From Constraint Programming to Heterogeneous Parallelism

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Abstract

The scaling limitations of multi-core processor development have led to a diversification of the processor cores used within individual computers. Heterogeneous computing has become widespread, involving the cooperation of several structurally different processor cores. Central processor (CPU) cores are most frequently complemented with graphics processors (GPUs), which despite their name are suitable for many highly parallel computations besides computer graphics. Furthermore, deep learning accelerators are rapidly gaining relevance.

Many applications could profit from heterogeneous computing but are held back by the surrounding software ecosystems. Heterogeneous systems are a challenge for compilers in particular, which usually target only the increasingly marginalised homogeneous CPU cores. Therefore, heterogeneous acceleration is primarily accessible via libraries and domain-specific languages (DSLs), requiring application rewrites and resulting in vendor lock-in.

This thesis presents a compiler method for automatically targeting heterogeneous hardware from existing sequential C/C++ source code. A new constraint programming method enables the declarative specification and automatic detection of *computational idioms* within compiler intermediate representation code. Examples of computational idioms are stencils, reductions, and linear algebra. Computational idioms denote algorithmic structures that commonly occur in performance-critical loops. Consequently, well-designed accelerator DSLs and libraries support computational idioms with their programming models and function interfaces. The detection of computational idioms in their middle end enables compilers to incorporate DSL and library backends for code generation. These backends leverage domain knowledge for the efficient utilisation of heterogeneous hardware.

The constraint programming methodology is first derived on an abstract model and then implemented as an extension to LLVM. Two constraint programming languages are designed to target this implementation: the Compiler Analysis Description Language (CAnDL), and the extended Idiom Detection Language (IDL). These languages are evaluated on a range of different compiler problems, culminating in a complete heterogeneous acceleration pipeline integrated with the Clang C/C++ compiler. This pipeline was evaluated on the established benchmark collections NPB and Parboil. The approach was applicable to 10 of the benchmark programs, resulting in significant speedups from 1.26× on “histo” to 275× on “sgemm” when starting from sequential baseline versions.

In summary, this thesis shows that the automatic recognition of computational idioms during compilation enables the heterogeneous acceleration of sequential C/C++ programs. Moreover, the declarative specification of computational idioms is derived in novel declarative programming languages, and it is demonstrated that constraint programming on Single Static Assignment intermediate code is a suitable method for their automatic detection.

Lay Summary

New computer processors used to improve over the speed of previous versions mostly because the transistors got smaller and more efficient. In recent years, the engineers have been unable to continue this process. Because the speed of processors is stagnating, companies try to improve other metrics. Specialised processors are now widespread, which cooperate with the central processor and complement its abilities. However, many software programs were not designed for this and ignore the specific features.

This thesis presents an approach to make existing programs use such specialised hardware efficiently. This approach was implemented in a software prototype. The prototype detects particular mathematical methods in other programs. Other researchers have already found the best ways to compute these methods on new hardware, and stored them in "libraries". By detecting that programs rely on the methods, the prototype can make the program use these efficient "libraries" instead. However, recognising the methods is hard. This thesis introduces new specification languages to express them. With the resulting precise formulations, it is possible to identify the mathematical methods automatically with algorithms.

The prototype software was evaluated by using it on standard test programs. Some of these programs are from NASA and imitate the calculations on their supercomputers. The prototype understood the essential parts of many programs. This allowed executing them on specialised processors and made five of them more than ten times faster.

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Finally, I am grateful to my parents, who have always supported and encouraged me.

Declaration

I declare that this thesis was composed by myself, that the work contained herein is my own except where explicitly stated otherwise in the text, and that this work has not been submitted for any other degree or professional qualification except as specified. Some of the material used in this thesis has been published in the following papers:

[1] Philip Ginsbach, Lewis Crawford and Michael F. P. O'Boyle.
CAnDL: A Domain Specific Language for Compiler Analysis.
Proceedings of the 27th International Conference on Compiler Construction (CC), 2018

[2] Philip Ginsbach and Michael F. P. O'Boyle.
Discovery and Exploitation of General Reductions: A Constraint Based Approach.
Proceedings of the 15th Annual International Symposium on Code Generation and Optimization (CGO), 2017

[3] Philip Ginsbach, Toomas Remmelg, Michel Steuwer, Bruno Bodin, Christophe Dubach and Michael F. P. O'Boyle.
Automatic Matching of Legacy Code to Heterogeneous APIs: An Idiomatic Approach.
Proceedings of the 23rd International Conference on Architectural Support for Programming Languages and Operating Systems (ASPLOS), 2018

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Table of Contents

List of Symbols and Notation

$\mathbb{N}$	The set of all positive integers, $\mathbb{N} = \{1, 2, 3, \dots\}$
$\mathbb{R}$	The set of all real numbers
$\emptyset$	The empty set, containing no elements
$s \in S$	Element-of relationship, s is in the set S
$A \subset B$	Subset-relationship, every element of A is also in B
$\lvert S \rvert$	Cardinality of S, the number of elements in the finite set S
$\{x \mid P(x)\}$	Set-builder notation, the set of all x that satisfy $P(x)$
$A \times B$	Cartesian product of sets, $A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$, corresponds to `pair<A,B>` in C++
A^n	Set of n-tuples in A, short for $A^n = A \times \cdots \times A$, corresponds to `array<A,n>` in C++
A^B	Generalisation of A^n, $x \in A^B$ has a value $x_b \in A$ for each $b \in B$, corresponds to `map<B,A>` in C++
$\mathscr{P}(S)$	Power set of S, the set of all subsets of S
$f : A \to B$	Function f that maps elements of A onto elements of B
$f : A \hookrightarrow B$	Injective function $x \neq y \implies f(x) \neq f(y)$, generalises $A \subset B$
$x \mapsto y$	The value x gets mapped onto the value y
$P \implies Q$	Logical consequence, "P implies Q"
$P \iff Q$	Logical equality, "P if and only if Q"
$P \wedge Q$	Logical conjunction, "P and Q"
$P \vee Q$	Logical disjunction, "P or Q"
$\neg P$	Logical negation, "not P"

Chapter 1

Introduction

1.1 The Emergence of Heterogeneous Computing

For several decades, from the 1970s until the early 2000s, advances in processor development followed Moore's Law [4] and Dennard Scaling [5]. The density of integrated circuits doubled every two years, and this allowed increased clock frequencies while maintaining steady power consumption. These conditions enabled continual processor improvements primarily via rising clock frequencies and microarchitectural refinements, enabling ever faster computations.

The physics-driven nature [6] of these advances in hardware has had important implications for software development. Not only did the performance of computers improve exponentially, but these performance gains were in the form of direct speedups, available to all the already existing programs. The primary interfaces between software and hardware - the instruction set architectures of processors - evolved gradually and with few paradigmatic changes [7]. This is best exemplified by the pervasive x86 instruction set architecture, which still retains backward compatibility with its initial version from 1978, and dominates desktop processors to this day. Therefore, software developers and users could rely on ever-increasing performance from hardware progress alone, without any intervention.

1.1.1 Via Multi-Processing to Heterogeneity

This continual progress started breaking down around 2005 with the apparent end of Dennard Scaling [8]. While the shrinking of transistors continued, this no longer enabled proportionally increased clock frequencies with the same power budget. Instead, processor designers started using the increasing transistor budget for additional processor cores. This shift toward multi-processing has left a deep mark on software development. New programming paradigms and languages, annotation systems, programming interfaces, libraries and compiler techniques are still being developed to address the challenges of parallel computing.

In recent years, transistor scaling has slowed significantly. In response to the breakdown of both Dennard Scaling and Moore's Law, the hardware industry has turned toward architectural innovation [9]. In particular, there is a trend toward specialised processors that work in tandem as heterogeneous systems. Specialised processor cores outperform general-purpose cores on specific tasks. Furthermore, as heat dissipation has become a major challenge, simultaneously powering all transistors is often unviable. This means that different processor cores have to be masked out dynamically at runtime ("dark silicon") [8]. This situation favours specialised cores, which can improve the overall system performance even if they are disabled most of the time and only used for the specific tasks at which they excel. By contrast, disabling a subset of a homogeneous multi-core system renders some of its cores redundant.

1.2 The Diminished Role of Traditional Compilers

Heterogeneous computing can help overcome the scaling limitations of homogeneous, general-purpose processors. However, it also poses a challenge to the associated software ecosystems. Existing software does not automatically benefit from entirely new accelerator designs in the way that it profited from the continuous improvements of established architectures. Where programs previously performed better on each succeeding hardware generation, heterogeneous accelerators arrive with novel and incompatible interfaces. This puts into question many of the achievements in portability and longevity of programs that are taken for granted in modern computing [10].

In particular, this new hardware landscape greatly diminishes the scope of responsibilities and impact that traditional compilers for languages such as C, C++ and Fortran can have. Such compilers used to be responsible for orchestrating program execution on the entirety of available computing resources. They are now generally limited to only targeting the relatively small homogeneous fraction of processor cores directly.

1.2.1 Libraries and Domain-Specific Languages

In order to reach peak performance on rapidly evolving and highly parallel hardware, new programming paradigms built around libraries and domain-specific languages have emerged. They succeed in utilising heterogeneous hardware in situations where traditional compilers fail.

Two examples show the diversity of these approaches. Firstly, Ragan-Kelley et al. [11] developed the domain-specific language Halide for image processing. The Halide toolchain is able to generate fast code for heterogeneous platforms by focusing on a well-understood class of computations, and by using a restrictive program representation. After tuning programs for particular platforms, it outperforms hand-optimised code, demonstrating the advantage of domain-specific compiler optimisation under circumstances of constrained semantics.

Secondly, Basic Linear Algebra Subprograms (BLAS) [12], a specification of function interfaces going back to the 1970s [13], remains the standard encapsulation for linear algebra and has been implemented for most accelerators. These implementations are widely used and offer unrivalled performance. Some versions are provided directly by hardware vendors to support accelerators [14, 15, 16, 17, 18], while others originated as academic projects [19]. These competing implementations use a plethora of approaches to achieve as close to peak performance as possible. These methods include manually written assembly code but also highly advanced code generation techniques, custom program representations, and many more.

1.2.2 The Consequences of the Decline of Compilers

Domain-specific languages and library interfaces make the full performance of heterogeneous systems accessible to programs. However, these success stories also leave significant problems unaddressed. The adoption costs are high, requiring application rewrites for accelerators. This coincides with often uncertain long-term prospects and minimal cross-platform portability. Even in the case of the agreed-upon BLAS standard – arguably the best case scenario – adoption of novel implementations is non-trivial in practice, due to the frequently encountered interface extensions for managing device handlers and memory synchronisation. For academic-backed domain-specific languages like Halide, on the other hand, complete rewrites are required in entirely novel software ecosystems, with an unclear future of support.

On homogeneous systems, programmers could rely on compilers for existing programming languages to evolve in lockstep with processor development. For example, even decades-old C++ source code compiles into efficient programs for the newest generation of x86 processors. Libraries and domain-specific languages are useful on homogeneous systems for achieving absolute maximum performance, but only in the context of heterogeneous computing do they become essential. Therefore, the pervasive requirement for domain-specific languages and libraries only arises because compilers are unable to map programs onto specialised cores. Instead, compilers are increasingly downgraded to merely coordinating the execution of core workloads as separate and opaque programs.

It may appear apparent that, for example, a C++ or Java compiler cannot be provided for a typical graphics processor. After all, most graphics processors have hardware limitations that prevent them from implementing the entire language standard. Indeed, many accelerators are further from Turing complete [20]. Nevertheless, this should not prevent a *partial compiler*. Such a partial compiler would compile fitting program parts for accelerators and utilise a fallback system for the remainder of the code. After all, that is precisely the result achieved with libraries and domain-specific languages, and likely the desirable outcome. Despite the apparent convenience of such an approach, the next section gives reasons why such a scheme has not become widespread so far.

1.3 Host Compilers and Kernel Compilers

While mainstream compilers for languages such as C, C++, Fortran or Java generally fail to exploit the full performance of heterogeneous systems, a specific class of compilers already plays an essential role in targeting heterogeneous hardware. Many of the kernel programs that remain opaque to the application compilers are themselves products of other, specialised compilers. This necessitates a distinction between *host compilers* and *kernel compilers*, which is mirrored by the differences between *host languages* and *kernel languages*. Host compilers translate full applications written in host languages – such as C, C++, Fortran and Java – while kernel compilers handle only succinct computational kernels that are expressed in dedicated domain-specific programming languages. These two classes of compilers have developed differently. Kernel compilers successfully apply many advanced techniques that are severely limited on host compilers [21, 22]. They reason automatically about parallelism [23], are more successful at accurately modelling data dependencies [11], and incorporate autotuning techniques [24].

This is made possible by a combination of factors that uniquely apply to kernel compilers. Smaller programs allow for more expensive compilation techniques; more restrictive languages and intermediate representations allow for stronger reasoning; abstractions in kernel compilers can be customised for the exact hardware architecture of accelerators; and domain knowledge from areas such as image processing can be directly embedded in custom compiler technology, without requiring their validity on generic programs. Moreover, kernel compilers often run in more controlled environments, with less need for predictability, reproducibility, and stability. The range of input programs might even be small enough to ship them in an already compiled form, making the compiler a behind-the-scenes tool in the library implementation process.

As host compilers operate under less forgiving conditions, it is unsurprising that they have lagged behind these developments. Because they cannot rely on such a restricted environment, host compilers are unable to match the optimisation capabilities of kernel compilers, even when a functional translation is possible. Straightforward approaches to targeting heterogeneous accelerators from host compilers, therefore, have intrinsic disadvantages when compared to domain-specific compilers. Host compilers that translate suitable program parts to accelerators without matching domain-specific performance are undesirable.

Hybrid approaches, such as OpenCL, reinforce this hypothesis. OpenCL is domain-specific in its expression of parallelism but otherwise designed to be general-purpose. Compilers are provided for many structurally different computing architectures, but as a consequence of being a relatively unrestricted language, the compilers often underperform, and OpenCL programs are notoriously lacking in performance portability [25]. Host compilers that require the same compromises as the OpenCL toolchain, being unable to leverage the restricted nature of input programs, would surely suffer the same shortcomings when targeting accelerators.

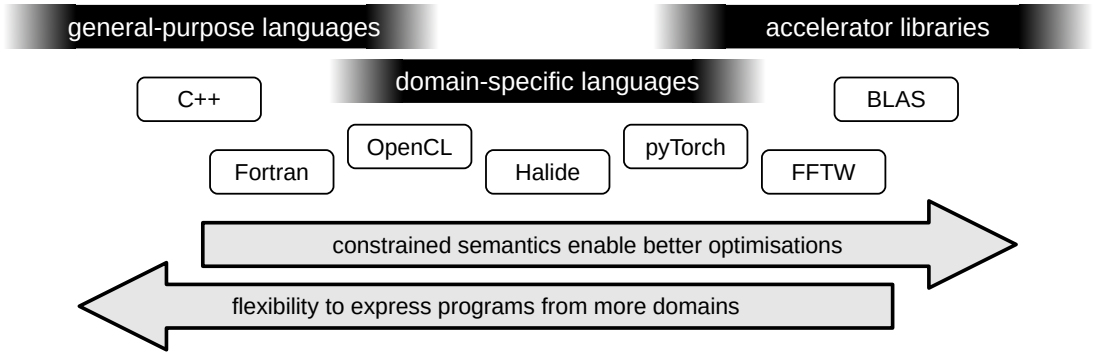


Figure 1.1: Domain-specific languages on a spectrum between general-purpose languages and libraries: Less flexibility allows for stronger reasoning and greater optimisation potential.

1.3.1 The Spectrum of Specialisation

General-purpose programming languages, domain-specific languages, and accelerator libraries exist on a spectrum of specialisation. Figure 1.1 provides some intuition about this spectrum by estimating the positions of popular languages and libraries in it. On the far left are versatile general-purpose languages, such as C++ and Fortran. Moving right, flexibility is gradually lost, with BLAS libraries at the end providing only fixed-function computations. However, the reduced scope allows for increased compiler optimisation capabilities in return.

It is necessary to clarify what is meant by compiler optimisation potential in this context and how it differs from raw performance. At the very left, C++ is renowned as a low-overhead, high-performance language, but it is difficult to apply profound structural changes to C++ programs in compilers. At the other extreme, the semantics of BLAS can be maintained with many structurally different implementations. The routines are so narrowly specified that they allow the library implementer to effectively operate as a compiler with almost unrestricted optimisation abilities. C++ is fast because of its low overhead, but BLAS is fast because its semantics are so constrained that implementations can be tuned to perfection.

More broadly, the superior optimisation potential of libraries and kernel compilers is a consequence of operating under more constrained conditions than host compilers. While expert programmers can often apply domain knowledge themselves when using versatile general-purpose languages, this potential is uncovered automatically and tuned for the target hardware by specialised tools. The restrictions on input programs imply domain knowledge that is leveraged to make stronger assumptions, use more powerful reasoning and generate better code.

Hypothesis: Restrictive program models expose additional optimisation opportunities and lead to increased portability. Compilers use program models that correspond closely to their input programming languages. This correspondence could be decoupled by recognising program parts that adhere to more constrained models, making the powerful domain-specific optimisation techniques of kernel compilers available to host compilers.

1.4 Moving on the Spectrum of Specialisation

Host compilers need methods to recognise restrictive program models within general-purpose code automatically. The term *program model* in this context means an internal representation of the program logic with associated assumptions and ways of reasoning about the behaviour. Reformulating program sections in restrictive models would make the superior domain-specific capabilities of kernel compilers applicable in host compilers. This would result in a compiler that can combine the positive aspects of both sides of the spectrum of specialisation.

Such an approach of detecting restricted models in general-purpose code has previously been used only for specific individual domains and required elaborate manually implemented compiler analysis functionality. For example, the Polly compiler by Grosser et al. [26] takes arbitrary C/C++ code as input and recognises whether parts of this code are expressible in the more restrictive polyhedral model [27, 28]. The compiler reformulates the relevant code in this model, enabling advanced optimisation techniques that use the domain knowledge of the polyhedral model for generating faster code [29, 30], substantiating the hypothesis.

1.4.1 Contributions of this Thesis

This thesis derives a systematic and configurable method for recognising constrained program sections that fit restrictive models during the compilation process. The approach emerges from the analysis of a standard program model for host compilers: Static Single Assignment (SSA) form. Intermediate representations based on SSA are accessible to mathematical reasoning, with their most relevant features built on graph and list structures. With this in mind, structural restrictions on intermediate representation code – corresponding to restricted domain-specific models – can be precisely formulated as constraints on a mathematical characterisation of SSA form programs. Constraint solver techniques then make the automatic recognition of adhering code sections feasible.

Instead of manually implementing sophisticated compiler analysis functionality, a flexible constraint programming language is introduced that can express many different such domain-specific models. An accompanying toolchain is developed that uses these specifications to automatically recognise code sections that satisfy them. This system captures the spectrum from Figure 1.1. On one extreme, sections of code are recognised as implementing specific algorithms, like matrix multiplication, that are parametrised with numeric values. Algorithms that allow parametrisation with an operator are next. Stencil kernels and reduction operations fall into this category. More generic again are programs that merely follow certain structural restrictions, such as code adhering to the polyhedral model. Chapters 4 to 6 cover each of these domains, culminating in a system for the automatic heterogeneous acceleration of sequential C/C++ code.

1.5 Structure of this Thesis

This thesis is divided into seven chapters.

Following the introduction, **Chapter 2** derives the underlying methodology of this thesis in the context of conventional compiler analysis. Based on a mathematical characterisation of programs in Static Single Assignment form, the chapter presents a new constraint programming method for expressing algorithmic program structures as constraint formulas. Later sections of the chapter discuss implementation decisions, the algorithmic complexity of the approach, and differences to established Satisfiability Modulo Theory (SMT) problems.

Chapter 3 gives an overview of the related work. This literature survey covers the four main areas of research relevant to this thesis. Firstly, *constraint programming* underpins the methodology of this thesis. Secondly, previous *compiler analysis and auto-parallelisation* approaches are used used to evaluate the results of this work. Thirdly, *heterogeneous computing* and the corresponding challenges motivate many of the compiler approaches that this thesis implements. Finally, the term "*computational idiom*" is used in this thesis to denote shared computational structures of performance bottlenecks. This term is established by comparison with literature on several overlapping concepts from different research disciplines.

Chapters 4 to 6 are each based on a published research article and elaborate on different applications of constraint programming in compilers.

Chapter 4 develops the Compiler Analysis Description Language (CAnDL) and presents its implementation in the LLVM compiler infrastructure. CAnDL is a specification language that generates compiler analysis passes from declarative descriptions via the CAnDL compiler. The chapter explores several compiler use cases with CAnDL, including the implementation of peephole optimisations and the prototyping of graphics shader optimisations. The chapter is based on published research [1].

Chapter 5 extends CAnDL into the Idiom Detection Langauge (IDL) and develops an auto-parallelising compiler for Complex Reduction and Histogram Computations (CReHCs) built on IDL-powered analysis functionality. CReHCs cover loops with indirect data accesses that are inaccessible to established approaches based on data flow and polyhedral analysis. The flexibility of constraint programming allows it to capture such irregular computations. The chapter is based on published research [2].

Chapter 6 applies IDL to the detection of algorithmic structures that go beyond the scope of traditional compiler analysis, including stencils, histograms, and sparse linear algebra. The recognition in LLVM enables automatic heterogeneous parallelisation of sequential code with domain-specific backends, resulting in significant speedups on benchmark programs from the established NPB and Parboil suites. The chapter is based on published research [3].

Finally, **Chapter 7** concludes this thesis with a summary of the core contributions, a critical analysis of the approaches, and a discussion of future work.

1.6 Summary

The end of Moore's Law and Dennard Scaling has resulted in a shift away from homogeneous multi-processing, toward architectural innovation and specialised processor cores. The rising heterogeneity of computing resources poses a challenge to established compiler toolchains, which often only access the increasingly marginal homogeneous fraction of processing power. Therefore, application programmers depend on domain-specific languages and libraries that leverage knowledge from their restricted operating domains to generate efficient code for the modern hardware landscape.

This domain knowledge can be made available to host compilers with approaches that automatically specialise general-purpose code and reformulate it in more restrictive program models. Previous work has established this on individual models, but this thesis develops a framework to formulate a wide range of restricted program models using a systematic approach based on constraint solving. To this purpose, a custom constraint programming language is developed and successively extended. Starting from a discussion of the underlying assumptions and features of SSA code, the methodology is developed into a complete system, implemented as part of the mature Clang C/C++ compiler.

In the later chapters, this core system is applied in combination with other techniques on a range of different compiler challenges. These include the rapid prototyping of compiler optimisations, the automatic parallelisation of programs with indirect memory accesses, a new formulation of the polyhedral model, and the detection of computational structures that occur widely in important bottlenecks of scientific programs.

Eventually, the techniques are combined into a fully integrated, extensible compiler tool for efficiently mapping sequential programs onto heterogeneous systems. This develops the thesis all the way from theoretical discussions of constraint programming approaches in compilers, to the experimental evaluation of real performance improvements on established benchmark suites and scientific applications.

Chapter 2

Constraint Programming on Static Single Assignment Code

The research contributions of this thesis are built on a new method for constraint programming on Static Single Assignment (SSA) compiler intermediate representation. This chapter derives and motivates the underlying methodology, which is then used as the basis for Chapters 4 to 6. The constraint programming approach is developed in three steps.

First, an overview of program representations during different compilation stages is given, and the particular features of SSA representations are highlighted. These features are used to derive a mathematical characterisation of the static structure of SSA programs, the *SSA model*. This is first derived generically and then demonstrated in more detail on LLVM IR, a specific SSA intermediate representation. Some mathematical notation is used in this section to build a reliable foundation for later parts of this thesis. The List of Symbols and Notation contains some of the notational conventions.

After the derivation of the SSA model, the concept of *SSA constraint problems* is defined. These are formulas that impose restrictions on parts of SSA code, formulated as constraints on the SSA model. SSA constraint problems can define code structures such as loops. Detecting adhering code sections in SSA code is then equivalent to finding solutions to the constraints. The structure of several classes of constraint formulas is discussed, reflecting conventional compiler analysis methods like data flow, dominance relationships, and data type restrictions.

Finally, efficient algorithms for solving SSA constraint problems are derived. Backtracking is used to find solutions quickly by incrementally extending partial solutions. The structure of specific classes of SSA constraint problems is analysed, and efficient backtracking solutions are derived individually, yielding composable building blocks for the solver. After deriving the algorithms, implementation considerations are discussed. Suitable data structures are selected, and the runtime complexity is analysed. To conclude the chapter, constraint programming on SSA intermediate representation is compared with Satisfiability Modulo Theory (SMT).

2.1 Background

Modern compilers for procedural languages such as C/C++, Fortran or JavaScript typically use a succession of different representations for the program during compilation. They reflect the requirements of the compilation stages in which they are used.

Front end representations are close to the source program and strongly influenced by the specific grammar of the programming language. Typically, they are built around an abstract syntax tree with additional annotations, such as type information. These representations are rich in information about syntactic and stylistic choices of the programmer, and they scale in complexity with the source language. Some advanced language features, such as overloaded operators, are not resolved in this class of representations. Therefore, the semantics of program parts is highly dependent on context.

Back end representations are based on a model of the target hardware. They typically approach an assembly-style format and expose the instruction set architecture of the hardware, making them platform-specific. Back end representations also encode decisions for problems that are removed from the algorithmic content of the user program, such as instruction selection and register allocation.

Middle end representations are designed to enable the analysis and transformation of code in order to apply optimisations. **Static Single Assignment** (SSA) representations have emerged as a common choice for the middle end in leading compilers. SSA abstracts away the complexities of the source language and the target architecture, focusing on a relatively simple description of the user program semantics. This enables reliable analysis and platform-independent reasoning.

Static Single Assignment was first proposed by Rosen et al. [31], and there are now many different compiler intermediate representations that implement the concept. Instruction sets, type systems and syntax (if a textual representation is even specified) of these representations vary considerably, depending on the requirements of the source languages (static or dynamic) and the operating constraints (just-in-time or ahead-of-time). Some prominent examples of compilers using SSA are **Clang** (LLVM IR), **GCC** (GIMPLE), **v8 Crankshaft** (Hydrogen), and **SpiderMonkey** (IonMonkey/MIR). Despite many differences, they share the same basic structure that is discussed in this chapter.

SSA form was developed as an improvement over previously used compiler intermediate representations. Specifically, the eponymous Static Single Assignment property applies to the more general Linear Intermediate Representations as an additional restriction, as described in Torczon and Cooper [32]. The following Section 2.1.1 gives an overview of the characteristic shared features of SSA representations that are relevant to this work.

2.1.1 Static Single Assignment Form

SSA representations are Linear Intermediate Representations, meaning that they represent each function as a single linear sequence of instructions. These instructions operate on an unlimited number of registers, using a well defined – but representation specific – instruction set. Branch instructions are used to redirect the execution conditionally or unconditionally. Consecutive instructions with no branching between them are grouped into basic blocks. Within these, instructions are executed in order of appearance. Basic blocks may have labels or be identified simply by enumerating them. Instruction arguments can be registers, constants, globals or function parameters. Additionally, branch instructions take basic block arguments as branch targets. Instructions may write their result into a single output register.

The SSA property stipulates that within a function, no register can be written at more than one static location. This means that registers can be identified directly with the instructions that define them. The registers can, therefore, be made implicit, with only the data flow between instructions required to recover them. In the presence of dynamic control flow, Φ-instructions are required to uphold the SSA property. These Φ-instructions are placed at the beginning of a basic block, and select one of several values dynamically, depending on the origin of incoming branches, i.e. depending on the previously executed basic block.

Figure 2.1 shows how SSA programs can be represented as a hierarchy of lists. Programs are lists of functions, which are lists of basic blocks, which are lists of instructions, which take lists of arguments. The ability to enumerate all these entities in linear sequences enables the identification of SSA values with integer indices in the later sections of this chapter.

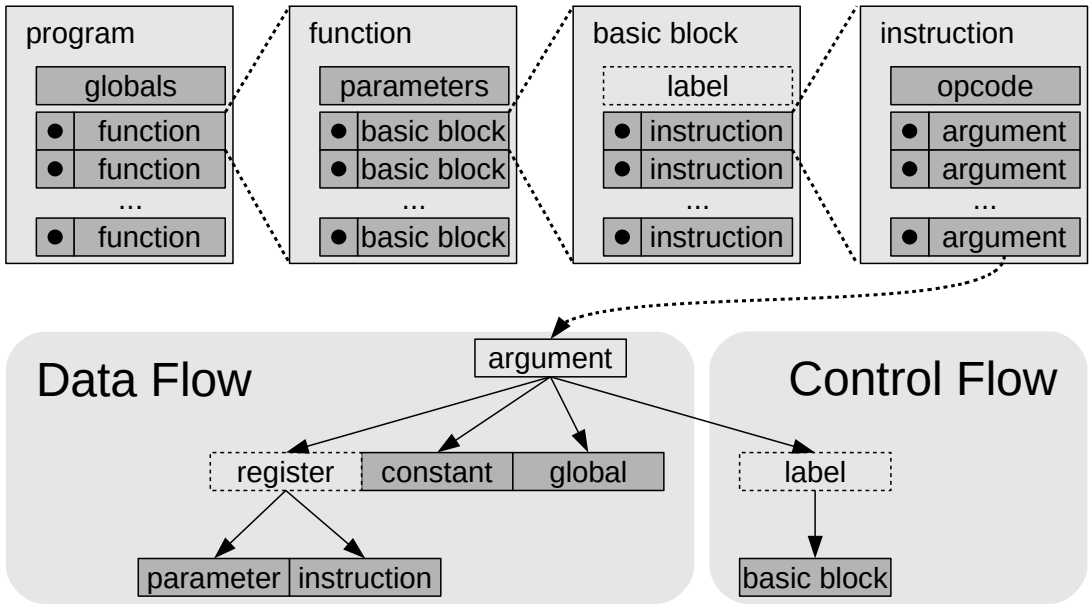


Figure 2.1: Structural overview of SSA: Programs are represented as hierarchies of lists. The SSA property makes registers implicit; values can be statically matched to defining instructions.

2.1.2 SSA Emerges During Compilation

Figure 2.2 shows how critical features of SSA form emerge from simplification steps that are applied to source code. This is demonstrated on the "`sqrt`" example function that approximates the square root of a double-precision floating-point value in C using the Babylonian method in Equation (2.1). Beginning with an initial guess x_0, the approximation is improved iteratively.

$$x_0 \approx \sqrt{S}, \qquad x_{n+1} = \frac{x_n + \frac{S}{x_n}}{2} \qquad \Longrightarrow \qquad \lim_{n\to\infty} x_n = \sqrt{S} \tag{2.1}$$

Starting from the source code **(a)** at the top left of the figure, the function is first modified by breaking down complex expressions. The expressions are turned into sequences of basic operations, shown at the top right **(b)**. Explicit variables appear for the previously implicit temporary values. This simplifies the program, with many of the operations directly mapping to individual processor instructions. Moreover, different input programs get mapped to the same, predictable output by this transformation, making it a normalisation.

In a second step, the structured control flow of the program is replaced with goto statements that coordinate the control flow between basic blocks. This is shown at the bottom right **(c)**. Although this does not ease the intuitive understanding of the program, it unifies several distinct control flow structures provided in the source language into a single mechanism. This simplifies program analysis. Importantly, no relevant information is lost by discarding the control flow structures. They can be reconstructed algorithmically.

Finally, the Static Single Assignment property is introduced at the bottom left **(d)**. Each variable that is assigned at more than one static location in the program is instead duplicated into multiple variables. Where necessary, these now distinct variables are bound together with Φ-instructions. This cannot be expressed in the C programming language syntax. Instead, the behaviour is documented by comments at lines 8–13.

The impact of the SSA property seems minor at first, but convenient implications can already be identified within the C language. As all local variables are written at exactly one static location, each variable can be declared and defined in the same place. This means that it is always known statically, which expression yielded the value of each variable. This immediately guarantees that the variable "`i`" in the example always has the value "`0`", and that the variable "`x`" always has the value "`1.0`".

In summary, this section developed an understanding of how SSA originated historically and how it emerges during the compilation process. The following section uses the observations from Figure 2.1 – that most parts of SSA code can be enumerated and represented as elements in lists – to derive a mathematical characterisation of the static structure of SSA programs. This characterisation then serves as the foundation of later sections, which define compiler analysis problems on it via constraints.

(a) C source function:

```
1  double sqrt(double S) {
2    double x = 1.0;
3    for(int i=0; i<N; i+=1)
4      x = 0.5 * (x + S / x);
5    return x;
6  }
```

(b) Complex expressions are broken down:

```
double sqrt(double S) {                1
  double x = 1.0;                      2
  for(int i=0; i<N; i+=1)              3
  {                                    4
    double t1 = S / x;                 5
    double t2 = x + t1;                6
    x = 0.5 * t2;                      7
  }                                    8
  return x;                            9
}                                      10
```

(c) Structured control flow is expanded:

```
double sqrt(double S) {                1
entry:                                 2
  double x = 1.0;                      3
  int i = 0;                           4
  goto header;                         5
                                       6
header:                                7
  bool test = i < N;                   8
  if(test) goto loop;                  9
      else goto exit;                  10
                                       11
loop:                                  12
  double t1 = S / x;                   13
  double t2 = x + t1;                  14
  x = 0.5 * t2;                        15
  i = i+1;                             16
  goto header;                         17
                                       18
exit:                                  19
    return x;                          20
}                                      21
```

(d) The SSA property is introduced:

```
1  double sqrt(double S) {
2  entry:
3    double x = 1.0;
4    int i = 0;
5    goto header;
6
7  header:
8    int i2 = /* if reached
9      from line  5: i,
10     from line 23: i3 */
11   int x2 = /* if reached
12     from line  5: x,
13     from line 23: x3 */
14   bool test = i2 < N;
15   if(test) goto loop;
16       else goto exit;
17
18 loop:
19   double t1 = S / x2;
20   double t2 = x2 + t1;
21   double x3 = 0.5 * t2;
22   int i3 = i2+1;
23   goto header;
24
25 exit:
26     return x2;
27 }
```

Figure 2.2: Static Single Assignment emerges from successive simplification and normalisation of source language features: Demonstration on an example C function that approximates the square root of an input value with the Babylonian method. The transformation results **(b-d)** are rendered in C. Real compilers typically operate on dedicated internal representations instead.

2.2 Deriving the SSA Model

This section develops a mathematical characterisation of SSA programs, denoted *SSA model*. The focus is on precise notation to express the static structure of existing SSA intermediate representations. The SSA model is unrelated to the operational semantics of programs and not a method for studying their behaviour at runtime. However, it contains all the information required to reconstruct a program that is semantically equivalent to the one from which it was extracted. The remainder of this section adheres to the naming conventions in Definition 2.1.

This notation already implies several decisions about the SSA model that is derived in this section. Firstly, the model captures only a single function at once. Secondly, basic blocks are not explicitly encoded. Instead, all instructions of the function are enumerated sequentially in depth-first order, starting from the function entry. Despite not being explicitly encoded, basic blocks can be reconstructed from the control flow, as discussed in Section 2.2.2. Thirdly, the argument structure of the instructions is modelled separately.

2.2.1 Data Flow and Control Flow

The data flow between instructions, as well as the control flow, is captured in graph structures. The Static Single Assignment property makes registers implicit, and the direct interaction between instructions becomes the natural model for data flow. Instruction arguments fall into four categories: function parameters ("*par*"), other instructions ("*ins*"), globals ("*glb*"), and constants ("*cst*"). Branching instructions also take basic block labels as branch targets, but those are treated separately in the control flow graph. All instruction arguments are therefore taken from the named lists introduced in Definition 2.1.

For each function, the sequences "*par*", "*ins*", "*glb*", "*cst*" can be statically determined. Individual instruction arguments can, therefore, be encoded by one integer each, indexing the *list of used values* as in Definition 2.2. The entire argument structure of the instructions in an SSA function can then be turned into a labelled multigraph, with edge labels accounting for the positional order of the arguments. This is the *data flow graph* in Definition 2.3. The source of a data flow edge can be any value, but the target is always an instruction ($L_{ins} \leq b \leq R_{ins}$).

Complementing the data flow graph is the *control flow graph* of the function, as introduced in Definition 2.4. The control flow graph is often defined with edges between basic blocks. By contrast, in this definition, the edges are directly between instructions. This is convenient later, in Sections 2.3 and 2.4, where both graphs can be treated identically.

The defining equation can be separated into several parts. The first line expresses that edges in the control flow graph are always between two instructions. The remainder of the equation gives two options, corresponding to different types of edges in the control flow graph. Firstly, there are trivial edges within basic blocks. Secondly, there are edges between basic blocks.

Definition 2.1: Features of Static Single Assignment Functions

For the remainder of this section, some function $\mathscr{F}$ in SSA form is assumed fixed. The following identifiers are then used to describe the features of this function:

- $|par|$ is the number of function parameters $par_1, \dots par_{|par|}$.
- $|ins|$ is the number of instructions $ins_1, \dots ins_{|ins|}$ that make up the function, in depth-first order, starting from the execution entry.
- $|glb|$ is the number of unique globals $glb_1, \dots glb_{|glb|}$ that are used as operands of any of the instructions.
- $|cst|$ is the number of unique constants $cst_1, \dots cst_{|cst|}$ that are used as operands of any of the instructions.

Definition 2.2: List of Used Values

The *list of used values* of the SSA function $\mathscr{F}$ is the tuple

$$val = (par_1, \dots, par_{|par|}, ins_1, \dots, ins_{|ins|}, glb_1, \dots, glb_{|glb|}, cst_1, \dots, cst_{|cst|}).$$

Furthermore, the following values are used to identify specific ranges in val:

$$\begin{array}{llll} L_{par} = 1 & R_{par} = |par| & L_{ins} = R_{par} + 1 & R_{ins} = R_{par} + |ins| \\ L_{glb} = R_{ins} + 1 & R_{glb} = R_{ins} + |glb| & L_{cst} = R_{glb} + 1 & R_{cst} = R_{glb} + |cst| \\ L_{val} = L_{par} & R_{val} = R_{cst} & & \end{array}$$

Definition 2.3: Data Flow Graph

The *data flow graph* of the SSA function $\mathscr{F}$ is the set $DFG_{\mathscr{F}} \subset \mathbb{N}^3$ such that

$$\begin{aligned} (n,a,b) \in DFG_{\mathscr{F}} \iff & (L_{val} \leq a \leq R_{val}) \wedge (L_{ins} \leq b \leq R_{ins}) \\ & \wedge (val_a \text{ is the } n\text{th argument of } ins_{b-L_{ins}+1}). \end{aligned}$$

For Φ-instructions, the incoming basic blocks are ordered according to the order on ins. The nth argument is the incoming value attached to the nth incoming basic blocks.

Definition 2.4: Control Flow Graph

The *control flow graph* of the SSA function $\mathscr{F}$ is the set $CFG_{\mathscr{F}} \subset \mathbb{N}^3$ such that

$$\begin{aligned} (n,a,b) \in CFG_{\mathscr{F}} \iff & (L_{ins} \leq a \leq R_{ins}) \wedge (L_{ins} \leq b \leq R_{ins}) \\ \wedge \Bigg(& (\neg(ins_{a-L_{ins}+1} \text{ terminates basic block}) \wedge (b = a+1) \wedge (n = 1)) \\ \vee (& (ins_{a-L_{ins}+1} \text{ terminates basic block}) \wedge (ins_{b-L_{ins}+1} \text{ first instruction in } n\text{th} \\ & \qquad \text{target basic block of } ins_{a-L_{ins}+1})) \Bigg). \end{aligned}$$

2.2.2 Identifying Remaining Structure

Section 2.2.1 introduced structures to model the control flow and data flow of SSA programs. This section identifies the remaining information that the SSA model needs in order to capture the program semantics fully. The benchmark for completion of the SSA model is the ability to recreate a semantically equivalent program in the SSA intermediate representation from it. Most of the program structure can already be recovered from $DFG_{\mathscr{F}}$ and $CFG_{\mathscr{F}}$:

1. The basic block boundaries are reconstructed by identifying all consecutive instructions A, B, where at least one of the following conditions is violated:
   - $\{(n,a,b) \in DFG^*_{\mathscr{F}} \mid a = A\} = \{(1,A,B)\}$
   - $\{(n,a,b) \in DFG^*_{\mathscr{F}} \mid b = B\} = \{(1,A,B)\}$.
2. Basic block labels and register names can be chosen freely without changing semantics.
3. The arguments of all instructions can be immediately filled in from $DFG_{\mathscr{F}}$. Similarly, $CFG_{\mathscr{F}}$ directly provides the target instructions for all goto statements.
4. The positional arguments of Φ-instructions in $DFG_{\mathscr{F}}$ are attached as incoming values to the incoming basic blocks after ordering those according to the order on *ins*. The original positional order of the incoming pairs is not recovered, but it is semantically equivalent.

The only part of the SSA representation that still needs modelling is per-value information. This includes the opcodes of instructions, the values of constants, and type information. This is demonstrated in Figure 2.3. At the top of the figure is a simple function in an abstract SSA representation, which calculates an approximation of the square root of a number using the Babylonian method. It has 11 instructions separated into four basic blocks, with the majority of the instructions in a loop that iteratively improves the result. The entire semantic information that is encoded in this SSA representation can be recovered from the structures at the bottom of the figure: per-instruction opcode information, lists of the parameters, globals, and constants used, the data flow graph as in Definition 2.3 and the control flow graph as in Definition 2.4.

Instruction sets and type systems differ between SSA representations, although they overlap significantly. This chapter aims to capture commonalities of SSA representations, the study of instruction sets and type systems is orthogonal to this. These structures are, therefore, modelled as opaque sets as in Definition 2.5.

Definition 2.5: Representation-Specific Sets

$Opcodes_L$ is the set of all opcodes available in the SSA language L, $Types_L$ is the set of all types in L, and $GlobalNames_L$ is the set of all available names for global values.

function Sqrt(S)

entry	goto *header*
header	$i \leftarrow \Phi(entry : 0,\ loop : i')$
	$x \leftarrow \Phi(entry : 1,\ loop : x')$
	$c \leftarrow i < N$
	if c goto *loop* else *exit*
loop	$t_1 \leftarrow S/x$
	$t_2 \leftarrow x + t_1$
	$x' \leftarrow t_2/2$
	$i' \leftarrow i + 1$
	goto *header*
exit	return x

$\cong$

parameters:

1: S

instructions:

2: goto □
3: $\Phi(\square : \square,\ \square : \square)$
4: $\Phi(\square : \square,\ \square : \square)$
5: $\square < \square$
6: if □ goto □ else □
7: $\square / \square$
8: $\square + \square$
9: $\square / \square$
10: $\square + \square$
11: goto □
12: return □

globals:

13: N

constants:

14: 1
15: 2
16: 0

$+$

$$
\begin{aligned}
DFG_{\mathscr{F}} = \{ & 10 \xrightarrow{2} 3,\ 16 \xrightarrow{1} 3,\\
& 9 \xrightarrow{2} 4,\ 14 \xrightarrow{1} 4,\\
& 3 \xrightarrow{1} 5,\ 13 \xrightarrow{2} 5,\\
& 5 \xrightarrow{1} 6\\
& 1 \xrightarrow{1} 7,\ 4 \xrightarrow{2} 7,\\
& 4 \xrightarrow{1} 8,\ 7 \xrightarrow{2} 8\\
& 8 \xrightarrow{1} 9,\ 15 \xrightarrow{2} 9\\
& 3 \xrightarrow{1} 10,\ 14 \xrightarrow{2} 10,\\
& 4 \xrightarrow{1} 12\}\\
CFG_{\mathscr{F}} = \{ & 2 \xrightarrow{1} 3,\\
& 3 \xrightarrow{1} 4,\\
& 4 \xrightarrow{1} 5,\\
& 5 \xrightarrow{1} 6,\\
& 6 \xrightarrow{1} 7,\ 6 \xrightarrow{2} 12,\\
& 7 \xrightarrow{1} 8,\\
& 8 \xrightarrow{1} 9,\\
& 9 \xrightarrow{1} 10,\\
& 10 \xrightarrow{1} 11,\\
& 11 \xrightarrow{1} 3\}
\end{aligned}
$$

Figure 2.3: SSA representation is decomposed into individual instructions, data flow and control flow. This is an equivalent representation of the function; no semantic information is lost. The example is a rendering of the Babylonian method in Figure 2.2, abstracting away the C syntax.

2.2.3 Putting the SSA Model Together

With separate mathematical structures in place to capture all the relevant information contained in SSA programs, the SSA model can now be assembled. Definition 2.6 shows the completed SSA model. The data flow graph $DFG_{\mathscr{F}}$ and the control flow graph $CFG_{\mathscr{F}}$ were discussed in detail previously, but some clarifications are provided for the remaining five structures.

The *type model*, *instruction model*, and *constant model* assign additional information from different domains to the values used in the function. Instead of attaching a single type to each value, the type model allows values to be linked with several elements in the set "*Types*". This is convenient to model subtyping hierarchies where, for example, an integer pointer value is a pointer in particular. The same is true for the instruction model, which enables it to express opcode categories, e.g. an arithmetic operation might be a subtraction in particular. The constant model is defined likewise as a subset of $\mathbb{R} \times \mathbb{N}$ but makes no use of the ability to assign multiple numeric values to the same constant.

Finally, the *parameter model* and *global model* encode which elements in the list of used values of $\mathscr{F}$ are parameters and globals, respectively. Parameter names are not significant, because their position already identifies parameters uniquely within the function signature. Therefore, the parameter model identifies all parameters but attaches no additional data. For global values, on the other hand, the names are attached by the global model as elements from the set "*GlobalNames*".

2.2.4 Additional Notation

The seven basic components of the SSA model are each expressed as a set of tuples. In order to conveniently manipulate these structures in later sections, Definition 2.7 introduces several functions and shorthand notation.

For a set of tuples, the function "*heads*" returns the set of all the first elements of the tuples. For example, "$heads(I_{\mathscr{F}})$" yields all the opcodes that are used in the function $\mathscr{F}$. In contrast, the function "*tails*" removes the first element of all tuples within a set. For example, "$tails(I_{\mathscr{F}})$" identifies the indices of all the instructions within the list of used values of $\mathscr{F}$ but removes the information about their opcodes. Finally, the "*select*" function is used to filter a set for only those tuples with a specific first element, and then returns the tails of all these tuples. For example, "$select(add, I_{\mathscr{F}})$" gives the indices of all additions within the list of used values of the function $\mathscr{F}$.

The representation of data flow and control flow as labelled multigraphs contains more information than is required for many tasks. The simplified versions $DFG^*_{\mathscr{F}}$ and $CFG^*_{\mathscr{F}}$ are constructed with the "*tails*" function, effectively removing the labels and resulting in ordinary graph structures. Similarly, the sets $I^*_{\mathscr{F}}$ and $C^*_{\mathscr{F}}$ are used for shorter notation.

Definition 2.6: Mathematical Characterisation of SSA Functions

The *SSA model* of the function $\mathscr{F}$ is the tuple

$$(DFG_{\mathscr{F}}, CFG_{\mathscr{F}}, T_{\mathscr{F}}, P_{\mathscr{F}}, I_{\mathscr{F}}, G_{\mathscr{F}}, C_{\mathscr{F}}),$$

where

- $DFG_{\mathscr{F}} \subset \mathbb{N}^3$ and $CFG_{\mathscr{F}} \subset \mathbb{N}^3$ are the data flow and control flow graph;
- $T_{\mathscr{F}} \subset Types \times \mathbb{N}$ is the *type model*, defined by the property

$$(t,k) \in T_{\mathscr{F}} \iff (L_{val} \leq k \leq R_{val}) \wedge (val_k \text{ has type } t);$$

- $P_{\mathscr{F}} \subset \mathbb{N}$ is the *parameter model*, defined by the property

$$k \in P_{\mathscr{F}} \iff L_{par} \leq k \leq R_{par};$$

- $I_{\mathscr{F}} \subset Opcodes \times \mathbb{N}$ is the *instruction model*, defined by the property

$$(c,k) \in I_{\mathscr{F}} \iff (L_{ins} \leq k \leq R_{ins}) \wedge (ins_{k-L_{ins}+1} \text{ has opcode } c);$$

- $G_{\mathscr{F}} \subset GlobalNames \times \mathbb{N}$ is the *global model*, defined by the property

$$(n,k) \in G_{\mathscr{F}} \iff \left(L_{glb} \leq k \leq R_{glb}\right) \wedge (glb_{k-L_{glb}+1} \text{ has name } n);$$

- $C_{\mathscr{F}} \subset \mathbb{R} \times \mathbb{N}$ is the *constant model*, defined by the property

$$(x,k) \in C_{\mathscr{F}} \iff (L_{cst} \leq k \leq R_{cst}) \wedge (cst_{k-L_{cst}+1} \text{ has numeric value } x).$$

Definition 2.7: Notation for Reducing Dimensionality

For a set A, any $a \in A$, and $S \subset A \times \mathbb{N}^k$ for some $k > 0$, the following are defined:

$$\begin{aligned} heads(S) &= \{a \in A \mid (a, b_1, \ldots, b_k) \in S \text{ for some } b_1, \ldots, b_k \in \mathbb{N}\} \\ tails(S) &= \{b \in \mathbb{N}^k \mid (a, b_1, \ldots, b_k) \in S \text{ for some } a \in A\} \\ select(a,S) &= \{b \in \mathbb{N}^k \mid (a, b_1, \ldots, b_k) \in S\}. \end{aligned}$$

Note that the case $A = \mathbb{N}$ is common.
In addition, $rev(S') = \{(a,b) \mid (b,a) \in S'\}$ is defined for $S' \subset \mathbb{N}^k$ and the following used:

$$\begin{aligned} DFG^*_{\mathscr{F}} &= tails(DFG_{\mathscr{F}}) & I^*_{\mathscr{F}} &= tails(I_{\mathscr{F}}) \\ CFG^*_{\mathscr{F}} &= tails(CFG_{\mathscr{F}}) & C^*_{\mathscr{F}} &= tails(C_{\mathscr{F}}) \end{aligned}$$

2.2.5 The LLVM Compiler Framework

The previously introduced SSA model is generic and applies to all SSA compiler intermediate representations. However, it needs to be specialised to a specific representation in order to use it on real compiler problems. LLVM intermediate representation (LLVM IR) is one of the most common languages in that class, because of its use in the popular LLVM framework started by Lattner and Adve [33]. It is used throughout this thesis for demonstration and evaluation.

LLVM is a comprehensive compiler infrastructure project, using LLVM IR as its central abstraction. The LLVM project was formerly named "Low Level Virtual Machine", alluding to a hypothetical machine that uses LLVM IR as its native assembly language. The instruction set of LLVM IR is roughly aligned to the semantics of C-style programming languages, but the project has matured beyond this background. It is now a widely influential framework with compiler front ends for a diverse set of languages, including C, C++, Haskell, Julia, Objective-C, Rust, Scala, and CUDA.

Other mainstream compilers, such as the GCC project, use very similar representations internally. Despite this, LLVM is unique in understanding the intermediate representation as an advertised and documented interface within the toolchain, as opposed to an obscure internal abstraction. This makes it very suitable for research implementations.

2.2.6 LLVM IR Example

Some crucial features of LLVM IR in the context of this thesis are demonstrated on an example in Figure 2.4. At the top **(a)** of the figure, a dot product is implemented as a function in C. This function takes as arguments two pointers "`a`", "`b`" to arrays of double-precision floating-point values representing the input vectors, and an unsigned integer "`n`" giving the size of the arrays. Within a single loop at lines 4–5, the dot product is accumulated in the variable "`d`", which is eventually returned as the result of the function.

At the bottom **(b)** of the figure is the corresponding LLVM IR code, captured from the middle end of the LLVM-based Clang compiler after optimisations. Additional comments were inserted manually. Lines 2–4 are the result of an optimisation called "loop inversion". If the arrays are empty, the loop is skipped entirely, and the program returns zero via lines 6–8. Otherwise, the loop at lines 13–24 is entered via the basic block at lines 10–11, which exists for normalisation purposes as the dedicated loop entry block.

Memory access in LLVM is expressed separately from index calculations. This is visible at lines 16–19. The "`getelementptr`" instruction is used to calculate the memory addresses of the array elements "`a[i]`" and "`b[i]`". The "`load`" instruction then reads the memory at the calculated addresses. This scheme simplifies the effectful memory access instructions, pushing the complexity of index calculations into the "`getelementptr`" instruction.

(a) C source code of a dot product function implementation:

```c
double dot(double* a, double *b, size_t n)
{
  double d = 0.0;
  for(int i = 0; i < n; i++)
    d += a[i]*b[i];
  return d;
}
```

(b) LLVM IR of the same dot product function:

```
define double @dot(double* %0, double* %1, i64 %2) {
; <label>:3:
  %4 = icmp eq i64 %2, 0  ; integer (i) comparison (cmp):
     check if register %2 is equal (eq) to constant zero
  br i1 %4, label %5, label %7 ; jump to line 6 if the
     comparison held, otherwise jump to line 10 instead

; <label>:5:
  %6 = phi double [ 0.0, %3 ], [ %16, %8 ] ; result is 0 if
     the phi node was reached from line 4, otherwise it was
     reached from line 24 and the result is taken from %16
  ret double %6

; <label>:7:
  br label %8

; <label>:8:
  %9 = phi i64 [ %17, %8 ], [ 0, %7 ]
  %10 = phi double [ %16, %8 ], [ 0.0, %7 ]
  %11 = getelementptr double, double* %0, i64 %9 ;
     getelementptr calculates memory addresses: here it
     computes the address of the %9-th value in the array %0
  %12 = load double, double* %11 ; loads a double precision
     floating point value from the calculated address
  %13 = getelementptr double, double* %1, i64 %9
  %14 = load double, double* %13
  %15 = fmul double %12, %14
  %16 = fadd double %10, %15
  %17 = add i64 %9, 1
  %18 = icmp eq i64 %17, %2
  br i1 %18, label %5, label %8
}
```

Figure 2.4: The correspondence between C and LLVM IR on an example function: The function computes the dot product of two vectors in a simple reduction loop. In the LLVM IR – generated by Clang – pointer calculations and memory accesses are visible within the SSA representation.

Figure 2.5 shows in detail how the SSA model is constructed from the LLVM intermediate representation of the program in Figure 2.4. At the top left, implementations of the dot product are shown in three programming languages that LLVM supports: Fortran, C and C++. At the top right is the corresponding LLVM IR code. While this is not precisely identical for different implementations, its basic structure is independent of the source language.

In the middle row, the structure of the LLVM IR code is separated into three components: labelled multigraphs for the data flow and control flow, as well as the per-instruction properties represented as a list. Together, they capture all the semantically significant information of the function, as previously demonstrated in Section 2.2.2. In the bottom row, the SSA model is shown, adhering to Definition 2.6. The labelled multigraphs for the control flow and data flow graphs are represented as sets of 3-tuples of integers.

2.3 Constraint Programming on the SSA Model

Properties of SSA programs can now be formulated as constraint problems on the SSA model. For this purpose, the *set of SSA models* is introduced in Definition 2.8, which Definition 2.9 then uses to formulate *SSA constraint problem*s.

Definition 2.8: Set of SSA Models

Given a specific SSA representation (LLVM, Hydrogen, MIR, ...), denote F the set of all valid functions that can be expressed in it.
The *set of SSA models* $\mathscr{M}$ is defined as

$$\mathscr{M} = \{M \mid M \text{ is the SSA model of some } \mathscr{F} \in F\}.$$

Definition 2.9: SSA constraint problem

An *SSA constraint problem* (V,C) is a pair of a finite set of variables V and a boolean predicate $C\colon \mathscr{M} \times \mathbb{N}^V \mapsto \{1,0\}$. The set of *constraint solutions* for an SSA constraint problem in the context of a specific SSA model $M \in \mathscr{M}$ is given as

$$S_M(V,C) = \{s \in \mathbb{N}^V \mid C(M,s) = 1\}.$$

The following intuition applies: The first argument of C is the SSA model of a function. The second argument of C is a solution candidate. $\mathbb{N}^V$ can be understood as abstractly rendering `map<string,unsigned>`, with elements assigning an integer to each constraint variable. These integers represent values in the SSA function by indexing into the list of used values. The predicate function determines whether these values have a specific relationship to each other. Finally, the set of constraint solutions lists all those tuples for which the predicate holds.

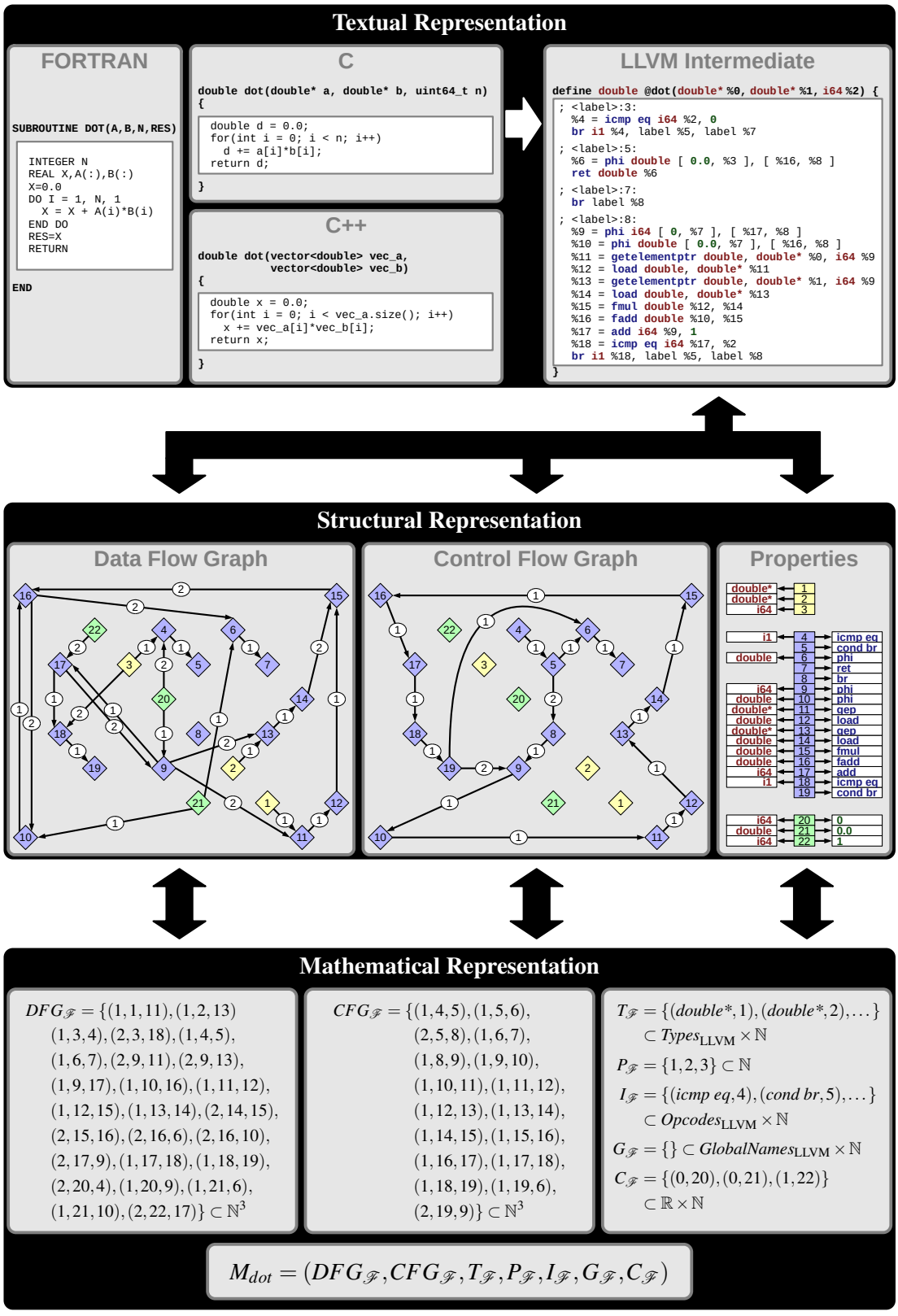


Figure 2.5: Compiler-generated LLVM IR code is decomposed into data flow, control flow and per-value attributes. Mathematical notations of the three components are shown at the bottom.

2.3.1 SSA Constraint Problem Example

Consider the task of detecting all simple loop iterators in a program. These are variables within a loop that are incremented by a constant value of one in each iteration. Figure 2.6 shows how this can be formulated as an SSA constraint problem and then demonstrates its application on the SSA model that was derived in Figure 2.5. The top **(a)** of the figure first gives an intuition about how such a compiler analysis task can be interpreted as a constraint problem to be solved in the context of an SSA model.

Simple loop iterators show up in LLVM IR as data flow cycles between a Φ-instruction and an addition. This is expressed as an SSA constraint problem at the top of the central part **(b)** of the figure. The formulation introduces the variables "phi", "update", "step" and a predicate C to describe the required conditions on the variables. This predicate is composed by logical conjunctions ("∧") of several *element-of* relationships that must hold simultaneously on the structures $C_{\mathscr{F}}$, $I_{\mathscr{F}}$, and $DFG_{\mathscr{F}}$ of the SSA model:

- The iterator is incremented in steps of one $[(1, x_{\text{step}}) \in C_{\mathscr{F}}]$.
- The updated iterator value is computed as an addition $[(add, x_{\text{update}}) \in I_{\mathscr{F}}]$ of the previous iterator value $[(x_{\text{phi}}, x_{\text{update}}) \in DFG^*_{\mathscr{F}}]$ and the step size $[(x_{\text{step}}, x_{\text{update}}) \in DFG^*_{\mathscr{F}}]$.
- The updated iterator is an incoming value $[(x_{\text{update}}, x_{\text{phi}}) \in DFG^*_{\mathscr{F}}]$ to the Φ-instruction that holds the current iterator value $[(phi, x_{\text{phi}}) \in I_{\mathscr{F}}]$.

Explicit control flow constraints for establishing the loop structure around the iterator are not required, because the data flow cycle $[(x_{\text{phi}}, x_{\text{update}}) \in DFG^*_{\mathscr{F}} \wedge (x_{\text{update}}, x_{\text{phi}}) \in DFG^*_{\mathscr{F}}]$ already implies the presence of a loop.

The lower section of Figure 2.6 **(b)** replicates the SSA model from Figure 2.5. Finally, the bottom **(c)** of the figure shows the constraint solutions $S_{M_{dot}}(V, C)$. This set contains only one tuple: $\{\text{phi} \mapsto 9, \text{update} \mapsto 17, \text{step} \mapsto 22\}$. The underlined items in the SSA model evidence the validity of this solution: $(1, 22) \in C_{\mathscr{F}}$, $(add, 17) \in I_{\mathscr{F}}$, $(1, 9, 17) \in DFG_{\mathscr{F}}$, $(2, 22, 17) \in DFG_{\mathscr{F}}$, $(2, 17, 9) \in DFG_{\mathscr{F}}$, and $(phi, 9) \in I_{\mathscr{F}}$. Therefore, all six *element-of* conditions that make up the SSA constraint problem hold.

Figure 2.5 identifies the integer values 9 and 17 with the instructions at lines 14 and 22 of the LLVM IR code from Figure 2.4 **(b)**. These two instructions correspond to the simple loop iterator "`i`" of the for-loop at lines 4–5 of the C source code in Figure 2.4 **(a)**, correctly identifying the only simple loop iterator in the program.

After the detailed derivation of the SSA model and some intuition about the nature of SSA constraint problems, only the solver S in Figure 2.6 remains unexplained. The next sections derive how backtracking can be used to compute the set of constraint solutions efficiently.

(a) Initial problem statement:

$$\underbrace{\text{Detect}}_{\text{solver } S}\ \underbrace{\text{simple loop iterators}}_{\text{SSA constraint problem } (V,C)}\ \text{in the}\ \underbrace{\text{dot product function}}_{\text{SSA model } M_{dot}}.$$

(b) Formulation as constraint problem:

S

SSA constraint problem

$$V = \{\text{phi}, \text{update}, \text{step}\}$$

$$\begin{aligned} C(M,x) = ((1,x_{\text{step}}) \in C_{\mathscr{F}} \wedge (add, x_{\text{update}}) \in I_{\mathscr{F}} \wedge \\ (x_{\text{phi}}, x_{\text{update}}) \in DFG^*_{\mathscr{F}} \wedge (x_{\text{step}}, x_{\text{update}}) \in DFG^*_{\mathscr{F}} \wedge \\ (x_{\text{update}}, x_{\text{phi}}) \in DFG^*_{\mathscr{F}} \wedge (phi, x_{\text{phi}}) \in I_{\mathscr{F}}) \end{aligned}$$

SSA model

$$\begin{aligned} DFG_{\mathscr{F}} = \{&(1,1,11),(1,2,13),(1,3,4),(2,3,18),(1,4,5),(1,6,7),\\ &(2,9,11),(2,9,13),\underline{(1,9,17)},(1,10,16),(1,11,12),(1,12,15),\\ &(1,13,14),(2,14,15),(2,15,16),(2,16,6),(2,16,10),\underline{(2,17,9)},\\ &(1,17,18),(1,18,19),(2,20,4),(1,20,9),(1,21,6),\underline{(1,21,10)},\\ &\underline{(2,22,17)}\} \end{aligned}$$

$$\begin{aligned} CFG_{\mathscr{F}} = \{&(1,4,5),(1,5,6),(2,5,8),(1,6,7),(1,8,9),(1,9,10),\\ &(1,10,11),(1,11,12),(1,12,13),(1,13,14),(1,14,15),\\ &(1,15,16),(1,16,17),(1,17,18),(1,18,19),(1,19,6),(2,19,9)\} \end{aligned}$$

$$T_{\mathscr{F}} = \{(double*,1),(double*,2),\dots\}$$

$$P_{\mathscr{F}} = \{1,2,3\}$$

$$\begin{aligned} I_{\mathscr{F}} = \{&(icmp\ eq,4),(cond\ br,5),(phi,6),(ret,7),(br,8),\underline{(phi,9)},\\ &(phi,10),(gep,11),(load,12),(gep,13),(load,14),(fmul,15),\\ &(fadd,16),\underline{(add,17)},(icmp\ eq,18),(cond\ br,19)\} \end{aligned}$$

$$G_{\mathscr{F}} = \{\}$$

$$C_{\mathscr{F}} = \{(0,20),(0,21),\underline{(1,22)}\}$$

(c) Resulting set of constraint solutions:

$$S_{M_{dot}}(V,C) = \{\{\text{phi} \mapsto 9, \text{update} \mapsto 17, \text{step} \mapsto 22\}\} \subset \mathbb{N}^V$$

Figure 2.6: Detection of simple loop iterators is formulated as a constraint problem and applied to the SSA model from Figure 2.5. A single solution corresponding to the C variable "`i`" is found.

2.4 Solving SSA Constraint Problems

The solver for SSA constraint problems should efficiently compute $S_M(V,C)$ for some concrete SSA constraint problem (V,C) and SSA model M. This is a search problem: all values in $\mathbb{N}^V$ that satisfy C in the context M need to be identified.

The search space $\mathbb{N}^V$ is infinitely large, but it can immediately be trimmed to only tuples that have all values $\leq |val|$. The case $|V| > 50$ is common in Chapters 4 to 6, and interesting functions often have $|val| > 100$. The remaining search space therefore has $> 100^{50} = 10^{100}$ elements. Brute-force search is unfeasible on such a large search space, even when assuming that the direct evaluation of the predicate for any potential solution can be performed efficiently. However, backtracking can be used to find partial solutions that are incrementally extended.

Definition 2.10: Backtracking Solution of Constraint Problems

Given an SSA constraint problem (V,C) and an enumeration of $V = \{v_1, \ldots, v_{|V|}\}$, any collection $(B_k)_{k=1\ldots|V|}$ of functions $B_k : \mathscr{M} \times \mathbb{N}^{k-1} \to \mathscr{P}(\mathbb{N})$ is denoted a *backtracking solution* of (V,C) if and only if the following is satisfied for all $M \in \mathscr{M}, x \in \mathbb{N}^V$:

$$C(M,x) = 1 \iff [x_{v_k} \in B_k(M, p_{k-1}(x)) \text{ for all } 1 \leq k \leq |V|], \tag{2.2}$$

where the projections $p_k \colon \mathbb{N}^V \to \mathbb{N}^k$ are defined by $x \mapsto (x_{v_1}, \ldots x_{v_k})$ for all $0 \leq k \leq |V|$.

Definition 2.10 defines the concept of a backtracking solution, but it does not show how such a backtracking solution could be constructed. That is explained in the next section. The concept is introduced in order to allow the definition of the backtracking search Algorithm 1, which works as follows: The variable x iterates over $\mathbb{N}^n$ ($n = |V|$, identified with $\mathbb{N}^V$ at line 7). The variable k tracks the number of currently considered dimensions of this partial solution. After the initialisation assignments at line 2, a single loop spans the remainder of the algorithm. In each iteration, the algorithm tries to assign a valid value to the latest considered element in the partial solution x (line 4). By convention, the minimum of the empty set is the symbol "∞". If this case occurs, the algorithm backtracks at lines 12–17. Otherwise, the solution is either complete (lines 6–8), or the dimension k of the partial solution is increased at line 10, and the algorithm continues by searching for the next element of x in the following iteration.

2.4.1 The Structure of SSA Constraint Problems

Basic construction rules for SSA constraint problems are the *element-of constraint problem* in Definition 2.11, and the *conjunction and disjunction constraint problems* in Definition 2.12. Definition 2.13 enables the composition of constraint problems that are not defined on the same set of variables. The example in Figure 2.6 can be constructed with only these rules.

Algorithm 1 Backtracking algorithm

```
1:  procedure DETECT(M, (B_k)_{k=1...n}, (v_k)_{k=1...n})
2:      k ← 1,   x ← (1,...,1) ∈ ℕ^n
3:      while true do
4:          x_k ← min{y ∈ B_k(M, p_{k-1}(x)) | y ≥ x_k}
5:          if x_k < ∞ then
6:              if k = n then
7:                  yield {v_k ↦ x_k}_{k=1...n} ∈ ℕ^V
8:                  x_k ← x_k + 1
9:              else
10:                 k ← k + 1
11:                 x_k ← 1
12:         else
13:             k ← k − 1
14:             if k ≥ 1 then
15:                 x_k ← x_k + 1
16:             else
17:                 exit
```

Definition 2.11: Element-of Constraint Problem

Given a set of tuples $S(M) \subset \mathbb{N}^V$ that may depend on $M \in \mathscr{M}$ (e.g. $S(M) \approx DFG^*_{\mathscr{F}}$), the *element-of constraint problem* (V, E_S) is defined by

$$E_S(M,x) = \begin{cases} 1 & \text{if } x \in S(M) \\ 0 & \text{otherwise.} \end{cases}$$

Definition 2.12: Conjunction and Disjunction Constraint Problems

Given SSA constraint problems (V,C) and (V,C'), the *conjunction constraint problem* $(V, C \wedge C')$ and the *disjunction constraint problem* $(V, C \vee C')$ are defined by

$$C \wedge C'(M,x) = \begin{cases} 1 & \text{if } C(M,x) = 1 \wedge C'(M,x) = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$C \vee C'(M,x) = \begin{cases} 1 & \text{if } C(M,x) = 1 \vee C'(M,x) = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Definition 2.13: Extension of a Constraint Problem

Given an SSA constraint problem (V,C) and an injection $i : V \hookrightarrow W$, the *extension of the constraint problem* (W, C^W) is defined by

$$C^W(M,x) = C\left(M, \left(x_{i(v)}\right)_{v \in V}\right).$$

Theorem 2.1: Backtracking Solution for Element-of Constraint Problems

For $S(M) \subset \mathbb{N}^V$, $n = |V|$, $V = \{v_1, \ldots, v_n\}$, the collection $(B_k[E_S])_{k=1\ldots n}$ of functions $B_k[E_S] \colon \mathscr{M} \times \mathbb{N}^{k-1} \to \mathscr{P}(\mathbb{N})$ is a backtracking solution of (V, E_S) when defined by

$$\begin{aligned} B_k[E_S](M,x) &= \mathit{heads}(R_k(M,x)) \\ R_1(M) &= \{p_n(s) \mid s \in S(M)\} \subset \mathbb{N}^n \\ R_{k+1}(M,x) &= \mathit{select}(x_k, R_k(M,(x_1,\ldots,x_{k-1}))) \subset \mathbb{N}^{n-k}. \end{aligned}$$

Proof: By definition, $E_S(M,x) = 1 \iff x \in S(M) \iff p_n(x) \in R_1(M)$.
It is also clear that for all $1 < k \leq n$ holds

$$p_n(x) \in \{p_n(s) \mid s \in S(M)\} \iff p_n(x) \in \{p_n(s) \mid s \in S(M), p_{k-1}(s) = p_{k-1}(x)\}.$$

For all $1 < k \leq n$, this gives

$$E_S(M,x) = 1 \implies x_{v_k} \in \{s_{v_k} \mid s \in S(M), p_{k-1}(s) = p_{k-1}(x)\} = B_k[E_S](M, p_{k-1}(x)).$$

This gives "$\implies$" for the equation in Definition 2.10. The reverse is true for $k = n$, as

$$x_{v_n} \in B_n[E_S](M, p_{n-1}(x)) = \{s_{v_n} \mid s \in S(M), p_{n-1}(s) = p_{n-1}(x)\} \implies x \in S(M).$$

Therefore, the equivalence from Definition 2.10 holds in both directions.

Theorem 2.2: Backtracking Solution for Conjunction Constraint Problems

For SSA constraint problems (V,C) and (V,C') with backtracking solutions $(B_k)_{k=1\ldots n}$, $(B'_k)_{k=1\ldots n}$, the collection $(B_k[C \wedge C'])_{k=1\ldots n}$ of functions $B_k[C \wedge C'] \colon \mathscr{M} \times \mathbb{N}^{k-1} \to \mathscr{P}(\mathbb{N})$ is a backtracking solution of $(V, C \wedge C')$ when defined by

$$B_k[C \wedge C'](M,x) = B_k(M,x) \cap B'_k(M,x).$$

Proof: The definition of $(V, C \wedge C')$ together with the assumption that Definition 2.10 holds for the two given backtracking solutions gives

$$\begin{aligned} C \wedge C'(M,x) = 1 \iff & \left[x_{v_k} \in B_k(M, p_{k-1}(x)) \text{ for all } 1 \leq k \leq n\right] \\ & \wedge \left[x_{v_k} \in B'_k(M, p_{k-1}(x)) \text{ for all } 1 \leq k \leq n\right]. \end{aligned}$$

With the definition of $B_k[C \wedge C']$, this immediately gives

$$C \wedge C'(M,x) = 1 \iff \left[x_{v_k} \in B_k[C \wedge C'](M, (x_{v_1}, \ldots, x_{v_{k-1}}), x_{v_k}) \text{ for all } 1 \leq k \leq n\right].$$

Theorem 2.3: Backtracking Solution for Disjunction Constraint Problems

For SSA constraint problems (V,C) and (V,C') with backtracking solutions $(B_k)_{k=1\dots n}$, $(B'_k)_{k=1\dots n}$, the collection $(B_k[C\vee C'])_{k=1\dots n}$ of functions $B_k[C\vee C']\colon \mathscr{M}\times\mathbb{N}^{k-1}\to\mathscr{P}(\mathbb{N})$ is a backtracking solution of $(V,C\vee C')$ when defined by

$$B_k[C\vee C'](M,x)=\left(\begin{cases} B_k(M,x) & \text{if } R_{k,1}(M,x)\\ \emptyset & \text{otherwise}\end{cases}\right)\cup\left(\begin{cases} B'_k(M,x) & \text{if } R_{k,2}(M,x)\\ \emptyset & \text{otherwise}\end{cases}\right)$$

$$R_1(M)=\begin{pmatrix}1\\1\end{pmatrix}\qquad R_{k+1}(M,x,x_k)=\begin{pmatrix}R_{k,1}(M,x)\wedge x_k\in B_k(M,x)\\ R_{k,2}(M,x)\wedge x_k\in B'_k(M,x)\end{pmatrix}.$$

Proof: The equivalence from Definition 2.10 holds for the backtracking solutions of (V,C), (V,C') by assumption. With $C\vee C'(M,x)=1 \iff C(M,x)=1\vee C'(M,x)=1$, this gives

$$\begin{aligned}C\vee C'(M,x)=1 \iff\ & x_{v_k}\in B_k(M,p_{k-1}(x)) \text{ for all } 1\le k\le n\\ & \vee\, x_{v_k}\in B'_k(M,p_{k-1}(x)) \text{ for all } 1\le k\le n.\end{aligned}$$

After expanding R_k, this directly corresponds to the definition of $B_n[C\vee C']$. Therefore

$$C\vee C'(M,x)=1 \iff x_{v_n}\in B_n[C\vee C'](M,p_{n-1}(x)).$$

This is sufficient for the equivalence in Definition 2.10 to hold in both directions, as for all $1\le k\le n$, the definition of $B_k[C\vee C']$ with expanded R_k directly gives

$$x_{v_n}\in B_n[C\vee C'](M,p_{n-1}(x)) \implies x_{v_k}\in B_k[C\vee C'](M,p_{k-1}(x)).$$

Theorem 2.4: Backtracking Solution for Extensions of Constraint Problems

For an SSA constraint problem (V,C) with a backtracking solution $(B_k)_{k=1\dots|V|}$ and a set W with an injection $i:V\hookrightarrow W$ and an enumeration of $W=\{w_1,\dots,w_{|W|}\}$ that is compatible with the enumeration of V such that $i(v_k)=w_{t(k)}$ with some $t:\mathbb{N}\to\mathbb{N}$ strictly increasing, the collection $(B_k[C^W])_{k=1\dots|W|}$ of functions $B_k[C^W]\colon \mathscr{M}\times\mathbb{N}^{k-1}\to\mathscr{P}(\mathbb{N})$ is a backtracking solution of (V,C^W) when defined by

$$B_k[C^W](M,x)=\begin{cases} B_k\left(M,\left(x_{t(1)},\dots,x_{t(k'-1)}\right)\right) & \text{if } k=t(k') \text{ for some } 1\le k'\le|V|\\ \mathbb{N} & \text{otherwise}\end{cases}.$$

Proof: This follows immediately from Definition 2.10 and Definition 2.13.

Theorem 2.1 introduces backtracking solutions for element-of constraint problems. They are constructed such that $B_k[E_S](M,x) = \{y_{v_k} \mid y \in S(M), y_{v_1} = x_1, \ldots, y_{v_{k-1}} = x_{k-1}\}$. This is optimal in the sense that the backtracking algorithm will only ever backtrack immediately after yielding a result. In the theorem, the backtracking solution is not defined directly. Instead, it uses a helper construct R_k. Section 2.4.3 derives an efficient implementation of R_k.

Theorem 2.2 introduces backtracking solutions for conjunction constraint problems. These are constructed in an obvious way, by taking the intersection set of the underlying backtracking solutions at each k. This is not possible for disjunction constraint problems, which are discussed in Theorem 2.3. The additional structure R_k is used to keep track of whether the current partial solution satisfies C or C' so far (this was a given for conjunctions). The backtracking solution at k is then the union only of those underlying backtracking solutions at k where the corresponding element in R_k signals validity. Finally, Theorem 2.3 shows that backtracking solutions for extensions of constraint problems can be constructed by skipping the additional variables.

2.4.2 Backtracking Example

Figure 2.7 demonstrates how backtracking can algorithmically determine the solution of the SSA constraint problem that was introduced in Figure 2.6 for recognising simple loop iterators. This SSA constraint problem is replicated at the top of the figure. Its construction follows the rules in Definitions 2.11 to 2.13, using conjunctions of element-of constraint problems.

The bottom left part of the figure shows the backtracking solution of the SSA constraint problem that is obtained by applying Theorems 2.1, 2.2 and 2.4. This construction, according to the theorems, guarantees that the defining condition in Definition 2.10 holds for $(B_k[C])_{k=1\ldots3}$ and that Algorithm 1 can be applied. The construction of one element in the backtracking solution is explained in detail as follows.

The condition $(x_{\text{update}}, x_{\text{phi}}) \in DFG^*_{\mathscr{F}}$ corresponds to the element-of constraint problem E_S with $S(M) = \{x \in \mathbb{N}^{\{\text{update,phi}\}} \mid (x_{\text{update}}, x_{\text{phi}}) \in DFG^*_{\mathscr{F}}\}$. Following Theorem 2.1, this gives

$$v_1 = \text{phi} \qquad v_2 = \text{update}$$
$$R_1(M) = rev(DFG^*_{\mathscr{F}}) \qquad R_2(M,x_1) = select(x_1, rev(DFG^*_{\mathscr{F}})).$$

Therefore, the backtracking solution is given by

$$B_1[E_S](M) = heads(rev(DFG^*_{\mathscr{F}}))$$
$$B_2[E_S](M,x_1) = heads(select(x_1, heads(rev(DFG^*_{\mathscr{F}})))).$$

This backtracking solution for E_S is extended to the entire set V using Theorem 2.4, yielding the additional function $B_3[E^V_S](M,x) = \mathbb{N}$. Finally, Theorem 2.2 embeds $B_1[S^V_S]$ in $B_1[C]$ and $B_2[S^V_S]$ in $B_2[C]$ using set intersections with the other terms. The third term $B_3[E^V_S] = \mathbb{N}$ disappears in the assembled backtracking solution, as the intersection with $\mathbb{N}$ is redundant.

Original SSA Constraint Problem

$$V = \{\text{phi}, \text{update}, \text{step}\}$$

$$\begin{aligned} C(M,x) = ((1, x_{\text{step}}) \in C_{\mathscr{F}} \wedge (add, x_{\text{update}}) \in I_{\mathscr{F}} \wedge \\ (x_{\text{phi}}, x_{\text{update}}) \in DFG^*_{\mathscr{F}} \wedge (x_{\text{step}}, x_{\text{update}}) \in DFG^*_{\mathscr{F}} \wedge \\ (x_{\text{update}}, x_{\text{phi}}) \in DFG^*_{\mathscr{F}} \wedge (phi, x_{\text{phi}}) \in I_{\mathscr{F}}) \end{aligned}$$

Backtracking Solution:

$k = 1 \quad v_1 = \text{phi}$

$$\begin{aligned} B_1[C](M) = heads(DFG^*_{\mathscr{F}}) \\ \cap\, heads(rev(DFG^*_{\mathscr{F}})) \\ \cap\, heads(select(phi, I_{\mathscr{F}})) \end{aligned}$$

$k = 2 \quad v_2 = \text{update}$

$$\begin{aligned} B_2[C](M,x) = \\ heads(select(add, I_{\mathscr{F}})) \\ \cap\, heads(select(x_1, DFG^*_{\mathscr{F}})) \\ \cap\, heads(rev(DFG^*_{\mathscr{F}})) \\ \cap\, heads(select(x_1, rev(DFG^*_{\mathscr{F}}))) \end{aligned}$$

$k = 3 \quad v_3 = \text{step}$

$$\begin{aligned} B_3[C](M,x) = select(1, C_{\mathscr{F}}) \\ \cap\, heads(select(x_2, rev(DFG^*_{\mathscr{F}}))) \end{aligned}$$

Backtracking Algorithm:

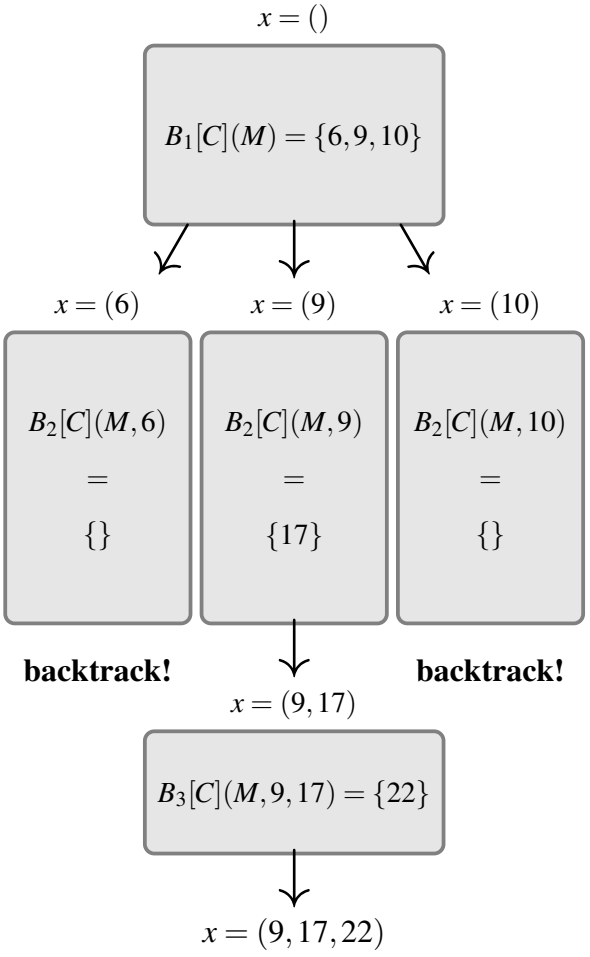


Figure 2.7: Backtracking is used to find the single solution of the SSA constraint problem for simple loop iterators from Figure 2.6. The backtracking solution $(B_k[C])_{k=1...3}$ is constructed with Theorems 2.1, 2.2 and 2.4. The partial solution x is extended in three steps, from top to bottom.

The bottom right of the figure shows the backtracking algorithm applied to the SSA model from Figure 2.5. Starting from the top with an empty partial solution, candidates for x_1 are determined, corresponding to the variable "phi". There are three Φ-instructions in the SSA model, all of which also satisfy the other conditions of being the source and destination of some edge in the data flow graph. Therefore, the algorithm continues with the partial solutions (6), (9) and (10) one level further down in the figure.

For $k = 2$, which corresponds to the variable "update", the backtracking solution requires x_2 to form a data flow cycle with x_1. This is expressed in the second and in the last line of $B_2[C](M,x)$. Furthermore, the value x_2 has to be an "*add*" instruction. The partial solution (6) corresponds to a Φ-instruction that is not part of any data flow cycle. Therefore, the algorithm backtracks. For the partial solution (10), the Φ-instruction is part of a data flow loop cycle with the value 16, but that value is an "*fadd*" instruction. The partial solution (9), however, can be extended according to the backtracking solution with the value 17. Therefore, the algorithm only continues at the bottom of the figure with this one partial solution $(9,17)$.

To complete this partial solution, $B_3[C]$ requires the value for the "step" variable to be a constant of value 1. Furthermore, it needs to be used as an argument to x_2, i.e. the "update" variable. Both of these conditions hold for the value 22, yielding a complete solution.

2.4.3 Implementation, Data Structures and Complexity

With the explicit construction rules for backtracking solutions in Theorems 2.1 to 2.4, suitable data structures can be identified in order to efficiently implement the backtracking algorithm. This requires a slight extension of Algorithm 1 to provide suitable interfaces for computing the supporting structures R_k from Theorems 2.1 and 2.3. Listing 2.1 shows how the backtracking algorithm with those hooks is implemented as a C++ function. Most of the "`solver`" function at lines 12–43 maps directly onto Algorithm 1 and needs no further explanation. However, some crucial details need elaboration, mostly relating to the iteration over the backtracking solution at line 4 of Algorithm 1.

The "`BacktrackingPart`" class provides a C++ interface for the individual functions B_k in backtracking solutions as in Definition 2.10. Four member functions must be implemented. The most important of them, "`skip_invalid`", gives $x_k \leftarrow min\{y \in B_k(M, p_{k-1}(x)) \mid y \geq x_k\}$ from line 4 of Algorithm 1 via "`B[k]->skip_invalid(x[k])`".

The other three member functions of "`BacktrackingPart`" manipulate shared state of "`BacktrackingPart`" objects that together comprise a backtracking solution. This allows computations that would otherwise have to be performed repeatedly by "`skip_invalid`", to be done ahead of time. More specifically, this corresponds to R_k in Theorems 2.1 and 2.4. Crucially, this distribution of the computations allows all previously discussed types of SSA constraint problems to be implemented efficiently with the "`BacktrackingPart`" interface.

```
1  // This class corresponds to Definition 2.10.
2  class BacktrackingPart {
3  public:
4    virtual SkipResult skip_invalid(unsigned& c) = 0;
5    virtual void begin() = 0;
6    virtual void fixate(unsigned c) = 0;
7    virtual void resume() = 0;
8  };
9
10 void yield(const vector<unsigned>& solution);
11
12 // This function corresponds to Algorithm 1.
13 void solver(vector<BacktrackingPart*> B) {
14     unsigned          k = 0;
15     vector<unsigned> x(B.size(), 0);
16     while(true) {
17         SkipResult result = B[k]->skip_invalid(x[k]);
18         if(result == SkipResult::CHANGE) continue;
19         if(result != SkipResult::FAIL) {
20             if(k + 1 == B.size())
21             {
22                 yield(x);
23                 B[k]->resume();
24                 x[k]++;
25             }
26             else {
27                 B[k]->fixate(x[k]);
28                 k++;
29                 [k]->begin();
30                 x[k] = 0;
31             }
32         }
33         else {
34             if(k > 0) {
35                 k = k - 1;
36                 B[k]->resume();
37                 x[k] = x[k] + 1;
38             }
39             else
40                 return;
41         }
42     }
43 }
```

Listing 2.1: Complete C++ implementation of Algorithm 1: The if-else statements at lines 19–41 precisely correspond to those at lines 5–17 of the algorithm. The "`BacktrackingPart`" objects iterate over $B_k[C](M,x)$ via the "`skip_invalid`" method. The remaining member function calls at lines 23,27,29,36 precompute structures used in "`skip_invalid`" for quick evaluation.

2.4.3.1 Data Structures for Constraint Classes

Element-of constraint problems require the structures R_k. These can be implemented with a tree, built around a sorted array of pairs, as shown with the definition of "`Tree`" on the first line of the top section of Listing 2.2. Each subtree in the array corresponds to a potential R_{k+1}. The instances of "`BacktrackingPart`" for this constraint keep an index (line 25) into the array (line 3), which "`begin`" initialises (line 18). Each time "`skip_invalid`" is called, the index is incremented until it either points to an entry $\geq$ the lower bound `c` or hits the end of the array (lines 9–10). Finally, "`resume`" does nothing (line 22), and "`fixate`" simply accesses the array at the current index for the subtree R_{k+1} (lines 20–21).

Conjunction constraint problems are simpler to implement, as shown in the lower section of Listing 2.2. The methods "`begin`", "`fixate`", and "`resume`" call the corresponding member functions of the underlying `BacktrackingPart` objects (line 20), and "`skip_invalid`" checks both of them with short-circuit evaluation in case of a "`FAIL`" result.

Disjunction constraints again require the structures R_k, consisting of an array of booleans, with each element corresponding to an underlying constraint option. Calls to "`skip_invalid`" are passed on to those underlying "`BacktrackingPart`" objects that are not disabled via R_k. If any succeed, then the smallest resulting value among them is selected. The "`fixate`" function determines R_{k+1} with "`skip_invalid`", disabling all of the underlying constraint options that do not return "`PASS`". The "`begin`", "`fixate`" and "`resume`" member calls are additionally passed on to all "`BacktrackingPart`" instances that are enabled in R_k.

2.4.3.2 Computational Complexity

The chosen data structures allow for efficient implementations of the individual functions that are used to search for solutions. However, SSA constraint problems are a generalisation of the subgraph isomorphism problem. Any subgraph can be specified using conjunctions of element-of constraints, and the solver will search for these within any provided SSA model. Cook [34] showed that this problem is NP-hard [35]. Therefore, the computational complexity has an exponential worst case (in the number of variables) when evaluated in general.

The exponential worst case is no impediment in practice due to the particular use case of SSA constraint problems. The nature of SSA constraint problems means that the same limited set of formulas is repeatedly solved in the context of many different SSA models. Therefore, it is viable to tune the formulas ahead of solving time to be efficiently solvable, in particular by choosing a good order in which to iterate over the variables. For specific SSA constraint problems, it can then be possible to make complexity assumptions.

```
struct Tree { vector<pair<int,Tree>> t; };
template<int n> // These are S(M) and (R_k(M,x))_{k=1...n} in Theorem 2.1.
struct ElementOfShared { Tree S; array<Tree*,n> R; };

template<int n, int k>
class ElementOfPart : public BacktrackingPart {
public:
    SkipResult skip_invalid(unsigned& c) override {
        while(index < shared->R[k]->t.size()
           && c > shared->R[k]->t[index].first) index++;
        if(index == shared->R[k]->t.size())
            return SkipResult::FAIL;
        if(shared->R[k]->t[index].first == c)
            return SkipResult::PASS;
        c = shared->R[k]->t[index].first;
        return SkipResult::SUCCESS;
    }
    void begin() override { index = 0;
        if(k == 0) shared->R[k] = &shared->S; }
    void fixate(unsigned) override {
        if(k < n) shared->R[k+1] = &shared->R[k][index]; }
    void resume() override {}
private:
    shared_ptr<ElementOfShared<n>> shared;
    size_t                         index;
};
```

```
class ConjunctionPart : public BacktrackingPart {
public:
    SkipResult skip_invalid(unsigned& c) {
        SkipResult r1 = parts[0]->skip_invalid(c);
        if(r1 == SkipResult::FAIL)   return r1;
        if(r1 == SkipResult::CHANGE) return r1;
        SkipResult r2 = parts[1]->skip_invalid(c);
        if(r2 == SkipResult::FAIL)   return r2;
        if(r2 != SkipResult::PASS)
            return SkipResult::CHANGE;
        return r1;
    }
    void begin() override {
        for(auto part: parts) part->begin(); }
    void fixate(unsigned c) override {
        for(auto part: parts) part->fixate(c); }
    void resume() override {
        for(auto part: parts) part->resume(); }
private:
    array<shared_ptr<BacktrackingPart>,2> parts;
};
```

Listing 2.2: "BacktrackingPart" is implemented for element-of constraints and conjunction constraints. The array "R" in "ElementOfShared" matches $(R_k(M,x))_{k=1...n}$ from Theorem 2.1.

2.4.4 Additional SSA Constraint Problems

Beyond Definitions 2.11 to 2.13, there are a number of supplementary construction rules for SSA constraint problems that are used in this thesis. Firstly, there are simple SSA constraint problems that operate directly on the integer values, without interpreting them as indices into the value list of the SSA model. These are listed in Definition 2.14. The first two, $(\{a,b\},C^{=})$ and $(\{a,b\},C^{\neq})$, enforce equality and inequality of the integer values, respectively. The third, $(\{a\},C^{unused})$, prevents the assignment of values from the SSA model to the variable. This is convenient only in disjunctions, for parts of solutions that may be omitted. For example, an offset may be optionally added when accessing an element in a structure. If the offset is not present, the corresponding variable is *unused* in that constraint solution. The value $R_{val}+1$ is the smallest integer that cannot be used as an index into the list of used values.

Generalised graph domination constraint problems as in Definition 2.15 are versatile tools for analysing SSA models. The special case of $|Orig| = |Dom| = |Dest| = 1$, with x_v for the unique $v \in Orig$ restricted to the control origin of the function, turns this definition into the established control flow domination concept. However, the generalisation encompasses a much larger set of interesting conditions. Importantly, generalised graph domination can also be applied to data flow, which Chapter 5 uses for defining kernel functions with restricted interfaces. In this interpretation, the set *Dom* represents an interface to the computation of the values in *Dest*, when data flow can originate from all the values in *Orig*.

The backtracking solution of generalised graph domination constraint problems is brute-force, meaning that $B_n[D_G]$ checks the condition as specified, and $B_k[D_G] \equiv \mathbb{N}$ for all $1 \leq k < n$. The checking of the condition in $B_n[D_G]$ is typically performed in $O(|val|)$ time, making this a computationally expensive constraint. The solver relies on this construction occurring only in conjunctions with other, more restrictive constraints. In addition, special cases can also be implemented as element-of constraint problems by pre-computing dominator trees.

Finally, *collect-all constraint problems* are introduced in Definition 2.16. Given a constraint problem (V,C), the variables V are divided into two subsets. The collect-all constraint problem identifies all possible solutions over one of the variable subsets, given that the other variable subset is fixed. The parameter n gives an upper limit to the number of these solutions, which is required to maintain a finite number of variables. Such collect-all constraint problems are used to approximate logical quantifiers. This is discussed in more detail in Chapter 4.

In order to efficiently calculate a backtracking solution of collect-all constraint problems, the enumeration of $V^{\langle U,n\rangle}$ must satisfy the ordering $u < (k,v)$ for all $u \in U, k = 1 \ldots n, v \in V \setminus U$. For values in U, the backtracking solution is the same as for the underlying constraint (V,C). Whenever a partial solution is determined for all variables in U, an invocation to the full solver is performed that identifies all possible partial solutions over $V \setminus U$. The remaining steps of the backtracking solution merely enforce this pre-computed list of solutions.

Definition 2.14: Simple Integer Constraint Problems

The SSA constraint problems $(\{a,b\}, C^{=})$, $(\{a,b\}, C^{\neq})$, and $(\{a\}, C^{unused})$ are given by

$$\begin{aligned} C^{=}(M,x) = 1 &\iff x_a = x_b \\ C^{\neq}(M,x) = 1 &\iff x_a \neq x_b \\ C^{unused}(M,x) = 1 &\iff x_a = R_{val} + 1. \end{aligned}$$

Definition 2.15: Generalised Graph Domination Constraint Problem

Given a graph $G(M) \subset \mathbb{N}^2$, a set of variables V, a set of origins $Orig \subset V$, a set of generalised dominators $Dom \subset V$, and a set of destinations $Dest \subset V$, the *generalised graph domination constraint problem* (V, D_G) is defined by

$$D_G(M,x) = 1 \iff (x_v \notin R_\infty(M,x) \text{ for all } v \in Dest \text{ with } L_{val} \leq x_v \leq R_{val}),$$

where $R_k(M,x) \subset \mathbb{N}$, the *set of vertices reachable in k steps*, is defined as

$$\begin{aligned} R_0(M,x) &= \{x_v \mid v \in Orig\} \setminus \{x_v \mid v \in Dom\} \\ R_{k+1}(M,x) &= \{b \mid (a,b) \in G(M), a \in R_k(M,x)\} \setminus \{x_v \mid v \in Dom\} \\ R_\infty(M,x) &= \textstyle\bigcup_{k=0}^{\infty} R_k(M,x). \end{aligned}$$

Definition 2.16: Collect-all Constraint Problem

Given a set of variables V, a subset $U \subset V$, and some $n \geq 1$, the set containing one copy of each variable in U and n copies of each value in $V \setminus U$ is denoted

$$V^{\langle U,n \rangle} = U \cup (\{1,\ldots,n\} \times (V \setminus U)).$$

For any $r \in \mathbb{N}^U$, the set of constraint solutions restricted to r in $\mathbb{N}^U$ is defined as

$$S_M(V,C)|_r = \{x \in S_M(V,C) \mid x_u = r_u \text{ for all } u \in U\}.$$

The functions $f_k \colon \mathbb{N}^{V^{\langle U,n \rangle}} \to \mathbb{N}^V$ and $g \colon \mathbb{N}^{V^{\langle U,n \rangle}} \to \mathbb{N}^U$ are defined as selecting the values for the kth copy of variables in $V \setminus U$ and directly projecting on $\mathbb{N}^U$.
For an SSA constraint problem (V,C), the *collect-all constraint problem* $(V^{\langle U,n \rangle}, C^{\langle U,n \rangle})$ is uniquely defined by the following conditions:

$$\begin{aligned} C^{\langle U,n \rangle}(M,x) = 1 \iff & \left|S_M(V,C)|_{g(x)}\right| \leq n \\ & \wedge \left\{f_k(x) \mid 1 \leq k \leq \left|S_M(V,C)|_{g(x)}\right|\right\} = S_M(V,C)|_{g(x)} \\ & \wedge f_k(x) \equiv R_{val} + 1 \text{ for all } k > \left|S_M(V,C)|_{g(x)}\right| \\ & \wedge p_n(f_1(x)), \ldots, p_n(f_n(x)) \text{ are in lexicographic order.} \end{aligned}$$

2.4.5 Satifiability Modulo Theory

There are some parallels between SSA constraint problems and other satisfiability problems, most importantly, Satisfiability Modulo Theory (SMT). Both methods construct formulas via constraints on variables that are combined with logical connectors, and backtracking is in both cases used as the basis for developing efficient solvers. This suggests that it could be possible to recast SSA constraint problems in the language of SMT. Where typical SMT problems might be formulated on the *theory of linear arithmetic* or the *theory of bit-vectors*, SSA constraint problems would operate on the *theory of SSA intermediate representation*. Powerful solvers have been developed for SMT problems over the last decade, making this appear worthwhile.

However, the structure and usage scenario of SSA constraint problems make them unfitting for this approach. The contextual nature of SSA constraint problems means that the constraint formulation in SMT would have to encode also the graph structure of the SSA function as context. This encoding is non-trivial and would introduce significant overhead.

Furthermore, one SSA constraint problem is typically evaluated with many different SSA function contexts, but an off-the-shelf SMT solver has no way of leveraging this knowledge. SSA constraint problems are crafted by hand, and some manual tuning is acceptable to enable quick solving times. In particular, the backtracking order for variables can be adjusted. Thus, it is a requirement for the solver to have consistently good performance across all SSA function contexts, but not necessarily across every possible constraint formula. Instead, it can dictate specific programming styles. The solver relying on such pre-processed formulas makes most of the sophisticated optimisations that are crucial for SMT redundant.

2.5 Summary

The chapter introduced an approach for applying constraint programming to SSA intermediate representation code. Based on a mathematical characterisation of the static structure of SSA functions, *SSA constraint problems* were defined. These are formulas that impose restrictions on compiler intermediate code, turning the detection of adhering program parts into a constraint satisfiability problem. The chapter derived efficient algorithms for solving such SSA constraint problems and discussed several significant types of constraint formulas, reflecting compiler analysis methods like data flow and dominator relationships.

Outlook Constraint programming on SSA intermediate representation forms the principal methodological basis of this thesis. Chapters 4 and 5 refer back to the definitions in this chapter for designing and implementing the languages CAnDL and IDL, which integrate constraint programming on SSA code into LLVM. Preceding that, the following chapter provides a review of related literature, putting the work into a broader context.

Chapter 3

Related Work

Four areas of research are of particular relevance to this thesis: **Constraint programming and specification languages** are central to the introduced methodology. The relevant literature includes the research into constraint programming in the context of program analysis and the design of specification languages. The survey of previous approaches to **compiler analysis and auto-parallelisation** establishes the baselines for the later evaluation sections. Related work on **heterogeneous computing** motivates the proposed approaches, by presenting the plethora of other programming paradigms to overcome the specific challenges of emerging hardware. Lastly, the diverse research landscape around concepts related to **computational idioms** puts the algorithmic structures that are detected in later chapters of this thesis into context.

3.1 Constraint Programming and Specification Languages

Declarative Languages, constraint programming, and the application of constraints to program analysis problems are well-established in the literature. Previous work covers query languages, logic programming, applications to software security and formal verification, model checking and SMT, but also more compiler-centric data flow analysis and type inference problems. The limited scope of this section requires a focus on work that is particularly relevant to this thesis.

For this research, constraint programming is most interesting within the context of research fields such as program analysis and model checking. Crucial background material for this thesis also comes from the programming language design community of declarative programming languages. Prolog and its many extensions and dialects particularly stand out as fully fledged logic programming languages, but parallels can also be drawn to querying languages that apply database techniques to static analysis.

These different fields vary significantly in their interests, motivations, and approaches, but the underlying challenges are often similar. Notably, the performance of backtracking solvers and the scalability to complex problems are a recurring theme.

3.1.1 Constraint Programming for Program Analysis

Constraint analysis on abstract languages Constraint systems have long been used for program analysis. Aiken [36] gives a comprehensive overview of earlier work, highlighting the crucial ability of constraint-based program analysis to separate *constraint specification* from *constraint resolution*. This separation is critical also for this thesis, as it enables the scalability of compiler analysis problems beyond what could reasonably be implemented with manual recognition routines. The constraint specification can be formulated briefly, by offloading the *constraint resolution* to a separate solver. However, the article does not present any techniques to capture higher-level algorithmic concepts like computational idioms. Instead, it focuses on more basic compiler analysis problems, such as data flow analysis and type inference.

More recent work on constraint-based program analysis by Gulwani et al. [37] leverages the advancements in modern off-the-shelf SAT/SMT solver technology. The analysis problems are lowered to bit-vector formulations, and the *constraint resolution* is entirely externalised to independently developed SMT solvers. The motivation of the approach is mainly to verify program properties, as opposed to the application of parallelising code transformation in this thesis. Furthermore, Section 2.4.5 showed that the confinement to conventional SMT solvers is inefficient for the resolution of SSA constraint problems.

Kundu et al. [38] propose constraints to verify the correctness of program transformations with their system for Parameterized Equivalence Checking (PEC). This system improves on previous work performing translation validation. Translation validation is the validity checking of transformations on concrete input programs by comparing the semantics before and after modification. PEC implements a hybrid approach that allows some aspects of the program to be underspecified, yet does not check the soundness of transformations in full generality. The checking is done via a custom solver for the generated constraint problems. The hybrid nature allows the system also to validate program transformations that significantly modify the control flow, e.g. loop unswitching. The system cannot discover transformation opportunities, only verify them after the transformation was applied.

Constraint analysis on compiler IR code There is previous work on using constraint-based program analysis for real compiler intermediate representations, mostly in the area of security and the formal verification of software systems. This includes investigations into using SMT on LLVM intermediate representation, which is also used for the implementations in this thesis. Zhao et al. [39] built a model of LLVM IR for such solvers. However, this model serves an entirely different purpose to the SSA model of this thesis. The model provides operational semantics, but cannot be used to detect large-scale algorithmic structures in user programs, as is required for automatic heterogeneous acceleration. Instead, the focus is on formally verifying the correctness of existing compiler transformations for all possible user input.

Recent domain-specific languages, such as Alive [40], operate on subsets of LLVM IR. The individual instructions are reformulated on bit-vectors, and the correctness of conditions is checked with an SMT solver. Alive only implements a subset of LLVM's integer and pointer arithmetic instructions. It has no support for control flow and does not scale to the applications that are used in this thesis for evaluation. Instead, it is designed for formally verifying already existing compiler optimisations that operate on only a handful of integer instructions at a time. Alive is meant to improve compilers, not user programs.

LifeJacket [41] proves the correctness of floating-point optimisations in LLVM, as does Alive-FP [42]. Both of these projects are extensions of the SMT-based Alive system. They extend the scope of the system to model a wider range of instructions as bit-vectors, enabling the verification of more compiler optimisations. LifeJacket and Alive-FP were successfully used to identify wrongly implemented optimising transformations in compilers. Nonetheless, the fundamental limitations of Alive remain, and control flow is not supported. Therefore, only peephole optimisations can be evaluated by these approaches.

The Alive-Infer system [43] also builds on Alive but goes beyond the verification of existing compiler optimisations. The tool uses an SMT solver to automatically generate preconditions that need to hold for transformations to be applied. This moves the Alive system away from verifying optimisations and closer to automatically detecting algorithmic structure in parts of user programs. However, Alive-Infer still requires the separate specification of the actual transformation. It only generates additional conditions and does not handle control flow.

Constraint analysis on other program models Other advanced approaches to extracting high-level code structures from programs that use constraints and verification systems have been proposed. Mendis et al. [44], Kamil et al. [45] suggest temporal logic as the foundation to formulate the necessary conditions for rephrasing well-structured Fortran and assembly code in restrictive models. These techniques leverage counter-example guided inductive synthesis to find provably correct translations into the high-level Halide language. The Halide compiler specialises the code again, exploring the optimisation space via powerful transformations that are enabled by its restrictive semantics. The focus is on a small class of computations with only dense memory accesses. This allows formal reasoning about correctness but is too restrictive for interesting computational idioms, such as sparse linear algebra.

Mullen et al. [46] study low-level program transformations that are implemented for the formally verified CompCert compiler [47], directly on x86 assembly. Instead of an automatic verification after modelling optimisations as SMT problems, the presented Peek system was checked with the interactive theorem prover Coq. This required approximately 30000 lines of manually written Coq code and proof lines. Some of the transformations consider rudimentary control flow constraints, but they cannot scale to computational idioms.

3.1.2 Declarative Programming Languages for Program Analysis

Languages for querying program properties From the perspective of language design, the declarative programming languages Prolog and SQL are perhaps most influential. The two languages differ fundamentally. Prolog ("programmation en logique") is a logic programming language that originated in academia for analysing natural language [48]. By contrast, SQL ("Structured Query Language") was developed at IBM for managing data in relational database management systems [49]. Nonetheless, specification languages for structures in program code have been designed taking inspiration from both backgrounds.

The first such specification language was the Omega system by Linton [50]. It uses a relational database to store all the relevant properties of a program. The captured information is based on the abstract syntax tree of programs that are implemented in a subset of the Ada programming language. Additional edges are inserted to connect shared variables of successive expressions, given some indication of data flow between instructions. The system then allows database-style queries formulated in QUEL [51], an SQL-style language.

The CodeQuest system [52] first combined the ideas of Omega and its database-oriented successors with the use of logic programming. The queries are translated into Datalog, a Prolog derivative that is implemented on top of SQL. This allows CodeQuest to be fundamentally more expressive, allowing recursive queries that are required for meaningful CFG inspection. Nevertheless, the approach is based on querying for source language features. This makes the detection of large-scale algorithmic structures in complex programming languages such as C++ infeasible, as demonstrated in Section 6.4.

Languages for generating compiler passes Custom specification languages for generating compiler analysis and transformation passes have been presented in the literature. Martin [53] introduced a specification language for program analysis functionality called PAG, based on abstract interpretation. The generated functionality was integrated into C and Fortran compilers via a well-specified interface and applied successfully to real benchmark codes. However, the tool is focused on relatively simple compiler optimisations such as constant propagation.

Domain-specific languages for compiler transformation passes were also studied by Olmos and Visser [54]. The proposed Stratego system uses rewrite rules to apply tree transformations to the abstract syntax tree of source programs. However, this was only evaluated on the Octave language, and the general applicability on large-scale programs remains unclear.

Lipps et al. [55] designed the domain-specific language OPTRAN for matching patterns in attributed abstract syntax trees of Pascal programs. Semantically equivalent, more efficient implementations can then automatically replace the matching code patterns. With the focus on Pascal, it remains unclear how the proposed concepts translate to the complex C++ programs with pointer calculations that were used for the evaluation of this thesis.

Another language for implementing compiler optimisations from declarative specifications is OPTIMIX [56, 57]. Similarly to the presented work in Chapter 4, OPTIMIX emphasises developer productivity. The system is based on graph rewrite rules. OPTIMIX programs are compiled into C code that performs the specified transformation. Such a domain-specific language for the generation of optimisation transformations was also used in the CoSy compiler [58]. Both OPTIMIX and the CoSy method are simple rewrite engines that have no knowledge of global program constraints.

Different code transformation techniques use LibASTMatchers and LibTooling [59] from the LLVM project. These tools do not provide a complete standalone language but are instead implemented in C++ as an embedded domain-specific language for pattern matching, relying heavily on other LLVM libraries. The approach is deeply integrated with the Clang compiler and exposes the abstract syntax tree (AST) of the compiler frontend directly. There are more than a thousand separate classes that implement types of AST nodes in Clang, introducing considerable complexity for any non-trivial pattern. Therefore, this is an entirely impractical approach for detecting complex algorithmic structures such as computational idioms.

Willcock et al. [60] designed a complex system for generating generic optimisation passes using concepts from generic programming. However, such schemes do not work at the IR level of established compiler frameworks. Instead, they require program rewrites by the user.

Whitfield and Soffa [61] praise Gospel, their framework and specification scripture for the exploration of the properties of code-improving transformations. The project furthermore includes the Genesis tool, which automatically generates transformers as specified in Gospel. Several standard optimisations were implemented with Gospel and Genesis, such as constant folding and common subexpression elimination. Similar approaches to generating compiler optimisations from specification languages include Rhodium [62]. The language expresses optimisations using explicit data flow facts, which are manipulated by local propagation and transformation rules. The transformations are applied to a custom intermediate language and can be proven correct with a theorem prover. Neither Gospel nor Rhodium provides means to tackle the issue of efficiently enabling large-scale program transformations.

3.2 Compiler Analysis and Auto-Parallelisation

The core motivation for the work in this thesis is the automatic heterogeneous parallelisation of sequential code. The derived methods are evaluated on two metrics: how broadly they apply to real code, and how significant their performance impact is when applied. These metrics are evaluated against other compiler analysis and parallelisation approaches. This section focuses on three areas of the vast research landscape that are particularly relevant for this: polyhedral compilation, the parallelisation of reductions, and dynamic approaches.

3.2.1 Compilation with the Polyhedral Model

The polyhedral model [27] is an established mathematical framework for modelling, analysing, and transforming well-behaved loop nests. Iterations in loop nests are treated as lattice points in a multi-dimensional grid. The iteration space can then be transformed with affine maps, potentially uncovering new parallelisation opportunities. This basic approach has been applied extensively in compilers. Furthermore, the required conditions have been relaxed in different ways, allowing the application of the approach to more input code.

Polly in LLVM Polyhedral optimisers have been integrated into mainstream C/C++ compilers. Most notably, Grosser et al. [26] implemented the Polly extensions for LLVM. Polly recognises parts of LLVM IR that are expressible in the polyhedral model and transforms them into that representation. Polyhedral optimisations can then be applied with PLuTo [63] before the model is translated back into optimised LLVM IR for further treatment by the core compiler. This enables the seamless application of polyhedral techniques on large-scale applications without source code changes via the many frontends of the LLVM infrastructure. However, this impacts only code that the tool can translate into a polyhedral representation.

Polly-ACC [64] is an extension of the Polly compiler that provides code generation for heterogeneous hardware. The tool uses the recognition functionality of standard Polly to detect code sections in LLVM IR that can be represented in the polyhedral model. These code sections are optimised with established polyhedral transformation techniques from Grosser et al. [26]. The optimised polyhedral code sections are then translated into CUDA code to be executed on the GPU. This results in significant speedups of some benchmark programs, but the impact remains limited to code that fits the polyhedral model.

Doerfert et al. [30] extended the applicability of polyhedral transformations within the Polly compiler to a broader set of input programs. Dependencies between iterations that originate from reduction variables cannot be eliminated with affine transformations. Therefore, they prohibit DOALL parallelism in a way that standard Polly is unable to resolve. By contrast, the reduction-enabled scheduling approach for Polly can parallelise such loop despite the reduction dependencies. This ability significantly improved the achieved speedup of Polly on benchmark programs that contain reductions.

Doerfert et al. [65] also investigated another method for widening the scope of polyhedral code transformations. This approach allows some conditions that are required for the legal application of transformations to remain unproven at compile time. These conditions are then checked at runtime, providing a fallback to the original code when assumptions are not met. The checks that this work allows to be delayed include the absence of aliasing, finite loop boundaries, and in-bounds memory accesses. This enabled Polly to cover 3.9× as many loops in the SPEC and NPB benchmarks at a negligible runtime overhead.

Other tools with automatic detection The Polyhedral Parallel Code Generator (PPCG) [66] is a source-to-source compiler that takes sequential C programs and generates optimised CUDA kernels to target GPU acceleration. The extraction of polyhedral code sections from the C input is based on the Polyhedral Extraction Tool [67]. This extraction system can automatically detect relevant code regions, but it is implemented on syntax level and relies on purpose-built C code with all arrays declared in variable-length C99 array syntax. This is not robust enough to reliably cover larger programs from benchmark collections such as NPB or Parboil, which are used for evaluation in this thesis.

C-to-CUDA [68] is another compiler that offers heterogeneous acceleration of sequential C code by representing it in the polyhedral model. However, the focus is on code generation and the application of optimising transformations. The automatic recognition in the abstract syntax tree of parallel loops that can be represented in the polyhedral model remains ad-hoc and handles only a small set of benchmarks.

Increased applicability of polyhedral transformations Recent work by Baghdadi et al. [69] has extended the polyhedral model beyond affine programs to some forms of sparsity. This is implemented in the Platform-Neutral Compute Intermediate Language, which is intended for heterogeneous systems and provides backends for accelerator programming. The platform provides extensions that can be used to model important features of sparse linear algebra, such as counted loops [70]. Such loops have dynamic, memory dependent bounds but statically known strides and are central to sparse linear algebra.

Tiramisu [71] is a polyhedral framework for targeting heterogeneous hardware, providing backends for CPUs, GPUs, distributed architectures, and FPGAs. Optimisations are performed on four layers of intermediate representation, resulting in performance that almost matches dedicated library functions. However, the tool does not detect polyhedral code sections within existing source code. Instead, it requires the programmer to implement the algorithms manually with a dedicated C++ API.

Zhang et al. [72] studied the extension of polyhedral approaches to allow the capturing of some sparse linear algebra calculations in the polyhedral model. The article introduces a novel non-affine split transformation for this purpose. Using the inspector-executor model, the approach achieved significant speedups when evaluated on some benchmark programs. The research does not address the automatic recognition of sparse linear algebra routines within existing programs. Besides, the approach was not evaluated against state-of-the-art library implementations such as Intel MKL and cuSPARSE.

Many approaches have been proposed for parallelising loop nests with reduction variables in the polyhedral model, among them Jouvelot and Dehbonei [73], Redon and Feautrier [74], Chi-Chung et al. [75], Gupta and Rajopadhye [76], Stock et al. [77].

3.2.2 Reduction Parallelism

Discovering and exploiting scalar reductions in programs has been studied for many years based on dependence analysis and idiom detection. Early work by Pottenger and Eigenmann [78], Suganuma et al. [79], Fisher and Ghuloum [80] focused on well-structured Fortran code and often paid little attention to robust detection in more complex programs. Rauchwerger and Padua [81] went beyond previous static approaches and developed a dynamic test to speculatively exploit reduction parallelism. Work by Gutiérrez et al. [82, 83, 84] has focused on the exploitation of reductions rather than discovery. Approaches to heterogeneous acceleration examined trade-offs in implementation [85] or exploitation of novel hardware [86, 87].

The treatment of more general reduction operations has received less attention. Das and Peng Wu [88] used dynamic profile analysis to guide manual analysis and show there is potential for finding generalised reductions. Kim [89] explored the use of dynamic analysis further but states in the article that detecting reductions on arrays remains challenging.

The difficulty in automatically detecting reductions has led to languages and annotation-based approaches, where it is the responsibility of the user to mark reductions in the program. Such a system was proposed by Deitz et al. [90]. Chandan Reddy and Cohen [91] also describe an annotation approach, based on the Platform-Neutral Compute Intermediate Language [69], incorporating the PPCG code generator to generate CUDA and OpenCL code for multiple computing platforms.

There has also been recent work extending on Rauchwerger and Padua [81] with more aggressive speculation and dynamic analysis [92] to exploit reduction parallelism. Han et al. [93] explain an approach for the parallelisation of a wide class of scalar reductions. They start from the observation that many reductions in real benchmark programs are not detected by current static analysis approaches. They propose a hardware-assisted speculative parallelisation approach for likely runtime reductions, denoted “partial reduction variables”. Candidates for speculative parallelisation are determined by searching for update-chains in the data flow graph. The approach was evaluated on some of the SPEC2000 benchmarks with a simulator. They achieve up to 46% speedup by including speculative reductions, but this approach requires hardware speculation support. Despite the hardware support, the system is unable to detect histogram reductions.

Privateer [94] is a complex system featuring combined compiler and runtime support to enable speculative parallelisation. The core approach is the privatisation of memory for each thread and an exception mechanism with recovery routines for accesses that violate parallelism. The authors explicitly allow for reduction parallelism involving only a single scalar associative and commutative operator. The evaluation only covered a set of five benchmark programs but yielded a geometric mean speedup of 11.4$\times$ on a 24-core machine. The runtime overhead was up to >50%. Despite this complexity, the approach only exploits simple scalar reductions.

3.2.3 Dynamic Analysis Approaches

There is an extensive body of work on complementing static analysis with profiling information from test runs. Other research uses runtime checks for unproven assumptions to allow unsound reasoning at compile time. Furthermore, functional mapping to heterogeneous systems with compilers has been combined with machine learning decision making at runtime for selecting the appropriate computing hardware to execute a given piece of code.

Tournavitis et al. [95] characterised the significant weaknesses of established static data dependence analysis techniques. Profile-driven parallelism detection and machine learning-based mapping approaches are suggested in order to improve on the state-of-art parallelising compilers. Wang et al. [96] implemented a system that automatically discovers parallelism based on profile-driven parallelism detection. The approach improves significantly over purely static approaches by replacing the traditional target-specific and inflexible mapping heuristics with a prediction mechanism that uses machine learning. The model is trained via an offline supervised learning scheme, using both static and dynamic features, such as cache miss rates and branch miss prediction rate. Dynamic approaches can eliminate spurious dependencies and profitability models based on powerful machine learning techniques greatly improve on simple heuristics. Nonetheless, such an approach cannot match the potential of domain-specific library backends and requires significant manual tuning effort.

Manilov et al. [97] present a dynamic approach to detecting a wide class of iterators using dynamic profiling data. The recognised iterators describe the traversal of data structures that are difficult to capture with traditional static techniques. This is an essential prerequisite for implementing compiler parallelisation approaches to pointer-based data structures. However, the approach only captures a small, if crucial, part of the calculations, and does not extend to full computational idioms such as sparse linear algebra.

Wen and O'Boyle [98] implemented a runtime framework for scheduling OpenCL kernels on a heterogeneous CPU/GPU system, improving on a previous approach by Wen et al. [99]. The presented machine learning-based predictive model decides at runtime whether kernels are merged or executed separately on appropriate devices. However, such a system can only be applied when the functional translation to accelerators is available. This is not the case for the sequential C code that this thesis is evaluated on. Moreover, the approach improves OpenCL performance but does not consider the impact of library backends, which often significantly outperform generic OpenCL implementations on appropriate tasks.

Ogilvie et al. [100] propose a system for performance prediction on heterogeneous systems that is based on active learning. This significantly reduces the tuning that is required for machine learning-based scheduling algorithms on CPU/GPU systems. This approach still requires the availability of functional mapping to heterogeneous devices, and cannot work on unchanged sequential C/C++ inputs.

3.3 Heterogeneous Computing

Heterogeneous computing has been a particularly active field of research since the widespread adoption of GPUs for general-purpose computations during the last decade. This field includes research from both software and hardware perspectives. The hardware research investigates the most promising directions of diversification for processors in heterogeneous systems [101].

However, the related work in the context of this research is from the software perspective. This section focuses on the different programming approaches that have been championed for targeting existing heterogeneous accelerators. These methods broadly fall into two categories: library approaches and domain-specific languages.

3.3.1 Libraries

Library interfaces are often the most performant path to exploiting heterogeneous computing. However, they provide narrow interfaces, accelerating only very particular computations. The established way of encapsulating fast linear algebra is via dedicated library implementations based on the BLAS interfaces [12]. Leading BLAS implementations are unmatched in speed on their target hardware platforms, but require application programmer effort and offer little portability. Implementations of dense linear algebra are available for many hardware platforms, such as cuBLAS [15] for NVIDIA GPUs, clBLAS [16] for AMD GPUs, and MKL [14] for Intel CPUs and accelerators.

While most individual library implementations focus on a single target hardware platform, some BLAS implementations attempt cross-platform acceleration and heterogeneity. Among them are systems by Wang et al. [102], Moreton-Fernandez et al. [103, 104].

Dense linear algebra is the best-supported class of calculations. Besides, implementations of sparse linear algebra also exist for the most important platforms, including cuSPARSE [105] for NVIDIA GPUs and clSPARSE [106] built on top of OpenCL.

More expressive computational idioms, such as reductions and stencils, are not suitable for library implementation. These idioms are parameterised with kernel functions, which could be implemented as callbacks but prevent a direct execution on heterogeneous hardware. Instead, domain-specific languages provide the appropriate abstraction level for these computations.

CPU-GPU data transfer optimisations Library implementations often require the manual management of CPU-GPU data transfers. These transfers have been studied extensively as bottlenecks for parallelisation efforts. Work by Jablin et al. [107] established a method for the automatic management of CPU-GPU communication. Similarly, Lee et al. [108] implemented a system to optimising data transfers using data flow analysis, although this was in the context of moving OpenMP code to GPUs.

3.3.2 Domain-Specific Languages

Many domain-specific languages have been proposed for the efficient and easy programming of heterogeneous systems. They allow implementers to restrict the compiler and runtime away from general-purpose programming concepts that are difficult to support on specific hardware. Domain-specific languages can be standalone with an entire toolchain and runtime ecosystem or be embedded in existing languages. Python and Scala are popular host languages for DSLs. DSLs range in complexity from only marginally more flexible than library interfaces to fully fledged programming languages such as OpenCL and CUDA.

Functional languages Lift [109] provides composable constructs that enable the functional implementation of data-parallel algorithms and operations. The language is especially suitable for dense linear algebra applications [110] and stencil codes [111], but extensions to support some forms of sparsity exist as well [112]. Lift performs optimisations by applying functional rewrite rules. This extensible set of rewrite rules allows the compiler to explore a vast space of possible program transformations. However, selecting the best of these many versions requires guidance from profiling runs. These profiling runs can be computationally expensive, taking approximately one day for the evaluation of a sufficient number of variants for tuning the matrix multiplication kernel [110]. Such user effort can be prohibitive but promises highly tuned OpenCL outputs.

There exist multiple other functional approaches to generating code for heterogeneous hardware. Among them, Chakravarty et al. [113], McDonell et al. [114] propose Accelerate, a domain-specific language that is embedded in Haskell. Accelerate applies sharing recovery and loop fusion optimisations to generates efficient GPU code. Many of these techniques target particular challenges that arise from the untypical nature of Haskell, especially the methods for interfacing heterogeneous accelerators from a lazily evaluated environment. This makes Accelerate unsuitable for evaluation in this thesis, which focuses on benchmarks that are provided as C/C++ programs.

Copperhead [115], is a data-parallel language embedded in Python. It exposes parallelism via higher-order functions such as *map*, *gather*, and *reduce*. However, Copperhead is unable to compile the formulated programs into standalone binaries, leaving the programs integrated tightly with the Python environment. This integration makes interfacing with C/C++ nontrivial and is unsuitable for the acceleration of existing benchmarks.

Collins et al. [116] introduced NOVA, a functional language targeted at code generation for GPUs. The evaluation showed comparable performance to dedicated library implementations on several important applications, including sparse matrix-vector multiplication. However, the work is highly focused on the generation of CUDA code and does not provide an OpenCL backend, which is crucial for evaluation on GPUs of multiple vendors.

Intermediate languages Delite [117] was presented as an intermediate representation that facilitates the rapid construction of domain-specific languages. The provided infrastructure targets heterogeneous platforms, with backends available for OpenMP, CUDA, and even MPI for cluster computing. Delite-based DSLs are proposed for machine learning, data querying, graph analysis, and scientific computing. However, the Delite approach is tightly integrated with the Scala language and does not offer a readily available end-to-end solution.

Halide, as proposed by Ragan-Kelley et al. [118] was designed for image processing, but is flexible enough also to allow the formulation of matrix multiplication and other computations. Suriana et al. [119] demonstrated that this extends to reduction computations as well. Halide's core design decision is the scheduling model, which allows the separation of the computation schedule and the actual computation. There has been follow-up work on automatically tuning the schedules, e.g. Mullapudi et al. [120], but by default, the burden of implementing efficient schedules is put on the application programmer.

Embedded languages Milk [121] is a pragma-based domain-specific language to annotate indirect memory accesses in C++. The approach is inspired by OpenMP and supported by modified versions of Clang and LLVM. This allows low-level optimisations that are particularly applicable to sparse linear algebra. The authors report performance gains of up to 3x, but the approach is unable to utilise the much greater potential of heterogeneous compute and requires detailed programmer intervention.

Other approaches Spatial [122] is a domain-specific language for high-level descriptions of application accelerators. It provides hardware-centric abstractions but also takes programmer productivity into consideration. The language does not target the established heterogeneous CPU-GPU systems. Instead, the focus is on Field Programmable Gate Arrays (FPGAs) and Coarse Grain Reconfigurable Architectures (CGRAs). This makes it unfit for comparative evaluation with the methods proposed in this thesis.

There have been multiple domain-specific libraries proposed specifically for linear algebra computations. Spampinato and Püschel [123, 124] introduced and extended the high-level LGen language, based on standard mathematical notation. The implemented routines are optimised with an autotuning compiler, exploring many transformations such as tiling, loop fusion, and vectorisation. The tool improves over Intel MKL on specific small-scale matrices but is unable to generate code for GPUs.

Recent research has highlighted the challenge of generating code that performs well across different heterogeneous hardware architectures. The PetaBricks language [125, 126] was one of the first to address this performance portability challenge by encoding algorithmic choices, which are empirically evaluated and decided on by the compiler. Similarly, Muralidharan et al. [127] explored the automatic selection of code variants using machine learning.

3.4 Computational Idioms

Notions that recognise the existence of *computational idioms* are known in several disciplines. The core observation is that existing software is not distributed evenly in the space of possible programs. Instead, programs tend to be clustered around design principles. Interestingly, this appears true in particular for performance-intensive programs and bottleneck computations.

The concrete concepts are partially overlapping and sometimes vague. In the discipline of software engineering, software design patterns describe program components as specialising implementations of a class of standard approaches.

Terms such as *map and reduce*, *stencil code*, and *linear algebra* are commonly used when designing libraries and domain-specific languages. Scientific computing is mostly concerned with the architectural implications of specific memory access patterns that are intrinsic to the choice of certain algorithmic approaches.

This section attempts to demarcate a meaningful conception of the term *computational idiom* by comparison with the existing literature of different domains with related concepts.

3.4.1 Higher-Order Functions

Many functional programming languages, such as OCaml and Haskell, encapsulate high-level algorithmic choices and common programming patterns as higher-order functions [128]. These are functions that are parameterised with other functions. Examples of higher-order functions are *map*, which applies a function to each element in a data structure, and *fold* / *reduce*, which accumulates the elements in a data structure with a reduction operator.

Many computational workloads can be expressed as instances of higher-order functions. For example, the popularity of the MapReduce framework [129] stems from the observation that many big data workloads exhibit characteristics that can be expressed efficiently with combinations of *map* and *reduce*. The framework provides an *idiomatic* approach to the development of big data applications, enabling shorter development times and more predictable performance.

The use of computational idioms for automatic heterogeneous acceleration requires a more restrictive view of types than what is common in functional programming languages. For example, the *reduce* operator allows the implementation of the insertion sort algorithm, as well as a simple sum over an array of floating-point values. These two algorithms do not share parallelisation opportunities. Therefore, the detection of *reduce*-instances is insufficient for enabling compiler parallelisation approaches. However, more restrictive versions of *reduce* are suitable for compiler detection. Chapter 5 studies the class of Complex Reduction and Histogram Computations, which is formulated as a computational idiom. This restricted class of *reduce*-calculations shares a common parallelisation approach.

3.4.2 Berkeley Parallel Dwarfs

The *Berkeley Dwarfs* are a collection of 13 computational methods that together comprise a large portion of the most common parallel computing workloads [130]. Each Dwarf is a computational pattern that appears in many such applications. The authors observed that these Dwarfs have persisted fundamentally unchanged for many years, even as concrete applications were repeatedly supplanted. The Dwarfs are informed by numerical computations that arise in the scientific computing community, but the authors suggest that the experience from this domain may prove useful in other areas as well.

The Berkeley Dwarfs were studied from the perspective of architecture requirements, not with automatic compiler recognition in mind. Therefore, some of the dwarfs are specified too broadly for use as computational idioms in this paper. However, dense linear algebra, sparse linear algebra, and structured grid computations (stencils) are essential idioms in Chapter 6.

3.4.3 Algorithmic Skeletons

Another abstraction that is related to computational idioms as used in this thesis is the notion of algorithmic skeletons [131]. This concept was introduced to classify the behaviour of parallel programs according to their organisation of workload distribution among threads. The motivation behind this classification was to enable the introduction of new, higher-level programming models and tools for parallel programming. The higher-order functions from functional programming were a major inspiration, observing a lack of similar abstractions on more mainstream programming languages. Among the established algorithmic skeletons are "Fixed Degree Divide & Conquer" and "Task Queue".

The concept of algorithmic skeletons has been used to implement many programming frameworks and libraries. The eSkel library was sketched on top of C and MPI by Cole [132], providing a higher-level programming model based on algorithmic skeletons. Skandium [133] is a parallel skeleton library that targets multi-core architectures. Eden [134] provides skeletons for parallel programming in Haskell. SkelCL [135] provides implementations of algorithmic skeletons that target GPUs via CUDA. This is implemented in C++, providing versions of higher-order functions such as *map*, *reduce*, and *zip* as templates. Finally, the Thread Building Blocks (TBB) library [136] was inspired by algorithmic skeletons.

The definitions for Algorithmic Skeletons are not specified formally to enable automated reasoning. They were instead drafted for human understanding and to guide the design of libraries and DSLs. This abstraction level is similar to the Berkley Dwarfs. The algorithmic skeletons were, however, heavily inspired by higher-order functions and similarly describe the algorithmic structure of computations. This distinguishes them from the Berkeley Dwarfs, which are more focused on mathematical domains and architectural requirements.

Chapter 4

The Compiler Analysis Description Language*

Chapter 2 derived an approach for constraint programming on Static Single Assignment (SSA) compiler intermediate representation. This chapter develops a novel domain-specific constraint programming language based on that methodology and presents an implementation within the production-quality LLVM compiler infrastructure.

In the first sections, the design of the Compiler Analysis Description Language (CAnDL) is motivated as an approach for simplifying the implementation of LLVM transformation passes. Optimising compilers have to use elaborate program transformations to exploit increasingly complex hardware. Implementing the required analysis functionality for such optimisations to be safely applied is a time-consuming and error-prone activity. This is a barrier to the rapid prototyping and evaluation of innovative new compiler optimisations. CAnDL automatically generates such compiler analysis functionality from constraint specifications.

The individual language constructs are introduced with their syntax and functionality in the third section of this chapter. The first introduced language features directly expose parts of the underlying SSA model from Chapter 2. Building on that, CAnDL provides higher-level constructs that allow for modularity in specifications and the reduction of repetitive constraints. Using these higher-level constructs, a collection of CAnDL specifications of standard compiler concepts is introduced as the CAnDL standard library. This collection of common building blocks includes single-entry single-exit regions, loops, and array-based memory accesses.

Finally, several case studies are presented for the experimental evaluation of CAnDL. They show that CAnDL scales to a wide range of compiler analysis tasks. These tasks range from the detection of peephole optimisation opportunities, over graphics shader optimisations, to fully capturing Static Control Parts (SCoPs) for polyhedral code analysis. All of them can be expressed more succinctly in CAnDL than with previous approaches.

*This chapter is based on published research: Ginsbach et al. [1].

4.1 Introduction

Compilers are intricate pieces of software responsible for the generation of efficient code. They transform input source code through several compilation stages, resulting in a binary program. In order to generate fast programs, state-of-the-art compilers rely on an elaborate middle end. At this stage, the user code is typically expressed in an SSA intermediate representation, and improved by successively applying a wide range of optimisations.

Most compiler optimisations require two steps: analysis and transformation. First, analysis routines find sections in user programs that enable the application of specific transformations. They further verify the necessary conditions to ensure the transformation can be applied legally without changing the program semantics. It is crucial that optimisations retain the semantics of the original program, as otherwise the resulting binary might be corrupted. Transformations are then applied to the analysis results in a second step. This often involves heuristic cost models to gauge the effect on runtime, code size, and other metrics.

The complexity of the necessary analysis is an impediment to the implementation of new compiler passes, preventing the rapid prototyping of new ideas. For example, simple peephole optimisations in the LLVM "`instcombine`" pass require approximately 30000 lines of C++ code, despite the transformations being simple. Menendez and Nagarakatte [43] showed that "`instcombine`" is an important source of bugs, and bugs in the middle end of a compiler are particularly pernicious [137]. They tamper with the user programs but can remain unnoticed and often only trigger in corner cases. Ideally, there would be a simpler way of implementing such analysis that reduces boilerplate code and opens the way for new compiler innovation.

This chapter presents the Compiler Analysis Description Language (CAnDL), a domain-specific language for compiler analysis. It is a constraint programming language, operating on the SSA intermediate representation of the LLVM compiler infrastructure (LLVM IR). Instead of writing compiler analysis code inside the main codebase of the compiler infrastructure, it lets compiler writers specify optimisation functionality external to the main C++ codebase. The CAnDL compiler then generates C++ functions that implement LLVM analysis passes, and are linked together with the Clang compiler binary. The formulation of optimising transformations in CAnDL is faster, simpler and less error-prone than writing them in C++. The language has a strong emphasis on modularity, which facilitates debugging and the formulation of highly readable code.

CAnDL is based on the constraint programming methodology introduced in Chapter 2. It uses a solver that is integrated into the LLVM codebase. CAnDL is developed as a complete programming language, with a full parser and code generator. The system is evaluated on a range of use cases from different domains, including standard LLVM optimisation passes, custom optimisations for graphics shader programs, and the detection of Static Control Parts (SCoPs) [26] for polyhedral program transformations [27].

4.2 Motivating Example

For an example of the CAnDL workflow, consider Equation (4.1). This basic algebraic equation can be interpreted as a recipe for a compiler optimisation: Assuming an environment without the particularities of floating-point arithmetic (i.e. assuming the "`-ffast-math`" flag is active), the compiler could use this equality to eliminate some square root invocations in user code. This is desirable, as the square root has to be approximated with relatively expensive numerical methods, whereas computing the absolute value is computationally cheap.

$$\forall a \in \mathbb{R}\colon\ \sqrt{a*a} = |a| \tag{4.1}$$

The compiler should use the equation left-to-right. It should analyse the user code in order to find segments that correspond to the left side of the equation and then transform all those occurrences analogous to the right side of the equation. The compiler, therefore, must detect $\sqrt{a*a}$ in the LLVM IR code and replace it with calls to the "`abs`" function. The generation of the new function call is trivial, but the detection of even this simple pattern requires some care when implemented manually in a sophisticated codebase such as LLVM.

The traditional approach in the Clang compiler is to integrate such optimisations into the previously mentioned "`instcombine`" pass. which already applies an extensive collection of peephole optimisations. However, this code makes heavy use of raw pointers and dynamic type casts, spans ∼30000 lines, and has been identified as a frequent source of bugs in Menendez and Nagarakatte [43], Yang et al. [137]. This is impractical and an impediment to compiler development.

Instead, CAnDL allows a declarative description of the analysis problem. It is easier to follow, has no interaction with other optimisations and is concise, as presented in Listing 4.1. The first line of the program assigns a name to the specification, which is then defined by the interaction of seven *atomic constraints*. These individual statements must simultaneously hold on the values of "`sqrt_call`", "`sqrt_fn`", "`square`" and "`a`". Lines 2–8 each stipulate one of these constraints, and they are joined together with logical conjunctions "∧".

```
1 Constraint SqrtOfSquare
2 ( opcode{sqrt_call} = call
3 ∧ {sqrt_fn} = {sqrt_call}.args[0]
4 ∧ function_name{sqrt_fn} = sqrt
5 ∧ {square} = {sqrt_call}.args[1]
6 ∧ opcode{square} = fmul
7 ∧ {a} = {square}.args[0]
8 ∧ {a} = {square}.args[1])
9 End
```

Listing 4.1: The left side of Equation (4.1) as specified in CAnDL

The CAnDL compiler translates the declarative program into a C++ function, which is then used for the analysis step in an LLVM optimisation pass. This is demonstrated in Figure 4.1, which shows the application of the analysis function generated from the CAnDL specification in Listing 4.1 to a user program. The input program (**a**) is a simple C function that calls the "`sqrt`" function twice with squares of floating-point values. This is translated using the Clang compiler into LLVM IR code (**b**), using standard optimisation passes during the compilation. The expression from the user program is here represented as a list of individual instructions and register assignments, with the occurrences of "`SqrtOfSquare`" clearly visible: the two "`fmul`" instructions (lines 4 and 6) compute squares via a floating-point multiplication, and these are then used as arguments to "`sqrt`" function invocations (lines 5 and 7).

The optimised LLVM IR (**b**) is used as the input to the generated analysis function, which detects two optimisation opportunities. These are identified as the first (**c**) and second (**d**) solution of the constraint problem. Each of the solutions assigns values from within the LLVM IR code to all the CAnDL variables in Listing 4.1, such that all constraints are satisfied. The validity of these solutions is demonstrated in the middle row of the figure (**e**-**f**). Substituting the variables in the CAnDL program with the concrete instances from the solutions, the individual atomic constraints can be checked individually:

- `%4` and `%6` are function calls, and their first argument (the function to be called) is `@sqrt`.
- `@sqrt` is the square root function. Note that it is identified by name.
- The second arguments of the function call instructions (the first function arguments) are `%3` and `%5`, respectively.
- `%3` and `%5` are square values, i.e. floating-point multiplications of a value with itself.

Using the solutions identified by the CAnDL system, a separate C++ function (**g**) applies the transformation. This function is easy to implement. The solutions to the constraint problem are internally provided as C++ dictionaries of type "`map<std::string,llvm::Value*>`", containing the information needed to apply code transformations. A new function call to "`abs`" is generated, with the value determined for "`a`" from Listing 4.1 as the only argument (line 6). This instruction replaces (line 4) the call instruction that was captured in "`sqrt_call`" (line 5). Conveniently, the LLVM infrastructure already provides all the necessary functions to create and replace instructions in the intermediate representation. After post-processing with standard dead code elimination, this results in the optimised code shown at the bottom of the figure (**h**).

Although this is a small example, it illustrates the main steps of the CAnDL scheme. In practice, the strength of the system is its ability to scale to very complex specifications, as demonstrated toward the end of the chapter. The following sections describe the CAnDL language in detail and outline how it is implemented on top of the constraint programming methodology from Chapter 2.

(a) C program code:

```
double example(double a, double b) {return sqrt(a*a) + sqrt(b*b); }
```

(b) Resulting LLVM IR:

```
define double @example(
 double %0,
 double %1) {
  %3 = fmul double %0, %0
  %4 = call double @sqrt(%3)
  %5 = fmul double %1, %1
  %6 = call double @sqrt(%5)
  %7 = fadd double %4, %6
  ret double %7 }
declare double @sqrt(double)
```

(c) First solution:

```
a = %0

square = %3
sqrt_call = %4

sqrt_fn = @sqrt
```

(d) Second solution:

```
a = %1

square = %5
sqrt_call = %6

sqrt_fn = @sqrt
```

(e) Validating the first solution:

```
( opcode{%4} = call
∧ {@sqrt} = {%4}.args[0]
∧ function_name{@sqrt} = sqrt
∧ {%3} = {%4}.args[1]
∧ opcode{%3} = fmul
∧ {%0} = {%3}.args[0]
∧ {%0} = {%3}.args[1])
```

(f) Validating the second solution:

```
( opcode{%6} = call
∧ {@sqrt} = {%6}.args[0]
∧ function_name{@sqrt} = sqrt
∧ {%5} = {%4}.args[1]
∧ opcode{%5} = fmul
∧ {%1} = {%5}.args[0]
∧ {%1} = {%5}.args[1])
```

(g) Complementing C++ transformation code:

```
using namespace std;
using namespace llvm;
void transform(map<string,Value*> solution, Function* abs) {
    ReplaceInstWithInst(
       dyn_cast<Instruction>(solution["sqrt_call"]),
       CallInst::Create(abs, {solution["a"]}));
}
```

(h) Transformed LLVM IR after dead code elimination:

```
define double @example(double %0, double %1) {
  %3 = call double @abs(double %0)
  %4 = call double @abs(double %1)
  %5 = fadd double %3, %4
  ret double %5 }
```

Figure 4.1: Demonstration of CAnDL specification in Listing 4.1 on an example C program (**a**): In the generated LLVM IR code (**b**), instances (**c**,**d**) of "`SqrtOfSquare`" are detected that fulfil all the constraints (**e**, **f**). Applying a transformation is simple (**g**) and results in efficient code (**h**).

4.3 Language Specification

The Compiler Analysis Description Language is a domain-specific programming language for the specification of compiler analysis problems. Individual CAnDL programs define specific computational structures that exist in user programs and can be exploited by applying code transformations. These structures are specified as constraint programs on the LLVM IR of user code. CAnDL builds on generic concepts from Chapter 2. These are independent of LLVM, so the methodology translates to other SSA representations.

The expressed structures can scale from simple instruction patterns that enable peephole optimisations, over control flow structures such as loops, to complex algorithmic concepts such as code regions that are suitable for polyhedral code transformations.

Like traditional constraint programs, CAnDL specifications have two fundamental features: **variables** and **constraints**. The basic constraint building blocks are well-established compiler analysis tools, such as constraints on data and control flow, data types and instruction opcodes. These are composed with logical connectors and several higher-level language features, such as range expressions, with finally a system of modularity and extensibility on top. This section introduces the language features, starting from the overall program structure.*

4.3.1 Top-Level Structure of CAnDL Programs

The following notational conventions are used for the description of CAnDL syntax in this section: terminal symbols are **bold**, non-terminals are *italic*, $\langle$**s**$\rangle$ is an identifier (alphanumeric string), and $\langle$**n**$\rangle$ is an integer literal. CAnDL uses Unicode characters such as "$\wedge$", "$\in$", "Φ" and is encoded as UTF-8. An individual CAnDL program contains constraint formulas that are bound to identifiers. As previously shown in Listing 4.1, the syntax for this is as follows:

$$\boxed{\textit{specification} ::= \textbf{Constraint}\ \langle\textbf{s}\rangle\ \textit{formula}\ \textbf{End}}$$

Listing 4.1 already demonstrated how logical conjunctions are used to combine simpler *formula*s. More generally, a *formula* can be any of the following:

$$\boxed{\textit{formula} ::= \textit{atomic} \mid \textit{conjunction} \mid \textit{disjunction} \mid \textit{conRange} \mid \textit{disRange} \mid \textit{include} \mid \textit{collect}}$$

The fundamental elements of every CAnDL program are *atomic* constraints. They are bound together by logical connectives "$\wedge$" and "$\vee$" (*conjunction* and *disjunction*), as well as other higher-level constructs. These include two kinds of range structures (*conRange*, *disRange*), and a system for modularity (*include*). Lastly, the *collect* construct allows for the formulation of more complex constraints that require quantifiers from first-order logic. The individual classes of atomic constraints are introduced next, followed by the higher-level constructs.

*The complete grammar file that was used to generate the parser of the CAnDL compiler is in Appendix A.

Syntax	SSA model formulation
data_type *variable* **=** ⟨**s**⟩	$(s,x) \in T_{\mathscr{F}}$
opcode *variable* **=** ⟨**s**⟩	$(s,x) \in I_{\mathscr{F}}$
ir_type *variable* **= literal**	$x \in C^*_{\mathscr{F}}$
ir_type *variable* **= argument**	$x \in P^*_{\mathscr{F}}$
ir_type *variable* **= instruction**	$x \in I^*_{\mathscr{F}}$
function_name *variable* **=** ⟨**s**⟩	$(s,x) \in G_{\mathscr{F}}$

Table 4.1: The simplest atomic constraints operate on a single variable and check element-of properties for the different sets in the SSA model. Function names are from the global model.

Syntax	SSA model formulation
variable **=** *variable***.args[** ⟨**n**⟩ **]**	$(n,x,y) \in DFG_{\mathscr{F}}$
variable $\in$ *variable***.args**	$(x,y) \in DFG^*_{\mathscr{F}}$
variable **=** *variable***.successors[** ⟨**n**⟩ **]**	$(n,y,x) \in CFG_{\mathscr{F}}$
variable $\in$ *variable***.successors**	$(y,x) \in CFG^*_{\mathscr{F}}$
variable **=** *variable*	$x = y$
variable **!=** *variable*	$x \neq y$

Table 4.2: The second class of atomic constraints operate on pairs of variables. The constraints check for graph edges in the data flow or control flow graphs, or directly for shallow equivalence.

4.3.2 Atomic Constraints

Based on the SSA model from Definition 2.6 in Chapter 2, CAnDL provides a wide range of atomic constraints. The simplest group consists of those that operate on only a single variable, listed in Table 4.1. These operate immediately on the underlying mathematical structures, testing element-of properties between variables and the sets. This can constrain data types ($T_{\mathscr{F}}$) and instruction opcodes ($I_{\mathscr{F}}$), or restrict variables to constant literals ($C^*_{\mathscr{F}}$), function parameters ($P^*_{\mathscr{F}}$), and instructions ($I^*_{\mathscr{F}}$). Ending the list is the "**function_name**" constraint. Function names can be statically determined only for direct function calls. In that case, the called object is necessarily a global value, and the function name can be identified in the global model $G_{\mathscr{F}}$.

There are additional atomic constraints that operate on pairs of variables. These are listed in Table 4.2. Most importantly, they check for specific edges in the control flow ($CFG_{\mathscr{F}}$) and data flow graphs ($DFG_{\mathscr{F}}$). CAnDL also provides weaker versions that do not enforce specific edge labels. Furthermore, two constraints are available for shallow comparisons of variables.

Besides those constraints that operate immediately on the sets of the SSA model, there are atomic constraints that enforce graph properties. Such graph properties include dominance relationships and the interaction of data flow and control flow for Φ-instructions.

4.3.2.1 Constraining Φ-Instructions

CAnDL provides the following syntax for expressing the data flow of Φ-instructions:

variable **->** *variable* Φ *variable*

The expression $\{A\}$**->**$\{B\}\Phi\{C\}$ means that C takes the value of A when reached from B. The underlying condition on the SSA model is more difficult to express than for the previous atomic constraints. Specifically, "reached from B" means that B was the last branch taken before arriving at C. Using the SSA model, this is equivalent to the following:

$$(C, phi) \in I_{\mathscr{F}} \wedge (B, h(C)) \in CFG^*_{\mathscr{F}} \wedge (A, C, n) \in DFG_{\mathscr{F}},$$
$$\text{where } h(C) := min\{c \mid (phi, x) \in I_{\mathscr{F}} \text{ for all } c \leq x \leq C\}$$
$$\text{and } n := \{b \leq B \mid (b, h(C)) \in CFG^*_{\mathscr{F}}\}.$$

Firstly, C is a Φ-instruction. Secondly, B has control flow to the basic block that contains C. The first instruction of this basic block is identified as $h(C)$, to account for the possibility of more than one Φ-instruction in the basic block. Thirdly, A is the nth argument of C, where n is the index of B in the list of jump instructions that target $h(C)$.

4.3.2.2 Identifying Graph Dominators

In order to express domination in the control flow graph [138], the following syntax is used:

domination(*variable*,*variable*)
strict_domination(*variable*,*variable*)

For both these constraints, the values are implicitly limited to instructions. The expression "**domination**({A},{B})" means that A is a dominator of B, i.e. any path through the control flow graph from the entry node to B must go through A. Strict domination additionally requires that A and B are distinct. Complementing these constructs, there are post-dominator versions "**post_domination**" and "**strict_post_domination**", as well as the following generalisation:

all control flow from *variable* **to** *variable* **passes through** *variable*

This constraint is similar to a standard control flow domination, but instead of taking paths from the control flow origin, a third variable is used for parametrisation, as required for Listing 4.6.

4.3.2.3 Additional Atomic Constraints

The set of atomic constraints that are supported by CAnDL can easily be extended. Possible additions include constraints on function attributes and value constraints on literals. This will be further explored in Chapters 5 and 6.

```
Constraint ValueChain
 {element[i] ∈ {element[i+1]}.args foreach i=0..4
End
```

```
Constraint ValueChain
( {element[0]} ∈ {element[1]}.args
∧ {element[1]} ∈ {element[2]}.args
∧ {element[2]} ∈ {element[3]}.args
∧ {element[3]} ∈ {element[4]}.args)
End
```

Listing 4.2: Example for the expansion of range constraints in CAnDL: The specification at the top can be "unrolled" manually, resulting in the equivalent, but more verbose, specification below.

4.3.3 Range Constraints

Building on top of the atomic constraints and the fundamental conjunction and disjunction constructs, there are range based constraints that operate on arrays of variables:

$$\begin{array}{c} \textit{conRange} ::= \textit{formula}\ \textbf{foreach}\ \langle \textbf{s} \rangle\ \textbf{=}\ \textit{index}\ \textbf{..}\ \textit{index} \\ \textit{disRange} ::= \textit{formula}\ \textbf{forany}\ \langle \textbf{s} \rangle\ \textbf{=}\ \textit{index}\ \textbf{..}\ \textit{index} \end{array}$$

These constructs allow the replication of a constraint formula over a range of indices. This is demonstrated in Listing 4.2, which shows two equivalent CAnDL programs, the first one formulated with *conRange* and the second one without. In both cases, the program specifies an array of five variables with data flow from each element to the next. This shows how *conRange* can be expanded by duplicating the contained formula, with logical conjunctions binding the replicas of the formula together. Otherwise identical, the *disRange* construct is based on logical disjunctions instead.

The syntactic structure of variable identifiers carries no semantic information for atomic constraints. Clearly, this is not true for range expressions, which rely on index calculations in order to evaluate the underlying variables. Therefore, it is important to introduce next the precise syntax for variable names, which are constructed as follows:

$$\begin{array}{c} \textit{variable} ::= \langle \textbf{s} \rangle \mid \textit{variable}\ \textbf{[}\ \textit{calculation}\ \textbf{]} \mid \textit{variable}\ \textbf{.}\ \langle \textbf{s} \rangle \\ \textit{calculation} ::= \langle \textbf{s} \rangle \mid \langle \textbf{n} \rangle \mid \textit{calculation}\ \textbf{+}\ \textit{calculation} \mid \textit{calculation}\ \textbf{-}\ \textit{calculation} \end{array}$$

The syntax of variable identifiers in CAnDL aligns closely to C/C++ conventions. They can contain simple index calculations to support the range constructs, as well as a hierarchical structure. This hierarchical structure corresponds to the modularity capabilities of CAnDL that are introduced in the next section.

4.3.4 Modularity

Modularity is central to CAnDL, and it is achieved using the *include* construct.

include ⟨**s**⟩ [(*variable* **->** *variable* { **,** *variable* **->** *variable* } **)**] [@ *variable*]

Note that the syntax in square brackets is optional and the syntax in curly brackets may be repeated. The basic version of *include*, using neither of the two optional structures, is simple. It copies the formula that corresponds to the identifier verbatim into the current specification. If "[@*variable*]" is specified, then all the variable names of the inserted constraint formula are prefixed with the given variable name, separated by a dot. This namespaces the inserted formula and prevents unwanted interactions with surrounding constraints. The other optional syntax is used to rename variables in the included formula. In contrast to the prefix syntax, this increases the interaction with surrounding constraints by injecting other variables. Accordingly, if both optional constructs are used, the prefix is only applied to variables that are not renamed.

Listing 4.3 illustrates this with two equivalent constraint programs. Both programs specify an addition of four values, first adding pairwise and then adding the intermediate results. In the code at the top, a formula for the addition of two values is bound to the name "`Sum`". This is then included three times in another formula named "`SumOfSums`". Using the optional grammatical constructs, the formula operates on a different set of variables each time, such that the third addition takes the results of the previous two as input.

```
Constraint Sum
( opcode{out} = add
∧ {in1} = {out}.args[0]
∧ {in2} = {out}.args[1])
End
Constraint SumOfSums
( include Sum@{sum1}
∧ include Sum@{sum2}
∧ include Sum({sum1.out}->{in1},{sum2.out}->{in2}))
End
```

```
Constraint SumOfSums
( opcode{sum1.out} = add
∧ {sum1.in1} = {sum1.out}.args[0]
∧ {sum1.in2} = {sum1.out}.args[1]
∧ opcode{sum2.out} = add
∧ {sum2.in1} = {sum2.out}.args[0]
∧ {sum2.in2} = {sum2.out}.args[1]
∧ opcode{out} = add
∧ {sum1.out} = {out}.args[0]
∧ {sum2.out} = {out}.args[1])
End
```

Listing 4.3: Example for expansion of the *include* construct: Both specifications are equivalent.

```
1 Constraint CollectArguments
2 ( ir_type{ins} = instruction
3 ∧ collect i 8 ({arg[i]} ∈ {ins}.args))
4 End
```

Listing 4.4: Simple *collect* example in CAnDL: Direct data dependencies of "ins" are collected.

4.3.4.1 Collect-all Constraints

The *collect* construct captures all possible solutions of a given formula, as in Definition 2.16.

collect ⟨**s**⟩ *index formula*

In Listing 4.4, the variables "arg[0]",...,"arg[7]" are specified to contain all of the direct data dependencies of "ins". In this example, solutions of "arg[i]" for a given value of "ins" are identified. The second argument gives an upper limit to the number of collected variables. The constant upper bound "8" is required here to keep the dimensionality of the search space finite. If "ins" has less than 8 arguments, the remaining "arg[]" are constrainted to *unused*. The first argument of *collect* specifies the name of an index variable that is used to detect which variables belong to the collected set.

This example is now extended to show how *collect* can be used to implement quantifiers. Consider the task of detecting instructions with only floating-point arguments. This involves the "∀" quantifier, as it is equivalent to the following equation:

$$\texttt{ins} \text{ has only } \textit{float} \text{ arguments} \iff [\forall x\colon\ (x,\texttt{ins}) \in DFG^*_{\mathscr{F}} \implies (\textit{float},x) \in I_{\mathscr{F}}] \qquad (4.2)$$

This can be rewritten to an equivalent formulation on sets:

$$S_1 = \textit{select}(\texttt{ins}, \textit{rev}(DFG^*_{\mathscr{F}})) \subset \textit{select}(\textit{float}, T_{\mathscr{F}}) = S_2.$$

Elementary set theory gives $S_1 \subseteq S_2 \iff S_1 = S_1 \cap S_2$. Therefore, if a set S is constrained to be equal to both S_1 and $S_1 \cap S_2$, then Equation (4.2) gives that this is satisfiable if and only if "ins" has only floating-point arguments. This condition can be expressed in CAnDL, as is shown in Listing 4.5. With the first *collect* statement at line 3, the set "arg" is constrained to be equal to S_1 and with the second one at lines 4-5, it is constrained to be equal to $S_1 \cap S_2$.

```
1 Constraint FloatingPointInstruction
2 ( ir_type{ins} = instruction
3 ∧ collect i 8 ( {ins} ∈ {arg[i]}.args)
4 ∧ collect i 8 ( {ins} ∈ {arg[i]}.args
5                 ∧ data_type{arg[i]} = float))
6 End
```

Listing 4.5: *Collect* restricts "ins" to instructions with only (up to 8) floating-point operands.

The exact same approach can be used for much more complex analysis tasks. For example, the final case study at the end of this chapter uses *collect* statements to restrict all array accesses within a loop to be affine in the loop iterators. First, *collect* all memory accesses in the loop (i.e. all "`load`" and "`store`" instructions), and then use a second *collect* statement to enforce affine calculations of the access indices that are used for these instructions.

4.3.5 Expressing Larger Structures

The modularity of CAnDL allows for the creation of building blocks that are used by multiple CAnDL specifications. This includes classic control flow structures, such as single-entry single-exit (SESE) regions and different classes of loops, as well as memory access patterns. These definitions a standard library that enables higher-level programming with CAnDL. This section gives an overview of how some of these standard building blocks are defined.

Listing 4.6 gives the specification of SESE regions in CAnDL. Such regions are spanned by the variables "`begin`" and "`end`", with an incoming control flow edge from "`precursor`" to "`begin`" (line 3) and an outgoing control flow edge from "`end`" to "`successor`" (line 5). Graph domination constraints at lines 8 and 9 guarantee that these are the only entry and exit of the region. Additional domination constraints at lines 6–7 and lines 10–13 make sure that there are no jumps from outside the region into an instruction of the region that is not "`begin`", and similarly that there are no early exits of the region. Finally, regions are restricted to align with basic block boundaries in lines 2 and 4.

Note that the use of variables "`precursor`" and "`successor`" prevents the detection of SESE regions without a precursor or successor. These two special cases could easily be captured with additional constraints that account for the possibility of an *unused* successor or precursor. However, this situation is only relevant in the case of basic blocks that end with a return statement or that are the entry point of a function. In the context of the loop-based algorithmic structures that this thesis focuses on, basic blocks can be assumed to have a precursor and successor.

Natural loops are easily defined in CAnDL, as shown in Listing 4.7. The specification includes "`SESE`" and adds only a single additional constraint for the back edge of the loop at line 3. Additional extensions to this specification are added to define more restricted loop structures, such as for-loops. This involves the identification of the loop iterator, the breaking condition, and the corresponding iteration space boundaries. In order to be a valid for-loop, the end of the iteration space must be determined before the loop is entered. This is expressed in Listing 4.8, restricting "`value`" to be either a function argument, a constant, or an instruction that strictly dominates the loop entry. Note that this formula is underspecified on its own. In order to get a useful constraint specification, it has to be included in larger CAnDL programs that also include "`Loop`".

```
Constraint SESE
( opcode{precursor} = branch
∧ {begin} ∈ {precursor}.successors
∧ opcode{end} = branch
∧ {successor} ∈ {end}.successors
∧ domination({begin}, {end})
∧ post_domination({end}, {begin})
∧ strict_domination({precursor}, {begin})
∧ strict_post_domination({successor}, {end})
∧ all control flow from {begin} to {precursor}
          passes through {end}
∧ all control flow from {successor} to {end}
          passes through {begin}) End
```

Listing 4.6: Single-entry single-exit region in CAnDL: The region spans "begin" to "end", with control flow "precursor" to "begin" as the entry, control flow "end" to "successor" as the exit.

```
Constraint Loop
( include SESE
∧  {begin} ∈ {end}.successors) End
```

Listing 4.7: Loops are defined in CAnDL as single-entry single-exit regions with a back edge. The back edge does not break the "single-exit" condition because it does not exit the region.

```
Constraint LocalConst
( ir_type{value} = literal
∨ ir_type{value} = argument
∨ strict_domination({value}, {scope.begin})) End
```

Listing 4.8: This specification restricts "value" to remain constant during loop execution. It is underspecified on its own and should be included in larger CAnDL programs that also include "Loop", renaming "value" to the specific variable that needs to be constrained in this way.

```
Constraint PointerAccess
( ( (opcode{access} = store ∧ {pointer} = {access}.args[1])
  ∨ (opcode{access} = load  ∧ {pointer} = {access}.args[0]))
∧ domination({scope.begin}, {access})
∧ post_domination({scope.end}, {access})) End

Constraint ArrayAccess
( include PointerAccess
∧ opcode{pointer} = gep
∧ {base_pointer} = {pointer}.args[0]
∧ include LocalConst({base_pointer}->{value})) End

Constraint LoopWithOnlyArrayAccesses
( include Loop @ {loop}
∧ collect i N (include PointerAccess({loop}->{scope})
                                        @{acc[i]})
∧ collect i N (include ArrayAccess  ({loop}->{scope})
                                        @{acc[i]})) End
```

Listing 4.9: Building blocks are combined to restrict the permitted memory access within a loop.

Another class of important building blocks is different categories of memory access. These form a hierarchy of restrictiveness and include multi-dimensional array access and array access that is affine in some loop iterators. Listing 4.9 combines several of the introduced building blocks to specify a loop in which all memory access locations are calculated as offsets from base pointers that are constant within the loop. This, for example, excludes loops with pointer-chasing. The "`ArrayAccess`" specification can be extended with more restrictive memory access patterns as previously mentioned. This is crucial to define advanced compiler analysis passes, such as the detection of regions suitable for polyhedral code analysis.

4.4 Implementation

CAnDL is integrated into the LLVM framework. CAnDL programs are read by the CAnDL compiler during LLVM build time, which then generates C++ source code to implement the specified LLVM analysis functionality. This code depends on the generic backtracking solver derived in Chapter 2, which is incorporated directly into the LLVM codebase. The generated code is compiled and linked together with the existing LLVM libraries. The Clang compiler, which is built on LLVM, then automatically invokes this solver during the compilation of user programs, after the optimisation passes.

The resulting Clang binary, built on the modified version LLVM, uses the solver to search for the specified computational structures and outputs the found instances into report files. It also makes the results available to ensuing transformation passes in the form of C++ structures.

4.4.1 Normalisation of LLVM IR

For reliable detection of code structures, the normalisation of LLVM IR via optimisation passes is critical. The promotion of memory to registers, loop-independent code motion, constant folding, loop normalisations, inlining, and other standard transformations result in predictable code structures that simplify the formulation of CAnDL specifications.

For example, elements of C++ `vector`s are accessed via the overloaded `operator[]`. Such operators appear in LLVM IR as opaque function calls and potentially hinder analysis with CAnDL specifications. However, these operators are inlined during optimisation, and parts of the resulting code are propagated out of loops by loop-independent code motion. The remaining array access can easily be captured by CAnDL using the same specifications that treat C code. Listing 4.10 demonstrates this normalising effect of optimisations. The top of the figure shows the C++ implementation of a function that computes a reduction over two sequences of integers. The first sequence is stored in a plain C array, whereas the second is in an instance of the `vector` class. In the optimised LLVM IR code, the difference between these implementations has disappeared from the reduction loops at lines 24–32 and 39–47.

```
int reduce(int* input1, size_t input1_size,
           std::vector<int> input2) {
    int result1 = 0, result2 = 0;
    for(size_t i = 0; i != input1_size; i++)
        result1 += input1[i];
    for(size_t i = 0; i != input2.size(); i++)
        result2 += input2[i];
    return result1 + result2;
}
```

```
define i32 @_Z23reducePimSt6vectorIiSaIiEE(i32*, i64,
                  %"class.std::vector"* ) {
  %4 = icmp eq i64 %1, 0
  br i1 %4, label %5, label %17

; <label>:5:
  %6 = phi i32 [ 0, %3 ], [ %22, %17 ]
  %7 = getelementptr inbounds %"class.std::vector",
                  %"class.std::vector"* %2,
                  i64 0, i32 0, i32 0, i32 1
  %8 = bitcast i32** %7 to i64*
  %9 = load i64, i64* %8, align 8
  %10 = bitcast %"class.std::vector"* %2 to i64*
  %11 = load i64, i64* %1
  %12 = icmp eq i64 %9, %11
  %13 = inttoptr i64 %11 to i32*
  br i1 %12, label %25, label %14

; <label>:14:
  %15 = sub i64 %9, %11
  %16 = ashr exact i64 %15, 2
  br label %28

; <label>:17:
  %18 = phi i64 [ %23, %17 ], [ 0, %3 ]
  %19 = phi i32 [ %22, %17 ], [ 0, %3 ]
  %20 = getelementptr inbounds i32, i32* %0, i64 %18
  %21 = load i32, i32* %20
  %22 = add nsw i32 %21, %19
  %23 = add nuw i64 %18, 1
  %24 = icmp eq i64 %23, %1
  br i1 %24, label %5, label %17

; <label>:25:
  %26 = phi i32 [ 0, %5 ], [ %33, %28 ]
  %27 = add nsw i32 %26, %6
  ret i32 %27

; <label>:28:
  %29 = phi i64 [ 0, %14 ], [ %34, %28 ]
  %30 = phi i32 [ 0, %14 ], [ %33, %28 ]
  %31 = getelementptr inbounds i32, i32* %13, i64 %29
  %32 = load i32, i32* %31
  %33 = add nsw i32 %32, %30
  %34 = add i64 %29, 1
  %35 = icmp eq i64 %34, %16
  br i1 %35, label %25, label %28
}
```

Listing 4.10: Standard LLVM optimisations as normalising passes: The same computation is expressed with a plain C array and a C++ `std::vector` respectively. During compilation, the interaction of function inlining and loop-independent code motion results in equivalent intermediate representation code for the reduction loops at lines 24–32 and at lines 39–47.

Most standard optimisations that LLVM provides have this implicit effect of normalising the code, but there are some notable exceptions. Most importantly, this involves optimisations that are not reliably applied uniformly on all code but utilise opaque cost heuristics or that require complex preconditions. Notably, this includes loop unrolling and vectorization, which are hence disabled. The CAnDL-enabled compiler invokes the detection functionality directly after optimisations and expects the pipeline to be configured with the command-line options "`-Os -fno-unroll-loops -fno-vectorize -fno-slp-vectorize -ffast-math`".

Some other LLVM optimisations also obfuscate the resulting LLVM IR but cannot be disabled via command-line options. Furthermore, there is some semantic information about multidimensional arrays that LLVM retains from the input language that effectively results in multiple distinct ways to express array accesses that are equivalent. To alleviate this, in addition to standard optimisations, the CAnDL-enabled compiler applies some custom transformations in order to pass on predictable LLVM IR code to the solver:

- Strength reductions result in special cases that would have to be covered explicitly by CAnDL specifications. To remove the requirement of considering these special cases, two strength reduction optimisations are reversed: `add`$\mapsto$`or` and `mul`$\mapsto$`shift`. Such instruction specialisations run counter to the goal of normalising the IR.

- Multi-dimensional array access can be expressed directly in LLVM IR. However, this only works for statically allocated arrays. As CAnDL should work on dynamically-sized arrays as well, this representation cannot be relied on and, in effect, introduces additional complexity. In order to remove this complication, complex "`getelementptr`" instructions are simplified to add an offset to a raw pointer. The index calculations are then performed entirely as integer arithmetic on the indices. This process effectively flattens multi-dimensional arrays. Chains of "`getelementptr`" instructions are merged, again resulting in integer arithmetic instead.

- Some interactions of "`select`" and "`getelementptr`" instructions are transformed as `(c?a[i]:a[j])`$\mapsto$`a[c?i:j]` to minimise the number of "`getelementptr`" instructions.

Finally, some structures in the LLVM IR are not modified by normalising transformations, but they are omitted in the SSA model that underpins the CAnDL specifications. This applies to all integer extension instructions and pointer conversions. These constructs are represented in LLVM IR as instructions with a single argument. In the SSA model, data flow edges skip these instructions, pointing from the argument of the instruction to any use of it. The conventional use of 32-bit integers on 64-bit systems obfuscates the SSA model without these omissions, particularly in the context of 32-bit loop interators that are automatically elevated to 64-bit integers during optimisation but then have to be downcast in every iteration before applying a comparison operator.

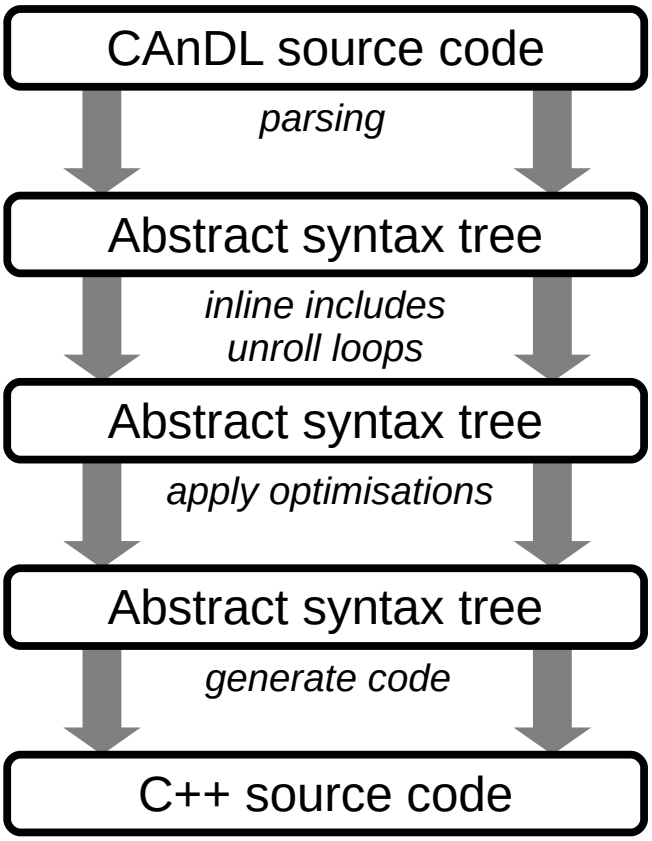


Figure 4.2: Flow within the CAnDL compiler: The CAnDL source code gets lowered in several steps to generated C++ source code.

4.4.2 The CAnDL Compiler

The CAnDL compiler is responsible for parsing CAnDL programs and generating C++ code from them. An overview of its flow is shown in Figure 4.2. The frontend reads in CAnDL source code and builds an abstract syntax tree. This syntax tree is simplified in two steps to eliminate some of the higher-order constructs of CAnDL. The *include* clauses are inlined and contained variables transformed accordingly. Furthermore, *conRange* and *disRange* are lowered to conjunction and disjunction constructs by duplicating the contained constraint code and renaming its variables appropriately for each iteration. The remaining core language consists only of atomics, conjunctions, disjunctions and collections.

The CAnDL compiler then applies optimisations to speed up the solving process using the later generated C++ code. For example, nested conjunctions and disjunctions are flattened wherever possible. Furthermore, if shallow equivalence of two variables is enforced in a conjunction, one of the two variables is chosen as having higher priority. All other occurrences of the other variable are then replaced with that one.

Finally, the compiler generates C++. This essentially means generating a function which at runtime constructs the constraint problem as a graph structure that is accessible to the solver.

```
Constraint SimpleAddition
( opcode{addition} = add
∧ {addition}.args[0] = {left}
∧ {addition}.args[1] = {right}) End
```

```
// Step 1: Instantiate Atomic Constraints
auto constr0 = make_shared<AddInstruction>(model);
auto constr1 = make_shared<FirstArgument> (model);
auto constr2 = make_shared<SecondArgument>(model);
// Step 2: Compose Higher-Level Constraints
auto constr3 = make_shared<Conjunction>(
                    constr0,
                    select<0>(constr1),
                    select<0>(constr2));
// Step 3: Assemble Backtracking Solution
vector<pair<string,shared_ptr<BacktrackingPart>>> result(3);
result[0] = make_pair("addition", constr3);
result[1] = make_pair("left",     select<1>(constr1));
result[2] = make_pair("right",    select<1>(constr2));
```

Listing 4.11: C++ code generation: The code is generated to first instantiate atomic constraints, then compose higher-level constructs, and finally assemble a backtracking solution for solving.

4.4.2.1 C++ Code Generation

The code generation process is demonstrated with an example in Listing 4.11. Every atomic constraint in CAnDL results in a line of C++ code that constructs an object of a corresponding class: In this case, the involved atomic constraints are implemented by "`AddInstruction`", "`FirstArgument`" and "`SecondArgument`". These objects are instantiated as shared pointers.

The compiler then generates analogous objects for the conjunction, disjunction and collect structures. In our example, this only affects the "`addition`", which is part of a conjunction clause. This results in an additional object construction that instantiates the "`Conjunction`" class corresponding to the "∧" operator in CAnDL.

Constraint classes that implement constraints operating on a single variable directly expose the "`BacktrackingPart`" interface introduced in Listing 2.1. In the example, this applies to "`AddInstruction`" and "`Conjunction`". For more complex constraints, the "`select<>`" template is used to specify which variable of a constraint is being considered. At lines 8–9, this is used to extract the parts of backtracking solutions referring to the "`addition`" variable, and to then pass them as arguments to the conjunction.

Finally, the generated objects are inserted into a vector, together with the corresponding variable names. The variables are in the order of appearance in the CAnDL code. This vector corresponds to the backtracking solution of the constraint problem and is passed to the solver.

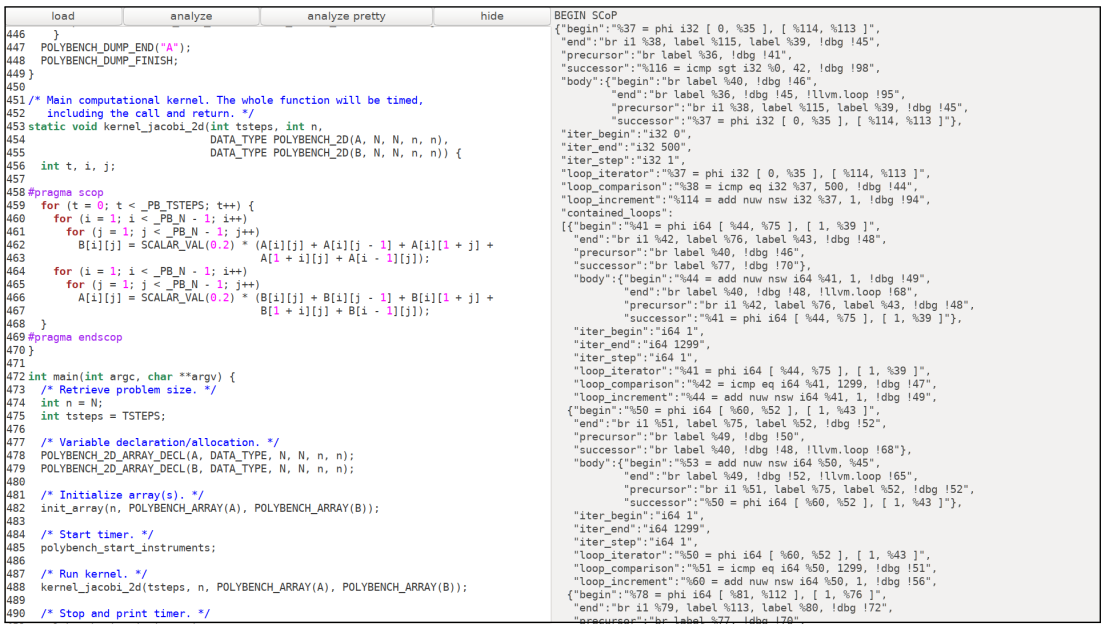


Figure 4.3: Interactive CAnDL test tool: The left hand panel shows a Static Control Part (SCoP) in Polybench jacobi-2d, the right hand panel shows the constraint solutions found by the solver.

4.4.3 Developer Tools

CAnDL simplifies the construction of compiler analysis functionality, but reasoning about the semantics of compiler intermediate representation remains difficult. The solver detects whatever the programmer specifies, without any additional effort, but it is difficult to ensure that the CAnDL code actually specifies the structures that the programmer intended. Generally, the accuracy of CAnDL programs can only be ensured with thorough testing, and it is important to keep in mind that CAnDL is targeted at expert compiler developers.

In order to make the debugging of CAnDL programs more feasible, supporting tools are provided. Most importantly, this includes an interactive GUI, where developers can test out corner cases of their CAnDL programs to find false positives and false negatives. This GUI is shown in Figure 4.3, with an example from one of the use cases presented in Section 4.5.

In the left half, part of a C program from the PolyBench benchmark suite is visible, which implements a two-dimensional Jacobi stencil. The GUI was configured to look for Static Control Parts (SCoPs), as described later. The user has clicked the “analyze” button, which triggered the analysis to run by invoking the modified Clang compiler. The GUI then read the report file and printed the results in the right-hand part of the figure.

The solver found a SCoP in the IR code (corresponding to lines 459–468 of the C program). The text on the right shows the hierarchical structure of the solution, with IR values assigned to every variable. The corresponding C entities can be recovered using the debug information that is contained in the generated LLVM IR code. By modifying the C code, the developer can test the detection and verify that no SCoP is detected if irregular control flow is introduced.

```
1  Constraint ComplexFactorisation
2  ( opcode{value} = add
3  ∧ {sum1.value} = {value}.args[0]
4  ∧ {sum2.value} = {value}.args[1]
5  ∧ include SumChain @ {sum1}
6  ∧ {product1.value} = {sum1.last_factor}
7  ∧ include MulChain @ {product1}
8  ∧ {product1.last_factor} = {product2.last_factor}
9  ∧ include SumChain @ {sum2}
10 ∧ {product2.value} = {sum2.last_factor}
11 ∧ include MulChain @ {product2}) End
```

Listing 4.12: Factorisation opportunities in CAnDL: This captures some opportunities that LLVM "`instcombine`" misses. "`SumChain`", "`MulChain`" are themselves specified in CAnDL (16 LoC).

4.5 Case Studies

The effectiveness of CAnDL was evaluated in three different use cases. Firstly, it was used for detecting opportunities to apply a simple peephole optimisation. Secondly, CAnDL was applied to graphics shader code optimisation. Finally, the detection of Static Control Parts (SCoPs) that are amenable for polyhedral code transformations was implemented in CAnDL. Where possible, the evaluation compares the number of lines of CAnDL code, the program coverage achieved and performance against prior approaches.

4.5.1 Case Study 1: Simple Optimisations

Arithmetic simplifications in LLVM are implemented in the "`instcombine`" pass. An example of this is the standard factorisation optimisation that uses the law of distributivity to simplify integer calculations, as shown in Equation (4.3). Within "`instcombine`", this is implemented in 203 lines of code (commit 7de5f26d, InstructionCombining.cpp lines 549-756 excluding lines 637-641), and additionally uses supporting functionality shared with other optimisations.

$$a*b+a*c \rightarrow a*(b+c) \tag{4.3}$$

This analysis problem can be formulated in CAnDL, as shown in Listing 4.12. Crucially, at lines 5,7,9,11, the specification makes use of "`SumChain`" and "`MulChain`", which allows the CAnDL program to capture a large, generalised class of opportunities for factorisation. The "`instcombine`" pass has limited support for this, and first requires the application of associative and commutative laws to reorder the values. For example, this is needed for Equation (4.4), and only partially supported by LLVM with the additional "`reassociate`" pass.

$$a*b+c+d*a*e->a*(b+d*e)+c \tag{4.4}$$

	LLVM	CAnDL
Lines of Code	203	12
Detected in NPB	1	1 + 2
Detected in Parboil	0	0 + 1
Detected in Rodinia	24	24 + 4
Total Compilation time	152.2s	152.2s+7.8s

Table 4.3: Factorisations enabled by LLVM vs CAnDL

4.5.1.1 Experimental Setup

The specification in Listing 4.12 was evaluated against the default factorisation optimisation in "`instcombine`" on three different benchmark collections: the sequential C versions of the NAS Parallel Benchmarks [139], as provided by Seo et al. [140]; the C/C++ Parboil programs by Stratton et al. [141]; and the OpenMP C/C++ programs of the Rodinia benchmark suite [142]. The existing LLVM "`instcombine`" pass was extended, so that it automatically logs every time that it successfully applied the "`tryFactorization`" function.

The individual benchmark programs in the three benchmark suites consist of 94915 lines of code in total. For each benchmark suite, the total number of reported factorisations, as well as the total compilation time, were measured. The standard LLVM optimisation was then disabled, and the CAnDL-generated detection functionality was used instead. The same application programs were compiled with the same version of Clang and identical compiler options, reporting the number of factorisations found and measuring the total compilation time again. This timing includes all the other passes within LLVM, plus the CAnDL code path.

4.5.1.2 Results

The results of the evaluation are shown in Table 4.3. In two of the benchmark collections – NPB and Parboil – there are only a limited number of factorisation opportunities. LLVM was unable to perform any factorisation in the entire Parboil suite. However, the Rodinia suite contains more opportunities, mostly in the "particlefilter" and "mummergpu" programs.

In all three benchmarks suites, the CAnDL system found all factorisation opportunities that the "`instcombine`" pass identified. In addition, it detected an additional 7 cases across all programs. Just 12 lines of CAnDL code were able to capture more factorisation opportunities than 200 lines of C++ code in LLVM.

Using CAnDL on large benchmark suites increased total compilation time by $\sim$5%. Given the small impact of individual peephole optimisations, an evaluation of the performance or code size impact vs "`instcombine`" is unlikely to yield significant results.

```
1  Constraint FloatingPointAssociativeReorder
2  ( include VectorMulChain
3  ∧ collect j N
4    ( {hoisted[k]} = {factors[i]} forany i=0..N
5    ∧ include ScalarHoist({hoisted[j]}->{out},
6                          {scalar[j]}->{in})@{hoist[j]})
7  ∧ collect j N
8    ( {nonhoisted[j]}  = {factors[i]} forany  i=0..N
9    ∧ {nonhoisted[j]} != {hoisted[i]} foreach i=0..N))
10 End
```

Listing 4.13: CAnDL defines multiplication chains with genuine vectors and hoisted scalars: After separating the two cases, some of the multiplications can be performed on scalars instead.

4.5.2 Case Study 2: Graphics Shader Optimisations

Graphics computations often involve arithmetic on vectors of single-precision floating-point values, which can represent vertex positions in space or colour values. Established graphics shader compilers utilise the LLVM intermediate representation internally [143].

In real shader code, there are often element-wise products of several floating-point vectors, where some of the factors are actually scalars that were hoisted to vectors. By reordering the factors and delaying the hoisting to vectors, some of the element-wise vector products can be simplified to products on scalars, as shown by example in the following equation.

$$\begin{aligned}\vec{x} &= \vec{a} *_v \vec{b} *_v \mathrm{vec3}(c) *_v \vec{d} *_v \mathrm{vec3}(e) \\ &= \mathrm{vec3}(c * e) * \vec{a} *_v \vec{b} *_v \vec{d}\end{aligned}$$

For general-purpose code, such reordering can be problematic. This is due to computation artefacts in floating-point arithmetic. However, this is generally no problem in the domain of graphics processing. Instead, associative reordering can result in performance improvements when combined with lowering to scalar multiplications as discussed above.

The required analysis functionality for this optimisation was implemented with CAnDL, as shown in Listing 4.13. Firstly, the specification uses "`VectorMulChain`" to detect chains of floating-point vector multiplications. At lines 4–6, all the factors that are hoisted from some scalar are collected into the array "`hoisted`". Correspondingly, all the other factors are collected into the array "`nonhoisted`" at lines 7–9.

"`VectorMulChain`" and "`ScalarHoist`" are in turn implemented as CAnDL programs. "`VectorMulChain`" discovers chains of floating-point vector multiplications in the IR code. It is defined very similarly to "`SumChain`" and "`MulChain`", which were used in the previous case study. It guarantees chains of maximal length by checking that neither of the first two factors is a multiplication itself and that the last factor is not used in any multiplication.

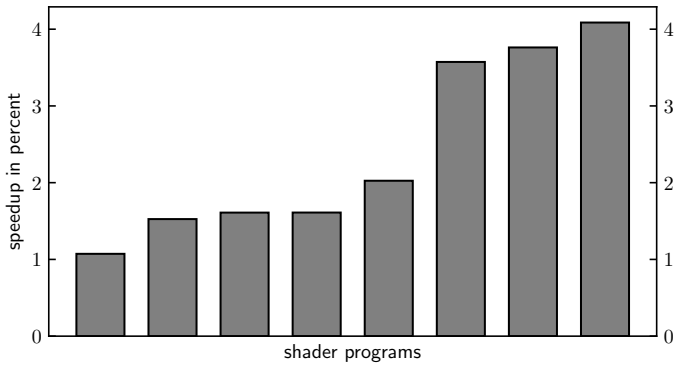


Figure 4.4: Speedup on Qualcomm Adreno 530 (evaluated on HTC 10, running Android 7.0)

"`ScalarHoist`" operates on "`in`" and "`out`", as well as some hidden internal variables. It specifies that "`out`" is a vector generated from "`in`" by setting all vector dimensions equal to "`in`". The precise implementation of this is highly specific to LLVM and involves combinations of the LLVM IR instructions "`insertelement`" and "`shufflevector`".

4.5.2.1 Experimental Setup

The LunarGLASS project [144] was used in this work to transform shaders into LLVM IR and back after optimisation. The CAnDL specification was applied to all fragment shaders in the GFXBench 4.0 suite taken from Kishonti [145]. A corresponding transformation pass was added to LLVM, which uses the detected solutions to implement the described optimisation. This was done by constructing the appropriate scalar and vector multiplications from the arrays "`hoisted`" and "`nonhoisted`", and then replacing the result of the original multiplication chain with the final multiplication in these newly generated instructions. Standard dead code elimination automatically removes the remnants of the original calculation.

The performance impact was evaluated on the Qualcomm Adreno 530 GPU. To measure the baseline of benchmark performance, all shaders were compiled with the default Qualcomm graphics stack. They were then compiled with LunarGLASS into LLVM, CAnDL was applied, and they were transformed back into GLSL code [146]. To evaluate the impact, the result was passed through the default graphics stack again, and the performance measured.

4.5.2.2 Results

There were 19 solutions to the specification across the benchmarks, and the transformation had an impact on the performance of 8 fragment shaders. The resulting performance impact is shown in Figure 4.4. Evidently, there are opportunities for such associative reordering that the default graphics stack misses. Although the performance impact was moderate with 1–4% speedup on 8 of the fragment shaders, it shows how new analysis can be rapidly prototyped and evaluated with only a few lines of code.

4.5.3 Case Study 3: Detection of Polyhedral SCoPs

The polyhedral model [27, 28] allows compilers to utilise powerful mathematical reasoning to detect parallelism opportunities in sequential code and to implement code transformations. However, this applies only for a restrictive class of well-structured loop nests. More precisely, conventional polyhedral code transformations are applicable to Static Control Parts (SCoPs). Detecting SCoPs is a fundamental and necessary first step for any later polyhedral optimisation.

Implementations of the polyhedral model may differ in their precise definition of SCoPs. The definition of Semantic SCoPs from the Polly compiler by Grosser et al. [26] was used for reference here. SCoP detection functionality was implemented in CAnDL and compared against Polly, which is also implemented as an extension to LLVM. The use of the same definition for SCoPS and the implementation in the same compiler infrastructure allow for a direct comparison between Polly and CAnDL. The specification of SCoPs is significantly more complex than the required CAnDL code for the previous case studies. However, it can be broken into several components, with some of them shown in Listing 4.14.* The integer constants "10" and "20" in the *collect* statements are required for the solver to restrict the search space to a finite set of variables.

Structured Control Flow SCoPs require well-structured control flow. This means that each conditional jump within the corresponding piece of LLVM IR is accounted for by for-loops and conditionals. This is ensured with the *collect* constraints, as in Listing 4.5. The construct is used with the CAnDL specifications "`For`" (lines 21–25) and "`IfBlock`" (lines 26–30) that describe the control flow of for-loops and conditionals. All the involved conditional jump instructions are extracted, and it is checked that these are indeed all conditional jumps within the potential SCoP (lines 14–24 and lines 31-33).

Once the control flow has been established, the iterators of the loops (lines 4–5) are used to define affine integer computations in the loop (lines 6–9). This is done in a brute-force fashion with a recursive constraint program "`AffineCalc`" (line 50). It is checked that the iteration domain of all the for-loops is well-behaved, i.e. the boundaries are affine in the loop iterators.

Affine Memory Access All memory accesses in the SCoP must be affine. For this to be true, it needs to be verified that for each "`load`" and "`store`" instruction, the base pointer is loop-invariant, and the index is calculated affinely. The loop-invariant base pointer is easily checked in "`MemoryAccess` with the "`LocalConst`" program from Listing 4.8.

Checking the index calculations is more involved and is again based on the method that was demonstrated in Listing 4.5. The *collect* construct is used to find all of the affine memory accesses in all the loop nests (lines 43–54). Another *collect* gathers all "`load`" and "`store`" instructions (36–42), guaranteeing that both collections are identical.

*The complete CAnDL code for this section is in Appendix B.

```
1  Constraint SCoP
2  ( include For @ {loop}
3  ∧ include StructuredControlFlow({loop}->{scope}) @ {control}
4  ∧ {inputs[0]} = {loop.iterator}
5  ∧ {inputs[i]} = {control.loop[i-1].iterator} foreach i=1..10
6  ∧ include AffineControlFlow({loop}->{scope},
7                              {inputs}->{inputs}) @ {control}
8  ∧ include AffineMemAccesses({loop}->{scope},
9                              {inputs}->{inputs}) @ {accesses}
10 ∧ include SideEffectFreeCalls({loop}->{scope}) @ {effects})
11 End
12
13 Constraint StructuredControlFlow
14 ( collect i 20 ( opcode{branch[i].value} = branch
15                 ∧ {branch[i].target1} =
16                       {branch[i].value}.successors[0]
17                 ∧ {branch[i].target2} =
18                       {branch[i].value}.successors[1]
19                 ∧ include ScopeValue({scope}->{scope},
20                              {branch[i].value}->{value}))
21 ∧ collect i 10 ( include For @ {loop[i]}
22                 ∧ domination({scope.begin},
23                              {loop[i].begin})
24                 ∧ strict_post_domination({scope.end},
25                                          {loop[i].end}))
26 ∧ collect i  10 ( include IfBlock @ {ifblock[i]}
27                 ∧ domination({scope.begin},
28                              {ifblock[i].precursor})
29                 ∧ strict_post_domination({scope.end},
30                                          {ifblock[i].successor}))
31 ∧ {loop[0..10].end,ifblock[0..10].precursor}
32       is the same set as {branch[0..20].value})
33 End
34
35 Constraint AffineMemAccesses
36 ( collect x 20 ( include MemoryAccess({scope}->{scope})
37                                          @ {newaccess[x]}
38                 ∧ opcode{newaccess[x].pointer} = gep
39                 ∧ domination({scope.begin},
40                       {newaccess[x].pointer})
41                 ∧ {newaffine[x].value} =
42                       {newaccess[x].pointer}.args[1])
43 ∧ collect x 20 ( include MemoryAccess({scope}->{scope})
44                                          @ {newaccess[x]}
45                 ∧ opcode{newaccess[x].pointer} = gep
46                 ∧ domination({scope.begin},
47                       {newaccess[x].pointer})
48                 ∧ {newaffine[x].value} =
49                       {newaccess[x].pointer}.args[1]
50                 ∧ include AffineCalc[M=10,N=6](
51                                           {scope}->{scope},
52                                         {inputs}->{input})
53                                           @ {newaffine[x]}))
54 End
```

Listing 4.14: Fragments of the specification of Scalar Control Parts (SCoPs) using CAnDL: SCoPs are defined at lines 1–11 by applying multiple restrictions to the containing loop. These restrictions are then individually implemented in CAnDL, using "`StructuredControlFlow`" and "`AffineMemAccesses`", shown in lines 13–33 and lines 35–54, respectively (cf. Appendix B).

	Polly	CAnDL
Lines of Code	1903	45
Detected in datamining	2	2
Detected in Linear-algebra	19	19
Detected in medley	3	3
Detected in stencils	6	6

Table 4.4: Polly and CAnDL detected all SCoPs.

4.5.3.1 Experimental Setup

The reliable detection of SCoPs was evaluated on the PolyBench suite [147], a collection of 31 benchmark programs that contain SCoPs of differing complexity from several application domains. For both the CAnDL-based approach and for the evaluation of Polly, it was counted how many of the computational kernels contained in the benchmark suite were captured in their entirety by the respective analysis.

Some post-processing of the generated constraint solutions was required to compare the results of CAnDL and Polly. This was needed because the output of CAnDL was not in the JSCoP format that Polly generates, but contained the raw constraint solution encoded as a JSON file. Furthermore, the CAnDL implementation did not merge consecutive outer level loops into a single SCoP of maximum size. Therefore, the detected loops from the CAnDL solver were extracted and grouped together. A Python script was then used to verify that they precisely covered the SCoPs detected by Polly.

4.5.3.2 Results

Table 4.4 shows that the CAnDL specification captured all the SCoPs that Polly detected. To measure the lines of code required, the CAnDL version was compared with the amount of code in `ScopDetection.cpp` of Polly. The same detection results were achieved with much fewer lines of code in CAnDL. Note that the line count that is given for the CAnDL program does not include all the CAnDL code involved in the detection of polyhedral regions. Code that is not specific to this idiom (such as loop structures) is considered as part of the CAnDL standard library. In the same way, the line count for Polly does not account for additional code that Polly relies on when detecting SCoPS, e.g. the expansive “ScalarEvolution” pass.

Detecting SCoPs with CAnDL incurred a significant overhead, but compile times remained below one second for each PolyBench program on an Intel i7-8665U processor. Detailed compile time results are presented in Table 4.5. The geomean overhead of enabling the CAnDL detection of SCoPS during compilation was 556%, mostly due to the large number of variables

	Clang compile times	Clang+CAnDL compile times
datamining/correlation	59	427
datamining/covariance	48	344
linear-algebra/2mm	53	599
linear-algebra/3mm	52	844
linear-algebra/atax	48	241
linear-algebra/bicg	51	246
linear-algebra/cholesky	52	245
linear-algebra/doitgen	46	424
linear-algebra/gemm	50	330
linear-algebra/gemver	53	329
linear-algebra/gesummv	46	170
linear-algebra/mvt	50	224
linear-algebra/symm	54	293
linear-algebra/syr2k	51	341
linear-algebra/syrk	49	327
linear-algebra/trisolv	49	187
linear-algebra/trmm	46	223
linear-algebra/durbin	54	309
linear-algebra/dynprog	47	291
linear-algebra/gramschmidt	54	704
linear-algebra/lu	48	245
linear-algebra/ludcmp	52	521
medley/floyd-warshall	47	204
medley/reg_detect	54	566
stencils/adi	54	753
stencils/fdtd-2d	52	500
stencils/fdtd-apml	61	977
stencils/jacobi-1d-imper	76	163
stencils/jacobi-2d-imper	56	299
stencils/seidel-2d	58	229

Table 4.5: Overhead of SCoP detection with CAnDL: Compile times are listed in milliseconds. The geomean increase in compile times due to SCoP detection with CAnDL was 556%.

that are required for expressing affine calculations. The compile time impact of all other idioms discussed in this thesis is much more moderate (compare Table 6.3). This overhead can be prohibitive during software development when frequent recompilation is required. However, it pales in comparison to autotuners, superoptimisation, and many other compiler techniques that involve constraint solvers and should be unproblematic in many contexts.

With a high-level representation of SCoPs, CAnDL allows polyhedral compiler researchers to explore the impact of relaxing or tightening the exact definition of SCoPs in a straightforward manner, enabling rapid prototyping. Simple commenting out of constraint statements in the CAnDL specification relaxes the conditions.

4.6 Conclusions

Optimising compilers require sophisticated program analysis in order to generate performant code. The state-of-the-art approach for implementing this functionality manually in C++ is not satisfactory, as exemplified by the complicated and error-prone "`instcombine`" pass in the LLVM compiler infrastructure.

The domain-specific Compiler Analysis Description Language (CAnDL) provides a more efficient approach. CAnDL specifications can automatically generate compiler analysis passes from a declarative description. They are easier to program and significantly reduce the code size and complexity when comparing against manual C++ implementations. Although CAnDL is based on a constraint programming paradigm and uses a backtracking solver to analyse the LLVM IR code, compile times never exceeded one second.

Many compiler analysis tasks are suitable for implementation with CAnDL. It can be used for the detection of standard peephole optimisation opportunities and the rapid prototyping of graphics shader optimisations. Despite its general approach, CAnDL scales to complex domain-specific code structures and can efficiently recognise large code regions suitable for polyhedral transformations.

Outlook The final case study showed that CAnDL can express the algorithmic structure of complex loop nests, not just peephole optimisation opportunities. This enables the capturing of entire loops at a time. The following chapters expand on this observation by investigating how the recognition of computational idioms can be leveraged for performance. While SCoPs were reported without informing additional compiler transformations, Chapter 5 applies auto-parallelisation techniques to a broad generalisation of reduction computations.

Chapter 5

Automatic Parallelisation of Reductions and Histograms*

The Compiler Analysis Description Language (CAnDL) makes the constraint methodology from Chapter 2 accessible to the Clang compiler within the LLVM compiler infrastructure. The previous chapter showed how this enables compiler analysis tools to be generated from declarative specifications, replicating and extending established compiler abilities. This chapter goes beyond restating and improving existing functionality. Instead, it implements automatic parallelisation methods for programs that were previously inaccessible to compiler reasoning.

Complex Reduction and Histogram Computations (CReHCs) are identified as a previously overlooked *computational idiom*. CReHCs constitute performance bottlenecks of important benchmark programs and share algorithmic structure that allows for targeted acceleration and parallelisation approaches. In contrast to the better-understood scalar reductions, CReHCs may contain indirect memory accesses and non-trivial control flow. Such loops have not typically been studied as a single class of calculations, but this chapter demonstrates that grouping them together allows for automatic detection and parallelisation using shared methods.

After introducing CReHCs, this chapter presents the Idiom Description Language (IDL) as an extension of CAnDL and uses it to implement the detection of this idiom. The additional language features of IDL enable capturing well-behaved kernel functions. CReHCs contain reduction operators that are modelled as kernel functions, but kernel functions are also required in the formulation of other computational idioms, such as stencil codes.

Grouping reductions and histograms together and formulating CReHCs in IDL enables their automatic recognition in C/C++ program code. The evaluation section shows results from benchmarking the outcomes of parallelisation routines that were implemented to complement the detection. Significant speedups were achieved on several programs from the NAS Parallel Benchmark suite, the Parboil Benchmarks, and Rodinia.

*This chapter is based on published research: Ginsbach and O'Boyle [2].

5.1 Introduction

Reductions occur widely in numerical applications. Parallelisation techniques for them were established by Rauchwerger and Padua [81], Yu and Rauchwerger [85], Jradi et al. [148], and others. Typical reductions successively apply an arithmetic operator over an array of numeric values in order to compute, for example, the sum of a set of floating-point numbers. Perhaps most prominently, a reduction makes up the innermost loop of the matrix multiplication kernel in the form of a dot product. In this form, reductions are critical to high performance computing workloads based on linear algebra, as well as embedded benchmarks and emerging machine learning and computer vision applications [91]. Even so, linear algebra allows for much more targeted optimisation methods. Interpreting the innermost loop of matrix multiplication as an arbitrary reduction is unlikely to achieve comparable performance.

However, there is a much larger class of computations that can be parallelised in much the same way. This is the class of Complex Reduction and Histogram Computations (CReHCs), a broad generalisation of conventional reductions that may also contain updates to dynamically selected elements in arrays. This manner of array access is characteristic for the calculation of histograms. CReHCs constitute an important set of standard program kernels. They typically have a higher arithmetic intensity and can be more profitably exploited on their own than simple scalar reductions.

Discovering and exploiting scalar reductions in programs has been studied for many years, by Pottenger and Eigenmann [78], among others. The treatment of generalisations of reduction operations has received less attention. While some work has been published on parallelising such computations, automatic detection mostly evades established compiler analysis methods. The reason is that histogram reductions intrinsically contain indirect memory accesses, posing a challenge to compilers that use standard data dependence [149] or the polyhedral model [28] as the basis of their analysis.

The constraint programming methodology derived in Chapter 2, however, is not inhibited by such indirect memory accesses. Chapter 4 developed the constraint programming language CAnDL using this methodology and showed that it could also recognise intricate control flow. This chapter presents the Idiom Detection Language (IDL) as an extension of CAnDL and uses it to implement the detection of CReHCs. The additional language features enable IDL to express well-behaved kernel functions, which capture the reduction operators of CReHCs. The IDL solver automatically identifies conforming subsets of LLVM IR during compilation.

This chapter focuses on deriving the detection of reductions and assumes that later compiler stages are responsible for mapping this to dedicated code generation backends. Nevertheless, to illustrate the potential performance of the scheme, it also introduces a complementing code generation pass. This transformation pass achieved significant program-wide speedups on those benchmarks where reductions were performance bottlenecks.

5.2 Motivation

Listing 5.1 shows a standard scalar reduction. Inside a single loop, the elements of the array "a" are successively accumulated in the variable "sum" using the "+=" operator. This accumulation creates dependencies between successive loop iterations. However, those are easy to break. Rather than sequentially accumulating "a" into "sum", it is possible to accumulate partial sums into thread-private copies of "sum" in parallel, which are then added to form the final value.

```
1  sum = 0;
2  for(i = 0; i < n; i++)
3    sum += a[i];
```

Listing 5.1: The most conventional example of a reduction is the adding up of values in an array: The reduction operator "+" *reduces* the array "a" to a single value – the reduction variable "sum".

Simple scalar reductions frequently occur, and existing compilers readily exploit them. Despite their abundance, scalar reductions rarely dominate execution time, and when they do, they are often part of more prominent algorithms, such as matrix multiplication. This often makes outer-scope parallelism more profitable to exploit. Therefore, the parallelisation of simple reductions alone has only a limited impact on program performance.

Listing 5.2 shows a more complex section of code that constitutes a performance bottleneck of the "Embarrassingly Parallel" benchmark program in the NAS Parallel Benchmarks. Note that the name of the benchmark refers to outer-loop parallelism. It is not directly evident that this computation can be treated as a reduction. However, it can be parallelised similarly to the simple sum. Creating thread-private copies of the reduction variables is again the critical step.

```
1  for(i = 0; i < NK; i++) {
2    x1 = 2.0 * x[2*i] - 1.0;
3    x2 = 2.0 * x[2*i+1] - 1.0;
4    t1 = x1 * x1 + x2 * x2;
5    if(t1 <= 1.0) {
6      t2   = sqrt(-2.0 * log(t1) / t1);
7      t3   = (x1 * t2);
8      t4   = (x2 * t2);
9      l    = MAX(fabs(t3), fabs(t4));
10     q[l] = q[l] + 1.0;
11     sx   = sx + t3;
12     sy   = sy + t4;
13   }
14 }
```

Listing 5.2: Example of a Complex Reduction and Histogram Computation: The bottleneck from the NAS Parallel Benchmarks can be parallelised as a reduction by privatising "sx", "sy", "q[]".

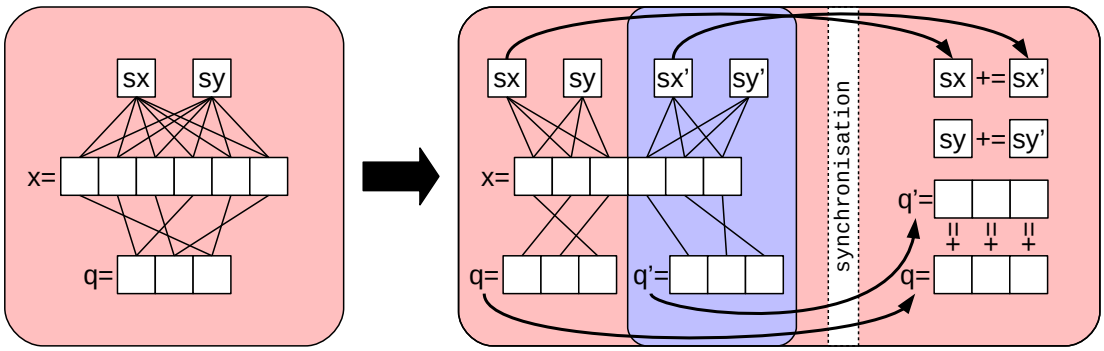


Figure 5.1: Parallelisation of Listing 5.2: The input "`x[]`" is split over two threads (red/blue) that operate on private copies of "`sx`", "`sy`", "`q[]`". These are merged after a synchronisation barrier.

The loop in Listing 5.2 is considered a CReHC for the following reasons: Firstly, there is an input array "`x[]`", from which values are read successively in each iteration. Secondly, the values "`sx`" and "`sy`" are scalar reduction variables, as is evident from lines 11–12. They are incremented conditionally, and the added value is not merely "`x[i]`" but the result of a complex calculation with control flow. Nonetheless, the calculation and the branching condition only depend on the input values "`x[2*i]`" and "`x[2*i+1]`". Thirdly, the array "`q[]`" is updated as a histogram array at line 10. The index "`l`" is not directly read from an input array, as in a conventional histogram, but again the calculation only depends on the input values.

As in the case of the simple sum, the parallelisation of this code requires the computation of partial results in separate memory locations before merging the partial results. The whole process is illustrated in Figure 5.1, with different colours on the right showing the distribution of the program over two threads. The scalar variables "`sx`", "`sy`" and the array "`q[]`" are privatised. Partial results are then accumulated in the local copies by partitioning the input array equally across the two threads. Finally, the threads are synchronised, and partial results merged. This is done by adding the local copies of "`sx`", "`sy`" and performing an element-wise addition of the local copies of "`q[]`".

Figure 5.2 gives an insight into why existing schemes do not detect such reductions by showing the compiler representation of this program. The histogram update occurs in the third basic block with the "load", assignment and "store" operations, but it is by no means obvious that this is a safe reduction. In fact, accurately detecting reductions is non-trivial. If in the original program, shown in Listing 5.2, the condition on line 5 was changed to "`t1 <= sx`", this would be no longer be a sound reduction, as there would now be a control dependence on an intermediate result. This, in turn, would manifest itself as an additional data dependence edge from block 3 to block 2 in Figure 5.2. Moreover, the code segment can only be classified as a reduction because all the function calls that are present are pure. Such details have to be checked to ensure correctness. What is needed is a way to specify these conditions precisely and to then automatically identify code regions that satisfy the constraints.

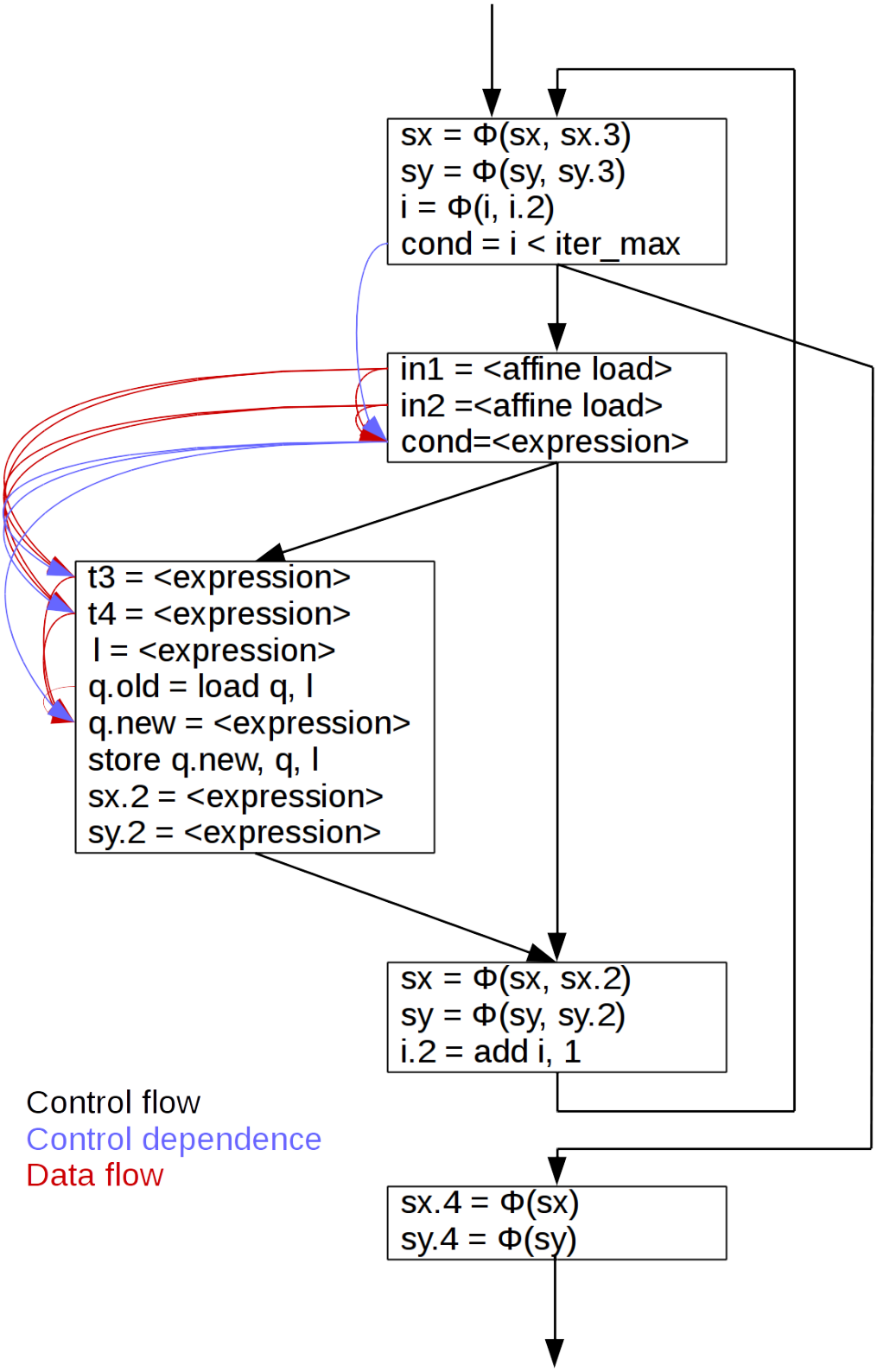


Figure 5.2: Fragments of the data flow, control flow, and control dependencies in the internal compiler representation of the source code in Listing 5.2: These interactions must be considered when searching for code that implements Complex Reduction and Histogram Computations.

5.3 Recognising CReHCs

There are three fundamental issues to address in exploiting CReHCs: detection, replacement, and profitability. This chapter focuses on the reliable detection of reductions. For evaluation purposes, a preliminary code generation phase that generates parallel code was implemented targeting the POSIX Threads interfaces. Smart profitability heuristics are essential in practice to determine whether or not to apply parallelising code transformations. For this work, a simple profile-based approach was used.

In the following sub-sections, the properties that loops need to satisfy to be interpreted as CReHCs are derived. Conditions for sound CReHCs are established and demonstrated on counterexamples. These observations then motivate extensions to CAnDL that culminate in the Idiom Detection Language (IDL). This language eventually enables the constraint-based specification of CReHCs for automatic compiler detection.

5.3.1 Constraint-Based Formulation

The motivation section described how CReHCs could be parallelised as reduction operations. For compilers to apply this parallelisation automatically, a precise specification of the suitable program loops is required. This section gives an overview of the necessary conditions for CReHCs, which are then precisely formulated in Sections 5.3.2 and 5.3.3.

5.3.1.1 Scalar Reductions

Informally, the following conditions are required to hold in a piece of source code for it to contain a computation that can be parallelised like a scalar reduction:

1. The code is contained in a for-loop, and the iteration space of the loop is known ahead of time (but not necessarily at compile time).
2. There is a scalar value x that is updated in every iteration.
3. One or multiple values $a_1, \ldots, a_n$ are read from arrays, and the access indices are affine in the loop iterator.
4. The updated value x' is computed as a term only of x, the array values $a_1, \ldots, a_n$, and of values that are constant within the loop.

This definition is broader than usual. In particular, it allows the reduction to encompass multiple input arrays. Furthermore, complex computations inside the reduction are possible, not just binary scalar operators. Note that condition 2 is enforced only at SSA level. Finally, this definition does not yet contain a commutativity condition that would be necessary to allow the parallelisation to work. Instead, it also captures intrinsically sequential scalar reductions.

```
1 sum = 0;
2 for(i = 0; i < n && sum < 10; i++)
3     sum += a[i];
```

```
1 sum = 0;
2 for(i = 0; i < n; i++)
3     b[i] += a[i];
```

```
1 chase = 0;
2 for(i = 0; i < n; i++)
3     chase = a[chase];
```

```
1 sum = 0;
2 active = true;
3 for(i = 0; i < n; i++) {
4     sum += active?a[i]:0;
5     active = active && (sum < 10);
6 }
```

Listing 5.3: Counterexamples to the four conditions: None of these computations can be parallelised as scalar reductions. The first and last example implement the same program.

Example Both "sx" and "sy" in Listing 5.2 satisfy the conditions. Firstly, the iteration space of the loop is bounded by "NK", which is constant. Secondly, within the SSA representation, both "sx" and "sy" are unconditionally updated via Φ-instructions, shown in Figure 5.2. This is despite the conditional statement in the original C representation of the program. Thirdly, two values are read from the single input array "x[]" in every iteration, and the indices "2*i" and "2*i+1" are affine in "i". Finally, the updated values depend only on their respective old values, the two values read from "x[]", and the constants "1.0" and "2.0". This also relies on the fact that all functions used in the computation are pure functions.

Counterexamples Listing 5.3 shows counterexamples to the four conditions, demonstrating why they are all needed in order to parallelise a given computation as a scalar reduction.

In the first example, the iteration space is not known in advance, as the computation can be terminated depending on the input data. This makes it impossible to compute partial sums in parallel. The second example is straightforward, as there is no reduction variable that could be privatised. The loop could still be computed in parallel – but not as a scalar reduction. The third example uses index calculations that are not affine in "i". This prevents a straightforward distribution of the input array across threads. In this particular case, the index calculation also involves values other than "i", preventing the computation of partial results entirely. The final example is equivalent to the first but breaks the fourth condition instead of the first.

5.3.1.2 Histogram Reductions

The conditions that apply to histogram reductions are similar to those for scalar reductions. However, instead of one function parameter, there are two:

1. The code is contained in a for-loop, and the iteration space of the loop is known ahead of time (but not necessarily at compile time).
2. One or multiple values $a_1, \ldots, a_n$ are read from arrays, and the access indices are affine in the loop iterator.
3. An integer k is computed as a term only of the array values $a_1, \ldots, a_n$, and of values that are constant within the loop.
4. A value x is read from an array at index k and a modified value x' is written at the same index. The writing may be control-dependent only on $a_1, \ldots, a_n$ and it may not be in a nested loop.
5. The updated value x' is computed as a term only of x, the array values $a_1, \ldots, a_n$, and of values that are constant within the loop.

Histogram reductions are only parallelisable if their update operator is commutative, just as is the case for scalar reductions. However, checking this condition requires a post-processing step, which is independent of the detection of the algorithmic structure. For this research, checking commutativity was considered the responsibility of code generation.

Example The first two conditions are the same as those in the previous discussion of scalar reductions and were shown to hold for the loop in Listing 5.2. The array "`q[]`" also satisfies the remaining conditions. Firstly, the variable "`l`" corresponds to the index k. Secondly, the element "`q[l]`" is read, modified and written. Lastly, the updated value is computed in an allowed fashion, as it is obtained by merely adding a constant "`1.0`" to the previous value.

Counterexamples Again, counterexamples are given in Listing 5.4 to develop an intuition about the significance of conditions 3–5.

In the first example, the index of the histogram is not computed from an input array but instead read from an input stream. This makes parallelisation as a histogram impossible, as the stream is only available sequentially. In the second and third examples, the computation effectively stops when an index beyond a threshold size occurs. This can only be accounted for by sequentially going through the array. Therefore, the parallel computation of partial results is not efficiently possible. This behaviour is evoked in two different ways, by breaking the conditions 4 and 5, respectively. This shows the need to restrict both the data flow and control flow of the histogram kernel.

```
for(i = 0; i < n; i++)
    hist[getchar()] += 1;
```

```
active = true;
for(i = 0; i < n; i++) {
    if(a[i] > 9)
        active = false;
    if(active)
        hist[a[i]] += 1;
}
```

```
active = true;
for(i = 0; i < n; i++) {
    if(a[i] > 9)
        active = false;
    hist[a[i]] += active?1:0;
}
```

Listing 5.4: Counterexamples to the last three conditions: None of these computations can be parallelised as histograms. The final two example loops implement equivalent functionality.

5.3.2 The Idiom Detection Language

For automatic detection during compilation, the conditions from Sections 5.3.1.1 and 5.3.1.2 are specified formally in a constraint language. CAnDL from Chapter 4 is taken as the basis for this formulation. However, additional language constructs are required in order to capture the kernel computations.

This extension of CAnDL leads to the definition of the Idiom Detection Language (IDL). IDL uses the solver infrastructure of CAnDL and retains the same high-level program structure. However, it extends the language and modifies the syntax to be more convenient for large-scale specifications, replacing uncommon Unicode characters. Table 5.1 shows syntax differences.*

CAnDL	IDL
∧, ∨	and, or
include Spec({A}->{B}, {C}->{D})@{E}	inherits Spec with {A} as {B} and {C} as {D} at {E}
{A}={B}, {A}≠{B}	{A} is [not] equal to {B}
domination({A},{B})	{A} control flow dominates {B}
opcode{A} = store	{A} is store instruction
{A} ∈ {B}.args	{A} has data flow to {B}

Table 5.1: The Idiom Detection Language (IDL) is derived from CAnDL. However, it uses a more descriptive syntax, without uncommon Unicode characters such as "∧", "∨", "∈", and "≠".

*The complete grammar file that was used to generate the parser of the IDL compiler is in Appendix C.

5.3.2.1 Kernel Functions as Generalised Domination

Complex Reduction and Histogram Computations can be expressed as a higher-order function. This means that the computational idiom is not just parameterised with numerical values, such as array dimensions, but contains kernel functions as parameters. To enable the capture of such higher-order functions in IDL, additional constraint expressions are required that go beyond what CAnDL provided. Specifically, IDL adds another atomic constraint based on generalised graph domination, as previously described in Definition 2.15, using the following syntax:

all flow from *variable_tuple* **or any origin to any of** *variable_tuple*
passes through at least one of *variable_tuple*

The syntax of this atomic constraint is quite descriptive, but some details require explanation. Firstly, "flow" in this constraint does not only capture data flow. To cover kernel functions with non-trivial control flow, the underlying graph on which this constraint operates is formed as an extension of the data flow graph $DFG^*_{\mathscr{F}}$. In addition to the data flow edges, it has an edge from each branch instruction to every instruction within all its targeted basic blocks. This encodes control dependence information and also guarantees that all relevant control flow is retained when flushing the graph in reverse direction from the result of a computation. This is needed for identifying the required instructions to implement a kernel as a separate function.

Secondly, as opposed to the control flow graph that is typically considered for dominance relationships, the resulting graph has no single "origin". Aside from the control entry, all non-pure function calls and reads from memory are considered graph origins.

Figure 5.3 demonstrates the significance of this definition. At the top is an SSA pseudocode representation of the complex reduction and histogram calculation from Listing 5.2, annotated in the right column with all the dependencies described above. It is now possible to check the validity of the kernel function that is used within the loop for the reduction on "`sx`" as follows:

```
1 all flow from {loop_carried[0..3]} or any origin
2      to any of {sx''} passes through at least one of
3      {sx,t2,t5,precursor,backedge,outside[0..N]}
```

This use of the new atomic constraint requires several additional conditions: "`precursor`" and "`backedge`" should be determined and "`outside[0..N]`" should contain all origins that lie outside the SESE region within which the kernel functions is considered. Furthermore, the array "`loop_carried`" needs to be constrained to contain all loop-carried Φ-instructions.

With this setup, it is straightforward to check that the constraint is satisfied. Control flow can only reach sx'' via the precursor at line 1 and the back edge at line 28. The other relevant graph origins for this example are the load instructions at lines 6,9,18 and the explicitly added Φ-instructions. The three of these that can reach sx'' through the graph are explicitly killed as generalised dominators "`sx`", "`t2`", and "`t5`".

Block		Operation	Dependencies
entry	**1:**	goto *loop*	**outside the considered region**
loop	**2:**	$i \leftarrow \Phi(entry : 0,\ loop : i')$	loop carried data flow
	3:	$sx \leftarrow \Phi(entry : 0.0,\ loop : sx'')$	**loop carried data flow**
	4:	$sy \leftarrow \Phi(entry : 0.0,\ loop : sy'')$	loop carried data flow
	5:	$t_1 \leftarrow 2 \cdot i$	control(**1, 28**), data(**2**)
	6:	$t_2 \leftarrow$ **load** $x[t_1]$	**origin of data flow**
	7:	$t_3 \leftarrow 2.0 \cdot t_2 - 1.0$	**control(1, 28), data(6)**
	8:	$t_4 \leftarrow 2 \cdot i + 1$	control(**1, 28**), data(**2**)
	9:	$t_5 \leftarrow$ **load** $x[t_4]$	**origin of data flow**
	10:	$t_6 \leftarrow 2.0 \cdot t_5 - 1.0$	**control(1, 28), data(9)**
	11:	$t_7 \leftarrow t_3 \cdot t_3 + t_6 \cdot t_6$	**control(1, 28), data(7, 10)**
	12:	$t_8 \leftarrow t_7 \leq 1.0$	**control(1, 28), data(11)**
	13:	if t_8 goto *ifblock* else *uncond*	**control(1, 28), data(12)**
ifblock	**14:**	$t_9 \leftarrow sqrt(-2.0 \cdot log(t_7)/t_7)$	**control(13), data(11)**
	15:	$t_{10} \leftarrow t_3 * t_9$	**control(13), data(7, 14)**
	16:	$t_{11} \leftarrow t_6 \cdot t_9$	control(**13**), data(**10, 14**)
	17:	$l \leftarrow MAX(fabs(t_{10}), fabs(t_{11}))$	control(**13**), data(**15, 16**)
	18:	$t_{12} \leftarrow$ **load** $q[l]$	origin of data flow
	19:	$t_{13} \leftarrow t_{12} + 1$	control(**13**), data(**18**)
	20:	**store** $q[l] \leftarrow t_{13}$	control(**13**), data(**19**)
	21:	$sx' \leftarrow sx + t_{10}$	**control(13), data(3, 15)**
	22:	$sy' \leftarrow sy + t_{11}$	control(**13**), data(**4, 16**)
	23:	goto *uncond*	**control(13)**
uncond	**24:**	$sx'' \leftarrow \Phi(loop : sx,\ ifblock : sx')$	**control(13, 23), data(3, 21)**
	25:	$sy'' \leftarrow \Phi(loop : sy,\ ifblock : sy')$	control(**13, 23**), data(**4, 22**)
	26:	$i' \leftarrow i + 1$	control(**13, 23**), data(**2**)
	27:	$t_{14} \leftarrow i' < NK$	control(**13, 23**), data(**26**)
	28:	if t_{14} goto *loop* else *exit*	**outside the considered region**

function kernel(sx, t_2, t_5)

entry	$t_3 \leftarrow 2.0 \cdot t_2 - 1.0$ $t_6 \leftarrow 2.0 \cdot t_5 - 1.0$ $t_7 \leftarrow t_3 \cdot t_3 + t_6 \cdot t_6$ $t_8 \leftarrow t_7 \leq 1.0$ if t_8 goto *ifblock* else *uncond*
ifblock	$t_9 \leftarrow sqrt(-2.0 \cdot log(t_7)/t_7)$ $t_{10} \leftarrow t_3 \cdot t_9$ $sx' \leftarrow sx + t_{10}$ goto *uncond*
uncond	$sx'' \leftarrow \Phi(entry : sx,\ ifblock : sx')$ return sx''

Figure 5.3: A kernel function is identified within the SSA pseudocode at the top (cf. Listing 5.2): The value of sx'' is calculated in the SESE region spanning lines 2–27 as a pure function of only sx, t_2, and t_5. This is determined by starting from sx'' (**blue**) and following the dependencies on the right, checking that all paths end in the predetermined function arguments sx, t_1 and t_5 (**red**). The kernel function can then be extracted and valid SSA reconstructed, as shown at the bottom.

```
Constraint KernelFunction
( collect i  4 ( {entries[i]} has control
                     flow to {scope.begin}) and
  collect i 24 ( inherits LocalConst
                     with {scope} as {scope}
                                  at {outside[i]} and
                 {outside[i].value}
                     is not a numeric constant and
                 {outside[i].value} has data
                     flow to {outside[i].use} and
                 {scope.begin} control flow
                     dominates {outside[i].use}) and
  collect i  8 ( {loop_carried[i].update} reaches
                     phi node {loop_carried[i].value}
                     from {scope.end} and
                 {scope.begin} control flow
                     dominates {loop_carried[i].value}) and
  all flow from {loop_carried[0..8].value} or any origin
       to any of {result} passes through at least one of
       {inputs[0..32],entries[0..4],outside[0..24].value})
End
```

```
Constraint ScalarPart
( {kernel.result} reaches phi node
       {old_value} from {loop.end} and
  inherits ScopeValue
       with {loop}      as {scope}
        and {old_value} as {value} and
  {kernel.result} has data flow to {final_value} and
  {loop.end} strictly control flow
       dominates {final_value} and
  inherits KernelFunction
       with {loop} as {scope} at {kernel} and
  inherits Concat(N1=31,N2=1)
       with {read_values}   as {in1}
        and {old_value}     as {in2}
        and {kernel.inputs} as {out})
End
```

Listing 5.5: IDL specifications of a kernel function and a scalar reduction within a CReHC: In "`ScalarPart`", the kernel function operates in a loop. Its input "`kernel.inputs`" is composed of "`read_values`" and the reduction value of the previous iteration, concatenated with "`Concat`".

5.3.2.2 Expressing Kernel Functions in IDL

Listing 5.5 encapsulates kernel functions as an IDL constraint specification in the top half. The specification is built around the previously introduced generalised graph domination constraint, which is invoked at lines 18–20. The arrays that are used in these final lines of the specification are filled with several collect-all statements at lines 2–17. Lines 2–3 collect all the entry points of the control flow. In the previous example, these were "`precursor`" and "`backedge`". All values from outside the scope that are used within the scope are collected at lines 4–12. These values can be considered the closure of the kernel function. Finally, lines 13–17 collect all the loop-carried Φ-instructions, which only exist in case the scope is a loop.

The bottom of Listing 5.5 shows the IDL specification of a scalar reduction component within a CReHC loop. This IDL specification is built around a kernel function, incorporating the specification at lines 10–11. The arguments to this kernel are set using the "`Concat`" specification at lines 12–15. This kernel function is connected with a loop-carried Φ-instruction at lines 2–6. Finally, lines 7–9 make sure that the reduction value eventually leaves the loop, excluding loop-carried iterators that are only used within the loop.

Listing 5.6 shows how histogram reductions are specified similarly. Two kernel functions calculate the index of the bin in the histogram (lines 7–9) and the updated value of its contents (lines 10–15). The histogram update in each iteration may be conditional, as expressed with "`ConditionalReadModifyWrite`" in lines 2–6. Crucially, the kernel functions use the scope of the loop and already restrict this conditional to only depend on the allowed values.

```
1  Constraint HistoPart
2  ( inherits ConditionalReadModifyWrite
3        with {loop}               as {scope}
4         and {idx_kernel.result}  as {address}
5         and {val_kernel.result}  as {new_value}
6                                  at {update} and
7    inherits KernelFunction
8        with {loop}         as {scope}
9         and {read_values}  as {inputs} at {idx_kernel} and
10   inherits KernelFunction
11       with {loop} as {scope} at {val_kernel} and
12   inherits Concat(N1=31,N2=1)
13       with {read_values}        as {in1}
14        and {update.old_value}   as {in2}
15        and {val_kernel.inputs}  as {out})
16 End
```

Listing 5.6: IDL specification of a histogram reduction in a Complex Reduction and Histogram Computation: Two kernel functions are present. Lines 7–9 calculate the index into the histogram array, lines 10–11 generate the updated value. The read-modify-write step may be conditional.

5.3.3 Specification of CReHCs in IDL

Listing 5.7 shows how the previously described IDL specifications can be assembled to define the class of Complex Reduction and Histogram Computations.*

Such computations are always encapsulated by a single for-loop, as stipulated at line 2. Lines 3–7 specify that input values "`read_values`" are read from one or several input arrays, according to the specification "`VectorRead`". These values are then passed at lines 12,17 to the "`HistoPart`" and "`ScalarPart`" invocations. Note that the seemingly redundant variable name assignments prevent prefixing with "`histo[k]`" and "`scalar[k]`", respectively.

Lines 19–24 make sure that there are no array writes aside from the histogram updates. Furthermore, lines 27–28 eliminate the possibility of effectful function calls within the loop. Together, these constraints rule out any unwanted side effects of the loop.

Lines 25–26 enforce that the loop contains at least one reduction or histogram computation. This is guaranteed indirectly because the two compared variables can only be the same if they are both *unused*.

5.4 Code Generation for CReHCs

Parallel code generation immediately follows the CAnDL-generated detection pass. It uses the detected constraint solutions to reimplement the corresponding loops with divide-and-conquer parallelism based on POSIX Threads.

For each Complex Reduction and Histogram Computation loop that is found, the relevant input arrays and closure variables are taken from the solution. They are packed into a data structure together with any histogram arrays and the iteration space boundaries.

A new function is generated that takes this data structure as its only parameter. Depending on the number of processors and recursion depth, the function decides whether to bisect its workload recursively. If not, the loop is executed as before, using the arguments in the structure. Otherwise, the function uses "`pthread_create`" to offload half of its workload onto another thread. For this, it copies its parameter array, replacing histogram arrays with newly allocated copies. After both threads finished their work, the copy is merged with the original histogram element-wise and then deallocated.

In general, determining the size of the histogram at compile time is not possible with static methods. All the branched-off threads, therefore, use dynamic boundary checking and reallocate the histogram array when necessary. This on-demand enlargement introduces some overhead but proved acceptable for many benchmark programs. The exceptions to this were "IS" and "histo", where the reduction arrays were large and the algorithmic intensity of the reduction operators low. For these two programs, the reduction array sizes had to be predefined

*The complete IDL code for this section is in Appendix D.

```
1  Constraint ComplexReductionsAndHistograms
2  ( inherits For at {loop} and
3    collect k 32 ( inherits VectorRead
4                        with {loop.iterator}  as {input_index}
5                         and {read_values[k]} as {value}
6                         and {loop}           as {scope}
7                                              at {read[k]}) and
8    collect k  2 ( inherits HistoPart
9                        with {loop.begin}  as {begin}
10                        and {read}         as {read}
11                        and {loop}         as {loop}
12                        and {read_values} as {read_values}
13                                           at {histo[k]}) and
14   collect k  2 ( inherits ScalarPart
15                       with {loop.begin}  as {begin}
16                        and {loop}         as {loop}
17                        and {read_values} as {read_values}
18                                           at {scalar[k]}) and
19   collect i  2 ( {stores[i]} is store instruction and
20                    inherits ScopeValue
21                       with {loop}      as {scope}
22                        and {stores[i]} as {value}) and
23   {stores[0..2]} is the same set as
24       {histo[0..2].update.store_instr} and
25   {scalar[0].kernel.result} is not the
26       same as {histo[0].update.store_instr} and
27   inherits SideEffectFreeCalls
28       with {loop} as {scope})
29 End
```

Listing 5.7: Complex Reduction and Histogram Computations (CReHCs) as IDL specification: The idiom comprises histogram (lines 8–13) and scalar (lines 14–18) reductions contained in a for-loop (line 2). These computations accumulate the values from the input array (lines 3–7). Additional conditions guarantee the absence of any further side effects in the loop (lines 19–28).

manually. As shown in the results section, this approach was sufficient to achieve significant speedups on the “IS” benchmark. However, the “histo” program achieved no parallel speedup despite this improvement.

Optimal code generation was not the main focus of this research, and more sophisticated methods for the parallelisation of reductions could be incorporated for better speedup results.

5.5 Experimental Setup

5.5.1 Benchmarks and Platform

The prototype idiom detection pass was applied to C versions of the NAS Parallel Benchmarks (NPB) [139], which were developed by the NASA Advanced Supercomputing Division (NAS). Specifically, the Seoul National University (SNU NPB) implementation by Seo et al. [140] was used, containing the original 8 NAS benchmarks plus 2 of the more recent unstructured components "UA" and "DC".

The approach was also evaluated on all of the Parboil [150] and Rodinia [142] benchmark programs. In total, this constitutes 40 programs of varying complexity, shown in Table 5.2. The "Embarrassingly Parallel" program in NPB, for instance, is a single file of 324 lines of code. By contrast, the source code of "Leukocyte" in Rodinia is distributed over more than 50 files. For each of the individual benchmark programs, the total number of scalar and histogram reductions found was captured.

To determine runtime coverage and performance, all of the benchmarks were evaluated on the same platform. This was a 64-core machine with 4 AMD Opteron 6376 processors and one terabyte of RAM.

5.5.2 Competing Approaches

To provide a useful comparison, the IDL approach was evaluated against two state-of-the-art competitors. The first is a recently-published approach that transforms reductions within the polyhedral framework; the second is a mature industrial compiler.

Polly-Reduction Doerfert et al. [30] developed a compiler analysis and transformation tool that detects reductions within code captured by the polyhedral model. This system augments Polly, an LLVM-based polyhedral compiler [26]. The polyhedral model is powerful when applicable, and Polly being based on LLVM IR allows for comparison of IDL against another approach that uses the same compiler infrastructure.

The sequential benchmark programs were compiled by version 3.9 of the Clang compiler, built with the Polly-enabled LLVM. The SCoPs that Polly detected with the compiler options "`-O3 -mllvm -polly -mllvm -polly-export`" were considered. Any Polly-based method only works within these SCoPs. Therefore, this gives an upper bound for the "Polly-Reduction" method. Reductions within the SCoPS were then manually identified. Any reduction within a SCoP was counted as a hit for "Polly-Reduction". This gives an optimistic estimate as to what coverage a polyhedral-based approach to reduction operations, such as the one presented in Doerfert et al. [30], can achieve.

NPB	BT	Block Tridiagonal Solver
	CG	Conjugate Gradient
	DC	Data Cube Operator
	EP	Embarrassingly Parallel Marsaglia Polar Method
	FT	Fast Fourier Transform
	IS	Small Integer Bucket Sort
	LU	Lower-Upper Symmetric Gauss-Seidel Solver
	MG	MultiGrid Approximation
	SP	Scalar Pentadiagonal Solver
	UA	Unstructured Adaptive Mesh
Parboil	bfs	Breadth-First Search
	cutcp	Distance-Cutoff Coulombic Potential
	histo	Saturating Histogram
	lbm	Lattice-Boltzmann Method Fluid Dynamics
	mri-gridding	Magnetic Resonance Imaging - Gridding
	mri-q	Magnetic Resonance Imaging - Q
	sad	Sum of Absolute Differences (part of MPEG encoding)
	sgemm	Dense Matrix-Matrix Multiply
	spmv	Sparse-Matrix Dense-Vector Multiplication
	stencil	Iterative 3D Jacobi Stencil
	tpacf	Two Point Angular Correlation Function
Rodinia	backprop	Back Propagation
	bfs	Breadth-First Search
	b+tree	Database B+Tree Search
	cfd	Computational Fluid Dynamics Solver
	heartwall	Mouse Heart Tracking on Ultrasound Images
	hotspot	2D Transient Thermal Differential Equation Solver
	hotspot3D	3D Transient Thermal Differential Equation Solver
	kmeans	K-Means Clustering Algorithm
	lavaMD	N-Body Simulation of Particles in 3D Space
	leukocyte	In Vivo Video Microscopy Leukocyte Tracking
	lud	Lower-Upper Matrix Decomposition
	mummergpu	Local DNA Sequence Alignment
	myocyte	Heart Muscle Cell Simulation
	nn	K-Nearest Neighbors from an Unstructured Data Set
	nw	Needleman-Wunsch Optimization for DNA Alignments
	particlefilter	Object Location Estimator for Noisy Measurements
	pathfinder	Find Minimal Cost Path through 2D Graph
	srad	Speckle Reducing Anisotropic Diffusion
	streamcluster	Clustering Algorithm from Parsec

Table 5.2: Overview of the 40 programs used for evaluation, grouped into three suites

ICC The Intel C/C++ compiler is a mature tool that incorporates auto-parallelisation and vectorisation. Rather than the polyhedral model, it uses simpler data dependence models as its fundamental analysis method. However, it is a very mature product with industry support.

For this evaluation, the benchmarks were compiled with the "`-parallel -qopt-report`" options. This generated optimisation reports. ICC does not explicitly report reductions, but any reduction that was within a loop that ICC considered to be parallelisable was counted. This also included in the detection results all those loops that ICC considered possible but inefficient.

5.6 Results

This section first presents the number of reductions found by the various different schemes. This is followed by an analysis of how significant each reduction was. Finally, the performance impact of the IDL-based parallelisation method was evaluated.

5.6.1 Discovery

The detection schemes were applied to the three benchmark suites, with the results shown in Figures 5.4 to 5.6. Across all the benchmark programs, the IDL specification captured a total of 90 reductions: 84 scalar reductions and 6 histogram reductions.

Discovered by IDL There were scalar reductions detected in nearly all of the individual NPB programs, with "UA" having the highest number at 11. Scalar reductions were less prevalent in Parboil with only 3 out of 11 benchmarks containing any. Furthermore, these were unevenly distributed: out of a total of 9 scalar reductions across all Parboil programs, 7 were in "cutcp". The Rodinia benchmark collection contained many more identifiable reductions than Parboil. The IDL solver detected scalar reductions in 15 out of the 19 benchmarks, with 9 reductions in the "particlefilter" program alone.

Every reduction captured by the other two competing approaches was also identified by the IDL solver. However, there were some regions that could be classified as reductions manually, but that did not adhere to the used IDL specification, and thus remained unrecognised. This was generally the case when the reduction loop was not the innermost loop, as is the case for the example in Listing 5.8. The three outer loops of this code section from the "SP" benchmark are reduction loops, but this is obfuscated by the innermost loop. None of the competing approaches were able to detect this reduction either.

```
1  for (k = 1; k <= nz2; k++) {
2    for (j = 1; j <= ny2; j++) {
3      for (i = 1; i <= nx2; i++) {
4        for (m = 0; m < 5; m++) {
5          add = rhs[k][j][i][m];
6          rms[m] = rms[m] + add*add;
7        }
8      }
9    }
10 }
```

Listing 5.8: Some reductions were missed by all detection approaches. In this loop from "SP", the outer loops are reduction loops, accumulating into scalar variables "`rms[0]`",...,"`rms[4]`", but the innermost loop is not a reduction, and it is therefore not captured by the IDL specification.

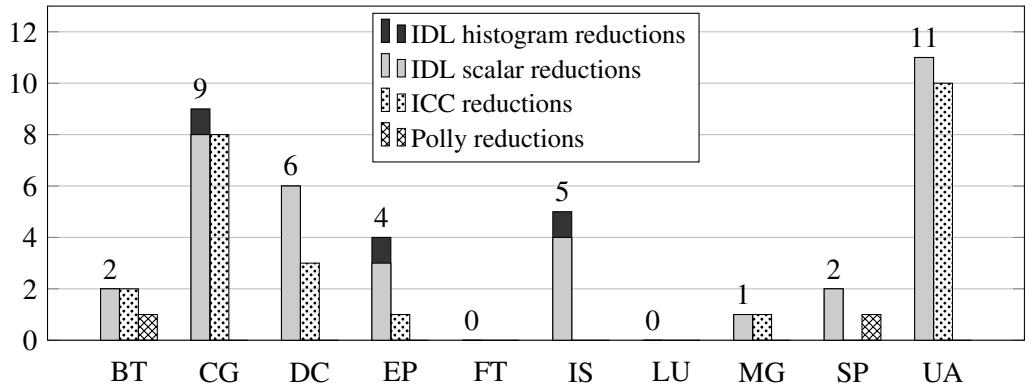


Figure 5.4: Reductions and histograms detected by the three competing approaches in **NPB**: Most programs contain scalar reductions, of which IDL consistently recognised more than ICC. Only IDL detected histograms. Most of the benchmarks were unsuitable for analysis with Polly.

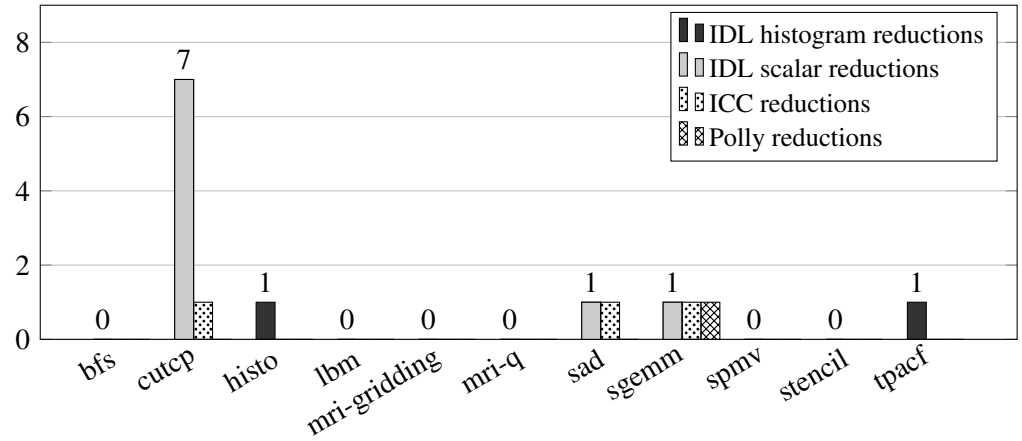


Figure 5.5: Reductions and histograms detected by the three competing methods in **Parboil**: In half of the benchmark components, no scheme detected any reductions. Nevertheless, IDL identified two histograms in “histo” and “tpacf”, and was not outperformed on scalar reductions.

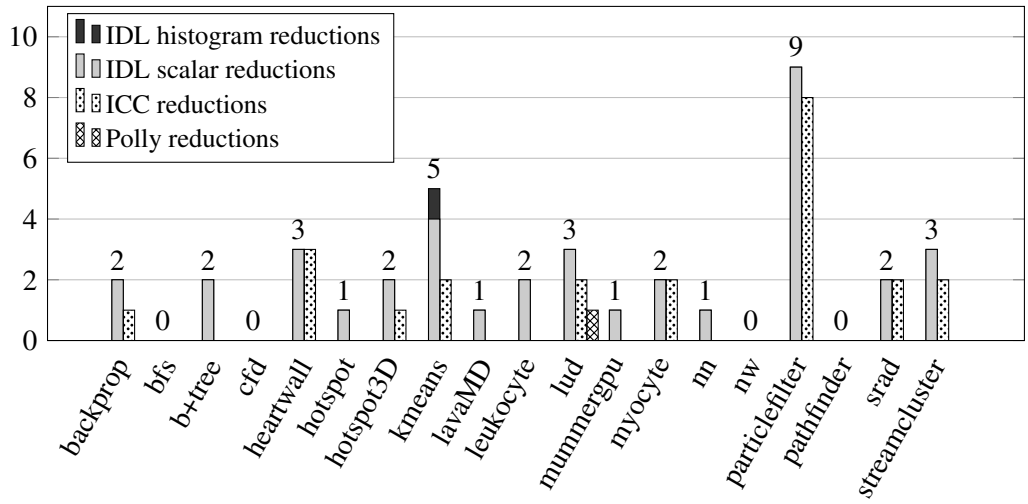


Figure 5.6: Reductions and histograms detected by the 3 competing approaches in **Rodinia**: Most programs contained scalar reductions. ICC matched by IDL only on “heartwall” and “srad”.

Histogram reductions were rarer than scalar reductions. However, the IDL solver was able to detect 3 in NAS, 2 in Parboil, and 1 in Rodinia. All of them eventually updated the bin value with an addition, but some contained complex expressions in the kernel function to compute the addend, as previously seen in Listing 5.2 for the "EP" program. Another interesting example was "tpacf" from the Parboil benchmarks. In this histogram reduction, the histogram index kernel function contained a binary search. At the other extreme, the performance bottleneck of "IS" was a plain histogram without any complications:

```
1 for( i=0; i<NUM_KEYS; i++ )
2   key_buff_ptr[key_buff_ptr2[i]]++;
```

Polly-Reduction The Polly compiler captured only four relevant code regions across all benchmark programs that contained a single scalar reduction each. These are in "BT" and "SP" from the NAS Parallel Benchmarks, "sgemm" from the Parboil Benchmarks, and "leukocte" from Rodinia. As expected, all loops that contain histogram computations were unsuitable for polyhedral analysis with Polly. Indirect memory access forms an integral part of any histogram computation but violates assumptions of the polyhedral model.

Aside from the fundamental limitation for histogram computations, Polly struggled with the more complex codebases of Rodinia and Parboil. This was not always due to fundamental limitations of the polyhedral model. Instead, it was often because of loop iteration boundaries that Polly could not statically determine and the use of flat array structures.

Figures 5.7 to 5.9 show the number of Static Control Parts (SCoPs) that Polly finds across the 40 individual programs. Detecting such SCoPs is the first part of any polyhedral analysis. As program code outside of SCoPs is left untouched by Polly, this provides an upper bound for the detection of reductions. In 23 of the 40 benchmarks, Polly found no SCoPs at all. This corresponded to 40% of NPB, 63.6% of the Parboil Benchmarks and 63.2% of Rodinia.

The majority of the SCoPs that Polly detected were in stencil computations. The stencil-based programs "LU", "BT", "SP" and "MG" in the NAS Parallel Benchmarks alone accounted for 37 of the 62 SCoPs that were found across all benchmarks (59.6%).

ICC The Intel C/C++ compiler was more successful than Polly at recognising reductions. It detected 25 out of 37 reductions in NPB, 3 out of 9 in Parboil and 23 out of 38 in Rodinia. However, these were all scalar reductions; no histograms were identified.

ICC was more robust and did not require SCoPs as a precondition for its analysis. On the well-structured NAS benchmarks, it performed well but failed to detect any reductions in "IS". Surprisingly, it also did not detect reductions in "SP", while Polly did. On closer inspection, this is again due to a deep perfectly nested loop where the reduction iterator is in the middle of the loop nest.

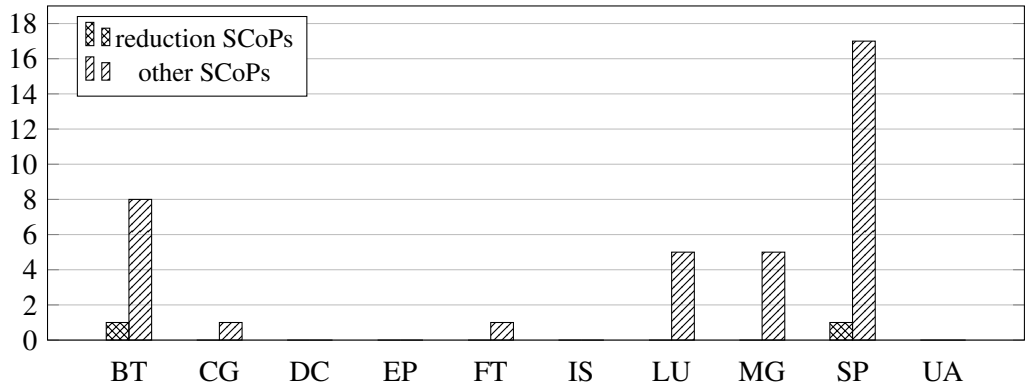


Figure 5.7: Static Control Parts (SCoPs) identified by Polly in **NPB**: Six of the programs had code regions that were representable in the polyhedral model; two of them contained reductions.

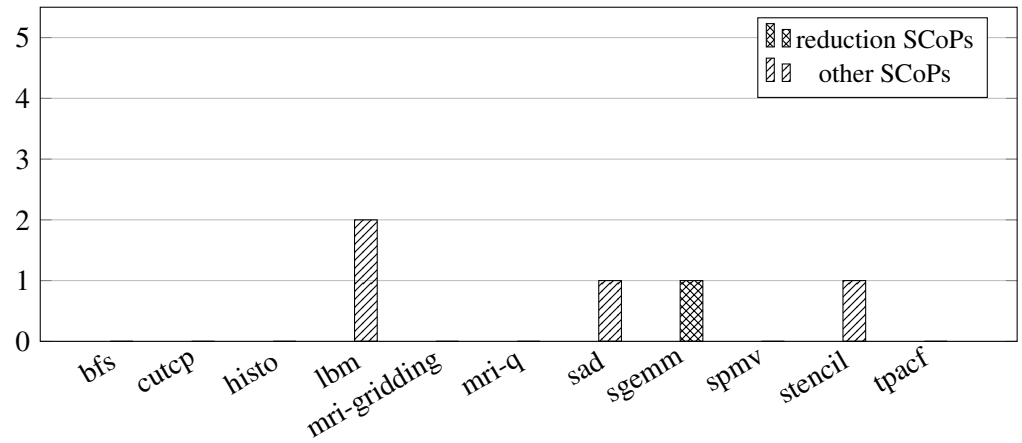


Figure 5.8: Static Control Parts (SCoPs) identified by Polly in **Parboil**: Most of the programs were unsuitable for Polly, only the reduction of the matrix multiplication in “sgemm” was captured.

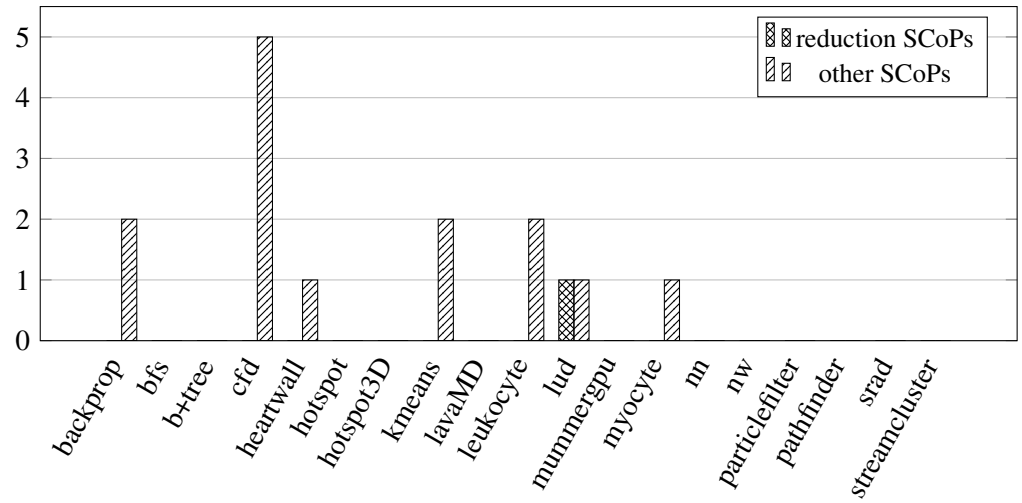


Figure 5.9: Static Control Parts (SCoPs) identified by Polly in **Rodinia**: Seven programs had code regions that were representable in the polyhedral model, but only one captured a reduction.

On the less well-structured Parboil benchmarks, ICC failed to detect many of the scalar reductions in "cutcp". This was because these reductions use the functions "`fmin`" and "`fmax`", which our system recognised as pure, but ICC did not. Finally, ICC recognised many of the scalar reductions in Rodinia but was outperformed on six of the programs, with IDL finding one additional reduction in each of them. It is clear that ICC did not attempt to detect histograms, as it missed all instances of them.

5.6.2 Runtime Coverage

Detecting large numbers of reductions is encouraging but does not address whether or not such detection is useful. To measure the potential impact, each program was profiled, examining the time spent in the different types of reduction loops. These measurements from these profiling runs are shown in Figures 5.10 and 5.11.

The two different classes of reductions behaved very differently. While there were many more scalar reductions than histogram reductions in total, histogram reductions were much more likely to constitute performance bottlenecks. In the benchmark programs that contained histogram reductions, they accounted for an average of 68% of the runtime.

Scalar reductions, on the other hand, were generally irrelevant to program runtime, with the one exception of the "sgemm" benchmark. Initial profiling already showed that none of the scalar reductions in Rodinia were significant. Therefore, precise measurements were omitted and are not shown in Figure 5.11.

It is evident that exploiting reduction parallelism for performance gains was promising only for histogram reductions. While the "sgemm" program was bottlenecked by a scalar reduction – the dot product inside the matrix multiplication – treating this loop as an arbitrary reduction was unlikely to yield optimal results. Instead, Chapter 6 shows how to accelerate this with a more targeted approach.

5.6.3 Performance

The speedup that was achieved by exploiting reduction parallelism on histograms is detailed in Figure 5.12. This was only evaluated for benchmarks with significant runtime coverage of reductions. The speedup from the IDL-based exploiting of reduction parallelism is compared against the manually optimised parallel benchmark versions that were shipped by the original implementors. The baseline is the sequential version of the respective benchmark program.

The IDL-based approach achieved 62% speedup on "EP" over the whole program. This was limited by the runtime coverage of the histogram. Following Amdahl's Law [151, 152], parallelisation on 48 cores has a theoretical limit of $1/((1-46\%)+46\%/48)-1 = 82\%$ speedup. The manually implemented parallel version, on the other hand, exploits outer-loop parallelism and outperforms the reduction-based method.

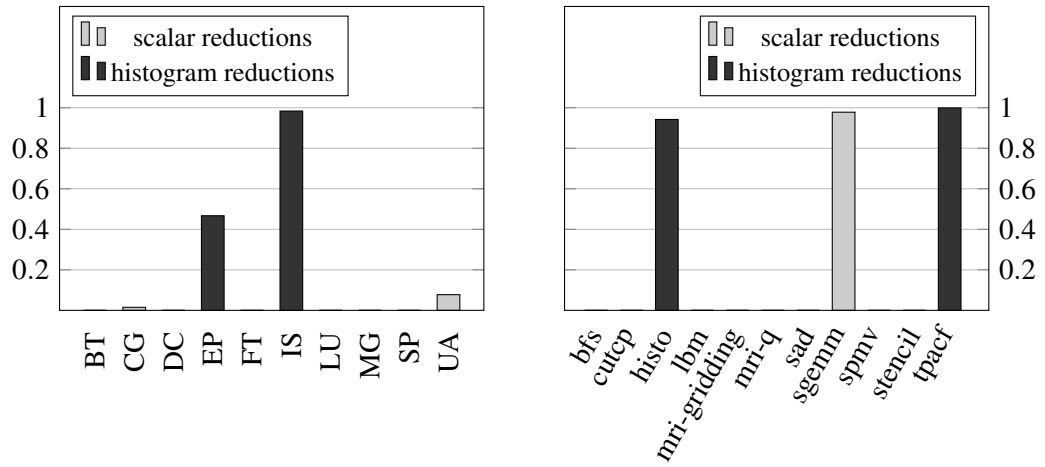


Figure 5.10: Runtime coverage of reductions in **NPB** and **Parboil**: Five of the programs spent a significant portion of their runtime performing Complex Reduction and Histogram Computations. Four of the five histograms were performance bottlenecks, but only one of the scalar reductions.

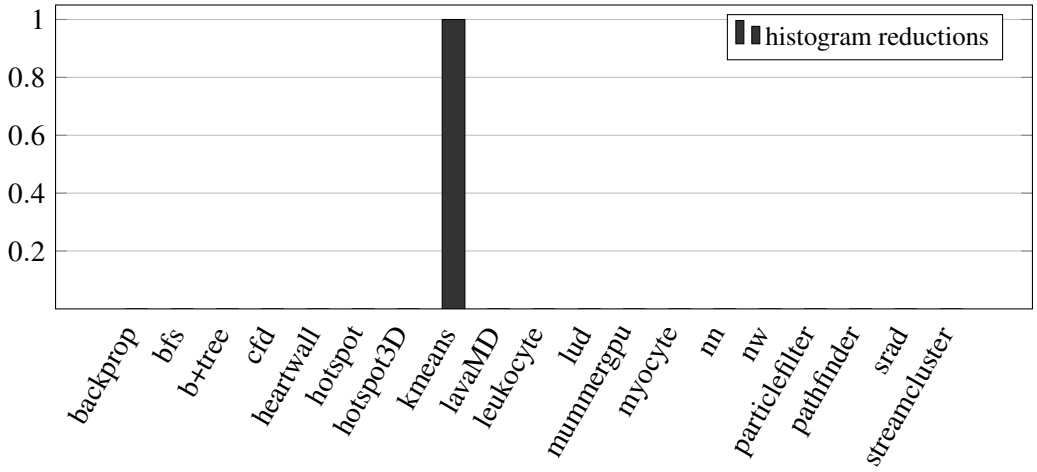


Figure 5.11: Runtime coverage of reductions in **Rodinia**: Scalar and histogram reductions had a performance impact only on “kmeans”, which spent almost the entire runtime on a histogram.

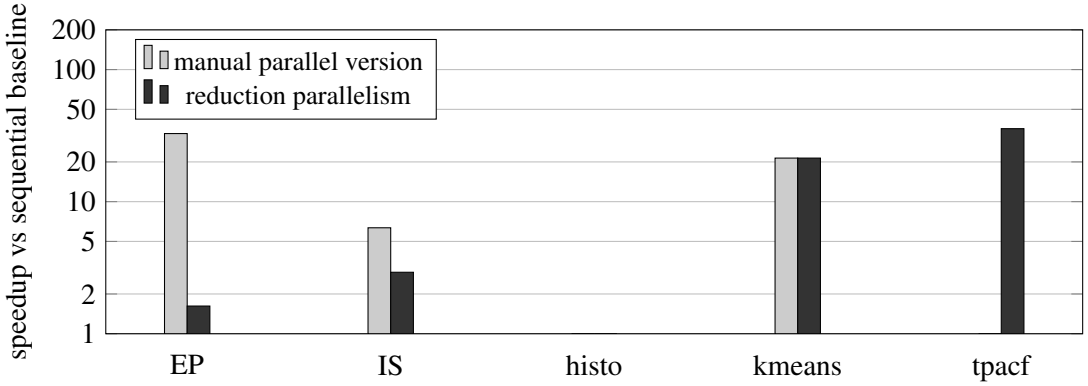


Figure 5.12: Speedup achieved with histogram parallelism on a logarithmic scale: Only “histo” did not profit from IDL-based parallelisation due to large data size and low arithmetic intensity.

On the "IS" benchmark, the IDL-based parallelisation results in 2.9× speedup, compared to 6.3× speedup of the manual parallel version. This discrepancy comes from the fact that the manual parallel version relies on knowledge about the distribution of the histogram keys at runtime that is not available to the IDL-based code transformation. As the keys are uniformly distributed, they can be efficiently sorted into even-sized bins before executing the actual histogram. This avoids the overhead of array privatisation. A smarter code generation approach for IDL could narrow this gap.

The "histo" benchmark of Parboil uses an unusually large amount of bins for the histogram. The array privatisation prevents any parallel speedup in the program. Interestingly, the parallel version that is provided by the implementers is also inefficient and achieves no speedup against sequential on our system. This suggests that there is no significant parallelisation potential available in the program.

The IDL-based reduction parallelism achieved an almost linear speedup of 35.7× on the "tpacf" program. The manual parallel version, which was provided by the original benchmark implementers as a reference to evaluate against, was implemented poorly with a critical section. This resulted in a slowdown compared to sequential execution on our highly parallel machine.

5.7 Conclusions

The chapter developed a new compiler-based approach to recognising a broad generalisation of reductions during compilation automatically. This detection was made possible by a constraint formulation in the Idiom Detection Language (IDL), an extension of the Compiler Analysis Description Language (CAnDL) from the previous chapter. Representing Complex Reduction and Histogram Computations (CReHCs) in this declarative language kept the specification separate from detection considerations, providing a modular and extendable approach to idiom recognition. Reduction parallelism in recognised CReHC loops was exploited by privatising the reduction variables and arrays.

This approach proved robust during the evaluation on C/C++ versions of three well-known benchmark suites: NPB, Parboil, and Rodinia. It detected more scalar reductions than other approaches and was alone in being able to detect computationally intense histogram reductions. Such reductions were shown to give significant performance improvements.

Outlook This chapter introduced a computational idiom in detail and provided the Idiom Detection Language (IDL) with the required tools to recognise this idiom during compilation. Chapter 6 uses similar methods to recognise a range of other common algorithmic structures, together resulting in significant coverage of benchmark bottlenecks.

Chapter 6

Heterogeneous Acceleration via Computational Idioms*

The previous chapter introduced a computational idiom – Complex Reduction and Histogram Computations (CReHCs) – and showed how the Idiom Detection Language (IDL) enabled its discovery and automatic parallelisation. This chapter takes a broader view of automatic idiom recognition and also includes common linear algebra computations and stencil codes. Instead of implementing bespoke parallelisation passes, this chapter follows the vision laid out in the introduction: the detected idioms are translated into stronger models, and external code generators are then used to leverage the implied domain knowledge.

To facilitate this, the chapter takes abstract algorithmic concepts that are conventionally explored outside the context of compiler analysis – computational idioms – and formalises them as IDL specifications. This enables detection and manipulation in optimising compilers. Heterogeneous acceleration serves as the motivation for this effort. As many scientific codes are structured around idiomatic performance bottlenecks, efforts that focus on computational idioms can greatly improve performance, especially on accelerators that were designed with such computations in mind. The focus is therefore on calculations that are well-supported by accelerators and their software ecosystems: linear algebra, stencil codes and CReHCs.

The IDL specifications were used for a prototype compiler that automatically detects the idioms and uses them to circumvent standard code generation via libraries and domain-specific languages: BLAS implementations, cuSPARSE, clSPARSE, Halide, Lift. This functionality is directly accessible from a modified version of the widely used Clang C/C++ compiler. The evaluation could, therefore, be performed on the well-established benchmark suites NAS and Parboil. 60 idiom instances were detected. In those cases where idioms were a significant part of the sequential execution time, the generated heterogeneous code achieved 1.26× to over 20× speedup on integrated and external GPUs.

*This chapter is based on published research: Ginsbach et al. [3].

6.1 Introduction

Heterogeneous accelerators provide the potential for superior performance. However, realising this potential in practice is difficult and requires significant programmer effort. Programs have to be partially rewritten to target heterogeneous systems, using a diverse range of broad and narrow interfaces. General-purpose languages such as OpenCL [153] provide some portability across heterogeneous devices, but the achieved performance often disappoints [154]. Despite the functional portability, rewrites are required in practice to achieve competitive performance. Optimised numerical libraries provide more reliable performance, but they are more specialised and often provided by hardware vendors without portability in mind [16, 15, 105, 106]. More narrow domain-specific languages (DSLs) have been proposed by Ragan-Kelley et al. [11], Franchetti et al. [155], Rompf and Odersky [156], among others in attempts to deliver both portability and performance.

Hardware is becoming increasingly heterogeneous, most recently with the development of deep neural network accelerators such as the Google TPU [20]. This means that library or DSL based programming is likely to become far more common. The problems with this trend that arise due to the aforementioned tradeoffs are evident. Firstly, application developers have to learn multiple specialised DSLs and vendor-specific libraries if they want good performance. Secondly, they have to rewrite their existing applications to use them. Thirdly, this ties code into ecosystems that might soon become obsolete. Accelerator DSLs are quickly evolving, and the adoption and long-term support of academic projects remain unclear. This situation is a severe impediment to the wide-spread efficient exploitation of heterogeneous hardware.

The ideal would be a compiler that automatically maps existing code to heterogeneous hardware, with full performance, and requiring no directions from the application programmer. While this remains an ambitious goal for the general case, this chapter presents a system that approximates such a general-purpose solution by utilising expertise that is already available and encapsulated in existing backend interfaces. Instead of implementing code generation for each heterogeneous accelerator, the system maps user code to heterogeneous hardware via existing libraries and domain-specific languages, effectively outsourcing the code generation to hardware and domain specialists.

The approach is based on detecting specific *computational idioms* in application code that correspond to the functionality of existing interfaces – libraries and DSLs – for heterogeneous acceleration. In addition to the Complex Reduction and Histogram Computations (CReHCs) introduced in Chapter 5, the focus is on sparse and dense linear algebra, as well as stencils. The covered idioms are a reflection of both the most relevant program bottlenecks and the available interfaces. Some computational idioms were more widely supported than others, potentially pointing to gaps in the accelerator landscape. Nonetheless, at least one backend existed per idiom. The Idiom Detection Language (IDL) then enabled the automatic detection of idioms.

Once detected, the idioms were translated into the matching DSL or replaced with an available library call. The optimised DSL output code, or the pre-built optimised library, was then linked into the original program.

The libraries cuSPARSE, clSPARSE, cuBLAS, clBLAS for sparse and dense linear algebra and the DSLs Halide [11] and Lift [157] were used as backends in the evaluation. The Lift language is a data-parallel functional language that supports generalised reductions as well as stencils and linear algebra. The wide range of backends allowed the freedom to target many APIs per idiom and pick the implementation that best suited the target platform.

New computational idioms could easily be added thanks to the flexibility of IDL. This also provides a powerful means of determining whether a proposed heterogeneous interface matches existing code, without touching the core compiler. The idioms addressed in this paper were expressed in less than 500 lines of IDL code. The approach was also highly robust, was applied to the entire NPB and Parboil benchmark suites and was evaluated on three platforms.

This chapter presents a novel approach that

- uses the Idiom Detection Language (IDL) for detecting idiomatic code sections that can be accelerated by domain-specific backends;
- implements several common computational idioms in IDL to automatically discover opportunities for accelerator exploitation;
- efficiently translates and maps the detected idioms to APIs for heterogeneous systems.

The work most similar in approach discovers stencil computations and maps them to the Halide DSL for acceleration. The Helium tool [44] recovers stencils from image-processing binaries. This requires large-scale dynamic analysis of binary traces and replacing them with Halide calls. This was significantly extended by Kamil et al. [45], detecting stencils in Fortran code. The focus of that work was on inferring post invariants based on syntax-guided synthesis in translation to Halide. However, it used a narrow approach to code snippet selection and relied on well-structured Fortran with occasional user annotations. The IDL approach is distinct in its use of an external programming language for the flexibility of describing arbitrary idioms. This allows an unbounded set of idioms to be considered and is not restricted to stencils.

To summarise, this chapter presents an automatic approach that discovers idioms in legacy code and maps them to heterogeneous platforms via libraries and DSLs. The tool was applied to 21 C/C++ programs from the NPB and Parboil benchmark suites, where it detected more reductions, stencils, matrix multiplications and sparse matrix-vector computations than existing schemes. For the programs where idioms dominate execution time, accelerator code was generated and evaluated on 3 platforms: a multi-core CPU, an integrated APU, and an external GPU. Overall, 60 idioms were detected, which dominated execution time in 10 programs. Speedup results for the accelerated code ranged from 1.26× to over 20×.

6.2 Overview

The approach is automatic and was implemented inside the LLVM compiler infrastructure. It takes arbitrary sequential C/C++ programs as input. Using the Clang compiler, the input source code is compiled into a Static Single Assignment (SSA) intermediate representation (LLVM IR). In this representation, the specified idioms are identified and replaced with calls to specific APIs. Finally, the code generated by the LLVM compiler and the output of the idiom specific code generators/libraries are linked together into a binary, producing an optimised program. LLVM was chosen as it is the best supported SSA-based compiler. The methodology could easily be transferred to other infrastructures, such as GCC.

6.2.1 Compiler Flow

Figure 6.1 gives a detailed overview of the compiler flow that underlies the approach of this chapter. The grey box at the bottom left is the IDL-enabled compiler. It takes two programs as inputs: the first is the source code of the user program, the second specifies the computational idioms that are to be detected. The same idioms can be discovered in many user programs. Therefore, the IDL program does not have to change from one run to the next.

The source code of the user program at the top left is compiled into optimised LLVM IR code. The idiom description is parsed and represented internally as a C++ object, as previously described in Chapter 2, Section 2.4.3. The solver takes the optimised LLVM IR code and the internal C++ representation of the constraint formula as inputs. It uses backtracking to recognises all idiomatic parts in the user program.

The recognised idioms and the LLVM IR code are then passed on to the transformation phase of the system. This phase extracts the idiomatic code sections from the user program and reformulates them for appropriate heterogeneous backends. For libraries, this means replacing the code covered by the idiom with a library call.

For DSL interfaces, the process is a little more involved. The user code captured by idioms is extracted and replaced with function calls to shim interfaces. The extracted code features are translated into the appropriate DSL. External DSL compilers are responsible for optimising this DSL code and generating library objects that implement the required function interfaces. The translation to DSL mainly involves representing the kernel functions. Idioms without kernel functions correspond to fixed DSL programs and require no additional work. The generated code is linked with the object code from the user program.

Determining the best heterogeneous API to target for a given platform, and the best of several overlapping idioms to exploit, will become an essential consideration as the number of idioms and APIs grows. For the results in this chapter, all applicable libraries and DSLs were evaluated, and the best-performing versions selected after profiling.

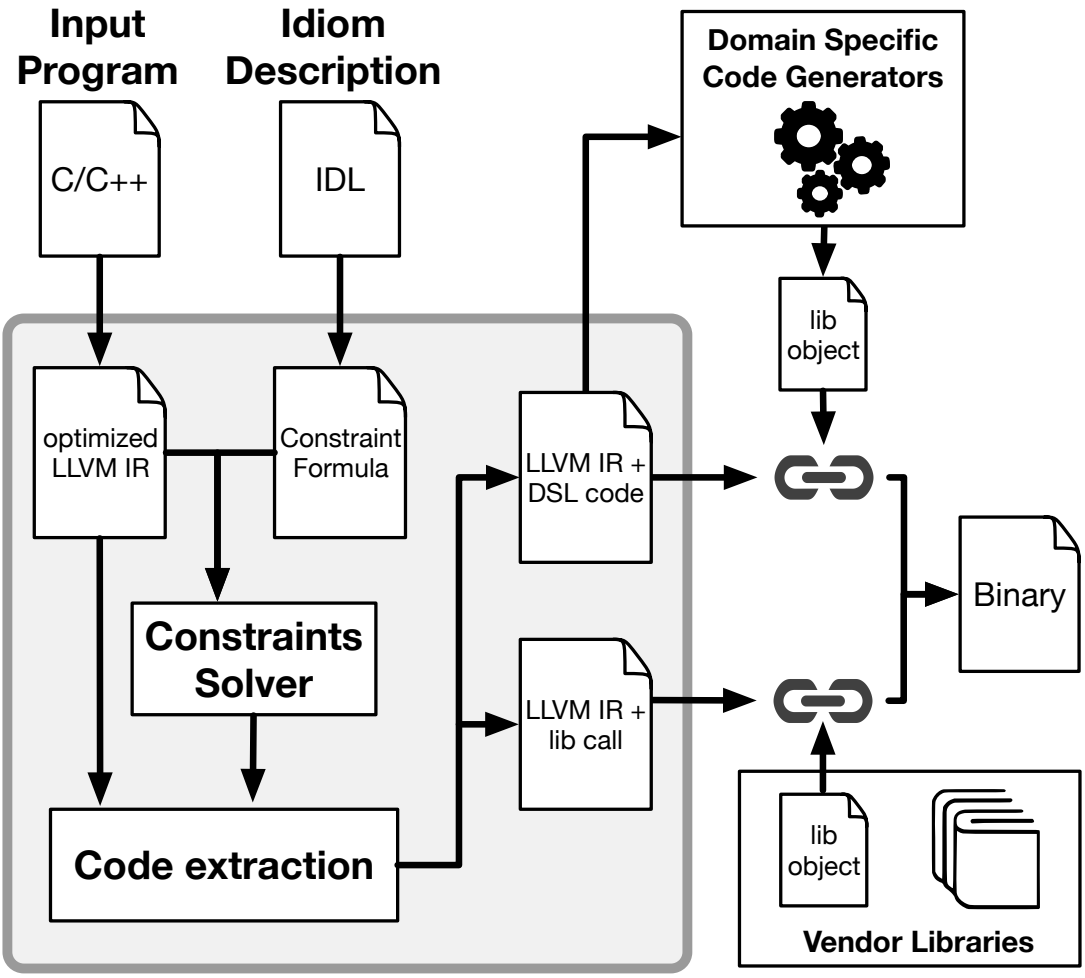


Figure 6.1: Workflow of the IDL acceleration system: The IDL solver recognises idiomatic loops in the optimised LLVM IR of user programs. These loops are extracted and replaced with shim function calls. Domain-specific code generators implement these calls as library objects, or they are taken directly from pre-generated vendor libraries (for idioms without kernel functions).

6.2.2 Accelerating Sparse Linear Algebra

Sparse linear algebra is central to many scientific codes and increasingly important as a basis for graph algorithms and data analytics [158], but it contains indirect data accesses that limit compiler optimisation. Instead of relying on tooling support, programmers use numerical libraries that are hand-optimised and target accelerator hardware. This reliance on libraries comes at a cost, however, as it ties programs into vendor-specific software ecosystems and results in non-portable code. The IDL approach offers an alternative by recognising sparse linear algebra kernels in compiler intermediate representation and incorporating the domain-specific backends without source code changes.

The code at the top of Listing 6.1 is the performance bottleneck of the "Conjugate Gradient" program from the NAS Parallel Benchmarks, with the corresponding LLVM IR code shown underneath. This bottleneck loop implements a standard operation from sparse linear algebra, namely the multiplication of a sparse matrix in Compressed Sparse Row (CSR) format with a dense vector. This computation is supported on accelerator hardware, using well-optimised libraries such as cuSPARSE and clSPARSE. However, compilers are unable to recognise and accelerate the computation automatically.

The structure of this sparse linear algebra computation has several features that make it unsuitable for most established compiler optimisations. Firstly, the iteration domain of the nested loop is memory-dependent (line 3). Secondly, there is indirect memory access (line 4). This makes the iteration domain of the loop nest non-polyhedral and the access structure to memory non-affine. Under these conditions, not only do simple data dependence models fail, but so do sophisticated analyses based on the polyhedral model.

IDL can express this sparse idiom, as derived in Section 6.3, Figure 6.3. The "Conjugate Gradient" LLVM IR code, together with the "`SPMV_CSR`" IDL specification, are input to the constraint solver, which outputs a constraint solution, as shown in Figure 6.2. In the solution, different values from the LLVM IR have been assigned to all IDL variables in the "`SPMV_CSR`" specification.

Listing 6.2 shows how this solution is used to generate a call to a cuSPARSE procedure. The individual solution variables are inserted into the "`cusparseDcsrmv`" code template as function arguments. The original code is then cut out and replaced with this function call. In practice, this involved a shim function that manages the device context and memory transfers from and to the GPU. Finally, the cuSPARSE library was linked with the object code produced by the Clang compiler, resulting in a speedup of 17$\times$ on a GPU as described in more detail in Section 6.8.

Central to this approach is the ability to detect computational idioms reliably. The next section derives in detail the formulation of sparse and dense linear algebra, as well as stencils, in the Idiom Detection Language.

```
for (j = 0; j < m; j++) {
  d = 0.0;
  for (k = rowstr[j]; k < rowstr[j+1]; k++)
    d = d + a[k]*z[colidx[k]];
  r[j] = d; }
```

```
; <label>:2:
  %j = phi i64 [ %j_next, %12 ], [ 0, %1 ]
  %j_cond = icmp slt i64 %j, %m
  br i1 %j_cond, label %3, label %13
; <label>:3:
  %4 = getelementptr i32, i32* %rowstr, i64 %j
  %5 = load i32, i32* %4
  %j_next = add nuw nsw i64 %j, 1
  %6 = getelementptr i32, i32* %rowstr, i64 %j_next
  %7 = load i32, i32* %6
  %k_begin = sext i32 %5 to i64
  %k_end = sext i32 %7 to i64
  br label %8
; <label>:8:
  %k = phi i64 [ %k_next, %9 ], [ %k_begin, %dnext ]
  %d = phi double [ 0.0, %3 ], [ %d_next, %9 ]
  %k_cond = icmp slt i64 %iv, %k_end
  br i1 %k_cond, label %9, label %12
; <label>:9:
  %a_addr = getelementptr double, double* %a, i64 %k
  %a_load = load double, double* %a_addr
  %cix_addr = getelementptr i32, i32* %colidx, i64 %k
  %cix_load = load i32, i32* %cix_addr
  %10 = sext i32 %cix_load to i64
  %z_addr = getelementptr double, double* %z, i64 %10
  %z_load = load double, double* %z_addr
  %11 = fmul double %a_load, %z_load
  %d_next = fadd double %d, %11
  %k_next = add nsw i64 %k, 1
  br label %8
; <label>:12:
  %r_addr = getelementptr double, double* %r, i64 %j
  store double %d, double* %r_addr
  br label %2
```

Listing 6.1: Sparse matrix-vector product shown in C at the top, and in LLVM IR at the bottom

Idiom Detection with IDL ⇓ **Program in Figure 6.3** ⇓

IDL Variable Name	Assigned Value
outer_loop.iterator	%j
inner_loop.iter_begin	%k_begin
inner_loop.iter_end	%k_end
inner_loop.iterator	%k
inner_loop.src1 (≈ val.value)	%z
col_ind.value	%cix_load
inner_loop.src2 (≈ vector.value)	%a_load

IDL Variable Name	Assigned Value
output.address	%r_addr
outer_loop.iter_begin	0
outer_loop.iter_end	%**m**
val.base_pointer	%**a**
col_ind.base_pointer	%**colidx**
vector.base_pointer	%**z**
...	...

Figure 6.2: Solution to "SPMV_CSR": Fitting LLVM IR values were assigned to all IDL variables.

Code Generation: Insert ⇓ **Arguments, Replace Code** ⇓

```
cusparseDcsrmv(context,
    CUSPARSE_OPERATION_NON_TRANSPOSE, m, n,
    rowstr[m+1]-rowstr[0], &gpu_1, descr, gpu_a,
    gpu_rowstr, gpu_colidx, gpu_z, &gpu_0, gpu_r);
```

Listing 6.2: GPU acceleration: Solution values are used to call "cusparseDcsrmv" backend.

6.3 Specification of Idioms in IDL

The specification of computational idioms in IDL requires careful handling of the arising complexity, using the modularity features that the language provides. Control flow constructs, memory access patterns, and additional data flow constraints are defined independently and then combined to form the complete specifications. This section extends upon the previously defined building blocks from Chapters 4 and 5.

6.3.1 Sparse Linear Algebra

The most frequently used performance-critical sparse linear algebra routines are variations on the multiplication of a sparse matrix with a dense vector (SPMV). Capturing the many different memory access patterns is crucial for these sparse linear algebra routines. On the other hand, the control flow is very rigid.

Listing 6.3 shows the basic skeleton of the SPMV idiom: there are two for-loops, the "`outer_loop`" and the "`inner_loop`". Naturally, "`outer_loop`" contains "`inner_loop`" (lines 4–7). The outer loop is a standard for-loop (line 2), while the inner loop is a for-loop contains a generalised dot product (line 3). Specifically, the "`DotProductFor`" specification requires that the loop contains a scalar reduction variable that is incremented by the result of a floating-point multiplication of two values "`src1`" and "`src2`" in every iteration. This skeleton is completed depending on the sparse matrix format, defining the specific computations and data flow that yields "`src1`" and "`src2`".

6.3.1.1 Compressed Sparse Row

The Compressed Sparse Row (CSR) format is one of the most widely used formats for sparse matrices [159]. An example matrix stored in CSR format is shown at the top of Figure 6.3. The 5×5 matrix at the top left is stored in the three separate arrays at the top right: "**val**", "**col_ind**", and "**row_ptr**".

All non-zero entries of the matrix are stored in a flat array "**val**" in a row-by-row order. The "**col_ind**" array stores the column position for each value. Finally, the "**row_ptr**" array stores the beginning of each row of the matrix as an offset into the other two arrays. The number of rows in the matrix is given directly by the length of the "**row_ptr**" array minus one. However, the number of columns is not explicitly stored.

The middle row of Figure 6.3 shows the corresponding SPMV computation in pseudocode. Finally, the bottom of the figure shows the continuation of the "`SPMV`" specification for CSR matrices in IDL. This continuation is immediately derived by walking through the pseudocode expressions and emitting "`inherits`" directives to the corresponding IDL subprograms.

```
1  Constraint SPMV
2  ( inherits For at {outer_loop} and
3    inherits DotProductFor at {inner_loop} and
4    {outer_loop.begin} strictly
5        control flow dominates {inner_loop.begin} and
6    {outer_loop.end} strictly
7        control flow post dominates {inner_loop.end} and
8  ...
```

Listing 6.3: Skeleton of the sparse matrix-vector product (SPMV) constraint specification in IDL: The precise sparse access patterns are specific to chosen storage formats for sparse matrices.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & -1 & 3 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

$$\mathbf{val} = \begin{bmatrix} 1 & 1 & 2 & 2 & -1 & 3 & 2 & 2 & -1 & 1 \end{bmatrix}$$

$$\mathbf{col\_ind} = \begin{bmatrix} 0 & 2 & 1 & 3 & 1 & 2 & 3 & 3 & 2 & 4 \end{bmatrix}$$

$$\mathbf{row\_ptr} = \begin{bmatrix} 0 & 2 & 4 & 7 & 8 & 10 \end{bmatrix}$$

```
COMPUTATION spmv_csr
  forall(0 <= i < rows) {
    output[i] = sum(row_ptr[i] <= j < row_ptr[i+1])
                  val[j] * vector[col_ind[j]]; }
```

```
7  ... # Set top-level expression: output[<1>]= <2> * <3>          stack=(1,2,3)
8    inherits VectorStore # 1: output[i]                           stack=(2,3)
9        with {outer_loop}          as {scope}
10        and {outer_loop.iterator} as {input_index} at {output} and
11   inherits VectorRead  # 2: val[j]                              stack=(3)
12       with {outer_loop}          as {scope}
13        and {inner_loop.src1}     as {value}#<-case <2>
14        and {inner_loop.iterator} as {input_index} at {val} and
15   inherits VectorRead  # 3: vector[col_ind[<4>]]                stack=(4)
16       with {outer_loop}      as {scope}
17        and {inner_loop.src2} as {value}#<-case <3>
18        and {col_ind.value}   as {input_index} at {vector} and
19   inherits VectorRead  # 4: col_ind[j]                          stack=()
20       with {outer_loop}          as {scope}
21        and {inner_loop.iterator} as {input_index} at {col_ind} and
22   inherits ReadForLoopRanges
23      with {outer_loop}          as {scope}
24       and {inner_loop}          as {for}
25       and {outer_loop.iterator} as {input_index} at {row_ptr})
26 End
```

Figure 6.3: Compressed Sparse Row in IDL: The top section of the figure shows the different arrays involved. The pseudocode in the middle of the figure informs the completion of Listing 6.3 at the bottom. Walking through the expressions and emitting IDL code one-by-one is sufficient.

6.3.1.2 Jagged Diagonal Storage

For Jagged Diagonal Storage (JDS) [160], the rows of the matrix are permuted such that the number of nonzeros per row decreases. The permutation is stored in a vector "**perm**", the number of nonzeros per row in "**nzcnt**". The nonzero entries are then stored in an array "**val**" in the following order: the first nonzero entry in each row, then the second nonzero entry in each row and so on. The array "**col_ind**" stores the column for each of the values and "**jd_ptr**" stores offsets into "**val**" and "**col_idx**".

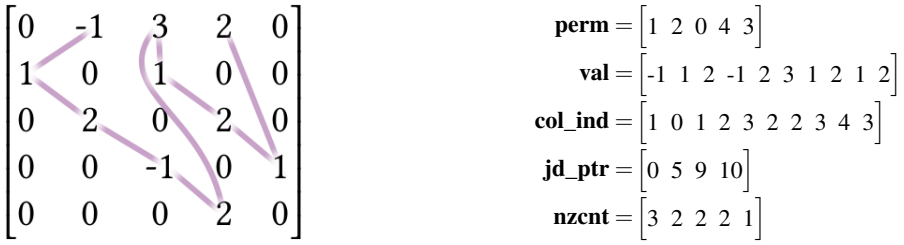
Figure 6.4 demonstrates this format on the example sparse matrix from Figure 6.3, and derives the corresponding IDL code for the "`SPMV_JDS`" idiom underneath. Note that the sparse matrix in this example is shown after the permutation operation that JDS requires.

6.3.2 Dense Linear Algebra

Describing dense linear algebra does not require storage format consideration aside from the simple *row-major* and *column-major* distinction. The generalised matrix multiplication idiom is shown in Listing 6.4. The control flow is captured by three nested for-loops. Inside these loops, the memory access is characterised by three matrix accesses, each with a different subset of the loop iterators. The corresponding "`MatrixRead`" and "`MatrixWrite`" idioms model generic access to matrices, allowing strides and transpositions. The floating-point calculations in the inner loop are encapsulated by the "`DotProductForAlphaBeta`" idiom. This extends the "`DotProductFor`" specification with the linear combination that is part of the generalised matrix multiplication idiom ($C \leftarrow \alpha AB + \beta C$).

```
1  Constraint GEMM
2  ( inherits ForNest(N=3) and
3    inherits MatrixStore
4        with {iterator[0]} as {col}
5         and {iterator[1]} as {row}
6         and {begin} as {begin} at {output} and
7    inherits MatrixRead
8        with {iterator[0]} as {col}
9         and {iterator[2]} as {row}
10        and {begin} as {begin} at {input1} and
11   inherits MatrixRead
12       with {iterator[1]} as {col}
13        and {iterator[2]} as {row}
14        and {begin} as {begin} at {input2} and
15   inherits DotProductForAlphaBeta
16       with {loop[2]}         as {loop}
17        and {input1.value}    as {src1}
18        and {input2.value}    as {src2}
19        and {output.address} as {update_address})
20 End
```

Listing 6.4: IDL specification of the generalised dense matrix-vector multiplication (GEMM)




```
COMPUTATION spmv_jds
  forall(0 <= i < rows) {
    output[perm[i]] = sum(0 <= j < nzcnt[i])
      val[jd_ptr[j]+i] * vector[col_ind[jd_ptr[j]+i]]; }
```

```
7  ... # Set top-level expression: output[<1>] = <2> * <3>          stack=(1,2,3)
8    inherits VectorStore # 1: output[perm[<4>]]                    stack=(4,2,3)
9        with {outer_loop} as {scope}
10        and {perm.value} as {input_index} at {output} and
11   inherits VectorRead  # 4: perm[i]                              stack=(2,3)
12       with {outer_loop}          as {scope}
13        and {outer_loop.iterator} as {input_index} at {perm} and
14   inherits VectorRead  # 2: val[(tmp1)=<5>]                      stack=(5,3)
15       with {outer_loop}      as {scope}
16        and {inner_loop.src1} as {value}#<-case <2>
17        and {tmp1.value}      as {input_index} at {val} and
18   inherits Addition    # 5: (tmp1)=jd_ptr[<6>]+i                 stack=(6,3)
19       with {jd_ptr.value}        as {input}
20        and {outer_loop.iterator} as {addend} at {tmp1} and
21   inherits VectorRead  # 6: jd_ptr[i]                           stack=(3)
22       with {outer_loop}          as {scope}
23        and {inner_loop.iterator} as {input_index} at {jd_ptr} and
24   inherits VectorRead  # 3: vector[<7>]                         stack=(7)
25       with {outer_loop}      as {scope}
26        and {inner_loop.src2} as {value}#<-case <3>
27        and {col_ind.value}   as {input_index} at {vector} and
28   inherits VectorRead  # 7: col_ind[(tmp1)=<5>]                  stack=()
29       with {outer_loop} as {scope}
30        and {tmp1.value} as {input_index} at {col_ind} and
31   inherits ReadForLoopIterations
32      with {outer_loop}          as {scope}
33       and {inner_loop}          as {for}
34       and {outer_loop.iterator} as {input_index} at {read_range}
```

Figure 6.4: Jagged Diagonal Storage in IDL: The top section of the figure shows the different arrays involved. The pseudocode in the middle of the figure informs the completion of Listing 6.3 at the bottom. Walking through the expressions and emitting IDL code one-by-one is sufficient.

```
1  Constraint Stencil
2  ( inherits ForNest and
3    inherits PermMultidStore
4        with {iterator} as {input}
5         and {begin}    as {begin} at {write} and
6    collect i 32
7    ( inherits StencilRead
8          with {write.input_index} as {input}
9           and {kernel.inputs[i]}  as {value}
10          and {begin} as {begin}  at {reads[i]}) and
11   {kernel.output} is first argument of {write.store} and
12   inherits KernelFunction
13       with {loop[0]} as {scope} at {kernel})
14 End
```

Listing 6.5: IDL specification of a basic stencil computation

6.3.3 Stencils

Listing 6.5 shows the base version of the stencil idiom. Stencils consist of a loop nest with multi-dimensional memory access (lines 3–5) to store the updated cell value. The updated value is computed by a kernel function (lines 12–13) using several values that are constrained by "`StencilRead`" (lines 6–10), which specifies multi-dimensional array access with only constant offsets in all dimensions.

6.4 Comparison to Syntactic Matching

The idiom descriptions may at first appear to be shallow syntactic pattern matching, but the approach is intrinsically more powerful. Because it operates on the IR level, the solver can detect idioms that are written in a superficially distinct style but are semantically equivalent.

For example, Listing 6.6 shows two syntactically distinct programs, which nevertheless are both implementations of general matrix multiplication. The solver – using Listing 6.4 – discovers that both of these loop nests are instances of GEMM and can be replaced with the same API call.

The reverse is also true: the solver distinguishes syntactically identical but semantically distinct programs. Listing 6.7 shows such an example. The loop nest in lines 7–11 appears to compute a standard GEMM, but in fact, the matrices are not stored contiguously, and acceleration with standard libraries is impossible.

There are limitations to this semantic matching. In particular, the use of low-level manual optimisations that circumvent the usual intermediate representation, e.g. SIMD intrinsics, may distort the algorithms beyond recognition. In practice, this is rarely encountered.

```
for (int mm = 0; mm < m; ++mm) {
  for (int nn = 0; nn < n; ++nn) {
    float c = 0.0f;
    for (int i = 0; i < k; ++i) {
      float a = A[mm + i * lda];
      float b = B[nn + i * ldb];
      c += a * b;
    }
    C[mm+nn*ldc] =
        C[mm+nn*ldc] * beta + alpha * c;
  }
}
```

```
double M1[1000][1000];
double M2[1000][1000];
double M2[1000][1000];

//...

for(int i = 0; i < 1000; i++)
    for(int j = 0; j < 1000; j++) {
        M3[i][j] = 0.0f;
        for(int k = 0; k < 1000; k++)
            M3[i][j]+=M1[i][k]*M2[k][j]; }
```

Listing 6.6: Two matching instances of "GEMM": Although both loop nests are implemented very differently, they both match the same IDL specification and can be accelerated identically.

```
double *M1[1000];
double *M2[1000];
double *M2[1000];

//...

for(int i = 0; i < 1000; i++)
    for(int j = 0; j < 1000; j++) {
        M3[i][j] = 0.0f;
        for(int k = 0; k < 1000; k++)
            M3[i][j]+=M1[i][k]*M2[k][j]; }
```

Listing 6.7: This C program that does not match "GEMM". Although the loop syntax is identical to the matching example from Listing 6.6, the different types of the matrices prevent detection. This is desirable: Established backends are incompatible with such non-contiguous memory layout.

6.5 Targeting Heterogeneous Backends

After idiom detection, user programs must be transformed to exploit the relevant APIs. Two types of heterogeneous APIs were targeted: libraries and domain-specific languages with their optimising compilers.

6.5.1 Domain-Specific Libraries

Libraries provide narrow interfaces but are often highly optimised. For example, the cuBLAS library is only suitable for a limited set of dense linear algebra operations and only works on Nvidia GPUs, but its implementation provides outstanding performance. For sparse linear algebra, the vendor-provided cuSPARSE, clSPARSE, and Intel MKL libraries were used. For dense BLAS routines, the available backends were cuBLAS, clBLAS, CLBlast, and MKL.

6.5.2 Domain-Specific Code Generators

Domain-Specific Languages provide wider interfaces than libraries and allow problems to be expressed as compositions of dedicated language constructs. Most importantly, this allows DSLs to capture kernel functions of idioms that are higher-order functions, e.g. stencils and reductions. The domain-specific optimising compiler then specialises the program for the target hardware.

The end result is a library object, which can then be treated identically to pre-generated vendor libraries. For this research, domain-specific code generators are, therefore, effectively used as on-demand library generators. The evaluation used Halide and Lift as domain-specific code generators.

Halide Ragan-Kelley et al. [11] introduced a language and optimising compiler targeted at image processing applications. Optimised code is generated for CPUs as well as GPUs. Halide separates the functional description of the problem from the description of the implementation. This involves a separate execution *schedule*. This separation allows retargeting of Halide programs to different platforms without touching the core program. Some of the stencil and linear algebra idioms were translated into Halide. However, stencils involving control flow in their kernel were not expressible in Halide and excluded for this backend.

Lift Steuwer et al. [109] introduced an optimising code generator based on rewrite rules [157, 161]. The Lift language consists of functional parallel patterns such as "`map`" and "`reduce`" that express a range of parallel applications. Stencil idioms, complex reductions and linear algebra idioms were translated to Lift.

6.6 Translating Computational Idioms

This section describes how the detected idioms are mapped to the previously described library APIs and domain-specific languages. The two types of APIs (library interfaces and domain-specific languages) are treated individually.

6.6.1 Domain-Specific Libraries

For library call interfaces, the original code is removed, and an appropriate function call is inserted. The solution that is generated by the solver using the IDL program contains both the IR instructions to remove as well as the arguments that are to be used for the function call.

For example, in the case of the "`GEMM`" program that was shown in Listing 6.4, the original code is removed by deleting the IR instruction at "`output.store_instr`" explicitly, which captures the store instruction of the "`MatrixStore`" subprogram. The remaining cleanup is left to the standard dead code elimination pass. The arguments that specify the matrix dimensions are taken from "`ForNest`" in combination with the stride and offset determined by "`MatrixRead`" and "`MatrixWrite`".

The mapping of solution variables to the arguments of the generated function call needs to be implemented individually for each backend, as it cannot be described using IDL itself. Once the code is replaced, LLVM continues with code generation as usual.

6.6.2 Domain-Specific Code Generators

For domain-specific code generators, the situation is a bit more involved than for libraries. Stencils and Complex Reduction and Histogram Computations are higher-order functions, containing kernel functions or reduction operators that have to be represented for the DSL.

For each combination of idiom and DSL, there is a parameterised skeleton program. This skeleton is then specialised for the appropriate data types and numeric parameters as well as the kernel function or reduction operator.

Numerical parameters are picked from the constraint solution in the same way as previously described for library call interfaces. Also, from the constraint solution, the loop body that contains the kernel function or reduction operator is accessible, as well as the input values and the result value used. This information is enough to cut out the kernel function, which is then used to generate code appropriate for the DSL backends.

Halide is a language embedded in C++. It requires a syntax tree of the kernel functions that built using a class hierarchy. However, Halide does not support control flow in the kernel functions, making code generation easier than for Lift in practice. The relevant kernel functions contain only a few LLVM IR instructions, which are easily assembled into Halide expressions.

Lift expects stencil kernels or reduction operators to be sequential C code with a specific function interface, which it requires for generating valid OpenCL code. As the kernel functions are only directly available as LLVM IR code from the constraint solution, the implementation of a rudimentary LLVM IR-to-C backend was necessary to generate input for the Lift compiler.

```
1  float mult(float x, float y) { return x*y; }
2  float add(float x, float y) { return x+y; }
3
4  gemm_in_lift(A, B, C, alpha, beta) {
5   map(fun(a_row, c_row) {
6    map(fun(b_col, c) {
7     map(fun(ab){ add(mult(alpha, ab), mult(beta, c))},
8      reduce(add, 0.0f, map(mult, zip(a_row, b_col))))
9    }, zip(transpose(B), c_row))
10  }, zip(A, C))
11 }
```

Listing 6.8: GEMM in Lift is expressed with the higher-order functions "zip", "map", "reduce".

Example After code for the DSLs was generated, it is passed to the DSL code generator. Listing 6.8 shows an example of the Lift code generated for GEMM ("gemm_in_lift"). It performs a dot product (expressed in line 8 using the Lift skeletons "zip", "map", and "reduce") for each row of the first matrix ("a_row") and column of the second ("b_col"). This code is compiled by Lift into optimised OpenCL code.

6.6.3 Pointer Aliasing

Since idiom detection works statically, pointer aliasing cannot be ruled out conclusively, which can make transformations unsound. For dense linear algebra, this is easily solved with simple runtime checks for non-overlapping memory. In case of any detected aliasing, the program falls back to the naive implementation.

However, for sparse linear algebra, this is not as straightforward. While aliasing can still be ruled out by dynamic checks, this involves additional overhead, as the indices of the indirect array access need to be monitored. This overhead can be amortised over multiple calls if the indirection array remains unchanged, which was the case for the programs involved in the evaluation.

Such techniques to rule out aliasing are important, but orthogonal to the detection and transformation methods that are the focus of this chapter. In practice, aliasing did not cause problems on any of the benchmark programs. However, detailed feedback was provided in the form of transformation reports that give the user control over the program modifications and allow the explicit disabling of the approach in uncertain cases.

NPB	BT	Block Tridiagonal Solver
	CG	Conjugate Gradient
	DC	Data Cube Operator
	EP	Embarrassingly Parallel Marsaglia Polar Method
	FT	Fast Fourier Transform
	IS	Small Integer Bucket Sort
	LU	Lower-Upper Symmetric Gauss-Seidel Solver
	MG	MultiGrid Approximation
	SP	Scalar Pentadiagonal Solver
	UA	Unstructured Adaptive Mesh
Parboil	bfs	Breadth-First Search
	cutcp	Distance-Cutoff Coulombic Potential
	histo	Saturating Histogram
	lbm	Lattice-Boltzmann Method Fluid Dynamics
	mri-gridding	Magnetic Resonance Imaging - Gridding
	mri-q	Magnetic Resonance Imaging - Q
	sad	Sum of Absolute Differences (part of MPEG encoding)
	sgemm	Dense Matrix-Matrix Multiply
	spmv	Sparse-Matrix Dense-Vector Multiplication
	stencil	Iterative 3D Jacobi Stencil
	tpacf	Two Point Angular Correlation Function

Table 6.1: Overview of the 21 programs used for evaluation, grouped into two suites

6.7 Experimental Setup

Benchmarks The approach was applied to all of the sequential C/C++ programs of the NAS Parallel Benchmarks, in the versions provided by Seoul National University (SNU NPB) [140]. Additionally, the Parboil benchmarks [150] were used, giving 21 evaluation programs in total.

Platform and Evaluation The evaluation platform used an AMD A10-7850K APU with a Radeon R7 integrated GPU (driver version 1912.5) and an Nvidia GTX Titan X external GPU (driver version 375.66). Execution times were measured as the median over ten runs.

Alternative Detection Approaches There were no readily available compilers performing idiom detection to compare against. Instead, two parallelising compilers were considered. The evaluation examined the ability to parallelise idiomatic code, without considering whether the idioms were recognised. Section 5.5.2 provided a detailed discussion of both competing tools: Polly and ICC. Therefore, they are only presented briefly here. It should be borne in mind that the competing compilers aim for parallelisation, not idiom detection itself.

Polly is an LLVM-based polyhedral compiler. The SCoPs that Polly detected with the options "`-O3 -mllvm -polly -mllvm -polly-export`" were collected during compilation. When Polly captured a SCoP with an idiom, it was counted as an idiom detection. This gives an optimistic estimate as to what idiom coverage a polyhedral based approach can achieve.

The Intel C++ Compiler (ICC) is a mature industry strength compiler that performs auto-parallelisation. All loops that were emitted as parallelisable by "`-parallel -qopt-report`" and that contained idioms were counted as idiom detections.

6.8 Results

The approach was evaluated in several steps. First, the number of detected idioms and their distribution over the benchmark programs was established. During this analysis, the runtime of the IDL-enabled Clang compiler was measured, and the compile time overhead of the solver over standard compilation evaluated. Next, the runtime coverage of the idioms was determined for each benchmark program to see where exploitation might be beneficial.

Where runtime coverage was substantial, speedups over the sequential C code are reported. Detailed results are given for the performance of each targeted backend interface. Finally, the evaluation included comparisons to the handwritten OpenMP and OpenCL implementations that were provided with the benchmark suites as reference implementations. These versions provide suitable estimates for the upper bound of available performance.

6.8.1 Idiom Detection

Figure 6.5 shows the different idioms detected across all the individual benchmark programs. IDL detected both scalar and histogram reductions, as well as stencils, dense matrix operations and sparse matrix-vector multiplication. 16 of the 21 programs contained computational idioms that IDL was able to identify. The most frequently occurring idiom was the scalar reduction, present in 10 of the programs. Five of the benchmarks contained histograms; three contained stencils; two contained SPMV; and only a single program calculated GEMM.

Table 6.2 summarises the number of computational idioms found by IDL, Polly, and ICC. Polly found three scalar reductions and five stencils, and also detected the GEMM operation as a generic SCoP. This was counted as a detected idiom, although Polly was unable to apply any domain-specific knowledge from linear algebra. ICC only recognised scalar reductions, but was much more successful at it than Polly, detecting a total of 28 instances.

Other approaches than IDL did not detect any histogram loops or sparse matrix operations. IDL detected 60 idioms overall with the impact on the compilation times shown in Table 6.3. On average, the compilation times were increased by 82%. This compilation overhead was due to the solver and could be reduced further by optimising its implementation.

6.8.2 Runtime Coverage

To determine if the detected idioms were impactful, Figure 6.6 shows the percentage of runtime spent in the detected computational idiom for each benchmark program. This data shows that either the detected idioms have a low runtime contribution or they dominate almost the entire execution. *EP* is the only exception with $\sim$50% runtime coverage. Heterogeneous acceleration was evaluated on the ten programs that spend a significant amount of time in the detected idioms. Only these can reasonably expect a performance gain.

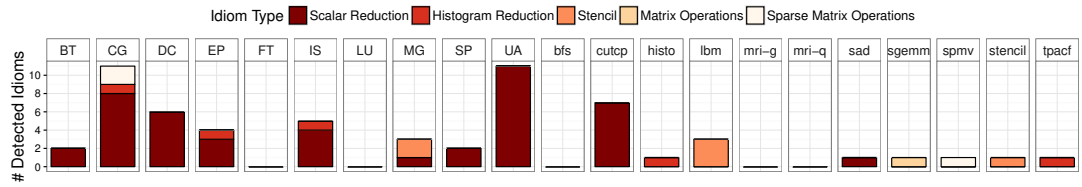


Figure 6.5: The different computational idioms found across the benchmark programs: Scalar reductions were the most common, with 10 out of 21 programs containing some. Other idioms were found in 1–5 programs each. Only five of the benchmarks contained no idiomatic code.

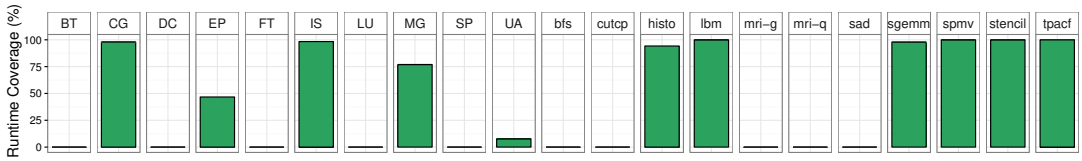


Figure 6.6: Runtime coverage: 10 of the 21 programs have idiomatic performance bottlenecks.

	Scalar Reduction	Histogram Reduction	Stencil	Matrix Op.	Sparse Matrix Op.
Polly	3	—	5	1	—
ICC	28	—	—	—	—
IDL	45	5	6	1	3

Table 6.2: Comparison of the number of idiom instances detected by IDL, ICC, and Polly: ICC detected 60% of IDL's reductions, but no other idioms. Polly focused mostly on stencil programs, but detected only 3 scalars. Both idioms with indirect memory access were exclusive to IDL.

	BT	CG	DC	EP	FT	IS	LU	MG	SP	UA	bfs	cutcp
without IDL	1.9	0.5	1.0	0.3	0.6	0.3	1.9	0.8	1.6	2.7	0.4	0.4
with IDL	4.0	0.8	1.6	0.6	1.2	0.5	3.9	4.5	3.2	7.3	0.5	0.6
overhead in %	116	77	57	77	93	62	103	484	97	169	30	65

	histo	lbm	mri-g	mri-q	sad	sgemm	spmv	stencil	tpacf
without IDL	0.2	0.3	0.2	0.2	0.4	0.6	0.3	0.2	0.2
with IDL	0.2	0.6	0.4	0.3	0.6	0.7	0.7	0.2	0.4
overhead in %	35	87	100	52	58	24	115	36	54

Table 6.3: Compile time cost of IDL solver in seconds: Compilation took 82% longer on average.

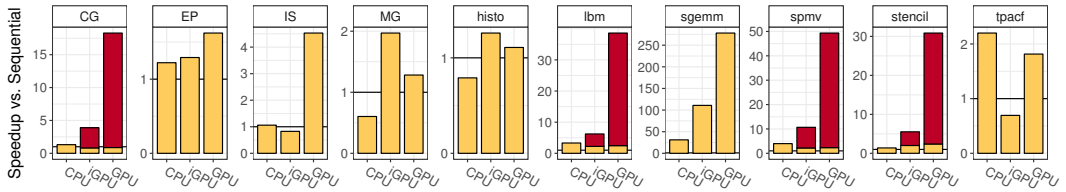


Figure 6.7: Speedup over sequential: Results for the best-performing backend on each platform are shown. The red bars indicate a manual modification for minimising redundant data transfers.

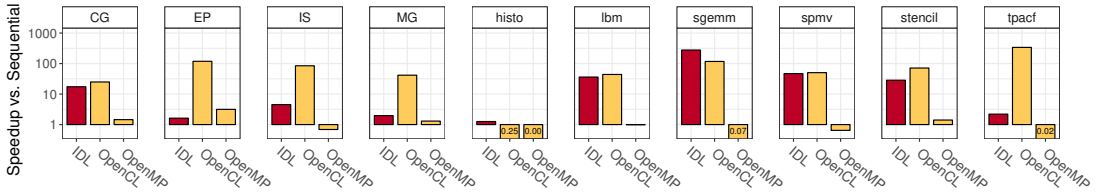


Figure 6.8: IDL versus manual expert parallelisation: Speedup over the sequential baseline was measured for IDL (selecting best performing backend; red bars) and the handwritten reference OpenCL and OpenMP implementations (provided by the benchmark developers; yellow bars).

6.8.3 Performance Results

Speedup over Sequential Figure 6.7 shows the end-to-end speedup obtained by accelerating idioms via heterogeneous APIs on a CPU, an integrated GPU, an external GPU, respectively. All of the results include data transfer overhead to and from the GPUs, where required. This overhead was intrinsic to the external GPU, which operates on distinct physical memory. The integrated GPU, on the other hand, only incurred this cost when APIs enforced additional data copies. Each bar shows the best-performing API for the given platform; Table 6.4 provides detailed results also for the other API backends.

Moderate speedups were obtained for five of the benchmarks, from 1.26× for "histo" up to 4.5× for 'IS". Besides "MG", all of these benchmarks had a scalar or histogram reduction as their performance bottleneck. Interestingly, it emerges that for different benchmarks, different hardware was beneficial. For "tpcaf", the CPU was the best platform, beating the GPU, for which the data transfer time dominated; for "MG" and "histo", the integrated GPU struck the right balance between computational power and avoiding data movement to the external GPU; and for "EP" and "IS", the data transfer to the GPU was amortised by its superior raw performance. The results emphasise the significance of flexible heterogeneous code generation. Committing to any one of the three hardware platforms in the source code would have always resulted in less-than-optimal performance for at least some of the programs.

For five of the benchmarks, IDL enabled significantly higher performance gains, from 17× for "CG" and up to over 275× for "sgemm". These benchmarks were computationally expensive, and the external GPU was always the fastest architecture by a considerable margin.

	CPU						iGPU					GPU			
	MKL	libSPMV	Halide	clBLAS	CLBlast	Lift	clSPARSE	libSPMV	clBLAS	CLBlast	Lift	cuSPARSE	libSPMV	cuBLAS	Lift
CG	**1504.21**	—	—	—	—	—	**644.02**	—	—	—	—	**113.51**	—	—	—
EP	—	—	—	—	—	**32762.50**	—	—	—	—	**30983.40**	—	—	—	**24680.70**
IS	—	—	**426.95**	—	—	1765.61	—	—	—	—	**547.28**	—	—	—	**99.95**
MG	—	—	—	—	—	**4699.63**	—	—	—	—	**1439.58**	—	—	—	**2211.56**
histo	—	—	—	—	—	**27.42**	—	—	—	—	**17.20**	—	—	—	**19.54**
lbm	—	—	—	—	—	**6457.93**	—	—	—	—	**5335.09**	—	—	—	**590.60**
sgemm	**53.50**	—	—	1661.75	660.44	1339.15	—	—	**14.73**	19.03	15.04	—	—	**5.99**	7.87
spmv	—	**218.17**	—	—	—	—	—	**102.233**	—	—	—	—	**18.437**	—	—
stencil	—	—	**5760.81**	—	—	21951.80	—	—	—	—	**2261.48**	—	—	—	**279.38**
tpacf	—	—	—	—	—	**19276.40**	—	—	—	—	**61111.90**	—	—	—	**23358.20**

Table 6.4: Detailed performance results for each heterogeneous backend interface: The runtime of each benchmark program was measured in milliseconds for every compatible combination on each of the three platforms. The fastest implementations per benchmark and target hardware are highlighted in **bold**.

The red highlighting in Figure 6.7 indicates an important runtime optimisation: Redundant data transfers for the iterative "CG", "lbm", "spmv" and "stencil" benchmarks were manually eliminated. All of these benchmarks executed computations inside a for-loop, and did not require access to the data on the CPU between iterations. A straightforward lazy copying technique was manually applied by flagging memory objects to avoid redundant transfers, similar to work by Jablin et al. [162]. This runtime optimisation was crucial for achieving high performance on some of the benchmark programs.

API Performance Comparison Table 6.4 shows a breakdown of the performance of each API on each program and platform. Not all APIs target all platforms, e.g. cuSPARSE only targets NVIDIA GPUs. The version of Halide that was used for this evaluation failed to generate valid GPU code for any of the benchmarks. Therefore, only the CPU platform was evaluated with Halide. The best-performing backend is highlighted in bold in the table entries. None of the previously existing backends supported "`SPMV-JDS`, which was detected in the "spmv" benchmark. The libSPMV library was implemented as an ad-hoc solution to this, providing straightforward parallelisation of the idiom.

On the multi-core CPU, Intel MKL gave the best linear algebra performance, outperforming the other libraries and Lift. Halide achieved good performance for the NPB "IS" and Parboil "stencil" benchmarks on the CPU, outperforming Lift due to its more advanced vectorisation capabilities. In programs that were dominated by scalar reductions, Lift performed well. On the internal GPU, clBLAS provided a better matrix multiplication implementation than CLBlast and Lift. On the external GPUs, library backends provided the best linear algebra implementations, while Lift performed well on stencils and reductions.

Speedup vs Handwritten Parallel Implementations Figure 6.8 shows the performance of the IDL approach compared to handwritten reference OpenMP and OpenCL implementations. For some of the benchmarks, the parallel versions were significantly modified beyond the reach of automation and used entirely different algorithms. For benchmarks where the parallel reference implementation did not make profound algorithmic changes ("CG", "histo", "lbm", "sgemm", "spmv", "stencil"), IDL enabled comparable – or better – performance. On four of the benchmarks ("EP", "IS", "MG", "tpacf") it was more beneficial to parallelise the entire application, which is beyond the scope of this work.

The handwritten versions of "sgemm" and "stencil' were outperformed by IDL. This was due to implementation flaws; a loop interchange already improved performance by almost 20×.

Summary 60 idioms were detected across the benchmark suites, and significant performance improvements were achieved by targeting different heterogeneous APIs for those benchmarks where idioms dominate execution time.

6.9 Conclusions

This chapter developed an approach for automatically detecting a broad class of computational idioms during compilation. These idioms are supported by existing numerical libraries and domain-specific languages that implement code generation for heterogeneous accelerators. Once detected, the idiomatic loops are replaced by function calls to code generated by these external tools.

The detection approach is based on the declarative Idiom Detection Language (IDL) that identifies program subsets that adhere to idiom specifications with a constraint solver. The evaluation showed that the ability of IDL to cover irregular idioms improved significantly over the state-of-the-art implemented by Polly and ICC. Of the 10 benchmark applications where IDL captured the majority of execution time as idiomatic, more than half implemented idioms with indirect memory accesses. The incorporation of specialised code generators for idiomatic code enabled reliable performance gains: speedups were achieved on all 10 programs. This shows that idiom detection is a promising path to heterogeneous acceleration.

Chapter 7

Conclusions

This thesis introduced a constraint programming methodology that operates on SSA compiler intermediate representation. The Compiler Analysis Description Language (CAnDL) and the extended Idiom Detection Language (IDL) were developed, and implemented in the LLVM framework. This made the constraint programming method available for program analysis.

Several computational idioms were specified using CAnDL and IDL, enabling automatic recognition of adhering user code sections during compilation. The well-studied kernels among these idioms were stencils and varied forms of sparse and dense linear algebra. Complementing these established kernels, Complex Reduction and Histogram Computations (CReHCs) were introduced as a new grouping of calculations. The evaluation on the established benchmark suites NPB and Parboil demonstrated that all of these computational idioms covered significant performance bottlenecks.

Recognising these computational idioms enabled generic compilers to apply idiom-specific optimising transformations, which are traditionally available only within domain-specific tools. Such transformations included the automatic parallelisation of programs that were inaccessible to previous analysis approaches.

Given this background, the idiom detection was performed on sequential C/C++ programs, achieving automatic heterogeneous parallelisation. Idiomatic loops in the code were redirected to domain-specific code generators. These specialised tools leveraged the domain knowledge that is available for code in restrictive idioms. This approach was applicable to 10 of the 21 benchmark programs from NPB and Parboil, resulting in speedups between 1.26× and 275× over the sequential baseline versions.

In summary, this thesis demonstrated that computational idioms are a suitable interface to heterogeneous acceleration, and developed approaches for automatically recognising them. The main contributions are the methodology of constraint programming on SSA intermediate representation, the design and implementation of CAnDL and IDL, and the identification of generalised reductions as a significant class of benchmark bottlenecks.

7.1 Contributions

Constraint Programming on Compiler Intermediate Representation Chapter 2 introduced constraint programming on SSA form intermediate code. Using a characterisation of the static structure of SSA programs, *SSA constraint problems* were defined. These are formulas that impose restrictions on compiler intermediate code, turning the detection of adhering program parts into a constraint satisfiability problem. This thesis derived efficient algorithms for solving SSA constraint problems and discussed significant classes of constraint formulas, reflecting compiler analysis methods such as data flow and dominance relationships.

The constraint programming methodology was derived starting from algebraic formulation through to developing the implementation in C++. It applies to any SSA compiler intermediate representation and is suitable for a wide range of analysis problems.

Specification Languages Chapters 4 and 5 designed and implemented two novel declarative specification languages: CAnDL (Compiler Analysis Description Language) and its extension IDL (Idiom Detection Langauge). These languages make the constraint programming method available in the LLVM framework. This enables constraint programming on the intermediate code of user programs during compilation with the Clang C/C++ compiler.

Complex Reduction and Histogram Computations Complex Reduction and Histogram Computations (CReHCs) were introduced in Chapter 5 as a computational idiom. CReHCs are a generalisation of the well-understood scalar reduction, that additionally covers the indirect array accesses typically found in histogram calculations. This type of loop was not previously studied, but this thesis showed that shared parallelisation opportunities exist. Moreover, an evaluation on established benchmark suites demonstrated that several performance bottlenecks in each of NPB, Parboil, and Rodinia are, in fact, CReHCs.

Specification of Computational Idioms Chapters 4 to 6 specified a range of computational idioms in CAnDL and IDL. These included stencils, different forms of sparse and dense linear algebra, polyhedral Static Control Parts (SCoPs), and CReHCs. The constraint formulations enabled the automatic recognition of high-level algorithmic structure during compilation. This enabled generic compilers to apply domain-specific reasoning.

Heterogeneous Acceleration Pipeline Chapter 6 implemented a heterogeneous acceleration pipeline. The user code sections recognised by IDL specifications were redirected to domain-specific code generators for heterogeneous accelerator hardware. The evaluation demonstrated significant heterogeneous parallelisation speedups on established benchmark programs.

7.2 Critical Analysis

The approaches of this thesis were built on the derivation of Chapter 2 and evaluated in several scenarios on C/C++ program code. Despite the effort of bridging between the algebraic formulation and the application in real-world scenarios, the work remains a prototype, and several issues need to be addressed before it becomes viable for productive use. Importantly, this includes questions about the prevalence of idiomatic code, the affordability of significant compile time overhead, and the ability to compensate for limitations of the underlying solver technology.

Universality of Computational Idioms Computational idioms were specified in Chapters 5 and 6 and evaluated successfully on a range of established benchmark collections. However, the NPB and Parboil collections are both from areas of scientific computing. It remains unclear how generally applicable these idioms are on codebases from other domains. Similar concerns are faced by competing approaches, including the polyhedral model.

More generally, it is unclear how much code in large-scale applications could be captured as "idiomatic" even with a more extensive set of idioms. Even some benchmark programs within NPB and Parboil contain idiosyncratic computations that are unlikely to reoccur in other contexts. An example is the "sad" program that computes the "Sum of Absolute Differences" algorithm used by the reference H.264 video encoder.

Compile Time Cost The compile time cost of idiom detection was evaluated in Chapter 6, showing that overheads between 35% and 115% occurred across the benchmark programs. Given that the approach is built on constraint satisfiability methods, this is a reassuring result, as the compile times remain within one order of magnitude. Nonetheless, from the perspective of compiler optimisations, this might be a prohibitive cost. Moreover, the compile time overhead is unevenly distributed, with disproportionately longer solver times on large functions and especially on code sections that are near misses to satisfying the specifications.

Bounded Number of Solver Variables The specification languages were restricted by the solver to a finite number of variables in the underlying constraint problems. Moreover, each additional variable incurred a slight overhead, introducing a tradeoff between solver speed and the generality of the specifications. This caused upper bounds for the number of features within many idiom specifications: The definition of CReHCs only allowed a maximum of two histograms, stencils only allowed a neighbourhood of up to 32 elements, and so on.

Similar shortcomings are well-known in other research disciplines using solvers, and some techniques for resolving them have been suggested by Krings and Leuschel [163] in the context of constraint logic programming.

7.3 Future Work

Directions for future work include the specification of more computational idioms, usability improvements of the specification languages, and the application of dynamic analysis and machine learning to complement the static analysis. Moreover, constraint specifications could eventually be generated from examples, eliminating the difficulty of writing them manually.

Complementing Dynamic and Machine Learning Approaches The static methods in this thesis could be enhanced with dynamic approaches and machine learning. Such methods would naturally complement the entirely static reasoning of constraint programming, but were outside the scope of this work.

Dynamic profiling could preselect candidate hot loops, drastically reducing the compile time overhead. More advanced dynamic analysis could collect program features to be fed into machine learning algorithms. The use of neural networks to guide compiler optimisations has been successfully studied in the relevant literature [164].

Dynamic methods are also suitable for automating the efficient memory transfers required between cooperating hardware in heterogeneous computing, and to rule out pointer aliasing.

Unbounded Number of Variables Related research disciplines found ways around some limitations of their underlying solvers. For example, Bounded Model Checking [165] uses SAT solvers and involves the unrolling of loops to generate formulas. In order to fit the underlying solver, this requires the introduction of a certain finite iteration limit k. However, the checking process can be repeated with increasing values for k, successively ruling out possible violations.

The same approach could be applied to constraint solving on SSA. The solver would be invoked repeatedly with a gradually increasing number of variables. When no collect constraint exceeds variable capacity anymore, the process would be terminated. This would ensure that small solutions are found without unnecessary overhead, yet solutions requiring many variables would not be discarded entirely. No upper limits for collect statements would be required with this scheme.

Pointer-Chasing Idioms The ability to recognise sparse linear algebra sets the methods of this work apart from previous approaches. Going beyond a single data access indirection, IDL could be used to capture data access patterns that involve pointer chases. This would allow it to analyse graph operations, such as depth-first graph traversal and the PageRank algorithm. Parboil implements the breadth-first search program “bfs” that could be accelerated this way.

Moreover, IDL could detect iterations over lists, and the insertion and removal of list items, extending work by Manilov et al. [97]. Such patterns are not usually performance-critical, but this would be another step toward an understanding of dynamic data structures in compilers.

Higher-level Specification Languages Compared to previous approaches, the specification languages CAnDL and IDL simplify the implementation of compiler analysis functionality and enable the detection of more sophisticated idiomatic structures. Nonetheless, knowledge of the compiler internals is still required to write correct specifications. Furthermore, the precise extent to which the customisation of the specification languages to LLVM IR was necessary has not been explored thoroughly.

Future work could investigate languages that abstract away the compiler-specific nature of CAnDL and IDL, providing an improved programming experience. The pseudocode and the corresponding specifications in Figures 6.3 and 6.4 suggest how this could be achieved. They show that for restricted domains (e.g. SPMV), generating IDL from high-level expressions is straightforward.

Phase-Ordering for Normalisation The presented methods for constraint programming on LLVM IR relied on preceding optimisation passes for the normalisation of the intermediate code generated from user programs. The relationship between normalisation and optimisation in compiler transformation passes poses interesting research questions.

To allow the specification of idioms on predictable intermediate code structures, the solver was invoked after the Clang optimisation pipeline. Optimisations such as loop-independent code motion helped eliminate the artefacts of superficial implementation decisions by mapping many different input programs onto the same optimised intermediate code.

While this approach was effective in practice, the existing LLVM optimisation passes were not designed for this purpose. Instead, they aim to maximise runtime performance and minimise code size. Incidentally, these two goals often coincide with normalising behaviour.

However, there are clear exceptions to this rule. The outcome of loop unrolling introduces unpredictability, as the decision to unroll is based on opaque heuristics and threshold values that depend on command-line options. Similarly, threshold values for loop unswitching interact with the loop inversion transformation in obfuscating ways. The surrounding conditionals of inverted loops within loop nests are sometimes – but not always – propagated out. Finally, strength reduction changes the opcodes of instructions in special cases.

These challenges were resolved pragmatically in this research. High threshold values were set, some optimisations were disabled (loop unrolling, vectorisation), and others were partially reversed (strength reduction).

Instead of pruning the "`-Os`" or "`-O2`" presets, future work could establish special-purpose normalisation compilers by re-evaluating the phase-ordering from scratch. According to a new metric, individual compiler optimisations could be evaluated on a spectrum from "distorting" to "normalising". The highly-scoring passes would then selected for a dedicated phase-ordering for intermediate code normalisation.

Generating Specifications from Examples The automatic generation of specifications from examples is another direction for follow-up research. Initial attempts were successful [166]. Using a graph-matching algorithm that operates on SSA code, implementation variations of the same idiom were overlayed, exposing the common structure. This was guided by a quality metric for the matching. Code structures that were shared between the examples were turned into constraints, and features that were unique to specific samples were discarded.

Suitable quality metrics for graph-matching are critical to the success of this approach and could eventually be tuned with machine learning approaches, such as neural networks. The manually implemented idioms from this thesis would provide suitable training data.

Eventually, the manual curation of example codes that group together into computational idioms could become redundant. Automatic profiling of large quantities of code would allow for the automatic identification of all the relevant hot loops. These loops could then be grouped into computational idioms by an automatic clustering algorithm using a distance metric. The success score for the previously discussed graph-matching approach would be a suitable basis for deriving such a metric for the distance between different candidate loops.

7.4 Summary

This chapter gave a brief overview of the thesis, outlining the development of the approaches and putting the achieved results into context. The core contributions of this work were listed individually in a dedicated section. Furthermore, the critical analysis discussed implementation issues and brought attention to details that could profit from further evaluation.

Interesting challenges remain for future work. The suggested topics include improvements of the basic methodology for constraint programming, enhanced usability of the specification languages, and the application to novel domains.

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Appendix A

Full Grammar of CAnDL

Listing A.1: Verbatim display of the grammar file that was used to generate the CAnDL parser: The file is in a custom version of Backus–Naur form. In the parse tree, all expressions that start with "@" are automatically expanded. The character "#" marks the top-level language construct. Any expression that does not ultimately become part of this construct constitutes a syntax error.

```
1  # ::= <specification>
2
3  specification ::= Constraint <s> <@formula> End
4
5  @formula ::= <atom> | <conjunction> | <disjunction>
6   | <rename> | <collect> | <if> | <default> | <@nested>
7
8  conjunction ::= '(' <@formula> ∧ <@formula>
9                                { ∧ <@formula> } ')'
10
11 disjunction ::= '(' <@formula> ∨ <@formula>
12                               { ∨ <@formula> } ')'
13
14 rename ::= ( <conRange> | <disRange> | <include> | <for> )
15            [ '(' <@variable> -> <@variable>
16            {  ,  <@variable> -> <@variable> } ')' ]
17            [  @  <@variable> ]
18
19 include ::= include <s> [ '[' <s> = <@index>
20                          {  , <s> = <@index> } ']' ]
21
22 conRange ::= <@formula> foreach <s> = <@index> .. <@index>
23 disRange ::= <@formula> forany  <s> = <@index> .. <@index>
24 for      ::= <@formula> for     <s> = <@index>
25
26 collect ::= collect <s> <n> <@formula>
27
28 if ::= if <@index> = <@index> then <@formula>
29                                 else <@formula> endif
30
31 default ::= <@formula> for <s> = <@index>
32             if not otherwise specified
```

```
@nested ::= '(' <@formula> ')'

atom ::= <IntegerType> | <FloatType>
  | <VectorType> | <PointerType>
  | <Unused> | <IntZero> | <FloatZero>
  | <Constant> | <Preexecution> | <Argument> | <Instruction>
  | <Same> | <Distinct>
  | <DFGEdge> | <CFGEdge> | <CDGEdge> | <PDGEdge>
  | <FirstOperand> | <SecondOperand>
  | <ThirdOperand> | <FourthOperand>
  | <FirstSuccessor> | <SecondSuccessor>
  | <ThirdSuccessor> | <FourthSuccessor>
  | <Dominate> | <DominateStrict>
  | <Postdom> | <PostdomStrict> | <Blocked>
  | <IncomingValue> | <FunctionAttribute>
  | <Opcode> | <SameSets>

IntegerType ::= data_type <@variable> = integer
FloatType   ::= data_type <@variable> = float
VectorType  ::= data_type <@variable> = vector
PointerType ::= data_type <@variable> = pointer

Unused ::= <@variable> is unused
Opcode ::= opcode <@variable> = <s>

IntZero   ::= <@variable> is integer zero
FloatZero ::= <@variable> is floating point zero

Constant     ::= ir_type <@variable> = constant
Preexecution ::= ir_type <@variable> = preexecution
Argument     ::= ir_type <@variable> = argument
Instruction  ::= ir_type <@variable> = instruction

Same     ::= <@variable> = <@variable>
Distinct ::= <@variable> ≠ <@variable>

DFGEdge ::= <@variable> ∈ <@variable> . args
CFGEdge ::= <@variable> ∈ <@variable> . successors
CDGEdge ::= <@variable> has control dominance to <@variable>
PDGEdge ::= <@variable> has dependence edge to <@variable>

FirstOperand  ::= <@variable> =
                  <@variable> . args '[' 0 ']'
SecondOperand ::= <@variable> =
                  <@variable> . args '[' 1 ']'
ThirdOperand  ::= <@variable> =
                  <@variable> . args '[' 2 ']'
FourthOperand ::= <@variable> =
                  <@variable> . args '[' 3 ']'
```

```
84  FirstSuccessor  ::= <@variable> =
85                      <@variable> . successors '[' 0 ']'
86  SecondSuccessor ::= <@variable> =
87                      <@variable> . successors '[' 1 ']'
88  ThirdSuccessor  ::= <@variable> =
89                      <@variable> . successors '[' 2 ']'
90  FourthSuccessor ::= <@variable> =
91                      <@variable> . successors '[' 3 ']'
92
93  Dominate        ::= domination
94                      '(' <@variable> , <@variable> ')'
95  Postdom         ::= post_domination
96                      '(' <@variable> , <@variable> ')'
97  DominateStrict  ::= strict_domination
98                      '(' <@variable> , <@variable> ')'
99  PostdomStrict   ::= strict_post_domination
100                     '(' <@variable> , <@variable> ')'
101
102 IncomingValue ::= <@variable> -> <@variable> Φ <@variable>
103
104 Blocked ::= all control flow from <@variable> to <@variable>
105             passes through <@variable>
106
107 FunctionAttribute ::= <@variable> has attribute pure
108
109 SameSets ::= <@variable> is the same set as <@variable>
110
111 @variable ::= <slottuple> | '{' <@openslot> '}'
112
113 slottuple ::= '{' <@openslot> , <@openslot>
114                               { , <@openslot> } '}'
115
116 @openslot ::= <slotmember> | <slotindex> | <slotrange>
117             | <slottuple>  | <slotbase>
118
119 slotbase   ::= <s>
120 slotmember ::= <@openslot> . <s>
121 slotindex  ::= <@openslot> '[' <@index> ']'
122 slotrange  ::= <@openslot> '[' <@index> .. <@index> ']'
123
124 @index ::= <basevar> | <baseconst>
125  | <addvar> | <addconst> | <subvar> | <subconst>
126
127 basevar   ::= <s>
128 baseconst ::= <n>
129 addvar    ::= <@index> + <s>
130 addconst  ::= <@index> + <n>
131 subvar    ::= <@index> - <s>
132 subconst  ::= <@index> - <n>
```

Appendix B

Polyhedral Code Sections in CAnDL

Listing B.1: Constraint specification in CAnDL of Scalar Control Parts (SCoPs): These regular loop nests enable polyhedral transformations to be applied, due to their structured control flow, side effect free function calls, and well-behaved memory access that is affine in loop iterators. The entire CAnDL code is displayed, including the relevant parts of the CAnDL standard library.

```
Constraint SCoP
( include For @ {loop}
∧ include StructuredControlFlow({loop}->{scope}) @ {control}
∧ {inputs[0]} = {loop.iterator}
∧ {inputs[i]} = {control.loop[i-1].iterator} foreach i=1..10
∧ include AffineControlFlow({loop}->{scope},
                            {inputs}->{inputs}) @ {control}
∧ include AffineMemAccesses({loop}->{scope},
                            {inputs}->{inputs}) @ {accesses}
∧ include SideEffectFreeCalls({loop}->{scope}) @ {effects})
End

Constraint StructuredControlFlow
( collect i 20 ( opcode{branch[i].value} = branch
                ∧ {branch[i].target1} =
                      {branch[i].value}.successors[0]
                ∧ {branch[i].target2} =
                      {branch[i].value}.successors[1]
                ∧ include ScopeValue({scope}->{scope},
                            {branch[i].value}->{value}))
∧ collect i 10 ( include For @ {loop[i]}
                ∧ domination({scope.begin},
                            {loop[i].begin})
                ∧ strict_post_domination({scope.end},
                                         {loop[i].end}))
∧ collect i  10 ( include IfBlock @ {ifblock[i]}
                ∧ domination({scope.begin},
                           {ifblock[i].precursor})
                ∧ strict_post_domination({scope.end},
                                       {ifblock[i].successor}))
∧ {loop[0..10].end,ifblock[0..10].precursor}
       is the same set as {branch[0..20].value})
End
```

```
Constraint AffineControlFlow
( include AffineCalc[M=10,N=1](
                            {scope}->{scope},
                {loop[i].iter_begin}->{value},
                            {inputs}->{input})
                  @ {forloop_affine_begin[i]} foreach i=0..10
∧ include AffineCalc[M=10,N=1](
                            {scope}->{scope},
                  {loop[i].iter_end}->{value},
                            {inputs}->{input})
                    @ {forloop_affine_end[i]} foreach i=0..10
∧ include AffineCalc[M=10,N=1](
                            {scope}->{scope},
           {ifblock[i].compare_left}->{value},
                            {inputs}->{input})
                   @ {ifblock_affine_left[i]} foreach i=0..10
∧ include AffineCalc[M=10,N=1](
                            {scope}->{scope},
         {ifblock[i].compare_right}->{value},
                            {inputs}->{input})
                  @ {ifblock_affine_right[i]} foreach i=0..10)
End

Constraint AffineMemAccesses
( collect x 20 ( include MemoryAccess({scope}->{scope})
                                        @ {newaccess[x]}
                ∧ opcode{newaccess[x].pointer} = gep
                ∧ domination({scope.begin},
                      {newaccess[x].pointer})
                ∧ {newaffine[x].value} =
                      {newaccess[x].pointer}.args[1])
∧ collect x 20 ( include MemoryAccess({scope}->{scope})
                                        @ {newaccess[x]}
                ∧ opcode{newaccess[x].pointer} = gep
                ∧ domination({scope.begin},
                      {newaccess[x].pointer})
                ∧ {newaffine[x].value} =
                      {newaccess[x].pointer}.args[1]
                ∧ include AffineCalc[M=10,N=6](
                                          {scope}->{scope},
                                        {inputs}->{input})
                                          @ {newaffine[x]}))
End

Constraint SideEffectFreeCalls
( collect i 20 ( opcode{callsite[i]} = call
                ∧ include ScopeValue({callsite[i]}->{value}))
∧ collect i 20 ( opcode{callsite[i]} = call
                ∧ include ScopeValue({callsite[i]}->{value})
                ∧ {function[i]} = {callsite[i]}.args[0]
                ∧ {function[i]} has attribute pure))
End
```

```
Constraint For
( include Loop
∧ {increment} -> {body.end} Φ {iterator}
∧ {increment} ∈ {comparison}.args
∧ opcode{comparison} = icmp
∧ {comparison} ∈ {end}.args
∧ {increment} ∈ {iterator}.args
∧ opcode{increment} = add
∧ {iterator} ∈ {increment}.args
∧ {iter_end} ∈ {comparison}.args
∧ include LocalConst({begin}->{scope.begin},
                     {iter_end}->{value})
∧ {iter_begin} ∈ {iterator}.args
∧ include LocalConst({begin}->{scope.begin},
                   {iter_begin}->{value})
∧ {iter_step} ∈ {increment}.args
∧ include LocalConst({begin}->{scope.begin},
                    {iter_step}->{value}))
End

Constraint Loop
( include SESE
∧ {begin} ∈ {end}.successors)
End

Constraint SESE
( opcode{precursor} = branch
∧ {begin} ∈ {precursor}.successors
∧ opcode{end} = branch
∧ {successor} ∈ {end}.successors
∧ domination({begin}, {end})
∧ post_domination({end}, {begin})
∧ strict_domination({precursor}, {begin})
∧ strict_post_domination({successor}, {end})
∧ all control flow from {begin} to {precursor}
          passes through {end}
∧ all control flow from {successor} to {end}
          passes through {begin})
End

Constraint IfBlock
( include PotentialSESE({truebegin}->{begin},
                        {trueend}->{end})
∧ {truebegin} = {precursor}.successors[0]
∧ {falsebegin} = {precursor}.successors[1]
∧ include PotentialSESE({falsebegin}->{begin},
                        {falseend}->{end})
∧ {trueend} ≠ {falseend}
∧ {condition} = {precursor}.args[0]
∧ opcode{condition} = icmp
∧ {compare_left} = {condition}.args[0]
∧ {compare_right} = {condition}.args[1])
End
```

```
Constraint AffineCalc
if N=0 then
( ( ( include LocalConst
    ∨ ( {value} is unused
      ∧ ir_type{scope.begin} = instruction))
  ∧ ( data_type{input[j]} = integer
    ∨ {input[j]} is unused) foreach j=0..M)
∨ ( ( {value} = {input[i]}
    ∧ ir_type{scope.begin} = instruction
    ∧ ( data_type{input[j]} = integer
      ∨ {input[j]} is unused) foreach j=0..M)
                            forany  i=0..M))
else
( ( ( include AffineCalc[M=M,N=0]
    ∧ {l.value} is unused
    ∧ {r.value} is unused)
  ∨ ( ( ( ( opcode{value} = add
          ∨ opcode{value} = sub)
        ∧ {l.value} = {value}.args[0]
        ∧ {r.value} = {value}.args[1])
      ∨ ( opcode{value} = select
        ∧ {l.value} = {value}.args[1]
        ∧ {r.value} = {value}.args[2])
      ∨ ( opcode{value} = mul
        ∧ include ArgumentsPermuted({l.value}->{src1},
                                    {r.value}->{src2},
                                      {value}->{dst})
        ∧ ir_type{l.value} = preexecution
        ∧ ir_type{r.value} = instruction))
    ∧ ir_type{scope.begin} = instruction
    ∧ domination({scope.begin}, {value})
    ∧ ( data_type{input[j]} = integer
      ∨ {input[j]} is unused) foreach j=0..M))
∧ include AffineCalc[M=M,N=N-1]({input}->{input},
                                {scope}->{scope}) @ {l}
∧ include AffineCalc[M=M,N=N-1]({input}->{input},
                                {scope}->{scope}) @ {r})
endif
End

Constraint ArgumentsPermuted
( ( {src1} = {dst}.args[0]
  ∧ {src2} = {dst}.args[1])
∨ ( {src2} = {dst}.args[0]
  ∧ {src1} = {dst}.args[1]))
End

Constraint LocalConst
( ( ir_type{scope.begin} = instruction
  ∧ ir_type{value} = preexecution)
∨ strict_domination({value}, {scope.begin}))
End
```

```
Constraint MemoryAccess
( ( ( opcode{access} = store
    ∧ {pointer} = {access}.args[1])
  ∨ ( opcode{access} = load
    ∧ {pointer} = {access}.args[0]))
∧ domination({scope.begin}, {access})
∧ post_domination({scope.end}, {access}))
End

Constraint ScopeValue
( domination({scope.begin}, {value})
∧ strict_post_domination({scope.end}, {value}))
End

Constraint PotentialSESE
( opcode{precursor} = branch
∧ {begin} ∈ {precursor}.successors
∧ opcode{end} = branch
∧ {successor} ∈ {end}.successors
∧ ( ( domination({begin}, {end})
    ∧ post_domination({end}, {begin})
    ∧ all control flow from {begin} to {precursor}
              passes through {end}
    ∧ all control flow from {successor} to {end}
              passes through {begin})
  ∨ ( {begin} = {end}
    ∧ {begin} ∈ {precursor}.successors
    ∧ {successor} ∈ {end}.successors)
  ∨ ( {precursor} = {end}
    ∧ {begin} = {successor})))
End
```

Appendix C

Full Grammar of IDL

Listing C.1: Verbatim display of the grammar file that was used to generate the parser for IDL: The file is in a custom version of Backus–Naur form. In the parse tree, all expressions that start with "@" are automatically expanded. The character "#" marks the top-level language construct. Any expression that does not ultimately become part of this construct constitutes a syntax error.

```
1  # ::= <specification>
2
3  specification ::= Constraint <s> <@formula> End
4
5  @formula ::= <atom> | <conjunction> | <disjunction>
6   | <rename> | <collect> | <if> | <default> | <@nested>
7
8  conjunction ::= '(' <@formula> and <@formula>
9                               { and <@formula> } ')'
10
11 disjunction ::= '(' <@formula> or <@formula>
12                              { or <@formula> } ')'
13
14 rename ::= ( <conRange> | <disRange> | <include> | <for> )
15            [ with <@variable> as <@variable>
16            {  and <@variable> as <@variable> } ]
17            [ at <@variable> ]
18
19 include ::= inherits <s> [ '(' <s> = <@index>
20                          {  ,  <s> = <@index> } ')' ]
21
22 conRange ::= <@formula> for all  <s> = <@index> .. <@index>
23 disRange ::= <@formula> for some <s> = <@index> .. <@index>
24 for      ::= <@formula> for      <s> = <@index>
25
26 collect ::= collect <s> <n> <@formula>
27
28 if ::= if <@index> = <@index> then <@formula>
29                                 else <@formula> endif
30
31 default ::= <@formula> for <s> = <@index>
32             if not otherwise specified
```

```
34 @nested ::= '(' <@formula> ')'
35
36 atom ::= <IntegerType> | <FloatType>
37  | <VectorType> | <PointerType>
38  | <Unused> | <IntZero> | <FloatZero>
39  | <Constant> | <Preexecution> | <Argument> | <Instruction>
40  | <Same> | <Distinct>
41  | <DFGEdge> | <CFGEdge> | <CDGEdge> | <PDGEdge>
42  | <FirstOperand> | <SecondOperand>
43  | <ThirdOperand> | <FourthOperand>
44  | <FirstSuccessor> | <SecondSuccessor>
45  | <ThirdSuccessor> | <FourthSuccessor>
46  | <Dominate> | <DominateStrict>
47  | <Postdom> | <PostdomStrict> | <Blocked>
48  | <IncomingValue> | <FunctionAttribute>
49  | <Opcode> | <SameSets>
50  | <GeneralizedDominance> | <NotNumericConstant> | <Block>
51
52 IntegerType ::= <@variable> is an integer
53 FloatType   ::= <@variable> is a float
54 VectorType  ::= <@variable> is a vector
55 PointerType ::= <@variable> is a pointer
56
57 Unused ::= <@variable> is unused
58 Opcode ::= <@variable> is <s> instruction
59
60 IntZero   ::= <@variable> is integer zero
61 FloatZero ::= <@variable> is floating point zero
62
63 Constant           ::= <@variable> is a constant
64 Preexecution       ::= <@variable> is preexecution
65 Argument           ::= <@variable> is an argument
66 Instruction        ::= <@variable> is instruction
67 NotNumericConstant ::= <@variable> is not a numeric constant
68
69 Same     ::= <@variable> is the same as <@variable>
70 Distinct ::= <@variable> is not the same as <@variable>
71 Block    ::= <@variable> spans block to <@variable>
72
73 DFGEdge ::= <@variable> has data flow to <@variable>
74 CFGEdge ::= <@variable> has control flow to <@variable>
75 CDGEdge ::= <@variable> has control dominance to <@variable>
76 PDGEdge ::= <@variable> has dependence edge to <@variable>
77
78 FirstOperand  ::= <@variable> is first
79                   argument of <@variable>
80 SecondOperand ::= <@variable> is second
81                   argument of <@variable>
82 ThirdOperand  ::= <@variable> is third
83                    argument of <@variable>
84 FourthOperand ::= <@variable> is fourth
85                    argument of <@variable>
```

```
FirstSuccessor  ::= <@variable> is first
                    successor of <@variable>
SecondSuccessor ::= <@variable> is second
                    successor of <@variable>
ThirdSuccessor  ::= <@variable> is third
                    successor of <@variable>
FourthSuccessor ::= <@variable> is fourth
                    successor of <@variable>

Dominate       ::= <@variable> control flow
                   dominates <@variable>
Postdom        ::= <@variable> control flow
                   post dominates <@variable>
DominateStrict ::= <@variable> strictly control flow
                   dominates <@variable>
PostdomStrict  ::= <@variable> strictly control flow
                   post dominates <@variable>

IncomingValue ::= <@variable> reaches phi node
                  <@variable> from <@variable>

Blocked ::= all control flow from <@variable> to <@variable>
            passes through <@variable>

FunctionAttribute ::= <@variable> has attribute pure

SameSets ::= <@variable> is the same set as <@variable>

GeneralizedDominance ::= all flow from <@variable> or any
                         origin to any of <@variable> passes
                         through at least one of <@variable>

@variable ::= <slottuple> | '{' <@openslot> '}'

slottuple ::= '{' <@openslot> , <@openslot>
                            { , <@openslot> } '}'

@openslot ::= <slotmember> | <slotindex> | <slotrange>
            | <slottuple>  | <slotbase>

slotbase   ::= <s>
slotmember ::= <@openslot> . <s>
slotindex  ::= <@openslot> '[' <@index> ']'
slotrange  ::= <@openslot> '[' <@index> .. <@index> ']'

@index ::= <basevar> | <baseconst>
 | <addvar> | <addconst> | <subvar> | <subconst>

basevar   ::= <s>
baseconst ::= <n>
addvar    ::= <@index> + <s>
addconst  ::= <@index> + <n>
subvar    ::= <@index> - <s>
subconst  ::= <@index> - <n>
```

Appendix D

Complex Reductions and Histograms in IDL

Listing D.1: Constraint specification of Complex Reduction and Histogram Computations in IDL: The complete code is displayed with all dependencies, including the relevant definitions from the IDL standard library, which is derived in part from the CAnDL standard library as in Appendix B.

```
Constraint ComplexReductionsAndHistograms
( inherits For at {loop} and
  collect k 32 ( inherits VectorRead
                      with {loop.iterator}  as {input_index}
                       and {read_values[k]} as {value}
                       and {loop}           as {scope}
                                            at {read[k]}) and
  collect k  2 ( inherits HistoPart
                      with {loop.begin}  as {begin}
                       and {read}        as {read}
                       and {loop}        as {loop}
                       and {read_values} as {read_values}
                                         at {histo[k]}) and
  collect k  2 ( inherits ScalarPart
                      with {loop.begin}  as {begin}
                       and {loop}        as {loop}
                       and {read_values} as {read_values}
                                         at {scalar[k]}) and
  collect i  2 ( {stores[i]} is store instruction and
                 inherits ScopeValue
                      with {loop}      as {scope}
                       and {stores[i]} as {value}) and
  {stores[0..2]} is the same set as
      {histo[0..2].update.store_instr} and
  {scalar[0].kernel.result} is not the
      same as {histo[0].update.store_instr} and
  inherits SideEffectFreeCalls
      with {loop} as {scope})
End
```

```
Constraint VectorRead
( {value} is load instruction and
  {address} is first argument of {value} and
  {base_pointer} is first argument of {address} and
  inherits LocalConst
      with {base_pointer} as {value} and
  {final_index} is second argument of {address} and
  ( ( {final_index} is add instruction and
      inherits ArgumentsPermuted
          with {final_index}   as {dst}
           and {strided_index} as {src1}
           and {offset}        as {src2} and
      inherits LocalConst
          with {offset} as {value} and
      {strided_index} is mul instruction and
      inherits ArgumentsPermuted
          with {strided_index} as {dst}
           and {input_index}   as {src1}
           and {stride}        as {src2} and
      inherits LocalConst
          with {stride} as {value}) or
    ( {final_index} is mul instruction and
      inherits ArgumentsPermuted
          with {final_index} as {dst}
           and {input_index} as {src1}
           and {stride}      as {src2} and
      inherits LocalConst
          with {stride} as {value} and
      {strided_index} is the same as {final_index} and
      {offset} is unused) or
    ( {final_index} is the same as {input_index} and
      {strided_index} is the same as {final_index} and
      {offset} is unused and
      {stride} is unused and
      {scope.begin} is instruction)))
End

Constraint ScalarPart
( {kernel.result} reaches phi node
      {old_value} from {loop.end} and
  inherits ScopeValue
      with {loop}      as {scope}
       and {old_value} as {value} and
  {kernel.result} has data flow to {final_value} and
  {loop.end} strictly control flow
      dominates {final_value} and
  inherits KernelFunction
      with {loop} as {scope} at {kernel} and
  inherits Concat(N1=31,N2=1)
      with {read_values}   as {in1}
       and {old_value}     as {in2}
       and {kernel.inputs} as {out})
End
```

```
Constraint HistoPart
( inherits ConditionalReadModifyWrite
      with {loop}               as {scope}
       and {idx_kernel.result} as {address}
       and {val_kernel.result} as {new_value}
                                 at {update} and
  inherits KernelFunction
      with {loop}         as {scope}
       and {read_values} as {inputs} at {idx_kernel} and
  inherits KernelFunction
      with {loop} as {scope} at {val_kernel} and
  inherits Concat(N1=31,N2=1)
      with {read_values}        as {in1}
       and {update.old_value}   as {in2}
       and {val_kernel.inputs} as {out})
End

Constraint ConditionalReadModifyWrite
( {store_instr} is store instruction and
  inherits MaxOnceInScope
      with {scope}        as {scope}
       and {store_instr} as {value} at {maxonce} and
  {address} is second argument of {store_instr} and
  {address} is gep instruction and
  {address} is first argument of {old_value} and
  {old_value} is load instruction and
  {new_value} is first argument of {store_instr})
End

Constraint MaxOnceInScope
( inherits ScopeValue and
  {value} has control flow to {value_after} and
  all control flow from {value_after} to
      {value} passes through {scope.end} and
  all control flow from {value_after} to
      {value} passes through {scope.begin})
End

Constraint ScopeValue
( {scope.begin} control flow dominates {value} and
  {scope.end} strictly control flow post dominates {value})
End

Constraint SideEffectFreeCalls
( collect i 20 ( {callsite[i]} is call instruction and
                 inherits ScopeValue
                     with {callsite[i]} as {value}) and
  collect i 20 ( {callsite[i]} is call instruction and
                 inherits ScopeValue
                     with {callsite[i]} as {value} and
                 {function[i]} is first
                     argument of {callsite[i]} and
                 {function[i]} has attribute pure))
End
```

```
Constraint For
( inherits Loop and
  {increment} reaches phi node {iterator} from {end} and
  {increment} is first argument of {comparison} and
  {comparison} is icmp instruction and
  {comparison} is first argument of {end} and
  {increment} is add instruction and
  {iterator} is first argument of {increment} and
  {iter_end} is second argument of {comparison} and
  inherits LocalConst
      with {begin}    as {scope.begin}
       and {iter_end} as {value} and
  {iter_begin} reaches phi node
      {iterator} from {precursor} and
  inherits LocalConst
      with {begin}      as {scope.begin}
       and {iter_begin} as {value} and
  {iter_step} is second argument of {increment} and
  inherits LocalConst
      with {begin}     as {scope.begin}
       and {iter_step} as {value})
End

Constraint Loop
( inherits SESE and
  {end} has control flow to {begin})
End

Constraint SESE
( {precursor} is branch instruction and
  {precursor} has control flow to {begin} and
  {end} is branch instruction and
  {end} has control flow to {successor} and
  {begin} control flow dominates {end} and
  {end} control flow post dominates {begin} and
  {precursor} strictly control flow dominates {begin} and
  {successor} strictly control flow post dominates {end} and
  all control flow from {begin} to {precursor}
         passes through {end} and
  all control flow from {successor} to {end}
         passes through {begin})
End

Constraint LocalConst
( ( {scope.begin} is instruction and
    {value} is preexecution) or
  {value} strictly control flow dominates {scope.begin})
End
```

```
Constraint KernelFunction
( collect i  4 ( {entries[i]} has control
                     flow to {scope.begin}) and
  collect i 24 ( inherits LocalConst
                     with {scope} as {scope}
                                  at {outside[i]} and
                 {outside[i].value}
                     is not a numeric constant and
                 {outside[i].value} has data
                     flow to {outside[i].use} and
                 {scope.begin} control flow
                     dominates {outside[i].use}) and
  collect i  8 ( {loop_carried[i].update} reaches
                     phi node {loop_carried[i].value}
                     from {scope.end} and
                 {scope.begin} control flow
                     dominates {loop_carried[i].value}) and
  all flow from {loop_carried[0..8].value} or any origin
       to any of {result} passes through at least one of
       {inputs[0..32],entries[0..4],outside[0..24].value})
End

Constraint ArgumentsPermuted
( ( {src1} is first  argument of {dst} and
    {src2} is second argument of {dst}) or
  ( {src2} is first  argument of {dst} and
    {src1} is second argument of {dst}))
End

Constraint Concat
( if N1=1 then
    {out[0]} is the same as {in1}
  else
    {out[i]} is the same as {in1[i]} for all i=0..N1
  endif and
  if N2=1 then
    {out[N1+0]} is the same as {in2}
  else
    {out[N1+i]} is the same as {in2[i]} for all i=0..N1
  endif and
  if N3=1 then
    {out[N1+N2+0]} is the same as {in3}
  else
    {out[N1+N2+i]} is the same as {in3[i]} for all i=0..N3
  endif)
for N2=0 if not otherwise specified
for N3=0 if not otherwise specified
End
```