

# EE2211 Introduction to Machine Learning

## Lecture 10

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# Course Contents

- Introduction and Preliminaries (Xinchao)
  - Introduction
  - Data Engineering
  - Introduction to Linear Algebra, Probability and Statistics
- Fundamental Machine Learning Algorithms I (Vincent)
  - Systems of linear equations
  - Least squares, Linear regression
  - Ridge regression, Polynomial regression
- Fundamental Machine Learning Algorithms II (Vincent)
  - Over-fitting, bias/variance trade-off
  - Optimization, Gradient descent
  - Decision Trees, Random Forest
- Performance and More Algorithms (Xinchao)
  - **Performance Issues**
  - K-means Clustering
  - Neural Networks

# EE2211: Learning Outcome

## A Summary of Module Content

- **I am able to understand the formulation of a machine learning task**
  - Lecture 1 (feature extraction + classification)
  - Lecture 4 to Lecture 9 (regression and classification)
  - Lecture 11 and Lecture 12 (clustering and neural network)
- **I am able to relate the fundamentals of linear algebra and probability to machine learning**
  - Lecture 2 (recap of probability and linear algebra)
  - Lecture 4 to Lecture 8 (regression and classification)
  - Lecture 12 (neural network)
- **I am able to prepare the data for supervised learning and unsupervised learning**
  - Lecture 1 (feature extraction), Page 26 to 31 [For supervised and unsupervised]
  - Lecture 2 (data wrangling) [For supervised and unsupervised]
  - Lecture 10 (Training/Validation/Test) [For supervised]
  - Programming Exercises in tutorials
- **I am able to evaluate the performance of a machine learning algorithm**
  - Lecture 5 to Lecture 9 (evaluate the difference between labels and predictions)
  - Lecture 10 (evaluation metrics)
- **I am able to implement regression and classification algorithms**
  - Lecture 5 to Lecture 9

# Outline

- Dataset Partition:
  - Training/Validation/Testing
- Cross Validation
- Evaluation Metrics
  - Evaluating the quality of a trained classifier

**We will talk about many metrics:  
It is OK you can't memorize them all  
But intuition is important!**

# A Real-world Scenario

- We would like to train a Random Forest for face classification (i.e., to tell an image is a human face or not)



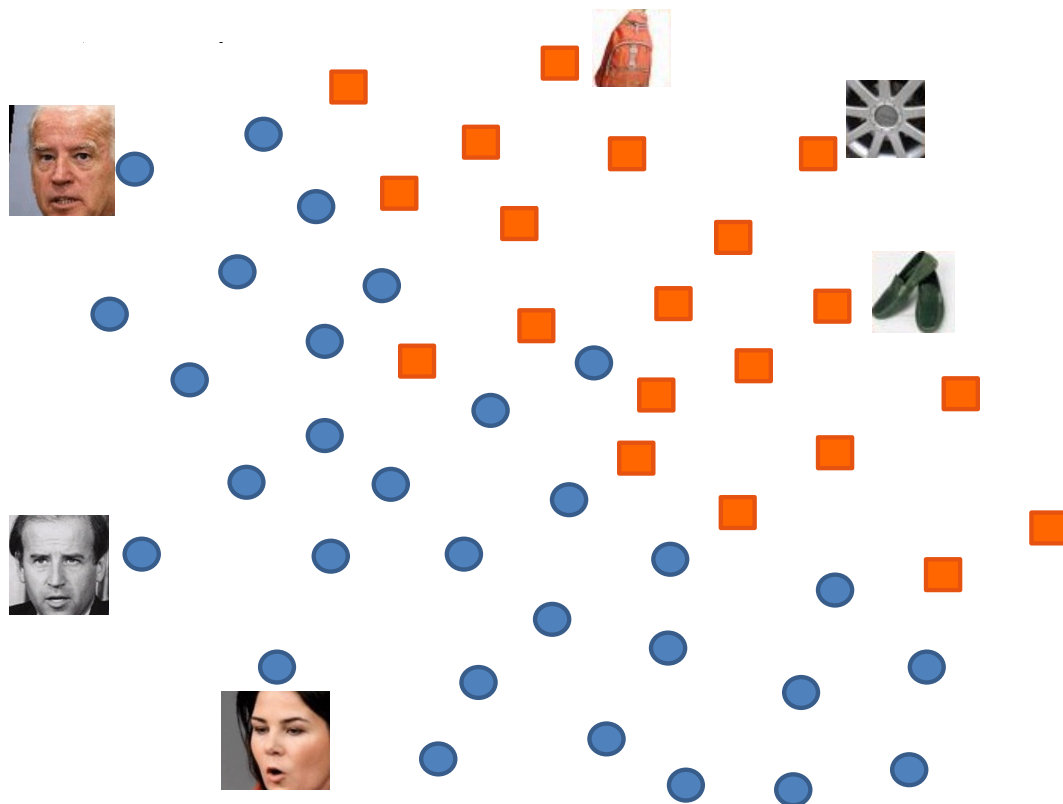
# Faces



## Non-faces

# A Real-world Scenario

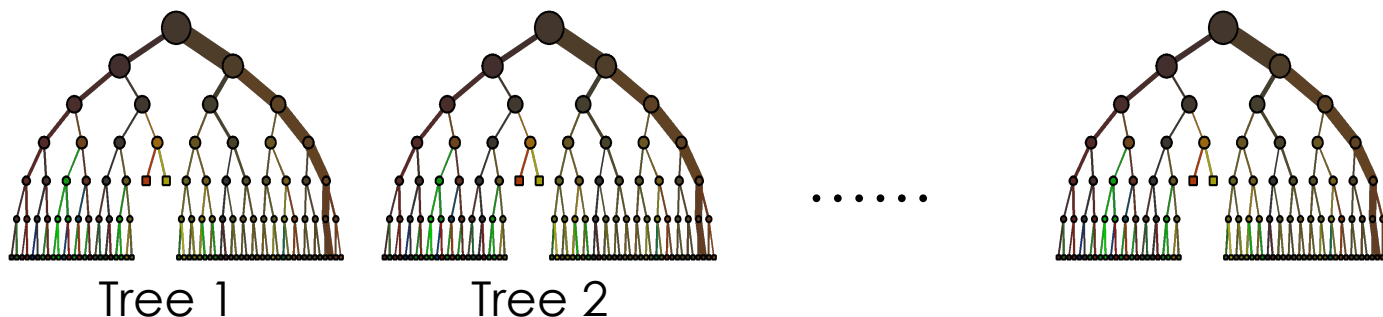
- We would like to train a Random Forest for face classification (i.e., to tell an image is a human face or not)
  - We will have one dataset to train the Random Forest



# A Real-world Scenario

- We would like to train a Random Forest for face classification (i.e., to tell an image is a human face or not)
  - We will have one dataset to train the Random Forest
  - We will have tunable (hyper)parameters for the Random Forest. For example, ***the number of trees*** in the Random Forest
    - Shall we use 100 trees?
    - Shall we use 200 trees?
    - ...

We need to decide on the parameter



# A Real-world Scenario

- We would like to train a Random Forest for face classification (i.e., to tell an image is a human face or not)
  - We will have **one dataset** to train the Random Forest
  - We will have tunable **(hyper)parameters** for the Random Forest. For example, ***the number of trees*** in the Random Forest
    - Shall we use 100 trees?
    - Shall we use 200 trees?
    - ...
- We need to decide on the parameter
  - Once we decide the number of trees, we will the Random Forest with the **selected parameter** on **unseen** test data.

Test Data



Yes!

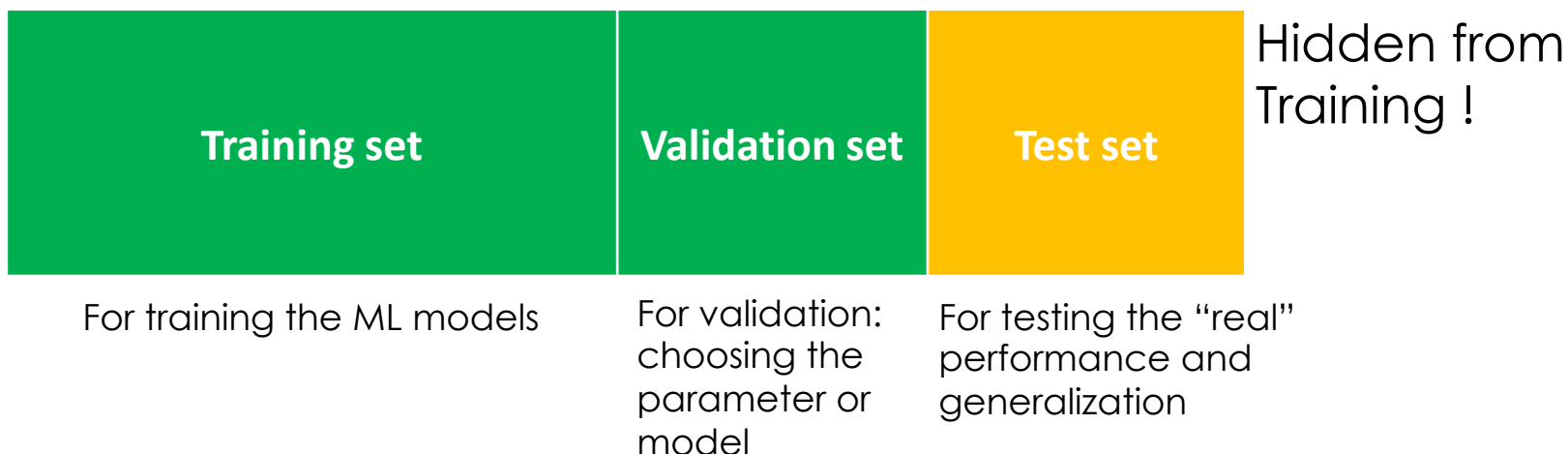


No!



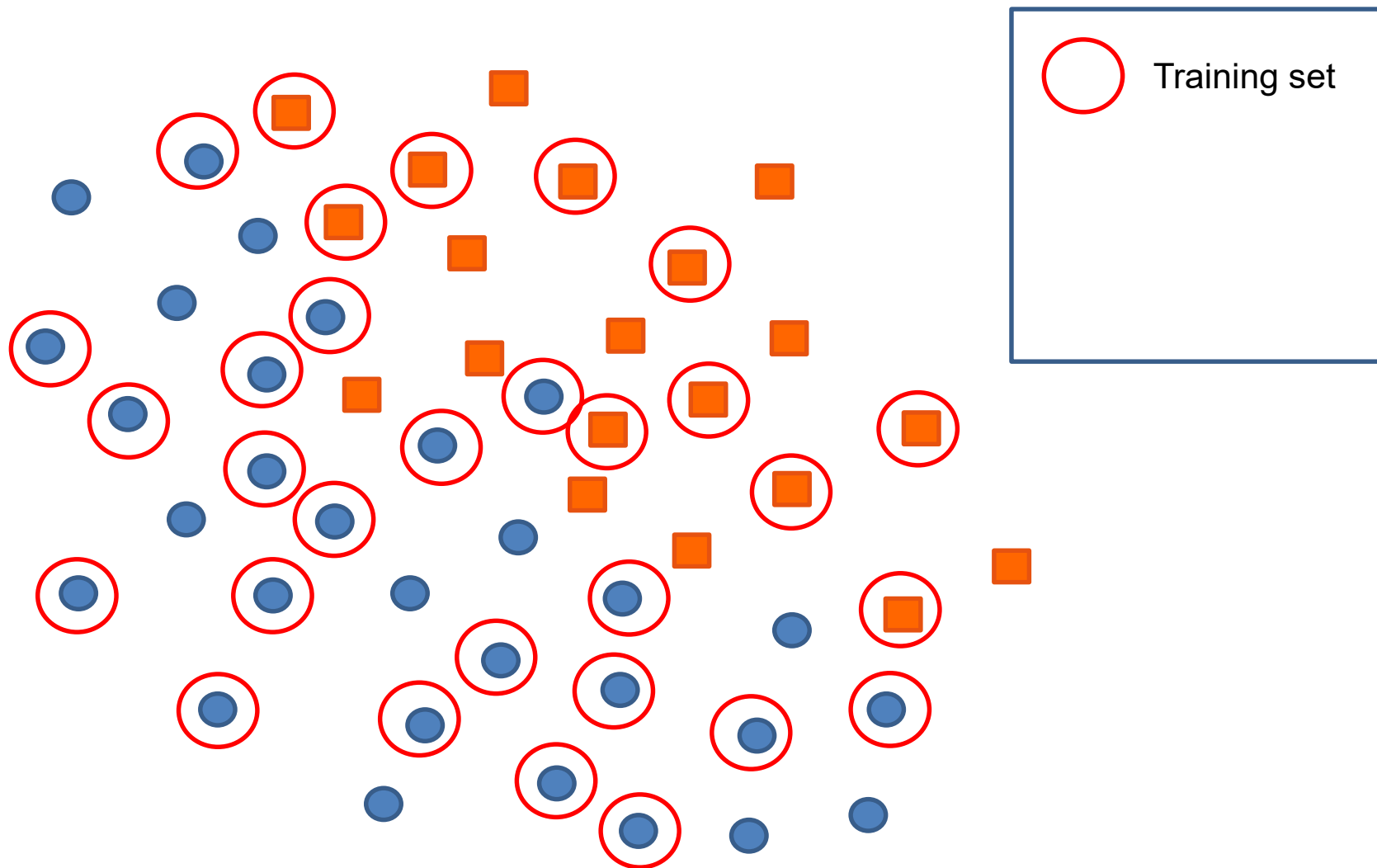
# Training, Validation, and Test

- In real-world application,
  - We don't have test data, since they are unseen
  - Imagine you develop a face detector app, you don't know whom you will test on
- In lab practice,
  - We divide the dataset into three parts

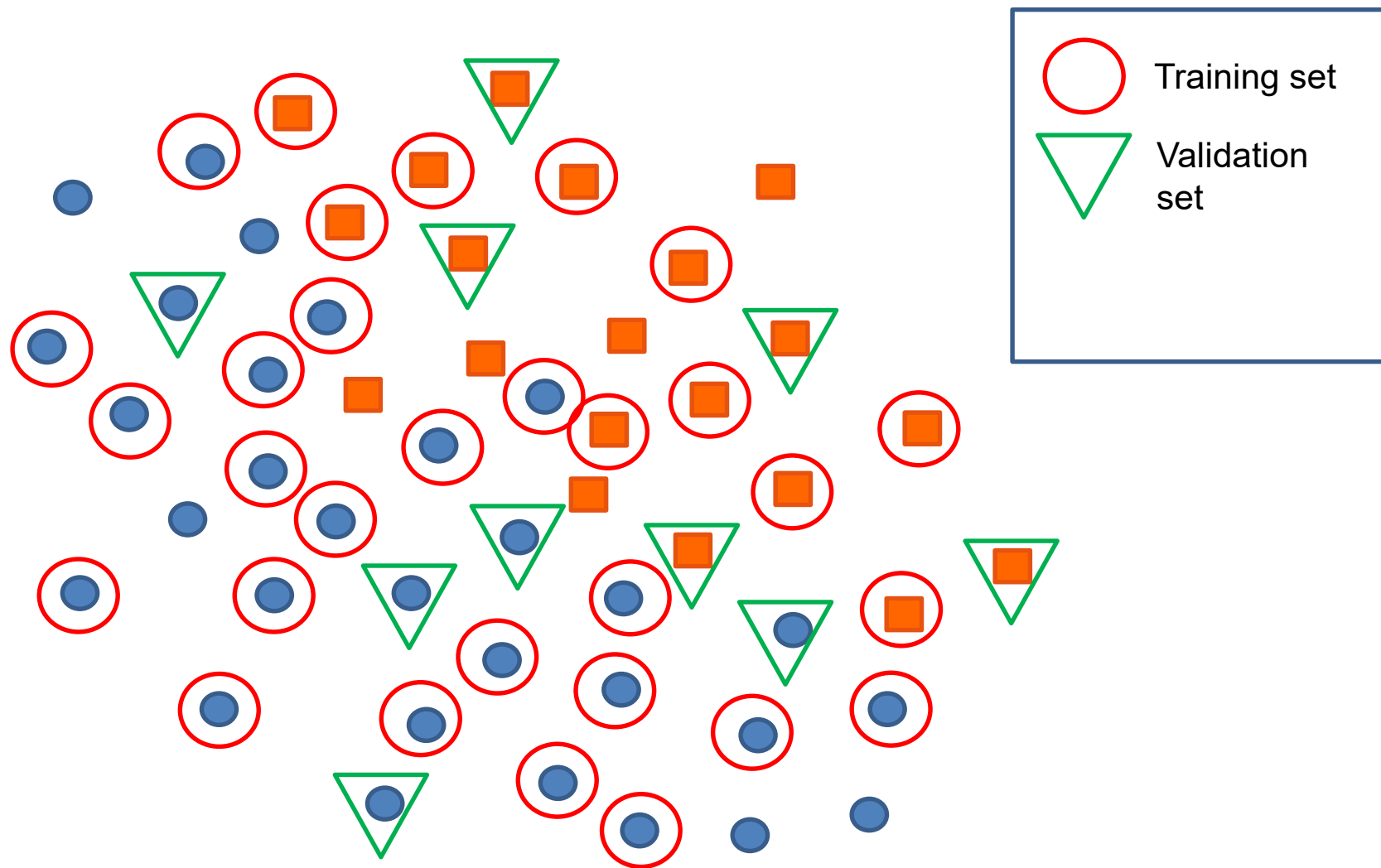


– **NEVER touch test data during training!!!**

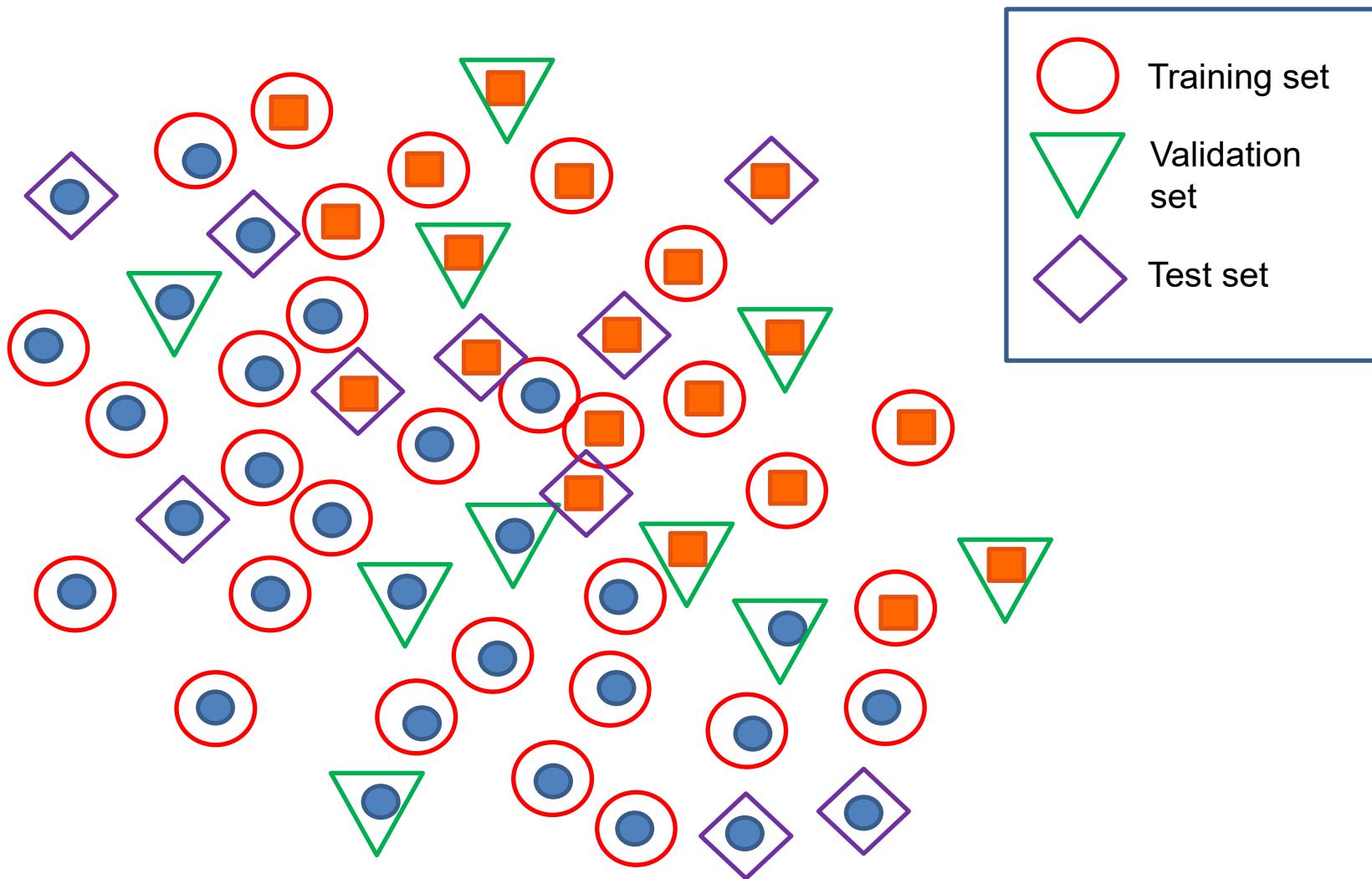
# Training, Validation, and Test



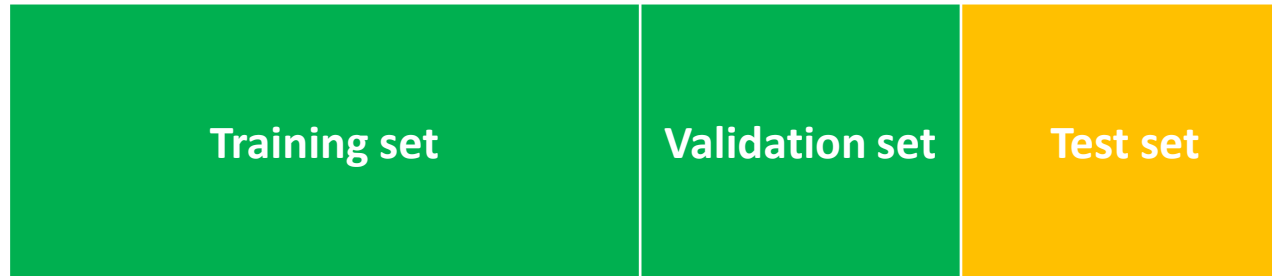
# Training, Validation, and Test



# Training, Validation, and Test



# Training, Validation, and Test



For training the ML models

For validation:  
choosing the  
parameter or  
model

For testing the “real”  
performance and  
generalization

Example: Assume I want to build a Random Forest. I have a parameter to decide: shall I have

- 100 Trees?
- 200 Trees?

What we do next is to use the training set to train two classifiers,

1)  $C_1$ : Random Forest with 100 trees, and 2)  $C_2$ : Random Forest with 200 trees

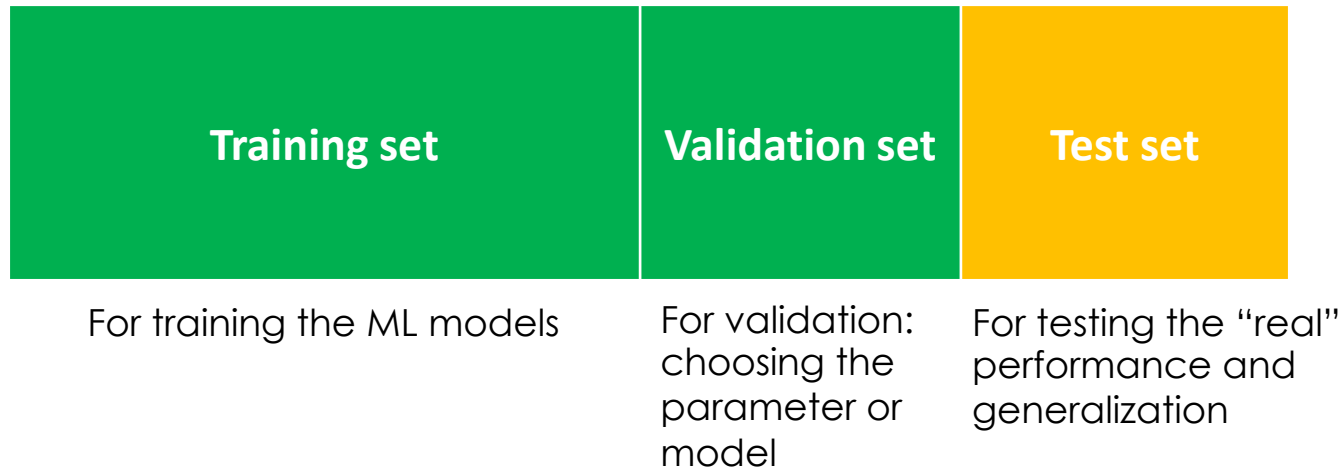
They have the following accuracy:

1.  $C_1$ : Random Forest with 100 trees: training accuracy 92%, validation accuracy 90%
2.  $C_2$ : Random Forest with 200 trees: training accuracy 95%, validation accuracy 88%

Which one to choose for real application, i.e., testing?

**The one with higher validation accuracy, i.e., Random Forest with 100 trees!**

# Python Demo: lec10.ipynb



- Problem Setup
  - Dataset used: IRISdataset
    - Link: [https://scikit-learn.org/stable/datasets/toy\\_dataset.html#iris-dataset](https://scikit-learn.org/stable/datasets/toy_dataset.html#iris-dataset)
  - Training/Validation/Test: 100/25/25
  - Machine Learning Task and Model: Polynomial regression
  - Parameters to select: Order 1 to 10

# Cross Validation

- In practice, we do the **k-fold cross validation**

Test

4-fold cross validation

Classifiers  
Trained

Fold 1

Train

Train

Train

Validation

$C_1^1$   $C_2^1$

Fold 2

Train

Train

Validation

Train

$C_1^2$   $C_2^2$

Fold 3

Train

Validation

Train

Train

$C_1^3$   $C_2^3$

Fold 4

Validation

Train

Train

Train

$C_1^4$   $C_2^4$

Example: which one to select for test?

	Fold 1 Accuracy on Validation Set 1	Fold 2 Accuracy on Validation Set 2	Fold 3 Accuracy on Validation Set 3	Fold 4 Accuracy on Validation Set 4	Average Accuracy on All Validation Sets
Classifier with Param1 (e.g. 100 trees)	88% $C_1^1$	89% $C_1^2$	93% $C_1^3$	92% $C_1^4$	90.5%
Classifier with Param2 (e.g. 200 trees)	90% $C_2^1$	88% $C_2^2$	91% $C_2^3$	91% $C_2^4$	90%

# Cross Validation

## Other common partitioning:

- 10-Fold CV
- 5-Fold CV
- 3-Fold CV

We may decide on the size of the test set, for example, 15%, 20%, 30% of the whole dataset, the rest for training/validation.

The **test set** contains the examples that the learning algorithm has **never seen before**, so if our model performs well on predicting the labels of the examples from the test set, we say that our model **generalizes well**.



# Training, Validation, and Test

- Validation is however not always used:
  - Validation is used when you need to **pick parameters or models**
  - If you have no models or parameters to compare, you may consider partition the data into only training and test

# Outline

- Dataset Partition:
  - Training/Validation/Testing
- Cross Validation
- Evaluation Metrics
  - Evaluating the quality of a trained classifier

**We will talk about many metrics:  
It is OK you can't memorize them all  
But intuition is important!**

# Evaluation Metrics

## Regression

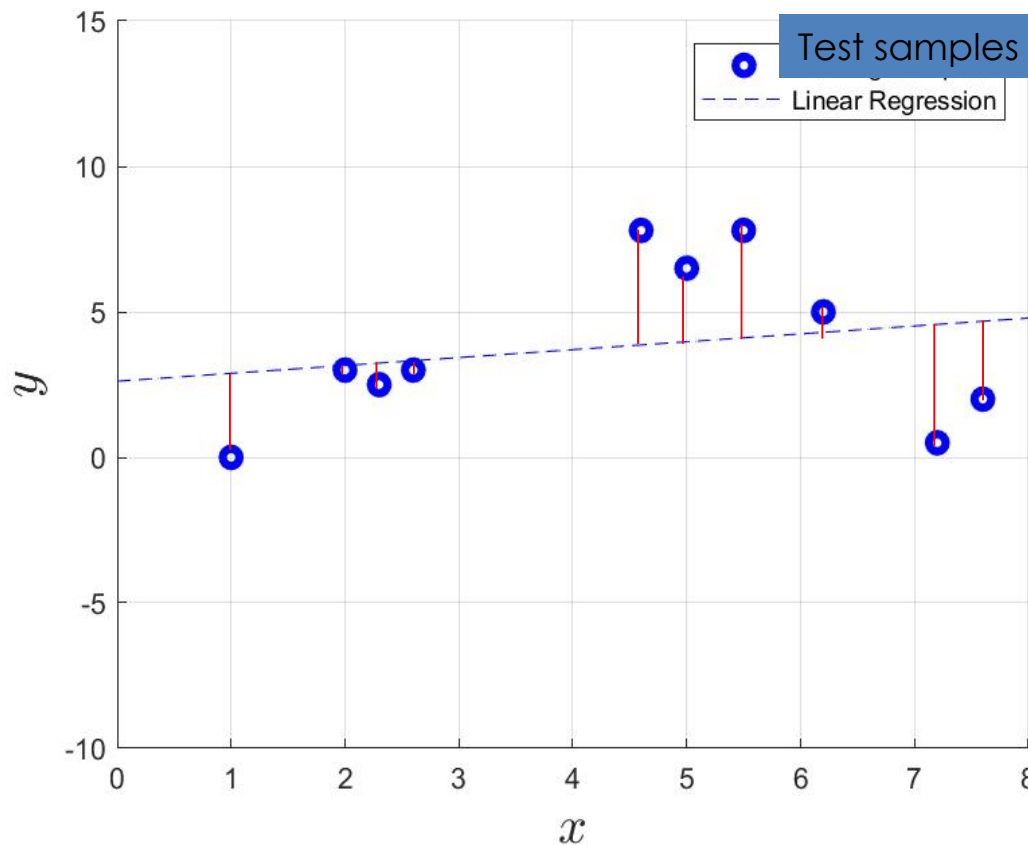
### Mean Square Error

$$(\text{MSE} = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n})$$

### Mean Absolute Error

$$(\text{MAE} = \frac{\sum_{i=1}^n |y_i - \hat{y}_i|}{n})$$

where  $y_i$  denotes the target output and  $\hat{y}_i$  denotes the predicted output for sample  $i$ .



# Evaluation Metrics

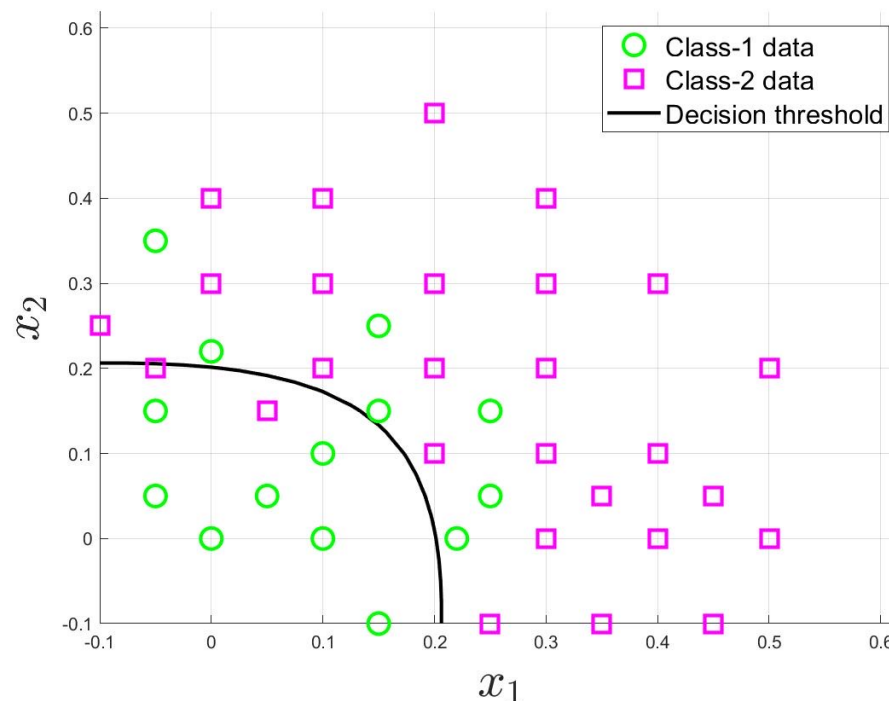
## Classification

Class-1: Positive Class

Class-2: Negative Class

**Confusion Matrix**

	Class-1 (predicted)	Class-2 (predicted)
Class-1 (actual)	7 (TP)	7 (FN)
Class-2 (actual)	2 (FP)	25 (TN)



TP: True Positive

FN: False Negative (i.e., **Type II Error**)

FP: False Positive (i.e., **Type I Error**)

TN: True Negative

# Evaluation Metrics

## Classification

- How many samples in the dataset have the real label of Class-2?
- How many samples are there in total?
- How many sample are correctly classified? How many are incorrectly classified?

	Class-1 (predicted)	Class-2 (predicted)
Class-1 (actual)	7 (TP)	7 (FN)
Class-2 (actual)	2 (FP)	25 (TN)

# Evaluation Metrics

## Classification

### Confusion Matrix for Binary Classification

	$\hat{P}$ (predicted)	$\hat{N}$ (predicted)	
$P$ (actual)	TP	FN	Recall $TP/(TP+FN)$
$N$ (actual)	FP	TN	
	Precision $TP/(TP+FP)$	Accuracy $(TP+TN)/(TP+TN+FP+FN)$	

# Evaluation Metrics

## Classification

### Cost Matrix for Binary Classification

	$\hat{P}$ (predicted)	$\hat{N}$ (predicted)
$P$ (actual)	$C_{p,p} * TP$	$C_{p,n} * FN$
$N$ (actual)	$C_{n,p} * FP$	$C_{n,n} * TN$

**Total cost:**

$$C_{p,p} * TP + C_{p,n} * FN + C_{n,p} * FP + C_{n,n} * TN$$

Main Idea: To assign different **penalties** for different entries. Higher penalties for more severe results.

Usually,  $C_{p,p}$  and  $C_{n,n}$  are set to 0;  $C_{n,p}$  and  $C_{p,n}$  may and may not equal

# Evaluation Metrics

- Example of cost matrix
  - Assume we would like to develop a self-driving car system
  - We have an ML system that detects the pedestrians using camera, by conducting a binary classification
    - When it detects a person (positive class), the car should stop
    - When no person is detected (negative class), the car keeps going

## True Positive (cost $C_{p,p}$ )

There is person, ML detects person and car stops

## True Negative (cost $C_{n,n}$ )

There is no person, car keeps going

## False Positive (cost $C_{n,p}$ )

There is no person, ML detects person and car stops

## False Negative (cost $C_{p,n}$ )

There is person, ML fails to detect person and car keeps going



Credit: automotiveworld.com

$$C_{n,p} \text{ ? } C_{p,n} (>, <, \text{ or } =)$$



# Evaluation Metrics

- Handling unbalanced data

- Assume we have 1000 samples, of which 10 are positive and 990 are negative

- Accuracy =  $990/1000=0.99$ !

- Yet, half of the Class-1 are Classified to Class-2!

	Class-1 (predicted)	Class-2 (predicted)
Class-1 (actual)	5 (TP)	5 (FN)
Class-2 (actual)	5 (FP)	985 (TN)

**The goal is to highlight the problems of the results!**

In this case, we shall

- 1) Use cost matrix, assign different costs for each entry
- 2) Use Precision and Recall! Precision = 0.5 and Recall = 0.5

# Evaluation Metrics

## Classification

(True Positive Rate)  $TPR = TP / (TP + FN)$  **Recall**  
 (False Negative Rate)  $FNR = FN / (TP + FN)$

(True Negative Rate)  $TNR = TN / (FP + TN)$   
 (False Positive Rate)  $FPR = FP / (FP + TN)$


$TPR + FNR = 1$  (100% of positive-class data)  
 $TNR + FPR = 1$  (100% of negative-class data)

	$\hat{P}$ (predicted)	$\hat{N}$ (predicted)
$P$ (actual)	TP	FN
$N$ (actual)	FP	TN

# Evaluation Metrics

## Classification

Prediction function  $y = f(x)$



sample	N1	N2	P1	N3	P2	P3
input x	-4	-3	-2.5	-2	-1.5	-0.5
Prediction y	-1.1	-0.5	-0.1	0.2	0.6	0.9
Label	-1	-1	1	-1	1	1

If threshold set to be  $y=0$ ,  
 N3, P2, P3 will be taken as +1  
 P1, N2, N1 will be taken as -1

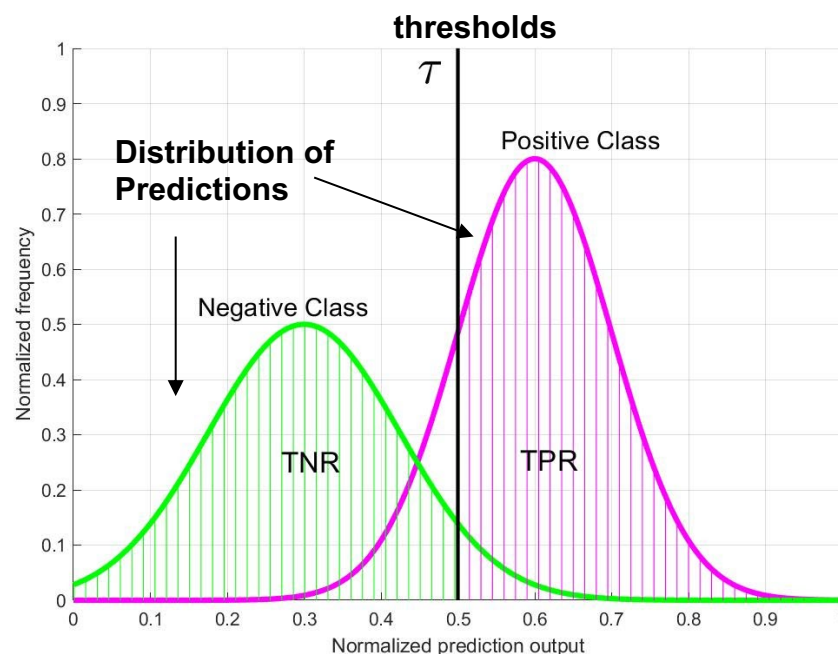
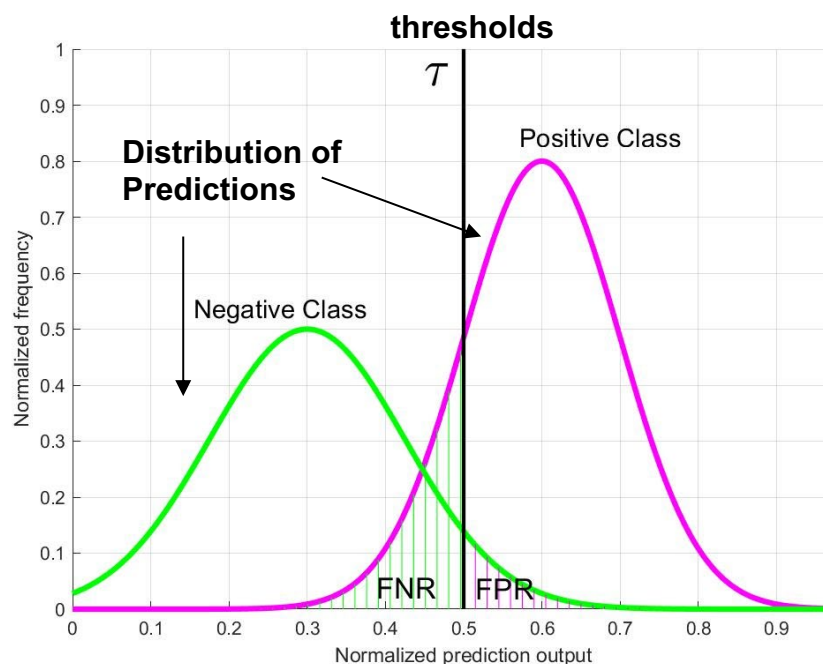
	$\hat{P}$ (predicted)	$\hat{N}$ (predicted)
P (actual)	$TP = 2$	$FN = 1$
N (actual)	$FP = 1$	$TN = 2$

# Evaluation Metrics

(True Positive Rate)  $TPR = TP / (TP + FN)$   
 (False Negative Rate)  $FNR = FN / (TP + FN)$   
 (True Negative Rate)  $TNR = TN / (FP + TN)$   
 (False Positive Rate)  $FPR = FP / (FP + TN)$

$TPR + FNR = 1$  (100% of positive-class data)  
 $TNR + FPR = 1$  (100% of negative-class data)

## Classification



Normalized prediction output: change the output to the range of [0,1]


Reference: Lec 2, Page 25

# Evaluation Metrics

## Classification

Prediction function  $y = f(x)$

We can change the threshold!



sample	N1	N2	P1	N3	P2	P3
input x	-4	-3	-2.5	-2	-1.5	-0.5
Prediction y	-1.1	-0.5	-0.1	0.2	0.6	0.9
Label	-1	-1	1	-1	1	1

If threshold set to be  $y=0.4$ ,  
P2, P3 will be taken as +1  
N3, P1, N2, N1 will be taken as -1

	$\hat{P}$ (predicted)	$\hat{N}$ (predicted)
P (actual)	$TP = 2$	$FN = 1$
N (actual)	$FP = 0$	$TN = 3$

# Evaluation Metrics

## Classification:

TP, FP, FN, TN will change wrt thresholds!

If threshold set to be  $y=0$ ,  
N3, P2, P3 will be taken as +1  
P1, N2, N1 will be taken as -1

	$\hat{P}$ (predicted)	$\hat{N}$ (predicted)
P (actual)	$TP = 2$	$FN = 1$
N (actual)	$FP = 1$	$TN = 2$

If threshold set to be  $y=0.4$ ,  
P2, P3 will be taken as +1  
N3, P1, N2, N1 will be taken as -1

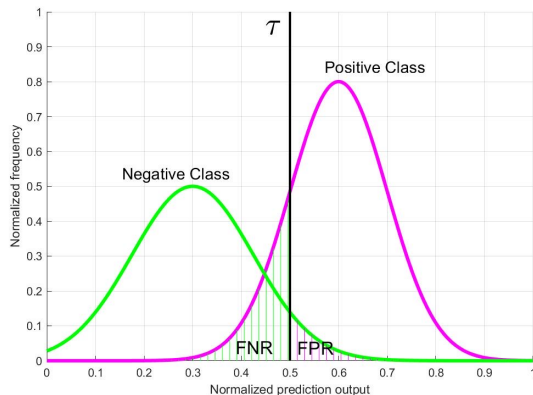
	$\hat{P}$ (predicted)	$\hat{N}$ (predicted)
P (actual)	$TP = 2$	$FN = 1$
N (actual)	$FP = 0$	$TN = 3$

Imagine we vary the thresholds at  $y$ !

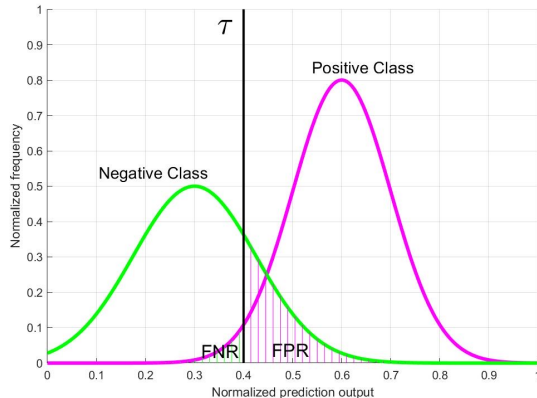
# Evaluation Metrics

## Classification

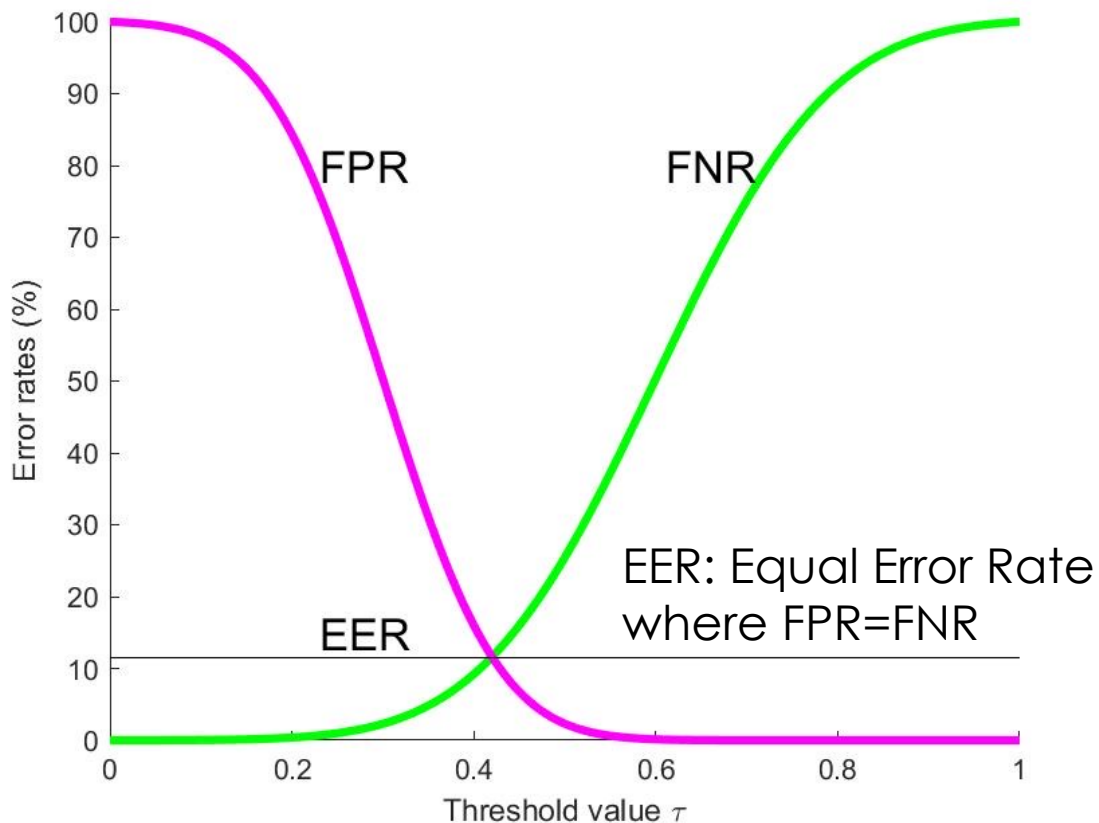
### Threshold 1



### Threshold 2

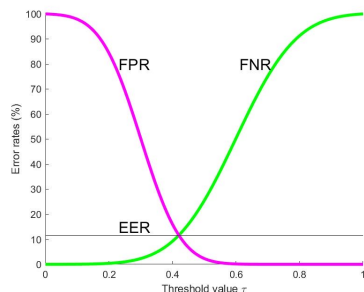


## Sliding the threshold



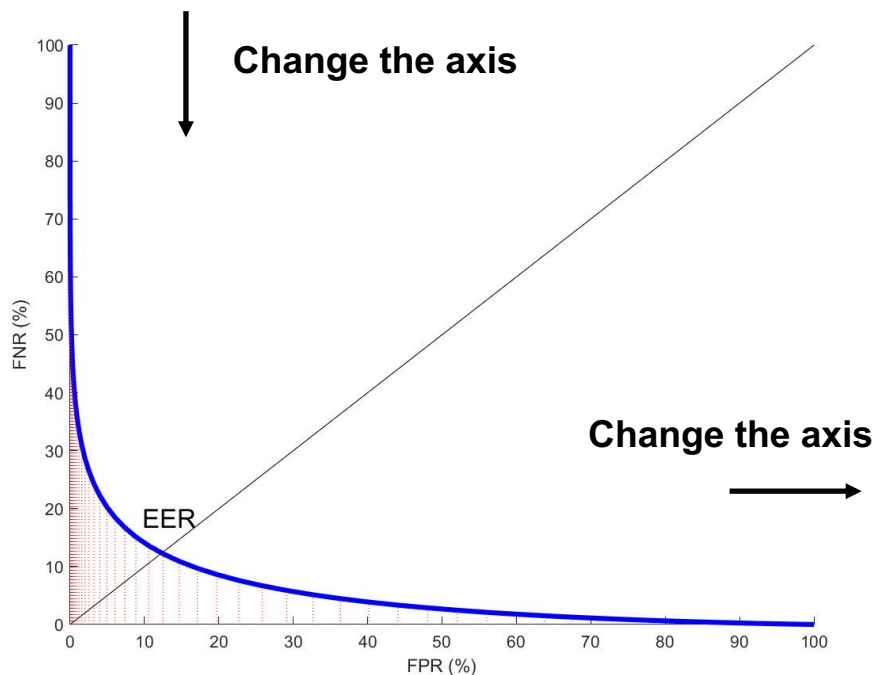
EER: Higher better or Lower better?

# Evaluation Metrics

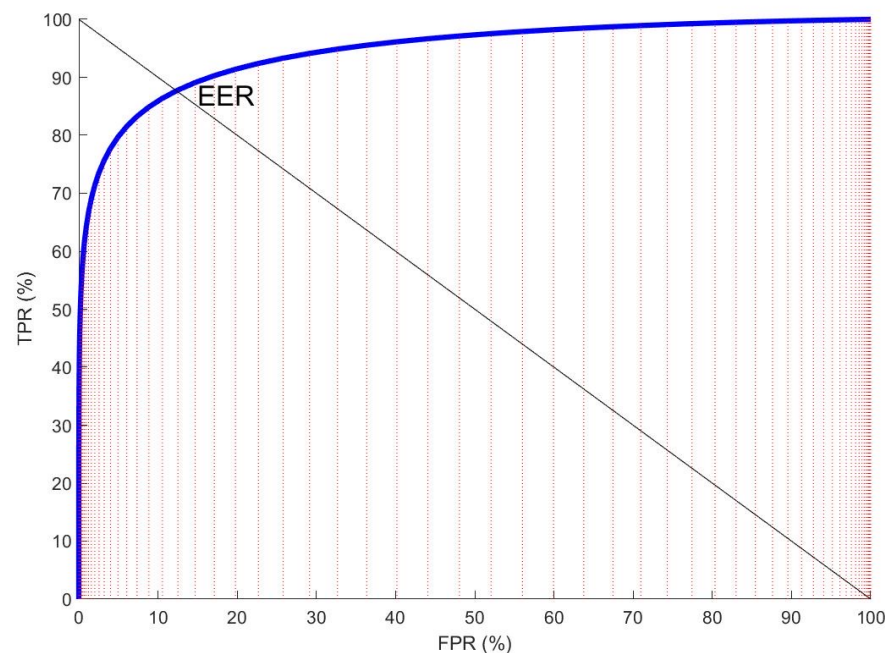


**ROC: Widely used**

$$\text{TPR} + \text{FNR} = 1$$



Detection Error Trade-off (DET) curve



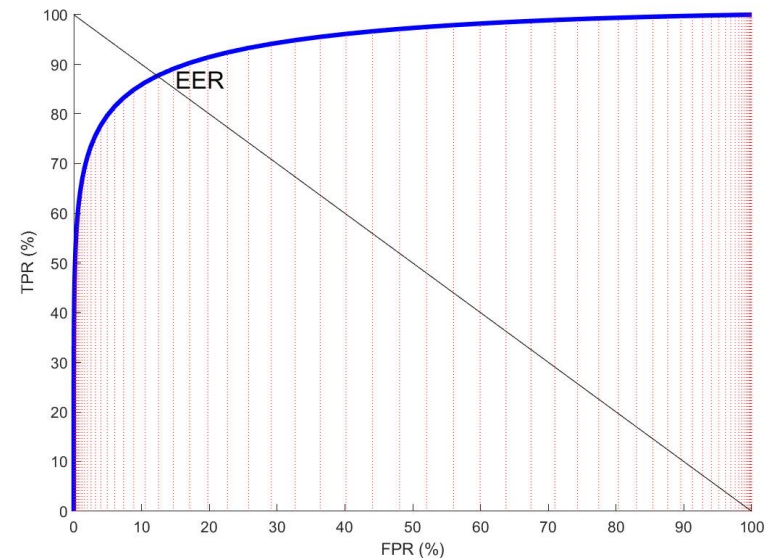
Receiver Operating Characteristic (ROC) Curve



# Evaluation Metrics

## Area Under the Curve (ROC curve)

AUC provides an aggregate measure of performance across all possible classification thresholds.

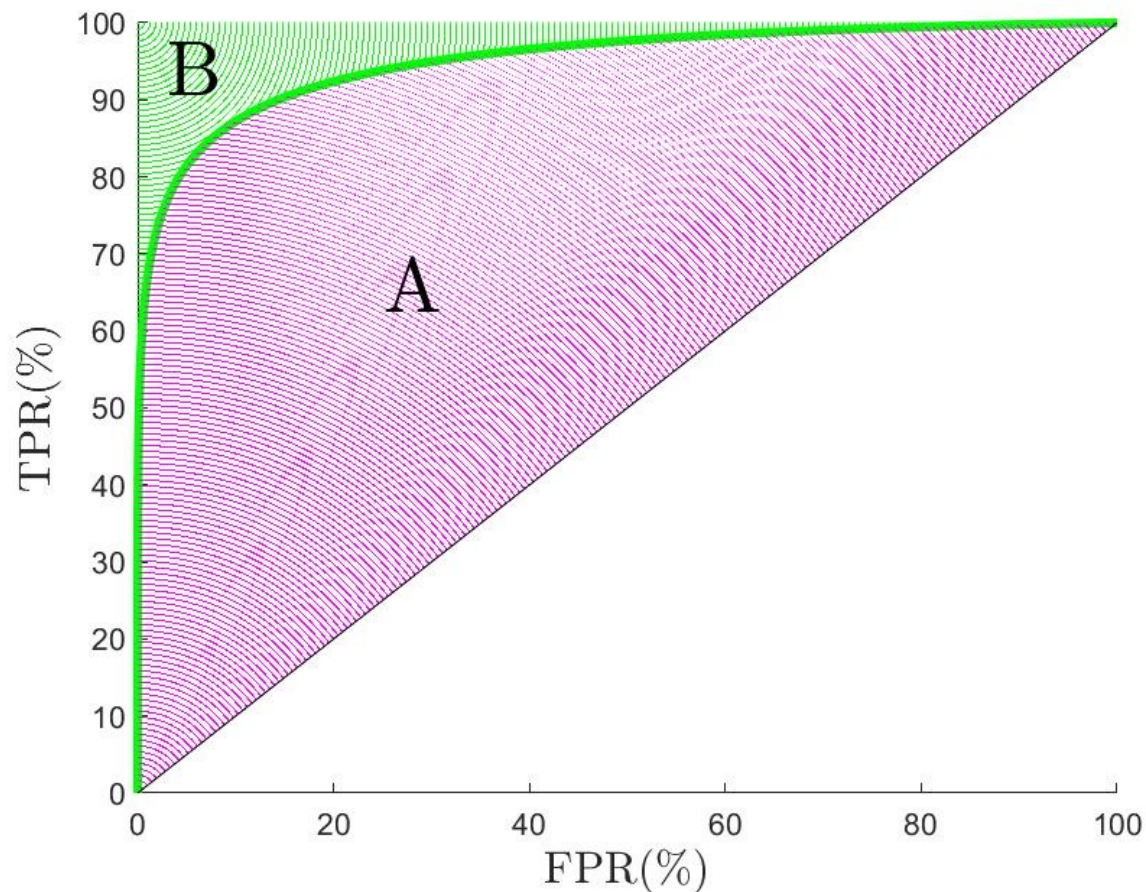


AUC ranges in value from 0 (100% wrong prediction) to 1 (100% correct prediction). It is classification-threshold-invariant.

# Evaluation Metrics

## Classification

Gini coefficient  
 $G = A / (A + B)$



# Evaluation Metrics

## Classification

### Confusion Matrix for Multicategory Classification

	$P_{\hat{1}}$ (predicted)	$P_{\hat{2}}$ (predicted)		$P_{\hat{C}}$ (predicted)
$P_1$ (actual)	$P_{1,\hat{1}}$	$P_{1,\hat{2}}$	...	$P_{1,\hat{C}}$
$P_2$ (actual)	$P_{2,\hat{1}}$	$P_{2,\hat{2}}$	...	$P_{2,\hat{C}}$
⋮	⋮	⋮	⋮	⋮
$P_C$ (actual)	$P_{C,\hat{1}}$	$P_{C,\hat{2}}$		$P_{C,\hat{C}}$

# Other Issues

- Computational speed and memory consumptions are also important factors
  - Especially for mobile or edge devices
- Other factors
  - Parallelable, Modularity, Maintainability
- Not focus of this module

