MIDTERM #1 REVIEW

JUNE 2ND, 5:30 – 6:45PM + EXTRA

- Combinatorics
- Propositional Logic
- Boolean Algebra
- Logic Circuits

ABOUT THE EXAM…

Detailed logistics
In Moodle Announcement

- CLOSED-BOOK, CLOSED NOTE
- 110/100
- Show up before 5:27pm, Camera ON, you and a clean desk
- Deliver via Moodle at 5:30pm, June 2nd
- 90 minutes in total (5:30 – 7:00pm)
  - 5 min for setting up, e.g., downloading or printing
  - 75 min for exam (by 6:50pm)
  - 10 min for scanning and submission (by 7:00pm)
- Submit via Moodle as assignment

SET

- Cardinality – size of the set
- Operations
  - Union – A ∪ B
  - Intersection – A ∩ B
  - Difference – A – B
  - Complement – $\bar{A}$ or $U - A$

COUNTING TECHNIQUES

- Product Rule
  - In subsequent tasks A and B, if there are x ways to do task A and y ways to do task B, then there are $x \cdot y$ ways to do A then B.
- Sum Rule
  - In separate tasks A and B, if there are x ways to do task A and y ways to do task B, then there are $x+y$ ways to do A and B.
- Counting by complement
  - All possible cases – invalid cases

Q: A 100-page document is being printed by four printers. Each page will be printed exactly once.

- Suppose that there are no restrictions on how many pages a printer can print. How many ways are there for the 100 pages to be assigned to the four printers? Printer = {a, b, c, d}, therefore, 4^{100}
- Suppose the first and the last page of the document must be printed in color, and only two printers are able to print in color. The two color printers can also print black-and-white. How many ways are there for the 100 pages to be assigned to the four printers? $2^2 * 4^{98} = 4^{99}$
- Suppose that all the pages are black-and-white, but each group of 25 consecutive pages (1-25, 26-50, 51-75, 76-100) must be assigned to the same printer. Each printer can be assigned 0, 25, 50, 75, or 100 pages to print. How many ways are there for the 100 pages to be assigned to the four printers? 4^4

COUNTING TECHNIQUES

- Permutations (counting sequences) – Ordered arrangement
  - a sequence of r items with no repetitions all taken from the same set
  - $P(n, r) = \frac{n!}{(n-r)!} = n(n-1)\ldots(n-r+1)$
  - $P(n, n) = n!$
- Combinations (counting subsets) – Unordered arrangement
  - The number of ways of selecting an r-subset from a set of size n
  - $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!}$

Q: Ten members of a wedding party are lining up in a row for a photograph.

- How many ways are there to line up the ten people?

  10!

- How many ways are there to line up the ten people if the groom must be to the immediate left of the bride in the photo?

  9!

- How many ways are there to line up the ten people if the bride must be next to the maid of honor and the groom must be next to the best man?

  2! * 2! * 8!

COUNTING TECHNIQUES

- Permutations with repetition
  - The number of distinct sequences with n_1 1's, n_2 2's, ..., n_k k's, where $n = n_1 + n_2 + n_3 + \cdots + nk$ is $\dfrac{n!}{n_1!n_2!n_3!\ldots nk!}$
- Combinations with repetition
  - Choosing r items from n varieties : $\binom{r+n-1}{n-1}$

Q: A school cook plans her calendar for the month of February in which there are 20 school days. She plans exactly one meal per school day. Unfortunately, she only knows how to cook ten different meals.

- How many ways are there for her to plan her schedule of menus for the 20 school days if there are no restrictions on the number of times she cooks a particular type of meal?

10^{20}

- How many ways are there for her to plan her schedule of menus if she wants to cook each meal the same number of times?

$$\binom{20}{2}\binom{18}{2}\cdots\binom{4}{2} = \frac{20!}{2!2!\ldots.2!} = \frac{20!}{2^{10}}$$

COUNTING TECHNIQUES

- Pigeonhole Principle
  - If $n + 1$ pigeons are placed in n boxes, then there must be at least one box with more than one pigeon.
  - Given #pigeon = N and #holes = k, #Collision = $\left\lceil \frac{N}{k} \right\rceil$
  - Given #holes = k, #Collision = b, #MinPigeon = $k(b - 1) + 1$

HW2 – Q10: Prove that at a party where there are at least two people, there are two people who know the same number of other people there.

Let n = total number of people at the party;

The number of people someone can know = 0, 1, 2, ..., n-1; (size = n)

Case 1: Everyone knows at least 1 people at the party,

so the number of people can know is 1, ..., n – 1. (size = n – 1)

Case 2: There is someone knows no one else at the party,

so the number of people you can know is 0, ..., n – 2. (size = n – 1)

Both cases have n – 1 people you can know (holes). With n people at the party, then there are 2 people that know each other because $\left\lceil \frac{n}{n-1} \right\rceil = 2$.

HW2 – Q10: Prove that at a party where there are at least two people, there are two people who know the same number of other people there.

Proof by contradiction:

Assume that everyone at the party knows different number of people. That is, each person knows a unique number of people from the set {0, 1, ..., n – 1}.

However, if one person knows no one at the party (0 person), then there is no one at the party knows everyone else (n – 1 persons), which contradicts the assumption that everyone knows a unique number of people from the set {0, 1, ..., n – 1}.

Therefore, the assumption is false. Hence, …

PROPOSITIONAL LOGIC

- A proposition is a declarative statement that is either true or false.
- Logical connectives & precedence
  - $\neg$, $\wedge$, $\vee$, $\oplus$, $\rightarrow$, $\leftrightarrow$
- Truth table
- Laws of propositional logic

Q: Express each English statement using logical operations $\vee$, $\wedge$, $\neg$ and the propositional variables M, F, and H defined below. The use of the word "or" means inclusive or.

M: The patient took the medication.

F: The patient had flu.

H: The patient had headache.

- The patient had flu and headache. $F \wedge H$
- The patient took the medication, but still had headache. $M \wedge H$
- The patient had flu or headache. $F \vee H$
- The patient did not have headache. $\neg H$
- Despite the fact that the patient took the medication, the patient had flu. $M \wedge F$
- There is no way that the patient took the medication. $\neg M$

Q: Prove that $(p \rightarrow q) \land (p \rightarrow r) \equiv p \rightarrow (q \land r)$

$(p \rightarrow q) \land (p \rightarrow r)$	
$\equiv (\neg p \lor q) \land (\neg p \lor r)$	Conditional identity / Implication (30)
$\equiv \neg p \lor (q \land r)$	Distributive (16)
$\equiv p \rightarrow (q \land r)$	Conditional identity / Implication (30)

BOOLEAN ALGEBRA

- Literals x, y, z ∈ {0,1}
- Logical operations
  - $\bar{x}, \mathrm{x} \cdot y, \mathrm{x} + y, \ =$
- Laws of Boolean algebra

BOOLEAN FUNCTION

- Boolean function
  - Expression: F(x, y, z): ….
  - I/O table
- Disjunctive Normal Form (DNF)
  - True cases, sum of products
- Conjunctive Normal Form (CNF)
  - False cases, product of sums

Q: Given the I/O table, find the Boolean function in

- DNF

$F(A, B, C) = ABC + AB\overline{C} + A\overline{B}C + \overline{A}BC + \overline{A}\overline{B}C$

- CNF

$F(A, B, C) = (\overline{A} + B + C)\ (A + \overline{B} + C)\ (A + B + C)$

A	B	C	$F(A, B, C)$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

BOOLEAN CIRCUIT

- Boolean circuit
  - Expression to Circuit
  - Circuit to expression
  - English specification to Circuit
- Logic gates
  - AND gate, OR gate, XOR gate, NOT
  - De Morgan’s law on logic gates