0613413 蘇松良. 財工 HW7.

買 賣

1. Show the arbitrage opportunity exist if $F_0 < (S_0 - I)e^{rT}$

At time 0:

long forward ($K = F_0$), borrow an underlying asset short it.

Initial Cash flow = 0

At time T: $\underline{S_T - F_0} - (S_T + Ie^{rT}) + S_0 e^{rT} = \underline{S_0e^{rT} - Ie^{rT} - F_0 > 0}$

arbitrage exist. #

2. underlying stock pays a continuous dividend yield at rate q:

Show that the forward price $= F_0 = S_0 e^{(r-q)T}$

forward contract at any time prior to T is:

$$f = Se^{-qT} - Xe^{-rT} \qquad f + Xe^{-rT} - Se^{-qT} = 0$$

- Porfolio: 1 long forward contract. cash amount Xe^{-rT}. short position in e^{-qT} units of the underlying asset.
- All dividend are paid for shorting additional units of the underlying assets.
- The cash will grow to X at maturity $\Rightarrow Xe^{-rT} \cdot e^{rT} = X$.
- The short position will grow to exactly 1 unit of the underlying asset.

$\Rightarrow$ sufficient fund to take delivery of the forward contract.

- This offsets the short position.
- value of the portfolio is 0 at maturity $\Rightarrow PV = 0$

$\Rightarrow$ Forward price $= F_0 = S_0 e^{(r-q)T}$