Laboratory 7

Generating different experiments with Markovian structure $\,$

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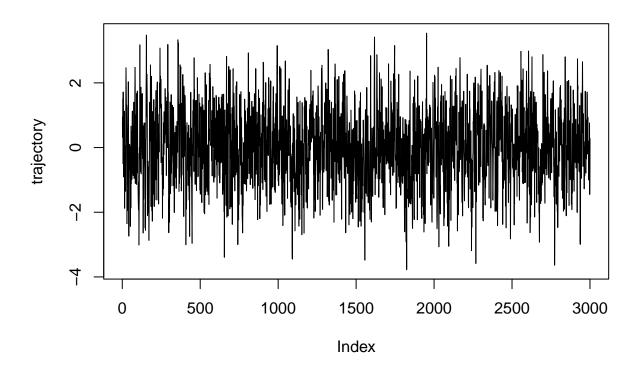
We are doing exercise 2.6 from lecture notes. Firstly, we are creating toy example. Then we define a function which simulates AR(1) process. We are investigating stationarity of our process and answer the question about distribution of X_n .

```
n_times <- 3000
X0 <- rnorm(1)
trajectory <- numeric(n_times)
alpha <- 1/2
trajectory[1] <- X0

for (i in (2:n_times)){
  trajectory[i] = alpha * trajectory[i-1] + rnorm(1)
}
head(trajectory)</pre>
```

```
## [1] 0.6885530 0.9413845 0.3143312 1.5102934 -0.5009041 1.7168632
```

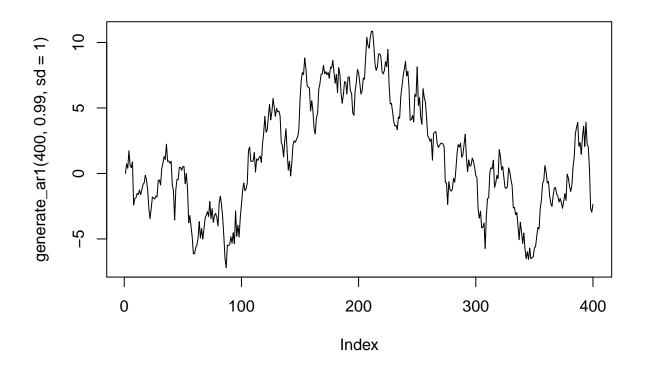
```
plot(trajectory, type = "l")
```



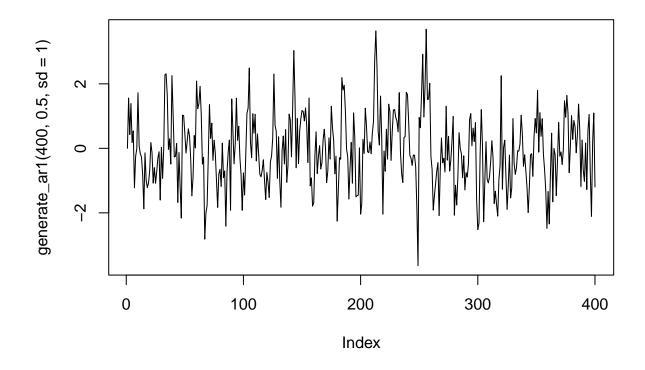
```
generate_ar1 <- function(n_times, alpha, sd){
  X0 <- 0
  trajectory <- numeric(n_times)
  randoms <- rnorm(n_times, mean = 0, sd = sd)

for (i in (2:n_times)){
  trajectory[i] = alpha * trajectory[i-1] + randoms[i]
  }
  return(trajectory)
}

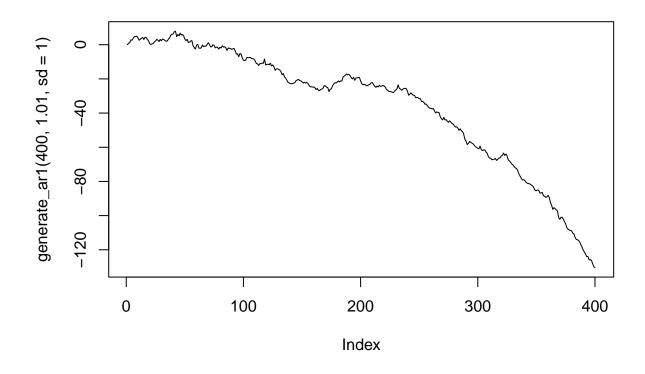
plot(generate_ar1(400, 0.99, sd = 1), type = "l")</pre>
```



plot(generate_ar1(400, 0.5, sd = 1), type = "1")

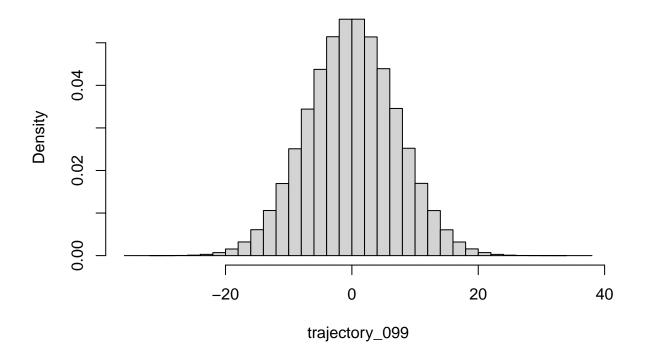


plot(generate_ar1(400, 1.01, sd = 1), type = "1")



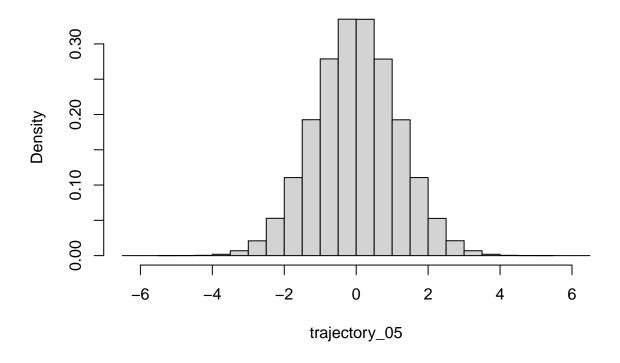
```
trajectory_099 <- generate_ar1(30000000, 0.99, sd = 1)
trajectory_05 <- generate_ar1(30000000, 0.5, sd = 1)
hist(trajectory_099, freq = FALSE)</pre>
```

Histogram of trajectory_099



hist(trajectory_05, freq = FALSE)

Histogram of trajectory_05



```
var(trajectory_099)
```

[1] 50.16248

```
var(trajectory_05)
```

[1] 1.332857

Let's clear the environment and proceed next task.

```
rm(list = ls())
```

Let's simulate Markov process with continuous time.

```
alpha <- 1
beta <- 3
start <- 1
n_jumps <- 200
time <- 0

step <- function(state, alpha, beta){
  if (state == 1){
    return(c(rexp(alpha)))
}</pre>
```

```
else if (state == 2){
    return(c(rexp(beta)))
}

trajectory <- vector()
for (i in 1:n_jumps){
    trajectory <- c(trajectory, step(i %% 2, alpha, beta))
}

# to consider</pre>
```

Clearing the environment.

```
rm(list = ls())
```