Financial Engineering and Computations

# Basic Financial Mathematics

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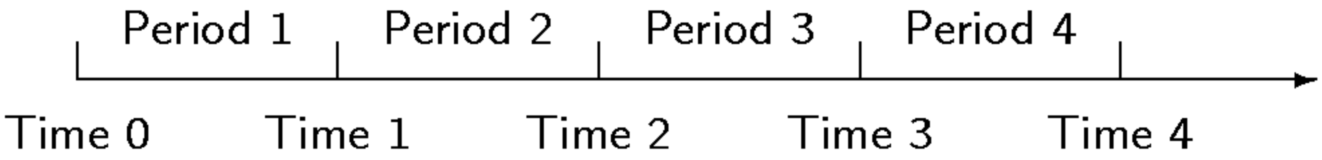
# 此章內容

- Financial Engineering & Computation教課書的第三章　Basic Financial Mathematics
- C++財務程式設計的第三章（3-4,3-5）

# Outline

- Time Value of Money
- Annuities
- Amortization
- Yields
- Bonds

# Time Value of Money

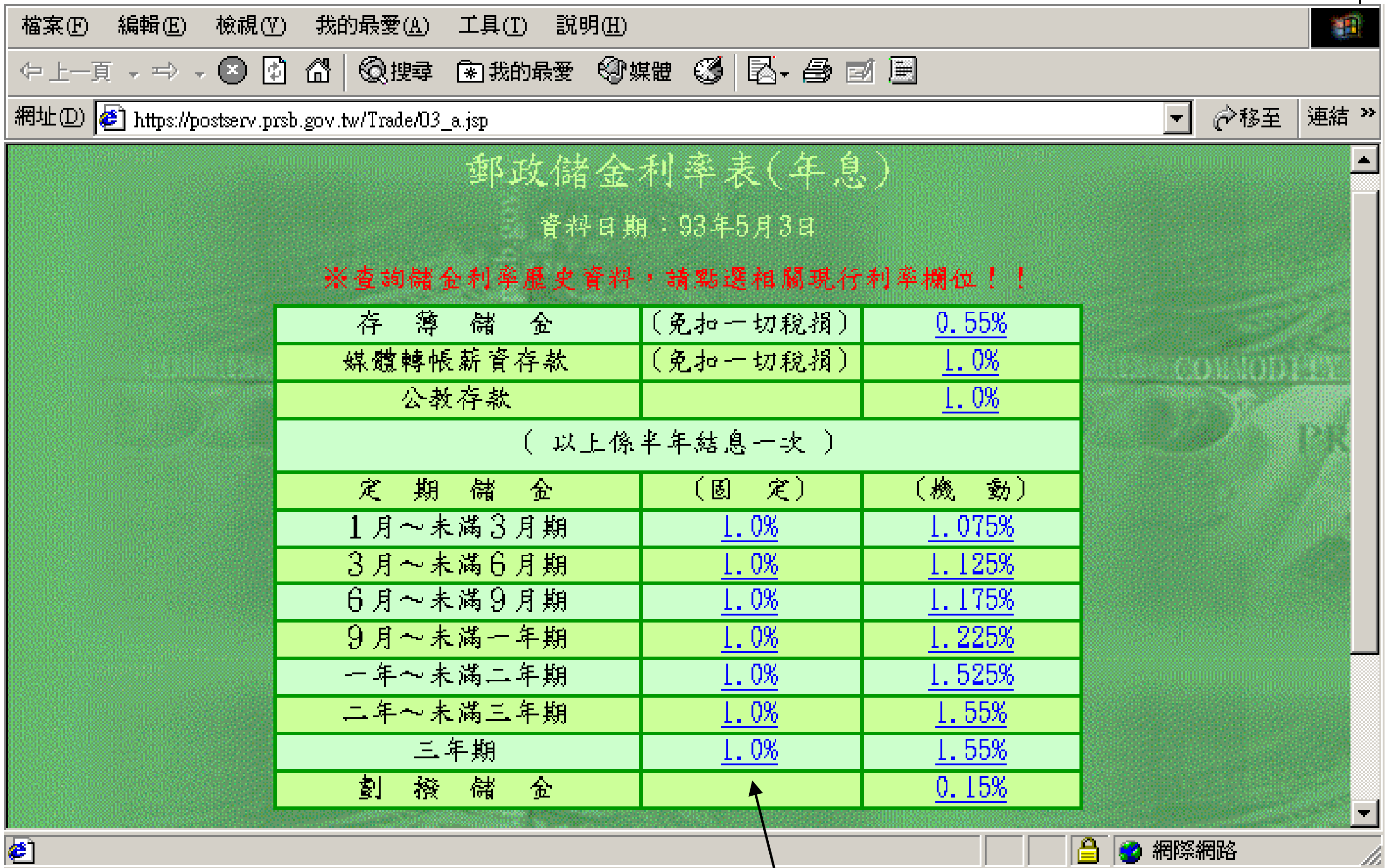



$$PV = FV(1+r)^{-n}$$

$$FV = PV(1+r)^{n}$$

- FV: future value
- PV: present value
- r: interest rate
- n: period terms

# Quotes on Interest Rates



郵政儲金利率表(年息)

資料日期：93年5月3日

※查詢儲金利率歷史資料，請點選相關現行利率欄位！！

| | | |
|---|---|---|
| 存　簿　儲　金 | （免扣一切稅捐） | 0.55% |
| 媒體轉帳薪資存款 | （免扣一切稅捐） | 1.0% |
| 公教存款 | | 1.0% |
| （以上係半年結息一次） | | |
| 定　期　儲　金 | （固　定） | （機　動） |
| 1月～未滿3月期 | 1.0% | 1.075% |
| 3月～未滿6月期 | 1.0% | 1.125% |
| 6月～未滿9月期 | 1.0% | 1.175% |
| 9月～未滿一年期 | 1.0% | 1.225% |
| 一年～未滿二年期 | 1.0% | 1.525% |
| 二年～未滿三年期 | 1.0% | 1.55% |
| 三年期 | 1.0% | 1.55% |
| 劃　撥　儲　金 | | 0.15% |

Annualized rate.

$r$ is assumed to be constant in this lecture.

# Time Value of Money

- Periodic compounding

(If interest is compounded *m* times per annum)

$$FV = PV\left(1+\frac{r}{m}\right)^{nm} \qquad (3.1)$$

- Continuous compounding

$$FV = PVe^{rn}$$

$$\lim_{t\to\infty}(1+\frac{1}{t})^t = e \rightarrow \lim_{m\to\infty}(1+\frac{r}{m})^{nm} = \lim_{m\to\infty}(1+\frac{1}{m/r})^{\frac{m}{r}rn} = e^{rn}$$

- Simple compounding

# Common Compounding Methods

- Annual compounding: $m = 1$.
- Semiannual compounding: $m = 2$.
  - Bond equivalent yield (BEY)

    --Annualize yield with semiannual compounding
- Quarterly compounding: $m = 4$.
- Monthly compounding: $m = 12$.
  - Mortgage equivalent yield (MEY)

    --Annualize yield with monthly compounding
- Weekly compounding: $m = 52$.
- Daily compounding: $m = 365$

# Equivalent Rate per Annum

- Annual interest rate is 10% compounded twice per annum.
- Each dollar will grow to be 1.1025 one year from now.

$$\left(1+(0.1/2)\right)^2 = 1.1025$$

- The rate is equivalent to an interest rate of 10.25% compounded once *per annum*.

# Conversion between compounding Methods

- Suppose $r_1$ is the annual rate with continuous compounding.
- Suppose $r_2$ is the equivalent compounded m times per annum.
- Then $\left(1+\frac{r_2}{m}\right)^m = e^{r_1}$
- Therefore $r_1 = m\ln\left(1+\frac{r_2}{m}\right) \Rightarrow r_2 = m\left(e^{\frac{r_1}{m}} - 1\right)$

## Are They Really "Equivalent"?

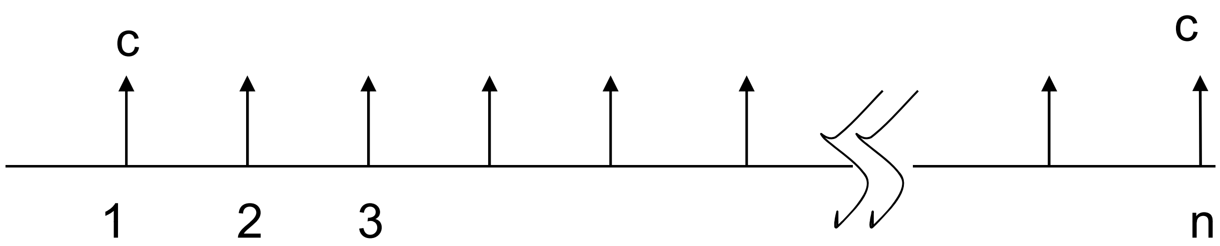
- Recall $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$ on the previous example.
- They are based on different cash flow.
- In what sense are they equivalent?

# Annuities

- An annuity pays out the same C dollars at the end of each year for years.
- With a rate or r , the FV at the end of the nth

year is

$$\sum_{i=0}^{n-1} C(1+r)^i = C\frac{(1+r)^n - 1}{r} \qquad (3.4)$$



# General Annuities

- If m payments of C dollars each are received per year, then Eq.(3.4) becomes

$$C\,\frac{\left(1+\frac{r}{m}\right)^{n\,m}-1}{\frac{r}{m}}$$

- The PV of a general annuity is

$$\sum_{i=1}^{nm} C\left(1+\frac{r}{m}\right)^{-i} = C\,\frac{1-\left(1+\frac{r}{m}\right)^{-n\,m}}{\frac{r}{m}} \qquad (3.6)$$

# Perpetual annuity

- An annuity that lasts forever is called a perpetual annuity. We can drive its *PV* from Eq.(3.6) by letting ***n*** go to infinity:

$$PV = \lim_{n\to\infty} \sum_{i=1}^{nm} C\left(1+\frac{r}{m}\right)^{-i} = \lim_{n\to\infty} C\,\frac{1-\left(1+\frac{r}{m}\right)^{-n\,m}}{\frac{r}{m}} = \frac{mC}{r}$$

- This formula is useful for valuing *perpetual fix-coupon debts.*

# Amortization

- It is a method of repaying a loan through regular payment of interest and principal.
- The size of the loan (the original balance) is reduced by the principal part of each payment.
- The interest part of each payment pays the interest incurred on the remaining principal balance.
- As the principal gets paid down over the term of the loan, the the interest part of the payment diminishes.

See next example!

# Example: Home mortgages

- Consider a 15-year, $250,000 loan at 8.0% interest rate, repay the interest 12 per month.
- Because *PV = 250,000*, *n = 15*, *m= 12*, and *r = 0.08* we can get a monthly payment *C is $2,389.13*.

$$\$250000 = \frac{C}{(1+\frac{0.08}{12})} + \frac{C}{(1+\frac{0.08}{12})^2} + \ldots\ldots + \frac{C}{(1+\frac{0.08}{12})^{12\times 15}}$$

$$= \sum_{i=1}^{180} C\left(1+\frac{0.08}{12}\right)^{-i} = C\left(\frac{1-(1+\frac{0.08}{12})^{-180}}{0.08/12}\right) \Rightarrow C = 2389.13$$

249277.536×(0.08/12)

Payment − Interest

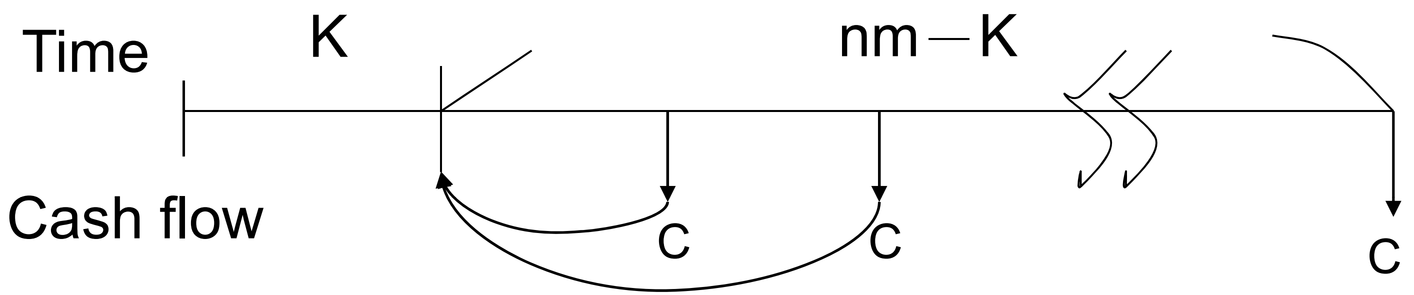
| Month | Payment | Interest | Principal | Remaining principal |
|---|---|---|---|---|
| | | | | 250,000.000 |
| 1 | 2,389.13 | 1,666.667 | 722.464 | 249,277.536 |
| 2 | 2,389.13 | 1,661.850 | 727.280 | 248,550.256 |
| 3 | 2,389.13 | 1,657.002 | 732.129 | 247,818.128 |
| | | ... | | |
| 178 | 2,389.13 | 47.153 | 2,341.980 | 4,730.899 |
| 179 | 2,389.13 | 31.539 | 2,357.591 | 2,373.308 |
| 180 | 2,389.13 | 15.822 | 2,373.308 | 0.000 |
| Total | 430,043.438 | 180,043.438 | 250,000.000 | |

We compute it in last page

# Calculating the Remaining Principal

- Right after the *k*th payment, the remaining principal is the PV of the future *nm-k* cash flows,

$$C\left(1+\frac{r}{m}\right)^{-1}+C\left(1+\frac{r}{m}\right)^{-2}+\ldots\ldots+C\left(1+\frac{r}{m}\right)^{-(nm-k)}=C\frac{1-\left(1+\frac{r}{m}\right)^{-nm+k}}{\frac{r}{m}}$$



# Yields

- The term **yield** denotes the return of investment.
- It has many variants.

  (1) Nominal yield (coupon rate of the bond)

  (2) Current yield

  (3) Discount yield

  (4) CD-equivalent yield

# Discount Yield

- U.S *Treasury bills* is said to be issue on a discount basis and is called a discount security.
- When the discount yield is calculated for short-term securities, a year is assumed to have **360 days**.
- The discount yield (discount rate) is defined as

$$\underbrace{\frac{\overbrace{par\ \ value - purchase\ \ price}^{\text{Interest}}}{par\ \ value}}_{\text{Interest rate}} \times \underbrace{\frac{360\ days}{number\ of\ \ days\ to\ maturity}}_{\text{Annualize}} \qquad (3.9)$$

# CD-equivalent yield

- It also called the money-market-equivalent yield.
- It is a simple annualized interest rate defined as

$$\frac{\text{par value} - \text{purchase price}}{\text{purchase price}} \times \frac{\mathbf{365}\ \text{days}}{\text{number of days to maturity}} \qquad (3.10)$$

# Example 3.4.1: Discount yield

- If an investor buys a U.S. $ 10,000, 6-month T-bill for U.S. $ 9521.45 with 182 days remaining to maturity.

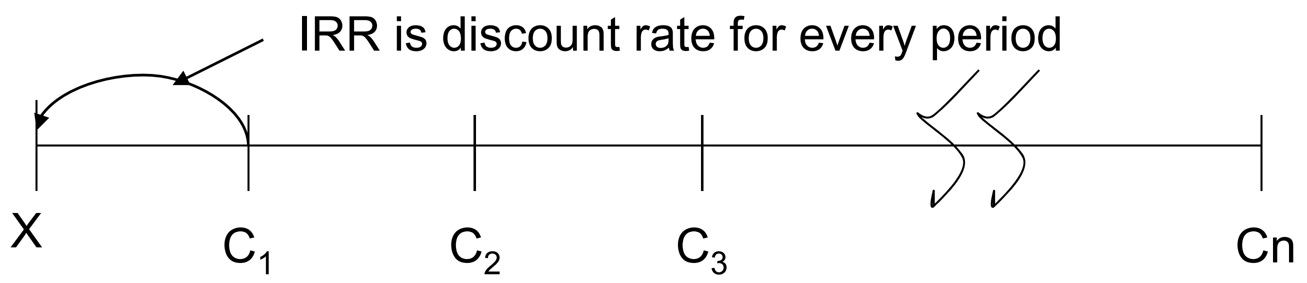
$$Discount\,yield = \frac{10000 - 9521.45}{10000} \times \frac{360}{182} = 0.0947$$

# Internal Rate of Return (IRR)

- It is the interest rate which equates an investment's PV with its price *X*.

$$X = C_1 \times (1+IRR)^{-1} + C_2 \times (1+IRR)^{-2} + \ldots + C_n \times (1+IRR)^{-n}$$

- IRR assumes all cash flows are reinvested at the same rate as the internal rate of return.
- It doesn't consider the reinvestment risk.



# Evaluating real investment with IRR

- Multiple IRR arise when there is more than one sign reversal in the cash flow pattern, and it is also possible to have no IRR.
- Evaluating real investment, IRR rule breaks down when there are multiple IRR or no IRR.
- Additional problems exist when the term structure of interest rates is not flat.

  →there is ambiguity about what the appropriate hurdle rate (cost of capital) should be.

# Class Exercise

- Assume that a project has cash flow as follow respectively, and initial cost is $1000 at date 0, please calculate the IRR. If cost of capital is 10%, do you think it is a good project?

| CF at date | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | IRR |
| -1000 | 800 | 1000 | 1300 | -2200 | ? |

# Class Exercise (Excel)

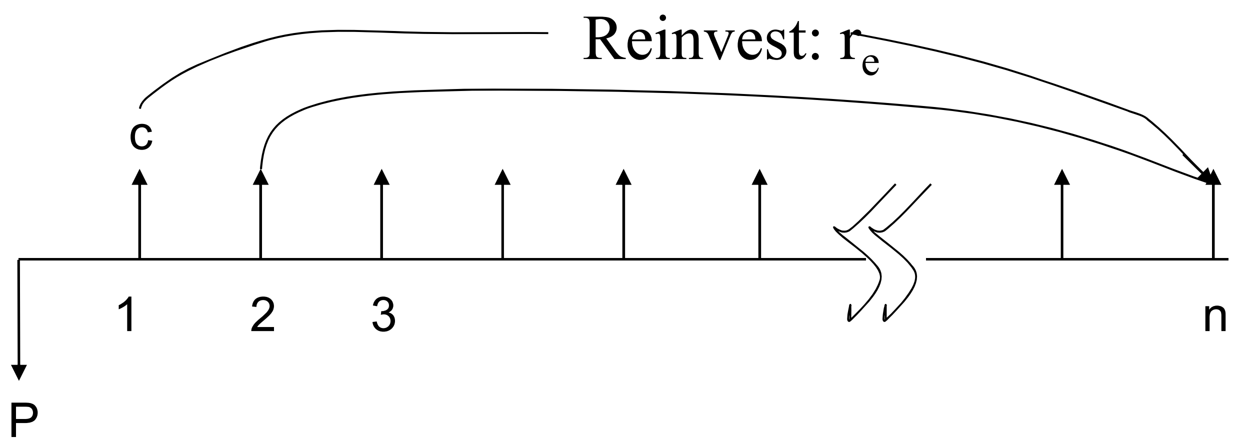
| | | |
|---|---|---|
| 12 | Time | CF |
| 13 | 0 | -1000 |
| 14 | 1 | 800 |
| 15 | 2 | 1000 |
| 16 | 3 | 1300 |
| 17 | 4 | -2200 |
| 18 | | 7% |
| 19 | | 37% |
| 20 | | |
| 21 | | |

=IRR(B13:B17,0.1)

=IRR(B13:B17,0.2)

Multiple IRR

# Holding Period Return




- The FV of investment in n period is $FV = P(1+y)^n$
- Let the reinvestment rates $r_e$, the FV of per cash income is

$$C\times(1+r_e)^{n-1} + C\times(1+r_e)^{n-2} + \ldots + C\times(1+r_e) + C \longrightarrow \text{Value is given}$$

- We define HPR (y) is

$$P(1+y)^n = C\times(1+r_e)^{n-1} + C\times(1+r_e)^{n-2} + \ldots + C\times(1+r_e) + C$$

# Methodology for the HPR($y$)

- Calculate the FV and then find the yield that equates it with the P
- Suppose the reinvestment rates has been determined to be $\boldsymbol{r_e}$.

| Step | **Periodic compounding** | **Continuous compounding** |
|---|---|---|
| (1)Calculate the future value | $FV=\sum_{t=1}^{n} C\,(1+r_e)^{n-t}$ | $FV = C\times\frac{(e^{r_e n}-1)}{e^{r_e}-1}$ |
| (2)Find the HPR | $y=\sqrt[n]{\frac{FV}{P}}-1$ | $y=\frac{1}{n}\ln(\frac{FV}{P})$ |

# Example 3.4.5:HPR

- A financial instrument promises to pay \$ 1,000 for the next 3 years and sell for \$ 2,500. If each cash can be put into a bank account that pays an effective rate of 5%.
- The FV is $\sum_{t=1}^{3} 1000 \times (1+0.05)^{3-t} = 3152.5$
- The HPR is $2500(1+HPR)^3 = 3125.5$

$$\Rightarrow HPR = \left(\frac{3152.5}{2500}\right)^{1/3} - 1 = 0.0804$$

# Numerical Methods for Yield

- Solve $f(r)=\sum_{t=1}^{n}\frac{C_t}{(1+r)^t}-x=0,\ for\ r\geq -1,\ x\ is\ market\ price$

$$Recall\ X=C_1\times(1+IRR)^{-1}+\ldots+C_n\times(1+IRR)^{-n}$$

$$\Rightarrow C_1\times(1+IRR)^{-1}+\ldots+C_n\times(1+IRR)^{-n}-X=0$$

$$Let\ f(r)=C_1\times(1+r)^{-1}+\ldots.C_n\times(1+r)-X$$

- The function *f(r)* is monotonic in *r*, if $C_t > 0$ for all t, hence a unique solution exists.

# The Bisection Method

- Start with $a$ and $b$ where $a < b$ and $f(a)\, f(b) < 0$.
- Then $f(r)$ must be zero for some $r \in (a\, ,\, b)$.
- If we evaluate $f$ at the midpoint $c \equiv (a + b) / 2$

  (1) $f(a)\, f(c) < 0 \rightarrow a<r<c$

  (2) $f(c)\, f(b) < 0 \rightarrow c<r<b$

- After $n$ steps, we will have confined $r$ within a bracket of length $(b - a) / 2^n$.

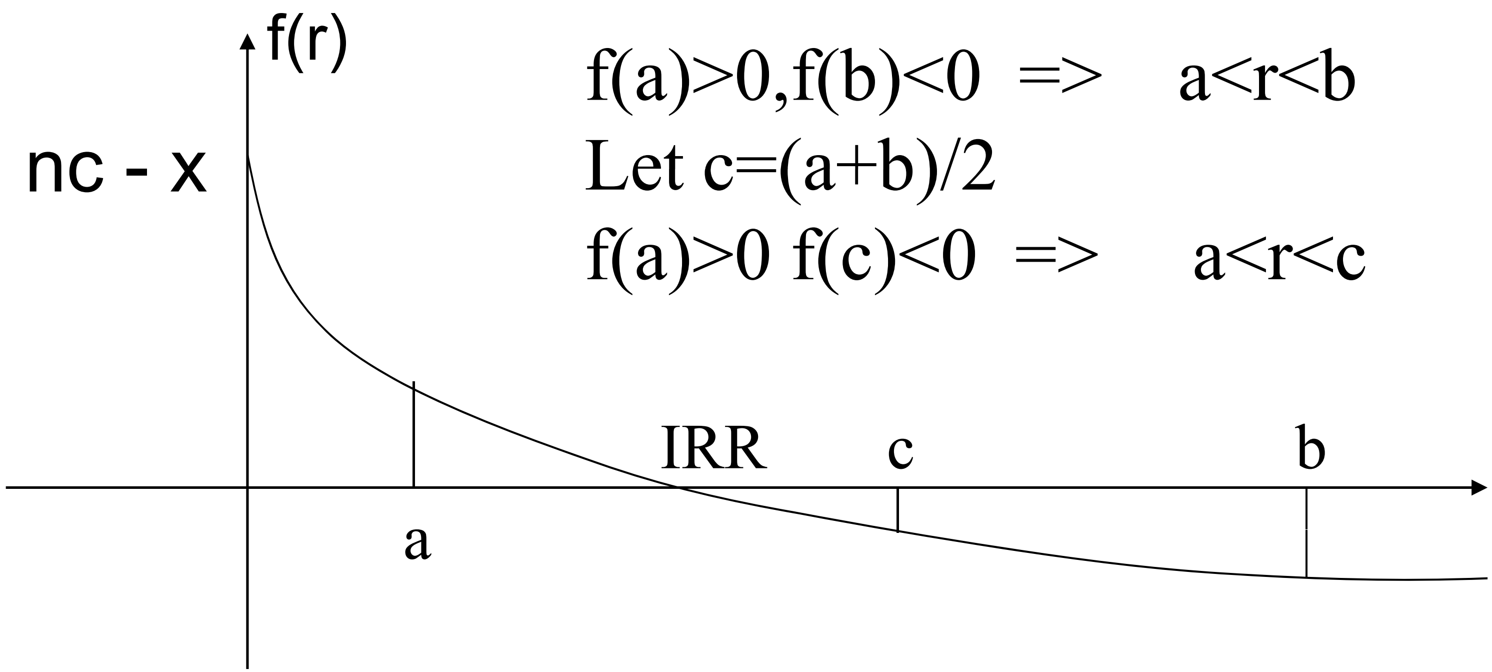
# Bisection Method

- Let $f(r)=C\times(1+r)^{-1}+C\times(1+r)^{-2}+\ldots+C\times(1+r)^{-n}-X$
- Solve $f(r)=0$

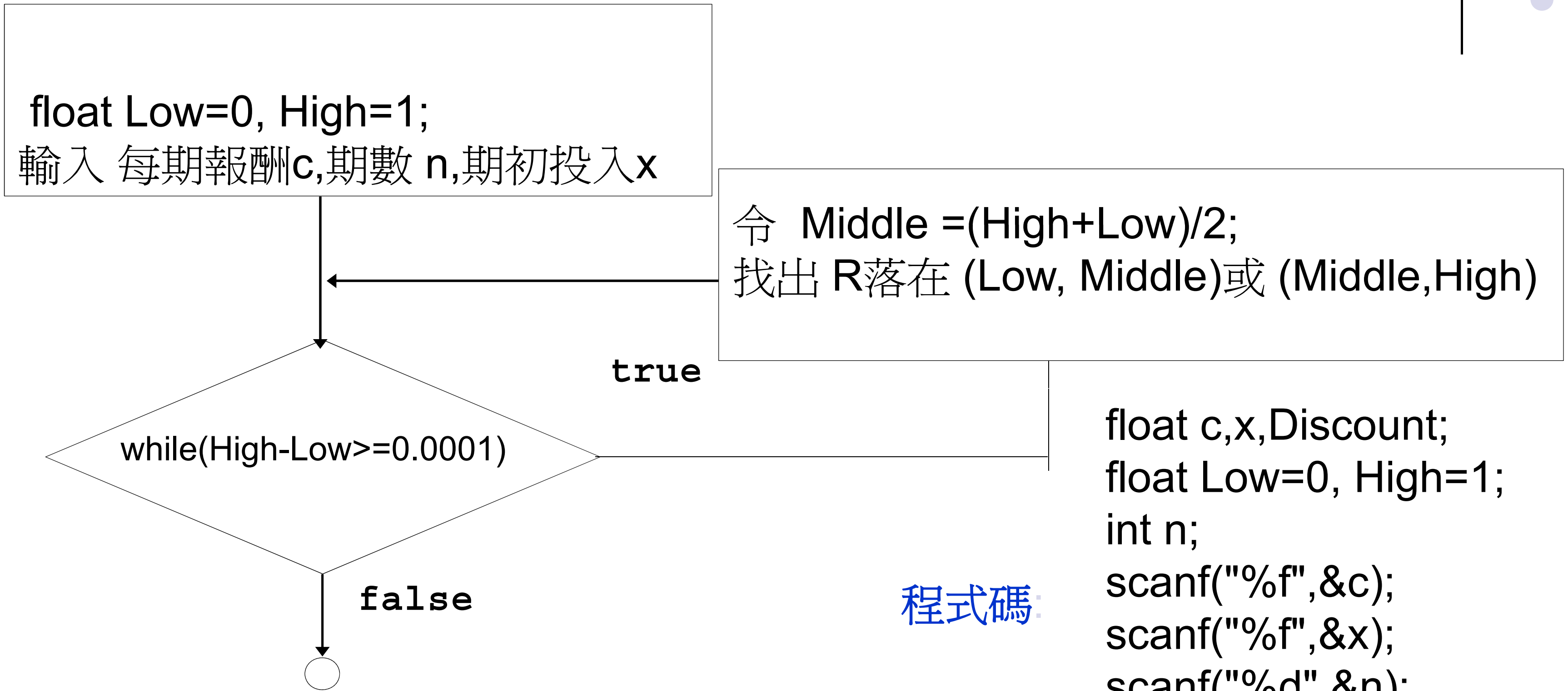
$f(a)>0, f(b)<0 \;\; => \;\; a<r<b$

Let $c=(a+b)/2$

$f(a)>0 \; f(c)<0 \;\; => \;\; a<r<c$



# C++:使用while 建構二分法




程式碼:

```
float c,x,Discount;
float Low=0, High=1;
int n;
scanf("%f",&c);
scanf("%f",&x);
scanf("%d",&n);
while(High-Low>=0.0001)
{

}
printf("Yield rate=%f",High);
```

# 用Bisection method縮小根的範圍

- 已知$f(r)=c\times(1+r)^{-1}+c\times(1+r)^{-2}+\ldots+c\times(1+r)^{-n}-x$
  - f(r)<0 → r >R
  - f(r)>0 → r<R
- 令 Middle=(High+Low)/2
  - 將根的範圍從(Low,High)縮減到
    - (Low,Middle)
    - (Middle,High)

```
float  Middle=(Low+High)/2;
float Value=0;
for(int i=1;i<=n;i=i+1)
{
 Discount=1;
 for(int j=1;j<=i;j++)
 {
          Discount=Discount/(1+Middle)
 }
  Value=Value+Discount*c;
}
Value=Value-x;
if(Value>0)
          { Low=Middle;}
  else
          {High=Middle;}
```

$$c\times(1+r)^{-1}+c\times(1+r)^{-2}+\ldots+c\times(1+r)^{-n}$$

用計算債券的公式計算

縮小根的範圍

# 計算 IRR (完整程式碼)

用while控制根的範圍

計算 $c\times(1+r)^{-1}+c\times(1+r)^{-2}+\ldots+c\times(1+r)^{-n}$

計算 $(1+r)^{-i}$

縮小根的範圍

```
float c,x,Discount;
float Low=0, High=1;
int n;
scanf("%f",&c);
scanf("%f",&x);
scanf("%d",&n);
while(High-Low>=0.0001)
{
float  Middle=(Low+High)/2;
float Value=0;
for(int i=1;i<=n;i=i+1)
{
 Discount=1;
 for(int j=1;j<=i;j++)
 {
            Discount=Discount/(1+Middle);
 }
  Value=Value+Discount*c;
}
Value=Value-x;
if(Value>0)
                { Low=Middle;}
  else
               {High=Middle;}
}
printf("Yield rate=%f",High);
```

# # Homework　1

- 第三章第十題

　假定有一個投資計畫，該投資計畫可在現在獲得9702元收益，在第一期結束時需支付19700元，第二期計畫結束時，可再獲得10000元，請仿照上述求內部收益率的程式，撰寫程式使用二分法求內部收益率，請問這種解法會不會碰到問題？

# The Newton-Raphson Method

- Converges faster than the bisection method.
- Start with a first approximation $X_0$ to a root of $f(x) = 0$.
- Then

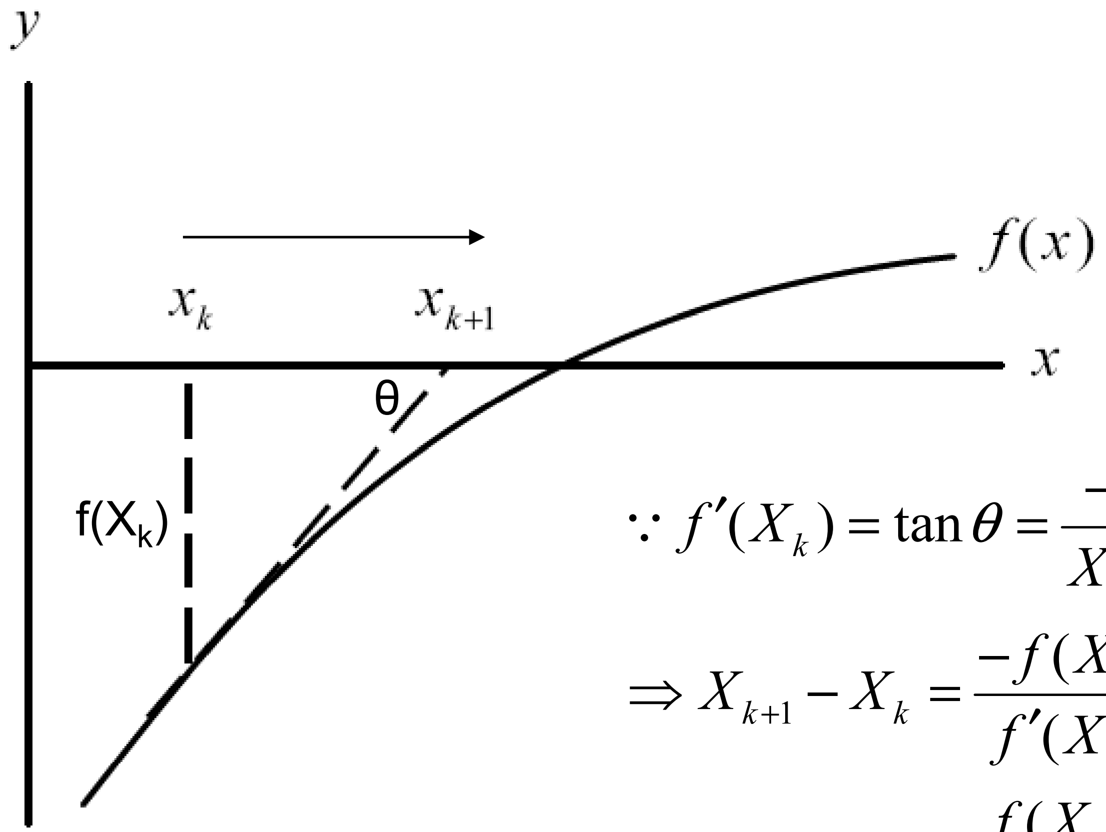
$$x_{k+1} \equiv x_k - \frac{f(x_k)}{f'(x_k)} \qquad (3.15)$$

- When computing yields,

$$f'(x) = -\sum_{t=1}^{n} \frac{t\,C_t}{(1+x)^{t+1}}$$

※ **Recall the bisection method, the X here is r (yield) in the bisection method!**

# Figure3.5: Newton-Raphson method




$$\because f'(X_k) = \tan\theta = \frac{-f(X_k)}{X_{k+1} - X_k}$$

$$\Rightarrow X_{k+1} - X_k = \frac{-f(X_k)}{f'(X_k)}$$

$$\Rightarrow X_{k+1} = X_k - \frac{f(X_k)}{f'(X_k)}$$

If $f(X_{k+1})=0$, we can obtain $X_{k+1}$ is yield

# Computed by Excel

- **Yield的計算**
  - **RATE( nper, pmt, pv, fv, type)**。
  - Nper：年金的總付款期數。
  - Pmt ：各期所應給付 (或所能取得) 的固定金額。
  - Pv ：期初應給付或取得的金額
  - Fv ：最後一次付款完成後，所應付出或獲得的現金餘額 。
  - Type 0=>期末支付 1=>期初支付

# Example

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | 某政府公債票面利率為5%，發行價格為$95，票面價格為$100，半年支付一次，到期期間為10年，求YTM? YTM=2.83%*2=5.66% | | | | | |
| 2 | Nper | 20 | | | | |
| 3 | Pmt | 2.5 | | | | |
| 4 | Pv | -95 | | | | |
| 5 | Fv | 100 | | | | |
| 6 | Type | 0 | | | | |
| 7 | | | | | | |
| 8 | YTM | 2.83% | | | | |
| 9 | | | | | | |

=RATE(B2,B3,B4,B,5B6)

# Bond

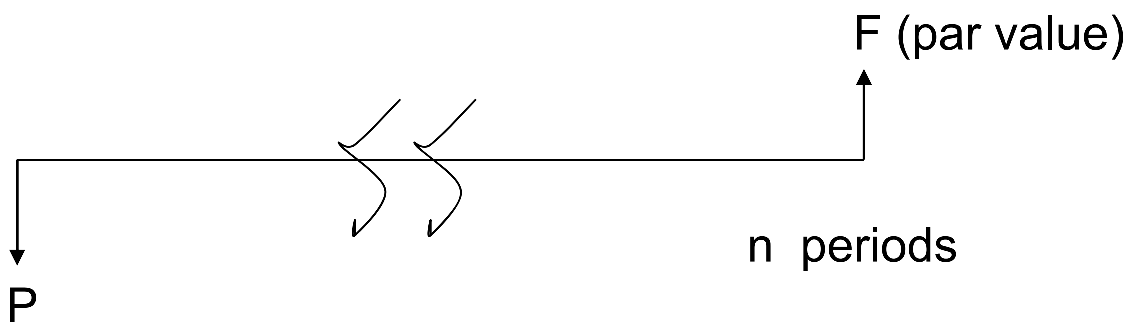
- A bond is a contract between the issuer (borrower) and the bondholder (lender).
- Bonds usually refer to long-term debts.
- Callable bond, convertible bond.
- Pure discount bonds vs. level-coupon bond

## Zero-Coupon Bonds (Pure Discount Bonds)

- The price of a zero-coupon bond that pays $F$ dollars in $n$ periods is $P = \dfrac{F}{(1+r)^n}$

  *where r is the interest rate per period*

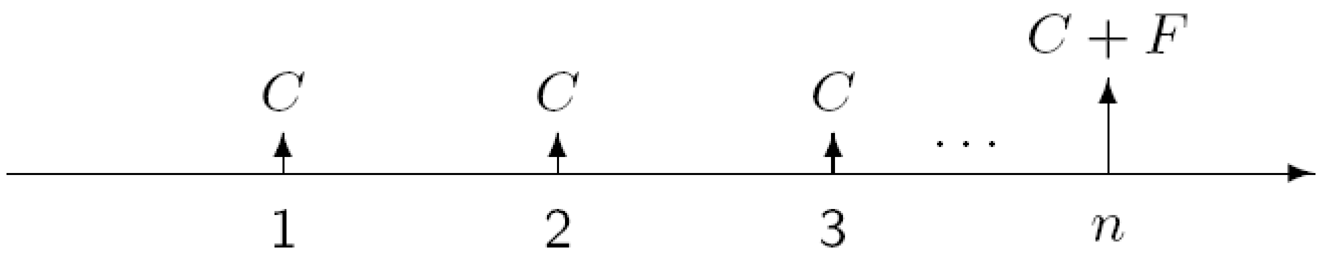
- No coupon is paid before bond mature.
- Can meet future obligations without reinvestment risk.



# Level-Coupon Bonds

- It pays interest based on coupon rate and the par value, which is paid at maturity.
- ***F*** denotes the par value and ***C*** denotes the coupon.

$$P = C\times(1+r)^{-1} + C\times(1+r)^{-2} + \ldots + C\times(1+r)^{-n} + F\times(1+r)^{-n}$$



# Pricing of Level-Coupon Bonds

$$P = \frac{C}{(1+\frac{r}{m})} + \frac{C}{(1+\frac{r}{m})^2} + \ldots\ldots + \frac{C}{(1+\frac{r}{m})^{nm}} + \frac{F}{(1+\frac{r}{m})^{nm}}$$

$$= \sum_{i=1}^{nm} \frac{C}{(1+\frac{r}{m})^i} + \frac{F}{(1+\frac{r}{m})^{nm}} = C\left(\frac{1-(1+\frac{r}{m})^{-nm}}{\frac{r}{m}}\right) + \frac{F}{(1+\frac{r}{m})^{nm}} \qquad (3.18)$$

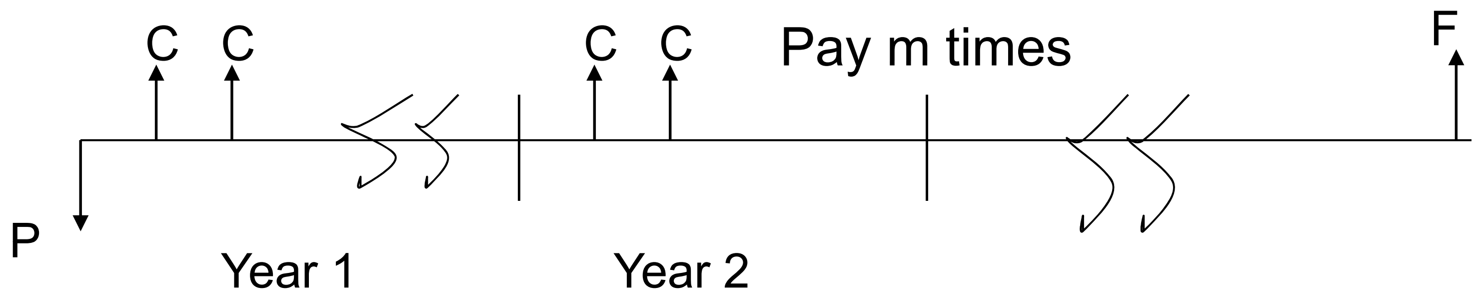
*where*

$n$: time to maturity (in years)

$m$ : number of payments per year.

$r$ : annual rate compounded $m$ times per annum.

$C = Fc/m$ where $c$ is the annual coupon rate.



# Yield To Maturity

- The YTM of a level-coupon bond is its IRR when the bond is held to maturity.

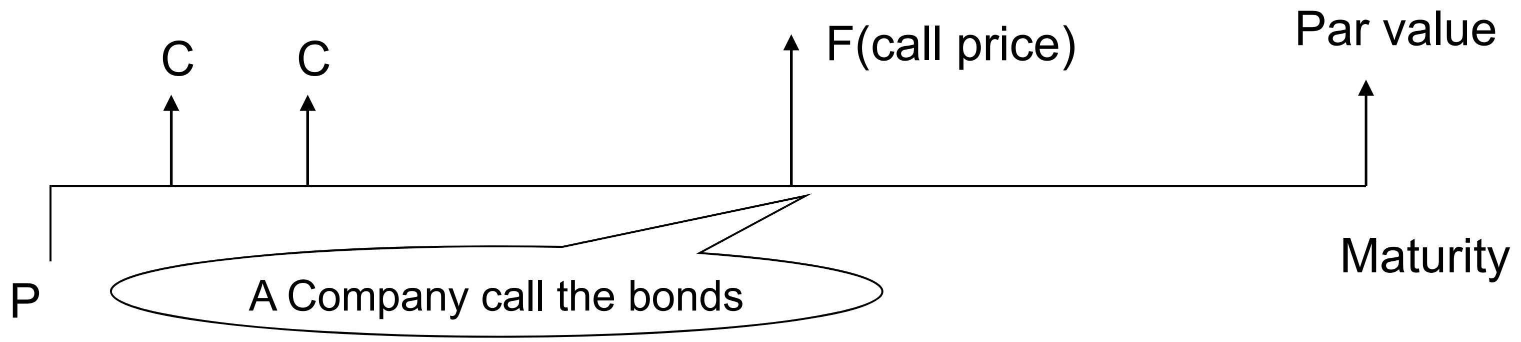
- For a 15% BEY, a 10-year bond with a coupon rate of 10% paid semiannually sells for

$$P = \frac{5}{(1+\frac{0.15}{2})} + \ldots\ldots \frac{5}{(1+\frac{0.15}{2})^{20}} + \frac{100}{(1+\frac{0.15}{2})^{20}}$$

$$= 5 \times \frac{1-\left(1+(0.15/2)\right)^{-2\times 10}}{0.15/2} + \frac{100}{\left(1+(0.15/2)\right)^{2\times 10}} = 74.5138$$

# Yield To Call

- For a callable bond, the yield to states maturity measures its yield to maturity as if were not callable.
- The yield to call is the yield to maturity satisfied by Eq(3.18), when ***n*** denoting the number of remaining coupon payments until the first call date and ***F*** replaced with call price.



# Price Behaviors

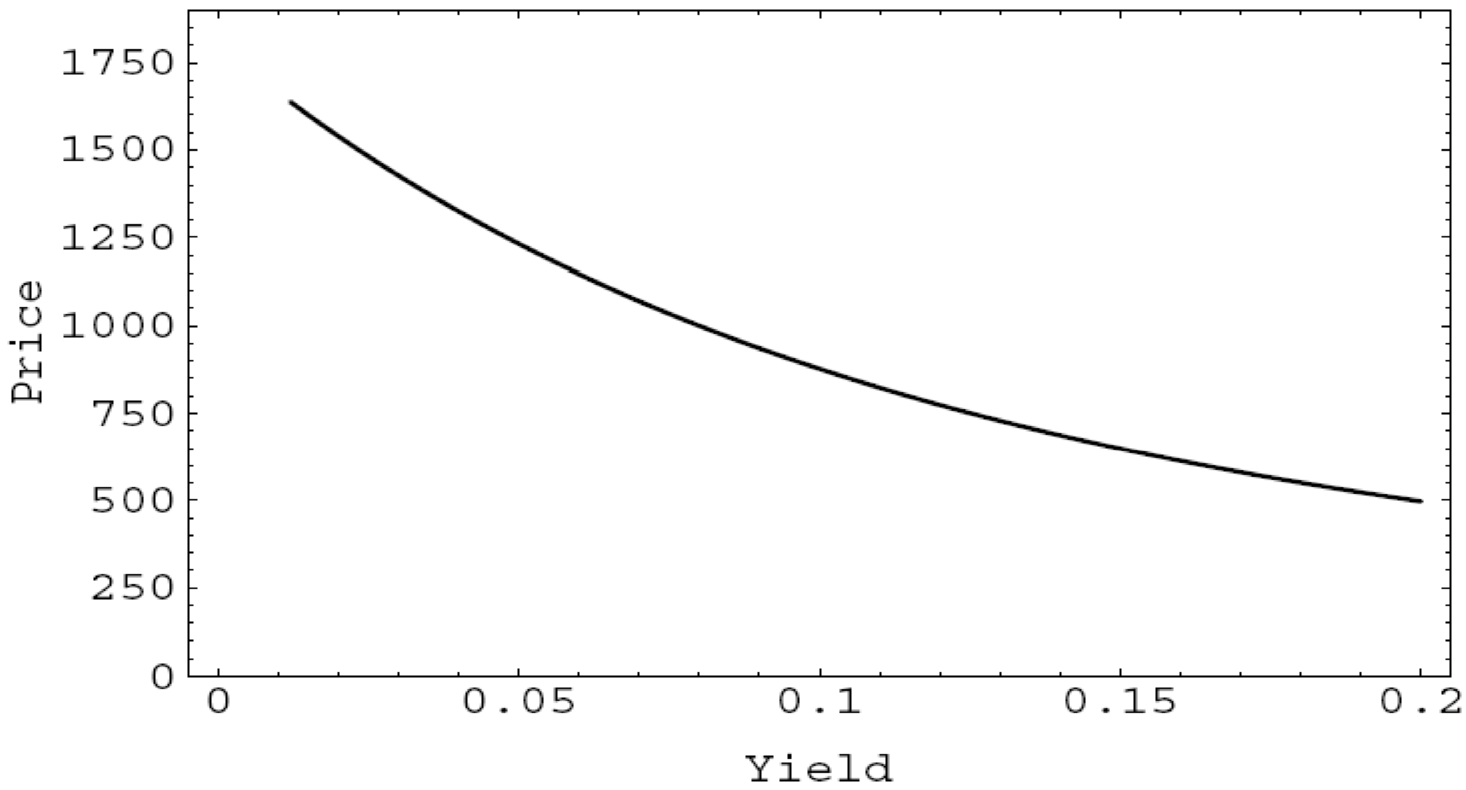
- Bond price falls as the interest rate increases, and vice versa.
- A level-coupon bond sells
  - at a premium (above its par value) when its coupon rate is above the market interest rate.
  - at par (at its par value) when its coupon rate is equal to the market interest rate.
  - at a discount (below its par value) when its coupon rate is below the market interest rate.

# Figure 3.8: Price/yield relations

| Yield (%) | Price (% of par) | |
|---|---|---|
| 7.5 | 113.37 | →Premium bond |
| 8.0 | 108.65 | |
| 8.5 | 104.19 | |
| 9.0 | 100.00 | →Par bond |
| 9.5 | 96.04 | →Discount bond |
| 10.0 | 92.31 | |
| 10.5 | 88.79 | |

# Figure 3.9: Price vs. yield.

Plotted is a bond that pays 8% interest on a par value of $1,000,compounding annually. The term is 10 years.



# Day Count Conventions: Actual/Actual

- The first "actual" refers to the actual number of days in a month.
- The second refers to the actual number of days in a year.
- Example: For coupon-bearing Treasury securities, the number of days between June 17, 1992, and October 1, 1992, is *106*.

  → 13 days (June), 31 days (July), 31 days (August), 30 days (September), and 1 day (October).

# Day Count Conventions:30/360

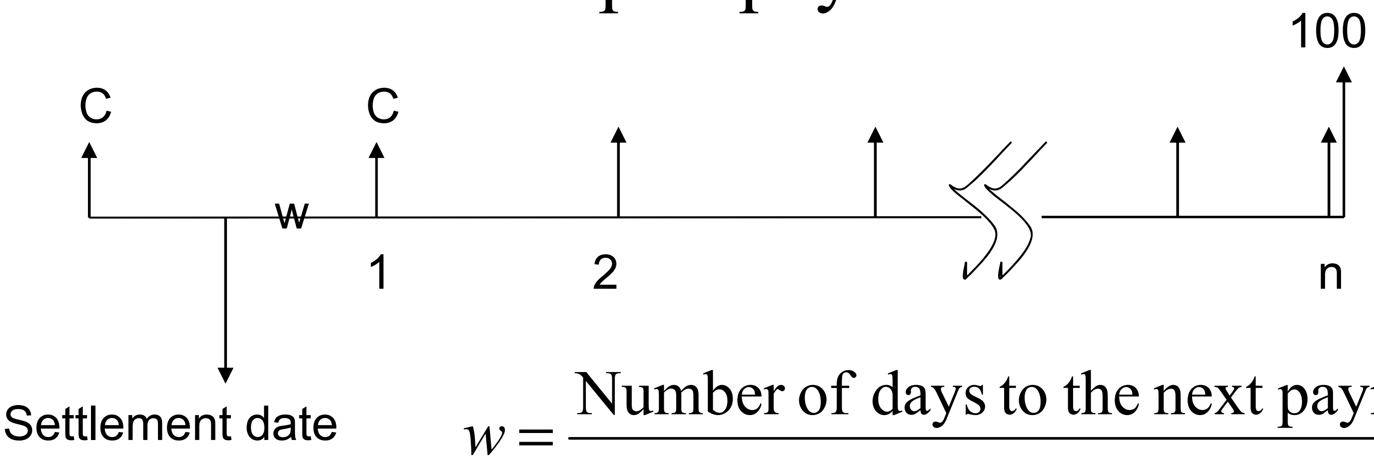
- Each month has 30 days and each year 360 days.
- The number of days between June 17, 1992, and October 1, 1992, is *104*.
  - – 13 days (June), 30 days (July), 30 days (August), 30 days (September), and 1 day (October).
- In general, the number of days from date1 to date2 is

$$360 \times (y2 - y1) + 30 \times (m2 - m1) + (d2 - d1)$$

*Where Date1 ≡ (y1,m1, d1)  Date ≡ (y2,m2, d2)*

# Bond price between two coupon date (Full Price, Dirty Price)

- In reality, the settlement date may fall on any day between two coupon payment dates.




$$w = \frac{\text{Number of days to the next payment date}}{\text{Number of days in the coupon period}}$$

$$\text{Dirty Price} = C\times(1+r)^{-\omega} + C\times(1+r)^{-\omega-1} + \ldots\ldots + C\times(1+r)^{-\omega-n+1} + 100\times(1+r)^{-\omega-n+1}$$

# Accrued Interest

- The original bond holder has to share accrued interest in 1-ω period
  - Accrued interest is $C \times (1-\omega)$
- The quoted price in the U.S./U.K. does not include the accrued interest; it is called the *clean price*.
- Dirty price= Clean price+ Accrued interest

# Example 3.5.3

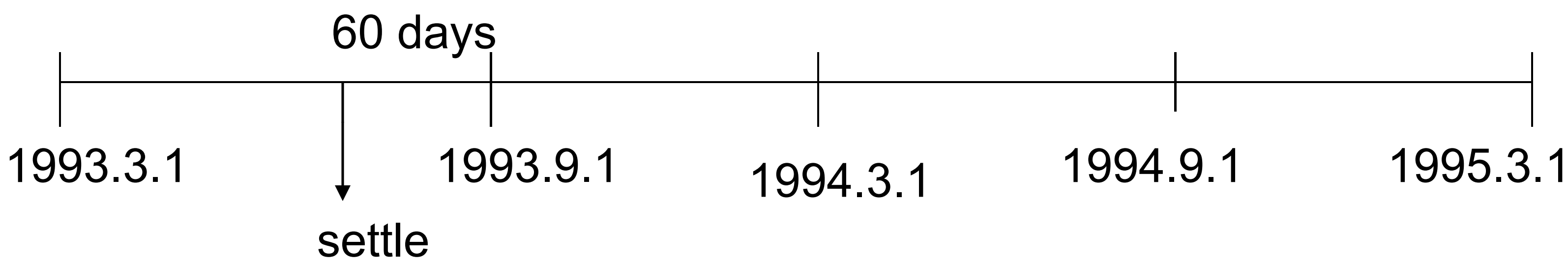
- Consider a bond with a 10% coupon rate, par value$100 and paying interest semiannually, with clean price 111.2891. The maturity date is March 1, 1995, and the settlement date is July 1, 1993. The yield to maturity is 3%.

# Example: solutions

- There are ***60*** days between July 1, 1993, and the next coupon date, September 1, 1993.
- The $\omega$= 60/180, C=5,and accrued interest is

  $5\times(1-(60/180)) = 3.3333$
- Dirty price=114.6224

  clean price = 111.2891



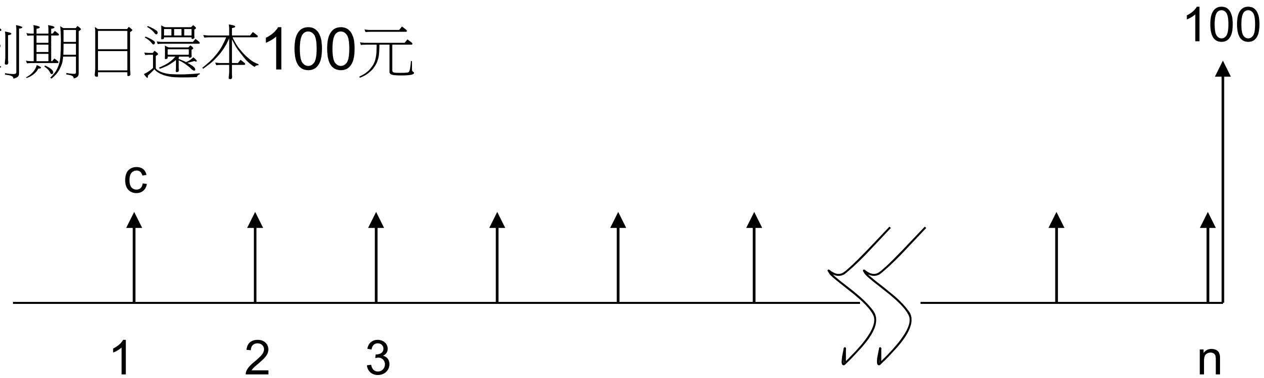
# Exercise 3.5.6

- Before: A bond selling at par if the yield to maturity equals the coupon rate. (But it assumed that the settlement date is on a coupon payment date).
- Now suppose the settlement date for a bond selling at par (i.e., the *quoted price* is equal to the par value) falls between two coupon payment dates.
- Then its yield to maturity is less than the coupon rate.

→The short reason: Exponential growth is replaced by linear growth, hence "overpaying" the coupon.

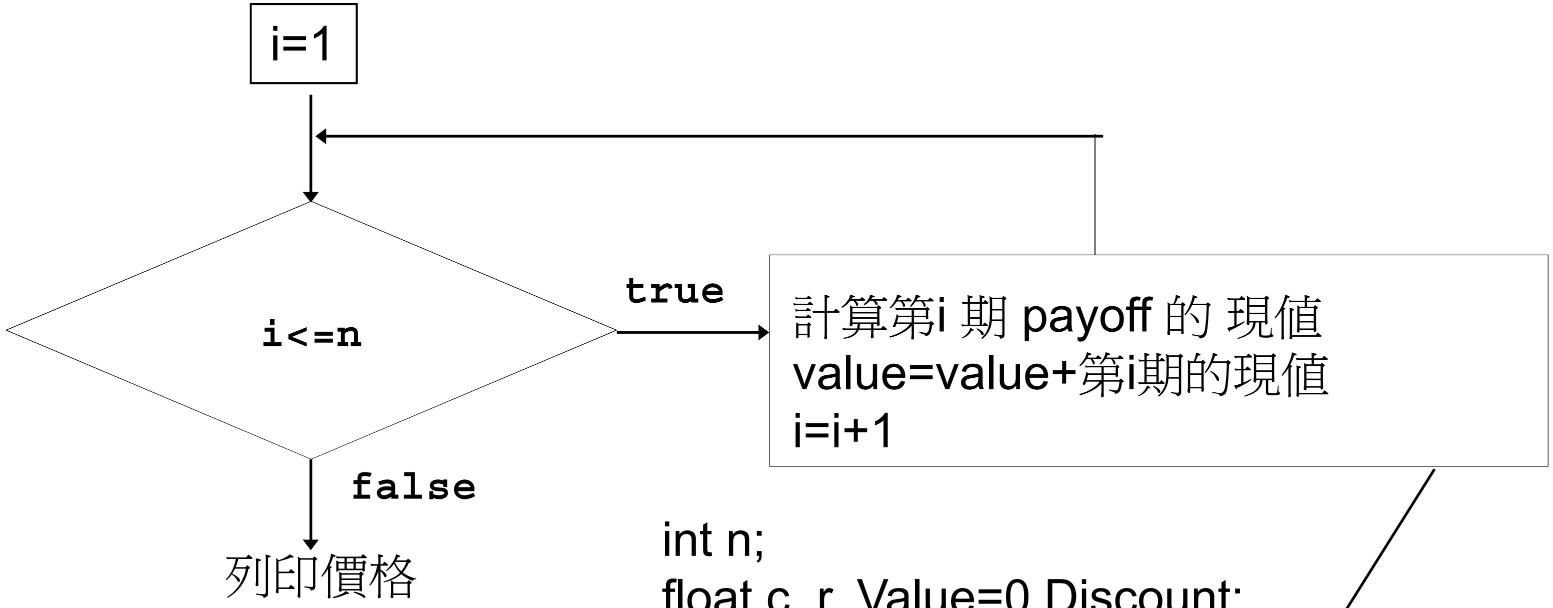
# C++:計算債券價格

- 考慮債券價格的計算
  - 假定單期利率爲r
  - 每一期支付coupon c,共付n期
  - 到期日還本100元




債券價格 $P = c\times(1+r)^{-1} + c\times(1+r)^{-2} + \ldots + c\times(1+r)^{-n} + 100\times(1+r)^{-n}$

# 程式想法




```
int n;
float c, r, Value=0,Discount;
scanf("%d",&n);
scanf("%f",&c);
scanf("%f",&r);
for(int i=1;i<=n;i=i+1)
{

}
printf("BondValue=%f",Value);
```

# 計算第i 次 payoff的 現值

- i<n 現值= $(1+r)^{-i}\times c$
- i=n 現值= $(1+r)^{-n}\times(c+100)$
- 用for計算 $(1+r)^{-i}$

計算第i 次 payoff的 現值

計算 $(1+r)^{-i}$

```
Discount=1;
for(int j=1;j<=i;j++)
{
 Discount=Discount/(1+r);
}
Value=Value+Discount*c;
if(i==n)
{
 Value=Value+Discount*100;
}
```

考慮最後一期本金折現

# 完整程式碼(包含巢狀結構)

```
#include <stdio.h>
void main()
{
        int n;
        float c, r, Value=0,Discount;
        scanf("%d",&n);
        scanf("%f",&c);
        scanf("%f",&r);
        for(int i=1;i<=n;i=i+1)
        {
         Discount=1;
         for(int j=1;j<=i;j++)
         {
          Discount=Discount/(1+r);
         }
         Value=Value+Discount*c;
         if(i==n)
         {
          Value=Value+Discount*100;
         }
        }
   printf("BondValue=%f",Value);
}
```

第i 次 payoff的 現值

Value爲前 i次payoff 現值

# # Homework 2

- Program exercise:

  Calculate the dirty and the clean price for a bond under actual/actual and 30/360 day count conversion.

  Input: Bond maturity date, settlement date, bond yield, and the coupon rate.

  The bond is assumed to pay coupons semiannually.