ON THE RADIATION EFFICIENCY OF THE VIOLIN

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ABSTRACT

Building upon previous research by Esteban Maestre and Julius Smith [1], the radiation efficiency function of a violin is considered. This is the ratio of the measured pressure that reaches a fixed microphone and the input force into the violin. We model the input based on the impedance of the bridge. Since radiation patterns are difficult to compute and the literature on musical instrument efficiency is quite limited, research is ongoing.

1. INTRODUCTION

We define efficiency η as a ratio of the work out versus the energy in to a system. By thermodynamics, η is in the range $0 \leq \eta \leq 1$. Extending this idea to the efficiency of a musical instrument, we say that

$$\eta = \frac{P_{rad}}{P_0}, \tag{1}$$

where P_{rad} is the *radiated power* received by a microphone at a radius r from the source, and P_0 is the power input to the instrument.

From a microphone recording of the violin at a known location, we remove the effects of attenuation due to distance as well as the effects of the recording environment, and define a measured radiation pressure P_{rad}. From Maestre's research, the bridge transmittance, admittance, and impedance transfer functions have been determined.

The modes can be determined by the poles of the impedance transfer function since the numerator is positive and real. We then prescribe these modes to a set of basis functions, and project the FFT of measurements taken by a microphone onto them.

Ideally, the radiation pattern is captured by a large number of microphones spaced a sampling interval apart, and locations carefully recorded. This research may indicate that more recordings should be taken to better understand the efficiency in three dimensions.

2. BACKGROUND

In [1], the input admittance bridge filter is described using a lumped string, positive real modal framework. Horizontal and vertical transfer functions Y_{xx} and Y_{yy} were measured from the bridge of a violin with a force hammer. Using (1) **direct linear projection** and (2) **iterative optimization**, the admittance filter was designed from initial estimation of modes via peak picking. We use a similar technique in the example below.

By (1) direct linear projection, poles p_i are extracted for each mode i by the following summand. The filter is paramaterized by f_{min}, f_{max}, and N, where N is the number of peaks (modes) over a bandwidth.

$$\hat{Y}(z) = \mathbf{g}\sum_{i=1}^{N} M_i(z), \tag{2}$$

$$M_i(z) = \frac{1-z^{-2}}{(1-p_i z^{-1})(1-p_i^* z^{-1})} \tag{3}$$

Then, $\mathbf{g}$ is minimized with least squares.

$$\min_{\mathbf{g}} = |\hat{Y}(f) - Y(f)|^2, \quad \mathbf{g} \geq 0 \tag{4}$$

Due to the relatively numerous modes in the low frequency region ($f_{max} = 1200$) and only one "broad" mode in the high frequencies (the "bridge hill"), Maestre et al call $f_{min} = 150 < f < 1200 = f_{max}$ the relevant bandwidth. Below 150 Hz, artifacts are present from the physical measurement setup, and above 1200 Hz, artifacts are scene due to processing and sensor noise. N can be any integer but ideally not in excess of the peaks counted by graphical inspection.

The results from (2) iterative optimization, where

$$Y(z) = g(1-z^{-2})\prod_{i=1}^{N+1}\frac{1}{(1-p_i z^{-1})(1-p_i^* z^{-1})} \tag{5}$$

$$\cdot\prod_{j=1}^{N}(1-q_j z^{-1})(1-q_j^* z^{-1}), \tag{6}$$

leads to an error function $\varepsilon(g, \mathbf{p}, \mathbf{q})$, computed as the least-squares (or alternatively, the log-magnitude) between the measured transfer function $Y(\omega)$ and modeled $\hat{Y}(\omega)$, and then minimized. Then, the poles are optimized by solving the linear system of the projections of the measured and modeled transfer functions onto the basis vectors of the modes.

F. G. Leppington gives a rigorous integral model of the radiation efficiency of acoustic panels in [2], equating it to

$$\begin{aligned}
\sigma &= (\rho_0 cab)^{-1} R_{rad} && (7)\\
&= \frac{2k^2}{\pi ab}\int_0^a\int_0^b \left\{(a-u)\cos\alpha ku + \frac{1}{\alpha k}\sin\alpha ku\right\} && (8)\\
&\quad \times \left\{(b-v)\cos\beta kv + \frac{1}{\beta k}\sin\beta kv\right\}\frac{\sin kr}{kr}\,dv\,du && (9)
\end{aligned}$$

where σ is the radiation efficiency, R_{rad} the radiation impedance, $\alpha = \frac{m\pi}{ka}$, $\beta = \frac{n\pi}{kb}$, (α, β non-dimensional wavenumbers, m, n integers), $r = \sqrt{u^2+v^2}$, and c the speed of sound. Though a lot to parse, integral models make sense for 3D applications.

Patricio de la Cuadra also explores integral models of radiation efficiency in his dissertation on flutes [3]. He states that

$$P_{rad} \leq \frac{1}{T}\int_T \frac{|p(r)|^2}{\rho_0 c} 4\pi r^2\,dt \qquad (10)$$

where P_{rad} is the radiated power, $p(r)$ is the radiated pressure (the raw microphone signal), T the period of oscillations, and c the speed of sound in the air.

Since Cuadra was concerned with sound generation in the flute, P_0 is the power of the jet flow, and is estimated as

$$\begin{aligned}
P_0 &= \frac{1}{T}\int_T S_m U_j P_m\,dt && (11)\\
&= \frac{1}{T}\int_T S_m \sqrt{\frac{2P_m}{\rho_0}} P_m\,dt && (12)
\end{aligned}$$

where S_m is the surface area of the hole formed by the lips, U_j is the average jet velocity (from Bernoulli's equation), $\rho_0 = 1.2$ kg/m^3, the mass density of ideal air, and P_m is the "slow-moving component" that does **not** oscillate at the frequency of the input pitch, representing the difference between the pressure traveling down towards the labium from the player's lips p_m and the pressure reflecting back into it p'_m.

In Bissinger [4], he defines *radiativity* $R(\omega)$ as a complex ratio of the pressure $P_{out}(\omega)$ over the input force $F_{in}(\omega)$. Then he states proportional relationships between the intensity and all of pressure, force, mobility, and radiation efficiency:

$$I(\omega) \propto P^2(\omega) \propto F^2(\omega) * Y^2_{corpus}(\omega)_{rms} * R_{eff}$$

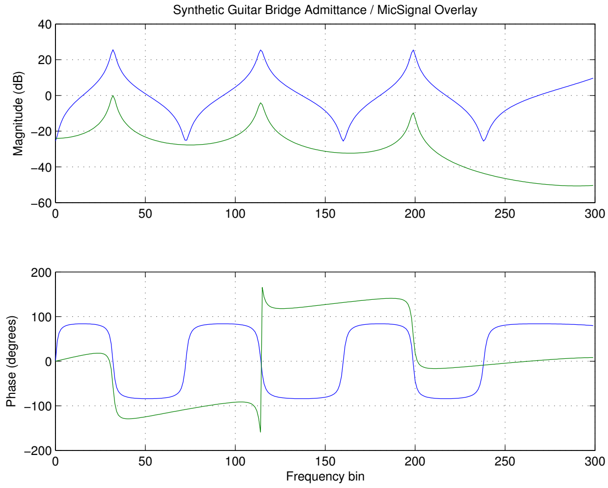

Figure 1. The amplitude and phase response of the synthetic guitar bridge admittance (blue) overlaid with the normalized, measured spectrum (green).

where $Y(\omega) = \frac{V(\omega)}{F(\omega)}$, the *mobility*, a complex ratio of surface velocity and force. However, explicit formulas for how to compute the input force are not given.

3. AN EXAMPLE: SYNTHETIC GUITAR BRIDGE IMPEDANCE

To illustrate this technique, we use a synthetic guitar example. Referring to Fig. 1, modes are clearly visible in both the green (microphone) spectrum and the blue (violin admittance) transfer function, with identical center frequencies but different resonant Q values and damping in the microphone case.

There are 4 modes in this example. Mode M_1 is centered at 32Hz, M_2 at 114Hz, M_3 at 199Hz, and M_4 at 330Hz.

There are also 3 antinodes in the bridge admittance at $f = 72.5$Hz, $f = 160$Hz, and $f = 238$Hz that are nearly local minima in the measurement curve, except for the third antinode.

The frequency response of a filter can be well represented as a linear combination of its modes. We define 4 basis vectors for the frequency responses of these modes and project the spectrum of the microphone's measurements to encapsulate the gain, bandwidth, and sharpness (Q) of just the peaks. First we obtain just the real component, and then we examine the complex part of this projection.

3.1. Real implementation

Because the admittance filter was designed in a positive real modal framework, we first proceed by ignoring the phase of the transfer functions in order to study only the magnitude of the power spectrum. Since the example is generated from MATLAB code wherein the coefficients of the filters are known, we extract our bases with the function `freqz()` from the poles of the biquads (the normalized denominator of the admittance), call it A_{adm}. Here is the code used to generate the admittance poles.

```
% Measured guitar body resonances (Hz)
F = [4.64 96.52 189.33 219.95];

nsec = length(F);

R = exp(-pi*B/fs);       % Pole radii
theta = 2*pi*F/fs;        % Pole angles
poles = R .* exp(j*theta);
A1 = -2*R.*cos(theta);    % 2nd-order coeff
A2 = R.*R;                % 2nd-order coeff
denoms = [ones(size(A1)); A1; A2]';
A = [1,zeros(1,2*nsec)]; % 8th order
for i=1:nsec,
    A = filter(denoms(i,:),1,A);
end;

g = 0.9;               % Uniform loss factor

B = g*fliplr(A);       % the desired Schur allpass

Badm = A - B;          % numerator: positive & real
Aadm = A + B;          % poles
Badm = Badm/Aadm(1);   % Renormalize / make monic
Aadm = Aadm/Aadm(1);
```

Then, the basis functions for each mode are derived from A_{adm}:

```
Bas = tf2sos(1, Aadm);
b1 = freqz(Bas(1,1:3),Bas(1,4:6),nfft,'whole',fs);
b2 = freqz(Bas(2,1:3),Bas(2,4:6),nfft,'whole',fs);
b3 = freqz(Bas(3,1:3),Bas(3,4:6),nfft,'whole',fs);
b4 = freqz(Bas(4,1:3),Bas(4,4:6),nfft,'whole',fs);
```

We take the power of these and normalize them to get 4 bandlimited filters of each mode.

Then, the measured spectrum from Fig. 1 is divided by the sum of these bases (and again normalized):

These graphs are not what we expected, with strong troughs and crests. I also attempted a “Bessel” exponent (3/2) which was of course a smoother result.

These have unitary gain at the center frequencies of the peaks in the microphone response. Examining the amplitude of the response in decibels, we see 3 bandpass filters at each of these f_c's, with $f_1 = f_c - 1$ Hz and $f_2 = f_c + 1$ Hz, rounding to the integer frequencies because that is the resolution of our bins. This implies our bandwidth $\beta = 2$Hz and $Q = f_c/2$.

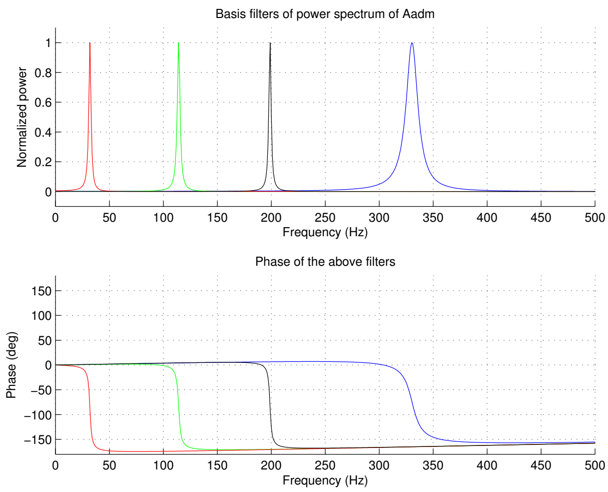

Figure 2. The bases are derived from the MATLAB functions used to generate the guitar IR, using `freqz()`, `infreqz()`, and `tf2sos()`. These are the bases' powers and phase.

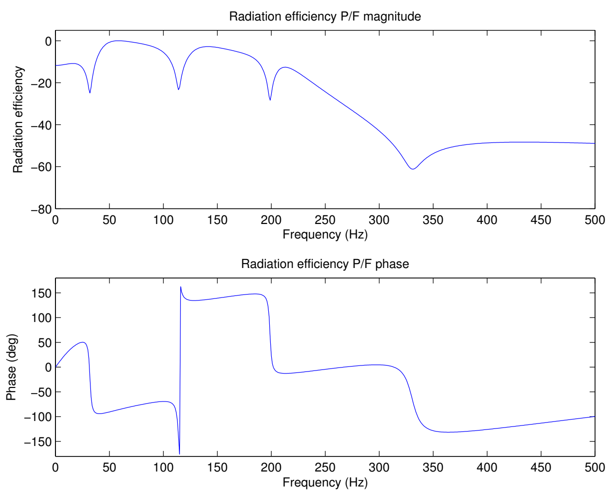

Figure 3. The mic spectrum divided by the sum of the power of the bases.

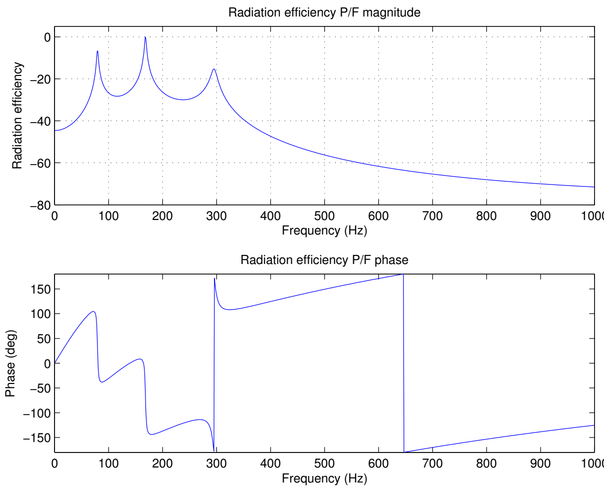

Figure 4. The mic spectrum divided by the sum of the bases (not power).

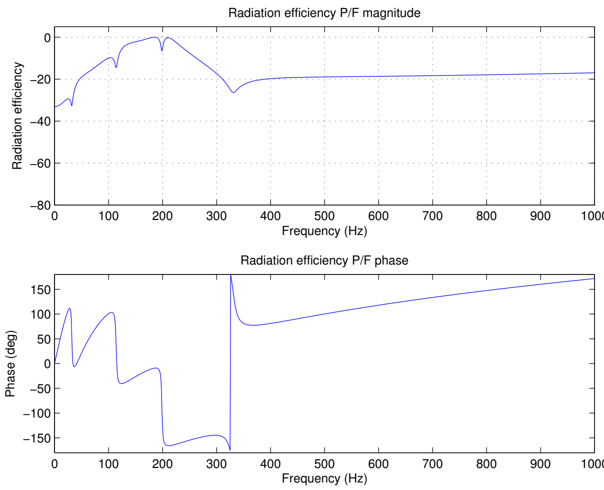

Figure 5. The mic spectrum divided by the sum of the bases (3/2 power).

The power spectrum of the microphone response M is a 8192×1 matrix whose ith element is the power of frequency bin i. Projecting M onto the bases, we get the real coefficients $\mathbf{g} = \{2.169 \times 10^{13}, 3.9727 \times 10^{12}, 5.8224 \times 10^{11}\}$ at our mode frequencies. These numbers are large due to the definition of the matrices A and B (see the attached MATLAB code). We observe that the gain decreases by $\approx 85\%$ as frequency increases in the microphone power spectrum.

These gains can be put into `invfreqz` to return the zeros and gains of our transfer function, but I am not sure of the proper order for B and A. With $|B| = 1$ and $|A| = 3$, we get $B = \{-0.4210, -4.6615\} \times 10^9$ and $A = \{1.000, -1.8565, 1.3409, -0.2154\}$.

3.2. Complex implementation

To address the phase of the transfer functions, we multiply the argument of the summand of the modeled transfer function with $e^{j\omega}$. This will allow us to simulate different microphone/violin positions, as these vary the locations of the nulls. All of the zeros and poles are real, but the gain $\mathbf{g}$ could be complex.

Using the same bases but retaining the imaginary component of the microphone response, we get

$$\mathbf{g} = \{1.6329 - j4.3616, -1.9787 + j0.2400, \quad (13)$$
$$0.67151 + j0.3627\} \times 10^6 \quad (14)$$

Again assuming that the order of B is 1 and the order of A is 3, `invfreqz` gives us

$$B = \{0.0682 - j0.2445, 0.0569 + j0.0056\} \times 10^{-11} \quad (15)$$
$$A = \{1.000, -0.9821 - j0.4546, 0.5768 - j0.9157, \quad (16)$$
$$-0.8391 + j0.5440\} \quad (17)$$

4. CONCLUSIONS & FUTURE WORK

From these efforts, the right path to pursue is still unclear. My intuitions say that things like bridge impedance and acoustic velocity could be all we need, but admittance does make sense in the context of radiation. Reading more literature in the realm of aeroacoustics (like [2]) and thermodynamics could be fruitful to pinning down the proper approach.

Then, we can apply these methods to Esteban's data, which I have inspected but not experimented with at this point.

5. REFERENCES

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[3] P. de la Cuadra, "The Sound of Oscillating Air Jets: Physics, Modeling and Simulation in Flute-Like Instruments," Doctoral dissertation, Stanford University, 2005.

[4] G. Bissinger, "Structural acoustics model of the violin radiativity profile," *J. of the Acous. Soc. of Amer.*, 4013–4023, 2008.