Project 3: The Building Blocks of Planet Formation

Key Topic: PDE Initial Value Problem

1 Background

Understanding how planets form around young stars is one of the biggest unsolved problems in modern astrophysics. With this problem, you will use a simulation to compare to new observations to understand this challenge first-hand. One of the biggest challenges is understanding how the building blocks of planets form: planetesimals (1-100 km sized rocky objects).

The environment in which planets form is a flattened disc-like structure that surrounds a young star. It should be no surprise to you that planets form in a disc, as all the Solar System planets lie in a plane that lies perpendicular to the angular momentum vector of the Sun's spin. This disc-like structure is known as an accretion disc. These discs are composed of mainly hydrogen and helium gas; however, they contain a small amount of "dust". These are small solid particles formed when a previous generation of stars died in supernova explosions (you can think of them essentially as sand particles). Measurements indicate that planetesimals are made by combining these small solid particles. This dust makes up 1% of the total mass of the disc, and the other 99% is the hydrogen/helium gas mixture. Planetesimals are thought to form through the "streaming instability" (see video1 & video2). However, this requires the ratio of dust density to gas density to become > 1 locally. One idea is that while the gas can be supported vertically in the disc against gravity by pressure, the dust cannot and should settle to the mid-plane of the disc, locally increasing the density.

We can describe the disc structure in cylindrical coordinates and assume axis-symmetry: R is the cylindrical radius from the star, z points out of the disc, orthogonal to its midplane, which occurs at $z = 0$. The midplane acts as a symmetry point where the disc structure is symmetrical above and below the mid-plane (i.e. $\rho_d(z) = \rho_d(-z)$, etc).

The gas in the disc is vertically in hydrostatic equilibrium and has the following density profile:

$$\rho_g = \rho_g(R, z = 0)\exp\left(-\frac{z^2}{2H(R)^2}\right)$$

where $H(R)$ is the "scale height" of the disc. However, the dust particles in the disc approximately[1] obey an flux-conservative advection equation:

$$\frac{\partial \rho_d}{\partial t} + \frac{\partial}{\partial z}\left(u_z \rho_d\right) = 0$$

where the dust's velocity u_z, is given by the terminal velocity approximation (assuming the disc is thin, $H \ll R$):

$$u_z = -\Omega^2 z t_s, \quad \text{where,} \quad t_s = \sqrt{\frac{\pi}{8}}\frac{\rho_{\rm in} a}{\rho_g c_s}$$

where $\Omega = \sqrt{GM/R^3}$ is the angular frequency of a circular Keplerian orbit at a radius R (with M the star's mass) and t_s is the "settling timescale" that depends on the dust particle's size a, its internal density $\rho_{\rm in} = 1000$ kg m^{-3} (i.e. the density of an individual dust grain) and $c_s = \sqrt{k_b T(R)/(2m_H)}$, the gas disc's sound-speed with $T(R)$ the temperature, k_b the Boltzmann constant and m_H the mass of the hydrogen atom. The disc's temperature is only a function of cylindrical radius and is constant in the $\hat{z}$ direction. The scale height of the disc is related to the sound speed and angular frequency through $H = c_s/\Omega$.

2 Problem Setup

Convert the problem into one in natural units, by using appropriate length, timescales etc, in the problem.

Using the dimensionless equation for the dust evolution, construct an update scheme that's first-order in space and time using the upwind method, making sure to choose an appropriate timestep correctly.

You should study this problem for discs around a 1 solar mass star (2×10^{30} kg).

[1] It's approximate because rather than modelling individual dust particles and all the forces on them, we treat the dust particles as a continuous media and just model its density distribution. Given one planetesimal contains $\sim 10^{30}$ dust particles, it would be impossible to model them as individual particles.

3 Dust settling

A disc's temperature obeys the form:

$$T(R) = 264\text{ K}\left(\frac{R}{1\text{ AU}}\right)^{-1/2}$$

where 1 AU is the Earth-Sun distance = 1.5×10^{11} m. The mid-plane gas density obeys:

$$\rho_g(R, z = 0) = 5 \times 10^{-7}\text{ kg m}^{-3}\left(\frac{R}{1\text{ AU}}\right)^{-3}$$

The dust and gas are initially uniformly mixed in the disc, such that $\rho_d(t = 0) = 0.01\rho_g$, and the dust-particles are 1μm in size. Write a computer code to evolve the dust density distribution and determine the time it takes to reach a local dust density to gas density > 1, allowing planetesimal formation to occur. Study this timescale as a function of radial distance in the disc.

Considering these discs are only around for 1 Myr, comment on your results in relation to the planets in the Solar System and the requirement that planetesimals are the initial planetary building blocks.

Think carefully about your domain size, boundary conditions and number of grid points, making sure your conclusions do not depend on these choices.

3.1 Turbulent Diffusion of The Dust

You will have seen the dust settle into a thin layer in the mid-plane of the disc. However, observations indicate that 1 μm dust particles have not settled to the mid-plane after 1 Myr at large radial distances. Figure 2 shows an observed disc demonstrating this phenomenon.

The explanation is these discs are turbulent, where the turbulence acts as an effective diffusion, preventing the dust from fully settling into the mid-plane. Our theories cannot predict the strength of the turbulence and it must be estimated from the observations. The effective diffusion constant given by the turbulence is approximated, using dimensionless arguments, as:

$$D = \alpha\frac{c_s^2}{\Omega}$$

where α is a dimensionless constant. Such that the dust actually obeys an advection-diffusion equation of the form:

$$\frac{\partial\rho_d}{\partial t} + \frac{\partial}{\partial z}(u_z\rho_d) - D\frac{\partial}{\partial z}\left[\rho_g\frac{\partial}{\partial z}\left(\frac{\rho_d}{\rho_g}\right)\right] = 0$$

By including diffusion in your code, use the observation to estimate the value of α, giving your result to an order-of-magnitude accuracy. To do this you should compare your results with (for different values of α) and without turbulence considering a radial distance of 100 AU.

Using your observationally determined value of α compare your results to those found without turbulence at $R = 100$ AU and comment of the ability to form planetesimals in the presence of turbulence.

4 Dust Particle Growth

You have shown that forming planetesimals with 1μm particles is difficult, especially in the presence of turbulence. However, observations have indicated that the dust particles can grow through mutual collisions and sticking (much like two snowballs sticking together when they collide). Use your code to determine how big the particles to have to grow too in order that they can overcome to the turbulence, settle to the mid-plane and form planetesimals.

Give your results for particle size at different radial distances from the Sun at radial distances where we find the giant planets.

4.1 Numerical diffusion

Your numerical scheme to solve the advection problem for the settling of the dust includes numerical (or artificial) diffusion. Demonstrate your results about the particle size required to form planetesimals are unaffected by numerical diffusion, by measuring the effective strength of numerical diffusion in your code.

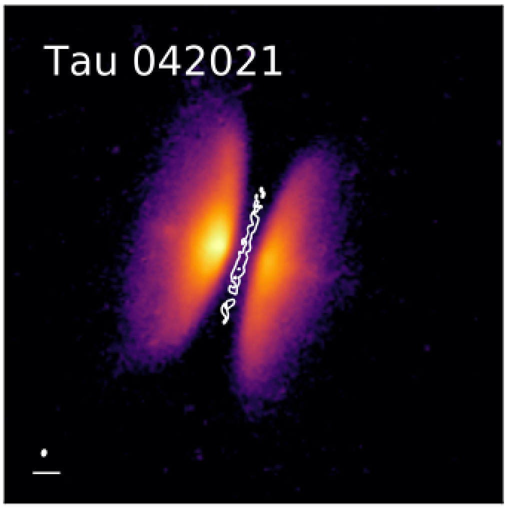

Figure 2: An image of a planet-forming disc in the Taurus star forming cluster, taken at a wavelength of $\sim 1\ \mu$m, showing the location of 1μm particles. The dark region corresponds to the location of dust particles, as they are blocking the light from the young star. The bar in the left corner represents a distance of 33 AU. The white contour shows the location of the disc's mid-plane. You can estimate the height at which the dust has settled by measuring the height above the mid-plane of the transition from the dark lane to the bright emission. For example at $R \sim 100$ AU this happens around $z \sim 0.1R$. Figure from Villenave et al. (2020).

References

Villenave M., Ménard F., Dent W. R. F., Duchêne G., Stapelfeldt K. R., Benisty M., Boehler Y., et al., 2020, A&A, 642, A164. doi:10.1051/0004-6361/202038087