EE2211 Tutorial 4

(Systems of Linear Equations)
Question 1:

Given $\mathbf{Xw} = \mathbf{y}$ where $\mathbf{X} = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(a) What kind of system is this? (even-, over- or under-determined?)
(b) Is **X** invertible? Why?
(c) Solve for **w** if it is solvable.

(Systems of Linear Equations)
Question 2:

Given $\mathbf{Xw} = \mathbf{y}$ where $\mathbf{X} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(a) What kind of system is this? (even-, over- or under-determined?)
(b) Is **X** invertible? Why?
(c) Solve for **w** if it is solvable.

(Systems of Linear Equations)
Question 3:

Given $\mathbf{Xw} = \mathbf{y}$ where $\mathbf{X} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 1 & -1 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 0 \\ 0.1 \\ 1 \end{bmatrix}$.

(a) What kind of system is this? (even-, over- or under-determined?)
(b) Is **X** invertible? Why?
(c) Solve for **w** if it is solvable.

(Systems of Linear Equations)
Question 4:

Given $\mathbf{Xw} = \mathbf{y}$ where $\mathbf{X} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

(a) What kind of system is this? (even-, over- or under-determined?)
(b) Is **X** invertible? Why?
(c) Solve for **w** if it is solvable.

(Systems of Linear Equations)
Question 5:

Given $\mathbf{w}^T\mathbf{X} = \mathbf{y}^T$ where $\mathbf{X} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(a) What kind of system is this? (even-, over- or under-determined?)
(b) Is **X** invertible? Why?
(c) Solve for **w** if it is solvable.

(Systems of Linear Equations)

Question 6:

Given $\mathbf{w}^T\mathbf{X} = \mathbf{y}^T$ where

$\mathbf{X} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 1 & -1 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(a) What kind of system is this? (even-, over- or under-determined?)
(b) Is **X** invertible? Why?
(c) Solve for **w** if it is solvable.

(Systems of Linear Equations)

Question 7:

This question is related to determination of types of system where an appropriate solution can be found subsequently. The following matrix has a left inverse.

$\mathbf{X} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

a) True
b) False

(Systems of Linear Equations)

Question 8:

MCQ: Which of the following is/are true about matrix **A** below? **There could be more than one answer.**

$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ & & \\ 4 & 5 & 6 \end{bmatrix}$

a) **A** is invertible
b) **A** is left invertible
c) **A** is right invertible
d) **A** has no determinant
e) None of the above

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Tutorial 4 (Additional Questions)

9. Consider the over-determined system $\mathbf{Xw} = \mathbf{y}$ where

$$\mathbf{X} = \begin{bmatrix} 1 & 2 \\ 5 & 10 \\ 3 & 7 \end{bmatrix}, \qquad \text{and} \qquad \mathbf{y} = \begin{bmatrix} 1 \\ 5 \\ c \end{bmatrix}. \tag{1}$$

   Find the set of values of c such that there is a unique solution to $\mathbf{Xw} = \mathbf{y}$.

10. Consider the under-determined system $\mathbf{Xw} = \mathbf{y}$ where

$$\mathbf{X} = \begin{bmatrix} 1 & 2 & -1 \\ -1 & -2 & d \end{bmatrix}, \qquad \text{and} \qquad \mathbf{y} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}. \tag{2}$$

   Find the set of values of d such that there is no solution to $\mathbf{Xw} = \mathbf{y}$.

11. Consider the matrix

$$\mathbf{X} = \begin{bmatrix} 1 & 2 \\ 5 & 10 \\ 3 & e \end{bmatrix}. \tag{3}$$

   Find the set of values of e such that the left-inverse of $\mathbf{X}$ exists.