A goal is to establish a continuity theorem for

$$(X^1, X^2, \dots, X^n) \mapsto \int_0^1 f(\bar{x}_t)\mathrm{d}x_t,$$

where f is C^∞, x_t is a C^1 path, $\bar{x}_t$ is a path with finite p variation for $p \in [n, n+1)$, and

$$X^k_{st} = \int_s^t (\bar{x}_u - \bar{x}_s)^{k-1}\mathrm{d}x_u \quad k = 1, 2, \dots, n.$$

A conjecture is that the integration is continuous w.r.t the norm

$$\sum_{i=1}^n \|X^i\|_{p/i}.$$

This holds at least if $x_t = \bar{x}_t$ and $n = 2$.