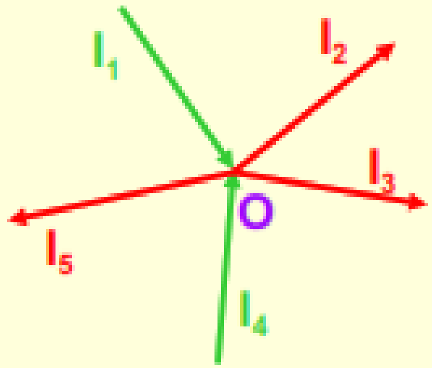
Kirchhoff's rules to electrical networks

P1

KIRCHHOFF'S LAWS:

I Law or Current Law or Junction Rule:

The algebraic sum of electric currents at a junction in any electrical network is always zero.



$$\Sigma I = 0$$

$$I_1 - I_2 - I_3 + I_4 - I_5 = 0$$

Sign Conventions:

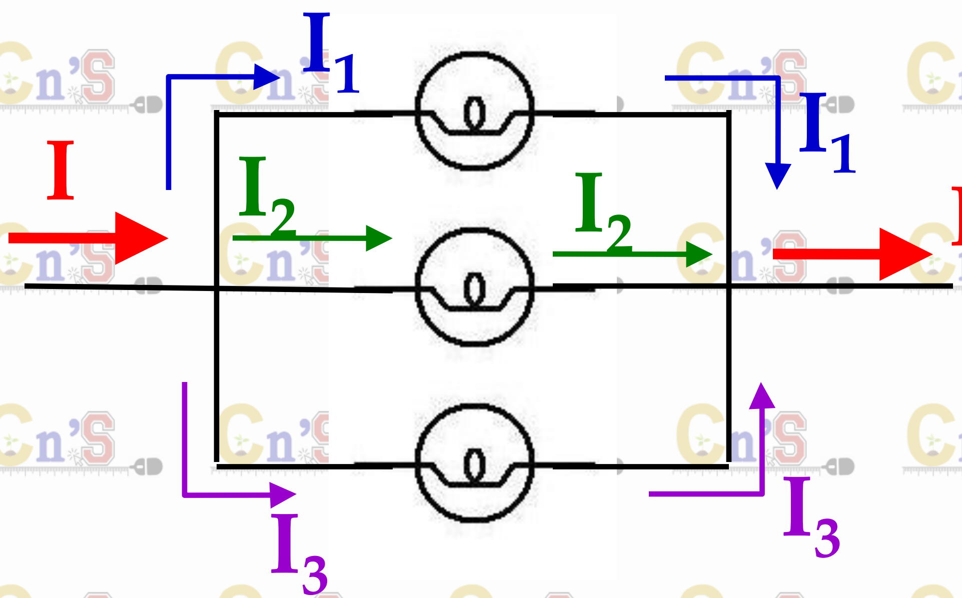
1. The incoming currents towards the junction are taken positive.
2. The outgoing currents away from the junction are taken negative.

Note: The charges cannot accumulate at a junction. The number of charges that arrive at a junction in a given time must leave in the same time in accordance with conservation of charges.

At circuit nodes (junctions), the current divides, and each path gets a fraction of it.
No charge is lost.

Kirchoff's Junction Rule

Current into node= Current out of node



$$I = I_1 + I_2 + I_3$$

$I_1 = V/R_1$

$I_2 = V/R_2$

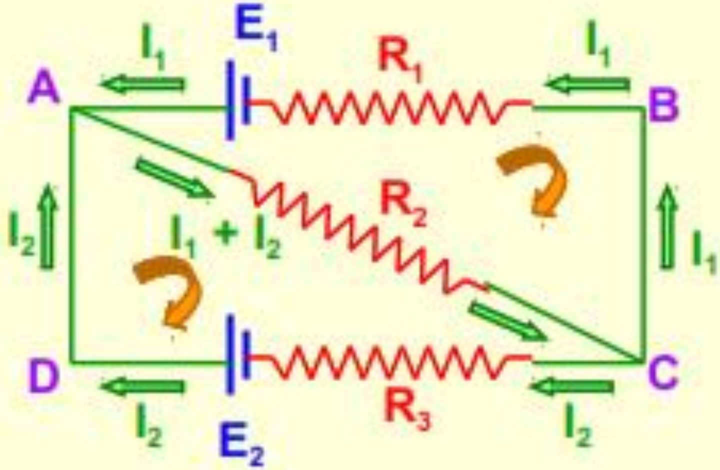
$I_3 = V/R_3$

The lower resistance the path, the greater the current.

II Law or Voltage Law or Loop Rule:

The algebraic sum of all the potential drops and emf's along any closed path in an electrical network is always zero.

$$\Sigma V = 0$$



Loop ABCA:

$- E_1 + I_1.R_1 + (I_1 + I_2).R_2 = 0$

Loop ACDA:

$- (I_1 + I_2).R_2 - I_2.R_3 + E_2 = 0$

Sign Conventions:

1. The **emf** is taken **negative** when we traverse from positive to negative terminal of the cell through the electrolyte.
2. The **emf** is taken **positive** when we traverse from negative to positive terminal of the cell through the electrolyte.

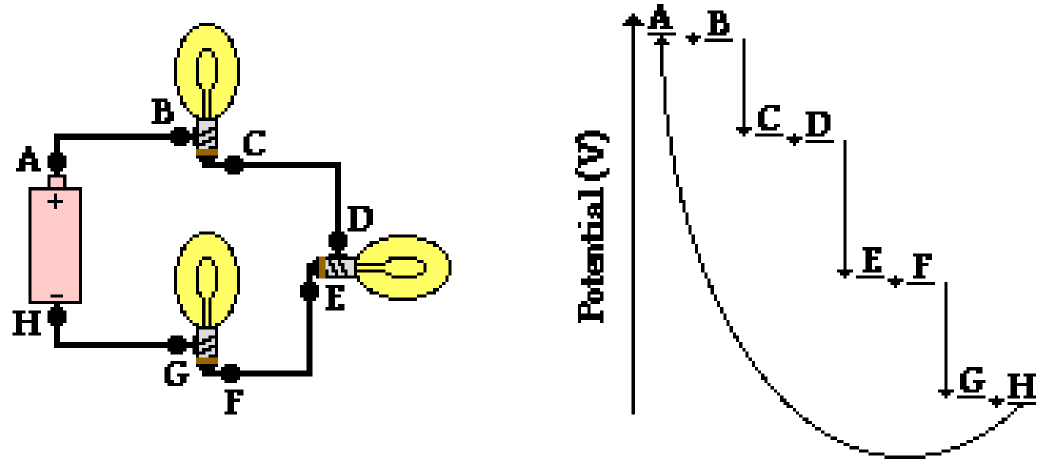
The potential **falls** along the direction of current in a current path and it rises along the direction opposite to the current path.

3. The potential **fall** is taken **negative**.
4. The potential **rise** is taken **positive**.

Note: The path can be traversed in clockwise or anticlockwise direction of the loop.

Kirchoff's Loop Rule

In a closed circuit, sum of all the voltage boosts = sum of all the voltage drops



A small amount of electric potential is lost in a wire. Most of the electric potential losses occur within the light bulbs. The total amount of electric potential loss in the external circuit is equal to the gain in electric potential which occurs within the battery.

$$\Delta V_{loop} = 0$$

$$\Delta V_{bat} = \sum \Delta V_{drop}$$

Energy is conserved as charge flows around a closed loop

Kirchhoff 's second law (voltage law)

Kirchhoff's law states that the algebraic sum of the products of resistance and current in each part of any closed circuit is equal to the algebraic sum of the emf's in that closed circuit.

This law is a consequence of conservation of energy.

In applying Kirchhoff's laws to electrical networks, the direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of current is not the actual direction, then on solving the problems, the current will be found to have negative sign. In the application of Kirchhoff's second law, we follow that the current in clockwise direction is taken as positive and the current in anticlockwise direction is taken as negative.

Considering the closed loop ABCDEFA,

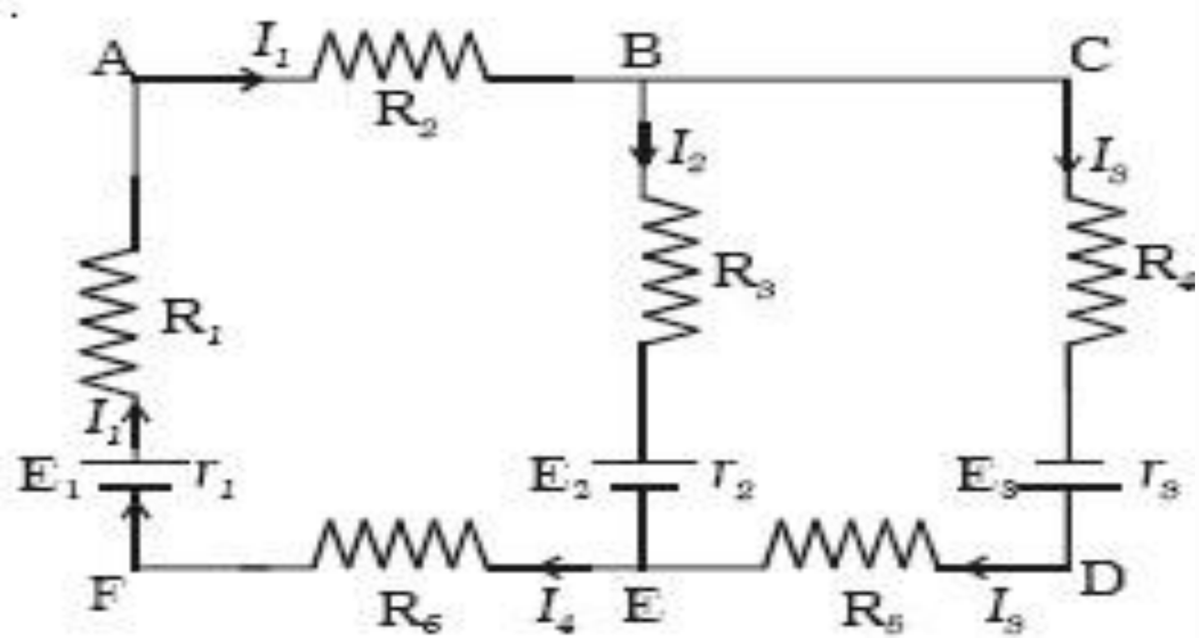
$I_1R_2 + I_3R_4 + I_3r_3 + I_3R_5 + I_4R_6 + I_1r_1 + I_1R_1 = E_1 + E_3$

Both cells E_1 and E_3 send currents in clockwise direction.

For the closed loop ABEFA

$I_1R_2 + I_2R_3 + I_2r_2 + I_4R_6 + I_1r_1 + I_1R_1 = E_1 – E_2$

Negative sign in E_2 indicates that it sends current in the anticlockwise direction.



Kirchoff's Rules

1. **Junction Rule** At any junction point in a circuit, the sum of all the currents entering the junction must equal the sum of all currents leaving the junction

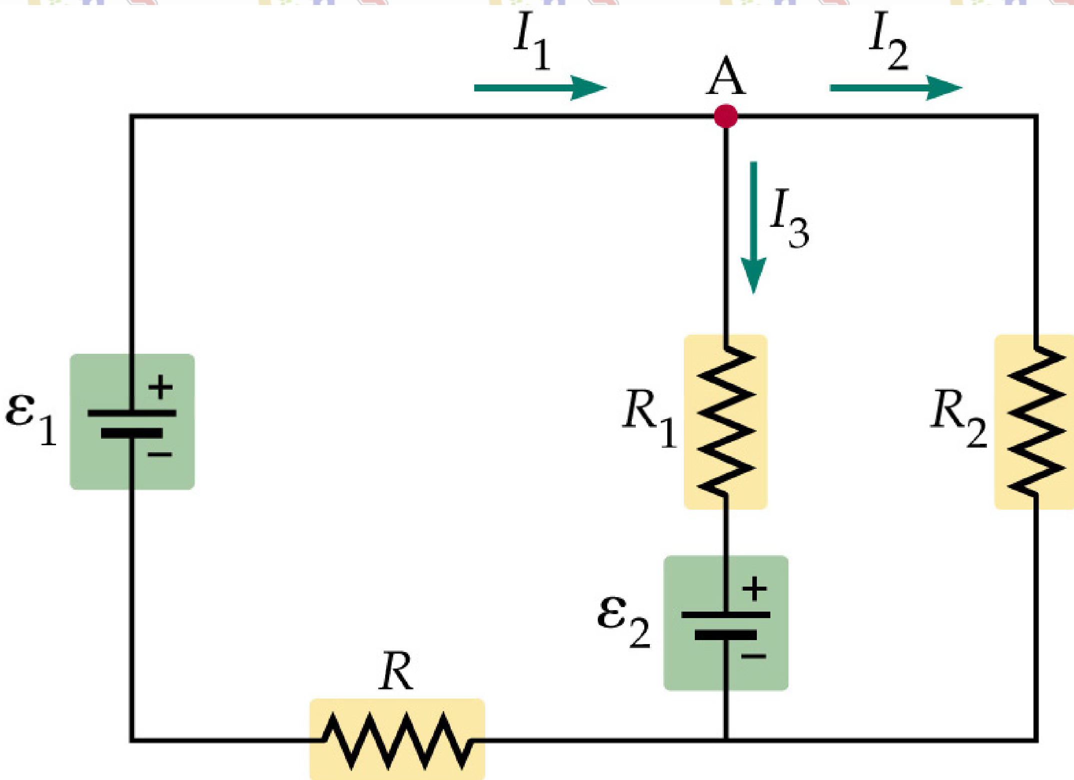
   Current into node= Current out of node

   (Conservation of charge)

2. **Loop Rule** The sum of the potential drops around any closed path of a circuit must be zero.

   (Conservation of energy)

Kirchhoff's junction rule

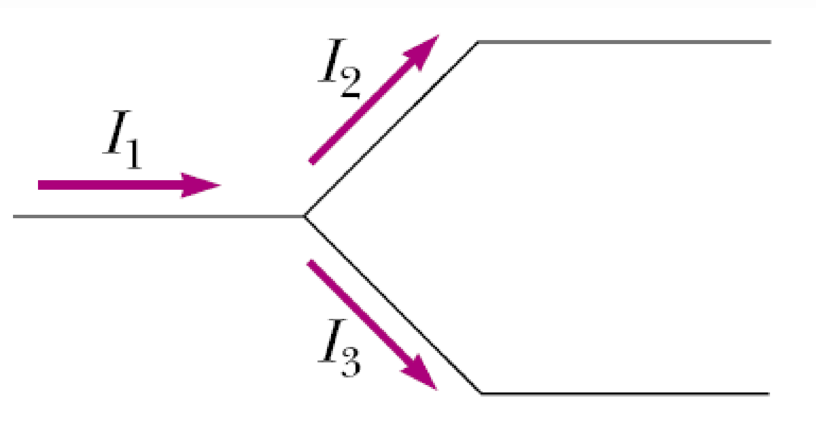


Kirchhoff's junction rule states that the sum of the currents entering a junction must equal the sum of the currents leaving the junction. In this case, for the junction labeled **A**:

$$I_1 = I_2 + I_3$$

or

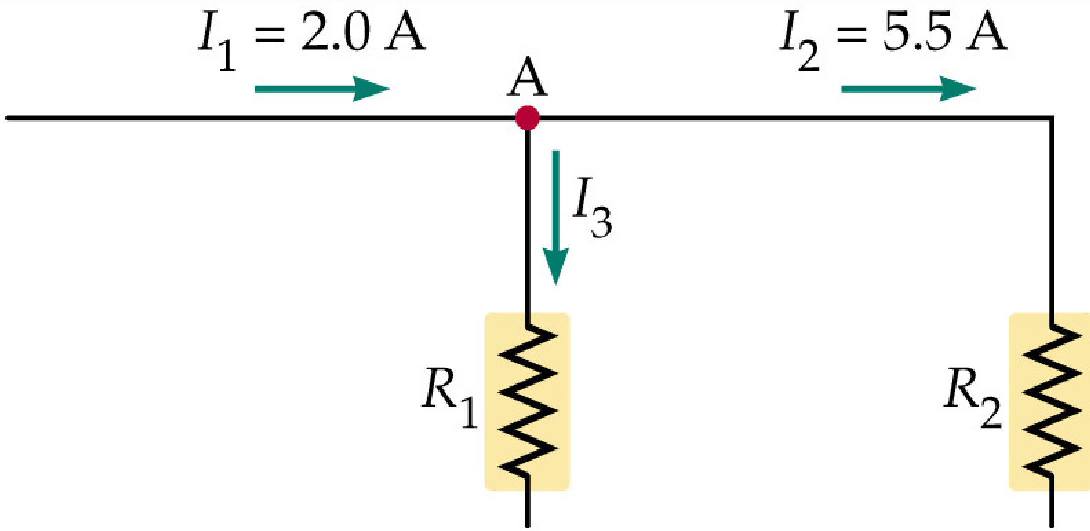
$$I_1 - I_2 - I_3 = 0$$



(a)

$$I_1 = I_2 + I_3$$

A specific application of Kirchhoff's junction rule



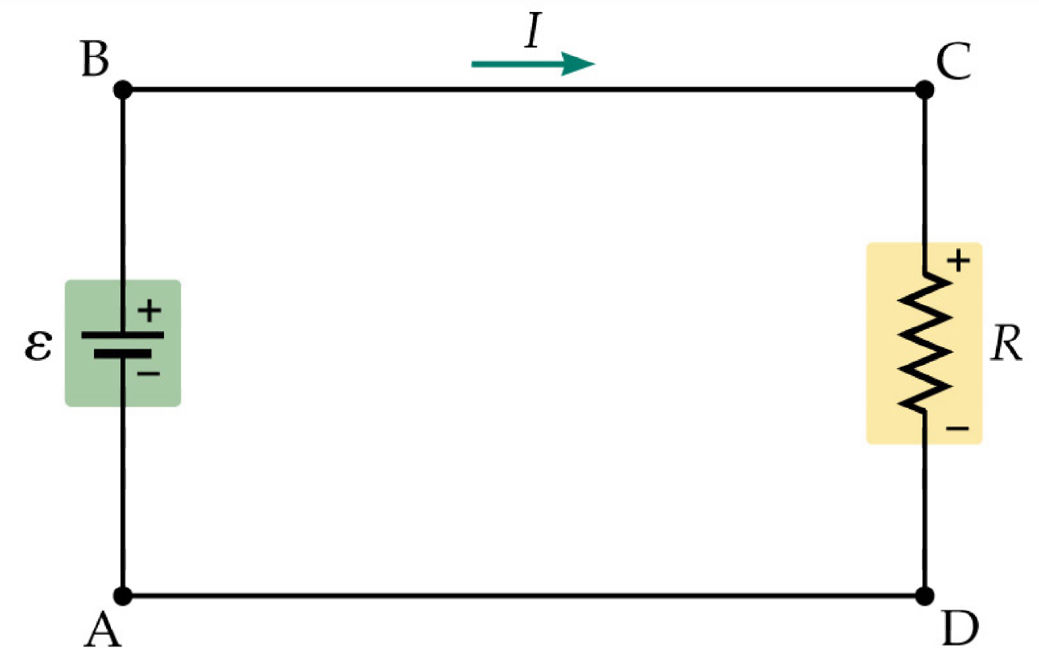
Applying Kirchhoff's junction rule to the junction **A:**

$$I_1 - I_2 - I_3 = 0$$

$$I_3 = (2.0 - 5.5)\ \text{A} = -3.5\ \text{A}$$

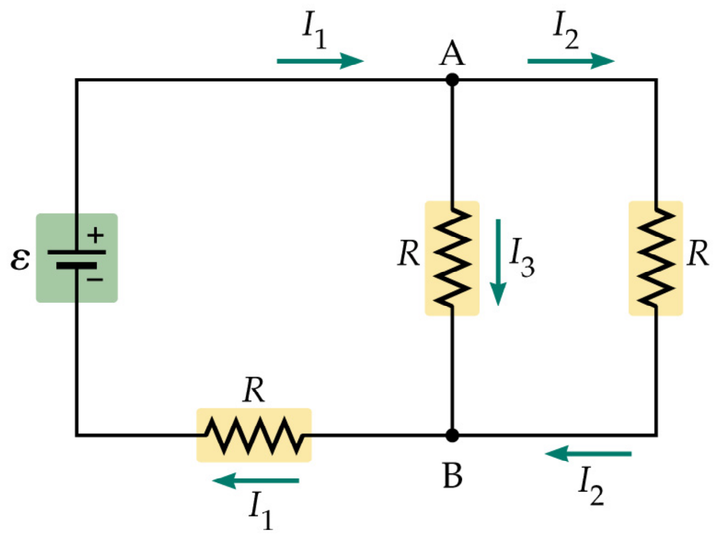
The minus sign indicates that I_3 flows opposite to the direction shown; that is, I_3 is upward.

Kirchhoff's loop rule



Kirchhoff's loop rule states that as one moves around a closed loop in a circuit the algebraic sum of all potential differences must be zero. The electric potential:

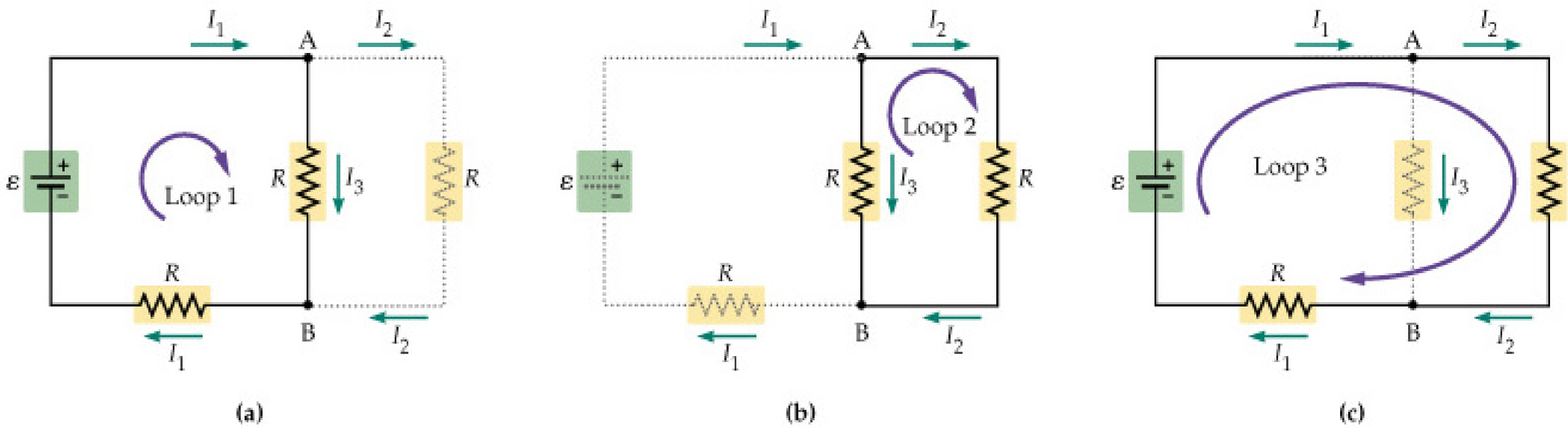
- increases as one moves from the *minus* to the *plus* plate of a battery
- decreases as one moves through a resistor in the direction of the current



Analyzing a simple circuit

Junction Rule: $I_1 = I_2 + I_3$

Loop Rule: *Use any two of these three loops*

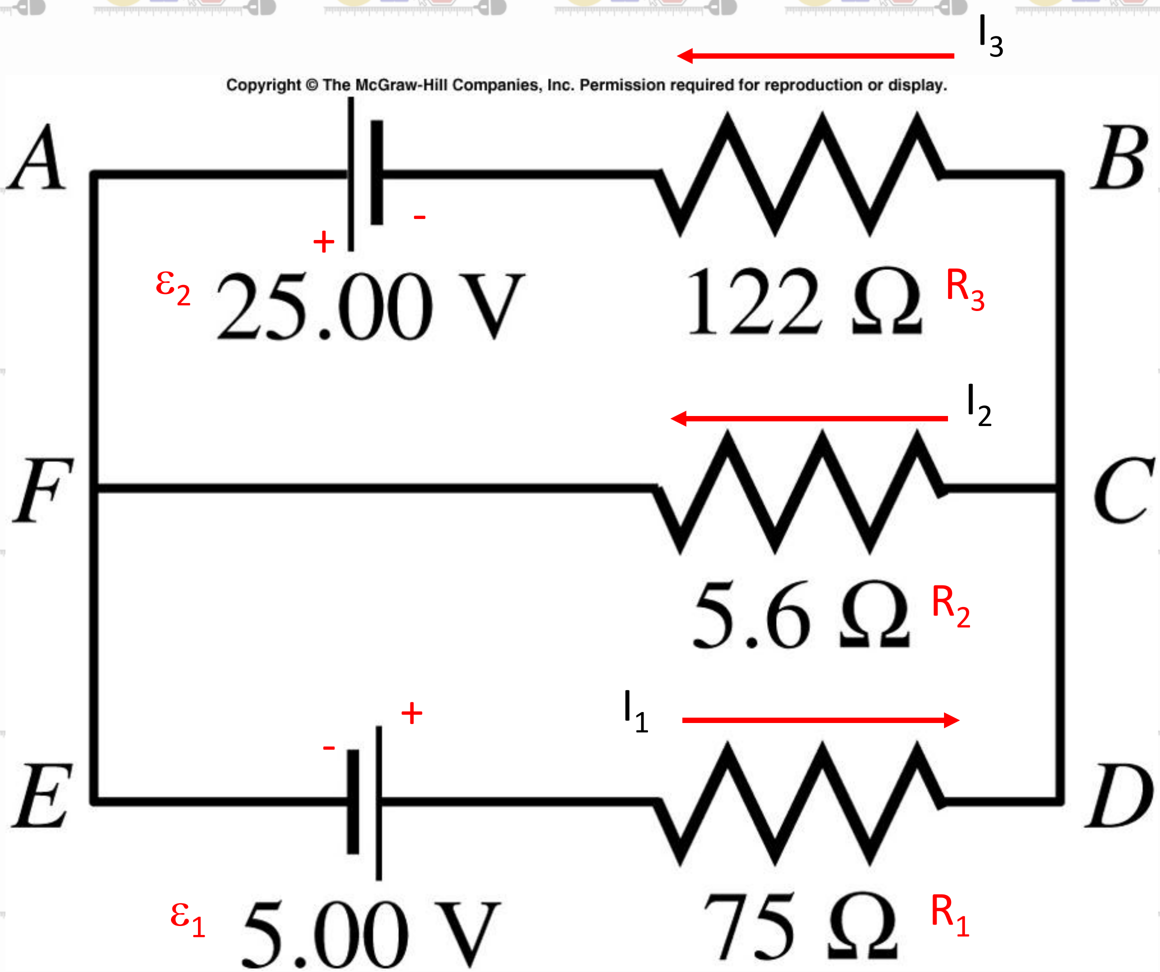


$\varepsilon - I_3R - I_1R = 0$

$I_3R - I_2R = 0$

What is the equation?

Find the three unknown currents (the current in each resistor).



Loop EDCFE: $$\varepsilon_1 - I_1 R_1 - I_2 R_2 = 0$$

Loop AFCBA: $$\varepsilon_2 + I_2 R_2 - I_3 R_3 = 0$$

Junction C: $$I_1 = I_2 + I_3$$

The point is to write down three equations for the three unknown currents.

(1) $$I_1R_1 + I_2R_2 = \varepsilon_1$$

(2) $$I_3R_3 - I_2R_2 = \varepsilon_2$$

(3) $$I_1 = I_2 + I_3$$

Substitute (3) into (1): $$\left(R_1 + R_2\right) I_2 + R_1 I_3 = \varepsilon_1 \quad (4)$$

$$R_3 I_3 - R_2 I_2 = \varepsilon_2 \quad (2)$$

Multiply the top equation by $-R_3$ and the bottom equation by $+R_1$, add the equations together, then solve for I_2.

$$-R_3\left(R_1 + R_2\right) I_2 - R_1 R_3 I_3 + R_1 R_3 I_3 - R_1 R_2 I_2 = R_1 \varepsilon_2 - R_3 \varepsilon_1$$

$$I_2 = \frac{R_1 \varepsilon_2 - R_3 \varepsilon_1}{-R_3\left(R_1 + R_2\right) - R_1 R_2}$$

$$I_2 = -0.123\,\text{amps}$$

Substitute I_2 = -0.123 amps in to (2):

$$I_3 = \frac{\varepsilon_2 + I_2 R_2}{R_3}$$

$$I_3 = +0.199 \text{ amps}$$

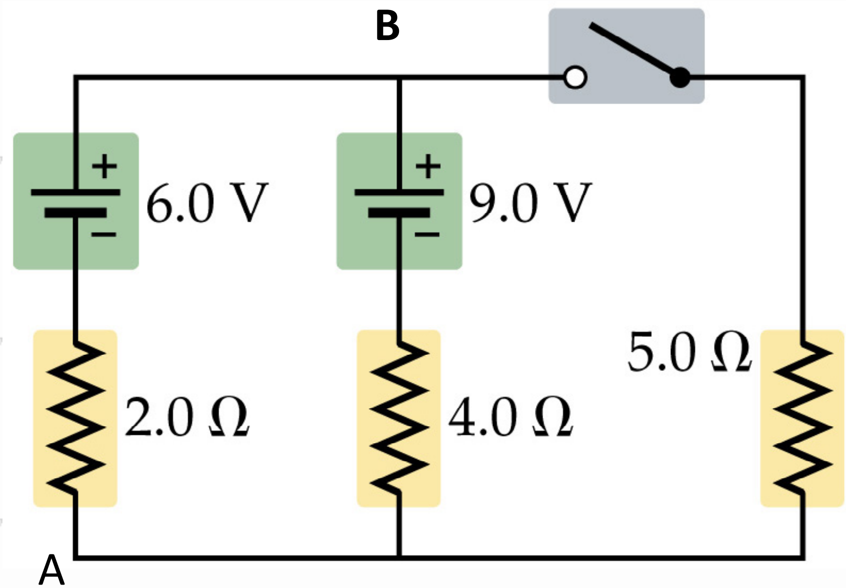
Now substitute the known values of I_2 and I_3 into (3):

$$I_1 = I_2 + I_3$$

$$= -0.123 \text{ amps} + 0.199 \text{ amps}$$

$$= +0.076 \text{ amps}$$

How much current flows through each battery when the switch is (a) closed and (b) open? (c) With the switch open, suppose that point A is grounded. What is the potential at point B?



Connections

Which circuit draws more current (how are I_1 and I_2 related)?
What is the order of bulb brightness?
How does charge flow in these circuits (how are I_2 and I_3 related)?
Does the charge get used up?