# MIDTERM #3 REVIEW

JULY $14^{TH}$, 5:30 – 6:45PM + EXTRA

- LHRR
- Recursive Form → Closed Form
- Closed Form → Recursive Form
- Big – O
- Binary Relations

# ABOUT THE EXAM…

Detailed logistics
In Moodle Announcement

- CLOSED-BOOK, CLOSED NOTE, 110/100
- Show up before 5:27pm, Camera ON, you and a clean desk
- Deliver via Moodle at 5:30pm, June 14th
- 90 minutes in total (5:30 – 7:00pm)
  - 5 min for setting up
  - 75 min for exam (by 6:50pm)
  - 10 min for scanning and submission (by 7:00pm)
- Submit via Moodle as assignment

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# RECURRENCE RELATION

A rule that defines a term $a_n$ as a function of previous terms in the sequence is called a recurrence relation.

# LINEAR HOMOGENOUS RECURSIVE RELATION (LHRR)

- Linear
  - There are **no product** of recursive terms
- Homogenous
  - There are **no additional terms** besides previous terms in the sequence

$$f_n = c_1 f_{n-1} + c_2 f_{n-2} + \cdots + c_k f_{n-k}$$

Q: Find the closed form

$s_0 = -2$

$s_1 = 20$

$s_n = s_{n-1} + 12\, s_{n-2}$ for $n \geq 2$

Step 0: Is it a LHRR?

Yes, LHRR, degree = 2

Step 1: Assume $s_n = c * x^n$

Step 2: Substitute $s_n = c * x^n$ to find x

Hence, $c * x^n = c * x^{n-1} + 12 * c * x^{n-2}$

$x^n = x^{n-1} + 12\, x^{n-2}$ (divide c for both side)

$x^2 = x + 12$ (divide $x^{n-2}$ for both side)

$x^2 - x - 12 = 0$

$(x + 3)(x - 4) = 0$

Therefore, we have $x = -3$ and $x = 4$.

Hence, $s_n = c_1 * (-3)^n + c_2 * 4^n$

Step 3: Use the base case to find c

$s_0 = c_1 * (-3)^0 + c_2 * 4^0 = -2 \rightarrow c_1 + c_2 = -2$

$s_1 = c_1 * (-3)^1 + c_2 * 4^1 = 20 \rightarrow -3\, c_1 + 4c_2 = 20$

Therefore, $c_1 = -4$, $c_2 = 2$

That is, $s_n = -4 * (-3)^n + 2 * 4^n$

Q: Find the recursive form given the closed form: $s_n = 5 * 2^n + 2 * 4^n$

Therefore, $x_1 = 2$, $x_2 = 4$

So we have $(x - 2)(x - 4) = 0$

$$x^2 - 6x + 8 = 0$$

$$x^2 = 6x - 8$$

Then we have $x^n = 6x^{n-1} - 8x^{n-2}$ (multiply both sides by $x^{n-2}$ )

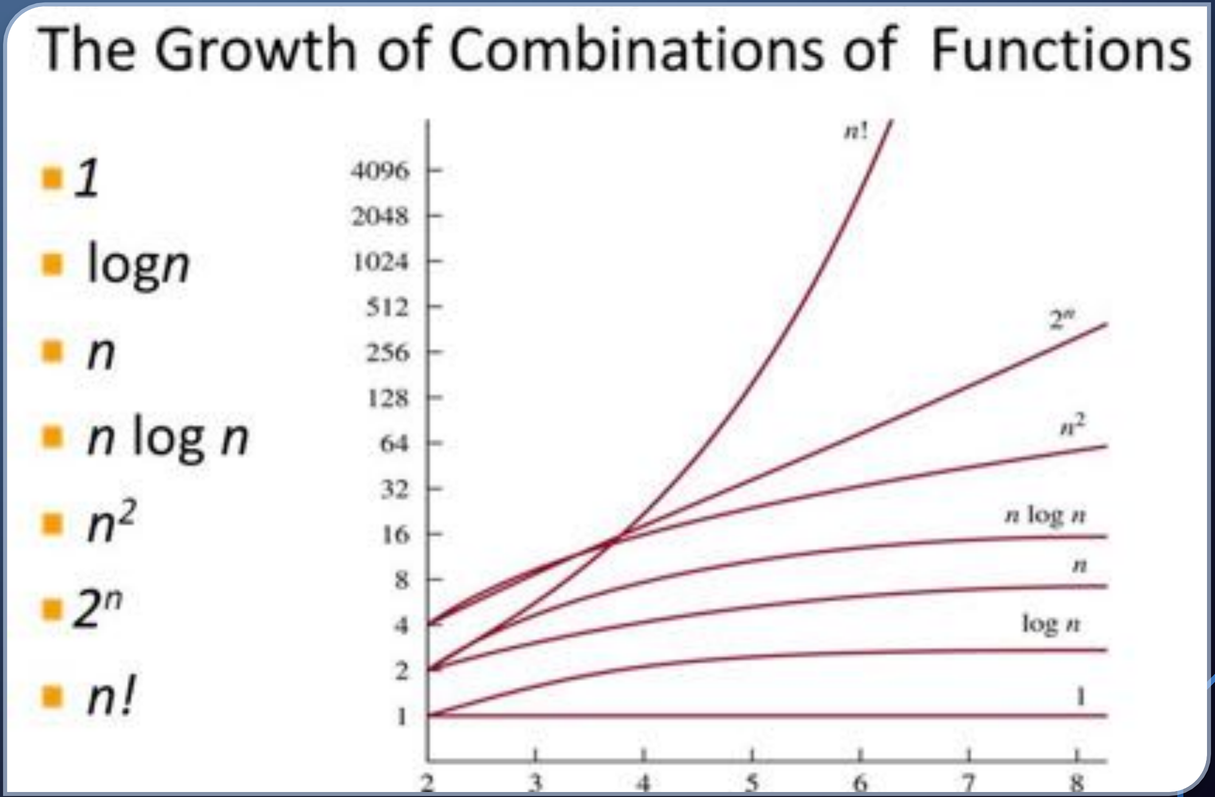
$s_n = 6 s_{n-1} - 8 s_{n-2}$ for $n \geq 2$

$s_0 = 5 * 2^0 + 2 * 4^0 = 7$

$s_1 = 5 * 2^1 + 2 * 4^1 = 18$

# STEPS TO FIND BIG – O OF A FUNCTION

1) Find the dominant term (as a function of the variable)
2) Replace all other functions with the variable with 1)
3) Simplify
4) Remove constants




$$n! > 2^n > n^2 > n\log n > n > \log n > 1$$

# SOME BIG – O PROPERTIES

$n! > 2^n > n^2 > n \log n > n > \log n > 1$

- When adding functions, the Big – O of the fastest growing function dominates
  - E.g., $3n \log n + 2000 \log n + 20n + 10000 \in O(n \log n)$
- When multiplying functions, the Big – O of the functions are multiplied
  - E.g., $n^3(n^2 \log n + n)$
  - $n^3 \in O(n^3)$
  - $n^2 \log n + n \in O(n^2 \log n)$
  - Therefore, $n^3(n^2 \log n + n) \in O(n^3 \cdot n^2 \log n) = O(n^5 \log n)$

# RECAP

- A binary relation between two sets A and B is a subset R of $A \times B$ ($aRb$).
- Representation
  - Set, Matrix, Directed Graph
- Reflexive: $\forall x \in S,\ xRx$
- Irreflexive / anti-reflexive $\forall x \in S,\ xRx$
- Transitive $\mathrm{x}Ry \wedge yRz \rightarrow xRz$
- Symmetric $\mathrm{x}Ry \Leftrightarrow yRx$
- Anti-symmetric $\mathrm{x}Ry \wedge yRx \rightarrow x = y$
- Asymmetric $\mathrm{x}Ry \rightarrow yRx$

# OPERATIONS ON BINARY RELATIONS

- Union
- Intersection
- XOR
- Composition
  - The pair $(a, c) \in S \circ R$ if and only if there is a b ∈ A such that (a, b) ∈ R and (b, c) ∈ S.