Matrix Decomposition

Singular Value Decomposition

$$X_{M\times N} = U_{M\times M}\Sigma_{M\times N}V^T_{N\times N}$$

$U^TU = UU^T = I$

$VV^T = V^TV = I$

$$\begin{bmatrix} \vdots & \vdots & & \vdots \\ u_1 & u_2 & \dots & u_m \\ \vdots & \vdots & & \vdots \end{bmatrix} \begin{bmatrix} \sigma_1 & & 0 \\ & \ddots & \\ 0 & & \sigma_n \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \vdots & \vdots & & \vdots \\ v_1 & v_2 & \dots & v_n \\ \vdots & \vdots & & \vdots \end{bmatrix}^T$$

$U_{ii} = u_i^T u_i = 1$

$U_{ij} = u_i^T u_j = 0$

Σ is a rectangular diagonal matrix with decreasing sigma values

$\sigma_1 > \sigma_2 > \dots > \sigma_n$

$$X = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \dots + \sigma_n u_n v_n T$$

For some $r < n$ we can approximate as

$$X \approx U_{m\times r}\Sigma_{r\times r}V^T_{n\times r}$$

$$XX^T = U\Sigma^2U^T$$

Σ^2 is the eigen vector matrix of XX^t

Ucaptures the spatial nodes

$$X^TX = V\Sigma^2V^T$$

Σ^2 is the eigen vector matrix of XX^t

V captures the temporal nodes

V

	U	Σ	V
SVD	$M \times M$	$M \times N$	$N \times N$
econ	$M \times N$	$N \times N$	$N \times N$
redn	$M \times r$	$r \times r$	$N \times r$