HW1 #2,4,5,12

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2. GCD (3141, 1592)

$3141 = 1592 \cdot 1 + 1549$

$1592 = 1 \cdot 1549 + 43$

$1549 = 36 \cdot 43 + 1$

$43 = 43 \cdot 1 + 0$

$= 1$ is GCD

relatively Prime

GCD (10001, 100083)

$100083 = 10001 \cdot 10 + 73$

$10001 = 73 \cdot 137 + 0$

$= 73$ is GCD

4. Find x and y such that $4144x + 7696y = 592$

(7696, 4144)

$7696 = 4144 \cdot 1 + 3552$

$4144 = 3552 \cdot 1 + \boxed{592}$

$3552 = 592 \cdot 6 + 0$

GCD is 592

$592 = 4144 + 3552$

$592 = 4144(2) + 7696(-1)$

$\boxed{x = 2, \; y = -1}$

5. Proof. Using the Euclid's Algorithm, we can infer that (N, a) can be written as $N = bc(a) + 1$. Using the Eclidean formula, $x = yq + r$, where all elements are integers, we can say $(x, b) = (y, r)$ $\therefore$ $(N, a) = (a, 1) = 1$. (N, b) and (N, c) follows the same way.

7. Proof. There are integers m + n such that $m|n$ and $n|m$ and $m = n$, Let $m = 1$ and $n = 1$. $m|n$ implies $n = mk$, Where as k is an integer. $1 = 1(1)$ where (1) is an integer, $\therefore$ $m|n$ and $n|m$ implies $m = nl$ where l is an integer

$-1 = -1(-1)$ where (-1) is an integer $\therefore$ $m = n$ or

$m = -n$ since $1 = 1$ and $-1 = 1$

(12) Proof. Suppose $a|b$ and $c|d$. It can be inferred that $b = k_1 a$ and $d = k_2 c$ for integers k_1 and k_2. This implies $bd = (k_1 a)(k_2 c) = (k_1 k_2)(ac)$. If we let m be an integer which equals $k_1 k_2$, we see that $bd = k(ac)$ $\therefore$ $bd \mid ac$