~~Let $\bar{x}_I$ be the~~ mea

Let $I = \{a_1 \ldots a_m\}$. Let $\mu = \frac{1}{n}\sum_{i=1}^{N} a_i$ $\qquad$ $Y_1(a_i) = \frac{1}{n}$ (uniform distribution)

Let $X_i(a_i)$ be a random variable assigned to a_i, with $E[X_i(a_i)] = \mu$ because the points are uniformly distributed.

Then,

$$X_I = \{X_1, X_2, \ldots, X_m\}, \quad \bar{X}_I = \frac{1}{m}\sum_{i=1}^{m} X_i$$

And we wish to prove:

$$E_I[\bar{X}_I] = \mu.$$

Then,

$$E_I[\bar{X}_I] = \frac{1}{m}\sum_{i=1}^{m} X_i$$

$$E_I[\bar{X}_I] = \frac{1}{m}(X_1 + X_2 + \cdots + X_m)$$

$$E_I[\bar{X}_I] = \frac{1}{m}X_1 + \frac{1}{m}X_2 + \cdots + \frac{1}{m}X_m$$

By linearity of expectation, regardless of X_i and X_j are independent or dependent, we have:

$$E_I[\bar{X}_I] = E[\tfrac{1}{m}X_1] + \cdots + E[\tfrac{1}{m}X_m]$$

$$= \frac{1}{m}E[X_1] + \cdots + \frac{1}{m}E[X_m]$$

$$= \frac{1}{m}\mu_{(1)} + \cdots + \frac{1}{m}E\,\mu_{(m)}$$

$$= \frac{m}{m}\mu$$

$$= \mu$$

$$= \frac{1}{n}\sum_{i=1}^{N} a_i$$

$$\mathbb{E}_I[\nabla L_I(x,y,\theta)] = \nabla \mathbb{E}_I[L_I(x,y,\theta)]$$

∇

$$L_I(x,y,\theta) = \frac{1}{m}\sum_{i\in I} \ell(x^{(i)}, y^{(i)}, \theta)$$

$$L(x,y,\theta) = \frac{1}{n}\sum_{i=1}^{n} \ell(x^{(i)}, y^{(i)}, \theta)$$

Thus

$$\begin{aligned}
\mathbb{E}_I[\nabla L_I(x,y,\theta)] &= \nabla \mathbb{E}_I[L_I(x,y,\theta)] \quad (*) \\
&= \nabla \mathbb{E}_I\Big[\frac{1}{m}\sum_{i\in I} \ell(x^{(i)}, y^{(i)}, \theta)\Big] \\
&= \nabla\Big(\mathbb{E}\Big[\frac{1}{m}\ell(x^{(1)}, y^{(1)}, \theta)\Big] + \cdots + \mathbb{E}\Big[\frac{1}{m}\ell(x^{(m)}, y^{(m)}, \theta)\Big]\Big) \\
&= \nabla\Big(\frac{1}{m}\mathbb{E}[\ell(x^{(1)}, y^{(1)}, \theta)] + \cdots + \frac{1}{m}\mathbb{E}[\ell(x^{(m)}, y^{(m)}, \theta)]\Big) \\
&= \nabla\Big(\frac{1}{m}\Big[\frac{1}{n}\sum_{i=1}^{n}\ell(x^{(i)}, y^{(i)}, \theta)\Big] + \cdots + \frac{1}{m}\Big[\frac{1}{n}\sum_{i=1}^{n}\ell(x^{(i)}, y^{(i)}, \theta)\Big]\Big) \\
&= \nabla\Big(\frac{\cancel{m}}{\cancel{m}}\Big[\frac{1}{n}\sum_{i=1}^{n}\ell(x^{(i)}, y^{(i)}, \theta)\Big]\Big) \\
&= \nabla\Big(\frac{1}{n}\sum_{i=1}^{n}\ell(x^{(i)}, y^{(i)}, \theta)\Big) \\
&= \nabla\big(\underline{L(x,y,\theta)}\big).
\end{aligned}$$

(*) : Based on the Leibniz integral rule:

$$\frac{d}{dx}\Big(\int_a^b f(x,t)\,dx\Big) = \int_a^b \frac{\partial}{\partial x} f(x,t)\,dt$$

Where we treat expectation as the discrete form of the above integral (~~and thus can be ex'~~ as:

$$\frac{d}{dx}\big[E(f)\big] = \frac{\partial}{\partial x}\big[x_1 f(x_1) + \cdots + x_n f(x_n)\big] = \Big[\frac{\partial}{\partial x}x_1 f(x_1) + \cdots + \frac{\partial}{\partial x}x_n f(x_n)\Big].$$

$(x$

$(x, y, \theta) = \nabla (y - w^T x)^2$

$(y - w^T \bar{x})(y - w^T \bar{x})$

$3 [\;] \; 1 [\;\;]$ $\qquad 3 \times$

$(y - \underline{w}^T \underline{x})(y - \underline{w}^T \underline{x})$ $\qquad (1 \times 3) \times (3 \times 1) = 1$

$[1 \times 1] = (y^2 - y \underline{w}^T \underline{x} - y \underline{w}^T \underline{x} + (\underline{w}^T \underline{x})(\underline{w}^T \underline{x}))$ $\qquad 3 [x] \cdot 3 [w]$

$(\cancel{y} - 2y\underline{x} + 2(\underline{w}^T \underline{x})\underline{x}) = 2(\underline{w}^T \underline{x})\underline{x} - 2y\underline{x}$

$2 [1 \times 1] \, 3 [\;] - 2 [1 \times 1] \, 3 [\;]$

$2 \hat{y} \underline{x} - 2 y \underline{x}$