

Since the refracted ray should be in the same plane as the incident ray and the normal we should be able to express it as:

$$\hat{r}_r = \alpha \hat{n} + \beta \hat{r}_i$$

α and β can be obtained from the noting that \hat{r}_i is a unit vector and Snell's law:

$$\begin{aligned}\hat{r}_i \cdot \hat{r}_i &= 1 \\ (\alpha \hat{n} + \beta \hat{r}_i) \cdot (\alpha \hat{n} + \beta \hat{r}_i) &= 1 \\ \alpha^2 + \beta^2 + 2\alpha\beta(\hat{n} \cdot \hat{r}_i) &= 1\end{aligned}$$

and from Snell

$$n_i \sin \theta_i = n_r \sin \theta_r \Rightarrow \theta_r = \arcsin \left(\frac{n_i}{n_r} \sin \theta_i \right)$$

where

$$\begin{aligned}\cos \theta_i &= -\hat{n} \cdot \hat{r}_i \\ \theta_i &= \arccos(-\hat{n} \cdot \hat{r}_i)\end{aligned}$$

And the expression for the angle on the refracted side:

$$\begin{aligned}-\hat{n} \cdot \hat{r}_r &= \cos \theta_r \\ (-\hat{n}) \cdot (\alpha \hat{n} + \beta \hat{r}_i) &= \cos \theta_r \\ -\alpha - \beta(\hat{n} \cdot \hat{r}_i) &= \cos \theta_r \\ \alpha &= -\cos \theta_r - \beta(\hat{n} \cdot \hat{r}_i)\end{aligned}$$

replacing this in the first equation

$$(-\cos \theta_r - \beta(\hat{n} \cdot \hat{r}_i))^2 + \beta^2 + 2(-\cos \theta_r - \beta(\hat{n} \cdot \hat{r}_i))\beta(\hat{n} \cdot \hat{r}_i) = 1$$

$$(1 - (\hat{n} \cdot \hat{r}_i)^2)\beta^2 = 1 - \cos^2 \theta_r$$

$$\beta = +\sqrt{\frac{1 - \cos^2 \theta_r}{1 - (\hat{n} \cdot \hat{r}_i)^2}} = \frac{\sin \theta_r}{\sin \theta_i}$$

So that \hat{r}_r can be expressed as:

$$\hat{r}_r = \left(-\cos \theta_r + \left(\frac{\sin \theta_r}{\sin \theta_i} \right) \cos \theta_i \right) \hat{n} + \frac{\sin \theta_r}{\sin \theta_i} \hat{r}_i$$