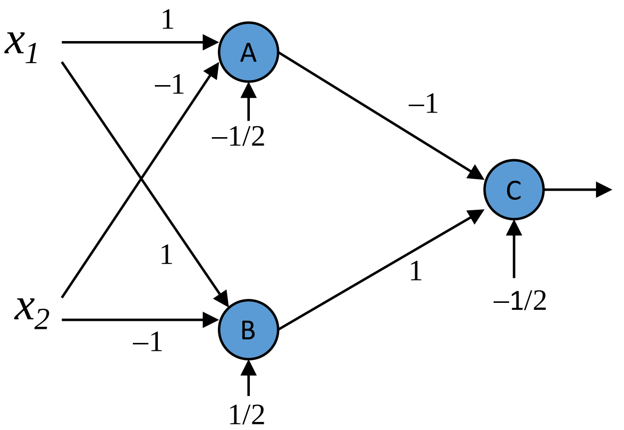
Introduction to AI NCTU Spring 2018 Homework #3 Solutions

1. For the following 2-layer feedforward neural network, compute the outputs for the following sets of inputs: (x_1=0, x_2=0), (x_1=1, x_2=0), (x_1=0, x_2=1), and (x_1=1, x_2=1). The activation function is also given.



Activation function:

$$\varphi(v) = \begin{cases} 1 & \text{for } v > 0 \\ 0 & \text{otherwise} \end{cases}$$

x_1	x_2	v_A	$\varphi(v_A)$	v_B	$\varphi(v_B)$	v_C	$\varphi(v_C)$
0	0	–1/2	0	1/2	1	1/2	1
1	0	1/2	1	3/2	1	–1/2	0
0	1	–3/2	0	–1/2	0	–1/2	0
1	1	–1/2	0	1/2	1	1/2	1

2. For each sentence below, determine whether it is valid, unsatisfiable, or neither. Briefly explain your answers. You can also use the equivalence rules or truth tables to prove your answers.

(a) $Smoke \Rightarrow Smoke$
valid: This is equivalent to $\neg Smoke \vee Smoke$, which is always true.

(b) $Smoke \Rightarrow Fire$
neither valid nor unsatisfiable

(c) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
neither valid nor unsatisfiable
(true if *Smoke* is true and *Fire* is true; false if *Smoke* is false and *Fire* is true)

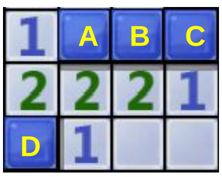
(d) $Smoke \vee Fire \vee \neg Fire$
valid

(e) $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$
valid: Both sides of the biconditional are equivalent to $\neg Smoke \vee Heat \vee Fire$

(f) $Big \vee Dumb \vee (Big \Rightarrow Dumb)$
valid: This is equivalent to $Big \vee Dumb \vee \neg Big \vee Dumb$

(g) $(Big \wedge Dumb) \vee \neg Dumb$
neither valid nor unsatisfiable
(true if *Dumb* is false; false if *Dumb* is true and *Big* is false)

3. Consider the snapshot (shown to the right) of the **Minesweeper** game. Let A, B, C and D be four propositional symbols representing the existence of mines in their locations.



(a) Try to write down propositional-logic sentences according to the game rule.
(b) Convert the sentences to CNF.
(c) Try to determine their truth values by repeatedly applying the resolution inference rule.

(a) A, D, $A \wedge D$, $A \wedge B \Leftrightarrow \neg D$, $A \wedge D \Leftrightarrow \neg B$, $B \wedge D \Leftrightarrow \neg A$,
$A \wedge B \Leftrightarrow \neg C$, $A \wedge C \Leftrightarrow \neg B$, $B \wedge C \Leftrightarrow \neg A$, $C \Leftrightarrow \neg B$, $B \Leftrightarrow \neg C$

(b)

Original sentence in (a)	CNF clauses
A	A
D	D
$A \wedge D$	A, D
$A \wedge B \Leftrightarrow \neg D$	$\neg A \vee \neg B \vee \neg D$, $A \vee D$, $B \vee D$
$A \wedge D \Leftrightarrow \neg B$	$\neg A \vee \neg B \vee \neg D$, $A \vee B$, $B \vee D$
$B \wedge D \Leftrightarrow \neg A$	$\neg A \vee \neg B \vee \neg D$, $A \vee B$, $A \vee D$
$A \wedge B \Leftrightarrow \neg C$	$\neg A \vee \neg B \vee \neg C$, $A \vee C$, $B \vee C$
$A \wedge C \Leftrightarrow \neg B$	$\neg A \vee \neg B \vee \neg C$, $A \vee B$, $B \vee C$
$B \wedge C \Leftrightarrow \neg A$	$\neg A \vee \neg B \vee \neg C$, $A \vee B$, $A \vee C$
$C \Leftrightarrow \neg B$	$\neg B \vee \neg C$, $B \vee C$
$B \Leftrightarrow \neg C$	$\neg B \vee \neg C$, $B \vee C$

(c) We have a total of 10 distinct CNF clauses

R1: A R2: D R3: $\neg A \vee \neg B \vee \neg D$ R4: $A \vee B$ R5: $A \vee C$ R6: $A \vee D$
R7: $B \vee C$ R8: $B \vee D$ R9: $\neg A \vee \neg B \vee \neg C$ R10: $\neg B \vee \neg C$

Resolution:

(R1, R3) $\rightarrow \neg B \vee \neg D$ (R11)
(R2, R11) $\rightarrow \neg B$ (R12)
(R12, R7) $\rightarrow C$ (R13)

According to the highlighted clauses (R1, R2, R12, R13), A, C, and D are *true*, and B is *false*.