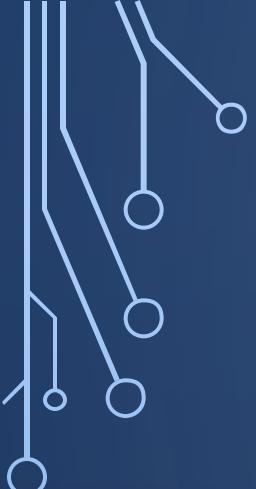


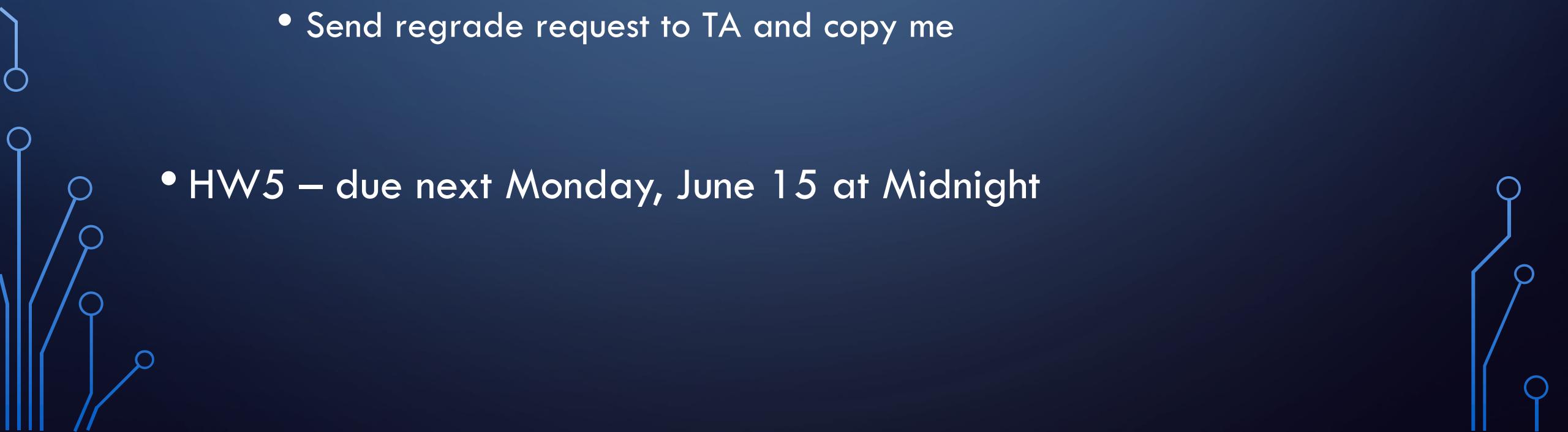
# PREDICATES & QUANTIFIERS

zyBooks Chapter: 4.6 ~ 4.13



# LOGISTICS



- Midterm 1
    - Submit regrade request by 11:00 PM Wednesday, June 10.
      - Send regrade request to TA and copy me
  - HW5 – due next Monday, June 15 at Midnight
- 

# RECAP – PREDICATES & QUANTIFIERS

- Predicates
  - A function that evaluates to a proposition
  - Domain of a variable: a set of ALL possible values
- Quantifiers – Size of the domain
  - Universal Quantifier –  $\forall xP(x)$
  - Existential Quantifier –  $\exists xP(x)$
- Bound and Free variables
  - A proposition does NOT contain free variable
- De Morgan's Law

# PREDICATES AND QUANTIFIERS

$D(x)$ :  $x$  likes dogs.

English	Existential	Universal
Someone does not like dogs	$\exists x \neg D(x)$	$\neg \forall x D(x)$
No one likes dogs	$\neg \exists x D(x)$	$\forall x \neg D(x)$
Someone likes dogs	$\exists x D(x)$	$\neg \forall x \neg D(x)$
Everyone likes dogs	$\neg \exists x \neg D(x)$	$\forall x D(x)$

De Morgan's Law

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$

## NESTED QUANTIFIERS

If a predicate has more than one variable, each variable must be bound by a separate quantifier. For example,  $\forall x \forall y L(x, y)$

A logical expression with more than one quantifier that bind different variables in the same predicate is said to have nested quantifiers.

# NESTED QUANTIFIERS

$F(x, y)$ :  $x$  flies to  $y$

- Every bird flies to everywhere

$$\forall x \forall y F(x, y)$$

- Some birds fly to somewhere

$$\exists x \exists y F(x, y)$$

- Every bird flies to somewhere

$$\forall x \exists y F(x, y)$$

- There is a place where every bird flies to

$$\forall x \exists y F(x, y)$$

# NESTED QUANTIFIERS

**Q:** Determine the truth value of each expression below. The domain is the set of all real numbers ([Exercise 4.9.3 in zyBooks](#)).

- $\forall x \exists y (xy > 0)$
- $\exists x \forall y (xy = 0)$
- $\forall x \forall y \exists z (z = (x - y)/3)$
- $\forall x \exists y \forall z (z = (x - y)/3)$
- $\forall x \forall y (xy = yx)$
- $\exists x \exists y \exists z (x^2 + y^2 = z^2)$
- $\forall x \exists y (\gamma^2 = x)$
- $\forall x \exists y (x < 0 \vee y^2 = x)$
- $\exists x \exists y (x^2 = y^2 \wedge x \neq y)$
- $\exists x \exists y (x^2 = y^2 \wedge |x| \neq |y|)$
- $\forall x \forall y (x^2 \neq y^2 \vee |x| = |y|)$

**Q:** Determine the truth value of each expression below. The domain is the set of all real numbers (Exercise 4.9.3 in zyBooks).

- $\forall x \exists y (xy > 0)$ 
  - False. If  $x = 0$ , there is no  $y$  such that  $xy > 0$ .
- $\exists x \forall y (xy = 0)$ 
  - True. If  $x = 0$ , then for all  $y$ ,  $xy = 0$ .
- $\forall x \forall y \exists z (z = (x - y)/3)$ 
  - True. For any  $x$  and  $y$ , set the value of  $z$  to be  $(x - y)/3$ .
- $\forall x \exists y \forall z (z = (x - y)/3)$ 
  - False. Once  $x$  and  $y$  are determined, then if  $z$  is any value besides  $(x - y)/3$ , the equality is false.
- $\forall x \forall y (xy = yx)$ 
  - True.

- $\exists x \exists y \exists z (x^2 + y^2 = z^2)$ 
  - True. One example is  $x = 3, y = 4, z = 5$ .
- $\forall x \exists y (y^2 = x)$ 
  - False. If  $x < 0$ , then there is no  $y$  such that  $y^2 = x$ .
- $\forall x \exists y (x < 0 \vee y^2 = x)$ 
  - True. If  $x \geq 0$ , then there is a  $y$  such that  $y^2 = x$ .
- $\exists x \exists y (x^2 = y^2 \wedge x \neq y)$ 
  - True. For example,  $x = 2$  and  $y = -2$ .
- $\exists x \exists y (x^2 = y^2 \wedge |x| \neq |y|)$ 
  - False.
- $\forall x \forall y (x^2 \neq y^2 \vee |x| = |y|)$ 
  - True. If the first part is false for some  $x$  and  $y$  (i.e.,  $x^2 = y^2$ ), then  $|x| = |y|$  is true.

# DE MORGAN'S ON NESTED QUANTIFIERS

$$\neg \forall x \exists y P(x, y) \equiv \exists x \forall y \neg P(x, y)$$

$$\neg \exists x \forall y P(x, y) \equiv \forall x \exists y \neg P(x, y)$$

# DE MORGAN'S LAW

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$

$$\neg \forall x \exists y P(x, y) \equiv \exists x \forall y \neg P(x, y)$$

$$\neg \exists x \forall y P(x, y) \equiv \forall x \exists y \neg P(x, y)$$

Q: Simplify to remove all  $\neg$  and  $\rightarrow$

$$\neg \forall x \exists y \exists z ((x > z) \rightarrow (x > y))$$

**Q:** Simplify to remove all  $\neg$  and  $\rightarrow$

$$\neg \forall x \exists y \exists z ((x > z) \rightarrow (x > y))$$

$$\equiv \exists x \forall y \forall z \neg ((x > z) \rightarrow (x > y))$$

De Morgan's

$$\equiv \exists x \forall y \forall z \neg (\neg(x > z) \vee (x > y))$$

Implication

$$\equiv \exists x \forall y \forall z \neg \neg(x > z) \wedge \neg(x > y)$$

De Morgan's

$$\equiv \exists x \forall y \forall z (x > z) \wedge (x \leq y)$$

Double Negation

**Q:** The domain for variables  $x$  and  $y$  is a group of people. The predicate  $F(x, y)$  is true if and only if  $x$  is a friend of  $y$ . For the purposes of this problem, assume that for any person  $x$  and person  $y$ , either  $x$  is a friend of  $y$  or  $x$  is an enemy of  $y$ . Therefore,  $\neg F(x, y)$  means that  $x$  is an enemy of  $y$ .

Steps:

- i. Translate each statement into a logical expression.
- ii. Negate the expression by adding a negation operation to the beginning of the expression.
- iii. Apply De Morgan's law until the negation operation applies directly to the predicate
- iv. Translate the logical expression back into English.
  - a) Everyone is a friend of everyone.
  - b) Someone is a friend of someone.
  - c) Someone is a friend of everyone.
  - d) Everyone is a friend of someone.

❖ Everyone is a friend of everyone.

- Logical expression:  $\forall x \forall y F(x, y)$
- Negation:  $\neg \forall x \forall y F(x, y)$
- Apply De Morgan's:  $\exists x \exists y \neg F(x, y)$
- English: Someone is an enemy of someone.

❖ Someone is a friend of someone.

- Logical expression:  $\exists x \exists y F(x, y)$
- Negation:  $\neg \exists x \exists y F(x, y)$
- Apply De Morgan's:  $\forall x \forall y \neg F(x, y)$
- English: Everyone is an enemy of everyone.

❖ Someone is a friend of everyone.

- Logical expression:  $\exists x \forall y F(x, y)$
- Negation:  $\neg \exists x \forall y F(x, y)$
- Apply De Morgan's:  $\forall x \exists y \neg F(x, y)$
- English: Everyone is an enemy of someone.

❖ Everyone is a friend of someone.

- Logical expression:  $\forall x \exists y F(x, y)$
- Negation:  $\neg \forall x \exists y F(x, y)$
- Apply De Morgan's:  $\exists x \forall y \neg F(x, y)$
- English: Someone is an enemy of everyone.