Problem Set 3: Reinforcement Learning and Constraint Satisfaction

Q1 Reinforcement Learning [Total: 40 points]

Q1a. [15 points] *Written RL problem*

For a simple cliff-walker Q-value problem, compute the Q-values at each state. The goal is the cell marked in green (with a reward of 0), and stepping on the red cells results in immediate failure with reward -100. All other states get a reward of -1.

The Q-value equation is given by:

$$Q(s, a) = r + \gamma \max_{a'} Q(s', a')$$

Assume a discount factor of 1.0 (i.e. $\gamma = 1$). As an example, Q-values for one cell have been computed for you.

(R = -1) U: D: L: R:	(R = -1) U: D: L: R:	(R = -1) U: D: L: R:	(R = -1) U: D: L: R:
(R = -1) U: D: L: R:	(R = -1) U: D: L: R:	(R = -1) U: D: L: R:	(R = -1) U: -2.0 D: 0.0 L: -2.0 R: -1.0
(R = -1) U: D: L: R:	Cliff (R = -100)		Goal (R = 0)

Q1b. [25 points] *Coding RL problem*

Let's start by reading about the [Cliff Walking Problem (https://medium.com/@lgvaz/understanding-q-learning-the-cliff-walking-problem-80198921abbc)](https://medium.com/@lgvaz/understanding-q-learning-the-cliff-walking-problem-80198921abbc)

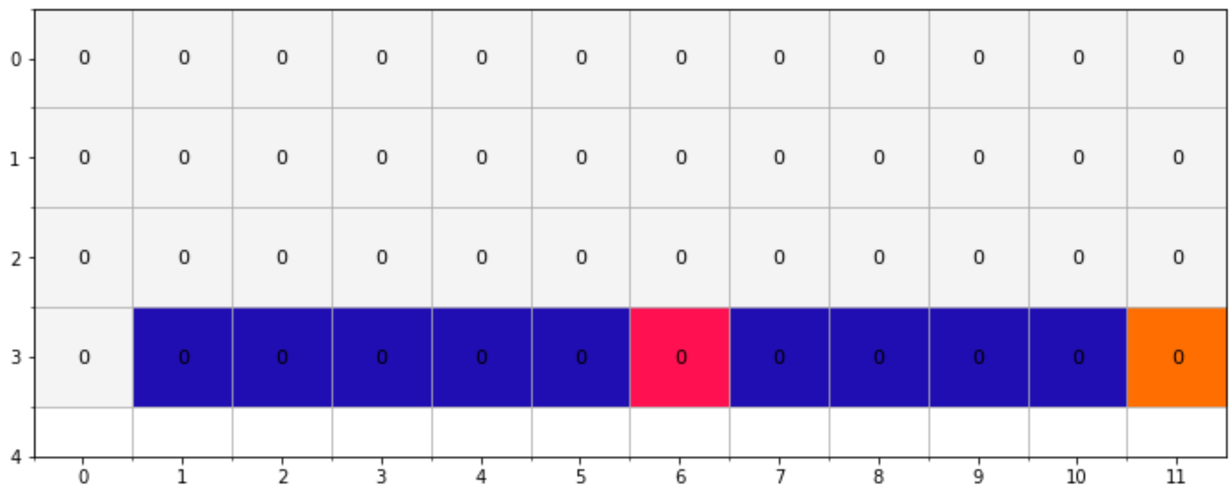
In [3]:
```python
import numpy as np
import matplotlib.pyplot as plt
from CliffWalker import GridWorld
```

We create a 4×12 grid, similar to the written problem in 1a. above on which you will implement a Q-learning algorithm.

In [4]:
```python
env = GridWorld()

# The number of states in simply the number of "squares" in our grid w
num_states = 4 * 12
# We have 4 possible actions, up, down, right and left
num_actions = 4

a = []

env.reset()
#print(a.append(env.reset()))
print(a.append(env.step(0)))
print(env.step(0))
print(env.step(0))
print(a.append(env.step(2)))
print(env.step(0))
print(env.step(2))
print(env.step(1))
print(env.step(2))
print(env.step(2))
print(env.step(1))
print(a.append(env.step(2)))
print(env.step(1))
print(env.step(2))
print(env.step(-1))
env.render()

print(max(a))

# STEPS / ACTION
# 0 = up
# 1 = down
# 2 = right
# 3 = left
```

None

```
(12, -1, False)
(0, -1, False)
None
(1, -1, False)
(2, -1, False)
(14, -1, False)
(15, -1, False)
(16, -1, False)
(28, -1, False)
None
(41, -100, True)
(42, -100, True)
(42, -100, True)
```



```
(29, -1, False)
```

Tasks

We ask you to implement two functions:

- an ϵ-greedy action picker
- a basic Q-learning algorithm

ϵ-greedy choices make the greedy choice most of the time but choose a random action ϵ fraction of the time. For example, for $\epsilon = 0.1$, if a random number is ≤ 0.1, then a random action is taken.

In [5]:
```
def egreedy_policy(q_values, state, epsilon=0.1):
    if np.random.random() < epsilon:
        return np.random.choice(4)
    else:
        return np.argmax(q_values[state])

# def egreedy_policy(q_values, state, epsilon=0.1):
#     '''
#     Choose an action based on a epsilon greedy policy.
#     A random action is selected with epsilon probability, else selec
#     '''
#     action = 0
#     n = np.random.random()
#     if n < epsilon:
#         action = np.random.choice(4)
#     else:
#         steps = [[0, 0, False]] * 4
#         current_state, reward, done = state.step(-1)
#         future_state = current_state
#         best_action = -1
#         for a in range(4):
#             next_state, reward, done = state.step(a)
#             steps[i] = [next_state, reward, done]
#         for b in range(len(steps)):
#             if steps[b][2] != True:
#                 if steps[b][0] > future_state:
#                     future_state = steps[b][0]
#                     best_action = steps[b][1]
#         action = best_action
#     return action

# print(str(np.random.choice(4)))

# b = [[0, 0, 0, 0]] * 4
# print(b)
```

Now, you can implement a basic Q-learning algorithm. For your reference, use the following:

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        Choose A from S using policy derived from Q (e.g., ϵ-greedy)
        Take action A, observe R, S'
        $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$
        $S \leftarrow S'$
    until S is terminal

We provide a skeleton code, leaving the Q-value update for you to implement.

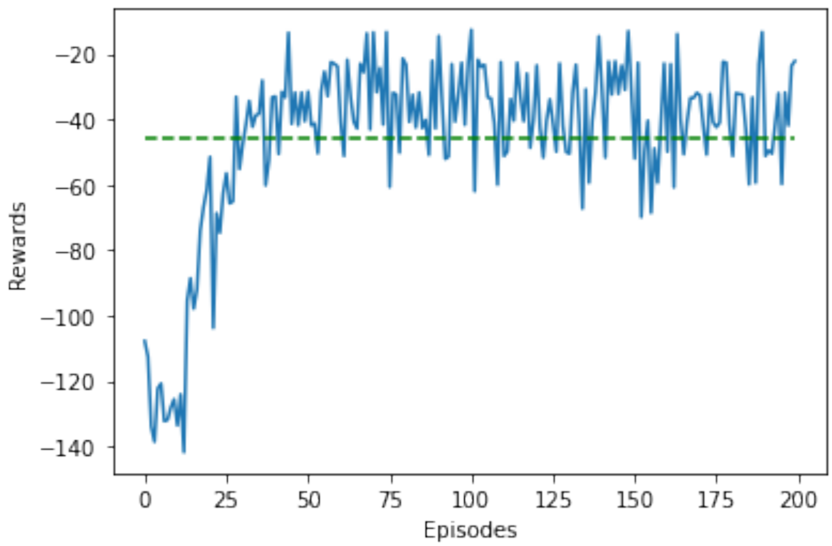
Note: learning rate α, exploration rate ϵ, and discount factor γ are provided as inputs to the function

In [6]:
```
def q_learning(env, num_episodes=200, render=True, epsilon=0.1,
               learning_rate=0.5, gamma=0.9):
    q_values = np.zeros((num_states, num_actions))
    ep_rewards = []

    for _ in range(num_episodes):
        state = env.reset()
        done = False
        reward_sum = 0

        while not done:
            # Choose action
            action = egreedy_policy(q_values, state, epsilon)
            # Do the action
            next_state, reward, done = env.step(action)
            reward_sum += reward

#             print("before:\n")
#             print(q_values)

            # Update Q-values
            # === STUDENT CODE GOES HERE ===
            td_target = reward + gamma * np.max(q_values[next_state])
            td_error = td_target - q_values[state][action]
            q_values[state][action] += learning_rate * td_error
            # ==============================

#             print("after:\n")
#             print(q_values)
            # Update state
            state = next_state

        ep_rewards.append(reward_sum)

    return ep_rewards, q_values

# print(42//12)
# print(42%12)
```

Now, let's the run Q-learning

In [7]:
```python
q_learning_rewards, q_values = q_learning(env, gamma=0.9, learning_rat

q_learning_rewards, _ = zip(*[q_learning(env, render=False, epsilon=0.
avg_rewards = np.mean(q_learning_rewards, axis=0)
mean_reward = [np.mean(avg_rewards)] * len(avg_rewards)

fig, ax = plt.subplots()
ax.set_xlabel('Episodes')
ax.set_ylabel('Rewards')
ax.plot(avg_rewards)
ax.plot(mean_reward, 'g--')

print('Mean Reward: {}'.format(mean_reward[0]))
```

```
Mean Reward: -45.6385
```



Visualization

Finally, let's look at the policy learned

In [8]:
```python
def play(q_values):
    env = GridWorld()
    state = env.reset()
    done = False

    while not done:
        # Select action
        action = egreedy_policy(q_values, state, 0.0)
        # Do the action
        next_state, reward, done = env.step(action)

        # Update state and action
        state = next_state

        env.render(q_values=q_values, action=action, colorize_q=True)
```

In [9]:
```python
%matplotlib
play(q_values)
```

```
Using matplotlib backend: MacOSX
```

Q2 Constraint Satisfaction [Total: 10 points]

Written CS problem

Suppose we want to schedule some final exams for CS courses with the following course numbers: 1007, 3137, 3157, 3203, 3261, 4115, 4118, 4156

Suppose also that there are no students in common taking the following pairs of courses:

1007-3137

1007-3157, 3137-3157

1007-3203

1007-3261, 3137-3261, 3203-3261

1007-4115, 3137-4115, 3203-4115, 3261-4115

1007-4118, 3137-4118

1007-4156, 3137-4156, 3157-4156

How many exam slots are necessary to schedule exams?