COUNTING

zyBooks Chapter: 2.1~2.3, 2.10, 2.11

LOGISTICS

HW1 is released!

Deadline: 11:59pm on May 20

Make sure you have access to WebAssign, zyBooks, and piazza

RECAP

- Set Definition – A collection of elements.
  - Order of elements does not matter. Repeated elements are listed once
- Set-builder Notation
  - **e.g.,** $\mathbb{Q} = \{ \frac{m}{n} \mid m, n \in \mathbb{Z}, n \neq 0 \}$
- Cardinality – size of set
- 4 Common Sets – $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$
- 4 Basic Set Operations
  - Union, Intersection, Difference, Complement

RECAP

A = { Apple Pie, Brownie, Cheesecake, Donut, Frozen Yogurt}

B = { Brownie, Cinnamon Roll, Donut, Ice Cream}

|A ∪ B| = ?   7

A ∩ B = ?   { Brownie, Donut }

A – B = ?   { Apple Pie, Cheesecake, Frozen Yogurt }   in A not in B

WHY COUNTING?

- How many passwords of 8 characters are there containing at least 1 special character (e.g. @, #, !) and 1 digit?
- How many tries an attacker needs to make to guess a user's password?
- How many ways are there to assign 20 programs to a quad-core CPU?
- If you flip a coin 3 times, what's the probability of observing exactly 2 heads? How about 30 times? Or even 300 times?

COUNTING TECHNIQUES

- Product & Sum Rule
- Counting by complement
- Permutations (counting sequences)
- Combinations (counting subsets)
- Perm/Com with replacement (multiset)
- Pigeonhole Principle

PRODUCT RULE

Let $A_1, A_2, \ldots, A_n$ be finite sets. Then,

$|A_1 \times A_2 \times \ldots \times A_n| = |A_1| \cdot |A_2| \cdot \ldots \cdot |A_n|$

Sequence of sets MATTERS!

Cartesian Product

$A = \{ 1, 2, 3 \}, \quad B = \{ x, y \}$

$A \times B = \{ (a, b) \mid a \in A \text{ and } b \in B \}$

$= \{ (1, x), (1, y), (2, x), (2, y), (3, x), (3, y) \}$

$B \times A = \{ (x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3) \}$

PRODUCT RULE

In subsequent tasks A and B, if there are x ways to do task A and y ways to do task B, then there are $x \cdot y$ ways to do A then B.

PRODUCT RULE

Q: How many bit strings are there of length 5?

$B = \{0, 1\}$

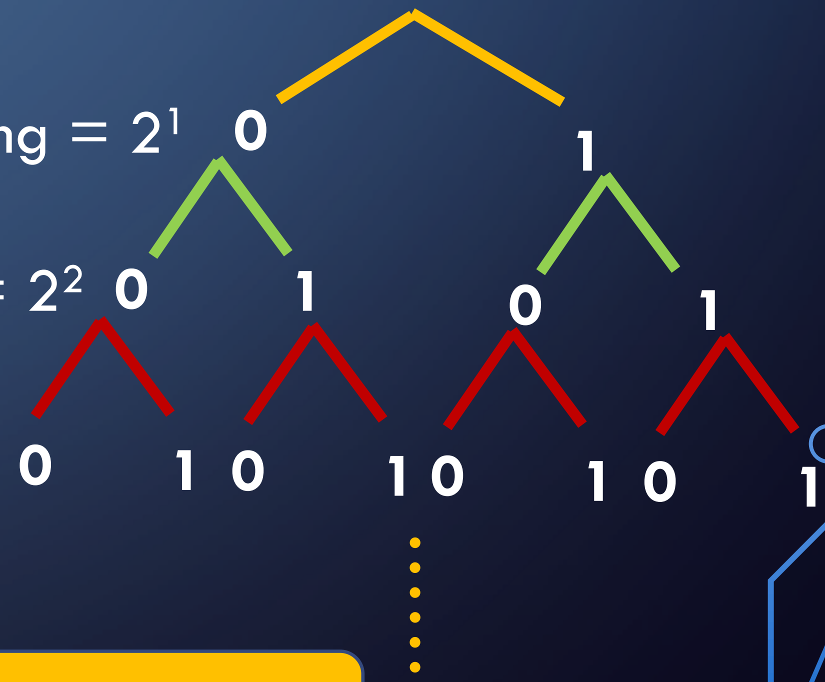
$|B| = 2$

$\#String = 2^5$

L = 1, $\#String = 2^1$

L = 2, $\#String = 2^2$

L = 3, $\#String = 2^3$



L = n?...

$\#String = 2^n = Cardinality^n$

PRODUCT RULE

Q: How many strings with upper case letters are there of length 5?

$L = \{ A, B, C, \ldots, X, Y, Z \}$

$|L| = 26$

$\#String = 26^5$

PRODUCT RULE

Q: How many license plates can be made if each plate consists of 3 upper case letters and 4 numbers?

$L = \{ A, B, C, \ldots, X, Y, Z \}$, $|L| = 26$

$N = \{ 0, 1, 2, \ldots, 8, 9 \}$, $|N| = 10$

$$\underline{26} \times \underline{26} \times \underline{26} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10} = 26^3 \times 10^4$$

PRODUCT RULE

Q: A lunch box consists of 1 main course, 2 sides and 1 drink. Let's assume we have 5 options for main course, 10 for sides and 5 for drinks. How many ~~different~~ lunch boxes one can make?

$|Main| = 5$

$|Side| = 10$

$|Drink| = 5$

$\underline{5} \times \underline{10} \times \underline{10} \times \underline{5} = 2500$

PRODUCT RULE

Q: How many bit strings are there of length 8 and start with 00?

Hint: the first 2 places are fixed; we only consider the remaining 6 places.

$|B| = 2$

$\#String = 2^6$

SUM RULE

Consider n sets, A1, A2,...,An. If the sets are mutually disjoint ($A_i \cap A_j = \emptyset$ for $i \neq j$), then

$$|A_1 \cup A_2 \cup \ldots \cup A_n| = |A_1| + |A_2| + \ldots + |A_n|$$

In separate tasks A and B, if there are x ways to do task A and y ways to do task B, then there are x+y ways to do A and B.

PRODUCT RULE VS. SUM RULE

Sequences

GoPack Bowl
(Greens, Tofu, …)

Wolfie Bowl
(Sushi Rice, tuna, …)

DIY Your Bowl

1. Pick a Base: Sushi Rice, Greens, Brown Rice
2. Pick a Fish: Tuna, Salmon
3. Pick three Toppings: …….
4. Pick a Dressing: …….

Pick from disjoints

SUM RULE

A student can choose from 3 sets of projects:

AI-Project = {AI_p1, AI_p2, …, AI_p20}
Crypto-Project = {C_p1, C_p2, …, C_p25}
Database-Project = {DB_p1, DB_p2, …, DB_p10}

Q: How many ways are there for a students to pick a project?

| AI-Project | = 20

| Crypto-Project | = 25

| Database-Project | = 10

20 + 25 +10 = 55

Q: How many passwords that consist of letters and digits are there between 6 and 8 characters?

Hint: a letter can be upper case or lower case.

$|C| = |L| + |D| = 26 \times 2 + 10 = 62$

P6: sets of pwd of length 6. $\rightarrow$ $|P6| = 62^6$

P7: sets of pwd of length 7. $\rightarrow$ $|P7| = 62^7$

P8: sets of pwd of length 8. $\rightarrow$ $|P8| = 62^8$

In total, there are $|P6| + |P7| + |P8|$ possible passwords

Q: How many 8-bit strings are there that either start with 1 or end with 00?

S1: Strings that start with 1. ➔ $|S1| = 2^7$

S2: Strings that end with 00. ➔ $|S2| = 2^6$

So we have $|S1| + |S2|$?

NO!!! There are OVERLAPS! ➔ 1_ _ _ _ _ 00

Duplicates: $|S1 \cap S2| = 2^5$

We have $|S1| + |S2| - |S1 \cap S2| = |S1 \cup S2|$

Principle of Inclusion-Exclusion

COMPLEMENT RULE

Q: How many passwords are there of length 6 that must contain at least 1 digit?

Hint: Count the INVALID strings, that is, the strings with NO digits

$|\text{BAD psw}| = 52^6$ ($|\text{letter}| = 52$)

$|\text{ALL Possible psw}| = 62^6$ ($|\text{letter}| + |\text{number}| = 62$)

Therefore, we have $|\text{ALL Possible psw}| - |\text{BAD psw}|$