Part II: Analysis Questions

For this, you will solve problems related to efficiency, and the Big O asymptotic notation. **SHOW YOUR WORK.**

The definition of Big O

Let f(n) and g(n) be functions mapping positive integers to positive real numbers.
We say that f(n) is O(g(n)) if there is a real constant c > 0 and an integer constant N ≥ 1 such that:

$f(n) \leq c * g(n)$, for $n \geq N$

Note that the constants c and n_0 are *not unique*. You just have to find a c and an *N* that satisfies the definition of Big O

3.1 Given a time function, $T(n) = 3x^2 + 5x + 2$, find constants **c** and **N** that prove that the big O of the growth function T(n) is n^2

$T(n) = 3n^2 + 5n + 2 \rightarrow 3n^2 + 5n + 2n \leq 3n^2 + 5n^2 + 2n^2$

$3n^2 + 5n + 2n \leq 10n^2$ $\quad c = 10$, $N = 1$

$O(n^2)$ quadratic

3.2 Find the Big O of $T(n) = 4x^3 + 12x^2 + 2x + 12$. Justify your answer by finding constants c and N

$T(n) = 4n^3 + 12n^2 + 2n + 12$

$4n^3 + 12n^2 + 2n + 12 \leq 4n^3 + 12n^3 + 2n^3 + 12n^3$

$4n^3 + 12n^2 + 2n + 12 \leq 30n^3$ $\quad c = 30$, $N = 1$

$O(n^3)$ cubic

3.3 Find the Big O of the following code:

```
for(int i = 0; i < n; i++)      2 + n + n
{
    for(int j = 0; j < n; j++)  2 + n^2 + n^2
    {
        System.out.println("Hello!");  2n^2
    }
}
```

$= 4n^2 + 2n + 4$

$4n^2 + 2n + 4 \leq 4n^2 + 2n^2 + 4n^2$

$4n^2 + 2n + 4 \leq 10n^2$ $\quad c = 10$, $N = 1$ $\quad O(n^2)$

You do not have to find a T(n), or c and N. Just give the Big O, and informally justify your answer.

$O(n^2)$. The nested for loop iterates n^2 times, therefore the Big O is quadratic