Equation Discovery

SDE ⇒ Stochastic Differential Equation

$$\frac{dm_X}{dt} = F_X(m_X, m_Y) + g_{XX}^{(m_X m_Y)} \eta_X$$

Use LASSO instead of regular regression modal to achieve a sparse solution for the model

$$\min(F - \hat{F})^T (F - \hat{F})$$

LASSO

$$\min ||F - \hat{F}||_2 + \lambda|\xi|_1$$

$$\min ||\dot{X} - \theta(X)\xi||_2 + \lambda|\xi|_1$$

candidate library matrix, Q

$$F = Q\xi$$

$$\dot{X} = \Theta(X)\Xi$$

$$\Theta(X) = \begin{bmatrix} 1 & X & X^{P_2} & X^{P_3} & \dots & \sin(X) & \cos(X) \end{bmatrix}$$

$$\Xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_{n-1} \\ \xi_n \end{bmatrix}$$

$$\begin{gathered} x = \sigma(y - x) \\ y = x(\rho - z) - y \\ z = xy - \beta z \end{gathered}$$

for a saddle graph

it is difficult to find null space solution when there is even a little bit of noise is present. Hence we introduce

SINDY solver

PyDADDY
PySR

SINDY

PI-SINDY

Auto-encoder SINDY

$$\left|\left|x-\psi(z)\right|\right|_2^2+\lambda_1\left|\left|\dot{x}-(\nabla_z\psi(z))(\Theta(z^T)\Xi)\right|\right|_2^2+\lambda_2\left|\left|(\nabla_x z)\dot{x}-\Theta(z^T)\Xi\right|\right|_2^2+\lambda_3\left|\left|\Xi\right|\right|_1$$

$$\dot{x}=\left(\frac{\partial x}{\partial z}\right)\left(\frac{\partial z}{\partial t}\right)=\left(\nabla_z\psi(z)\right)\left(\Theta(z^T)\Xi\right)$$

$$\dot{z}=\left(\frac{\partial z}{\partial x}\right)\left(\dot{x}\right)=\left(\nabla_x z\right)\left(\dot{x}\right)$$

SVD

$$X_{M\times N}=U\Sigma U^T$$

where $U^TU=UU^T=I$

Σ is a diagonal matrix