Lab12

Gibbs Sampler

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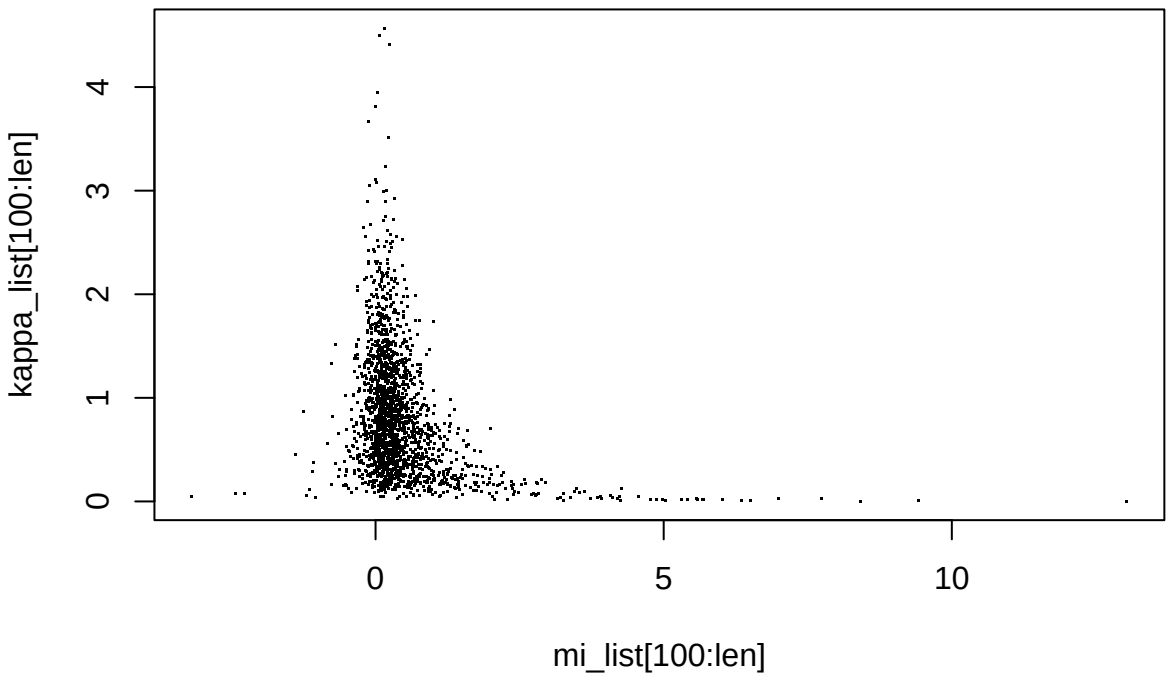
Today we are going to see some examples of usage of Gibbs sampler.

```
len <-2000

n <- 5
s2 <- 5
y_bar <- 0
m <- 5
v2 <- 0.2


mi_list <- numeric(len)   # mi
kappa_list <- numeric(len)  # kappa
kappa_list[1] <- 1

for(i in (2:len)){
  kappa <- kappa_list[i - 1]
  random_first <- rnorm(1, mean = (n*kappa*y_bar/(n*kappa + v2)) + ( (v2/(n*kappa + v2)) * m), sd = 1/(
  mi_list[i] <- random_first

  mi <- mi_list[i]
  random_second <- rgamma(1, shape = n/2, rate = s2/2 + (n/2)*(mi - y_bar)*(mi - y_bar))
  kappa_list[i] <- random_second
}


plot(mi_list[100:len], kappa_list[100:len], pch =".")
```




```
mean(mi_list[100:len])
```

```
## [1] 0.4394195
```

```
mean(kappa_list[100:len])
```

```
## [1] 0.82503
```

```
sd(mi_list[100:len])
```

```
## [1] 0.882925
```

```
sd(kappa_list[100:len])
```

```
## [1] 0.5970128
```

Next, we consider autologistic model.

```
# Parameters
n_sim <- 2000
burn  <- 500
d     <- 5
```

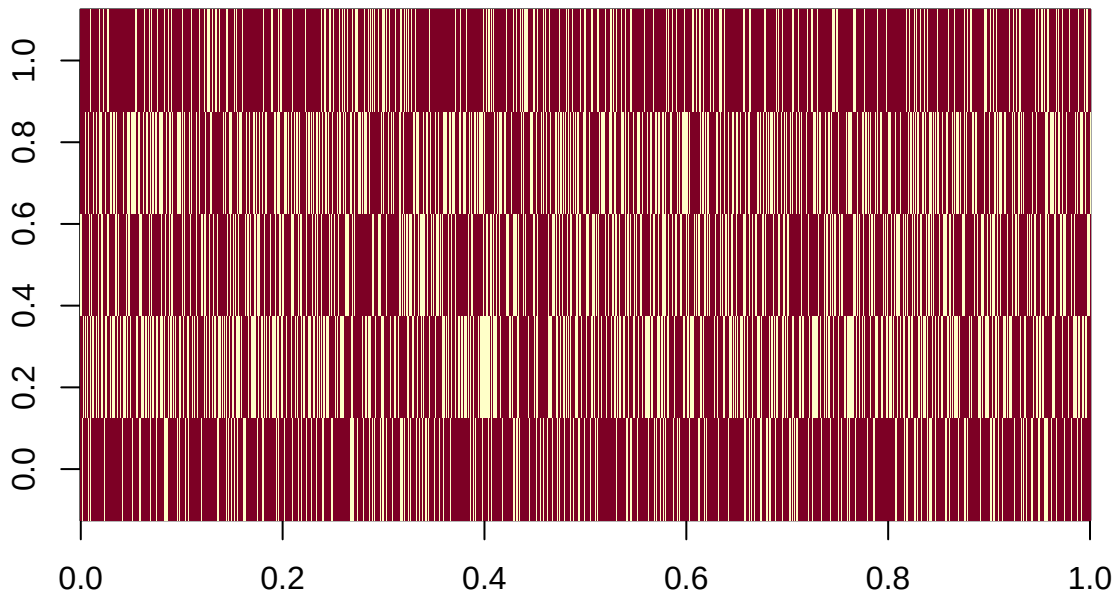
```r
theta <- matrix(0, d, d)
theta[row(theta) == col(theta) - 1] <- -1
theta[row(theta) == col(theta) + 1] <- -1
mu <- rep(2, d)

# Experiment
X <- matrix(0, n_sim, d)

for (i in 2:n_sim) {
  x <- X[i - 1, ]
  for (j in 1:d) {
    eta <- mu[j] + sum(theta[j, ] * x)
    p   <- 1 / (1 + exp(-eta))
    x[j] <- rbinom(1, 1, p)
  }
  X[i, ] <- x
}

# Simple statistics
colMeans(X[(burn + 1):n_sim, ])
```

```
## [1] 0.8026667 0.6113333 0.6906667 0.6146667 0.8013333
```

```r
cov(X[(burn + 1):n_sim, ])
```

```
##               [,1]         [,2]          [,3]        [,4]         [,5]
## [1,]  0.1584985546 -0.032051590 -0.0003753614  0.00329642  0.006800978
## [2,] -0.0320515899  0.237763398 -0.0469255059  0.01290905 -0.003884367
## [3,] -0.0003753614 -0.046925506  0.2137887481 -0.05323193  0.008551479
## [4,]  0.0032964198  0.012909050 -0.0532319324  0.23700956 -0.039245719
## [5,]  0.0068009784 -0.003884367  0.0085514788 -0.03924572  0.159304425
```

```r
image(X[(burn + 1):n_sim, ])
```



Let's pack it into function.

```
simulate_autologistic <- function(n_sim, burn, theta){

d <- ncol(theta)
X <- matrix(0, n_sim, d)

mu <- diag(theta)
diag(theta) <- 0

for (i in 2:n_sim) {
  x <- X[i - 1, ]
  for (j in 1:d) {
    eta <- mu[j] + sum(theta[j, ] * x)
    p   <- 1 / (1 + exp(-eta))
    x[j] <- rbinom(1, 1, p)
  }
  X[i, ] <- x
}
return (X[(burn + 1):n_sim, ])
}
```

```
theta <- matrix(0, d, d)
theta[row(theta) == col(theta) - 1] <- -1
theta[row(theta) == col(theta) + 1] <- -1
diag(theta) <- 2
```

```r
simulations <- simulate_autologistic(20000, 500, theta)
colMeans(simulations)
```

```
## [1] 0.7913333 0.6204103 0.6616410 0.6222051 0.7926154
```

```r
cov(simulations)
```

```
##               [,1]          [,2]         [,3]         [,4]          [,5]
## [1,]  0.1651333573 -0.0350043934  0.008267980 -0.002012787  0.0003155034
## [2,] -0.0350043934  0.2355134478 -0.051106884  0.011003762 -0.0001056938
## [3,]  0.0082679796 -0.0511068839  0.223883660 -0.048755863  0.0068555468
## [4,] -0.0020127870  0.0110037622 -0.048755863  0.235077962 -0.0338890436
## [5,]  0.0003155034 -0.0001056938  0.006855547 -0.033889044  0.1643846667
```

```r
image(simulations)
```