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ex6.4：

則 $E(\bar{x}) = \mu$

$V(\bar{x}) = \frac{\sigma^2}{n} = E(\bar{x}^2) - \mu^2$

$E(\hat{\theta}_1) =$

$$E\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}\right) = \frac{1}{n}E\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \frac{n-1}{n}\sigma^2$$

$E(\hat{\theta}_2) =$

$$E\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}\right) = \frac{1}{n-1}E\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n-1}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \sigma^2$$

因此，$\hat{\theta}_2 = \sum_{i=1}^{n}(x_i-\bar{x})^2/(n-1)$ 為母体变異数 σ^2 之不偏估計量，

而 $\hat{\theta}_1 = \sum_{i=1}^{n}(x_i-\bar{x})^2/n$ 為母体变異数 σ^2 之偏誤估計量。