Conduction

OBJECTIVES

- Introduce the three ways of transferring of heat – conduction, convection and radiation.
- Explain conduction and its mechanism. Explain the mechanism of convection.
- Make a brief account of radiation of heat in the infra-red band of the electromagnetic spectrum.
- Discuss the difference of radiation when compared with conduction and convection.
- Explain the steady state of flow of heat.
- Introduce the graphs of temperature distribution along a rod when it is conducting heat in the steady state. Consider where the rod is lagged and where the rod is not lagged.

- Use the above graphs to focus on the idea of temperature gradient.
- How logging leads to the axial flow of heat along a rod.
- Discuss the factors on which the rate of conduction of heat along a rod depends. -The cross sectional area and the temperature gradient
- Introduce and discuss the equation $\frac{dQ}{dt} = kA\frac{dT}{dx}$
- Define thermal conductivity and obtain its units and dimensions.
- Determination of thermal conductivity of a metal using Searle’s method.

- **Heat** is defined as *the **energy** that is **transferred from** a body at a **higher temperature** to one at **a lower temperature** , by conduction, convection or radiation.*

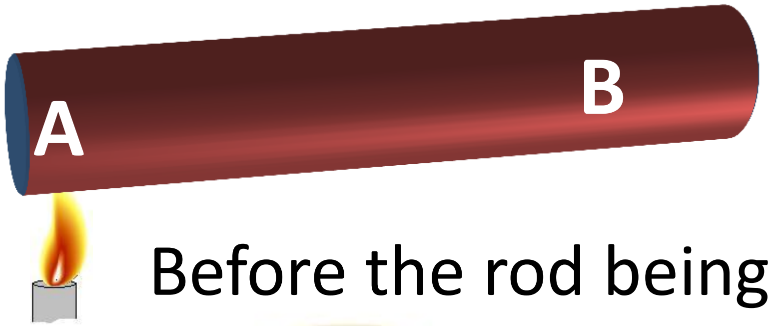
- Heat always transferred from a hot region (higher temperature) to a cool region (lower temperature) until thermal equilibrium is achieved.

    - Heat is transferred by three mechanisms,

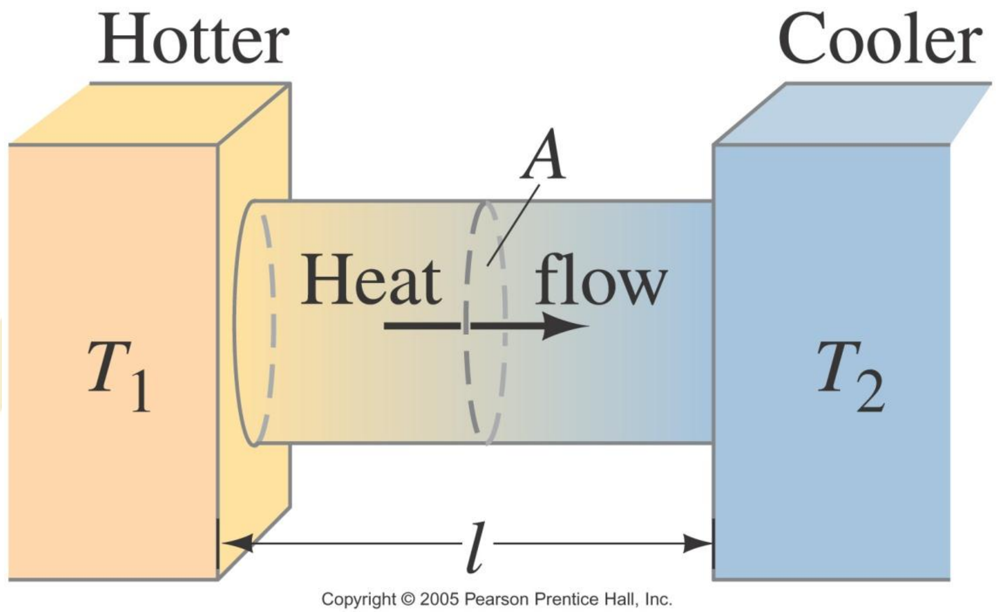
        1) Conduction
        2) Convection
        3) Radiation

- Thermal Conduction is defined as **the process whereby heat is transferred through a substance from a region of high temperature to a region of lower temperature.**

The mechanism of heat conduction through solid material



- Suppose a rod is heated at one end (A).
- Before the rod being heated all the molecules vibrate about their equilibrium position.
- As the rod is heated the molecules at the hot end (A) vibrate with increasing amplitude, thus the kinetic energy increases.
- While vibrating the hot molecules collide with the neighbouring colder molecules result in transfer of kinetic energy to the colder molecules.
- This transfer of energy will continue until the cold end (B) of the rod become hot. This way energy of thermal motion is passed along from one molecule to the next keeping their original position fixed.

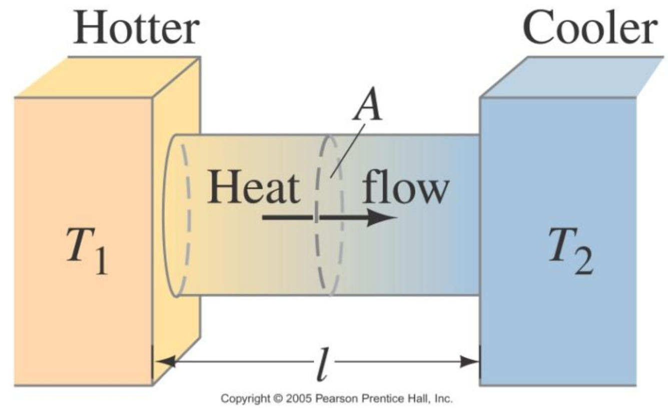


$$\boxed{T_1 > T_2}$$

- Consider a uniform cylinder conductor of length l with temperature T_1 at one end and T_2 at the other end as shown in figure above.
- The heat flows to the right because T_1 is greater than T_2.

- The **rate of heat flow** , $\dfrac{dQ}{dt}$

through the conductor is given by:



$$\boxed{\frac{dQ}{dt} = kA\frac{dT}{dx}}$$

$\dfrac{dQ}{dt}$: rate of heat flow

A : cross sectional area

k : thermal conductivity

$\dfrac{dT}{dx}$: temperature gradient

: the ratio of the temperature difference between two points to the distance between these points.

: $\dfrac{T_1 - T_2}{l}$

$$\boxed{\frac{dQ}{dt} = kA\frac{dT}{dx}}$$

- The rate of of heat flow through an object depends on :
  - **Thermal conductivity.**
  - **Cross-sectional area** through which the heat flow.
  - **Thickness** of the material.
  - **Temperature difference** between the two sides of the material.

- The rate of heat flow is a **scalar quantity** and its unit is J s^{-1} or Watt (W).

$$\boxed{\frac{Q}{t} = kA\frac{\Delta T}{dx}}$$

k : thermal conductivity

$(\mathrm{Wm}^{-1\,o}\mathrm{C}^{-1})$

$$\boxed{k = \frac{\frac{dQ}{dt}}{A\frac{dT}{dx}}}$$

Thermal conductivity , k is defined as ***the rate of heat flows perpendicularly through unit cross sectional area of a solid , per unit temperature gradient along the direction of heat flow.***

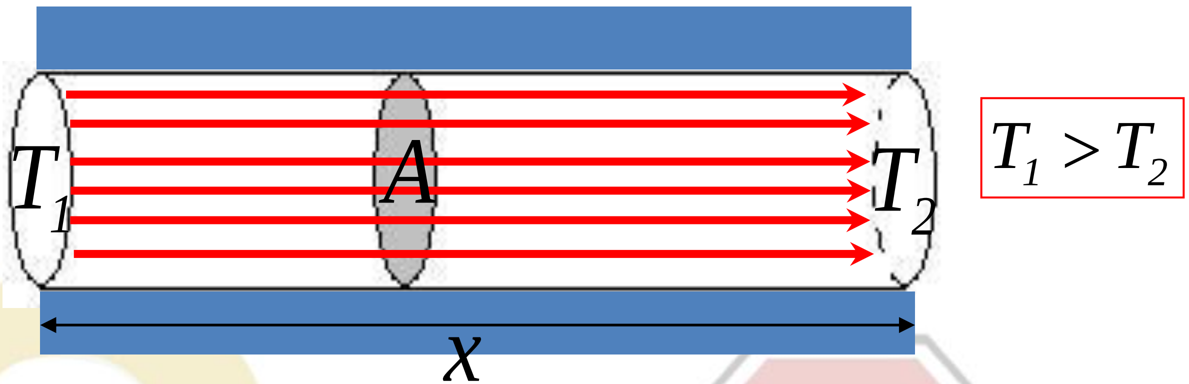
Thermal conductivity is a property of conducting material. (the ability of the material to conduct heat) where good conductors will have higher values of k compared to poor conductors.

TABLE 14–4
Thermal Conductivities

Substance	Thermal Conductivity, k: $\frac{\text{kcal}}{(\text{s}\cdot\text{m}\cdot\text{C}^\circ)}$	Thermal Conductivity, k: $\frac{\text{J}}{(\text{s}\cdot\text{m}\cdot\text{C}^\circ)}$
Silver	10×10^{-2}	420
Copper	9.2×10^{-2}	380
Aluminum	5.0×10^{-2}	200
Steel	1.1×10^{-2}	40
Ice	5×10^{-4}	2
Glass	2.0×10^{-4}	0.84
Brick	2.0×10^{-4}	0.84
Concrete	2.0×10^{-4}	0.84
Water	1.4×10^{-4}	0.56
Human tissue	0.5×10^{-4}	0.2
Wood	0.3×10^{-4}	0.1
Fiberglass	0.12×10^{-4}	0.048
Cork	0.1×10^{-4}	0.042
Wool	0.1×10^{-4}	0.040
Goose down	0.06×10^{-4}	0.025
Polyurethane	0.06×10^{-4}	0.024
Air	0.055×10^{-4}	0.023

Materials with large k are called **conductors**; those with small k are called **insulators**.

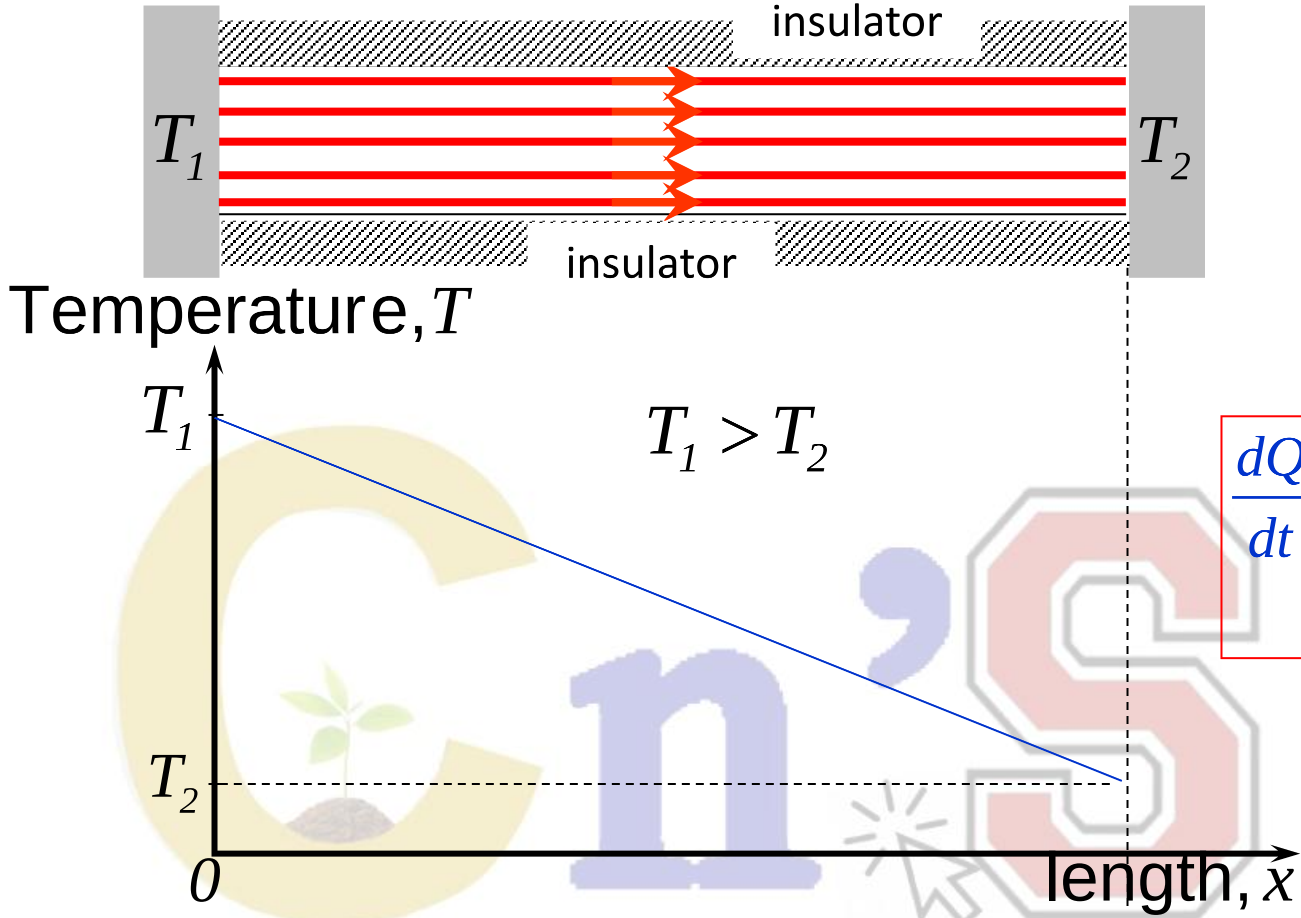
Heat conduction through insulated rod



- Consider heat conduction through an insulated rod which has cross sectional area A and length x as shown above.

- If the rod is completely covered with a good insulator, no heat loss from the sides of the rod.

- By assuming no heat is lost to the surroundings, therefore heat can only flow through the cross sectional area from higher temperature region, T_1 to lower temperature region, T_2.

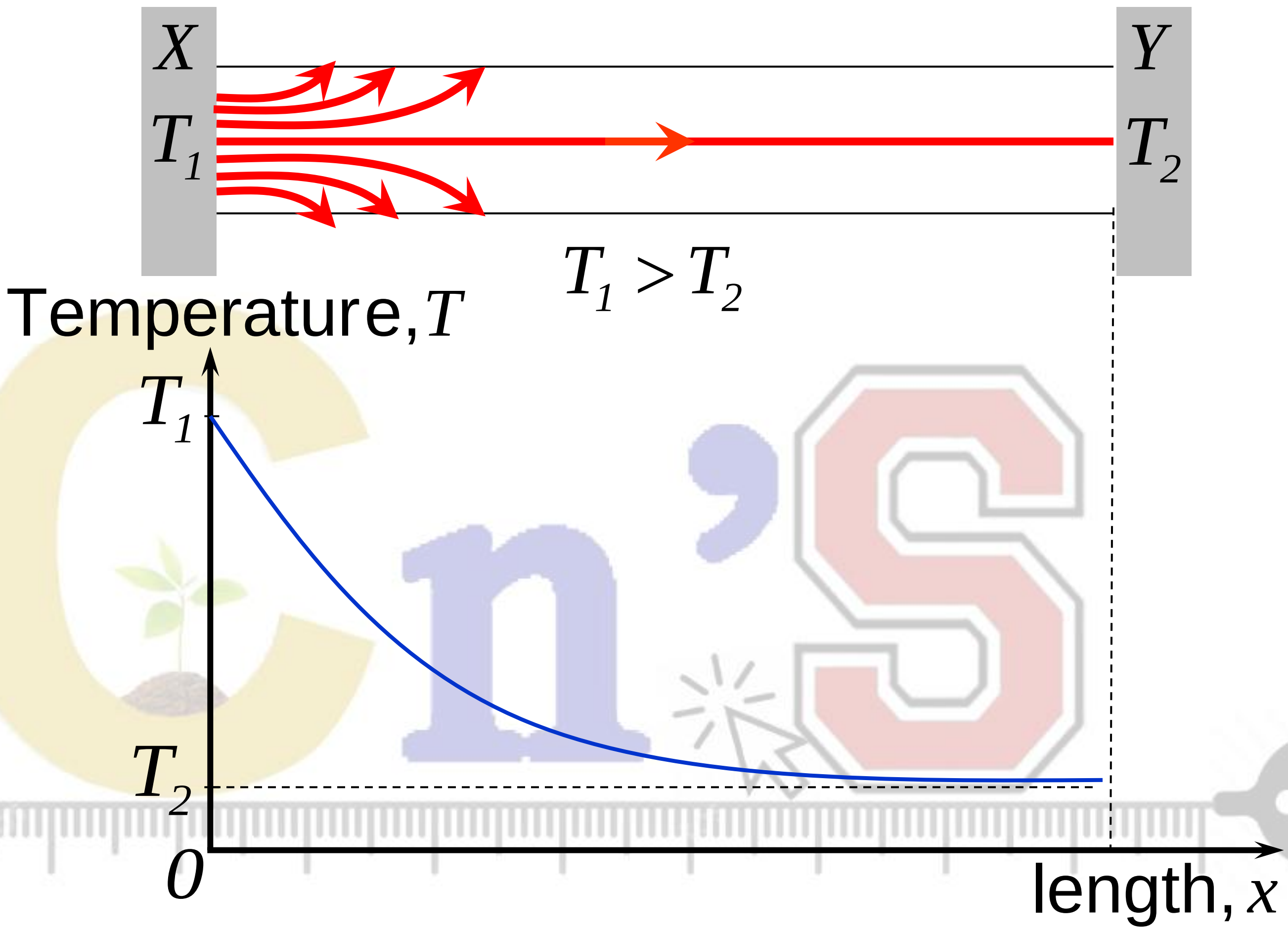


$$\frac{dQ}{dt} = \text{constant}$$

along the rod

- The red lines (arrows) represent the direction of heat flow.
- When the rod is in steady state (the temperature falls at a constant rate) thus the **rate of heat flows** is **constant** along the rod. This causes the **temperature gradient** will be **constant** along the rod.

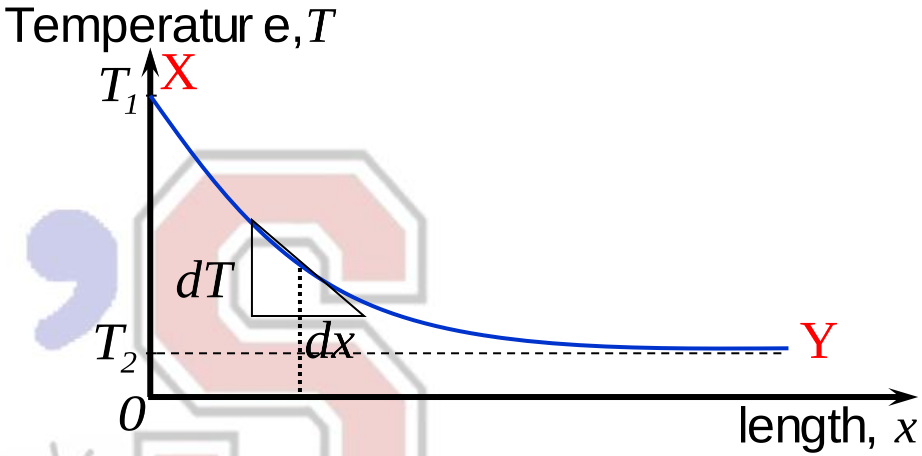
Heat conduction through non-insulated rod



- The metal is not covered with an insulator, thus heat is lost to the surroundings from the sides of the rod.
- The lines of heat flow are divergent and the temperature fall faster near the hotter end than that near the colder end.

- Less heat is transferred to Y.

- This causes the **temperature gradient** gradually decreases along the rod and result a **curve graph** where the temperature gradient at X higher than that at Y .



- From $\boxed{\dfrac{dQ}{dt} = kA\dfrac{dT}{dx}}$

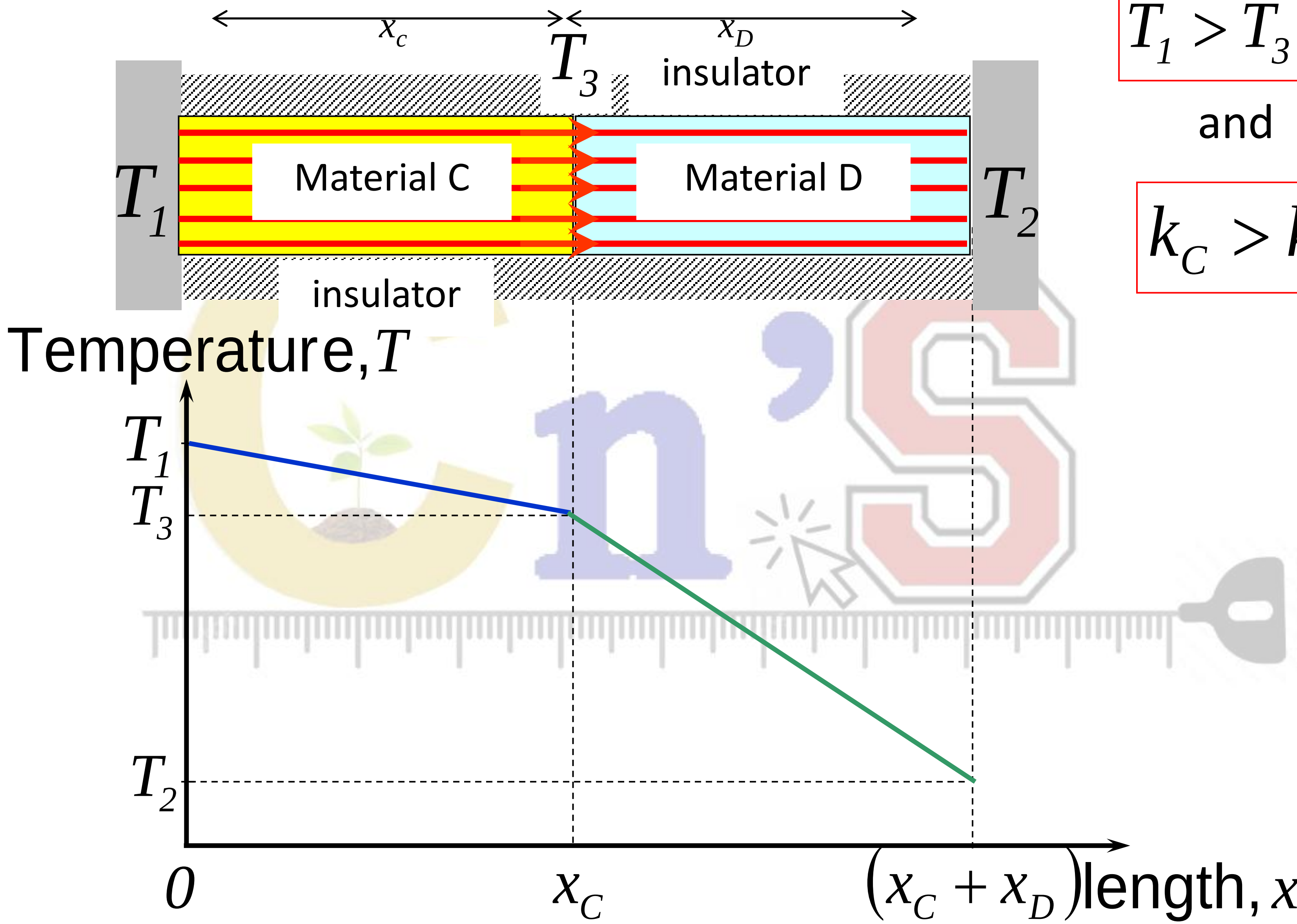
- And from the graph

$$\boxed{\frac{dT}{dx}\text{ at X} > \frac{dT}{dx}\text{ at Y}}$$

- Thus $\boxed{\dfrac{dQ}{dt}\text{ at X} > \dfrac{dQ}{dt}\text{ at Y}}$ where A and k are the same along the rod.

Temperature gradient , $\dfrac{dT}{dx}$ at any point on the rod is given by the **slope of the tangent at that point.**

Combination 2 metals in series



$$T_1 > T_3 > T_2$$

and

$$k_C > k_D$$

- When steady state is achieved , the **rate of heat flow** through both materials is same (**constant**).

$$\boxed{\frac{dQ}{dt_C} = \frac{dQ}{dt_D}} \qquad \boxed{A_C = A_D}$$

- From the equation of thermal conductivity, we get

$$\boxed{k \propto \frac{1}{\left(\frac{dT}{dx}\right)}} \qquad \boxed{k = \frac{\frac{dQ}{dt}}{A\frac{dT}{dx}}}$$

But $k_C > k_D$

$$\boxed{\left(\frac{dT}{dx}\right)_C < \left(\frac{dT}{dx}\right)_D}$$

Example 13.1

A metal cube have a side of 8 cm and thermal conductivity of 250 W m^{-1} K^{-1}. If two opposite surfaces of the cube have the temperature of 90 °C and 10 °C, respectively. Calculate

a) the temperature gradient in the metal cube.

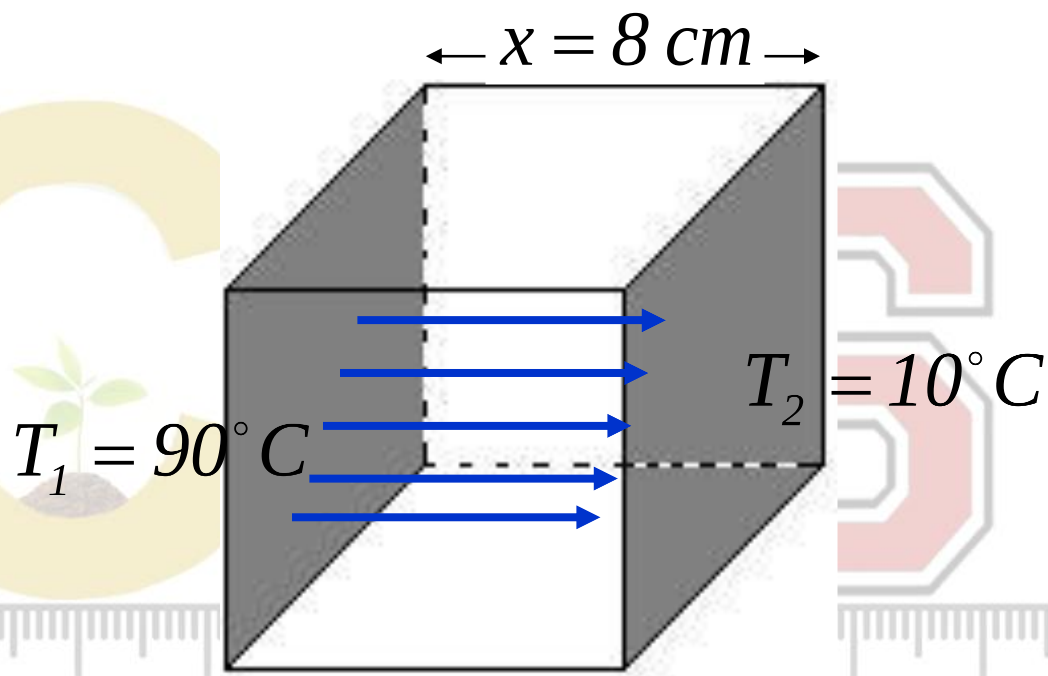
b) the quantity of heat flow through the cube in 10 minutes.

(Assume the heat flow is steady and no energy is lost to the surroundings)

Solution 13.1

$A = l^2 = (8\text{x}10^{-2})^2 = 64\ \text{x}10^{-4}\ \text{m}^2$, $k = 250\ \text{W m}^{-1}\ \text{K}^{-1}$,

$T_1 = 90°\text{C}$, $T_2 = 10°\text{C}$



a) Temperature gradient

b) Given $t = 10 \times 60 = 600$ s

Example 13.2

A 5 mm thick copper plate is sealed to a 10 mm thick aluminium plate and both have the same cross sectional area of 1 m^2.The outside face of the copper plate is at 100 °C, while the outside face of the aluminium plate is at 80 °C.

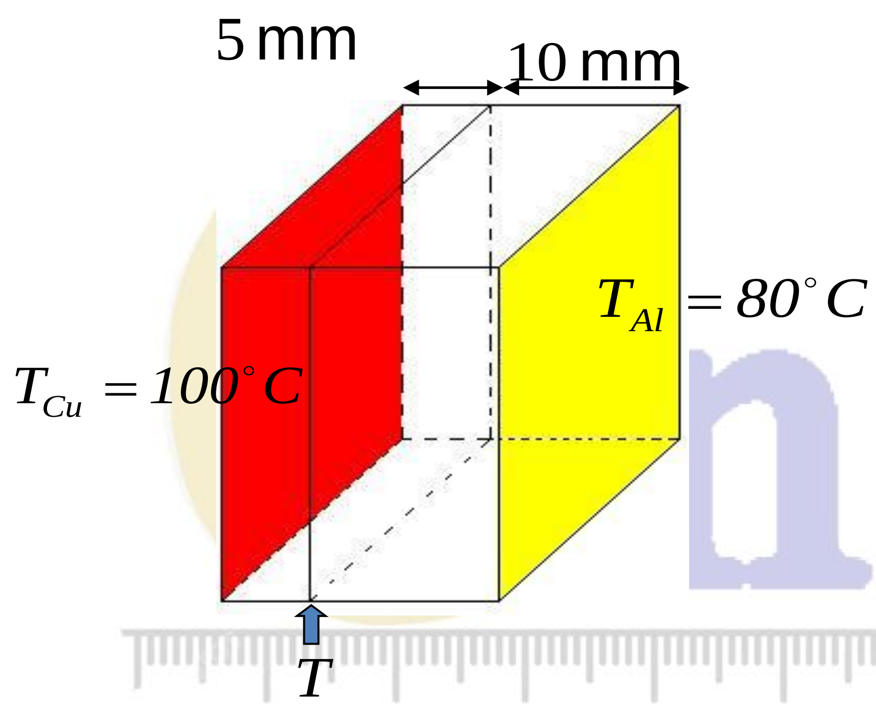
a) Find the temperature at the copper-aluminium interface.

b) Calculate the rate of heat flow through the cross sectional area if heat flow is steady and no energy is lost to the surroundings.

(Use k_{Cu} = 400 W m^{-1}°C^{-1} and k_{Al} = 200 W m^{-1}°C^{-1})

Solution 13.2

x_{Cu}= 5x10^{-3}m, x_{Al}= 10x10^{-3}m, A= 1 m^2, T_{cu} = 100°C, T_{Al} = 80°C

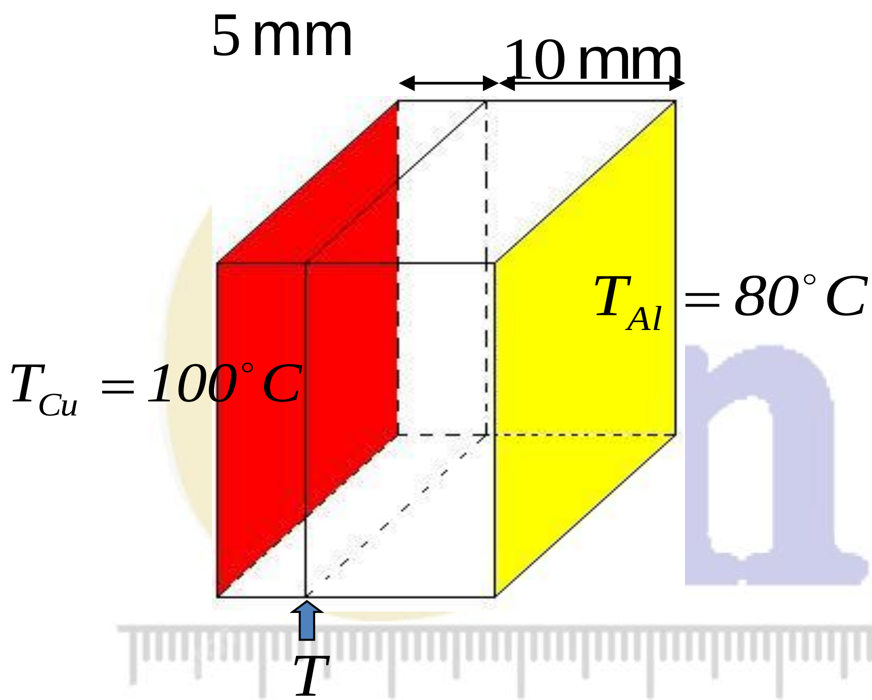


a) The **rate of heat flow** through the copper and aluminium plate is **same**, therefore

$$\left(\frac{dQ}{dt}\right)_{Cu} = \left(\frac{dQ}{dt}\right)_{Al}$$

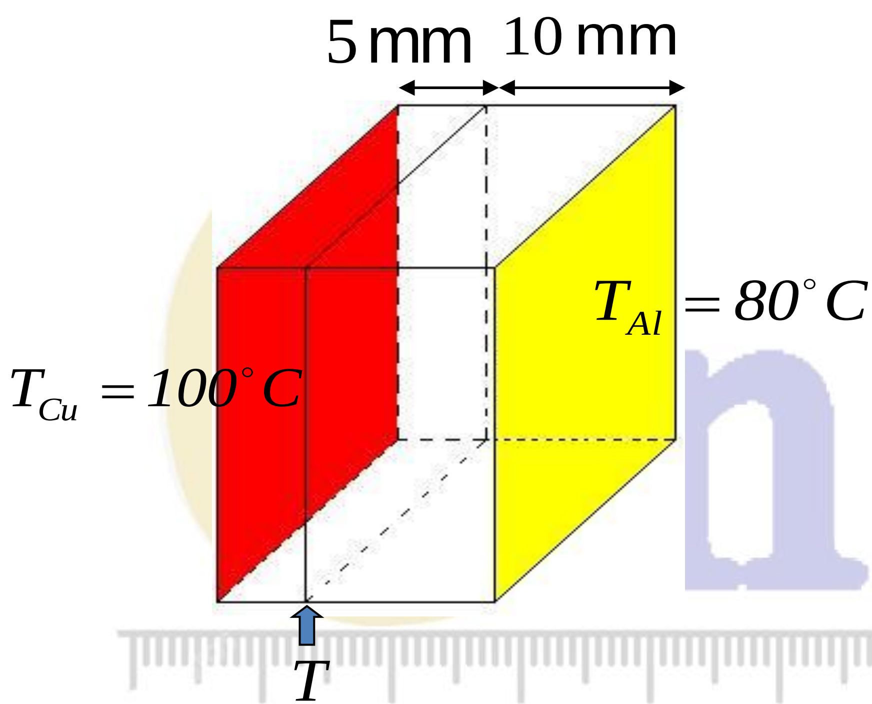
Solution 13.2

x_{Cu}= 5x10^{-3}m, x_{Al}= 10x10^{-3}m, A= 1 m^2, T_{cu} = 100°C, T_{Al} = 80°C



Solution 13.2

x_{Cu}= 5x10^{-3}m, x_{Al}= 10x10^{-3}m, A= 1 m^2, T_{cu} = 100°C, T_{Al} = 80°C



b) By applying the equation for **rate of heat flow** through the copper plate, hence

$$\left(\frac{dQ}{dt}\right)_{Cu} = -k_{Cu} A \left(\frac{T - T_{Cu}}{x_{Cu}}\right)$$

Exercise

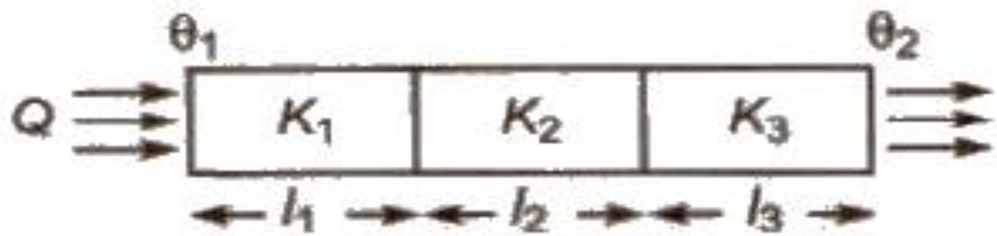
1. A metal plate 5.0 cm thick has a cross sectional area of 300 cm^2. One of its face is maintained at 100°C by placing it in contact with steam and another face is maintained at 30°C by placing it in contact with water flow. Determine the thermal conductivity of the metal plate if the rate of heat flow through the plate is 9 kW. (Assume the heat flow is steady and no energy is lost to the surroundings).**(214 W $m^{-1}K^{-1}$)**

2. A rod 1.300 m long consists of a 0.800 m length of aluminium joined end to end to a 0.500 m length of brass. The free end of the aluminium section is maintained at 150.0°C and the free end of the brass piece is maintained at 20.0°C. No heat is lost through the sides of the rod. At steady state, find the temperature of the point where the two metal are joined.(Use k of aluminium = 205 W m^{-1}°C^{-1} and k of brass = 109 W m^{-1}°C^{-1}) **(90.2°C)**

When Conducting Rods are Connected in Series

The amount of heat flow

$$Q = \frac{A(\theta_1 - \theta_2)\,t}{\frac{l_1}{K_1} + \frac{l_2}{K_2} + \frac{l_3}{K_3}}$$



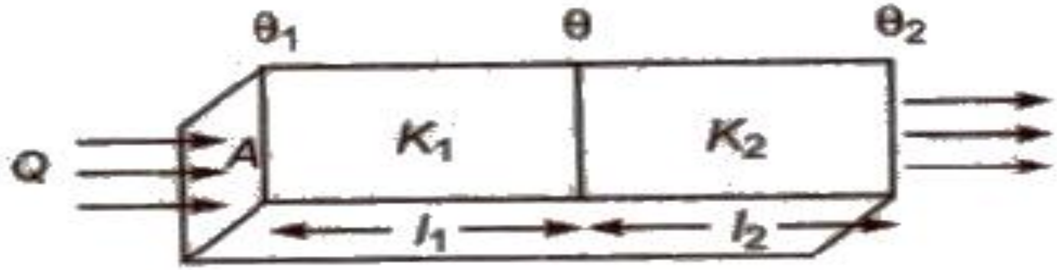
Equivalent thermal conductivity

$$H = \frac{l_1 + l_2 + l_3}{\frac{l_1}{K_1} + \frac{l_2}{K_2} + \frac{l_3}{K_3}}$$

When Two Conducting Rods are Connected in Series

Rate of heat flow

$$H = \frac{Q}{t} = \frac{K_1 A(\theta_1 - \theta)}{l_1} = \frac{K_2 A(\theta - \theta_2)}{l_2}$$



Temperature of contact surface

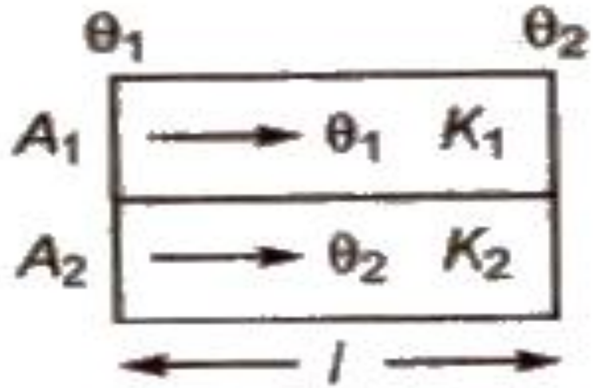
$$\theta = \frac{\frac{K_1\theta_1}{l_1} + \frac{K_2\theta_2}{l_2}}{\frac{K_1}{l_1} + \frac{K_2}{l_2}} = \frac{K_1\theta_1 l_2 + K_2\theta_2 l_1}{K_1 l_2 + K_2 l_1}$$

Equivalent thermal conductivity H = $l_1 + l_2 / l_1 / K_1 + l_2 / k_2$

When Conducting Rods are Connected in Parallel

Rate of heat flow

$$H = \frac{Q}{t} = \left(\frac{K_1 A_1}{l} + \frac{K_2 A_2}{l}\right)(\theta_1 - \theta_2)$$

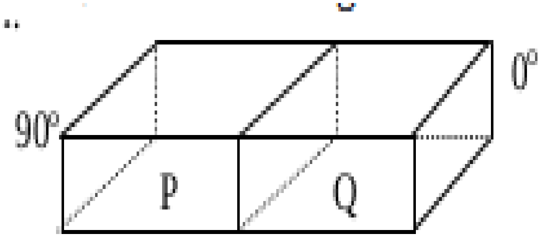


Equivalent thermal conductivity

$K = K_1 A_1 + K_2 A_2 / A_1 + A_2$

1. Two metals have the same size but different type. The thermal conductivity of P = 2 times the thermal conductivity of Q. What is the temperature between the two metals, as shown in the figure below.

$$\frac{Q}{t} = kA\frac{T_1 - T_2}{l}$$



$$\left(\frac{Q}{t}\right)_P = \left(\frac{Q}{t}\right)_Q$$

$$k_P A \frac{(T_1 - T_2)}{l} = k_Q A \frac{(T_1 - T_2)}{l}$$

$$k_P (T_1 - T_2) = k_Q (T_1 - T_2)$$

$$2k(T_1 - T_2) = k(T_1 - T_2)$$

$$2k(90 - T) = k(T - 0)$$

$$2(90 - T) = (T - 0)$$

$$180 - 2T = T - 0$$

$$180 - 2T = T$$

$$180 = T + 2T$$

$$180 = 3T$$

$$T = 180/3$$

$$T = 60$$

Q/t = The rate of the heat conduction,
k = thermal conductivity,
A = the cross-sectional area of the object
, T_1 = high temperature,
T_2 = low temperature,
l = the length of metal.

$k_Q = k$
$k_P = 2k$

2. Metal A and metal B have the same length and the same cross-sectional. If the thermal conductivity of metal A = ¼ times the thermal conductivity of metal B. Both metals heated at its end and the change in temperature of both metals are the same. The ratio of the rate of the heat conduction of metal A to metal B.

Known :

The cross-sectional of A (A) = The cross-sectional B (A)

The length of A (l) = the length of B (l)

The thermal conductivity of metal B (k_B) = k

The thermal conductivity of metal A (k_A) = ¼ k

The change in temperature of metal A (ΔT) = the change in temperature of metal B (ΔT)

Wanted: The ratio of the rate of the heat conduction

$$\frac{Q}{t}=kA\frac{T_2-T_1}{l}=kA\frac{\Delta T}{l}$$

The rate of the heat conduction for metal A :

$$\frac{Q}{t}=kA\frac{\Delta T}{l}$$

The rate of the heat conduction for metal B :

$$\frac{Q}{t}=kA\frac{\Delta T}{l}=0.25kA\frac{\Delta T}{l}$$

The ratio of the rate of the heat conduction :

$$\frac{Q}{t}A:\frac{Q}{t}B$$

$$kA\frac{\Delta T}{l}:0.25kA\frac{\Delta T}{l}$$

$$1:0.25$$

$$1(4):0.25(4)$$

$$4:1$$