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1 Relevant Theory

1.1 The Spectral Theorem

$$S = Q\Sigma Q^T \tag{1}$$

1.2 Singular Value Decomposition (SVD)

$$A = U\Sigma V^T \tag{2}$$

1.3 Principle Component Analysis

From the previous section on SVD, we know that any matrix can be expressed as a product of an orthogonal, a diagonal, and another orthogonal matrix:

$$A = U\Sigma V^T$$

maybe explain how rest of sigmas/singvals are zero leading to only R pieces
also not entirely clear myself on why $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_R$

This can be further expressed as a sum of R rank-1 pieces, R being the original matrix A's rank.

$$= \sigma_1 u_1 v_1^T + \ldots + \sigma_R u_R v_R^T$$

These pieces are individually know as *principal components*, and are, as the name suggests, key to PCA.

Picking any $k \leq R$, A_k is defined as the sum of the first k principal components:

$$A_k = \sigma_1 u_1 v_1^T + \ldots + \sigma_k u_k v_k^T$$

We claim that this matrix A_k is the closest possible approximation of A with rank k. ... *maybe explain norms and def. of "approxomation", add proofs for Eckart-Young*

This claim is neatly defined in the *Eckart-Young Theorem*: *italics?*

$$\text{rank}(B) = k \implies \|A - B\| \geq \|A - A_k\| \tag{3}$$

1.4 Phylogenetic PCA

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2 Demonstrative Work

References

Akhøj, Morten, Xavier Pennec, and Stefan Sommer (2023). "Tangent Phylogenetic PCA". In: DOI: 10.1007/978-3-031-31438-4_6.