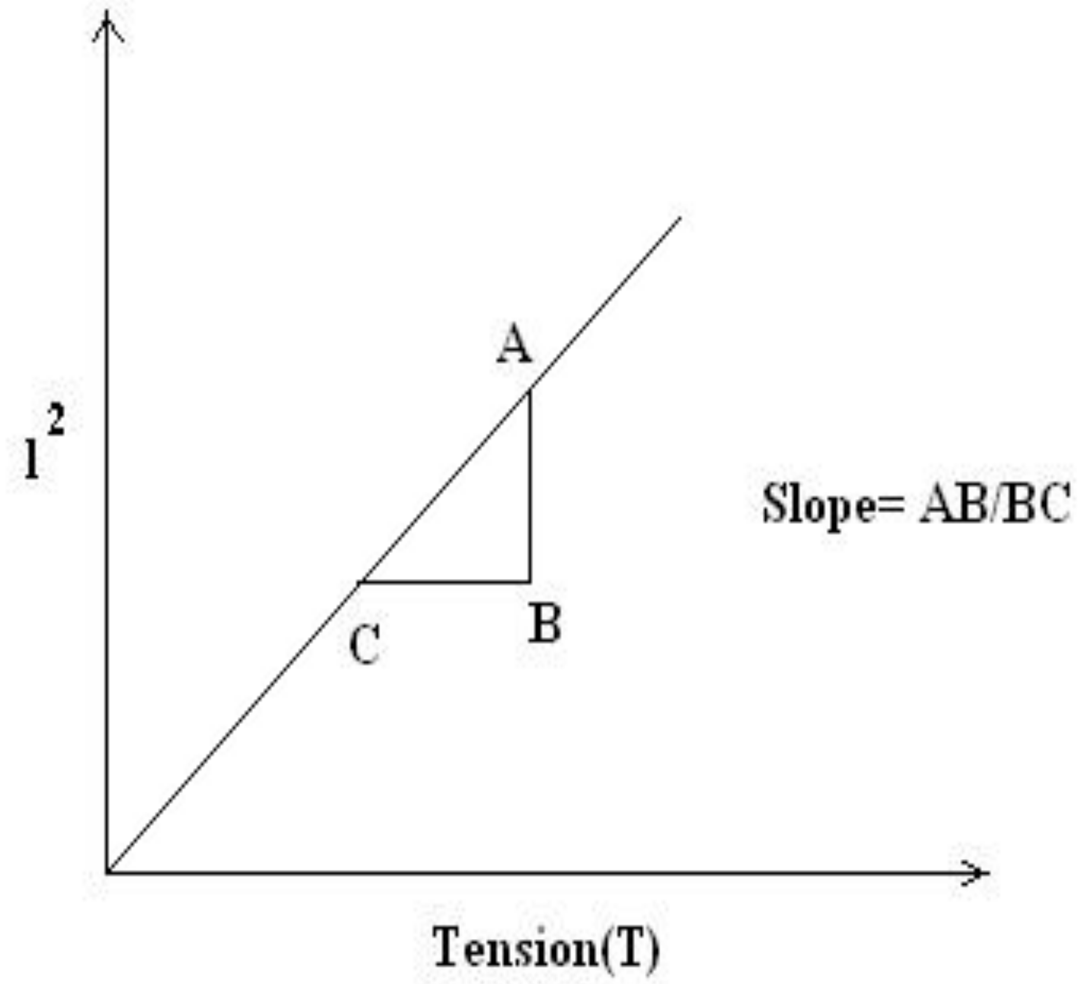

1/ l
A
B
C
Slope= AB/BC
Frequency (f)

Use of Sonometer to Verify the Law of Tension:

- If the vibrating length and mass per unit length of a wire remain constant then, the frequency of transverse vibration of a stretched string is directly proportional to the square root of the tension in the string.
- To verify the law, the vibrating length of given wire and linear density 'm' is constant. A set of tuning forks having different frequencies n_1, n_2, n_3, n_4 etc. is taken.
- By adjusting the tension T, each fork is made to vibrate in unison with the fixed length of the wire, one after the other. Let the tensions corresponding to the frequencies n_1, n_2, n_3, n_4 etc. be T_1, T_2, T_3, T_4 etc. respectively.
- Then it is found that, within limits of experimental error.

$$\frac{n_1}{\sqrt{T_1}}=\frac{n_2}{\sqrt{T_2}}=\frac{n_3}{\sqrt{T_3}}=\frac{n_4}{\sqrt{T_4}} \text{ i.e. } \frac{n}{\sqrt{T}}=\text{ constant or } n \;\alpha\; \sqrt{T}$$

- A graph of n^2 against T comes out as a straight line.



l^2
A
B
C
Slope= AB/BC
Tension(T)

Q8. Explain the use of sonometer to verify the law of mass.

- if the vibrating length and tension in the string remain constant then, the frequency of a transverse vibration of a stretched string is inversely proportional to the square root of its mass per unit length.
- To verify when *l* and T are kept constant, This law can not be verified directly, as neither 'n' nor 'm' can be varied continuously as in the case of *l* or T. Therefore, this law is verified indirectly as follows. The relation can be written as

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore n^2 = \frac{1}{4l^2} \times \frac{T}{m}$$

$$\therefore m = \frac{T}{4n^2} \times \frac{1}{l^2}$$

- Then, to verify the law, we must show that when n and T are kept constant. A number of wires having linear densities m_1, m_2, m_3, m_4 etc. are taken. Each one of them is subjected to the same tension T. Then, using a given tuning fork (n = constant) each wire is made to vibrate in unison with the fork, by adjusting its length. Let l_1, l_2, l_3, l_4 etc. be the vibrating length corresponding to linear densities m_1, m_2, m_3, m_4 etc. respectively.
- Then it is found that, within the limits of experimental error.

$$m_1 l_1^2 = m_2 l_2^2 = m_3 l_3^2 = m_4 l_4^2$$

Hence the law is indirectly verified.

The speed of a sound wave in a liquid or gas depends on the medium's compressibility and inertia. If the fluid has a bulk modulus of B and an equilibrium density of ρ, the speed of sound is

$$v = \sqrt{\frac{B}{\rho}}$$

The speed of a longitudinal wave in a solid rode is

$$v = \sqrt{\frac{Y}{\rho}}$$

where Y is the Young's modulus of the solid, and ρ is the density of the solid.

The speed of sound also depends on the temperature of the medium. For example traveling through air, the relationship between the speed of sound and tepmerature θ in degrees Celsius is

$$v = (331\ \mathrm{m/s})\sqrt{1 + \frac{\theta}{273}}$$

•State that the speed of a transverse wave in stretched string is given by

where *T* is the tension of the string and m is the mass of a unit length of the string.

•Deduce by using

$$= \sqrt{\frac{T}{m}}$$

$$= f\ \lambda$$

$$= \sqrt{\frac{T}{m}}$$

and

that the frequency of the fundamental mode of a string of length is,

$$= \frac{1}{2l}\sqrt{\frac{T}{m}}$$

•Give the equation, frequency of the overtone is,

$$= \frac{n+1}{2l}\sqrt{\frac{T}{m}}$$

Notes:

- Frequency of p^{th} overtone = $(p+1)n$
- Frequency of p^{th} harmonic = pn
- Mode of vibration of string gives frequency n, 2n, 3n, …… so on.
- The stretched string vibrates with all harmonics.

Expression for the Frequency of the Fundamental Mode of a Vibration of a String in Terms of its Radius of Cross-section and Density of the Material:

- The frequency (n) of the fundamental mode of transverse vibration of a stretched string is given by

$$n = \frac{1}{2\,l}\sqrt{\frac{T}{m}}$$

$$\text{Now,}\quad m = \frac{\text{mass of wire}}{\text{length of wire}}$$

$$m = \frac{\text{Volume} \times \text{density}}{Length}$$

$$\therefore m = \frac{\text{Area of cross section} \times Length \times \text{density}}{Length}$$

$$\therefore \mathrm{m} = \text{area of crosssection of wire} \times \text{density}$$

$$\therefore \mathrm{m} = \pi \mathrm{r}^2 \rho$$

$$\therefore n = \frac{1}{2\,l}\sqrt{\frac{T}{\pi r^2 \rho}}$$

$$\therefore n = \frac{1}{2l\,\mathrm{r}}\sqrt{\frac{T}{\pi \rho}}$$

This is an expression for the fundamental mode of a transverse vibration of a string

in terms of its radius of cross-section and density of the material.

- For a given material of wire, having a constant length and same tension, the frequency of transverse vibration is inversely proportional to its radius.
- For a wire having the same radius, same length and same tension, the frequency of transverse vibration is inversely proportional to the square root of the density of its material.

Expression for the Frequency of the Fundamental Mode of a Transverse Vibration of a String in Terms of Young's Modulus of Elasticity of the Material:

- The frequency (n) of the fundamental mode of transverse vibration of a stretched string is given by

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad \text{............} \ (1)$$

$$\text{Now,} \quad m = \frac{mass\ of\ \ wire}{\text{length of wire}}$$

$$m = \frac{\text{Volume} \times \text{density}}{Length}$$

$$\therefore m = \frac{\text{Area of cross section} \times Length \times \text{density}}{Length}$$

$$\therefore \text{m} = \text{area of crosssection of wire} \times \text{density}$$

$$\therefore \text{m} = \pi r^2 \rho \quad \text{............} \ (2)$$

Now, T = F = tension in the wire

By definition of Young's modulus of Elasticity

$$Y = \frac{Stress}{Strain} = \frac{F / A}{l / L}$$

$$\therefore Y = \frac{FL}{Al}$$

$$\therefore F = \frac{YAl}{L}$$

$$\therefore T = \frac{YAl}{L} \quad \text{............} \ (3)$$

Substituting value of equation (2) and (3) in (1)

$$\therefore n = \frac{1}{2L}\sqrt{\frac{\frac{YAl}{L}}{\pi r^2 \rho}}$$

$$\therefore n = \sqrt{\frac{\frac{YAl}{L}}{4L^2 \pi r^2 \rho}} = \sqrt{\frac{Y \pi r^2 l}{4L^3 \pi r^2 \rho}}$$

$$\therefore n = \sqrt{\frac{Yl}{4L^3 \rho}}$$

$$\therefore n = \frac{1}{2L}\sqrt{\frac{Yl}{\rho L}}$$

- This is an expression for the fundamental mode of a transverse vibration of a string in terms of Young's modulus of elasticity of the material.

Expression for the Frequency of the Fundamental Mode of a Transverse Vibration of a String in Terms of Coefficient of Thermal Expansion of the Material:

- The frequency (n) of the fundamental mode of transverse vibration of a stretched string is given by

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad \text{..............} \quad (1)$$

$$\text{Now,} \quad m = \frac{mass\ of\ wire}{\text{length of wire}}$$

$$m = \frac{\text{Volume} \times \text{density}}{Length}$$

$$\therefore m = \frac{\text{Area of cross section} \times Length \times \text{density}}{Length}$$

m = area of the cross-section of wire × density

$\therefore$ m = $\pi r^2 \rho$ .... (2)

Now, T = F = tension in the wire

Substituting these values in equation (1)

$$n = \frac{1}{2l}\sqrt{\frac{F}{\pi r^2 \rho}}$$

$$\therefore n = \frac{1}{2l}\sqrt{\frac{F / \pi r^2}{\rho}}$$

$$\therefore n = \frac{1}{2l}\sqrt{\frac{Stress}{\rho}}$$

But thermal stress = $Y\alpha t$

Where,

α = Coefficient of thermal expansion

Y = Young's modulus of Elasticity

$$\therefore n = \frac{1}{2L}\sqrt{\frac{Y\alpha t}{\rho}}$$

Effect of dipping Stretching Load in a Liquid on Frequency of Vibrating Wire:

- The specific gravity of a liquid is defined as the ratio of the weight of the body in the air to the loss of the weight of the body when immersed in the water. Thus, the specific gravity of liquid is given by

$$\sigma = \frac{\text{Weight of body in air}}{\text{Loss of weight of body in liquid}}$$

$$\therefore \sigma = \frac{T_1}{T_1 - T_2}$$

$$\therefore \frac{1}{\sigma} = \frac{T_1 - T_2}{T_1}$$

$$\therefore \frac{1}{\sigma} = 1 - \frac{T_2}{T_1}$$

$$\therefore \frac{T_2}{T_1} = 1 - \frac{1}{\sigma}$$

$$\therefore \frac{T_2}{T_1} = \frac{\sigma - 1}{\sigma}$$

$$\therefore \frac{T_1}{T_2} = \frac{\sigma}{\sigma - 1}$$

Now, by law of tension

$$n \ \alpha \ \sqrt{T} \quad \text{or} \quad \frac{n}{\sqrt{T}} = \text{constant}$$

$$\therefore \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\therefore \frac{n_1}{n_2} = \sqrt{\frac{\sigma}{\sigma - 1}}$$

This relation gives the new frequency of vibration of wire on immersing the load in a liquid of specific gravity σ.

- Find the speed of the sound in water, which has a bulk modulus of about $2.1 \cdot 10^9$ Pa and a density of about 10^3 kg/m^3.

$$v_w = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.1 \cdot 10^9 \text{ Pa}}{10^3 \text{ kg/m}^3}} \approx 1500 \text{ m/s}$$

- If a solid bar is struck at one end with a hammer, a longitudinal pulse propagates down the bar. Find the speed of sound in a bar of aluminium, which has a Young's modulus of $7 \cdot 10^{10}$ Pa and a density of $2.7 \cdot 10^3$ kg/m^3.

$$v_{Al} = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{7 \cdot 10^9 \text{ Pa}}{2.7 \cdot 10^3 \text{ kg/m}^3}} \approx 5100 \text{ m/s}$$

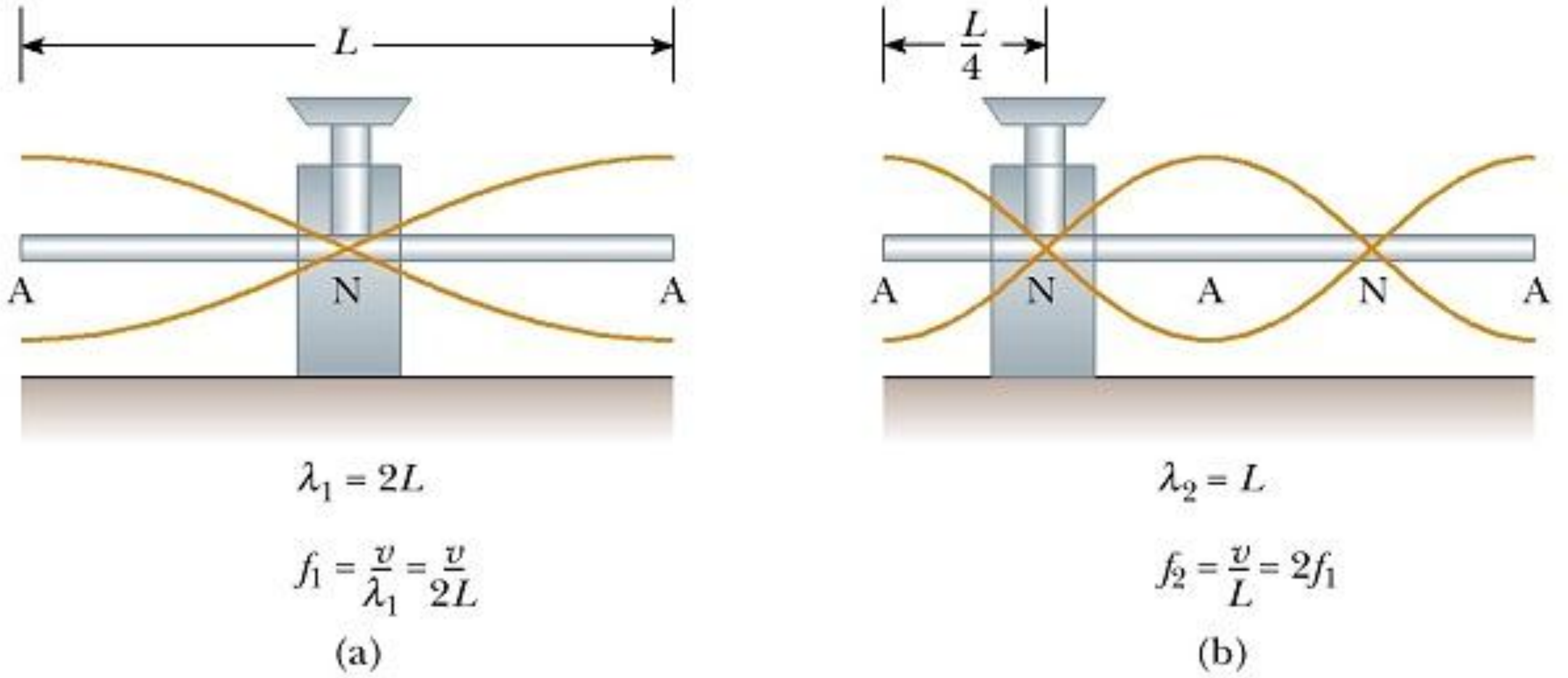


Figure 18.16 Normal-mode longitudinal vibrations of a rod of length L (a) clamped at the middle to produce the first normal mode and (b) clamped at a distance $L/4$ from one end to produce the second normal mode. Note that the dashed lines represent amplitudes parallel to the rod (longitudinal waves).

Standing waves

The wave seems to be stationary, hence the name "standing wave".

A **standing wave/stationary wave** is the result of two waves of the same **wavelength**, **frequency**, and **amplitude** travelling in opposite directions through the same medium.

Stationary (Standing) Wave) is one
whose waveform/wave profile does not advance {move},
where there is no net transport of energy, and where the positions of antinodes and nodes do not change (with time).

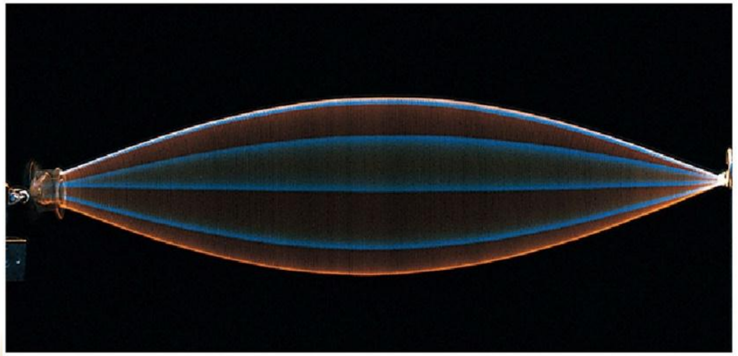
A stationary wave is formed when two <u>progressive</u> waves of the same <u>*frequency*</u>, <u>*amplitude*</u> and <u>*speed*</u>, travelling in <u>*opposite directions*</u> are superposed.
{Assume boundary conditions are met}

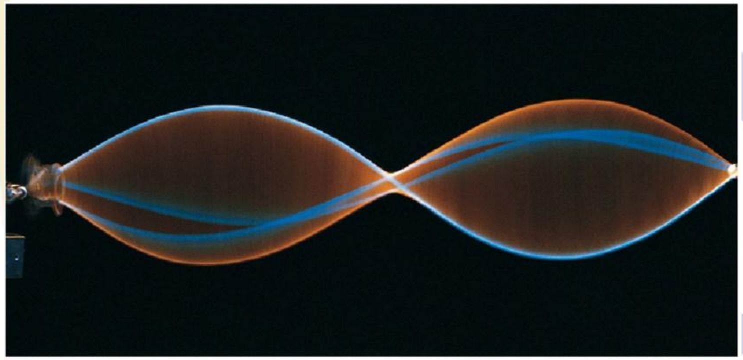
Stationary Waves can be both Transverse and Longitudinal.

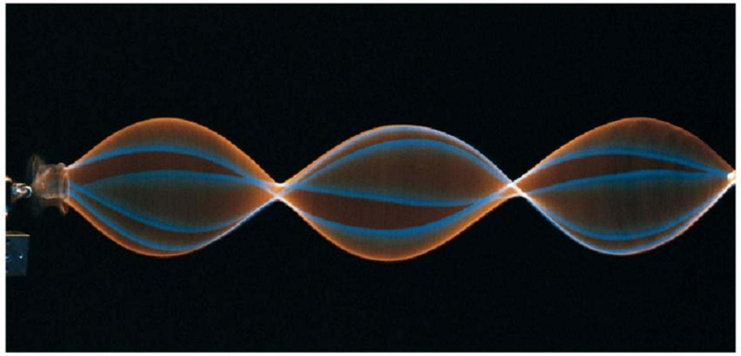
- Transverse Stationary Waves are formed in Strings.

- Longitudinal Stationary Waves are formed in Air columns i.e. in an Organ pipe.

Resonance on string







- First three natural vibrational modes of a string fixed at both ends (e.g. a guitar string).
- A normal pluck excites primarily the first vibrational mode.

In Longitudinal Stationary Waves, there is maximum variation of Pressure, hence also of Density, at the Nodes.

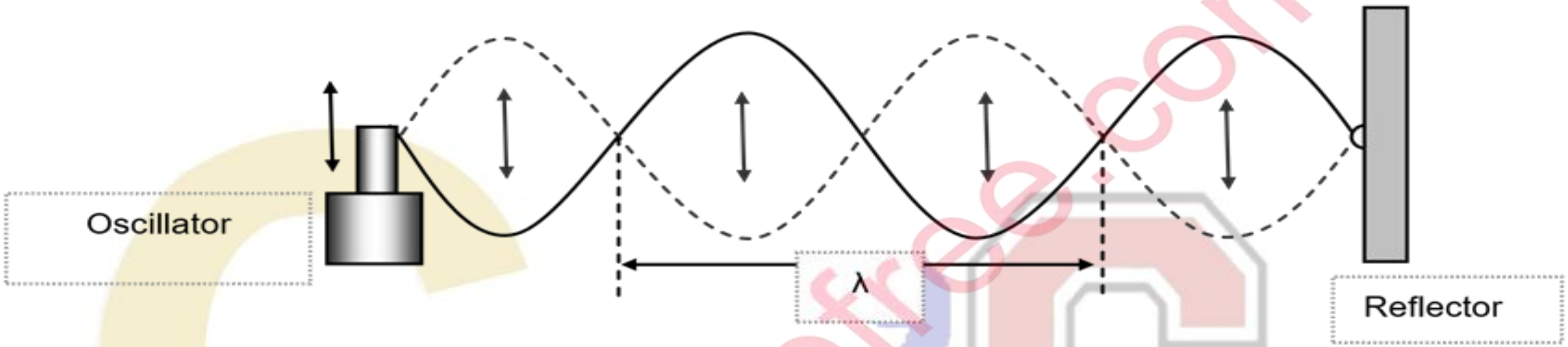
There is NO variation of Pressure, and Density, at the Antinodes.

Therefore, Antinodes may be called Pressure Nodes

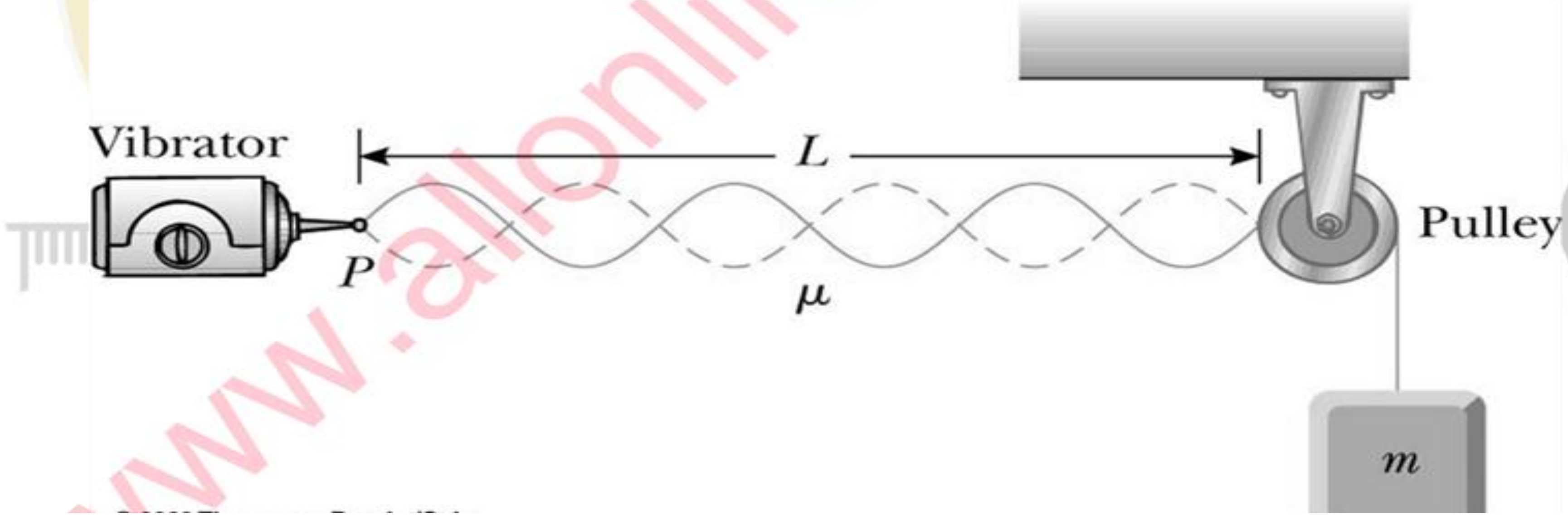
And Nodes may be called Pressure Antinodes.

Stretched String

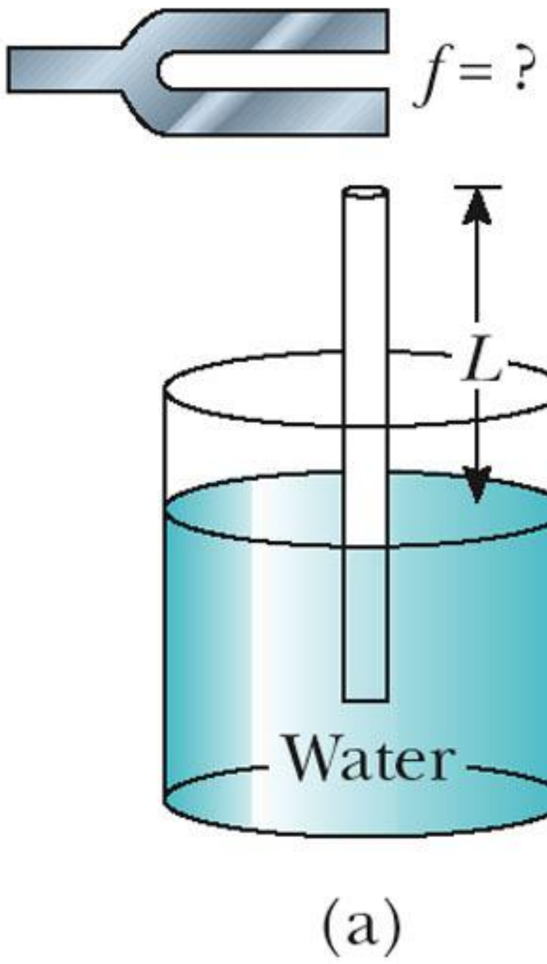
A horizontal rope with one end fixed and another attached to a vertical oscillator. Stationary waves will be produced by the direct and reflected waves in the string.

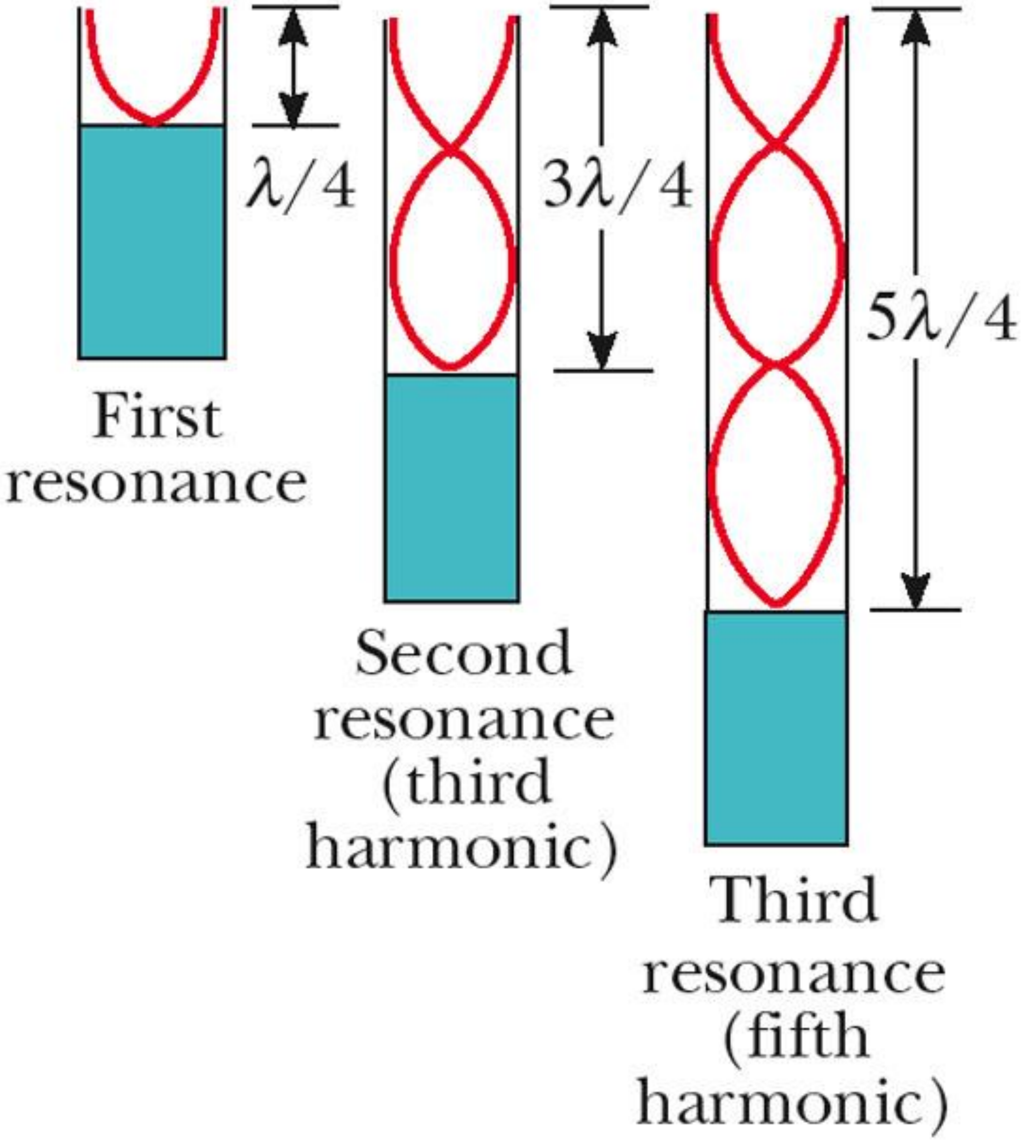


Or we can have the string stopped at one end with a pulley as shown below.



Closed tube resonance

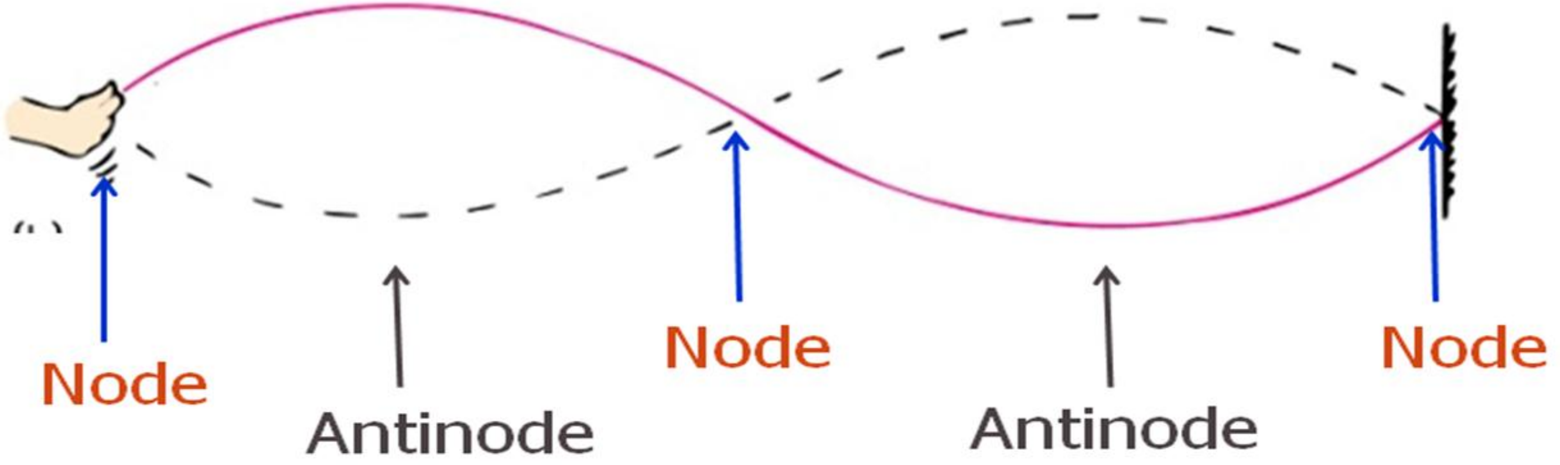


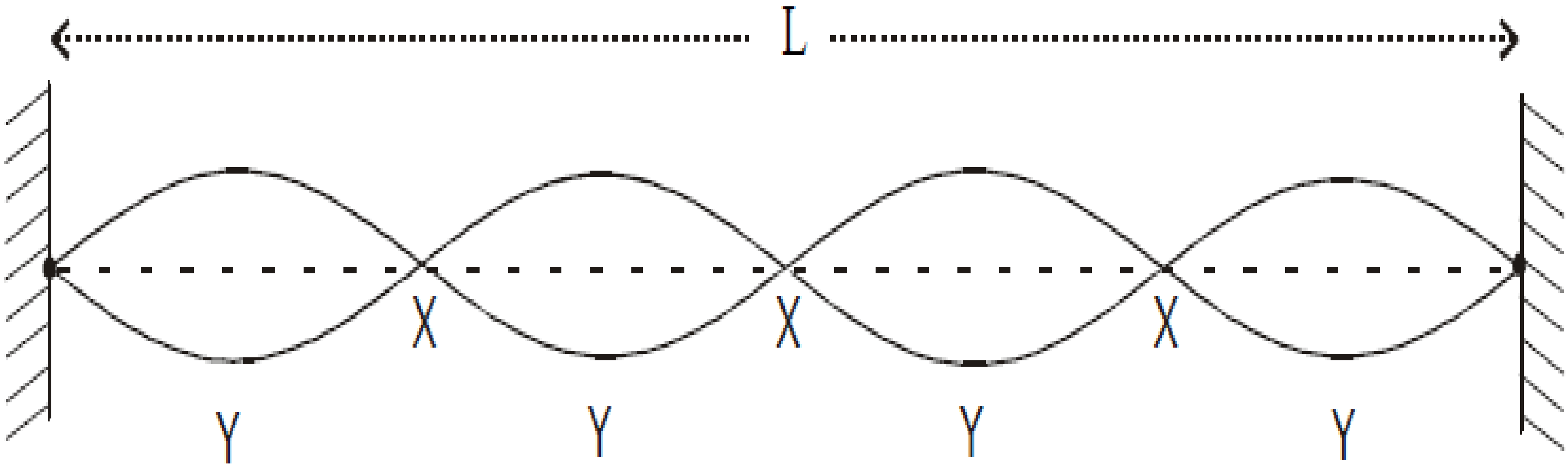


Standing Waves

- A **Node -** point of zero amplitude /displacement due to maximum destructive interference.

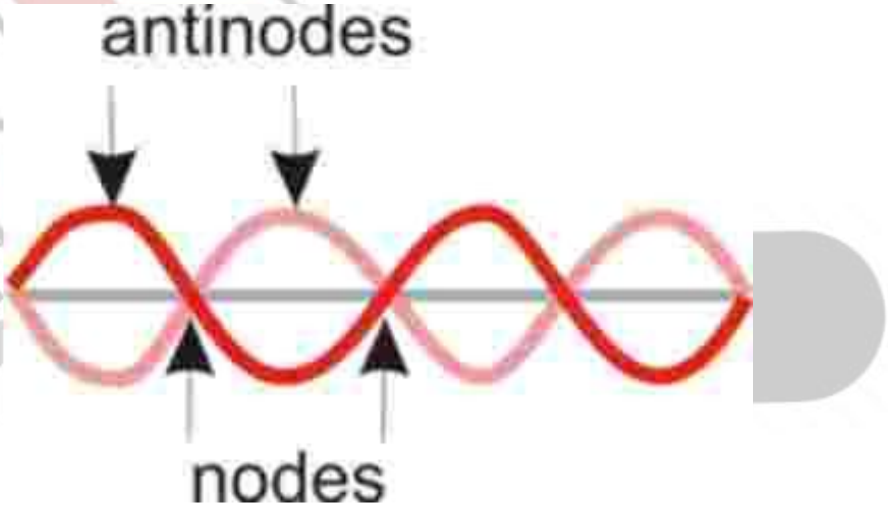
- An *antinode* occurs where the standing wave vibrates at maximum amplitude





From the diagram we can see that:

1. The distance between two consecutives nodes is λ/2
2. The distance between two consecutive antinodes is λ/2
3. The distance between an anti-node and the next node is λ/4



Standing Waves

When waves on a rope hits a fixed end, it reflects and is inverted. This reflected waves then combine with oncoming incident waves. At certain frequencies the resulting superposition yields a standing wave, in which some points on the rope called nodes never move at all, and other points called antinodes have an amplitude twice as big as the original wave.

- Particles, except nodes, perform SHM with the same period as the component waves.

- No energy is transferred from particle to particle in stationary waves.

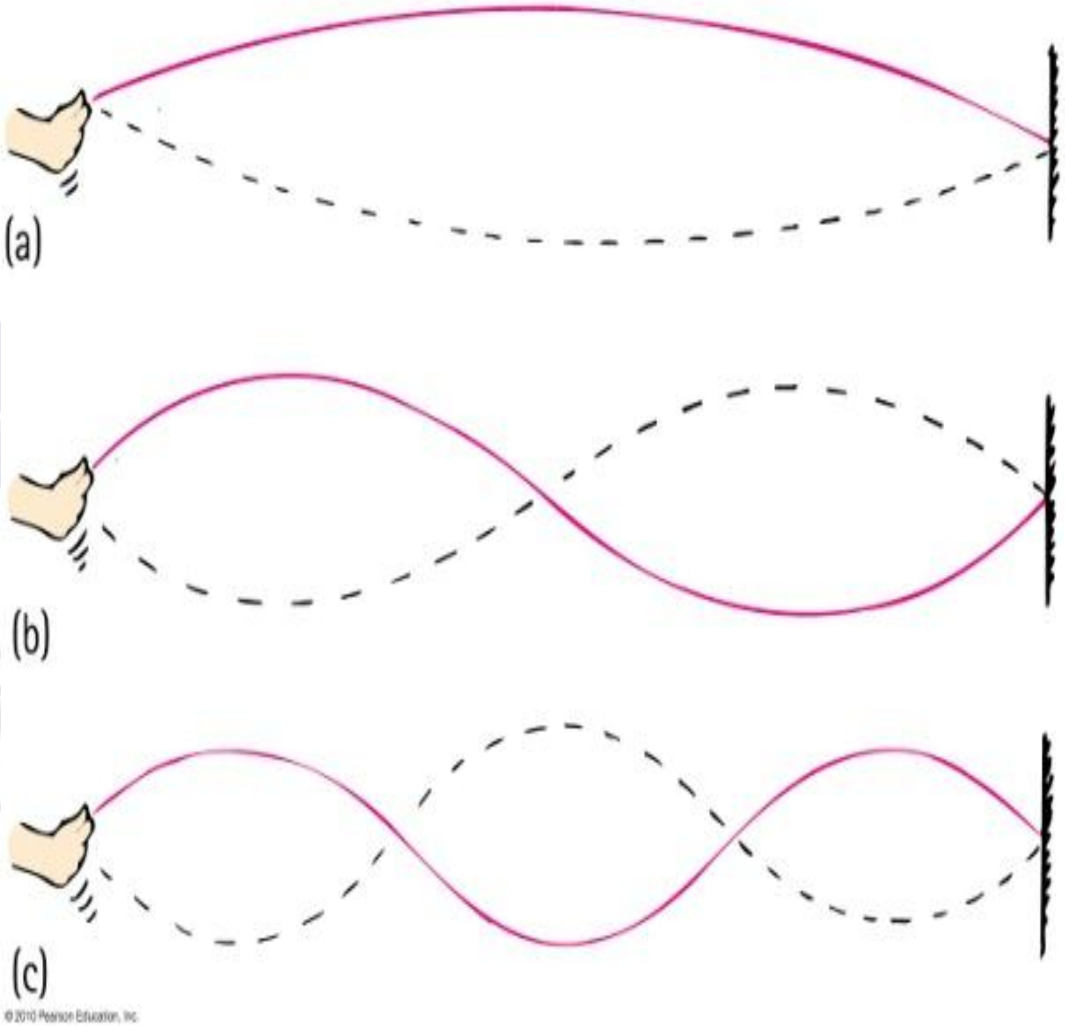
- Node is a region of destructive superposition where the waves always meet out of phase by radians. Hence displacement here is permanently zero {or minimum}.

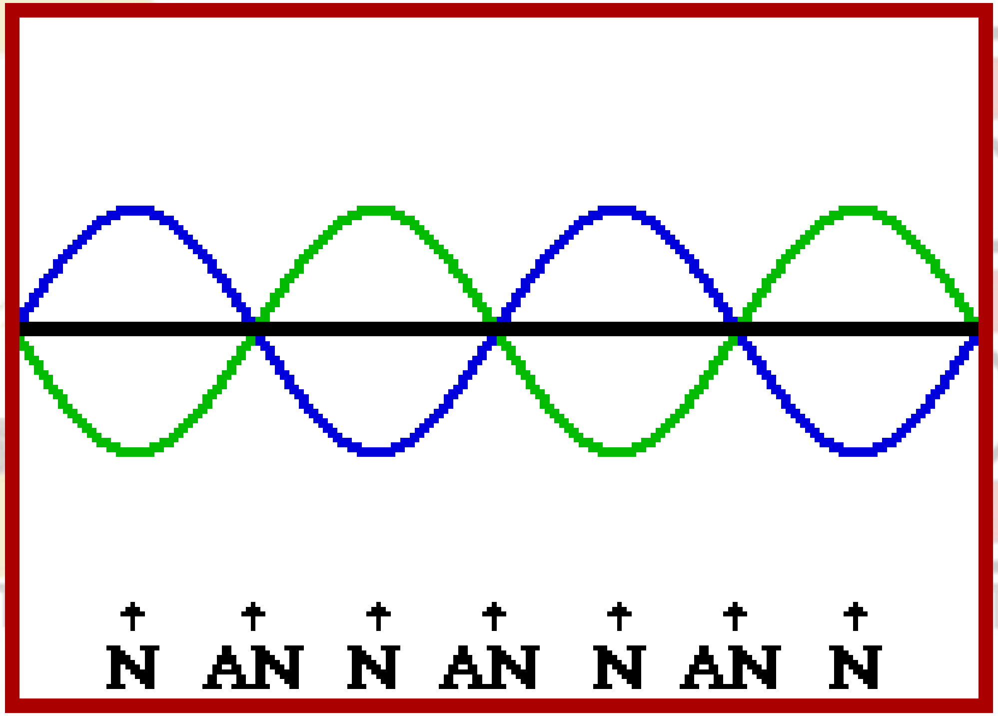
- Antinode is a region of constructive superposition where the waves always meet in phase. Hence a particle here vibrates with maximum amplitude {but it is NOT a point with a permanent large displacement!}

- Max pressure change occurs at the nodes {NOT the antinodes} because every node changes from being a point of compression to become a point of rarefaction {half a period later} (a)Has ___ nodes and ___ antinodes

(b)Has ___ nodes and ___ antinodes

(c)Has ___ nodes and ___ antinodes

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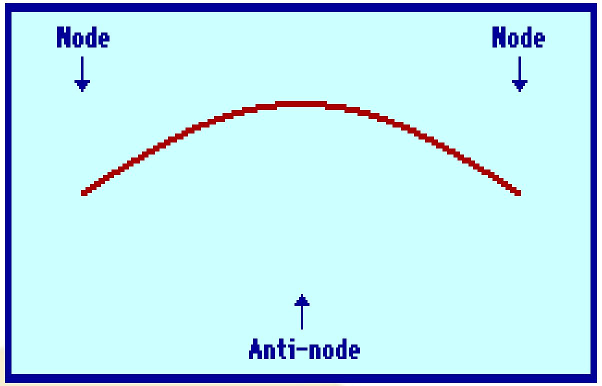
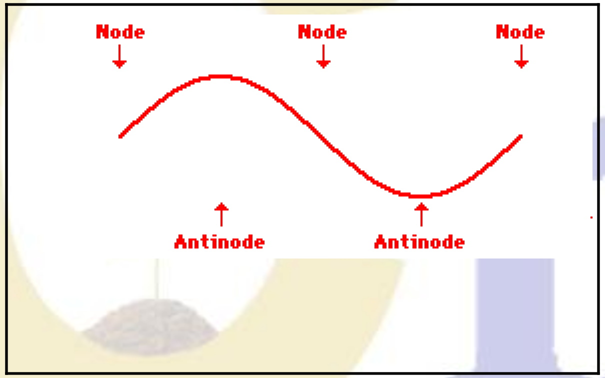
Overtones (Harmonics)

- Waves on string and wind instruments resonate at their natural frequencies when disturbed.
- A rope of given length can support standing waves of many different frequencies, called harmonics, which are named based on the number of antinodes.
- Overtones (Harmonics) are the multiples of a natural frequency and will cause an instrument to resonate.



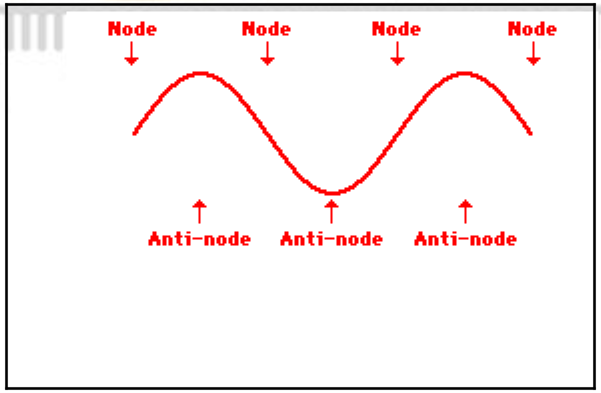
First Harmonic Standing Wave Pattern

(The Fundamental)



Second Harmonic Standing Wave Pattern

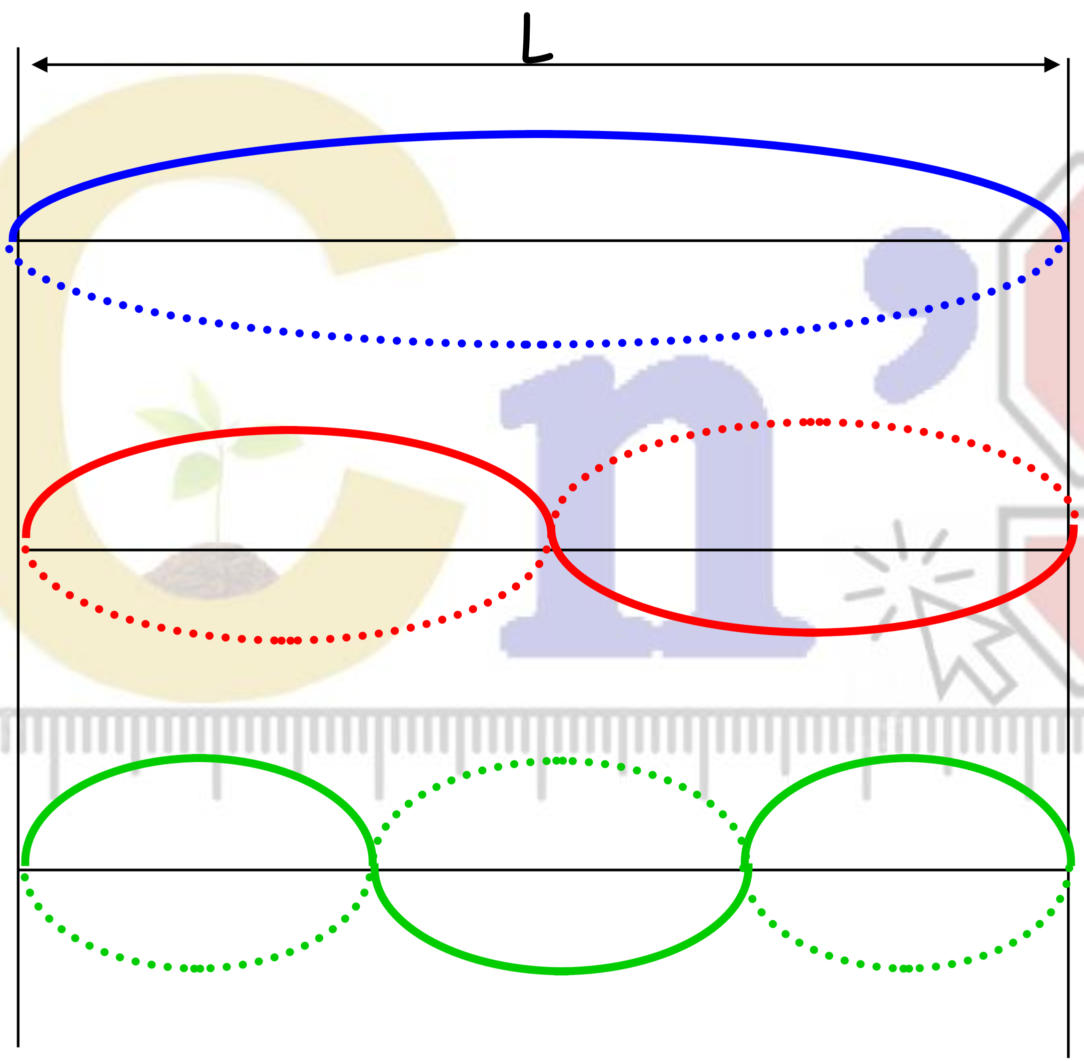
First Overtone



Third Harmonic Standing Wave Pattern

Second Overtone

Fixed-end standing waves (violin string)



Fundamental
First harmonic
$\lambda = 2L$

First Overtone
Second harmonic
$\lambda = L$

Second Overtone
Third harmonic
$\lambda = 2L/3$

What is an organ pipe?

An organ pipe is a wind instrument.

It consist of a long tube in which air is forced from one end and sound is produced by means of a vibrating air column. When air is forced in the pipe, the air inside is set into vibrations and stationary waves are produced in the pipe.

Types of Organ Pipes

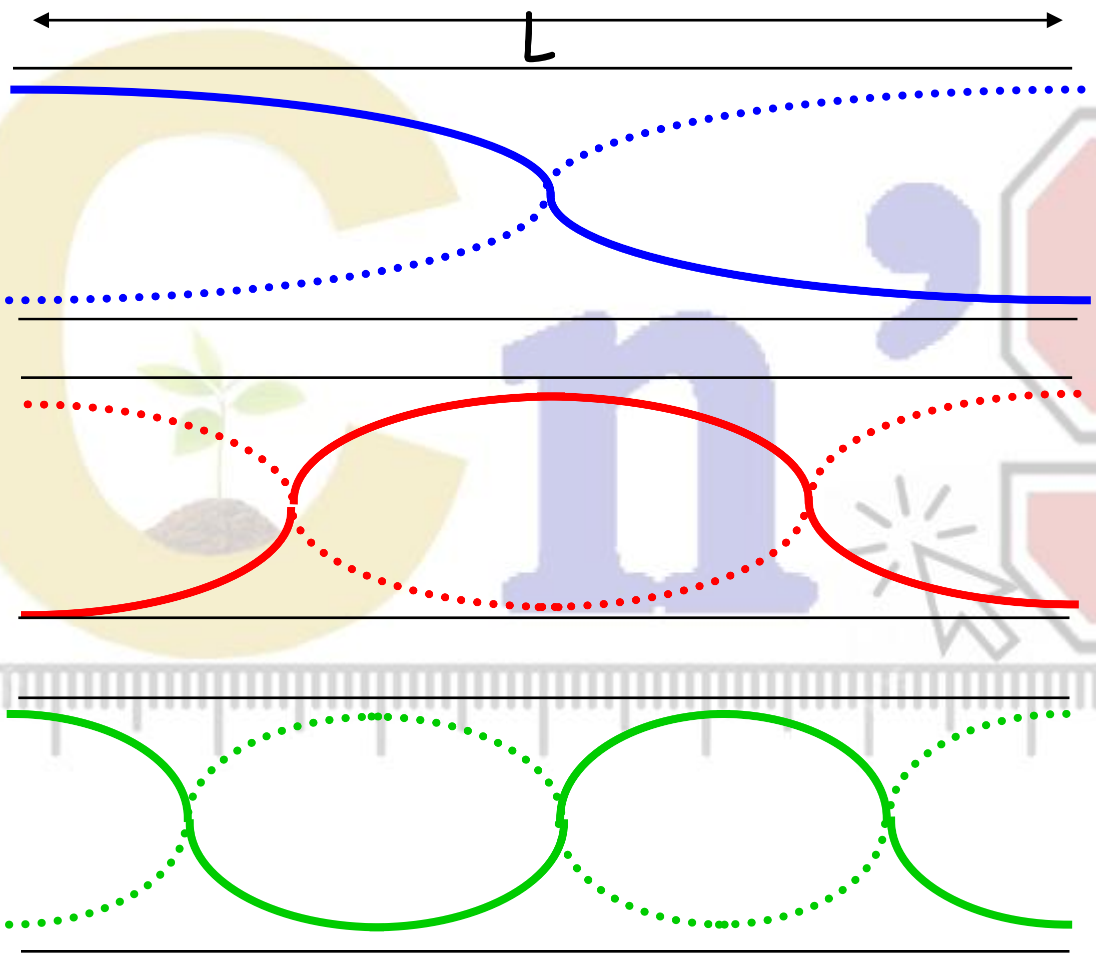
(i) Closed Pipe

If one end of the organ pipe is closed, it is called closed pipe.

(ii) Open Pipe

If both ends of the organ pipe are open, it is called open pipe.

Open-end standing waves (organ pipes)

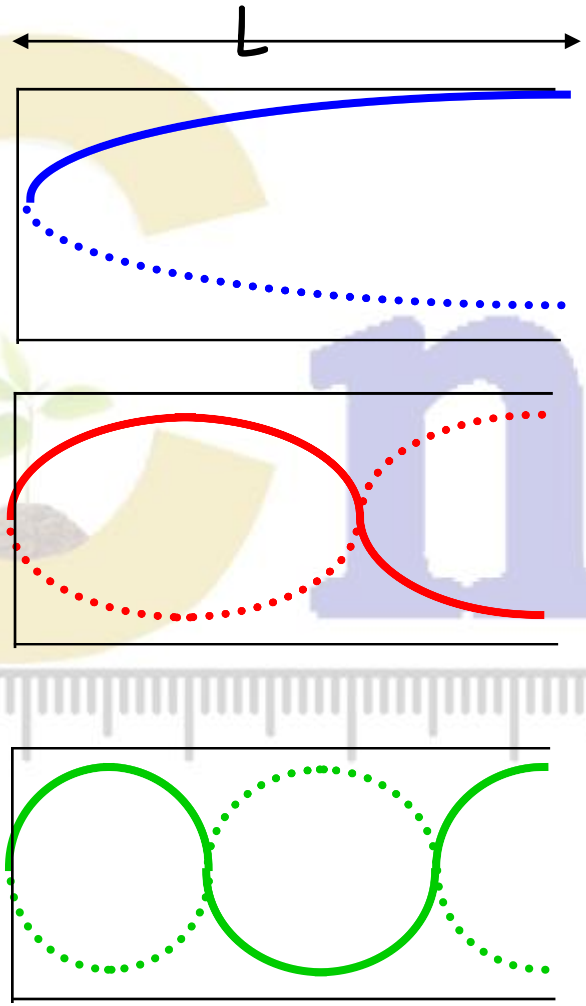


Fundamental
First harmonic
$\lambda = 2L$

First Overtone
Second harmonic
$\lambda = L$

Second Overtone
Third harmonic
$\lambda = 2L/3$

Closed End standing waves (some organ pipes)



First harmonic
$\lambda = 4L$

Second harmonic
$\lambda = (4/3)L$

Third harmonic
$\lambda = (4/5)L$

Progressive Wave:

- The wave advancing in given direction continuously is called as a progressive wave.
- e.g. sound waves.
- In an unbounded (infinite)) medium, the progressive wave travels in a given direction continuously until it gets dissipated.

Characteristics of Progressive Waves:

- They are produced when a disturbance is created in an elastic medium.
- They continuously travel away from the source.
- They transfer energy through the medium.
- All the particles of the medium perform SHM with the same amplitude.
- Every particle of the medium is set into vibrations.
- Different particles have got different phase at a given instant.

	Stationary Waves	Progressive Waves
Amplitude	Varies from maximum at the anti-nodes to zero at the nodes.	Same for all particles in the wave (provided no energy is lost).
Wavelength	Twice the distance between a pair of adjacent nodes or anti-nodes.	The distance between two consecutive points on a wave, that are in phase.
Phase	Particles in the same segment/ between 2 adjacent nodes, are in phase. Particles in adjacent segments are in anti-phase.	All particles <u>within one wavelength</u> have different phases.
Wave Profile	The wave profile does not advance.	The wave profile advances.With a constant speed
Energy	No energy is transported by the wave. Energy is associated with the wave but there is no transfer of energy across any section of the medium	Energy is transported in the direction of the wave.

Free Oscillation:

An oscillating system is said to be undergoing free oscillations when it oscillates without the interference of an external force. The system oscillates at its natural frequency.

For example, a simple pendulum when slightly displaced from its mean position vibrates freely with its natural frequency that depends only upon the length of pendulum.

- Forced Oscillation: In contrast to free oscillations, an oscillating system is said to undergo forced oscillations if it is subjected to an external periodic force. For example, the mass of a vibrating pendulum is struck repeatedly, the forced vibrations are produced. ie. no longer at its own natural frequency.

Free Vibrations:

- A body or a system capable of vibrating, when displaced from its position of rest, vibrates with a certain definite frequency. This frequency is characteristic of the body or the system. Such oscillations are called free oscillations or free vibrations and the frequency of such oscillations is called natural frequency of the body or the system.
- **Example – 1:** When a wire under tension, which is fixed at its ends, is plucked and released, it vibrates with a frequency which depends on the length of the string, its mass per unit length and tension in the string.
- **Example – 2:** When the bob of a simple pendulum oscillates, its frequency of oscillation depends on the length of the pendulum.
- Due to frictional force, the amplitude of oscillation decreases continuously and finally, the body stops vibrating.

Characteristics of Free Vibrations:

- They are produced when a body capable of vibrating is disturbed from its normal equilibrium position and then released.
- The frequency of vibration depends on the body and is called natural frequency.
- The frequency of vibration is same as the natural frequency of the body.
- The amplitude of vibration is large.
- Vibration continues for a little more time after the external force is removed.
- **Example:** Oscillations of bob of pendulum

Forced Vibrations:

- Forced vibrations are the vibrations produced in a body by applying an external periodic force having a frequency, normally different from the natural frequency of the body.

- A body or a system, capable of vibrating can also be made to vibrate at any desired frequency. The body can be made to vibrate with the same frequency as the frequency of the applied periodic force.
- Suppose that the natural frequency of a metal vessel is 200 Hz. If a tuning fork of frequency 256 Hz is set up into vibrations and its stem is placed in contact with the vessel, then the vessel will be forced to vibrate at a frequency of 256 Hz. In such a case, the vessel is said to perform forced vibrations.
- Initially, the body tends to vibrate with its natural frequency. But very soon, the natural vibrations die out and it begins to vibrate with the frequency of the applied periodic force.
- The amplitude of forced vibrations depends on:
  - The difference in frequencies of the external force and the natural frequency of the body.
  - The amplitude of the applied force.
  - damping.

Characteristics of Forced Vibrations:

- They are produced when an external periodic force acts on the body.
- The frequency of vibration is same as the frequency of external periodic force.
- The frequency of vibration is different from the natural frequency of the body.
- The amplitude of vibration is small.
- Amplitude becomes zero as soon as the external force is removed.
- **Example:** A vibrating tuning fork on a wooden box, a musical instrument having soundboard or box.

Resonance:

- The amplitude of the forced vibrations depends on the difference between the natural frequency of the body and the frequency of the applied periodic force.
- When the difference between the two frequencies is large, the response of the body is poor or the forced vibrations are of small amplitude. When the frequency difference becomes smaller, the body vibrates more readily or the amplitude of the forced vibrations increases.
- Finally, when the frequency (f) of the applied periodic force becomes the same as the natural frequency fo of the body, the amplitude of the forced vibrations becomes maximum and the phenomenon is known as resonance.
- If anybody is made to vibrate, by an external periodic force, with a frequency which is same as the natural frequency of the body, the body begins to vibrate with a very large amplitude. This phenomenon is called resonance.

Distinguishing Between Free Vibrations and Resonance:

Forced vibrations

- They are produced by the external periodic force of any frequency.
- The frequency of vibration is different from the natural frequency of the body.
- The amplitude of vibration is small.
- Vibration stops as soon as the external force is removed.

Resonance :

- They are produced by an external periodic force whose frequency is equal to the natural frequency of the body.
- The frequency of vibration is same as the natural frequency of the body.
- The amplitude of vibration is large.
- Vibration continues for a little more time after the external force is removed

Advantages of Resonance:

- Resonance is useful to determine an unknown frequency.
- Resonance is useful to increase the intensity of sound in musical instruments.
- Resonance is useful to tune a radio receiver to any desired frequency.
- Resonance is useful to analyse musical notes.

Disadvantages of Resonance:

- Soldiers are asked to break steps when crossing a bridge. It can be explained as follows
  - Soldiers marching on a bridge take steps with definite frequency and force the bridge to vibrate with the frequency of the steps.
  - If the forced frequency on the bridge is equal to the natural frequency of vibration of the bridge, the bridge is set into resonant vibrations.
  - Due to the resonance, the bridge vibrates with higher amplitude and due to this, it may collapse.
- Due to rhythmic clapping of the audience, the roof of the stadium may collapse. It can be explained as follows
  - When audience claps rhythmically they do so with a certain frequency and force the roof of a stadium to

- When audience claps rhythmically they do so with a certain frequency and force the roof of a stadium to vibrate with the frequency of the clap.
- If the forced frequency on the roof of a stadium is equal to the natural frequency of vibration of the roof of a stadium, the roof of a stadium is set into resonant vibrations.
- Due to the resonance, the roof of a stadium vibrate with higher amplitude and due to this, it may collapse.
- When the speed of an aircraft increases, different parts are forced to vibrate. which is dangerous for the structure of the aircraft.

Resonance:

- The vibration of a body at its natural frequency due to the action of a vibrating source of the same frequency.

- A phenomenon whereby the amplitude of a system undergoing forced oscillations increases to a maximum. It occurs when the frequency of the periodic driving force is equal to the natural frequency of the system.

- When the frequency of the applied force is equal to the natural frequency of simple harmonic oscillator, the amplitude of the motion may become extraordinary large. This phenomenon is called resonance.

Examples of Useful Purposes of Res

- (a) Oscillation of a child's swing. A swing is the good example of mechanical resonance. It is like a pendulum with a single natural frequency depending on its length. If a series of regular pushes are given to the swing, its motion can be built up enormously. If pushes are given irregularly, the swing will hardly vibrate.
- (b) Tuning of musical instruments.
- (c) **Electrical Resonance in Tuning of a Radio**: - Natural frequency of the radio is adjusted so that it responds resonantly to a specific broadcast frequency. Tuning of a radio is the best example of electrical resonance. When we turn the knob of a radio, to tune a station, we are changing the natural frequency of electrical circuit of receiver, to make it equal to the transmission frequency of the radio station. When the two frequencies match, energy absorption is maximum and this is the only station we hear

- (d) . Cooking of a Food in Microwave Oven: Another good example of resonance is the heating and cooking of food very efficiently and evenly by microwave oven. - Microwave ovens produce microwaves of a frequency 2450 MHz, which is equal to the natural frequency of water and fat molecules, thus causing the water molecules in the food to vibrate more violently. (At this frequency the waves are absorbed due to resonance by water and fat molecules in the food.)

- (e) Magnetic Resonance Imaging (MRI) is used in hospitals to create images of the human organs.

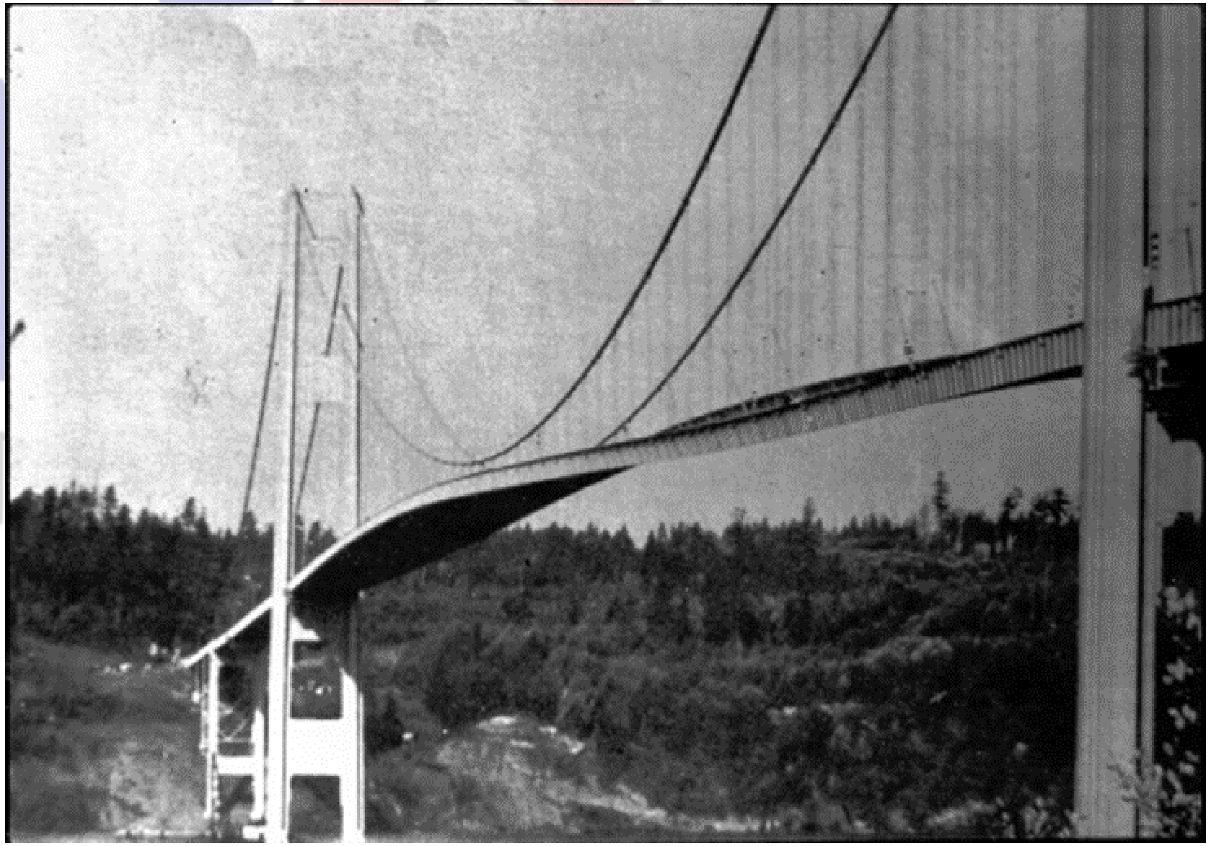
- (f) Seismography - the science of detecting small movements in the Earth‟s crust in order to locate centres of earthquakes.

Examples of Destructive Nature of Resonance

- (a) An example of a disaster that was caused by resonance occurred in the United States in 1940. The Tarcoma Narrows Bridge in Washington was suspended by huge cables across a valley. Shortly after its completion, it was observed to be unstable.

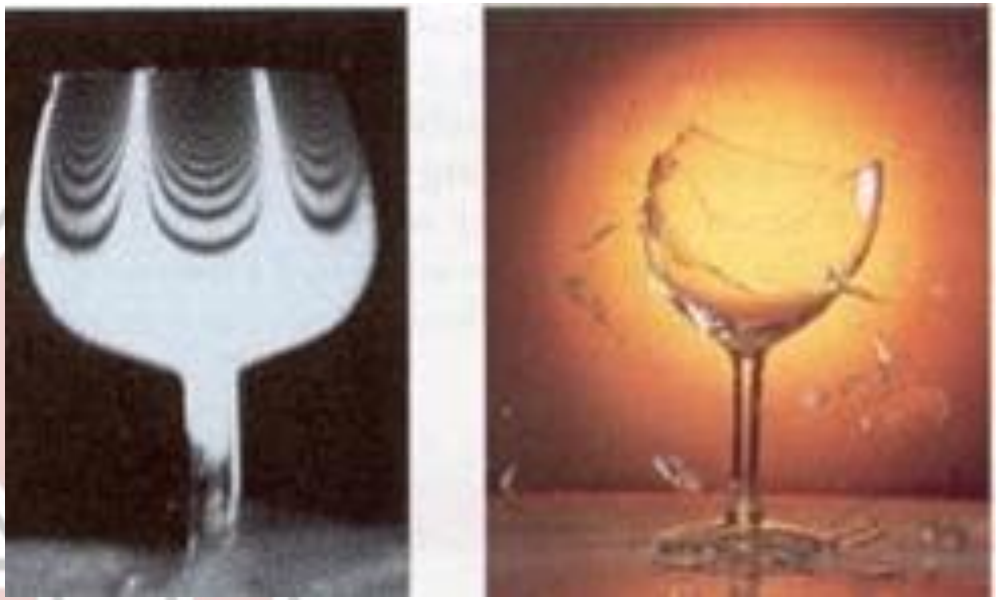
On a windy day four months after its official opening, the bridge began vibrating at its resonant frequency. The vibrations were so great that the bridge collapsed.

(Even a non-resonant drive can transfer energy. Driven by 40 mph wind Causes vibration of bridge at its natural (resonant) frequency.)



- (b) High-pitched sound waves can shatter fragile objects, an example being the shattering of a wine glass when a soprano hits a high note.

Can tune a speaker to the fundamental resonant frequency of the wine glass (1210 Hz).



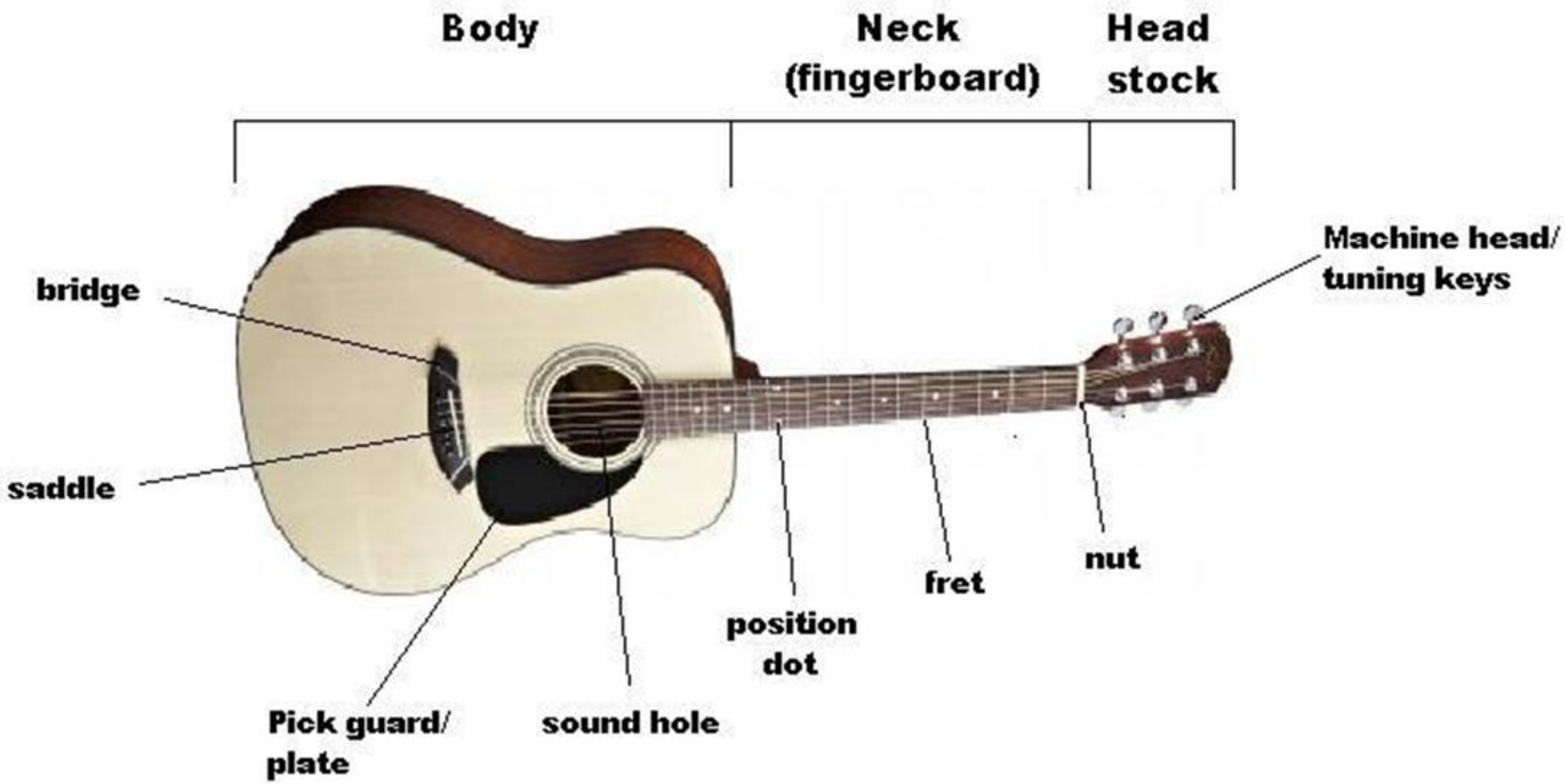
- (c) Buildings that vibrate at natural frequencies close to the frequency of seismic waves face the possibility of collapse during earthquakes.
- (d) March of soldiers on bridge: The column of soldiers, while marching on a bridge of long span is advised to break their steps. Their rhythmic march might set up oscillation of dangerously large amplitude in the bridge structure.






Musical Instrument

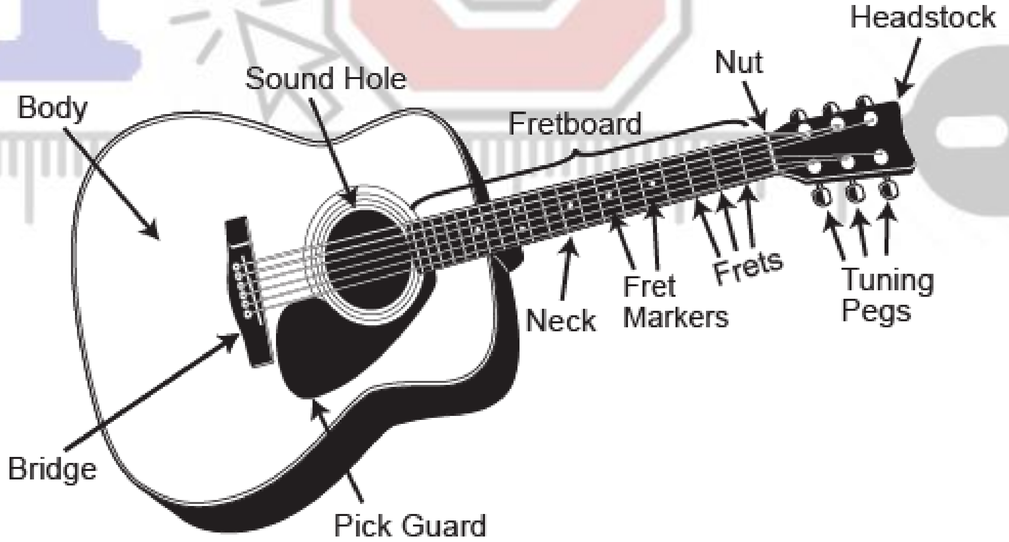
Musical Instruments



- Musical instruments are played by physically altering the natural frequency of the instrument.

Musical Instruments

- The instrument forms a standing wave and resonates amplifying the amplitude of the played frequency.
- The body of the instrument undergoes forced vibrations further amplifying the amplitude of the frequency.
- Energy is added to the instrument by the musician by strumming, striking, or blowing.



- Sounds will be more intense if additional material is made to vibrate
  - i.e. A guitar would not be audible if the sound was not transmitted through its wooden body

Speed of Sound

Sound travels at **330 metres per second** (330m/s), or 760 mph.

Lightning travels much faster than the **sound** of thunder.

A **3 second gap** between the **flash** of lightning and the **sound** of thunder means that the storm is about a **kilometre** away.

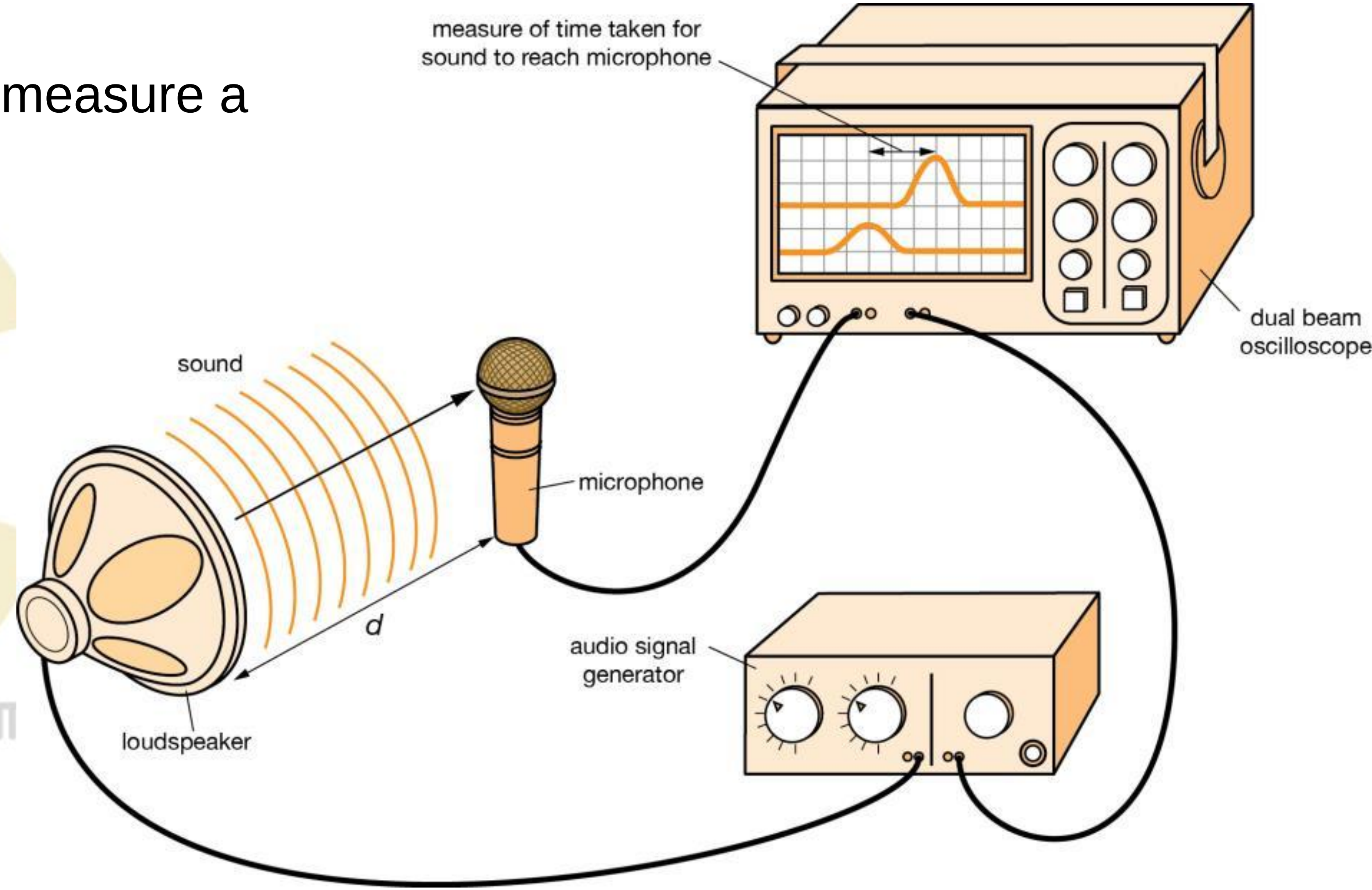
Depends upon the temperature of the air. Sound travels faster through hot air than through cold air.

Does not depend upon the pressure of the air. If atmospheric pressure changes, speed does not.

Is different through different materials. Eg. Fastest through solids, then liquids, then gases.

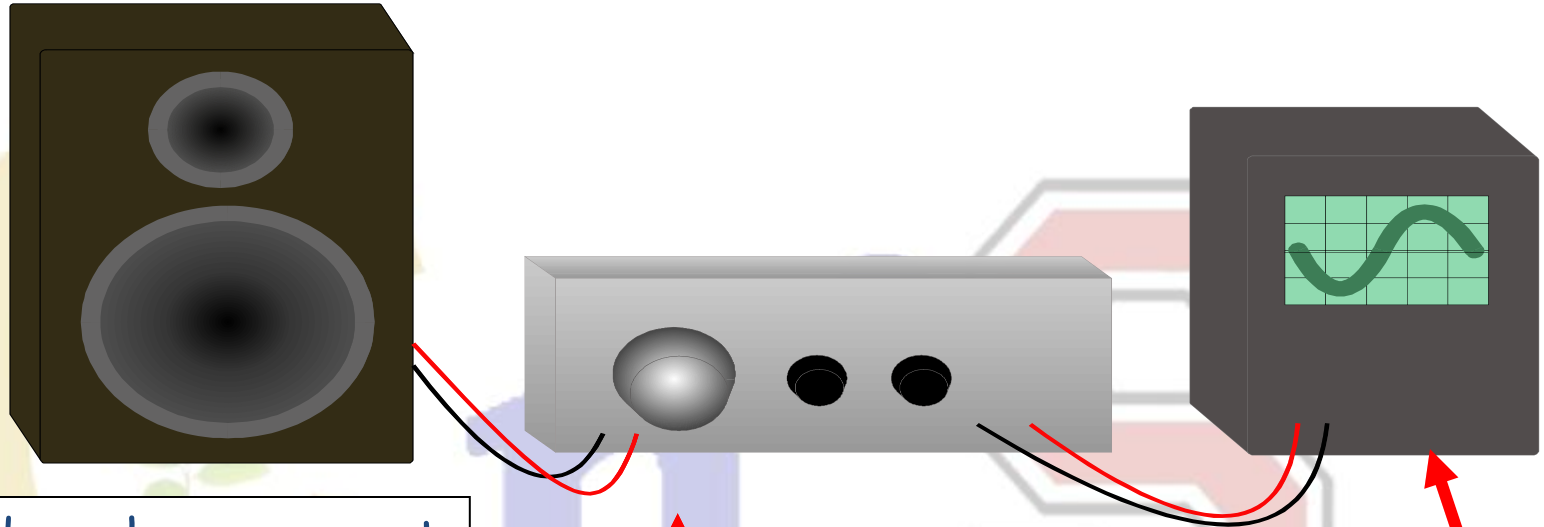
Measure the speed of sound … with a double beam oscilloscope

To find the speed of sound, measure a distance and a time.



…or use echoes from an exterior school wall.

Seeing the sound



Loudspeakers convert the signal from the signal generator into sound waves.

Signal generators can produce signals over a range of frequencies and of varying amplitudes.

The oscilloscope allows us to study the frequency and loudness of a sound.

A longitudinal wave travels in a gas when a vibration is set up in the gas.

The speed of sound / the speed of longitudinal wave in a medium is:

- $v = \sqrt{\frac{E}{\rho}}$

where E is Modulus of Elasticity and ρ is Density of medium.

The speed of the wave is given by the equation $\quad \mathbf{V} = \sqrt{\frac{\gamma P}{\rho}}$

Using the Ideal gas equation,

derive the equation, $\mathbf{V} = \sqrt{\frac{\gamma RT}{M}}$

- SOUND PROPAGATES UNDER ADIABATIC CONDITIONS, NOT UNDER ISOTHERMAL CONDITIONS.
- SINCE $E_{adiabatic} = \sqrt{\frac{\gamma P}{\rho}}$

$\gamma = C_P / C_V$

FOR AIR GAMMA IS 1.4

THEREFORE V=280 √ 1.4=330m/s

(a) Effect of pressure

If the temperature of the gas remains constant, then by Boyle's law PV = constant

That is, $P\,(m/\rho)$ = constant

p/ρ is a constant, when mass (m) of a gas is constant. If the pressure changes from P to P ′ then the corresponding density also will change from ρ to ρ ′ such that P/ρ is a constant.

In Laplace's formula $\sqrt{\gamma P/\rho}$ is also a constant. Therefore the in a gas is independent of the change in pressure provided the temperature remains constant. The velocity of sound waves is independent of its pressure at constant temperature.

Effect of temperature

For a gas, PV = RT

P (m/ρ) = RT

Or, P/ρ = RT/m

where m is the mass of the gas, T is the absolute temperature and R is the gas constant.

Therefore v = √γRT/m

V= k √T

It is clear that the velocity of sound in a gas is directly proportional to the square root of its absolute temperature.

+Explain why sound travels faster in warm air than in cold air.

Ans. The speed of sound varies directly as the square root of absolute temperature, i.e., $V = k\sqrt{T}$ It means that greater the temperature of air, more will be the speed of sound in it. That's why sound travel faster in warm air than in cold air.

- Show that one degree Celsius rise in temperature produces approximately 0.61 ms-1 increases the speed of sound. (use the Charles Law too)

The change in the speed of sound caused by a 10 degree Temperature change

$$v_1 = AT_1^{1/2} \qquad v_2 = AT_2^{1/2}$$

Take the ratio $\dfrac{v_1}{v_2} = \dfrac{AT_1^{1/2}}{AT_1^{1/2}} = \sqrt{\dfrac{T_1}{T_2}}$

Now we put in temperatures

$$\sqrt{\frac{293}{303}} = 0.983$$

2% change

(c) Effect of density

- Consider two different gases at the same temperature and pressure with different densities. The velocity of sound in two gases are given by,
- $v_1 = \sqrt{(\gamma_1 P/\rho_1)}$ and $v_2 = \sqrt{(\gamma_2 P/\rho_2)}$
- So, $v_1/v_2 = \sqrt{(\gamma_1/\gamma_2)(\rho_2/\rho_1)}$
- For two gases having same value of γ,

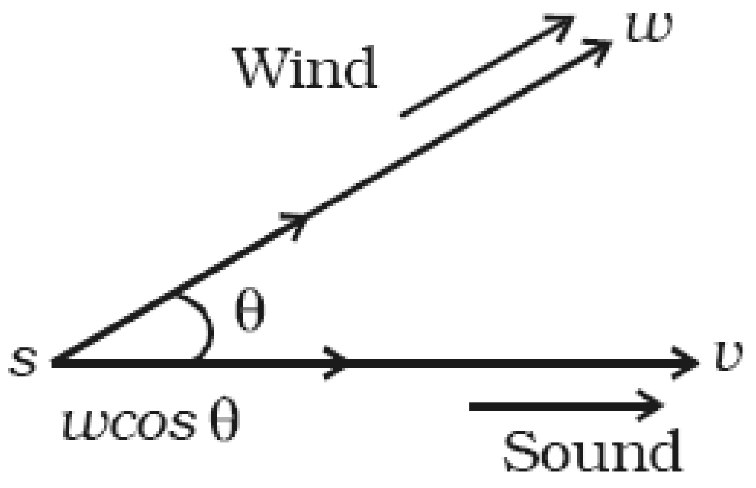
$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_1}{\rho_2}}$$

- The velocity of sound in a gas is inversely proportional to the square root of the density of the gas.
- Thus the speed of sound in hydrogen is four time its speed in oxygen as the density of oxygen is 16 times that of hydrogen.

(d) Effect of humidity

- When the humidity of air increases, the amount of water vapour present in it also increases and hence its density decreases, because the density of water vapour is less than that of dry air. Since velocity of sound is inversely proportional to the square root of density, the sound travels faster in moist air than in dry air. Due to this reason it can be observed that on a rainy day sound travels faster

(e) Effect of wind



Effect of wind

The velocity of sound in air is affected by wind. If the wind blows with the velocity w along the direction of sound, then the velocity of sound increases to v + w. If the wind blows in the opposite direction to the direction of sound, then the velocity of sound decreases to v – w. If the wind blows at an angle θ with the direction of sound, the effective velocity of sound will be (v + w cos θ).

Note: In a medium, sound waves of different frequencies or wavelengths travel with the same velocity. Hence there is no effect of frequency on the velocity of sound.

Speed of sound in a solid > speed in a liquid >speed in a gas

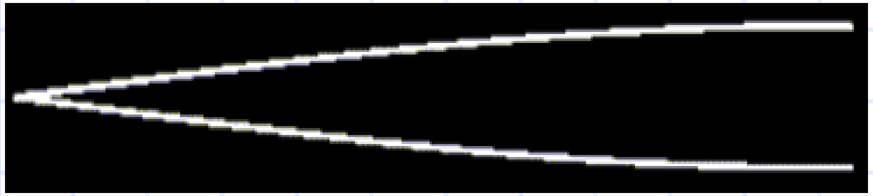
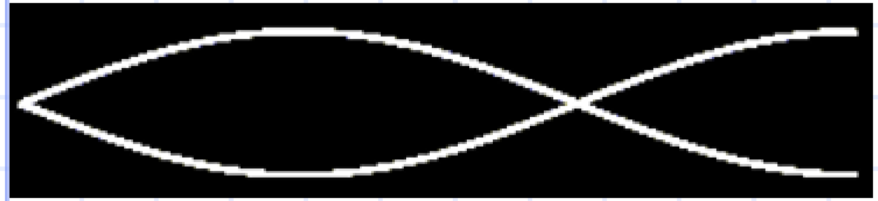
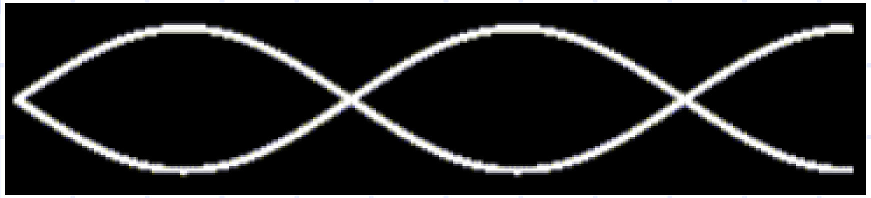
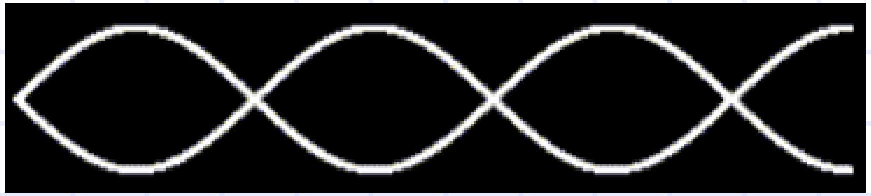
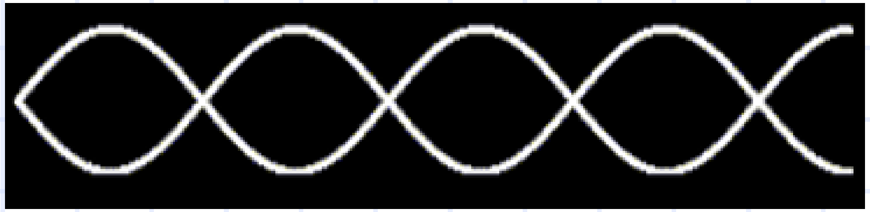
Why should sound travel faster in solids than in gases?

Ans. The formula for speed of sound is:

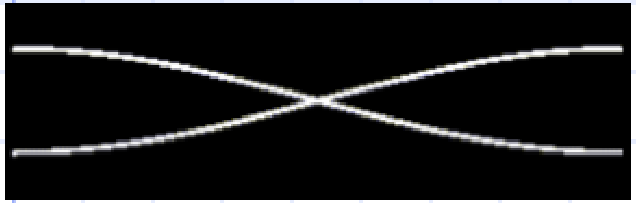
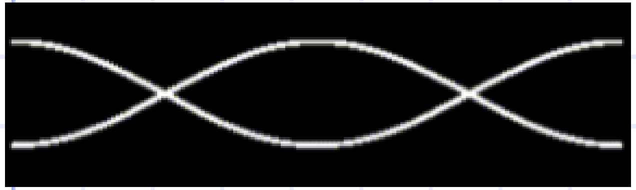
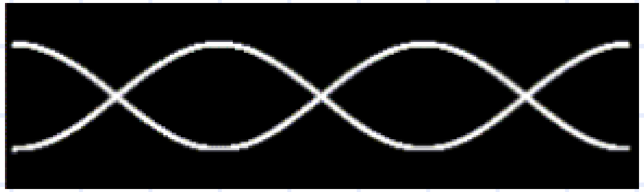
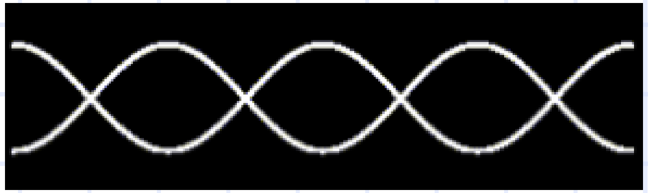
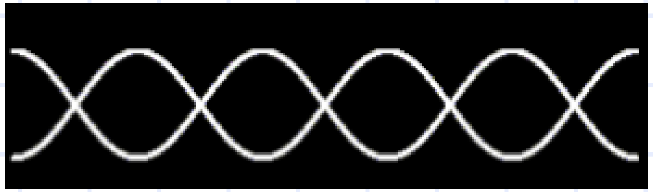
- $v = \sqrt{\frac{E}{\rho}}$

where E is Modulus of Elasticity and ρ is Density of medium. Although the density of solids is greater than the density of gases but the modulus of elasticity for solids is much greater than gases. Hence, sound travel faster in solids than in gases.

Modes of vibration for a closed tube

Picture of Standing Wave		Name	Structure
	$L = \frac{1}{4}\lambda_1$ $f_1 = v/4L$	1st Harmonic or Fundamental	1 Antinode 1 Node
	$L = \frac{3}{4}\lambda_3$ $f_3 = 3v/4L$	3rd Harmonic or 1st Overtone	2 Antinodes 2 Nodes
	$L = 1\frac{1}{4}\lambda_5$ $f_5 = 5v/4L$	5th Harmonic or 2nd Overtone	3 Antinodes 3 Nodes
	$L = 1\frac{3}{4}\lambda_7$ $f_7 = 7v/4L$	7th Harmonic or 3rd Overtone	4 Antinodes 4 Nodes
	$L = 2\frac{1}{4}\lambda_9$ $f_9 = 9v/4L$	9th Harmonic or 4th Overtone	5 Antinodes 5 Nodes

Modes of vibration for an open tube

Picture of Standing Wave		Name	Structure
	$L = \frac{1}{2}\lambda_1$ $f_1 = v/2L$	1st Harmonic or Fundamental	2 Antinodes 1 Node
	$L = \lambda_2$ $f_2 = v/L$	2nd Harmonic or 1st Overtone	3 Antinodes 2 Nodes
	$L = 1\frac{1}{2}\lambda_3$ $f_3 = 3v/2L$	3rd Harmonic or 2nd Overtone	4 Antinodes 3 Nodes
	$L = 2\lambda_4$ $f_4 = 2v/L$	4th Harmonic or 3rd Overtone	5 Antinodes 4 Nodes
	$L = 2\frac{1}{2}\lambda_5$ $f_5 = 5v/2L$	5th Harmonic or 4th Overtone	6 Antinodes 5 Nodes

Organ Pipes

Organ pipes are those cylindrical pipes which are used for produe musical (longitudial) sounds. Organ pipes are of two types

1. **Open Organ Pipe** Cylindrical pipes open at both ends.
2. **Closed Organ Pipe** Cylindrical pipes open at one end closed at other end.

Fundamental Note

It is the sound of lowest frequency produced in fundamental note., vibration of a system.

Overtones Tones having frequencies greater than the runoamen note are called overtones.

Harmonics When the frequencies of overtone are integral multiples of the fundamental, then they are known as **harmonics**. Thus note of lowest frequency n is called fundamental note or **first harmonics**. The note of frequency 2n is called **second harmonic or first overtone.**

Vibrations of Air Column in a Pipe Closed at One End:

- Consider a pipe closed at one end and open at the other. If a vibrating tuning fork of proper frequency is held near its open end, it sends longitudinal waves in the pipe. They are reflected from the closed end. The incident and reflected waves interfere and stationary waves are formed.
- The closed end acts as a rigid wall. The air at the closed end is not free to vibrate. Hence a node is formed at the closed end. Air at the open end is free to vibrate with maximum amplitude and antinode is formed at the open end. This is the simplest mode of vibration of an air column closed at one end and is called the fundamental mode.

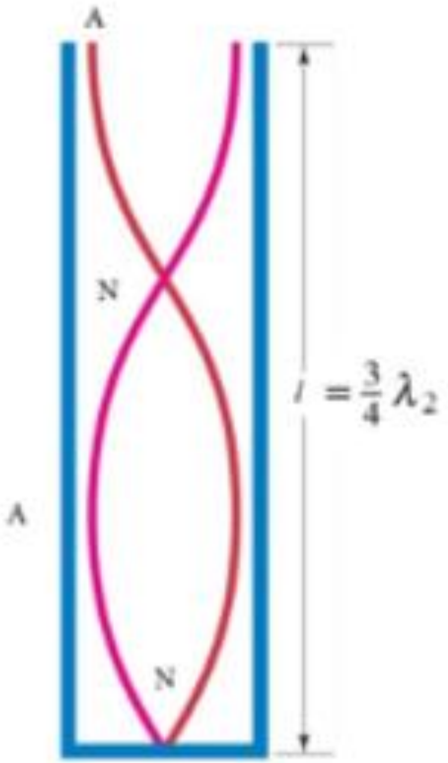
- $V = n\lambda$
- $L = \lambda/4$
- $\lambda = 4L$
- $n = V/4L$

If v is the velocity of sound in air at room temperature, then

$$\therefore n = v/4l$$

This is the frequency of the fundamental mode. Where is length of vibrating air column

- Now consider the following figure. The same air column can also be made to vibrate so that there is a node at the closed end, an antinode at the open end, with one more node and antinode in between. In this case, the air column is said to be emitted the first overtone. If n_1 is the frequency and λ_1 the wavelength for the first overtone,



$$l = \frac{3\lambda_1}{4} \qquad \therefore \lambda_1 = \frac{4l}{3}$$

But $V = n_1\lambda_1$

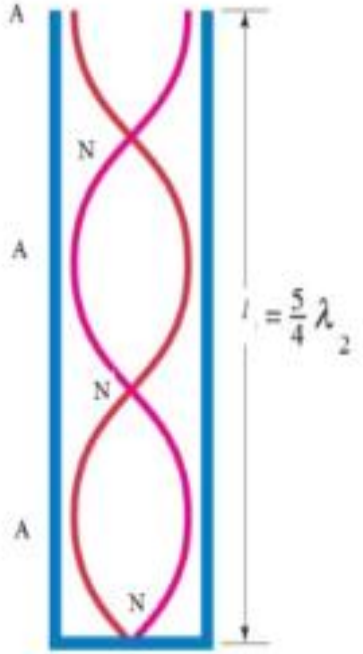
$$\therefore V = n_1 \frac{4l}{3}$$

$$\therefore n_1 = 3\left(\frac{V}{4l}\right)$$

$$\therefore n_1 = 3n$$

Thus the first overtone is the third harmonic.

- Now consider the following figure. In this case, the air column is shown emitting the second overtone. If n_2 is the frequency and λ_2 the wavelength for the second overtone, then



$$l = \frac{5\lambda_2}{4} \qquad \therefore \lambda_2 = \frac{4l}{5}$$

But $V = n_2\lambda_2$

$$\therefore V = n_2 \cdot \frac{4l}{5}$$

$$\therefore n_2 = 5\left(\frac{V}{4l}\right)$$

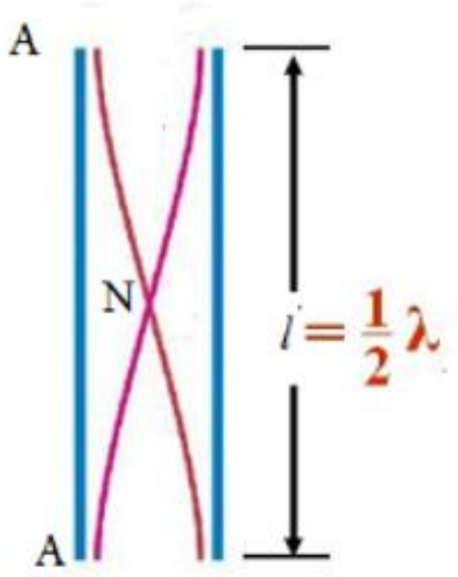
$$\therefore n_2 = 5n$$

Therefore, the second overtone is the fifth harmonic.

- Thus, for an air column closed at one end, only the odd harmonics are present as overtones.
- For air column in a pipe closed at one end p^{th} overtone is $(2p + 1)$ th harmonic.
- Thus frequency of p^{th} overtone = $(2p+1)n$.

Vibrations of Air Column in a Pipe Open at Both Ends:

- Consider a tube open at both ends. A vibrating tuning fork held near anyone end produces longitudinal waves in the air column. They are reflected from the other end. The incident and reflected waves interface with each other and stationary waves are formed.
- When an air column open at both ends is set up into vibrating, both the ends become antinodes. In the fundamental mode as shown in following figure, there is only one node midway between the two antinodes. Therefore, the length of the air column,



$$l = \lambda/2 \qquad \therefore \lambda = 2l$$

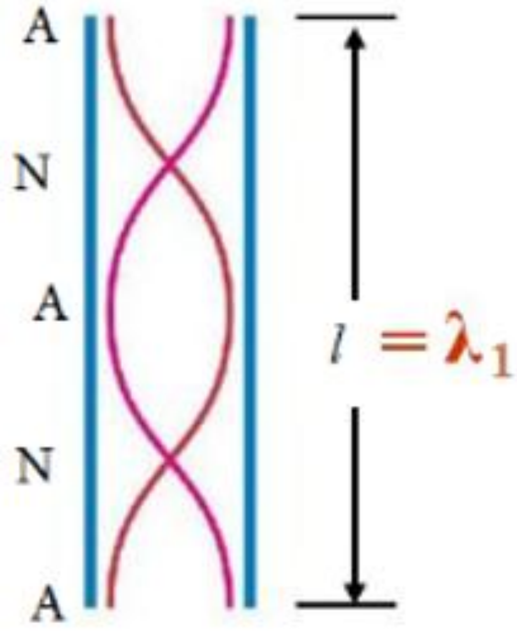
$$\text{But } V = n\lambda$$

$$\therefore \quad V = n \cdot 2l$$

$$\therefore \; n = \frac{v}{2l}$$

This is the fundamental frequency.

- Now consider the following figure. When the air column is emitting the first overtone, there are two nodes and one antinode in between the two antinodes at the ends. Therefore, if n_1 is the frequency and λ_1 is the wavelength for the first overtone,



Then $l = \lambda_1$

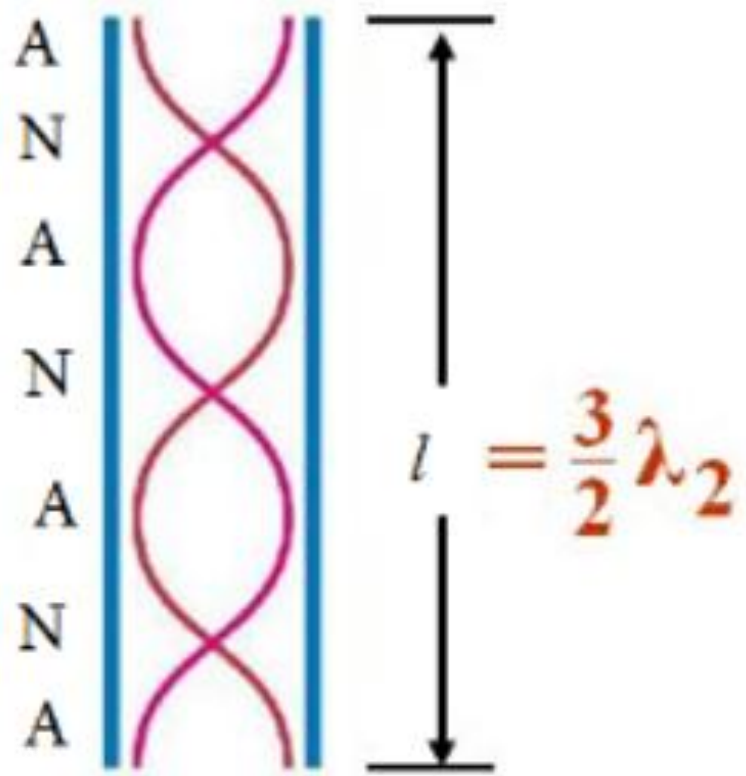
But $v = n_1\lambda_1$ $\quad\quad \therefore V = n_1 l$

$$\therefore n_1 = \frac{V}{l} = 2\left(\frac{V}{2l}\right)$$

$$\therefore n_1 = 2n$$

Hence, in this case, the first overtone is the second harmonic.

- In the following figure, the air column is shown emitting the second overtone. If n_2 is the frequency and λ_2 the wavelength for the second overtone, then



$$l = \frac{3\lambda_2}{2} \qquad \therefore \lambda_2 = \frac{2l}{3}$$

$$\text{But } v = n_2\lambda 2 \qquad \therefore v = n_2 . \frac{2l}{3}$$

$$\therefore n_2 = 3\left(\frac{v}{2l}\right) = 3n$$

$$\therefore n_2 = 3n$$

The second overtone is the third harmonic.

- Thus for an air column open at both ends, both odd, as well as even harmonics, are present as overtones. i.e. all harmonics are present.
- For air column in a pipe open at both ends p^{th} overtone is $(p + 1)^{th}$ harmonic
- Thus frequency of p^{th} overtone = $(p+1)n$.

Note:

- The frequency of the fundamental note of an open pipe is double that for a closed pipe of the same length.

End correction :

- It was shown by Regnault, that the antinode is not formed exactly at the open end but at a distance 0.3 d above the open end where d is the internal diameter of the tube. This additional distance called the end correction,
- End correction is necessary, as the wave spreads out slightly, just above the open end, and the air particles just outside the open end are also set into vibrations. For a pipe open at one end and closed at the other

corrected length = $l + 0.3d$

End Correction

Antinode is not obtained at exact open end but slightly above it. The distance between open and antinode is called end correction.

It is denoted by e.

- Effective length of an open organ pipe = $(l + 2e)$
- Effective length of a closed organ pipe = $(1 + e)$
- If r is the radius of organ pipe, then $e = 0.6\,r$

Factors Affecting Frequency of Pipe

1. Length of air column, $n \propto (1/l)$
2. Radius of air column, $n \propto (1/r)$
3. Temperature of air column, $n \propto \sqrt{T}$
4. Pressure of air inside air column, $n \propto \sqrt{p}$
5. Density of air, $n \propto (1/\sqrt{\rho})$
6. Velocity of sound in air column, $n \propto v$

Hence, the velocity of sound in air at room temperature is

$$\therefore \; v = n\lambda = 4n\,(l + 0.3d)$$

Hence, knowing n, l and d, the velocity of sound in air is determined.

For a pipe open at both ends, end correction = $2 \times 0.3\,d = 0.6\,d$

Corrected length = $l + 0.6\ d$

Hence, the velocity of sound in air at room temperature is

$$\therefore\ v = n\lambda = 2n\ (l + 0.6d)$$

Elimination of end correction :

- By the usual method, we find the first resonating length (l_1). Similarly, we find the second resonating length (l_2) for which a loud sound is heard once again. If "e" is the end correction, then

$$l_1 + e = \lambda/4 \quad ....\ (1)$$

$$l_2 + e = 3\lambda/4 \quad ....\ (1)$$

Subtracting equation (1) from (2)

$$l_2 - l_1 = 3\lambda/4 - \lambda/4$$

$$\therefore \; l_2 - l_1 = \lambda/2$$

$$\therefore \; \lambda = 2(l_2 - l_1)$$

Now, $v = n\lambda = 2n(l_2 - l_1)$

End Correction for a Pipe Closed at One End:

- If v is the velocity of sound in air, λ is the wavelength of sound. is the vibrating length of a pipe closed at one end, n is the frequency of vibration in the fundamental mode which is equal to the frequency of the tuning fork at time of resonance. Let 'e' be the end correction. This end correction depends on the diameter of the tube. we have

$$V = n\lambda = 4n\,(l + e)$$

For the first case

$$V = 4n_1\,(l_1 + e) \quad .... (1)$$

Where $l1$ is a length of vibrating air column vibrating in unison with a tuning fork of frequency n_1.

For the second case

$$V = 4n_2 (l_2 + e) \quad .... (2)$$

Where $l2$ is a length of vibrating air column vibrating in unison with a tuning fork of frequency n_2.

From (1) and (2)

$$4n_1 (l_1 + e) = 4n_2 (l_2 + e)$$

$$\therefore \quad l_1 n_1 + en_1 = l_2 n_2 + en_2$$

$$\therefore \quad l_1 n_1 - l_2 n_2 = en_2 - en_1$$

$$\therefore \quad l_1 n_1 - l_2 n_2 = e(n_2 - n_1)$$

$$e = \frac{n_1 l_1 - n_2 l_2}{n_2 - n_1}$$

End cor. for a pipe op at both ends

- If v is the velocity of sound in air, λ is the wavelength of sound. is the vibrating length of a pipe open at both the ends, n is the frequency of vibration in the fundamental mode which is equal to the frequency of the tuning fork at time of resonance. Let 'e' be the end correction. This end correction depends on the diameter of the tube. we have

$$V = n\lambda = 2n\,(l + 2e)$$

For the first case

$$V = 2n_1\,(l_1 + 2e) \quad \text{............} \ (1)$$

Where $l1$ is a length of vibrating air column vibrating in unison with a tuning fork of frequency n_1.

For the second case

$$V = 2n_2\,(l_2 + 2e) \quad \text{............} \ (2)$$

Where $l2$ is a length of vibrating air column vibrating in unison with a tuning fork of frequency n_2.

From (1) and (2)

$$2n_1\,(l_1 + 2e) = 2n_2\,(l_2 + 2e)$$

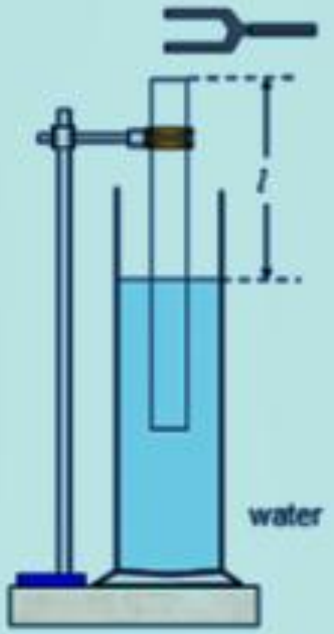
$$\therefore \ l_1n_1 + 2en_1 = l_2n_2 + 2en_2$$

$$\therefore \ l_1n_1 - l_2n_2 = 2en_2 - 2en_1$$

$$\therefore \ l_1n_1 - l_2n_2 = 2e(n_2 - n_1)$$

$$e = \frac{n_1l_1 - n_2l_2}{2(n_2 - n_1)}$$

Resonance Tube Experiment to Determine Velocity of Sound in Air:



- A metal tube, open at both the ends is immersed in a tall glass jar filled with water. An air column is thus formed between the open end of the metal tube and the surface of the water. The length of the air column, which is closed by the water surface at the lower end, can be varied by raising or lowering the metal tube.
- A tuning fork is set up into vibrations and held near the mouth of the tube so that its arms vibrate parallel to the axis of the tube. The longitudinal wave, starting from the tuning fork, travels along the length of the air column and is reflected back from the surface of the water. The incident and reflected waves interfere to produce a stationary wave.
- The molecules of air, in contact with the water surface, remain at rest. Therefore, the closed end becomes a node. The molecules of air, near the mouth of the tube, vibrate with maximum amplitude. Therefore, the open end becomes an antinode.

- The frequency of the air column can be changed by adjusting its length. When its frequency becomes the same as the frequency of the fork, resonance takes place, and a loud sound is heard.
- If the length of the air column is increased from a small value, the first resonance occurs when there is a node at the closed end and an antinode at the open end, with no other nodes or antinodes in between. Therefore, the length of the air column $l = \lambda/4$

$$\therefore \lambda = 4l$$

Due to resonance the frequency of the air column is the same as that of the fork

Now velocity of sound is given by $v = n\lambda$

$$\therefore v = 4nl$$

End correction :

- It was shown by Regnault, that the antinode is not formed exactly at the open end but at a distance 0.3 d above the open end where d is the internal diameter of the tube. This additional distance, called the end correction, is necessary, as the wave spreads out slightly, just above the open end, and the air particles just outside the open end are also set into vibrations.

$$\therefore \text{ corrected length} = l + 0.3d$$

Hence, the velocity of sound in air at room temperature is

$$V = n\lambda = 4n\,(l + 0.3d)$$

Q # 6. A church organ consists of pipes, each open at one end, of different lengths. The minimum length is 30 mm and the longest is 4 m. Calculate the frequency range of the fundamental notes. (Speed of sound = 340 ms^{-1})

Given Data: Speed of Sound $v = 340\frac{m}{s}$, **(a)** Minimum Length $l = 30\ mm = 0.03\ m$, **(b)** Maximum Length $l' = 4\ m$
To Determine: Frequency Range
Calculations: (a) Fundamental Frequency for Length l: $f_1 = \frac{v}{4l} = \frac{340}{4\times0.03} = 2833\ Hz$ **(b)** Fundamental Frequency for Length l': $f_1' = \frac{v}{4l'} = \frac{340}{4\times4} = 21\ Hz$ So the Frequency Range is from 21 Hz to 2833 Hz

Q # 5. An organ pipe has a length of 50 cm. Find the frequency of its fundamental note and the next harmonic when it is (a) open at both ends, (b) closed at one end.

Given Data: Length of organ pipe $l = 50\ cm = 0.5\ m$, Speed of Sound $v = 350 \frac{m}{s}$
To Determine: (a) <u>For Open Organ Pipe</u>: Fundamental Frequency $f_1 = ?$ and $f_2 = ?$ (b) <u>For Close Organ Pipe</u>: Fundamental Frequency $f_1' = ?$ and $f_2' = ?$
Calculations: (a) <u>For Open Organ Pipe</u>: Fundamental Frequency $f_1 = \frac{v}{2l} = \frac{350}{2\times0.5} = 350\ Hz$ $f_2 = 2f_1 = 2 \times 350 = 700\ Hz$ (b) <u>For Close Organ Pipe</u>: Fundamental Frequency $f_1' = \frac{v}{4l} = \frac{350}{4\times0.5} = 175\ Hz$ $f_2' = 3f_1' = 3 \times 175 = 525\ Hz$

The speed of a sound wave in a liquid or gas depends on the medium's compressibility and inertia. If the fluid has a bulk modulus of B and an equilibrium density of ρ, the speed of sound is

$$v = \sqrt{\frac{B}{\rho}}$$

The speed of a longitudinal wave in a solid rode is

$$v = \sqrt{\frac{Y}{\rho}}$$

where Y is the Young's modulus of the solid, and ρ is the density of the solid.

The speed of sound also depends on the temperature of the medium. For example traveling through air, the relationship between the speed of sound and tepmerature θ in degrees Celsius is

$$v = (331 \text{ m/s})\sqrt{1 + \frac{\theta}{273}}$$

- Find the speed of the sound in water, which has a bulk modulus of about $2.1 \cdot 10^9$ Pa and a density of about 10^3 kg/m^3.

$$v_w = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.1 \cdot 10^9 \text{ Pa}}{10^3 \text{ kg/m}^3}} \approx 1500 \text{ m/s}$$

- If a solid bar is struck at one end with a hammer, a longitudinal pulse propagates down the bar. Find the speed of sound in a bar of aluminium, which has a Young's modulus of $7 \cdot 10^{10}$ Pa and a density of $2.7 \cdot 10^3$ kg/m^3.

$$v_{Al} = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{7 \cdot 10^9 \text{ Pa}}{2.7 \cdot 10^3 \text{ kg/m}^3}} \approx 5100 \text{ m/s}$$

- Find the first four harmonics of a 1-m-long string if the string has a mass per unit length of $2 \cdot 10^{-3}$ kg/m and is under a tension of 80 N.

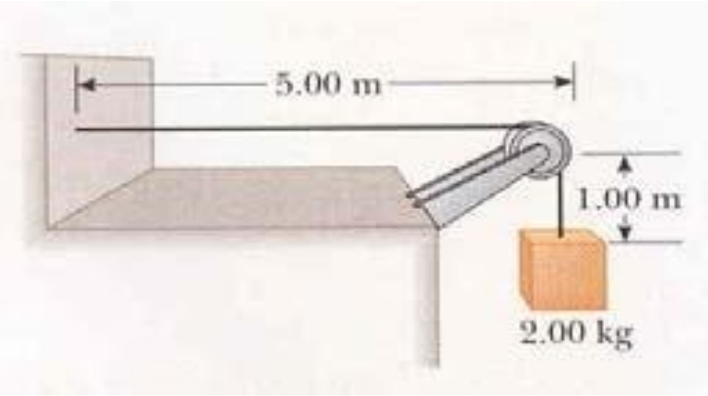
$$
\begin{aligned}
f_n &= \frac{n}{2L}\sqrt{\frac{F_T}{\mu}} \qquad n = 1, 2, 3, 4 \\
f_1 &= \frac{1}{2(1\text{ m})}\sqrt{\frac{80\text{ N}}{2 \cdot 10^{-3}\text{ kg/m}}} = 100\text{ Hz} \\
f_2 &= 200\text{ Hz} \\
f_3 &= 300\text{ Hz} \\
f_4 &= 400\text{ Hz}
\end{aligned}
$$

- A simple 2-m-long pendulum oscillates in a location where g = 9.8 m/s². How many complete oscillations does it make in 5 min?

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{2\text{ m}}{9.8\text{ m/s}^2}} = 2.84\text{ s}$$

$$n = \frac{t}{T} = \frac{300\text{ s}}{2.84\text{ s}} = 105.6 \qquad \text{(105 complete oscilations)}$$

- A uniform string has a mass of 0.3 kg and a length of 6 m. Tension is maintained in the spring by suspending a 2-kg block from one end. Find the speed of a pulse on this string.



$$F_T = mg = (2\text{ kg})(9.8\text{ m/s}^2) = 19.6\text{ N}$$

$$\mu = \frac{m_s}{l} = \frac{0.3\text{ kg}}{6\text{ m}} = 0.05\text{ kg/m}$$

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{19.6\text{ N}}{0.05\text{ kg/m}}} = 19.8\text{ m/s}$$