NO. 財工

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0613413 蔡協宏 HW5

-The MD of Floating-rate instruments

$\underbrace{C, C, \dots C}_{\bar{i}}, \underbrace{y, \dots y+1}_{n-\bar{j}}$ fixed coupon rate = C

market yield = y

$$MD_{fix} = \frac{1}{P}\left(\sum_{i=1}^{n} \frac{iC}{(1+y)^i} + \frac{n}{(1+y)^n}\right)$$

$$MD_{floating} = -(1+y)\frac{\partial P/P}{\partial y}$$

$$\frac{\partial y}{\partial (1+y)^{\bar{j}+1}} = \frac{(1+y)^{\bar{j}+1} \cdot 1 - y \cdot (\bar{j}+1)(1+y)^{\bar{j}}}{[(1+y)^{\bar{j}+1}]^2}$$

$\frac{1}{1+y}$

代入

$$P = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \dots \frac{C}{(1+y)^{\bar{j}}} + \frac{y}{(1+y)^{\bar{j}+1}} + \dots \frac{y}{(1+y)^n} + \frac{1}{(1+y)^n}$$

$$\partial P/\partial y = \frac{-C}{(1+y)^2} + \frac{-2C}{(1+y)^3} + \dots \frac{-\bar{j}C}{(1+y)^{\bar{j}+1}} + \sum_{\bar{i}=\bar{j}+1}^{n}\left[\frac{1}{(1+y)^{\bar{i}}} - \frac{(\bar{i}+1)y}{(1+y)^{\bar{i}+1}}\right] + \frac{-n}{(1+y)^{n+1}}$$

$$-(1+y)\frac{\partial P}{\partial y} = \frac{C}{(1+y)} + \frac{2C}{(1+y)^2} + \dots \frac{\bar{j}C}{(1+y)^{\bar{j}}} + \sum_{\bar{i}=\bar{j}+1}^{n}\left[\frac{\bar{i}y}{(1+y)^{\bar{i}}} - \frac{1}{(1+y)^{\bar{i}-1}}\right] + \frac{n}{(1+y)^n}$$

$\sum_{\bar{i}=1}^{\bar{j}} \frac{\bar{i}C}{(1+y)^{\bar{i}}}$ ⇒ 這裡的C是y!(用fixed-rate=y的bond來看)

$$\sum_{\bar{i}=1}^{\bar{j}} \frac{\bar{i}y}{(1+y)^{\bar{i}}} + \sum_{\bar{i}=\bar{j}+1}^{n}\left[\frac{\bar{i}y}{(1+y)^{\bar{i}}} - \frac{1}{(1+y)^{\bar{i}-1}}\right] + \frac{n}{(1+y)^n}$$

⇒ $MD_{fixed\ rate}$

⇒ $MD_{floating\ rate} = MD_{fixed\ rate} - \sum_{\bar{i}=\bar{j}+1}^{n} \frac{1}{(1+y)^{\bar{i}-1}}$

prove it!

SEASON