BINARY RELATIONS

zyBooks Chapter: 10

LOGISTICS

- HW8 – Due Friday, July 10 at 11:59pm
- HW9 – Released, due next Monday, July 13 at 11:59pm

- Midterm Review 3 this Thursday
- Midterm 3 on next Tuesday, July 14.

BINARY RELATIONS

A binary relation between two sets A and B is a subset R of $A \times B$.

The two sets A and B may or may not be equal. For $a \in A$ and $b \in B$, the fact that $(a, b) \in R$ is denoted by aRb.

Recap (lecture1):

$A = \{ 1, 2, 3 \}$, $B = \{ x, y \}$

$A \times B = \{ (a, b) \mid a \in A \text{ and } b \in B \}$

$= \{ (1, x), (1, y), (2, x), (2, y), (3, x), (3, y) \}$

$B \times A = \{ (x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3) \}$

REPRESENTATION

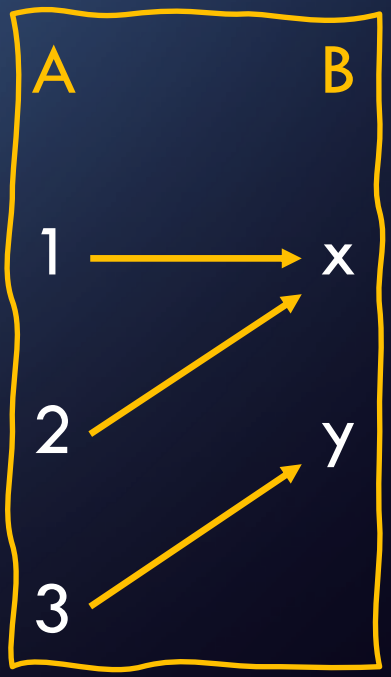
- Set
- Matrix
- Graph – directed graph

A binary relation between two sets A and B is a subset R of $A \times B$.

$A = \{ 1, 2, 3 \}$, $B = \{ x, y \}$

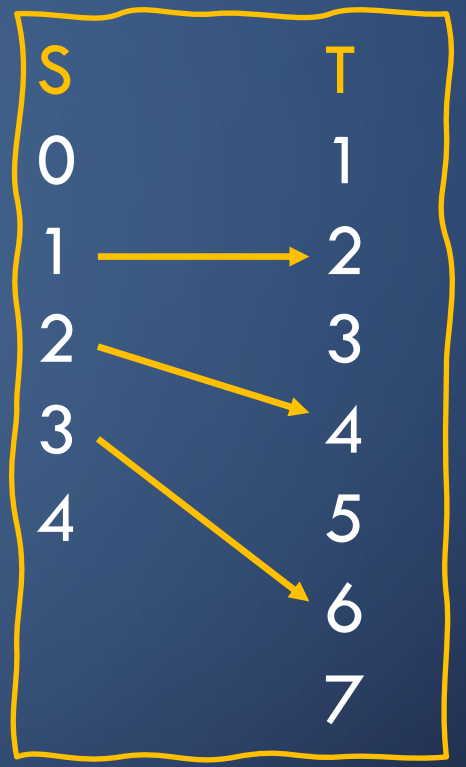
Define: $R = \{ (1, x), (2, x), (3, y) \}$

	x	y
1	1	0
2	1	0
3	0	1



Example:

1. Enumerate the relationship
2. Draw the directed graph
3. Create the matrix



S = { 0, 1, 2, 3, 4 }, T = { 1, 2, 3, 4, 5, 6, 7 }

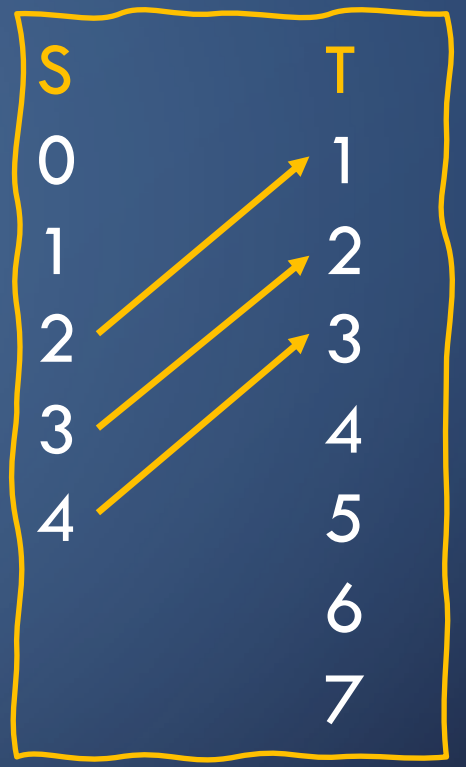
xRy if 2x = y, where x ∈ S and y ∈ T

R = { (1, 2), (2, 4), (3, 6) }

	1	2	3	4	5	6	7
1	0	1	0	0	0	0	0
2	0	0	0	1	0	0	0
3	0	0	0	0	0	1	0
4	0	0	0	0	0	0	0

Example:

1. Enumerate the relationship
2. Draw the directed graph
3. Create the matrix



S = { 0, 1, 2, 3, 4 }, T = { 1, 2, 3, 4, 5, 6, 7 }

xRy if x = y + 1, where x ∈ S and y ∈ T

R = { (2, 1), (3, 2), (4, 3) }

	1	2	3	4	5	6	7
1	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0
3	0	1	0	0	0	0	0
4	0	0	1	0	0	0	0

PROPERTIES OF BINARY RELATIONS

- Reflexive
- Irreflexive / anti-reflexive
- Transitive
- Symmetric
- Anti-symmetric
- Asymmetric

REFLEXIVE RELATIONS

$$\forall x \in S,\ xRx.$$

ALL elements are related to themselves

Example 1

Example 2

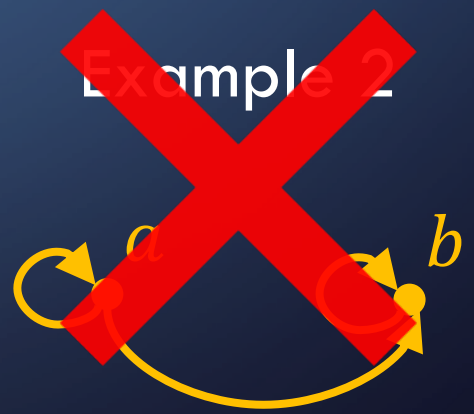
IRREFLEXIVE/ANTI-REFLEXIVE RELATIONS

$$\forall x \in S,\ x \not R x$$

NONE of the elements in the set relate to themselves

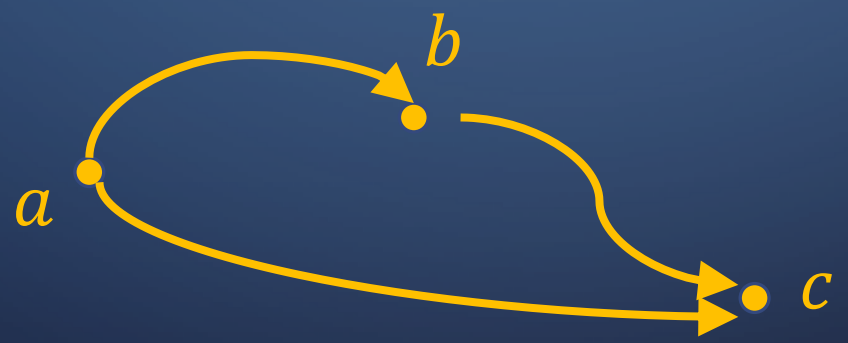
Example 1

Example 2



TRANSITIVE RELATIONS

$$\mathrm{x}Ry \wedge yRz \rightarrow xRz.$$



SYMMETRIC RELATIONS

$$\mathrm{x}Ry \rightarrow yRx.$$

Example 1

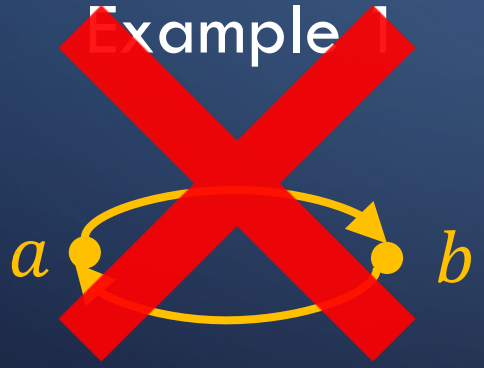
Example 2

ANTI-SYMMETRIC RELATIONS

Only zero or one of xRy and yRx is true when $x \neq y$

In other words, $xRy \wedge yRx \rightarrow x = y$

Example 1

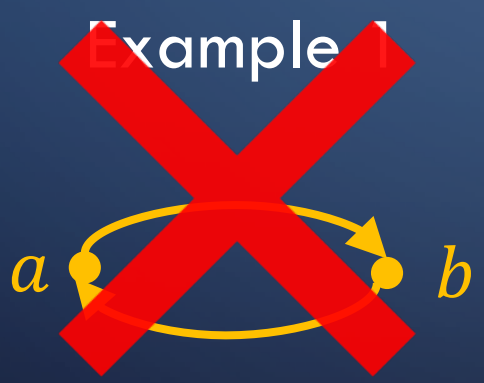


Example 2

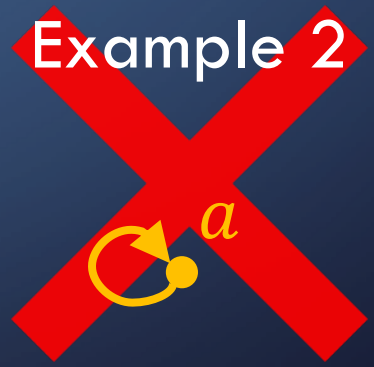
ASYMMETRIC RELATIONS

$$xRy \rightarrow y\not{R}x.$$

Example 1



Example 2

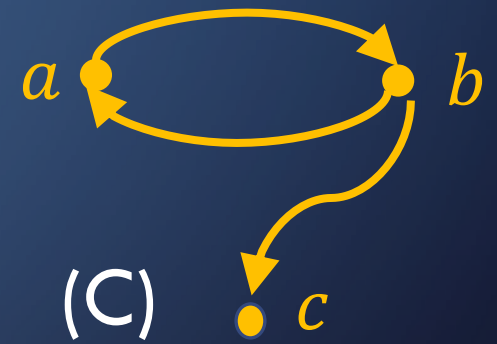


Q: Determine if following binary relations are …

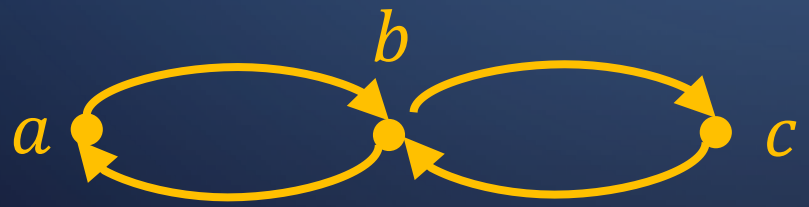
1) Reflexive, 2) Irreflexive, 3) Transitive, 4) Symmetric, 5) Anti-symmetric, 6) Asymmetric

(A)

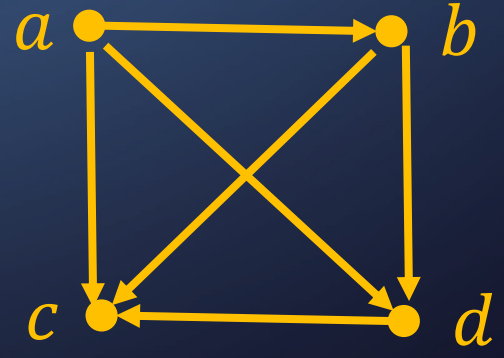
(B)



(C)



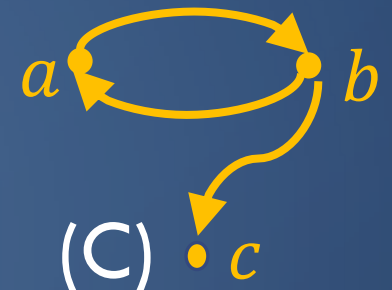
(D)



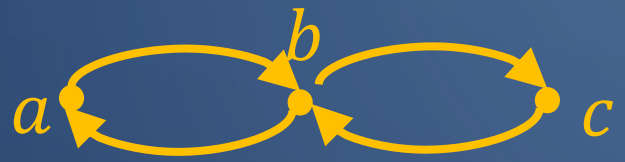
(E)

(A)

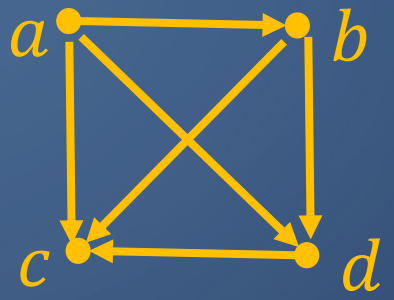
(B)



(C)



(D)



(E)

	Reflexive	Irreflexive	Transitive	Symmetric	Anti-symmetric	Asymmetric
A	✓		✓	✓	✓	
B			✓	✓	✓	
C		✓				
D		✓		✓		
E		✓	✓		✓	✓