ID5130 - Assignment 1

ME21B065
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Q1] Representing the equations in a matrix-vector form

$$\begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{Bmatrix}$$

$a_1 = 0, \quad a_i = -1 \quad \forall\, i \in \{2,3,4\}$

$c_4 = 0, \quad c_i = -1 \quad \forall\, i \in \{1,2,3\}$

$b_i = 3, \quad \forall\, i \in \{1,2,3,4\}$

Iteration 1

$i=1 \Rightarrow \quad \beta_1^{(1)} = -c_1/b_2 = 1/3$

$b_1^{(1)} = b_1 + \beta_1^{(1)} a_2 = 3 + 1/3(-1) = 8/3$

$c_1^{(1)} = \beta_1^{(1)} c_2 = -1/3$

$y_1^{(1)} = y_1 + \beta_1^{(1)} y_2 = 2 + 1/3(1) = 7/3$

$i=2 \Rightarrow \quad \alpha_2^{(1)} = -a_2/b_1 = 1/3 \qquad \beta_2^{(1)} = -c_2/b_3 = 1/3$

$a_2^{(1)} = \alpha_2^{(1)} a_1 = 0$

$b_2^{(1)} = b_2^{(1)} + \alpha_2^{(1)} c_1 + \beta_2^{(1)} a_3 = 3 + 1/3(-1) + 1/3(-1) = 7/3$

$c_2^{(1)} = \beta_2^{(1)} c_3 = -1/3 \qquad y_2^{(1)} = y_2 + \alpha_2^{(1)} y_1 + \beta_2^{1} y_3 = 2$

$i=3 \Rightarrow \quad \alpha_3^{(1)} = -a_3/b_2 = 1/3 \qquad \beta_3^{(1)} = -c_3/b_4 = 1/3$

$a_3^{(1)} = \alpha_3^{(1)} \cdot a_2 = -1/3$

$b_3^{(1)} = b_3 + \alpha_3^{(1)} c_2 + \beta_3^{(1)} a_4 = 3 + 1/3(-1) + 1/3(-1) = 7/3$

$$c_3^{(1)} = \beta_3^{(1)} c_4 = 0$$

$$y_3^{(1)} = y_3 + \alpha_3^{(1)} y_2 + \beta_3^{(1)} y_4 = 2$$

$i=4 \Rightarrow$ $$\alpha_4^{(1)} = -a_4 / b_3 = 1/3$$

$$a_4^{(1)} = \alpha_4^{(1)} a_3 = -1/3$$

$$b_4^{(1)} = b_4 + \alpha_4^{(1)} c_3 = 3 + 1/3(-1) = 8/3$$

$$y_4^{(1)} = y_4 + \alpha_4^{(1)} y_3 = 2 + 1/3(1) = 7/3$$

$$A^{(1)} x = y^{(1)} \Rightarrow \begin{bmatrix} 8/3 & 0 & -1/3 & 0 \\ 0 & 7/3 & 0 & -1/3 \\ -1/3 & 0 & 7/3 & 0 \\ 0 & -1/3 & 0 & 8/3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 7/3 \\ 2 \\ 2 \\ 7/3 \end{Bmatrix}$$

Iteration - 2

$i=1 \Rightarrow$ $$\beta_1^{(2)} = -c_1^{(1)} / b_3^{(1)} = 1/7$$

$$b_1^{(2)} = b_1^{(1)} + \beta_1^{(2)} a_3^{(1)} = 55/21$$

$$c_1^{(2)} = \beta_1^{(2)} c_3^{(1)} = 0$$

$$y_1^{(2)} = y_1^{(1)} + \beta_1^{(2)} \cdot y_3^{(1)} = 55/21$$

$i=2 \Rightarrow$ $$\beta_2^{(2)} = -c_2^{(1)} / b_4^{(1)} = 1/8$$

$$b_2^{(2)} = b_2^{(1)} + \beta_2^{(2)} a_4^{(1)} = 55/24$$

$$c_2^{(2)} = \beta_2^{(2)} c_4^{(1)} = 0$$

$$y_2^{(2)} = y_2^{(1)} + \beta_2^{(2)} y_4^{(1)} = 55/24$$

$i=3 \Rightarrow \qquad \alpha_3^{(2)} = -a_3^{(1)}/b_1^{(1)} = 1/8$

$$a_3^{(2)} = \alpha_3^{(2)} \cdot a_1^{(1)} = \underline{\underline{0}}$$

$$b_3^{(2)} = b_3^{(1)} + \alpha_3^{(2)} \cdot c_1^{(1)} = \underline{\underline{55/24}}$$

$$y_3^{(2)} = y_3^{(1)} + \alpha_3^{(2)} \cdot y_1^{(1)} = \underline{\underline{55/24}}$$

$i=4 \Rightarrow \qquad \alpha_4^{(2)} = a_4^{(1)}/b_2^{(1)} = 1/7$

$$a_4^{(2)} = \alpha_4^{(2)} \cdot a_2^{(1)} = \underline{\underline{0}}$$

$$b_4^{(2)} = \cancel{4}\, b_4^{(1)} + \alpha_4^{(2)} \cdot c_2^{(1)} = \underline{\underline{55/21}}$$

$$y_4^{(2)} = y_4^{(1)} + \alpha_4^{(2)} \cdot y_2^{(1)} = \underline{\underline{55/21}}$$

$$A^{(2)} x = y^{(2)} \Rightarrow \begin{bmatrix} 55/21 & 0 & 0 & 0 \\ 0 & 55/24 & 0 & 0 \\ & 0 & 55/24 & 0 \\ 0 & 0 & 0 & 55/21 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 55/21 \\ 55/24 \\ 55/24 \\ 55/21 \end{Bmatrix}$$

$$x_1 = y_1^{(2)}/b_1^{(2)} = \underline{\underline{1}}$$

$$x_2 = y_2^{(2)}/b_2^{(2)} = \underline{\underline{1}}$$

$$x_3 = y_3^{(2)}/b_3^{(2)} = \underline{\underline{1}}$$

$$x_4 = y_4^{(2)}/b_4^{(2)} = \underline{\underline{1}}$$