BINARY RELATIONS

zyBooks Chapter: 10

RECAP

- A binary relation between two sets A and B is a subset R of A × B (aRb).
- Representation
  - Set, Matrix, Directed Graph
- Reflexive: $\forall x \in S,\ xRx$
- Irreflexive / anti-reflexive $\forall x \in S,\ x\cancel{R}x$
- Transitive $xRy \wedge yRz \rightarrow xRz$
- Symmetric $xRy \Leftrightarrow yRx$
- Anti-symmetric $xRy \wedge yRx \rightarrow x = y$
- Asymmetric $xRy \rightarrow y\cancel{R}x$

S = { 2, 4, 8, 16, 32, 64}

xRy if $x^k = y$ for some int k, where x, y ∈ S.

Q: Enumerate the relation (set format)

Q: Draw a matrix for this relation

Q: Draw an arrow diagram

	2	4	8	16	32	64
2	1	1	1	1	1	1
4	0	1	0	1	0	1
8	0	0	1	0	0	1
16	0	0	0	1	0	0
32	0	0	0	0	1	0
64	0	0	0	0	0	1

R = { (2, 2), (2, 4), (2, 8), (2, 16), (2, 32), (2, 64),

(4, 4), (4, 16), (4, 64)

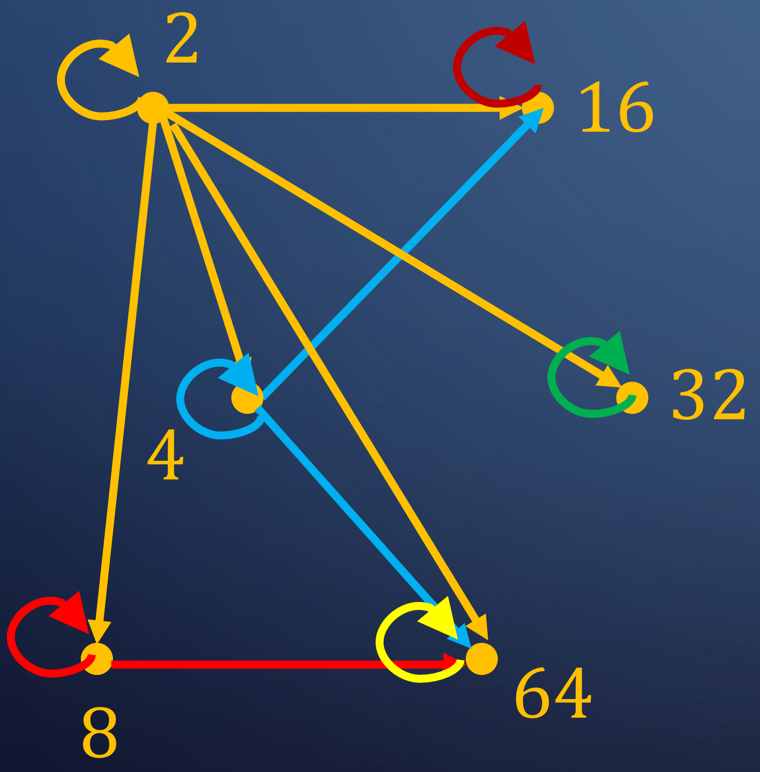
(8, 8), (8, 64),

(16, 16), (32, 32), (64, 64) }

Q: Draw an arrow diagram

Q: Determine if this binary relations is …

1) Reflexive, 2) Irreflexive, 3) Transitive, 4) Symmetric, 5) Anti-symmetric, 6) Asymmetric



	2	4	8	16	32	64
2	1	1	1	1	1	1
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64	0	0	0	0	0	1

Reflexive

Transitive

Anti-symmetric

OPERATIONS ON BINARY RELATIONS

- Union
- Intersection
- XOR
- Composition

Let $R = \{ (1, 1), (2, 1) \}$ $\rightarrow$ $\begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \end{array}$

and $S = \{ (1, 1), (1, 2) \}$ $\rightarrow$ $\begin{array}{cc} \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \end{array}$

- Union
  - $R \cup S = \{ (1, 1), (1, 2), (2, 1) \}$ $\rightarrow$ $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$
- Intersection
  - $R \cap S = \{ (1, 1) \}$ $\rightarrow$ $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
- XOR
  - $R \oplus S = \{ (1, 2), (2, 1) \}$ $\rightarrow$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

COMPOSITION – $S \circ R$

The pair $(a, c) \in S \circ R$ if and only if there is a b ∈ A such that (a, b) ∈ R and (b, c) ∈ S.

Read “S composed with R”, “S after R”…

Let R = { (1, 1), (2, 1) } $\rightarrow$ $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

and S = { (1, 1), (1, 2) } $\rightarrow$ $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$S \circ R$

Step 1: Look at elements in R

(1, 1)

Step 2: Search for pairs in S that starting with 1

(1, 1), (1, 2)

Step 3: "Compose"

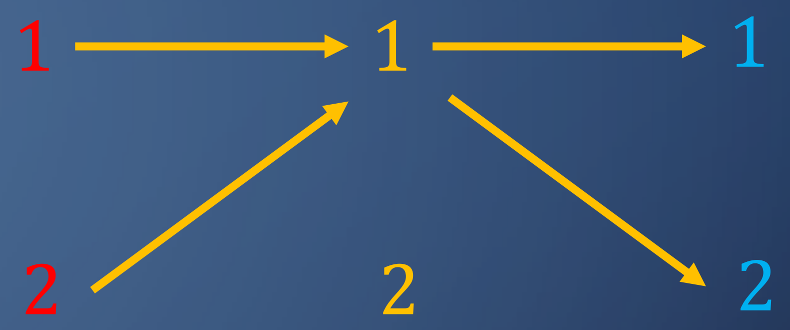
(1, 1), (1, 2)

Step 4: Repeat for all elements in R

$S \circ R$ = { (1, 1), (1, 2), (2, 1), (2, 2) }

Let $R = \{ (1, 1), (2, 1) \}$

and $S = \{ (1, 1), (1, 2) \}$



$S \circ R$

$S \circ R = \{ (1, 1), (1, 2), (2, 1), (2, 2) \}$

Step 1: Draw the directed graph for R

Step 2: Draw the directed graph for S

Step 3: “Compose”

Let R = { (1, 1), (2, 1) } → $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

and S = { (1, 1), (1, 2) } → $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$S \circ R$

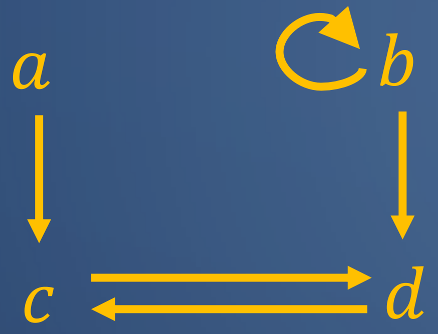
Matrix Multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

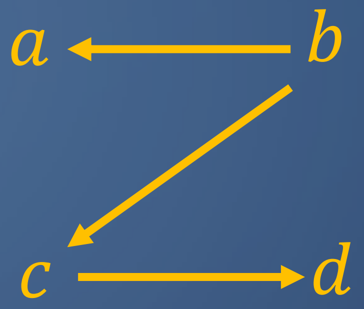
$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Q: Draw directed graph for the following binary relations

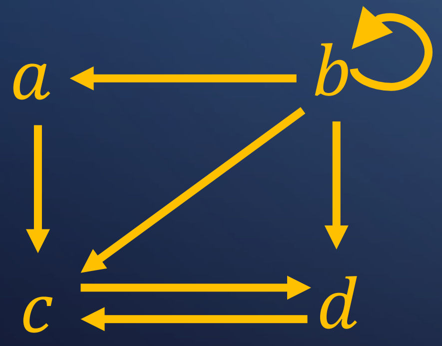
R =



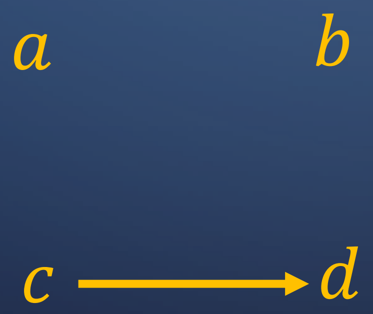
S =



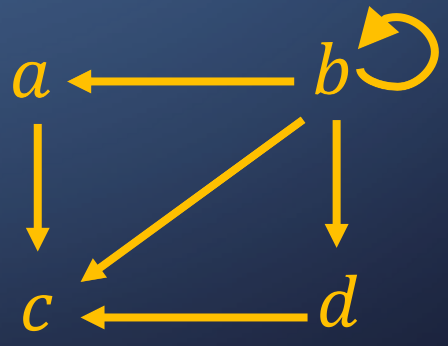
R ∪ S



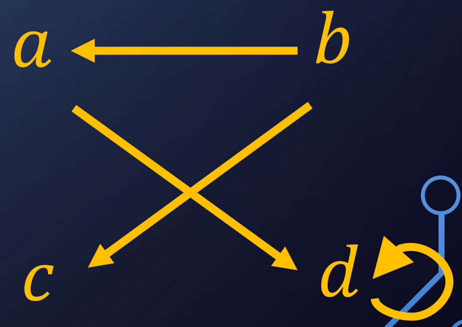
R ∩ S



R ⊕ S



S ∘ R



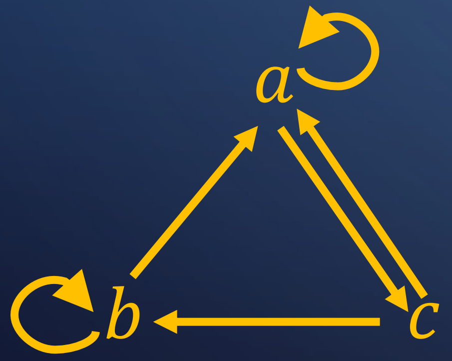
Q:

$R_1 = \{ (a, c), (b, a), (b, b), (c, b) \}$

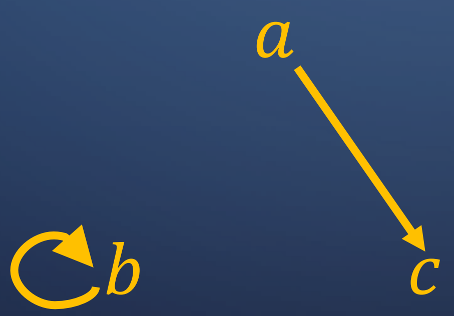
$R_2 = \{ (a, a), (a, c), (b, b), (c, a) \}$

Draw the directed graph for …

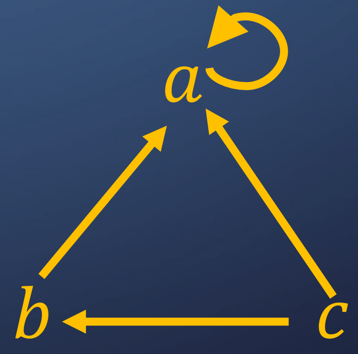
$R_1 \cup R_2$



$R_1 \cap R_2$



$R_1 \oplus R_2$



$R_2 \circ R_1$