$$E = \sum_{i=1}^{m} e_i^2 = \sum_{i=1}^{m} (y_i - \underline{x}_i^T \underline{w})^2 \qquad e_i = y_i - \underline{x}_i^T \underline{w}$$

Claim: $E = (y - Xw)^T (y - Xw) = \|y - Xw\|^2$

$$\underline{a} = \underline{y} - X\underline{w}$$

Show this claim now: $\|a\|^2 = \underline{a}^T \underline{a} = \sum_{i=1}^{m} a_i^2$

Start with $E = \underbrace{(y - Xw)^T}_{a^T} \underbrace{(y - Xw)}_{a}$

$$\underline{a}^T\underline{a} = \begin{bmatrix} a_1 & \cdots & a_m \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_m \end{bmatrix}$$

$$= \sum_{i=1}^{m} (\underline{y} - X\underline{w})_i^2$$

$$(\underline{a} - \underline{b})_i = a_i - b_i$$

$$= \sum_{i=1}^{m} (y_i - (X\underline{w})_i)^2$$

Left to show that $(X\underline{w})_i = \underline{x}_i^T \underline{w}$

$$\begin{bmatrix} x_{11} & \cdots & x_{1d} \\ x_{i1} & x_{i2} \cdots & x_{id} \\ x_{m1} & \cdots & x_{md} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_d \end{bmatrix} = \begin{bmatrix} \\ \square \\ \\ \end{bmatrix} \leftarrow i^{th} \text{ row}$$

$$x_{i1} w_1 + x_{i2} w_2 + \cdots + x_{id} w_d$$

$$= \underline{w}^T \underline{x}_i \qquad \underline{x}_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ \vdots \\ x_{id} \end{bmatrix}$$

$$\{(\underline{x}_i, y_i)\}_{i=1}^{n} \xrightarrow{\text{Learning}} \hat{w}_{LS} = (X^T X)^{-1} X^T y$$

$$\hat{w}_{LS} \xrightarrow{+\ \underline{x}_{new}} \text{Prediction } y_{new}$$

MCQ

FIB