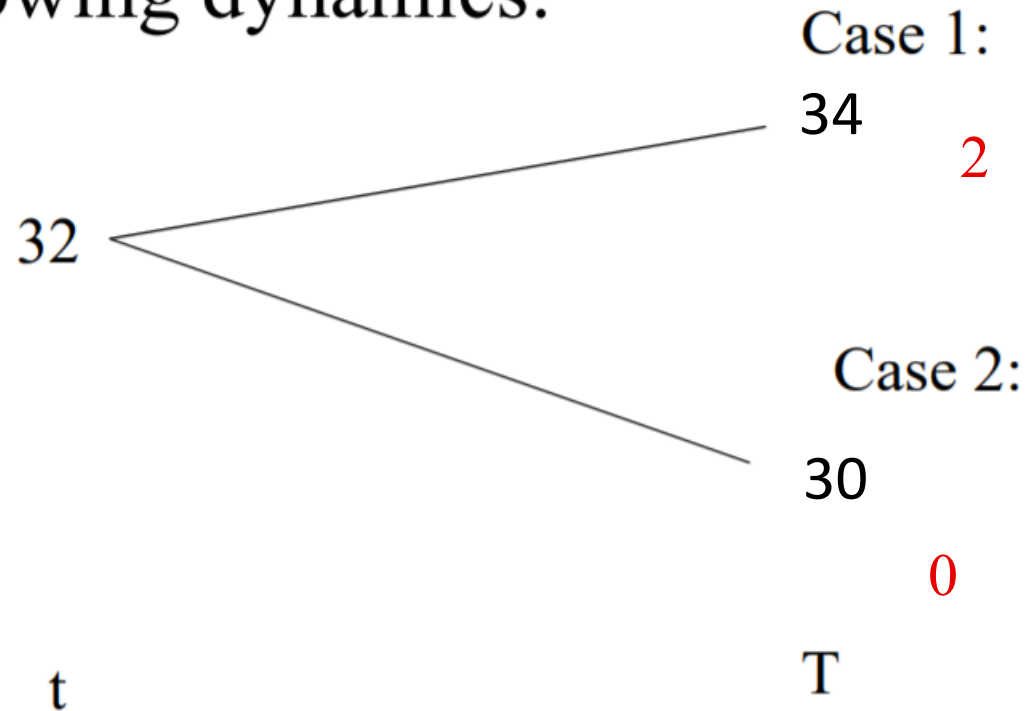


財務工程導論

HW1

How to Determine the Option Price (Arbitrage-Base Pricing Theorem)

- Assume that the exchange rate follows the following dynamics:



Arbitrage-Base Pricing Theorem

Replicate the Option

- Replication: Construct a portfolio that has the same payoff as the option at maturity.
- This call option can be replicated as follows:
 - We buy x TWDs and y USDs at time t
 - We hope that this portfolio generates the same payoff as the option at time T .
 - At case 1: $1x+34y=2$
 - At case 2: $1x+30y=0$
 - Solve the equations, we have $x=\underline{-15}$, $y=\underline{0.5}$

Arbitrage-Base Pricing Theorem

Replicate the Option and Determine the Option Price

- A foreign exchange option can be constructed as follows:
 - Borrow 15 TWD
 - Buy 0.5USD
 - The total cost= $-15+32*0.5=1$
- At case 1:
 - The value of portfolio= $-15+34*0.5=2$
- At case 2:
 - The Value of portfolio= $-15+30*0.5=0$
- The Value of the option is 1

Arbitrage-Based Pricing Theorem

Condition of Arbitrage Opportunity

- Arbitrage opportunity exists if the option value is **not** 1 TWDs.
- Let the option value $P > \underline{1}$
 - Sell a call option for P dollars.
 - Construct a replication portfolio
 - Borrow 15 TWD and buy 0.5 USD
 - Benefit at time t = $P - \underline{1} > 0$.
 - No loss will be introduced at either case.

		TWDs	USDs	Option	Total
USD = 34	Case 1	<u>-15</u>	<u>0.5*34</u>	<u>-2</u>	<u>0</u>
USD = 30	Case 2	<u>-15</u>	<u>0.5*30</u>	<u>0</u>	<u>0</u>

用Portfolio去賠給Option loss掉的\$\$

Arbitrage-Based Pricing Theorem

Determine the Option Value by No Arbitrage Assumption

- Similar case is applied for the case option value $P < \underline{1}$
 - Buy a call option for P dollars.
 - Construct a replication portfolio
 - Borrow 0.5 USD and buy 15 TWD
 - Benefit at time $t = \underline{1}$ - $P > 0$.
 - No loss will be introduced at either case.

	TWDs	USDs	Option	Total
Case 1	<u>15</u>	<u>-0.5*34</u>	<u>2</u>	<u>0</u>
Case 2	<u>15</u>	<u>-0.5*30</u>	<u>0</u>	<u>0</u>