

Lab 5

Poisson process

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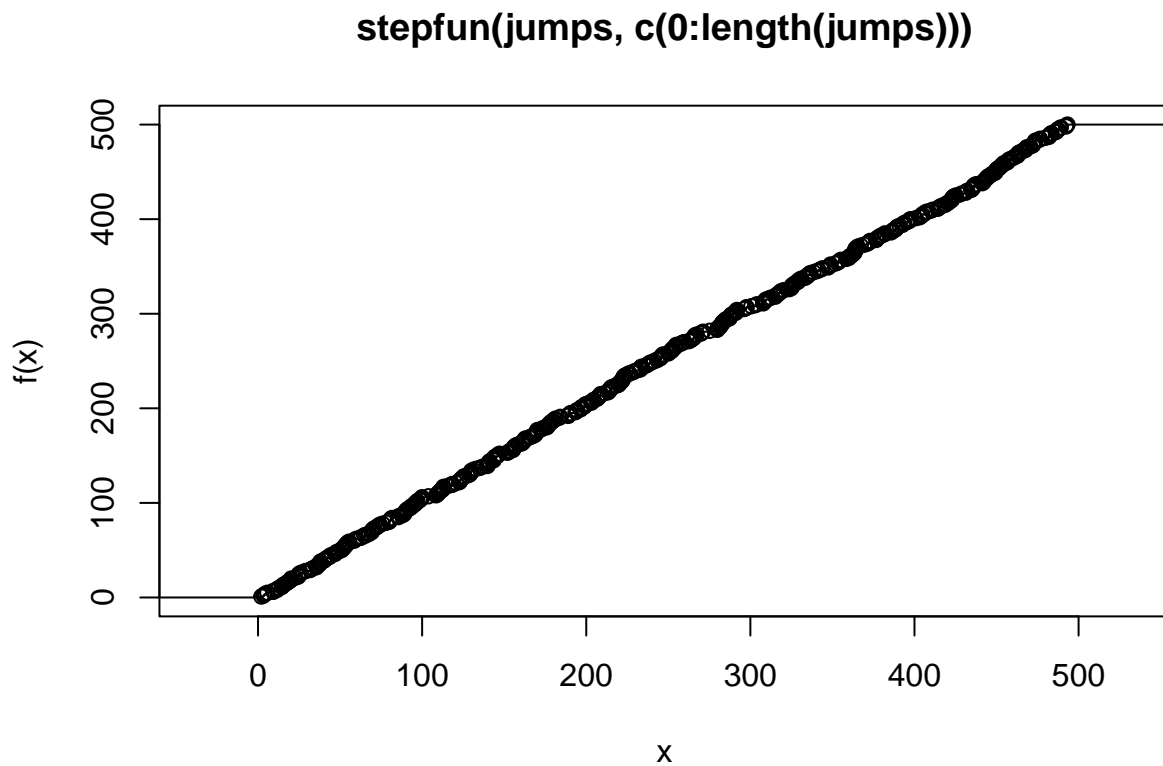
Today we are going to generate Poisson process.

```
n <- 500
lambda <- 1

Y <- rexp(n, rate = lambda) # generating Poisson process from definition
jumps <- cumsum(Y)

trajectory <- stepfun(jumps, c(0:length(jumps)))

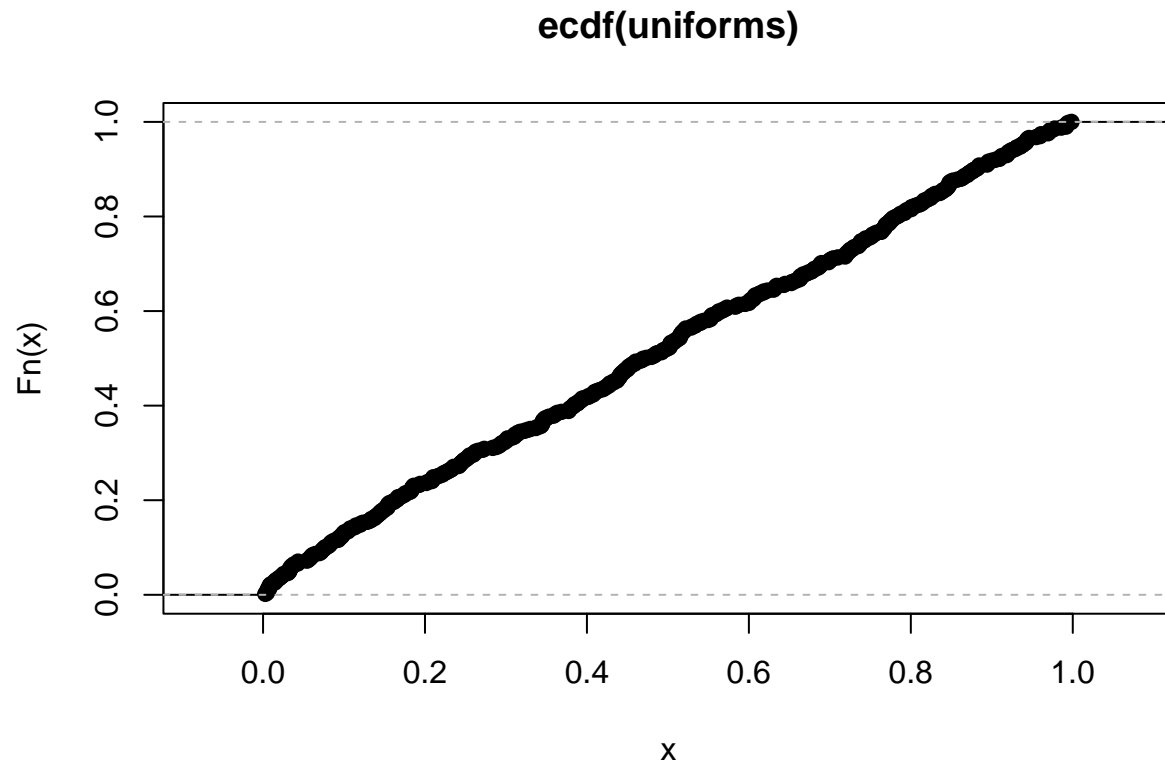
plot(trajectory)
```



Another way to generate Poisson process.

```
n <- 500

uniforms <- runif(n - 1)
plot(ecdf(uniforms))
```



Let's solve exercise 2.2 from lecture notes. We are waiting until waiting time is less than t_0 . Then we are going to consider next and prev time of jump and we are considering distribution of them. In next chunks, we are simulating Poisson process and estimate densities for different parameters (3 different chunks = 3 sets of parameters).

```
lambda <- 1
t0 <- 2
n_simulations <- 20000
all_diffs <- vector()
delta_plus <- vector()
delta_minus <- vector()

for (i in 1:n_simulations){

  waiting_time <- 0

  while (waiting_time < t0){
    Y <- rexp(1, rate = lambda)

    prev_waiting_time <- waiting_time
    waiting_time <- waiting_time + Y
```

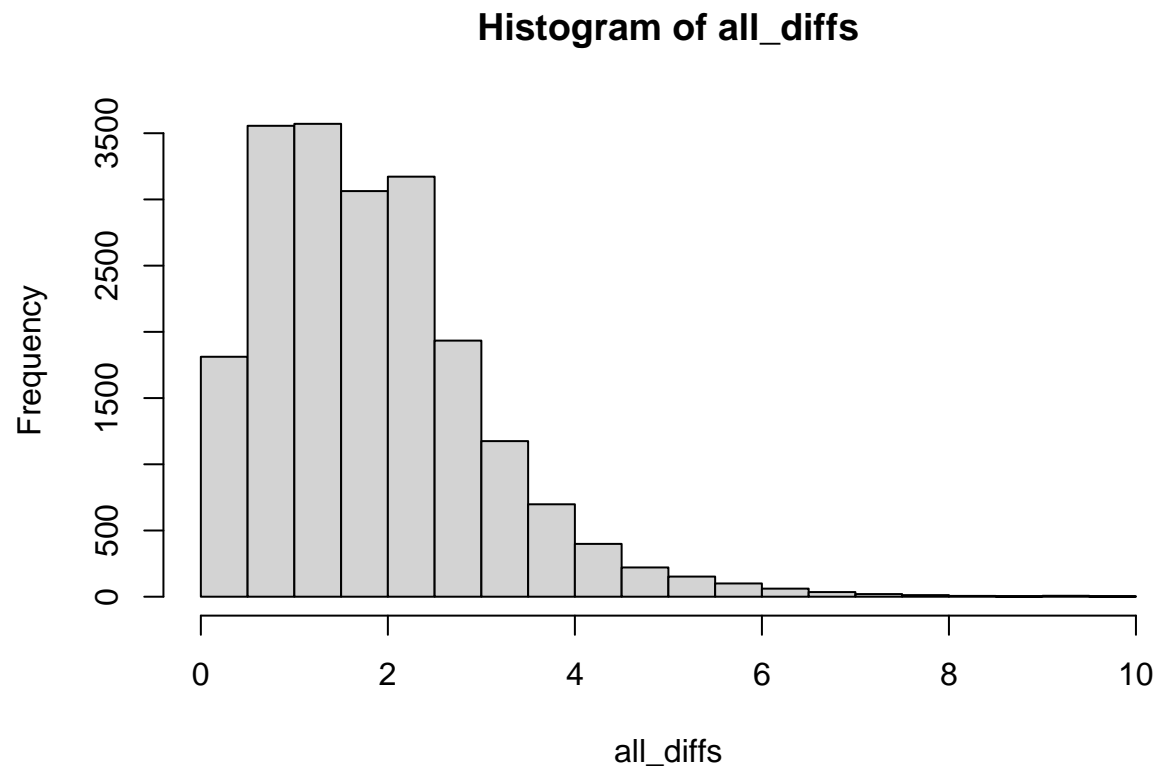
```

}

all_diffs <- c(all_diffs, waiting_time - prev_waiting_time)
delta_plus <- c(delta_plus, waiting_time - t0)
delta_minus <- c(delta_minus, t0 - prev_waiting_time)
}

hist(all_diffs)

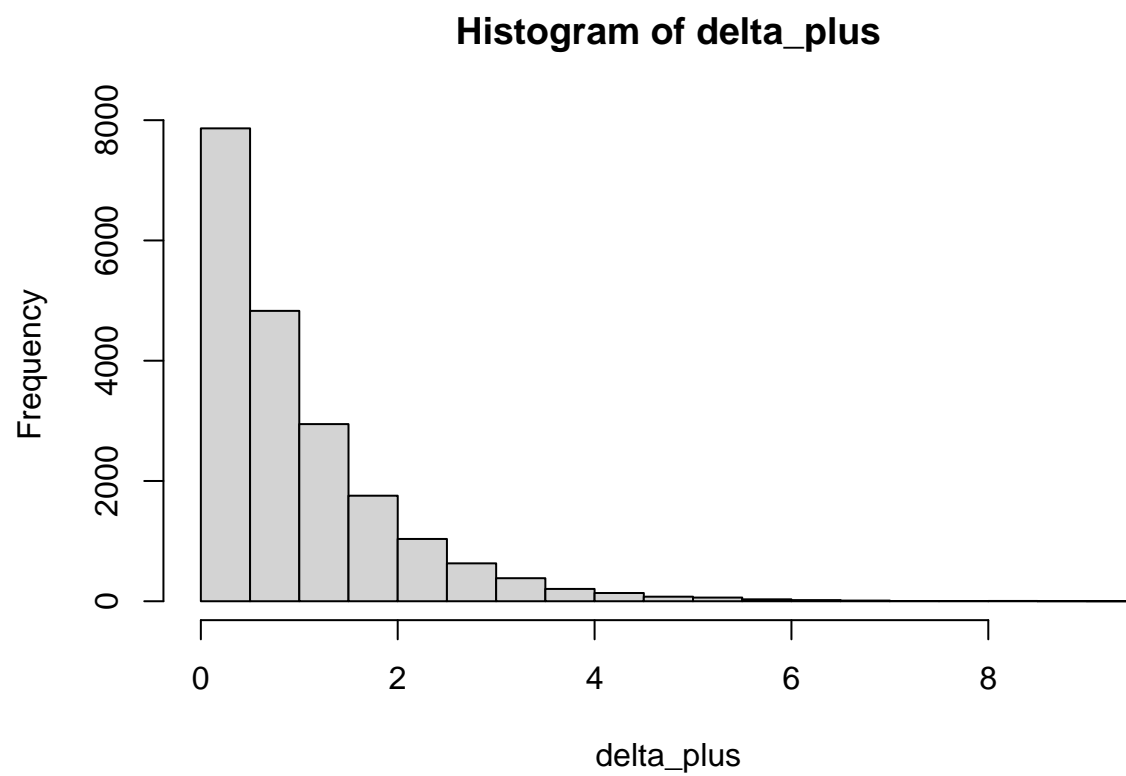
```



```

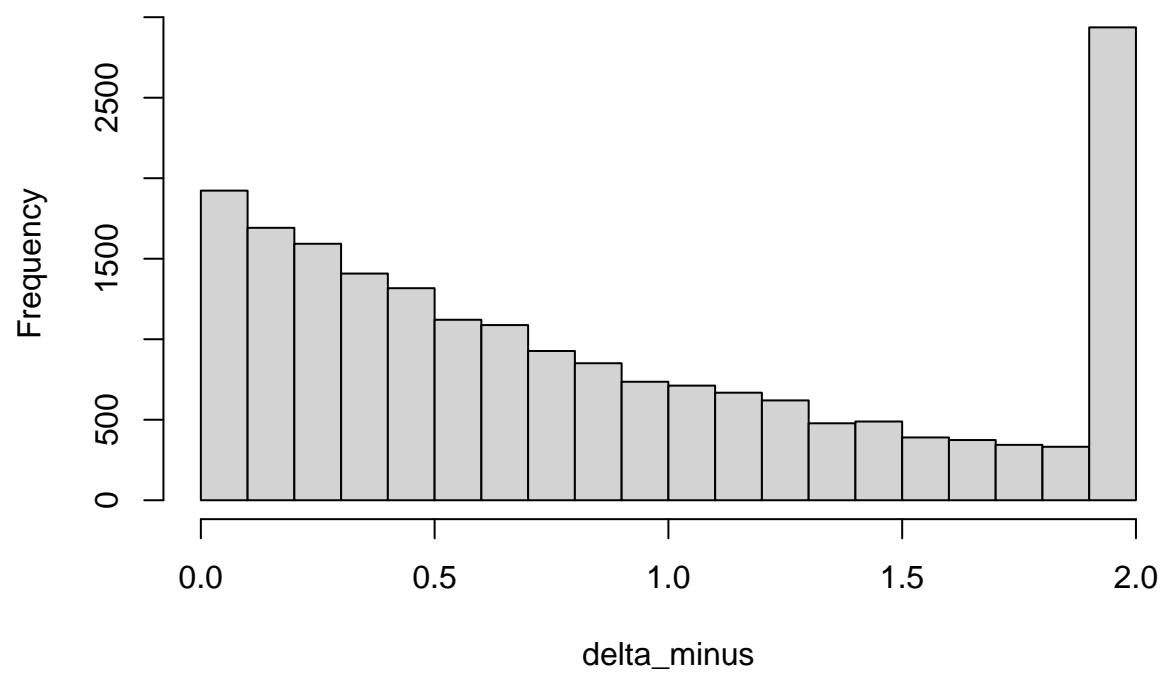
hist(delta_plus)

```



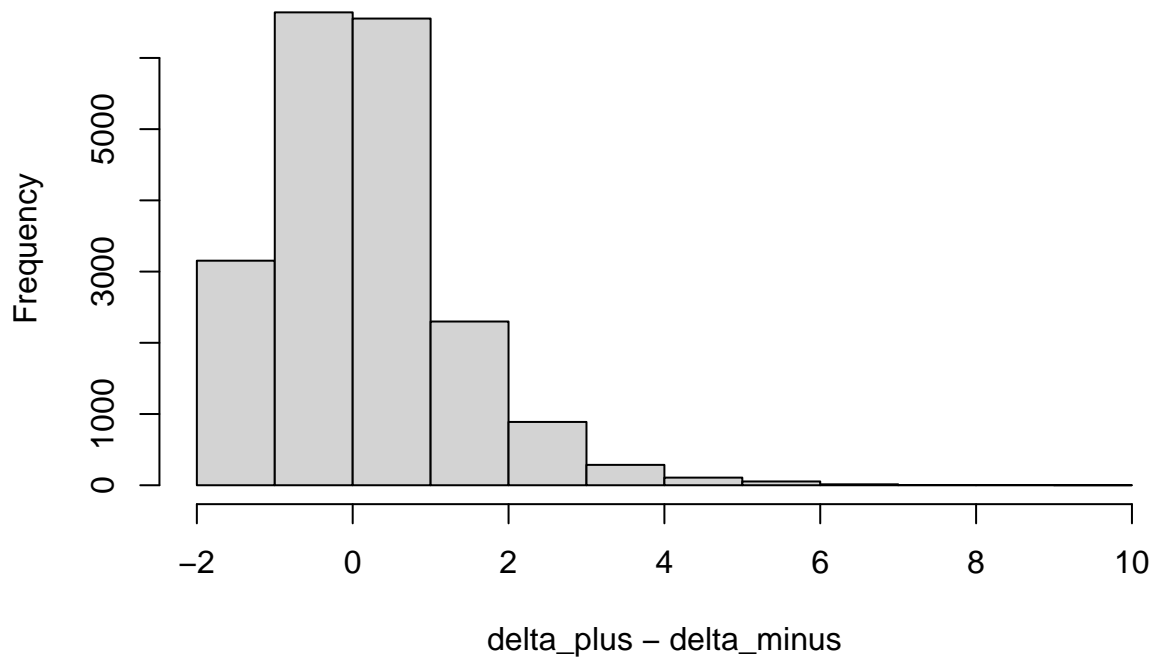
```
hist(delta_minus)
```

Histogram of delta_minus



```
hist(delta_plus - delta_minus)
```

Histogram of delta_plus – delta_minus



```
lambda <- 1
t0 <- 0.001
n_simulations <- 20000
all_diffs <- vector()
delta_plus <- vector()
delta_minus <- vector()

for (i in 1:n_simulations){

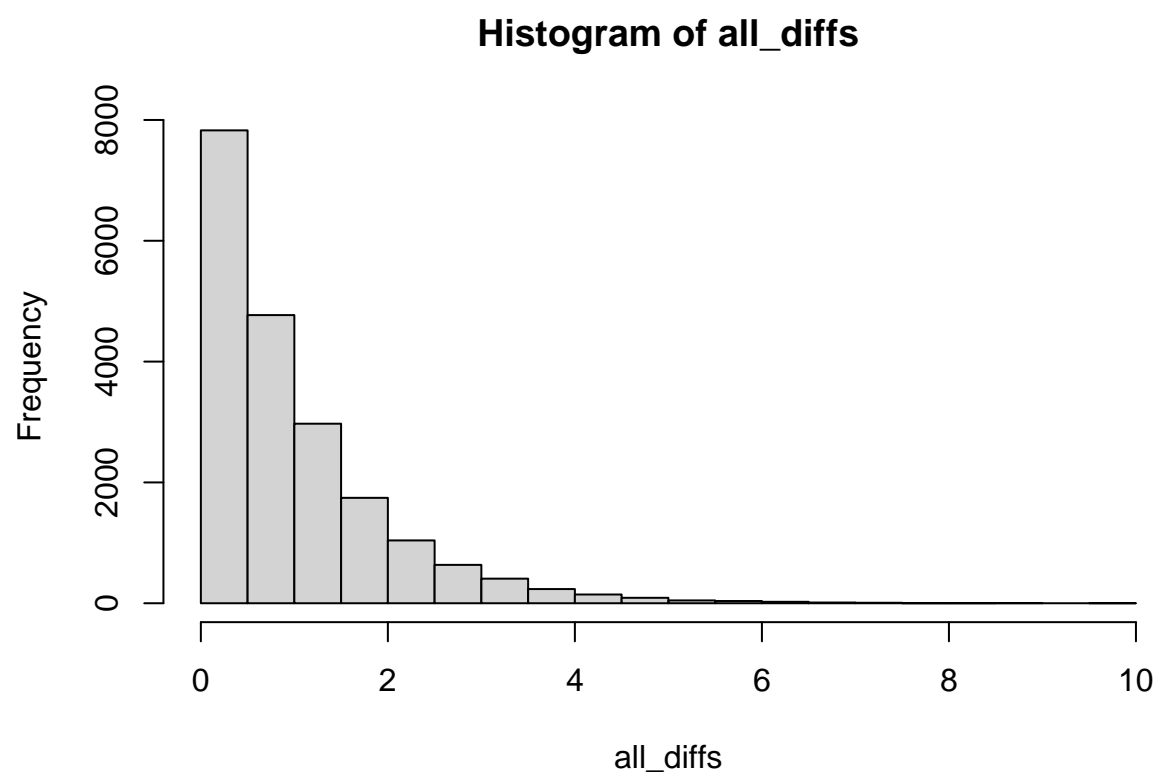
  waiting_time <- 0

  while (waiting_time < t0){
    Y <- rexp(1, rate = lambda)

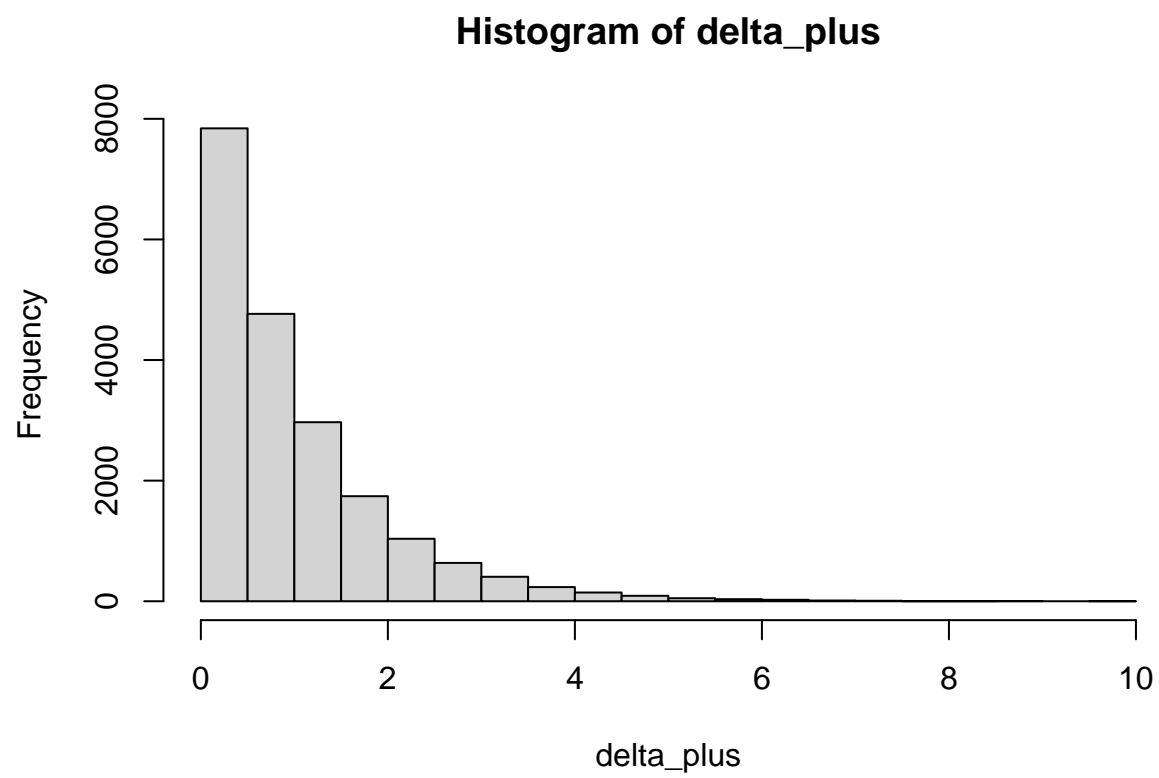
    prev_waiting_time <- waiting_time
    waiting_time <- waiting_time + Y
  }

  all_diffs <- c(all_diffs, waiting_time - prev_waiting_time)
  delta_plus <- c(delta_plus, waiting_time - t0)
  delta_minus <- c(delta_minus, t0 - prev_waiting_time)
}

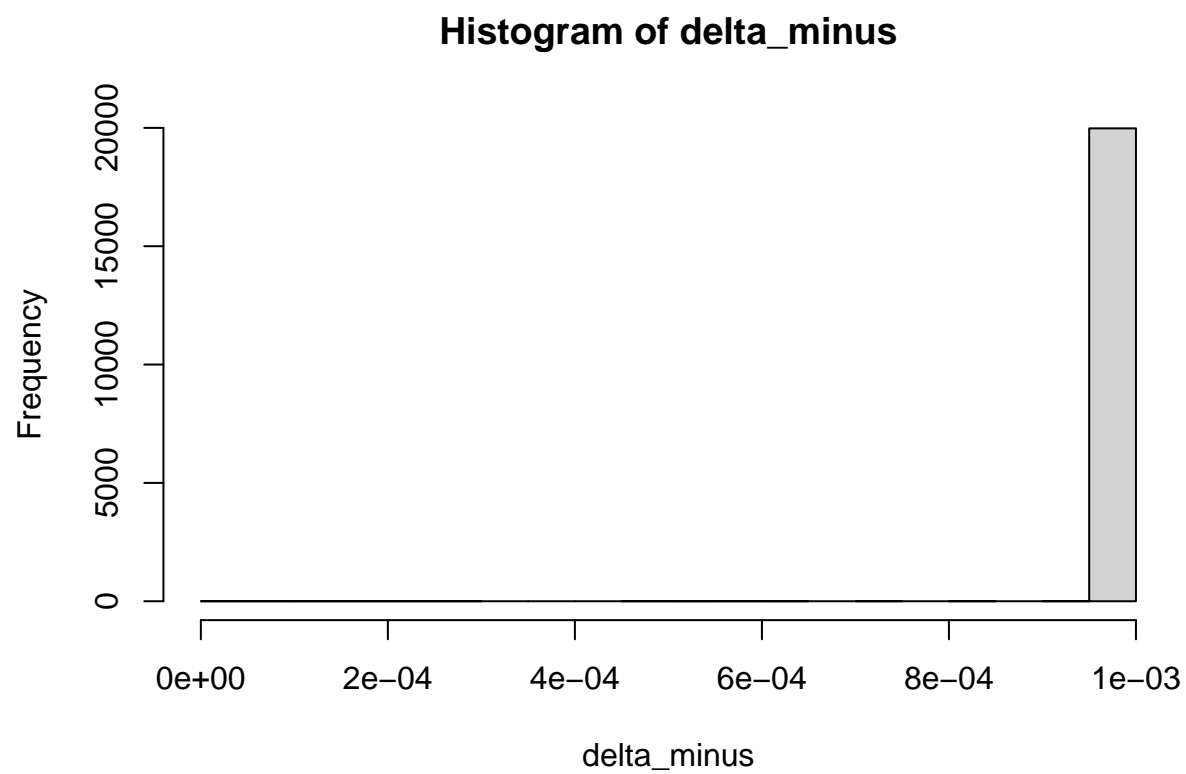
hist(all_diffs)
```



```
hist(delta_plus)
```

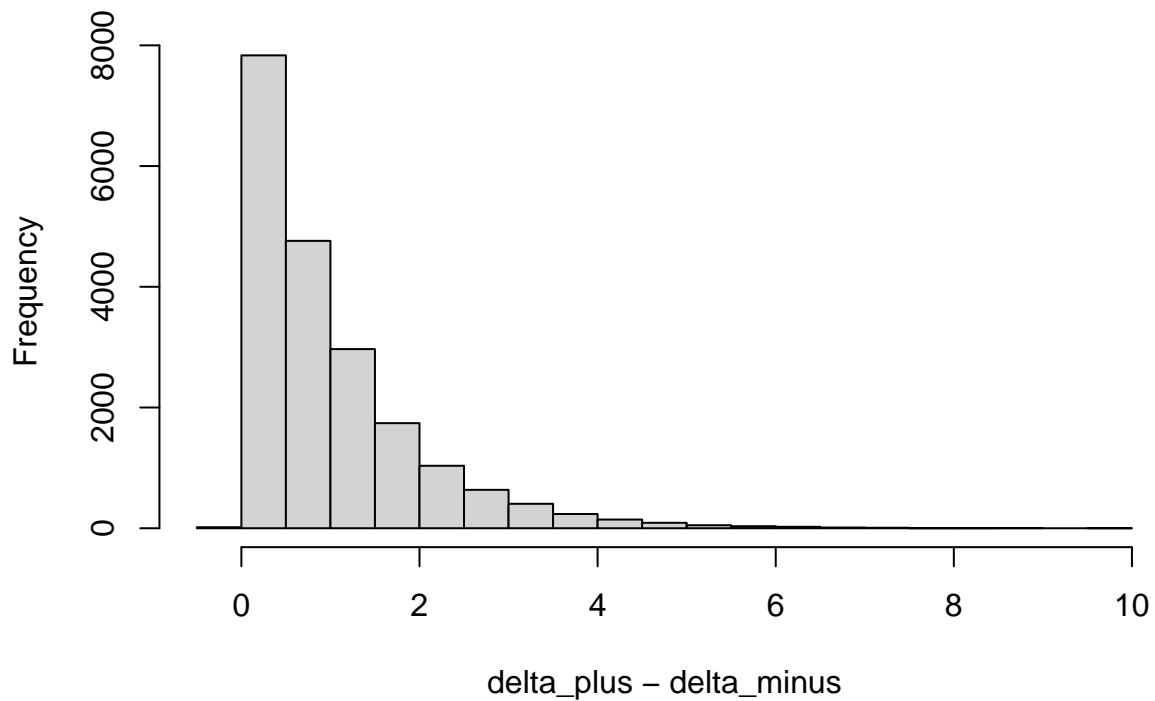


```
hist(delta_minus)
```

```
hist(delta_plus - delta_minus)
```

Histogram of delta_plus – delta_minus



```
lambda <- 1
t0 <- 200
n_simulations <- 20000
all_diffs <- vector()
delta_plus <- vector()
delta_minus <- vector()

for (i in 1:n_simulations){

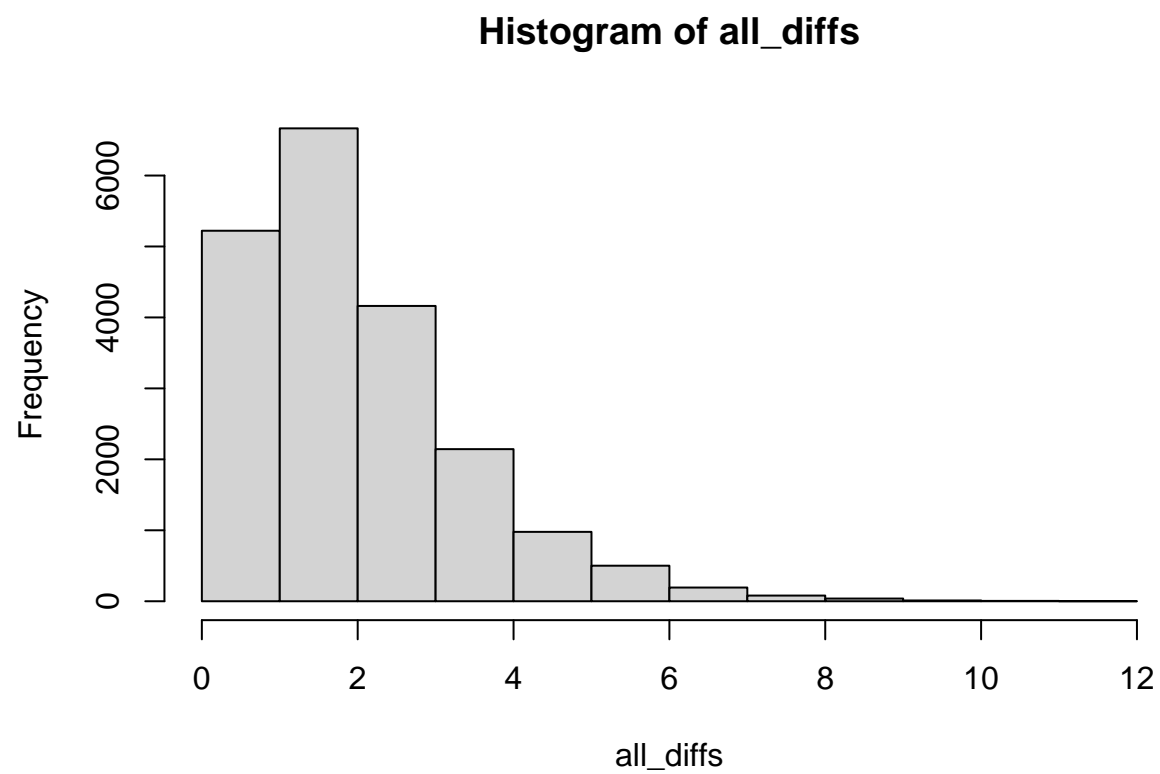
  waiting_time <- 0

  while (waiting_time < t0){
    Y <- rexp(1, rate = lambda)

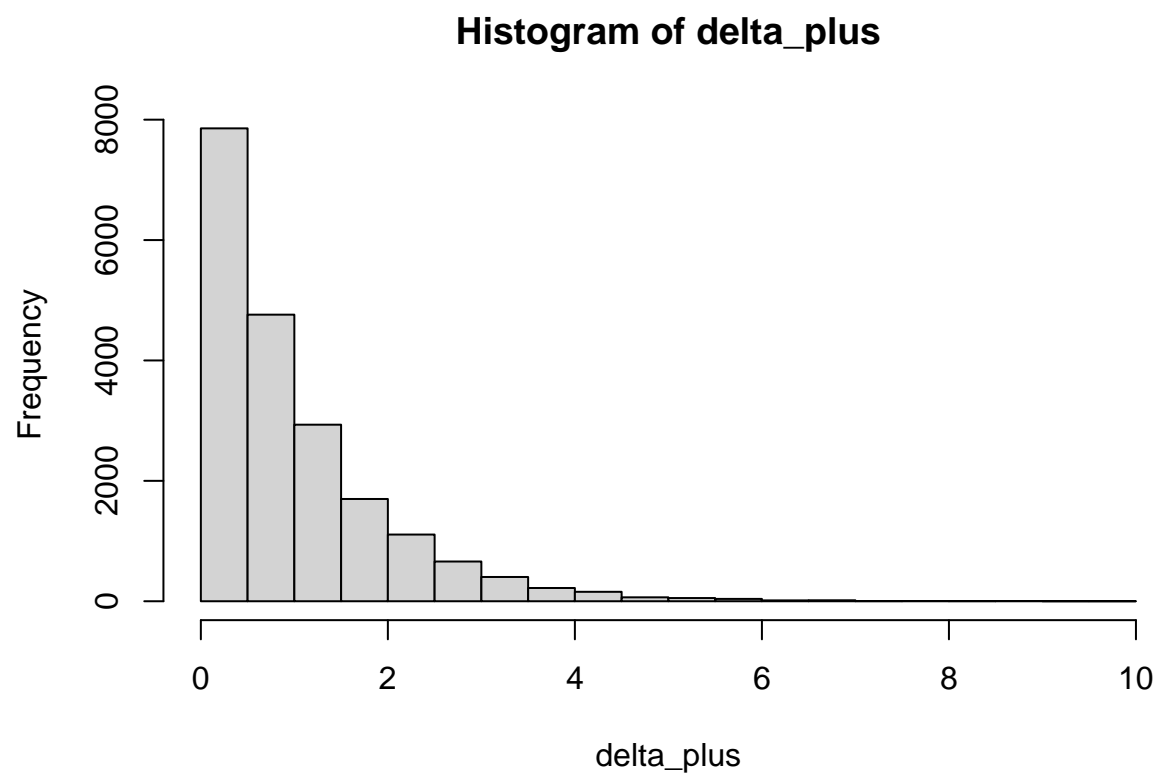
    prev_waiting_time <- waiting_time
    waiting_time <- waiting_time + Y
  }

  all_diffs <- c(all_diffs, waiting_time - prev_waiting_time)
  delta_plus <- c(delta_plus, waiting_time - t0)
  delta_minus <- c(delta_minus, t0 - prev_waiting_time)
}

hist(all_diffs)
```

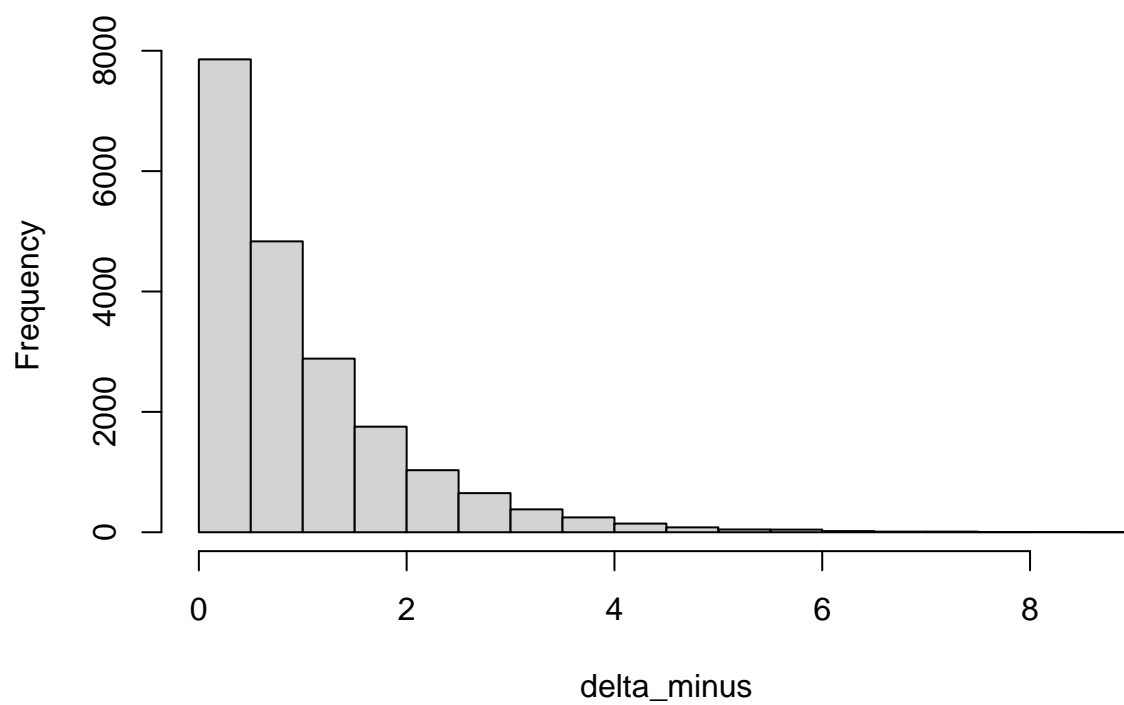


```
hist(delta_plus)
```



```
hist(delta_minus)
```

Histogram of delta_minus



```
hist(delta_plus - delta_minus)
```

Histogram of delta_plus – delta_minus

