

Lab11

Asymptotic Variance

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Today we are going to calculate asymptotic variance.

Let's consider AR(1) model: $X_{n+1} = \alpha X_n + W_{n+1}$

```
create_ar1 <- function(n, alpha){
  process <- numeric(n)
  noise <- rnorm(n)
  for(i in (2:n)){
    process[i] <- alpha*process[i - 1] + noise[i]
  }
  return (process)
}
```

```
ar1_simulations <- function(n, alpha, n_sim){
  simulations <- numeric(n_sim)
  for (i in (1:n_sim)){
    process <- create_ar1(n, alpha)
    simulations[i] <- mean(process)
  }
  return (simulations)
}
```

```
n <- 2000
alpha <- 0
n_sim <- 10000

realizations <- ar1_simulations(n, alpha, n_sim)
estimator <- n*var(realizations)
estimator
```

```
## [1] 1.011907
```

```
n*mean(realizations^2)
```

```
## [1] 1.011837
```

```
2*sd(n*(realizations^2)) / sqrt(n_sim)
```

```
## [1] 0.02861803
```

```
n <- 2000
alpha <- 0.9
n_sim <- 10000

realizations <- ar1_simulations(n, alpha, n_sim)
estimator <- n*var(realizations)
estimator
```

```
## [1] 98.70553
```

```
n*mean(realizations^2)
```

```
## [1] 98.71668
```

```
2*sd(n*(realizations^2)) / sqrt(n_sim)
```

```
## [1] 2.770451
```

```
n <- 2000
alpha <- -0.9
n_sim <- 10000

realizations <- ar1_simulations(n, alpha, n_sim)
estimator <- n*var(realizations)
estimator
```

```
## [1] 0.2750511
```

```
2*sd(n*(realizations^2)) / sqrt(n_sim)
```

```
## [1] 0.007852465
```

```
n <- 2000
alpha <- 0.99
n_sim <- 100000

realizations <- ar1_simulations(n, alpha, n_sim)
estimator <- n*var(realizations)
estimator
```

```
## [1] 9230.695
```

```
2*sd(n*(realizations^2)) / sqrt(n_sim)
```

```
## [1] 82.1651
```

```
n <- 2000
alpha <- -0.99
n_sim <- 10000

realizations <- ar1_simulations(n, alpha, n_sim)
estimator <- n*var(realizations)
estimator
```

```
## [1] 0.2581493
```

```
2*sd(n*(realizations^2)) / sqrt(n_sim)
```

```
## [1] 0.007349645
```