EE2211 Introduction to Machine Learning

Lecture 12

Wang Xinchao
xinchao@nus.edu.sg

Course Contents

- Introduction and Preliminaries (Xinchao)
  - Introduction
  - Data Engineering
  - Introduction to Linear Algebra, Probability and Statistics
- Fundamental Machine Learning Algorithms I (Vincent)
  - Systems of linear equations
  - Least squares, Linear regression
  - Ridge regression, Polynomial regression
- Fundamental Machine Learning Algorithms II (Vincent)
  - Over-fitting, bias/variance trade-off
  - Optimization, Gradient descent
  - Decision Trees, Random Forest
- Performance and More Algorithms (Xinchao)
  - Performance Issues
  - K-means Clustering
  - Neural Networks

About this week's lecture…

- Neural Network (NN) is a very big topic
  - In NUS we have multiple full-semester modules to discuss NN
    - EE4305 Fuzzy/Neural Systems for Intelligent Robotics
    - EE5934/EE6934 Deep Learning
    - …
  - In EE2211, we only give a very gentle introduction

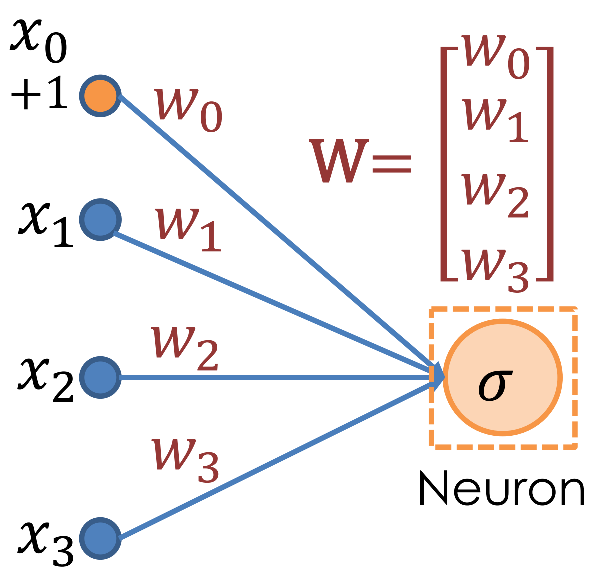
- Understanding at conceptual level is sufficient
  - In final exam, we have only 1 True/False + 1 MCQ about NN
  - No computation is required

- You will do some computation in tutorial, but final exam will be much simpler than the questions in tutorial

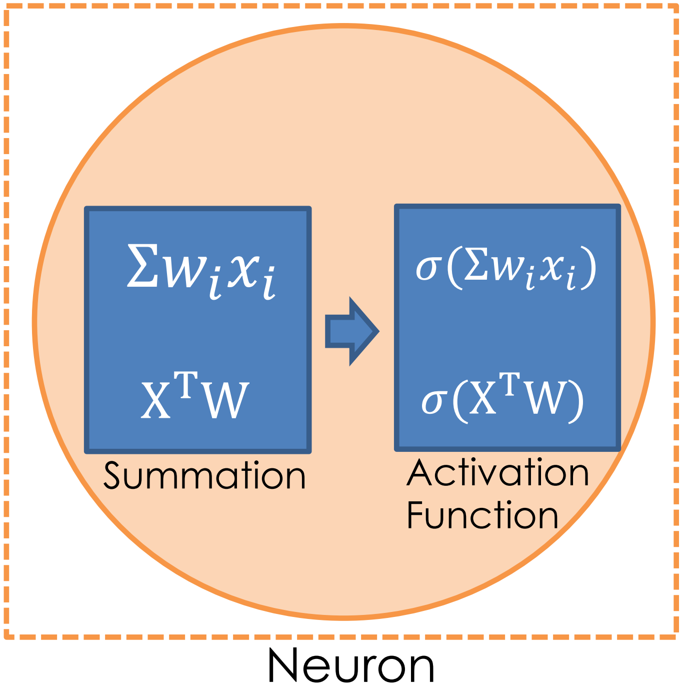
Outline

- Introduction to Neural Networks
  - Perceptron
  - Activation Functions
  - Multi-layer Perceptron
- Training and Testing of Neural Networks
  - Training: Forward and Backward
  - Testing: Forward
- Convolutional Neural Networks

Perceptron



$$\mathbf{W} = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \end{bmatrix}$$



$$\mathbf{X} = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Output of Neuron: $\sigma(X^T W)$ or $\sigma(\Sigma w_i x_i)$

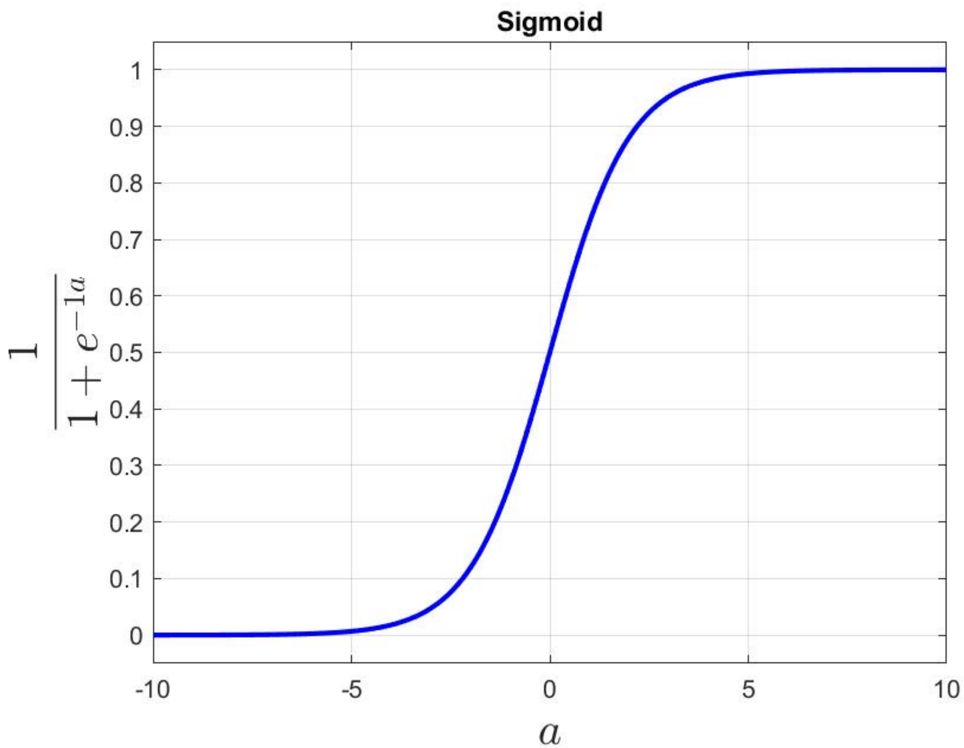
Activation Function: non-linear function to introduce non-linearity into the neural networks!

Goal of training: to learn W!

Activation Functions

Sigmoid Activation Function

$$\sigma(a) = \frac{1}{1 + e^{-\beta a}},$$

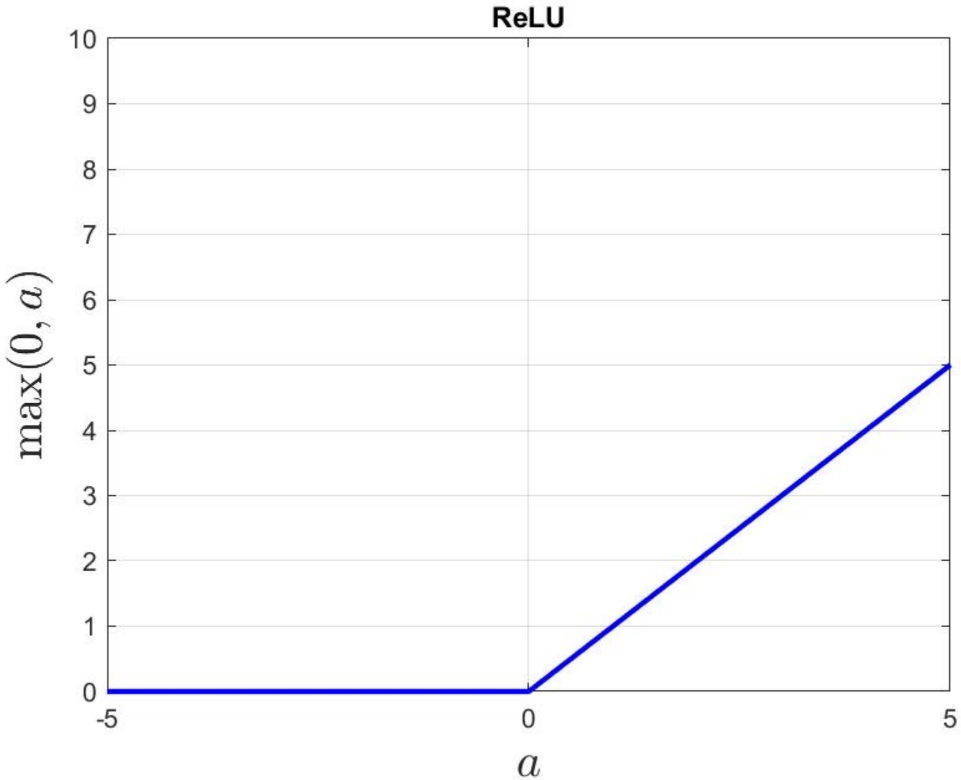


Activation Functions

ReLU Activation Function

$$\sigma(a) = \max(0, a)$$

Rectified Linear Unit (ReLU)

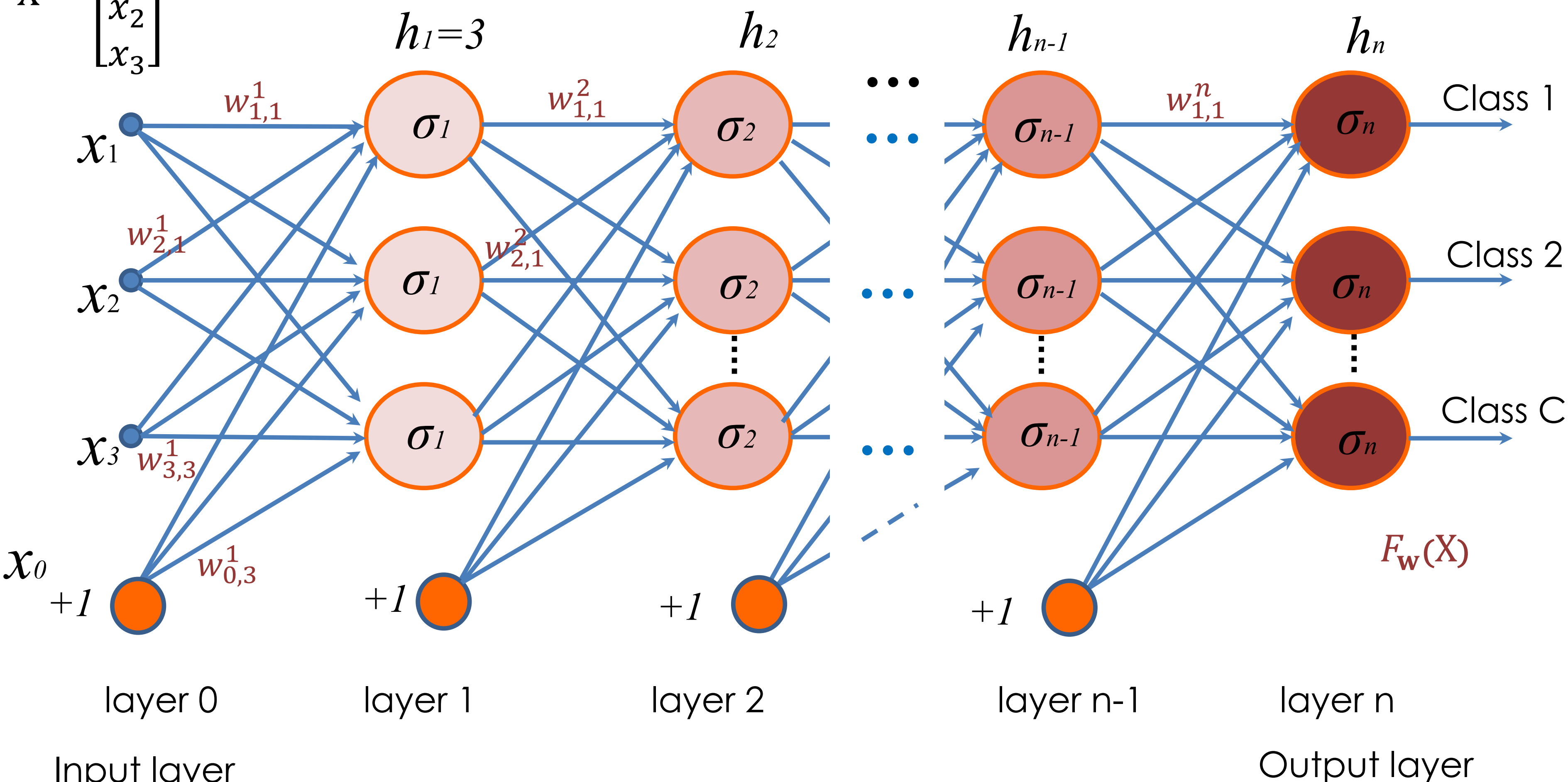


Multilayer Perceptron (Neural Network)

Goal of training: to learn W!

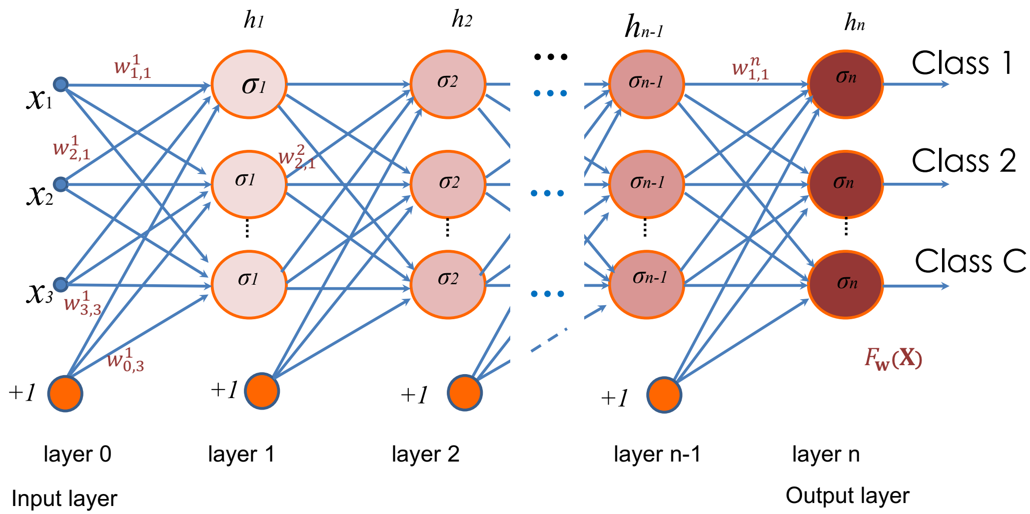
Nested function!

$$X = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



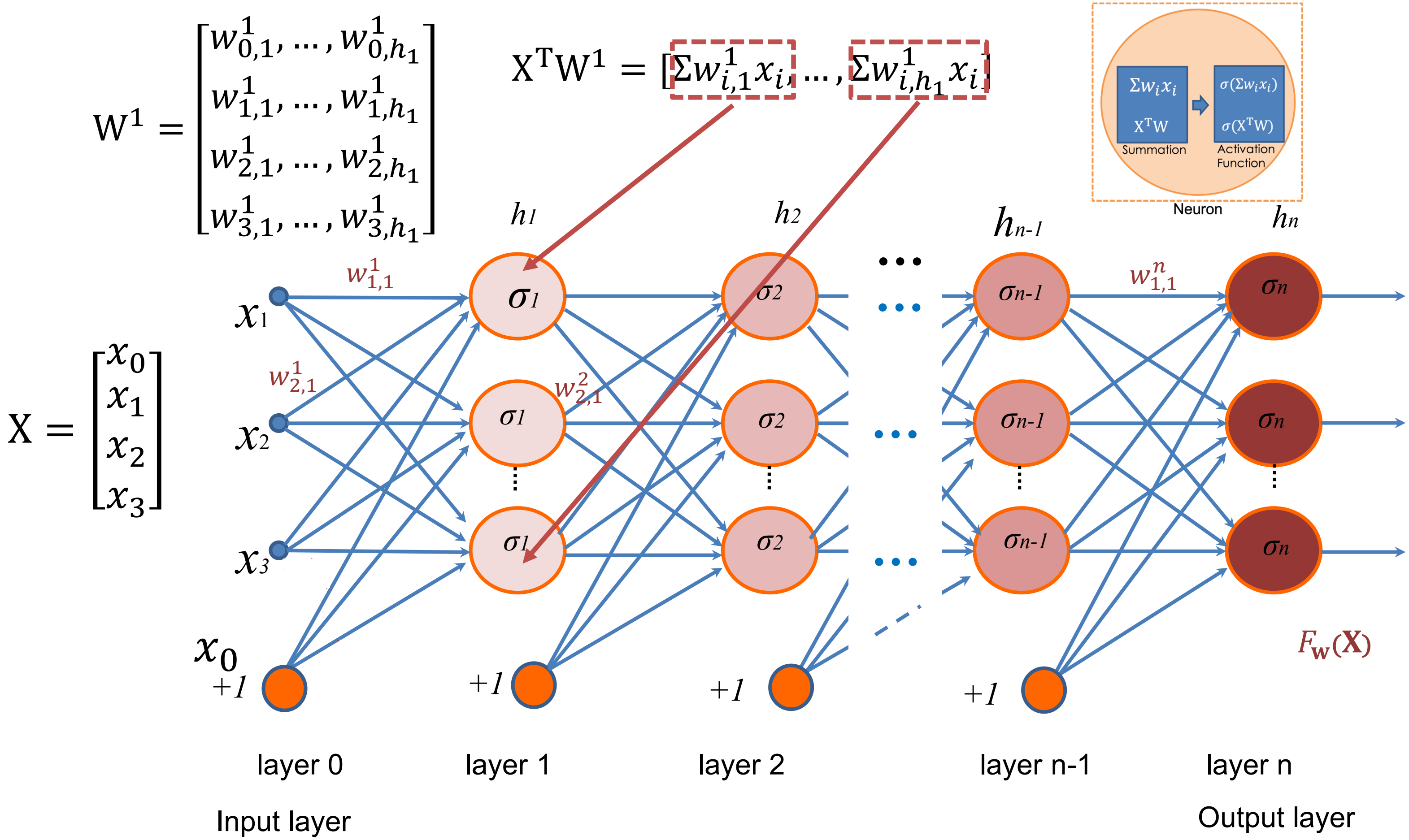
Note: h_n denotes the number of hidden neurons in layer n.

Things to Note



1. The number of hidden neurons in different layers may differ, i.e., h_1 don't have to be equal to h_2.
2. For **classification** task, the number of neurons in the last layer equals to the number of classes.
3. We can treat the whole network as a function $F_{\mathbf{w}}(\mathrm{X})$, where $\mathbf{w}$ is to be learned.

Multilayer Perceptron (Neural Network)



A **neural network** is essentially a **nested function**.

$$F_W(X) = \sigma([1, \dots \sigma([1, \sigma(X^T W^1)] \; W^2) \dots] \; W^n)$$

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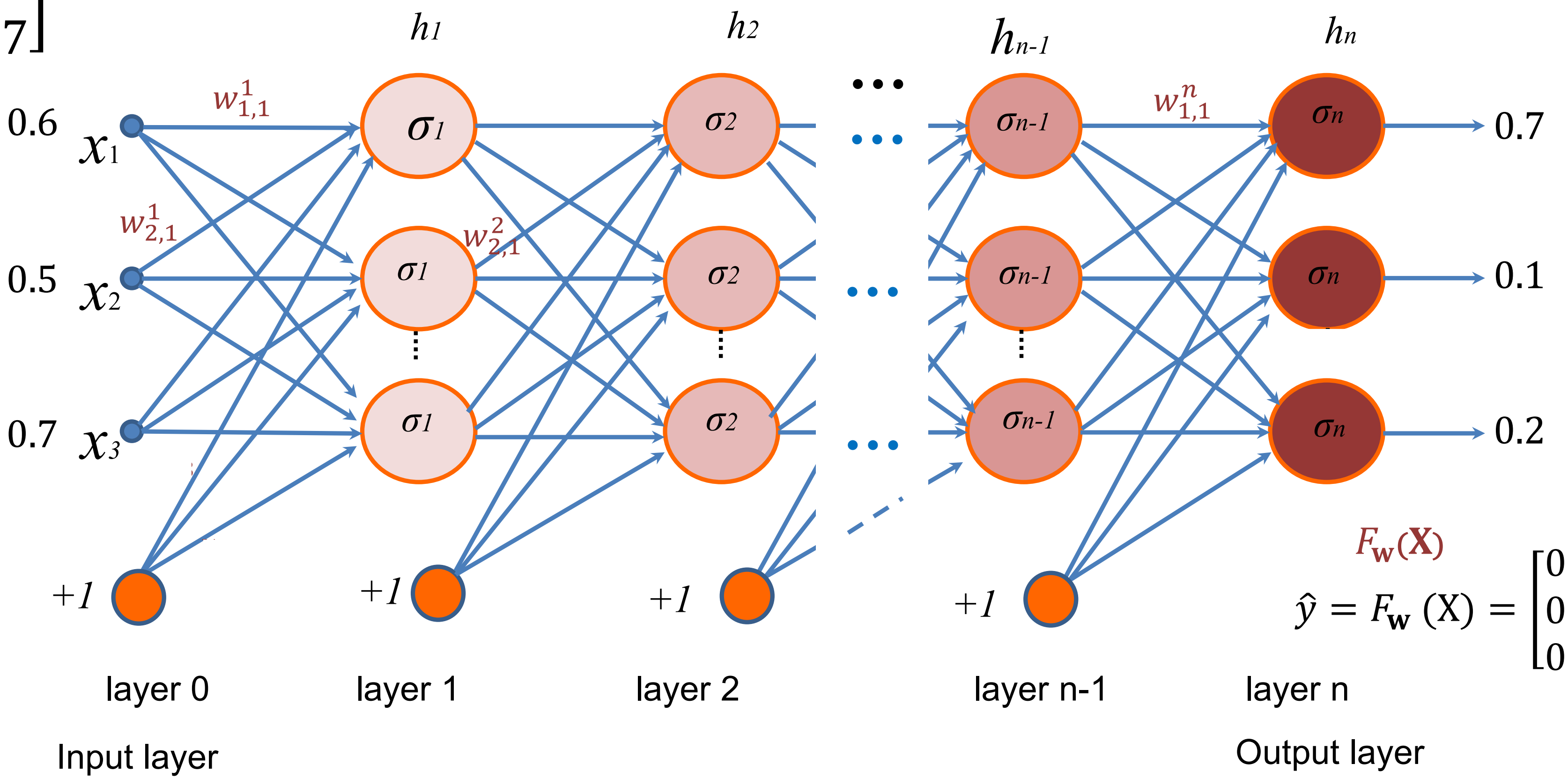
Goal of Neural Network Training: to Learn W

$$X = \begin{bmatrix} 0.6 \\ 0.5 \\ 0.7 \end{bmatrix}$$



$$\hat{y} = F_{\mathbf{W}}(X) = \begin{bmatrix} 0.7 \\ 0.1 \\ 0.2 \end{bmatrix}$$

Specifically, **W** is learned through

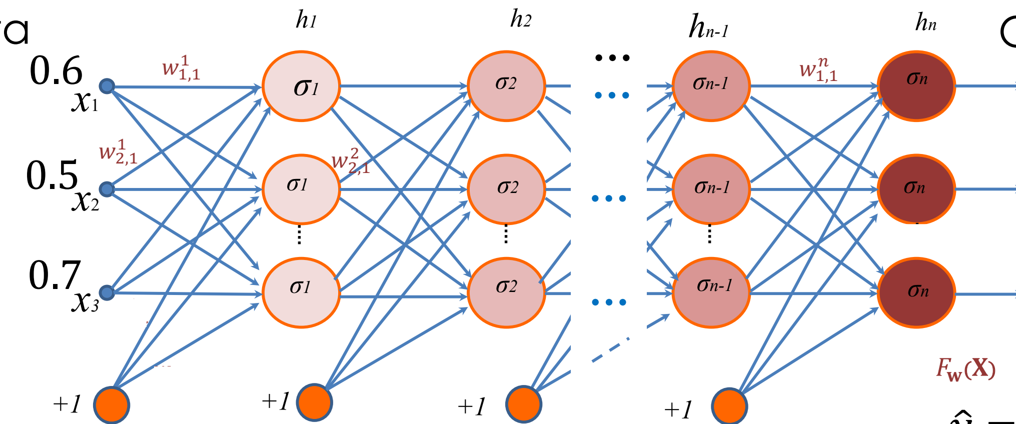
1. Random initialization
2. **Backpropagation**

Neural Network Training: Backpropagation

Assume we train a NN for 3-class classification

Input data

$$X = \begin{bmatrix} 0.6 \\ 0.5 \\ 0.7 \end{bmatrix}$$



Output	One-hot Class Labels
0.7	0
0.1	0
0.2	1

$F_{\mathbf{w}}(\mathbf{X})$

$$\hat{y} = F_{\mathbf{w}}(\mathrm{X}) = \begin{bmatrix} 0.7 \\ 0.1 \\ 0.2 \end{bmatrix} \qquad y = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

1. Forward: (weights are fixed)
To compute network responses
To compute the errors at each output

2. Backward: (weights are updated)
To pass back the error from the output to the hidden layers
To update all weights to optimize the network

A loss function for a single sample:

$$\min_{\mathbf{w}} \sum_{i=1}^{C} (\hat{y}_i - y_i)^2$$

or

$$\min_{\mathbf{w}} ||\hat{y} - y||^2$$

Update W!

Neural Network Training: Backpropagation

- Recall that the parameters W are randomly initialized.
- We use **Backpropagation** to update W.
- In essence, **Backpropagation** is gradient descent!
- Assume we have N samples, each sample denoted by X^j and the output of NN by $\hat{y}^j$, loss function is then

$$J = \sum_{j=1}^{N} \left\| \hat{y}^j - y^j \right\|^2, \quad \min_{\mathbf{w}} J$$

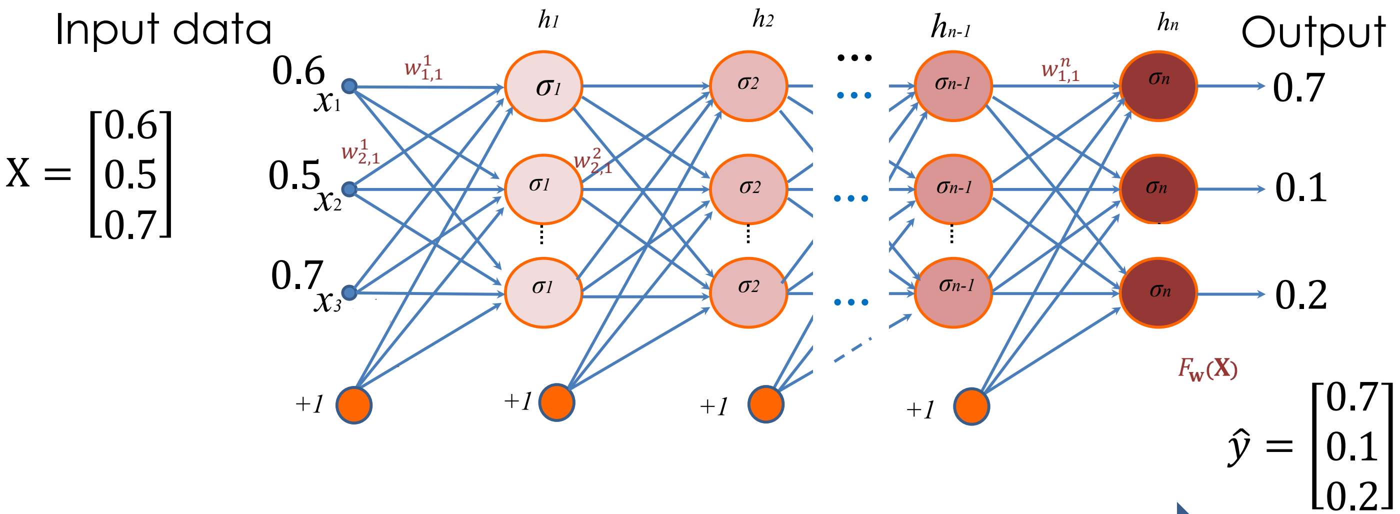
Recall gradient descent in Lec 8: $\mathrm{w} \leftarrow \mathrm{w} - \eta \nabla_{\mathrm{w}} J$

- We would therefore like to compute $\nabla_{\mathrm{w}} J$!
  - J is a function of $\hat{y}$, and $\hat{y}$ is a function of $\mathbf{w}$, i.e., $\hat{y} = F_{\mathbf{w}}(\mathrm{X})$
  - Use gradient descent and chain rule!

Being aware of the concept is sufficient for exam. No calculation needed.

Neural Network Testing

Once all network is trained and parameters are updated

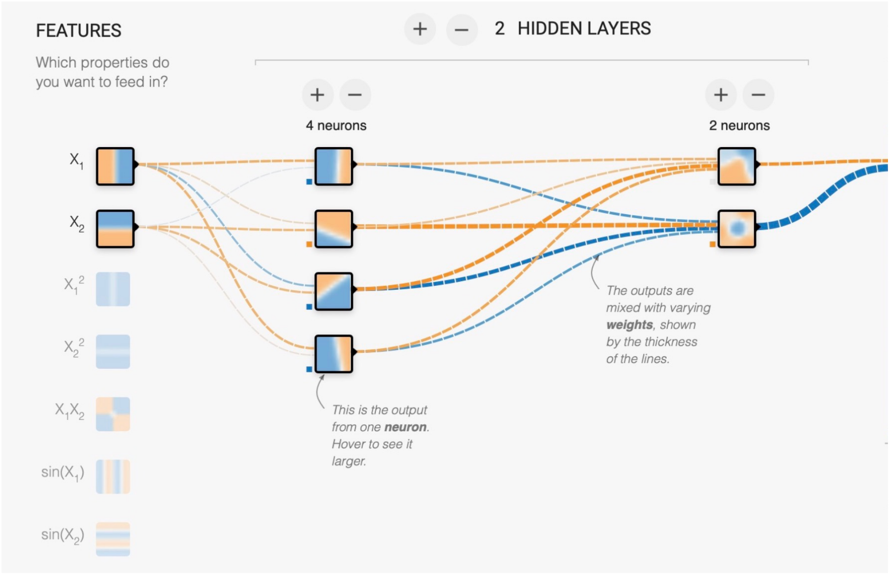


1. Forward: (weights are fixed)
To estimate compute network responses
To predict the output labels given novel inputs

Python Demo

```
[ ]  from sklearn.neural_network import MLPClassifier
     # Train the neural network classifier with 3 hidden layers of 200 neurons each
     clf = MLPClassifier(hidden_layer_sizes=(200, 200, 200), activation="relu", learning_rate="invscaling", verbose=True)
     clf.fit(X, y)
     plot_clf(clf, X, y)
```

- Python Code (A simple multi-layer neural network classifier)
  - lec12.ipynb
- Demo
  - https://playground.tensorflow.org/



Supplementary materials
(Not required for exam)

1) https://www.youtube.com/watch?v=tIeHLnjs5U8

This video series includes animations that explain backpropagation calculus.

2) https://www.youtube.com/playlist?list=PLQVvvaa0QuDcjD5BAw2DxE6OF2tius3V3

This video series includes hands-on coding examples in Python.

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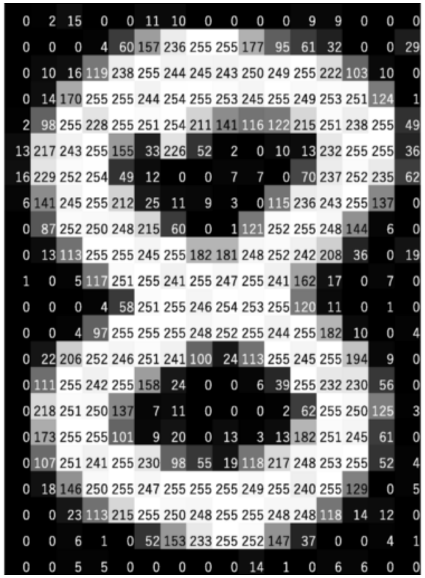
Convolutional Neural Network (CNN)

- A convolutional neural network (CNN) is a special type of neural network that significantly reduces the number of parameters in a deep neural network.
- Very popular in image-related applications
- Each image is stored as a matrix in a computer

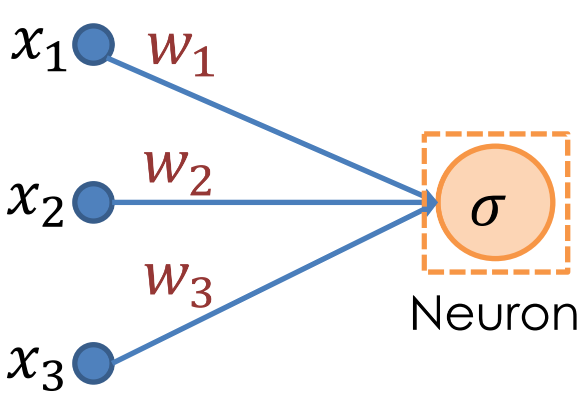


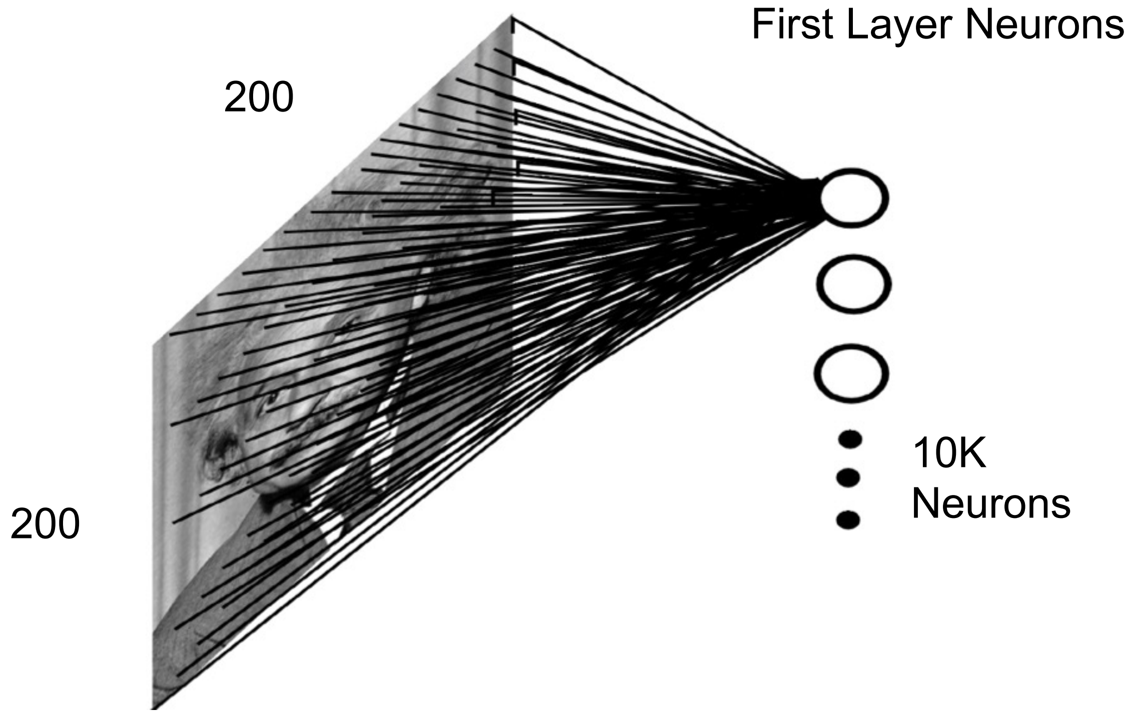
0	2	15	0	0	11	10	0	0	0	0	9	9	0	0	0
0	0	0	4	60	157	236	255	255	177	95	61	32	0	0	29
0	10	16	119	238	255	244	245	243	250	249	255	222	103	10	0
0	14	170	255	255	244	254	255	253	245	255	249	253	251	124	1
2	98	255	228	255	251	254	211	141	116	122	215	251	238	255	49
13	217	243	255	155	33	226	52	2	0	10	13	232	255	255	36
16	229	252	254	49	12	0	0	7	7	0	70	237	252	235	62
6	141	245	255	212	25	11	9	3	0	115	236	243	255	137	0
0	87	252	250	248	215	60	0	1	121	252	255	248	144	6	0
0	13	113	255	255	245	255	182	181	248	252	242	208	36	0	19
1	0	5	117	251	255	241	255	247	255	241	162	17	0	7	0
0	0	0	4	58	251	255	246	254	253	255	120	11	0	1	0
0	0	4	97	255	255	255	248	252	255	244	255	182	10	0	4
0	22	206	252	246	251	241	100	24	113	255	245	255	194	9	0
0	111	255	242	255	158	24	0	0	6	39	255	232	230	56	0
0	218	251	250	137	7	11	0	0	0	2	62	255	250	125	3
0	173	255	255	101	9	20	0	13	3	13	182	251	245	61	0
0	107	251	241	255	230	98	55	19	118	217	248	253	255	52	4
0	18	146	250	255	247	255	255	255	249	255	240	255	129	0	5
0	0	23	113	215	255	250	248	255	255	248	248	118	14	12	0
0	0	6	1	0	52	153	233	255	252	147	37	0	0	4	1
0	0	5	5	0	0	0	0	0	14	1	0	6	6	0	0

https://medium.com/lifeandtech/convert-csv-file-to-images-309b6fdb8c49

Convolutional Neural Network (CNN)

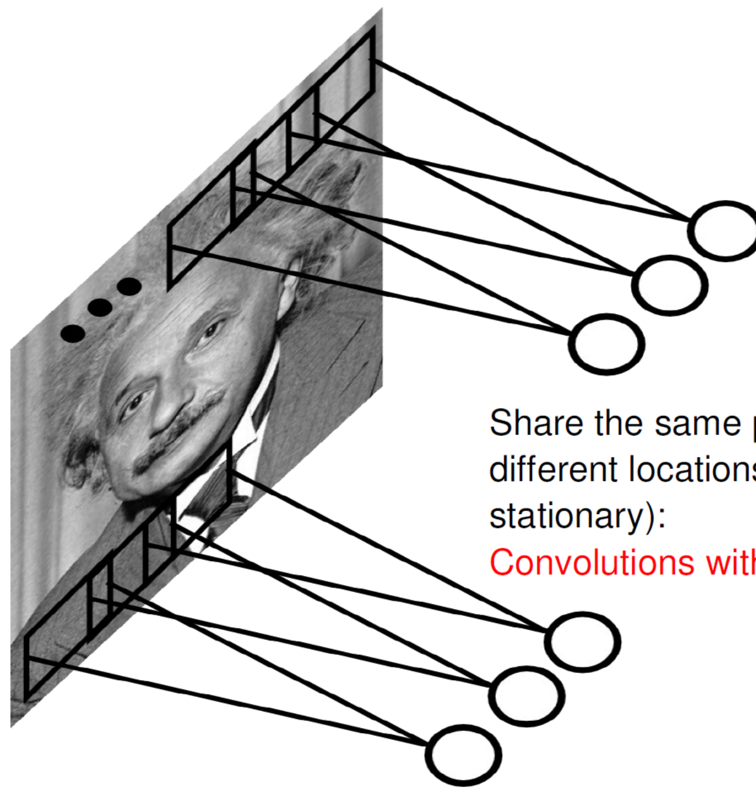
- If we model all matrix entries as inputs all at once
  - Assume we have an image/matrix size of 200x200
  - Assume we have 10K neuros in the first layer
  - We already have 200x200x10K=400 Million parameters to learn!





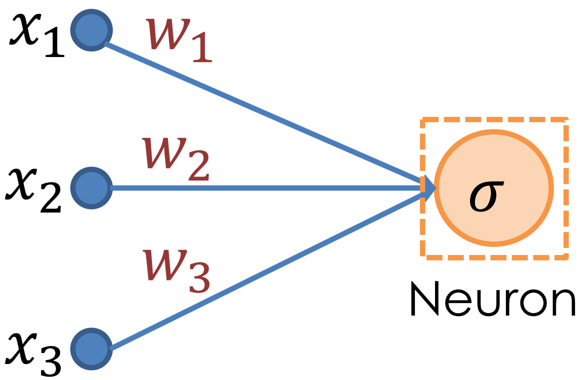
Convolutional Neural Network (CNN)

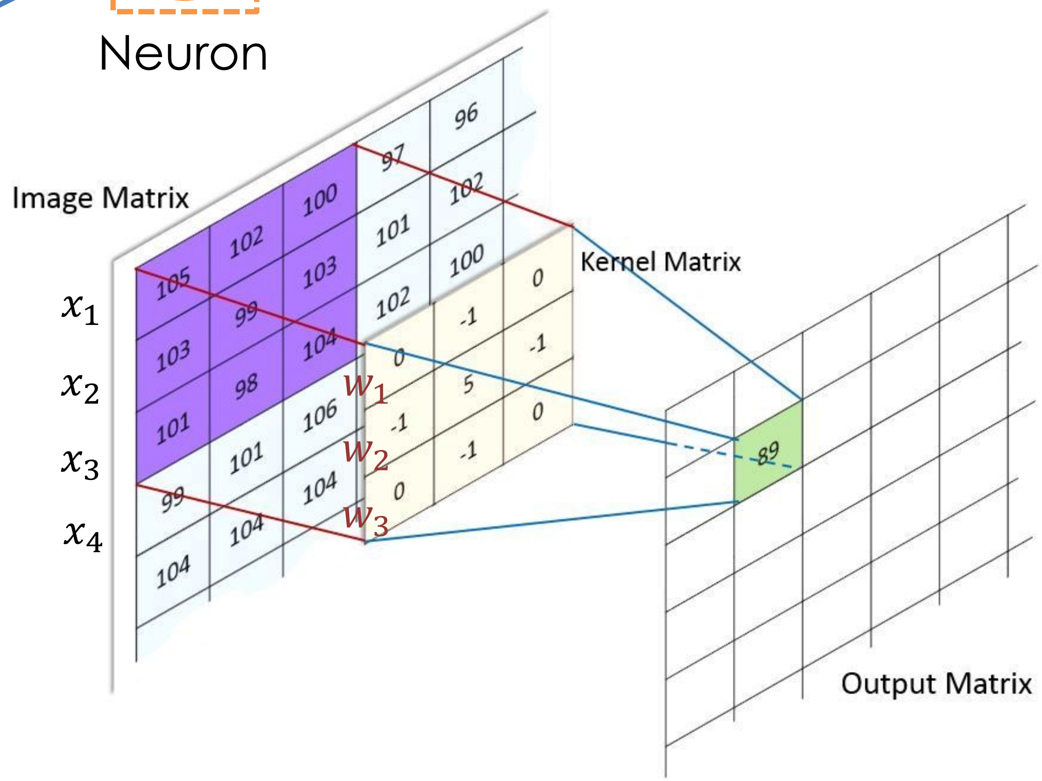
- Hence, we introduce CNN to reduce the number of parameters.
- Works in **a sliding-window manner**!



Share the same parameters across different locations (assuming input is stationary):
Convolutions with learned kernels

Convolutional Neural Network (CNN)





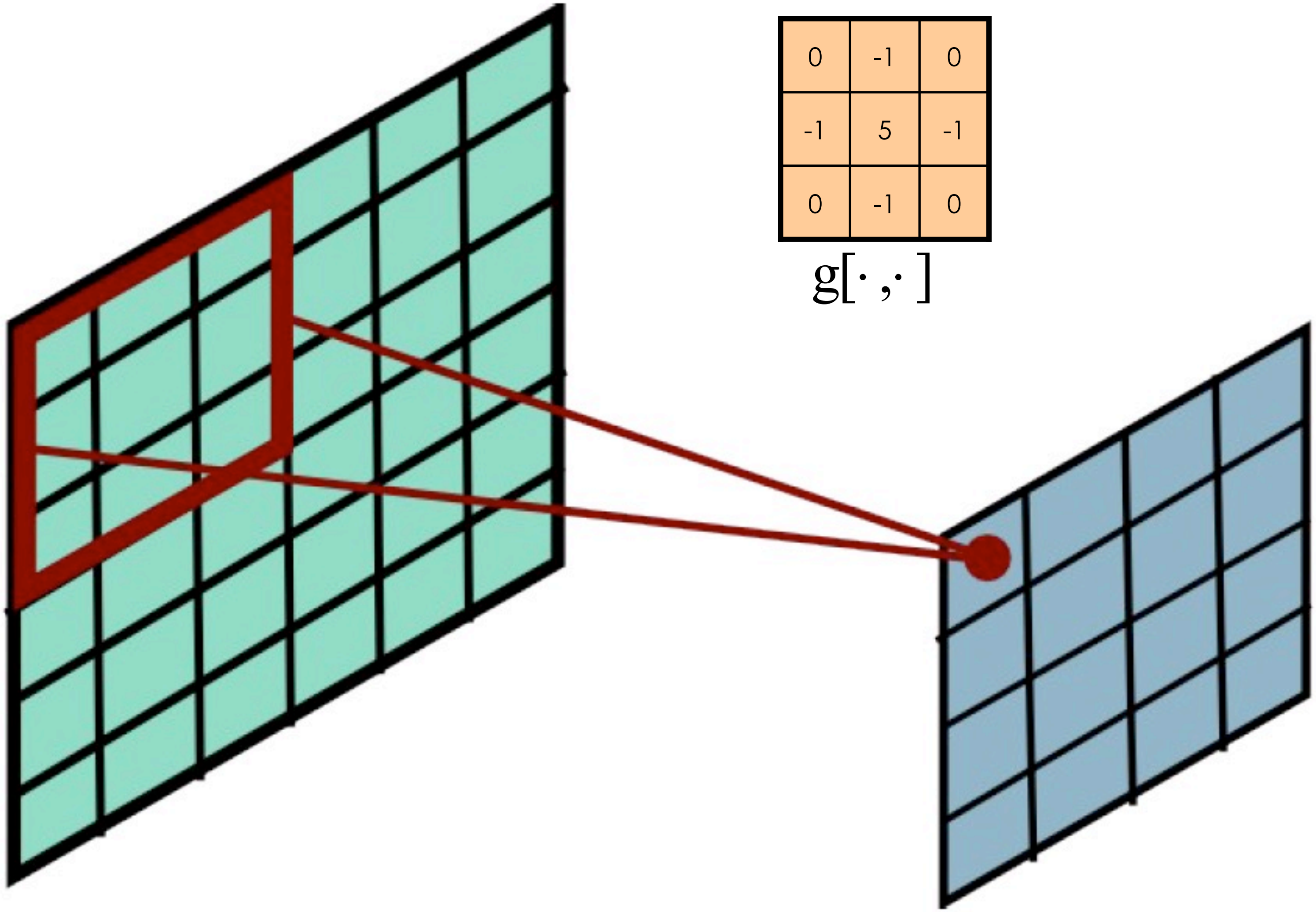
0	-1	0
-1	5	-1
0	-1	0

$g[\cdot,\cdot]$

Kernels to be learned

Image source: https://brilliant.org/wiki/convolutional-neural-network/

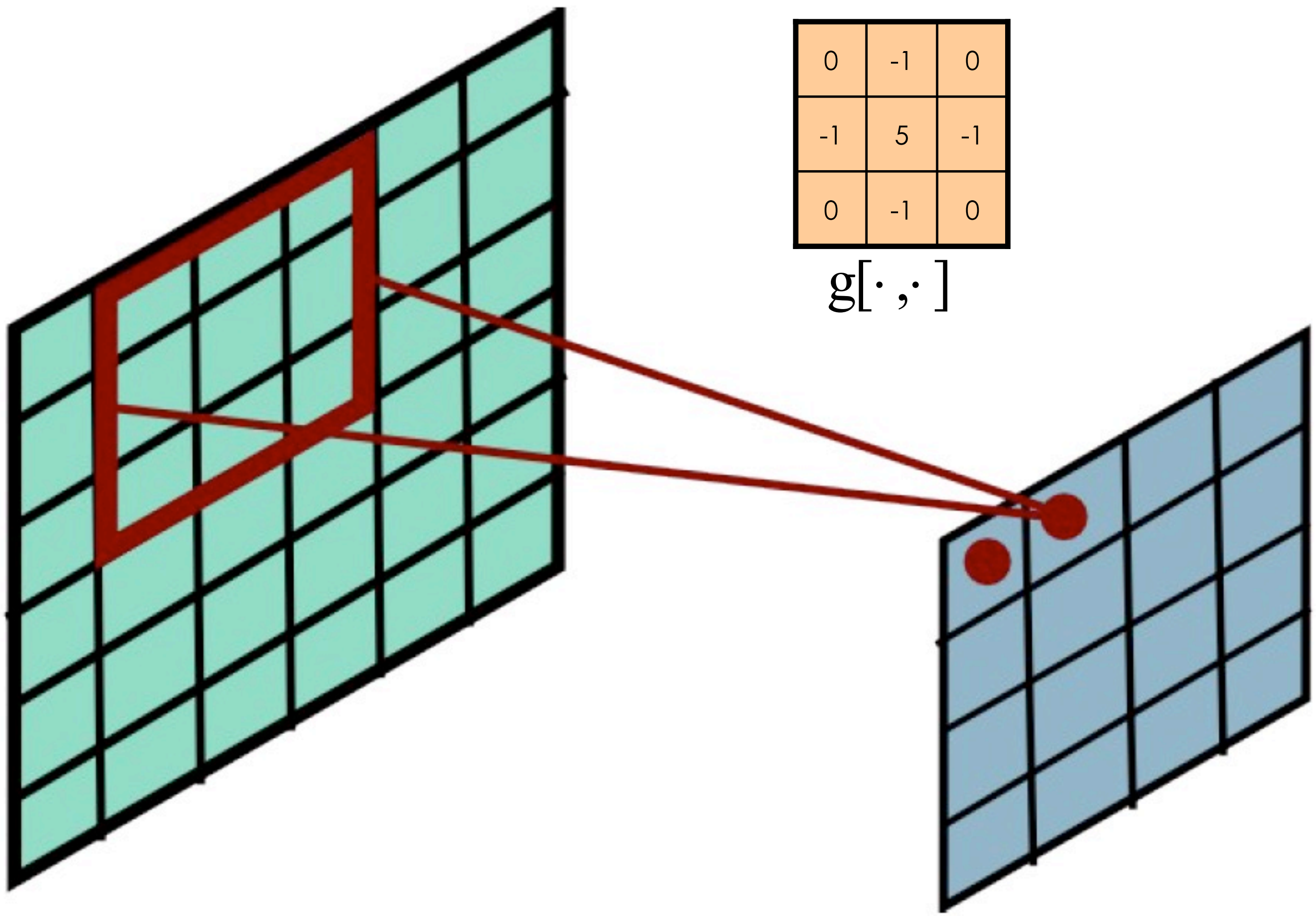
Convolutional Neural Network (CNN)



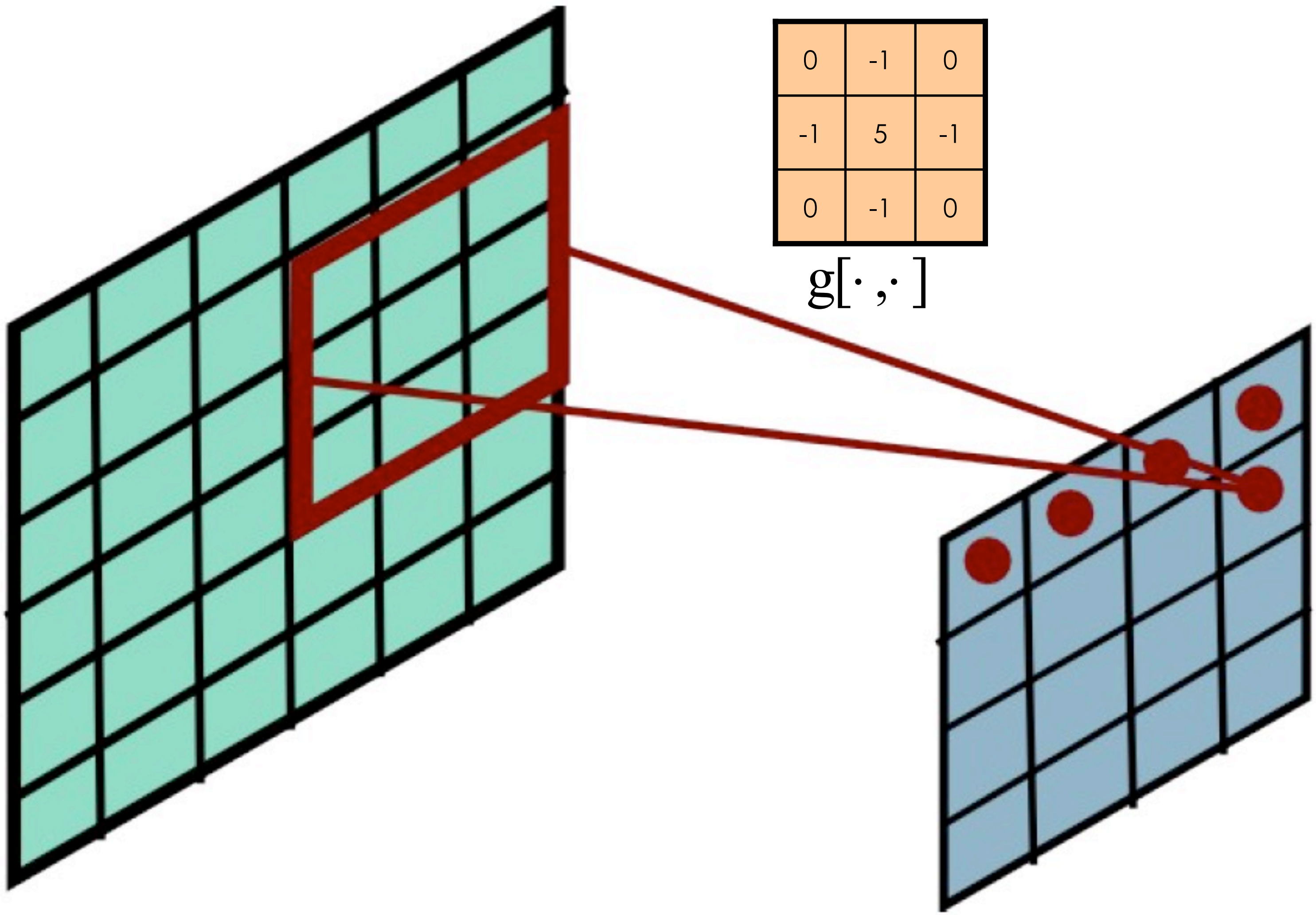
0	-1	0
-1	5	-1
0	-1	0

$g[\cdot,\cdot]$

Convolutional Neural Network (CNN)



Convolutional Neural Network (CNN)



0	-1	0
-1	5	-1
0	-1	0

$g[\cdot,\cdot]$

Convolutional Neural Network (CNN)

0	-1	0
-1	5	-1
0	-1	0

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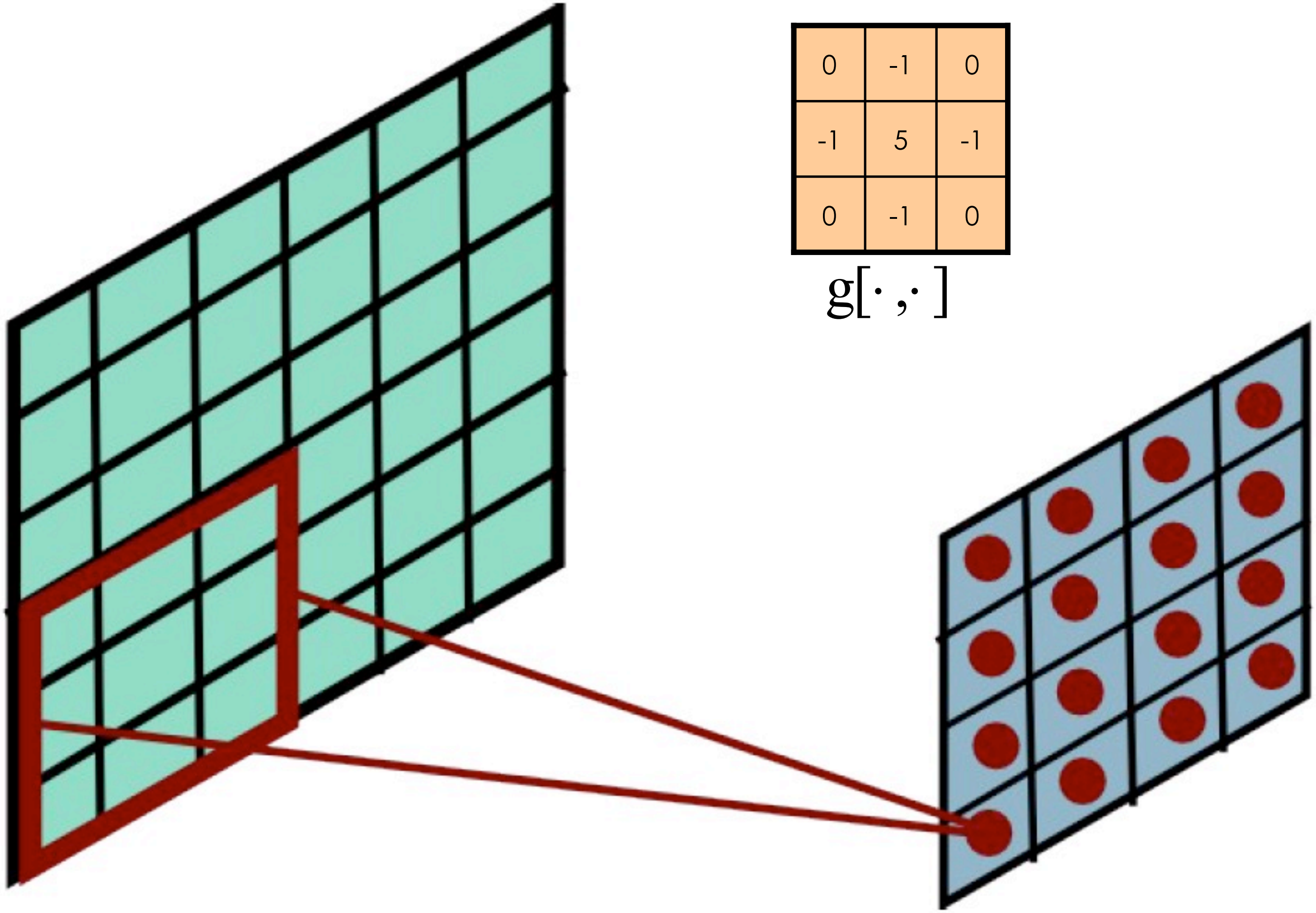
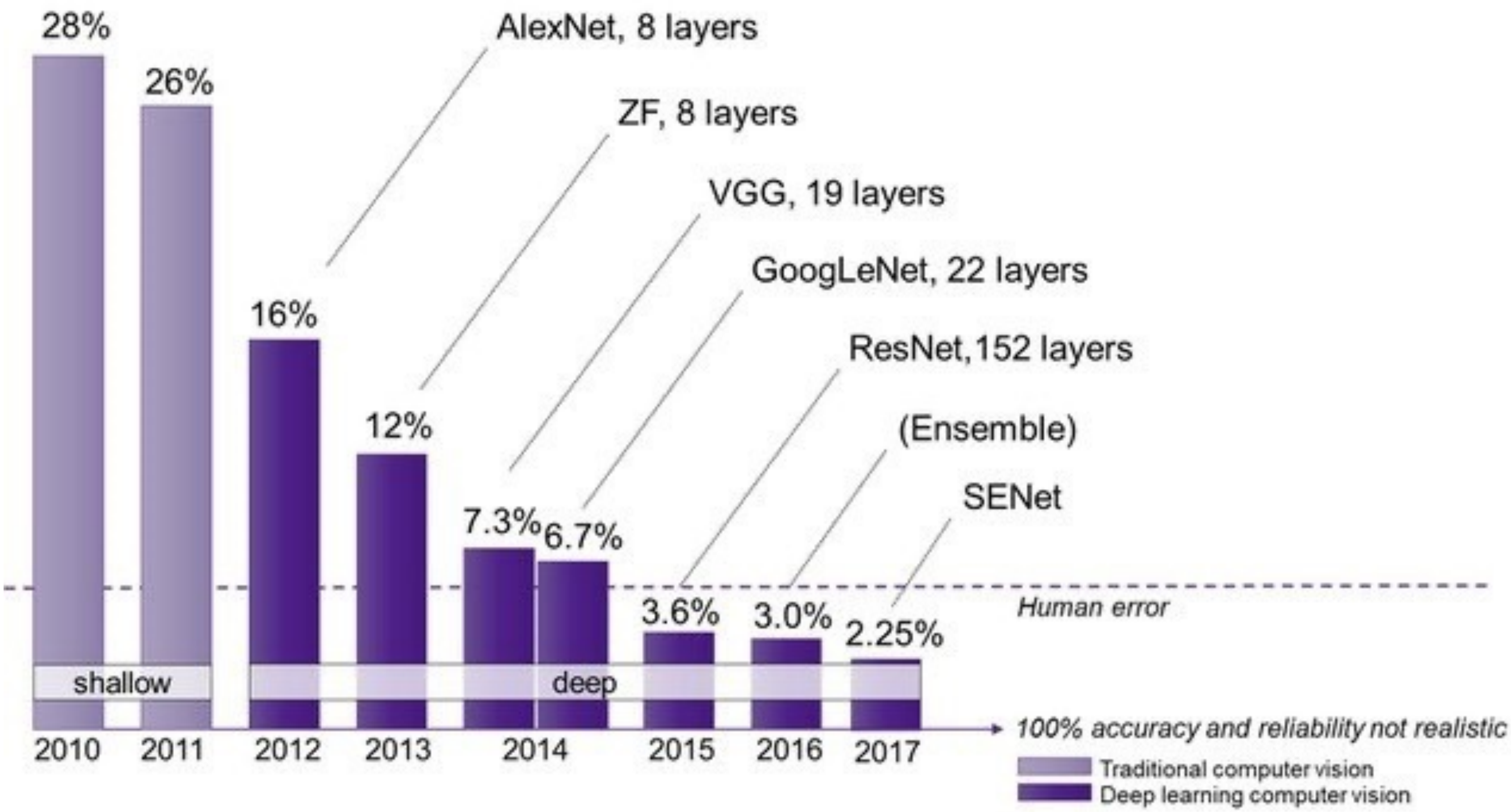


Neural Networks are Effective



Summary

- Introduction to Neural Networks
  – Multi-layer perceptron
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Thanks everyone, for your time and effort throughout the semester!