

Since the refracted ray should be in the same plane as the incident ray and the normal we should be able to express it as:

$$r_r = \alpha n + \beta r_i$$

α and β can be obtained from the noting that r is a unit vector and Snell's law:

$$\begin{aligned} r_i \cdot r_i &= 1 \\ (\alpha n + \beta r_i) \cdot (\alpha n + \beta r_i) &= 1 \\ \alpha^2 + \beta^2 + 2\alpha\beta(n \cdot r_i) &= 1 \end{aligned}$$

and from Snell

$$n_i \sin \theta_i = n_r \sin \theta_r \Rightarrow \theta_r = \arcsin \left(\frac{n_i}{n_r} \sin \theta_i \right)$$

where

$$\theta_i = \arccos(-n \cdot r_i)$$

And the expression of the angle on the refracted side:

$$\begin{aligned} -n \cdot r_r &= \cos \theta_r \\ (-n) \cdot (\alpha n + \beta r_i) &= \cos \theta_r \\ -\alpha - \beta(n \cdot r_i) &= \cos \theta_r \\ \alpha &= -\cos \theta_r - \beta(n \cdot r_i) \end{aligned}$$

replacing this in the first equation

$$(-\cos \theta_r - \beta(n \cdot r_i))^2 + \beta^2 + 2(-\cos \theta_r - \beta(n \cdot r_i))\beta(n \cdot r_i) = 1$$

$$(1 - (n \cdot r_i)^2)\beta^2 = 1 - \cos^2 \theta_r$$

$$\beta = +\sqrt{\frac{1 - \cos^2 \theta_r}{1 - (n \cdot r_i)^2}}$$