HW8_151B

Elina Ding

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Problem 1 1.

```
x=c(0,0.1,0.2,0.3,0.4,0.5)
y=c(2, 2.20254,2.40715,2.61592,2.83096,3.05448)
```

$$\left[\text{\$\$ 1 \quad 01 \quad 0.11 \quad 0.21 \quad 0.31 \quad 0.41 \quad 0.5} \right] \left[\text{c1c2} \right] = \left[\text{22.202542.407152.615922.830963.05448} \right]$$

$$

To solve the function, we need to find vector c such that $A^{tAc=A}ty$ A is the above matrix, so A^t=

$$\left[1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 10 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \right]$$

$$ We solve the matrix

```
x=c(0,0.1,0.2,0.3,0.4,0.5)
x2=c(rep(1,6),x)
A=matrix(x2, nrow=6)

At=t(A)
left=At%*%A
right=At%*%y
c=solve(left, right)
ans1=A%*%c
c
```

```
##          [,1]
## [1,] 1.992335
## [2,] 2.104694
```

```
ans1
```

```
##          [,1]
## [1,] 1.992335
## [2,] 2.202804
## [3,] 2.413274
## [4,] 2.623743
## [5,] 2.834212
## [6,] 3.044682
```

$$\left[\text{2.} \quad 1 \quad 1 \quad 11 \quad e^{0.1} \quad e^{-0.1}1 \quad e^{0.2} \quad e^{-0.2}1 \quad e^{0.3} \quad e^{-0.3}1 \quad e^{0.4} \quad e^{-0.4}1 \quad e^{0.5} \quad e^{-0.5} \right] \left[\text{c1c2c3} \right] = \left[\text{22.202542.407152.615922.830963.05448} \right]$$

Similarly, we find vector c such that $A^{tAc=A}ty$

```
x
```

```
## [1] 0.0 0.1 0.2 0.3 0.4 0.5
```

```
A=matrix(c(rep(1,6),exp(x),exp(-x)), nrow=6)

At=t(A)
left=At%*%A
right=At%*%y
c=solve(left, right)
ans2=A%*%c
c
```

```
##           [,1]
## [1,]  1.990191
## [2,]  1.015467
## [3,] -1.005639
```

```
ans2
```

```
##          [,1]
## [1,] 2.000018
## [2,] 2.202515
## [3,] 2.407137
## [4,] 2.615931
## [5,] 2.830989
## [6,] 3.054461
```

3. c has been computed, we find the least square residual now

```
sqrt(sum(y-ans1)^2)/sqrt(6)
```

```
## [1] 2.71948e-16
```

```
sqrt(sum(y-ans2)^2)/sqrt(6)
```

```
## [1] 9.064933e-16
```

The residual for problem 1 is 2.71948e-16, and the residual for problem 2 is 9.064933e-16 We can see that the residual for the first approximation is smaller.

Problem 2

1. the system of linear equations is $$\begin{bmatrix} 1 & -1 & 4 \ 1 & 4 & -2 \ 1 & 4 & 2 \ 1 & -1 & 0 \

\end{bmatrix} [c1c2c3] = [10592]

$$

2. We solve c such that A$^{tAc=A}$ty

```
v1=c(1,1,1,1)
v2=c(-1,4,4,-1)
v3=c(4,-2,2,0)
y=c(10,5,9,2)
A=matrix(c(v1,v2,v3), nrow=4)
At=t(A)
left=At%*%A
right=At%*%y
solve(left,right)
```

```
##      [,1]
## [1,]  3.8
## [2,]  0.8
## [3,]  1.5
```

[4 6 46 34 −44 −4 24] [c1c2c3] = [264448]

3. See attached
4.

```
R=matrix(c(2,0,0,3,5,0,2,-2,4) ,nrow=3)
R
```

```
##      [,1] [,2] [,3]
## [1,]    2    3    2
## [2,]    0    5   -2
## [3,]    0    0    4
```

```
Q=t(matrix(c(0.5,0.5,0.5,0.5,-0.5,0.5,0.5,-0.5,0.5,-0.5,0.5,-0.5) , nrow=4))
solve(R, Q%*%y)
```

```
##      [,1]
## [1,]  3.8
## [2,]  0.8
## [3,]  1.5
```

The system is [2 3 20 5 −20 0 4] [c1c2c3] = [1316] To solve this system, we solve $2c1 + 3c2 + 2c3 = 135c2 - 2c3 = 14c3 = 6$ So we get

c3=1.5, substitute c3 in the second equation we get c2=0.8, and so c1=3.8

Problem 3

```
     p=1                  2                   3                4                 5
```

$$M=10:1.489558431e-01 ; 1.38520639e-02 ; 1.348430402e-02 ; 3.91663169e-03 ; 7.17531346e-04 \

20: 1.40623714344279e-01 ; 6.47574688211460e-02 ; 4.60745564203011e-02 ; 1.66756012406856e-02 4.17428049102780e-03 \ 40: 3.98691464527726e-01 ;3.98647324986986e-01; 2.78744176871092e-01 ; 1.39098498917987e-01 6.01839502738770e-02\ 80: 3.61176387466276e-01 ; 3.60044180030240e-01 ; 2.65663636332380e-01; 1.39558228489277e-01 5.59567168058997e-02\ $$

2. M= 10| 4.70332276427008e-01 5.78876771950981e-01 5.72438992717738e-01 4.64315949263320e-01 3.33867561065075e-01

20| 4.83223328819196e-01 5.06433824332837e-01 5.24632617608203e-01 5.44844803807371e-01 4.85670103142444e-01

40| 3.55545856239515e-01 3.56121367483524e-01 2.59430758916691e-01 1.31836184257742e-01 5.16275171949400e-02

80| 3.48122138677863e-01 3.45368503639675e-01 2.59357261115823e-01 1.33177098799108e-01 5.22119286339277e-02

3. As the above information, we find taht the smallest combination is with m=10 and p=5

value of the relative residual of the fit does not correlate with the value of the relative residual of the solution of the linear system since they have diff[illegible]ends e.g. the residual of solution has a smaller residual with smaller M and larger P

Processing math: 100%