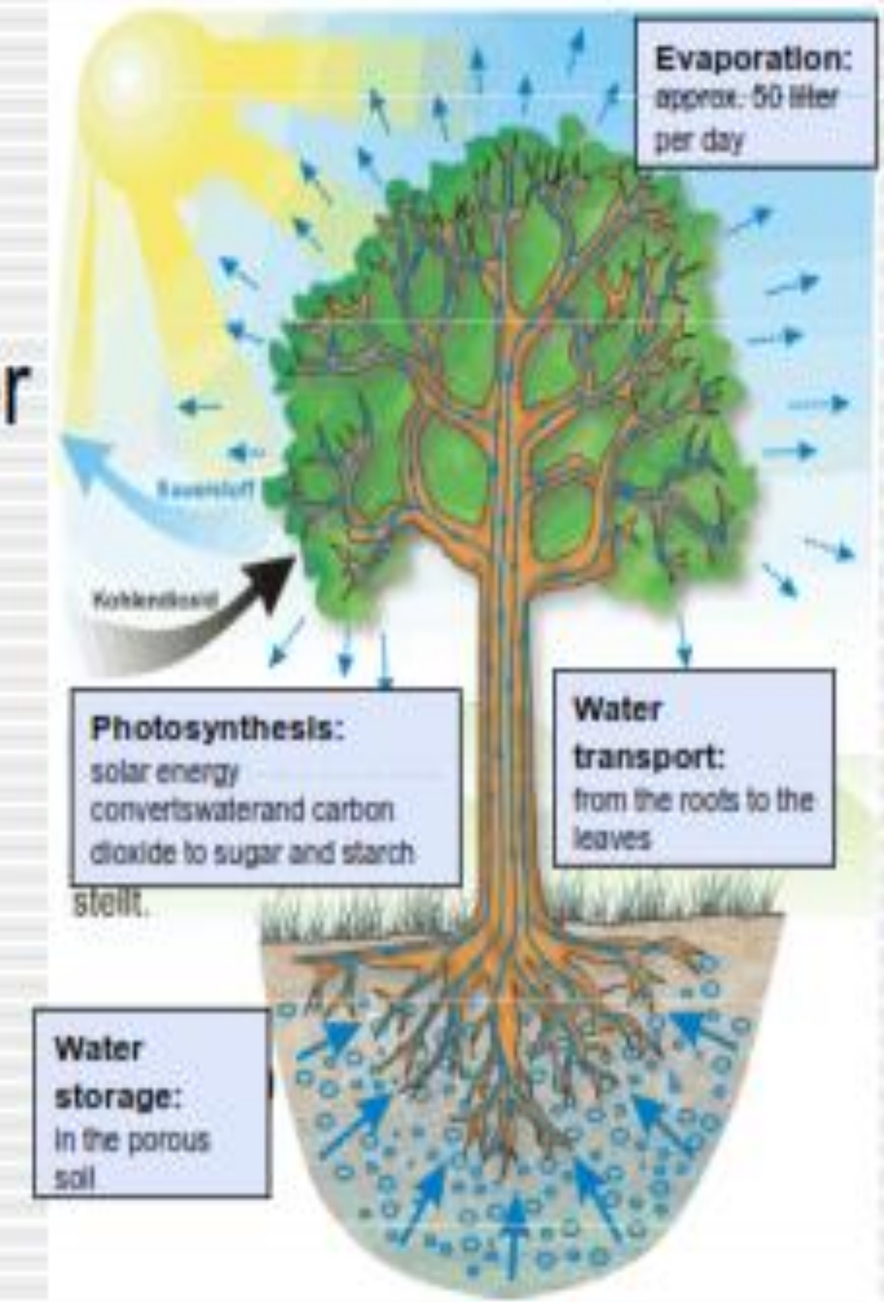
SURFACE TENSION

Small Dimensions:

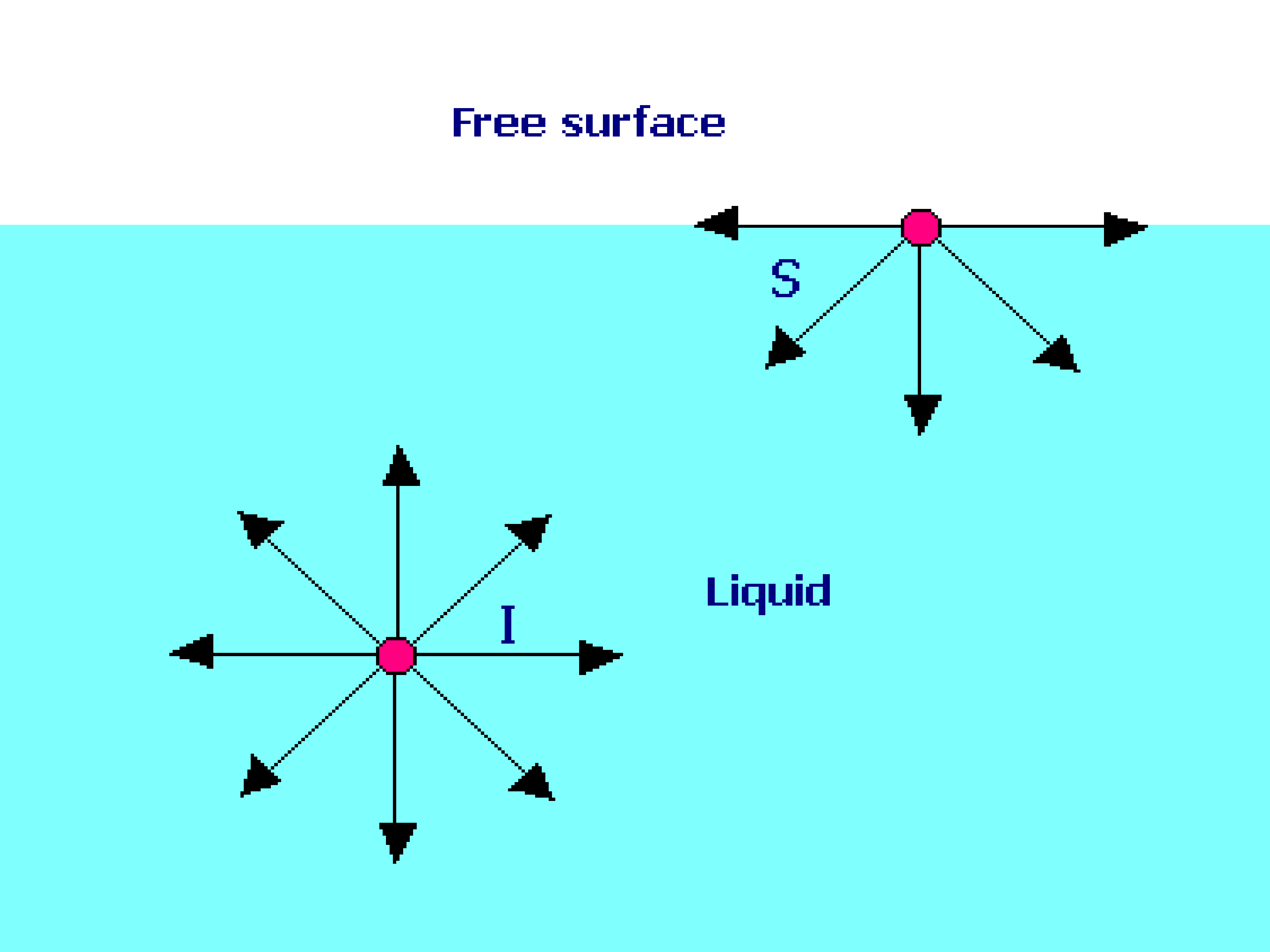
- Surface tension dominates over other forces ...
- Trees know it !
- Transpiration up to 200l/h
- Velocities up to 15m/h



OBJECTIVES

- Demonstrate the nature of free liquid surface using simple activities.
- Explain the nature of free liquid surface using suitable examples.
- Describe the nature of free liquid surface using the inter-molecular forces.
- Define surface tension and give units.
- Introduce free surface energy considering the work done to increase the surface area isothermally.
- Obtain the relationship between surface tension and free surface energy.
- Introduce angle of contact for a liquid meniscus.
- Explain the instances where the angle is less than 90°, 90° and greater than 90° considering cohesive forces and adhesive forces.
- Explain capillary rise using surface tension.
- Guide students to derive the equation, $h\rho g = \frac{2T\cos\theta}{r}$ considering force equilibrium.

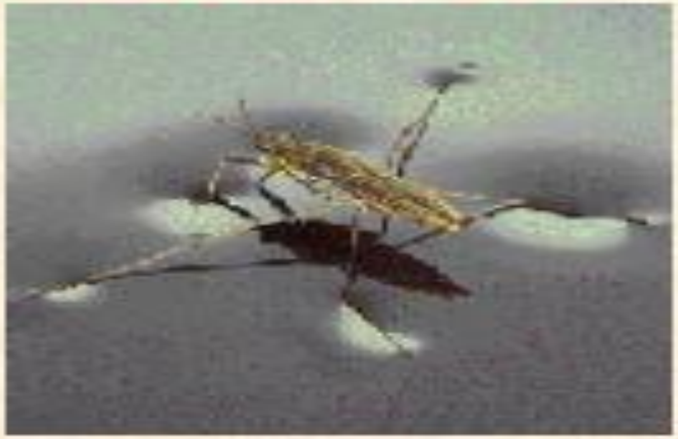
- Guide students to obtain the expression, $P_{in} - P_{out} = \frac{2T}{r}$ for a spherical meniscus.
- Show that the pressure difference is large when the radius is small.
- Guide students to obtain the expression considering pressure difference method.
- Show that the pressure difference of an air bubble in a liquid and a liquid drop is given by *2T/r*.
- Show that the pressure difference of a soap bubble is given by *4T/r* because there are two liquid-air interfaces.
- Guide students to conduct the following experiments to find surface tension.
    - Microscope-slide method.
    - Capillary-rise method.
    - Jaeger method.
- Guide students to solve problem related to surface tension.
- Conduct a discussion to describe the uses of surface tension.


Free surface
S
Liquid
I

This is a property for liquids and arises from *intermolecular forces of attraction* . you can notice that a molecule in the interior of the liquid is attracted equally from all directions by the molecules around it, while a molecule at the surface of the liquid is attracted only sideways and toward the interior. *(figure 1)* shows the two types of attractions as discussed. The forces on the sides being counterbalanced the surface molecule is pulled only inward the liquid . so the molecules have tendency to go into the bulk of the liquid. Now the surface of the liquid is under tension and in order to have the minimum number of molecules at the surface of the liquid so it contracts to have the smallest possible area. Then this is the reason why the drops of a liquid in air is assumed to be spherical, because for a given volume the spherical shape has the smallest possible area.

Thus all molecules lying in surface film experiences a net downward force. Therefore, free surface of the liquid behaves like a stretched membrane.

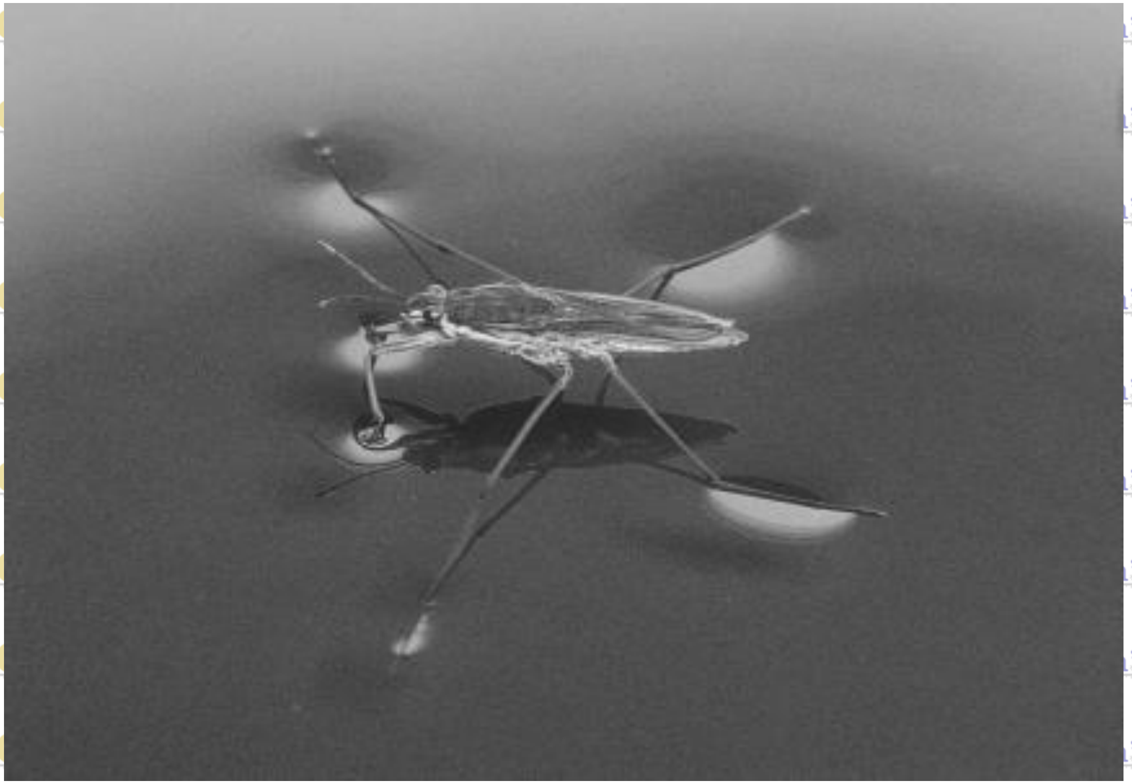
Insects can walk on water



Depression in water surface
(increases surface area)

Surface tension opposes this, which results in an upwards force that tends to bring back surface to original flat shape.

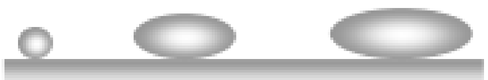
Liquid surface behaves like a rubber membrane under tension



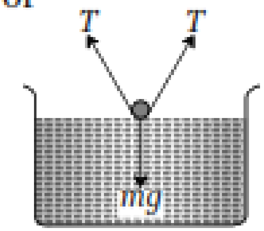
Surface tension is a property that allows the surface of a liquid to behave somewhat as a trampoline does. When a person stands on a trampoline, the trampoline stretches downward a bit and, in so doing, exerts an upward elastic force on the person. This upward force balances the person's weight. The surface of the water behaves in a similar way. The indentations in the water surface made by the feet of an insect known as a water strider, because it can stride or walk on the surface just as a person can walk on a trampoline.

Examples of Surface Tension

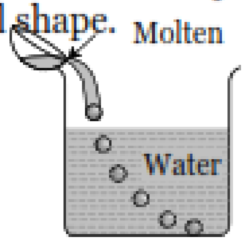
(1) When mercury is split on a clean glass plate, it forms globules. Tiny globules are spherical on the account of surface tension because force of gravity is negligible. The bigger globules get flattened from the middle but have round shape near the edges, figure



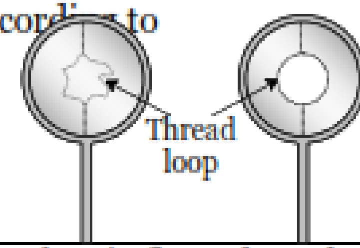
(2) When a greased iron needle is placed gently on the surface of water at rest, so that it does not prick the water surface, the needle floats on the surface of water despite it being heavier because the weight of needle is balanced by the vertical components of the forces of surface tension. If the water surface is pricked by one end of the needle, the needle sinks down.



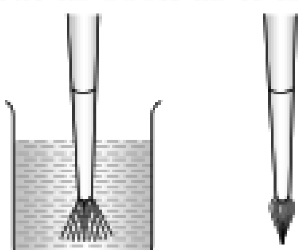
(3) When a molten metal is poured into water from a suitable height, the falling stream of metal breaks up and the detached portion of the liquid in small quantity acquire the spherical shape.



(4) Take a frame of wire and dip it in soap solution and take it out, a soap film will be formed in the frame. Place a loop of wet thread gently on the film. It will remain in the form, we place it on the film according to figure. Now, piercing the film with a pin at any point inside the loop, It immediately takes the circular form as shown in figure.



(5) Hair of shaving brush/painting brush when dipped in water spread out, but as soon as it is taken out, its hair stick together.

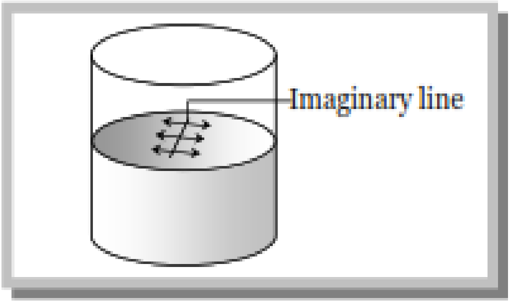


(6) If a small irregular piece of camphor is floated on the surface of pure water, it does not remain steady but dances about on the surface. This is because, irregular shaped camphor dissolves unequally and decreases the surface tension of the water locally. The unbalanced forces make it move haphazardly in different directions.

(7) Rain drops are spherical in shape because each drop tends to acquire minimum surface area due to surface tension, and for a given volume, the surface area of sphere is minimum.

(8) Oil drop spreads on cold water. Whereas it may remain as a drop on hot water. This is due to the fact that the surface tension of oil is less than that of cold water and is more than that of hot water.

The property of a liquid due to which its free surface tries to have minimum surface area and behaves as if it were under tension some what like a stretched elastic membrane is called surface tension. A small liquid drop has spherical shape, as due to surface tension the liquid surface tries to have minimum surface area and for a given volume, the sphere has minimum surface area.



Surface tension of a liquid is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of liquid, the direction of this force being perpendicular to the line and tangential to the free surface of liquid. So if F is the force acting on one side of imaginary line of length L, then $T = (F/L)$

(1) It depends only on the nature of liquid and is independent of the area of surface or length of line considered.

(2) It is a scalar as it has a unique direction which is not to be specified.

(3) Dimension : $[MT^{-2}]$.

(4) Units : N/m (S.I.)

(5) It is a molecular phenomenon and its root cause is the electromagnetic forces.

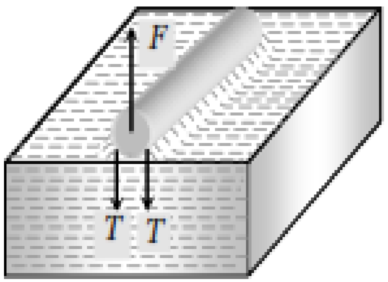
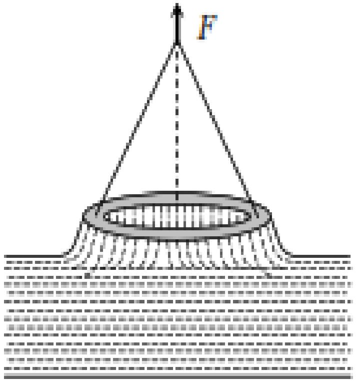
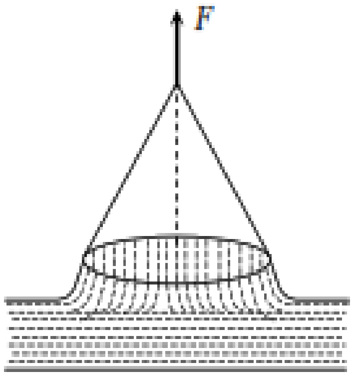
Intermolecular Force

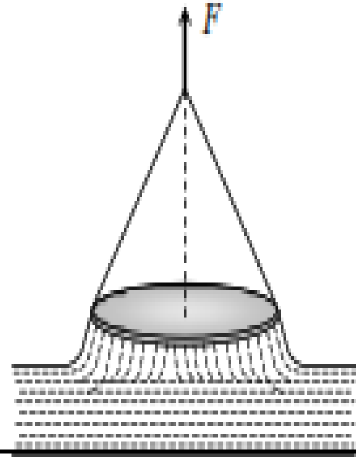
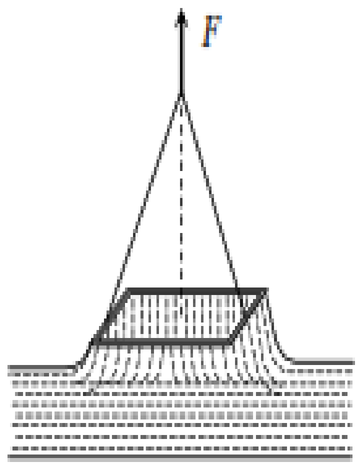
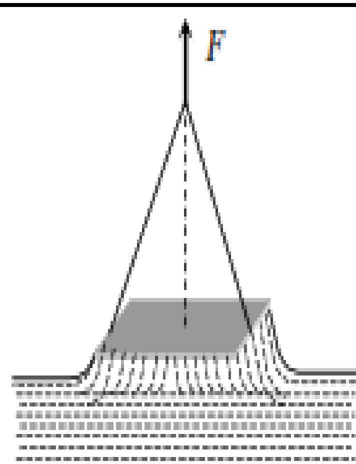
The force of attraction or repulsion acting between the molecules are known as intermolecular force. The nature of intermolecular force is electromagnetic.

The intermolecular forces of attraction may be classified into two types.

Cohesive force	Adhesive force
The force of attraction between molecules of same substance is called the force of cohesion. This force is lesser in liquids and least in gases.	The force of attraction between the molecules of the different substances is called the force of adhesion.
Ex. (i) Two drops of a liquid coalesce into one when brought in mutual contact. (ii) It is difficult to separate two sticky plates of glass welded with water. (iii) It is difficult to break a drop of mercury into small droplets because of large cohesive force between the mercury molecules.	Ex. (i) Adhesive force enables us to write on the blackboard with a chalk. (ii) A piece of paper sticks to another due to large force of adhesion between the paper and gum molecules. (iii) Water wets the glass surface due to force of adhesion.

If a body of weight W is placed on the liquid surface, whose surface tension is T. If F is the minimum force required to pull it away from the water then value of F for different bodies can be calculated by the following table.

Body	Figure	Force
Needle (Length = l)		$F = 2l\,T + W$
Hollow disc (Inner radius = r_1 Outer radius = r_2)		$F = 2\pi(r_1 + r_2)T + W$
Thin ring (Radius = r)		$F = 2\pi(r + r)T + W$ $F = 4\pi rT + W$

Circular plate or disc (Radius = r)		$F = 2\pi r T + W$
Square frame (Side = l)		$F = 8l\,T + W$
Square plate		$F = 4l\,T + W$

(1) **Temperature :** The surface tension of liquid decreases with rise of temperature. The surface tension of liquid is zero at its boiling point and it vanishes at critical temperature. At critical temperature, intermolecular forces for liquid and gases becomes equal and liquid can expand without any restriction. For small temperature differences, the variation in surface tension with temperature is linear and is given by the relation

$$T_t = T_0(1 - \alpha t)$$

where T_t, T_0 are the surface tensions at $t^{o}C$ and $0^{o}C$ respectively and α is the temperature coefficient of surface tension.

Examples : (i) Hot soup tastes better than the cold soup.

(ii) Machinery parts get jammed in winter.

(2) **Impurities :** The presence of impurities either on the liquid surface or dissolved in it, considerably affect the force of surface tension, depending upon the degree of contamination. A highly soluble substance like sodium chloride when dissolved in water, increases the surface tension of water. But the sparingly soluble substances like phenol when dissolved in water, decreases the surface tension of water.

Factors Affecting Surface Tension.

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Q: Describe the effect of nature of liquid and temperature on surface tension.

Solution: Surface tension is a property that arises due to the intermolecular forces of attraction among the molecules of the liquid. Greater are the intermolecular forces of attraction, higher is the surface tension of that liquid. Now, let us explain the effect of temperature on surface tension.

The surface tension of liquid generally decreases with the increase in temperature and becomes zero at the critical temperature. The decrease in surface tension with an increase of temperature is due to the fact that with an increase in temperature, the kinetic energy of the molecules increases. Therefore, the intermolecular attraction decreases.

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Applications of Surface Tension

(1) The oil and grease spots on clothes cannot be removed by pure water. On the other hand, when detergents (like soap) are added in water, the surface tension of water decreases. As a result of this, wetting power of soap solution increases. Also the force of adhesion between soap solution and oil or grease on the clothes increases. Thus, oil, grease and dirt particles get mixed with soap solution easily. Hence clothes are washed easily.

(2) The antiseptics have very low value of surface tension. The low value of surface tension prevents the formation of drops that may otherwise block the entrance to skin or a wound. Due to low surface tension, the antiseptics spreads properly over wound.

(3) Surface tension of all lubricating oils and paints is kept low so that they spread over a large area.

(4) Oil spreads over the surface of water because the surface tension of oil is less than the surface tension of cold water.

(5) A rough sea can be calmed by pouring oil on its surface.

(6) In soldering, addition of 'flux' reduces the surface tension of molten tin, hence, it spreads.

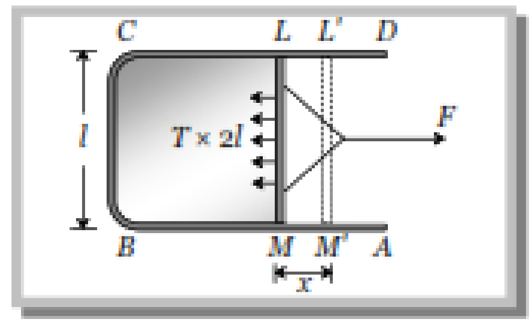
Surface Energy

The molecules on the liquid surface experience net downward force. So to bring a molecule from the interior of the liquid to the free surface, some work is required to be done against the intermolecular force of attraction, which will be stored as potential energy of the molecule on the surface. The potential energy of surface molecules per unit area of the surface is called surface energy.

Unit : $Joule/m^2$ (S.I.)

Dimension : $[MT^{-2}]$

If a rectangular wire frame $ABCD$, equipped with a sliding wire LM dipped in soap solution, a film is formed over the frame. Due to the surface tension, the film will have a tendency to shrink and thereby, the sliding wire LM will be pulled in inward direction. However, the sliding wire can be held in this position under a force F, which is equal and opposite to the force acting on the sliding wire LM all along its length due to surface tension in the soap film.



If T is the force due to surface tension per unit length, then $F = T \times 2l$

Here, l is length of the sliding wire LM. The length of the sliding wire has been taken as $2l$ for the reason that the film has got two free surfaces.

Suppose that the sliding wire LM is moved through a small distance x, so as to take the position $L'M'$. In this process, area of the film increases by $2l \times x$ (on the two sides) and to do so, the work done is given by

$$W = F \times x = (T \times 2l) \times x = T \times (2lx) = T \times \Delta A$$

$\therefore$ $W = T \times \Delta A$ [ΔA = Total increase in area of the film from both the sides]

If temperature of the film remains constant in this process, this work done is stored in the film as its surface energy.

From the above expression $T = \dfrac{W}{\Delta A}$ or $T = W$ [If $\Delta A = 1$]

Surface Tension may be defined as the amount of work done in increasing the area of the liquid surface by unity against the force of surface tension at constant temperature.

Work Done in Blowing a Liquid Drop or Soap Bubble

(1) If the initial radius of liquid drop is r_1 and final radius of liquid drop is r_2 then

$W = T \times$ Increment in surface area

$W = T \times 4\pi[r_2^2 - r_1^2]$ [drop has only one free surface]

(2) In case of soap bubble

$W = T \times 8\pi[r_2^2 - r_1^2]$ [Bubble has two free surfaces]

Splitting of Bigger Drop

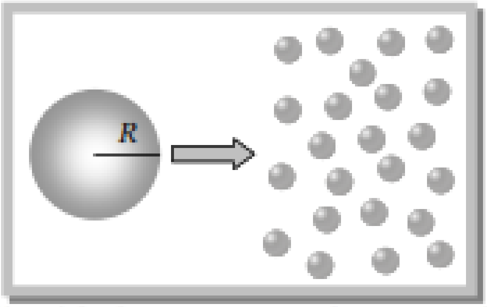
When a drop of radius R splits into n smaller drops, (each of radius r) then surface area of liquid increases. Hence the work is to be done against surface tension.

Since the volume of liquid remains constant therefore $\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3 \quad \therefore \; R^3 = nr^3$

Work done = $T \times \Delta A$ = T [Total final surface area of n drops – surface area of big drop] = $T[n4\pi r^2 - 4\pi R^2]$

Various formulae of work done

$4\pi T[nr^2 - R^2]$	$4\pi R^2 T[n^{1/3} - 1]$	$4\pi T r^2 n^{2/3}[n^{1/3} - 1]$	$4\pi T R^3\left[\frac{1}{r} - \frac{1}{R}\right]$



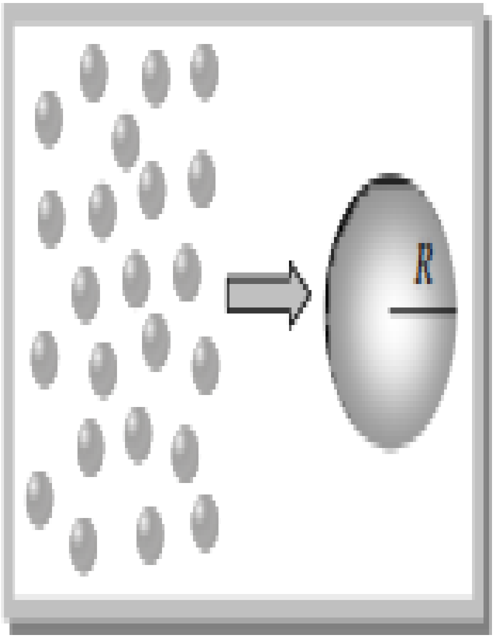
ENERGY REQUIRED= WORK DONE= INCREASE IN SURFACE ENERGY

Formation of Bigger Drop

If n small drops of radius r coalesce to form a big drop of radius R then surface area of the liquid decreases.

Amount of surface energy released = Initial surface energy – final surface energy

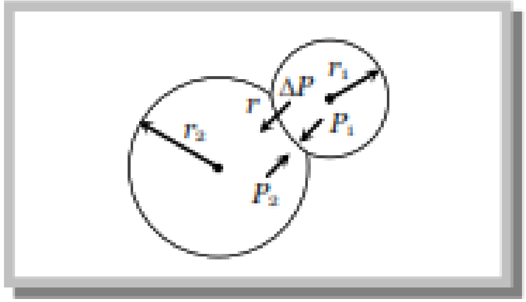
$$E = n4\pi r^2 T - 4\pi R^2 T$$



Various formulae of released energy			
$4\pi T[nr^2 - R^2]$	$4\pi R^2 T(n^{1/3} - 1)$	$4\pi T r^2 n^{2/3}(n^{1/3} - 1)$	$4\pi T R^3\left[\frac{1}{r} - \frac{1}{R}\right]$

ENERGY RELEASED= DECREASE IN SURFACE ENERGY

(1) Formation of double bubble : If r_1 and r_2 are the radii of smaller and larger bubble and P_0 is the atmospheric pressure, then the pressure inside them will be $P_1 = P_0 + \frac{4T}{r_1}$ and $P_2 = P_0 + \frac{4T}{r_2}$.



Now as $r_1 < r_2 \therefore P_1 > P_2$

So for interface $\Delta P = P_1 - P_2 = 4T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$ .....(i)

As excess pressure acts from concave to convex side, the interface will be concave towards the smaller bubble and convex towards larger bubble and if r is the radius of interface.

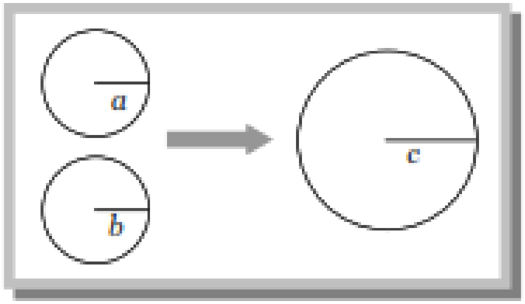
$$\Delta P = \frac{4T}{r} \qquad .....(ii)$$

From (i) and (ii) $\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$

$\therefore$ Radius of the interface $r = \frac{r_1 r_2}{r_2 - r_1}$

(2) Formation of a single bubble

(i) Under isothermal condition two soap bubble of radii '*a*' and '*b*' coalesce to form a single bubble of radius '*c*'.



If the external pressure is P_0 then pressure inside bubbles

$$P_a = \left(P_0 + \frac{4T}{a}\right),\ P_b = \left(P_0 + \frac{4T}{b}\right) \text{ and } P_c = \left(P_0 + \frac{4T}{c}\right)$$

and volume of the bubbles

$$V_a = \frac{4}{3}\pi a^3,\ V_b = \frac{4}{3}\pi b^3,\ V_c = \frac{4}{3}\pi c^3$$

Now as mass is conserved $\mu_a + \mu_b = \mu_c \Rightarrow \dfrac{P_a V_a}{RT_a} + \dfrac{P_b V_b}{RT_b} = \dfrac{P_c V_c}{RT_c}$ $\left[\text{As } PV = \mu RT,\ i.e., \mu = \dfrac{PV}{RT}\right]$

$\Rightarrow \quad P_a V_a + P_b V_b = P_c V_c$ .....(i) [As temperature is constant, *i.e.*, $T_a = T_b = T_c$]

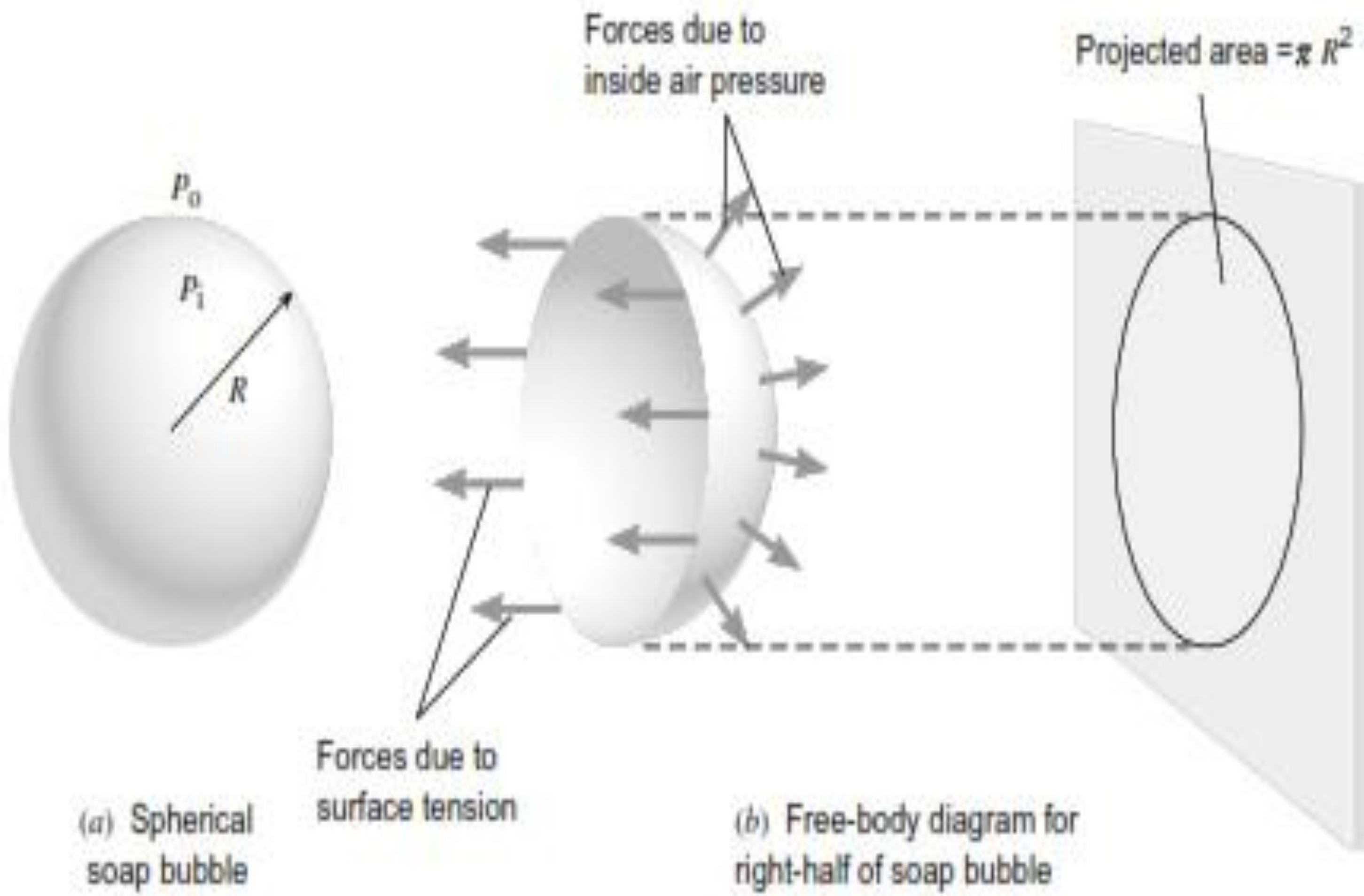
Substituting the value of pressure and volume

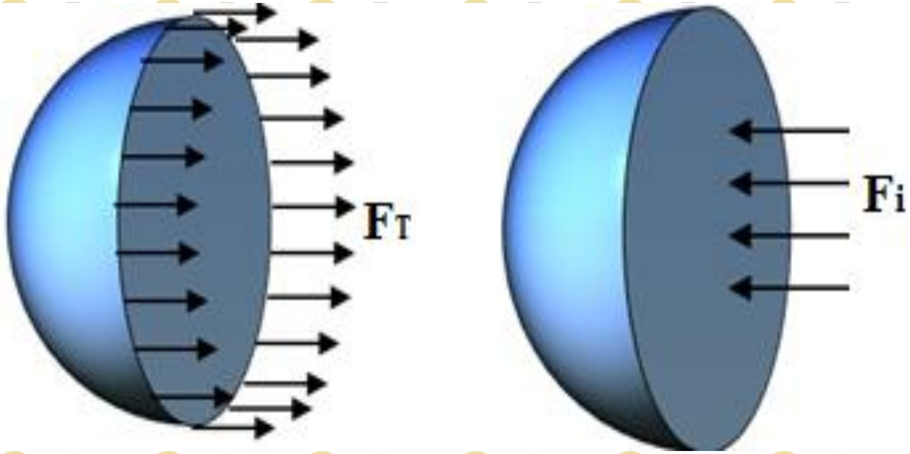
$$\Rightarrow \quad \left[P_0 + \frac{4T}{a}\right]\left[\frac{4}{3}\pi a^3\right] + \left[P_0 + \frac{4T}{b}\right]\left[\frac{4}{3}\pi b^3\right] = \left[P_0 + \frac{4T}{c}\right]\left[\frac{4}{3}\pi c^3\right]$$

$$\Rightarrow \quad 4T(a^2 + b^2 - c^2) = P_0(c^3 - a^3 - b^3)$$

$\therefore$ Surface tension of the liquid $T = \dfrac{P_0(c^3 - a^3 - b^3)}{4(a^2 + b^2 - c^2)}$

Excess pressure in BUBBLES





Consider the equilibrium of a hemispherical portion of a liquid bubble of radius R and surface tension T. For the equilibrium of the liquid bubble.

$F_0 - F_i + F_T = 0 = 0$ where F_0 = force due to the outside pressure.

F_i = force due to the inside pressure

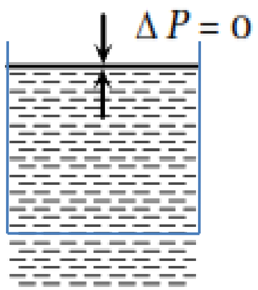
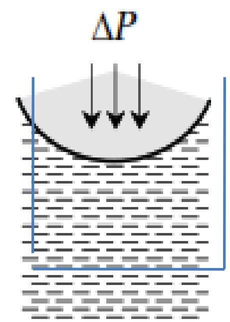
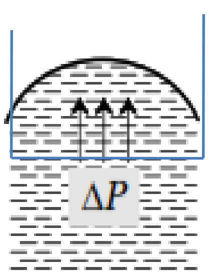
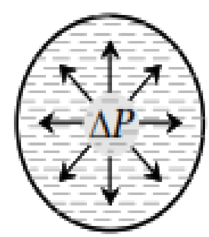
F_T = force due to the surface tension

$P_0 \Pi R^2 - P_i \Pi R^2 + 2(2\Pi RT) = 0$

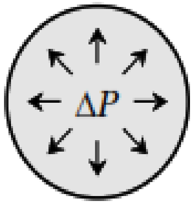
=> $P_i - P_0 = 4T/R$

where R is the radius of the bubble.

Excess pressure in different cases is given in the following table :

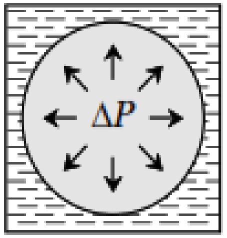
Plane surface	Concave surface
 $\Delta P = 0$	 $\Delta P = \frac{2T}{R}$
Convex surface	**Drop**
 $\Delta P = \frac{2T}{R}$	 $\Delta P = \frac{2T}{R}$

Bubble in air



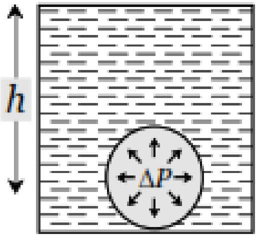
$$\Delta P = \frac{4T}{R}$$

Bubble in liquid



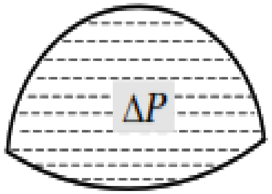
$$\Delta P = \frac{2T}{R}$$

Bubble at depth h below the free surface of liquid of density d



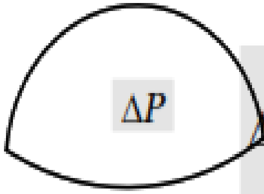
$$\Delta P = \frac{2T}{R} + hdg$$

Liquid surface of unequal radii



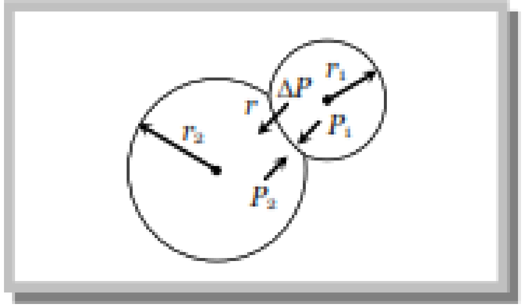
$$\Delta P = T\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

Liquid film of unequal radii



$$\Delta P = 2T\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

(1) Formation of double bubble : If r_1 and r_2 are the radii of smaller and larger bubble and P_0 is the atmospheric pressure, then the pressure inside them will be $P_1 = P_0 + \frac{4T}{r_1}$ and $P_2 = P_0 + \frac{4T}{r_2}$.



Now as $r_1 < r_2 \therefore P_1 > P_2$

So for interface $\Delta P = P_1 - P_2 = 4T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$ .....(i)

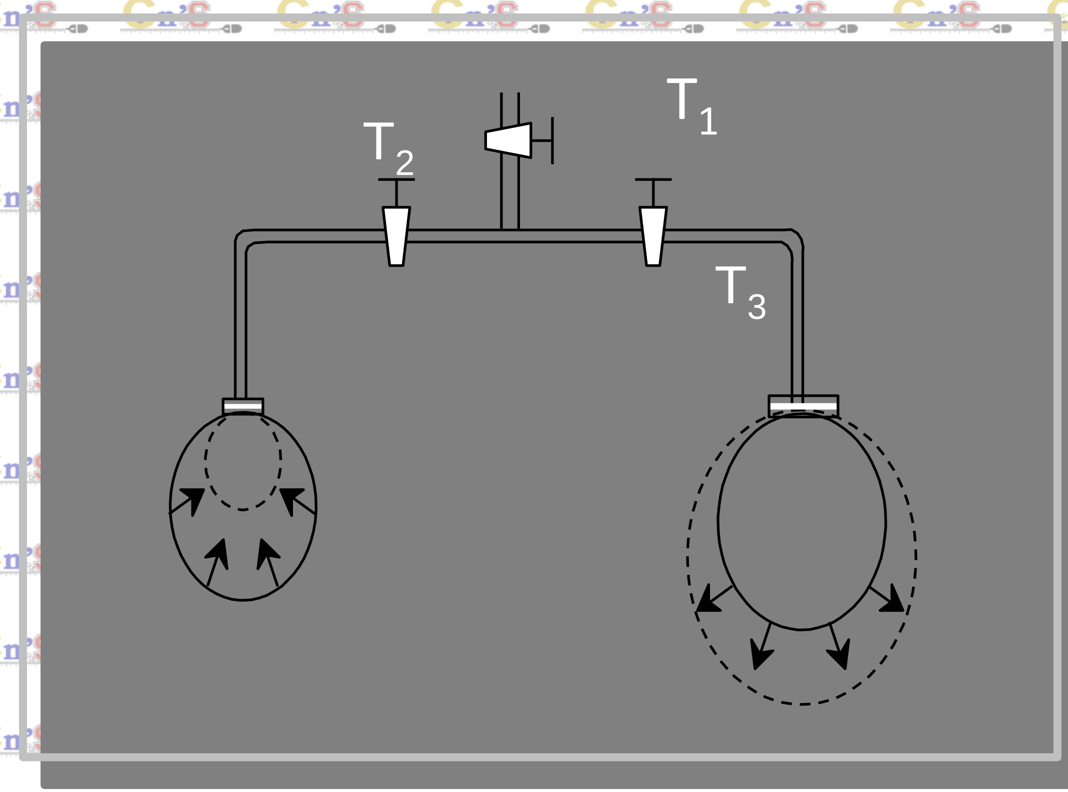
As excess pressure acts from concave to convex side, the interface will be concave towards the smaller bubble and convex towards larger bubble and if r is the radius of interface.

$$\Delta P = \frac{4T}{r} \quad .....(ii)$$

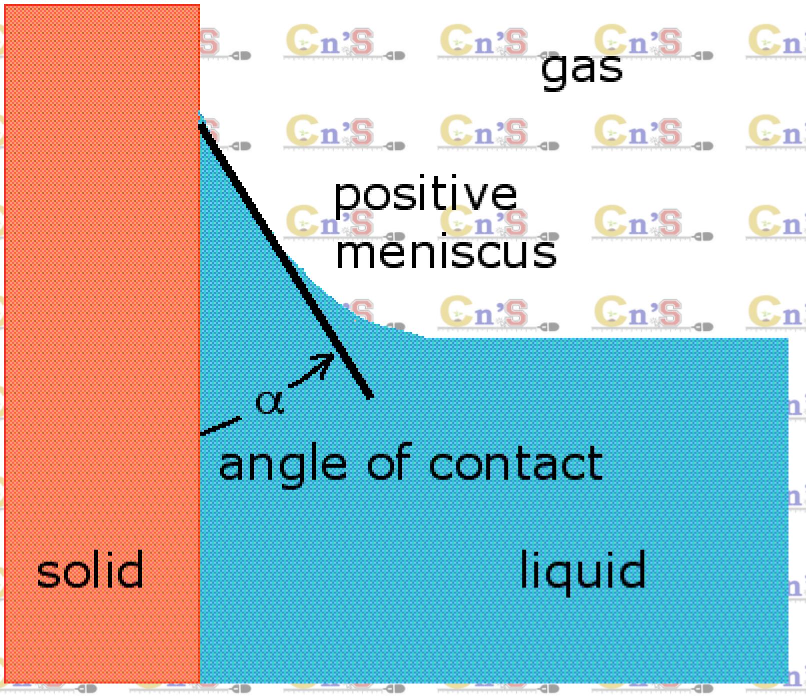
From (i) and (ii) $\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$

$\therefore$ Radius of the interface $r = \frac{r_1 r_2}{r_2 - r_1}$

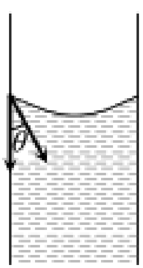
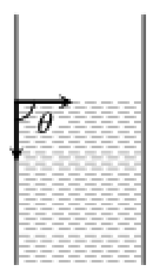
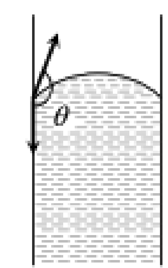
Excess pressure is inversely proportional to the radius of bubble (or drop), i.e., pressure inside a smaller bubble (or drop) is higher than inside a larger bubble (or drop). This is why when two bubbles of different sizes are put in communication with each other, the air will rush from smaller to larger bubble, so that the smaller will shrink while the larger will expand till the smaller bubble reduces to droplet, with the same radius, cos excess P inside the tube is 4T/r



Angle of contact



Angle of contact between a liquid and a solid is defined as the angle enclosed between the tangents to the liquid surface and the solid surface inside the liquid, both the tangents being drawn at the point of contact of the liquid with the solid.

$\theta < 90^o$  concave meniscus. Liquid wets the solid surface	$\theta = 90^o$  plane meniscus. Liquid does not wet the solid surface.	$\theta > 90^o$  convex meniscus. Liquid does not wet the solid surface.

Angle of Contact

When a liquid is in contact with a solid, the angle between the surface of the liquid and the tangent drawn to the surface of the liquid, at the point of contact, on the side of the liquid is called “angle of contact” of the given solid liquid pair.

Features of Angle of Contact

1. For a given solid liquid pair the angle of contact is constant.
2. If the liquid partially wets the solid, the angle of contact is acute.
3. If the liquid doesn't wet the solid at all, the angle of contact is obtuse.

4. If the liquid wets the solid completely, the angle of contact is approximately zero.

5. Any small contamination of the liquid can change the angle of contact largely.

6. The angle of contact depends on the magnitudes of adhesive forces between solid and liquid molecules and cohesive forces between liquid molecules.

Angle of contact

(i) Its value lies between 0° and 180°

$\theta = 0^o$ for pure water and glass, $\theta = 8^o$ for tap water and glass, $\theta = 90^o$ for water and silver

$\theta = 138^o$ for mercury and glass, $\theta = 160^o$ for water and chromium

(ii) It is particular for a given pair of liquid and solid. Thus the angle of contact changes with the pair of solid and liquid.

(iii) It does not depends upon the inclination of the solid in the liquid.

(iv) On increasing the temperature, angle of contact decreases.

(v) Soluble impurities increases the angle of contact.

(vi) Partially soluble impurities decreases the angle of contact.

Capillarity.

If a tube of very narrow bore (called capillary) is dipped in a liquid, it is found that the liquid in the capillary either ascends or descends relative to the surrounding liquid. This phenomenon is called capillarity. The root cause of capillarity is the difference in pressures on two sides of (concave and convex) curved surface of liquid.

Examples of capillarity :

(i) Ink rises in the fine pores of blotting paper leaving the paper dry.

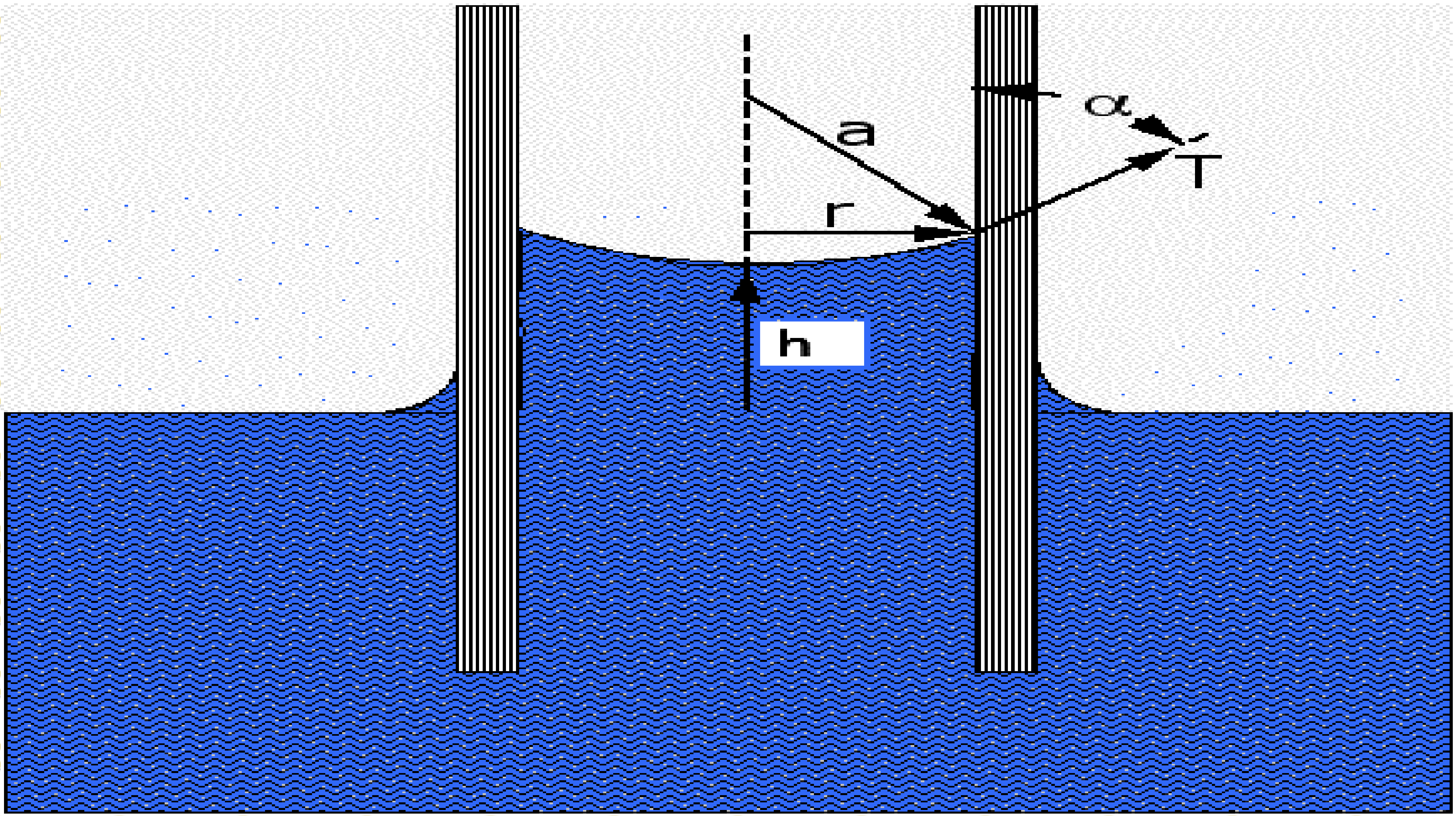
(ii) A towel soaks water.

(iii) Oil rises in the long narrow spaces between the threads of a wick.

(iv) Wood swells in rainy season due to rise of moisture from air in the pores.

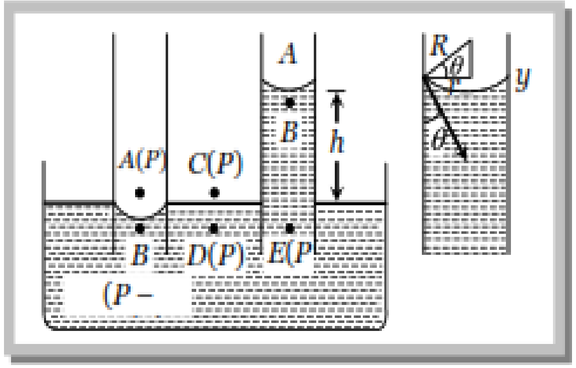
(v) Ploughing of fields is essential for preserving moisture in the soil.

(vi) Sand is drier soil than clay. This is because holes between the sand particles are not so fine as compared to that of clay, to draw up water by capillary action.


a
α
T
r
h

Ascent Formula:

When one end of capillary tube of radius r is immersed into a liquid of density d which wets the sides of the capillary tube (water and capillary tube of glass), the shape of the liquid meniscus in the tube becomes concave upwards.



R = radius of curvature of liquid meniscus.

T = surface tension of liquid

P = atmospheric pressure

Pressure at point $A = P$, Pressure at point $B = P - \frac{2T}{R}$

Pressure at points C and D just above and below the plane surface of liquid in the vessel is also P (atmospheric pressure). The points B and D are in the same horizontal plane in the liquid but the pressure at these points is different.

In order to maintain the equilibrium the liquid level rises in the capillary tube upto height h.

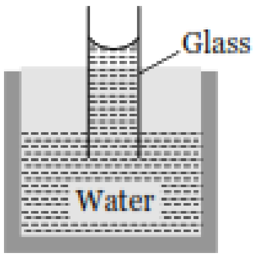
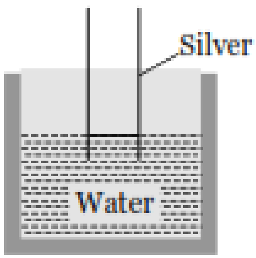
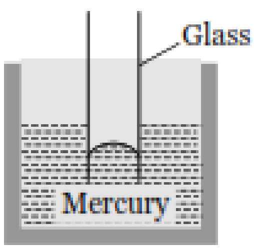
Pressure due to liquid column = pressure difference due to surface tension

$$\Rightarrow \qquad hdg = \frac{2T}{R}$$

$$\therefore \qquad h = \frac{2T}{Rdg} = \frac{2T\cos\theta}{rdg} \qquad \left[\text{As } R = \frac{r}{\cos\theta}\right]$$

(i) The capillary rise depends on the nature of liquid and solid both *i.e.* on T, d, θ and R.

(ii) Capillary action for various liquid-solid pair.

	Meniscus	**Angle of contact**	**Level**
	Concave	$\theta < 90^o$	Rises
	Plane	$\theta = 90^o$	No rise no fall
	Convex	$\theta > 90^o$	Fall

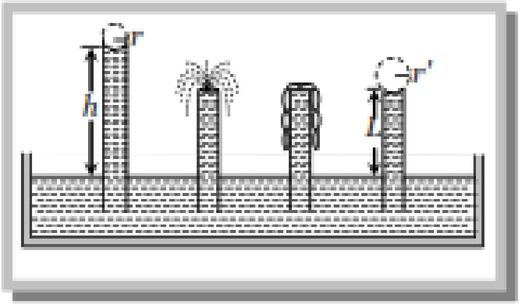
(iii) For a given liquid and solid at a given place

$$h \propto \frac{1}{r} \qquad \text{[As } T, \theta, d \text{ and } g \text{ are constant]}$$

i.e. lesser the radius of capillary greater will be the rise and vice-versa. This is called Jurin's law.

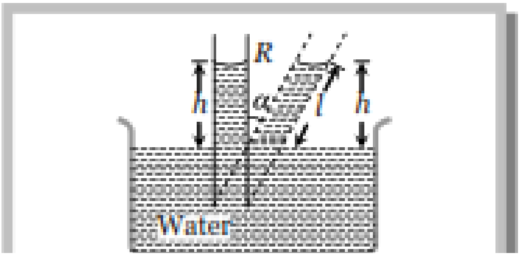
(iv) If the weight of the liquid contained in the meniscus is taken into consideration then more accurate ascent formula is given by

$$h = \frac{2T\cos\theta}{rdg} - \frac{r}{3}$$



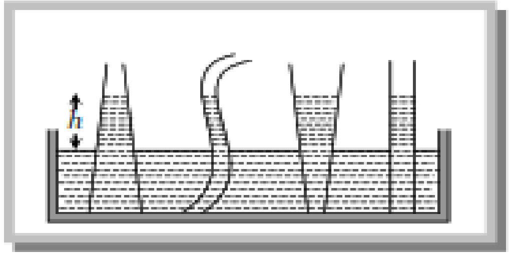
(v) In case of capillary of insufficient length, *i.e.*, $L < h$, the liquid will neither overflow from the upper end like a fountain nor will it tickle along the vertical sides of the tube. The liquid after reaching the upper end will increase the radius of its meniscus without changing nature such that :

$$hr = Lr' \quad \because\ L < h \qquad \therefore r' > r$$



(vi) If a capillary tube is dipped into a liquid and tilted at an angle α from vertical, then the vertical height of liquid column remains same whereas the length of liquid column (l) in the capillary tube increases.

$$h = l\cos\alpha \ \text{ or } \ l = \frac{h}{\cos\alpha}$$



(vii) It is important to note that in equilibrium the height h is independent of the shape of capillary if the radius of meniscus remains the same. That is why the vertical height h of a liquid column in capillaries of different shapes and sizes will be same if the radius of meniscus remains the same.

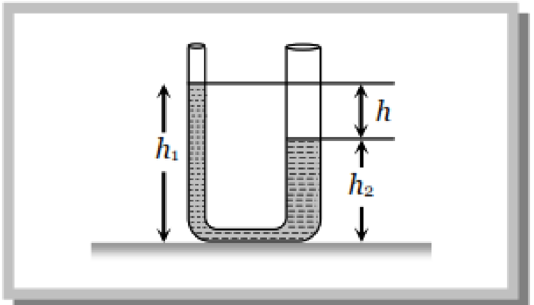
Q: The radii of the two columns is U-tube are r_1 and r_2. When a liquid of density ρ (angle of contact is 0º) is filled in it, the level difference of liquid in two arms is h. Find out the surface tension of the liquid.

Solution: We know that, $h = 2T/r\rho g$

So, $h_1 = 2T/r_1\rho g$ & $h_2 = 2T/r_2\rho g$

$h_1 - h_2 = h = 2T/\rho g(1/r_1 - 1/r_2)$

Thus, $T = h\rho g r_1 r_2 / 2(r_2 - r_1)$



(3) The difference of levels of liquid column in two limbs of u-tube of unequal radii r_1 and r_2 is

$$h = h_1 - h_2 = \frac{2T\cos\theta}{dg}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

(4) A large force (F) is required to draw apart normally two glass plate enclosing a thin water film because the thin water film formed between the two glass plates will have concave surface all around. Since on the concave side of a liquid surface, pressure is more, work will have to be done in drawing the plates apart.

$F = \dfrac{2AT}{t}$ where T= surface tension of water film, t= thickness of film, A = area of film.

Q: The radii of the two columns is U-tube are r_1 and r_2. When a liquid of density ρ (angle of contact is 0º) is filled in it, the level difference of liquid in two arms is h. Find out the surface tension of the liquid.

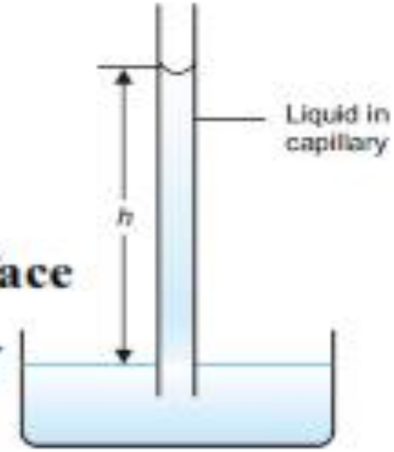
Solution: We know that, $h = 2T/r\rho g$

So, $h_1 = 2T/r_1\rho g$ & $h_2 = 2T/r_2\rho g$

$h_1 - h_2 = h = 2T/\rho g(1/r_1 - 1/r_2)$

Thus, $T = h\rho g r_1 r_2 / 2(r_2 - r_1)$

1-Capillary-rise Method :



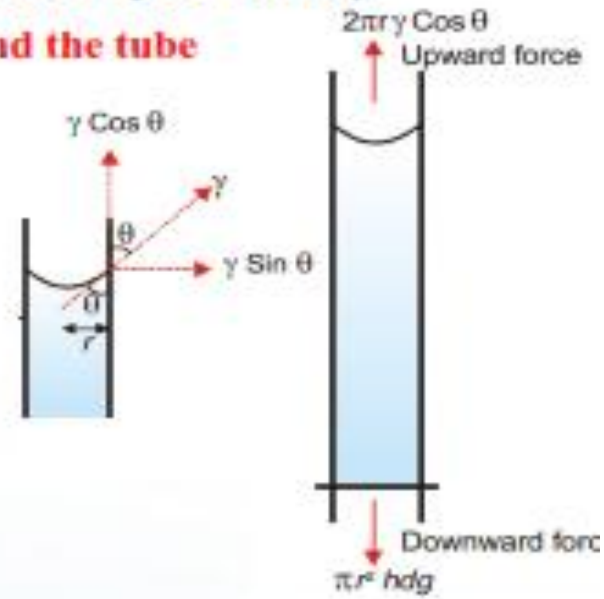
In this method a capillary tube with radius r is inserted vertically into a liquid, the liquid rises to height h the surface tension (γ) is acting along the inner circumference exactly supports the weight of the liquid column.

- **From the definition of the surface tension discussed previously**

θ:is the angle between the tangent line to the meniscus surface and the tube

γ:is the surface tension of the liquid

r:is the radius of the capillary tube



then the total force (upward force) = $2\pi r\gamma\cos\theta$

θ for most liquids equals zero and $\cos\theta = 1$

the total downward force (Mass x Gravity) = $\pi r^2 hdg$

where d : is the density of the liquid

g : is equal to (980 cm/s)

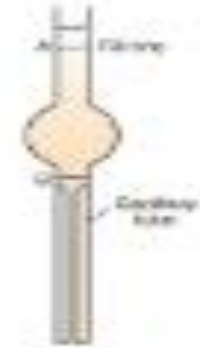
upward force = downward force

$$2\pi r\gamma = \pi r^2 hdg$$

$$\gamma = \frac{hrdg}{2}$$

2- Drop Formation Method:

In this method a drop of water is allowed to form at the lower end of a capillary tube as shown in the opposite figures.



. the drop of water is supported by two forces, the first one is the *Surface tension* force, the second is its *weight which pulls it downward.* When the two forces are balanced the drop breaks thus at the point of breaking.

$$mg = 2\pi r\gamma$$

Where **m : the mass of the drop**
r : the radius of the tube
γ : the tension surface

Experimental Steps

The apparatus is filled with the liquid with the unknown surface tension, then a 20 drops of water is allowed to fall in a weighing bottle and weighed. Thus the of one drop is found. The apparatus is cleaned and dried and then filled with another liquid, say*(Water)* and weight one drop of this liquid with the same method .

Thus from the previous equation :

$$m_1 g = 2\pi r\gamma_1$$

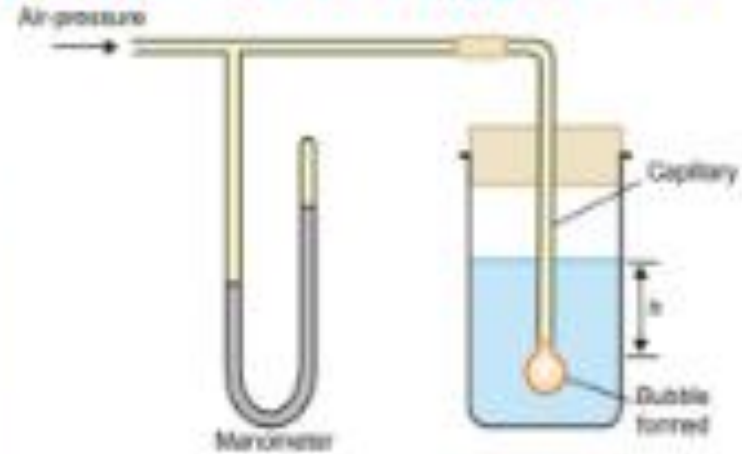
$$m_2 g = 2\pi r\gamma_2$$

$$\frac{\gamma_1}{\gamma_2} = \frac{m_1}{m_2}$$

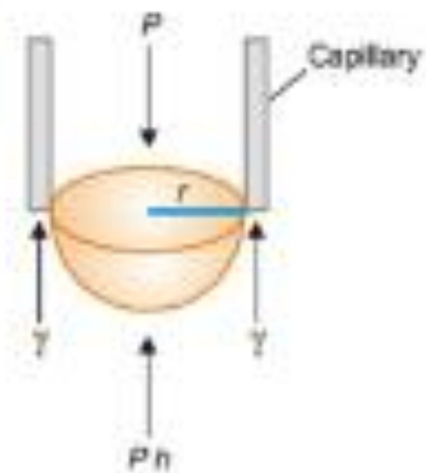
Knowing the surface tension of the reference liquid from tables, that of the liquid under study can be found.

2- Maximum bubble pressure method

In this method air-pressure is applied slowly through a capillary tube dipping in the experimental liquid



A bubble is formed at the end of the capillary. Slowly the bubble grows and becomes hemispherical. Then it breaks away when the pressure recorded by the manometer is noted. This is the maximum pressure required tomake a bubble at the end of the capillary. At the moment of breaking, the forces due to maximum pressure *P* equals that of the opposing hydrostatic pressure P_h and the surface tension γ at the circumference of the capillary. Thus



$$P\pi r^2 = P_h \pi r^2 + 2\pi r\gamma$$

$$P = P_h + \frac{2\gamma}{r}$$

$$P = h d g + \frac{2\gamma}{r}$$

where r= radius of capillary; d= density of the liquid; h = depth of liquid. Knowing the value of P, h, d and r, γ can be found.