Homework 4.

4.1.

x	y
0	2
1	-1
2	3

a) Determine the Fourier coefficients m, a and b such that the model

$$y = f(x) = m + a\cdot\cos\left(\frac{2\pi x}{L}\right) + b\cdot\sin\left(\frac{2\pi x}{L}\right)$$

fits the measurement data exactly.

$$\left.\begin{aligned} y_0 &= m + a\cos(0) + b\sin(0) \\ y_1 &= m + a\cos\left(\tfrac{2\pi}{3}\right) + b\sin\left(\tfrac{2\pi}{3}\right) \\ y_2 &= m + a\cos\left(\tfrac{4\pi}{3}\right) + b\sin\left(\tfrac{4\pi}{3}\right) \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} &① \; 2 = m + a \\ &② \; -1 = m - \tfrac{a}{2} + b\tfrac{\sqrt{3}}{2} \\ &③ \; 3 = m - \tfrac{a}{2} - b\tfrac{\sqrt{3}}{2} \end{aligned}\right\} \Rightarrow \quad m = 2 - a \quad ①$$

$$\left.\begin{aligned} ② + ③ &\rightarrow 2 = 2m - a \\ ① &\rightarrow m = 2 - a \end{aligned}\right\} \Rightarrow 2 = 4 - 3a \Rightarrow \boxed{a = \frac{2}{3}} \qquad \boxed{m = \frac{4}{3}}$$

$$-1 = \frac{4}{3} - \frac{1}{3} + b\frac{\sqrt{3}}{2} \Rightarrow -2 = b\frac{\sqrt{3}}{2} \Rightarrow \boxed{b = -\frac{4}{\sqrt{3}}}$$

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