ME579 – NONLINEAR VIBRATIONS

TERM PROJECT

MIDDLE EAST TECHNICAL UNIVERSITY

DEPARTMENT OF MECHANICAL ENGINEERING

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This report consists of two parts: user manual for Nonlinear Equation Solver and solved cases by using solver. *(All units are in metric system)*

User Manual

This Nonlinear Equation Solver was written for term project of ME579-Nonlinear Vibrations by using DFM. In the solver, Arc-length method is used. In order to use the program, below steps should be followed.

- To start solver, open “proje_v1.m” file and run it.
- After program started, user should import K, M, H and C matrices and enter all forces act on the system. (K, M, H and C matrices can be fully squared, upper-lower triangular matrix, column, or row matrix, and ready to use in .mat format)

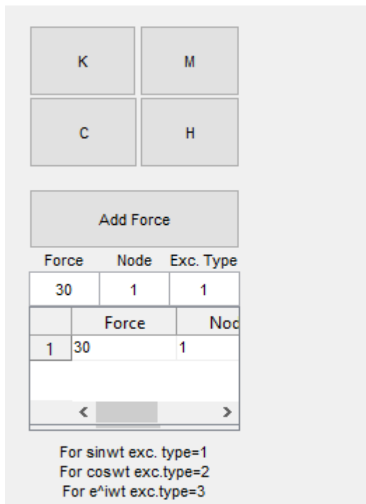


Figure 1

- User can calculate the natural frequencies (rad/s) at this step by pressing the “Calculate Natural Frequencies” button.

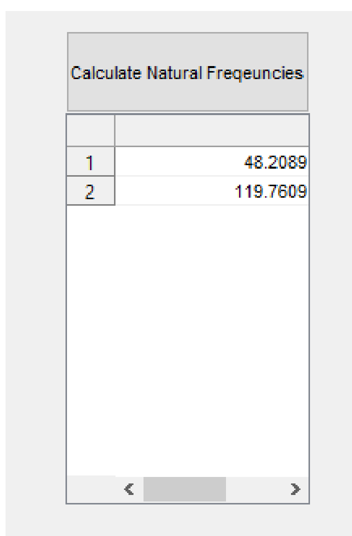


Figure 2

- Now, user can add nonlinear elements to the system as in showed in Figure 3.

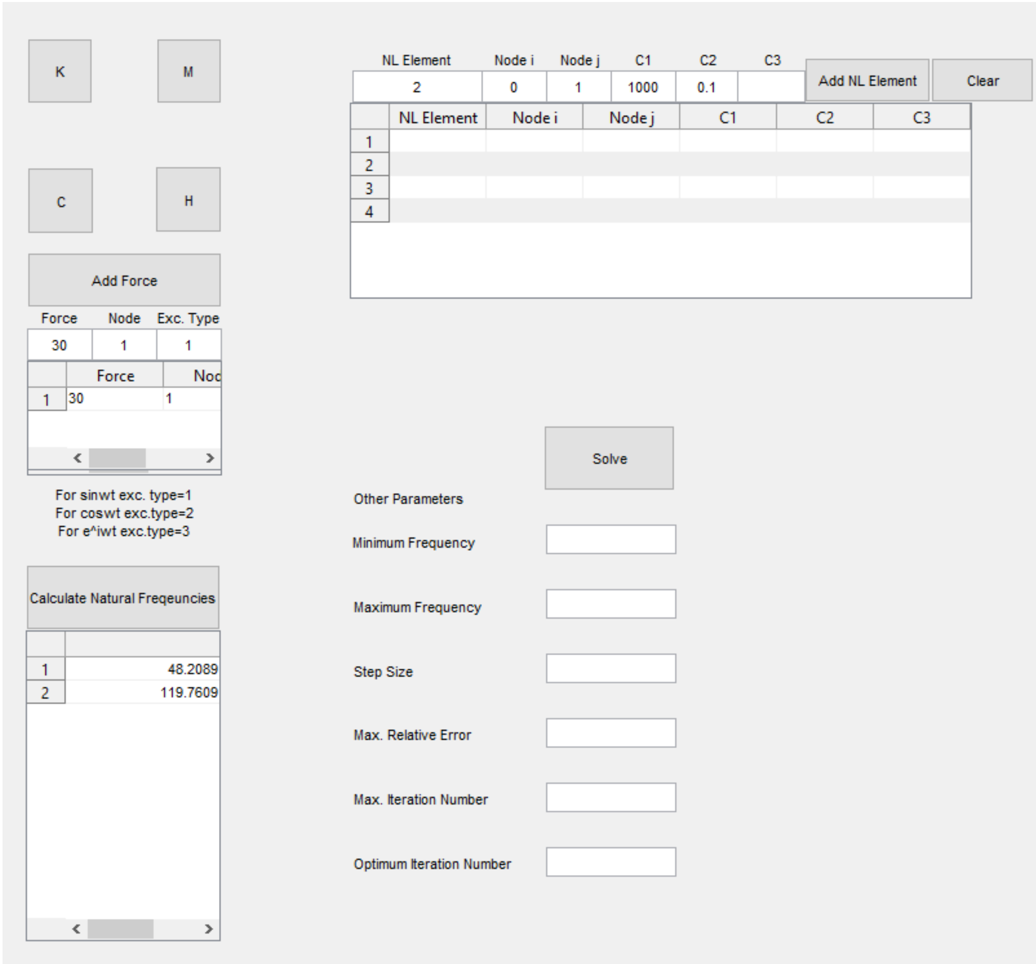


Figure 2

- For adding nonlinear elements and related parameters, Table 1. should be followed.

	NL ELEMENT	**C1**	**C2**	**C3**
CUBIC STIFFNESS	1	kc		
GAP NONLINEARITY	2	kg	delta	
PIECEWISE STIFFNESS	3	k1	k2	delta
DRY FRICTION	4	kd	muN	
CUBIC DAMPING	5	c_c		
SQUARED DAMPING	6	c_s		
SQUARED STIFFNESS	7	ks		

Table 1. Available Nonlinear Elements and Parameters

- After adding nonlinear elements, user should enter remaining inputs as in showed in Figure 3 and press solve. (**Step Size=0.005, Max. Iteration Number=30 and N_opt=2 are recommended values**)

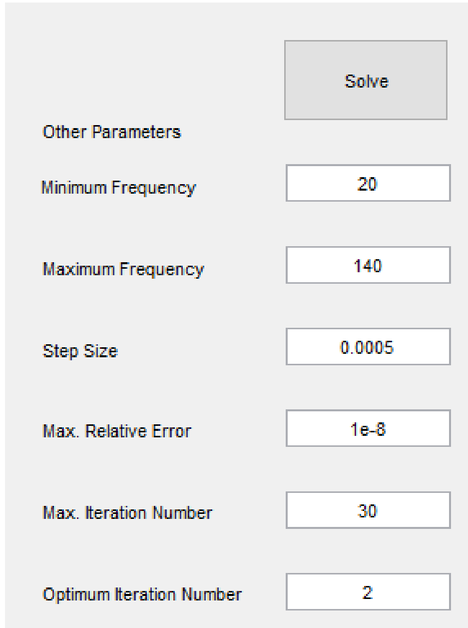


Figure 4

- When solution is done, plotting section will be appeared. User can enter mass coordinate, force coordinate and plot type for desired force normalized response before pressing "Plot" button. Also, responses of all masses (format is xs1 xs2 ... xsn xc1 xc2 … xcn) (responseresults.txt) and frequency interval of solution (frequencyresult.txt) can be exported to root directory of solver in .txt format by using "Export Response Matrix" and "Export Frequency Matrix" buttons.

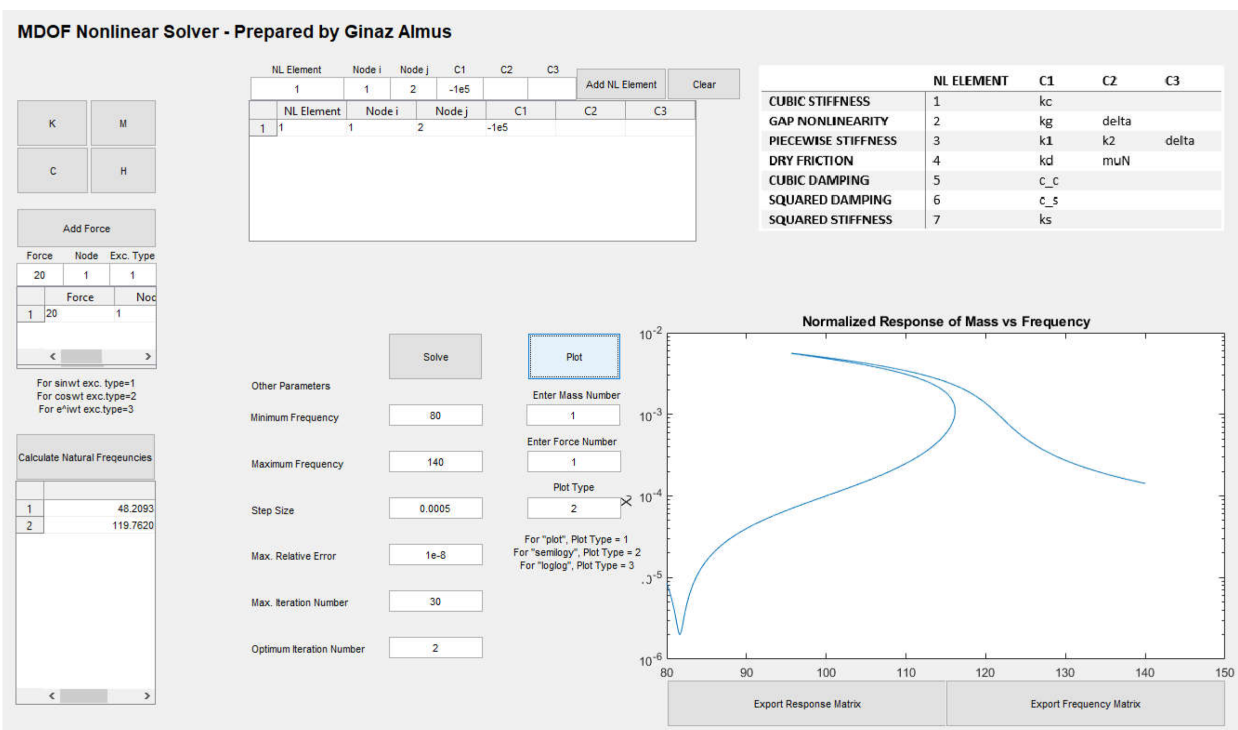
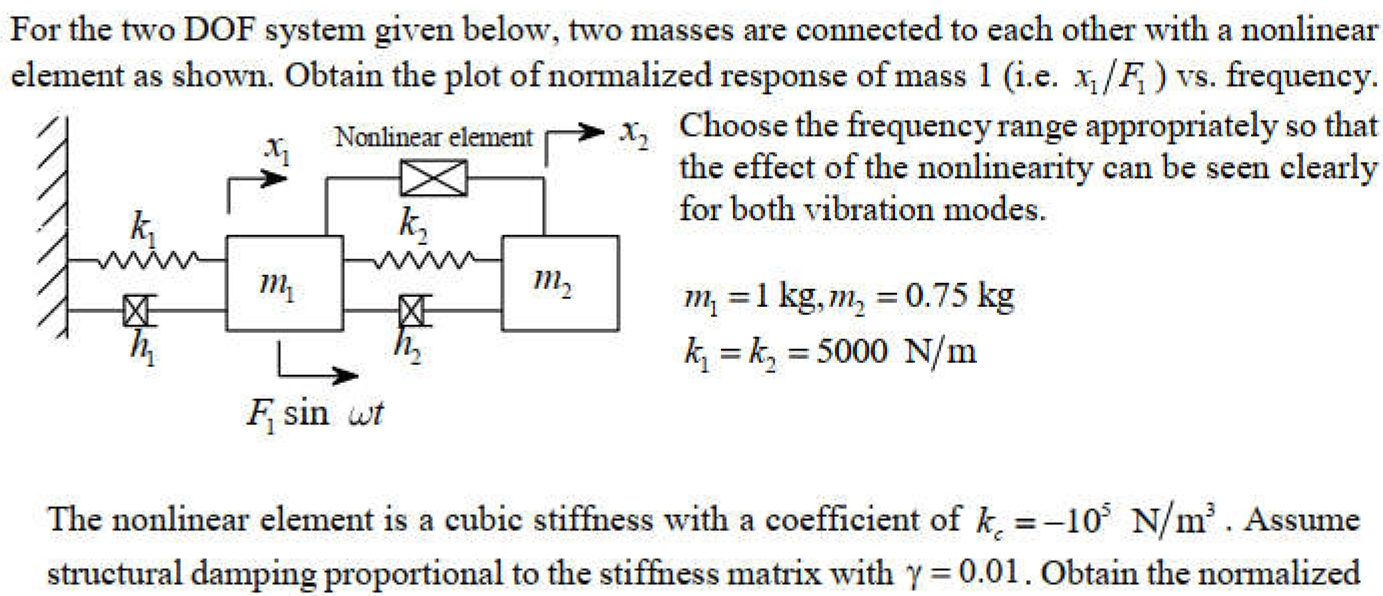


Figure 5

Case 1: Cubic Stiffness Problem (TH2 -Problem 1a):

For this case, cubic stiffness problem (Figure 1) which is taken from TH2 exam is solved by using solver. For this problem, normalized response of Mass 1 is plotted (Figure 2.) for first two modes where F1=20N. All input matrices can be found in Case1 file. Also, other input parameters are available in Figure 2.



For the two DOF system given below, two masses are connected to each other with a nonlinear element as shown. Obtain the plot of normalized response of mass 1 (i.e. x_1/F_1) vs. frequency.

Choose the frequency range appropriately so that the effect of the nonlinearity can be seen clearly for both vibration modes.

$m_1 = 1$ kg, $m_2 = 0.75$ kg

$k_1 = k_2 = 5000$ N/m

The nonlinear element is a cubic stiffness with a coefficient of $k_c = -10^5$ N/m^3. Assume structural damping proportional to the stiffness matrix with $\gamma = 0.01$. Obtain the normalized

Figure 1. Cubic Stiffness Problem where F1=20N

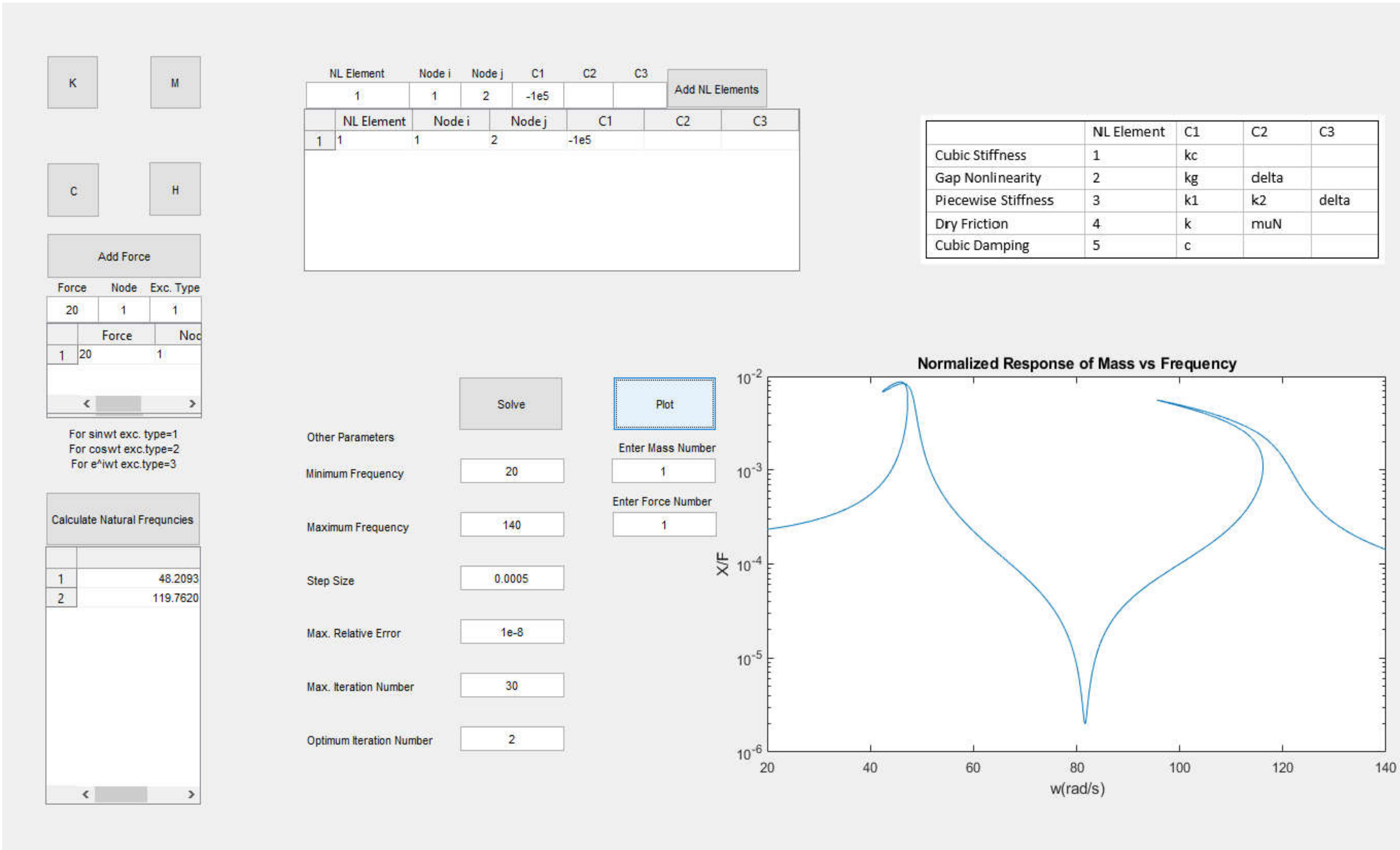
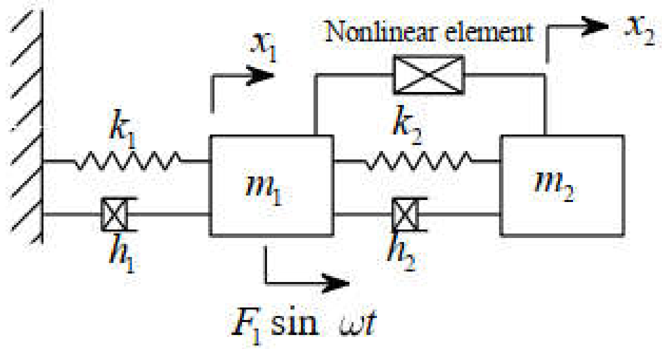


Figure 2. Normalized response of Mass 1 and other parameters

Case 2: Gap Nonlinearity Problem

For this case, previous case (Case 1) is modified where only nonlinear element is gap nonlinearity. For this problem, normalized response of mass 1 is plotted (Figure 4.) for first and second mode where kg values are 500 and 1000 and gap is 0.1. All input matrices can be found in Case2 file. Also, other input parameters are available in Figure 4.

For the two DOF system given below, two masses are connected to each other with a nonlinear element as shown. Obtain the plot of normalized response of mass 1 (i.e. x_1/F_1) vs. frequency.



Choose the frequency range appropriately so that the effect of the nonlinearity can be seen clearly for both vibration modes.

$m_1 = 1$ kg, $m_2 = 0.75$ kg

$k_1 = k_2 = 5000$ N/m

Figure 3. Gap Nonlinearity Problem where kg=[500 2000] Nm^{-1}, γ=0.005 and F1=30N

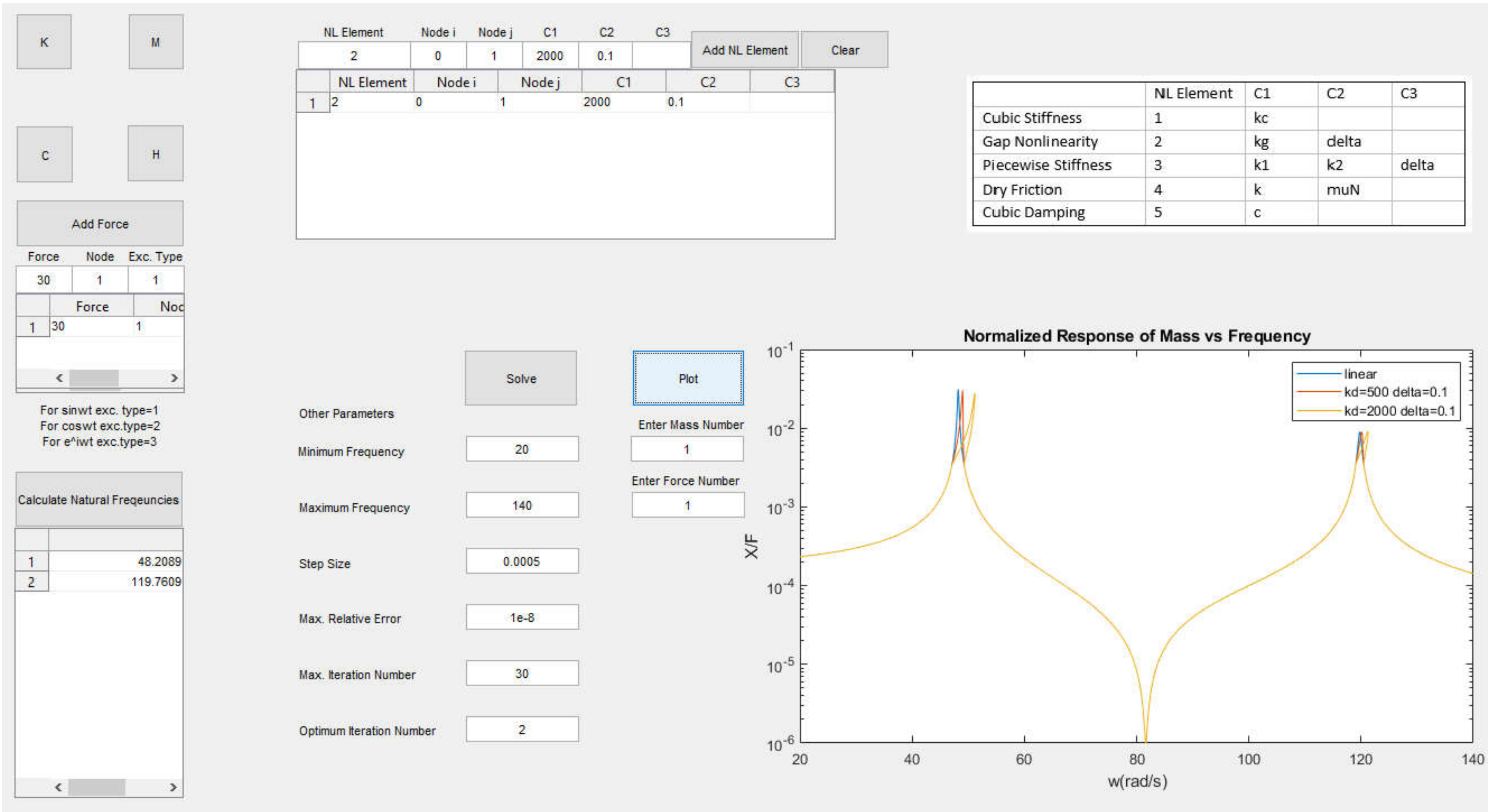
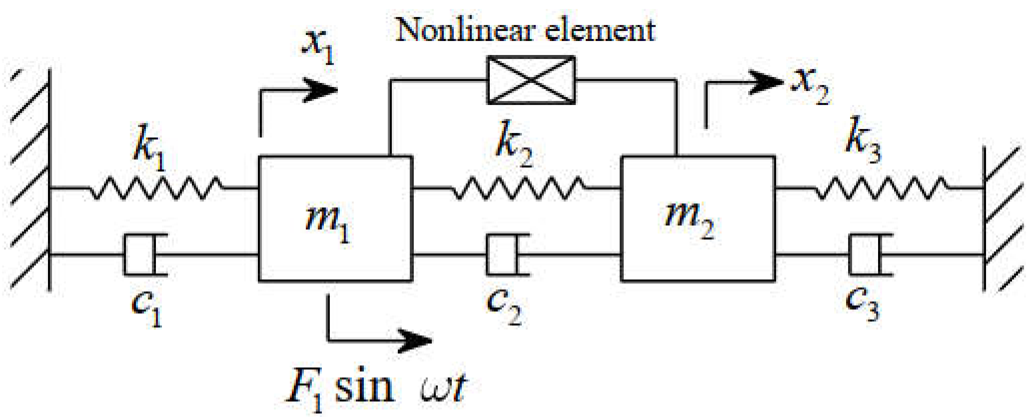


Figure 4. Normalized response of Mass 1 and other parameters

Case 3: Piecewise Stiffness Problem (TH1-Problem 4)

For this case, piecewise linear stiffness problem (Figure 5) which is taken from TH1 exam is solved by using solver. For this problem, normalized response of Mass 1 is plotted (Figure 6) for second mode. All input matrices can be found in Case3 file. Also, other input parameters are available in Figure 6.

For the two DOF system given below, the first and second coordinates are connected to each other with a piecewise linear stiffness element having $k_1 = 10$ N/m, $k_2 = 100$ N/m.



$m_1 = 1$, $m_2 = 0.75$ kg
$k_1 = k_2 = 1000$ N/m
$k_3 = 50$ N/m
$c_1 = c_2 = 0.05$ Ns/m
$c_3 = 0.00025$ Ns/m

On <u>the same plot</u>, obtain the normalized response of the first mass vs. frequency between 3-11Hz with no nonlinear element and with nonlinear element having the following break values. $\delta = 0.001, 0.005, 0.01, 0.03, 0.05, 0.08, 0.15$ m (Amplitude of the forcing is $F_1 = 1$ N).

Figure 5. Piecewise Linear Stiffness Problem

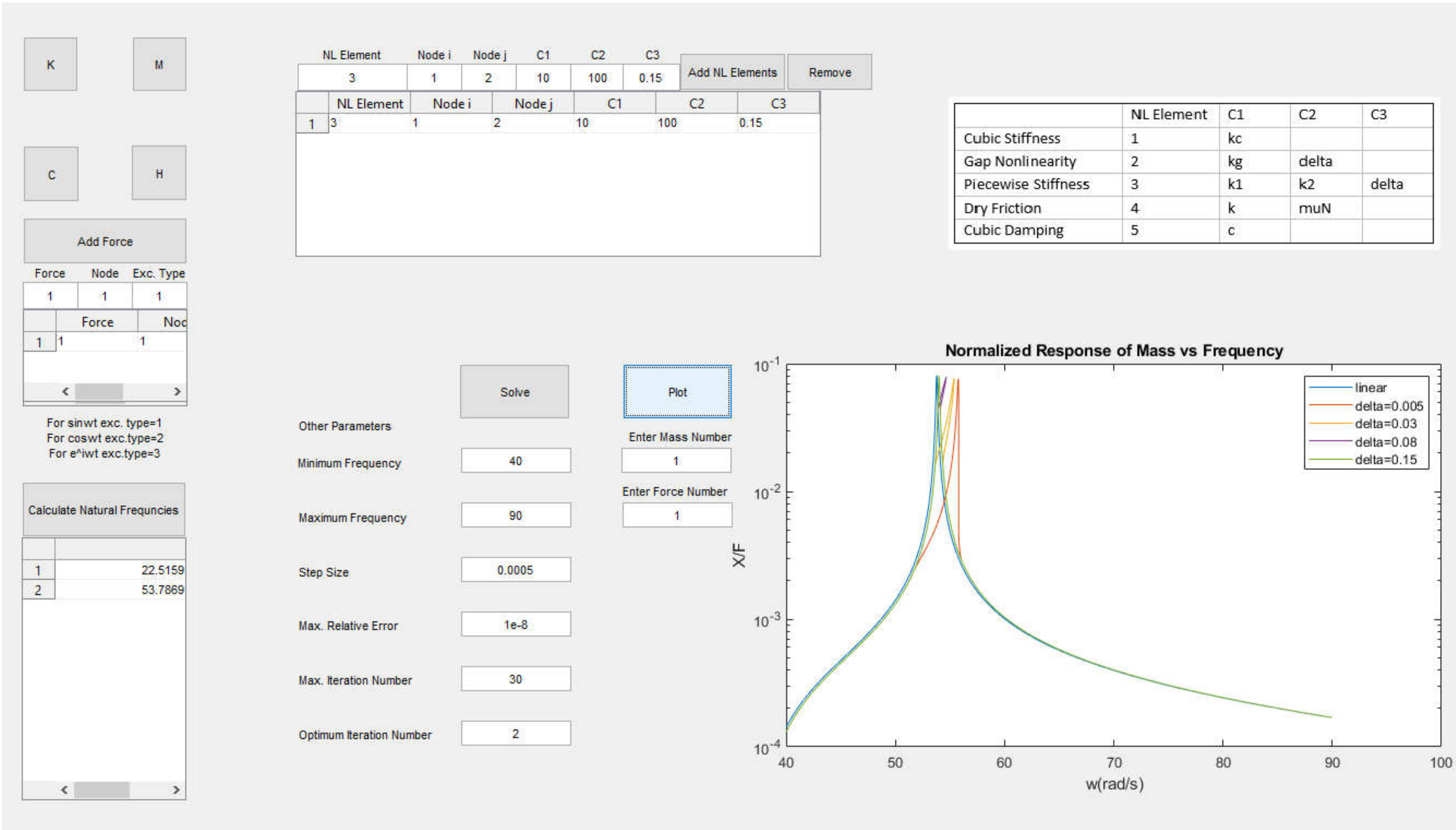
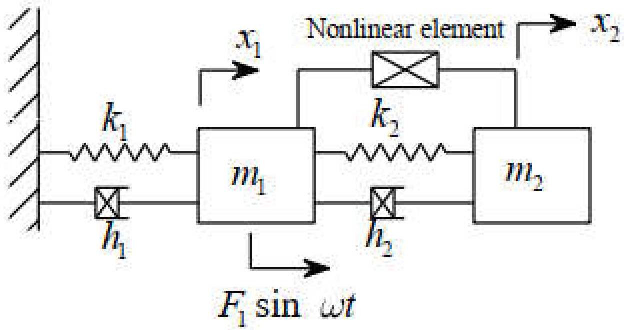


Figure 6. Normalized response of Mass 1 around second natural frequency and other parameters

Case 4: Dry Friction Problem (TH2 -Problem 1b):

For this case, dry friction problem (Figure 7) which is taken from TH2 exam is solved by using solver. For this problem, normalized response of mass 1 is plotted (Figure 8) for first mode. All input matrices can be found in Case4 file. Also, other input parameters are available in Figure 8.

For the two DOF system given below, two masses are connected to each other with a nonlinear element as shown. Obtain the plot of normalized response of mass 1 (i.e. x_1/F_1) vs. frequency.



Choose the frequency range appropriately so that the effect of the nonlinearity can be seen clearly for both vibration modes.

$m_1 = 1$ kg, $m_2 = 0.75$ kg
$k_1 = k_2 = 5000$ N/m

The nonlinear element is a dry friction damper with stiffness of $k = 2000\ N/m$. The excitation force amplitude is $F_1 = 30N$ and loss factor is $\gamma = 0.005$. Obtain the normalized response for $\mu N = 5,\ 10,\ 15,\ 20,\ 30,\ 50,\ 100,\ 200,\ 1000N$ on the same plot together with the normalized response of the linear system and comment on the results.

Figure 7. Dry Friction Problem

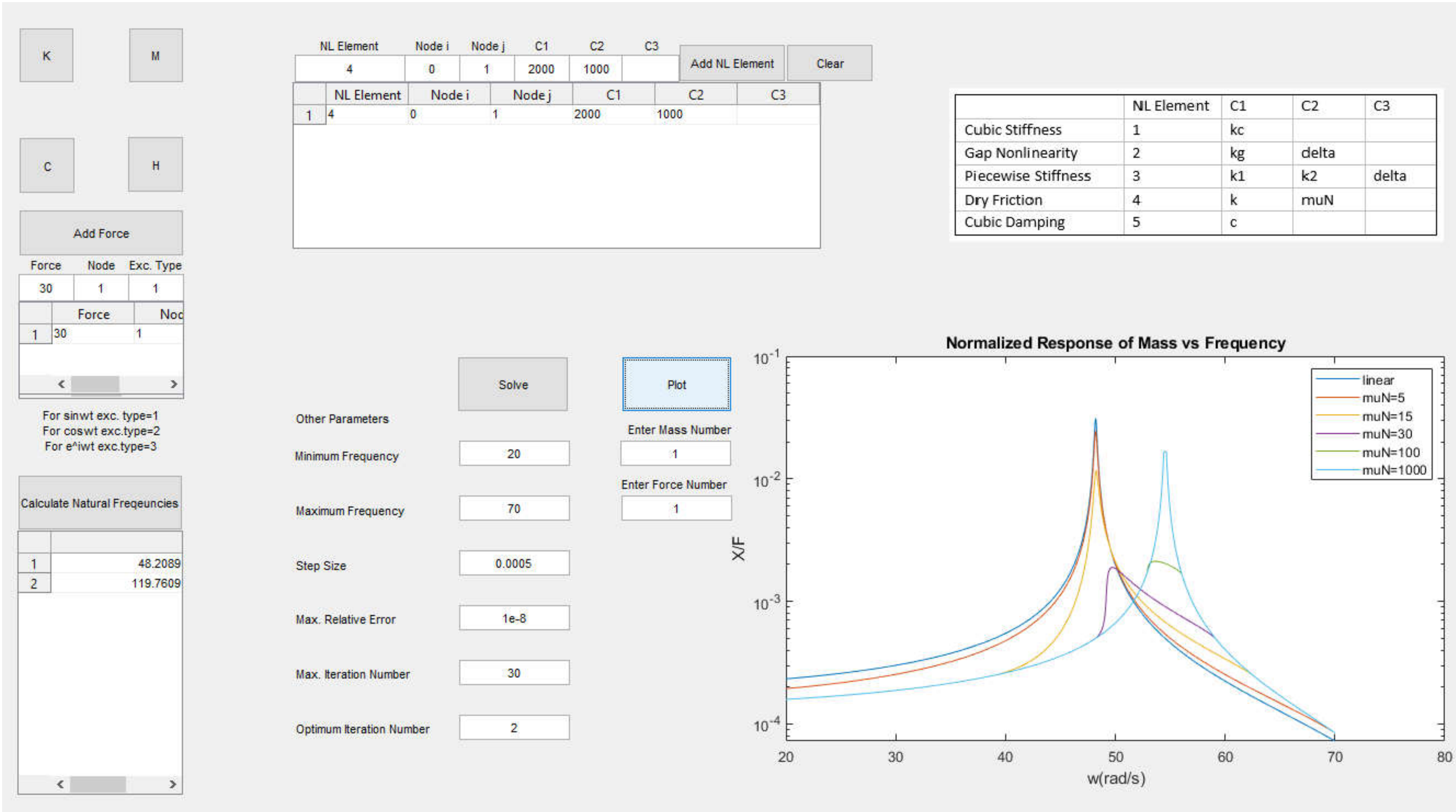
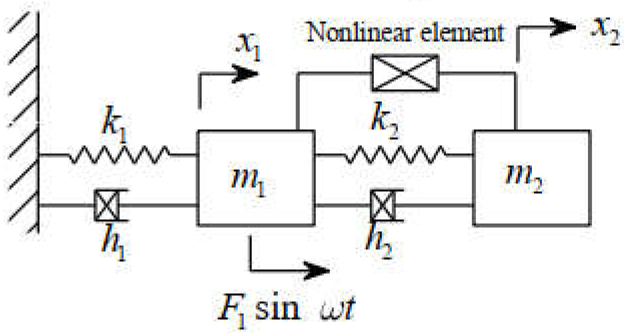


Figure 8. Normalized response of Mass 1 around first natural frequency and other parameters

Case 5: Cubic Damping Problem: For this case, previous case (Case 4) is modified where only nonlinear element is cubic damping. For this problem, normalized response of mass 1 is plotted (Figure 9) for first and second mode where cubic damping values are 0.1, 0.5, 1, 5 and 10. All input matrices can be found in Case5 file. Also, other input parameters are available in Figure 9. Describing function for cubic damping nonlinearity is taken from Elliott's study [2].

For the two DOF system given below, two masses are connected to each other with a nonlinear element as shown. Obtain the plot of normalized response of mass 1 (i.e. x_1/F_1) vs. frequency.



Choose the frequency range appropriately so that the effect of the nonlinearity can be seen clearly for both vibration modes.

$m_1 = 1\ \text{kg}, m_2 = 0.75\ \text{kg}$

$k_1 = k_2 = 5000\ \text{N/m}$

Figure 9. Cubic Damping Problem where c_c=[0.1 0.5 1 5 20] Ns^3m^{-3}, γ=0.005 and F1=30N

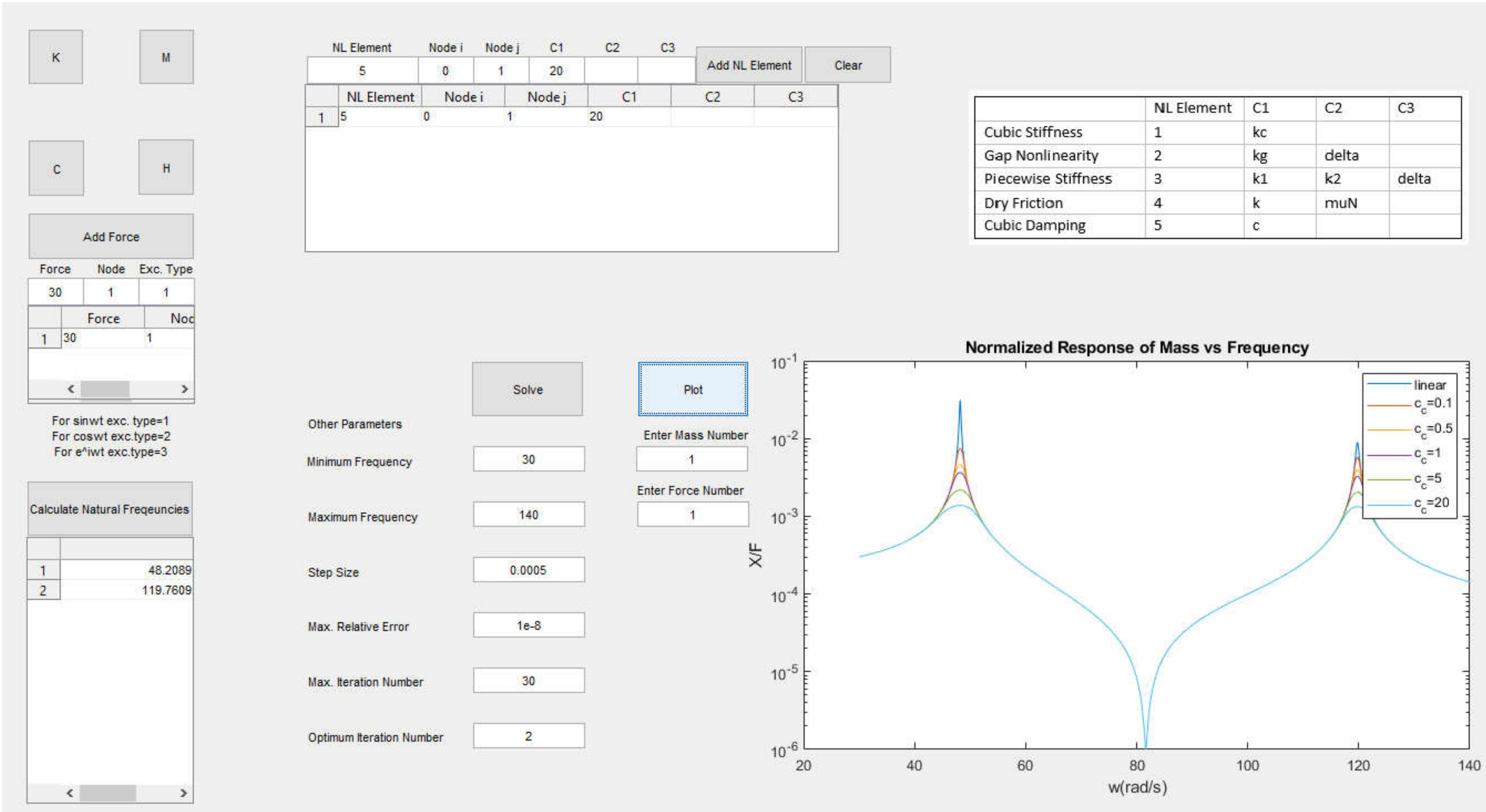


Figure 10. Normalized response of Mass 1 and other parameters

References

[1] ME579 Lecture Notes

[2] Elliott SJ, Ghandchi Tehrani M, Langley RS. 2015 Nonlinear damping and quasi-linear modelling. *Phil. Trans. R. Soc. A.* **373:** 20140402

[3] Ferhatoglu, E., Cigeroglu, E., & Özgüven, H. N. (2018). A new modal superposition method for nonlinear vibration analysis of structures using hybrid mode shapes. Mechanical Systems and Signal Processing, 107, 317–342.