

Computer vision and pattern recognition

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[moodle](#)

## Final exam

- ☒ project (1 week before the exam)
- ☐ scritto di 1 ora
- ☐ orale subito dopo lo scritto, di 30 minuti in media

exam dates: **14/01, 28/01 e 12/02.**

## Strategia per l'esame

- ☒ presentare gli argomenti diverse volte di fila, prima con le slides davanti finché non ho capito tutto, poi senza come se fossi all'orale
- ☒ non avere dubbi su nessun argomento
- ☒ sapere le formule a memoria
- ☒ sapere dire almeno qualcosa di ogni possibile argomento
- ☒ rispondere alle vecchie domande da esame
- ☒ sapere a memoria tutti gli schemi degli argomenti
- ☐ il giorno prima dell'esame dormire benissimo: camomilla/ latte caldo, melatonina, meditazione,...

## Scritto del 14 gennaio

Domanda lunga era su bag of words

Due domande su ransac

Domanda su eigenfaces (se c'è una permutazione dei pixel cambia qualcosa)

Una domanda sul valore C di SVM

Due domande su matrice essenziale e fondamentale

Una domanda su epipoli

Un'immagine in cui chiedeva cos'era tra linear perceptron, SVM primale o duale, nessuna di queste

## Orali

- Domande molto dettagliate
- Non chiede nulla del progetto, casomai cose sbagliate nello scritto

### Consigli utili per l'orale:

- parlare piano e non dire stronzate a caso
- ragionare prima di parlare
- respirare lentamente

### Tecnica per imparare efficacemente:

Applicare in maniera ricorsiva:

- leggere recap dell'indice di un argomento
- ripasso dettagliato consultando le slides e segnando altrove i dubbi
- ripetizione dettagliata senza guardare gli appunti
- recap dell'indice senza guardarlo
- risolvere i dubbi scritti
- a fine giornata ripetere i dubbi risolti e i punti chiave dell'indice

### Orale Marco, 40 minuti, 25

- reti convoluzionali applicate alle immagini
  - perché si chiama convoluzionale
  - quali sono le funzioni di attivazione nei primi strati
  - grafico della rectified linear unit
  - addestramento di una rete, cosa si apprende
  - quale funzione viene allenata, gradiente rispetto a cosa e di cosa...
- viola jones detector
  - struttura del classificatore a cascata
  - la sequenzialità dei weak learners si una in fase di training o test o entrambe

### Orale Alex, 15 minuti, 29

- approccio scale space
  - a cosa si riduce la normalizzazione
- kernel gaussiano generalizzato

- differenza tra istogramma e signature
  - EMD
  - che problema di ottimizzazione è, complessità computazionale

## Errori da segnalare al prof

- pag 86 SVM kernel map ha valori in R
- pag 62 image processing  $h(-x)$
- pag 21 stereopsis:  $m'$  nella definizione di coordinate normalizzate

## Indice degli argomenti

### Image formation

- **pinhole camera**
  - perspective projection
  - aperture problem
    - thin lens
      - moving sensor
      - field of view
      - blur circles
      - depth of view
  - telecentric camera
    - orthographic projection
  - thick camera
    - radial distortion
    - chromatic aberration
    - vignetting
  - sensing
    - integrator
    - sampler
    - quantizer
- **camera model**
  - non linear perspective projection
    - projective space
      - augmented vector
  - affine transformations
    - degrees of freedom

- perspective projection matrix
  - pixelization (intrinsic)
  - rigid transform (extrinsic)
  - characterization
  - center of projection coordinates
  - optical ray
  - depth of a point
- **camera calibration**
  - extrinsic and intrinsic parameters
  - direct and indirect methods
  - direct linear transform for estimating  $P$ 
    - least squares system (algebraic residual)
    - alternative derivation
      - cross product
      - kronecker product
        - vector operator
    - degenerate configuration of points
      - non coplanar points
  - iterative non-linear method
    - first step: DTL
    - then: minimizing the reprojection error (geometric residual)
    - get extrinsic and intrinsic parameters from  $P$ 
      - QR factorization of  $Q^{-1}$
  - iterative radial distortion compensation
    1. estimate  $P$  from correspondences
    2. estimate  $K$
    3. estimate distortion parameters  $k_1, k_2$
    4. correct the coordinates  $m'$
    5. go back to 1.
  - zhang method (for extr+intr parameters)
    - start from  $>3$  plane correspondences
    - estimate the homography  $H$
    - estimate  $K$ 
      - compute  $V$
      - solve  $Vb = 0$  with least squares (SVD)
      - apply Cholesky to  $B$
      - recover  $K$  from Cholesky
    - estimate  $R, t$ 
      - compute  $r_1, r_2, t$  from  $K$
      - compute  $r_3$
      - get closest orthogonal  $R$  (Frobenius norm)
    - alternative derivation of  $H$

## Image processing

- **image processing**
  - digital image
    - sampling
    - quantizing
  - local operators
    - linear operators
      - correlation
      - convolution
        - impulse response
        - properties of conv only: commutative, associative, is a product in frequency domain
      - properties of both:
        - shift invariant
        - linear
  - padding
- **linear filtering examples**
  - low pass filters
    - box
    - gaussian
    - bilinear
  - band pass filters
    - sobel
    - corner
    - LoG
    - directional derivative
      - steerable filters
  - unsharp masking
  - separable filtering
    - $W^2$  to  $2W$  pp operations
- **Non-linear filters**
  - Median filtering
    - $\alpha$ -trimmed mean
    - weighted median
  - bilateral filtering
    - Anisotropic diffusion
  - morphological operations
- **image warping**
  - forward warp
  - inverse warp
- **Fourier transforms**
  - 1D signal continuous + discrete

- 2D signal
- change of basis (magnitude-phase)
- frequency filtering
- convolution theorem
- duality theorem
  - box sinc duality
  - gaussian duality
- aliasing
  - Nyquist theorem
  - low pass filter
- **multi-resolution representations**
  - upsampling
    - interpolation kernel
    - bilinear and bicubic kernels
  - downsampling
    - two steps:
      - low pass filter (aliasing)
      - sampling
    - image pyramid

## Feature detection

- **tracking vs matching**
- **corner detection**
  - Harris detector
    - possibile keypoint  $(x, y)$  + intorno specificato  $W$
    - response function  $R(A)$  dalla second moment matrix  $A$ 
      - sum of square differences  $E(W, \Delta u)$
      - gaussian weighting window  $w(x, y)$
      - Taylor expansion and gradient for intensity
    - local maxima of the response function above a ts
  - Hessian detector
    - hessian matrix (curvature)
    - local maxima of  $|\det H|$  above a ts
  - invariance and covariance
    - rotational covariance
- **scale-space representation**
  - principle of scale selection
  - $\gamma$  normalized derivatives
    - Decide which image features you are interested in (e. g. blobs, corners, edges)
    - Choose a detector and compute the normalized derivatives

- Find local extrema of the detector function over the whole scale space
- Scale-space blob detection
  - LoG
    - procedure
      - normalized LoG
      - local extrema in the scale-space
    - properties
      - band pass filter
      - rotational and scale covariance
      - affine covariance
  - SIFT
    - multiresolution pyramid
    - DoG as LoG approximation
    - local scale-space extrema of DoG
    - spatial location interpolation (scale-space Taylor expansion)
      - $\text{gradient}(\text{DoG}) = 0$  and update location
    - low contrast rejection ( $|\text{DoG}| < t_s$ )
    - edges rejection (Hessian response of DoG  $< t_s$ )
- descriptors (vectors associated to points)
  - SIFT
    - scale invariant: size neighbor
    - rotationally invariant: histogram of gradients
    - $4 \times 4$  patch
    - dominant direction
    - histograms of gaussian weighted gradients
    - normalized (contrast invariance) vector of gradient histograms (with  $t_s$ )
  - MOPS
    - $8 \times 8$  patch at the scale
    - intensity of sampled neighbor
    - standardization
    - dominant direction
  - PCA-SIFT
    - $41 \times 41$  patch at the scale
    - dominant direction
    - vector of x and y derivatives
    - PCA dimensionality reduction
  - GLOH (similar to pca-sift)
    - 17 log polar binning
    - 16 gradient orientations
    - PCA
  - steerable filters
    - dimension = n filters
- matching using
  - distances
    - euclidean

- earth mover's
  - strategies
    - threshold
    - NN
    - NNDR
- **edge detection**
  - principles:
    - robustness to noise (low pass filter)
    - good localization
    - single response
  - derivative methods
    - LoG (zeros second derivative)
    - Canny (local maxima first derivatives)
      - local maxima of gradient of gaussian
      - $\sigma$  scale of edges
      - non maximum suppression (good localization)
      - hysteresis thresholding (single response)
  - signatures for color edges detection
    - EMD
      - oriented ( $\theta$ ) circular mask
      - optimal flow
      - distance between signatures
      - local maxima on  $\theta$
- **fitting geometric primitives**
  - voting techniques
    - Hough transform
      - steps:
        - line detection
        - line intersection
        - peak detection
      - polar representation
 
$$\rho = x \cos \theta + y \sin \theta$$
      - generalized Hough
        - circles case
      - restrict the search
        - bounded angle
        - known orientation
        - known radius
    - line fitting
      - least squares (quadratic)
      - total least squares (quadratic+constraint)
      - sensitivity to outliers
        - M estimators
          - sub quadratic loss functions (non linear)



- Huber loss function (quadratic+linear)
- RANSAC
  - consensus ts
  - fixed number of iterations
    - $z = (1 - w^n)^k$  failure probability
- comparison bw the two

## Stereopsis

- **Triangulation**
  - Normal case
    - depth of a point
    - disparity error
  - General case
    - perspective proj system (linear)
    - minimization problem with geometric residual (nonlinear)
- **conjugate points correspondences**
  - Epipolar geometry
    - equation of epipolar lines (as a projection)
    - fundamental matrix and Longuet Higgins equation
  - Epipolar rectification
    - same R, K but different t
      - t from the optical centers
      - arbitrary K
      - orthogonal basis construction for R
- **Relative pose**
  - essential matrix
    - Normalized coordinates
    - epipolar constraint
    - Longuet Higgins equation for  $E$
  - Factorization of the essential matrix  $E = SR$ 
    - admissible configuration
    - Depth-speed ambiguity
  - matrix estimation
    - Eight point algorithm for E
    - Structure from motion
    - seven point algorithm for F
    - normalized eight point algorithm

## Support vector machines

- supervised learning

- risk functional
  - loss function
- empirical risk minimization
  - consistency of ERM
  - generalization error
    - bias variance dilemma
    - overfitting
    - VC dimension (classification problems)
      - why richness of  $H$
      - consistency of ERM
      - empirical risk bound
      - SRM principle
- binary classification
  - linear decision function
  - maximal margin hyperplane
    - maximal margin formulation
    - canonical hyperplanes and QP minimization formulation
    - properties
      - robustness to parameter and pattern noise
      - margin and SRM
    - dual Lagrangian formulation (QP and convex)
      - Karush-Kuhn-Tucker conditions
      - sparse solution and support vectors
      - mechanical interpretation
    - decision function in dual form
  - non separable case
    - soft margin hyperplane
      - role of  $C$  (regularization constraint)
      - bounded support vectors
      - soft margin bound theorem
    - feature mapping
      - perceptron
        - training algorithm
        - problems
        - dual formulation
        - potential functions
        - polynomial machines
      - kernel trick
        - kernel characterization
        - Mercer's theorem
        - ray of smallest enclosing sphere
  - SVMs
    - generalization capability and computational tractability
    - LOO method

- examples of kernels
  - choice of degree
  - SVM and transfer learning
- multiclass SVM
  - one vs all (decision tree)
  - one vs one (decision tree)
  - all at once
- SVM data augmentation
  - virtual support vectors
    - training time
    - scales quadratically in #transf
  - kernel jittering
    - kernel computation time
    - scales linearly in #transf

## Recognition

- window based detection
  - Viola Jones (face detection)
    - Haar features and integral images
    - Boosting method
      - weak learners
    - cascade training
      - set target rates F,D
      - feature extraction (integral images)
      - feature selection (boosting)
      - rates evaluation
  - HoG descriptor (pedestrian detection)
    - histogram of gradients
    - normalized subwindows
    - train SVM on the feature vector
    - deformable part model
- face recognition in the space of faces
  - affine subspace modeling
  - eigenfaces
    - differences from the mean face
    - PCA
      - variance maximization (SVD)
    - kNN face classification
  - Fisherfaces
    - LDA
      - $S_B, S_W$  scatter matrices
      - relative variance maximization problem (generalized eigenvalue)
  - Singularity of the within class scatter matrix

- two step procedure: PCA+LDA
- Instance recognition
  - from local features
    - invariant local features detection
    - match features with euclidean distance bw descriptors
    - geometric consistency check
      - RANSAC
      - GHT voting
  - Visual vocabulary for large databases
    - local feature detection
    - SIFT descriptors
    - cluster the feature space using kNN
    - visual words
    - $f_{id}$  relative frequency and BOW
    - similarity between documents
  - database construction
    - build a BOW for each document
    - compute the inverted idxs
    - keep track of locations
  - image retrieval
    - find BOW of the image
    - use inverted idx to find the best matches
    - check with spatial consistency
- Category recognition
  - Bag of words + SVM
    - bow histograms for each training image
    - SVM with selected generalized kernel
      - distance between descriptors = signatures
        - EMD (visual category)
        - $\chi^2$  (texture)
      - distance between feature vectors
        - pyramid match kernel  $\rightarrow O(\#match)$
      - distance in the image space
        - spatial pyramid kernel
    - NO consistency check!!!
  - CNN
    - general ANN architecture
      - layers
      - activation functions
    - iterative minimization of the loss (or error)
      - loss function
      - regularization term
    - gradient descent method
      - SGD

- backpropagation
- CNNs
  - layers
    - pooling
    - bank of filters
  - properties
    - sparse connectivity
    - parameter sharing
    - equivariance to translation
  - examples
- transfer learning

## Confronto tra i metodi

algorithm	decision functions (single layers)
Perceptron	$\Theta(wz + b) = \Theta(\sum_i \alpha_i y_i (z_i * z) + b)$ where $z = \phi(x)$
SVM	$\Theta(w\phi(x) + b) = \Theta(\sum_{SV} \alpha_i y_i k(x_i, x) + b)$
ANN (single layer)	$f(wx + b)$
RBF NN	$f(\sum_{i=1, \dots, l} w_i k(x_i, x) + b)$

- gaussian SVM = RBF NN with proper weights
- sigmoidal SVM = NN with sigmoid activation function