PREDICATES & QUANTIFIERS

zyBooks Chapter: 4.8, 4.9, 4.13

RECAP

- Nested Quantifiers
- De Morgan's Law
  - $\neg\forall x\exists y\ P(x, y) \equiv \exists x\forall y\ \neg P(x, y)$
  - $\neg\exists x\forall y\ P(x, y) \equiv \forall x\exists y\ \neg P(x, y)$

RULES OF INFERENCE WITH QUANTIFIERS

Existential Instantiation

Existential Elimination

$\exists x\, P(x)$

$\therefore P(c)$ for some c

Existential Generalization

Existential Introduction

$P(c)$ for some c

$\therefore \exists x\, P(x)$

RULES OF INFERENCE WITH QUANTIFIERS

Universal Instantiation

Universal Elimination

$\forall x\ P(x)$

$\therefore P(c)$

Universal Generalization

Universal Introduction

$P(c)$ for every c

$\therefore \forall x\ P(x)$

Example 1:

All W. Shakespeare books are famous.

The library has a book written by W. Shakespeare.

∴ The library has some famous books.

Step 1: Define predicates

S(x): x is a W. Shakespeare book

F(x): x is a famous book

L(x): x is in the library

Step 2: Translate

$\forall x \, (S(x) \rightarrow F(x))$

$\exists x \, (S(x) \land L(x))$

$\therefore \exists x \, (F(x) \land L(x))$

Step 2: Translate

$\forall x \, (S(x) \rightarrow F(x))$

$\exists x \, (S(x) \wedge L(x))$

$\therefore \exists x \, (F(x) \wedge L(x))$

Step 3: Construct the proof

1	$\forall x \, (S(x) \rightarrow F(x))$		Given
2	$\exists x \, (S(x) \wedge L(x))$		Given
3	$S(c) \wedge L(c)$	2	Existential Elimination, $x = c$
4	$S(c)$	3	Simplification
5	$L(c)$	3	Simplification
6	$S(c) \rightarrow F(c)$	1	Universal Elimination, $x = c$
7	$F(c)$	4, 6	Modus Ponens
8	$F(c) \wedge L(c)$	5, 7	Conjunction
9	$\exists x \, (F(x) \wedge L(x))$	8	Existential Introduction on c

Example 2:

Linda is making a cheesecake.

All cheesecakes are delicious.

∴ Linda is making something delicious.

Step 1: Define predicates

$C(x)$: x is a cheesecake

$D(x)$: x is delicious

$M(x)$: Linda is making x

Step 2: Translate

$\exists x\ (\ M(x) \wedge C(x)\ )$

$\forall x\ (\ C(x) \rightarrow D(x)\ )$

$\therefore \exists x\ (\ M(x) \wedge D(x)\ )$

Step 2: Translate

$\exists x\ (\ M(x) \wedge C(x)\ )$

$\forall x\ (\ C(x) \rightarrow D(x)\ )$

$\therefore\ \exists x\ (\ M(x) \wedge D(x)\ )$

Step 3: Construct the proof

1	$\exists x\ (\ M(x) \wedge C(x)\ )$		Given
2	$\forall x\ (\ C(x) \rightarrow D(x)\ )$		Given
3	$M(a) \wedge C(a)$	1	Existential Elimination, $x = a$
4	$M(a)$	3	Simplification
5	$C(a)$	3	Simplification
6	$C(a) \rightarrow D(a)$	2	Universal Elimination, $x = a$
7	$D(a)$	5, 6	Modus Ponens
8	$M(a) \wedge D(a)$	4, 7	Conjunction
9	$\exists x\ (\ M(x) \wedge D(x)\ )$	8	Existential Introduction on a

Example 3: Prove the following argument

$$\forall x \, (P(x) \vee Q(x))$$

$$\forall x \, \neg P(x)$$

$$\exists x \, (Q(x) \rightarrow S(x))$$

$$\therefore \exists x \, S(x)$$

Example 3:

$\forall x\ (\ P(x) \vee Q(x)\ )$

$\forall x\ \neg P(x)$

$\exists x\ (\ Q(x) \rightarrow S(x)\ )$

$\therefore\ \exists x\ S(x)$

1	$\forall x\ (\ P(x) \vee Q(x)\ )$		Given
2	$\forall x\ \neg P(x)$		Given
3	$\exists x\ (\ Q(x) \rightarrow S(x)\ )$		Given
4	$Q(a) \rightarrow S(a)$	3	Existential Elimination, $x = a$
5	$P(a) \vee Q(a)$	1	Universal Elimination, $x = a$
6	$\neg P(a)$	2	Universal Elimination, $x = a$
7	$Q(a)$	5, 6	Disjunctive Syllogism
8	$S(a)$	4, 7	Modus Ponens
9	$\exists x\ S(x)$	8	Existential Introduction on a