

1. [25%] A propositional KB contains these sentences:

$A, B, P \Rightarrow Q, A \wedge M \Rightarrow P, L \wedge B \Rightarrow M, A \wedge B \Rightarrow L$

(a) Convert them into CNF.

(b) Use resolution to prove Q .

(a) [R1] A [R2] B

[R3] $P \Rightarrow Q \rightarrow \neg P \vee Q$

[R4] $A \wedge M \Rightarrow P \rightarrow \neg(A \wedge M) \vee P \rightarrow \neg A \vee \neg M \vee P$

[R5] $L \wedge B \Rightarrow M \rightarrow \neg(L \wedge B) \vee M \rightarrow \neg L \vee \neg B \vee M$

[R6] $A \wedge B \Rightarrow L \rightarrow \neg(A \wedge B) \vee L \rightarrow \neg A \vee \neg B \vee L$

(b) [R7: R1+R4] $\neg M \vee Q$

[R8: R1+R6] $\neg B \vee L$

[R9: R2+R5] $\neg L \vee M$

[R10: R2+R8] L

[R11: R9+R10] M

[R12: R7+R11] Q

[R12 + $\neg Q$ (negated query)] false \rightarrow contradiction $\rightarrow Q$ is true.

2. [30%] Convert the following English sentences to first-order logic sentences. Whenever possible, use the words in the original sentence as the names of predicates, functions, and constant terms.

(a) Some friends of Bob like baseball.

$\exists x \text{ Friend}(\text{Bob}, x) \wedge \text{Like}(x, \text{Baseball})$

(b) A soccer fan is one who likes soccer.

$\forall x \text{ Fan}(x, \text{Soccer}) \Rightarrow \text{Like}(x, \text{Soccer})$ [It is ok to use " \Leftrightarrow " here.]

(c) Bob is the only one in his class who likes soccer. [You can use *Classmate* as a predicate here.]

$\forall x \text{ Classmate}(\text{Bob}, x) \Rightarrow \neg \text{Like}(x, \text{Soccer})$

3. [25%] Given the FOL sentences in Prob. 2(b) and (c), and the following fact:

Jane is Bob's classmate.

Try to prove

Jane is not a soccer fan.

You can use any sound inference method. Specify the substitutions here.

We have the additional ground term: $\text{Classmate}(\text{Bob}, \text{Jane})$

The queried term Q is: $\neg \text{Fan}(\text{Jane}, \text{Soccer})$

The following procedure uses the resolution rule.

The CNF forms:

[R1] $\text{Classmate}(\text{Bob}, \text{Jane})$

[R2] $\text{Fan}(x, \text{Soccer}) \Rightarrow \text{Like}(x, \text{Soccer}) \rightarrow \neg \text{Fan}(y, \text{Soccer}) \vee \text{Like}(y, \text{Soccer})$

(The "standardizing apart" step is applied to avoid variable name clash.)

[R3] $\text{Classmate}(\text{Bob}, x) \Rightarrow \neg \text{Like}(x, \text{Soccer}) \rightarrow \neg \text{Classmate}(\text{Bob}, x) \vee \neg \text{Like}(x, \text{Soccer})$

Resolution:

[R4: R1+R3] $\neg \text{Like}(\text{Jane}, \text{Soccer})$ (substitution: $\{x/\text{Jane}\}$)

[R5: R2+R4] $\neg \text{Fan}(\text{Jane}, \text{Soccer})$ (substitution: $\{y/\text{Jane}\}$)

[R5 + $\neg Q$] false \rightarrow contradiction $\rightarrow Q$ is true.

4. [20%] Which of the following terms are suitable to be used as Functions in FOL, and which are not?

Brother, Mother, Head, Hat.

Give a brief explanation of your criterion.

Mother and *Head* are suitable as functions, and *Brother* and *Hat* are not.

This is because it is expected that only a single object can be someone's mother (or head), but an object can have multiple brothers (or hats).