Homework 5.

5.1. Symmetry relations.

Fill in the following table as they relate to the sums and products to symmetric functions.
Code: E = even, O = odd, H = Hermitition, A = asymmetric.

Sums.

- $E + E = E$: $f(x) = E(x) + E(x) \longrightarrow f(-x) = E(-x) + E(-x) = E(x) + E(x) = f(x)$

- $E + O = A$: $f(x) = E(x) + O(x) \longrightarrow f(-x) = E(x) - O(x) \begin{cases} \neq f(x) & \text{(Par)} \\ \neq -f(x) & \text{(Impar)} \end{cases}$

- $O + O = O$: $f(x) = O(x) + O(x) \longrightarrow f(-x) = -O(x) - O(x) = -f(x)$

- $H + H = H$: $f(x) = H(x) + H(x) \longrightarrow f^*(-x) = H(x) + H(x) = f(x)$

$$f(-x) = f^*(x)$$

Products.

- $E \cdot E = E$: $f(x) = E(x) \cdot E(x) \longrightarrow f(-x) = E(x) \cdot E(x) = f(x)$

- $E \cdot O = O$: $f(x) = E(x) \cdot O(x) \longrightarrow f(-x) = -E(x) \cdot O(x) = -f(x)$

- $O \cdot O = E$: $f(x) = O(x) \cdot O(x) = (O(x))^2 \longrightarrow f(-x) = (-O(x))^2 = (O(x))^2 = f(x)$

- $H \cdot H = H$: $H(x) = a(x) + i\,b(x)$, siendo $a(x)$ par y $b(x)$ impar, es decir,

$$\begin{cases} a(x) = a(-x) \\ b(x) = -b(-x) \end{cases}$$

$$(H(x))^2 = a^2(x) - b^2(x) + 2a(x)b(x)i$$

Entonces: $(H^*(-x))^2 = a^2(-x) - b^2(-x) - 2i\,a(-x)\,b(-x) = a^2(x) - b^2(x) + 2a(x)b(x)i = (H(-x))^2$

5.2. Integrals of symmetric functions.

Hint: $\int_{-a}^{a} E(x)\,dx \neq 0$; $\int_{-a}^{a} O(x)\,dx = 0$

$$\begin{cases} \cos(x) \longrightarrow \text{par} \\ \sin(x) \longrightarrow \text{impar} \end{cases}$$

a) $\int_{-a}^{a} \underbrace{\underbrace{E(x)}_{\text{Par}}\underbrace{\cos(x)}_{\text{Par}}}_{\text{Par}}\,dx = 2\int_{0}^{a} E(x)\cos(x)\,dx$; b) $\int_{-a}^{a} \underbrace{\underbrace{O(x)}_{\text{Impar}}\underbrace{\cos(x)}_{\text{Par}}}_{\text{Impar}}\,dx = 0$

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c) $\int_{-a}^{a} \underbrace{\underbrace{E(x)}_{\text{Par}} \underbrace{\sin(x)}_{\text{Impar}}}_{\text{Impar}} dx = 0$

d) $\int_{-a}^{a} \underbrace{\underbrace{O(x)}_{\text{Impar}} \underbrace{\sin(x)}_{\text{Impar}}}_{\text{Par}} dx = 2\int_{0}^{a} O(x) \sin(x)\, dx$

Homework 6.

6.2. Determine the Fourier coefficients of:

$$I(x) = 2\sin(x) - \sin(2x)$$

$$\boxed{a_K = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{\underbrace{(2\sin(x) - \sin(2x))}_{\text{Impar}} \underbrace{\cos(Kx)}_{\text{Par}}}_{\text{Impar}} dx = 0}$$

$$b_K = \frac{1}{\pi}\left\{\int_{-\pi}^{\pi} 2\sin(x)\sin(Kx) - \int_{-\pi}^{\pi}\sin(2x)\sin(Kx)\,dx\right\}$$

Teniendo en cuenta que: $\int_{-\pi}^{\pi} \sin(mx)\sin(Kx) = \pi\,\delta_{mK}$

Entonces, nos queda que:

$$\boxed{b_K = \frac{1}{\pi}\left[2\pi\,\delta_{1K} - \pi\,\delta_{2K}\right]} \Rightarrow \boxed{\begin{cases} K=1 \rightarrow b_1 = 2 \\ K=2 \rightarrow b_2 = -1 \\ K\neq 1,2 \rightarrow b_K = 0 \end{cases}}$$

6.3. If a function $f(x)$ has the set of Fourier coefficients a_K, b_K, what are the Fourier coefficients for the function $f(-x)$? If the coefficients of $f(x)$ are expressed in complex form as c_K, what are the coefficients for $f(-x)$?

- Serie de Fourier para $f(x)$: $$f(x) = \frac{a_0}{2} + \sum_{K=1}^{\infty}\left(a_K\cos\left(\frac{2\pi Kx}{L}\right) + b_K\sin\left(\frac{2\pi Kx}{L}\right)\right)$$

Para $f(-x)$: $$\boxed{f(-x) =} \frac{a_0}{2} + \sum_{K=1}^{\infty}\left(a_K\cos\left(-\frac{2\pi Kx}{L}\right) + b_K\sin\left(-\frac{2\pi Kx}{L}\right)\right) =$$

$$= \boxed{\frac{a_0}{2} + \sum_{K=1}^{\infty}\left(a_K\cos\left(\frac{2\pi Kx}{L}\right) - b_K\sin\left(\frac{2\pi Kx}{L}\right)\right)} \Rightarrow \begin{cases} a_K \text{ queda igual} \\ b_K \text{ cambia de signo}\end{cases}$$

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En forma compleja:

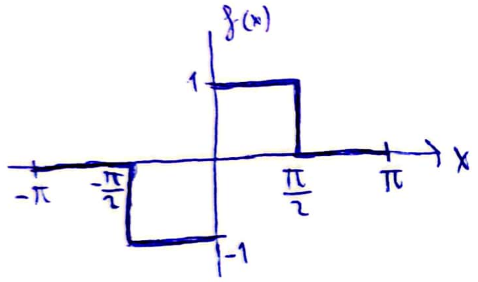
$$f(x) = \sum_{K=-\infty}^{\infty} c_K e^{iKx} \longrightarrow \boxed{f(-x) = \sum_{K=-\infty}^{\infty} c_K e^{-iKx} = \sum_{-K=-\infty}^{\infty} c_{-K} e^{iKx}}$$

Los c_K se intercambian con los c_{-K}

Otra forma de obtener los coeficientes de Fourier complejos es, una vez calculados a_K y b_K, hallar los c_K mediante su relación.

$$c_K = \frac{a_K - i b_K}{2} \qquad c_{-K} = \frac{a_K + i b_K}{2} \quad \xRightarrow{f(-x)} \quad c_K = \frac{a_K + i b_K}{2} \hat{=} c_{-K} \qquad c_{-K} = \frac{a_K - i b_K}{2} \hat{=} c_K$$

6.4. Determine the Fourier series for the "dipole" function $f(x)$.



Vemos que es una función Impar, por tanto:

$$\boxed{a_K = 0}$$

$$b_K = \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{f(x) \sin(Kx)}_{\text{Par}} dx = \frac{2}{\pi} \int_0^{\pi} f(x) \sin(Kx)\, dx = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \sin(Kx)\, dx = \frac{2}{K\pi} \left(1 - \cos\left(\frac{\pi}{2} K\right)\right)$$

$$\cos\left(\frac{\pi}{2} K\right) = \begin{cases} 0 & \text{si } K \text{ es impar} \\ (-1)^{K/2} & \text{si } K \text{ es par} \end{cases}$$

Por tanto:

$$\boxed{\begin{cases} b_K = \frac{2}{K\pi} & \text{si } K \text{ impar} \\ b_K = \frac{2}{K\pi}\left(1 - (-1)^{K/2}\right) & \text{si } K \text{ par} \begin{cases} \text{si } K = 2 + 4n,\ n \in \mathbb{N} \longrightarrow b_K = \frac{4}{K\pi} \\ \text{si } K = 4 + 4n,\ n \in \mathbb{N} \longrightarrow b_K = 0 \end{cases} \end{cases}}$$

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